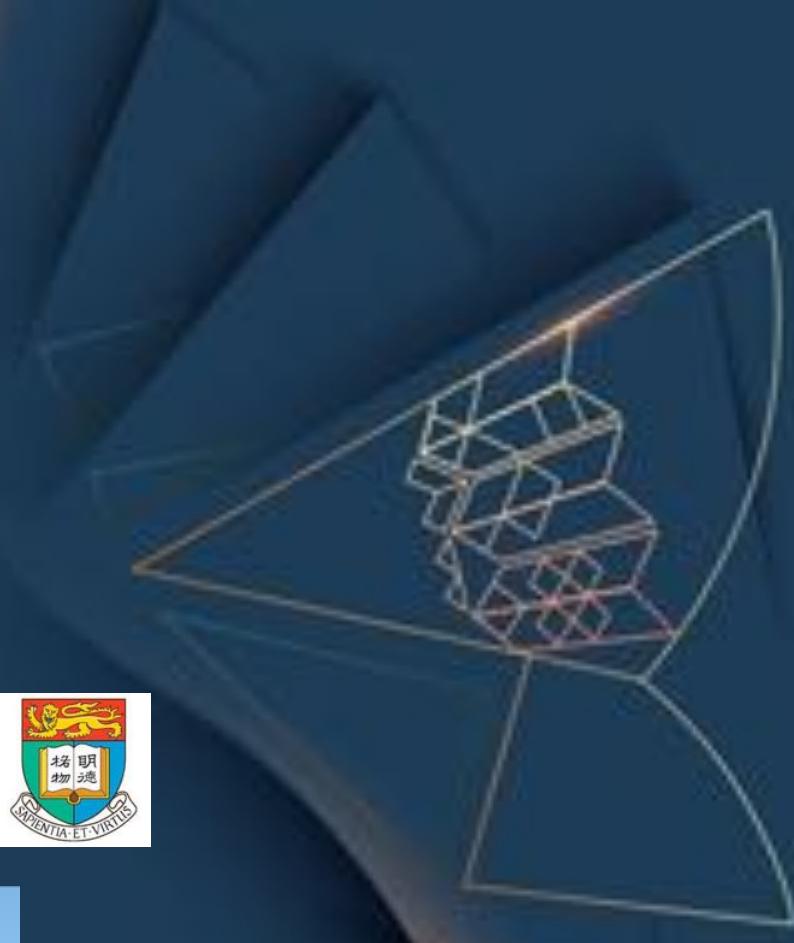
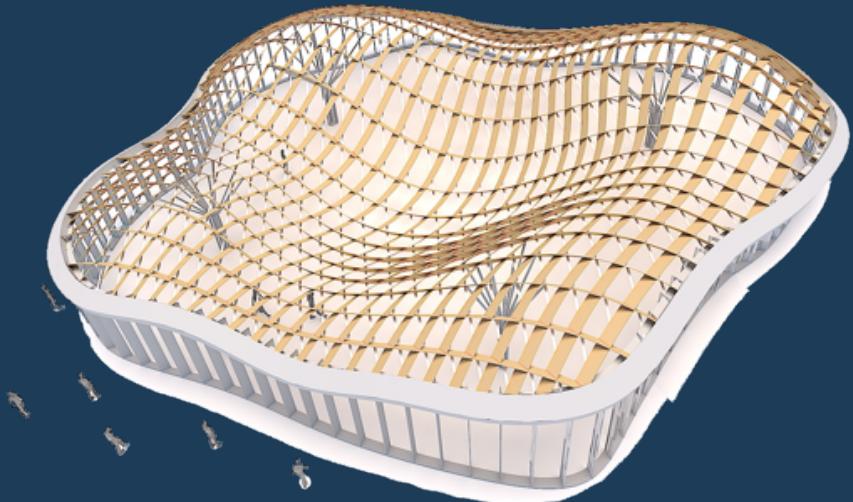




Rectifying Strip Patterns

Bolun Wang¹, Hui Wang¹,

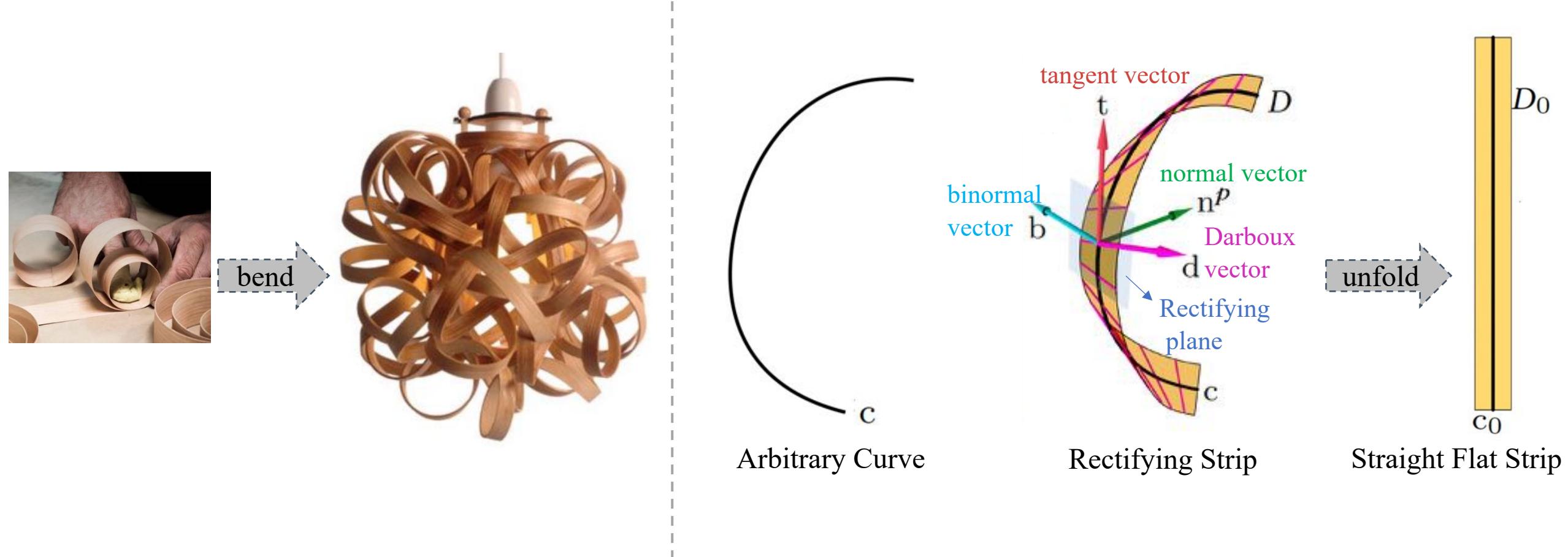
Eike Schling², Helmut Pottmann¹



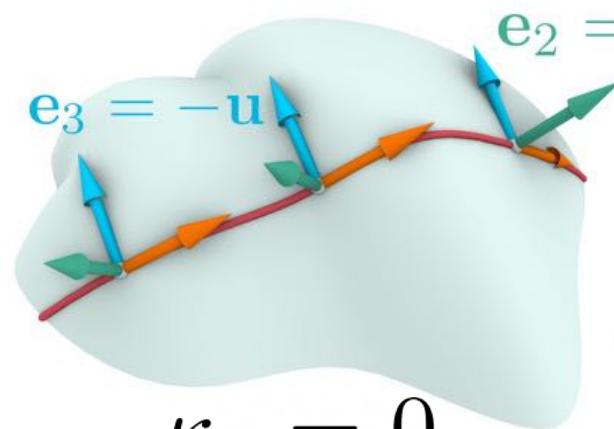
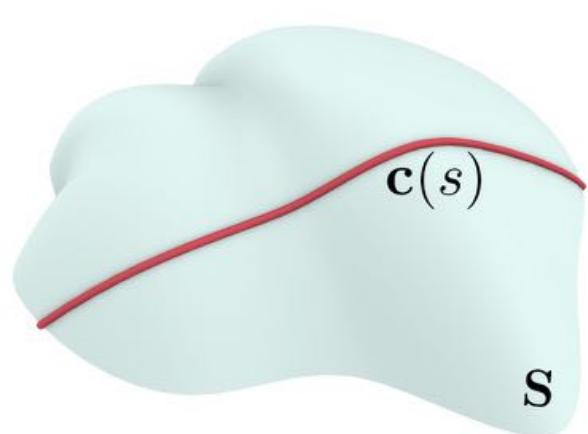
Project Page

Motivation

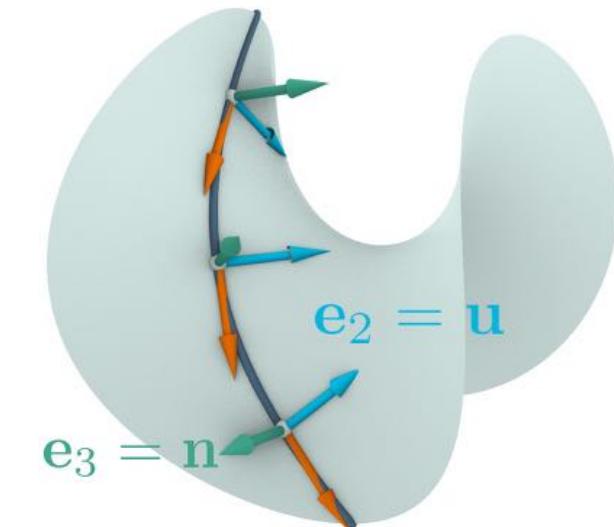
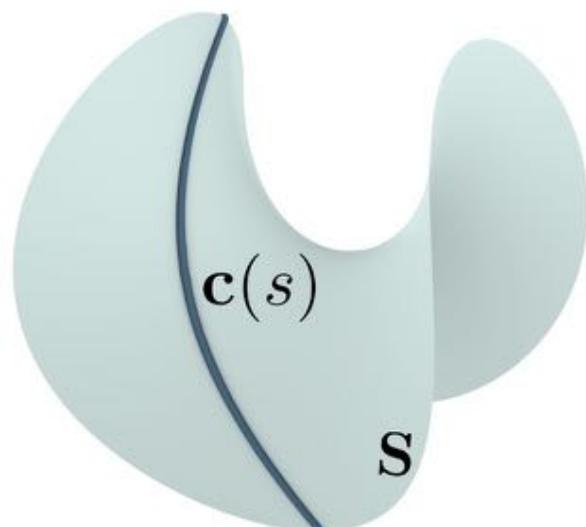
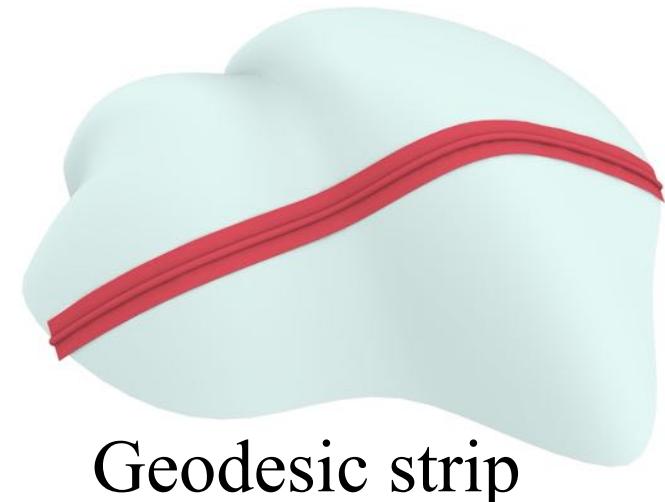
Rectifying Strips in Differential Geometry



