

# Designing asymptotic geodesic hybrid gridshells

Curves and Surfaces 2022

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June 23, Arcachon

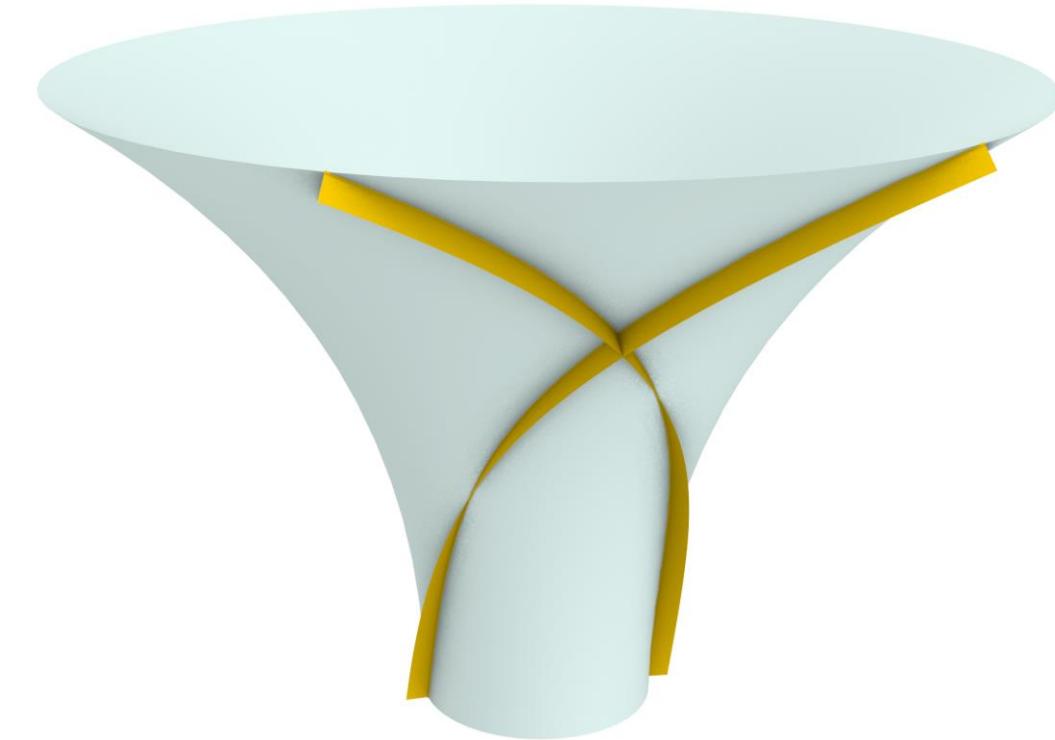
# Introduction



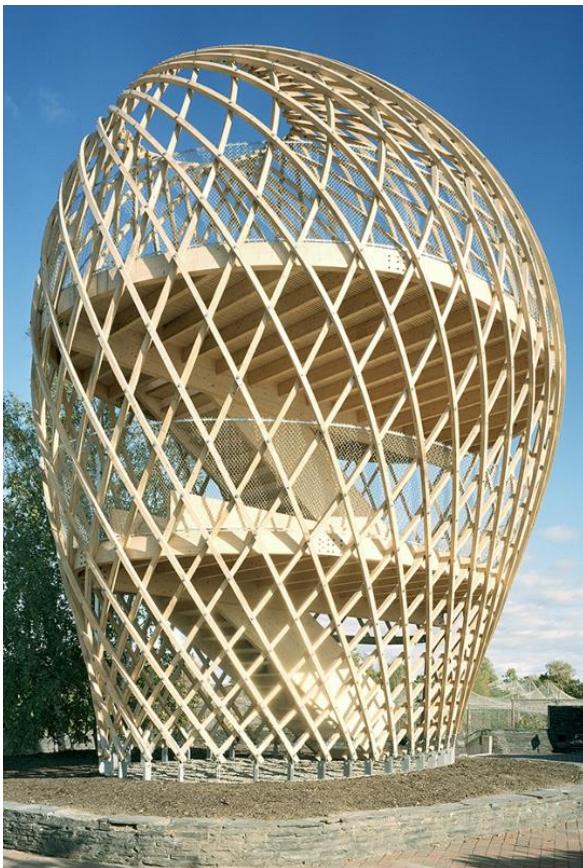
Geodesic strips



Asymptotic strips



# Introduction

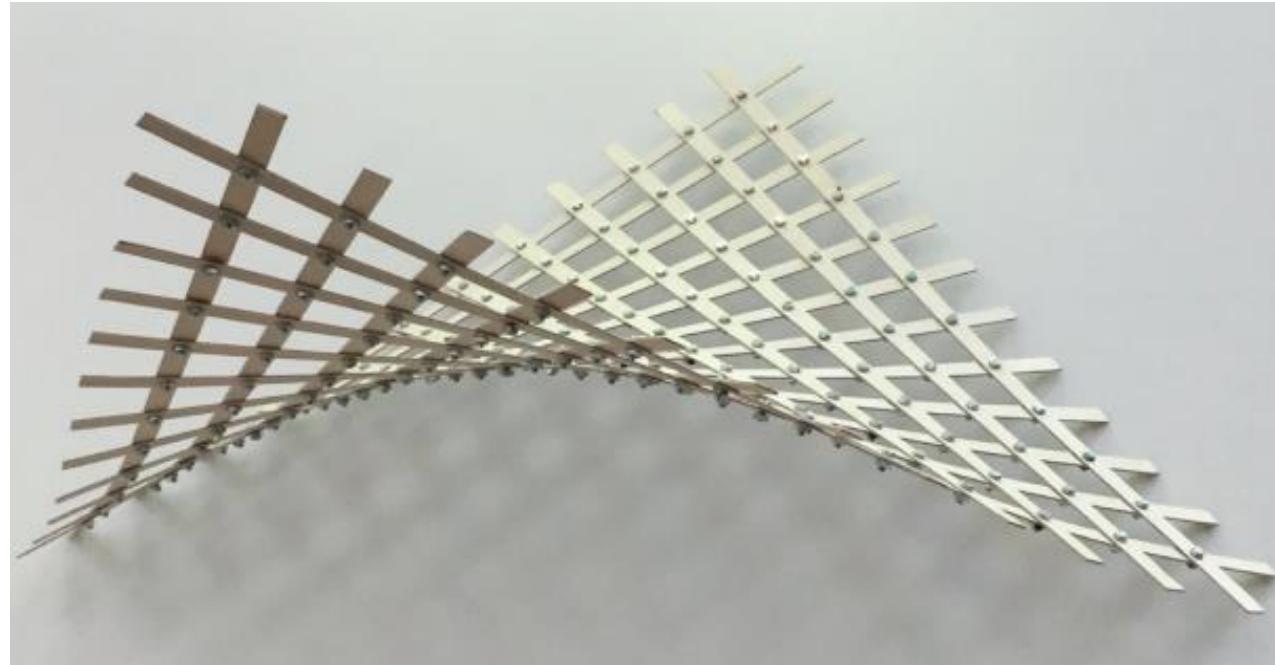


Geodesic gridshell

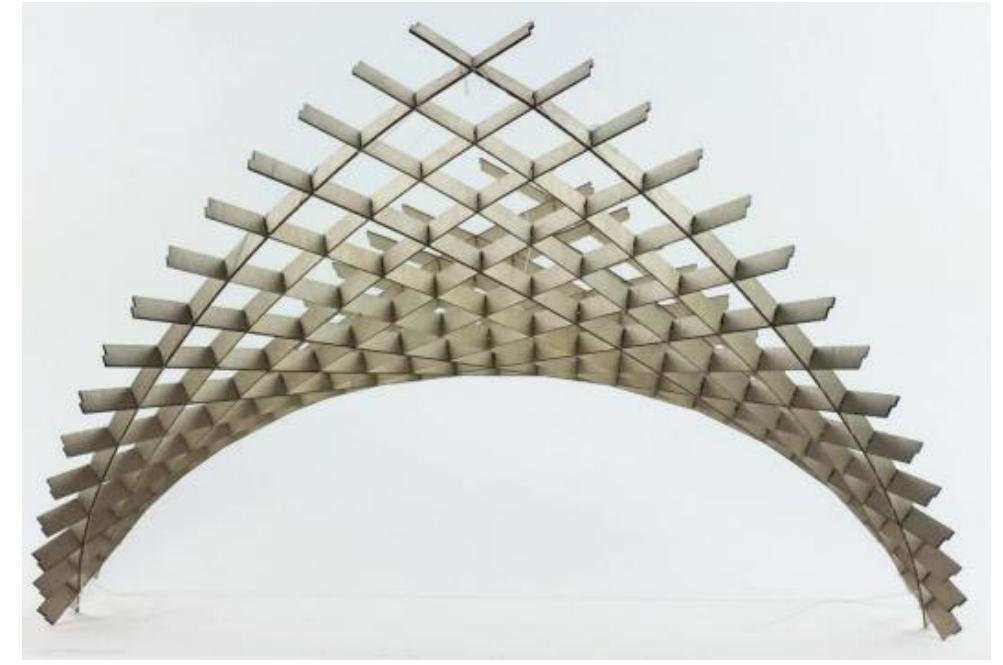


Asymptotic gridshell

# Introduction



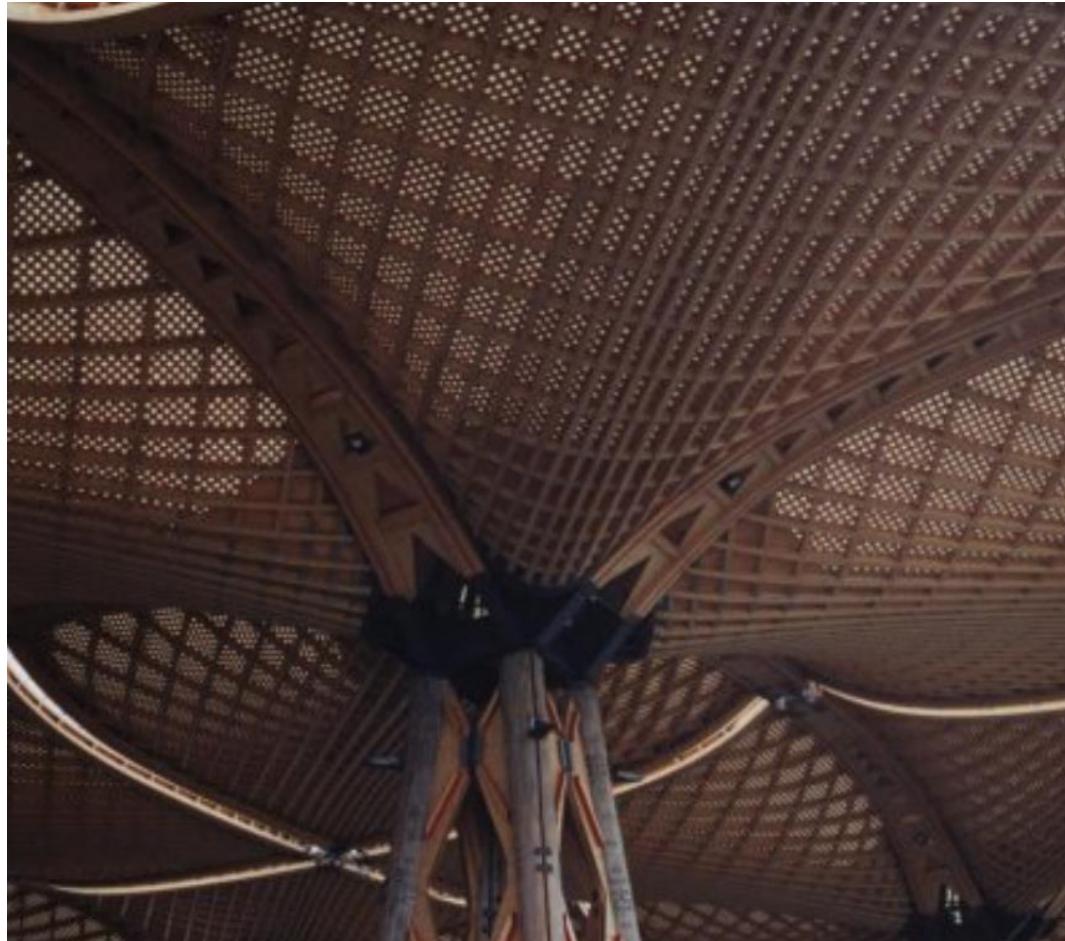
Geodesic gridshell



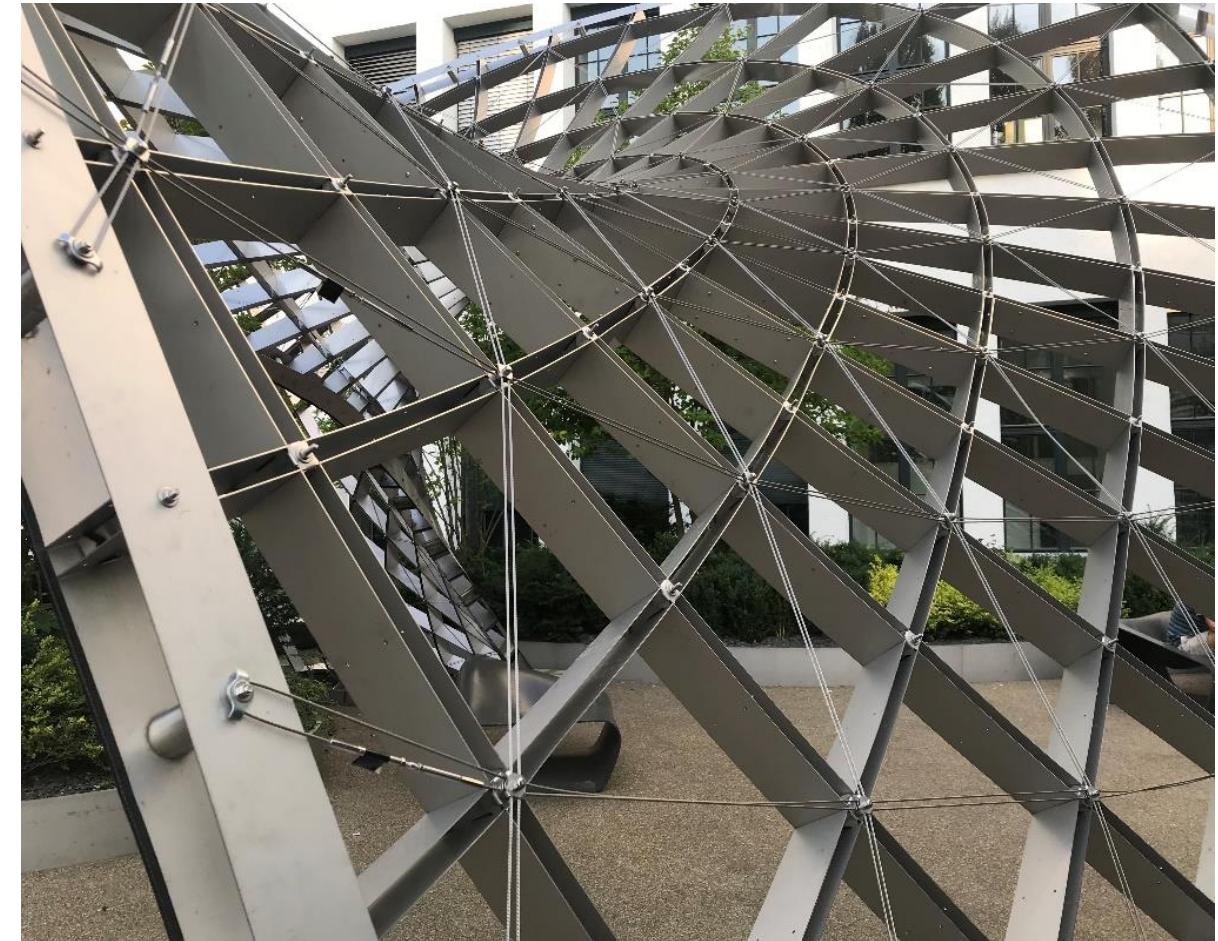
Asymptotic gridshell



# Motivation

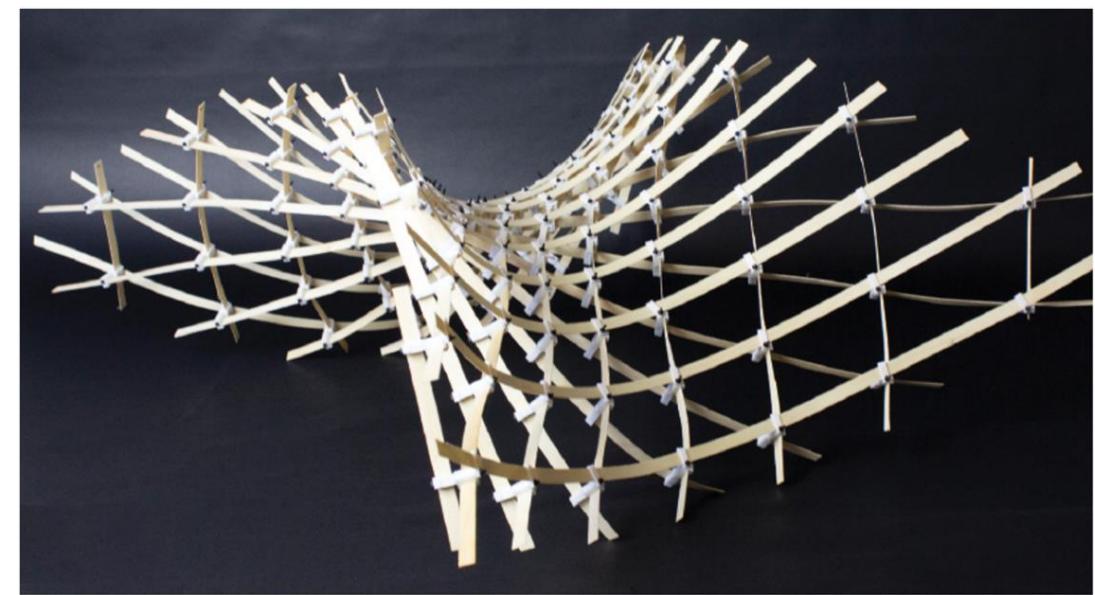
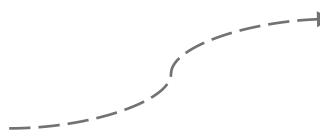
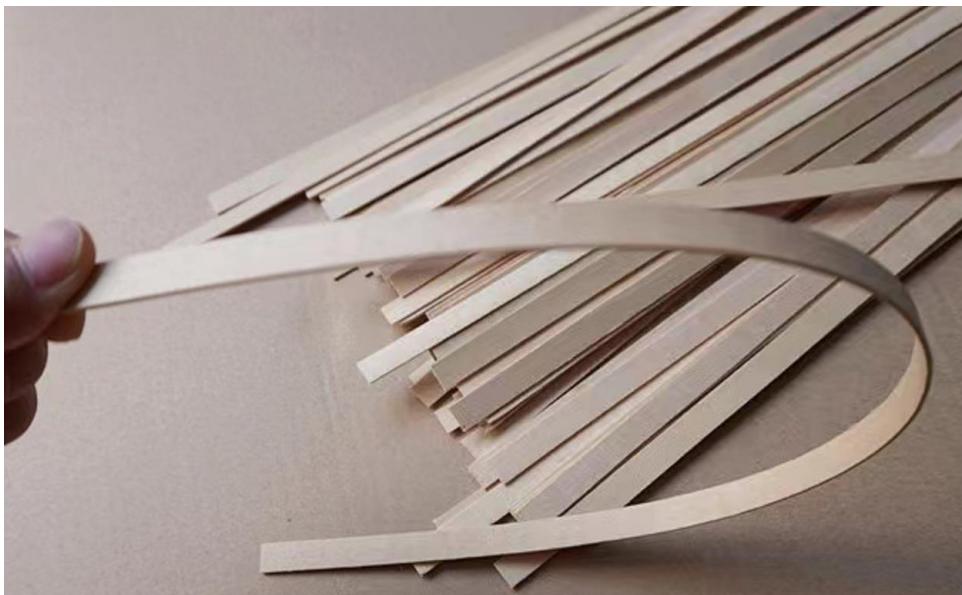


