# 20) Regression and Classification Trees

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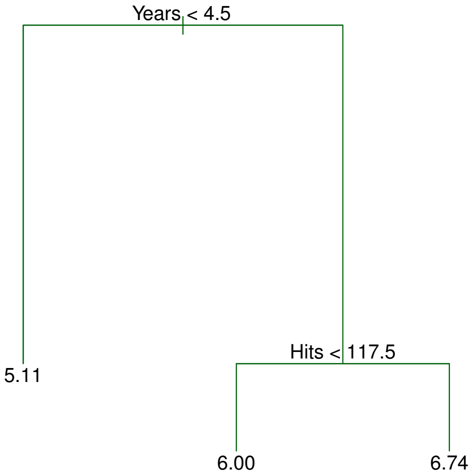
# Reference

Tables, Graphics, and Figures from:

1) Hastie et al. (2017): Ch 9.2

2) James et al. (2017): Ch 8.1

# Hitters Data

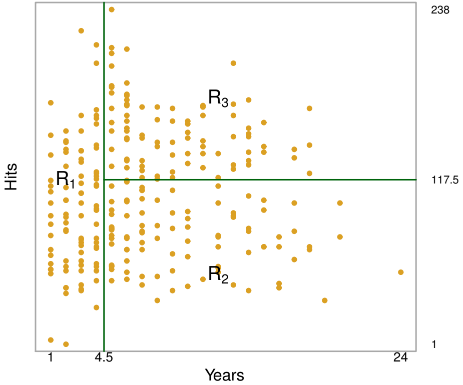


$$e^{5.11} \cong \$165K, \; e^{6} \cong \$402K, \; e^{6.74} \cong \$845K$$

# Three-Region Partition

$$R_3 = \{X | Years \geq 4.5, Hits \geq 117.5\}$$

$$R_1 = \{X | Years < 4.5\}$$



$$R_2 = \{X | Years \geq 4.5, Hits < 117.5\}$$

# OLS vs Trees (Impurity Measure)

$$f(x) = \beta_0 + \sum_{j=1}^{p} \beta_j x_j$$

$$f(x) = \sum_{m=1}^{M} c_m I(x \in R_m)$$

$$\hat{c}_m = \frac{1}{N_m} \sum_{x_i \in R_m} y_i$$

$$Q_m(T) = \frac{1}{N_m} \sum_{x_i \in R_m} (y_i - \hat{c}_m)^2$$

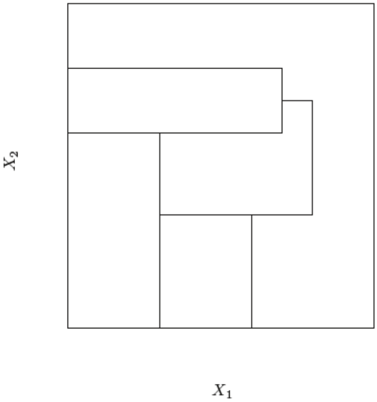
# Top-down Greedy (Recursive Binary Splitting)

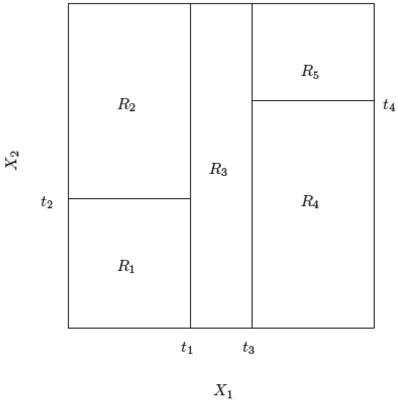
$$\sum_{j=1}^{J} \sum_{i \in R_j} (y_i - \hat{y}_{R_j})^2$$

$$R_1(j, s) = \{X | X_j < s\}$$
$$R_2(j, s) = \{X | X_j \geq s\}$$

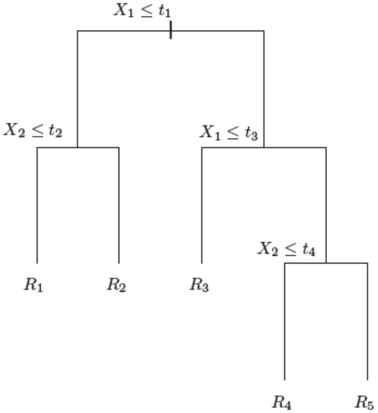
$$\sum_{i: x_i \epsilon R_1(j,s)} (y_i - \hat{y}_{R_1})^2 + \sum_{i: x_i \epsilon R_2(j,s)} (y_i - \hat{y}_{R_2})^2$$

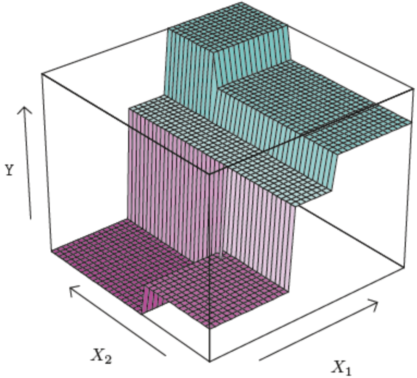
# No Recursive Binary Splitting vs Recursive Binary Splitting