Attaching Rectifying Strips on the Surface



$$\begin{aligned} \mathbf{c}'' &= \kappa_n \mathbf{n} \\ \mathbf{e}_2 &= \mathbf{n}, \mathbf{e}_3 = -\mathbf{u} \end{aligned}$$



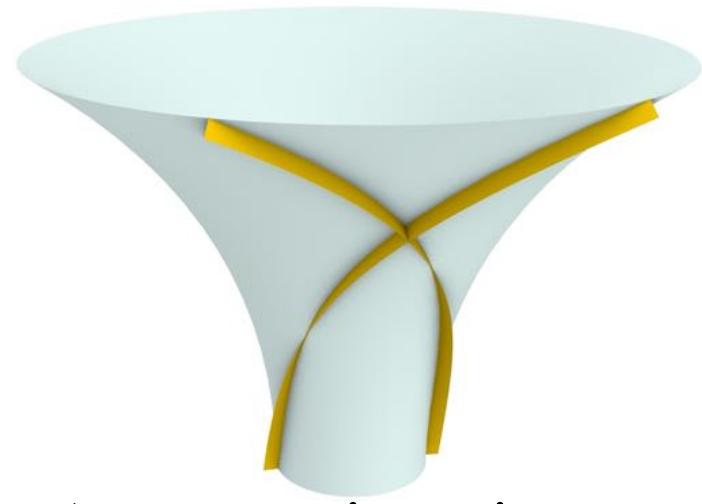
$$\begin{aligned} \mathbf{c}'' &= \kappa_g \mathbf{u} \\ \mathbf{e}_2 &= \mathbf{u}, \mathbf{e}_3 = \mathbf{n} \end{aligned}$$



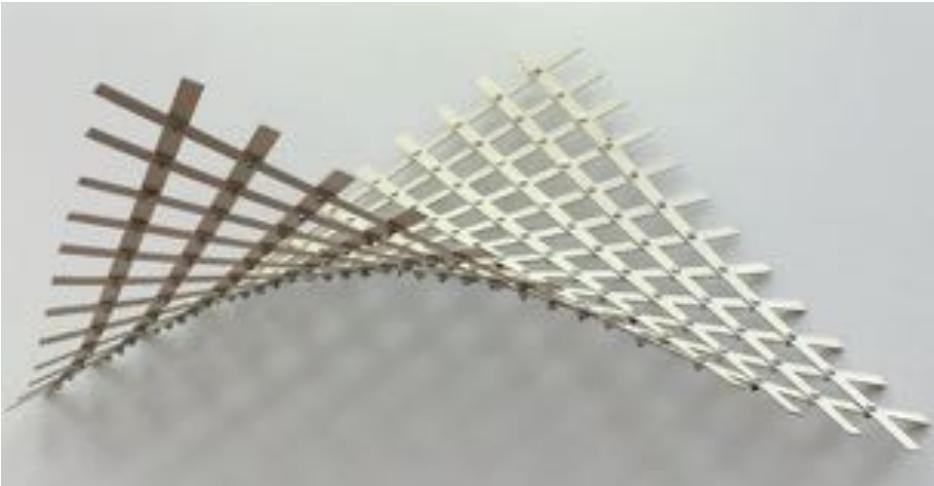
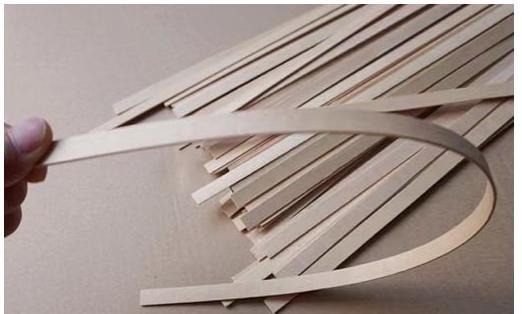
Motivations: Gridshell Structures



Geodesic strips



Asymptotic strips



Geodesic gridshell



Asymptotic gridshell

Motivations: Gridshell Structures



Fabrication



Formation



Transportation



Fabrication: Simplification of construction elements, massively produced

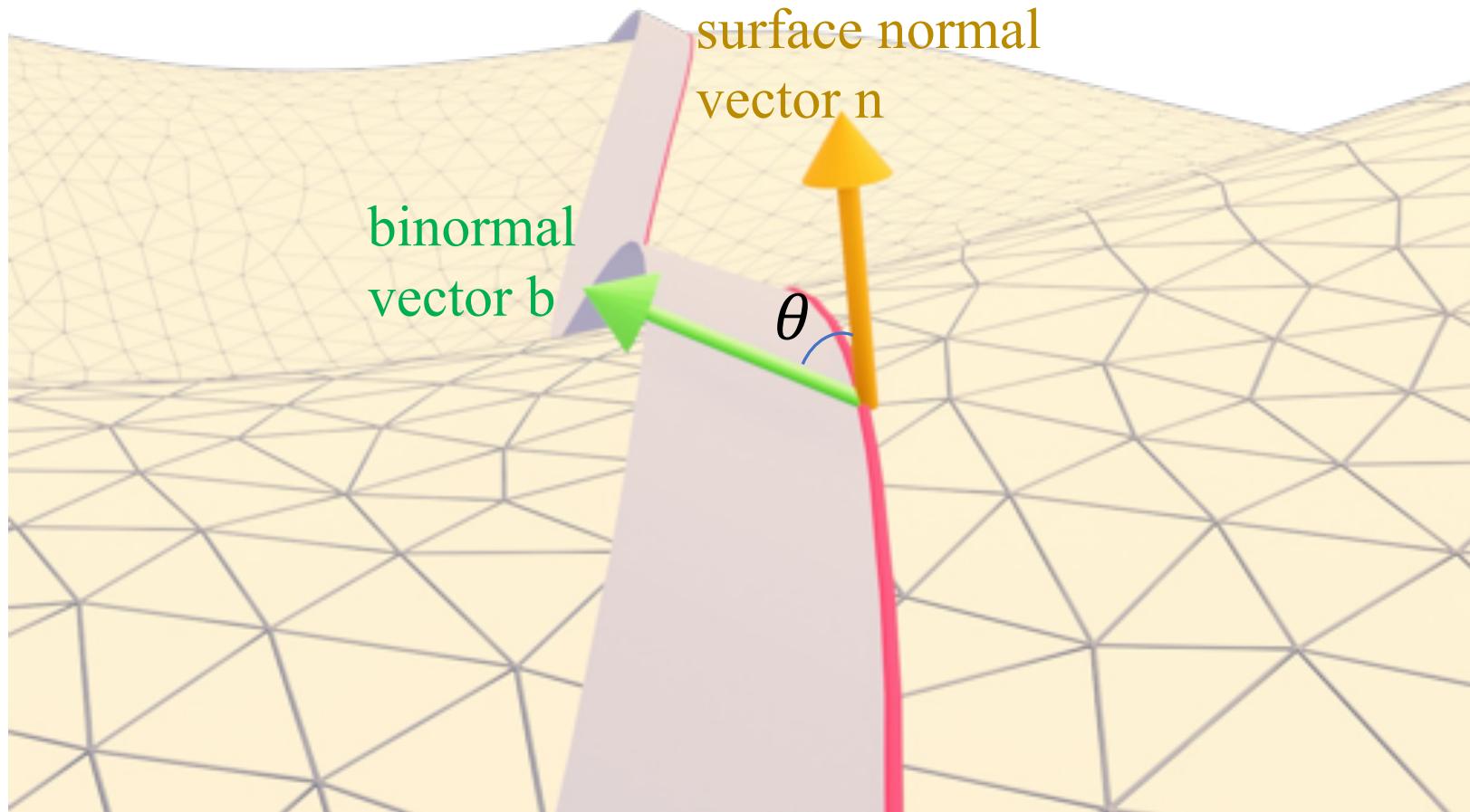
Formation: bending originally flat straight strips

Transportation: easily move

Motivations: Gridshell Structures

Pseudo-geodesic curves:

The signed angle θ between b and n is **constant**. [W. Wunderlich, 1950]



$\theta = 90^\circ$, geodesic

$\theta = 0^\circ$, asymptotic

Motivations: Gridshell Structures

$$\begin{cases} x = a \cos t \\ y = a \sin t \\ z = a \tan \theta \operatorname{ch}\left(\frac{t}{\tan \theta}\right) \end{cases}$$

