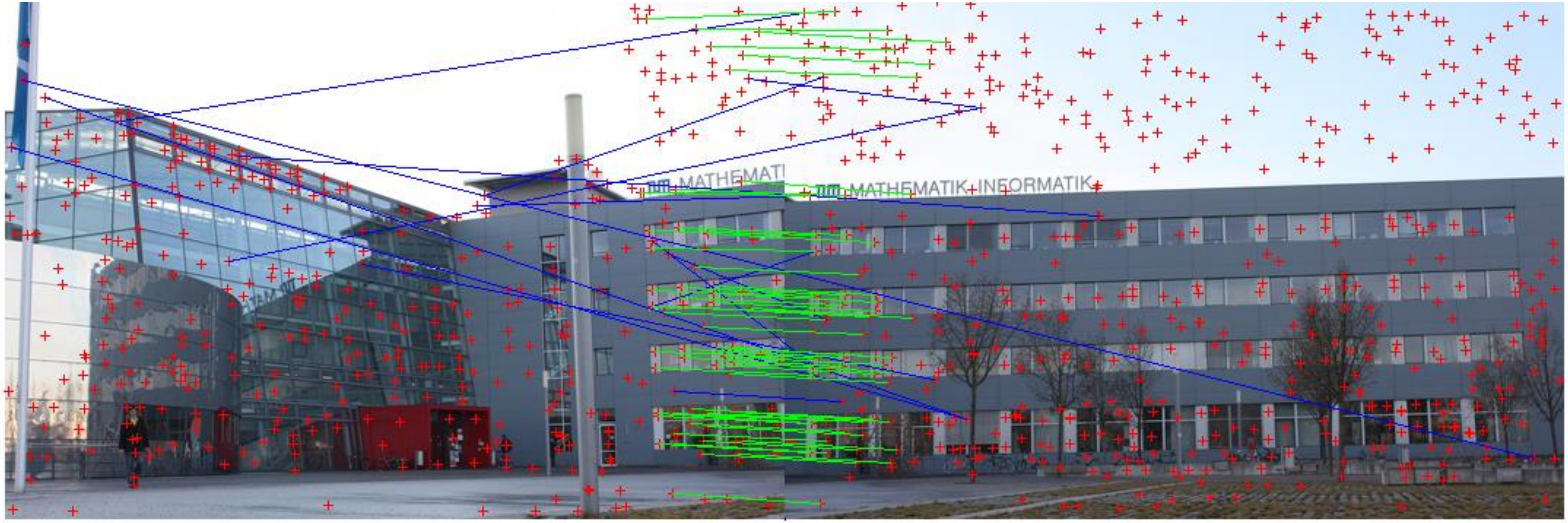
Features



References

- http://szeliski.org/Book/
- http://www.cs.cornell.edu/courses/cs5670/2019sp/lectures/lectures.html
- http://www.cs.cmu.edu/~16385/
- https://towardsdatascience.com/sift-scale-invariant-feature-transform-c7233dc60f37

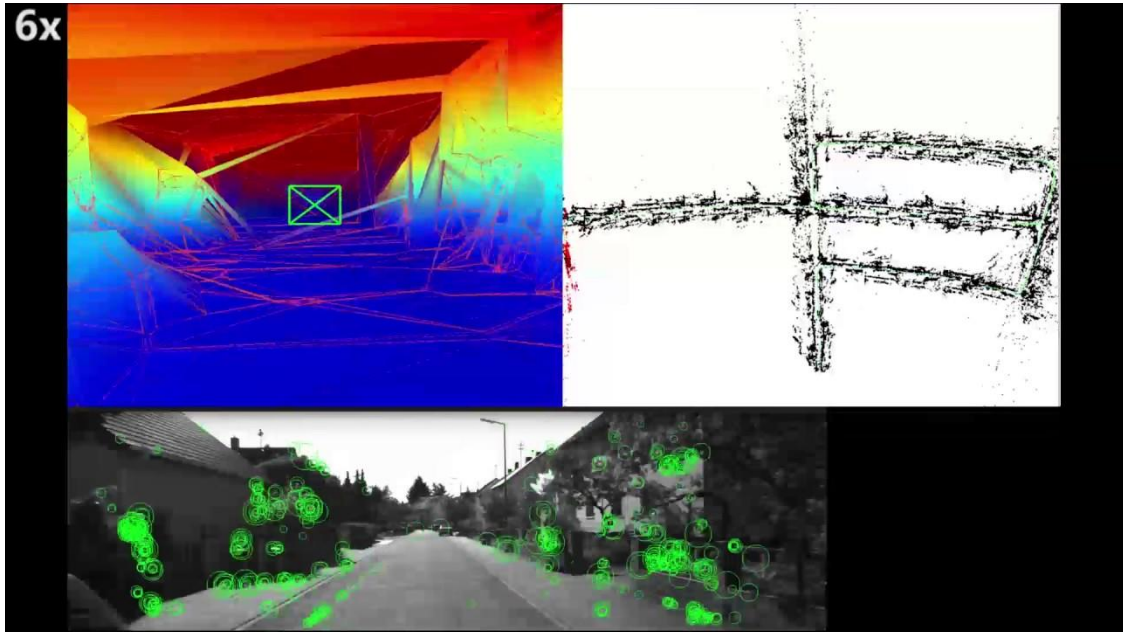
contents

- **What and why we need features detection?**
- Feature detection
  - Blob detection
  - Harris corner detection
  - SIFT detector
- Feature description
  - Template matching
  - HOG
  - SIFT descriptor
- SIFT feature matching
- Panoramas

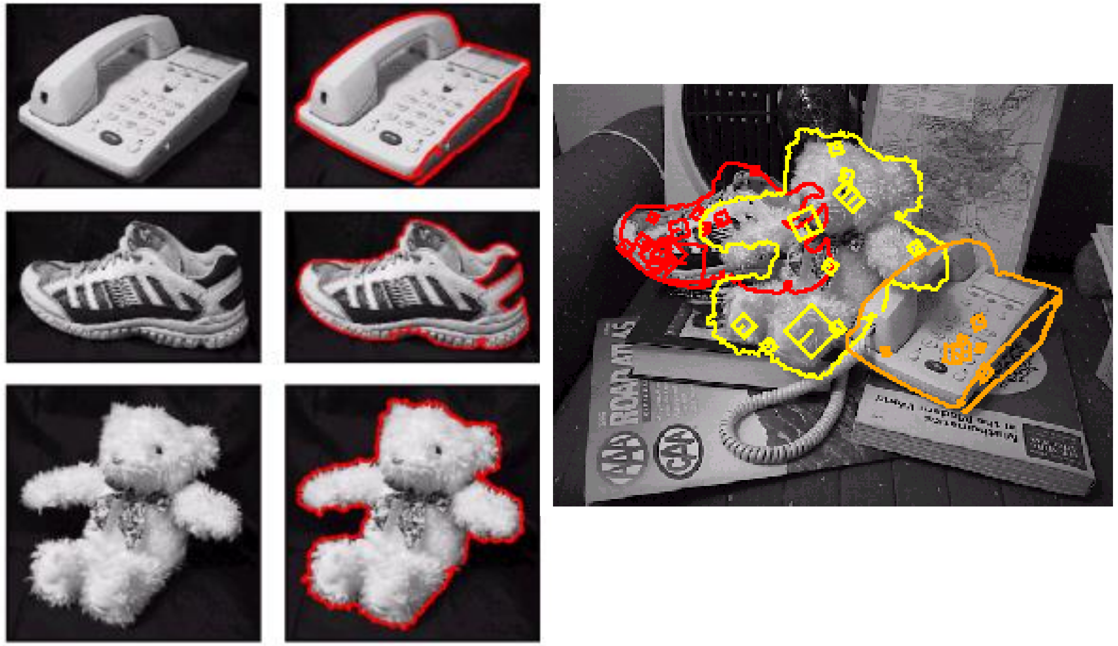
What is a feature?

- There is no universal or exact definition of what constitutes a feature, and the exact definition often depends on the problem or the type of application. Given that, a feature is defined as an **"interesting"** part of an image.
  - [from: Wikipedia]

What can we do with features?



Robot navigation

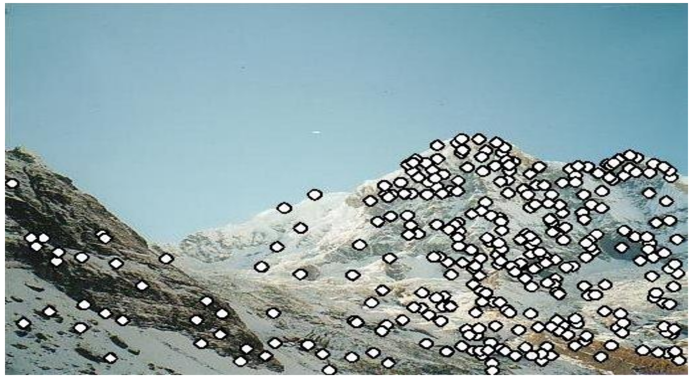


Object recognition

Panorama stitching

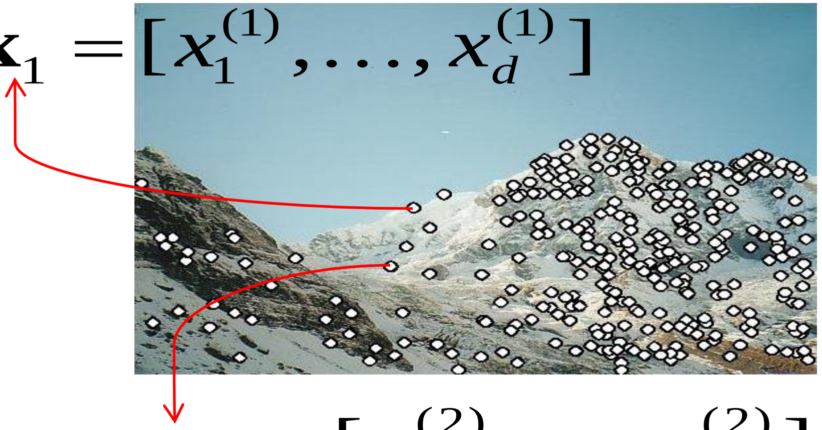
Local features: main components

1. Detection: Identify the interest points (also called **keypoints**).



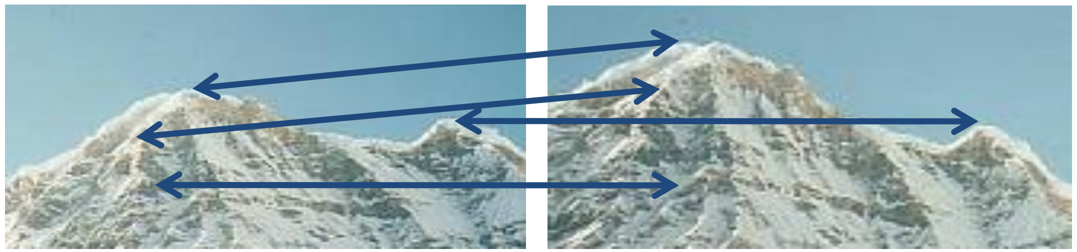
2. Description: Extract vector feature descriptor surrounding each interest point.

$$\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$$



$$\mathbf{x}_2 = [x_1^{(2)}, \dots, x_d^{(2)}]$$

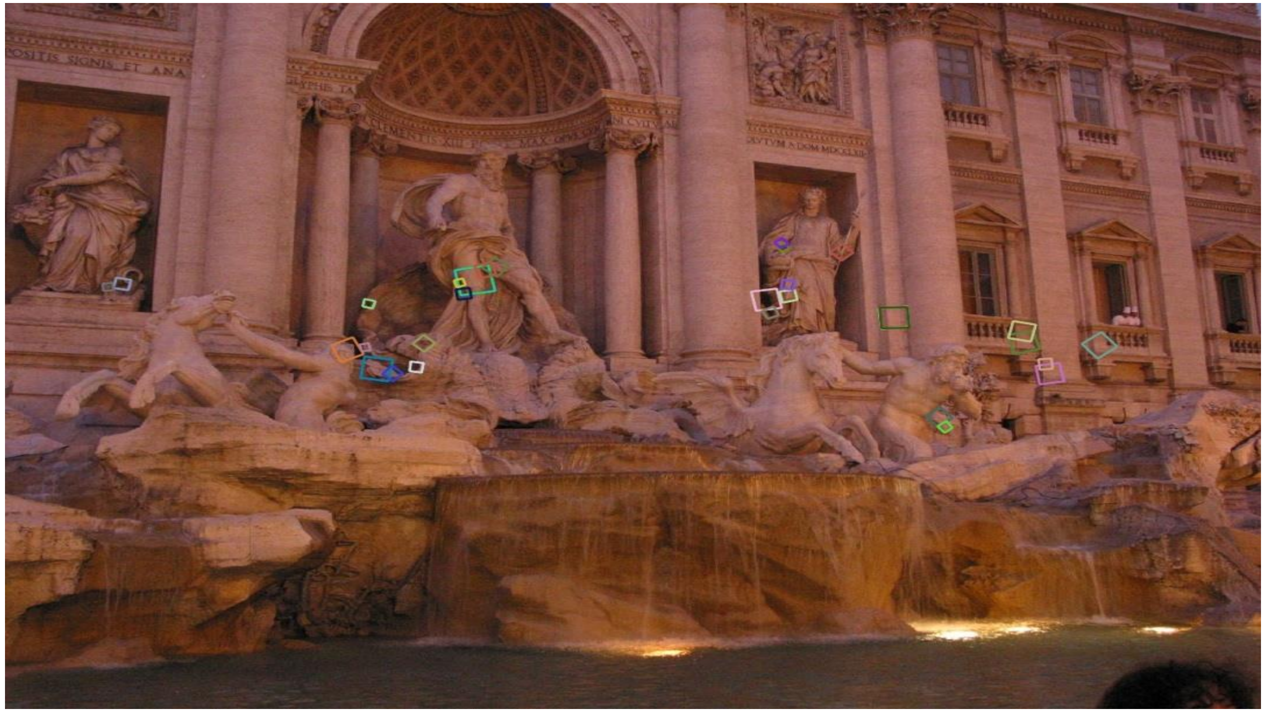
3. Matching: Determine correspondence between descriptors in two views.



