**The algorithms to derive the frequency and weight of different interconnection motifs with intra-guild interactions**

# Notation

## 1. For binary networks

The two subnetworks are represented by $P$ and $Q$ and three intra-guild association matrices $A$, $B$ and $C$:

$$P_{ab} = \begin{cases} 1 & \text{if } a \text{ interact to } b \\ 0 & \text{if } a \text{ not interact to } b \end{cases}$$

$$Q_{bc} = \begin{cases} 1 & \text{if } b \text{ interact to } c \\ 0 & \text{if } b \text{ not interact to } c \end{cases}$$

$$A_{aa'} = \begin{cases} 1 & \text{if } a \text{ interact to } a' \\ 0 & \text{if } a \text{ not interact to } a' \end{cases}$$

$$B_{bb'} = \begin{cases} 1 & \text{if } b \text{ interact to } b' \\ 0 & \text{if } b \text{ not interact to } b' \end{cases}$$

$$C_{cc'} = \begin{cases} 1 & \text{if } c \text{ interact to } c' \\ 0 & \text{if } c \text{ not interact to } c' \end{cases}$$

We define $A$ represents the number of rows of $P$, and $B$ represents both the number of columns of $P$ and the number of rows of $Q$; $C$ represents the number of columns of $Q$.

$\mathcal{A}_{aa'} = \begin{cases} 1 & if\ A_{aa'} = 0 \\ 0 & if\ A_{aa'} > 0 \end{cases}$ is the complement of $A$, and $\mathcal{A}_{aa'} = 0$ while $a = a'$.

$\mathcal{B}_{bb'} = \begin{cases} 1 & if\ B_{bb'} = 0 \\ 0 & if\ B_{bb'} > 0 \end{cases}$ is the complement of $B$, and $\mathcal{B}_{bb'} = 0$ while $b = b'$.

$\mathcal{C}_{cc'} = \begin{cases} 1 & if\ C_{cc'} = 0 \\ 0 & if\ C_{cc'} > 0 \end{cases}$ is the complement of $C$, and $\mathcal{C}_{cc'} = 0$ while $c = c'$.

$O_b = \sum_{a=1}^{A} P_{ab}$ defines a vector of length $B$, in which each element is the sum of each column of $P$ and represents the weighted degree of the nodes of group $b$ in the $P$ subnetwork.

$R_b = \sum_{c=1}^{C} Q_{bc}$ defines a vector of length $B$, in which each element is the sum of each row of $Q$ and represents the weighted degree of the nodes of group $b$ in the $Q$ subnetwork.

$G_b = \frac{1}{2}\sum_{a=1}^{A} (P^T \cdot A)_{ba} * P_{ab}{}^T$ defines a vector of length $B$, in which each element represents the sum of pairwise connected intra-guild degrees of the nodes of group $b$ in the $P$ subnetwork.

$\mathcal{G}_b = \frac{1}{2}\sum_{a=1}^{A} (P^T \cdot \mathcal{A})_{ba} \mathcal{P}_{ab}{}^T$ defines a vector of length $B$, in which each element represents the sum of pairwise disconnected intra-guild degrees of the nodes of group $b$ in the $P$ subnetwork.

$W_b = \frac{1}{2}\sum_{c=1}^{C} (Q \cdot C)_{bc} * Q_{bc}$ defines a vector of length $B$, in which each element represents the sum of pairwise connected intra-guild degrees of the nodes of group $b$ in the $Q$ subnetwork.

$\mathcal{W}_b = \frac{1}{2}\sum_{c=1}^{C} (Q \cdot \mathcal{C})_{bc} * \mathcal{Q}_{bc}$ defines a vector of length $B$, in which each element represents the sum of pairwise disconnected intra-guild degrees of the nodes of group $b$ in the $Q$ subnetwork.

**Explanation: contributing to matrix $P$.**

$\mathcal{P}_{ab} = \begin{cases} 1 & if\ P_{ab} = 0 \\ 0 & if\ P_{ab} > 0 \end{cases}$ is the complement of $P$

$$U_{bb'} = \sum_{a}^{A} P_{ab} P_{ab'} = (P^T P)_{bb'}$$

$U$: this is a matrix of dimension $B \times B$. For two columns $b$, $b'$ in $P$, i.e. two nodes $b$, $b'$ in the $b$-node group, entry $bb'$ gives the following: it counts the sum of products of pairwise weighted degrees of nodes in the $a$-node group, which are adjacent to both $b$ and $b'$.

$$D = [\text{vec}(P_b \cdot P_b^T \circ A)^T]_{b=1}^{B} \in \mathbb{R}^{b \times a^2}$$

$$E = [\text{vec}(P_b \cdot \mathcal{P}_b^T \circ A)^T]_{b=1}^{B} \in \mathbb{R}^{b \times a^2}$$

$$F = [\text{vec}(\mathcal{P}_b \cdot P_b^T \circ A)^T]_{b=1}^{B} \in \mathbb{R}^{b \times a^2}$$

$$\mathcal{D} = [\text{vec}(P_b \cdot P_b^T \circ \mathcal{A})^T]_{b=1}^{B} \in \mathbb{R}^{b \times a^2}$$

$$\mathcal{E} = [\text{vec}(P_b \cdot \mathcal{P}_b^T \circ \mathcal{A})^T]_{b=1}^{B} \in \mathbb{R}^{b \times a^2}$$

$$\mathcal{F} = [\text{vec}(\mathcal{P}_b \cdot P_b^T \circ \mathcal{A})^T]_{b=1}^{B} \in \mathbb{R}^{b \times a^2}$$

$$H_{bb'} = (EF^T)_{bb'}$$

$$\mathcal{H}_{bb'} = (\mathcal{E}\mathcal{F}^T)_{bb'}$$

$$I_{bb'} = (DF^T)_{bb'}$$

$$\mathcal{I}_{bb'} = (\mathcal{D}\mathcal{F}^T)_{bb'}$$

$$J_{bb'} = I_{bb'}{}^T$$

$$\mathcal{J}_{bb'} = \mathcal{I}_{bb'}{}^T$$

$$K_{bb'} = \frac{1}{2}(DD^T)_{bb'}$$

$$\mathcal{K}_{bb'} = \frac{1}{2}(\mathcal{D}\mathcal{D}^T)_{bb'}$$

**Explanation: contributing to matrix $Q$.**

$$\mathcal{Q}_{bc} = \begin{cases} 1 & if\ Q_{bc} = 0 \\ 0 & if\ Q_{bc} > 0 \end{cases}$$ is the complement of $Q$

$$V_{bb'} = \sum_{c}^{C} Q_{bc} Q_{b'c} = (QQ^T)_{bb'}$$

$V$: this is a matrix of dimension $B \times B$. For two rows in $Q$, $b$, $b'$, i.e. two nodes $b$, $b'$ in the $b$-node group, entry $bb'$ gives the number of columns, or nodes in the $c$-node group, which are adjacent to both $b$ and $b'$.