Parametrization

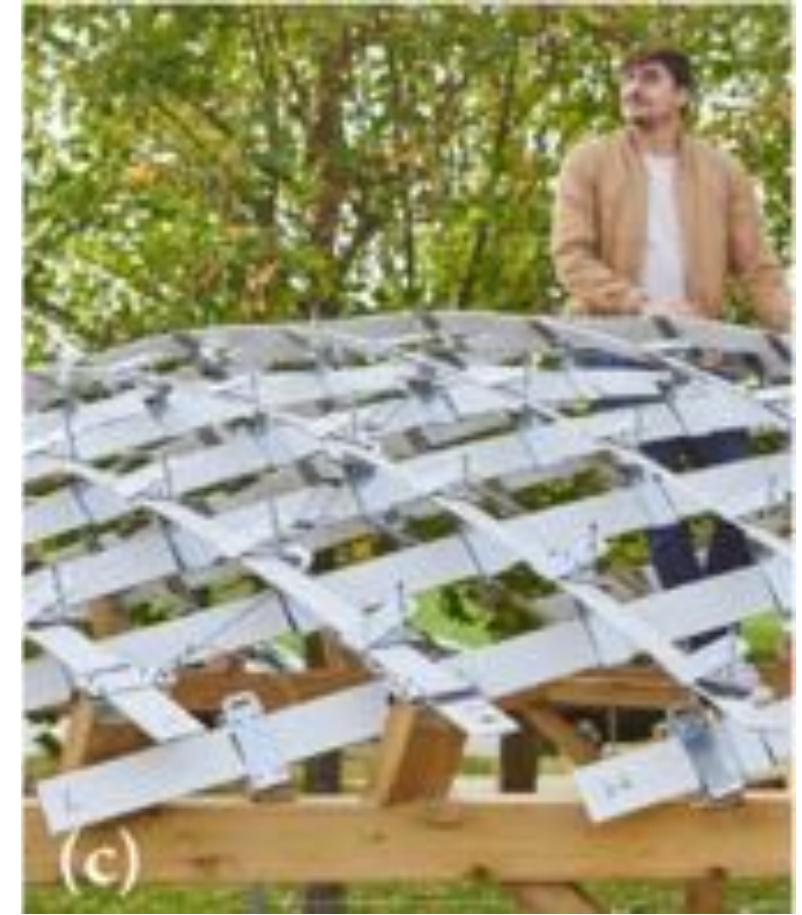


Given t



Changing t

Pseudo-geodesic curve on a cylinder

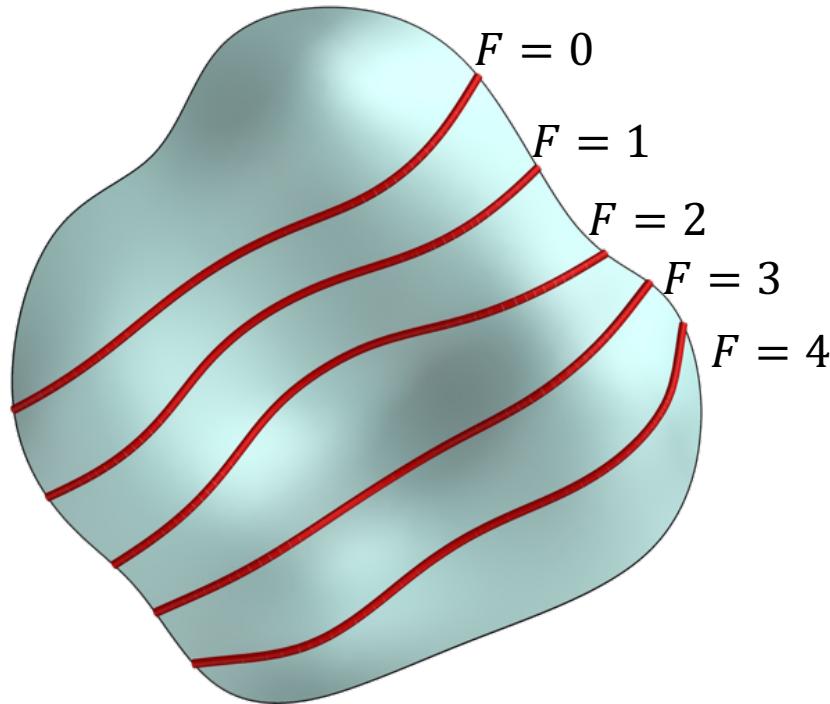


Strips holding a constant angle to the surface
[Mesnil and Baverel 2023]

Method and Results

Method: A Level-Set Based Framework

Assign function values for the curves



A robust version of
[Jiang et al. 2019]'s tracing algorithm.

Optimize the level sets to fit the curves

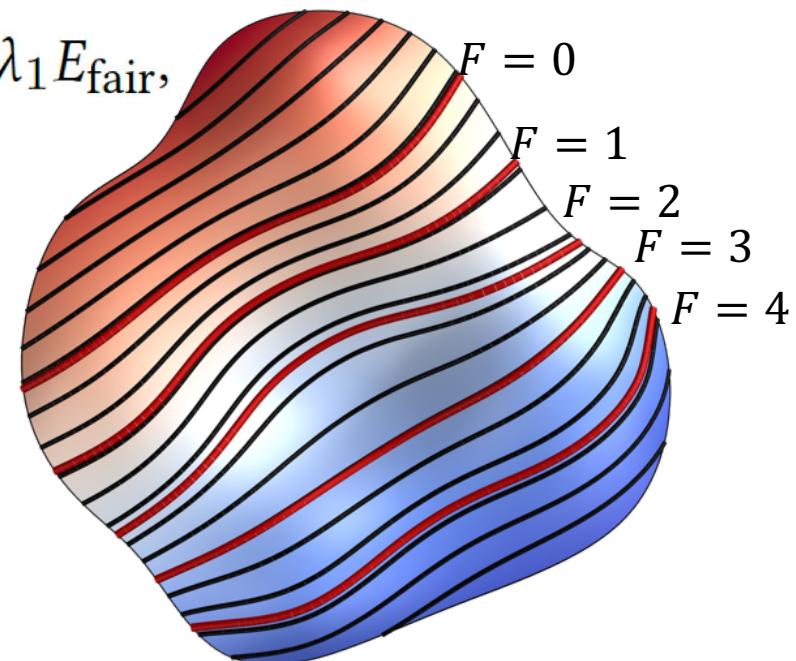
Linear interpolation:

$$E_{\text{trace}} = \sum_{c^i \in C_t} \sum_{p \in c^i} \left(\frac{F^i - F_0}{F_1 - F_0} - \frac{\|p - v_0\|}{\|v_1 - v_0\|} \right)^2.$$

Fairness:

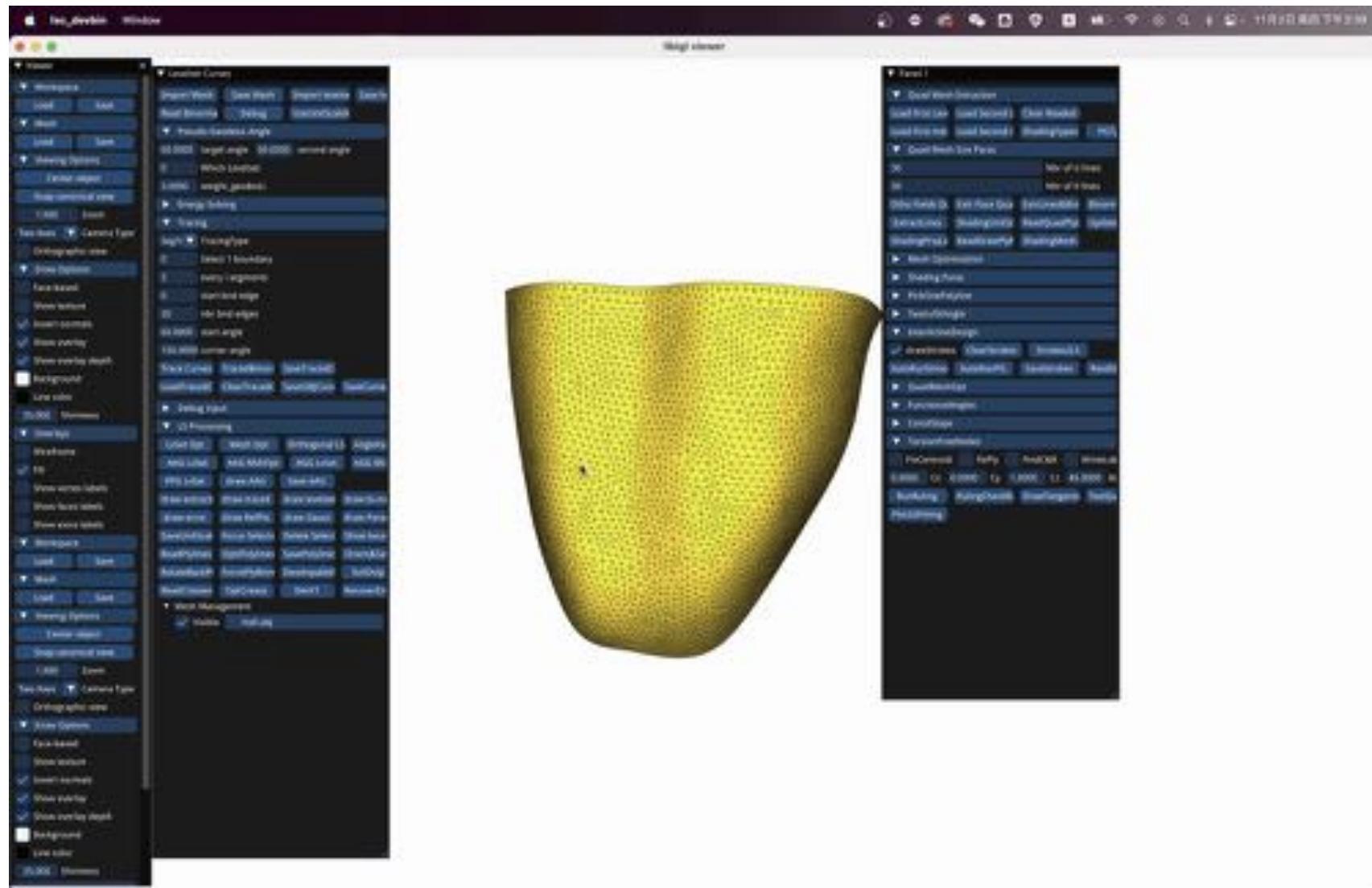
$$E_{\text{fair}} = \sum_{v \in \mathcal{V}} \|H(v)\|^2 \mathcal{A}(v).$$

$$\min E_{\text{init}} = \lambda_0 E_{\text{trace}} + \lambda_1 E_{\text{fair}},$$



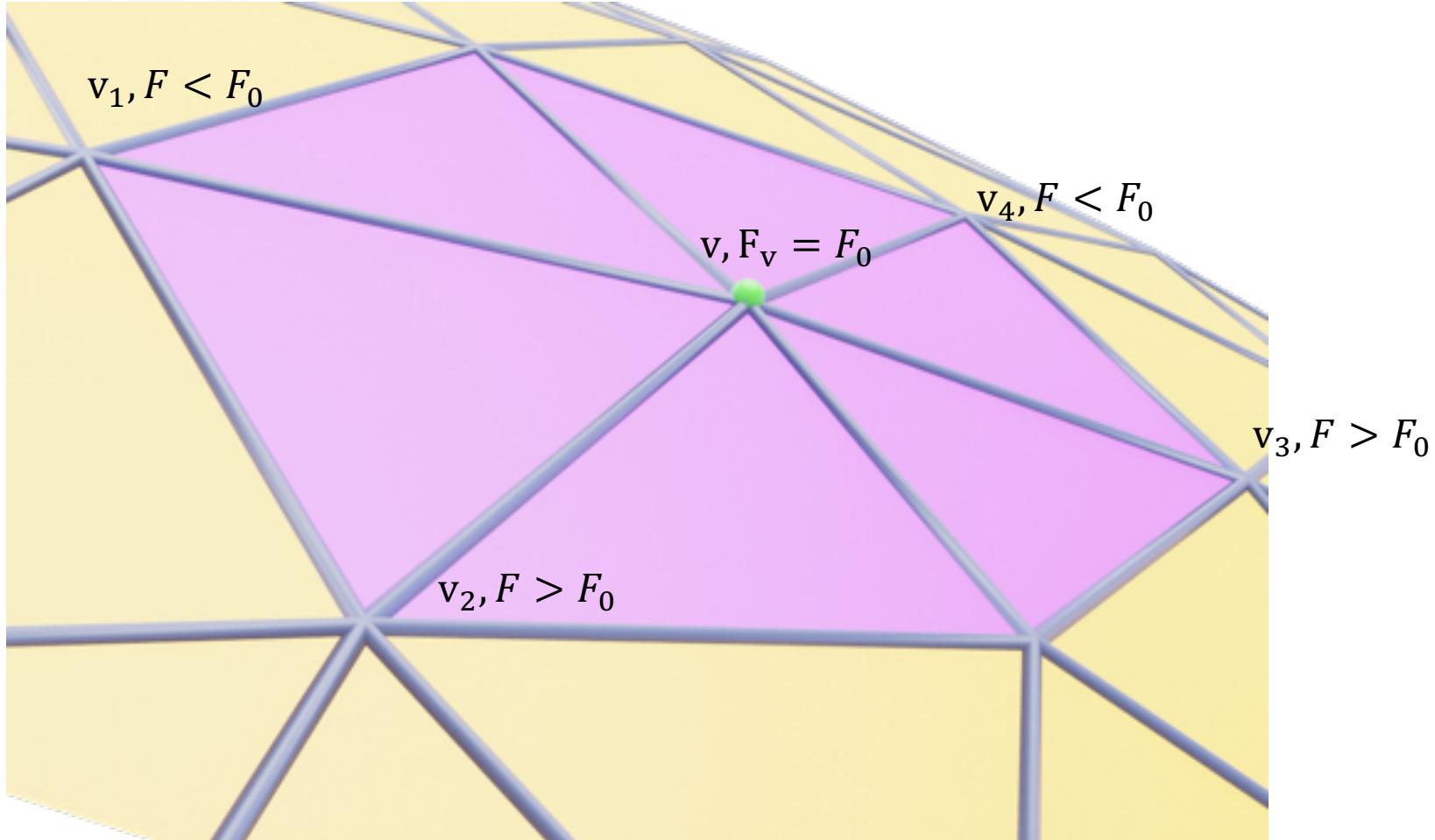
Method: A Level-Set Based Framework

An interactive method (optional initialization):



