

# Discrete Geodesic Parallel Coordinates

Hui Wang Davide Pellis Florian Rist Helmut Pottmann Christian Müller



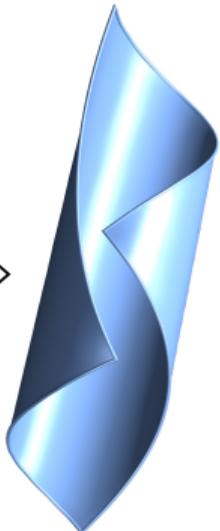
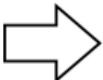
# Motivation



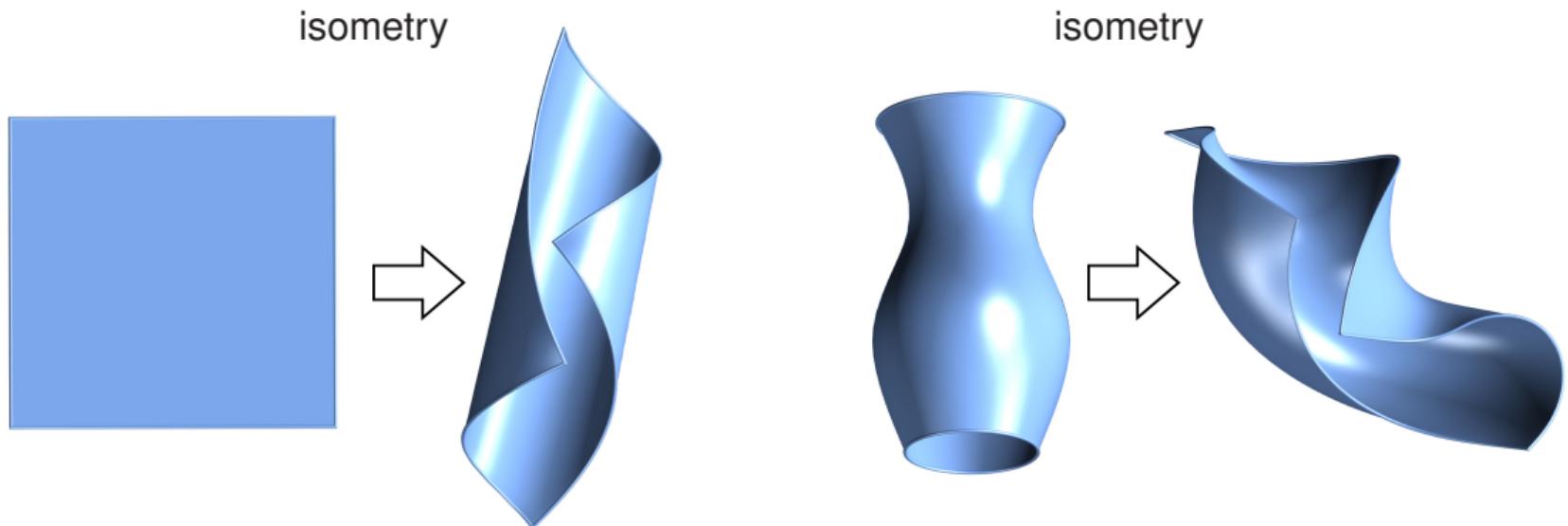
**Walt Disney Concert Hall**  
Frank O. Gehry

# Motivation

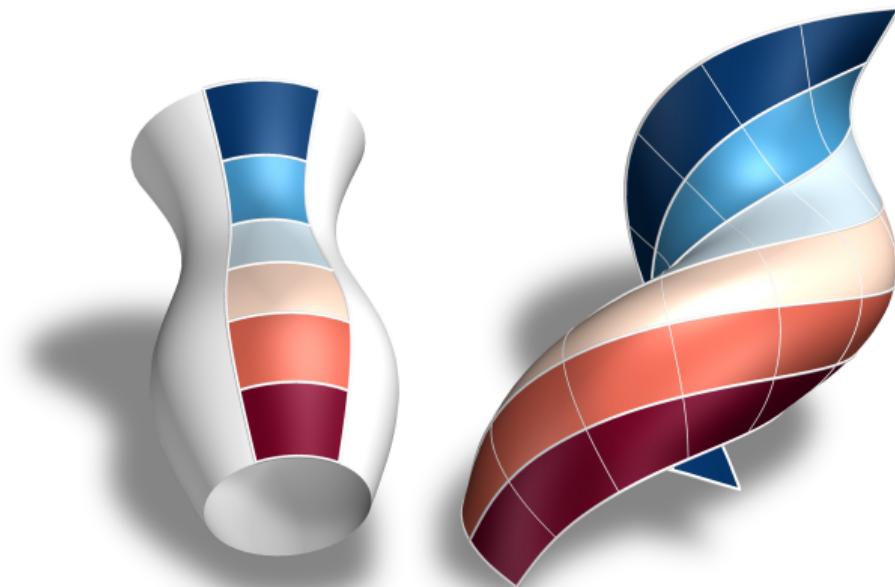
isometry



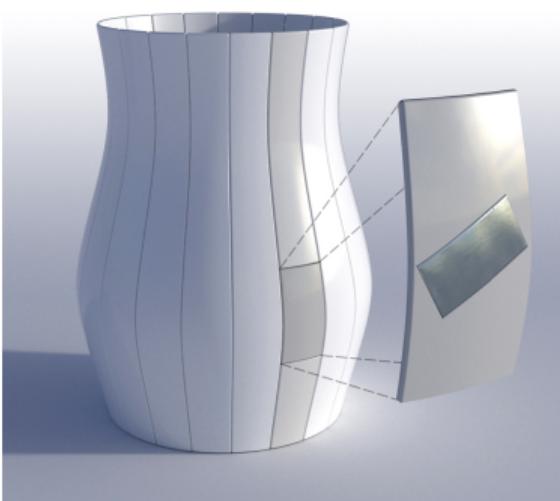
# Motivation



# Motivation


$$\text{molds} \approx \sqrt{\text{panels}}$$

# Motivation



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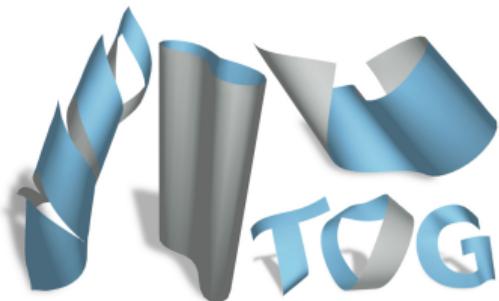


# Motivation

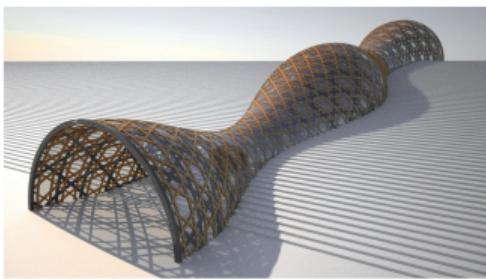


straight congruent flat strips

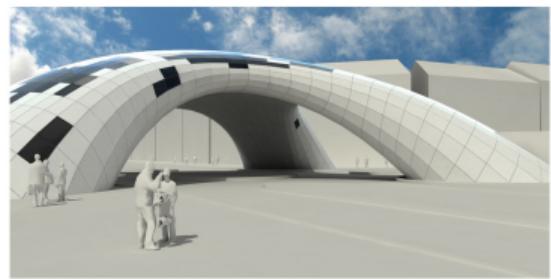
## Related Work



[Rabinovich et al. 2018]

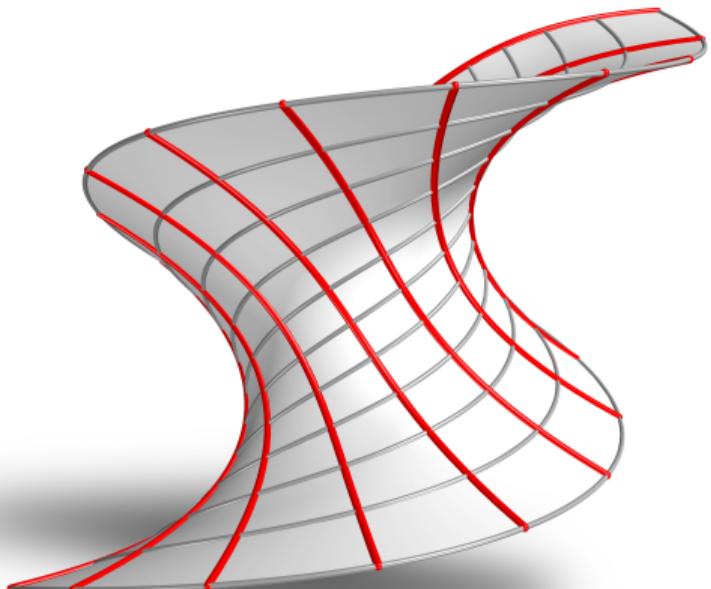


[Pottmann et al. 2010]



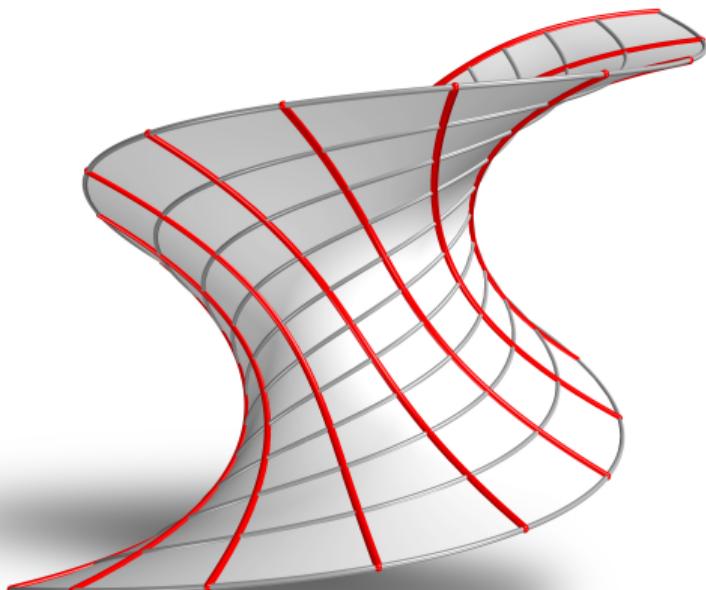
[Tang et al. 2014]

# Geodesic parallel coordinates



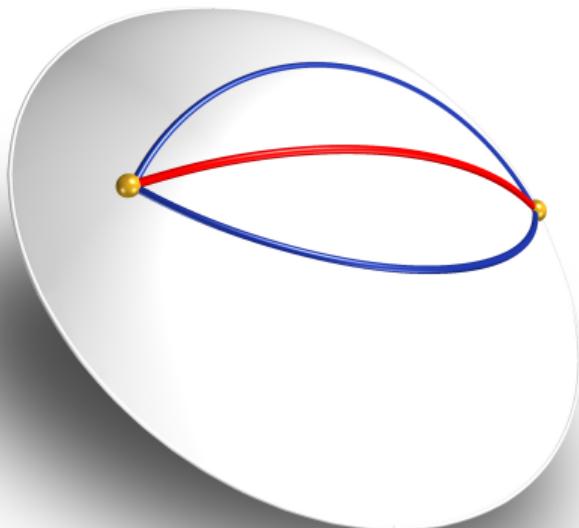
- Orthogonal parameter lines
- The parameter lines of one family are geodesics (red)

# Geodesic parallel coordinates



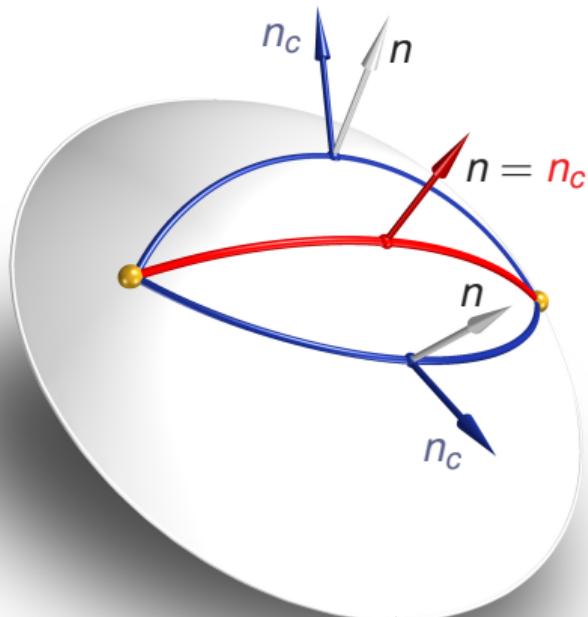
- Orthogonal parameter lines
- The parameter lines of one family are geodesics (red)
  - ↓
- The parameter lines of the other family are 'parallel' (gray)

