

# Random Permutation Maxout (RPM) transform

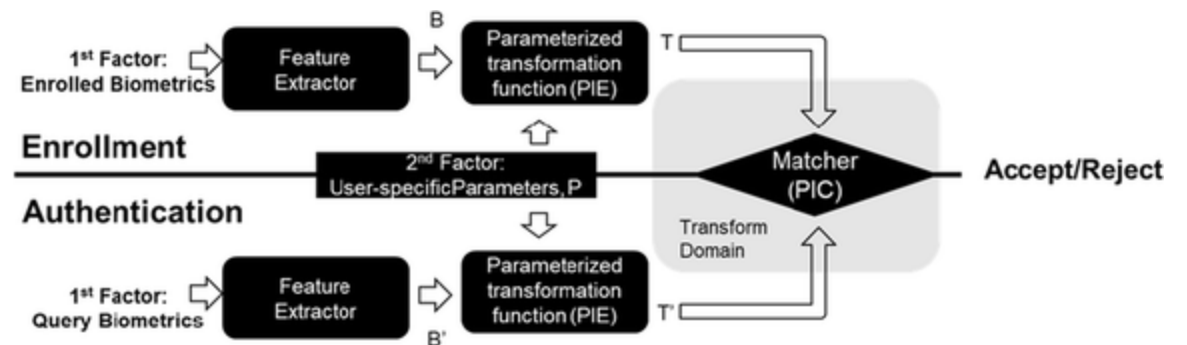
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# Motivation

- 生物特征是不变的，表达主体丰富的生物信息
- 如果主体的生物特征被盗，可能会发生冒名顶替和隐私入侵的情况

# Cancellable biometrics construct

- Pseudonymous Identifier Encoder (PIE)
  - Generate the corresponding protected template  $T = PIE(B, P)$
  - Parameter: a biometric signal or template  $B$ , an user-specific parameter  $P$
- Pseudonymous Identifier Comparator (PIC)
  - Compare pairs of  $T$ 's during authentication



# Methodology description (Overview)

Suppose  $\mathbf{x} \in \mathbb{R}^d$  be a biometric feature vector with length  $d$ , RPM transform is carried out in the following steps:

1. Generate  $p$  user-specific token-seeded stacked  $m$  permutation array  $P_i \in \{0, 1\}^{d \times d \times m}, i = 1, \dots, p$  where  $m$  is the desired length of resulting RPM transformed vector. Note that a permutation matrix is a square binary matrix that has exactly one entry of 1 in each row and each column and 0s elsewhere.
2. Multiplying  $\mathbf{x}$  to  $P_i$ , yield a matrix,  $V_i = \mathbf{x}P_i \in \mathbb{R}^{d \times m}, i = 1, \dots, p$ .
3. Perform Hadamard product (element-wise product) yields  $\mathbf{Y} = V_1 \circ \dots \circ V_p = \prod_{i=1}^p V_i \in \mathbb{R}^{d \times m}$  where  $p$  is known as Hadamard product order.
4. Discard last  $d - k$  **column** vectors from  $\mathbf{Y}$ , yield  $\mathbf{Y}' \in \mathbb{R}^{k \times m}$ , where we named  $k$  as truncation size.
5. For each **row** vectors of  $\mathbf{Y}'$ , the indices of the largest magnitude entry are recorded and form a discrete RPM transformed vector,  $\mathbf{z} \in [1 \ k]^m$ .

# Methodology description (Step 0)

Suppose  $\boldsymbol{x} \in \mathbb{R}^d$  be a biometric feature vector with length  $d$ .  
 $d = 5$