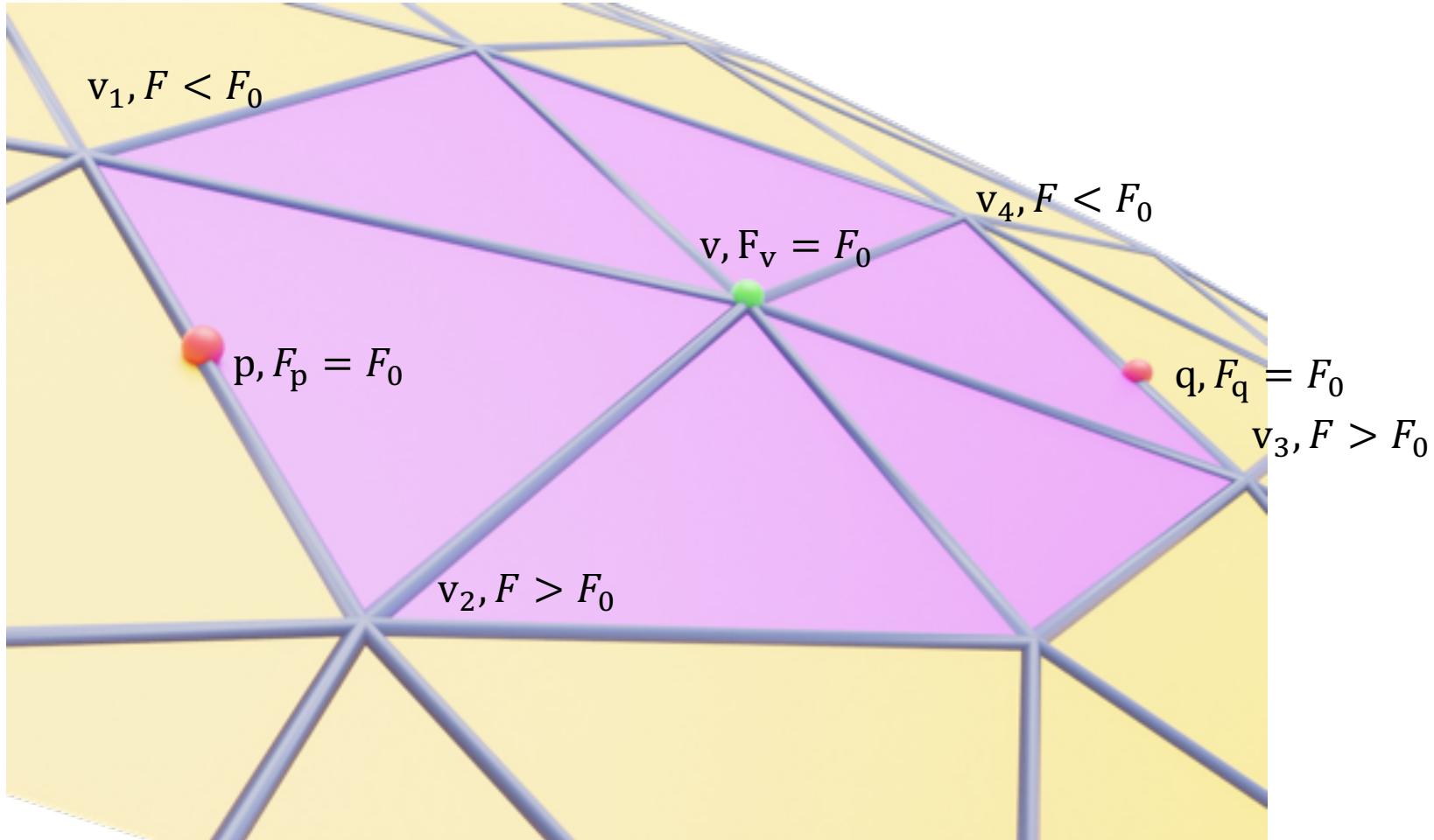
Optimizing Pseudo-geodesics

Control the inclination angles \leftrightarrow Controlling the binormals on each vertex star



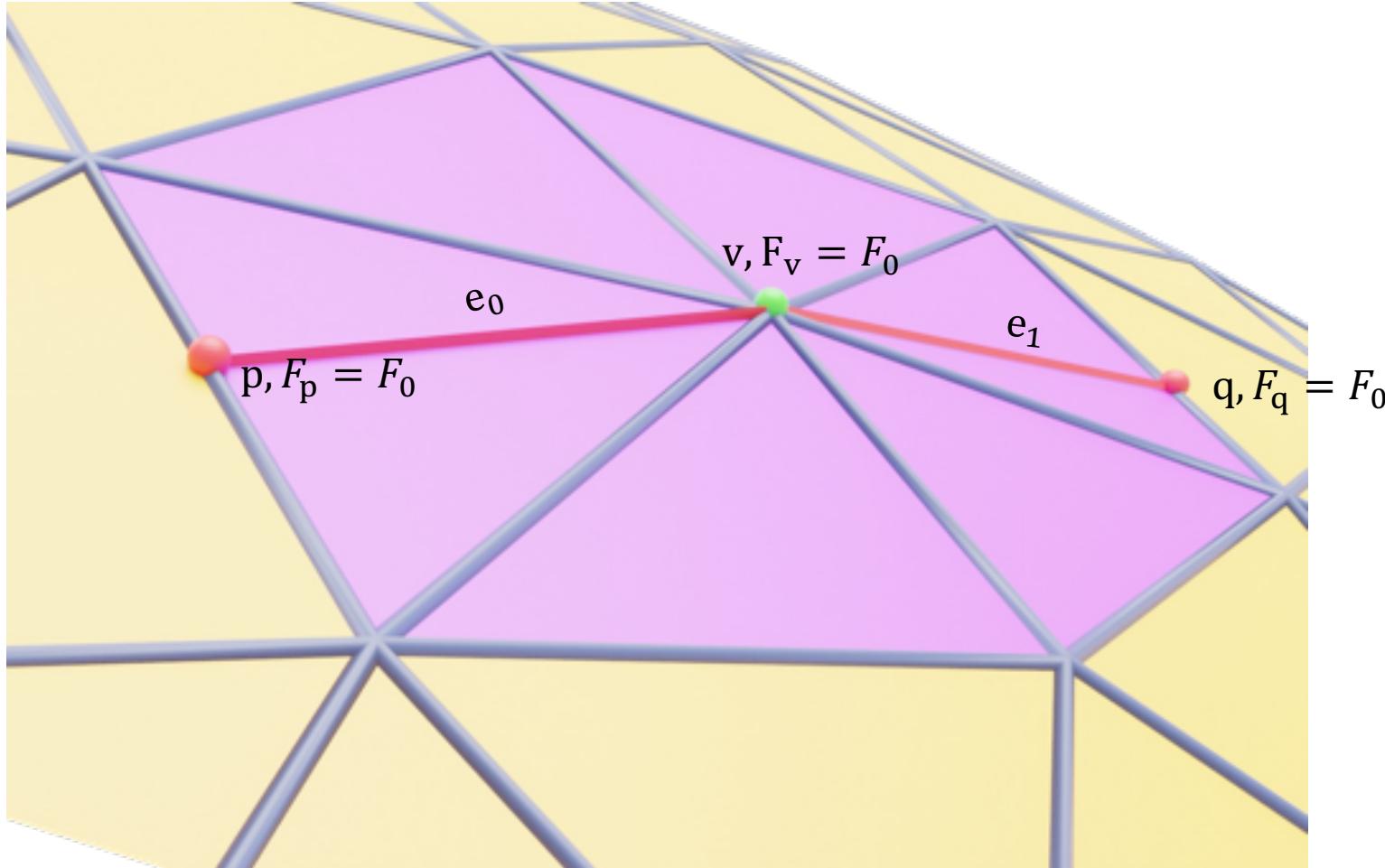
Optimizing Pseudo-geodesics

Control the inclination angles \leftrightarrow Controlling the binormals on each vertex star



Optimizing Pseudo-geodesics

Control the inclination angles \leftrightarrow Controlling the binormals on each vertex star