# Geodesic curves



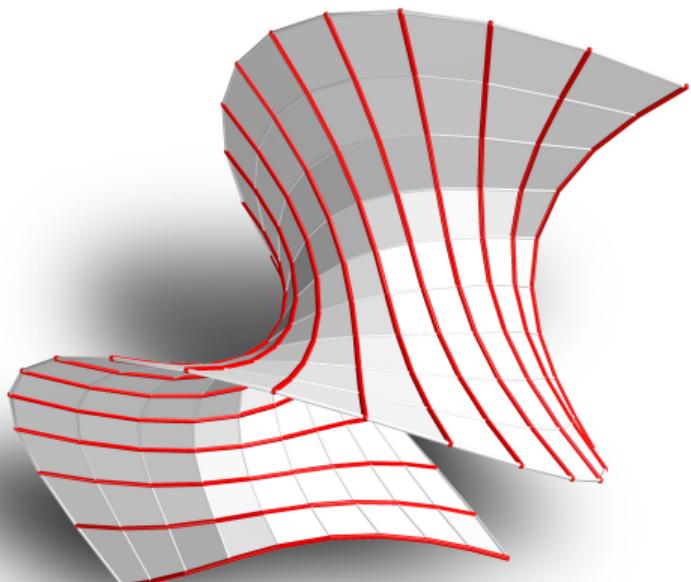
- (Locally) shortest paths on surfaces

# Geodesic curves



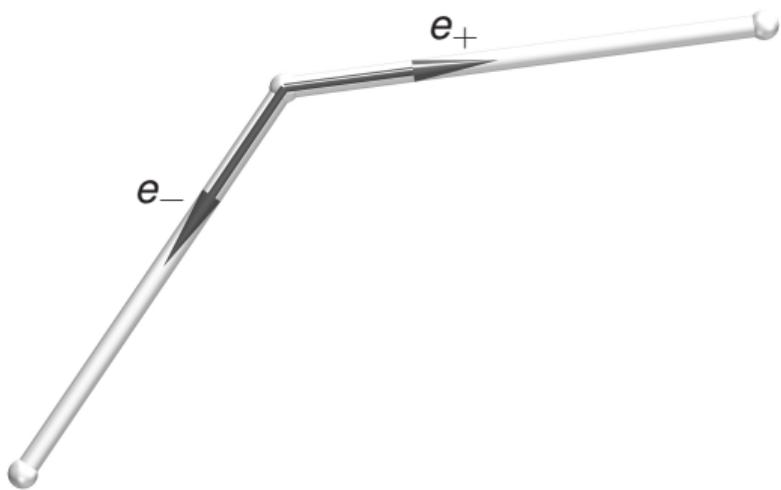
- (Locally) shortest paths on surfaces
- Principal curve normal  $n_c$  and surface normal  $n$  coincide

# Discrete geodesic parallel coordinates

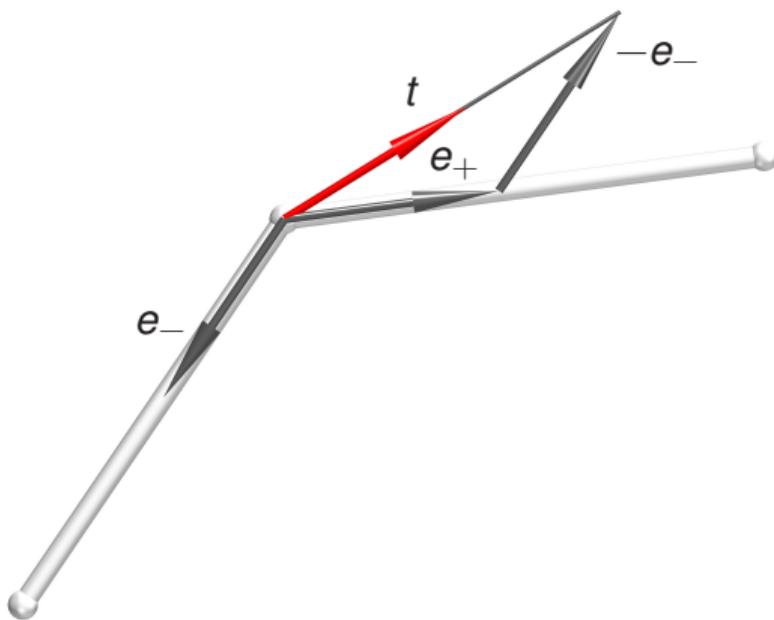


- Discrete orthogonal mesh polylines
- One family of polylines are discrete geodesics (**red**)

# Discrete curves

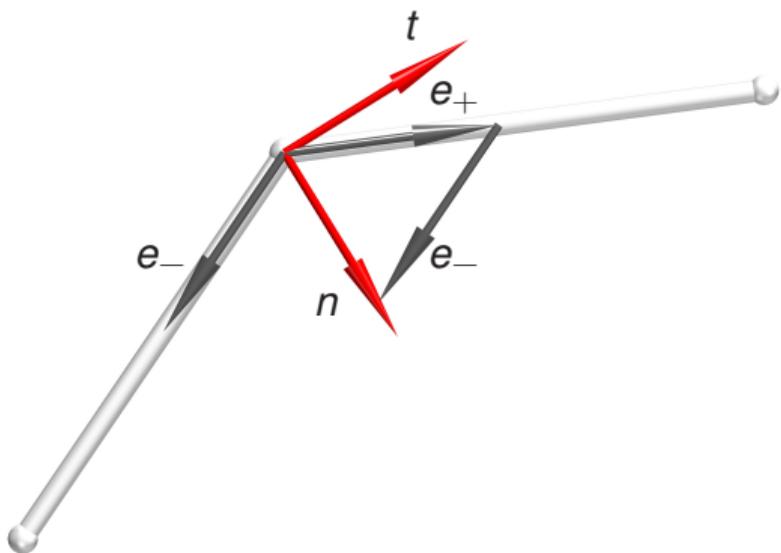


# Discrete curves



$$t = (e_+ - e_-) / \|e_+ - e_-\|$$

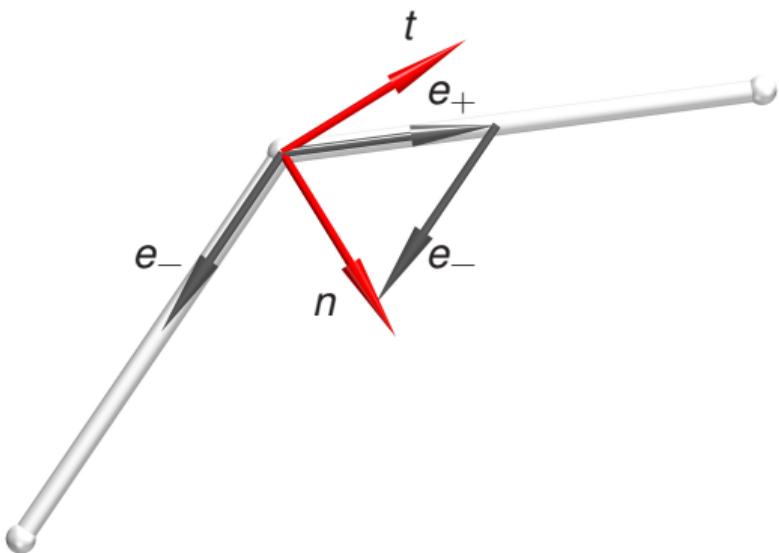
# Discrete curves



$$t = (e_+ - e_-) / \|e_+ - e_-\|$$

$$n = (e_+ + e_-) / \|e_+ + e_-\|$$

# Discrete curves

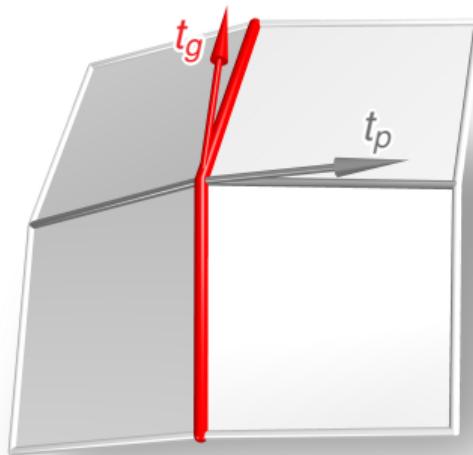


$$t = (e_+ - e_-) / \|e_+ - e_-\|$$

$$n = (e_+ + e_-) / \|e_+ + e_-\|$$

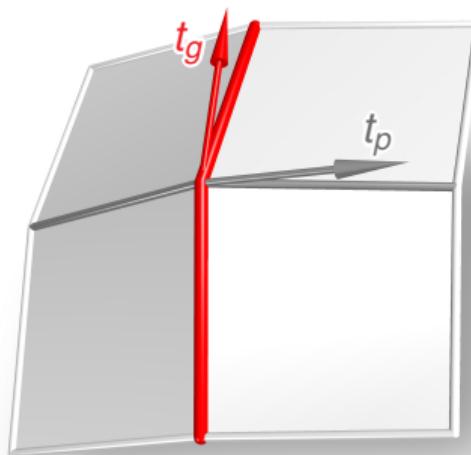
$$t \perp n$$

# Discrete geodesic parallel coordinates



# Discrete geodesic parallel coordinates

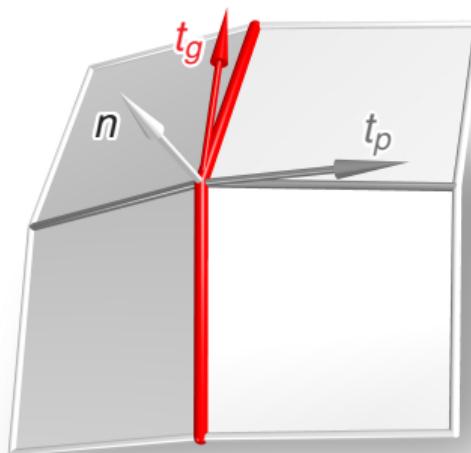
Geodesic:



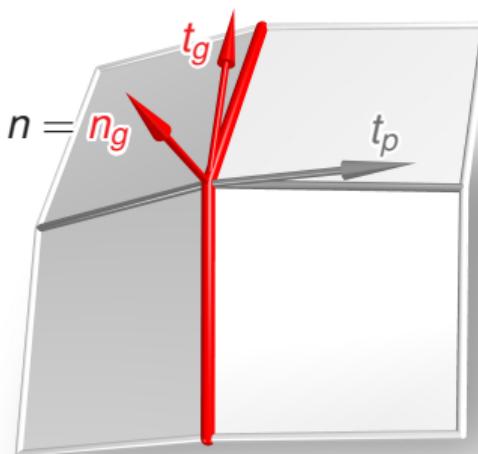
# Discrete geodesic parallel coordinates

Geodesic:

surface normal:  $n = t_p \times t_g$



# Discrete geodesic parallel coordinates

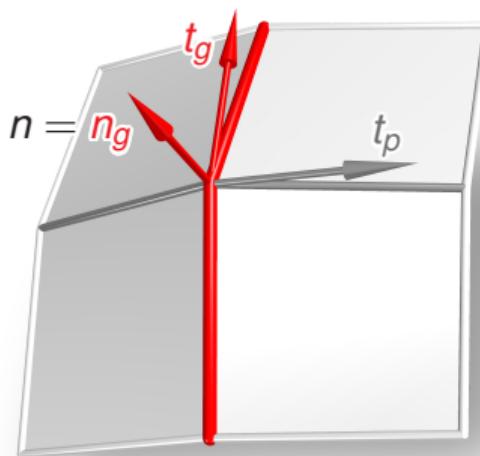


## Geodesic:

surface normal:  $n = t_p \times t_g$

$$n \parallel n_g \Leftrightarrow n_g \perp t_g, t_p$$

# Discrete geodesic parallel coordinates



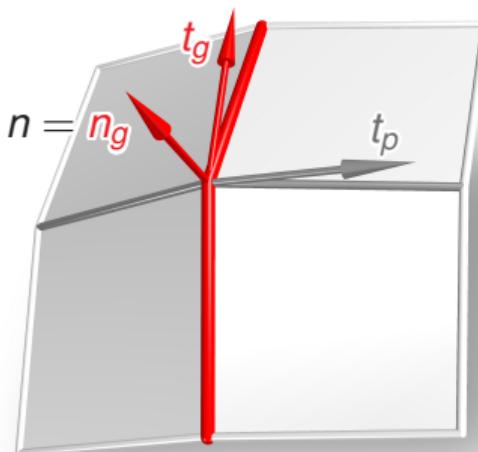
## Geodesic:

surface normal:  $n = t_p \times t_g$

$$n \parallel n_g \Leftrightarrow n_g \perp t_g, t_p$$

$$n_g \perp t_p \Leftrightarrow \langle n_g, t_p \rangle = 0$$

# Discrete geodesic parallel coordinates



## Geodesic:

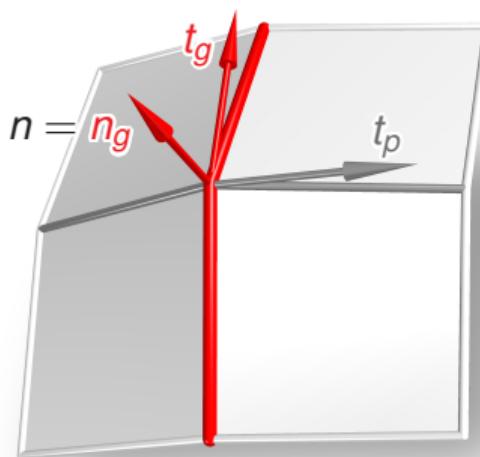
surface normal:  $n = t_p \times t_g$

$$n \parallel n_g \Leftrightarrow n_g \perp t_g, t_p$$

$$n_g \perp t_p \Leftrightarrow \langle n_g, t_p \rangle = 0$$

$$= \langle e_+ + e_-, e_+ - e_- \rangle$$

# Discrete geodesic parallel coordinates



## Geodesic:

surface normal:  $n = t_p \times t_g$

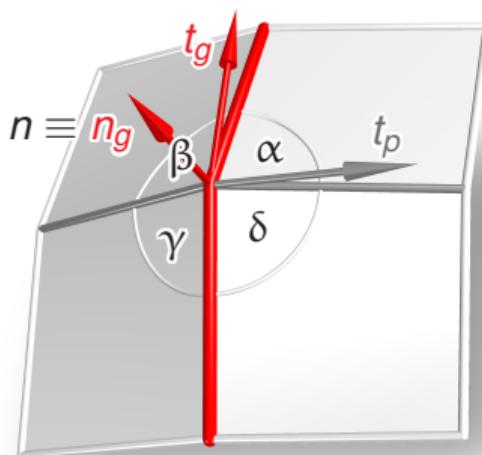
$$n \parallel n_g \Leftrightarrow n_g \perp t_g, t_p$$

$$n_g \perp t_p \Leftrightarrow \langle n_g, t_p \rangle = 0$$

$$= \langle e_+ + e_-, e_+ - e_- \rangle$$

$$= \langle e_+, e_+ \rangle - \langle e_+, e_- \rangle + \langle e_-, e_+ \rangle - \langle e_-, e_- \rangle$$