Properties of SIFT

- **SIFT: scale invariant feature transform.**
- Extraordinarily robust matching technique for local keypoint detection description and matching.
- Can handle changes in viewpoint: 3D change of POV, scale, rotation and translation.
  - Up to about 60 degree out of plane rotation.
- Can handle significant changes in illumination.
  - Sometimes even day vs. night.
- Fast and efficient—can run in real time.

SIFT example



contents

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Advantages of local keypoints

Locality:

- features are local, so robust to occlusion and clutter => invariance to 3D change of viewpoint.

Quantity:

- hundreds or thousands in a single image

Distinctiveness:

- can differentiate a large database of objects

Efficiency

- real-time performance achievable

keypoints

- Keypoints should be a unique point of the image where all close neighbors are very different from.
- First attempt: lets take a blob in the image which has a very different surrounding, and this will be our keypoint.

contents

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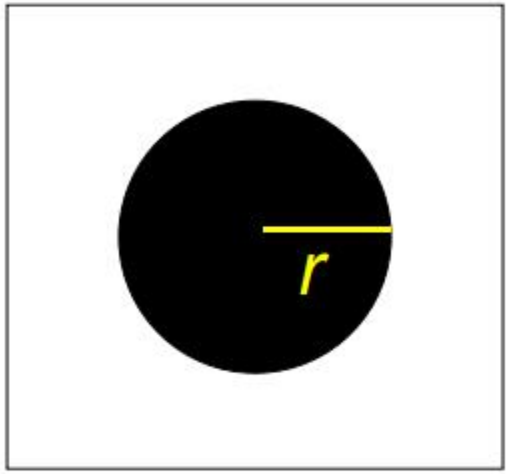
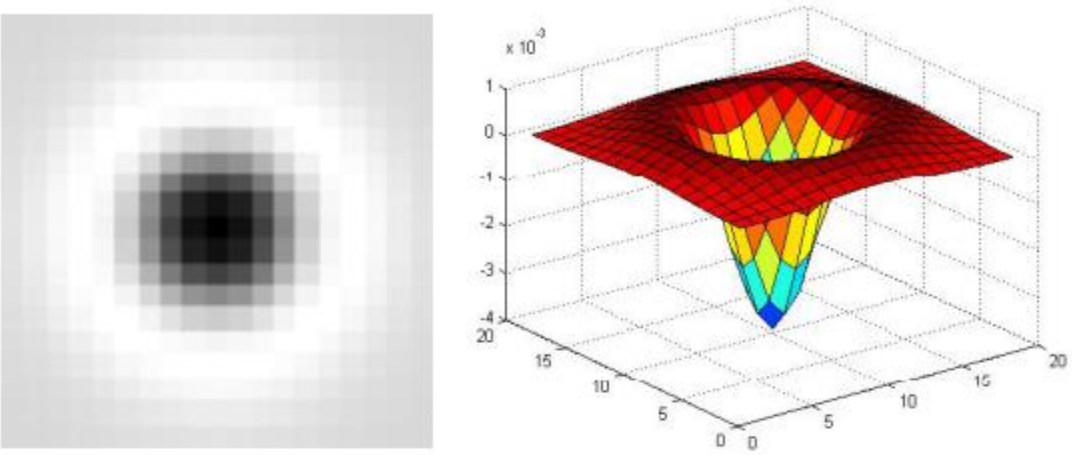
Blob detection

- Essentially cross correlation- we are convolving a signal with a template of a LoG (Laplacian of Gaussian) to get the highest response when the template matches the signal.

$$\mathrm{LoG}(x,y;\sigma) = \Delta_{(x,y)}G(x,y;\sigma) =$$
$$\frac{\partial^2 G(x,y;\sigma)}{\partial x^2} + \frac{\partial^2 G(x,y;\sigma)}{\partial y^2} = \frac{1}{\pi\sigma^4}\left(\frac{x^2+y^2}{2\sigma^2} - 1\right)\mathrm{e}^{-\frac{x^2+y^2}{2\sigma^2}}$$

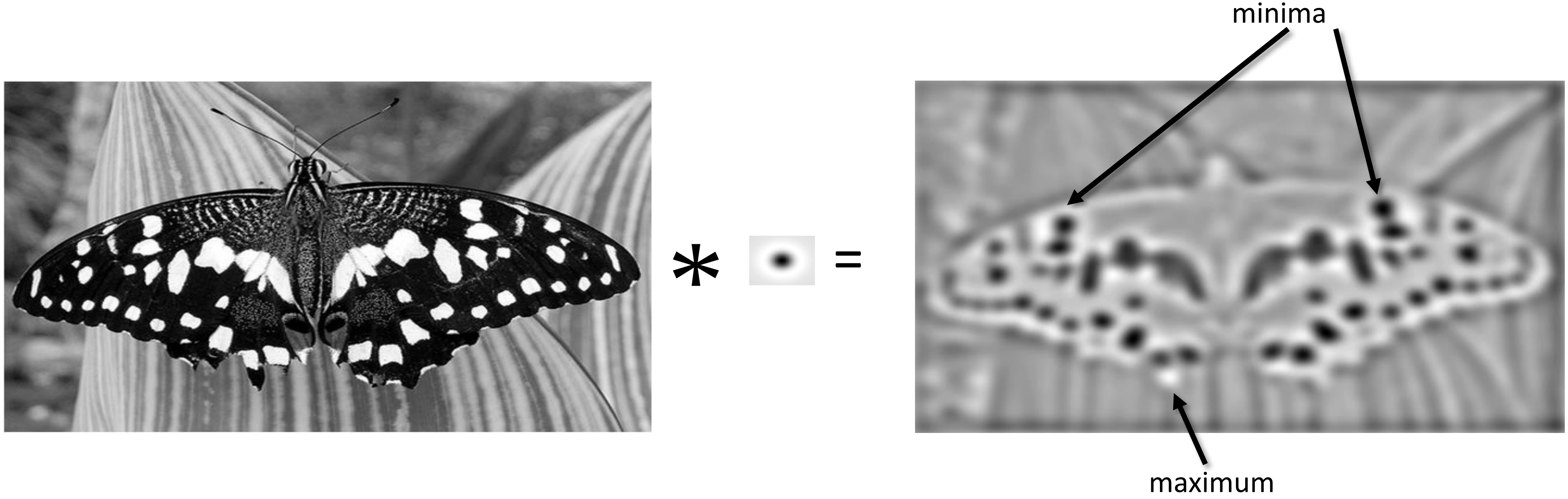


image



Laplacian

Blob detection

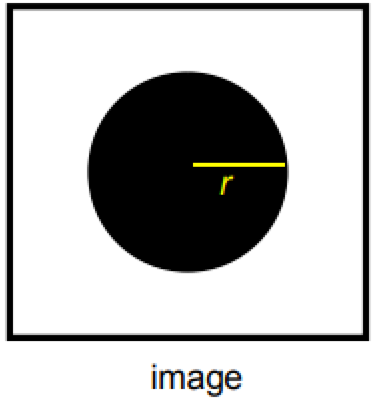


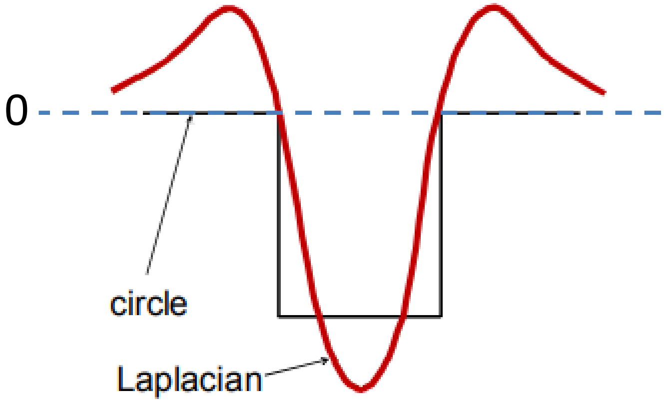
- Find maxima *and minima* of LoG operator in space and scale

Blob detection

$$\mathrm{LoG}(x,y;\sigma) = \Delta_{(x,y)}G(x,y;\sigma) = \frac{\partial^2 G(x,y;\sigma)}{\partial x^2} + \frac{\partial^2 G(x,y;\sigma)}{\partial y^2} = \frac{1}{\pi\sigma^4}\left(\frac{x^2+y^2}{2\sigma^2} - 1\right)\mathrm{e}^{-\frac{x^2+y^2}{2\sigma^2}}$$

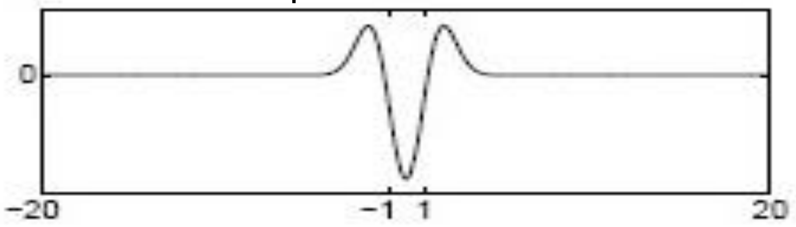
- Highest response occurs when the signal is exactly the width of the negative part of the LoG => search for all $\{(x,y)\}_\mathrm{i}$ where the LoG is exactly zero (the boarder between negative and positive => the result is a circle: $\sqrt{x^2+y^2} = r = \sqrt{2}\sigma$.

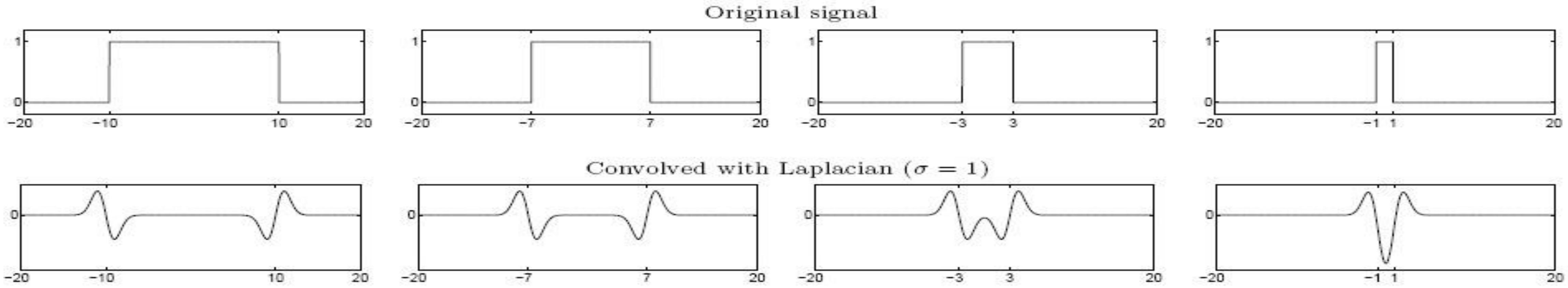




Blob detection

Laplacian filter

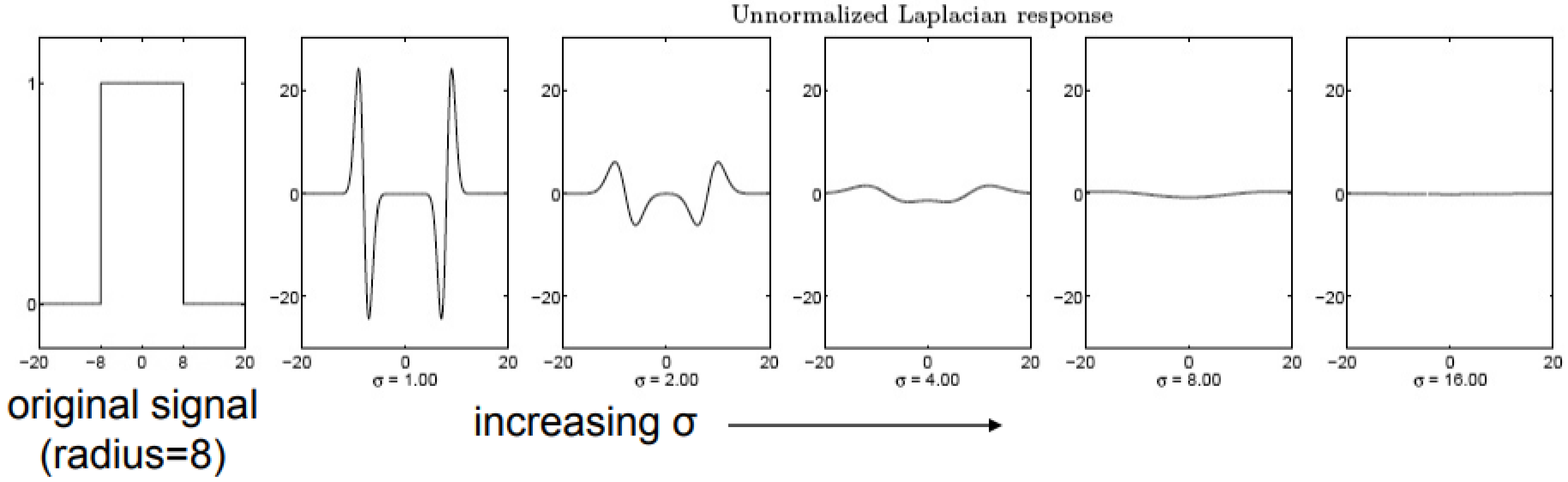




Highest absolute response when the signal has the same **characteristic scale** as the filter

Blob detection- normalized LoG

- We want to find the characteristic scale of the blob by convolving it with LoGs at several scales and looking for the maximum response.
- However, LoG response decays as scale increases.



