A Closed-Form Location Estimator for Use with Room Environment Microphone Arrays

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***Abstract*— The *linear intersection* (LI) estimator, which is a closed-form method for the localization of source positions given sensor array time-delay estimate information, is presented. The LI estimator is shown to be robust and accurate, to closely model the search-based ML estimator, and to outperform a benchmark algorithm. The computational complexity of the LI estimator is suitable for use in real-time microphone-array applications where search-based location algorithms may be infeasible.**

I. Introduction

MICROPHONE-ARRAY systems can be used to determine the positions of active talkers and can be electronically steered to provide spatially-selective speech acquisition. Since it is steered electronically, a microphone array's directivity pattern can be updated rapidly to follow a moving talker or to switch between several alternating or simultaneous talkers. These features make microphone arrays a desirable alternative to single-microphone systems for hands-free speech acquisition, especially those involving multiple or moving sources. Furthermore, the ability of microphone-array systems to determine talker location makes them attractive for use in multimedia teleconferencing systems, where the location of the talker can be used not only for steering the directivity of the microphone array but also for pointing cameras or determining binaural cues for stereo imaging.

In microphone-array systems, a directly observable signal characteristic is the time difference of arrival (TDOA) of a source signal relative to a pair of microphones. Extensive literature exists on the general topic of intersensor delay estimation and subsequent source localization [1], and there are several works devoted to the specific problem of delay estimation of speech signals [2]–[4]. As presented in Section II, given a set of TDOA estimates, the maximum likelihood (ML) estimate of the source location involves the minimization of an error measure that is a nonlinear function of the potential source location. As a result, the estimator normally requires a numerical search of a potential location space (a subset of $\mathcal{R}^3$). Although the utility of these objective spaces for minimization by efficient search algorithms that rapidly converge to the desired location estimate has been demonstrated [5], there are applications where a full search is not feasible due to limited computational resources. This is particularly true for real-time situations requiring a high update rate and/or many sensors. These circumstances necessitate the development of closed-form location estimators that, although they provide suboptimal localization data, are computationally inexpensive. For those circumstances where the optimal estimate is required, the closed-form solution may be used as an intermediate solution, providing the initial starting point for a less burdensome, partial search.

A closed-form solution to the source localization problem, which is termed the *linear intersection* (LI) method, is detailed in Section III. For the closed-form location estimator presented here and the many others found in the literature [6]–[18], the requirement of a closed-form solution necessitates the development of alternative error criteria. These alternative error criteria take several forms and vary in the degree to which they approximate the ML error criterion and perform in comparison with the search-based estimators. A discussion of several of these algorithms as well as a relative performance evaluation is presented in [7]. Smith and Abel found their proposed estimation procedure—an approach that linearizes the TDOA differences and obtains an estimate through a linear least-squares matrix solution—to exhibit an rms error below that of the estimators presented in [12] and [6]. Their estimation procedure is termed the *spherical interpolation* (SI) method and will be employed in Section IV as a benchmark for evaluating the proposed algorithm.

Existing closed-form solutions to this type of localization problem have been developed with different situations in mind. Radar, sonar, and global positioning are the most common examples. These applications differ from the speech-source localization problem addressed here in several respects. Primarily, the TDOA estimates for these other scenarios are evaluated relative to an absolute time-scale or a single reference sensor. The localization technique detailed in this work requires only that TDOA estimates be found between isolated pairs of sensors. This generalization has been imposed out of necessity. Given a general placement of sensors within an environment and realistic speech sources possessing nonideal radiation patterns, there is no assurance that the received signal coherence across the span of the sensors will be appropriate to allow precise TDOA estimation relative to a single reference sensor. In practice, these conditions

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restrict the intrapair separation distance. The closed-form location estimator presented in the following section is derived specifically from the context of an autonomous sensor-pair geometry and is designed to closely approximate the ML estimator.