|      |
|------|
| 2.1  |
| -1.2 |
| 0.3  |
| 1.7  |
| -1.4 |

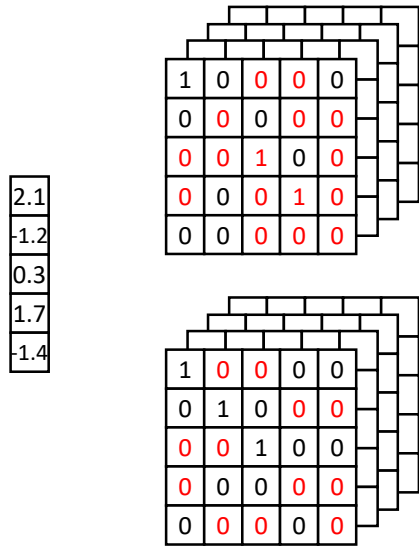
$\boldsymbol{x} \in \mathbb{R}^5$

# Methodology description (Step 1)

$$p = 2$$

$$m = 4$$

Generate  $p$  user-specific token-seeded stacked  $m$  permutation array  $P_i \in \{0, 1\}^{d \times d \times m}$ ,  $i = 1, \dots, p$  where  $m$  is the desired length of resulting RPM transformed vector. **Note that a permutation matrix is a square binary matrix that has exactly one entry of 1 in each row and each column and 0s elsewhere.**



$$x \in \mathbb{R}^5$$

$$P_i \in \{0, 1\}^{5 \times 5 \times 4} (i = 1, 2)$$

# Methodology description (Step 2)

Multiplying  $\mathbf{x}$  to  $P_i$ , yield a matrix,  $\mathbf{V}_i = \mathbf{x}P_i \in \mathbb{R}^{d \times m}, i = 1, \dots, p$ .

The diagram illustrates the multiplication of a vector  $\mathbf{x} \in \mathbb{R}^5$  by a stack of matrices  $\mathcal{P}_i \in \{0, 1\}^{5 \times 5 \times 4}$  (for  $i = 1, 2$ ) to produce a stack of matrices  $\mathbf{V}_i = \mathbf{x}\mathcal{P}_i \in \mathbb{R}^{5 \times 4}$ .

**Vector  $\mathbf{x}$ :**

|      |
|------|
| 2.1  |
| -1.2 |
| 0.3  |
| 1.7  |
| -1.4 |

**Matrix  $\mathcal{P}_1$  (top):**

|   |   |   |   |   |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 |

**Matrix  $\mathcal{P}_2$  (bottom):**

|   |   |   |   |   |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

**Resulting Matrix  $\mathbf{V}_1$  (top):**

|     |      |     |     |      |
|-----|------|-----|-----|------|
| 2.1 | 0    | 0.3 | 1.7 | 0    |
| 0   | -1.2 | 0   | 1.7 | -1.4 |
| 2.1 | -1.2 | 1   | 0   | -1.4 |
| 2.1 | 0    | 0.3 | 0   | -1.4 |

**Resulting Matrix  $\mathbf{V}_2$  (bottom):**

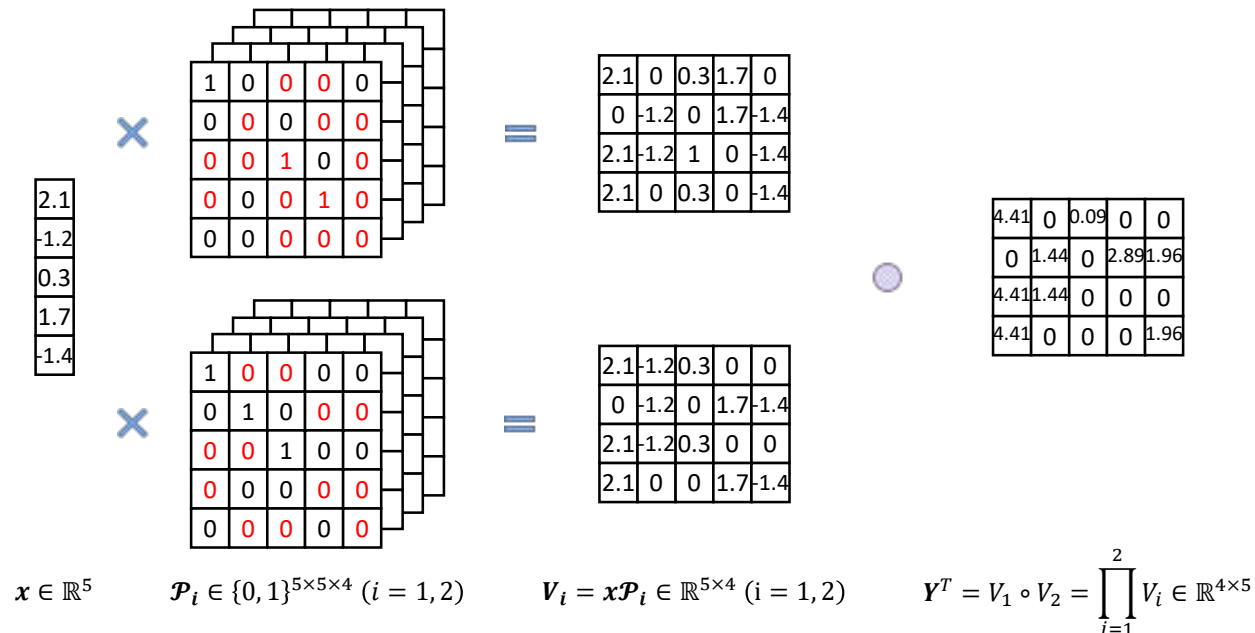
|     |      |     |     |      |
|-----|------|-----|-----|------|
| 2.1 | -1.2 | 0.3 | 0   | 0    |
| 0   | -1.2 | 0   | 1.7 | -1.4 |
| 2.1 | -1.2 | 0.3 | 0   | 0    |
| 2.1 | 0    | 0   | 1.7 | -1.4 |