Expo Hannover Pavillion



The Inside/Out Gridshell, TUM

# Goal



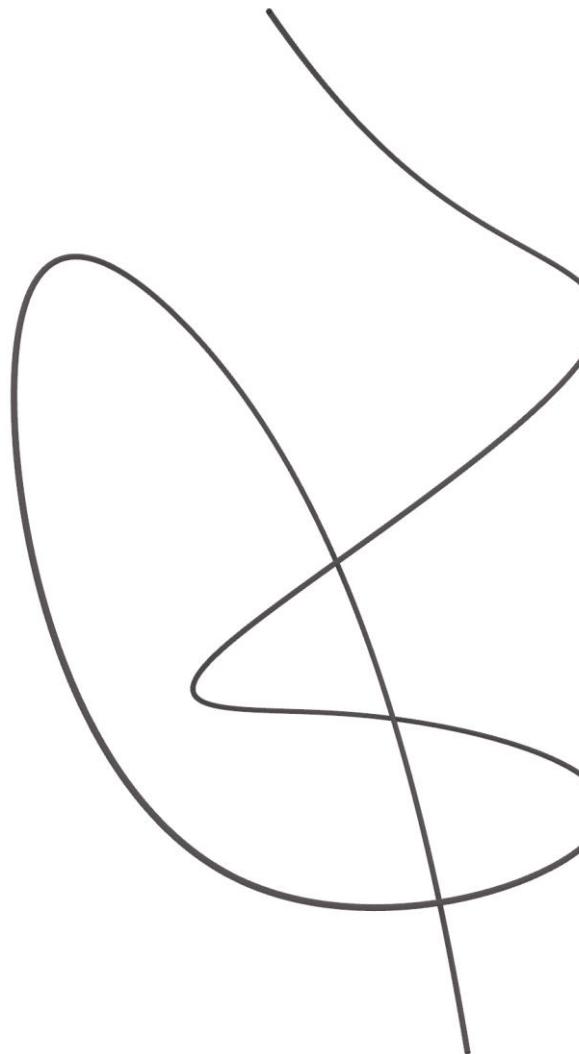
# Elementary Differential Geometry



جامعة الملك عبد الله  
لله العلوم والتكنولوجيا  
King Abdullah University of  
Science and Technology

| VCC VISUAL  
COMPUTING  
CENTER

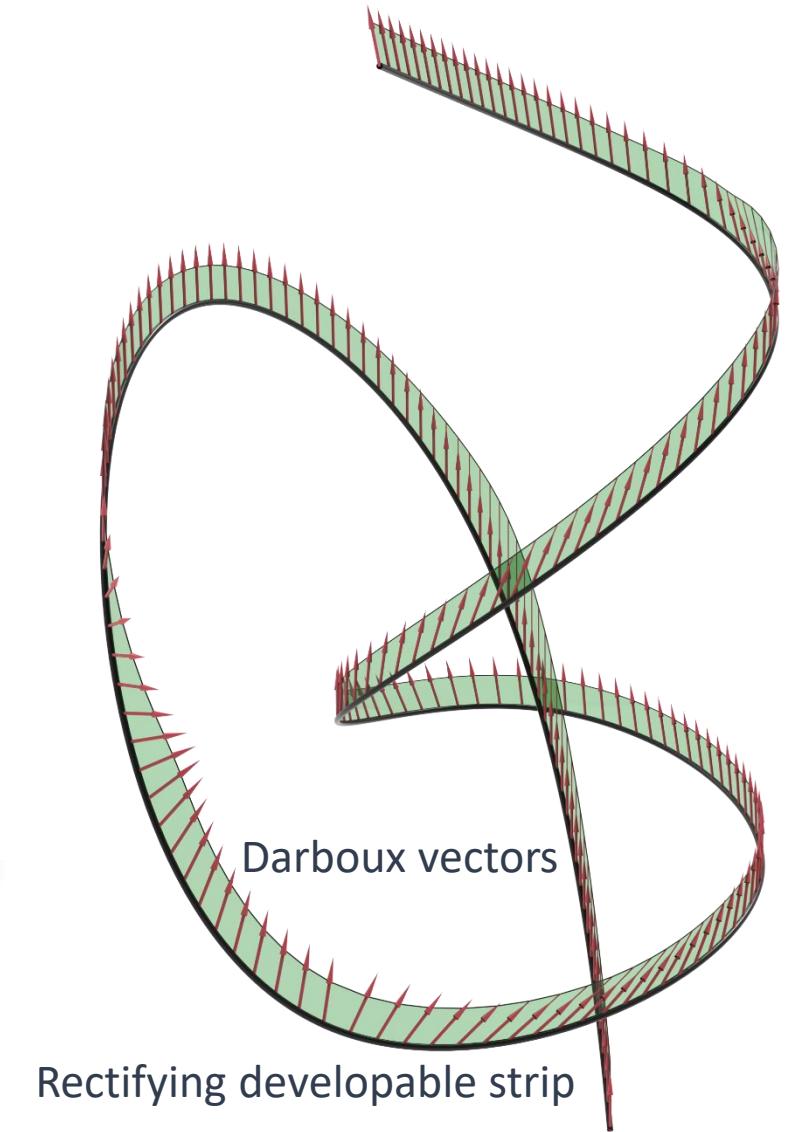
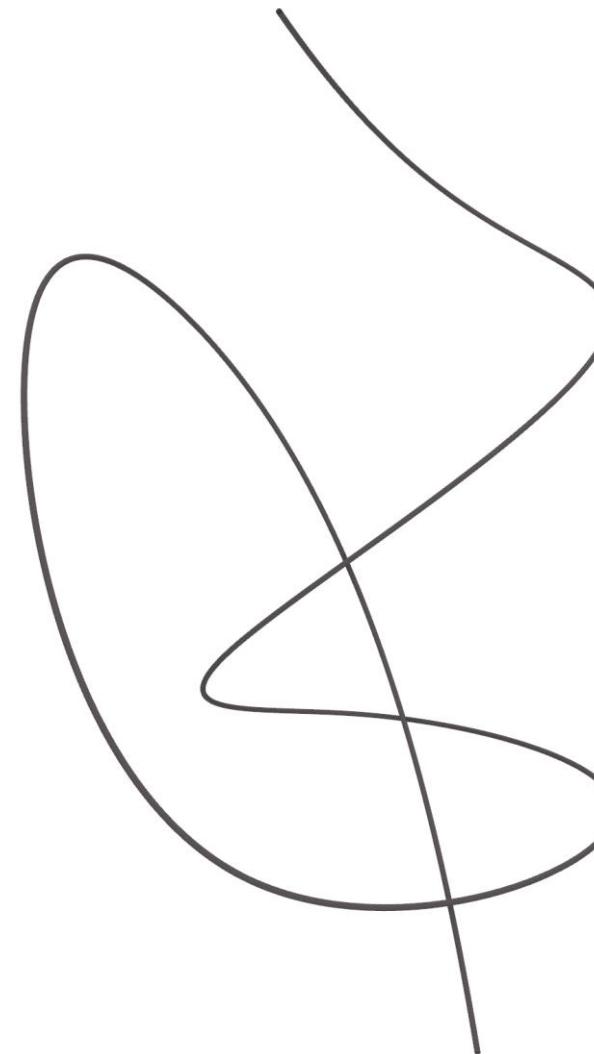
# Straight development



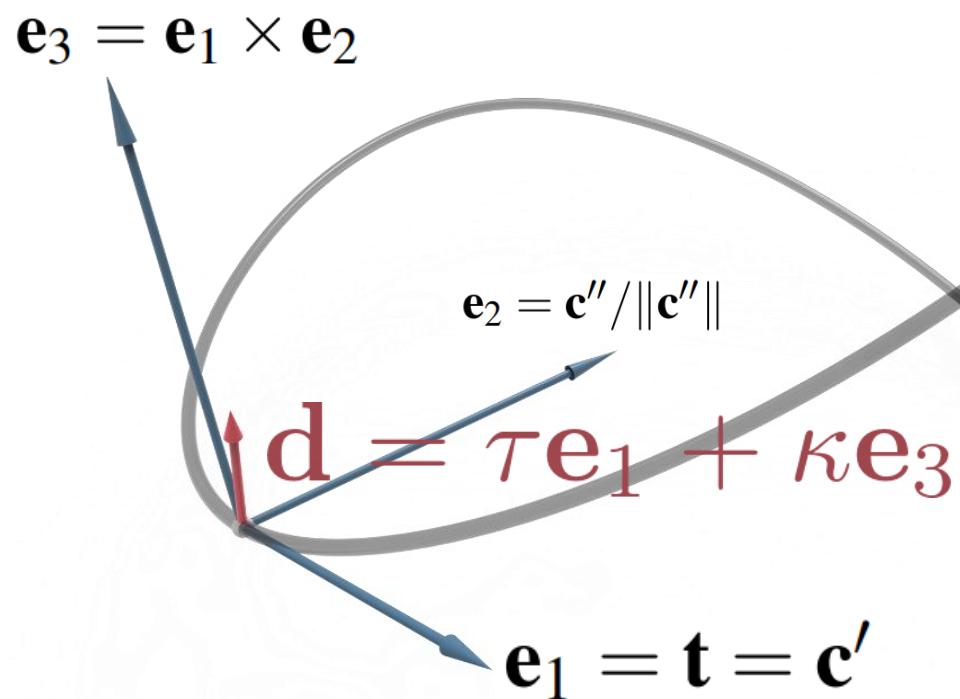
?

- Developable surface
- Pass through it
- Straight development

# Straight development



# Darboux vector



$$(\mathbf{e}_1(s), \mathbf{e}_2(s), \mathbf{e}_3(s))$$

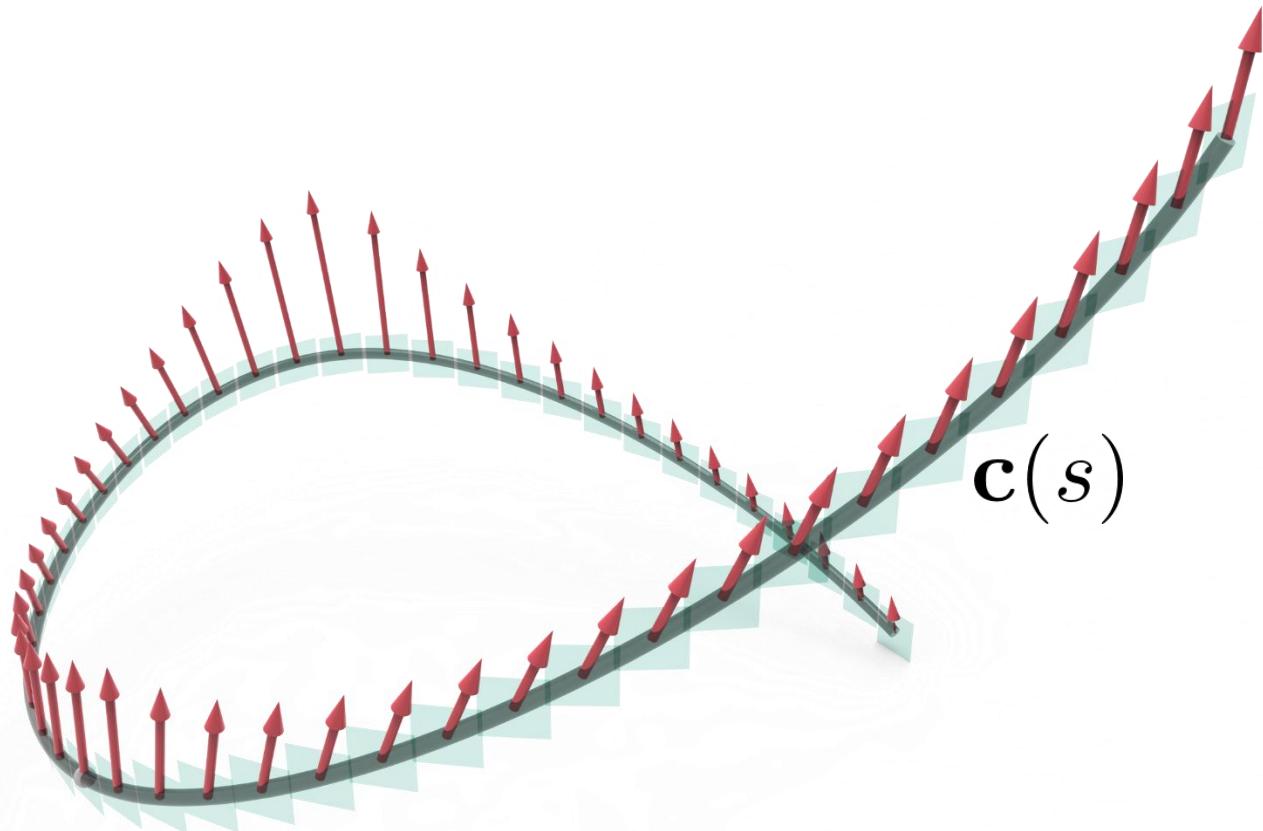
$$\begin{bmatrix} \mathbf{e}'_1 \\ \mathbf{e}'_2 \\ \mathbf{e}'_3 \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{bmatrix}$$