II. Source Localization Problem

The localization problem addressed here may be stated as follows. There are N pairs of sensors m_{i1} and m_{i2} for $i \in [1, N]$. The ordered triplet (x, y, z) of spatial coordinates for the sensors will be denoted by $\mathbf{m}_{i1}$ and $\mathbf{m}_{i2}$, respectively. For each sensor pair, a time difference of arrival (TDOA) estimate τ_i for a signal source located at $\mathbf{s}$ is available. The true TDOA associated with a source $\mathbf{s}$ and the ith sensor pair is given by

$$T(\{\mathbf{m}_{i1}, \mathbf{m}_{i2}\}, \mathbf{s}) = \frac{|\mathbf{s} - \mathbf{m}_{i1}| - |\mathbf{s} - \mathbf{m}_{i2}|}{c} \tag{1}$$

where c is the speed of propagation in the medium. In practice, τ_i is corrupted by noise, and in general, $\tau_i \neq T(\{\mathbf{m}_{i1}, \mathbf{m}_{i2}\}, \mathbf{s})$. In addition to the τ_i, a variance estimate σ_i^2 associated with each TDOA is also assumed to be available as a byproduct of the time-delay estimation procedure. This variance estimate is generally a function of the signal content and sensor SNR conditions as well as the source radiation pattern and the physical conditions of the room. Throughout this discussion, τ_i is assumed to be an unbiased estimate of the true TDOA and possesses a unimodal probability distribution. These assumptions are reasonable under a broad range of light to moderate reverberation conditions [19].

Given these N sensor pairs and TDOA estimate combinations

$$\{\mathbf{m}_{i1}, \mathbf{m}_{i2}\}, \tau_i, \sigma_i^2 \qquad \text{for } i = 1, \cdots, N$$

it is desired to obtain an estimate $\hat{\mathbf{s}}$ of the source location. If the TDOA estimates are assumed to be independently corrupted by additive zero-mean, uncorrelated Gaussian noise, the ML estimate $\hat{\mathbf{s}}_{\mathrm{ML}}$ is found through minimization of a least-squares error criterion [20] denoted here by $J_{\mathrm{ML}}(\mathbf{s})$

$$\hat{\mathbf{s}}_{\mathrm{ML}} = \arg\min_{\mathbf{s}}(J_{\mathrm{ML}}(\mathbf{s})) \tag{2}$$

where

$$J_{\mathrm{ML}}(\mathbf{s}) = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \cdot [\tau_i - T(\{\mathbf{m}_{i1}, \mathbf{m}_{i2}\}, \mathbf{s})]^2. \tag{3}$$

Since $T(\{\mathbf{m}_{i1}, \mathbf{m}_{i2}\}, \mathbf{s})$ is a nonlinear function of $\mathbf{s}$, (2) does not possess a closed-form solution.

III. The Linear Intersection Algorithm

Given a specific sensor pair $\{\mathbf{m}_{i1}, \mathbf{m}_{i2}\}$ and their associated TDOA estimate τ_i, the locus of potential source locations in three-space forms half of a hyperboloid of two sheets. This hyperboloid is centered about the midpoint of $\mathbf{m}_{i1}$ and $\mathbf{m}_{i2}$ and has the directed line segment $\overline{\mathbf{m}_{i1}\mathbf{m}_{i2}}$ as its axis of symmetry. For sources with a large source-range to sensor-separation ratio, the hyperboloid may be well approximated by a cone its with vertex at the sensors' midpoint, with $\overline{\mathbf{m}_{i1}\mathbf{m}_{i2}}$ as the axis of symmetry, and a constant direction angle relative to this axis. According to this approximation, the direction angle θ_i for a sensor-pair TDOA-estimate combination is given by

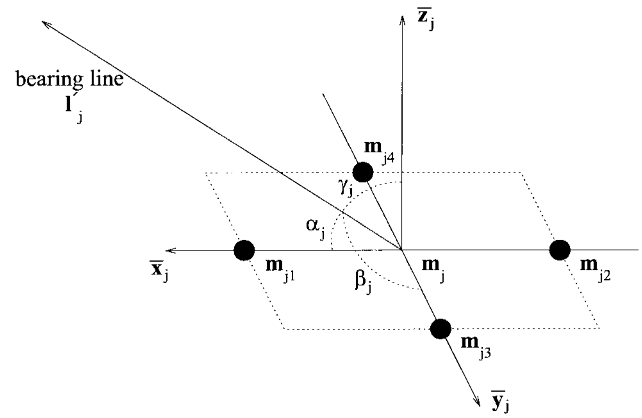

Fig. 1. Quadruple sensor arrangement and local Cartesian coordinate system.