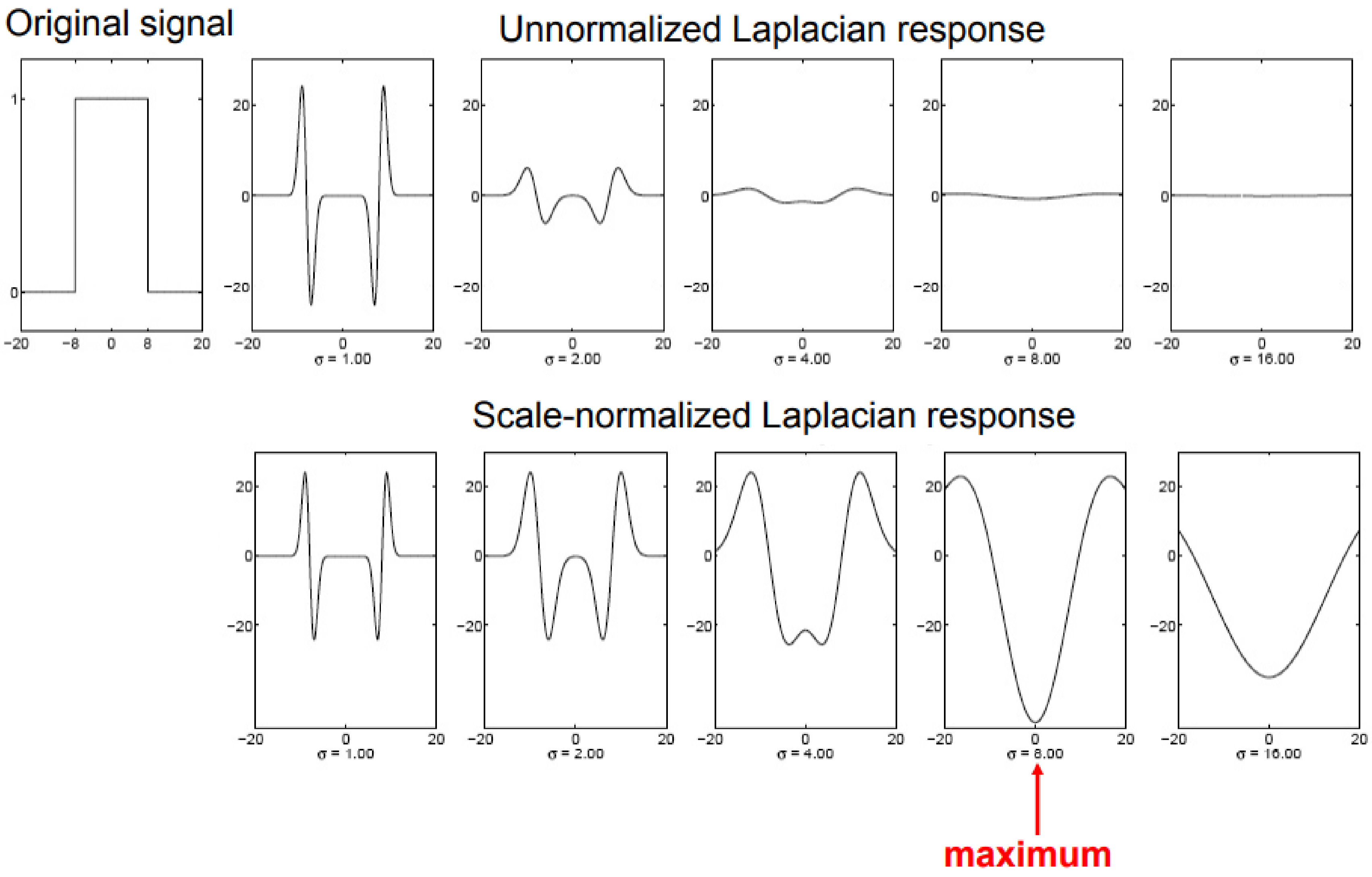
Blob detection- normalized LoG

- We want that the maximum of the LoG will be always at the same value, so we are using the scale normalized LoG:

$$\mathrm{normLoG}(x,y;\sigma) = \sigma^2 \Delta_{(x,y)} G(x,y;\sigma) = \sigma^2 \left(\frac{\partial^2 G(x,y;\sigma)}{\partial x^2} + \frac{\partial^2 G(x,y;\sigma)}{\partial y^2} \right)$$

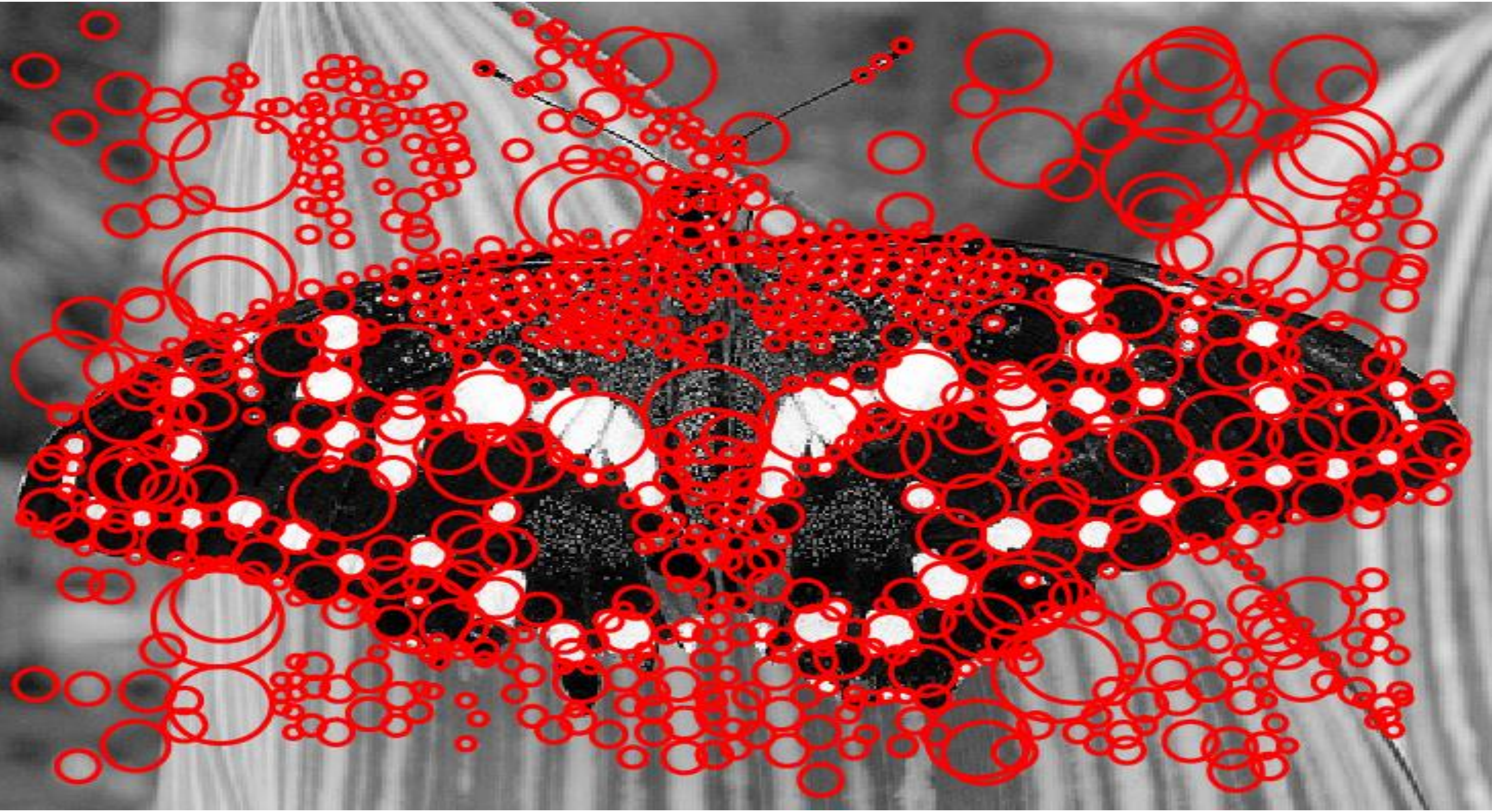
- Full derivation is available here: http://www.cim.mcgill.ca/~langer/558/2009/lecture11.pdf

Blob detection- normalized LoG



Blob detection- normalized LoG

- An example of max responses:

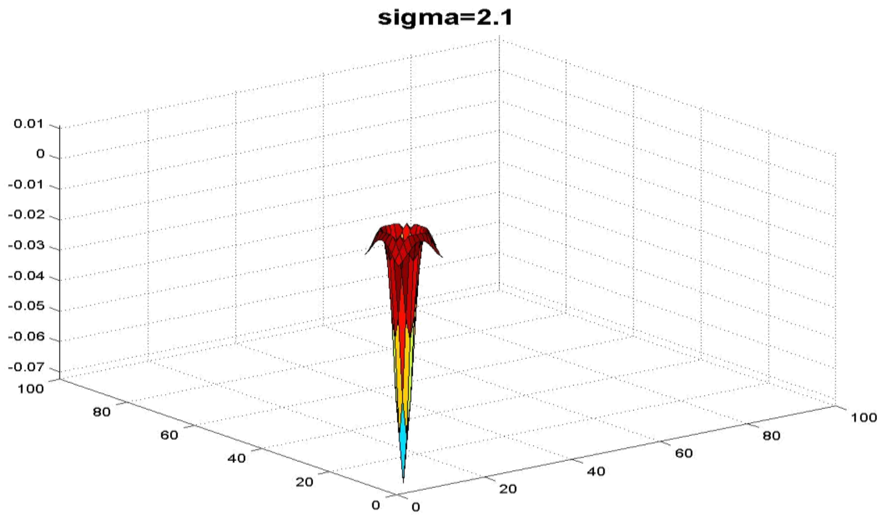


Normalized LoG – scale invariance

- Blob detection is invariance to scale.

Full size

3/4 size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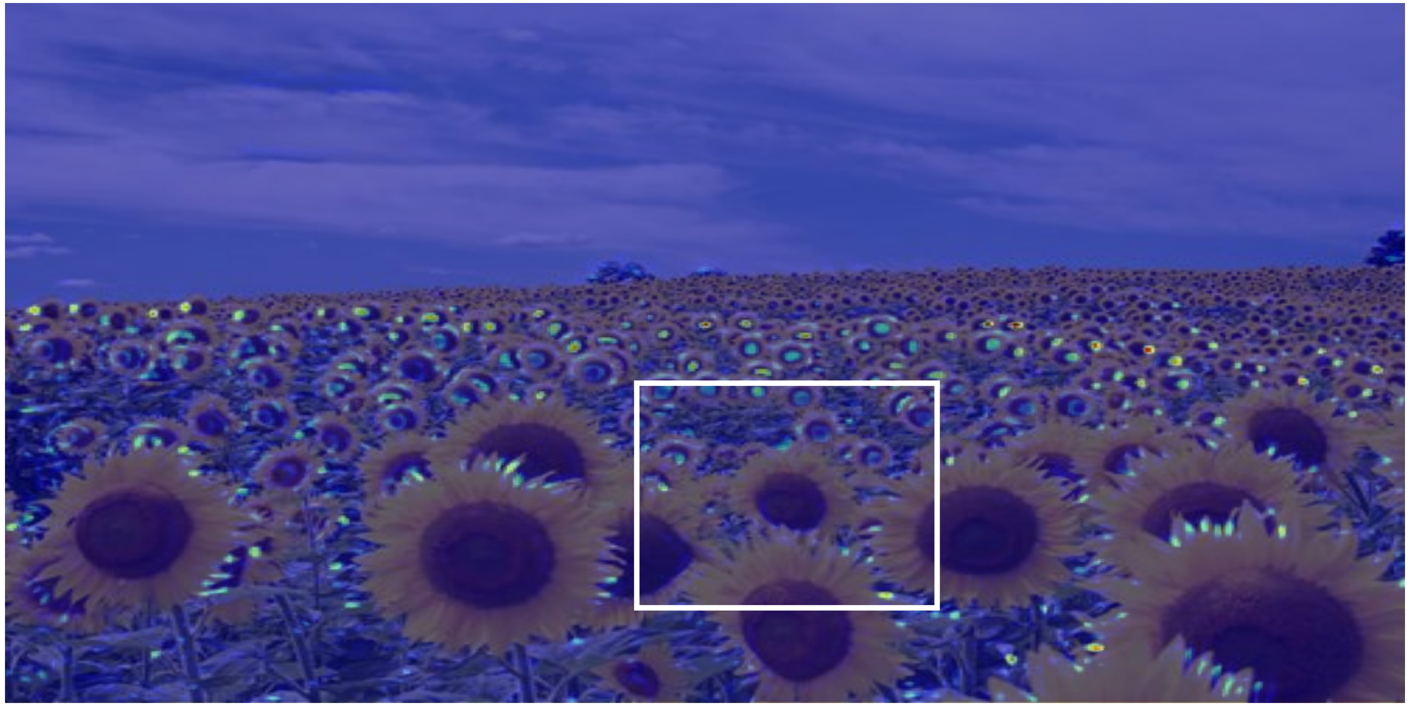