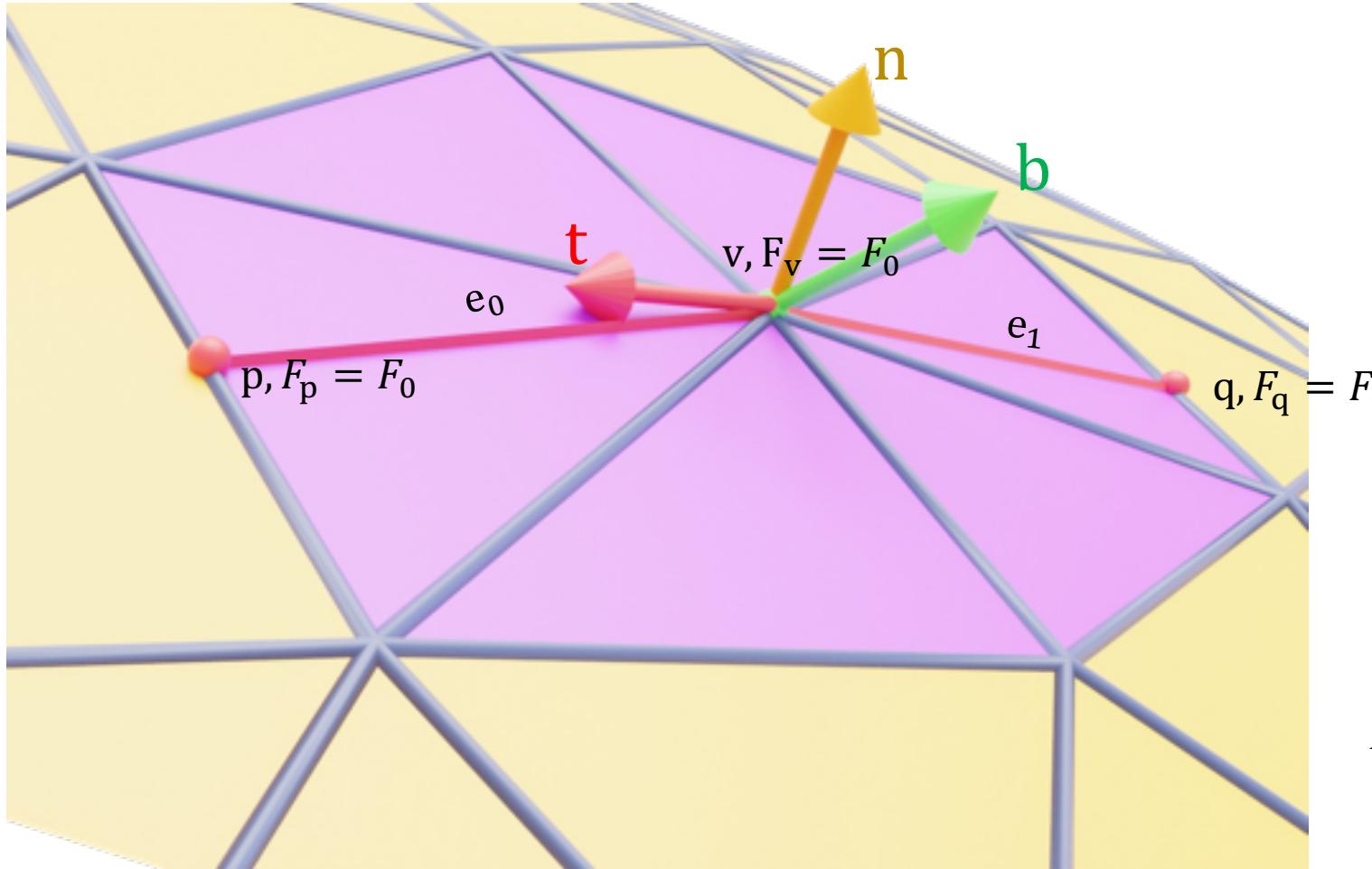


$$e_0 = \frac{p - v}{\|p - v\|},$$

$$e_1 = \frac{v - q}{\|v - q\|},$$

Optimizing Pseudo-geodesics

Control the inclination angles \leftrightarrow Controlling the binormals on each vertex star



$$e_0 = \frac{p - v}{\|p - v\|},$$

$$e_1 = \frac{v - q}{\|v - q\|},$$

$$b = \frac{e_0 \times e_1}{\|e_0 \times e_1\|}$$

Angle constraint:

$$\min (b \cdot n - \cos \theta)^2$$

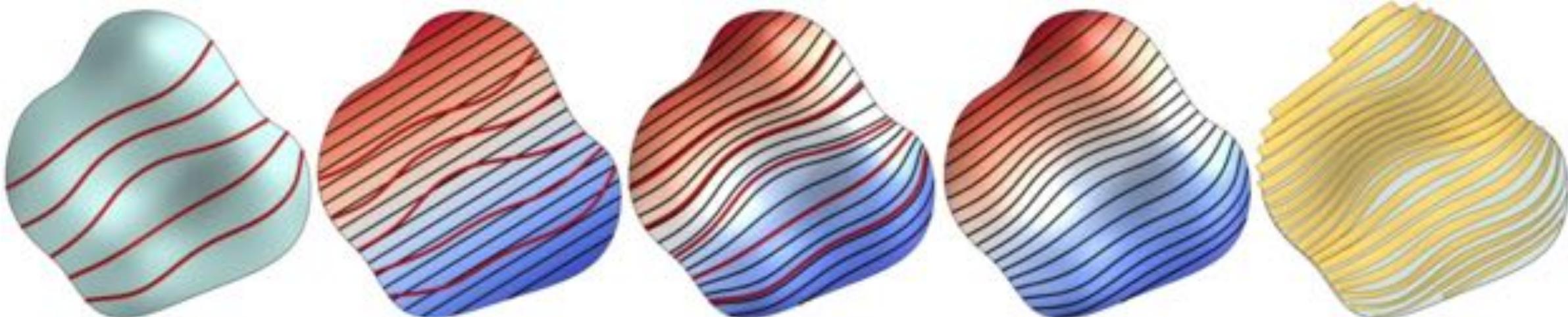
Optimizing Pseudo-geodesics

Angle constraints: $E_{\text{angle}} = \sum_{v \in \mathcal{V}} ((b \cdot n)^2 - \cos^2 \theta)^2 \mathcal{A}(v) + \sum_{v \in \mathcal{V}} ((b \cdot n)(b \cdot u) - \sin \theta \cos \theta)^2 \mathcal{A}(v),$

Preventing vanishing gradients: $E_{\text{grad}} = \sum_{f \in \mathcal{F}} (\|\nabla F(f)\| - r)^2 \mathcal{A}(f),$

Fairness: $E_{\text{fair}} = \sum_{v \in \mathcal{V}} \|H(v)\|^2 \mathcal{A}(v).$

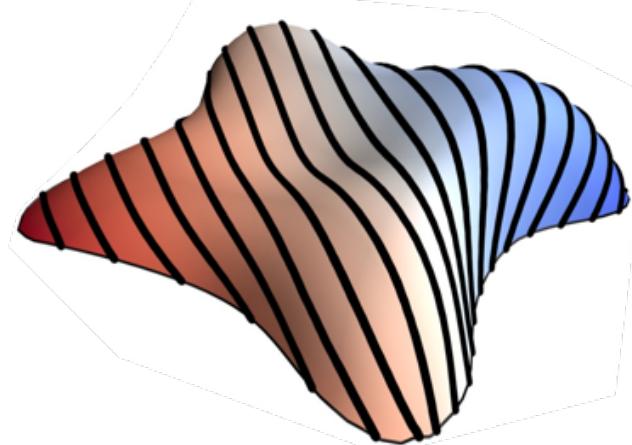
$$\min E_{\text{pg}} = \lambda_{\text{fair}} E_{\text{fair}} + \lambda_{\text{grad}} E_{\text{grad}} + \lambda_{\text{angle}} E_{\text{angle}}.$$



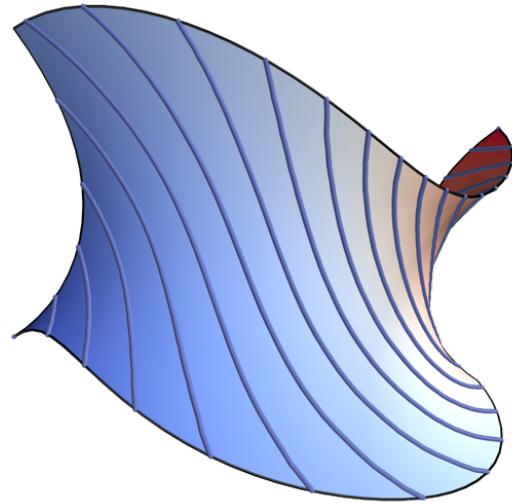
Input surface + target angle θ

Pseudo-geodesics of angle θ

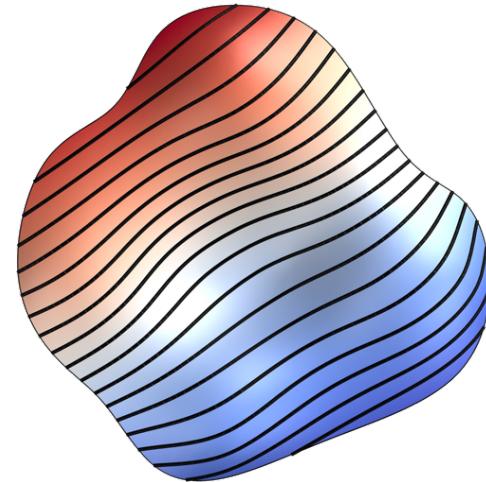
Optimizing 1-family of Pseudo-geodesics



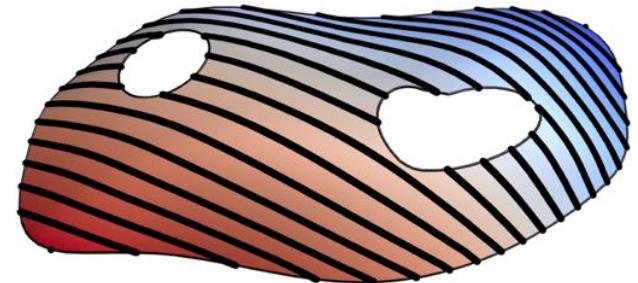
$\theta = 90^\circ$
Geodesic



$\theta = 0^\circ$
Asymptotic



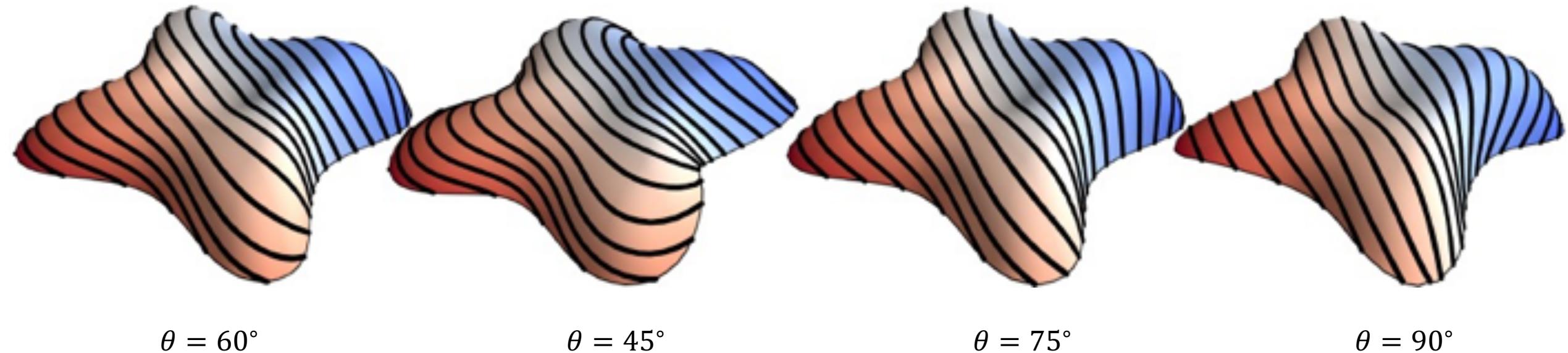
$\theta = 60^\circ$
Pseudo-geodesic



$\theta = 75^\circ$
Pseudo-geodesic

Optimizing 1-family of Pseudo-geodesics

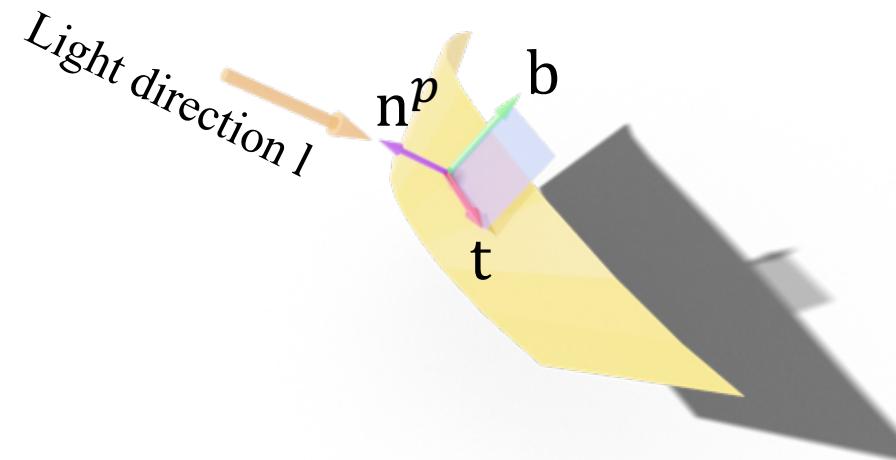
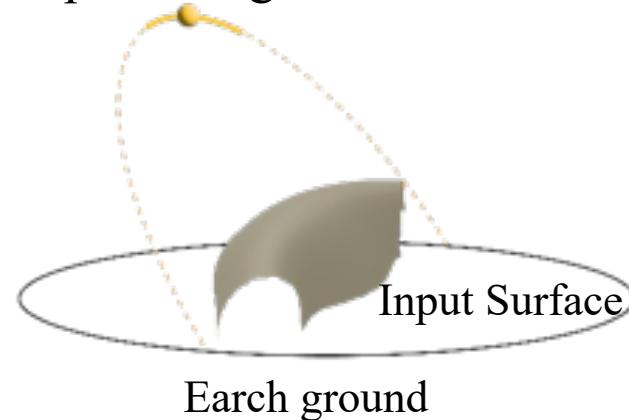
Keeping F fixed and optimize the underlying surface