# Discrete geodesic parallel coordinates



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surface normal:  $n = t_p \times t_g$

$$n \parallel n_g \Leftrightarrow n_g \perp t_g, t_p$$

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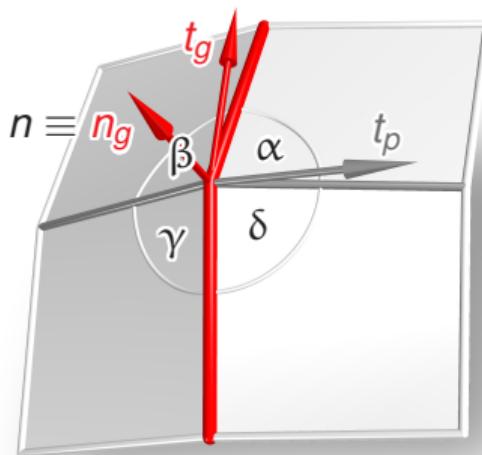
$$= \langle e_+ + e_-, e_+ - e_- \rangle$$

$$= \langle e_+, e_+ \rangle - \langle e_+, e_- \rangle + \langle e_-, e_+ \rangle - \langle e_-, e_- \rangle$$

$$= \cos \alpha - \cos \beta + \cos \delta - \cos \gamma$$

# Discrete geodesic parallel coordinates

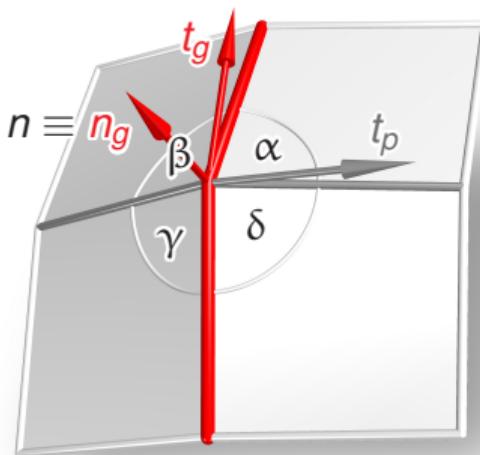
**Geodesic:**  $\cos \alpha + \cos \delta = \cos \beta + \cos \gamma$



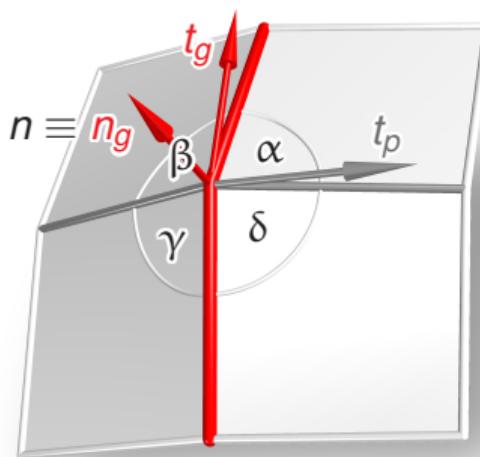
# Discrete geodesic parallel coordinates

**Geodesic:**  $\cos \alpha + \cos \delta = \cos \beta + \cos \gamma$

**Parallel:**



# Discrete geodesic parallel coordinates

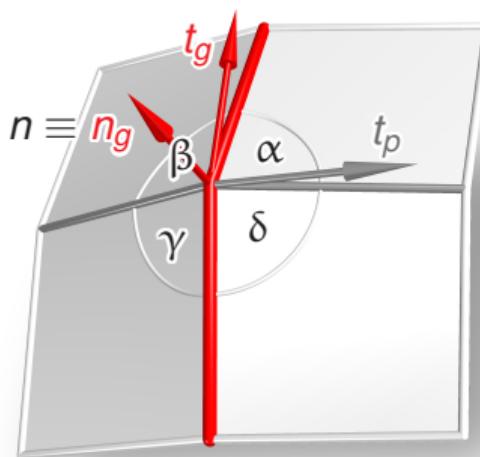


**Geodesic:**  $\cos \alpha + \cos \delta = \cos \beta + \cos \gamma$

**Parallel:**

$$t_g \perp t_p \Leftrightarrow \langle t_g, t_p \rangle = 0$$

# Discrete geodesic parallel coordinates



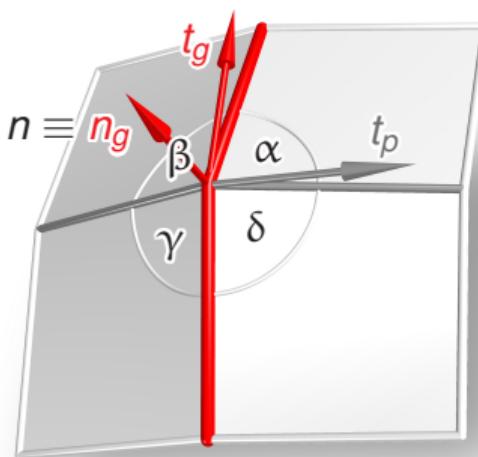
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$$t_g \perp t_p \Leftrightarrow \langle t_g, t_p \rangle = 0$$

$$= \langle e_+ - e_-, e_+ - e_- \rangle$$

# Discrete geodesic parallel coordinates



**Geodesic:**  $\cos \alpha + \cos \delta = \cos \beta + \cos \gamma$

**Parallel:**

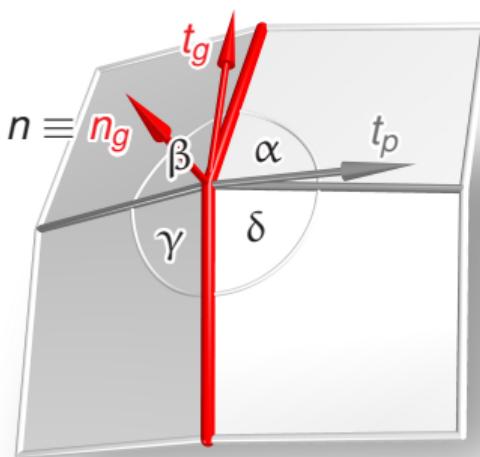
$$t_g \perp t_p \Leftrightarrow \langle t_g, t_p \rangle = 0$$

$$= \langle e_+ - e_-, e_+ - e_- \rangle$$

$$= \langle e_+, e_+ \rangle - \langle e_+, e_- \rangle + \langle e_-, e_+ \rangle - \langle e_-, e_- \rangle$$

$$= \cos \alpha - \cos \beta - \cos \gamma + \cos \delta$$

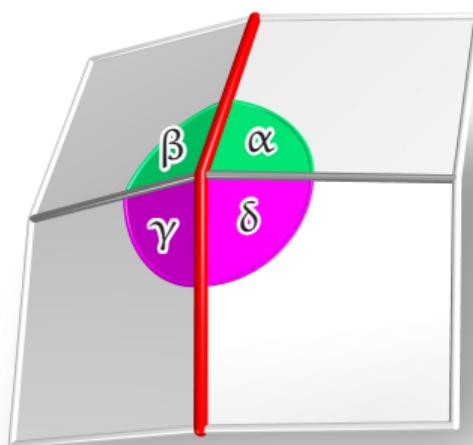
# Discrete geodesic parallel coordinates



**Geodesic:**  $\cos \alpha + \cos \delta = \cos \beta + \cos \gamma$

**Parallel:**  $\cos \alpha + \cos \gamma = \cos \beta + \cos \delta$

# Discrete geodesic parallel coordinates



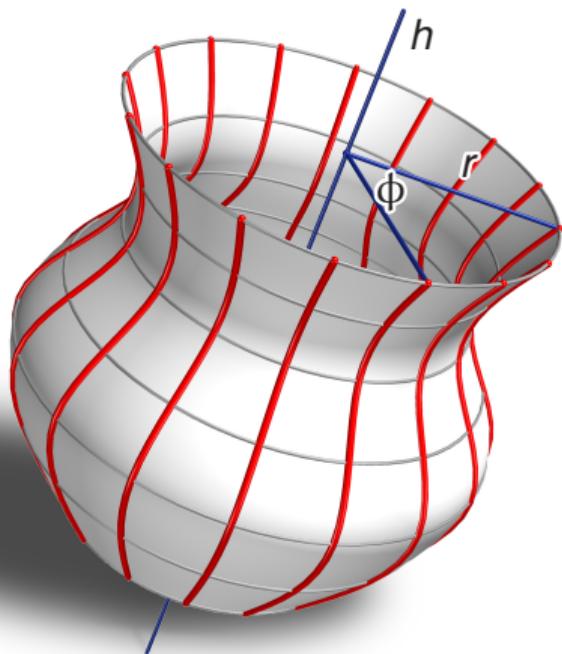
**Geodesic:**  $\cos \alpha + \cos \delta = \cos \beta + \cos \gamma$

**Parallel:**  $\cos \alpha + \cos \gamma = \cos \beta + \cos \delta$

**Geodesic parallel:**

$$\alpha = \beta, \gamma = \delta$$

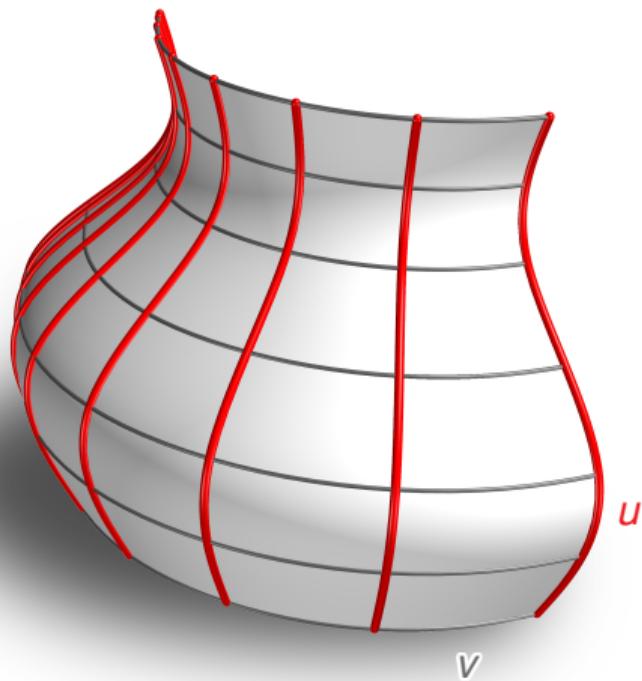
# Rotational surfaces



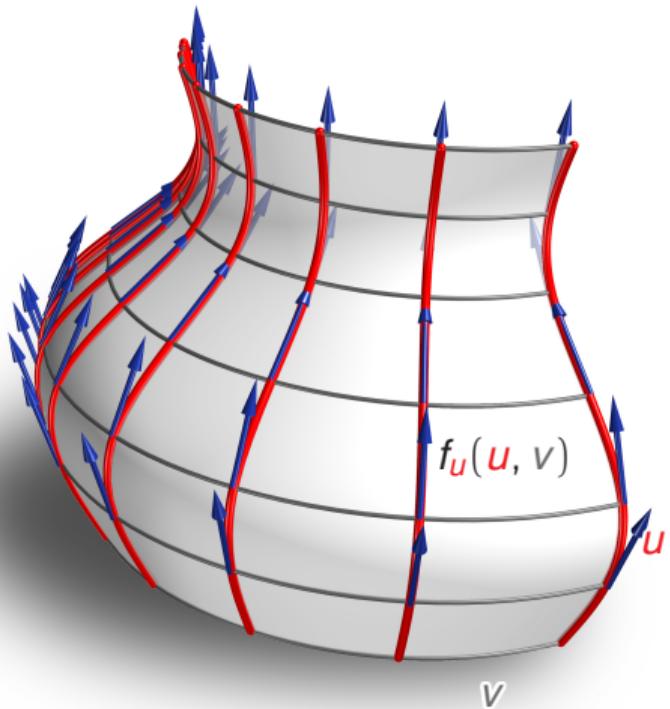
**Geodesic parallel parametrization:**

$$f = \left( r(\textcolor{red}{u}) \cos \phi(\textcolor{red}{v}), r(\textcolor{red}{u}) \sin \phi(\textcolor{red}{v}), h(\textcolor{red}{u}) \right)$$

# Rotational isometric surfaces

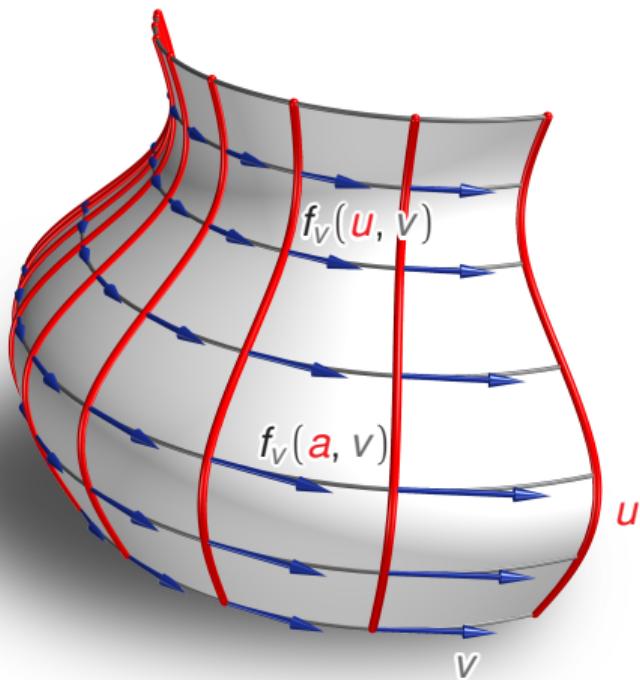


# Rotational isometric surfaces



$$\partial_v \|f_{\color{red}u}(u, v)\| = 0$$

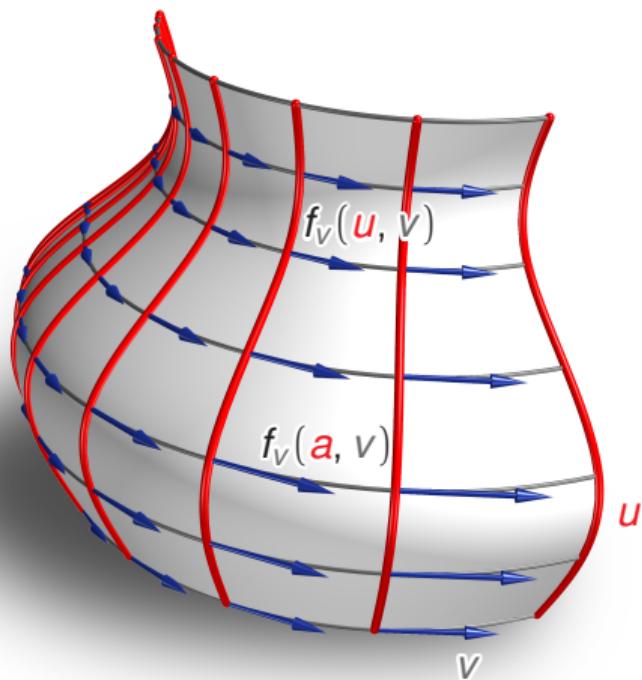
# Rotational isometric surfaces



$$\partial_v \|f_{\textcolor{red}{u}}(\textcolor{red}{u}, v)\| = 0$$

$$r(\textcolor{red}{u}) = \frac{\|f_v(\textcolor{red}{u}, v)\|}{\|f_v(\textcolor{red}{a}, v)\|}$$