$\mathbf{x} \in \mathbb{R}^5$        $\mathcal{P}_i \in \{0, 1\}^{5 \times 5 \times 4} (i = 1, 2)$        $\mathbf{V}_i = \mathbf{x}\mathcal{P}_i \in \mathbb{R}^{5 \times 4} (i = 1, 2)$

# Methodology description (Step 3)

Perform Hadamard product (element-wise product) yields  $\mathbf{Y} = \mathbf{V}_1 \circ \dots \circ \mathbf{V}_p = \prod_{i=1}^p \mathbf{V}_i \in \mathbb{R}^{d \times m}$  where  $p$  is known as Hadamard product order.

$$p = 2$$



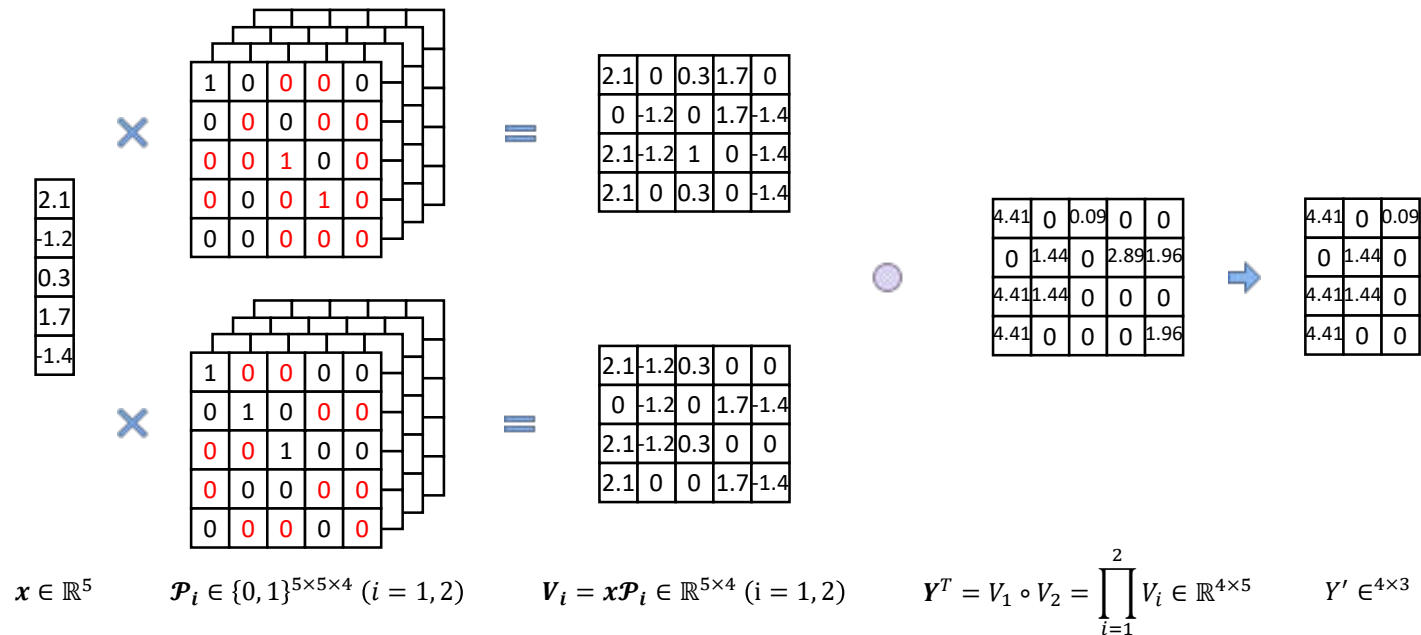


# Methodology description (Step 4)

$$d - k = 5 - 3 = 2$$

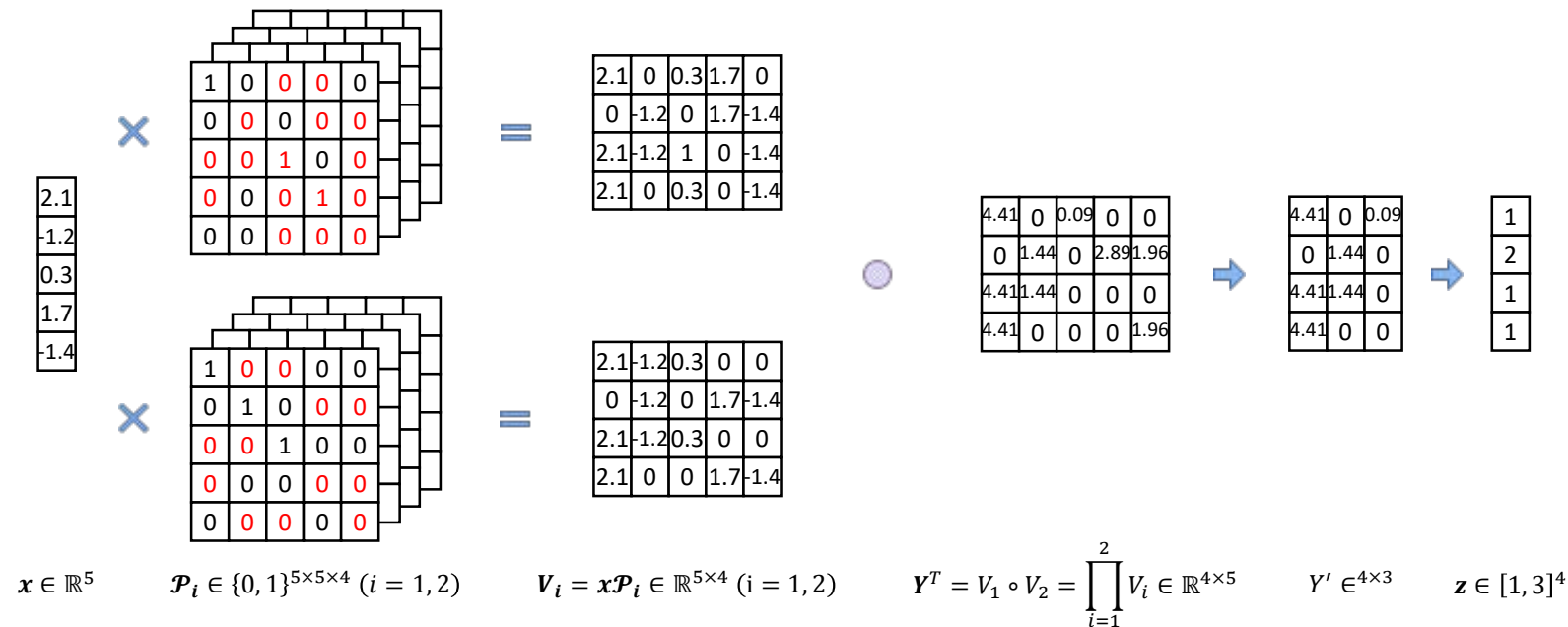
$$k = 3$$

Discard last  $d - k$  **column** vectors from  $Y$ , yield  $Y' \in \mathbb{R}^{k \times m}$ , where we named  $k$  as truncation size.