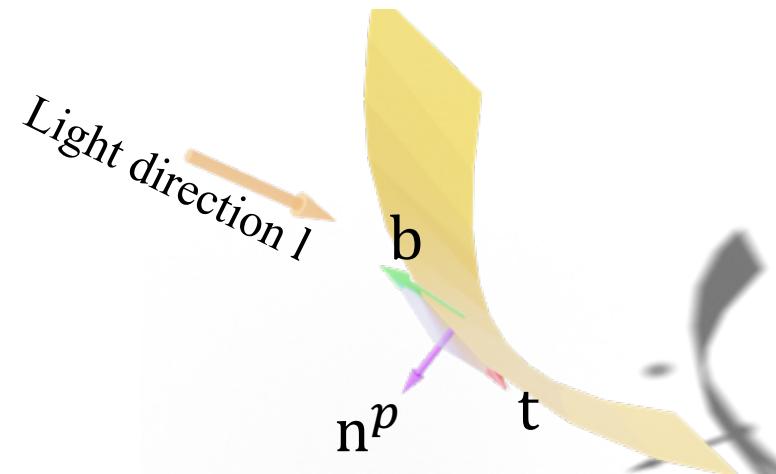
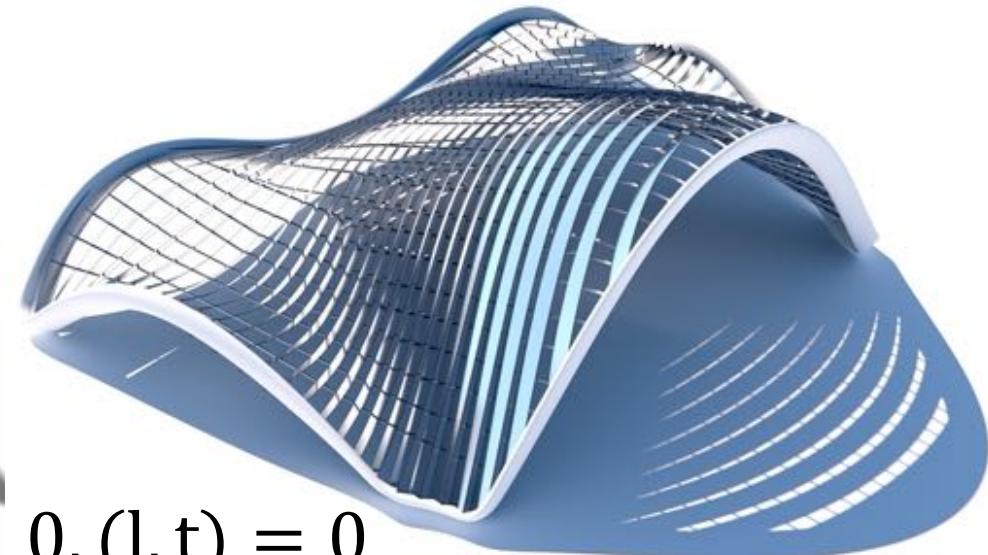
Applications: Shading Systems

Makkah, 12:00, Dec 1st.

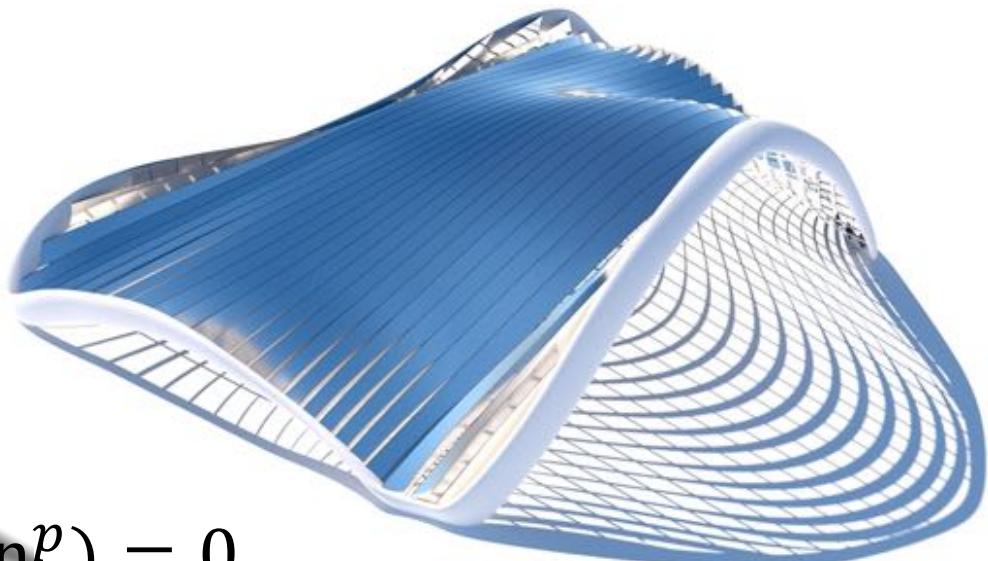
Input sunlight direction



Blocking the light: $(l, b) = 0, (l, t) = 0$



Letting the light through: $(l, n^p) = 0$



Applications: Shading Systems

Vienna, Aug 1st



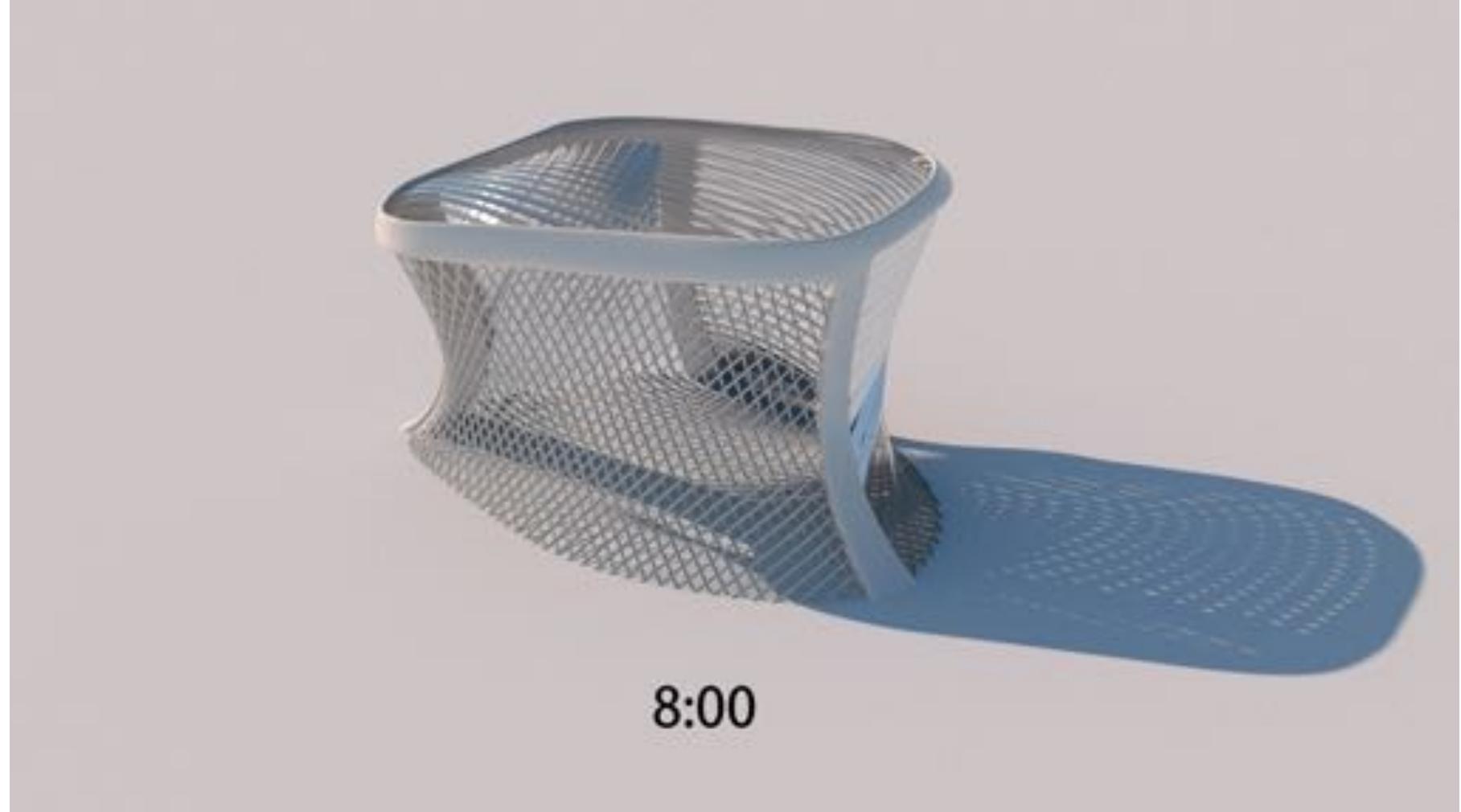
Light through



Light blocked



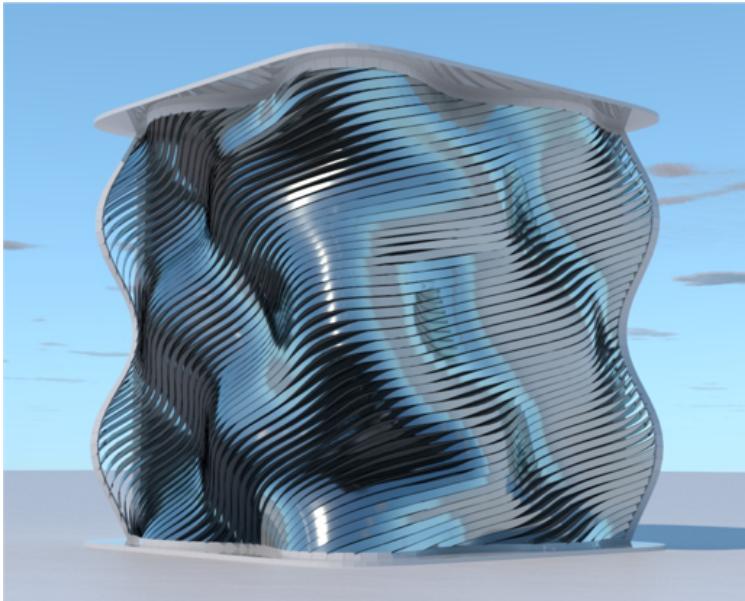
Light blocked



Applications: Shading Systems

London, Aug 15th

Sunlight through in the morning,
and sunlight blocked in the afternoon.