$\kappa$ : curvature

$\tau$ : torsion

$$\mathbf{e}'_i = \mathbf{d} \times \mathbf{e}_i, \quad i = 1, 2, 3$$

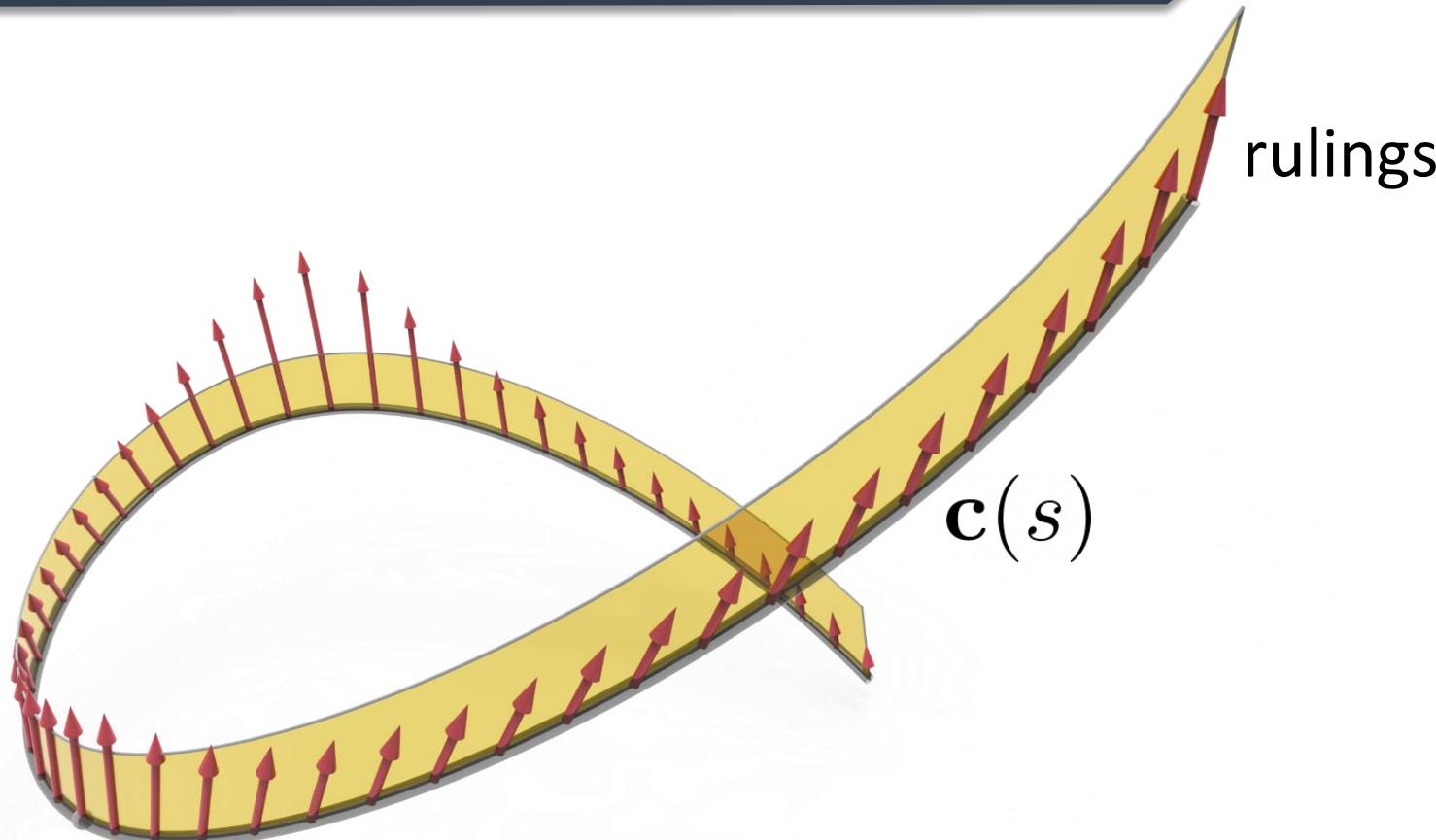
# Darboux vector



rectifying planes

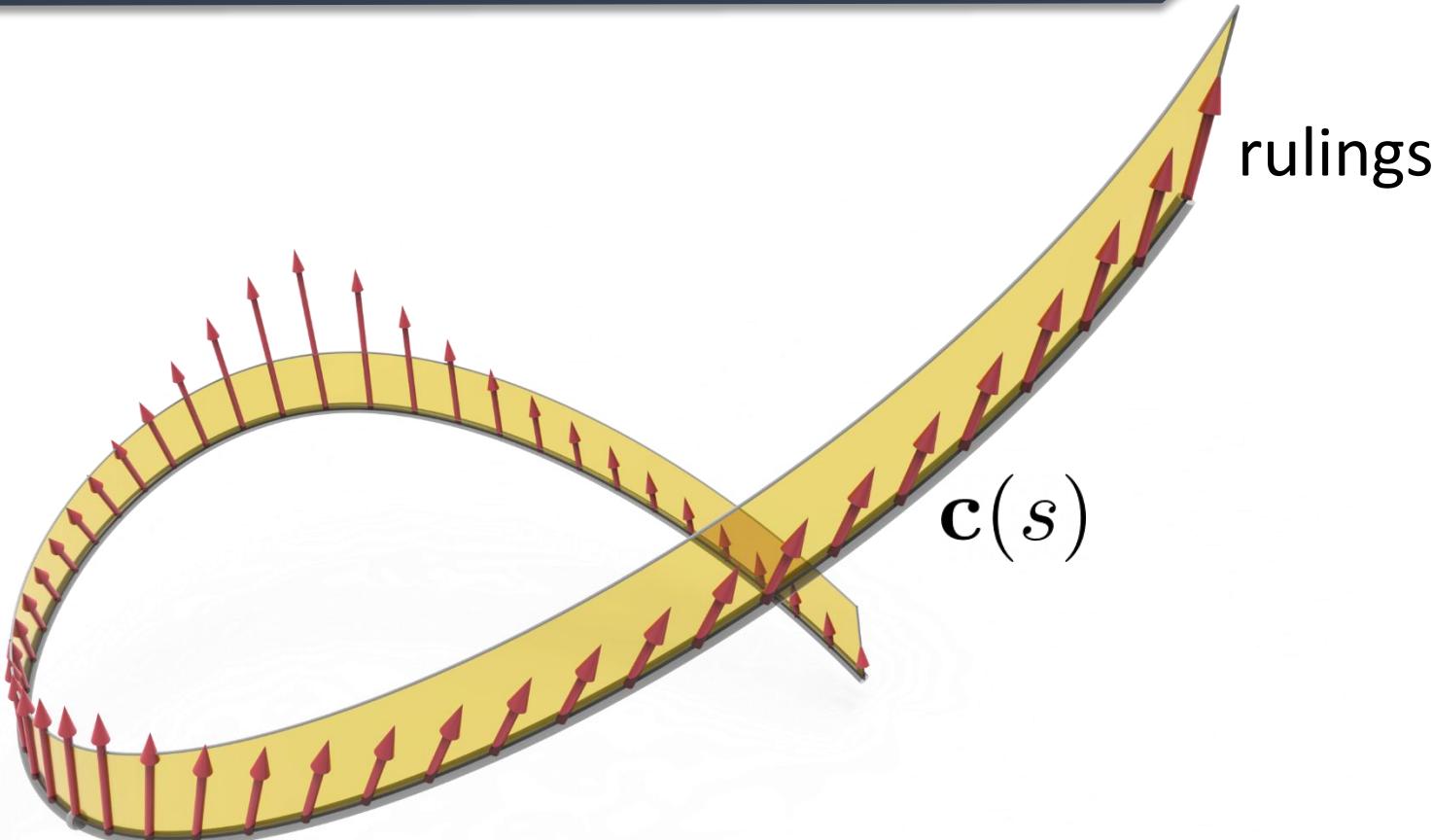
$$\mathbf{d} = \tau \mathbf{e}_1 + \kappa \mathbf{e}_3$$

# Rectifying developable surface



rulings

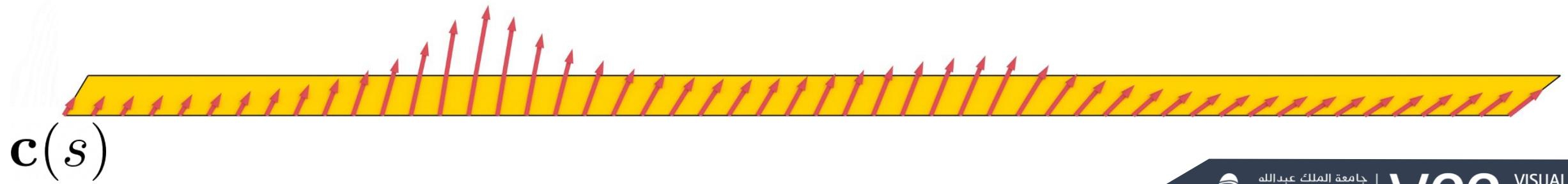
# Rectifying developable surface



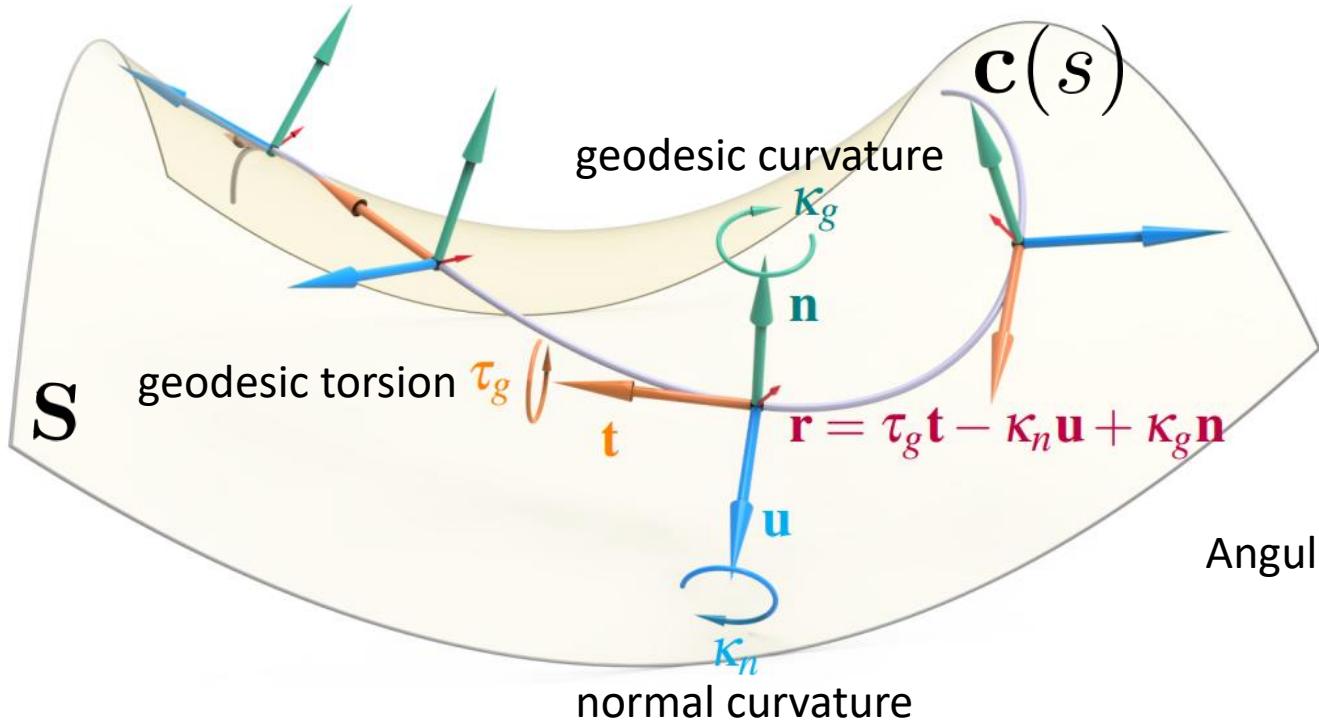
$c(s)$

rulings

- Developable surface
- Pass through it
- Straight development



# Darboux Frame



$$(\mathbf{t}(s), \mathbf{u}(s), \mathbf{n}(s))$$

$$\mathbf{t}' = \mathbf{r} \times \mathbf{t}$$

$$\mathbf{u}' = \mathbf{r} \times \mathbf{u}$$

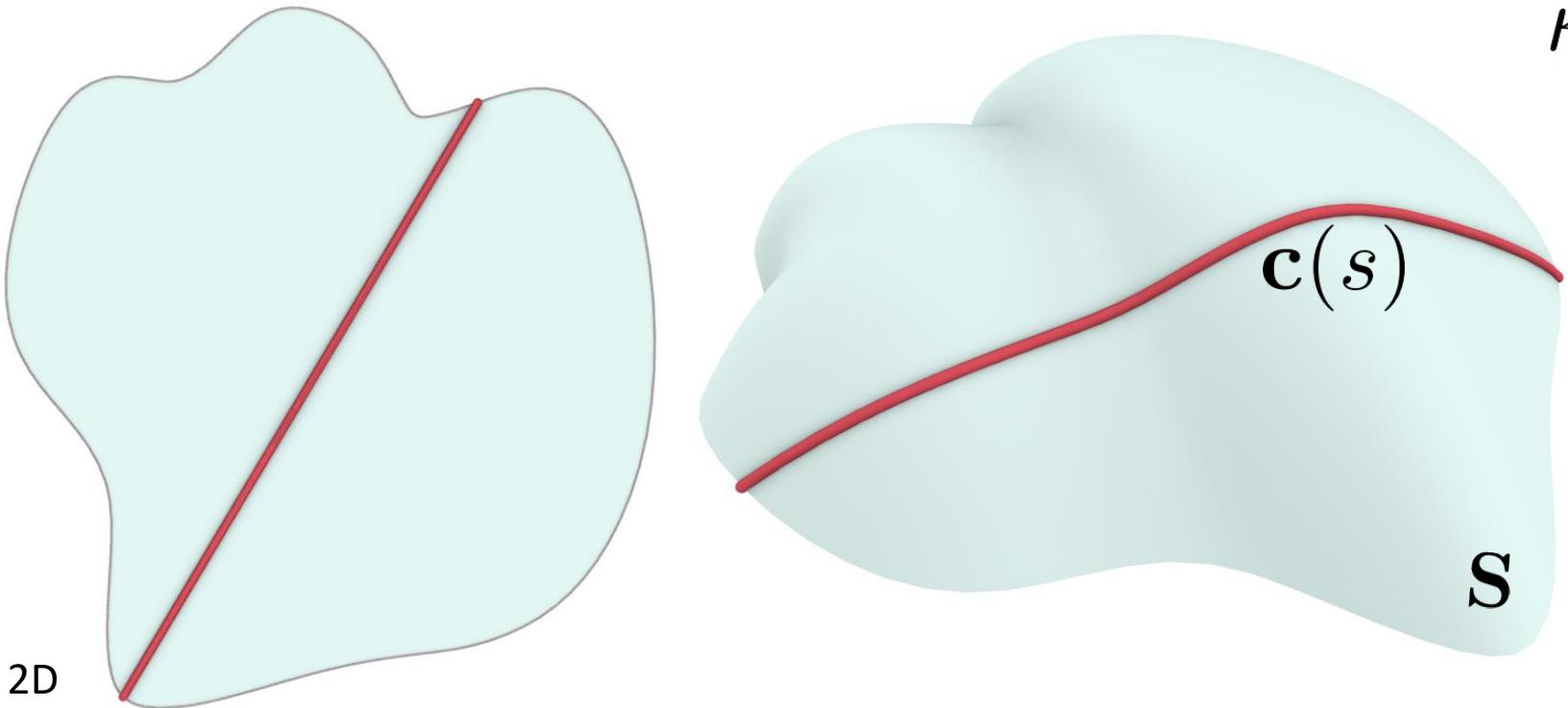
$$\mathbf{n}' = \mathbf{r} \times \mathbf{n}$$

Angular velocity vector:

$$\boldsymbol{\tau} = \tau_g \mathbf{t} - \kappa_n \mathbf{u} + \kappa_g \mathbf{n}$$