$$\theta_i = \cos^{-1}\left(\frac{c \cdot \tau_i}{|\mathbf{m}_{i1} - \mathbf{m}_{i2}|}\right). \tag{4}$$

Now, consider two pairs of sensors $\{\mathbf{m}_{j1}, \mathbf{m}_{j2}\}$ and $\{\mathbf{m}_{j3}, \mathbf{m}_{j4}\}$, where j is used to index the sets of sensor quadruples, along with their associated TDOA estimates τ_{j12} and τ_{j34}, respectively. The sensors are constrained to lie on the midpoints of a rectangle, and as a result $\overline{\mathbf{m}_{j1}\mathbf{m}_{j2}}$ and $\overline{\mathbf{m}_{j3}\mathbf{m}_{j4}}$ are orthogonal and mutually bisecting. A local Cartesian coordinate system is established with unit vectors defined as $\overline{\mathbf{x}_j} = \frac{\mathbf{m}_{j1} - \mathbf{m}_{j2}}{|\mathbf{m}_{j1} - \mathbf{m}_{j2}|}$, $\overline{\mathbf{y}_j} = \frac{\mathbf{m}_{j3} - \mathbf{m}_{j4}}{|\mathbf{m}_{j3} - \mathbf{m}_{j4}|}$, and $\overline{\mathbf{z}_j} = \overline{\mathbf{x}_j} \times \overline{\mathbf{y}_j}$ with the origin at the common midpoint of the two sensor pairs denoted by $\mathbf{m}_j$. This geometry is depicted in Fig. 1. The first sensor pair TDOA estimate approximately determines a cone with constant direction angle α_j relative to the $\overline{\mathbf{x}_j}$ axis, as given by (4). The second specifies a cone with constant direction angle β_j relative to the $\overline{\mathbf{y}_j}$ axis. Each cone has a vertex at the local origin. If the potential source location is restricted to the positive-z halfspace, the locus of potential source points common to these two cones is the bearing line $\mathbf{l}'_j$ in three-space. The remaining direction angle γ_j may be calculated from the identity

$$\cos^2\alpha_j + \cos^2\beta_j + \cos^2\gamma_j = 1$$

with $0 \le \gamma_j \le \frac{\pi}{2}$, and the line may be expressed in terms of the local coordinate system by the parametric equation

$$\mathbf{l}'_j = [x_j \quad y_j \quad z_j]^{\mathrm{T}} = r_j \mathbf{a}'_j$$

where r_j is the range of a point on the line from the local origin at $\mathbf{m}_j$ and $\mathbf{a}'_j$ is the vector of direction cosines

$$\mathbf{a}'_j \equiv [\cos\alpha_j \quad \cos\beta_j \quad \cos\gamma_j]^{\mathrm{T}}.$$

The line $\mathbf{l}'_j$ may then be expressed in terms of the global Cartesian coordinate system via the appropriate translation and rotation, namely,

$$\mathbf{l}_j = r_j \mathbf{R}_j \mathbf{a}'_j + \mathbf{m}_j$$

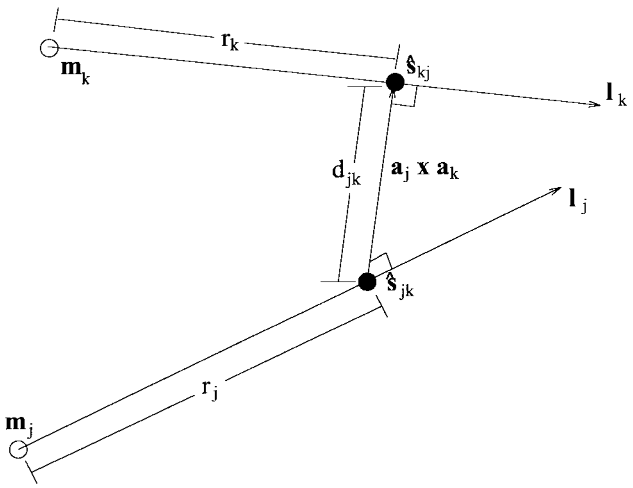

Fig. 2. Illustration of the process used to calculate the points of closest intersection $\mathbf{t}_{jk}$ and $\mathbf{t}_{kj}$ for a pair of bearing lines $\mathbf{l}_j$ and $\mathbf{l}_k$ in three-space.