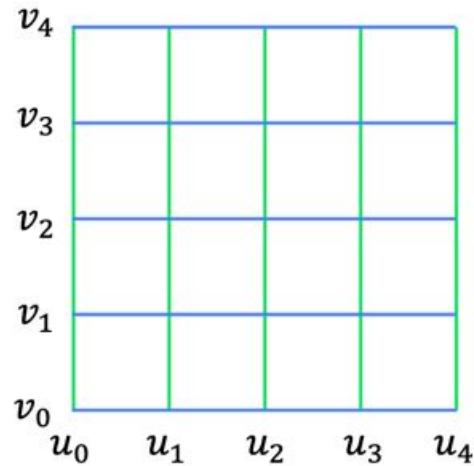


Outside view

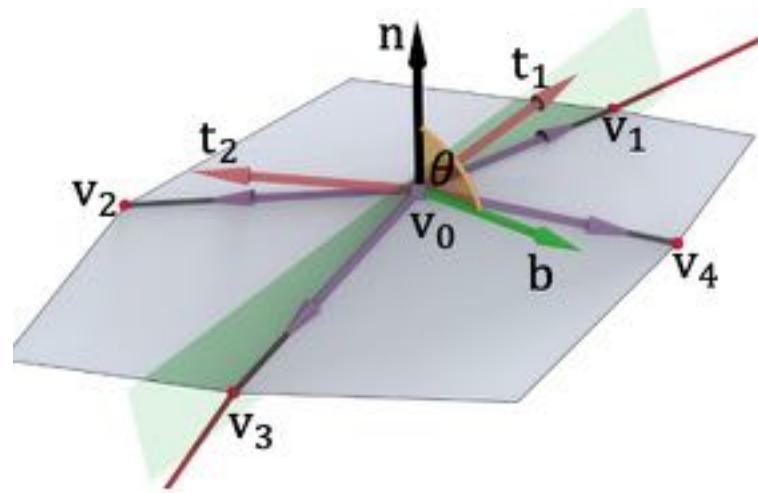


Inside view

Optimizing 2-family of Pseudo-geodesics



Parametric net



Vertex star

$$E_{\text{angle}} = \sum_{v \in \mathcal{V}} ((b \cdot n)^2 - \cos^2 \theta)^2 + \\ \sum_{v \in \mathcal{V}} ((b \cdot n)(b \cdot u) - \sin \theta \cos \theta)^2$$

