Identity 1. *Let A and B be sets. Show that*

$$A \cup (B - A) = A \cup B$$

Proof.

$$\begin{aligned} A \cup (B - A) &= A \cup (B \cap A^c) && \text{set difference} \\ &= A \cup (A^c \cap B) && \text{commutative} \\ &= (A \cup A^c) \cap (A \cup B) && \text{distributive} \\ &= U \cap (A \cup B) && \text{complement} \\ &= A \cup B && \text{identity} \end{aligned}$$

□

Proof. Let $x \in A \cup (B - A)$. Then $x \in A$ or $x \in (B - A)$ by definition of union. So $x \in B$ and $x \notin A$ (by set difference). But $x \in A$ by previous statement, so $x \in A$ or $x \in B$. By definition of union, $x \in (A \cup B)$. □

Identity 2. *Let A and B be sets. Show that*

$$(A \cap B^c)^c \cup B = A^c \cup B$$

Proof.

$$\begin{aligned} (A \cap B^c)^c \cup B &= (A^c \cup (B^c)^c) \cup B && \text{de Morgan's} \\ &= (A^c \cup B) \cup B && \text{double complement} \\ &= A^c \cup (B \cup B) && \text{associative} \\ &= A^c \cup B && \text{idempotent} \end{aligned}$$

□

Identity 3. *Let A, B and C be sets. Show that*

$$(A - B) - C = A - (B \cup C)$$

Proof.

$$\begin{aligned} (A - B) - C &= (A \cap B^c) - C && \text{set difference} \\ &= (A \cap B^c) \cap C^c && \text{set difference} \\ &= A \cap (B^c \cap C^c) && \text{associative} \\ &= A \cap (B \cup C)^c && \text{de Morgan's} \\ &= A - (B \cup C) && \text{set difference} \end{aligned}$$

□

Proof. Let $x \in (A - B) - C$. Then $x \in (A - B)$ and $x \notin C$ by definition of set difference. Further, $x \in A$ and $x \notin B$ also by definition of set difference. Thus $x \in A$ and $x \notin B$ and $x \notin C$, which implies $x \notin (BorC)$. Hence, $x \notin (B \cup C)$ by definition of union. Thus, given $x \in A$ we have $x \in A - (B \cup C)$ by definition of set difference. □

Identity 4. *Let A, B and C be sets. Show that*

$$(B - A) \cup (C - A) = (B \cup C) - A$$

Proof.

$$\begin{aligned}(B - A) \cup (C - A) &= (B \cap A^c) \cup (C \cap A^c) && \text{set difference}\\ &= (B \cup C) \cap A^c && \text{distibutive}\\ &= (B \cup C) - A && \text{set difference}\end{aligned}$$

□

Proof. Let $x \in (B - A) \cup (C - A)$. Then $x \in (B - A)$ or $x \in (C - A)$. If $x \in (B - A)$, then $x \in B$ and $x \notin A$ by definition of set difference. If $x \in (C - A)$, then $x \in C$ and $x \notin A$ by definition of set difference. Thus, $x \in B$ or $x \in C$ and $x \notin A$. By definition of union, $x \in (B \cup C)$. By definition of set difference, if $x \in (B \cup C)$ and $x \notin A$ then $x \in (B \cup C) - A$. □