1 General

run.sh

```
g++ -g -O2 -std=gnu++17 -static prog.cpp
./a.exe
```

test.sh

```
# compile and test all *.in and *.ans
g++ -g -O2 -std=gnu++17 -static prog.cpp
for i in *.in; do
 f=${i%.in}
 ./a.exe < $i > "$f.out"
 diff -b -q "$f.ans" "$f.out"
done
```

Header

```
// use better compiler options
#pragma GCC optimize("Ofast","unroll-loops")
#pragma GCC target("avx2,fma")
// include everything
#include <bits/stdc++.h>
#include <bits/extc++.h>
#include <sys/resource.h>
// namespaces
using namespace std;
using namespace __gnu_cxx; // rope
using namespace __gnu_pbds; // tree/trie
// common defines
#define fastio ios_base::sync_with_stdio(0);cin.tie(0);
#define nostacklim rlimit RZ;getrlimit(3,&RZ);RZ.rlim_cur=-1;setrlimit(3,&RZ);
#define DEBUG(v) cerr<<__LINE__<<": "<<#v<<" = "<<v<<'\n';
#define TIMER cerr<<1.0*clock()/CLOCKS_PER_SEC<<"s\n";
#define ll long long
#define ull unsigned ll
#define i128 __int128
#define u128 unsigned i128
#define ld long double
// global variables
mt19937 rng((uint32_t)chrono::steady_clock::now().time_since_epoch().count());
```

Fast IO

```
#ifdef _WIN32
#define getchar_unlocked() _getchar_nolock()
#define putchar_unlocked(x) _putchar_nolock(x)
#endif
void read(unsigned int& n) {
 char c; n = 0;
 while ((c=getchar_unlocked())!=' '&&c!='\n')
  n = n * 10 + c - '0';
}
void read(int& n) {
 char c; n = 0; int s = 1;
 if ((c=getchar_unlocked())=='-') s = -1;
 else n = c - '0';
 while ((c=getchar_unlocked())!=' '&&c!='\n')
  n = n * 10 + c - '0';
 n *= s;
}
void read(ld& n) {
 char c; n = 0;
 ld m = 0, o = 1; bool d = false; int s = 1;
 if ((c=getchar_unlocked())=='-') s = -1;
 else if (c == '.') d = true;
 else n = c - '0';
 while ((c=getchar_unlocked())!=' '&&c!='\n') {
  if (c == '.') d = true;
  else if (d) { m=m*10+c-'0'; o*=0.1; }
  else n = n * 10 + c - '0';
 }
 n = s * (n + m * o);
}
void read(double& n) {
 ld m; read(m); n = m;
}
void read(float& n) {
 ld m; read(m); n = m;
}
void read(string& s) {
 char c; s = "";
 while((c=getchar_unlocked())!=' '&&c!='\n')
  s += c;
}
bool readline(string& s) {
 char c; s = "";
 while(c=getchar_unlocked()) {
  if (c == '\n') return true;
  if (c == EOF) return false;
  s += c;
 }
 return false;
}
void print(unsigned int n) {
 if (n / 10) print(n / 10);
 putchar_unlocked(n % 10 + '0');
}
void print(int n) {
 if (n < 0) { putchar_unlocked('-'); n*=-1; }
 print((unsigned int)n);
}
```

Common Structs

```
// n-dimension vectors
// Vec<2, int> v(n, m) = arr[n][m]
// Vec<2, int> v(n, m, -1) default init -1
template<int D, typename T>
struct Vec : public vector<Vec<D-1, T>> {
 template<typename... Args>
 Vec(int n=0, Args... args) : vector<Vec<D-1, T>>(n, Vec<D-1, T>(args...)) {}
};
template<typename T>
struct Vec<1, T> : public vector<T> {
 Vec(int n=0, T val=T()) : vector<T>(n, val) {}
};
```

2 Algorithms

Binary Search

```
// search for k in [p,n)
template<typename T>
int binsearch(T x[], int k, int n, int p = 0) {
 for (int i = n; i >= 1; i /= 2)
  while (p+i < n && x[p+i] <= k) p += i;
 return p; // bool: x[p] == k;
}
```

Min/Max Subarray

```
// max - compare = a < b, reset = a < 0
// min - compare = a > b, reset = a > 0
// returns {sum, {start, end}}
pair<int, pair<int, int>>
    ContiguousSubarray(int* a, int size,
    bool(*compare)(int, int),
    bool(*reset)(int), int defbest = 0) {
 int best = defbest, cur = 0, start = 0, end = 0, s = 0;
 for (int i = 0; i < size; i++) {
  cur += a[i];
  if ((*compare)(best, cur)) { best = cur; start = s; end = i; }
  if ((*reset)(cur)) { cur = 0; s = i + 1; }
 }
 return {best, {start, end}};
}
```

Quickselect

```
#define QSNE -999999
int partition(int arr[], int l, int r)
{
 int x = arr[r], i = l;
 for (int j = l; j <= r - 1; j++)
  if (arr[j] <= x)
   swap(arr[i++], arr[j]);
 swap(arr[i], arr[r]);
 return i;
}
// find k'th smallest element in unsorted array, only if all distinct
int qselect(int arr[], int l, int r, int k)
{
 if (!(k > 0 && k <= r - l + 1)) return QSNE;
 swap(arr[l + rng() % (r-l+1)], arr[r]);
 int pos = partition(arr, l, r);
 if (pos-l==k-1) return arr[pos];
 if (pos-l>k-1) return qselect(arr,l,pos-1,k);
 return qselect(arr, pos+1, r, k-pos+l-1);
}
// TODO: compare against std::nth_element()
```

Saddleback Search

```
// search for v in 2d array arr[x][y], sorted on both axis
pair<int, int> saddleback_search(int** arr, int x, int y, int v) {
 int i = x-1, j = 0;
 while (i >= 0 && j < y) {
  if (arr[i][j] == v) return {i, j};
  (arr[i][j] > v)? i--: j++;
 }
 return {-1, -1};
}
```

Ternary Search

```
// < max, > min, or any other unimodal func
#define TERNCOMP(a,b) (a)<(b)
int ternsearch(int a, int b, int (*f)(int)) {
 while (b-a > 4) {
  int m = (a+b)/2;
  if (TERNCOMP((*f)(m), (*f)(m+1))) a = m;
  else b = m+1;
 }
 for (int i = a+1; i <= b; i++)
  if (TERNCOMP((*f)(a), (*f)(i)))
   a = i;
 return a;
}
#define TERNPREC 0.000001
double ternsearch(double a, double b, double (*f)(double)) {
 while (b-a > TERNPREC * 4) {
  double m = (a+b)/2;
  if (TERNCOMP((*f)(m), (*f)(m + TERNPREC))) a = m;
  else b = m + TERNPREC;
 }
 for (double i = a + TERNPREC; i <= b; i += TERNPREC)
     if (TERNCOMP((*f)(a), (*f)(i)))
   a = i;
 return a;
}
```

Golden Section Search

```
// < max, > min, or any other unimodal func
#define TERNCOMP(a,b) (a)<(b)
double goldsection(double a, double b, double (*f)(double)) {
 double r = (sqrt(5)-1)/2, eps = 1e-7;
 double x1 = b - r*(b-a), x2 = a + r*(b-a);
 double f1 = f(x1), f2 = f(x2);
 while (b-a > eps)
  if (TERNCOMP(f2,f1)) {
   b = x2; x2 = x1; f2 = f1;
   x1 = b - r*(b-a); f1 = f(x1);
  } else {
   a = x1; x1 = x2; f1 = f2;
   x2 = a + r*(b-a); f2 = f(x2);
  }
 return a;
}
```

3 Structures

Fenwick Tree

```
// Fenwick tree, array of cumulative sums - O(log n) updates, O(log n) gets
struct Fenwick {
 int n; ll* tree;
 void update(int i, int val) {
  ++i;
  while (i <= n) {
   tree[i] += val;
   i += i & (-i);
  }
 }
 Fenwick(int size) {
  n = size;
  tree = new ll[n+1];
  for (int i = 1; i <= n; i++)
   tree[i] = 0;
 }
 Fenwick(int* arr, int size) : Fenwick(size) {
  for (int i = 0; i < n; i++)
   update(i, arr[i]);
 }
 ~Fenwick() { delete[] tree; }
 ll operator[](int i) {
  if (i < 0 || i > n) return 0;
  ll sum = 0;
  ++i;
  while (i>0) {
   sum += tree[i];
   i -= i & (-i);
  }
  return sum;
 }
 ll getRange(int a, int b) { return operator[](b) - operator[](a-1); }
};
```

Hashtable

```
// similar to unordered_map, but faster
struct chash {
 const uint64_t C = (ll)(2e18 * M_PI) + 71;
 ll operator()(ll x) const { return __builtin_bswap64(x*C); }
};
int main() {
 gp_hash_table<ll,int,chash> hashtable({},{},{},{},{1<<16});
 for (int i = 0; i < 100; i++)
  hashtable[i] = 200+i;
 if (hashtable.find(10) != hashtable.end())
  cout << hashtable[10];
}
```

Ordered Set

```
template <typename T>
using oset = tree<T,null_type,less<T>,rb_tree_tag,tree_order_statistics_node_update>;
template <typename T, typename D>
using omap = tree<T,D,less<T>,rb_tree_tag,tree_order_statistics_node_update>;
int main()
{
 oset<int> o_set;
 o_set.insert(5); o_set.insert(1); o_set.insert(3);
 // get second smallest element
 cout << *(o_set.find_by_order(1));
 // number of elements less than k=4
 cout << ' ' << o_set.order_of_key(4) << '\n';
 // equivalent with ordered map
 omap<int,int> o_map;
 o_map[5]=1;o_map[1]=2;o_map[3]=3;
 cout << (*(o_map.find_by_order(1))).first;
 cout << ' ' << o_map.order_of_key(4) << '\n';
}
```

Rope

```
// O(log n) insert, delete, concatenate
int main() {
 // generate rope
 rope<int> v;
 for (int i = 0; i < 100; i++)
  v.push_back(i);
 // move range to front
 rope<int> copy = v.substr(10, 10);
 v.erase(10, 10);
 v.insert(copy.mutable_begin(), copy);
 // print elements of rope
 for (auto it : v)
  cout << it << " ";
}
```