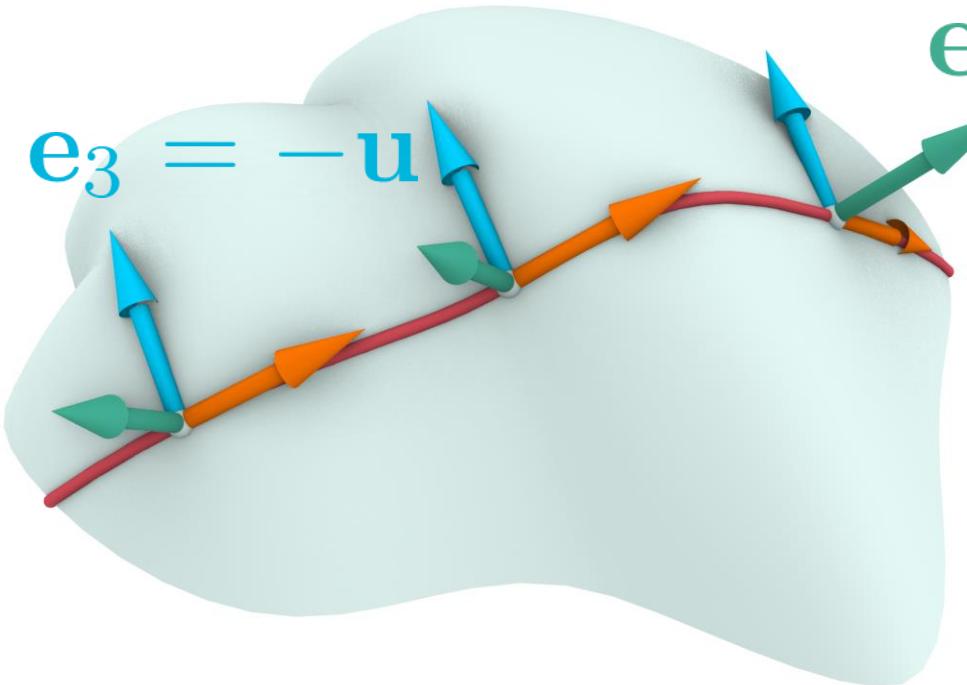


# Geodesic curve



$$\kappa_g = 0$$

# Geodesic curve



$$\kappa_g = 0$$

$$\mathbf{c}'' = \kappa_n \mathbf{n}$$

$$\mathbf{e}_2 = \mathbf{n}, \mathbf{e}_3 = -\mathbf{u}$$

# Geodesic strip



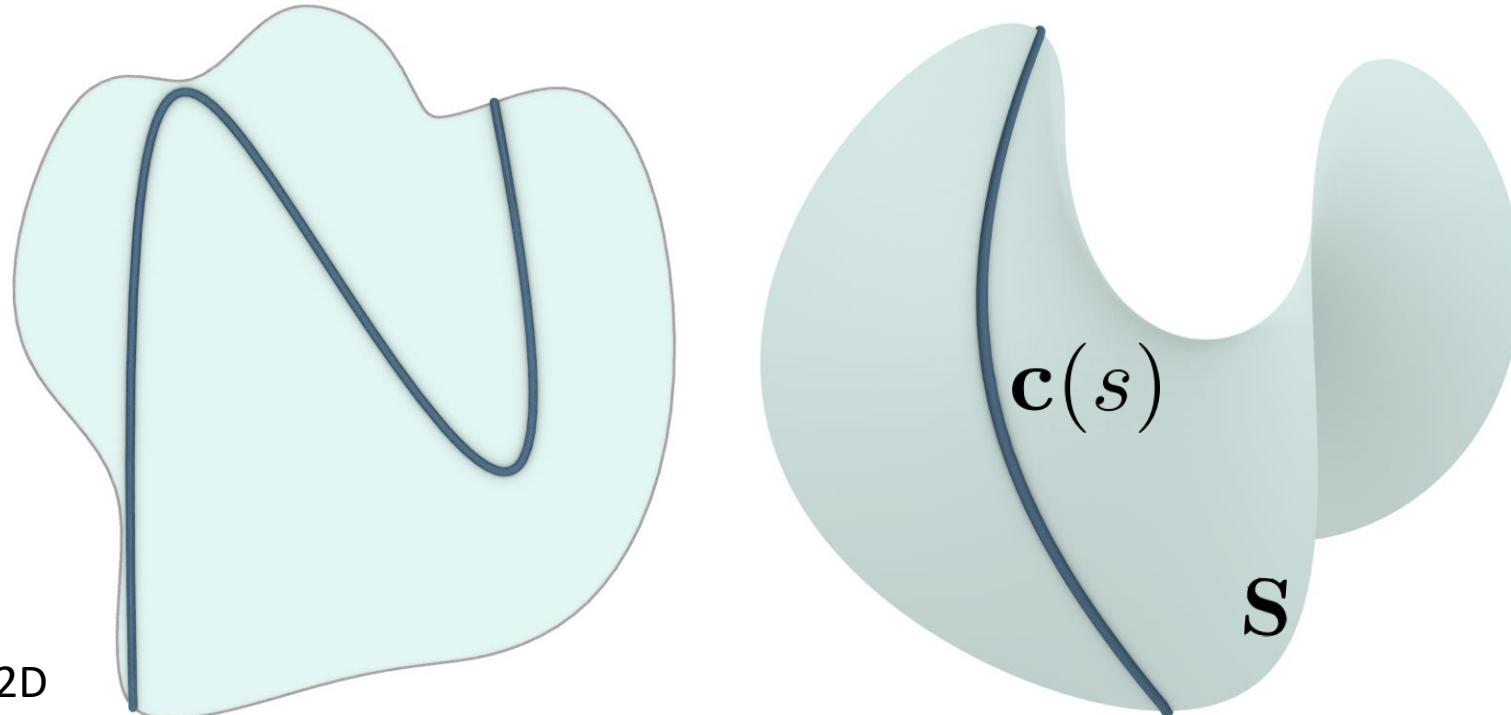
$$\kappa_g = 0$$

$$\mathbf{c}'' = \kappa_n \mathbf{n}$$

$$\mathbf{e}_2 = \mathbf{n}, \mathbf{e}_3 = -\mathbf{u}$$

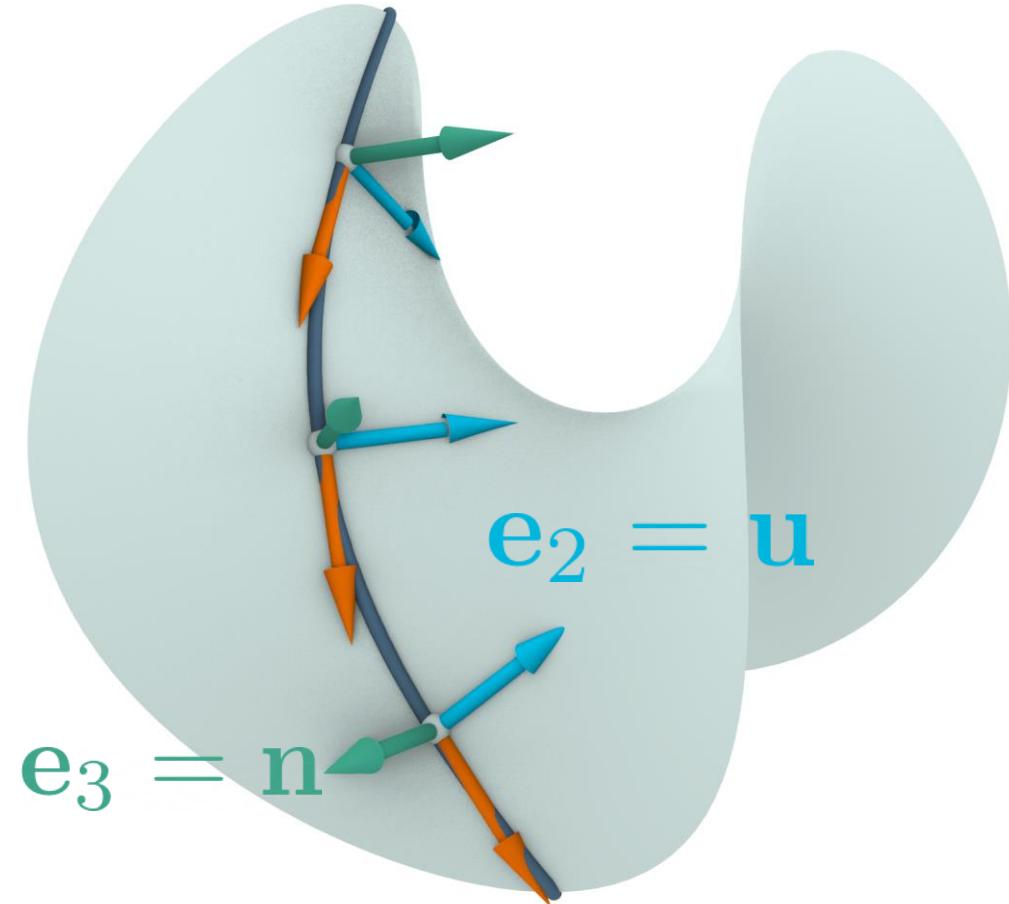


# Asymptotic curve



$$\kappa_n = 0$$

# Asymptotic curve

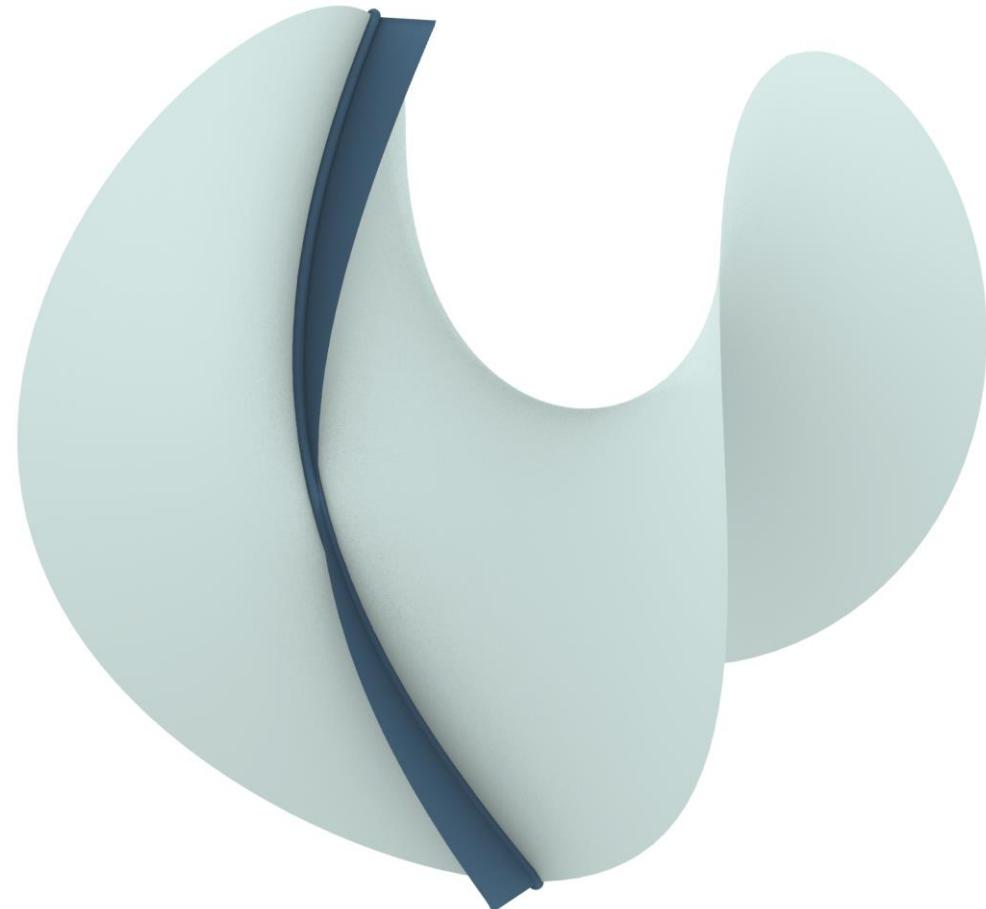


$$\kappa_n = 0$$

$$\mathbf{c}'' = \kappa_g \mathbf{u}$$

$$\mathbf{e}_2 = \mathbf{u}, \mathbf{e}_3 = \mathbf{n}$$

# Asymptotic strip



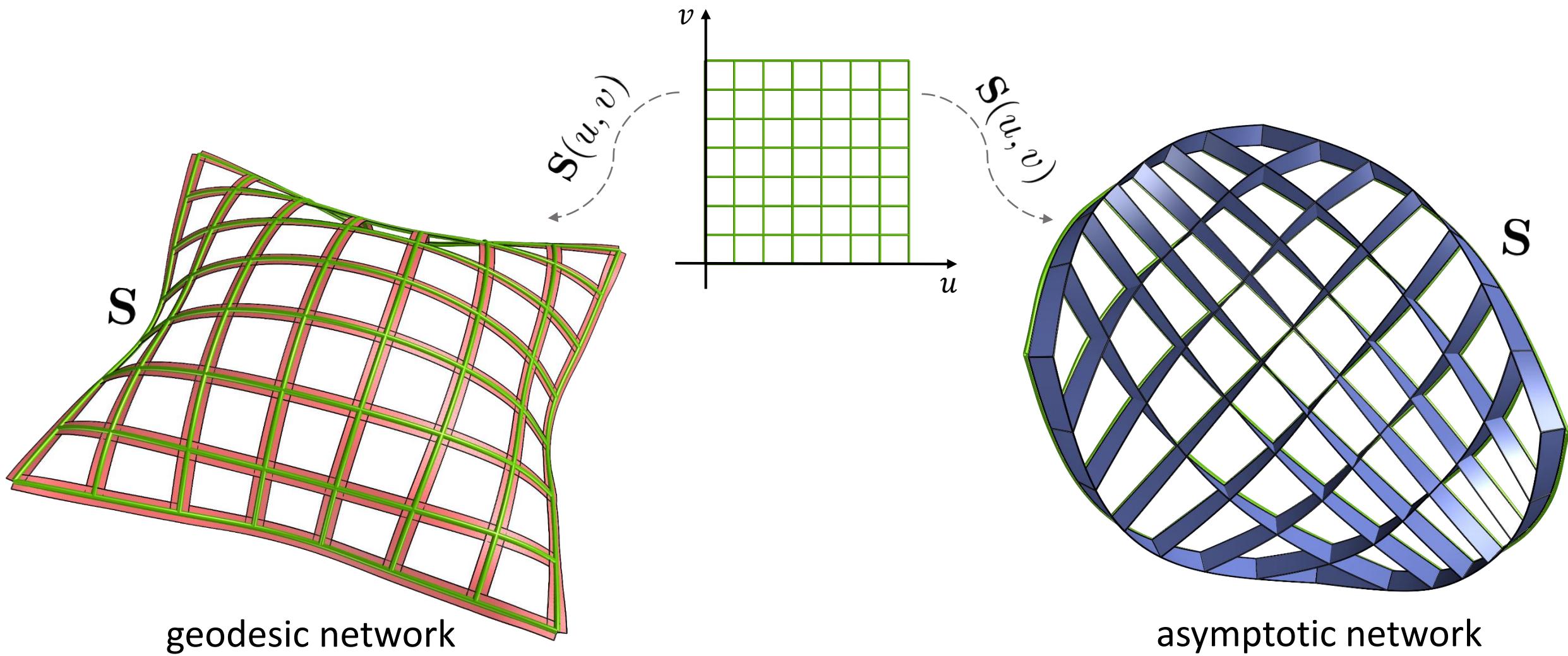
$$\kappa_n = 0$$

$$\mathbf{c}'' = \kappa_g \mathbf{u}$$

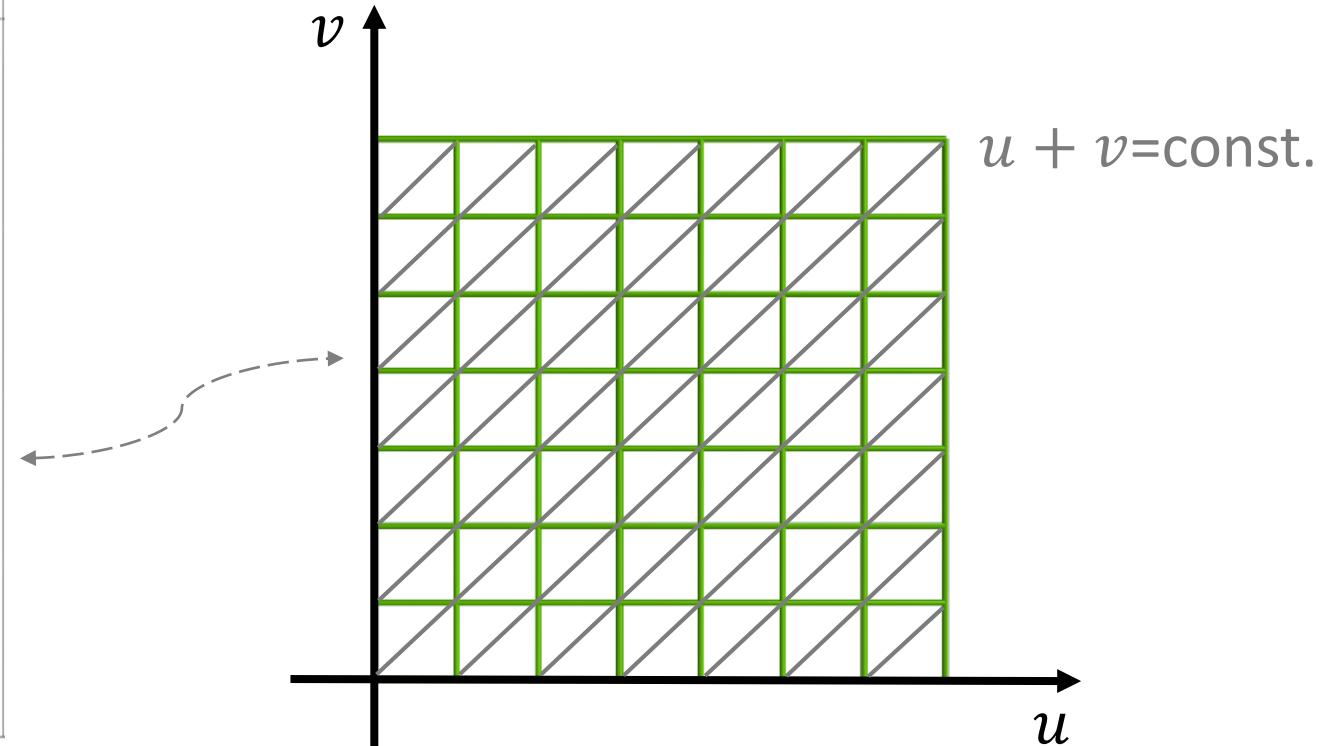
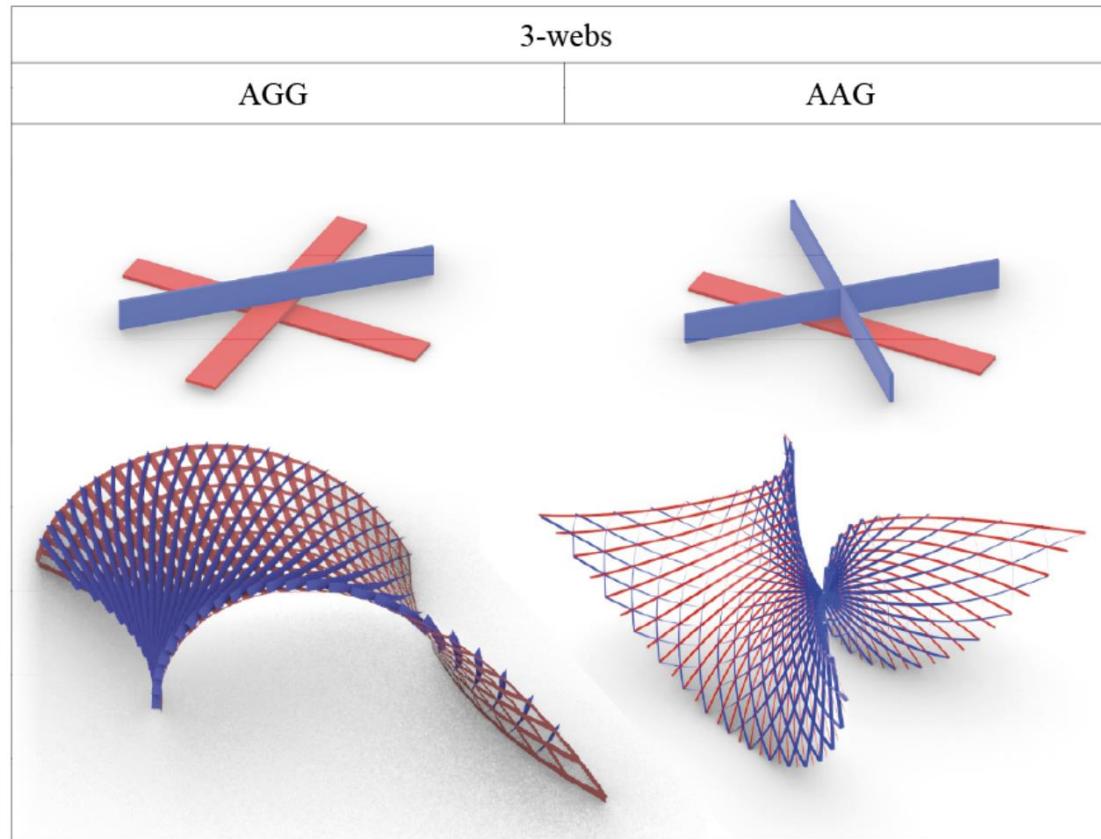
$$\mathbf{e}_2 = \mathbf{u}, \mathbf{e}_3 = \mathbf{n}$$