$$L = [\mathrm{vec}(Q_b Q_b{}^T \circ C)^T]_{b=1}^{B} \in \mathbb{R}^{b \times c^2}$$

$$M = [\mathrm{vec}(Q_b \mathcal{Q}_b{}^T \circ C)^T]_{b=1}^{B} \in \mathbb{R}^{b \times c^2}$$

$$N = [\mathrm{vec}(\mathcal{Q}_b Q_b{}^T \circ C)^T]_{b=1}^{B} \in \mathbb{R}^{b \times c^2}$$

$$\mathcal{L} = [\mathrm{vec}(Q_b Q_b{}^T \circ \mathcal{C})^T]_{b=1}^{B} \in \mathbb{R}^{b \times c^2}$$

$$\mathcal{M} = [\mathrm{vec}(Q_b \mathcal{Q}_b{}^T \circ \mathcal{C})^T]_{b=1}^{B} \in \mathbb{R}^{b \times c^2}$$

$$\mathcal{N} = [\mathrm{vec}(\mathcal{Q}_b Q_b{}^T \circ \mathcal{C})^T]_{b=1}^{B} \in \mathbb{R}^{b \times c^2}$$

$$S_{bb'} = (MN^T)_{bb'}$$

$$\mathcal{S}_{bb'} = (\mathcal{M}\mathcal{N}^T)_{bb'}$$

$$T_{bb'} = (LN^T)_{bb'}$$

$$\mathcal{T}_{bb'} = (\mathcal{L}\mathcal{N}^T)_{bb'}$$

$$Z_{bb'} = \frac{1}{2}(LL^T)_{bb'}$$

$$\mathcal{Z}_{bb'} = \frac{1}{2}(\mathcal{L}\mathcal{L}^T)_{bb'}$$

**All the formulae for counting motifs**

M111:

$$\sum_{b=1}^{B} O_b R_b$$

M112-1:

$$\sum_{b=1}^{B} O_b \mathcal{W}_b$$

M112-2:

$$\sum_{b=1}^{B} O_b W_b$$

M211-1:

$$\sum_{b=1}^{B} \mathcal{G}_b R_b$$

M211-2:

$$\sum_{b=1}^{B} G_b R_b$$

M212-1:

$$\sum_{b=1}^{B} \mathcal{G}_b \mathcal{W}_b$$

M212-2:

$$\sum_{b=1}^{B} G_b \mathcal{W}_b$$

M212-3:

$$\sum_{b=1}^{B} \mathcal{G}_b W_b$$

M212-4:

$$\sum_{b=1}^{B} G_b W_b$$

M121-1:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B} U_{bb'} * V_{bb'} * \mathcal{B}_{bb'}$$

M121-2:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B} U_{bb'} * V_{bb'} * B_{bb'}$$

M122-1:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}U_{bb'}*\mathcal{B}_{bb'}*\mathcal{S}_{bb'}$$

M122-2:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}U_{bb'}*B_{bb'}*\mathcal{S}_{bb'}$$

M122-3:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}U_{bb'}*\mathcal{B}_{bb'}*S_{bb'}$$

M122-4:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}U_{bb'}*B_{bb'}*S_{bb'}$$

M122-5:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}U_{bb'}*\mathcal{B}_{bb'}*\mathcal{T}_{bb'}$$

M122-6:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}U_{bb'}*B_{bb'}*\mathcal{T}_{bb'}$$

M122-7:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}U_{bb'}*\mathcal{B}_{bb'}*T_{bb'}$$

M122-8:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}U_{bb'}*B_{bb'}*T_{bb'}$$

M122-9:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}U_{bb'}*\mathcal{B}_{bb'}*\mathcal{Z}_{bb'}$$

M122-10:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}U_{bb'}*B_{bb'}*\mathcal{Z}_{bb'}$$

M122-11:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}U_{bb'}*\mathcal{B}_{bb'}*Z_{bb'}$$

M122-12:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}U_{bb'}*B_{bb'}*Z_{bb'}$$

M221-1:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{H}_{bb'}*\mathcal{B}_{bb'}*V_{bb'}$$

M221-2:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}H_{bb'}*\mathcal{B}_{bb'}*V_{bb'}$$

M221-3:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{H}_{bb'}*B_{bb'}*V_{bb'}$$

M221-4:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}H_{bb'}*B_{bb'}*V_{bb'}$$

M221-5:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{I}_{bb'}*\mathcal{B}_{bb'}*V_{bb'}$$

M221-6:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}I_{bb'}*\mathcal{B}_{bb'}*V_{bb'}$$

M221-7:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{I}_{bb'}*B_{bb'}*V_{bb'}$$

M221-8:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}I_{bb'}*B_{bb'}*V_{bb'}$$

M221-9:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{K}_{bb'}*\mathcal{B}_{bb'}*V_{bb'}$$

M221-10:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}K_{bb'}*\mathcal{B}_{bb'}*V_{bb'}$$

M221-11:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{K}_{bb'}*B_{bb'}*V_{bb'}$$

M221-12:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}K_{bb'}*B_{bb'}*V_{bb'}$$

M222-1:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{I}_{bb'}*\mathcal{B}_{bb'}*\mathcal{S}_{bb'}$$

M222-2:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}I_{bb'}*\mathcal{B}_{bb'}*\mathcal{S}_{bb'}$$

M222-3:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{I}_{bb'}*B_{bb'}*\mathcal{S}_{bb'}$$

M222-4:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{I}_{bb'}*\mathcal{B}_{bb'}*S_{bb'}$$

M222-5:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}I_{bb'}*B_{bb'}*\mathcal{S}_{bb'}$$

M222-6:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}I_{bb'}*\mathcal{B}_{bb'}*S_{bb'}$$

M222-7:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{I}_{bb'}*B_{bb'}*S_{bb'}$$

M222-8:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}I_{bb'}*B_{bb'}*S_{bb'}$$

M222-9:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{K}_{bb'}*\mathcal{B}_{bb'}*\mathcal{S}_{bb'}$$

M222-10:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}K_{bb'}*\mathcal{B}_{bb'}*\mathcal{S}_{bb'}$$

M222-11:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{K}_{bb'}*B_{bb'}*\mathcal{S}_{bb'}$$

M222-12:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{K}_{bb'}*\mathcal{B}_{bb'}*S_{bb'}$$

M222-13:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}K_{bb'}*B_{bb'}*\mathcal{S}_{bb'}$$

