Problem Statement

In probability theory and statistics, variance is the expectation of the squared deviation of a random variable from its mean. As a special case, we can compute the variance of a nonempty finite set X = { x_1, ..., x_n } as follows:

1. Let mu = (x_1 + ... + x_n) / n be the mean of the set.
2. Let y_i = (x_i - mu)^2 be the square of the difference between x_i and the mean.
3. The variance of X, denoted var(X), can now be computed as the average of all y_i. (In other words, as the sum of all y_i, divided by n.)

For example, if X = { 0, 1 }, we have mu = 1/2, then y_1 = y_2 = 1/4, and finally var(X) = (1/4 + 1/4) / 2 = 1/4.

The range of a nonempty finite set is the difference between its maximum and its minimum. For example, the range of the set { 40, 51, 67, 70 } is 70 - 40 = 30.

You are given a **s** that contains a set of distinct positive integers. You are also given an **R**.

Consider all nonempty subsets of **s** with range less than or equal to **R**. Alice computed the variance of each of those subsets. Bob took all Alice's results and computed their average. Compute and return the number computed by Bob.

Definition

Class:

AverageVarianceSubset
Method:

average
Parameters:

vector <int>, int
Returns:

double
Method signature:

double average(vector <int> s, int R)
(be sure your method is public)

Limits

Time limit (s):

2.000
Memory limit (MB):

256

Notes

- The returned value must have an absolute or relative error less than 1e-9.

Constraints

- **s** will contain between 1 and 50 elements, inclusive.
- Elements in **s** will be distinct.
- Each element in **s** will be between 1 and 1,000,000,000, inclusive.
- **R** will be between 0 and 1,000,000,000, inclusive.

Examples

0)
{1,2,3}
1
Returns: 0.1
This set has seven nonempty subsets. Out of those, five have range at most 1: {1}, {2}, {3}, {1,2}, and {2,3}.
Alice computed the variance of each of these subsets:

- The variance of {1} is 0.
- The variance of {2} is 0.
- The variance of {3} is 0.
- The variance of {1,2} is 1/4.
- The variance of {2,3} is 1/4.

Bob took Alice's results and computed their average: (0 + 0 + 0 + 1/4 + 1/4) / 5 = 1/10.
1)
{1,2,3}
3
Returns: 0.3095238095238096
This time Alice will consider all seven nonempty subsets. The two new subsets:

- The variance of {1,3} is 1.
- The variance of {1,2,3} is 2/3.

Bob will then compute the value (0+0+0+1/4+1/4+1+2/3)/7 = 13/42.
2)
{5,1,3,2}
1000000000
Returns: 1.2476851851851847
3)
{1,11,111,1111,11111,111111,1111111,11111111,111111111}
123456
Returns: 1.1349430459217174E9
4)
{1,11,111,1111,11111,111111,1111111,11111111,111111111}
999999999
Returns: 9.989198236452121E14

5)
{1,1000000000}
1000000000
Returns: 8.3333333166666672E16
Note that the answer can be very large.
6)
{1,1000000000}
1
Returns: 0.0