# Surface parametrization



# AGG-,AAG-web

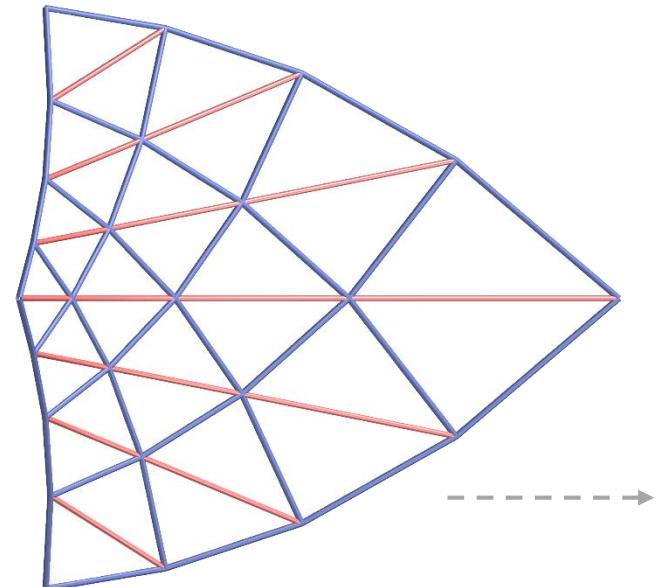


- Geodesic
- Asymptote

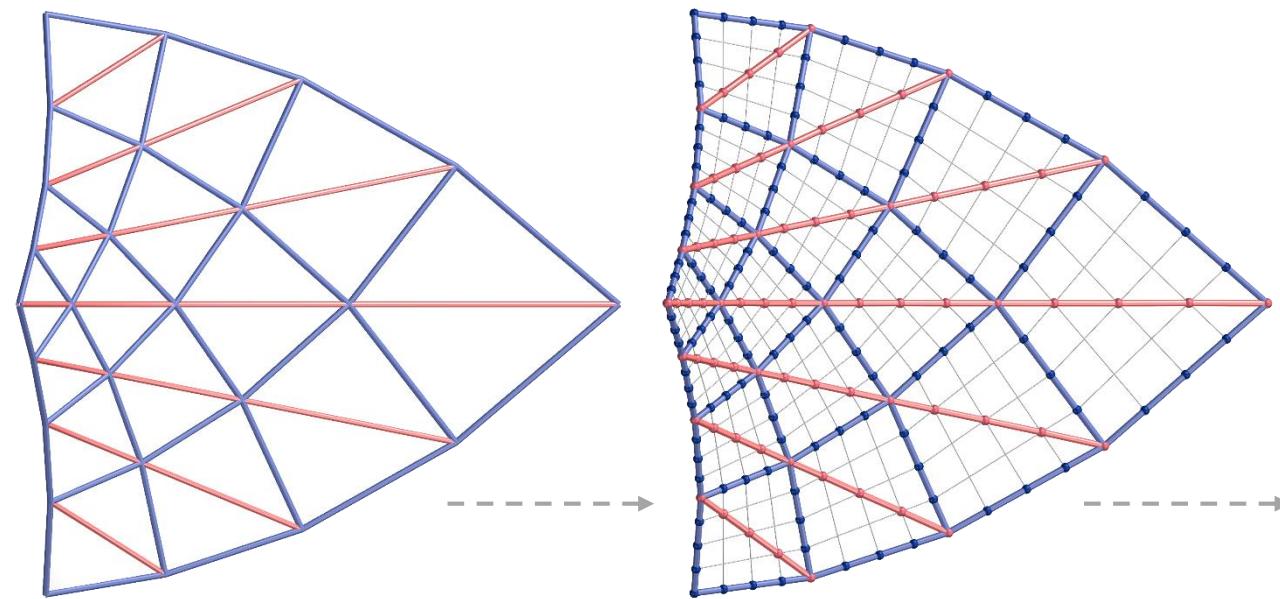
# Discretization and optimization



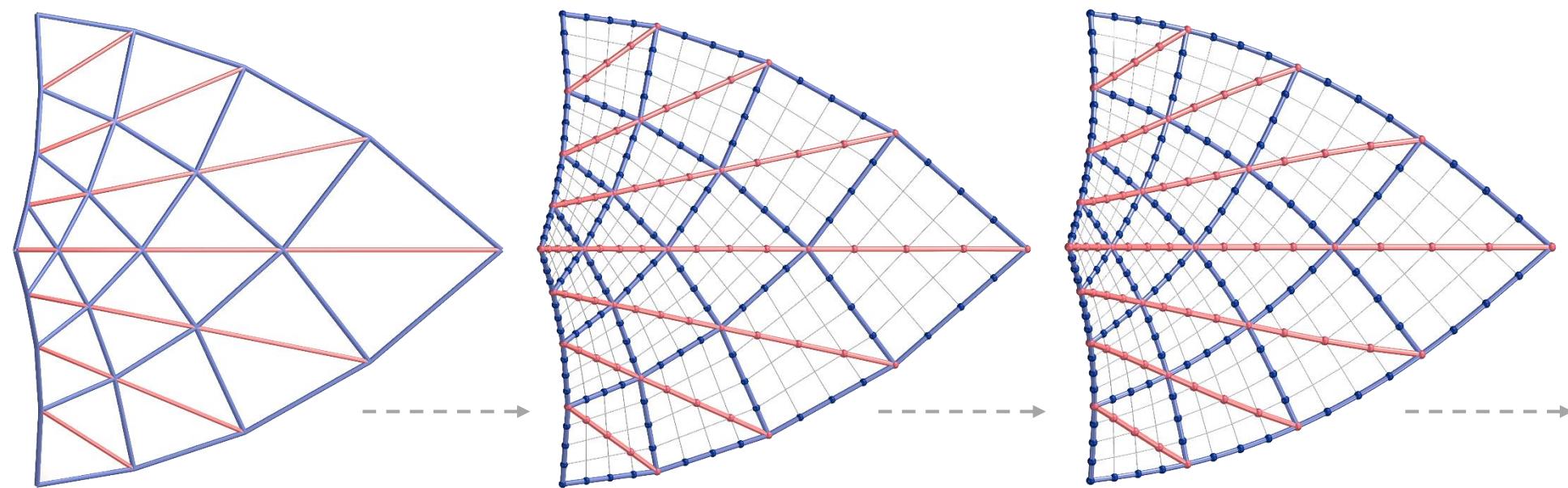
# Computational approach



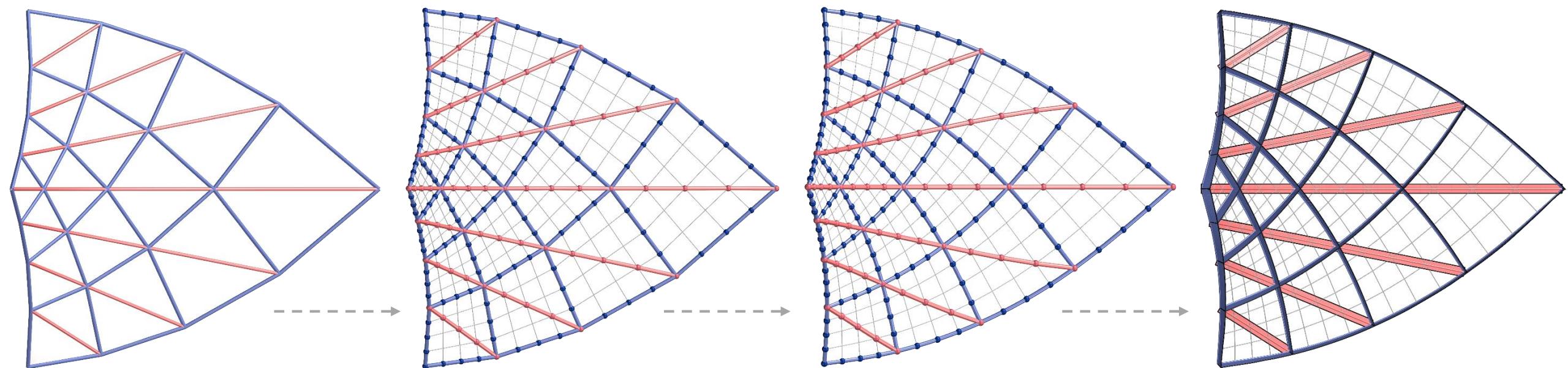
# Computational approach



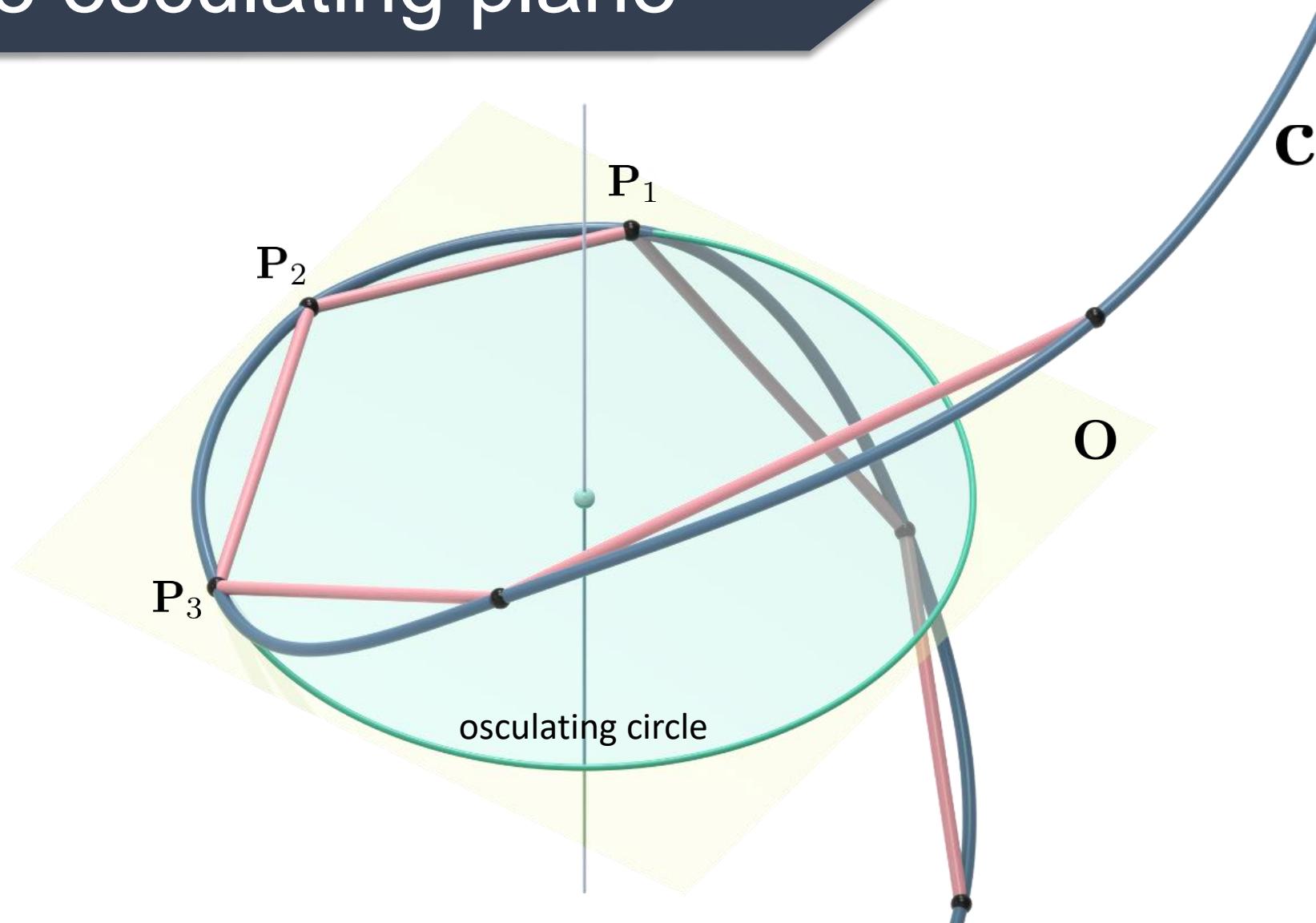
# Computational approach



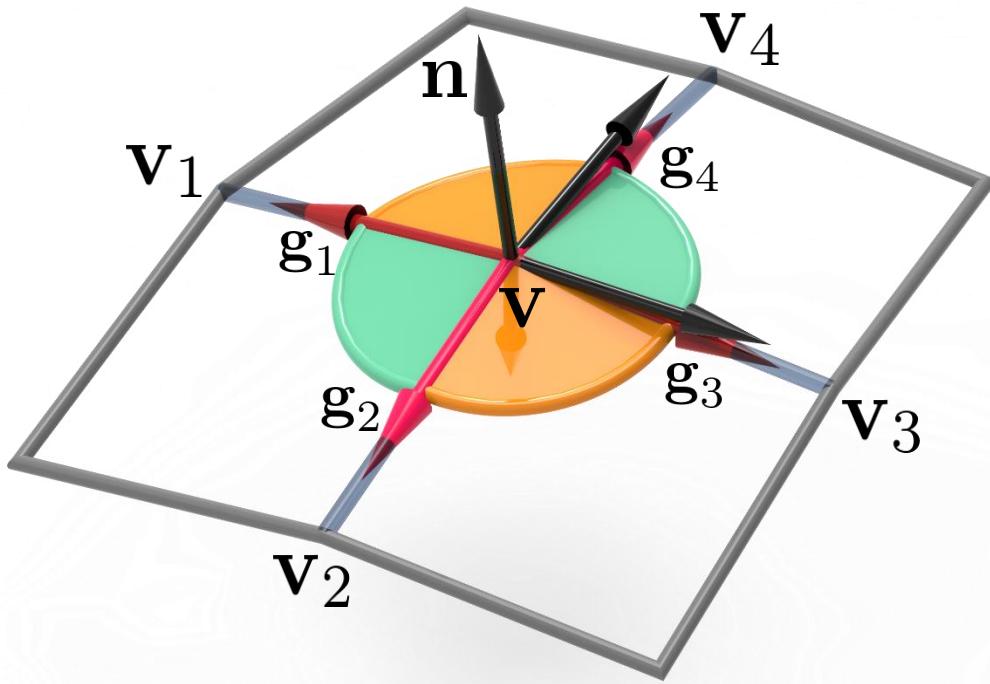
# Computational approach



# Discrete osculating plane



# Constraints: G-net



[Wunderlich 1951, Rabinovich et al. 2018]

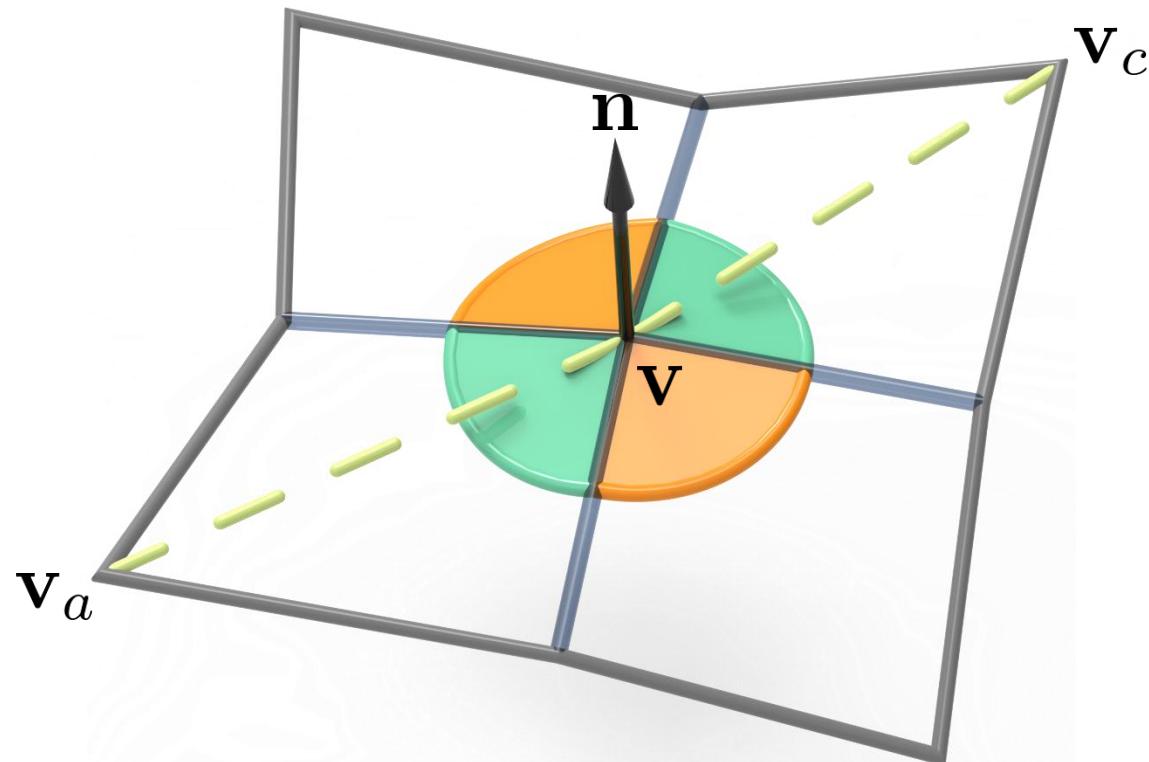
$$\mathbf{g}_i = (\mathbf{v}_i - \mathbf{v}) / \|\mathbf{v}_i - \mathbf{v}\|, \quad i = 1, \dots, 4,$$

$$\mathbf{g}_1 \cdot \mathbf{g}_2 = \mathbf{g}_3 \cdot \mathbf{g}_4, \quad \mathbf{g}_2 \cdot \mathbf{g}_3 = \mathbf{g}_4 \cdot \mathbf{g}_1.$$

Vertex normal:

$$\mathbf{n} \parallel \mathbf{g}_1 + \mathbf{g}_3 \parallel \mathbf{g}_2 + \mathbf{g}_4$$

# Constraints: AGG-web



$$\mathbf{g}_i = (\mathbf{v}_i - \mathbf{v}) / \|\mathbf{v}_i - \mathbf{v}\|, \quad i = 1, \dots, 4,$$

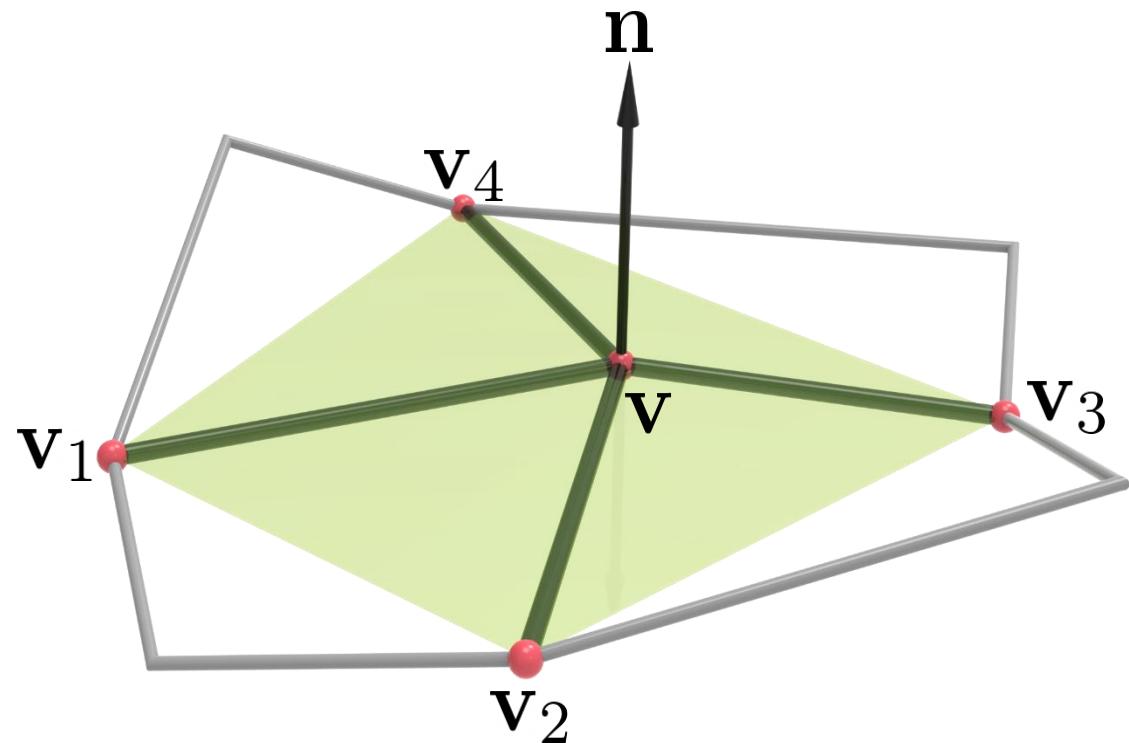
$$\mathbf{g}_1 \cdot \mathbf{g}_2 = \mathbf{g}_3 \cdot \mathbf{g}_4, \quad \mathbf{g}_2 \cdot \mathbf{g}_3 = \mathbf{g}_4 \cdot \mathbf{g}_1.$$

Vertex normal:

$$\mathbf{n} \parallel \mathbf{g}_1 + \mathbf{g}_3 \parallel \mathbf{g}_2 + \mathbf{g}_4$$

$$\mathbf{n} \cdot (\mathbf{v}_a - \mathbf{v}) = 0, \quad \mathbf{n} \cdot (\mathbf{v}_c - \mathbf{v}) = 0.$$

# Constraints: A-net

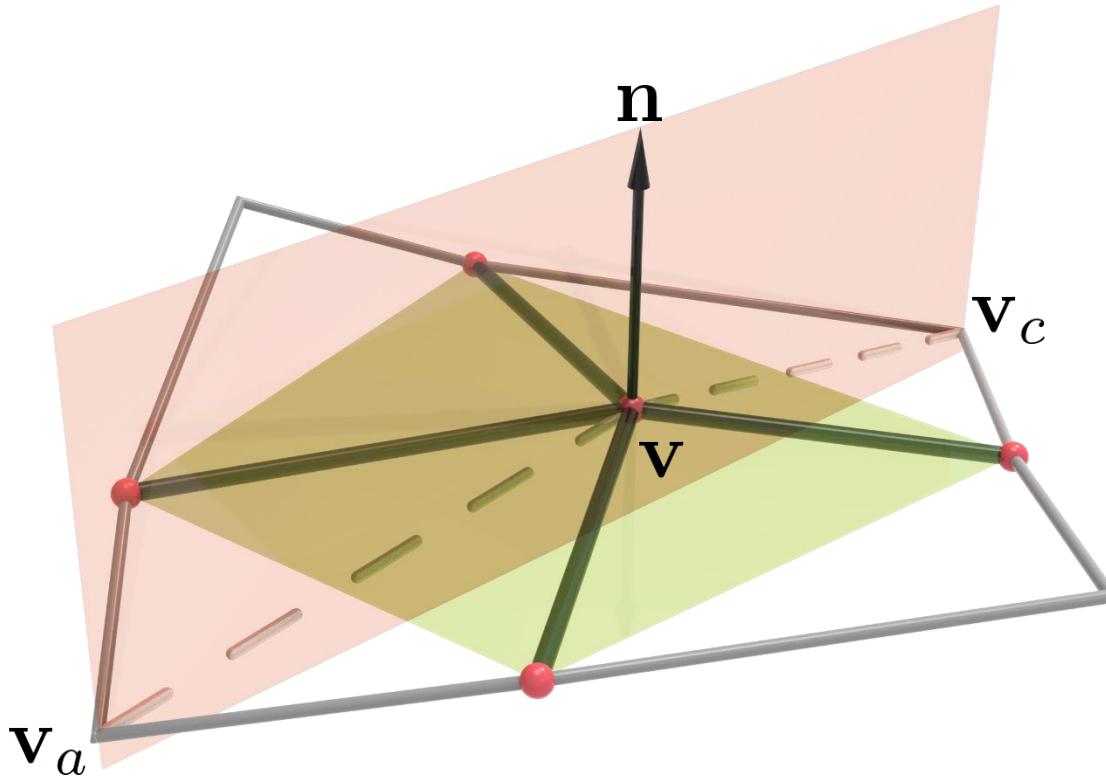


$$\mathbf{n} \cdot (\mathbf{v}_i - \mathbf{v}) = 0, \quad i = 1, \dots, 4.$$

$$\|\mathbf{n}\|^2 = 1.$$

[Bobenko et al. 2008]