Segment Tree

```
//max(a,b), min(a,b), a+b, a*b, gcd(a,b), a^b
struct SegmentTree {
 typedef int T;
 static constexpr T UNIT = INT_MIN;
 T f(T a, T b) {
  if (a == UNIT) return b;
  if (b == UNIT) return a;
  return max(a,b);
 }
 int n; vector<T> s;
 SegmentTree(int n, T def=UNIT) : s(2*n, def),
 ↪ n(n) {}
 SegmentTree(vector<T> arr) :
 ↪ SegmentTree(arr.size()) {
  for (int i=0;i<arr.size();i++)
 ↪ update(i,arr[i]);
 }
 void update(int pos, T val) {
  for (s[pos += n] = val; pos /= 2;)
   s[pos] = f(s[pos * 2], s[pos*2+1]);
 }
 T query(int b, int e) { // query [b, e)
  T ra = UNIT, rb = UNIT;
  for (b+=n, e+=n; b<e; b/=2, e/=2) {
   if (b % 2) ra = f(ra, s[b++]);
   if (e % 2) rb = f(s[--e], rb);
  }
  return f(ra, rb);
 }
 T get(int p) { return query(p, p+1); }
};
```

Sparse Table

```
template<class T> struct SparseTable {
 vector<vector<T>> m;
 SparseTable(vector<T> arr) {
  m.push_back(arr);
  for (int k = 1; (1<<(k)) <= size(arr); k++) {
   m.push_back(vector<T>(size(arr)-(1<<k)+1));
   for (int i = 0; i < size(arr)-(1<<k)+1; i
++)
    m[k][i] = min(m[k-1][i],
 ↪ m[k-1][i+(1<<(k-1))]);
  }
 }
 // min of range [l,r]
 T query(int l, int r) {
  int k = __lg(r-l+1);
  return min(m[k][l], m[k][r-(1<<k)+1]);
 }
};
```

Trie

```
typedef trie<string, null_type,
↪ trie_string_access_traits<>,
 pat_trie_tag, trie_prefix_search_node_update>
↪ trie_type;
int main() {
 // generate trie
 trie_type trie;
 for (int i = 0; i < 20; i++)
  trie.insert(to_string(i)); // true if new,
↪ false if old
 // print things with prefix "1"
 auto range = trie.prefix_range("1");
 for (auto it = range.first; it !=
↪ range.second; it++)
  cout << *it << " ";
}
```

Wavelet Tree

```
using iter = vector<int>::iterator;
struct WaveletTree {
 Vec<2, int> C; int s;
 // sigma = highest value + 1
 WaveletTree(vector<int>& a, int sigma) :
↪ s(sigma), C(sigma*2, 0) {
  build(a.begin(), a.end(), 0, s-1, 1);
 }
 void build(iter b, iter e, int L, int U, int
↪ u) {
  if (L == U) return;
  int M = (L+U)/2;
  C[u].reserve(e-b+1); C[u].push_back(0);
  for (auto it = b; it != e; ++it)
   C[u].push_back(C[u].back() + (*it<=M));
  auto p = stable_partition(b, e, [=](int
↪ i){return i<=M;});
  build(b, p, L, M, u*2);
  build(p, e, M+1, U, u*2+1);
 }
 // number of occurences of x in [0,i)
 int rank(int x, int i) {
  int L = 0, U = s-1, u = 1, M, r;
  while (L != U) {
   M = (L+U)/2;
   r = C[u][i]; u*=2;
   if (x <= M) i = r, U = M;
   else i -= r, L = M+1, ++u;
  }
  return i;
 }
 // number of occurences of x in [l,r)
 int count(int x, int l, int r) {
  return rank(x, r) - rank(x, l);
 }
 // kth smallest in [l, r)
 int kth(int k, int l, int r) const {
  int L = 0, U = s-1, u = 1, M, ri, rj;
  while (L != U) {
   M = (L+U)/2;
   ri = C[u][l]; rj = C[u][r]; u*=2;
   if (k <= rj-ri) l = ri, r = rj, U = M;
   else k -= rj-ri, l -= ri, r -= rj,
   L = M+1, ++u;
  }
  return U;
 }
 // # elements between [x,y] in [l, r)
 mutable int L, U;
 int range(int x, int y, int l, int r) const {
  if (y < x or r <= l) return 0;
  L = x; U = y;
  return range(l, r, 0, s-1, 1);
 }
 int range(int l, int r, int x, int y, int u)
↪ const {
  if (y < L or U < x) return 0;
  if (L <= x and y <= U) return r-l;
  int M = (x+y)/2, ri = C[u][l], rj = C[u][r];
  return range(ri, rj, x, M, u*2) + range(l-ri,
↪ r-rj, M+1, y, u*2+1);
 }
 // # elements <= x in [l, r)
 int lte(int x, int l, int r) {
  return range(INT_MIN, x, l, r);
 }
};
```

4 Strings

Aho Corasick

```
// range of alphabet for automata to consider
// MAXC = 26, OFFC = 'a' if only lowercase
const int MAXC = 256;
const int OFFC = 0;
struct aho_corasick {
 struct state
 {
  set<pair<int, int>> out;
  int fail; vector<int> go;
  state() : fail(-1), go(MAXC, -1) {}
 };
 vector<state> s;
 int id = 0;
 aho_corasick(string* arr, int size) : s(1) {
  for (int i = 0; i < size; i++) {
   int cur = 0;
   for (int c : arr[i]) {
    if (s[cur].go[c-OFFC] == -1) {
     s[cur].go[c-OFFC] = s.size();
     s.push_back(state());
    }
    cur = s[cur].go[c-OFFC];
   }
   s[cur].out.insert({arr[i].size(), id++});
  }
  for (int c = 0; c < MAXC; c++)
   if (s[0].go[c] == -1)
    s[0].go[c] = 0;
  queue<int> sq;
  for (int c = 0; c < MAXC; c++) {
   if (s[0].go[c] != 0) {
    s[s[0].go[c]].fail = 0;
    sq.push(s[0].go[c]);
   }
  }
  while (sq.size()) {
   int e = sq.front(); sq.pop();
   for (int c = 0; c < MAXC; c++) {
    if (s[e].go[c] != -1) {
     int failure = s[e].fail;
     while (s[failure].go[c] == -1)
      failure = s[failure].fail;
     failure = s[failure].go[c];
     s[s[e].go[c]].fail = failure;
     for (auto length : s[failure].out)
      s[s[e].go[c]].out.insert(length);
     sq.push(s[e].go[c]);
    }
   }
  }
 }
 // list of {start pos, pattern id}
 vector<pair<int, int>> search(string text)
 {
  vector<pair<int, int>> toret;
  int cur = 0;
  for (int i = 0; i < text.size(); i++) {
   while (s[cur].go[text[i]-OFFC] == -1)
    cur = s[cur].fail;
   cur = s[cur].go[text[i]-OFFC];
   if (s[cur].out.size())
    for (auto end : s[cur].out)
     toret.push_back({i - end.first + 1,
↪ end.second});
  }
  return toret;
 }
};
```

Boyer Moore

```
struct defint { int i = -1; };
vector<int> boyermoore(string txt, string pat)
↪ {
 vector<int> toret; unordered_map<char, defint>
↪ badchar;
 int m = pat.size(), n = txt.size();
 for (int i = 0; i < m; i++) badchar[pat[i]].i
↪ = i;
 int s = 0;
 while (s <= n - m) {
  int j = m - 1;
  while (j >= 0 && pat[j] == txt[s + j]) j--;
  if (j < 0) {
   toret.push_back(s);
   s += (s + m < n) ? m - badchar[txt[s +
↪ m]].i : 1;
  } else
   s += max(1, j - badchar[txt[s + j]].i);
 }
 return toret;
}
```

English Conversion

```
const string ones[] = {"", "one", "two",
↪ "three", "four", "five", "six", "seven",
↪ "eight", "nine"};
const string teens[] ={"ten", "eleven",
↪ "twelve", "thirteen", "fourteen",
↪ "fifteen", "sixteen", "seventeen",
↪ "eighteen", "nineteen"};
const string tens[] = {"twenty", "thirty",
↪ "forty", "fifty", "sixty", "seventy",
↪ "eighty", "ninety"};
const string mags[] = {"thousand", "million",
↪ "billion", "trillion", "quadrillion",
↪ "quintillion", "sextillion",
↪ "septillion"};
string convert(int num, int carry) {
 if (num < 0) return "negative " +
↪ convert(-num, 0);
 if (num < 10) return ones[num];
 if (num < 20) return teens[num % 10];
 if (num < 100) return tens[(num / 10) - 2] +
↪ (num%10==0?"":" ") + ones[num % 10];
 if (num < 1000) return ones[num / 100] +
↪ (num/100==0?"":" ") + "hundred" +
↪ (num%100==0?"":" ") + convert(num % 100,
↪ 0);
 return convert(num / 1000, carry + 1) + " " +
↪ mags[carry] + " " + convert(num % 1000,
↪ 0);
}
string convert(int num) {
 return (num == 0) ? "zero" : convert(num, 0);
}
```

Knuth Morris Pratt

```
vector<int> kmp(string txt, string pat) {
  vector<int> toret;
 int m = txt.length(), n = pat.length();
 int next[n + 1];
 for (int i = 0; i < n + 1; i++)
  next[i] = 0;
 for (int i = 1; i < n; i++) {
  int j = next[i + 1];
  while (j > 0 && pat[j] != pat[i])
   j = next[j];
  if (j > 0 || pat[j] == pat[i])
   next[i + 1] = j + 1;
 }
 for (int i = 0, j = 0; i < m; i++) {
  if (txt[i] == pat[j]) {
   if (++j == n)
    toret.push_back(i - j + 1);
  } else if (j > 0) {
   j = next[j];
   i--;
  }
 }
 return toret;
}
```

Longest Common Prefix (array)

```
// longest common prefix of strings in array
string lcp(string* arr, int n, bool sorted =
↪ false) {
 if (n == 0) return "";
 if (!sorted) sort(arr, arr + n);
 string r = ""; int v = 0;
 while (v < arr[0].length() && arr[0][v] ==
↪ arr[n-1][v])
  r += arr[0][v++];
 return r;
}
```

Longest Common Subsequence

```cpp
string lcs(string a, string b) {
 int m = a.length(), n = b.length();
 int L[m+1][n+1];
 for (int i = 0; i <= m; i++) {
  for (int j = 0; j <= n; j++) {
   if (i == 0 || j == 0) L[i][j] = 0;
   else if (a[i-1] == b[j-1]) L[i][j] = L[i-1][j-1]+1;
   else L[i][j] = max(L[i-1][j], L[i][j-1]);
  }
 }
 // return L[m][n]; // length of lcs
 string out = "";
 int i = m - 1, j = n - 1;
 while (i >= 0 && j >= 0) {
  if (a[i] == b[j]) {
   out = a[i--] + out;
   j--;
  }
  else if (L[i][j+1] > L[i+1][j]) i--;
  else j--;
 }
 return out;
}
```