M222-14:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}K_{bb'}*\mathcal{B}_{bb'}*S_{bb'}$$

M222-15:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{K}_{bb'}*B_{bb'}*S_{bb'}$$

M222-16:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}K_{bb'}*B_{bb'}*S_{bb'}$$

M222-17:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{H}_{bb'}*\mathcal{B}_{bb'}*\mathcal{T}_{bb'}$$

M222-18:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}H_{bb'}*\mathcal{B}_{bb'}*\mathcal{T}_{bb'}$$

M222-19:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{H}_{bb'}*B_{bb'}*\mathcal{T}_{bb'}$$

M222-20:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{H}_{bb'}*\mathcal{B}_{bb'}*T_{bb'}$$

M222-21:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} H_{bb'} * B_{bb'} * \mathcal{T}_{bb'}$$

M222-22:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} H_{bb'} * \mathcal{B}_{bb'} * T_{bb'}$$

M222-23:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{H}_{bb'} * B_{bb'} * T_{bb'}$$

M222-24:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} H_{bb'} * B_{bb'} * T_{bb'}$$

M222-25:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{I}_{bb'} * \mathcal{B}_{bb'} * \mathcal{T}_{bb'}$$

M222-26:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} I_{bb'} * \mathcal{B}_{bb'} * \mathcal{T}_{bb'}$$

M222-27:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{I}_{bb'} * B_{bb'} * \mathcal{T}_{bb'}$$

M222-28:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{I}_{bb'} * \mathcal{B}_{bb'} * T_{bb'}$$

M222-29:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} I_{bb'} * B_{bb'} * \mathcal{T}_{bb'}$$

M222-30:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} I_{bb'} * \mathcal{B}_{bb'} * T_{bb'}$$

M222-31:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{I}_{bb'} * B_{bb'} * T_{bb'}$$

M222-32:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} I_{bb'} * B_{bb'} * T_{bb'}$$

M222-33:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{K}_{bb'} * \mathcal{B}_{bb'} * \mathcal{T}_{bb'}$$

M222-34:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} K_{bb'} * \mathcal{B}_{bb'} * \mathcal{T}_{bb'}$$

M222-35:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{K}_{bb'} * B_{bb'} * \mathcal{T}_{bb'}$$

M222-36:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{K}_{bb'} * \mathcal{B}_{bb'} * T_{bb'}$$

M222-37:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} K_{bb'} * B_{bb'} * \mathcal{T}_{bb'}$$

M222-38:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} K_{bb'} * \mathcal{B}_{bb'} * T_{bb'}$$

M222-39:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{K}_{bb'} * B_{bb'} * T_{bb'}$$

M222-40:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} K_{bb'} * B_{bb'} * T_{bb'}$$

M222-41:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{H}_{bb'} * \mathcal{B}_{bb'} * \mathcal{Z}_{bb'}$$

M222-42:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B} H_{bb'} * \mathcal{B}_{bb'} * \mathcal{Z}_{bb'}$$

M222-43:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{H}_{bb'}*B_{bb'}*\mathcal{Z}_{bb'}$$

M222-44:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{H}_{bb'}*\mathcal{B}_{bb'}*Z_{bb'}$$

M222-45:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}H_{bb'}*B_{bb'}*\mathcal{Z}_{bb'}$$

M222-46:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}H_{bb'}*\mathcal{B}_{bb'}*Z_{bb'}$$

M222-47:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{H}_{bb'}*B_{bb'}*Z_{bb'}$$

M222-48:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B}H_{bb'}*B_{bb'}*Z_{bb'}$$

M222-49:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{I}_{bb'}*\mathcal{B}_{bb'}*\mathcal{Z}_{bb'}$$

M222-50:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}I_{bb'}*\mathcal{B}_{bb'}*\mathcal{Z}_{bb'}$$

M222-51:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{I}_{bb'}*B_{bb'}*\mathcal{Z}_{bb'}$$

M222-52:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{I}_{bb'}*\mathcal{B}_{bb'}*Z_{bb'}$$

M222-53:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}I_{bb'}*B_{bb'}*\mathcal{Z}_{bb'}$$

M222-54:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} I_{bb'} * \mathcal{B}_{bb'} * Z_{bb'}$$

M222-55:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{I}_{bb'} * B_{bb'} * Z_{bb'}$$

M222-56:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B} I_{bb'} * B_{bb'} * Z_{bb'}$$

M222-57:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{K}_{bb'} * \mathcal{B}_{bb'} * \mathcal{Z}_{bb'}$$

M222-58:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B} K_{bb'} * \mathcal{B}_{bb'} * \mathcal{Z}_{bb'}$$

M222-59:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{K}_{bb'} * B_{bb'} * \mathcal{Z}_{bb'}$$

M222-60:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{K}_{bb'} * \mathcal{B}_{bb'} * Z_{bb'}$$

M222-61:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B} K_{bb'} * B_{bb'} * \mathcal{Z}_{bb'}$$

M222-62:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B} K_{bb'} * \mathcal{B}_{bb'} * Z_{bb'}$$

M222-63:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B} \mathcal{K}_{bb'} * B_{bb'} * Z_{bb'}$$

M222-64:

$$\frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B} K_{bb'} * B_{bb'} * Z_{bb'}$$

M222-65:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{J}_{bb'}*\mathcal{B}_{bb'}*\mathcal{T}_{bb'}$$

M222-66:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}J_{bb'}*\mathcal{B}_{bb'}*\mathcal{T}_{bb'}$$

M222-67:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{J}_{bb'}*B_{bb'}*\mathcal{T}_{bb'}$$

M222-68:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{J}_{bb'}*\mathcal{B}_{bb'}*T_{bb'}$$

M222-69:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}J_{bb'}*B_{bb'}*\mathcal{T}_{bb'}$$

M222-70:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}J_{bb'}*\mathcal{B}_{bb'}*T_{bb'}$$

M222-71:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}\mathcal{J}_{bb'}*B_{bb'}*T_{bb'}$$

M222-72:

$$\sum_{b=1}^{B}\sum_{b'=1}^{B}J_{bb'}*B_{bb'}*T_{bb'}$$

## 2. For weighted networks

$$PW_{ab} = \begin{cases} \mathbb{R}^+ & \text{if } a \text{ interact to } b \\ 0 & \text{if } a \text{ not interact to } b \end{cases}$$

$$QW_{bc} = \begin{cases} \mathbb{R}^+ & \text{if } b \text{ interact to } c \\ 0 & \text{if } b \text{ not interact to } c \end{cases}$$

$$AW_{aa'} = \begin{cases} \mathbb{R}^+ & \text{if } a \text{ interact to } a' \\ 0 & \text{if } a \text{ not interact to } a' \end{cases}$$

$$BW_{bb'} = \begin{cases} \mathbb{R}^+ & \text{if } b \text{ interact to } b' \\ 0 & \text{if } b \text{ not interact to } b' \end{cases}$$

$$CW_{cc'} = \begin{cases} \mathbb{R}^+ & \text{if } c \text{ interact to } c' \\ 0 & \text{if } c \text{ not interact to } c' \end{cases}$$

We also define $A$ represents the number of rows of $PW$, and $B$ represents both the number of columns of $PW$ and the number of rows of $QW$; $C$ represents the number of columns of $QW$.