# Constraints: AAG-web



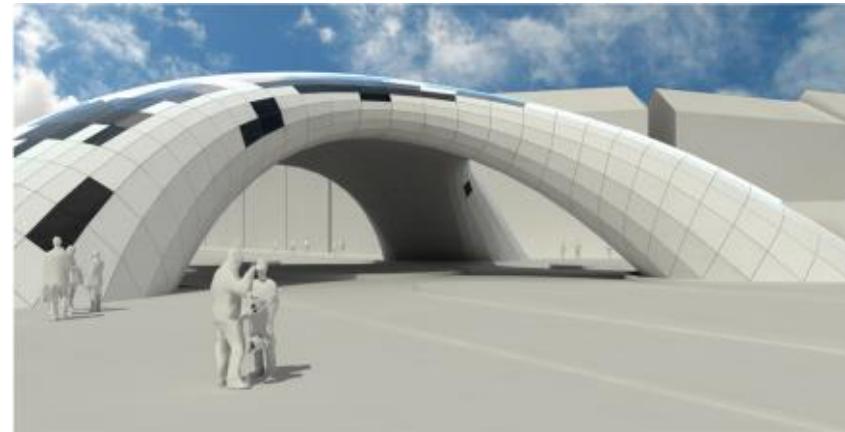
$$\mathbf{n} \cdot (\mathbf{v}_i - \mathbf{v}) = 0, \quad i = 1, \dots, 4.$$

$$\|\mathbf{n}\|^2 = 1.$$

$$\mathbf{n} \cdot [(\mathbf{v}_a - \mathbf{v}) \times (\mathbf{v}_c - \mathbf{v})] = 0.$$



# Target function



[Tang et al. 2014]

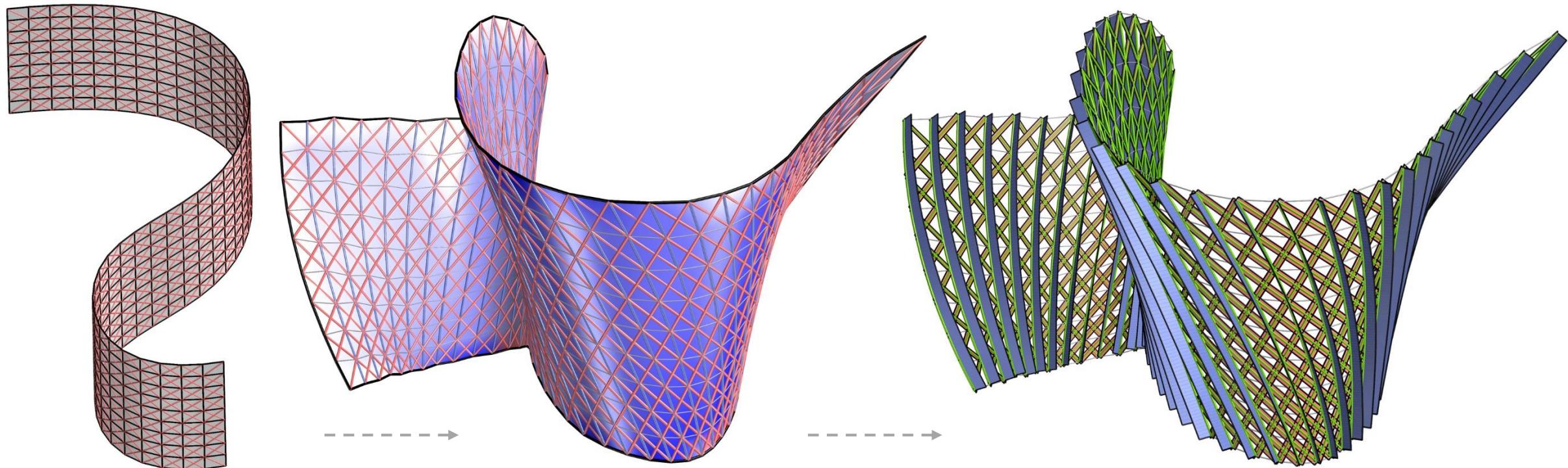
$$F(X) = \sum(C_i(X)^2 + \omega_1 K(X)^2 + \omega_2 V(X)^2 + \omega_3 P(X)^2 + \varepsilon(X - X_c)^2)$$

Fairness:  $\mathbf{v}_{i+1} - 2\mathbf{v}_i + \mathbf{v}_{i-1} = 0$

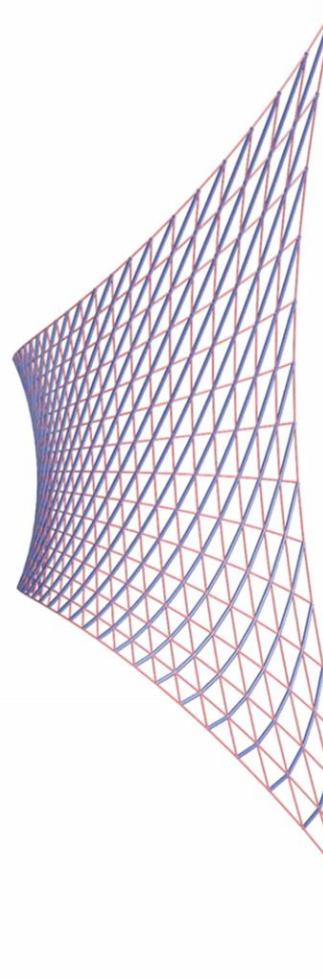
Proximity to curve or surfaces:  $(\mathbf{v}_p - \mathbf{p}) \cdot \mathbf{n}_p = 0$

Control the change of vertices:  $\mathbf{v}^{(j-1)} - \mathbf{v}^{(j)} = 0$

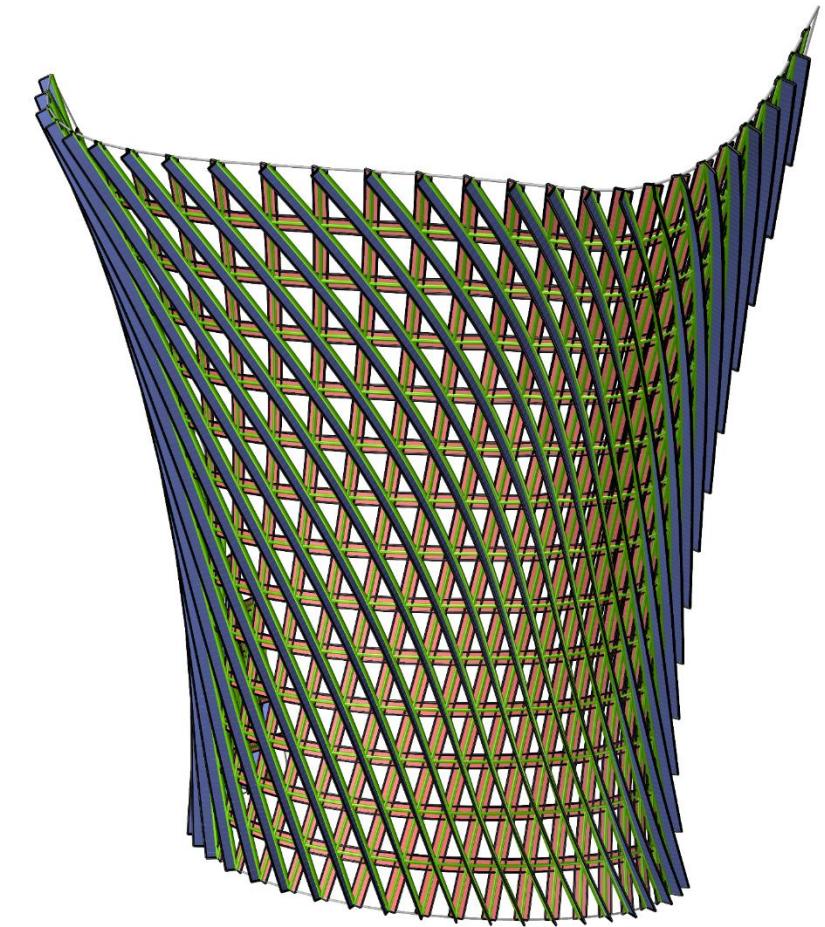
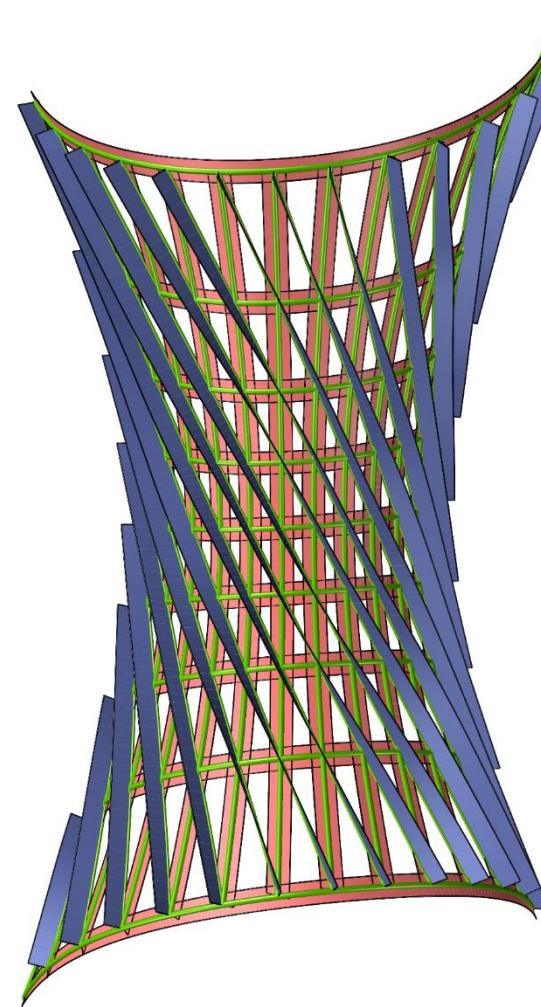
# Initialization: AGG-web



# AGG-webs

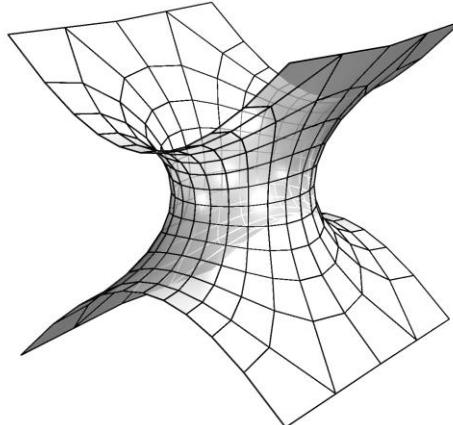


3X

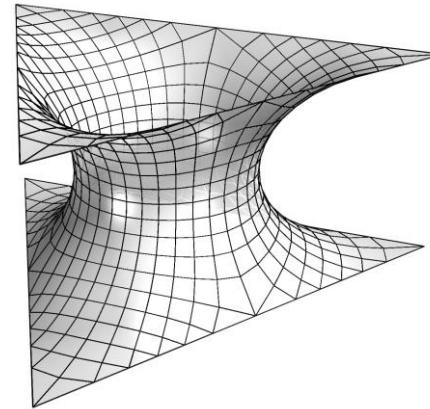


# Initialization: AAG-web

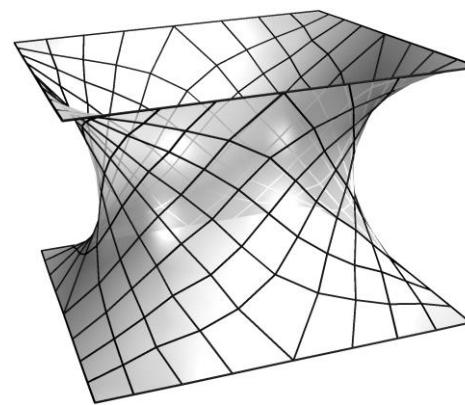
Initial minimal surfaces:



Scherk tower surface

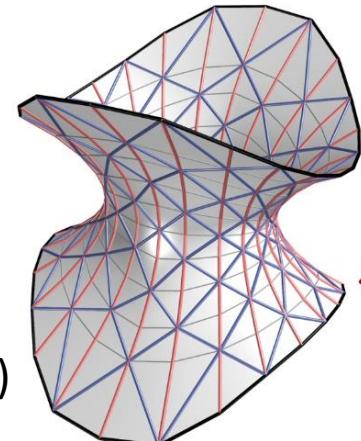


Schwarz H surface

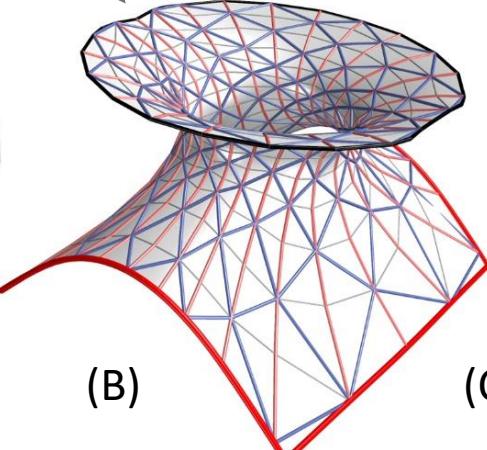


Schwarz P surface

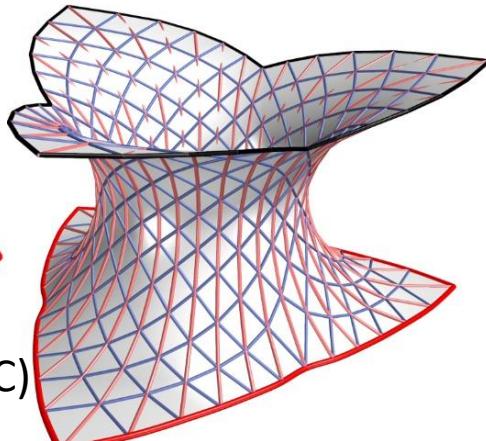
Optimized webs:



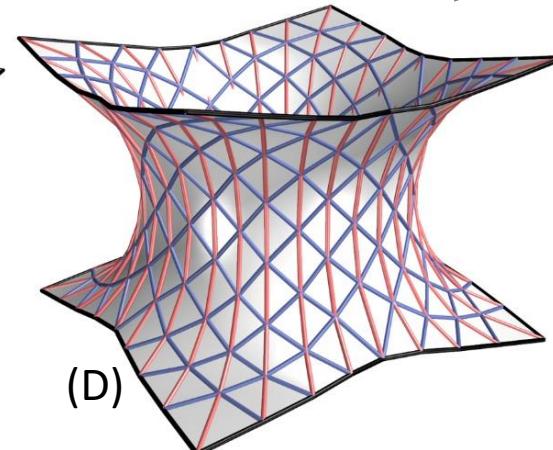
(A)



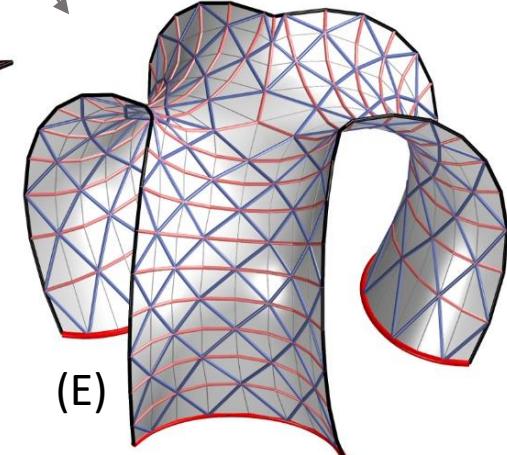
(B)



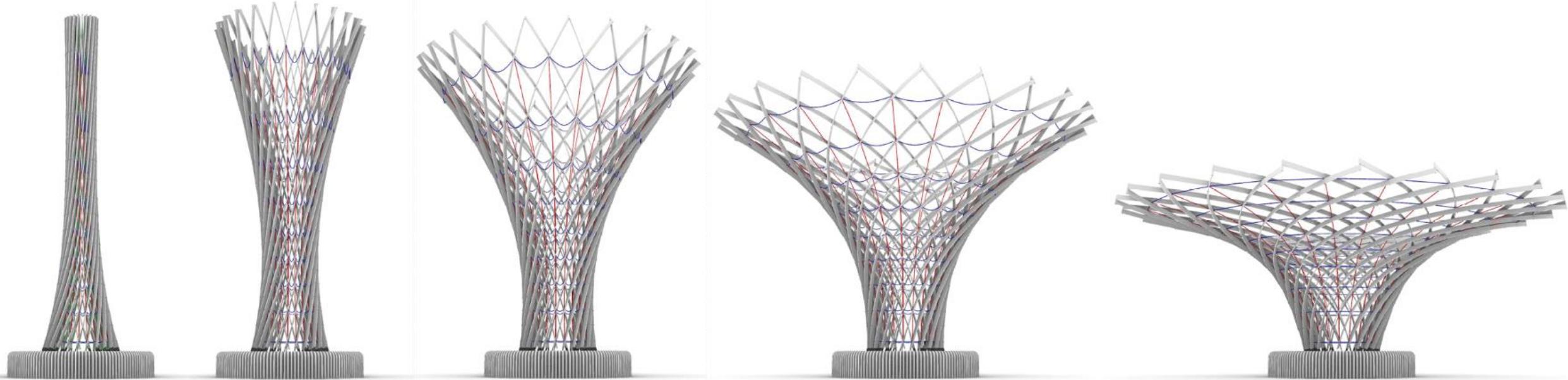
(C)