where $\mathbf{R}_j$ is the 3×3 rotation matrix from the jth local coordinate system to the global coordinate system. Alternatively, if $\mathbf{a}_j$ represents the rotated direction cosine vector, then

$$\mathbf{l}_j = r_j \mathbf{a}_j + \mathbf{m}_j.$$

Given that the N pairs of sensors may be arranged into M groupings of mutually orthogonal and mutually bisecting sensor quadruples with bearing lines

$$\mathbf{l}_j = r_j \mathbf{a}_j + \mathbf{m}_j \qquad \text{for } j = 1, \cdots, M$$

the problem of estimating a specific source location remains. The approach taken here will be to calculate a number of potential source locations from the points of closest intersection for all pairs of bearing lines and use a weighted average of these locations to generate a final source-location estimate. Fig. 2 illustrates the process used to determine the points of closest intersection. Specifically, given two bearing lines

$$\mathbf{l}_j = r_j \mathbf{a}_j + \mathbf{m}_j \tag{5}$$
$$\mathbf{l}_k = r_k \mathbf{a}_k + \mathbf{m}_k \tag{6}$$

the shortest distance between the lines is measured along a line parallel to their common normal and is given by [21]

$$d_{jk} = \frac{|(\mathbf{a}_j \times \mathbf{a}_k) \cdot (\mathbf{m}_j - \mathbf{m}_k)|}{|\mathbf{a}_j \times \mathbf{a}_k|}.$$

Accordingly, the point on $\mathbf{l}_j$ with the closest intersection to $\mathbf{l}_k$ (which is denoted by $\hat{\mathbf{s}}_{jk}$) and the point on $\mathbf{l}_k$ with closest intersection to $\mathbf{l}_j$ (which is denoted by $\hat{\mathbf{s}}_{kj}$) may be found by first solving for the local ranges r_j and r_k and substituting these values into (5) and (6). The local ranges are found by subtracting (5) from (6) at the closest intersections ($\mathbf{l}_j = \hat{\mathbf{s}}_{jk}$ and $\mathbf{l}_k = \hat{\mathbf{s}}_{kj} = \hat{\mathbf{s}}_{jk} + d_{jk}(\mathbf{a}_j \times \mathbf{a}_k)$), giving the overconstrained matrix equation

$$r_j \mathbf{a}_j - r_k \mathbf{a}_k = \mathbf{m}_k - \mathbf{m}_j - d_{jk}(\mathbf{a}_j \times \mathbf{a}_k).$$

Each of the potential source locations is weighted based on its probability conditioned on the observed set of N sensor pair, TDOA-estimate combinations. The TDOA estimates are assumed to be independent, normal distributions with mean given by the estimate itself. The weight W_{jk} associated with the potential source location $\hat{\mathbf{s}}_{jk}$ is calculated from

$$W_{jk} = \prod_{i=1}^{N} P\big[T(\{\mathbf{m}_{i1}, \mathbf{m}_{i2}\}, \hat{\mathbf{s}}_{jk}), \tau_i, \sigma_i^2\big] \tag{7}$$

where $P(x, m, \sigma^2)$ is the value of a Gaussian probability distribution function with mean m and variance σ^2 evaluated at x, i.e.,

$$P(x, m, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-m)^2}{2\sigma^2}\right].$$

In situations where the TDOA estimates possess a known, non-Gaussian distribution, $P(x, m, \sigma^2)$ may be altered accordingly to reflect this knowledge.