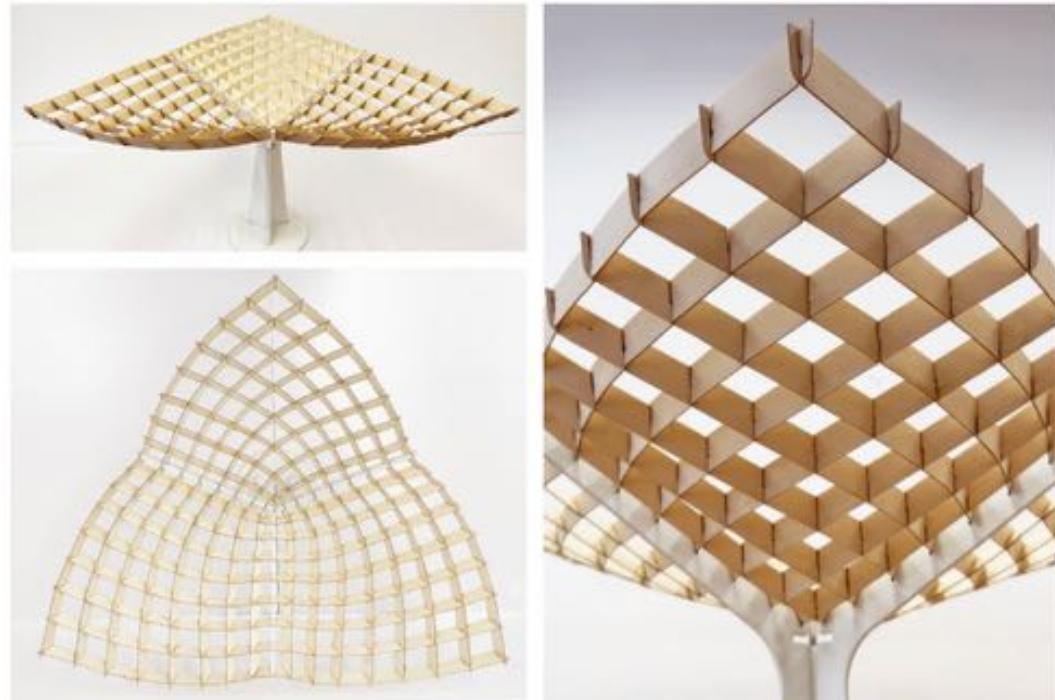


Pseudo-geodesic net (PP-net) with
 $\theta_1 = \theta_2 = 60^\circ$

Applications: Gridshell Structures

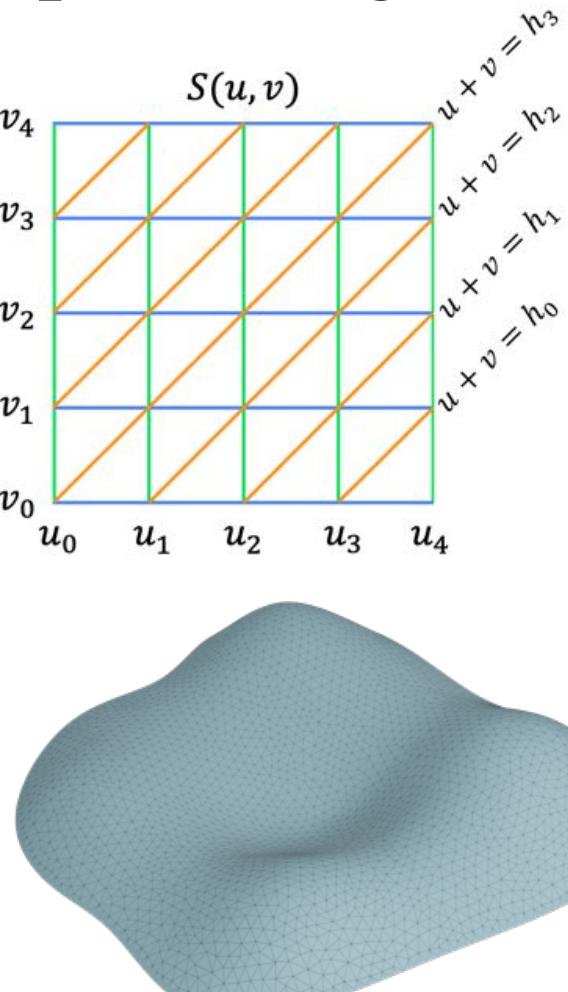


PP-net
 $\theta_1 = \theta_2 = 50^\circ$

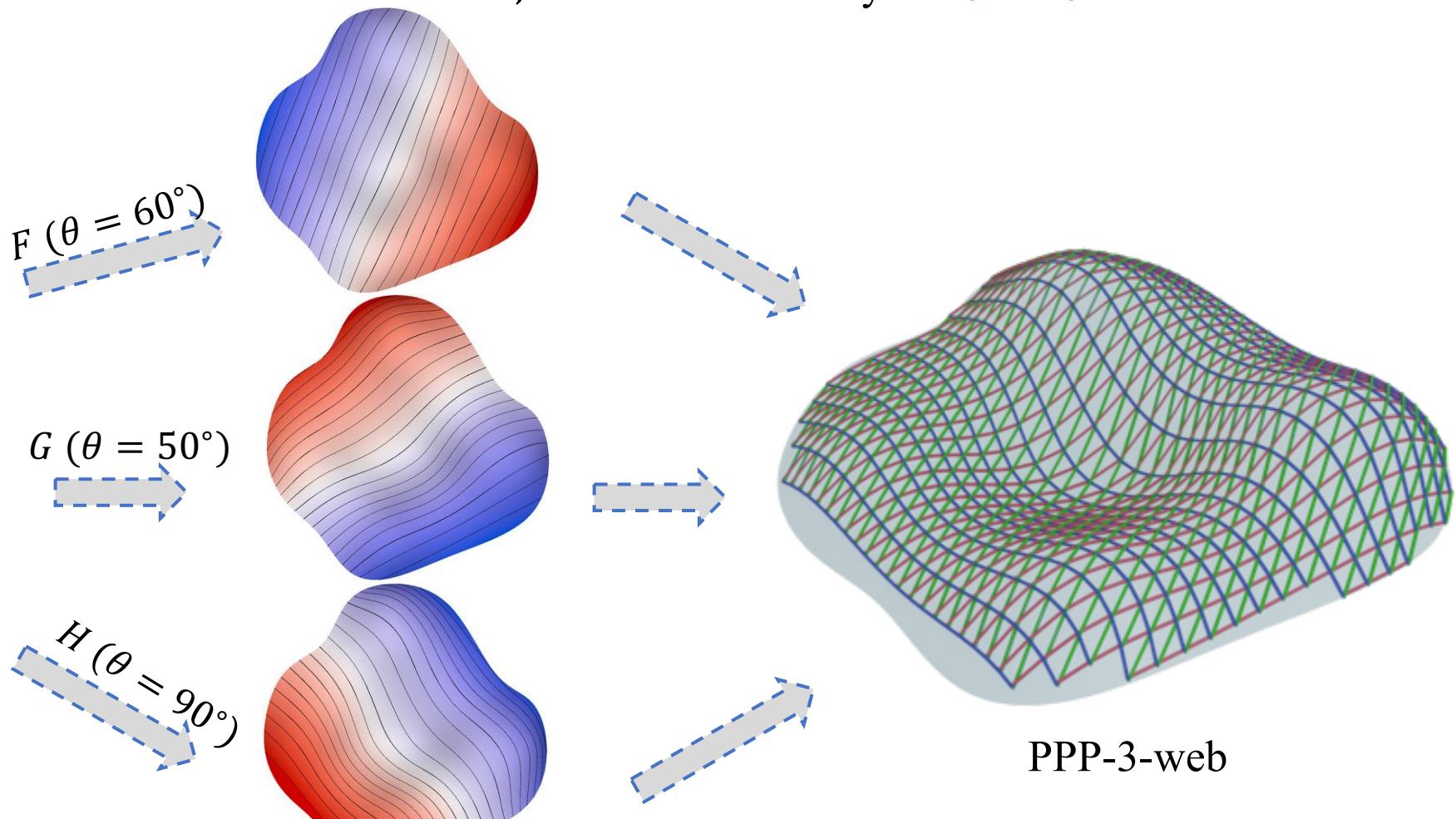


Physical model

Optimizing 3-family of Pseudo-geodesics

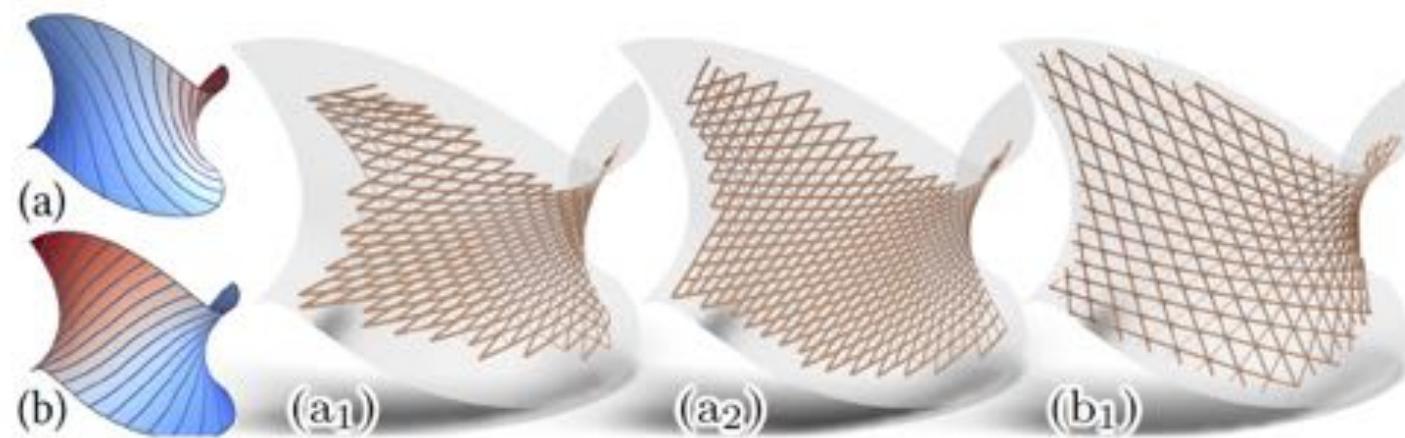


Geometry of webs [Blaschke and Bol 1938]: The 3-web on S can be represented by the level-sets of 3 functions F , G and H that satisfy $F + G + H = 0$

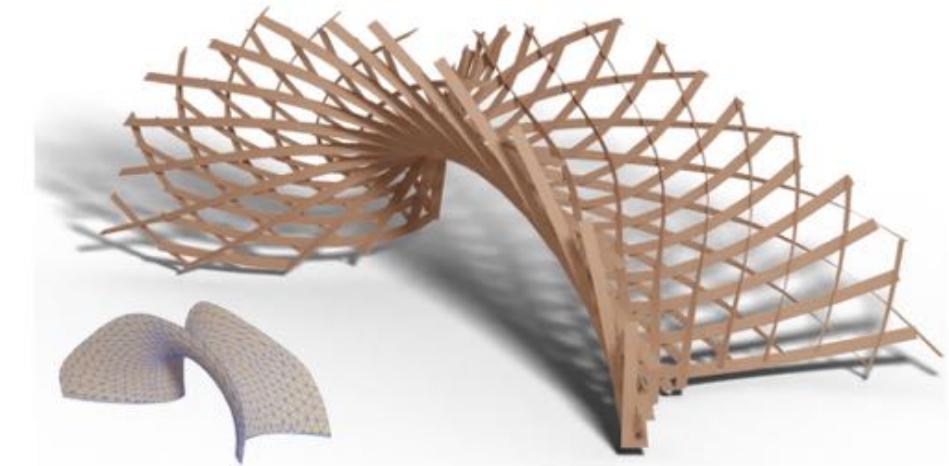


Angle constraints + Geometry of webs ($F + G + H = 0$)

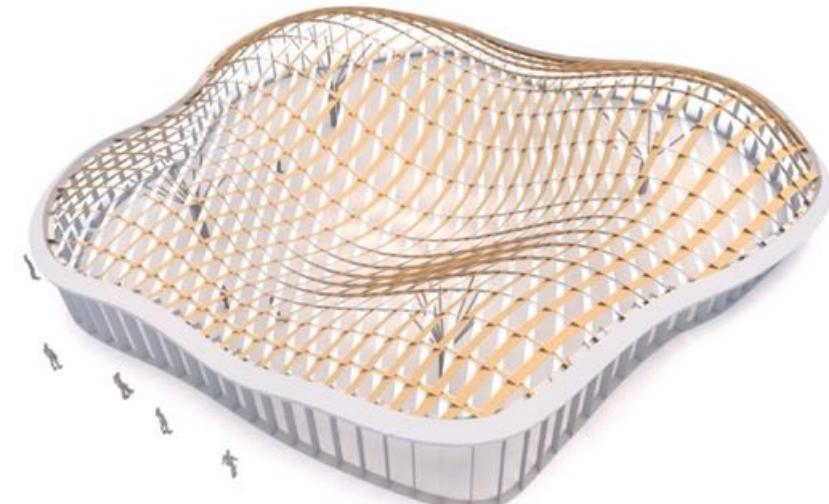
Applications: Gridshell Structures



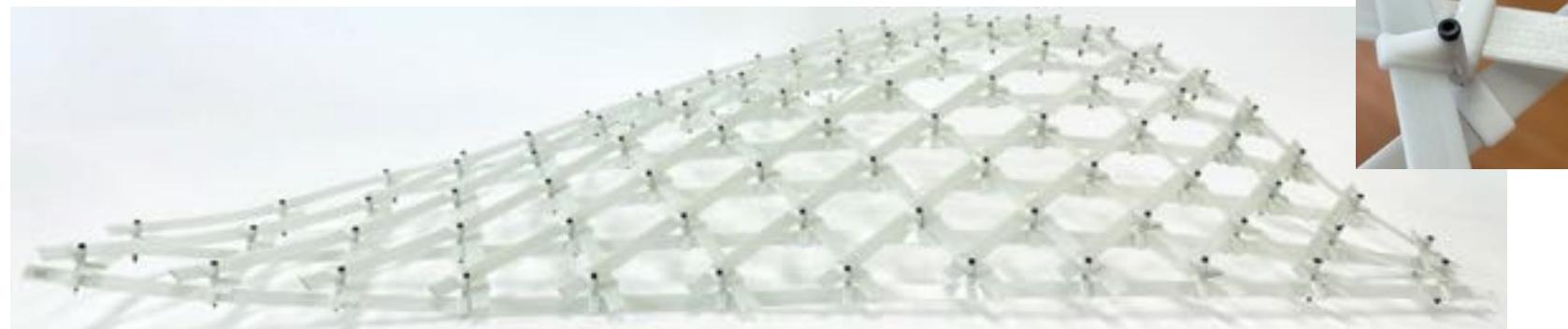
Asymptotic Geodesic Geodesic-web



AAG-web



PPG-web $\theta_1 = 30^\circ, \theta_2 = 45^\circ$



Physical model: PPG-web $\theta_1 = \theta_2 = 60^\circ$

Contribution and Conclusion

- Computational design of shapes from rectifying strips (straight flat strips)
- Controllable inclinations of rectifying strips along level-set curves
- Various rectifying strip patterns applied in shading system and gridshell structures



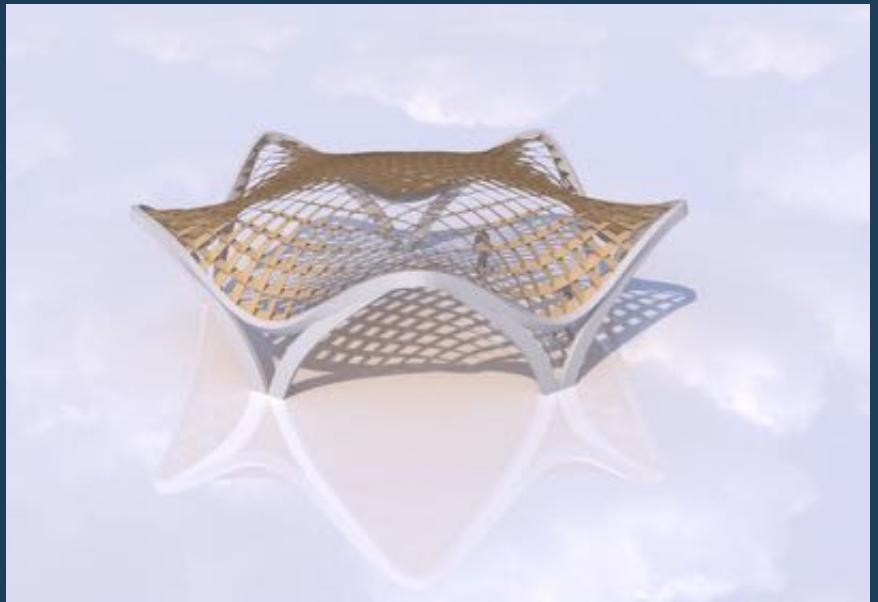
Github Code



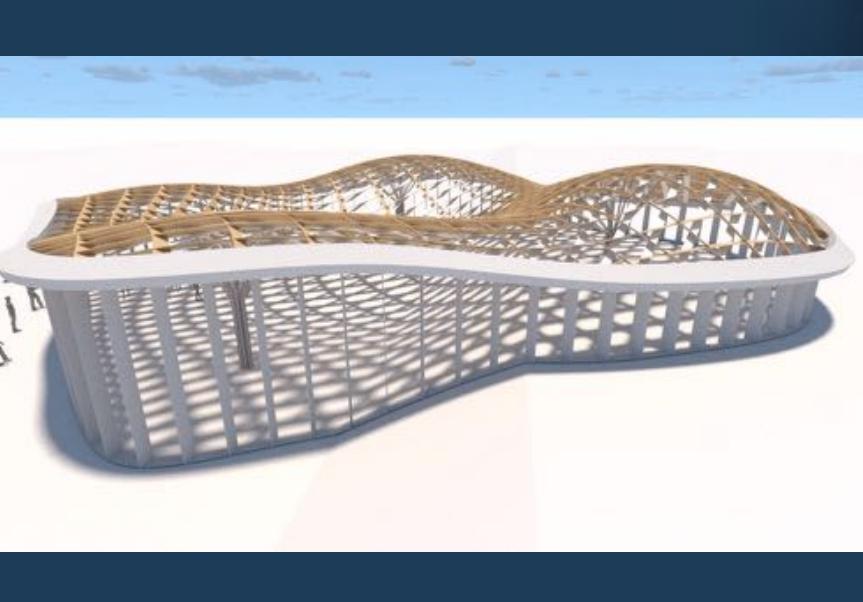
Project Page



1-family
Shading system



2-family
PP-gridshell



3-family
PPG-gridshell