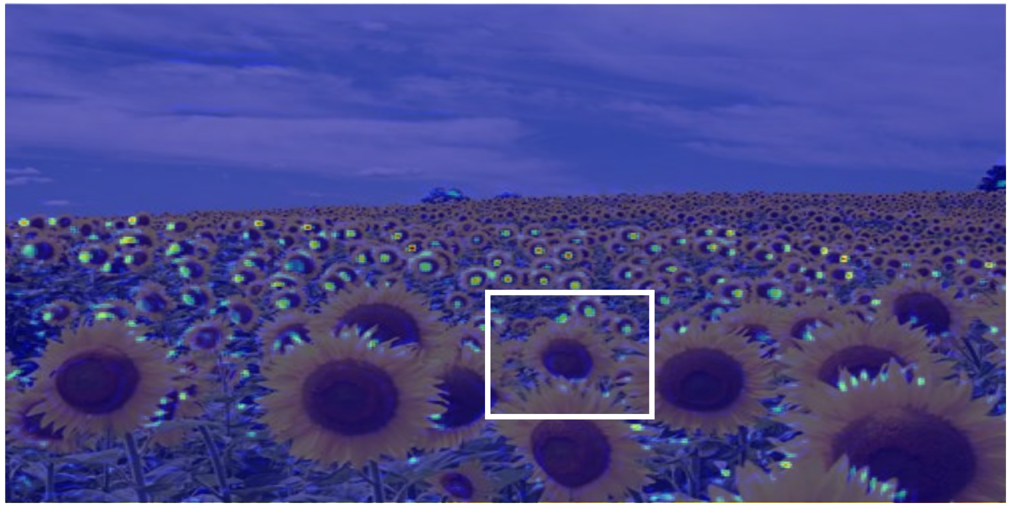

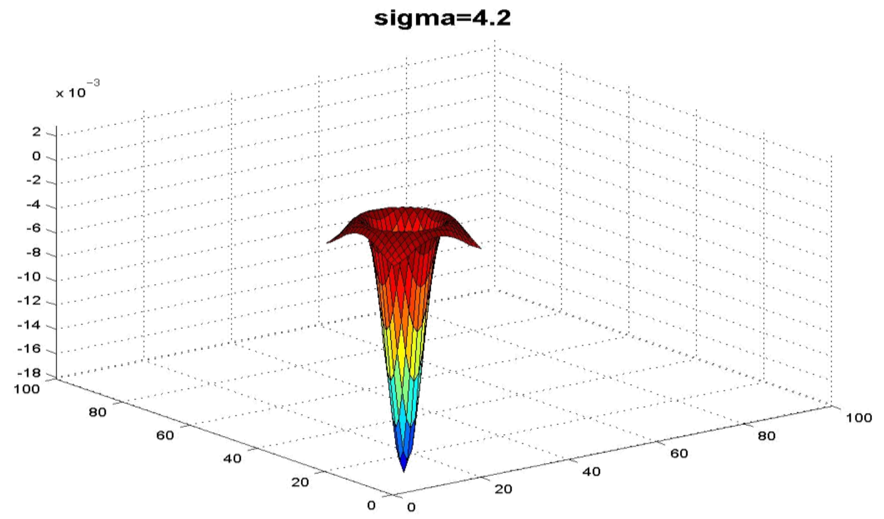
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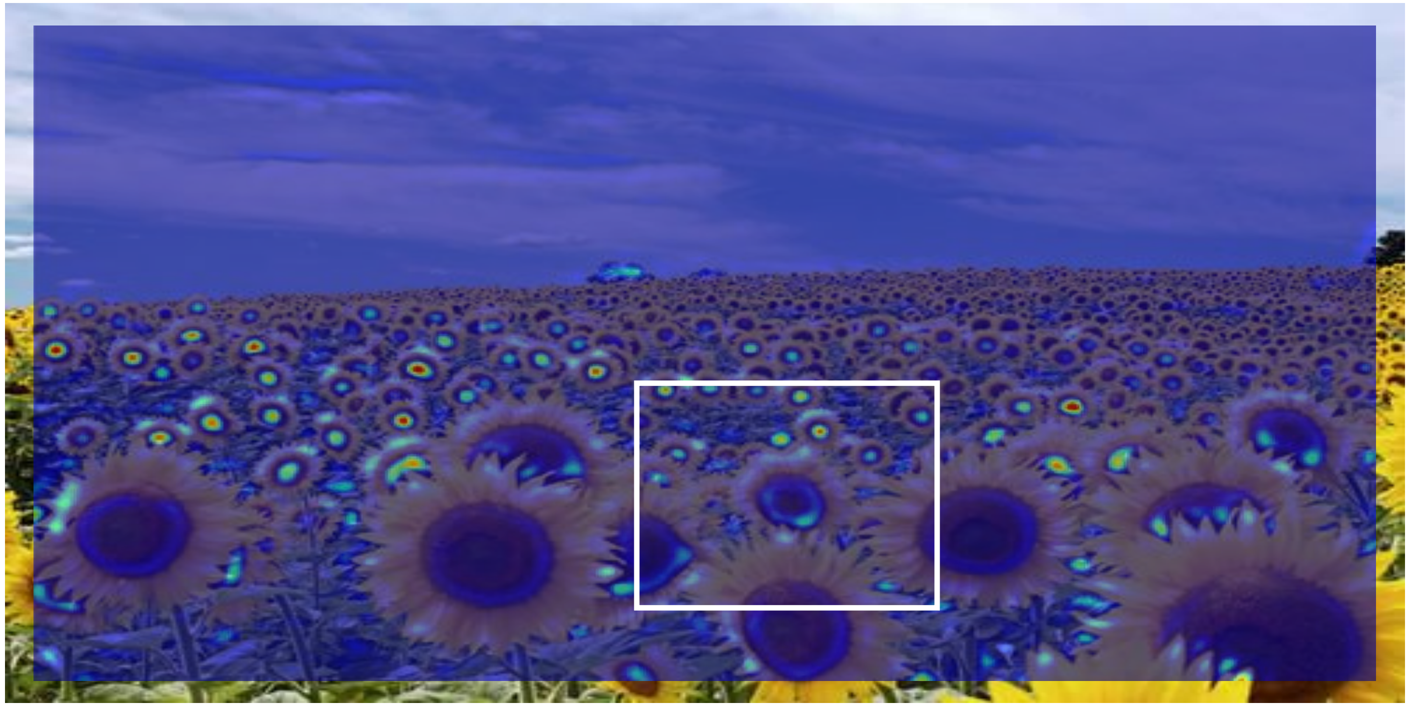

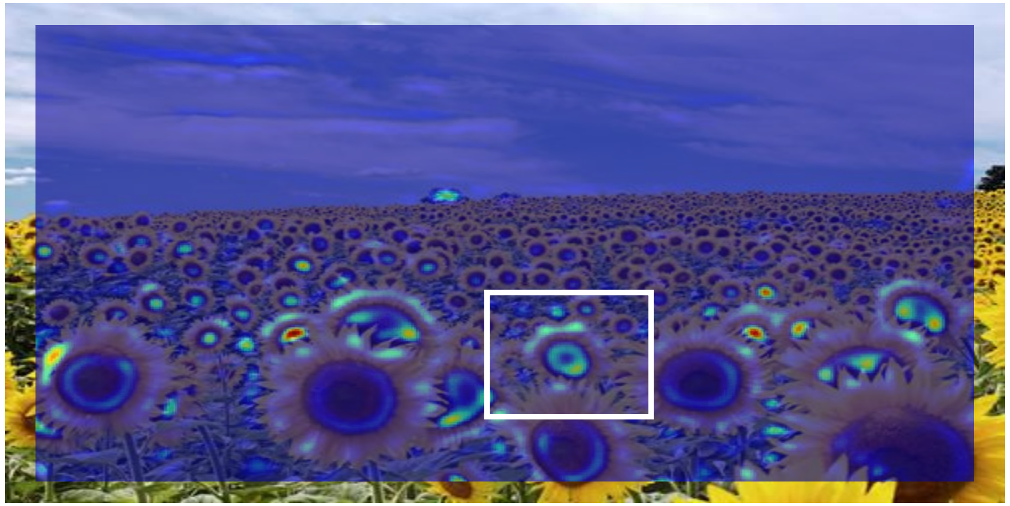

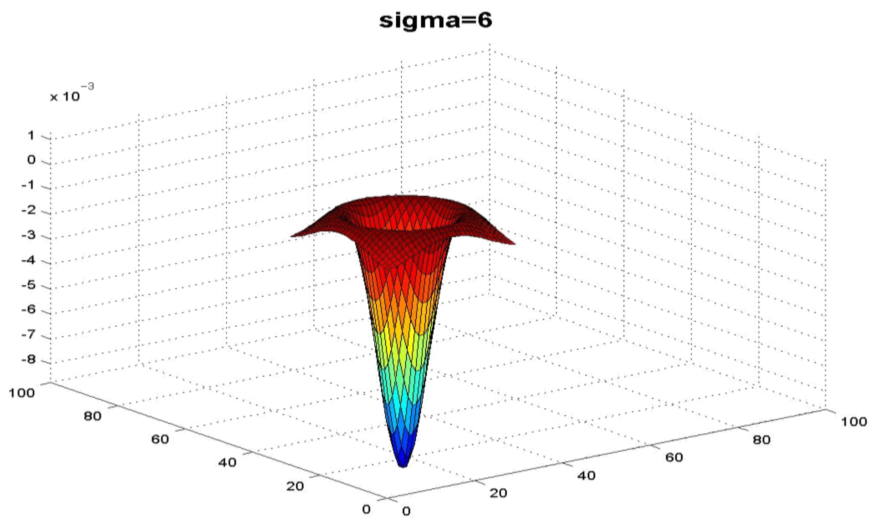
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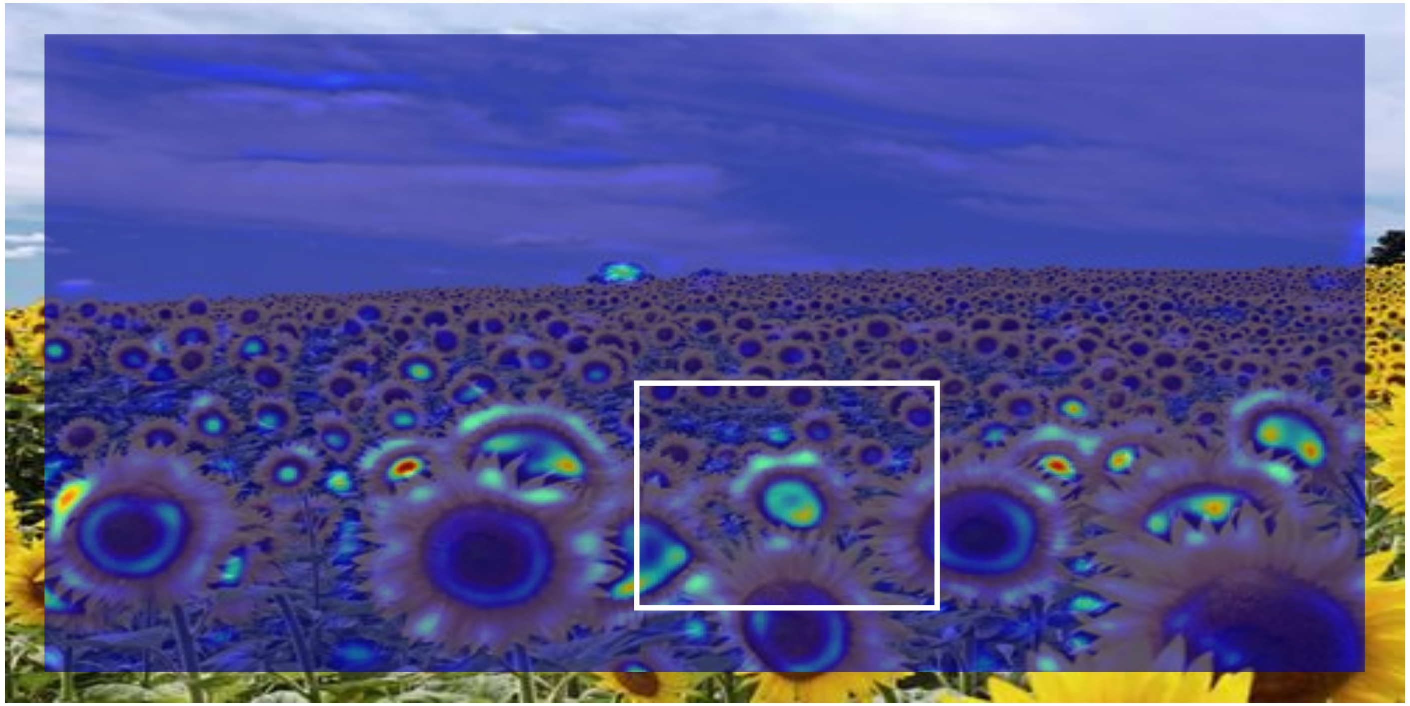