The final location estimate, which will be referred as the *linear intersection estimate* ($\hat{\mathbf{s}}_{\mathrm{LI}}$), is then calculated as the weighted average of the potential source locations

$$\hat{\mathbf{s}}_{\mathrm{LI}} = \frac{\displaystyle\sum_{j=1}^{M} \sum_{\substack{k=1,\\ k\neq j}}^{M} W_{jk} \hat{\mathbf{s}}_{jk}}{\displaystyle\sum_{j=1}^{M} \sum_{\substack{k=1,\\ k\neq j}}^{M} W_{jk}}. \tag{8}$$

Evaluated in this manner, $\hat{\mathbf{s}}_{\mathrm{LI}}$ represents the expected value of a partially known random variable. The points of closest intersection $\hat{\mathbf{s}}_{jk}$ are assumed to be points of high probability clustered about the peak of a symmetrical probability distribution. In this sense, the LI algorithm attempts to model the ML estimate that searches for the maximum in the joint probability distribution of the TDOA estimate set. By associating each potential source location with a probabilistic value, the weighting terms W_{jk} serve as a means for excluding outlier locations related to radically errant TDOA estimates and their subsequently incorrect bearing lines. Essentially, the weighting term, as calculated from (7) for an aberrant $\hat{\mathbf{s}}_{jk}$, is sufficiently small so that the potential location plays little or no role in the final location estimation.

Fig. 3 depicts the LI localization method for a simulated 6 m $\times$ 6 m $\times$ 4 m rectangular room with four sets of 0.5 m $\times$ 0.5 m square arrays, where one is centered on each of the walls. Note that each of the four-element arrays satisfies the LI sensor positional constraint when sensor-pair locations are selected as the diagonal elements at the vertices of each square. The top graph in the figure displays the bearing lines projecting from the quadruple units for a simulated source at location (2 m, 4 m, 3 m). To generate this situation, the true TDOA values have been corrupted by additive noise with a standard deviation of 0.01 m when scaled by the propagation speed of sound in air. This is intended to represent a moderate noise condition. The points of closest intersection $\hat{\mathbf{s}}_{jk}$ and the final LI estimate are shown along with their projections onto the xy, xz, and yz planes. The bottom graph in Fig. 3 presents an enlarged overhead view of the intersection region alone. The individual $\hat{\mathbf{s}}_{jk}$ locations are now visible and denoted by 'x,' and the corresponding normal vectors are plotted as