$OW_b = \sum_{a=1}^{A} PW_{ab}$ defines a vector of length $B$, in which each element is the sum of each column of $P_{ab}$ and represents the weighted degree of the nodes of group $b$ in the P subnetwork.

$RW_b = \sum_{c=1}^{C} QW_{bc}$ defines a vector of length $B$, in which each element is the sum of each row of $Q_{bc}$ and represents the weighted degree of the nodes of group $b$ in the Q subnetwork.

$$GW_b = \frac{1}{2}\sum_{a=1}^{A}\sum_{a'=1}^{A}(PW_{ab} + PW_{a'b} + AW_{aa'})^*P_{ab}{}^*P_{a'b}{}^*A_{aa'}$$

$$\mathcal{G}W_b = \frac{1}{2}\sum_{a=1}^{A}\sum_{a'=1}^{A}(PW_{ab} + PW_{a'b})^*P_{ab}{}^*P_{a'b}{}^*\mathcal{A}_{aa'}$$

$$WW_b = \frac{1}{2}\sum_{c=1}^{C}\sum_{c'=1}^{C}(QW_{bc} + QW_{bc'} + CW_{cc'})*Q_{bc}*Q_{bc'}*C_{cc'}$$

$$\mathcal{W}W_b = \frac{1}{2}\sum_{c=1}^{C}\sum_{c'=1}^{C}(QW_{bc} + QW_{bc'})*Q_{bc}*Q_{bc'}*\mathcal{C}_{cc'}$$

$$\mathcal{P}_{ab} = \begin{cases} 1 & if\ PW_{ab} = 0 \\ 0 & if\ PW_{ab} > 0 \end{cases} \quad \text{is the complement of } PW$$

$$UW_{bb'} = \sum_{a}^{A}(PW_{ab} + PW_{ab'})*P_{ab}*P_{ab'}$$

$$DW = \left[\mathrm{vec}\left(P_b \cdot PW_b^T * PW_b\right)^T\right]_{b=1}^{B} \in \mathbb{R}^{b\times a^2}$$

$$EW = \left[\mathrm{vec}\left(PW_b \cdot \mathcal{P}_b{}^T\right)^T\right]_{b=1}^{B} \in \mathbb{R}^{b\times a^2}$$

$$FW = \left[\mathrm{vec}\left(\mathcal{P}_b \cdot PW_b^T\right)^T\right]_{b=1}^{B} \in \mathbb{R}^{b\times a^2}$$

$$\mathcal{D}W = \left[\mathrm{vec}\left(\left(P_b \cdot PW_b^T * PW_b\right) \circ \mathcal{A}\right)^T\right]_{b=1}^{B} \in \mathbb{R}^{b\times a^2}$$

$$\mathcal{E}W = \left[\mathrm{vec}\left(PW_b \cdot \mathcal{P}_b{}^T \circ \mathcal{A}\right)^T\right]_{b=1}^{B} \in \mathbb{R}^{b\times a^2}$$

$$\mathcal{F}W = \left[\mathrm{vec}\left(\mathcal{P}_b \cdot PW_b^T \circ \mathcal{A}\right)^T\right]_{b=1}^{B} \in \mathbb{R}^{b\times a^2}$$

$$HW_{bb'} = \sum_{a=1}^{A^2}(EW_{ba} + FW_{b'a} + \mathrm{vec}(\mathrm{AW})_a)*E_{ba}*F_{b'a}*\mathrm{vec}(\mathrm{A})_a$$

$$\mathcal{H}W_{bb'} = \sum_{a=1}^{A^2}(\mathcal{E}W_{ba} + \mathcal{F}W_{b'a})*\mathcal{E}_{ba}*\mathcal{F}_{b'a}$$

$$IW_{bb'} = \sum_{a=1}^{A^2}(DW_{ba} + FW_{b'a} + \mathrm{vec}(\mathrm{AW})_a)*D_{ba}*F_{b'a}*\mathrm{vec}(\mathrm{A})_a$$

$$\mathcal{I}W_{bb'} = \sum_{a=1}^{A^2}(\mathcal{D}W_{ba} + \mathcal{F}W_{b'a})*\mathcal{D}_{ba}*\mathcal{F}_{b'a}$$

$$JW_{bb'} = IW_{bb'}{}^T$$

$$\mathcal{J}W_{bb'} = \mathcal{I}W_{bb'}{}^T$$

$$KW_{bb'} = \frac{1}{2}\sum_{a=1}^{A^2}(DW_{ba} + DW_{b'a} + \mathrm{vec}(\mathrm{AW})_a)*D_{ba}*D_{b'a}*\mathrm{vec}(\mathrm{A})_a$$

$$\mathcal{K}W_{bb'} = \frac{1}{2}\sum_{a=1}^{A^2}(\mathcal{D}W_{ba} + \mathcal{D}W_{b'a})*\mathcal{D}_{ba}*\mathcal{D}_{b'a}$$

$$\mathcal{Q}_{bc} = \begin{cases} 1 & if\ QW_{bc} = 0 \\ 0 & if\ QW_{bc} > 0 \end{cases} \text{ is the complement of } QW$$

$$VW_{bb'} = \sum_{c}^{C}(QW_{bc} + QW_{b'c})*Q_{bc}*Q_{b'c}$$

$$LW = [\text{vec}(Q_b \cdot QW_b{}^T * QW_b)^T]_{b=1}^{B} \in \mathbb{R}^{b \times c^2}$$

$$MW = [\text{vec}(QW_b \cdot \mathcal{Q}_b{}^T)^T]_{b=1}^{B} \in \mathbb{R}^{b \times c^2}$$

$$NW = [\text{vec}(\mathcal{Q}_b \cdot QW_b^T)^T]_{b=1}^{B} \in \mathbb{R}^{b \times c^2}$$

$$\mathcal{L}W = [\text{vec}((Q_b \cdot QW_b{}^T * QW_b) \circ \mathcal{C})^T]_{b=1}^{B} \in \mathbb{R}^{b \times c^2}$$

$$\mathcal{M}W = [\text{vec}(QW_b \cdot \mathcal{Q}_b{}^T \circ \mathcal{C})^T]_{b=1}^{B} \in \mathbb{R}^{b \times c^2}$$