# Methodology description (Step 5)

For each **row** vectors of  $\mathbf{Y}'$ , the indices of the largest magnitude entry are recorded and form a discrete RPM transformed vector,  $\mathbf{z} \in [1 \ k]^m$ .



# Experimental setup (Benchmark)

- AR dataset
  - 99 subjects
    - 6 samples from expression and illumination groups
    - 12 samples from occlusion group
    - Size  $128 \times 128$
- FERET
  - 1196 individuals
    - 5 images
    - Size  $128 \times 128$

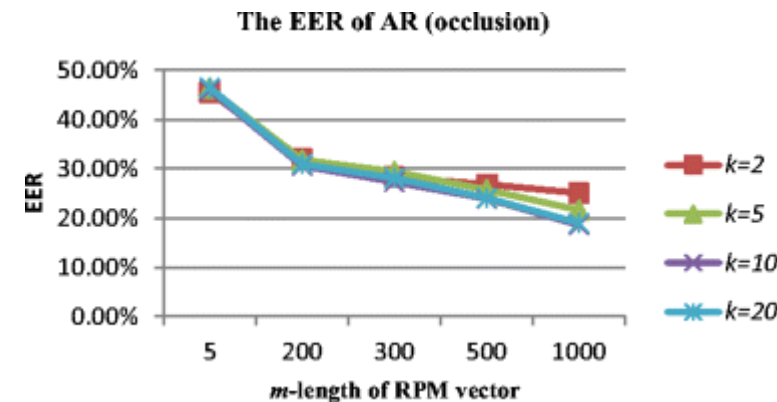
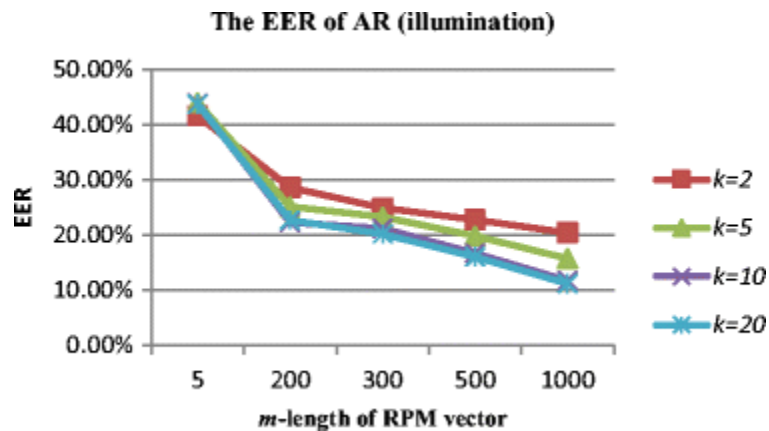
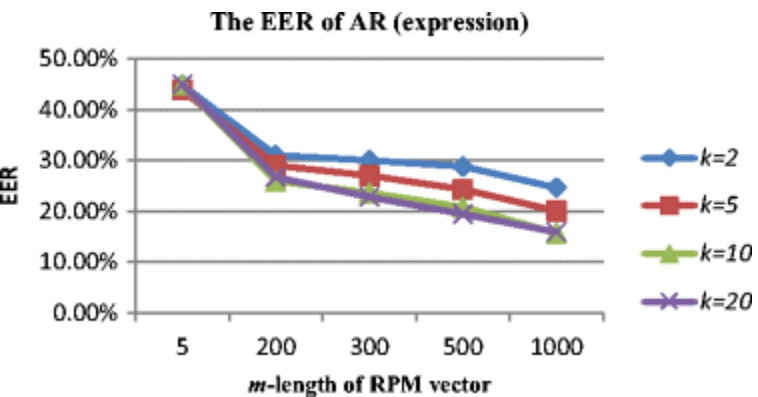
# Experimental setup (Indicator)

- Equal Error Rate (EER)
  - Estimate False Acceptance Rate (FAR) and False Rejection Rate (FRR)