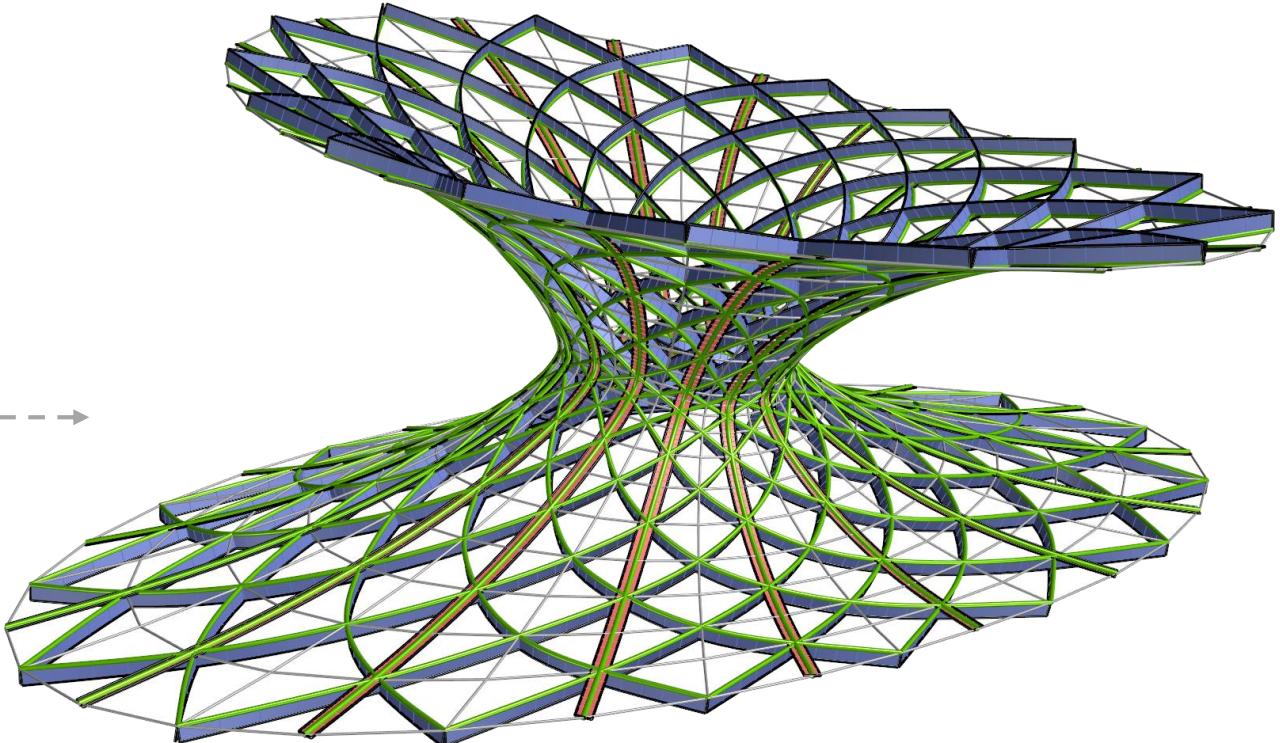
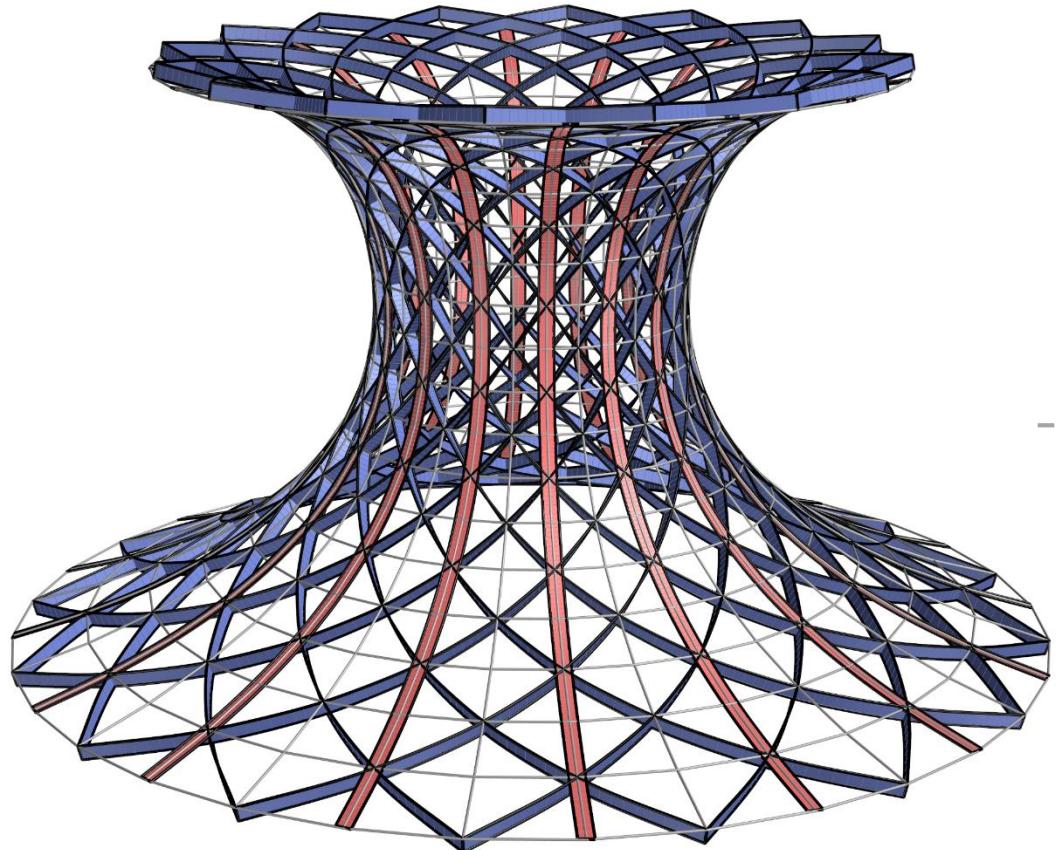
(D)



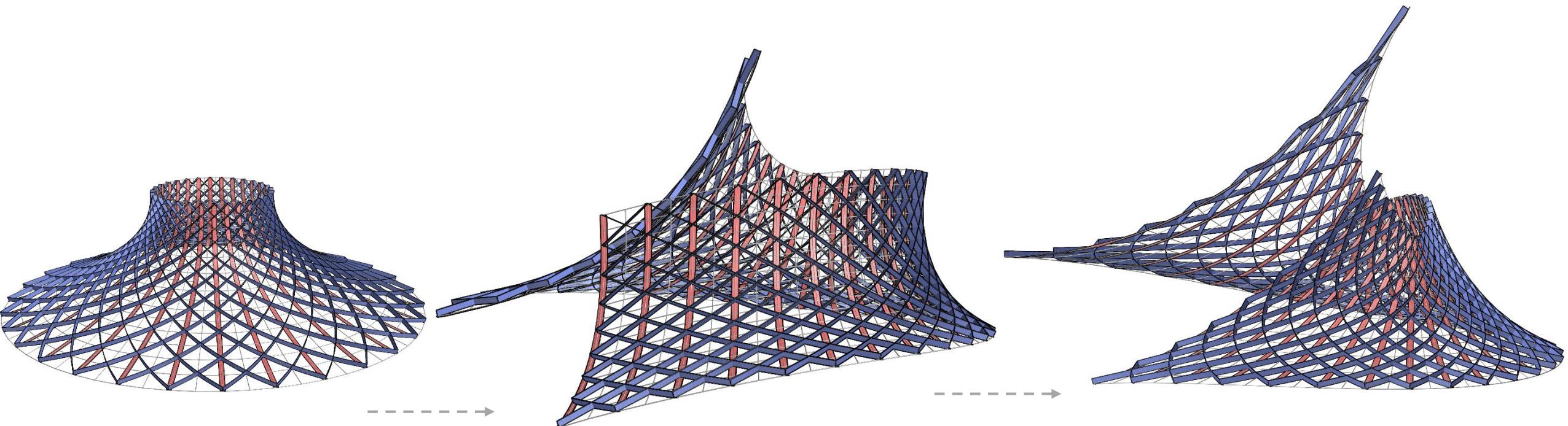
(E)