$$\mathcal{N}W = [\text{vec}(\mathcal{Q}_b \cdot QW_b^T \circ \mathcal{C})^T]_{b=1}^{B} \in \mathbb{R}^{b \times c^2}$$

$$SW_{bb'} = \sum_{c=1}^{C^2}(MW_{bc} + NW_{b'c} + \text{vec}(C\text{W})_c)*M_{bc}*N_{b'c}*\text{vec}(C)_c$$

$$\mathcal{S}W_{bb'} = \sum_{c=1}^{C^2}(\mathcal{M}W_{bc} + \mathcal{N}W_{b'c})*\mathcal{M}_{bc}*\mathcal{N}_{b'c}$$

$$TW_{bb'} = \sum_{c=1}^{C^2}(LW_{bc} + MW_{b'c} + \text{vec}(C\text{W})_c)*L_{bc}*M_{b'c}*\text{vec}(C)_c$$

$$\mathcal{T}W_{bb'} = \sum_{c=1}^{C^2}(\mathcal{L}W_{bc} + \mathcal{M}W_{b'c})*\mathcal{L}_{bc}*\mathcal{M}_{b'c}$$

$$ZW_{bb'} = \frac{1}{2}\sum_{c=1}^{C^2}(LW_{bc} + LW_{b'c} + \text{vec}(C\text{W})_c)*L_{bc}*L_{b'c}*\text{vec}(C)_c$$

$$\mathcal{Z}W_{bb'} = \frac{1}{2}\sum_{c=1}^{C^2}(\mathcal{L}W_{bc} + \mathcal{L}W_{b'c})*\mathcal{L}_{bc}*\mathcal{L}_{b'c}$$

**All the formulae for the weights of all the cases of a particular motif**

M111:

$$\frac{1}{2}\sum_{b=1}^{B} OW_b R_b + RW_b O_b$$

M112-1:

$$\frac{1}{3}\sum_{b=1}^{B} OW_b \mathcal{W}_b + \mathcal{W}W_b O_b$$

M112-2:

$$\frac{1}{4}\sum_{b=1}^{B} OW_b W_b + WW_b O_b$$

M211-1:

$$\frac{1}{3}\sum_{b=1}^{B} \mathcal{G}W_b R_b + RW_b \mathcal{G}_b$$

M211-2:

$$\frac{1}{4}\sum_{b=1}^{B} GW_b R_b + RW_b G_b$$

M212-1:

$$\frac{1}{4}\sum_{b=1}^{B} \mathcal{G}W_b \mathcal{W}_b + \mathcal{W}W_b \mathcal{G}_b$$

M212-2:

$$\frac{1}{5}\sum_{b=1}^{B} GW_b \mathcal{W}_b + \mathcal{W}W_b G_b$$

M212-3:

$$\frac{1}{5}\sum_{b=1}^{B} \mathcal{G}W_b W_b + WW_b \mathcal{G}_b$$

M212-4:

$$\frac{1}{6}\sum_{b=1}^{B} GW_b W_b + WW_b G_b$$

M121-1:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B} (UW_{bb'} * V_{bb'} + VW_{bb'} * U_{bb'}) * \mathcal{B}_{bb'}$$

M121-2:

$$\frac{1}{10}\sum_{b=1}^{B}\sum_{b'=1}^{B}(UW_{bb'}{*}V_{bb'}+VW_{bb'}{*}U_{bb'}){*}B_{bb'}+BW_{bb'}{*}U_{bb'}{*}V_{bb'}$$

M122-1:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(UW_{bb'}{*}\mathcal{S}_{bb'}+\mathcal{S}W_{bb'}{*}U_{bb'}){*}\mathcal{B}_{bb'}$$

M122-2:

$$\frac{1}{10}\sum_{b=1}^{B}\sum_{b'=1}^{B}(UW_{bb'}{*}\mathcal{S}_{bb'}+\mathcal{S}W_{bb'}{*}U_{bb'}){*}B_{bb'}+U_{bb'}{*}\mathcal{S}_{bb'}{*}BW_{bb'}$$

M122-3:

$$\frac{1}{10}\sum_{b=1}^{B}\sum_{b'=1}^{B}(UW_{bb'}{*}S_{bb'}+SW_{bb'}{*}U_{bb'}){*}\mathcal{B}_{bb'}$$

M122-4:

$$\frac{1}{12}\sum_{b=1}^{B}\sum_{b'=1}^{B}(UW_{bb'}{*}S_{bb'}+SW_{bb'}{*}U_{bb'}){*}b_{bb'}+U_{bb'}{*}S_{bb'}{*}BW_{bb'}$$

M122-5:

$$\frac{1}{5}\sum_{b=1}^{B}\sum_{b'=1}^{B}(UW_{bb'}{*}\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}{*}U_{bb'}){*}\mathcal{B}_{bb'}$$

M122-6:

$$\frac{1}{6}\sum_{b=1}^{B}\sum_{b'=1}^{B}(UW_{bb'}{*}\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}{*}U_{bb'}){*}B_{bb'}+U_{bb'}{*}\mathcal{T}_{bb'}{*}BW_{bb'}$$

M122-7:

$$\frac{1}{6}\sum_{b=1}^{B}\sum_{b'=1}^{B}(UW_{bb'}{*}T_{bb'}+TW_{bb'}{*}U_{bb'}){*}\mathcal{B}_{bb'}$$

M122-8:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(UW_{bb'}{*}T_{bb'}+TW_{bb'}{*}U_{bb'}){*}b_{bb'}+U_{bb'}{*}T_{bb'}{*}BW_{bb'}$$

M122-9:

$$\frac{1}{12}\sum_{b=1}^{B}\sum_{b'=1}^{B}(UW_{bb'}{*}\mathcal{Z}_{bb'}+\mathcal{Z}W_{bb'}{*}U_{bb'}){*}\mathcal{B}_{bb'}$$

M122-10:

$$\frac{1}{14}\sum_{b=1}^{B}\sum_{b'=1}^{B}(UW_{bb'}{*}\mathcal{Z}_{bb'}+\mathcal{Z}W_{bb'}{*}U_{bb'}){*}B_{bb'}+U_{bb'}{*}\mathcal{Z}_{bb'}{*}BW_{bb'}$$

M122-11:

$$\frac{1}{14}\sum_{b=1}^{B}\sum_{b'=1}^{B}(UW_{bb'}{*}Z_{bb'}+ZW_{bb'}{*}U_{bb'}){*}\mathcal{B}_{\ bb'}$$