# Rotational isometric surfaces



$$\partial_v \|f_{\textcolor{red}{u}}(\textcolor{red}{u}, v)\| = 0$$

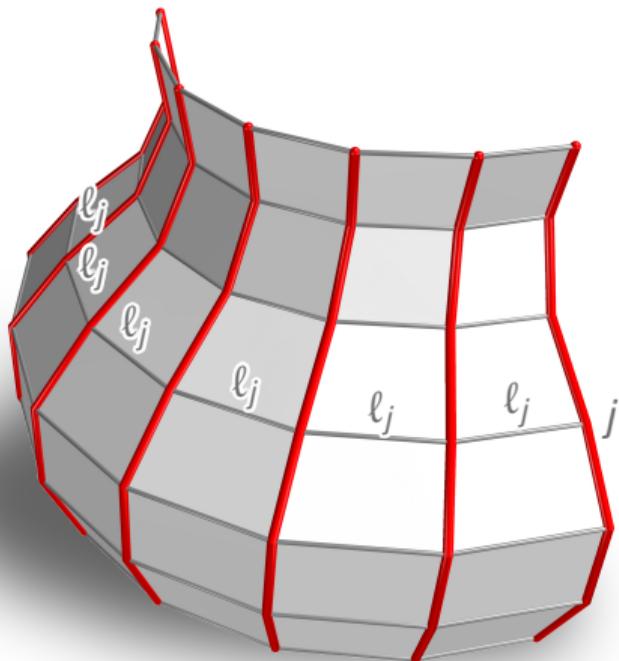
$$r(\textcolor{red}{u}) = \frac{\|f_v(\textcolor{red}{u}, v)\|}{\|f_v(\textcolor{red}{a}, v)\|}$$

↓

$$\hat{f} = \left( r(\textcolor{red}{u}) \cos \phi(v), r(\textcolor{red}{u}) \sin \phi(v), h(\textcolor{red}{u}) \right)$$

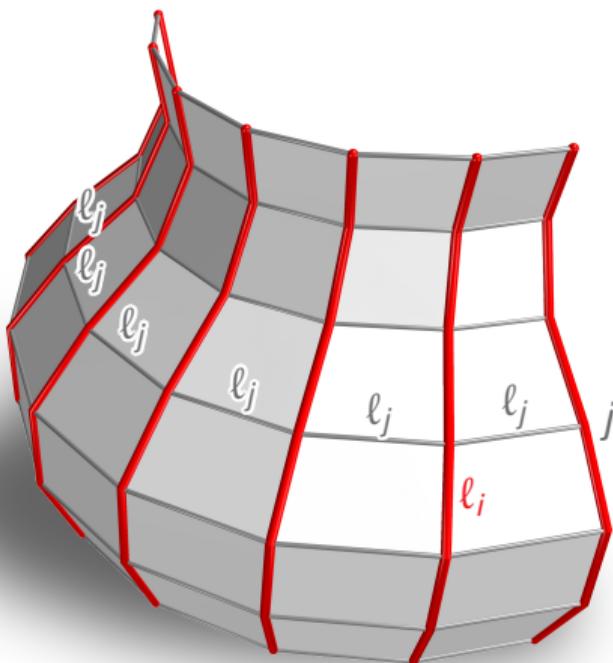
$$h(\textcolor{red}{u}) = \int_{u_0}^{\textcolor{red}{u}} \sqrt{\|f_{\textcolor{red}{u}}(t, v)\| + r'(t)} dt$$

# Discrete rotational isometric surfaces



Equal edge length along parallel  
polylines

# Discrete rotational isometric surfaces



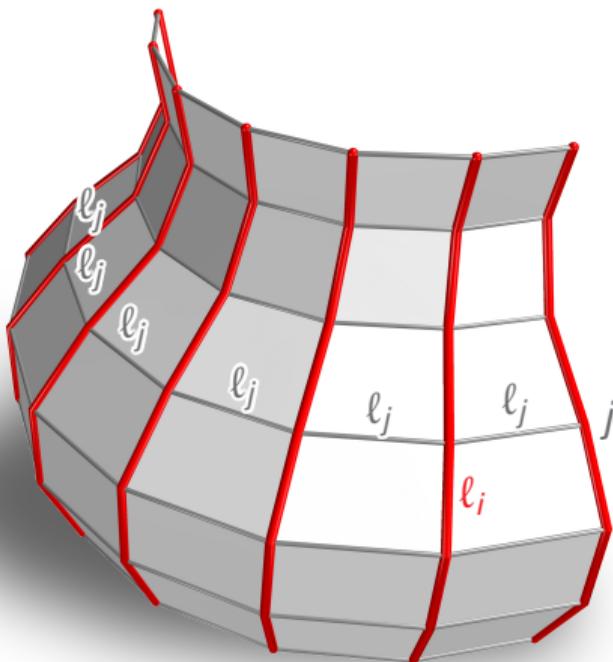
Equal edge length along parallel  
polylines



Isometric rotational surface:

$$r_j = \frac{\ell_j}{2 \sin(\pi/n_j)}$$

# Discrete rotational isometric surfaces



Equal edge length along parallel polylines

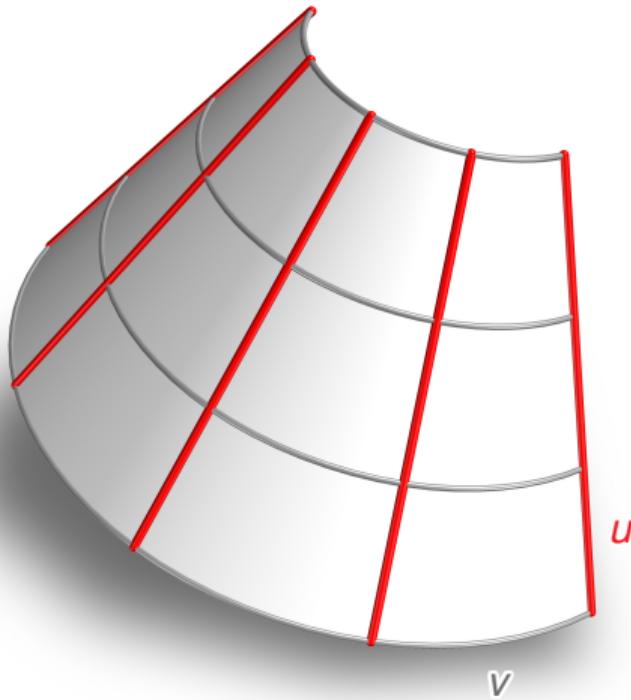


Isometric rotational surface:

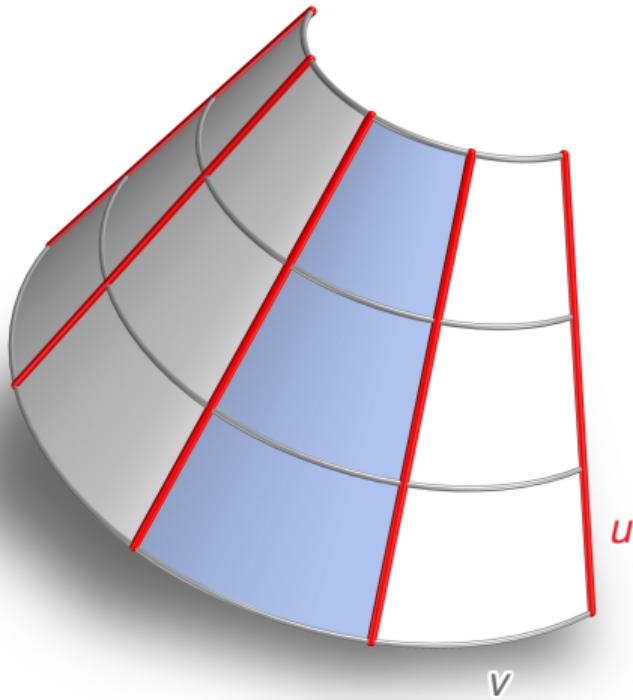
$$r_j = \frac{\ell_j}{2 \sin(\pi/n_j)}$$

$$\Delta h_j = \sqrt{\ell_i^2 - (r_{j-1} - r_j)^2}$$

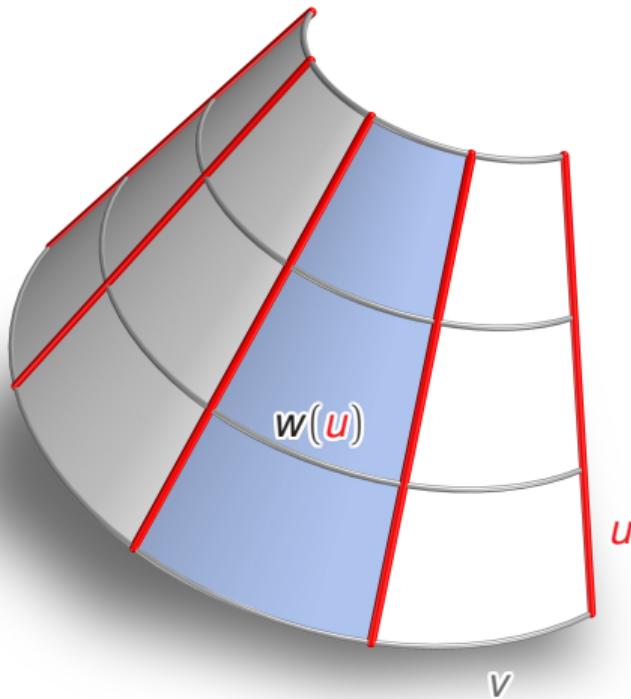
# Developable surfaces



# Developable surfaces



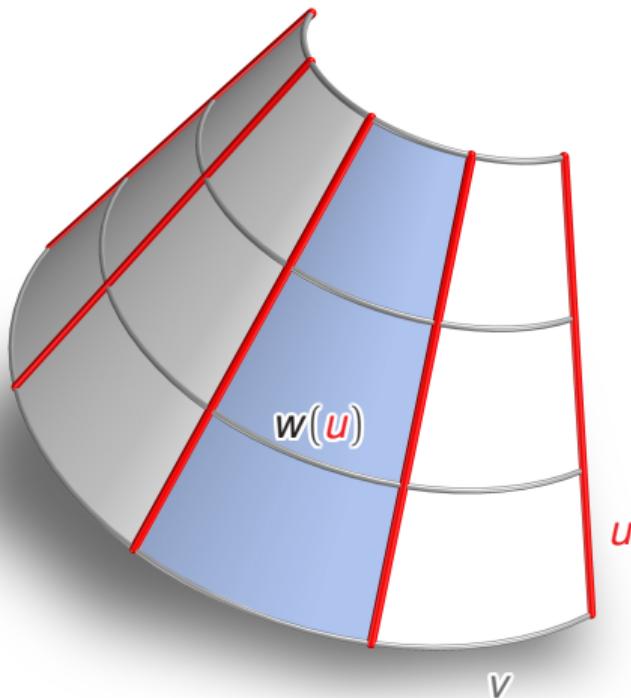
# Developable surfaces



Strip width:

$$w(u) = \int_v^{v+\epsilon} \|f_v(u, t)\| dt$$

# Developable surfaces



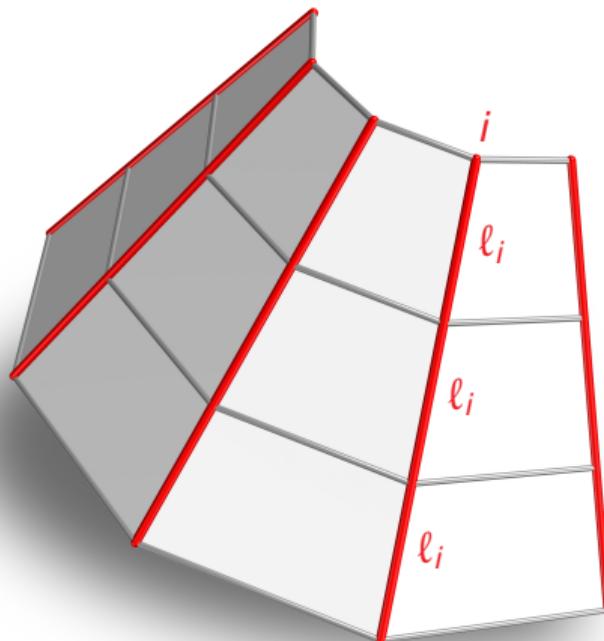
Strip width:

$$w(\textcolor{red}{u}) = \int_v^{v+\epsilon} \|f_v(\textcolor{red}{u}, t)\| dt$$

Jacobi equation:  
(constant speed parametrization along  
geodesics)