# Experimental setup (Parameter)

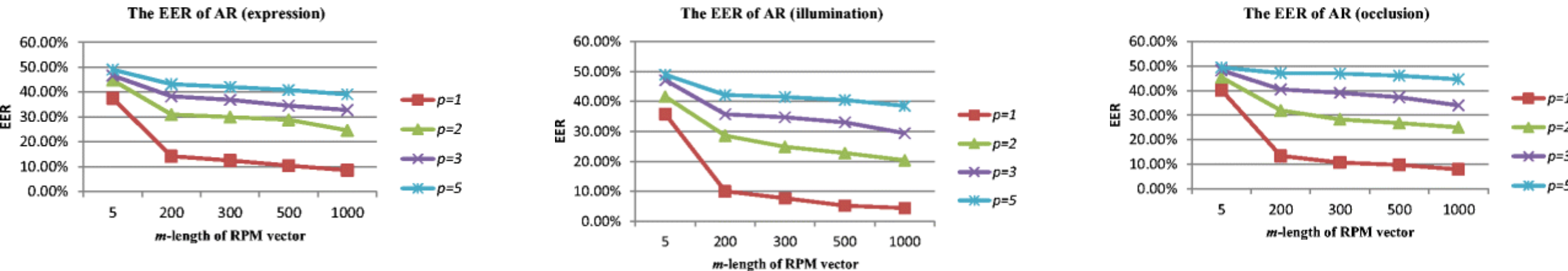
- Discrete Cosine Transform based Region Log-Tied Rank Covariance Matrices (DCT-RLTCM)
  - Filter size  $k = 11$
  - # of filter  $T = 30$
  - Region size  $r = 16$
  - WPCA dimension  $d = 300$