M122-12:

$$\frac{1}{16}\sum_{b=1}^{B}\sum_{b'=1}^{B}(UW_{bb'}{*}Z_{bb'}+ZW_{bb'}{*}U_{bb'}){*}b_{bb'}+U_{bb'}{*}Z_{bb'}{*}BW_{bb'}$$

M221-1:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{H}W_{bb'}{*}V_{bb'}+VW_{bb'}{*}\mathcal{H}_{\ bb'}){*}\mathcal{B}_{\ bb'}$$

M221-2:

$$\frac{1}{10}\sum_{b=1}^{B}\sum_{b'=1}^{B}(HW_{bb'}{*}V_{bb'}+VW_{bb'}{*}H_{bb'}){*}\mathcal{B}_{\ bb'}$$

M221-2:

$$\frac{1}{10}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{H}W_{bb'}{*}V_{bb'}+VW_{bb'}{*}\mathcal{H}_{\ bb'}){*}B_{bb'}+\mathcal{H}_{bb'}{*}V_{bb'}{*}BW_{bb'}$$

M221-4:

$$\frac{1}{12}\sum_{b=1}^{B}\sum_{b'=1}^{B}(HW_{bb'}{*}V_{bb'}+VW_{bb'}{*}H_{bb'}){*}B_{bb'}+H_{bb'}{*}V_{bb'}{*}BW_{bb'}$$

M221-5:

$$\frac{1}{5}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{I}W_{bb'}{*}V_{bb'}+VW_{bb'}{*}\mathcal{I}_{\ bb'}){*}\mathcal{B}_{\ bb'}$$

M221-6:

$$\frac{1}{6}\sum_{b=1}^{B}\sum_{b'=1}^{B}(IW_{bb'}{*}V_{bb'}+VW_{bb'}{*}I_{bb'}){*}\mathcal{B}_{\ bb'}$$

M221-7:

$$\frac{1}{6}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{I}W_{bb'}{*}V_{bb'}+VW_{bb'}{*}\mathcal{I}_{\ bb'}){*}B_{bb'}+\mathcal{I}_{bb'}{*}V_{bb'}{*}BW_{bb'}$$

M221-8:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(IW_{bb'}{*}V_{bb'}+VW_{bb'}{*}I_{bb'}){*}B_{bb'}+I_{bb'}{*}V_{bb'}{*}BW_{bb'}$$

M221-9:

$$\frac{1}{12}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{K}W_{bb'}{*}V_{bb'}+VW_{bb'}{*}\mathcal{K}_{\ bb'}){*}\mathcal{B}_{\ bb'}$$

M221-10:

$$\frac{1}{14}\sum_{b=1}^{B}\sum_{b'=1}^{B}(KW_{bb'}*V_{bb'}+VW_{bb'}*K_{bb'})*\mathcal{B}_{bb'}$$

M221-11:

$$\frac{1}{14}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{K}W_{bb'}*V_{bb'}+VW_{bb'}*\mathcal{K}_{bb'})*B_{bb'}+\mathcal{K}_{bb'}*V_{bb'}*BW_{bb'}$$

M221-12:

$$\frac{1}{16}\sum_{b=1}^{B}\sum_{b'=1}^{B}(KW_{bb'}*V_{bb'}+VW_{bb'}*K_{bb'})*B_{bb'}+K_{bb'}*V_{bb'}*BW_{bb'}$$

M222-1:

$$\frac{1}{5}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{I}W_{bb'}*\mathcal{S}_{bb'}+\mathcal{S}W_{bb'}*\mathcal{I}_{bb'})*\mathcal{B}_{bb'}$$

M222-2:

$$\frac{1}{6}\sum_{b=1}^{B}\sum_{b'=1}^{B}(IW_{bb'}*\mathcal{S}_{bb'}+\mathcal{S}W_{bb'}*I_{bb'})*\mathcal{B}_{bb'}$$

M222-3:

$$\frac{1}{6}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{I}W_{bb'}*\mathcal{S}_{bb'}+\mathcal{S}W_{bb'}*\mathcal{I}_{bb'})*B_{bb'}+\mathcal{I}_{bb'}*\mathcal{S}_{bb'}*BW_{bb'}$$

M222-4:

$$\frac{1}{6}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{I}W_{bb'}*S_{bb'}+SW_{bb'}*\mathcal{I}_{bb'})*\mathcal{B}_{bb'}$$

M222-5:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(IW_{bb'}*\mathcal{S}_{bb'}+\mathcal{S}W_{bb'}*I_{bb'})*B_{bb'}+I_{bb'}*\mathcal{S}_{bb'}*BW_{bb'}$$

M222-6:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(IW_{bb'}*S_{bb'}+SW_{bb'}*I_{bb'})*\mathcal{B}_{bb'}$$

M222-7:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{I}W_{bb'}*S_{bb'}+SW_{bb'}*\mathcal{I}_{bb'})*B_{bb'}+\mathcal{I}_{bb'}*S_{bb'}*BW_{bb'}$$

M222-8:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(IW_{bb'}*S_{bb'}+SW_{bb'}*I_{bb'})*B_{bb'}+I_{bb'}*S_{bb'}*BW_{bb'}$$

M222-9:

$$\frac{1}{12}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{K}W_{bb'}*\mathcal{S}_{bb'}+\mathcal{S}W_{bb'}*\mathcal{K}_{bb'})*\mathcal{B}_{bb'}$$

M222-10:

$$\frac{1}{14}\sum_{b=1}^{B}\sum_{b'=1}^{B}(KW_{bb'}*\mathcal{S}_{bb'}+\mathcal{S}W_{bb'}*K_{bb'})*\mathcal{B}_{bb'}$$

M222-11:

$$\frac{1}{14}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{K}W_{bb'}*\mathcal{S}_{bb'}+\mathcal{S}W_{bb'}*\mathcal{K}_{bb'})*B_{bb'}+\mathcal{K}_{bb'}*\mathcal{S}_{bb'}*BW_{bb'}$$

M222-12:

$$\frac{1}{14}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{K}W_{bb'}*S_{bb'}+SW_{bb'}*\mathcal{K}_{bb'})*\mathcal{B}_{bb'}$$

M222-13:

$$\frac{1}{16}\sum_{b=1}^{B}\sum_{b'=1}^{B}(KW_{bb'}*\mathcal{S}_{bb'}+\mathcal{S}W_{bb'}*K_{bb'})*B_{bb'}+K_{bb'}*\mathcal{S}_{bb'}*BW_{bb'}$$

M222-14:

$$\frac{1}{16}\sum_{b=1}^{B}\sum_{b'=1}^{B}(KW_{bb'}*S_{bb'}+SW_{bb'}*K_{bb'})*\mathcal{B}_{bb'}$$