# Tree and Perspective Plot



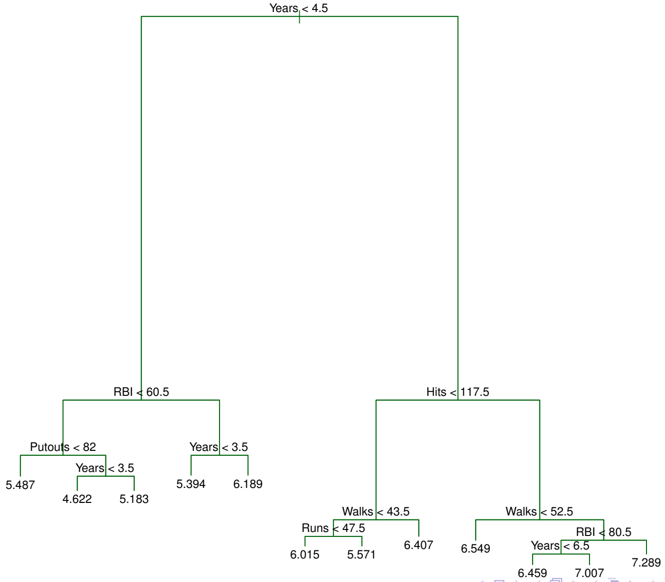


# Cost Complexity Pruning (Weakest Link Pruning)

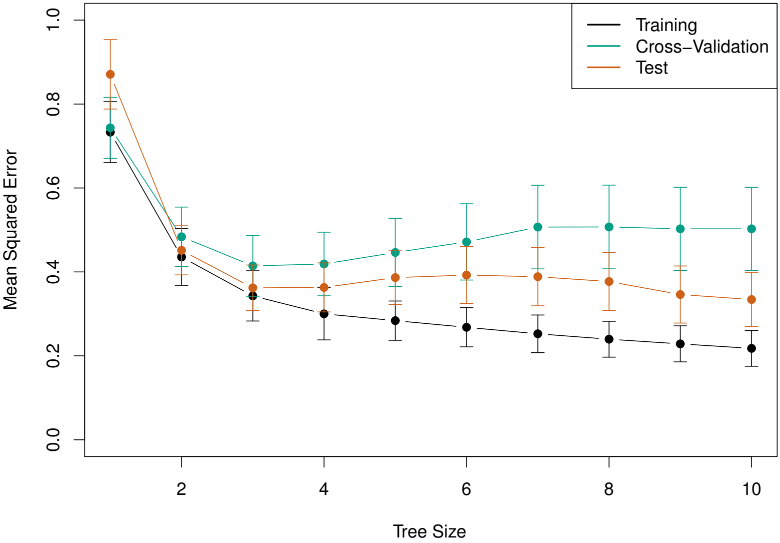
$$\sum_{m=1}^{|T|} \sum_{x_i \in R_m} (y_i - \hat{y}_{R_m})^2 + \alpha |T|$$

$|T| = \#$ of terminal nodes of the tree

# Unpruned Tree (Top-down Greedy Splitting)



# Six-Fold Cross-Validation for Pruning Tree



# Training Error Rate, Gini Index, and Entropy

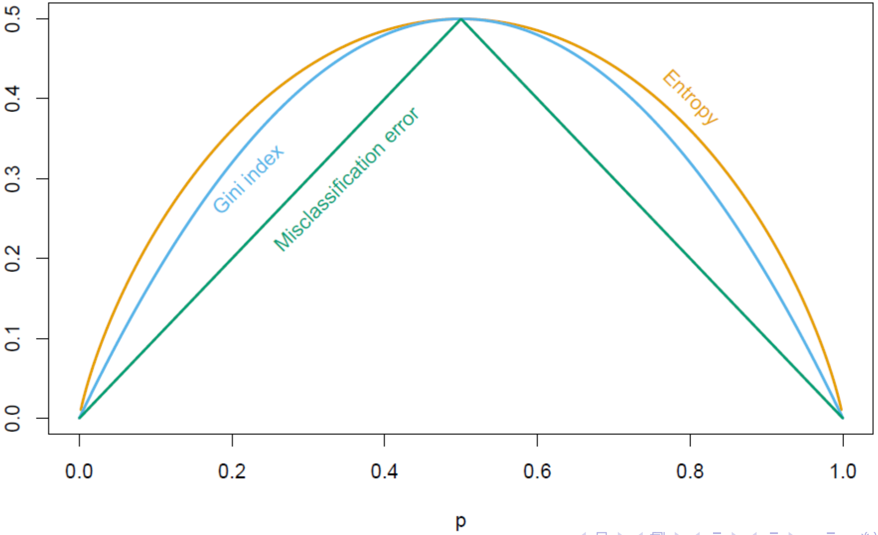
$$\hat{p}_{mk} = \frac{1}{N_m} \sum_{x_i \in R_m} I(y_i = k)$$