Longest Common Substring

```cpp
// l is array of palindrome length at that index
int manacher(string s, int* l) {
 int n = s.length() * 2;
 for (int i = 0, j = 0, k; i < n; i += k, j = max(j-k, 0)) {
  while (i >= j && i + j + 1 < n && s[(i-j)/2] == s[(i+j+1)/2]) j++;
  l[i] = j;
  for (k = 1; i >= k && j >= k && l[i-k] != j-k; k++)
   l[i+k] = min(l[i-k], j-k);
 }
 return *max_element(l, l + n);
}
```

Cyclic Rotation (Lyndon)

```cpp
// simple strings = smaller than its nontrivial suffixes
// lyndon factorization = simple strings factorized
// "abaaba" -> "ab", "aab", "a"
vector<string> duval(string s) {
 int n = s.length();
 vector<string> lyndon;
 for (int i = 0; i < n;) {
  int j = i+1, k = i;
  for (; j < n && s[k] <= s[j]; j++)
   if (s[k] < s[j]) k = i;
   else k++;
  for (; i <= k; i += j - k)
   lyndon.push_back(s.substr(i,j-k));
 }
 return lyndon;
}
// lexicographically smallest rotation
int minRotation(string s) {
 int n = s.length(); s += s;
 auto d = duval(s); int i = 0, a = 0;
 while (a + d[i].length() < n) a += d[i++].length();
 while (i && d[i] == d[i-1]) a -= d[i--].length();
 return a;
}
```

Minimum Word Boundary

```cpp
// minimum word boundary
// compose string s using words from dict
// NOTE: can reuse words from dict
unsigned int mwb(string s, set<string> dict) {
 int l = s.size();
 vector<unsigned int> arr(l+1, -1);
 arr[0] = 0;
 for (int i = 0; i < l; i++) {
  if (arr[i] != -1) {
   for (auto e : dict) {
    int L = e.size();
    if (l >= i + L) {
     bool isGood = true;
     for (int j = 0; isGood && j < L; j++)
      if (s[i+j] != e[j])
       isGood = false;
     if (isGood)
      arr[i+L] = min(arr[i]+1, arr[i+L]);
    }
   }
  }
 }
 return arr[l];
}
```

Hashing

```cpp
#define HASHER 27
ull basicHash(string s) {
 ull v = 0;
 for (auto c : s) v = (c - 'a' + 1) + v * HASHER;
 return v;
}
const int MAXN = 1000001;
ull base[MAXN] = {1};
void genBase(int n) {
 for (int i = 1; i <= n; i++)
  base[i] = base[i-1] * HASHER;
}
struct advHash {
 ull v, l; vector<ull> wip;
 advHash(string& s): v(0) {
  wip = vector<ull>(s.length()+1);\
  wip[0] = 0;
  for (int i = 0; i < s.length(); i++)
   wip[i+1] = (s[i] - 'a' + 1) + wip[i] * HASHER;
  l = s.length(); v = wip[l];
 }
 ull del(int pos, int len) {
  return v - wip[pos+len]*base[l-pos-len] + wip[pos]*base[l-pos-len];
 }
 ull substr(int pos, int len) {
  return del(pos+len, (l-pos-len)) - wip[pos]*base[len];
 }
 ull replace(int pos, char c) {
  return v - wip[pos+1]*base[l-pos-1] + ((c - 'a' + 1) + wip[pos] * HASHER)*base[l-pos-1];
 }
 ull replace(int pos, string s) {
  // can't increase total string size
  ull r = v - wip[pos+s.size()]*base[l-pos-s.size()], c = wip[pos];
  for (int i = 0; i < s.size(); i++)
   c = (s[i]-'a'+1) + c * HASHER;
  return r + c * base[l-pos-s.size()];
 }
};
```

Subsequence Count

```cpp
// "banana", "ban" >> 3 (ban, ba..n, b..an)
ull subsequences(string body, string subs) {
 int m = subs.length(), n = body.length();
 if (m > n) return 0;
 ull** arr = new ull*[m+1];
 for (int i = 0; i <= m; i++) arr[i] = new ull[n+1];
 for (int i = 1; i <= m; i++) arr[i][0] = 0;
 for (int i = 0; i <= n; i++) arr[0][i] = 1;
 for (int i = 1; i <= m; i++)
  for (int j = 1; j <= n; j++)
   arr[i][j] = arr[i][j-1] + ((body[j-1] == subs[i-1])? arr[i-1][j-1] : 0);
 return arr[m][n];
}
```

Suffix Array + LCP

```cpp
struct SuffixArray {
 vector<int> sa, lcp;
 SuffixArray(string& s, int lim=256) {
  int n = s.length() + 1, k = 0, a, b;
  vector<int> x(begin(s), end(s)+1), y(n), ws(max(n, lim)), rank(n);
  sa = lcp = y;
  iota(begin(sa), end(sa), 0);
  for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
   p = j; iota(begin(y), end(y), n - j);
   for (int i = 0; i < (n); i++)
    if (sa[i] >= j)
     y[p++] = sa[i] - j;
   fill(begin(ws), end(ws), 0);
   for (int i = 0; i < (n); i++) ws[x[i]]++;
   for (int i = 1; i < (lim); i++) ws[i] += ws[i - 1];
   for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
   swap(x, y); p = 1; x[sa[0]] = 0;
   for (int i = 1; i < (n); i++) {
    a = sa[i - 1]; b = sa[i];
    x[b] = (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
   }
  }
  for (int i = 1; i < (n); i++) rank[sa[i]] = i;
  for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
   for (k && k--, j = sa[rank[i] - 1];
     s[i + k] == s[j + k]; k++);
 }
 // smallest cyclic shift
 int cyclic() { return sa[0]; }
 // longest repeated substring
 pair<int,int> lrs() {
  int length = -1, index = -1;
  for (int i = 0; i < lcp.size(); i++) {
   if (lcp[i] > length) {
    length = lcp[i];
    index = sa[i];
   }
  }
  return {index,length};
 }
 // count distinct substrings, excluding empty
 int distincts() {
  int n = sa.size() - 1, r = n - sa[0];
  for (int i = 1; i < lcp.size(); i++)
   r += (n - sa[i]) - lcp[i - 1];
  return r;
 }
 // count repeated substrings, excluding empty
 int repeateds() {
  int r = 0;
  for (int i = 1; i < lcp.size(); i++)
   r += max(lcp[i] - lcp[i-1], 0);
  return r;
 }
};
```

Suffix Tree (Ukkonen's)

```cpp
struct SuffixTree {
 // n = 2*len+10 or so
 enum { N = 50010, ALPHA = 26 };
 int toi(char c) { return c - 'a'; }
 int t[N][ALPHA],l[N],r[N],p[N],s[N],v=0,q=0,m=2;
 string a;
 void ukkadd(int i, int c) { suff:
  if (r[v]<=q) {
   if (t[v][c]==-1) { t[v][c]=m; l[m]=i;
    p[m++]=v; v=s[v]; q=r[v]; goto suff; }
   v=t[v][c]; q=l[v];
  }
  if (q==-1 || c==toi(a[q])) q++; else {
   l[m+1]=i; p[m+1]=m; l[m]=l[v]; r[m]=q;
   p[m]=p[v]; t[m][c]=m+1; t[m][toi(a[q])]=v;
   l[v]=q; p[v]=m; t[p[m]][toi(a[l[m]])]=m;
   v=s[p[m]]; q=l[m];
   while (q<r[m]) { v=t[v][toi(a[q])]; q+=r[v]-l[v]; }
   if (q==r[m]) s[m]=v; else s[m]=m+2;
   q=r[v]-(q-r[m]); m+=2; goto suff;
  }
 }
 SuffixTree(string a) : a(a) {
  fill(r,r+N,(int)(a).size());
  memset(s, 0, sizeof s);
  memset(t, -1, sizeof t);
  fill(t[1],t[1]+ALPHA,0);
  s[0]=1;l[0]=l[1]=-1;r[0]=r[1]=p[0]=p[1]=0;
  for(int i=0;i<a.size();i++) ukkadd(i,toi(a[i]));
 }
 // Longest Common Substring between 2 strings
 // returns {length, offset from first string}
 pair<int, int> best;
 int lcs(int node, int i1, int i2, int olen) {
  if (l[node] <= i1 && i1 < r[node]) return 1;
  if (l[node] <= i2 && i2 < r[node]) return 2;
  int mask=0, len=node?olen+(r[node]-l[node]):0;
  for(int c=0; c<ALPHA; c++) if (t[node][c]!=-1)
   mask |= lcs(t[node][c], i1, i2, len);
  if (mask==3) best=max(best,{len,r[node]-len});
  return mask;
 }
 static pair<int, int> LCS(string s, string t) {
  SuffixTree st(s+(char)('z'+1)+t+(char)('z'+2));
  st.lcs(0, s.size(), s.size()+t.size()+1, 0);
  return st.best;
 }
};
```

String Utilities

```cpp
void lowercase(string& s) {
 transform(s.begin(), s.end(), s.begin(), ::tolower);
}
void uppercase(string& s) {
 transform(s.begin(), s.end(), s.begin(), ::toupper);
}
void trim(string &s) {
 s.erase(s.begin(),find_if_not(s.begin(),s.end(),[](int c){return isspace(c);}));
 s.erase(find_if_not(s.rbegin(),s.rend(),[](int c){return isspace(c);}).base(),s.end());
}
vector<string> split(string& s, char token) {
    vector<string> v; stringstream ss(s);
    for (string e;getline(ss,e,token);)
        v.push_back(e);
    return v;
}
```

5 Greedy

Interval Cover

```cpp
// L,R = interval [L,R], in = {{l,r}, index}
// does not handle case where L == R
vector<int> intervalCover(double L, double R, vector<pair<pair<double,double>,int>> in) {
```

```
  int i = 0; pair<double,int> pos = {L,-1};
  vector<int> a;
  sort(begin(in), end(in));
  while (pos.first < R) {
    double cur = pos.first;
    while (i < (int)in.size() && in[i].first.first <= cur)
      pos = max(pos,{in[i].first.second,in[i].second}), i++;
    if (pos.first == cur) return {};
    a.push_back(pos.second);
  }
  return a;
}
```

6 Math

Catalan Numbers

```
ull* catalan = new ull[1000000];
void genCatalan(int n, int mod) {
  catalan[0] = catalan[1] = 1;
  for (int i = 2; i <= n; i++) {
    catalan[i] = 0;
    for (int j = i - 1; j >= 0; j--) {
      catalan[i] += (catalan[j] * catalan[i-j-1]) % mod;
      if (catalan[i] >= mod)
        catalan[i] -= mod;
    }
  }
}
// TODO: consider binomial coefficient method
```