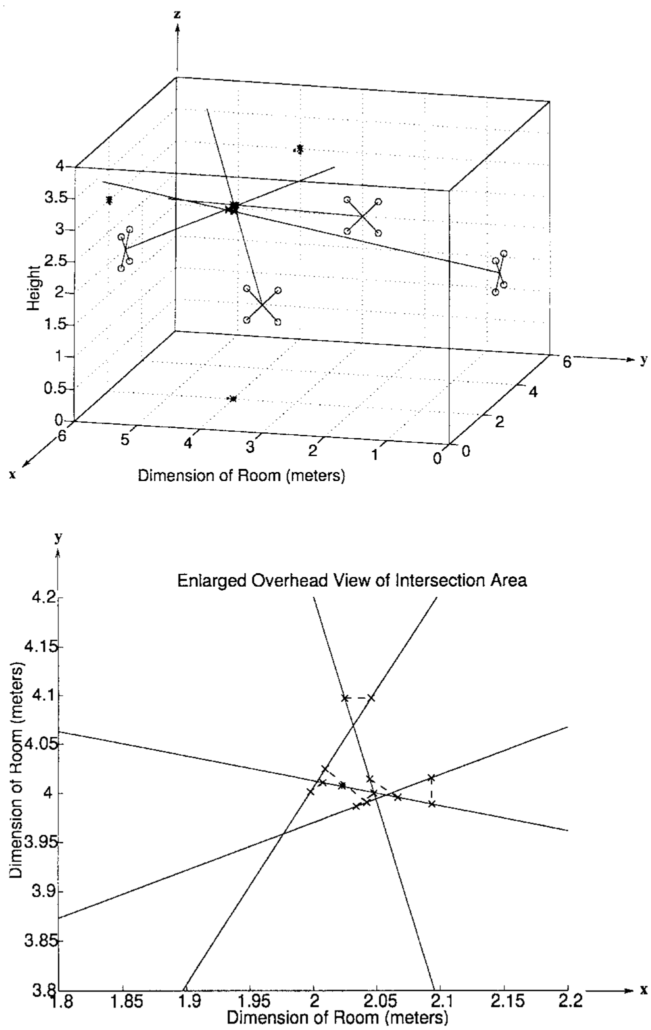

Fig. 3. Illustration of linear intersection algorithm. The top graph shows the bearing lines projecting from the quadruple units for a simulated source at location $(2, 4, 3)$ along with the $\hat{\mathbf{s}}_{\rm LI}$ and $\hat{\mathbf{s}}_{jk}$ locations and their projections onto the xy, xz, and yz planes. The bottom graph is an enlarged overhead view of the intersection region. The individual $\hat{\mathbf{s}}_{jk}$ locations are now visible and denoted by 'x's, their corresponding normal vectors by dashed lines, and the final LI estimate by a '$*$.'

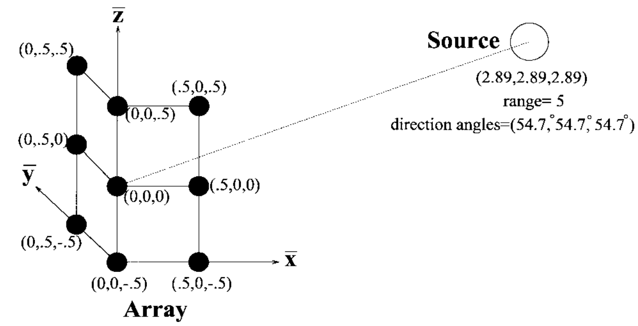

Fig. 4. Closed-form estimator comparison: The nine-element orthogonal array used for the comparison simulations. All distances are in meters.

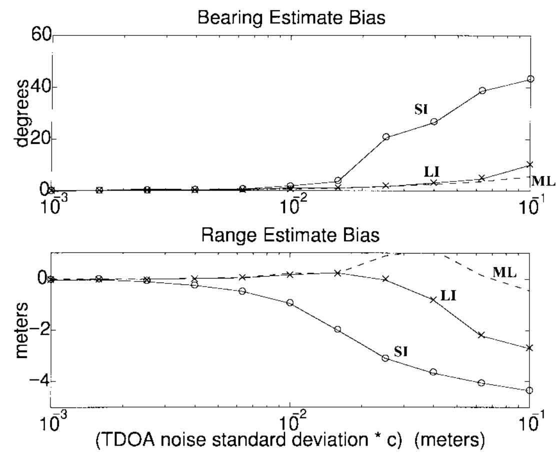

Fig. 5. Closed-form estimator comparison: Sample bias plots for the three estimation procedures—LI, SI, and ML—as a function of the level of noise added to the true TDOA values.

dashed lines. For these four bearing lines, there are a total of $\binom{4}{2} = 6$ normal vectors and 12 $\hat{\mathbf{s}}_{jk}$ locations. The final location estimate $\hat{\mathbf{s}}_{\rm LI}$ is indicated by the '*' near the center of the graph surrounded by the intermediate points of closest intersection. In this example, the final estimate was found to deviate from the actual location by less than 3 cm in any dimension, which is well within the physical boundaries of a human talker.

IV. Closed-Form Estimator Comparison

As a means of evaluating the LI location estimator, the statistical characteristics of the LI and spherical interpolation (SI) localization methods [7] were compared through a series of Monte Carlo simulations modeled after those conducted in [7]. The experimental setup, which was a nine-sensor orthogonal array with half-meter spacings and a source located at a range of 5 m with equal direction angles, is depicted in Fig. 4. The true TDOA values (1) were corrupted by additive white Gaussian noise. One hundred trials were performed at each of 11 noise levels ranging from a standard deviation the equivalent of 10^{-3}m to 10^{-1}m when scaled by the propagation speed of sound in air (c). The LI method partitioned the array into four square sensor quadruples and required the evaluation of eight TDOA estimates: one for each diagonal sensor pair. The SI method required that all the TDOA values be relative to a reference sensor. The sensor at the origin was chosen for this purpose, and the TDOA for each of the remaining eight sensors relative to the reference were calculated. In addition to calculating the LI and SI estimates, the ML estimate was computed via a search method with the initial guess set equal to the true location.