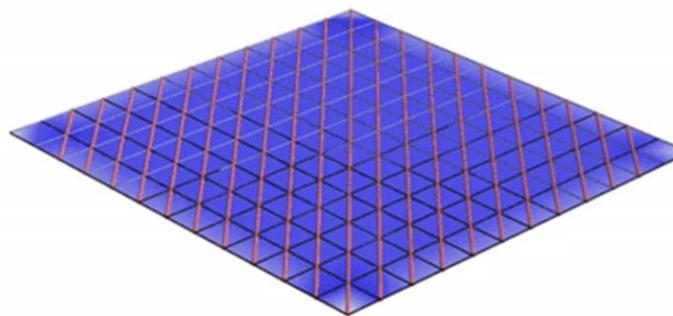


# Initialization: AAG-web



# Initialization: AAG-web

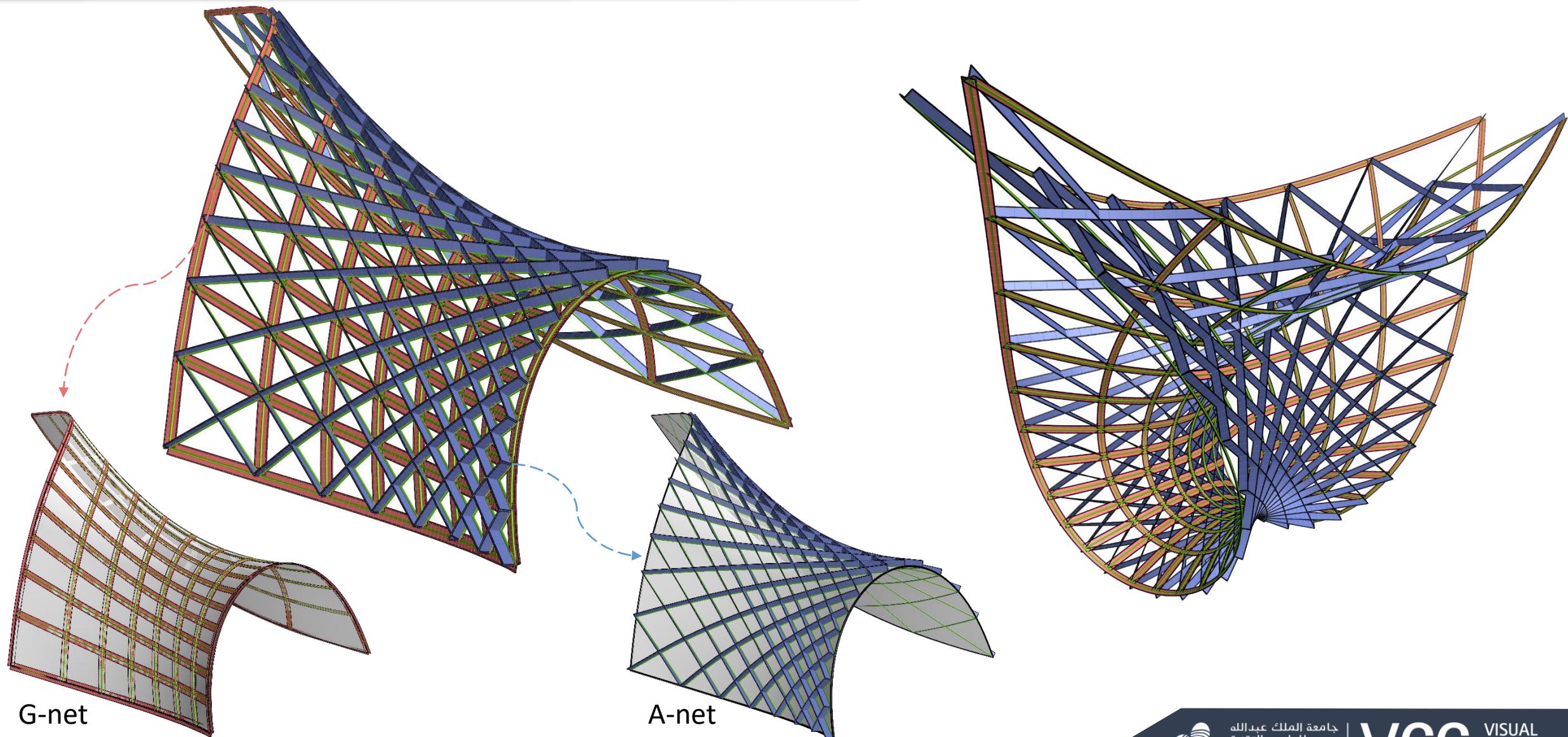




3X



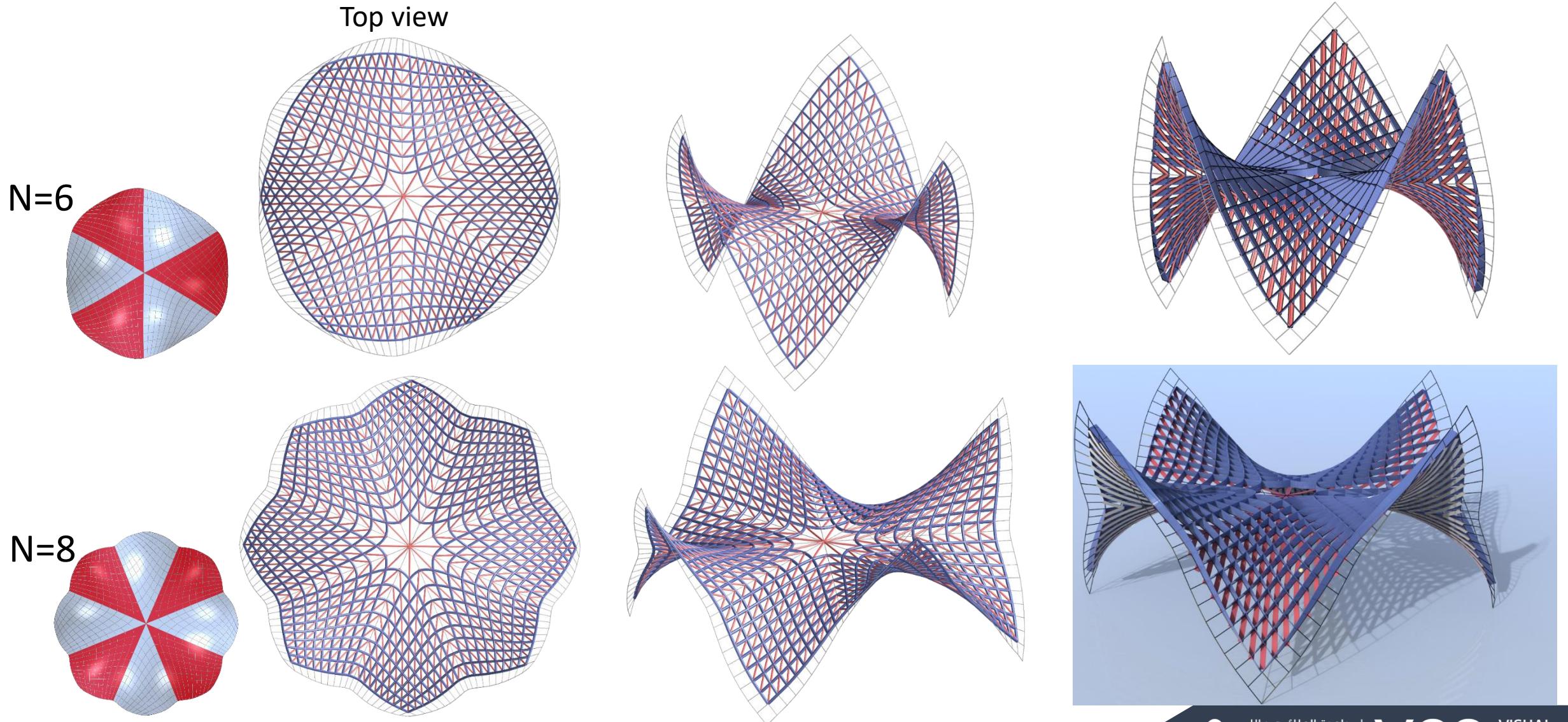
# AGAG-webs



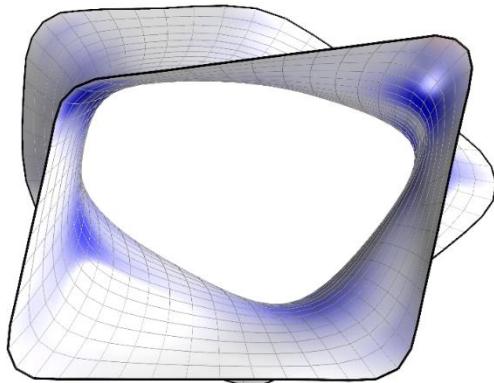
G-net

A-net

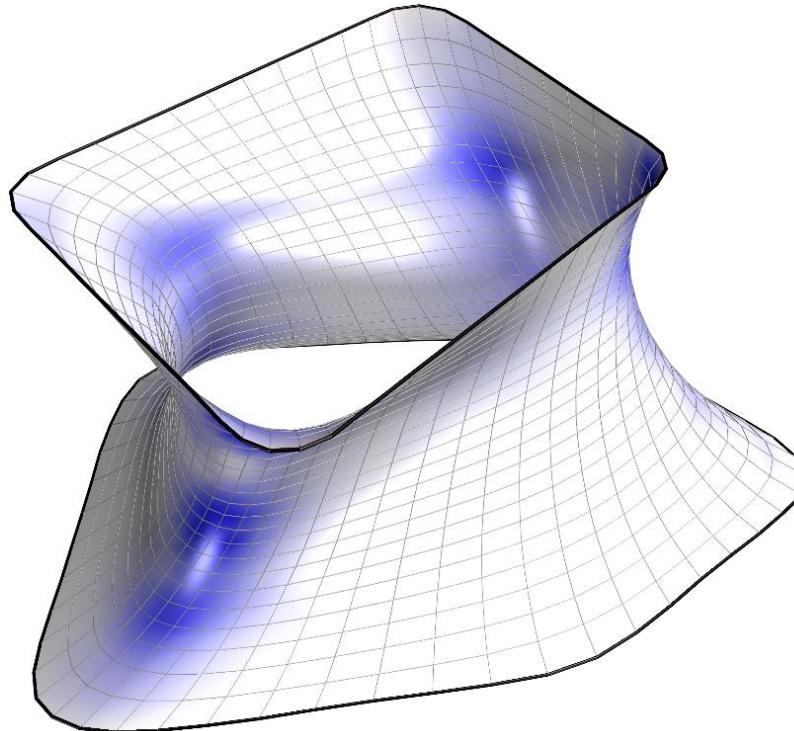
# With combinatorial singularity



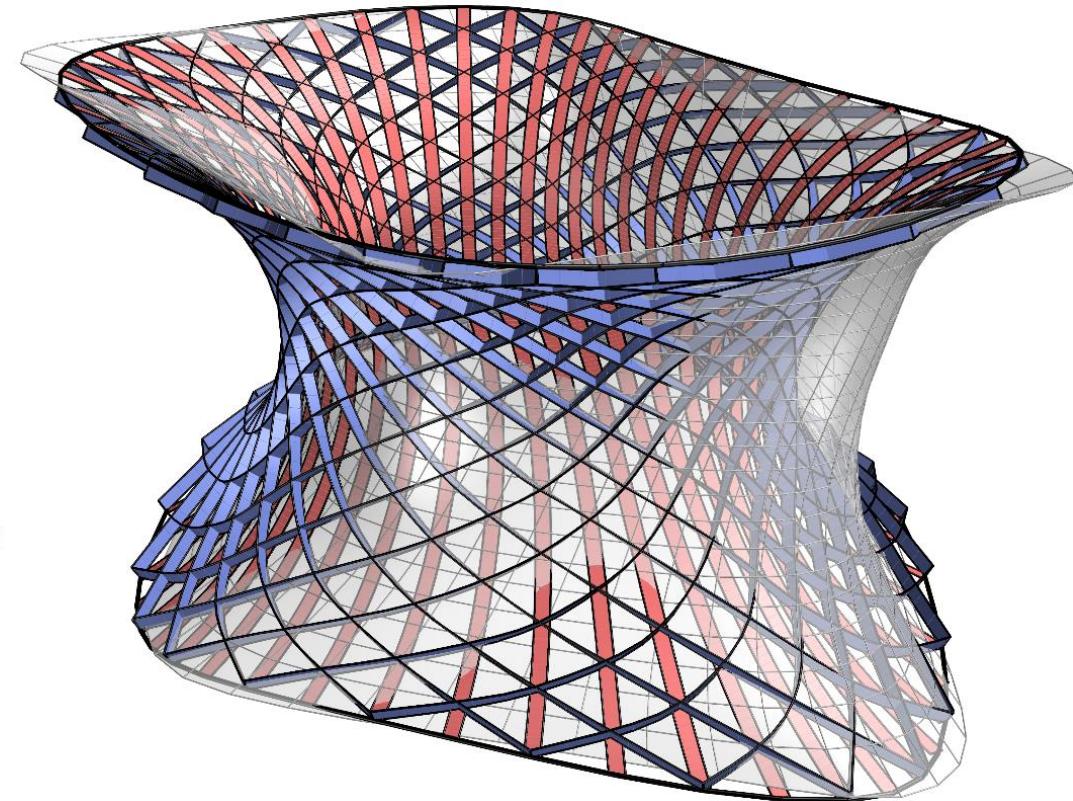
# Shape implementation



Top view



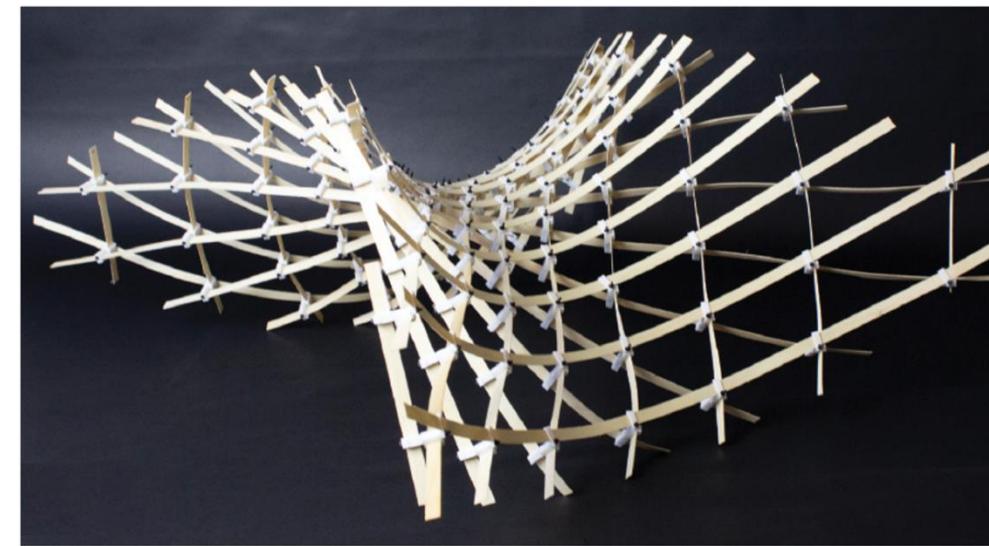
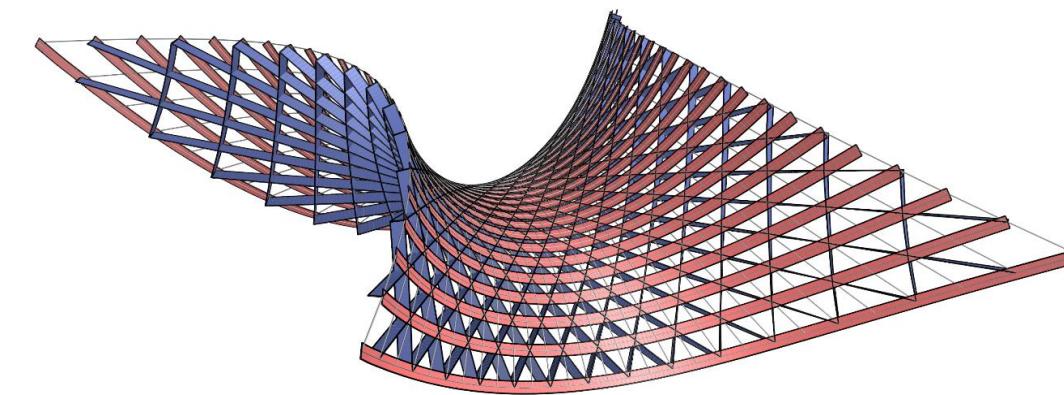
Soumaya Museum



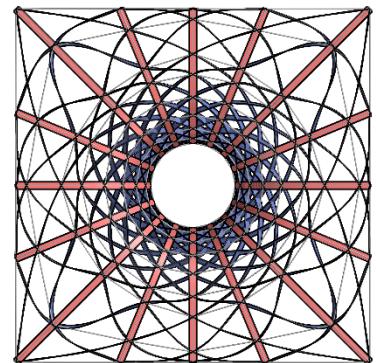
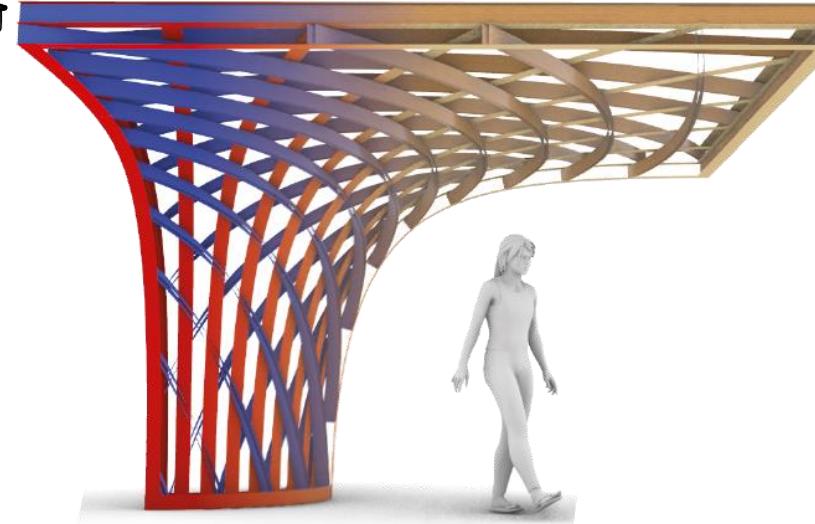
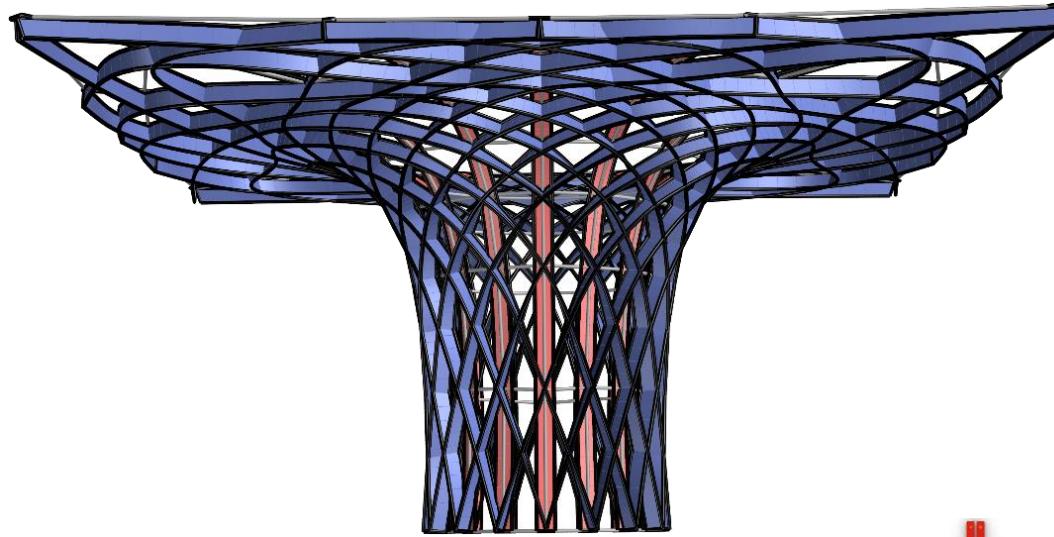
# Application



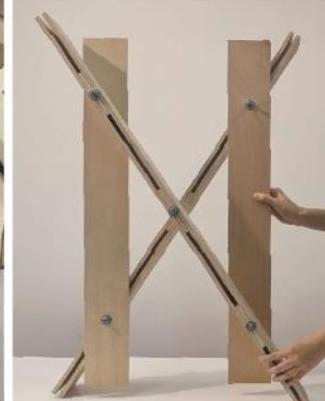
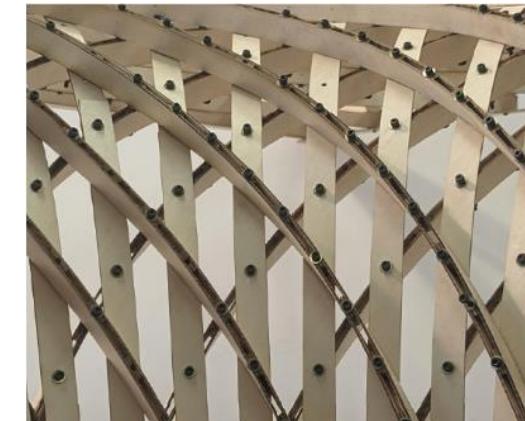
# AAG gridshell



# AAG gridshell



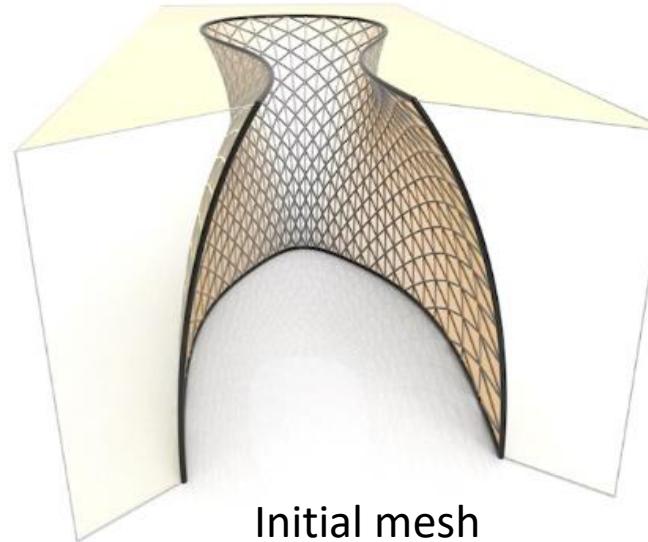
Top view



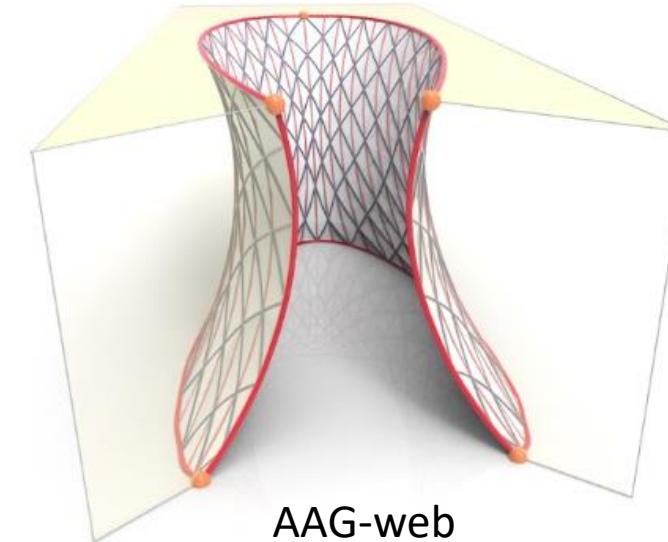
# AAG gridshell



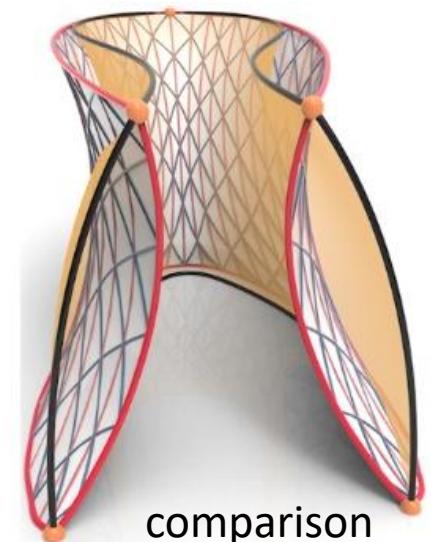
Emporia shopping center façade



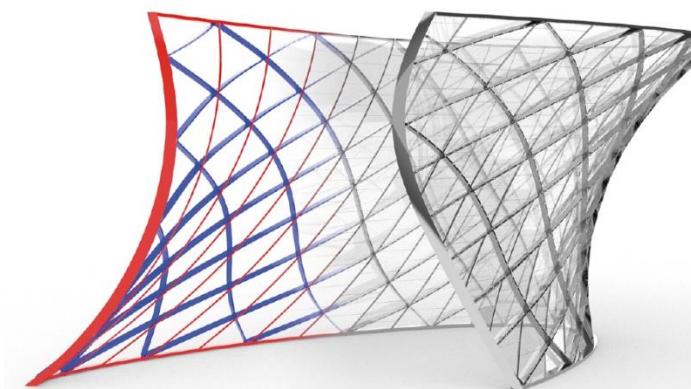
Initial mesh



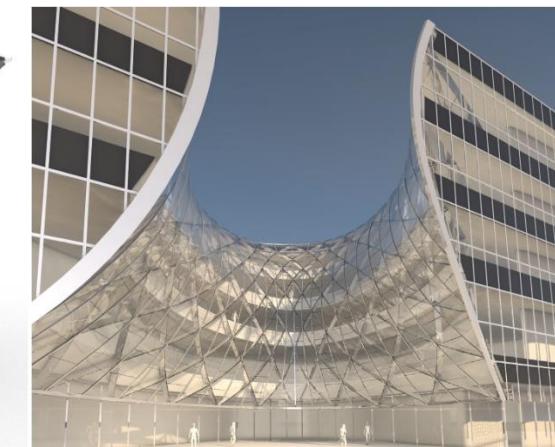
AAG-web



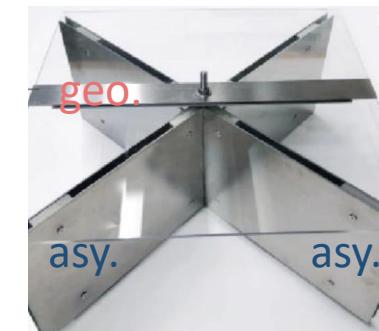
comparison



AAG gridshell

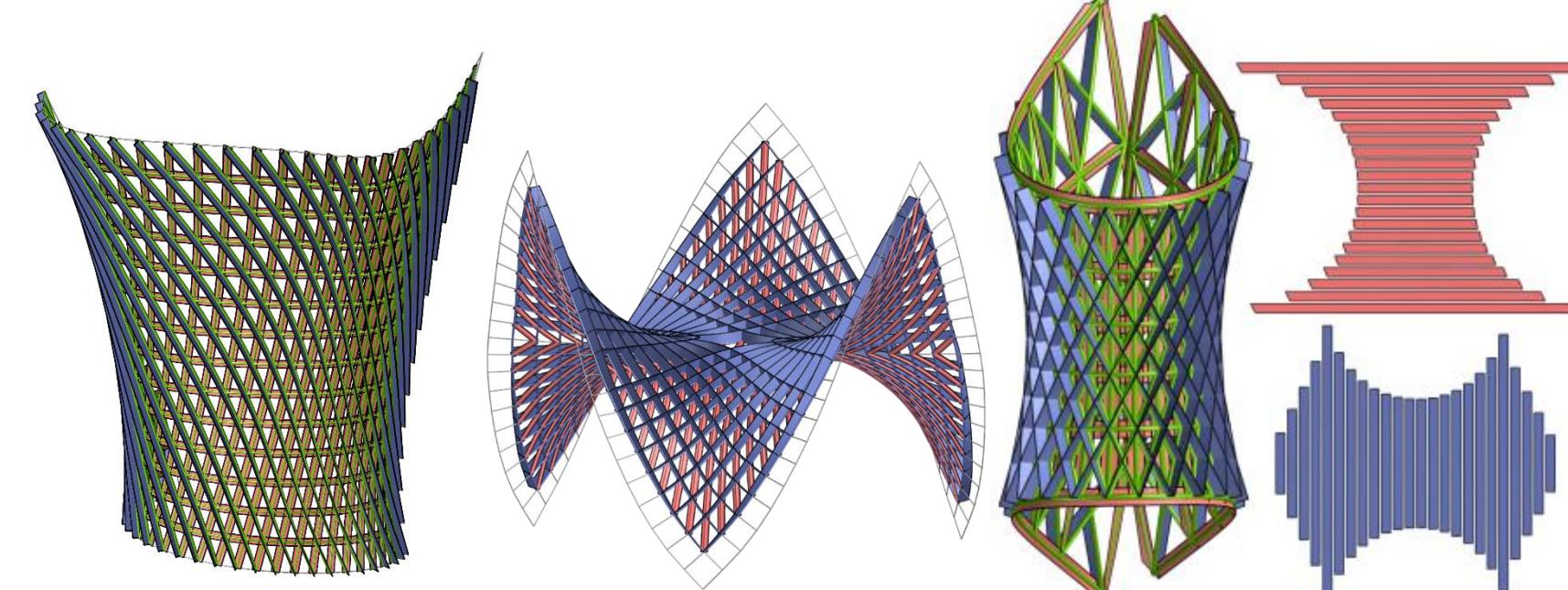


rendering



element

# Conclusion



AGG-web

AAG-web

AGAG-web and unrolled strips

AAG timber model



Thank you!