12.3) Bootstrap

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December 2019

Reference

Tables, Graphics, and Figures from

An Introduction to Statistical Learning

James et al. (2017): Ch 5.2

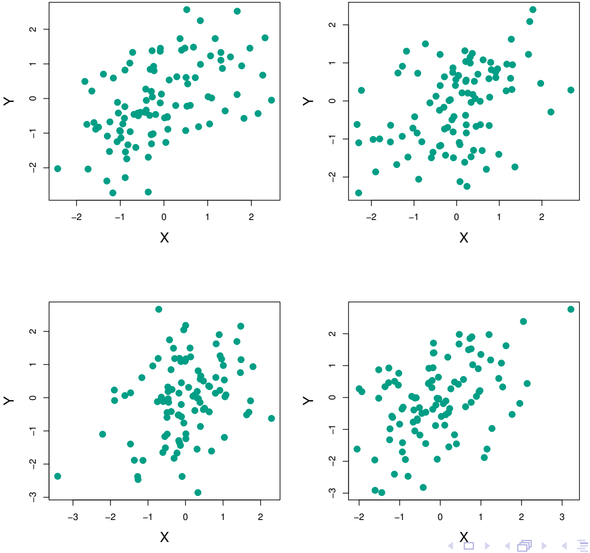
100 Simulated Returns for Investments

$$Var[\alpha X + (1-\alpha)Y]$$

$$\alpha = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}}$$

$$\hat{\alpha} \in [53\% \text{ to } 65\%]$$

57.6%, 53.2%, 65.7%, and 65.1%



1000 Estimates for α

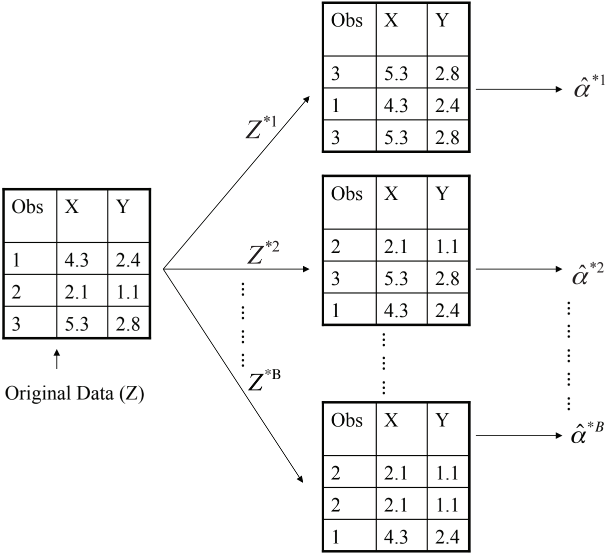
$$\sigma_X^2 = 1,\ \sigma_Y^2 = 1.25,\ \sigma_{XY} = 0.5$$

$$\therefore\ \alpha = 0.6$$

$$\bar{\alpha} = \frac{1}{1000}\sum_{r=1}^{1000}\hat{\alpha}_r = 0.5996$$

$$SE(\hat{\alpha}) = \sqrt{\frac{1}{999}\sum_{r=1}^{1000}(\hat{\alpha}_r - \bar{\alpha})^2} = 0.083$$

Bootstrap Approach (Sampling from Data)



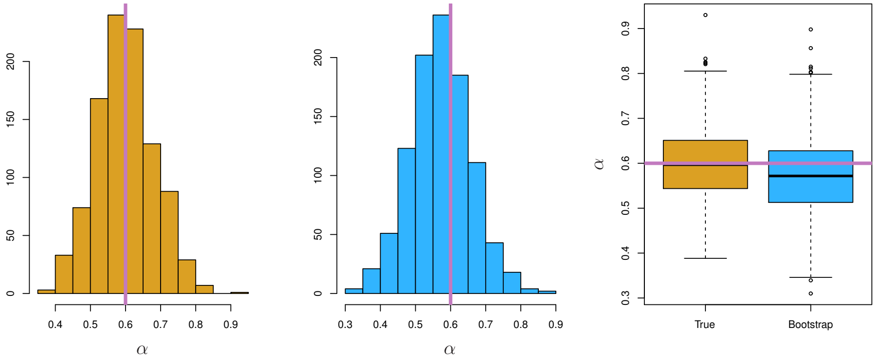
Obs	X	Y
1	4.3	2.4
2	2.1	1.1
3	5.3	2.8

Obs	X	Y
3	5.3	2.8
1	4.3	2.4
3	5.3	2.8

Obs	X	Y
2	2.1	1.1
3	5.3	2.8
1	4.3	2.4

Obs	X	Y
2	2.1	1.1
2	2.1	1.1
1	4.3	2.4

1000 Simulated Data Sets from the True Population vs 1000 Bootstrap Samples from a Single Data Set



1000 Simulated Data Sets

$$SE(\hat{\alpha}) = 0.087$$

$$\sqrt{\frac{1}{B-1}\sum_{r=1}^{B}\left(\hat{\alpha}^{*r} - \frac{1}{B}\sum_{r'=1}^{B}\hat{\alpha}^{*r'}\right)^2}$$

$$SE_B(\hat{\alpha}) = 0.083$$