Combinatorics (nCr, nPr)

```
// can optimize by precomputing factorials, and fact[n]/fact[n-r]
ull nPr(ull n, ull r) {
  ull v = 1;
  for (ull i = n-r+1; i <= n; i++)
    v *= i;
  return v;
}
ull nPr(ull n, ull r, ull m) {
  ull v = 1;
  for (ull i = n-r+1; i <= n; i++)
    v = (v * i) % m;
  return v;
}
ull nCr(ull n, ull r) {
  long double v = 1;
  for (ull i = 1; i <= r; i++)
    v = v * (n-r+i) /i;
  return (ull)(v + 0.001);
}
// requires modulo math
// can optimize by precomputing mfac and minv-mfac
ull nCr(ull n, ull r, ull m) {
  return mfac(n, m) * minv(mfac(k, m), m) % m * minv(mfac(n-k, m), m) % m;
}
```

Multinomials

```
ll multinomial(vector<int>& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  for(int i = 1; i < v.size(); i++)
    for (int j = 0; j < v[i]; j++)
      c = c * ++m / (j+1);
  return c;
}
```

Chinese Remainder Theorem

```
bool ecrt(ll* r, ll* m, int n, ll& re, ll& mo) {
  ll x, y, d; mo = m[0]; re = r[0];
  for (int i = 1; i < n; i++) {
    d = egcd(mo, m[i], x, y);
    if ((r[i] - re) % d != 0) return false;
    x = (r[i] - re) / d * x % (m[i] / d);
    re += x * mo;
    mo = mo / d * m[i];
    re %= mo;
  }
  re = (re + mo) % mo;
  return true;
}
```

Count Digit Occurences

```
/*count(n,d) counts the number of occurences of a digit d in the range [0,n]*/
ll digit_count(ll n, ll d) {
  ll result = 0;
  while (n != 0) {
    result += ((n%10) == d ? 1 : 0);
    n /= 10;
  }
  return result;
}
ll count(ll n, ll d) {
  if (n < 10) return (d > 0 && n >= d);
  if ((n % 10) != 9) return digit_count(n, d) + count(n-1, d);
  return 10*count(n/10, d) + (n/10) + (d > 0);
}
```

Discrete Logarithm

```
int discretelog(int a, int b, int m) {
  ll n = sqrt(m) + 1, an = 1;
  for (ll i = 0; i < n; ++i)
    an = (an * a) % m;
  unordered_map<ll, ll> vals;
  for (ll q = 0, cur = b; q <= n; q++) {
    vals[cur] = q;
    cur = (cur * a) % m;
  }
  for (ll p = 1, cur = 1; p <= n; p++) {
    cur = (cur * an) % m;
    if (vals.count(cur)) {
      int ans = n * p - vals[cur];
      return ans;
    }
  }
  return -1;
}
```

Euler Phi / Totient

```
int phi(int n) {
  int r = n;
  for (int i = 2; i * i <= n; i++) {
    if (n % i == 0) r -= r / i;
    while (n % i == 0) n /= i;
  }
  if (n > 1) r -= r / n;
  return r;
}
#define n 100000
ll phi[n+1];
void computeTotient() {
  for (int i=1; i<=n; i++) phi[i] = i;
  for (int p=2; p<=n; p++) {
    if (phi[p] == p) {
      phi[p] = p-1;
      for (int i = 2*p; i<=n; i += p) phi[i] = (phi[i]/p) * (p-1);
    }
  }
}
```

Factorials

```
// digits in factorial
#define kamenetsky(n) (floor((n * log10(n / M_E)) + (log10(2 * M_PI * n) / 2.0)) + 1)
// approximation of factorial
#define stirling(n) ((n == 1) ? 1 : sqrt(2 * M_PI * n) * pow(n / M_E, n))
// natural log of factorial
#define lfactorial(n) (lgamma(n+1))
```

Prime Factorization

```
// do not call directly
ll pollard_rho(ll n, ll s) {
  ll x, y;
  x = y = rand() % (n - 1) + 1;
  int head = 1, tail = 2;
  while (true) {
    x = mult(x, x, n);
    x = (x + s) % n;
    if (x == y) return n;
    ll d = __gcd(max(x - y, y - x), n);
    if (1 < d && d < n) return d;
    if (++head == tail) y = x, tail <<= 1;
  }
}
// call for prime factors
void factorize(ll n, vector<ll> &divisor) {
  if (n == 1) return;
  if (isPrime(n)) divisor.push_back(n);
  else {
    ll d = n;
    while (d >= n) d = pollard_rho(n, rand() % (n - 1) + 1);
    factorize(n / d, divisor);
    factorize(d, divisor);
  }
}
```

Factorize Factorials

```
// NOTE: count distinct divisors of n by
// computing (q1+1)*(q2+1)*...*(qk+1)
// where qi are powers of primes pi dividing n
// use that and this code to solve https://open.kattis.com/problems/divisors
// max power of a prime p dividing n!
// O(log(n))
int legendre(int n, int p) {
  int mx = 0;
  while(n>0) n/=p, mx+=n;
  return mx;
}
bitset<10000> sieve;
vector<int> primes;
// get all primes O(n log n)
// if dealing with small numbers
void genPrimes(int n) {
  sieve[0] = sieve[1] = 1;
  primes.push_back(2);
  for (int i = 3; i <= n; i+=2)
    if (i%2 != 0 && !sieve[i]) {
      primes.push_back(i);
      for (int j = i * 3; j <= n; j += i*2)
        sieve[j] = 1;
    }
}
// make sure you call genPrimes first
// return vector of prime factor powers as vector v of size pi(n)
// so that v[i] = power of primes[i] dividing n!
// O(pi(n) * log(n)) where pi(n) is prime counting fn
// so basically O(n) since pi(n) = O(n/log(n))
vector<int> factorize_factorial(int n) {
  vector<int> factorization(primes.size(), 0);
  for(int i=0;i<primes.size() && primes[i] <= n;i++) {
    factorization[i] = legendre(n, primes[i]);
  }
  return factorization;
}
// same thing but for C(n,k)
vector<int> factorize_binom(int n, int k) {
  vector<int> factorization(primes.size(), 0);
  for(int i=0;i<primes.size() && primes[i] <= n;i++) {
    factorization[i] = legendre(n, primes[i]) - legendre(k, primes[i]) - legendre(n-k, primes[i]);
  }
  return factorization;
}
```

Farey Fractions

```
// generate 0 <= a/b <= 1 ordered, b <= n
// farey(4) = 0/1 1/4 1/3 1/2 2/3 3/4 1/1
// length is sum of phi(i) for i = 1 to n
vector<pair<int, int>> farey(int n) {
  int h = 0, k = 1, x = 1, y = 0, r;
  vector<pair<int, int>> v;
  do {
    v.push_back({h, k});
    r = (n-y)/k;
    y += r*k; x += r*h;
    swap(x,h); swap(y,k);
    x = -x; y = -y;
  } while (k > 1);
  v.push_back({1, 1});
  return v;
}
```

Fast Fourier Transform

```
#define cd complex<double>
const double PI = acos(-1);
void fft(vector<cd>& a, bool invert) {
  int n = a.size();
  for (int i = 1, j = 0; i < n; i++) {
    int bit = n >> 1;
    for (; j & bit; bit >>= 1) j ^= bit;
    j ^= bit;
    if (i < j) swap(a[i], a[j]);
  }
  for (int len = 2; len <= n; len <<= 1) {
    double ang = 2 * PI / len * (invert ? -1 : 1);
    cd wlen(cos(ang), sin(ang));
    for (int i = 0; i < n; i += len) {
      cd w(1);
      for (int j = 0; j < len / 2; j++) {
        cd u = a[i+j], v = a[i+j+len/2] * w;
        a[i+j] = u + v;
        a[i+j+len/2] = u - v;
        w *= wlen;
      }
    }
  }
  if (invert)
    for (auto& x : a)
      x /= n;
}
vector<int> fftmult(vector<int> const& a, vector<int> const& b) {
  vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
  int n = 1 << (32 - __builtin_clz(a.size() + b.size() - 1));
  fa.resize(n); fb.resize(n);
  fft(fa, false); fft(fb, false);
  for (int i = 0; i < n; i++) fa[i] *= fb[i];
  fft(fa, true);
  vector<int> toret(n);
  for (int i = 0; i < n; i++) toret[i] = round(fa[i].real());
  return toret;
}
```

Greatest Common Denominator

```
ll egcd(ll a, ll b, ll& x, ll& y) {
  if (b == 0) { x = 1; y = 0; return a; }
  ll gcd = egcd(b, a % b, x, y);
  x -= a / b * y;
  swap(x, y);
  return gcd;
}
```

Kth Root (floor)

```
struct KthRoot {
  vector<ull> pow[65]; // pow[k][x] = pow(x+2,k)  (k >= 4)
  KthRoot() {
    for (ull t = 2; t < (1<<16); t++) {
      ull s = t*t; s = s*s;
      for (int k = 4; ; k++) {
        pow[k].push_back(s);
```

```
    if (__builtin_umulll_overflow(s,t,&s))
      break;
    }
  }
  ull sqrt(ull n) const {
    if (n == -1ull) return (unsigned int)(-1);
    ull x = std::sqrt((double)n);
    return x*x > n ? x-1 : x;
  }
  ull cbrt(ull n) const {
    ull x = 0, y = 0;
    for (int s = 63; s >= 0; s -= 3) {
      x <<= 1;
      y = 3*x*(x+1)+1;
      if (y <= (n>>s)) n -= y<<s, x++;
    }
    return x;
  }
  // returns floor(n^(1/k)), k >= 1
  ull operator()(ull n, int k) {
    if (k == 1 || n == 0) return n;
    if (k == 2) return sqrt(n);
    if (k == 3) return cbrt(n);
    auto ub = upper_bound(pow[k].begin(),
      pow[k].end(), n);
    return (ub-pow[k].begin())+1;
  }
};
```

Josephus Problem

```
// 0-indexed, arbitrary k
int josephus(int n, int k) {
  if (n == 1) return 0;
  if (k == 1) return n-1;
  if (k > n) return (josephus(n-1,k)+k)%n;
  int res = josephus(n-n/k,k)-n%k;
  return res + ((res<0)?n:res/(k-1));
}
// fast case if k=2, traditional josephus
int josephus(int n) {
  return 2*(n-(1<<(32-__builtin_clz(n)-1)));
}
```