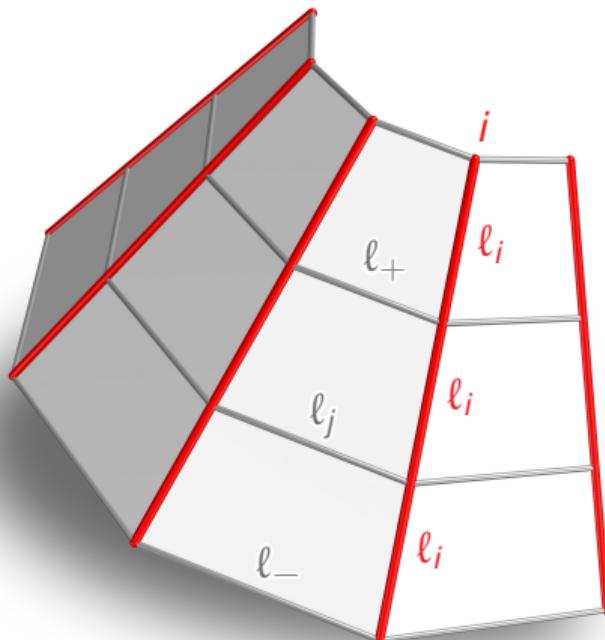
$$K = 0 \Rightarrow \partial_{uu} w(\textcolor{red}{u}) = 0$$

# Discrete developable surfaces



Equal edge length along geodesic  
polylines

# Discrete developable surfaces



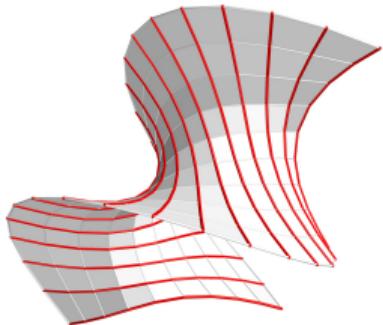
Equal edge length along geodesic  
polylines



Discrete Jacobi equation:

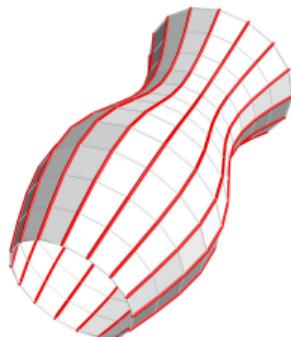
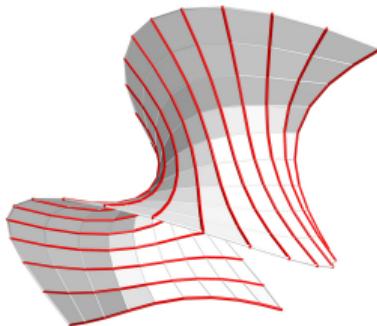
$$\ell_j = \frac{\ell_- + \ell_+}{2}$$

# Modeling with geodesic parallel meshes



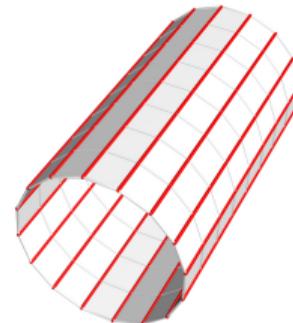
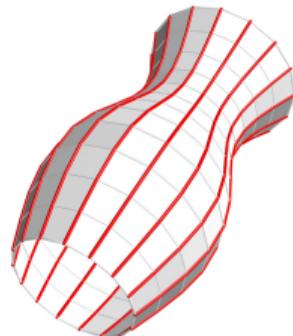
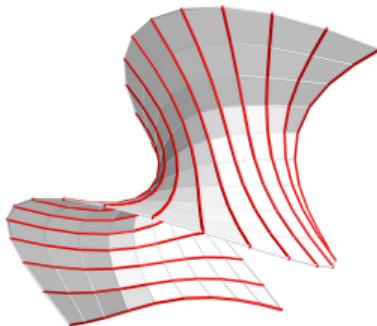
- Geodesic parallel angles

# Modeling with geodesic parallel meshes



- Geodesic parallel angles
- Geodesic parallel angles
- Equal edge length along parallel polylines

# Modeling with geodesic parallel meshes

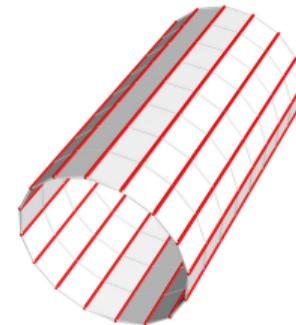
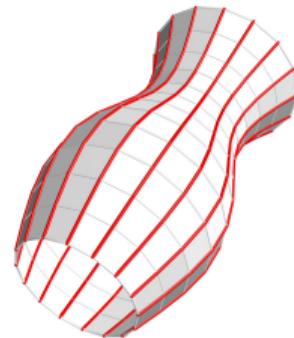
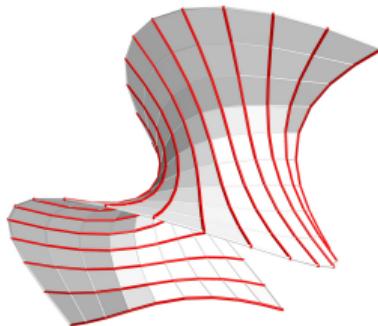


- Geodesic parallel angles

- Geodesic parallel angles
- Equal edge length along parallel polylines

- Geodesic parallel angles
- Equal edge length along geodesic polylines
- Discrete Jacobi equation

# Modeling with geodesic parallel meshes



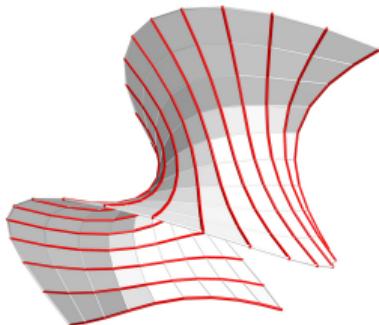
- Geodesic parallel angles

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Guided projection [Tang et al. 2014]

# Geodesic parallel meshes



- Geodesic parallel angles



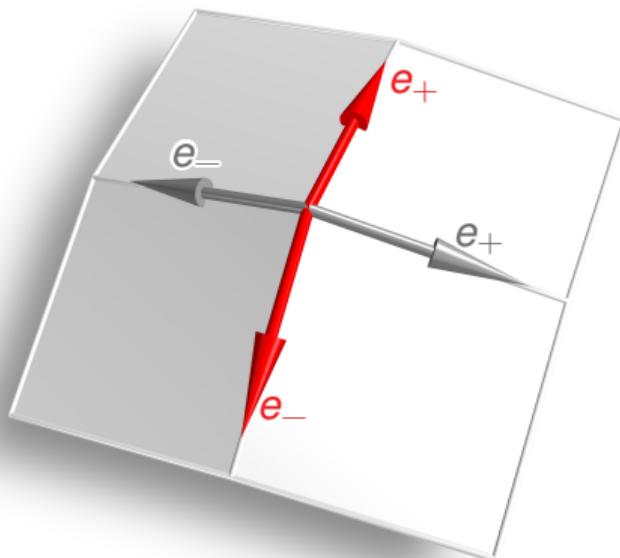
- Geodesic parallel angles
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- Geodesic parallel angles
- Equal edge length along geodesic polylines
- Discrete Jacobi equation

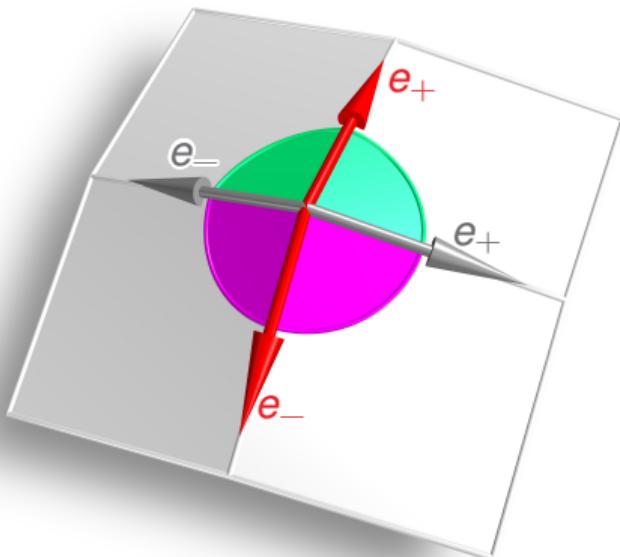
# Geodesic parallel meshes

## Constraints



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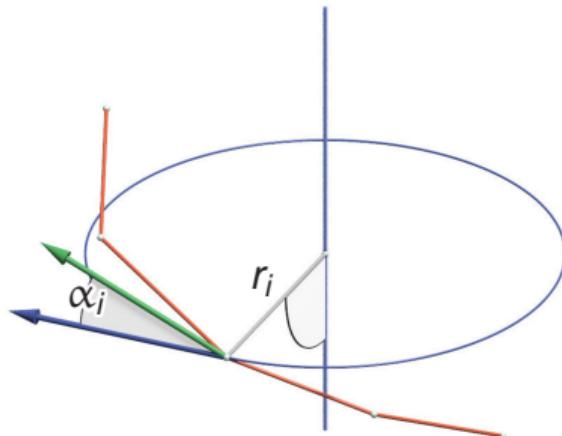
**Geodesic parallel angles:**

$$\langle e_+, e_- \rangle - \langle e_+, e_+ \rangle = 0,$$

$$\langle e_-, e_- \rangle - \langle e_-, e_+ \rangle = 0.$$

# Geodesic parallel meshes

## Verification

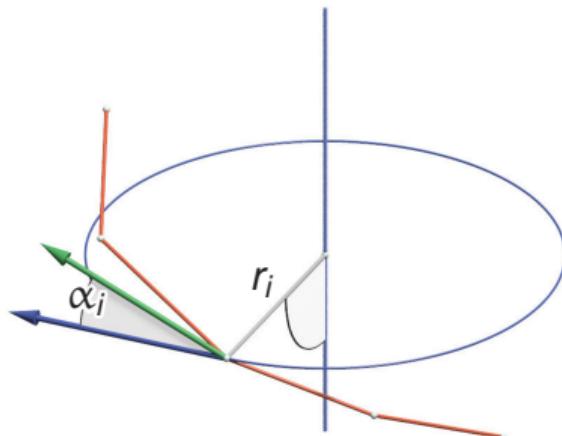


**Clairaut's relation:**

$$r_i \cos \alpha_i = \text{const}$$

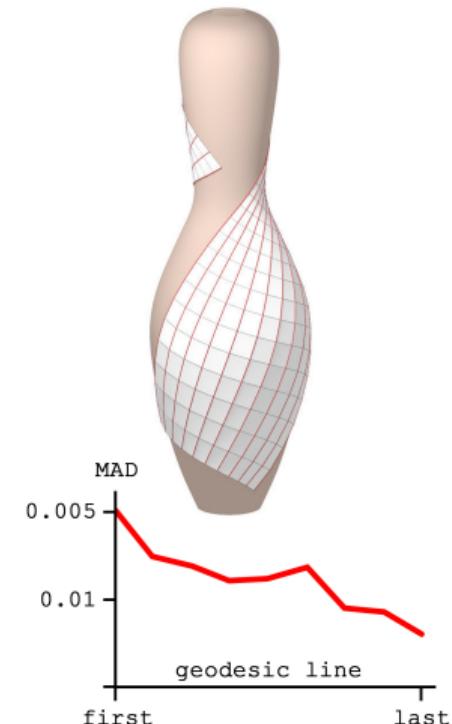
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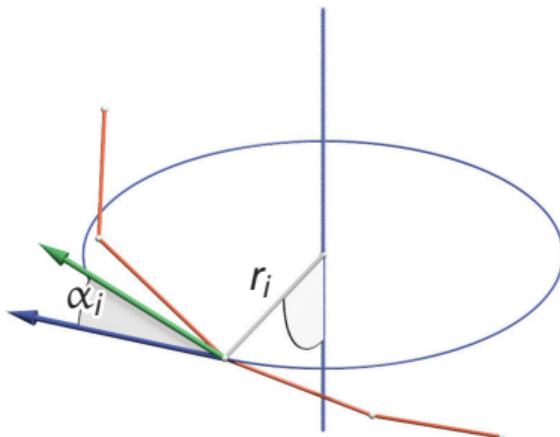
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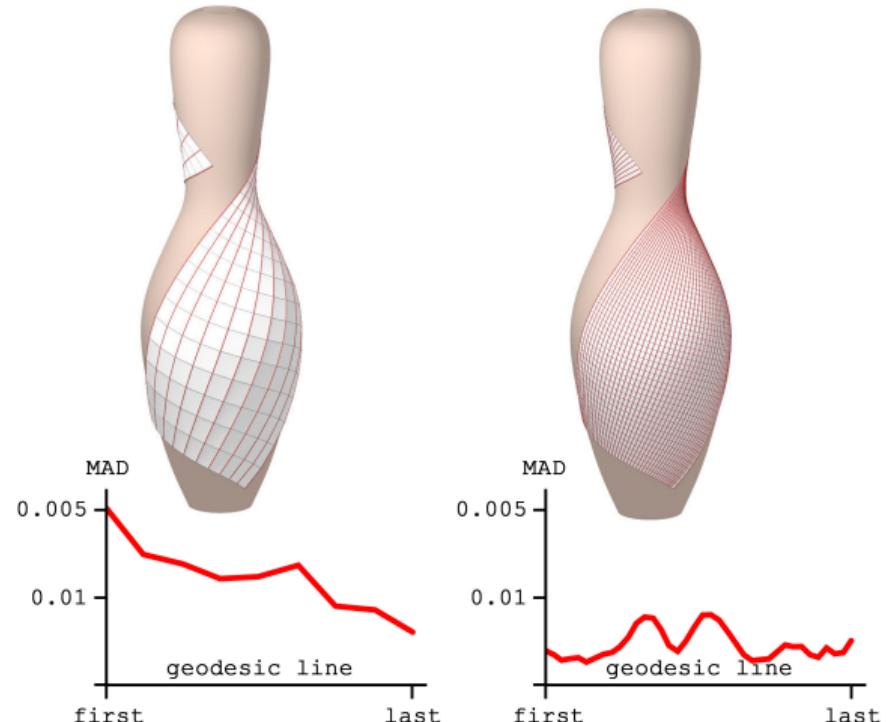
# Geodesic parallel meshes