# Accuracy performance ( $m$ and $k$ fixing $p = 2$ )



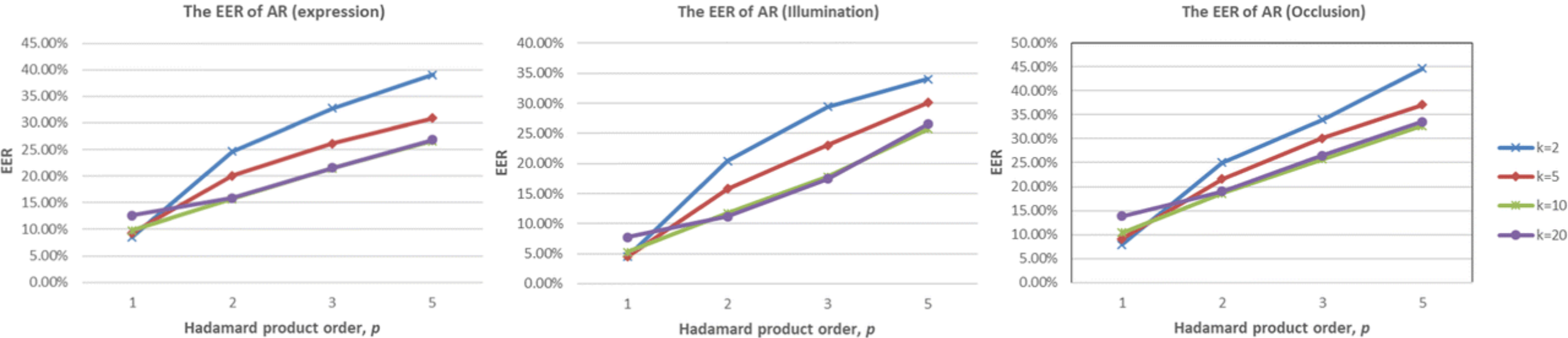
$k$  越小, max ranked feature 的信息越少

# Accuracy performance ( $p$ and $m$ fixing $k = 2$ )



$p$  越大, Hadamard product 的次数越多, 引入了噪声, 引起失真

# Accuracy performance ( $p$ and $k$ fixing $m = 1000$ )



最优参数  $\{p = 2, m = 1000, k = 20\}$



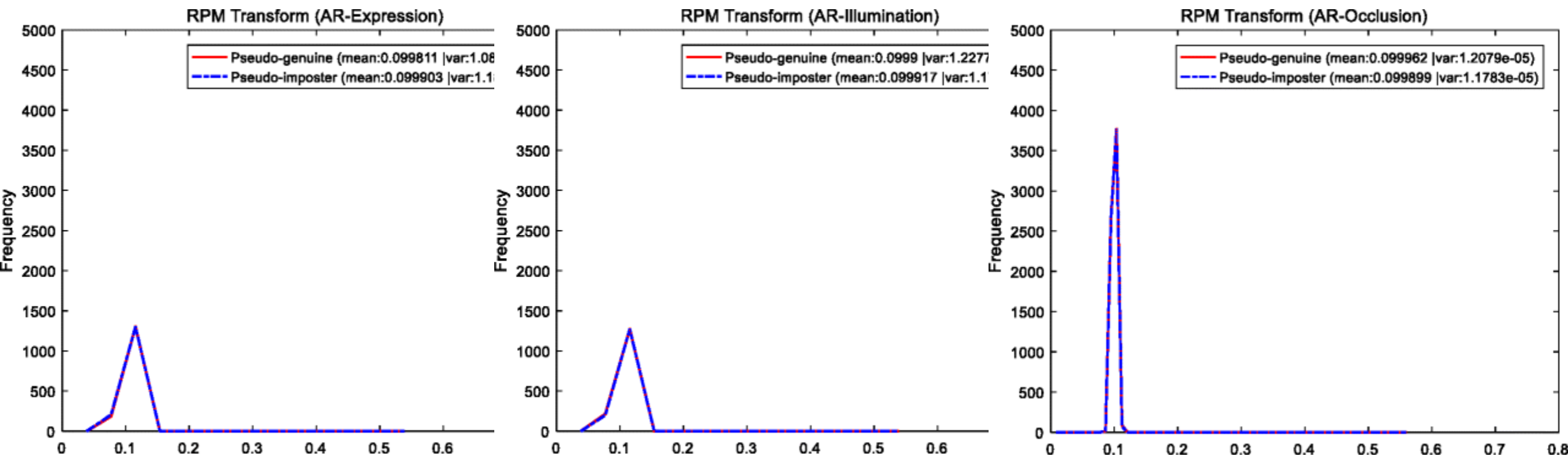
# Justification

Suppose  $x \in \mathbb{R}^d$  be a biometric feature vector with length  $d$ , RPM transform is carried out in the following steps:

用户改变用户特定的种子

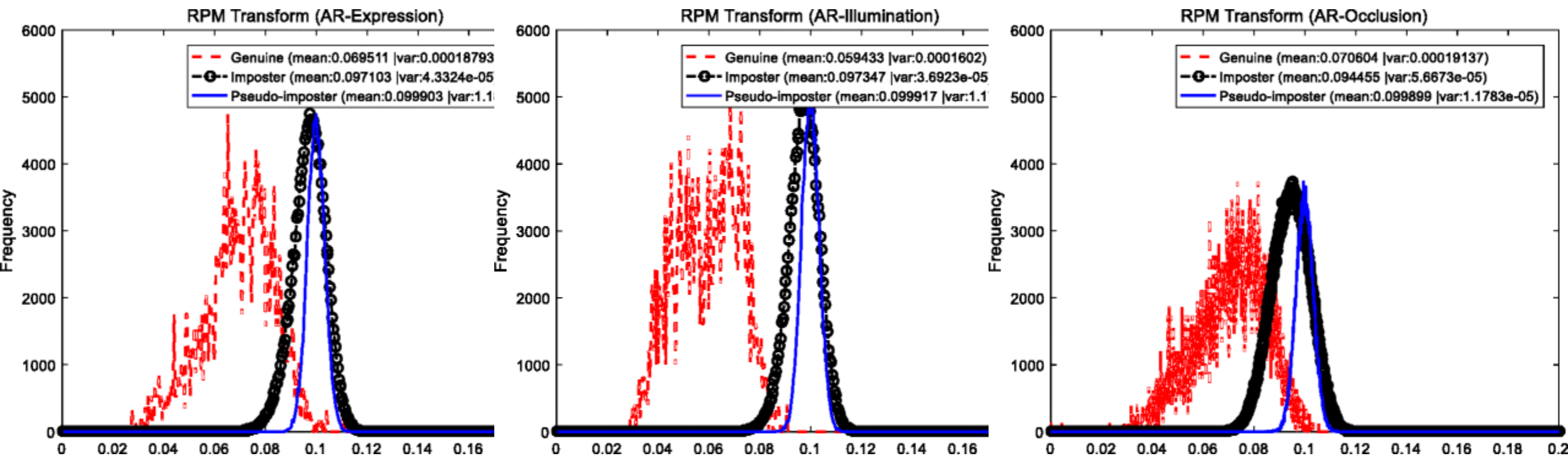
1. Generate  $p$  user-specific token-seeded stacked  $m$  permutation array  $P_i \in \{0, 1\}^{d \times d \times m}, i = 1, \dots, p$  where  $m$  is the desired length of resulting RPM transformed vector. Note that a permutation matrix is a square binary matrix that has exactly one entry of 1 in each row and each column and 0s elsewhere.   
salting
2. Multiplying  $x$  to  $P_i$ , yield a matrix,  $V_i = xP_i \in \mathbb{R}^{d \times m}, i = 1, \dots, p$ . revocation & unlikability
3. Perform Hadamard product (element-wise product) yields  $Y = V_1 \circ \dots \circ V_p = \prod_{i=1}^p V_i \in \mathbb{R}^{d \times m}$  where  $p$  is known as Hadamard product order.
4. Discard last  $d - k$  column vectors from  $Y$ , yield  $Y' \in \mathbb{R}^{k \times m}$ , where we named  $k$  as truncation size.
5. For each row vectors of  $Y'$ , the indices of the largest magnitude entry are recorded and form a discrete RPM transformed vector,  $z \in [1 \ k]^m$ .