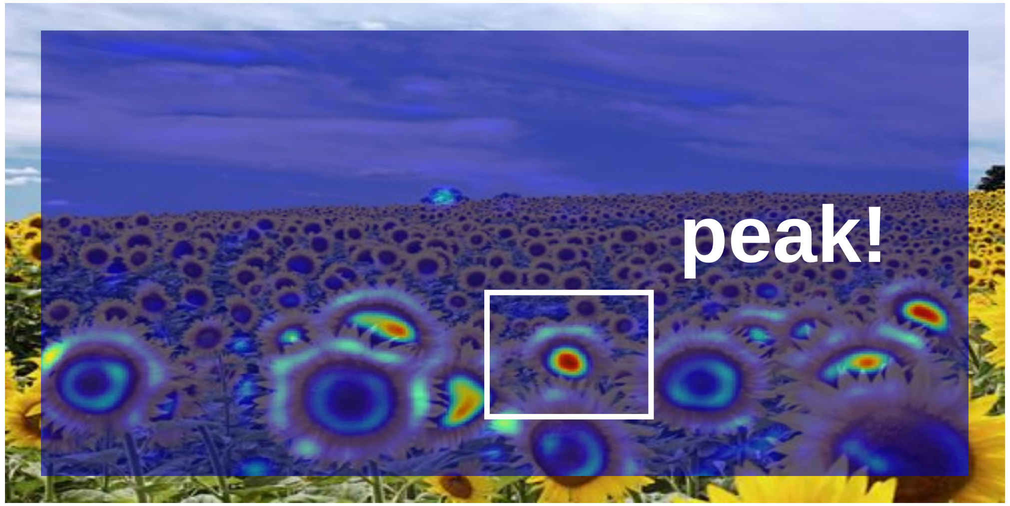
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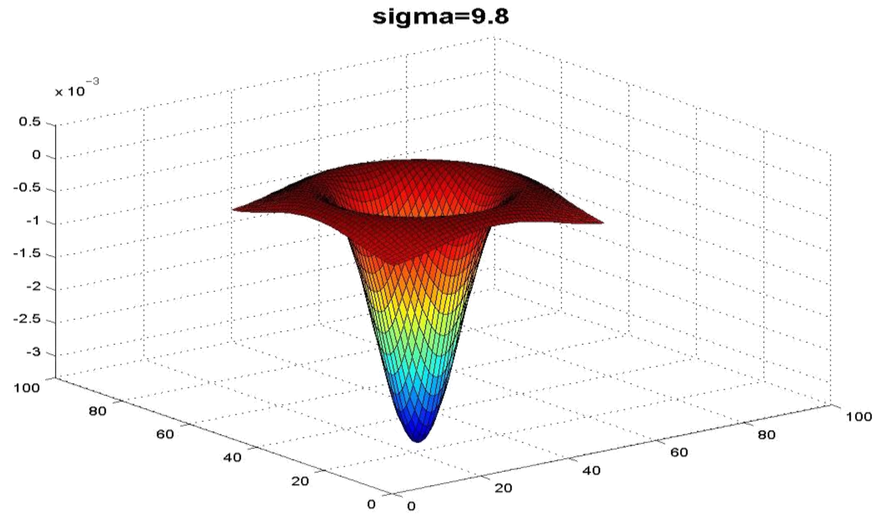
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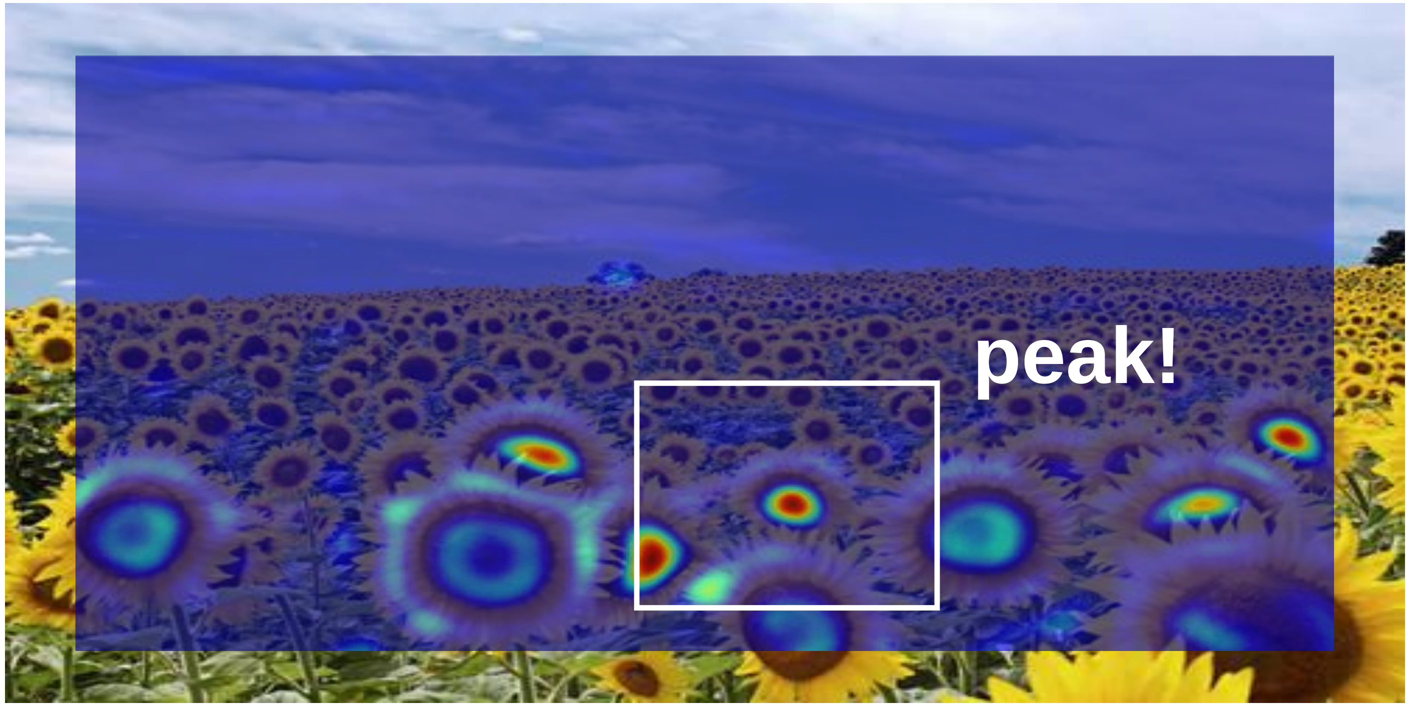
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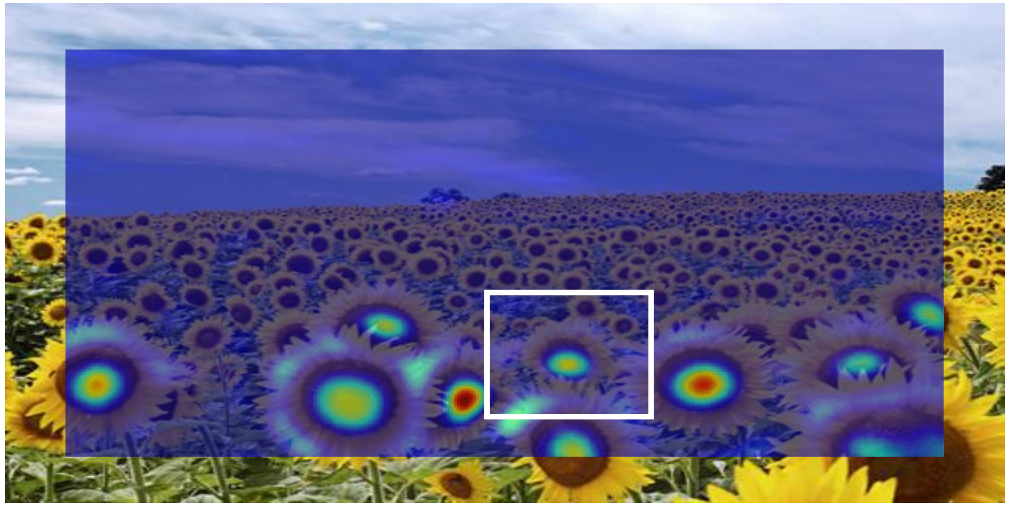

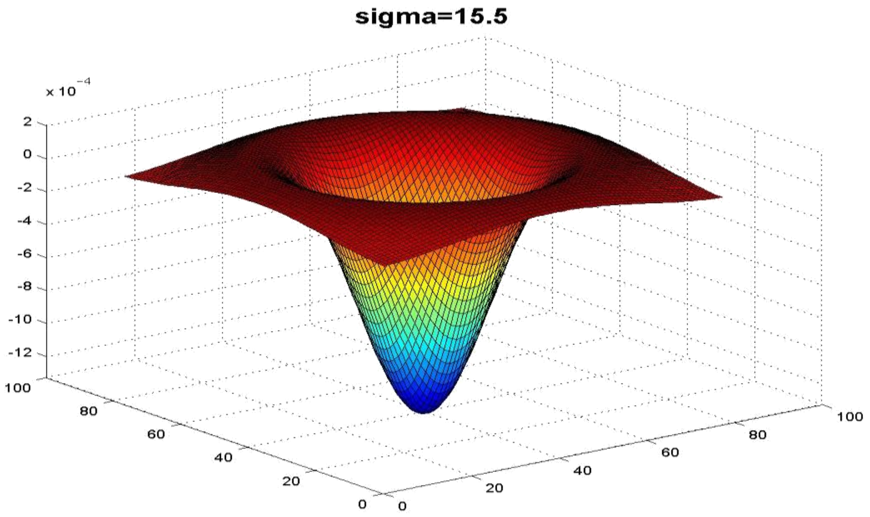
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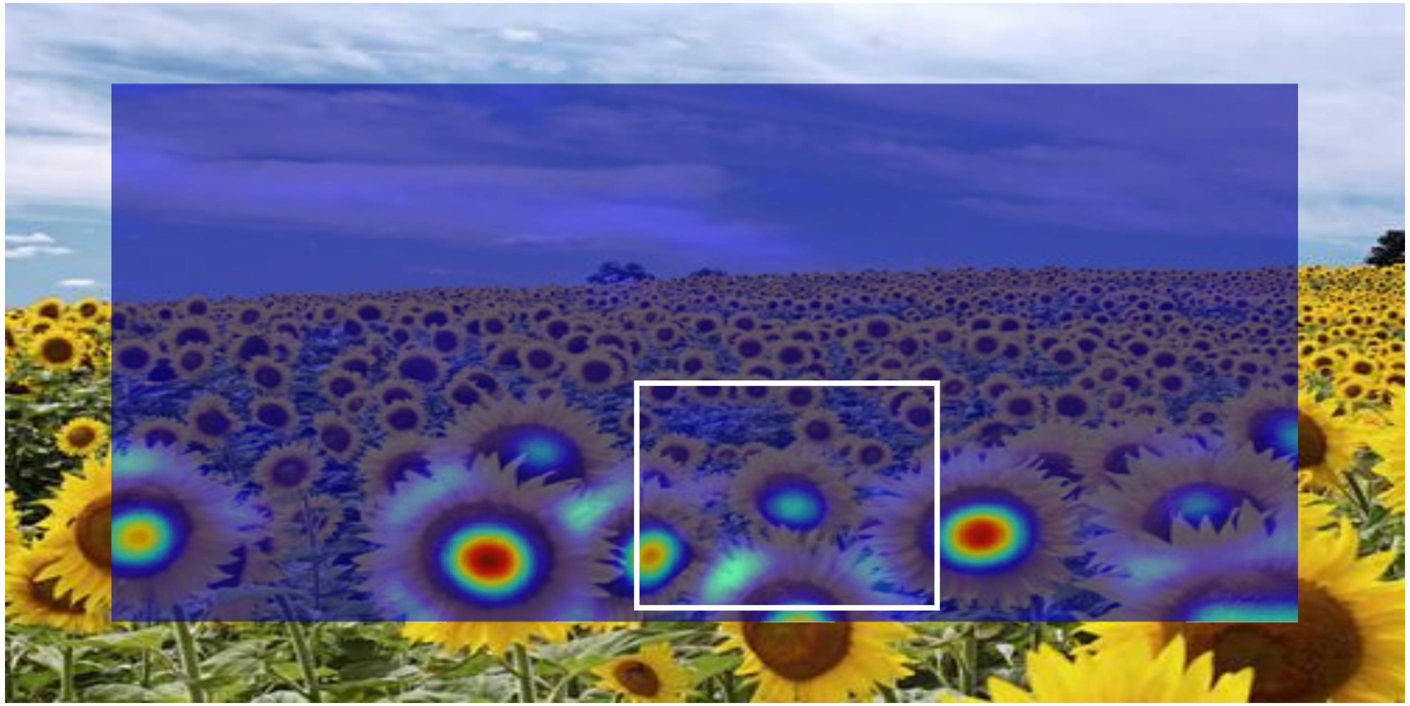