Figs. 5–7 summarize the results of these simulations. Fig. 5 presents the sample bias for the estimated source bearing and range for each of the estimation methods as a function of the level of noise added to the true TDOA values. While each of the methods exhibits some degree of bias in the noisier trials, the situation is most extreme for the SI method. This tendency for the SI method to consistently bias its estimates toward the origin was noted by the authors of [7]. The LI method performs comparably to the ML estimate for all but the most extreme noise conditions. Fig. 6 shows the sample standard deviations. For the standard deviation of the bearing estimates, a trend similar to the bearing bias is observed. The SI method's performance degrades rapidly for noise levels above 10^{-2}m. However, in terms of the range, each of the closed-form

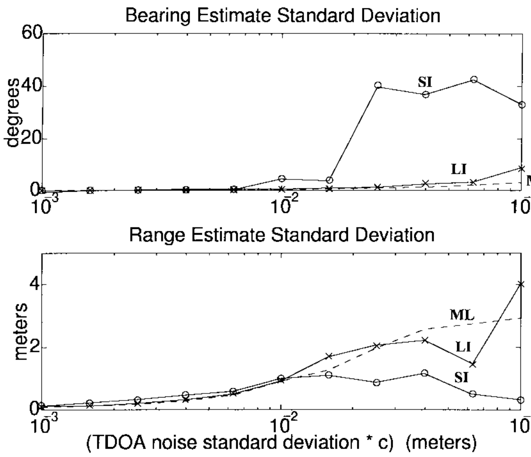

Fig. 6. Closed-form estimator comparison: Standard deviation plots for the three estimation procedures—LI, SI, and ML—as a function of the level of noise added to the true TDOA values.

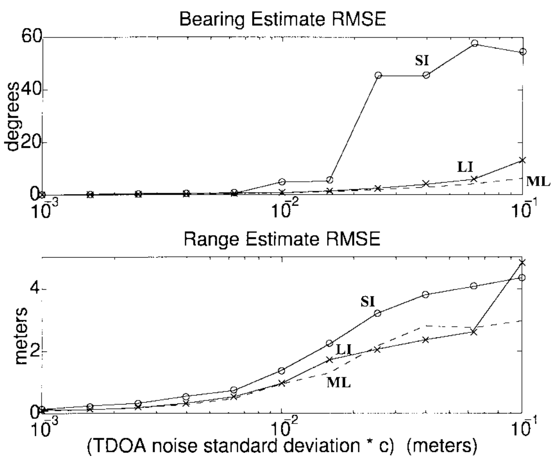

Fig. 7. Closed-form estimator comparison: Root mean square error plots for the three estimation procedures—LI, SI, and ML—as a function of the level of noise added to the true TDOA values.

estimators displays a smaller variance than the ML estimator at the higher noise conditions. This is a consequence of the estimator biases observed previously. Finally, in Fig. 7, the root-mean-square errors (RMSE) are illustrated, which combine the bias and standard deviation. Once again, the LI method closely tracks the ML estimator in all but the most extreme condition, whereas the SI method exhibits a marked performance decrease in both bearing and range for moderate and large noise levels.

Simulations performed over a broad range of source positions exhibit trends similar to those in Figs. 5–7. The LI estimator is consistently less sensitive to noise conditions and possesses a significantly smaller bias in both its range and bearing estimates when compared with the SI estimator.

V. Discussion

A closed-form method for the localization of source positions given only TDOA information has been presented. It was shown to be a robust and accurate estimator, closely modeling the ML estimator, and clearly outperforming a representative algorithm.

From an implementation standpoint, the constraint that the array be composed of rectangular four-element subarrays is not problematic for typical room-oriented microphone-array applications. It is an advantage of the LI method that localization in three-space can be performed with a 2–D array. The SI method, as well as many similar approaches, requires that the matrix of sensor locations be full rank. This necessitates the use of a 3-D sensor placement for localization in three-space. In addition, since the LI method does not require the estimation of delays between sensors other than those in the local subarray, the subarrays can be placed far apart, and delay-estimation processing can be performed locally. The SI algorithm evaluates all TDOA estimates relative to a single reference sensor.

In [5], the *linear intersection* method was used in conjunction with several real microphone-array systems and was shown to be an effective source localization procedure when used alone or as a means of providing initial search conditions to the more computationally demanding search-based algorithms.

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