## Verification



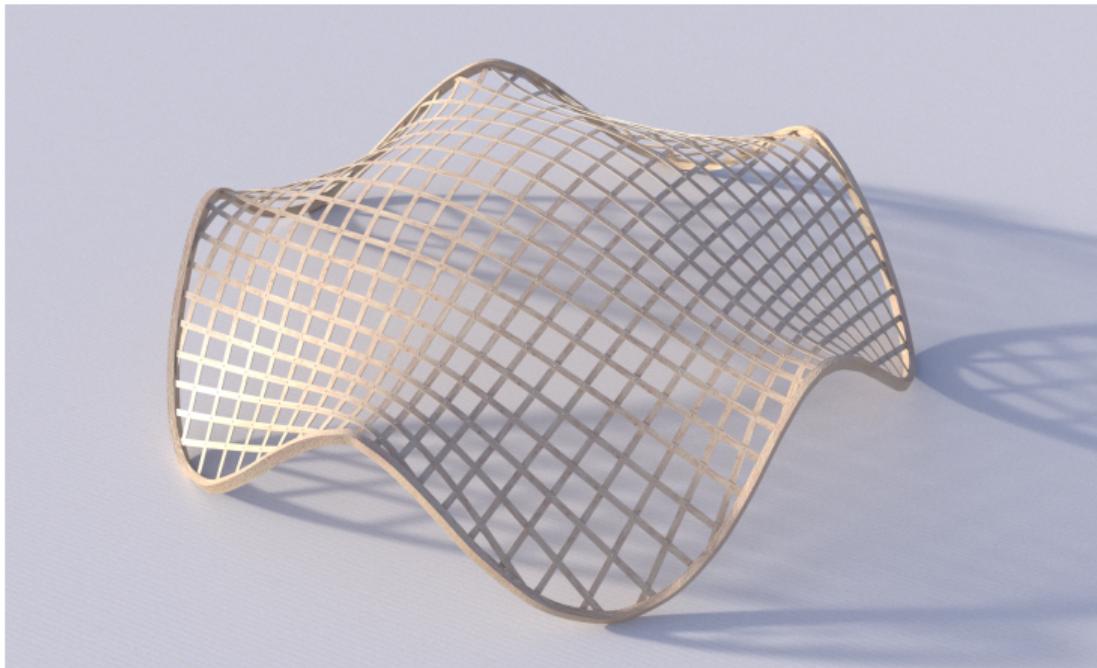
**Clairaut's relation:**

$$r_i \cos \alpha_i = \text{const}$$



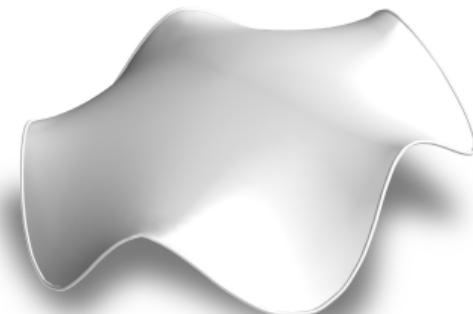
# Geodesic parallel meshes

## Geodesic gridshells



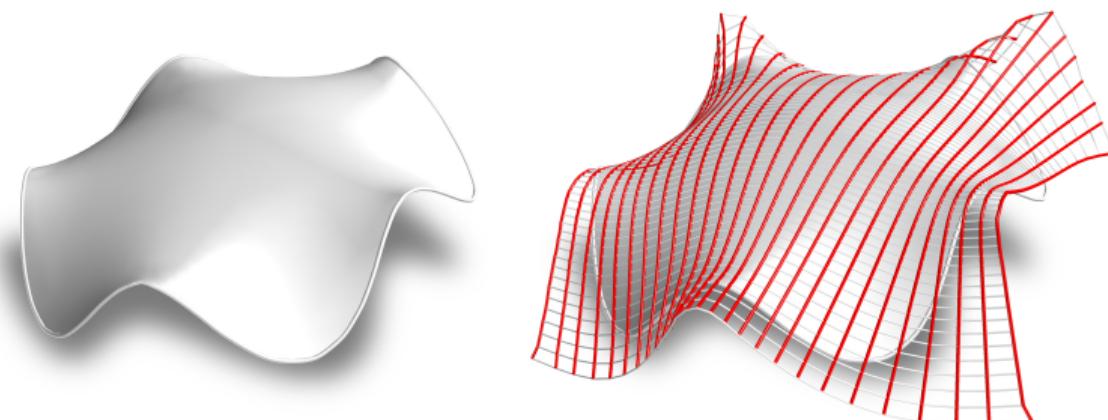
# Geodesic parallel meshes

## Geodesic gridshells



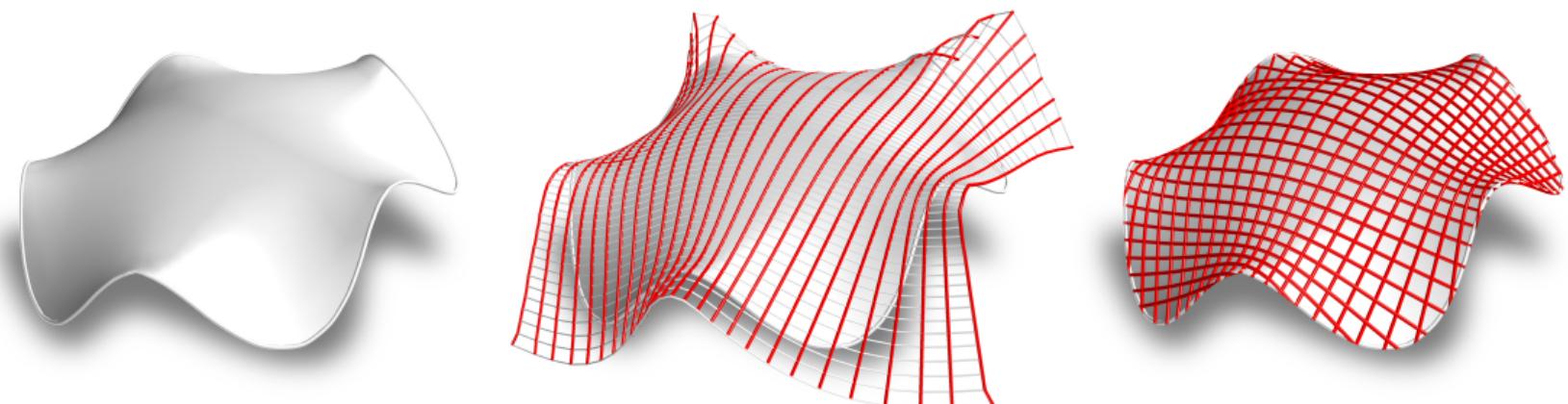
# Geodesic parallel meshes

## Geodesic gridshells



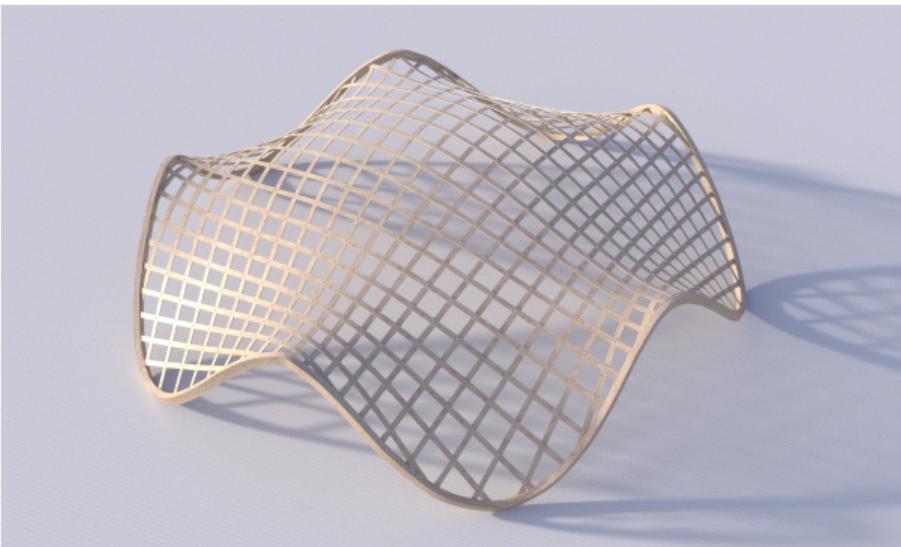
# Geodesic parallel meshes

## Geodesic gridshells

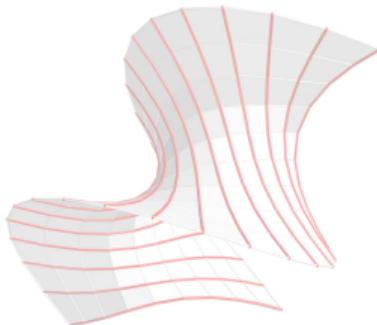


# Geodesic parallel meshes

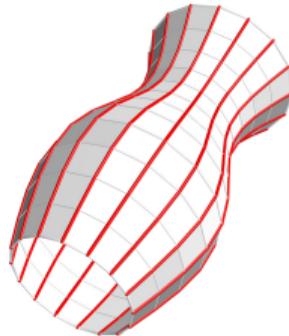
## Geodesic gridshells



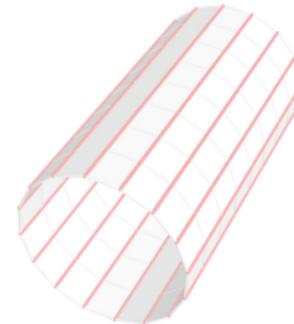
# Rotational isometric meshes



- Geodesic parallel angles



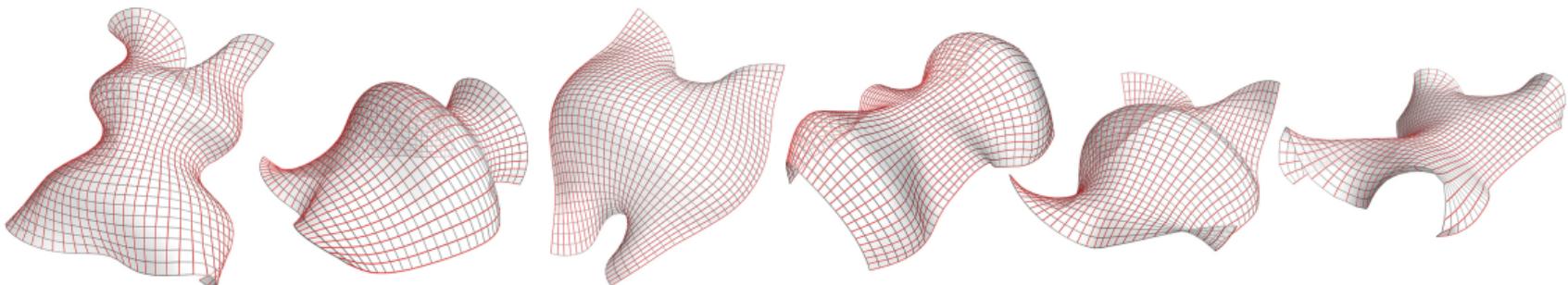
- Geodesic parallel angles
- Equal edge length along parallel polylines



- Geodesic parallel angles
- Equal edge length along geodesic polylines
- Discrete Jacobi equation