Least Common Multiple

```
#define lcm(a,b) ((a*b)/__gcd(a,b))
```

Modulo Operations

```
#define MOD 1000000007
#define madd(a,b,m) (a+b-((a+b-m>=0)?m:0))
#define mult(a,b,m) ((ull)a*b%m)
#define msub(a,b,m) (a-b+((a<b)?m:0))
ll mpow(ll b, ll e, ll m) {
  ll x = 1;
  while (e > 0) {
    if (e % 2) x = (x * b) % m;
    b = (b * b) % m;
    e /= 2;
  }
  return x % m;
}
ull mfac(ull n, ull m) {
  ull f = 1;
  for (int i = n; i > 1; i--)
    f = (f * i) % m;
  return f;
}
// if m is not guaranteed to be prime
ll minv(ll b, ll m) {
  ll x = 0, y = 0;
  if (egcd(b, m, x, y) != 1) return -1;
  return (x % m + m) % m;
}
ll mdiv_compmod(int a, int b, int m) {
  if (__gcd(b, m) != 1) return -1;
  return mult(a, minv(b, m), m);
}
// if m is prime (like 10^9+7)
ll mdiv_primemod (int a, int b, int m) {
  return mult(a, mpow(b, m-2, m), m);
}
```

```
// tonelli shanks = sqrt(n) % m, m is prime
ll legendre(ll a, ll m){
  if (a % m==0) return 0;
  if (m == 2) return 1;
  return mpow(a,(m-1)/2,m);
}
ll msqrt(ll n, ll m) {
  ll s = __builtin_ctzll(m-1), q = (m-1ll)>>s,
    z = rand()%(m-1)+1;
  if (m == 2) return 1;
  if (s == 1) return mpow(n,(m+1)/4ll,m);
  while (legendre(z,m)!=m-1) z = rand()%(m-1)+1;
  ll c = mpow(z,q,m), r = mpow(n,(q+1)/2,m), t
    = mpow(n,q,m), M = s;
  while (t != 1){
    ll i=1, ts = (t * t) % m;
    while (ts != 1) i++, ts = (ts * ts) % m;
    ll b = c;
    for (int j = 0; j < M-i-1; j++) b = (b * b) %
      m;
    r = r * b % m; c = b * b % m; t = t * c % m;
      M = i;
  }
  return r;
}
```

Modulo Tetration

```
ll tetraloop(ll a, ll b, ll m) {
  if(b == 0 || a == 1) return 1;
  ll w = tetraloop(a,b-1,phi(m)), r = 1;
  for (;w;w/=2) {
    if (w&1) {
      r *= a; if (r >= m) r -= (r/m-1)*m;
    }
    a *= a; if (a >= m) a -= (a/m-1)*m;
  }
  return r;
}
int tetration(int a, int b, int m) {
  if (a == 0 || m == 1) return ((b+1)&1)%m;
  return tetraloop(a,b,m) % m;
}
```

Matrix

```
template<typename T>
struct Mat : public Vec<2, T> {
  int w, h;
  Mat(int x, int y) : Vec<2, T>(x, y), w(x),
    h(y) {}
  static Mat<T> identity(int n) { Mat<T> m(n,n);
    for (int i=0;i<n;i++) m[i][i] = 1; return
    m; }
  Mat<T>& operator+=(const Mat<T>& m) {
    for (int i = 0; i < w; i++)
      for (int j = 0; j < h; j++)
        (*this)[i][j] += m[i][j];
    return *this;
  }
  Mat<T>& operator-=(const Mat<T>& m) {
    for (int i = 0; i < w; i++)
      for (int j = 0; j < h; j++)
        (*this)[i][j] -= m[i][j];
    return *this;
  }
  Mat<T> operator*(const Mat<T>& m) {
    Mat<T> z(w,m.h);
    for (int i = 0; i < w; i++)
      for (int j = 0; j < h; j++)
        for (int k = 0; k < m.h; k++)
          z[i][k] += (*this)[i][j] * m[j][k];
    return z;
  }
  Mat<T> operator+(const Mat<T>& m) { Mat<T>
    a=*this; return a+=m; }
  Mat<T> operator-(const Mat<T>& m) { Mat<T>
    a=*this; return a-=m; }
  Mat<T>& operator*=(const Mat<T>& m) { return
    *this = (*this)*m; }
  Mat<T> power(int n) {
    Mat<T> a = Mat<T>::identity(w),m=*this;
    for (;n;n/=2,m*=m) if (n&1) a *= m;
```

```
    return a;
  }
};
```

Matrix Exponentiation

```
// F(n) = c[0]*F(n-1) + c[1]*F(n-2) + ...
// b is the base cases of same length c
ll matrix_exponentiation(ll n, vector<ll> c,
  vector<ll> b) {
  if (nth < b.size()) return b[nth-1];
  Mat<ll> a(c.size(), c.size()); ll s = 0;
  for (int i = 0; i < c.size(); i++) a[i][0] =
    c[i];
  for (int i = 0; i < c.size() - 1; i++)
    a[i][i+1] = 1;
  a = a.power(nth - c.size());
  for (int i = 0; i < c.size(); i++)
    s += a[i][0] * b[i];
  return s;
}
```

Matrix Subarray Sums

```
template<class T> struct MatrixSum {
  Vec<2, T> p;
  MatrixSum(Vec<2, T>& v) {
    p = Vec<2,T>(v.size()+1, v[0].size()+1);
    for (int i = 0; i < v.size(); i++)
      for (int j = 0; j < v[0].size(); j++)
        p[i+1][j+1] = v[i][j] + p[i][j+1] +
          p[i+1][j] - p[i][j];
  }
  T sum(int u, int l, int d, int r) {
    return p[d][r] - p[d][l] - p[u][r] + p[u][l];
  }
};
```

Mobius Function

```
const int MAXN = 10000000;
// mu[n] = 0 iff n has no square factors
// 1 = even number prime factors, -1 = odd
short mu[MAXN] = {0,1};
void mobius(){
  for (int i = 1; i < MAXN; i++)
    if (mu[i])
      for (int j = i + i; j < MAXN;j += i)
        mu[j] -= mu[i];
}
```

Minimum Excluded

```
int mex(set<int>& s) {
  auto i = s.begin(); int v = 0;
  while (i != s.end() && *i == val) i++, v++;
  return v;
}
```

Nimber Arithmetic

```
#define nimAdd(a,b) ((a)^(b))
ull nimMul(ull a, ull b, int i=6) {
  static const ull M[]={INT_MIN>>32,
    M[0]^(M[0]<<16), M[1]^(M[1]<<8),
    M[2]^(M[2]<<4), M[3]^(M[3]<<2),
    M[4]^(M[4]<<1)};
  if (i-- == 0) return a&b;
  int k=1<<i;
  ull s=nimMul(a,b,i), m=M[5-i],
    t=nimMul(((a^(a>>k))&m)|(s&~m),
    ((b^(b>>k))&m)|(m&(~m>>1))<<k, i);
  return ((s^t)&m)<<k|((s^(t>>k))&m);
}
```

Permutation

```
// c = array size, n = nth perm, return index
vector<int> gen_permutation(int c, int n) {
  vector<int> idx(c), per(c), fac(c); int i;
  for (i = 0; i < c; i++) idx[i] = i;
  for (i = 1; i <= c; i++) fac[i-1] = n%i, n/=i;
  for (i = c - 1; i >= 0; i--)
    per[c-i-1] = idx[fac[i]],
    idx.erase(idx.begin() + fac[i]);
  return per;
}
```

```
// get what nth permutation of vector
int get_permutation(vector<int>& v) {
  int use = 0, i = 1, r = 0;
  for (int e : v) {
    r = r * i++ + __builtin_popcount(use &
      -(1<<e));
    use |= 1 << e;
  }
  return r;
}
```

Permutation (string/multiset)

```
string freq2str(vector<int>& v) {
  string s;
  for (int i = 0; i < v.size(); i++)
    for (int j = 0; j < v[i]; j++)
      s += (char)(i + 'A');
  return s;
}
// nth perm of multiset, n is 0-indexed
string gen_permutation(string s, ll n) {
  vector<int> freq(26, 0);
  for (auto e : s) freq[e - 'A']++;
  for (int i = 0; i < 26; i++) if (freq[i] > 0)
    {
    freq[i]--; ll v = multinomial(freq);
    if (n < v) return (char)(i+'A') +
      gen_permutation(freq2str(freq), n);
    freq[i]++; n -= v;
  }
  return "";
}
```

Miller-Rabin Primality Test

```
// Miller-Rabin primality test - O(10 log^3 n)
bool isPrime(ull n) {
  if (n < 2) return false;
  if (n == 2) return true;
  if (n % 2 == 0) return false;
  ull s = n - 1;
  while (s % 2 == 0) s /= 2;
  for (int i = 0; i < 10; i++) {
    ull temp = s;
    ull a = rand() % (n - 1) + 1;
    ull mod = mpow(a, temp, n);
    while (temp!=n-1&&mod!=1&&mod!=n-1) {
      mod = mult(mod, mod, n);
      temp *= 2;
    }
    if (mod!=n-1&&temp%2==0) return false;
  }
  return true;
}
```

Sieve of Eratosthenes

```
bitset<100000001> sieve;
// generate sieve - O(n log n)
void genSieve(int n) {
  sieve[0] = sieve[1] = 1;
  for (ull i = 3; i * i < n; i += 2)
    if (!sieve[i])
      for (ull j = i * 3; j <= n; j += i * 2)
        sieve[j] = 1;
}
// query sieve after it's generated - O(1)
bool querySieve(int n) {
  return n == 2 || (n % 2 != 0 && !sieve[n]);
}
```

Compile-time Prime Sieve

```
const int MAXN = 100000;
template<int N>
struct Sieve {
  bool sieve[N];
  constexpr Sieve() : sieve() {
    sieve[0] = sieve[1] = 1;
    for (int i = 2; i * i < N; i++)
      if (!sieve[i])
        for (int j = i * 2; j < N; j += i)
          sieve[j] = 1;
  }
```

```cpp
};
bool isPrime(int n) {
  static constexpr Sieve<MAXN> s;
  return !s.sieve[n];
}
```

Simpson's / Approximate Integrals

```cpp
// integrate f from a to b, k iterations
// error <= (b-a)/18.0 * M * ((b-a)/2k)^4
// where M = max(abs(f````(x))) for x in [a,b]
// "f" is a function "double func(double x)"
double Simpsons(double a, double b, int k, double (*f)(double)) {
  double dx = (b-a)/(2.0*k), t = 0;
  for (int i = 0; i < k; i++)
    t += ((i==0)?1:2)*(*f)(a+2*i*dx) + 4 * (*f)(a+(2*i+1)*dx);
  return (t + (*f)(b)) * (b-a) / 6.0 / k;
}
```