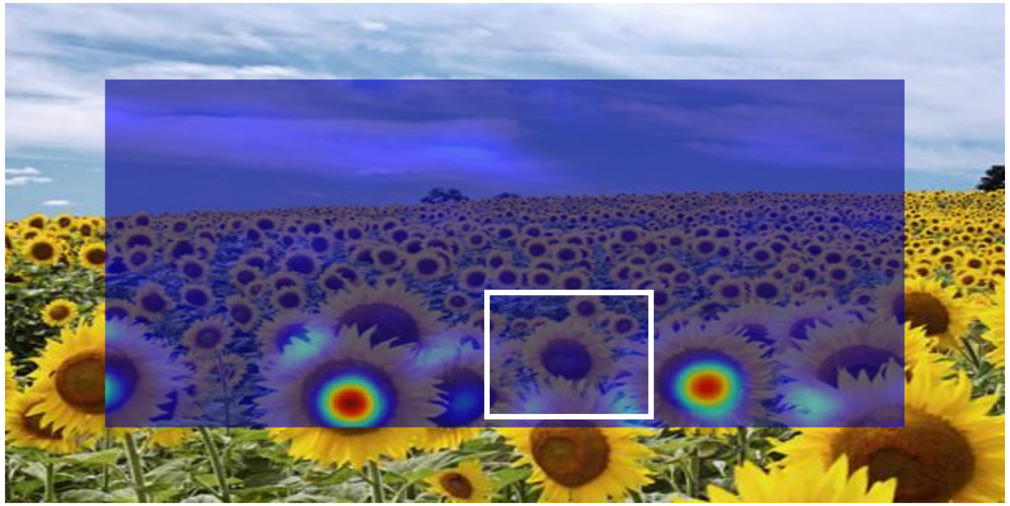

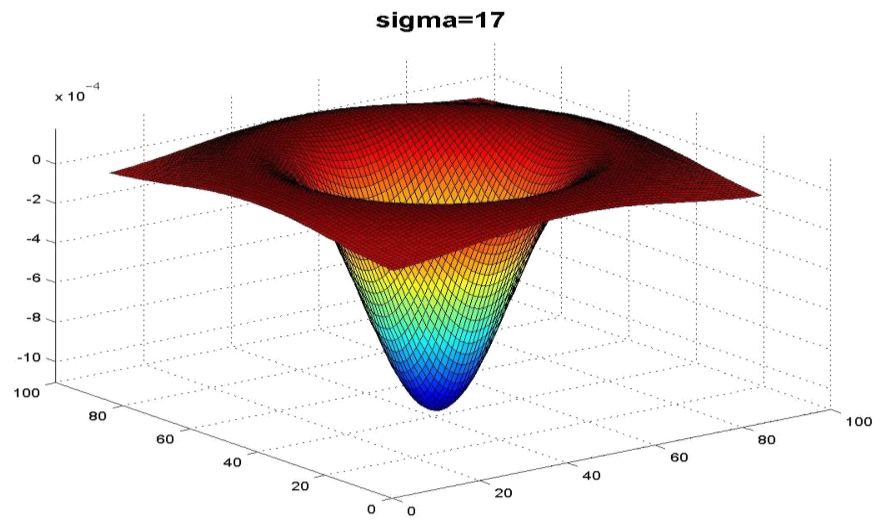
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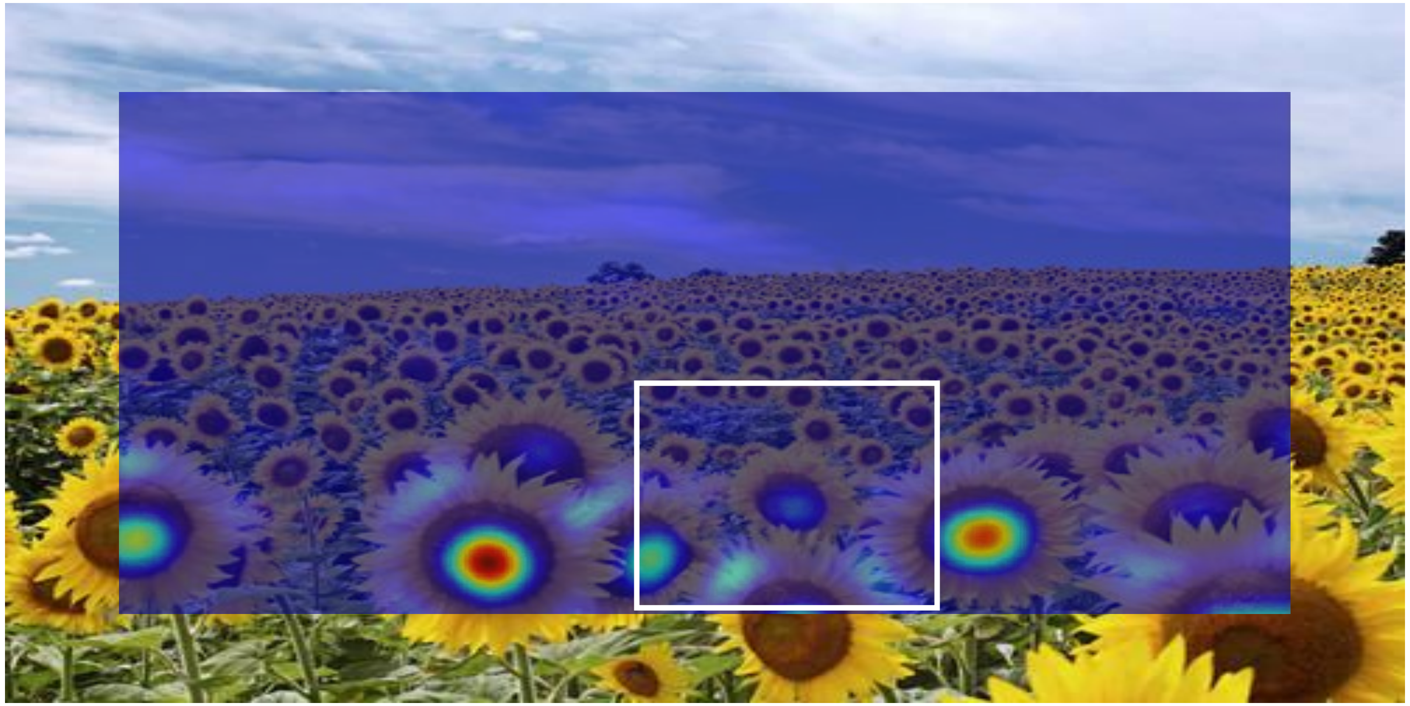
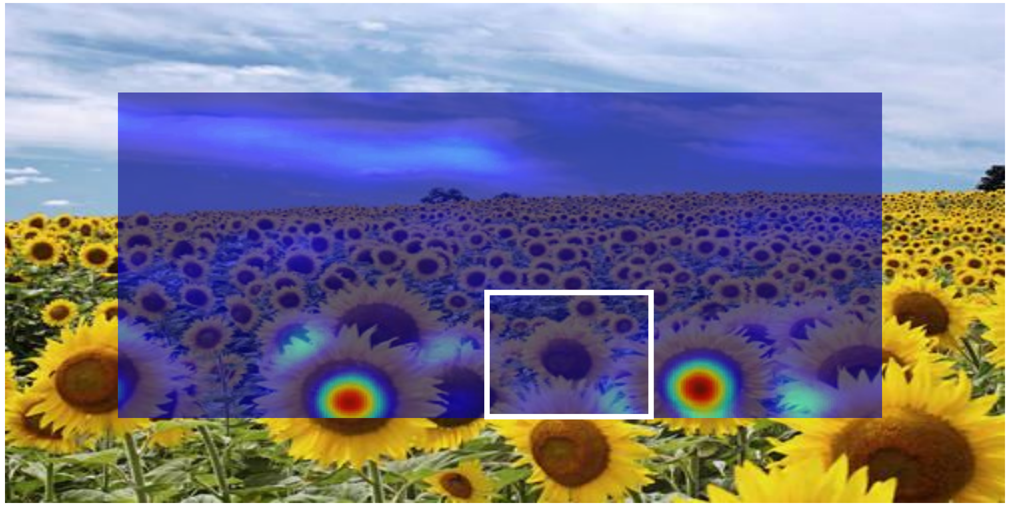
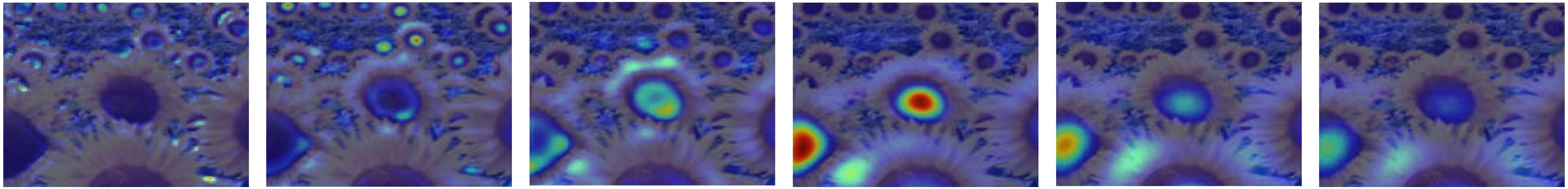