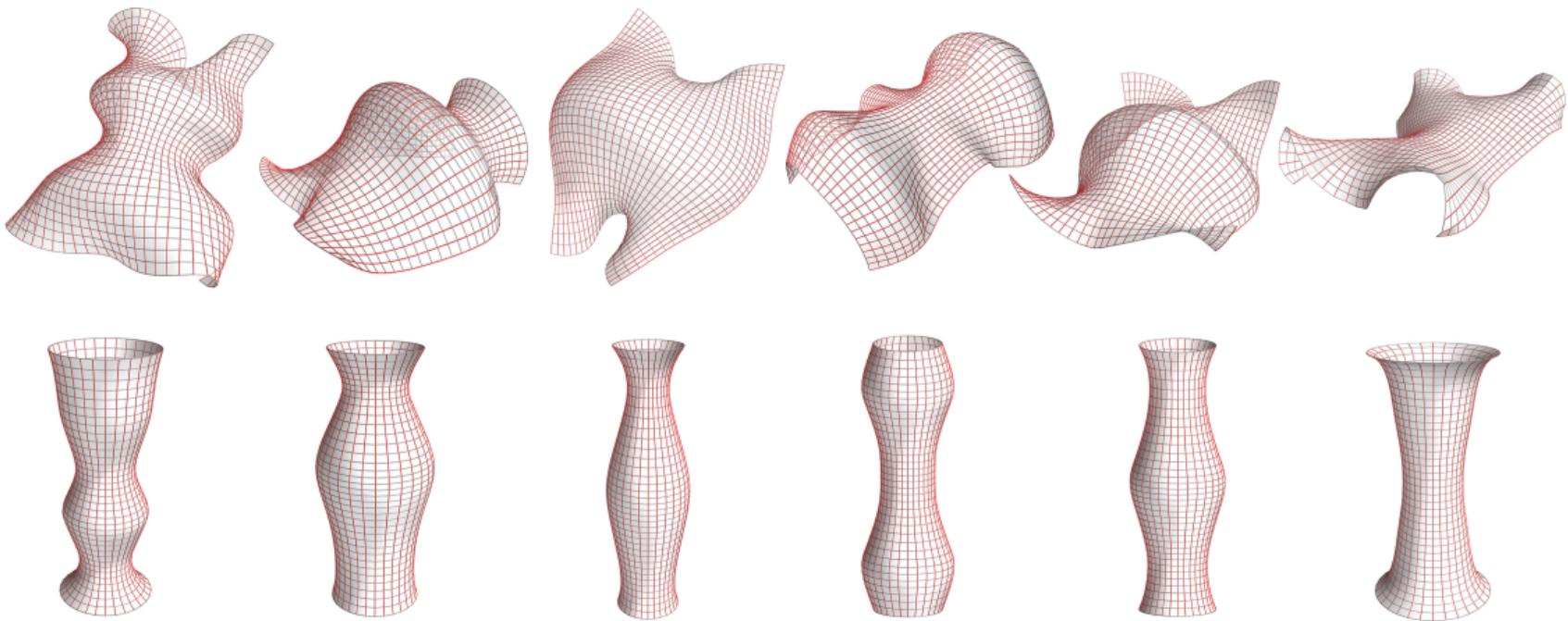
# Rotational isometric meshes

## Modeling



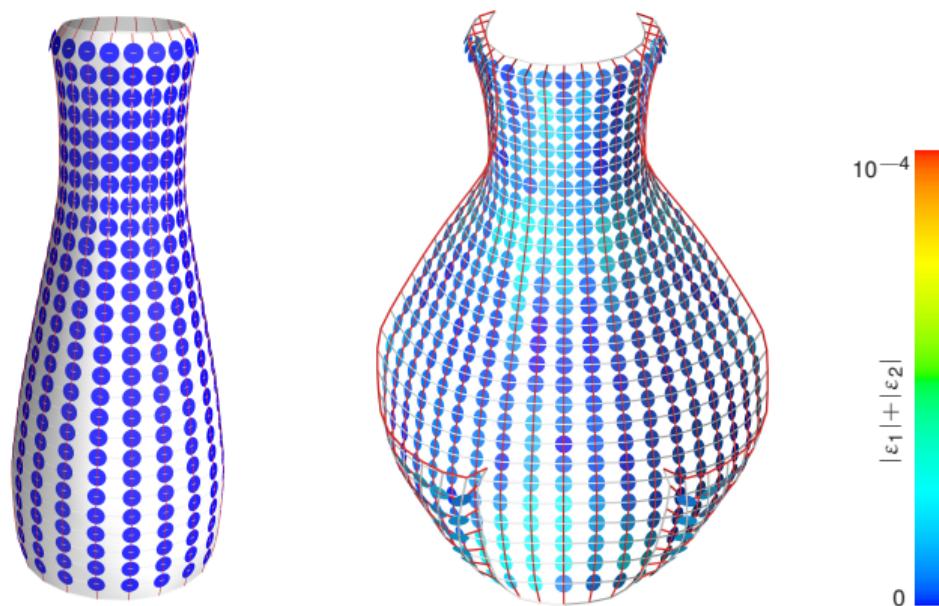
# Rotational isometric meshes

## Modeling



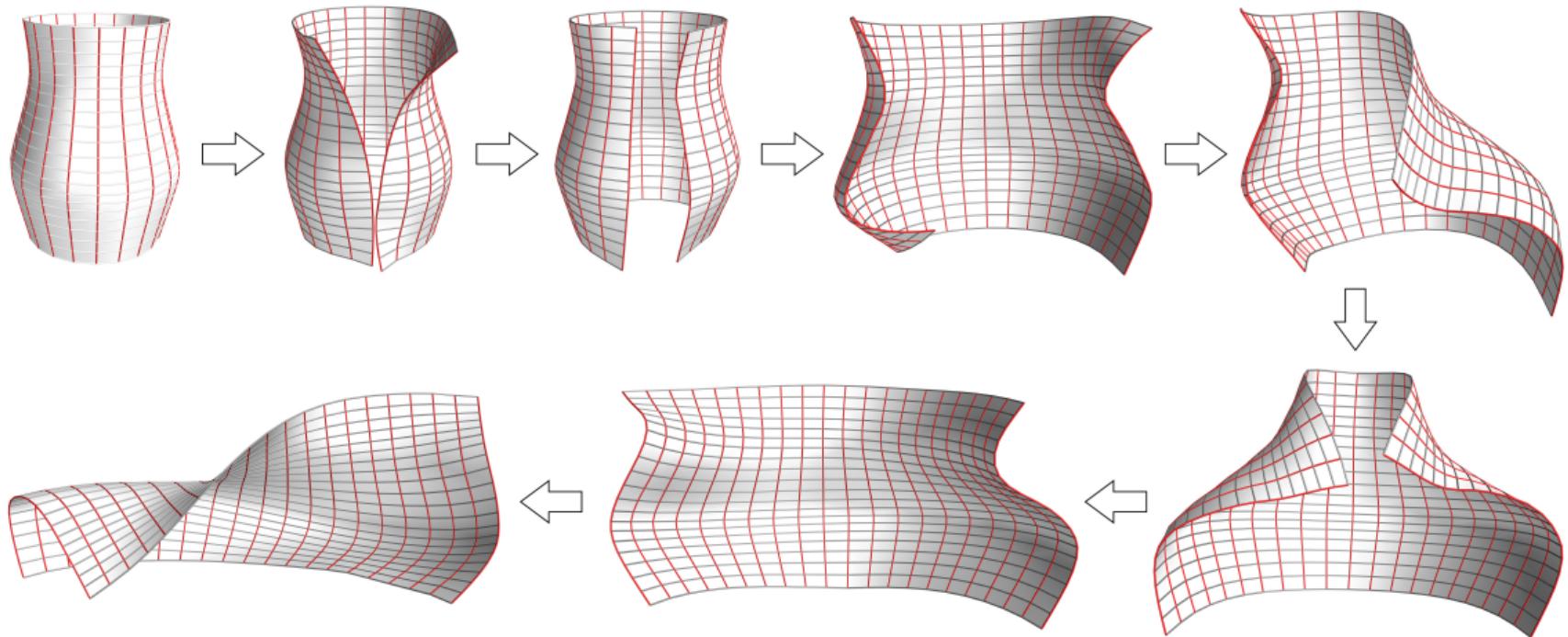
# Rotational isometric meshes

Isometry error estimation



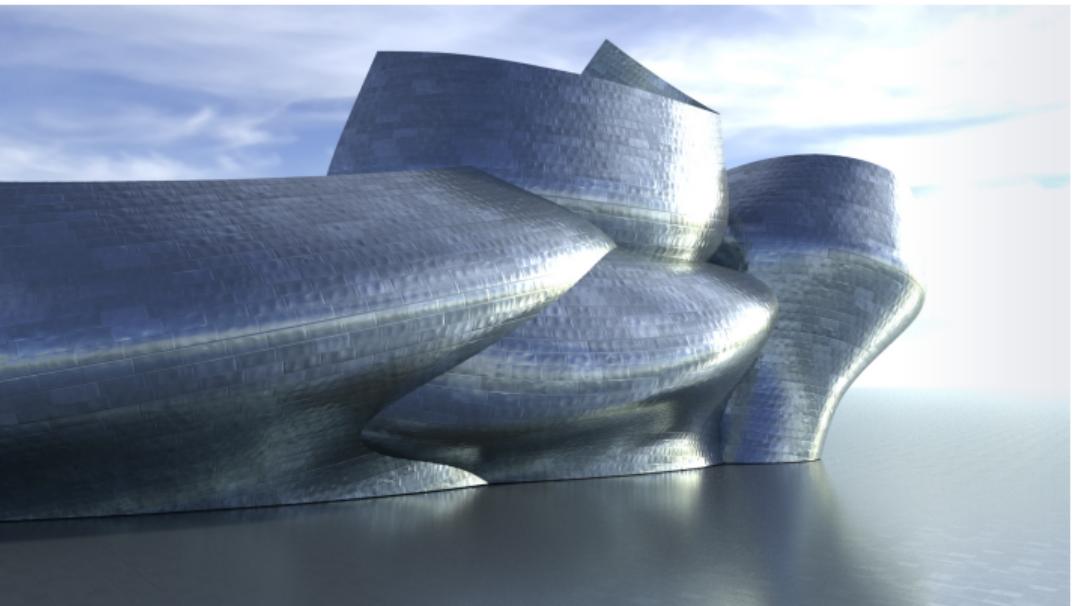
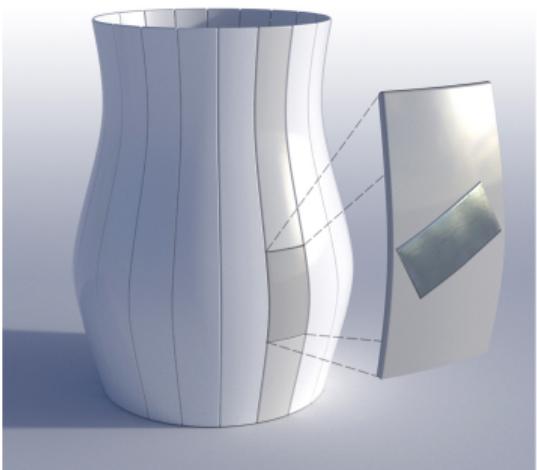
# Rotational isometric meshes

Isometric deformation



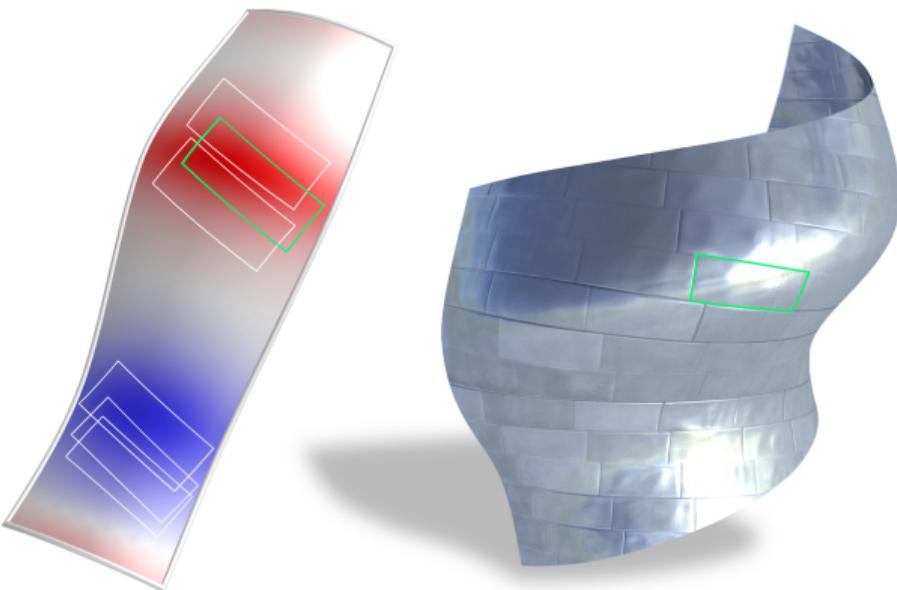
# Rotational isometric meshes

Surfaces of revolution as molds



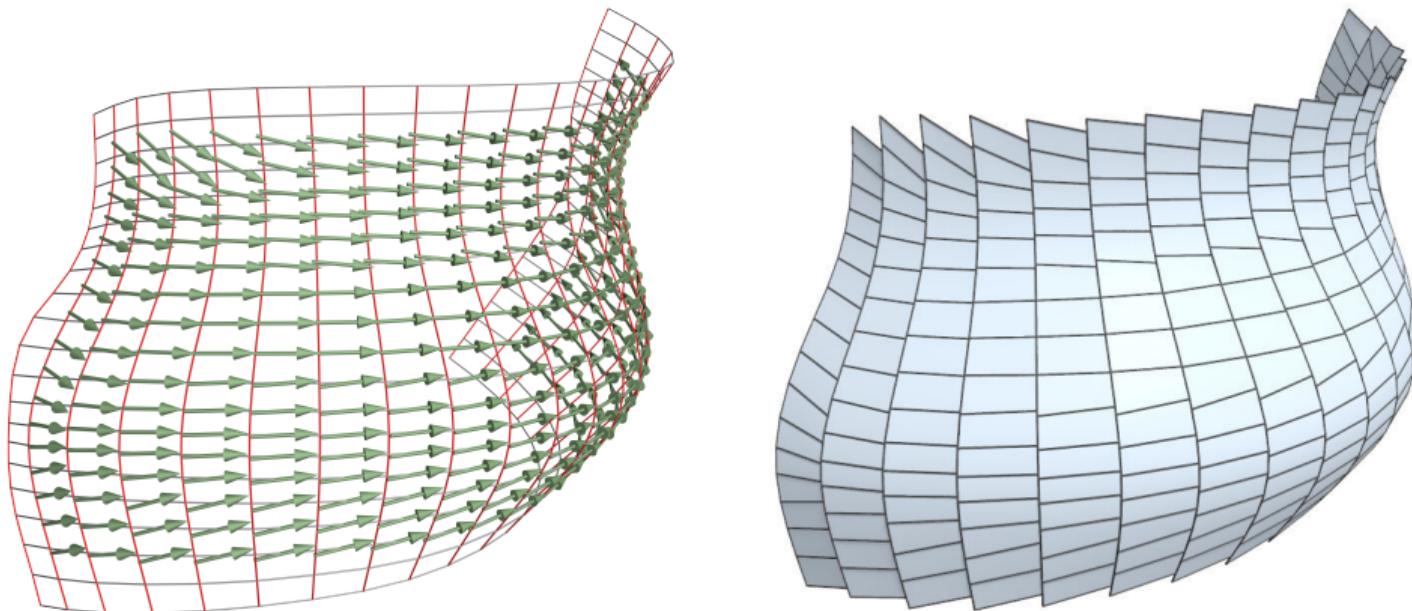
# Rotational isometric meshes

Surfaces of revolution as molds



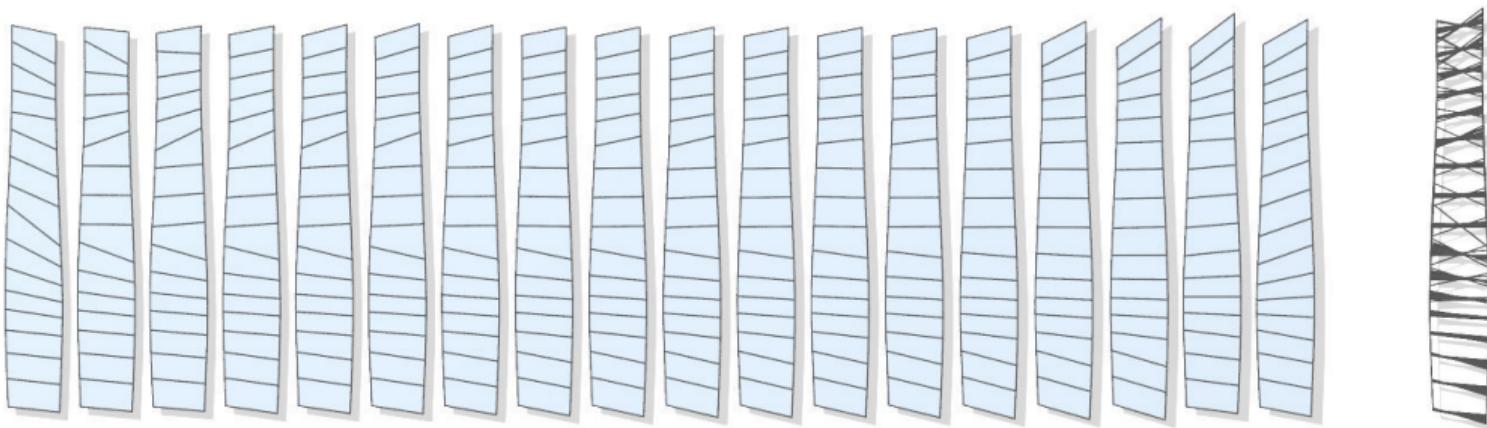
# Rotational isometric meshes

Repetitive strip models



# Rotational isometric meshes

Repetitive strip models

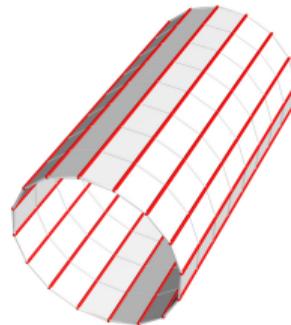
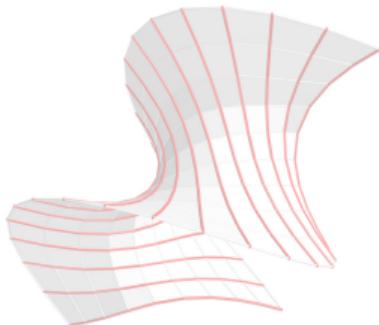