M222-15:

$$\frac{1}{16}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{K}W_{bb'}*S_{bb'}+SW_{bb'}*\mathcal{K}_{bb'})*B_{bb'}+\mathcal{K}_{bb'}*S_{bb'}*BW_{bb'}$$

M222-16:

$$\frac{1}{18}\sum_{b=1}^{B}\sum_{b'=1}^{B}(KW_{bb'}*S_{bb'}+SW_{bb'}*K_{bb'})*B_{bb'}+K_{bb'}*S_{bb'}*BW_{bb'}$$

M222-17:

$$\frac{1}{5}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{H}W_{bb'}*\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}*\mathcal{H}_{bb'})*\mathcal{B}_{bb'}$$

M222-18:

$$\frac{1}{6}\sum_{b=1}^{B}\sum_{b'=1}^{B}(HW_{bb'}*\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}*H_{bb'})*\mathcal{B}_{bb'}$$

M222-19:

$$\frac{1}{6}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{H}W_{bb'}*\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}*\mathcal{H}_{bb'})*B_{bb'}+\mathcal{H}_{bb'}*\mathcal{T}_{bb'}*BW_{bb'}$$

M222-20:

$$\frac{1}{6}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{H}W_{bb'}*T_{bb'}+TW_{bb'}*\mathcal{H}_{bb'})*\mathcal{B}_{bb'}$$

M222-21:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(HW_{bb'}*\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}*H_{bb'})*B_{bb'}+H_{bb'}*\mathcal{T}_{bb'}*BW_{bb'}$$

M222-22:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(HW_{bb'}*T_{bb'}+TW_{bb'}*H_{bb'})*\mathcal{B}_{bb'}$$

M222-23:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{H}W_{bb'}*T_{bb'}+TW_{bb'}*\mathcal{H}_{bb'})*B_{bb'}+\mathcal{H}_{bb'}*T_{bb'}*BW_{bb'}$$

M222-24:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(HW_{bb'}*T_{bb'}+TW_{bb'}*H_{bb'})*B_{bb'}+H_{bb'}*T_{bb'}*BW_{bb'}$$

M222-25:

$$\frac{1}{6}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{I}W_{bb'}*\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}*\mathcal{I}_{bb'})*\mathcal{B}_{bb'}$$

M222-26:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(IW_{bb'}*\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}*I_{bb'})*\mathcal{B}_{bb'}$$

M222-27:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{I}W_{bb'}*\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}*\mathcal{I}_{bb'})*B_{bb'}+\mathcal{I}_{bb'}*\mathcal{T}_{bb'}*BW_{bb'}$$

M222-28:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{I}W_{bb'}*T_{bb'}+TW_{bb'}*\mathcal{I}_{bb'})*\mathcal{B}_{bb'}$$

M222-29:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(IW_{bb'}*\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}*I_{bb'})*B_{bb'}+I_{bb'}*\mathcal{T}_{bb'}*BW_{bb'}$$

M222-30:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(IW_{bb'}*T_{bb'}+TW_{bb'}*I_{bb'})*\mathcal{B}_{bb'}$$

M222-31:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{I}W_{bb'}*T_{bb'}+TW_{bb'}*\mathcal{I}_{bb'})*B_{bb'}+\mathcal{I}_{bb'}*T_{bb'}*BW_{bb'}$$

M222-32:

$$\frac{1}{9}\sum_{b=1}^{B}\sum_{b'=1}^{B}(IW_{bb'}*T_{bb'}+TW_{bb'}*I_{bb'})*B_{bb'}+I_{bb'}*T_{bb'}*BW_{bb'}$$

M222-33:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{K}W_{bb'}*\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}*\mathcal{K}_{bb'})*\mathcal{B}_{bb'}$$

M222-34:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(KW_{bb'}*\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}*K_{bb'})*\mathcal{B}_{bb'}$$

M222-35:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{K}W_{bb'}*\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}*\mathcal{K}_{bb'})*B_{bb'}+\mathcal{K}_{bb'}*\mathcal{T}_{bb'}*BW_{bb'}$$

M222-36:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{K}W_{bb'}*T_{bb'}+TW_{bb'}*\mathcal{K}_{bb'})*\mathcal{B}_{bb'}$$

M222-37:

$$\frac{1}{9}\sum_{b=1}^{B}\sum_{b'=1}^{B}(KW_{bb'}*\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}*K_{bb'})*B_{bb'}+K_{bb'}*\mathcal{T}_{bb'}*BW_{bb'}$$

M222-38:

$$\frac{1}{9}\sum_{b=1}^{B}\sum_{b'=1}^{B}(KW_{bb'}*T_{bb'}+TW_{bb'}*K_{bb'})*\mathcal{B}_{bb'}$$

M222-39:

$$\frac{1}{9}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{K}W_{bb'}*T_{bb'}+TW_{bb'}*\mathcal{K}_{bb'})*B_{bb'}+\mathcal{K}_{bb'}*T_{bb'}*BW_{bb'}$$

M222-40:

$$\frac{1}{10}\sum_{b=1}^{B}\sum_{b'=1}^{B}(KW_{bb'}*T_{bb'}+TW_{bb'}*K_{bb'})*B_{bb'}+K_{bb'}*T_{bb'}*BW_{bb'}$$

M222-41:

$$\frac{1}{12}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{H}W_{bb'}*\mathcal{Z}_{bb'}+\mathcal{Z}W_{bb'}*\mathcal{H}_{bb'})*\mathcal{B}_{bb'}$$

M222-42:

$$\frac{1}{14}\sum_{b=1}^{B}\sum_{b'=1}^{B}(HW_{bb'}*\mathcal{Z}_{bb'}+\mathcal{Z}W_{bb'}*H_{bb'})*\mathcal{B}_{bb'}$$

M222-43:

$$\frac{1}{14}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{H}W_{bb'}*\mathcal{Z}_{bb'}+\mathcal{Z}W_{bb'}*\mathcal{H}_{bb'})*B_{bb'}+\mathcal{H}_{bb'}*\mathcal{Z}_{bb'}*BW_{bb'}$$

M222-44:

$$\frac{1}{14}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{H}W_{bb'}*Z_{bb'}+ZW_{bb'}*\mathcal{H}_{bb'})*\mathcal{B}_{bb'}$$

M222-45:

$$\frac{1}{16}\sum_{b=1}^{B}\sum_{b'=1}^{B}(HW_{bb'}*\mathcal{Z}_{bb'}+\mathcal{Z}W_{bb'}*H_{bb'})*B_{bb'}+H_{bb'}*\mathcal{Z}_{bb'}*BW_{bb'}$$

M222-46:

$$\frac{1}{16}\sum_{b=1}^{B}\sum_{b'=1}^{B}(HW_{bb'}*Z_{bb'}+ZW_{bb'}*H_{bb'})*\mathcal{B}_{bb'}$$