$$E = 1 - \underset{k}{max} \left(\hat{p}_{mk}\right)$$

$$G = \sum_{k=1}^{K} \hat{p}_{mk}(1 - \hat{p}_{mk})$$

$$D = - \sum_{k=1}^{K} \hat{p}_{mk} log \hat{p}_{mk}$$

# Gini Index and Entropy are more sensitive to changes in the node probabilities



# Heart Data Set

303 patients

**AHD**: Yes for heart disease based on an angiographic test

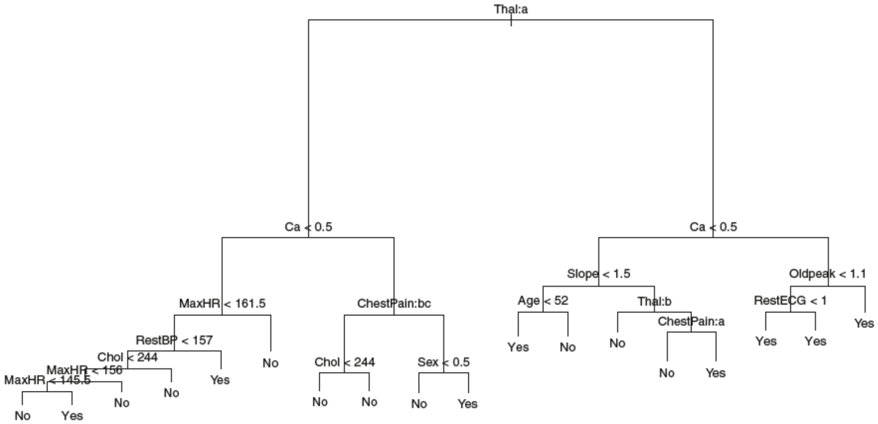
**Thal**: Thallium stress test, nuclear imaging shows how blood flows into heart

**ChestPain**: angina, atypical angina, non-anginal pain, and asymptomatic

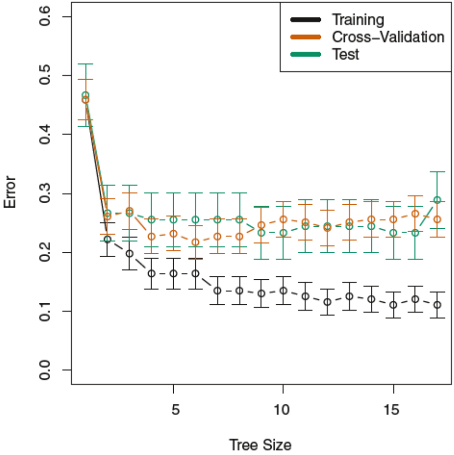
**RestECG**: Electrocardiograms

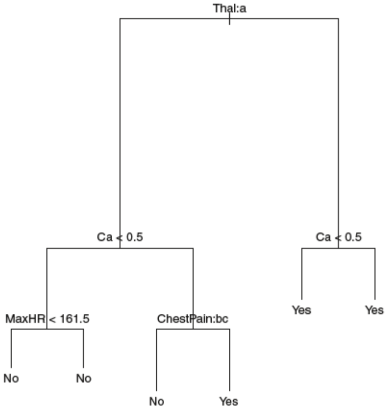
# Heart Data: Unpruned Tree

Normal $<$ $|$Thal:a$|$ $<$ Fixed or Reversible Defects

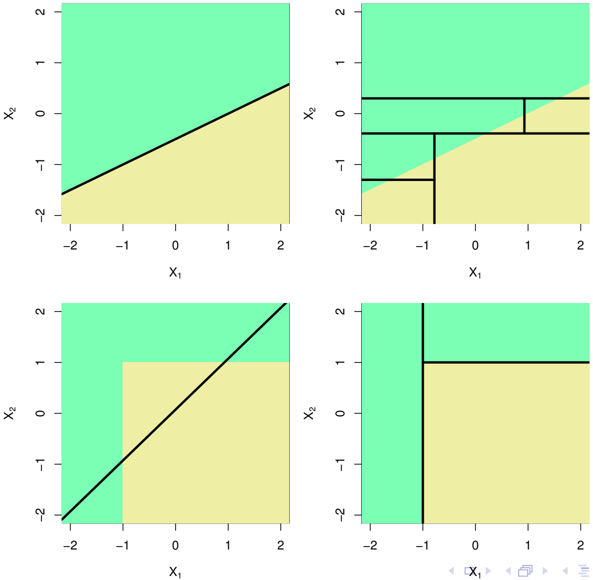


# Pruned Tree (Minimal Cross-Validation Error)





# Linear vs Non-linear True Decision Boundary



$X_2$
2
1
0
−1
−2
−2
−1
0
1
2
$X_1$
$X_2$
2
1
0
−1
−2
−2
−1
0
1
2
$X_1$
$X_2$
2
1
0
−1
−2
−2
−1
0
1
2
$X_1$
$X_2$
2
1
0
−1
−2
−2
−1
0
1
2
$X_1$