Exercice 6

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$$\langle f, g \rangle = \int_0^{\pi/2} f(t)\, g(t)\, dt$$

$F = \text{Vect}(\cos, \sin)$ $f : \mathbb{R} \to \mathbb{R}$

$\forall t \in \mathbb{R}$ $f(t) = t+1$.

F : $\cos(x)$ $\sin(x)$

$$\| \cos(t) \|^2 = \langle \cos(t), \cos(t) \rangle$$

$$I = \int_0^{\frac{\pi}{2}} \cos(t) \cos(t)\, dt = \Big[\sin(t) \cos(t) \Big]_0^{\frac{\pi}{2}}$$

$$+ \int_0^{\frac{\pi}{2}} \sin^2(t) = \int_0^{\frac{\pi}{2}} (1 - \cos^2(t))\, dt$$

$$= \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \cos^2(t)\, dt$$

On a alors $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2(t)\, dt = \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \cos^2(t)\, dt$

$$= \frac{\pi}{2}$$

$2 \times I = \frac{\pi}{2}$ et $I = \frac{\pi}{4}$

$$\mathcal{E}_1 = \frac{\cos(x)}{\|\cos(x)\|} = \frac{\cos(x)}{\sqrt{\frac{\pi}{4}}}$$

$$\tilde{\mathcal{E}}_2 = \sin(x) - \langle \sin(x), \mathcal{E}_1 \rangle \mathcal{E}_1$$

$$\tilde{\mathcal{E}}_2 = \sin(x) - \left(\int_0^{\frac{\pi}{2}} \sin(x) \frac{\cos(x)}{\sqrt{\frac{\pi}{4}}} \, dt \right) \frac{\cos(x)}{\sqrt{\frac{\pi}{4}}} dt$$

Utilisation de wolfram pour l'intégrale

$$\tilde{\mathcal{E}}_2 = \sin(x) - \frac{2}{\pi} \cos(x)$$

$$\mathcal{E}_2 = \frac{\tilde{\mathcal{E}}_2}{\|\tilde{\mathcal{E}}_2\|}$$

$$\sqrt{\langle \tilde{\mathcal{E}}_2, \tilde{\mathcal{E}}_2 \rangle} = \sqrt{\int_0^{\frac{\pi}{2}} \left(\sin(x) - \frac{2}{\pi}\cos(x)\right)^2 dx}$$

Par wolfram on trouve

$$\|\tilde{\mathcal{E}}_2\| = \frac{\pi}{4} - \frac{1}{\pi}$$

$$\mathcal{E}_2 = \frac{\sin(x) - \frac{2}{\pi}\cos(x)}{\frac{\pi}{4} - \frac{1}{\pi}}$$

Une base orthonormale de

F est $(\varepsilon_1, \varepsilon_2)$

avec $\varepsilon_1 = \dfrac{\cos(x)}{\sqrt{\frac{\pi}{4}}}$

et $\varepsilon_2 = \dfrac{\sin(x) - \frac{2}{\pi}\cos(x)}{\frac{\pi}{4} - \frac{1}{\pi}}$