# Rotational isometric meshes

Repetitive strip models



# Nearly developable meshes



- Geodesic parallel angles

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- Equal edge length along parallel polylines

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- Equal edge length along geodesic polylines
- Discrete Jacobi equation

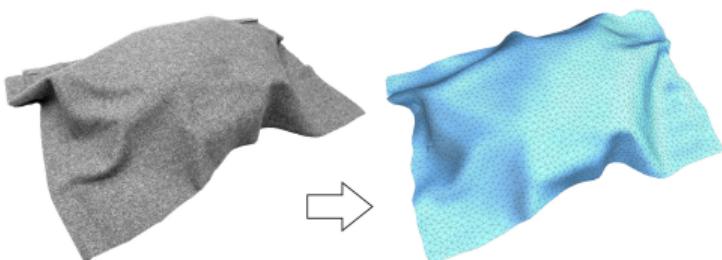
# Nearly developable meshes

## Approximation



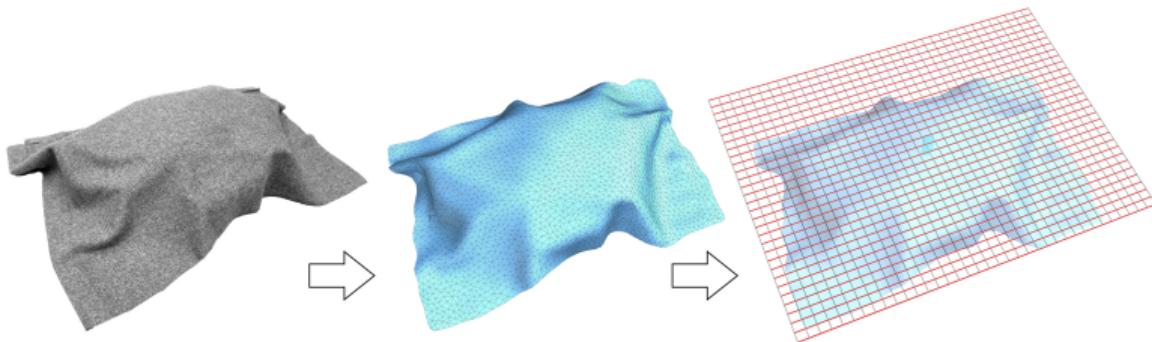
# Nearly developable meshes

## Approximation



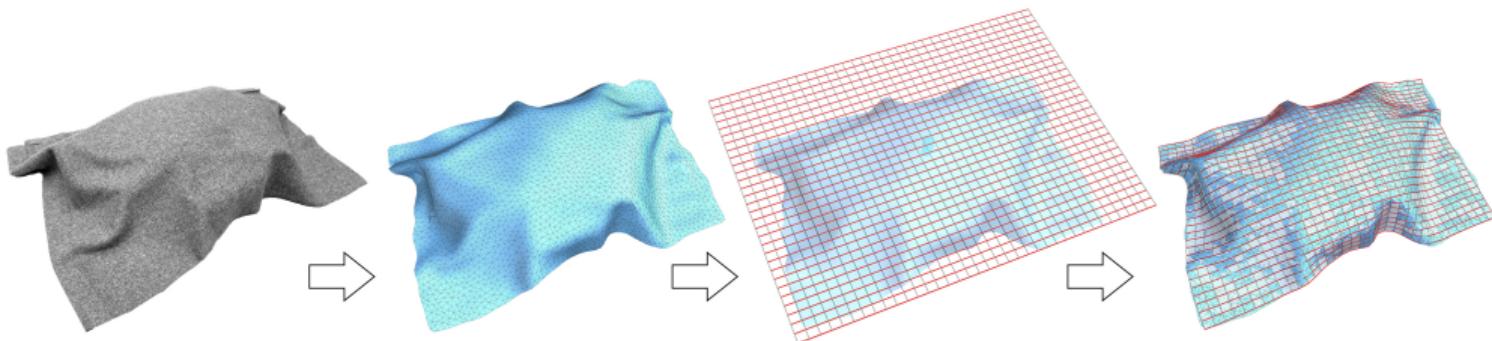
# Nearly developable meshes

Approximation



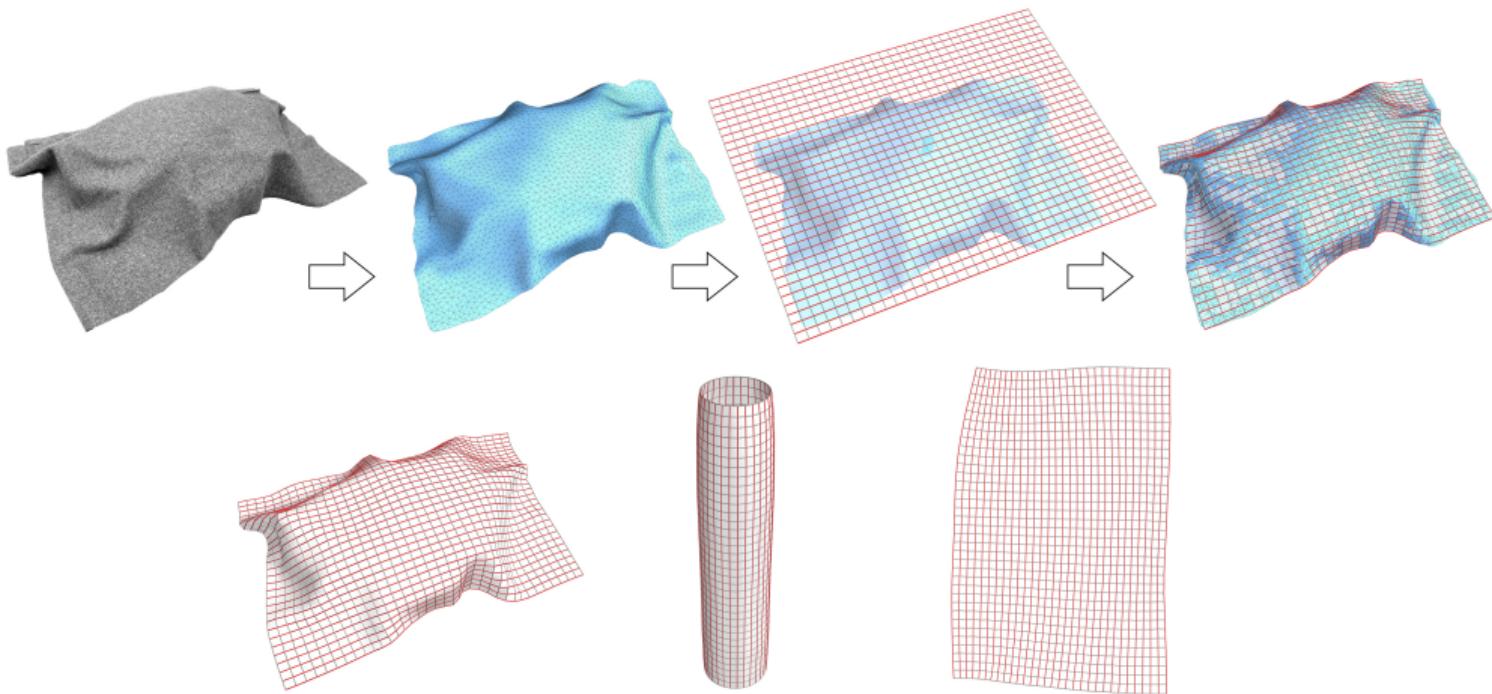
# Nearly developable meshes

## Approximation



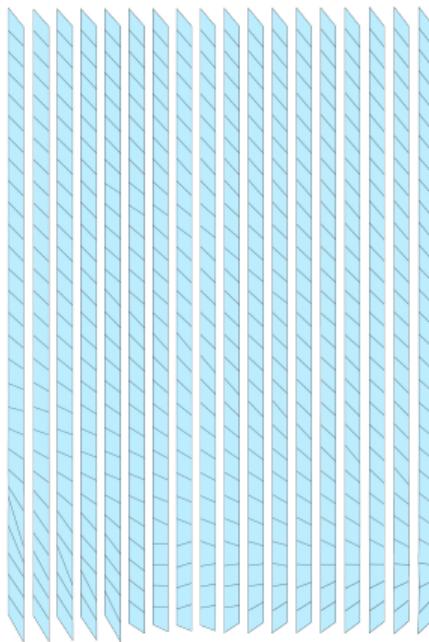
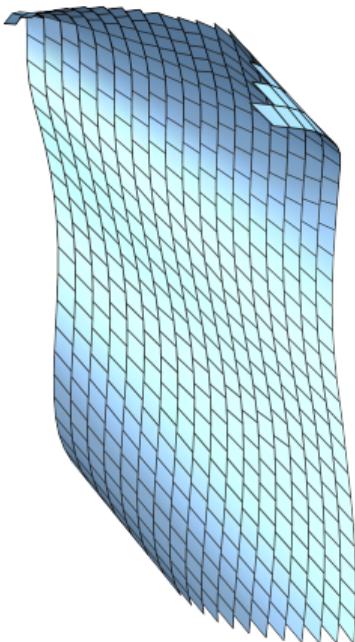
# Nearly developable meshes

## Approximation



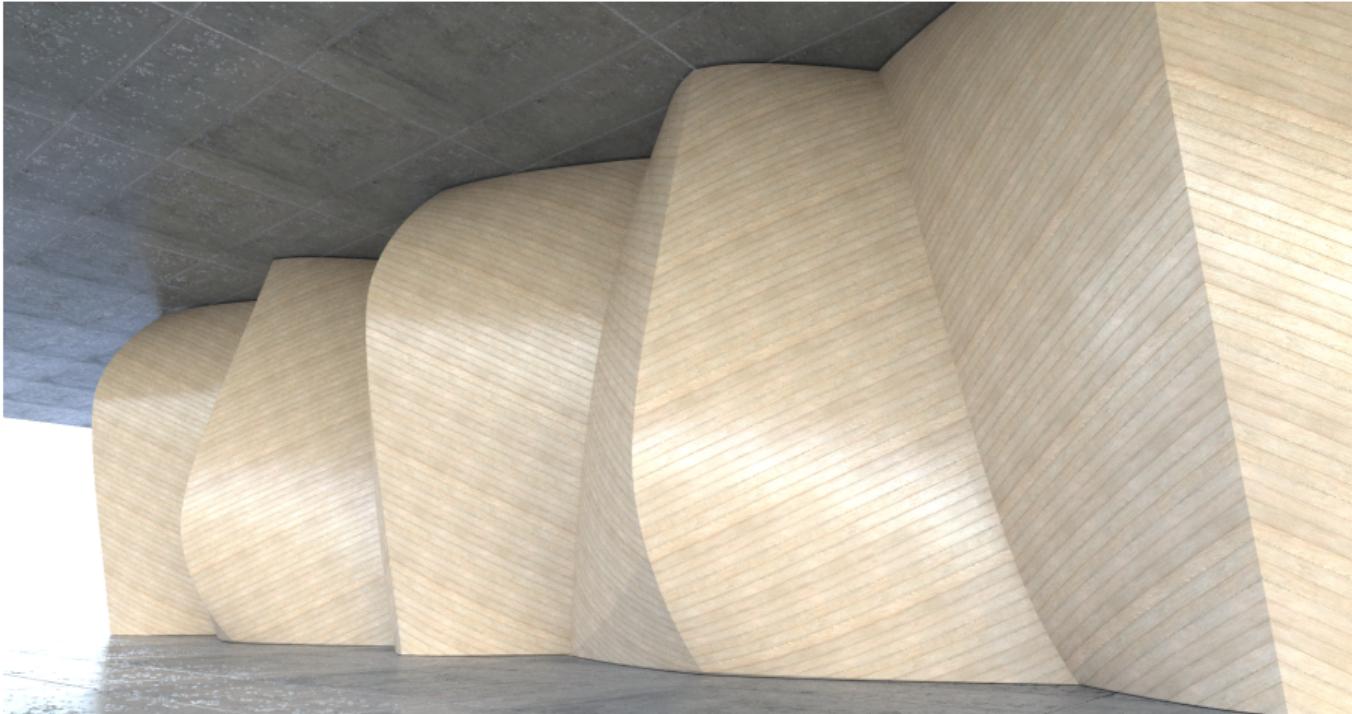
# Nearly developable meshes

## Verification



# Nearly developable meshes

## Cladding





Thank You!



Discretization  
in Geometry  
and Dynamics