Common Equations Solvers

```cpp
// ax^2 + bx + c = 0, find x
vector<double> solveEq(double a, double b, double c) {
  vector<double> r;
  double z = b * b - 4 * a * c;
  if (z == 0)
    r.push_back(-b/(2*a));
  else if (z > 0) {
    r.push_back((sqrt(z)-b)/(2*a));
    r.push_back((sqrt(z)+b)/(2*a));
  }
  return r;
}
// ax^3 + bx^2 + cx + d = 0, find x
vector<double> solveEq(double a, double b, double c, double d) {
  vector<double> res;
  long double a1 = b/a, a2 = c/a, a3 = d/a;
  long double q = (a1*a1 - 3*a2)/9.0, sq = -2*sqrt(q);
  long double r = (2*a1*a1*a1 - 9*a1*a2 + 27*a3)/54.0;
  long double z = r*r-q*q*q, theta;
  if (z <= 0) {
    theta = acos(r/sqrt(q*q*q));
    res.push_back(sq*cos(theta/3.0) - a1/3.0);
    res.push_back(sq*cos((theta+2.0*PI)/3.0) - a1/3.0);
    res.push_back(sq*cos((theta+4.0*PI)/3.0) - a1/3.0);
  }
  else {
    res.push_back(pow(sqrt(z)+fabs(r), 1/3.0));
    res[0] = (res[0] + q / res[0]) * ((r<0)?1:-1) - a1 / 3.0;
  }
  return res;
}
// linear diophantine equation ax + by = c, find x and y
// infinite solutions of form x+k*b/g, y-k*a/g
bool solveEq(ll a, ll b, ll c, ll &x, ll &y, ll &g) {
  g = egcd(abs(a), abs(b), x, y);
  if (c % g) return false;
  x *= c / g * ((a < 0) ? -1 : 1);
  y *= c / g * ((b < 0) ? -1 : 1);
  return true;
}
// m = # equations, n = # variables, a[m][n+1] = coefficient matrix
// a[i][0]x + a[i][1]y + ... + a[i][n]z = a[i][n+1]
// find a solution of some kind to linear equation
const double eps = 1e-7;
bool zero(double a) { return (a < eps) && (a > -eps); }
vector<double> solveEq(double **a, int m, int n) {
  int cur = 0;
  for (int i = 0; i < n; i++) {
    for (int j = cur; j < m; j++) {
      if (!zero(a[j][i])) {
        if (j != cur) swap(a[j], a[cur]);
        for (int sat = 0; sat < m; sat++) {
          if (sat == cur) continue;
          double num = a[sat][i] / a[cur][i];
          for (int sot = 0; sot <= n; sot++)
            a[sat][sot] -= a[cur][sot] * num;
        }
        cur++;
        break;
      }
    }
  }
  for (int j = cur; j < m; j++)
    if (!zero(a[j][n])) return vector<double>();
  vector<double> ans(n,0);
  for (int i = 0, sat = 0; i < n; i++)
    if (sat < m && !zero(a[sat][i]))
      ans[i] = a[sat][n] / a[sat++][i];
  return ans;
}
// solve A[n][n] * x[n] = b[n] linear equation
// rank < n is multiple solutions, -1 is no solutions
// `alls` is whether to find all solutions, or any
const double eps = 1e-12;
int solveEq(Vec<2, double>& A, Vec<1, double>& b, Vec<1, double>& x, bool alls=false) {
  int n = A.size(), m = x.size(), rank = 0, br, bc;
  vector<int> col(m); iota(begin(col), end(col), 0);
  for(int i = 0; i < n; i++) {
    double v, bv = 0;
    for(int r = i; r < n; r++)
      for(int c = i; c < n; c++)
        if ((v = fabs(A[r][c])) > bv)
          br = r, bc = c, bv = v;
    if (bv <= eps) {
      for(int j = i; j < n; j++)
        if (fabs(b[j]) > eps)
          return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    for(int j = 0; j < n; j++)
      swap(A[j][i], A[j][bc]);
    bv = 1.0 / A[i][i];
    for(int j = (alls)?0:i+1; j < n; j++) {
      if (j != i) {
        double fac = A[j][i] * bv;
        b[j] -= fac * b[i];
        for(int k = i+1; k < m; k++)
          A[j][k] -= fac*A[i][k];
      }
    }
    rank++;
  }
  if (alls) for (int i = 0; i < m; i++) x[i] = -DBL_MAX;
  for (int i = rank; i--;) {
    bool isGood = true;
    if (alls)
      for (int j = rank; isGood && j < m; j++)
        if (fabs(A[i][j]) > eps)
          isGood = false;
    b[i] /= A[i][i];
    if (isGood) x[col[i]] = b[i];
    if (!alls)
      for(int j = 0; j < i; j++)
        b[j] -= A[j][i] * b[i];
  }
  return rank;
}
```

Graycode Conversions

```cpp
ull graycode2ull(ull n) {
  ull i = 0;
  for (; n; n = n >> 1) i ^= n;
  return i;
}
ull ull2graycode(ull n) {
  return n ^ (n >> 1);
}
```

Date Utilities

```cpp
// handles -4799-01-01 to 1465001-12-31
int date2int(int y, int m, int d){
  return 1461*(y+4800+(m-14)/12)/4+367*(m-2-(m-14)/12*12)/12-3*((y+4900+(m-14)/12)/100)/4+d-32075;
}
pair<int,pair<int,int>> int2date(int x){
  int n,i,j;
  x+=68569;
  n=4*x/146097;
  x-=(146097*n+3)/4;
  i=(4000*(x+1))/1461001;
  x-=1461*i/4-31;
  j=80*x/2447;
  return {100*(n-49)+i+j/11, {j+2-12*(j/11), x-2447*j/80}};
}
int dayOfWeek(int y, int m, int d){ //0=sunday
  static int cal[]={0,3,2,5,0,3,5,1,4,6,2,4};
  y-=m<3;
  return (y+y/4-y/100+y/400+cal[m-1]+d)%7;
}
```

Unix/Epoch Time

```cpp
// 0-indexed month/time, 1-indexed day
// minimum 1970, 0, 1, 0, 0, 0
ull toEpoch(int year, int month, int day, int hour, int minute, int second) {
  struct tm t; time_t epoch;
  t.tm_year = year - 1900; t.tm_mon = month;
  t.tm_mday = day; t.tm_hour = hour;
  t.tm_min = minute; t.tm_sec = second;
  t.tm_isdst = 0; // 1 = daylights savings
  epoch = mktime(&t);
  return (ull)epoch;
}
vector<int> toDate(ull epoch) {
  time_t e=epoch; struct tm t=*localtime(&e);
  return {t.tm_year+1900,t.tm_mon,t.tm_mday,t.tm_hour,t.tm_min,t.tm_sec};
}
int getWeekday(ull epoch) {
  time_t e=epoch; struct tm t=*localtime(&e);
  return t.tm_wday; // 0-6, 0 = sunday
}
int getDayofYear(ull epoch) {
  time_t e=epoch; struct tm t=*localtime(&e);
  return t.tm_yday; // 0-365
}
const int months[] = {31,28,31,30,31,30,31,31,30,31,30,31};
bool validDate(int year, int month, int day) {
    bool leap = !(year%(year%25?4:16));
    if (month >= 12) return false;
    return day <= months[month] + (leap && month == 1);
}
```

Theorems and Formulae

Montmort Numbers count the number of derangements (permutations where no element appears in its original position) of a set of size n. $!0 = 1$, $!1 = 0$, $!n = (n+1)(!(n-1)+!(n-2))$, $!n = n!\sum_{i=0}^{n}\frac{(-1)^i}{i!}$, $!n = [\frac{n!}{e}]$

In a partially ordered set, a chain is a subset of elements that are all comparable to eachother. An antichain is a subset where no two are comparable.

Dilworth's theorem states the size of a maximal antichain equals the size of a minimal chain cover of a partially ordered set S. The width of S is the maximum size of an antichain in S, which is equal to the minimum number of chains needed to cover S, or the minimum number of chains such that all elements are in at least one chain.

Rosser's Theorem states the nth prime number is greater than $n*ln(n)$ for $n > 1$.

Nicomachi's Theorem states $1^3 + 2^3 + ... + n^3 = (1 + 2 + ... + n)^2$ and is equivalent to $(n\frac{n+1}{2})^2$.

Lagrange's Four Square Theorem states every natural number is the sum of the squares of four non-negative integers. This is a special case of the **Fermat Polygonal Number Theorem** where every positive integer is a sum of at most n s-gonal numbers. The nth s-gonal number $P(s,n) = (s-2)\frac{n(n-1)}{2} + n$

7 Graphs

```cpp
struct edge {
  int u,v,w;
  edge (int u,int v,int w) : u(u),v(v),w(w) {}
  edge () : u(0), v(0), w(0) {}
};
bool operator < (const edge &e1, const edge &e2) { return e1.w < e2.w; }
bool operator > (const edge &e1, const edge &e2) { return e1.w > e2.w; }
struct subset {
  int p, rank, sz;
  subset(int p) : p(p), rank(0), sz(1) {}
  subset() : p(0), rank(0), sz(0) {}
  void make_set(int _p) { p=_p, rank=0, sz=1; }
};
```

BFS

```cpp
// adjacency list named 'graph'
int visited[MAX];
int parent[MAX];
int vc = 0;
vector<int> bfs(int start, int end) {
  vc++;
  visited[start] = vc;
  parent[start] = -1;
  queue<int> q;
  q.push(start);
  while (!q.empty()) {
    int v = q.front(); q.pop();
    for (auto e : graph[v]) {
      if (visited[e] != vc) {
        visited[e] = vc;
        q.push(e);
        parent[e] = v;
        if (e == end) goto DONE;
      }
    }
  }
DONE:
  // path reconstruction
  if (visited[end] != vc) return {};
```

```
  vector<int> path;
  for (int v = end; v != -1; v = parent[v])
    path.push_back(v);
  return path;
}
```