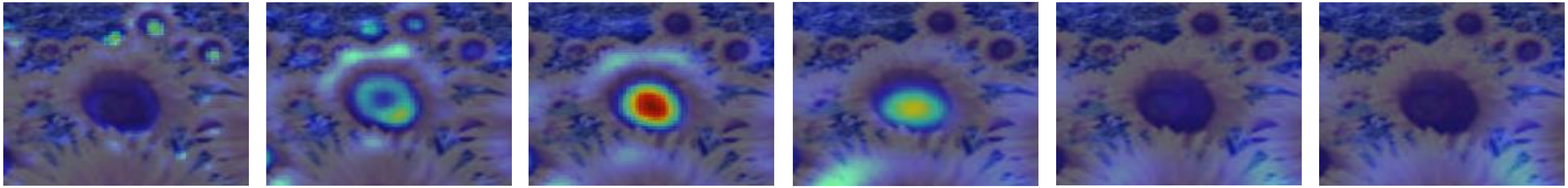
Normalized LoG – optimal scale

2.1 4.2 6.0 9.8 15.5 17.0



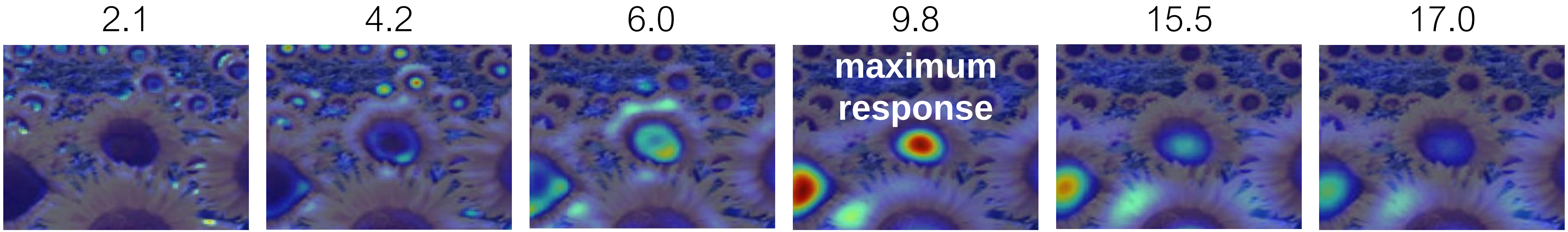
Full size image

2.1 4.2 6.0 9.8 15.5 17.0

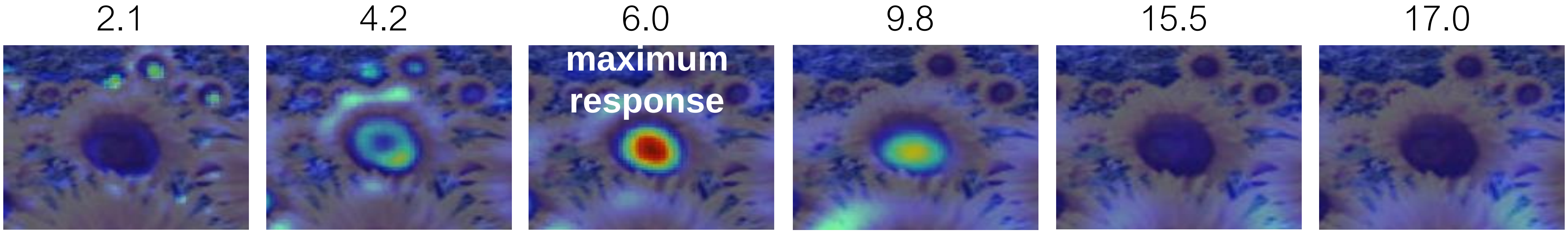


3/4 size image

Normalized LoG – optimal scale



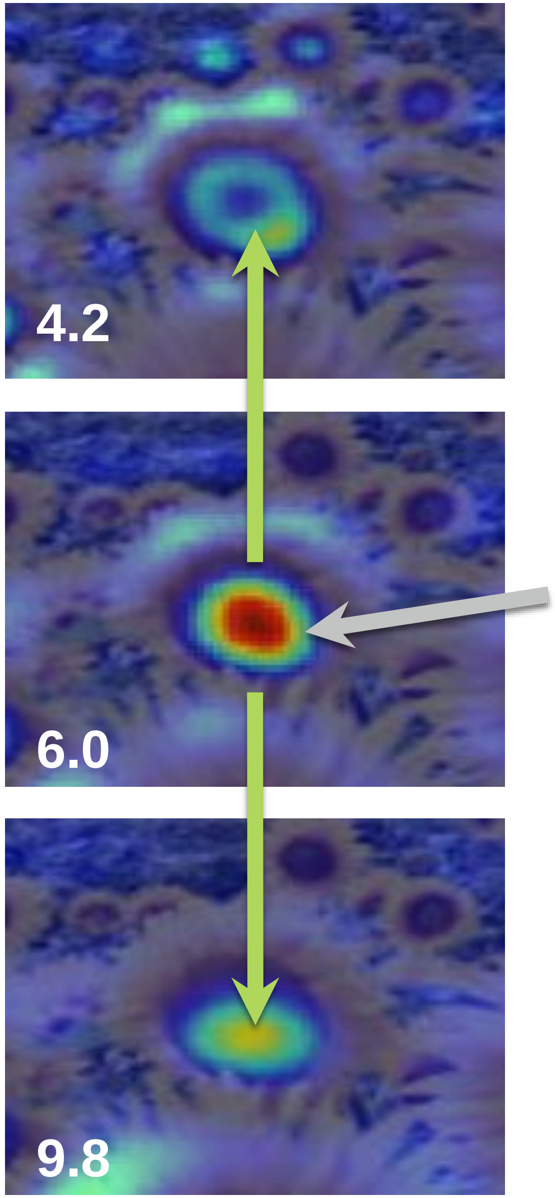
Full size image



3/4 size image

Normalized LoG – optimal scale

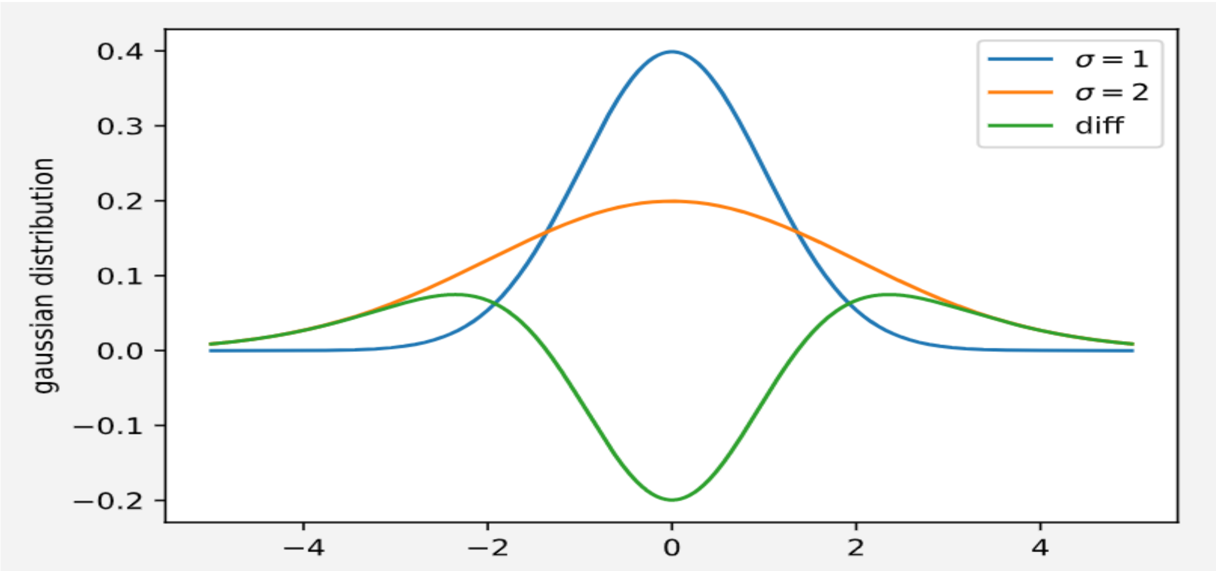
cross-scale maximum

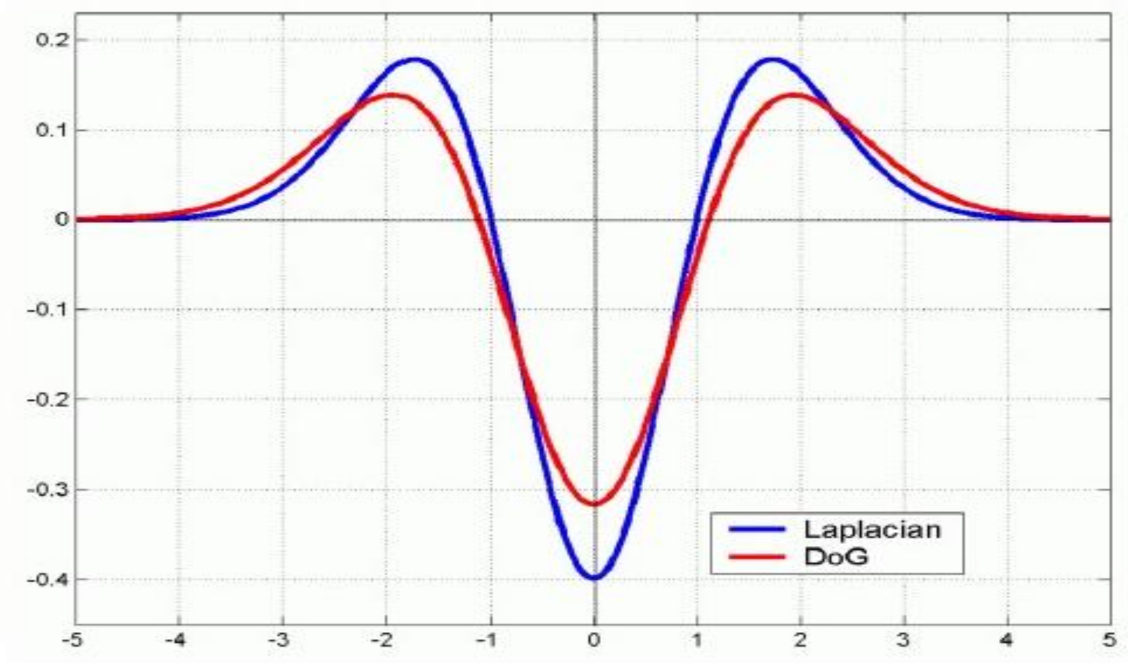


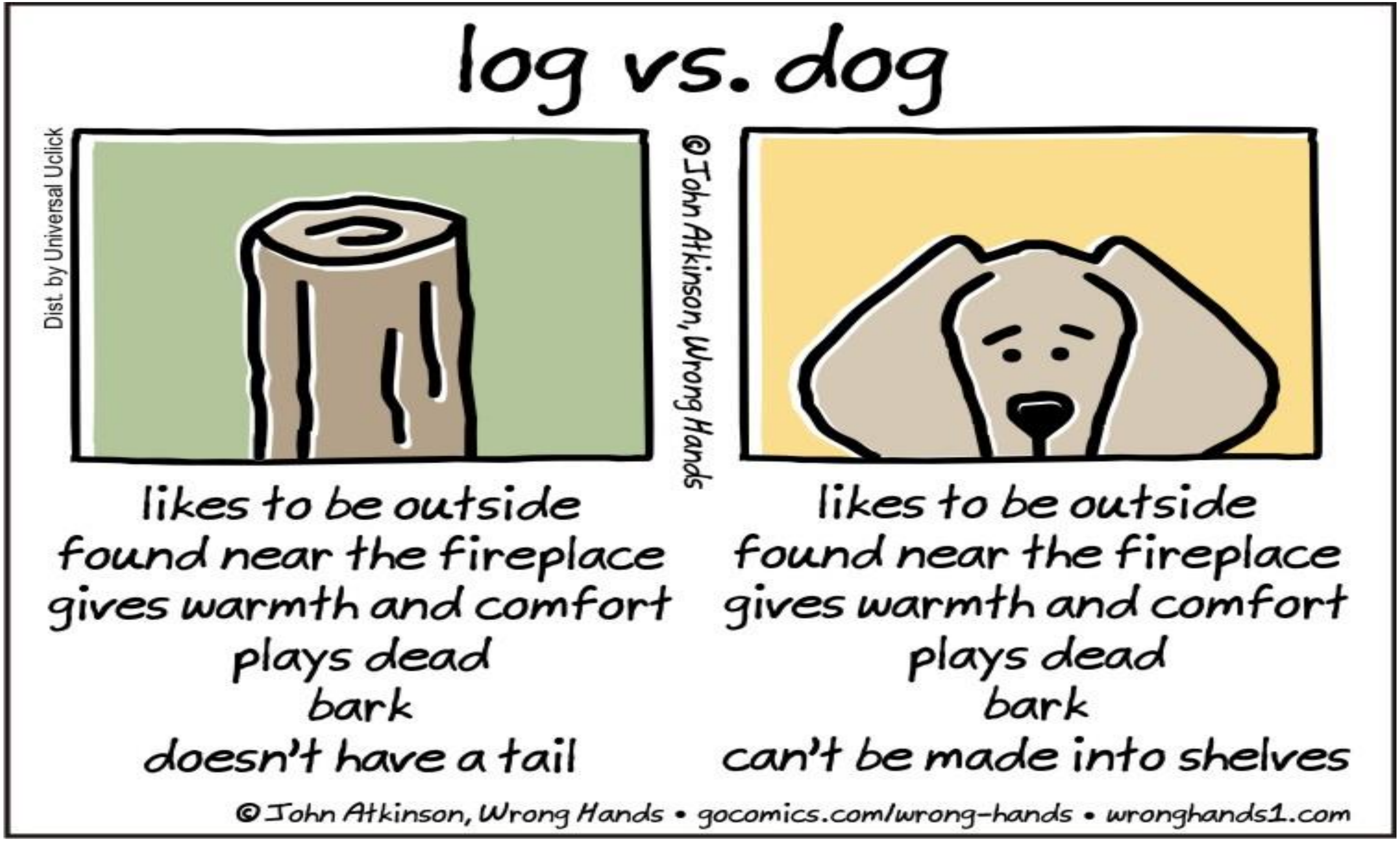
local maximum

As seen in class- edges: DoG

- Can also use difference of Gaussians (DoG) to mimic LoG.
- Why do we want to do this? Faster computationally (explained here: https://dsp.stackexchange.com/a/37675)



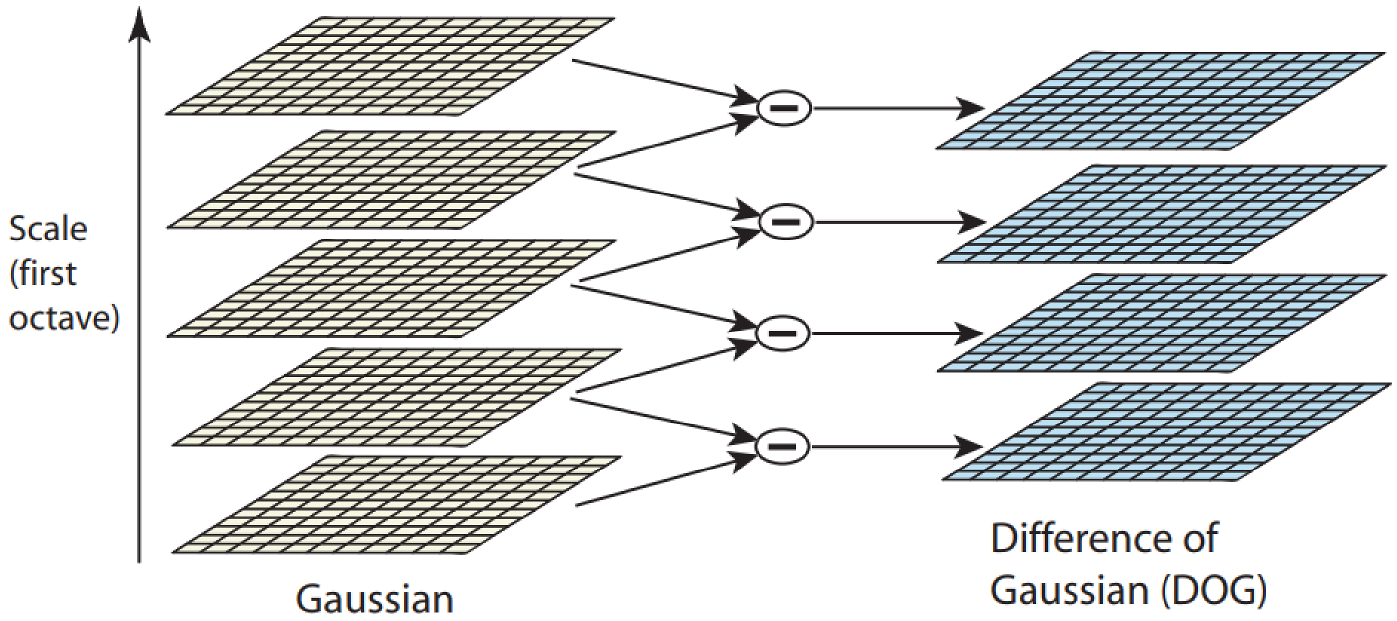


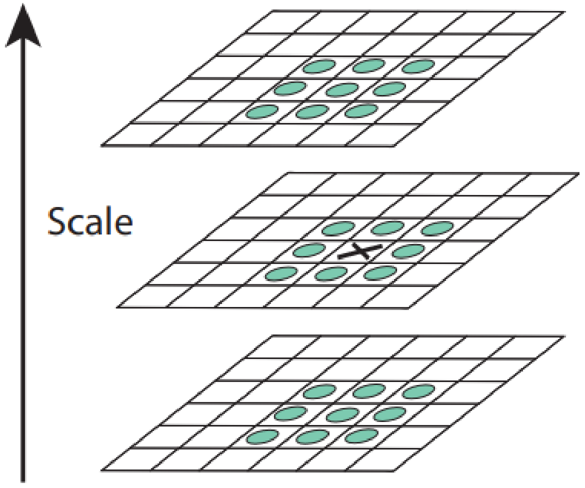


log vs. dog
Dist. by Universal Uclick
© John Atkinson, Wrong Hands
likes to be outside
found near the fireplace
gives warmth and comfort
plays dead
bark
doesn't have a tail
likes to be outside
found near the fireplace
gives warmth and comfort
plays dead
bark
can't be made into shelves
© John Atkinson, Wrong Hands • gocomics.com/wrong-hands • wronghands1.com