# Non-linkability analysis



如果假阳率和假阴率的分布不同，敌手能够区分两个转换后的向量是否属于相同的主体

# Revocability analysis



给定 50 个随机扰动种子, 相同的源 DCT-RLTCM 向量, 生成的 RPM 向量是不同的

# Justification

Suppose  $\mathbf{x} \in \mathbb{R}^d$  be a biometric feature vector with length  $d$ , RPM transform is carried out in the following steps:

1. Generate  $p$  user-specific token-seeded stacked  $m$  permutation array  $P_i \in \{0, 1\}^{d \times d \times m}, i = 1, \dots, p$  where  $m$  is the desired length of resulting RPM transformed vector. Note that a permutation matrix is a square binary matrix that has exactly one entry of 1 in each row and each column and 0s elsewhere.
2. Multiplying  $\mathbf{x}$  to  $P_i$ , yield a matrix,  $V_i = \mathbf{x}P_i \in \mathbb{R}^{d \times m}, i = 1, \dots, p$ .
3. Perform Hadamard product (element-wise product) yields  $\mathbf{Y} = V_1 \circ \dots \circ V_p = \prod_{i=1}^p V_i \in \mathbb{R}^{d \times m}$  where  $p$  is known as Hadamard product order. Non-invertibility
4. Discard last  $d - k$  column vectors from  $\mathbf{Y}$ , yield  $\mathbf{Y}' \in \mathbb{R}^{k \times m}$ , where we named  $k$  as truncation size.
5. For each row vectors of  $\mathbf{Y}'$ , the indices of the largest magnitude entry are recorded and form a discrete RPM transformed vector,  $\mathbf{z} \in [1 \ k]^m$ .

# Privacy attack (Non-invertibility analysis)

- 假设敌手获取转换后的向量、令牌, RPM 算法及其参数
- 在 min-max 范围内枚举生成伪 RPM 向量

| AR           | Min value with 4 decimal precision | Max value with 4 decimal precision | (Max-Min)1000            | Trial numbers                       |
|--------------|------------------------------------|------------------------------------|--------------------------|-------------------------------------|
| Expression   | -0.1571                            | 0.1561                             | 3132( $\approx 2^{12}$ ) | $\approx (2^{12})^{300} = 2^{3600}$ |
| Illumination | -0.1156                            | 0.1374                             | 2530( $\approx 2^{11}$ ) | $\approx (2^{11})^{300} = 2^{3300}$ |
| Occlusion    | -0.1116                            | 0.1157                             | 2273( $\approx 2^{11}$ ) | $\approx (2^{11})^{300} = 2^{3300}$ |