Dijkstra's

```
const int inf = 20000001; // change as needed
// use add_edge(..., true) for digraphs
void add_edge(Vec<2, edge> &graph, int u, int v, int w, bool directed=true) {
  graph[u].push_back({u,v,w});
  if(!directed) graph[v].push_back({v,u,w});
}
vector<int> dijkstra(Vec<2, edge> &graph, int src) {
  vector<int> D(graph.size(), inf);
  priority_queue<edge, vector<edge>, greater<edge>> pq;
  pq.push({src,src,0});
  D[src]=0;
  while(!pq.empty()) {
    edge e = pq.top(); pq.pop();
    int v = e.v;
    for(int i=0;i<graph[v].size();i++) {
      int u = graph[v][i].v;
      if(D[v] + graph[v][i].w < D[u]) {
        D[u] = D[v] + graph[v][i].w;
        pq.push({src,u,D[u]});
      }
    }
  }
  return D;
}
```

Eulerian Path

```
#define edge_list vector<edge>
#define adj_sets vector<set<int>>
struct EulerPathGraph {
  adj_sets graph; // actually indexes incident edges
  edge_list edges; int n; vector<int> indeg;
  EulerPathGraph(int n): n(n) {
    indeg = *(new vector<int>(n,0));
    graph = *(new adj_sets(n, set<int>()));
  }
  void add_edge(int u, int v) {
    graph[u].insert(edges.size());
    indeg[v]++;
    edges.push_back(edge(u,v,0));
  }
  bool eulerian_path(vector<int> &circuit) {
    if(edges.size()==0) return false;
    stack<int> st;
    int a[] = {-1, -1};
    for(int v=0;v<n;v++) {
      if(indeg[v]!=graph[v].size()) {
        bool b = indeg[v] > graph[v].size();
        if (abs(((int)indeg[v])-((int)graph[v].size())) > 1) return false;
        if (a[b] != -1) return false;
        a[b] = v;
      }
    }
    int s = (a[0]!=-1 && a[1]!=-1 ? a[0] : (a[0]==-1 && a[1]==-1 ? edges[0].u : -1));
    if(s==-1) return false;
    while(!st.empty() || !graph[s].empty()) {
      if (graph[s].empty()) {
        circuit.push_back(s); s = st.top(); st.pop(); }
      else {
        int w = edges[*graph[s].begin()].v;
        graph[s].erase(graph[s].begin());
        st.push(s); s = w;
      }
    }
    circuit.push_back(s);
    return circuit.size()-1==edges.size();
  }
};
```

Floyd Warshall

```
const ll inf = 1LL << 62;
#define FOR(i,n) for (int i = 0; i < n; i++)
void floydWarshall(Vec<2, ll>& m) {
  int n = m.size();
  FOR(i,n) m[i][i] = min(m[i][i], 0LL);
  FOR(k,n) FOR(i,n) FOR(j,n) if (m[i][k] != inf && m[k][j] != inf) {
    auto newDist = max(m[i][k] + m[k][j], -inf);
    m[i][j] = min(m[i][j], newDist);
  }
  FOR(k,n) if (m[k][k] < 0) FOR(i,n) FOR(j,n)
    if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -inf;
}
```

Bellman Ford

```
const int inf = 20000001;
vector<ll> bellman_ford(vector<edge> edges, int src, int V) {
  vector<ll> D(V,inf);
  D[src] = 0;
  for (int i=1;i<=V-1;i++)
    for (edge e : edges)
      if (D[e.u] != inf && D[e.u] + e.w < D[e.v])
        D[e.v] = D[e.u] + e.w;
  // detect negative cycles: *typically* 2 is as good as V-1 for this
  for (int i=1;i<=V-1;i++)
    for (edge e : edges)
      if (D[e.u] != inf && D[e.u] + e.w < D[e.v])
        D[e.v] = -inf;
  return D;
}
```

Minimum Spanning Tree

```
// returns vector of edges in the mst
// graph[i] = vector of edges incident to vertex i
// places total weight of the mst in &total
// if returned vector has size != n-1, there is no MST
vector<edge> mst(Vec<2, edge> graph, ll &total) {
  total = 0;
  priority_queue<edge, vector<edge>, greater<edge>> pq;
  vector<edge> MST;
  bitset<20001> marked; // change size as needed
  marked[0] = 1;
  for (edge ep : graph[0]) pq.push(ep);
  while(MST.size()!=graph.size()-1 && pq.size()!=0) {
    edge e = pq.top(); pq.pop();
    int u = e.u, v = e.v, w = e.w;
    if(marked[u] && marked[v]) continue;
    else if(marked[u]) swap(u, v);
    for(edge ep : graph[u]) pq.push(ep);
    marked[u] = 1;
    MST.push_back(e);
    total += e.w;
  }
  return MST;
}
```

Union Find

```
int uf_find(subset* s, int i) {
  if (s[i].p != i) s[i].p = uf_find(s, s[i].p);
  return s[i].p;
}
void uf_union(subset* s, int x, int y) {
  int xp = uf_find(s, x), yp = uf_find(s, y);
  if (s[xp].rank > s[yp].rank) s[yp].p = xp, s[xp].sz += s[yp].sz;
  else if (s[xp].rank < s[yp].rank) s[xp].p = yp, s[yp].sz += s[xp].sz;
  else s[yp].p = xp, s[xp].rank++, s[xp].sz += s[yp].sz;
}
int uf_size(subset *s, int i) {
  return s[uf_find(s, i)].sz;
}
```

Bipartite Graph

```
/*
A bipartite graph has "left" and "right" set of nodes
Every edge has an endpoint in each set (L/R)
A matching is a subset of all edges
Such that each vertex is an endpoint
Of at most one edge in the subset
sqrt(V)*E time
tested on "piano lessons"
sourced from https://codeforces.com/blog/entry/58048
*/
#define MAXNODES 1001
bitset<MAXNODES> V;
bool match(int node, Vec<2,int> &G, vector<int> &R, vector<int> &L) {
  if (V[node]) return false;
  V[node] = 1;
  for(auto vec : G[node]) {
    if (R[vec] == -1 || match(R[vec], G, R, L)) {
      L[node] = vec; R[vec] = node;
      return true;
    }
  }
  return false;
}
vector<pair<int, int>> bipartite_match(Vec<2, int> &G, int m) {
  vector<int> L(G.size(), -1), R(m, -1);
  V.reset();
  bool running = true;
  while (running) {
    running = false;
    V.reset();
    for (int i=0;i<L.size();i++)
      if (L[i] == -1)
        running |= match(i, G, R, L);
  }
  vector<pair<int,int>> ret;
  for (int i = 0; i < L.size(); ++i)
    if(L[i]!=-1) ret.push_back({i, L[i]});
  return ret;
}
```

2D Grid Shortcut

```
#define inbound(x,n) (0<=x&&x<n)
#define fordir(x,y,n,m) for(auto[dx,dy]:dir)if(inbound(x+dx,n)&&inbound(y+dy,m))
const pair<int,int> dir[] = {{1,0},{0,1},{-1,0},{0,-1}};
```

8 2D Geometry

```
#define point complex<double>
#define EPS 0.0000001
#define sq(a) ((a)*(a))
#define cb(a) ((a)*(a)*(a))
double dot(point a, point b) { return real(conj(a)*b); }
double cross(point a, point b) { return imag(conj(a)*b); }
struct line { point a, b; };
struct circle { point c; double r; };
struct segment { point a, b; };
struct triangle { point a, b, c; };
struct rectangle { point tl, br; };
struct convex_polygon {
  vector<point> points;
  convex_polygon(vector<point> points) : points(points) {}
  convex_polygon(triangle a) {
    points.push_back(a.a); points.push_back(a.b); points.push_back(a.c);
  };
  convex_polygon(rectangle a) {
    points.push_back(a.tl);
      points.push_back({real(a.tl), imag(a.br)});
    points.push_back(a.br);
      points.push_back({real(a.br), imag(a.tl)});
  }
};
struct polygon {
  vector<point> points;
  polygon(vector<point> points) : points(points) {}
  polygon(triangle a) {
    points.push_back(a.a); points.push_back(a.b); points.push_back(a.c);
  }
  polygon(rectangle a) {
    points.push_back(a.tl);
      points.push_back({real(a.tl), imag(a.br)});
    points.push_back(a.br);
      points.push_back({real(a.br), imag(a.tl)});
  }
  polygon(convex_polygon a) {
    for (point v : a.points)
      points.push_back(v);
  }
};
// triangle methods
double area_heron(double a, double b, double c) {
  if (a < b) swap(a, b);
  if (a < c) swap(a, c);
  if (b < c) swap(b, c);
  if (a > b + c) return -1;
  return sqrt((a+b+c)*(c-a+b)*(c+a-b)*(a+b-c)/16.0);
}
// segment methods
double lengthsq(segment a) { return sq(real(a.a) - real(a.b)) + sq(imag(a.a) - imag(a.b)); }
double length(segment a) { return sqrt(lengthsq(a)); }
// circle methods
double circumference(circle a) { return 2 * a.r * M_PI; }
double area(circle a) { return sq(a.r) * M_PI; }
// rectangle methods
double width(rectangle a) { return abs(real(a.br) - real(a.tl)); }
double height(rectangle a) { return abs(imag(a.br) - imag(a.tl)); }
double diagonal(rectangle a) { return sqrt(sq(width(a)) + sq(height(a))); }
double area(rectangle a) { return width(a) * height(a); }
double perimeter(rectangle a) { return 2 * (width(a) + height(a)); }
// check if `a` fit's inside `b`
// swap equalities to exclude tight fits
bool doesFitInside(rectangle a, rectangle b) {
  int x = width(a), w = width(b), y = height(a), h = height(b);
  if (x > y) swap(x, y);
  if (w > h) swap(w, h);
  if (w < x) return false;
```

```cpp
 if (y <= h) return true;
 double a=sq(y)-sq(x), b=x*h-y*w, c=x*w-y*h;
 return sq(a) <= sq(b) + sq(c);
}
// polygon methods
// negative area = CCW, positive = CW
double area(polygon a) {
 double area = 0.0; int n = a.points.size();
 for (int i = 0, j = 1; i < n; i++, j = (j + 1) % n)
  area += (real(a.points[j]-a.points[i]))*(imag(a.points[j]+a.points[i]));
 return area / 2.0;
}
// get both unsigned area and centroid
pair<double, point> area_centroid(polygon a) {
 int n = a.points.size();
 double area = 0;
 point c(0, 0);
 for (int i = n - 1, j = 0; j < n; i = j++) {
  double v = cross(a.points[i], a.points[j]) / 2;
  area += v;
  c += (a.points[i] + a.points[j]) * (v / 3);
 }
 c /= area;
 return {area, c};
}
```