M222-47:

$$\frac{1}{16}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{H}W_{bb'}*Z_{bb'}+ZW_{bb'}*\mathcal{H}_{bb'})*B_{bb'}+\mathcal{H}_{bb'}*Z_{bb'}*BW_{bb'}$$

M222-48:

$$\frac{1}{18}\sum_{b=1}^{B}\sum_{b'=1}^{B}(HW_{bb'}*Z_{bb'}+ZW_{bb'}*H_{bb'})*B_{bb'}+H_{bb'}*Z_{bb'}*BW_{bb'}$$

M222-49:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{I}W_{bb'}*\mathcal{Z}_{bb'}+\mathcal{Z}W_{bb'}*\mathcal{I}_{bb'})*\mathcal{B}_{bb'}$$

M222-50:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(IW_{bb'}*\mathcal{Z}_{bb'}+\mathcal{Z}W_{bb'}*I_{bb'})*\mathcal{B}_{bb'}$$

M222-51:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{I}W_{bb'}*\mathcal{Z}_{bb'}+\mathcal{Z}W_{bb'}*\mathcal{I}_{bb'})*B_{bb'}+\mathcal{I}_{bb'}*\mathcal{Z}_{bb'}*BW_{bb'}$$

M222-52:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{I}W_{bb'}*Z_{bb'}+ZW_{bb'}*\mathcal{I}_{bb'})*\mathcal{B}_{bb'}$$

M222-53:

$$\frac{1}{9}\sum_{b=1}^{B}\sum_{b'=1}^{B}(IW_{bb'}*\mathcal{Z}_{bb'} + \mathcal{Z}W_{bb'}*I_{bb'})*B_{bb'} + I_{bb'}*\mathcal{Z}_{bb'}*BW_{bb'}$$

M222-54:

$$\frac{1}{9}\sum_{b=1}^{B}\sum_{b'=1}^{B}(IW_{bb'}*Z_{bb'} + ZW_{bb'}*I_{bb'})*\mathcal{B}_{bb'}$$

M222-55:

$$\frac{1}{9}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{I}W_{bb'}*Z_{bb'} + ZW_{bb'}*\mathcal{I}_{bb'})*B_{bb'} + \mathcal{I}_{bb'}*Z_{bb'}*BW_{bb'}$$

M222-56:

$$\frac{1}{10}\sum_{b=1}^{B}\sum_{b'=1}^{B}(IW_{bb'}*Z_{bb'} + ZW_{bb'}*I_{bb'})*B_{bb'} + I_{bb'}*Z_{bb'}*BW_{bb'}$$

M222-57:

$$\frac{1}{16}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{K}W_{bb'}*\mathcal{Z}_{bb'} + \mathcal{Z}W_{bb'}*\mathcal{K}_{bb'})*\mathcal{B}_{bb'}$$

M222-58:

$$\frac{1}{18}\sum_{b=1}^{B}\sum_{b'=1}^{B}(KW_{bb'}*\mathcal{Z}_{bb'} + \mathcal{Z}W_{bb'}*K_{bb'})*\mathcal{B}_{bb'}$$

M222-59:

$$\frac{1}{18}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{K}W_{bb'}*\mathcal{Z}_{bb'} + \mathcal{Z}W_{bb'}*\mathcal{K}_{bb'})*B_{bb'} + \mathcal{K}_{bb'}*\mathcal{Z}_{bb'}*BW_{bb'}$$

M222-60:

$$\frac{1}{18}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{K}W_{bb'}*Z_{bb'} + ZW_{bb'}*\mathcal{K}_{bb'})*\mathcal{B}_{bb'}$$

M222-61:

$$\frac{1}{20}\sum_{b=1}^{B}\sum_{b'=1}^{B}(KW_{bb'}*\mathcal{Z}_{bb'} + \mathcal{Z}W_{bb'}*K_{bb'})*B_{bb'} + K_{bb'}*\mathcal{Z}_{bb'}*BW_{bb'}$$

M222-62:

$$\frac{1}{20}\sum_{b=1}^{B}\sum_{b'=1}^{B}(KW_{bb'}*Z_{bb'} + ZW_{bb'}*K_{bb'})*\mathcal{B}_{bb'}$$

M222-63:

$$\frac{1}{20}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{K}W_{bb'}*Z_{bb'} + ZW_{bb'}*\mathcal{K}_{bb'})*B_{bb'} + \mathcal{K}_{bb'}*Z_{bb'}*BW_{bb'}$$

M222-64:

$$\frac{1}{22}\sum_{b=1}^{B}\sum_{b'=1}^{B}(KW_{bb'}{*}Z_{bb'}+ZW_{bb'}{*}K_{bb'}){*}B_{bb'}+K_{bb'}{*}Z_{bb'}{*}BW_{bb'}$$

M222-65:

$$\frac{1}{6}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{J}W_{bb'}{*}\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}{*}\mathcal{J}_{bb'}){*}\mathcal{B}_{bb'}$$

M222-66:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(JW_{bb'}{*}\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}{*}J_{bb'}){*}\mathcal{B}_{bb'}$$

M222-67:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{J}W_{bb'}{*}\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}{*}\mathcal{J}_{bb'}){*}B_{bb'}+\mathcal{J}_{bb'}{*}\mathcal{T}_{bb'}{*}BW_{bb'}$$

M222-68:

$$\frac{1}{7}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{J}W_{bb'}{*}T_{bb'}+TW_{bb'}{*}\mathcal{J}_{bb'}){*}\mathcal{B}_{bb'}$$

M222-69:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(JW_{bb'}{*}\mathcal{T}_{bb'}+\mathcal{T}W_{bb'}{*}J_{bb'}){*}B_{bb'}+J_{bb'}{*}\mathcal{T}_{bb'}{*}BW_{bb'}$$

M222-70:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(JW_{bb'}{*}T_{bb'}+TW_{bb'}{*}J_{bb'}){*}\mathcal{B}_{bb'}$$

M222-71:

$$\frac{1}{8}\sum_{b=1}^{B}\sum_{b'=1}^{B}(\mathcal{J}W_{bb'}{*}T_{bb'}+TW_{bb'}{*}\mathcal{J}_{bb'}){*}B_{bb'}+\mathcal{J}_{bb'}{*}T_{bb'}{*}BW_{bb'}$$

M222-72:

$$\frac{1}{9}\sum_{b=1}^{B}\sum_{b'=1}^{B}(JW_{bb'}{*}T_{bb'}+TW_{bb'}{*}J_{bb'}){*}B_{bb'}+J_{bb'}{*}T_{bb'}{*}BW_{bb'}$$