Intersection

```cpp
// -1 coincide, 0 parallel, 1 intersection
int intersection(line a, line b, point& p) {
 if (abs(cross(a.b - a.a, b.b - b.a)) > EPS) {
  p = cross(b.a - a.a, b.b - b.a) / cross(a.b - a.a, b.b - b.a) * (b - a) + a;
  return 1;
 }
 if (abs(cross(a.b - a.a, a.b - b.a)) > EPS) return 0;
 return -1;
}
// area of intersection
double intersection(circle a, circle b) {
 double d = abs(a.c - b.c);
 if (d <= b.r - a.r) return area(a);
 if (d <= a.r - b.r) return area(b);
 if (d >= a.r + b.r) return 0;
 double alpha = acos((sq(a.r) + sq(d) - sq(b.r)) / (2 * a.r * d));
 double beta = acos((sq(b.r) + sq(d) - sq(a.r)) / (2 * b.r * d));
 return sq(a.r) * (alpha - 0.5 * sin(2 * alpha)) + sq(b.r) * (beta - 0.5 * sin(2 * beta));
}
// -1 outside, 0 inside, 1 tangent, 2 intersection
int intersection(circle a, circle b, vector<point>& inter) {
 double d2 = norm(b.c - a.c), rS = a.r + b.r, rD = a.r - b.r;
 if (d2 > sq(rS)) return -1;
 if (d2 < sq(rD)) return 0;
 double ca = 0.5 * (1 + rS * rD / d2);
 point z = point(ca, sqrt(sq(a.r) / d2 - sq(ca)));
 inter.push_back(a.c + (b.c - a.c) * z);
 if (abs(imag(z)) > EPS) inter.push_back(a.c + (b.c - a.c) * conj(z));
 return inter.size();
}
// points of intersection
vector<point> intersection(line a, circle c) {
 vector<point> inter;
 c.c -= a.a;
 a.b -= a.a;
 point m = a.b * real(c.c / a.b);
 double d2 = norm(m - c.c);
 if (d2 > sq(c.r)) return 0;
 double l = sqrt((sq(c.r) - d2) / norm(a.b));
 inter.push_back(a.a + m + l * a.b);
 if (abs(l) > EPS) inter.push_back(a.a + m - l * a.b);
 return inter;
}
// area of intersection
double intersection(rectangle a, rectangle b) {
 double x1 = max(real(a.tl), real(b.tl)), y1 = max(imag(a.tl), imag(b.tl));
 double x2 = min(real(a.br), real(b.br)), y2 = min(imag(a.br), imag(b.br));
 return (x2 <= x1 || y2 <= y1) ? 0 : (x2-x1)*(y2-y1);
}
```

Convex Hull

```cpp
bool cmp(point a, point b) {
 if (abs(real(a) - real(b)) > EPS) return real(a) < real(b);
 if (abs(imag(a) - imag(b)) > EPS) return imag(a) < imag(b);
 return false;
}
convex_polygon convexhull(polygon a) {
 sort(a.points.begin(), a.points.end(), cmp);
 vector<point> lower, upper;
 for (int i = 0; i < a.points.size(); i++) {
  while (lower.size() >= 2 && cross(lower.back() - lower[lower.size() - 2], a.points[i] - lower.back()) < EPS)
   lower.pop_back();
  while (upper.size() >= 2 && cross(upper.back() - upper[upper.size() - 2], a.points[i] - upper.back()) > -EPS)
   upper.pop_back();
  lower.push_back(a.points[i]);
  upper.push_back(a.points[i]);
 }
 lower.insert(lower.end(), upper.rbegin() + 1, upper.rend());
 return convex_polygon(lower);
}
```

Maximum Colinear Points

```cpp
const ll range = 10000;
struct Slope { // a rational number with unsigned infinity (1,0)
 ll p, q;
 Slope(ll pP=0, ll qP=0) {
  if(qP==0) {
   p = 1, q = 0;
   return;
  }
  ll g = __gcd(pP, qP);
  pP /= g, qP /= g;
  if(qP < 0) pP *= -1, qP *= -1;
  p = pP, q = qP;
 }
 bool operator== (const Slope &other) const {
  return other.p == p && other.q == q;
 }
};
namespace std {
 template<>
 struct hash<Slope> { // typical rectangular/lattice hash
  size_t operator() (const Slope &r) const {
   return (2*range+1) * (r.p + range) + r.q + range;
  }
 };
}
// n points in [-range, range]
// compute the largest colinear subset
int max_colinear_points(vector<pair<ll,ll>> &points) {
 if(points.size() <= 2) return points.size();
 int best = 0;
 unordered_map<Slope, int> counter;
 for(int i=0;i<points.size();i++) {
  for(int j=i+1;j<points.size();j++) {
   Slope slope(points[i].second-points[j].second,points[i].first-points[j].first);
   best = max(best, ++counter[slope]+1);
  }
  if(i != points.size()-1) counter.clear();
 }
 return best;
}
```

9 3D Geometry

```cpp
struct point3d {
 double x, y, z;
 point3d operator+(point3d a) const { return {x+a.x, y+a.y, z+a.z}; }
 point3d operator*(double a) const { return {x*a, y*a, z*a}; }
 point3d operator-() const { return {-x, -y, -z}; }
 point3d operator-(point3d a) const { return *this + -a; }
 point3d operator/(double a) const { return *this * (1/a); }
 double norm() { return x*x + y*y + z*z; }
 double abs() { return sqrt(norm()); }
 point3d normalize() { return *this / this->abs(); }
};
double dot(point3d a, point3d b) { return a.x*b.x + a.y*b.y + a.z*b.z; }
point3d cross(point3d a, point3d b) { return {a.y*b.z - a.z*b.y, a.z*b.x - a.x*b.z, a.x*b.y - a.y*b.x}; }
struct line3d { point3d a, b; };
struct plane { double a, b, c, d; } // a*x + b*y + c*z + d = 0
struct sphere { point3d c; double r; };
#define sq(a) ((a)*(a))
#define cb(a) ((a)*(a)*(a))
double surface(circle a) { return 4 * sq(a.r) * M_PI; }
double volume(circle a) { return 4.0/3.0 * cb(a.r) * M_PI; }
```

10 Optimization

Snoob

```cpp
// SameNumberOfOneBits, next permutation
int snoob(int a) {
 int b = a & -a, c = a + b;
 return c | ((a ^ c) >> 2) / b;
}
// example usage
int main() {
 char l1[] = {'1', '2', '3', '4', '5'};
 char l2[] = {'a', 'b', 'c', 'd'};
 int d1 = 5, d2 = 4;
 // prints 12345abcd, 1234a5bcd, ...
 int min = (1<<d1)-1, max = min << d2;
 for (int i = min; i <= max; i = snoob(i)) {
  int p1 = 0, p2 = 0, v = i;
  while (p1 < d1 || p2 < d2) {
   cout << ((v & 1) ? l1[p1++] : l2[p2++]);
   v /= 2;
  }
  cout << '\n';
 }
}
```

Powers

```cpp
bool isPowerOf2(ll a) {
 return a > 0 && !(a & a-1);
}
bool isPowerOf3(ll a) {
 return a>0&&!(12157665459056928801ull%a);
}
bool isPower(ll a, ll b) {
 double x = log(a) / log(b);
 return abs(x-round(x)) < 0.00000000001;
}
```

11 Python

Recursion Limit Removal (Basic)

```python
import sys
sys.setrecursionlimit(10**6)
```

Recursion Limit Removal (Advanced)

```python
# @bootstrap over recursive function
# replace 'return' with 'yield'
# for when sys method does not work
from types import GeneratorType
def bootstrap(f, stack=[]):
 def wrappedfunc(*args, **kwargs):
  if stack:
   return f(*args, **kwargs)
  else:
   to = f(*args, **kwargs)
   while True:
    if type(to) is GeneratorType:
     stack.append(to)
     to = next(to)
    else:
     stack.pop()
     if not stack:
      break
     to = stack[-1].send(to)
   return to
 return wrappedfunc
# EXAMPLE recursive fibonacci
@bootstrap
def f(n):
 if (n < 2):
  yield n
 yield (yield f(n-1)) + (yield f(n-2))
```

Python 3 Compatibility

```python
import sys
from __future__ import division, print_function
if sys.version_info[0] < 3:
 from __builtin__ import xrange as range
 from future_builtins import ascii, filter, hex, map, oct, zip
```

12 Additional

Judge Speed

```cpp
// kattis: 0.50s
// codeforces: 0.421s
// atcoder: 0.455s
#include <bits/stdc++.h>
using namespace std;
int v = 1e9/2, p = 1;
int main() {
 for (int i = 1; i <= v; i++) p *= i;
 cout << p;
}
```

Judge Pre-Contest Checks

- __int128 and __float128 support?
- does extra or missing whitespace cause WA?
- documentation up to date?
- printer usage available and functional?

```cpp
// each case tests a different fail condition
// try them before contests to see error codes
struct g { int arr[1000000]; g(){}};
vector<g> a;
// 0=WA 1=TLE 2=MLE 3=OLE 4=SIGABRT 5=SIGFPE 6=SIGSEGV 7=recursive MLE
int judge(int n) {
 if (n == 0) exit(0);
 if (n == 1) while(1);
 if (n == 2) while(1) a.push_back(g());
 if (n == 3) while(1) putchar_unlocked('a');
 if (n == 4) assert(0);
 if (n == 5) 0 / 0;
 if (n == 6) *(int*)(0) = 0;
 return n + judge(n + 1);
}
```

GCC Builtin Docs

```
// 128-bit integer
__int128 a;
unsigned __int128 b;
// 128-bit float
// minor improvements over long double
__float128 c;
// log2 floor
__lg(n);
// number of 1 bits
// can add ll like popcountll for long longs
__builtin_popcount(n);
// number of trailing zeroes
__builtin_ctz(n);
// number of leading zeroes
__builtin_clz(n);
// 1-indexed least significant 1 bit
__builtin_ffs(n);
// parity of number
__builtin_parity(n);
```

Limits

int	±2147483647	$\pm 2^{31}-1$	10^9
uint	4294967295	$2^{32}-1$	10^9
ll	±9223372036854775807	$\pm 2^{63}-1$	10^{18}
ull	18446744073709551615	$2^{64}-1$	10^{19}
i128	±170141183460469231...	$\pm 2^{127}-1$	10^{38}
u128	340282366920938463...	$2^{128}-1$	10^{38}

Complexity classes input size (per second):

$O(n^n)$ or $O(n!)$	$n \leq 10$
$O(2^n)$	$n \leq 30$
$O(n^3)$	$n \leq 1000$
$O(n^2)$	$n \leq 30000$
$O(n\sqrt{n})$	$n \leq 10^6$
$O(n \log n)$	$n \leq 10^7$
$O(n)$	$n < 10^9$