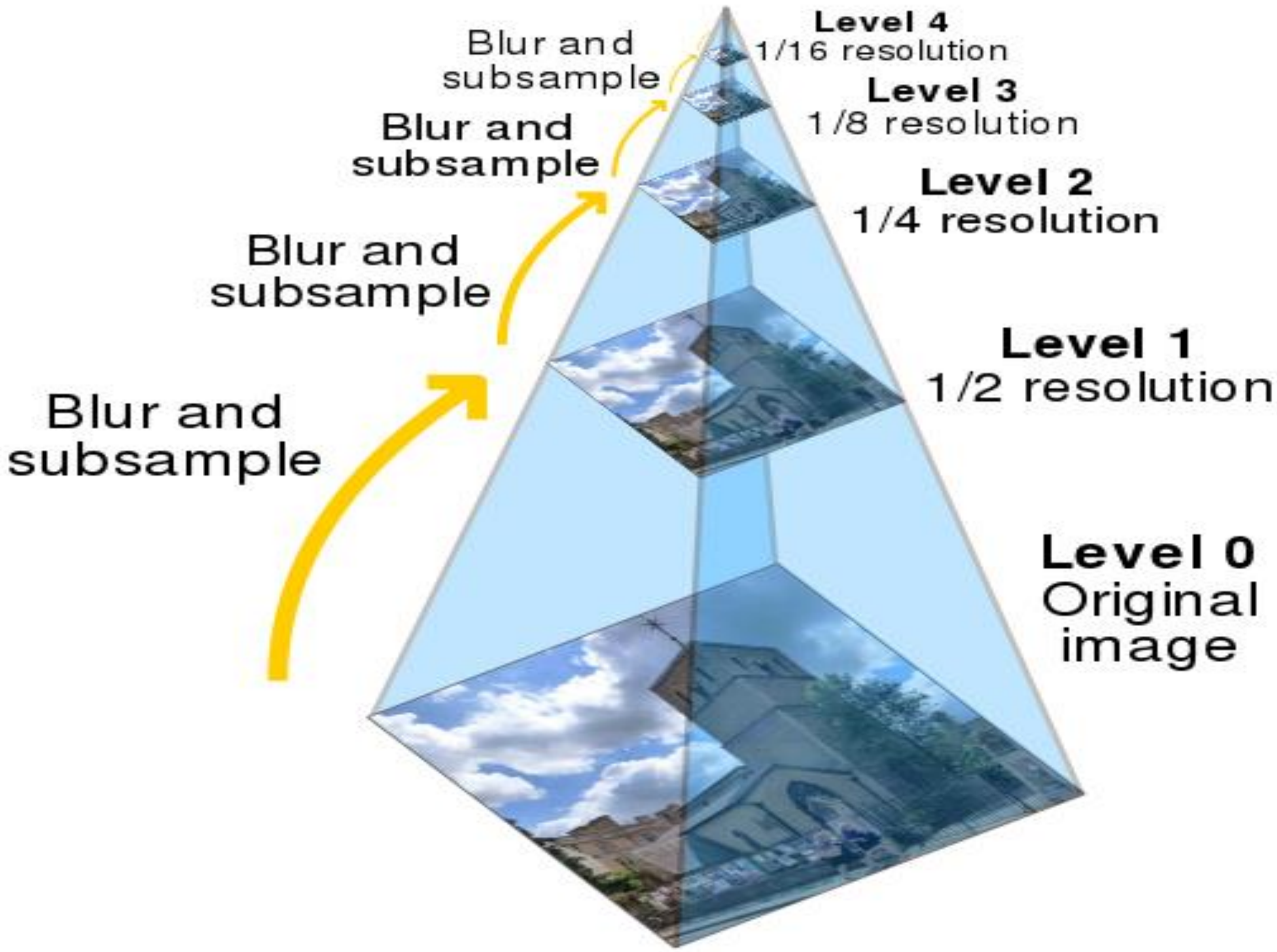
Blob detection algorithm

- Build DoG images.
- Search across different image scales for the max response.



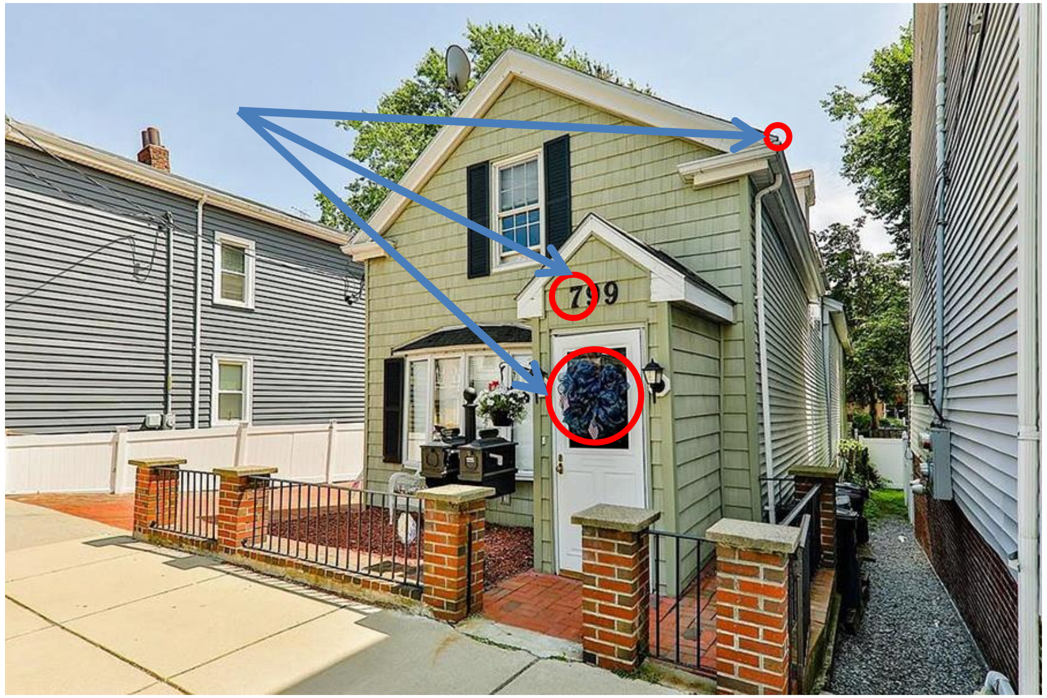


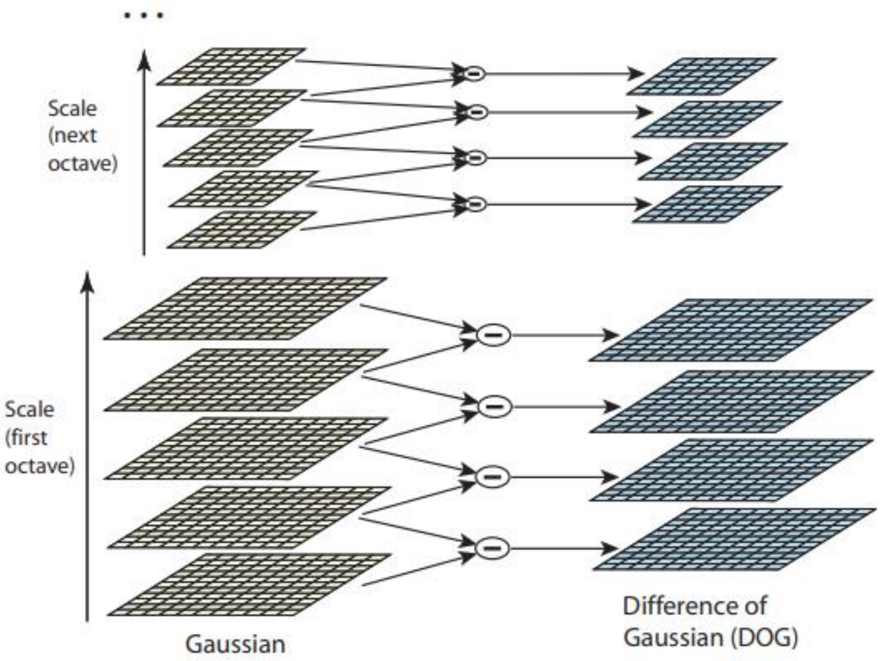
Reminder: Gaussian pyramid



Improved LoG blob detector

- Since the images is low-passed filter so much, we can decimate the image and not loose data in the process!
- We can build a Gaussian pyramid (as taught in image processing recap class) and run the entire algorithm on the different **octave scales.**
- **When using different octave scales we can get features from different image scale.**





Blob detection: summary

- Advantages:
  - Invariant to translation, rotation, scale and intensity shift $I \to I + b$ (because we use only the derivatives: $(\nabla G) * I = \nabla(G * I) = G * (\nabla \mathrm{I})$).
- Disadvantages:
  - Can also match edges, not just corners (why it's matter? Now)

contents

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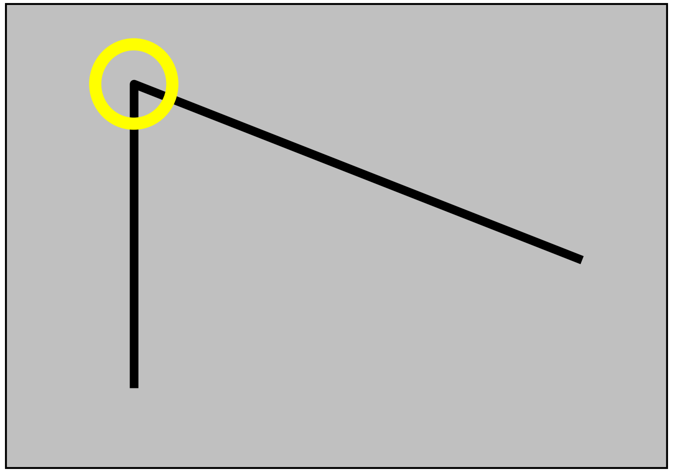
- SIFT feature matching
- Panoramas

keypoints

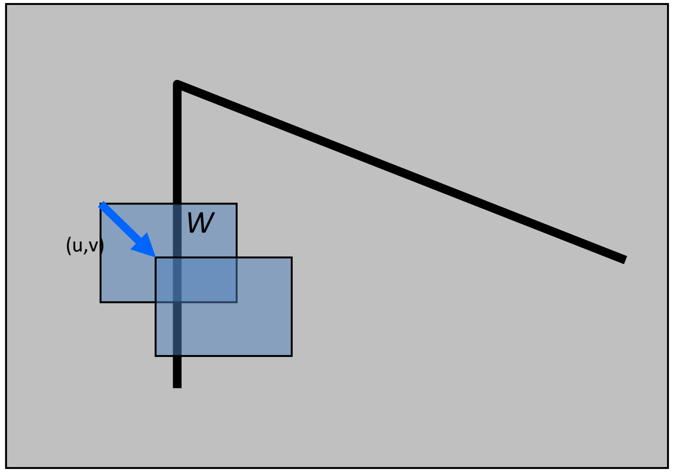
- Keypoints should be a unique point of the image where all close neighbors are very different from.
- Atempt 2: lets find corners in edge image- they are unique because their surrounding are very different.



Harris corner detection

- Consider shifting the window W by (u,v)
  - compare each pixel before and after by summing up the squared differences (SSD).
  - this defines an SSD "error" $E(u,v)$:

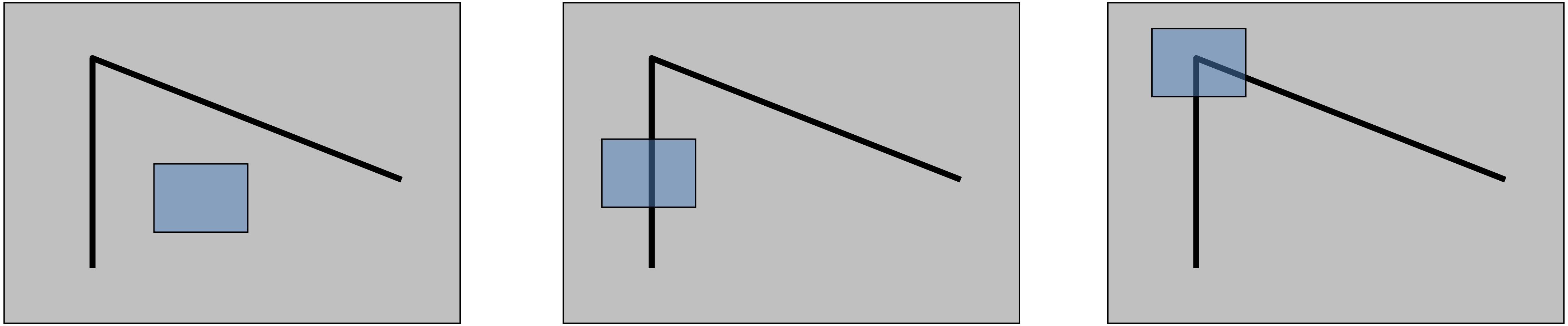
$$E(u,v) = \sum_{(x,y)\in W} \left[I(x+u,y+v) - I(x,y)\right]^2$$



- **We are happy if this error is high for all $(\boldsymbol{u},\boldsymbol{v}) \neq (\mathbf{0},\mathbf{0})$**

Local measures of uniqueness

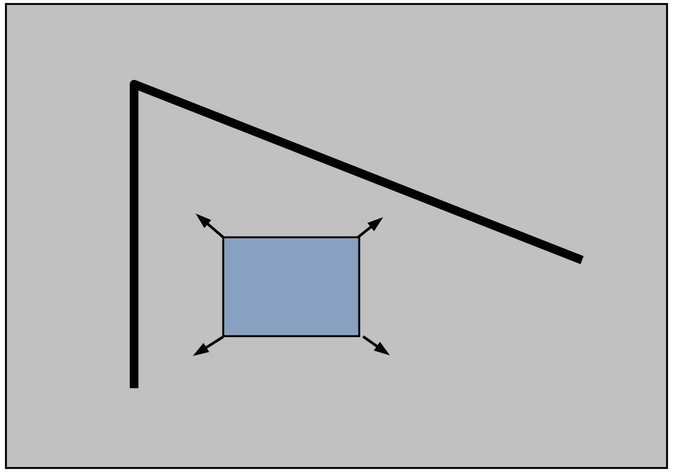
- Suppose we only consider a small window of pixels.
- How does the window change when you shift it?



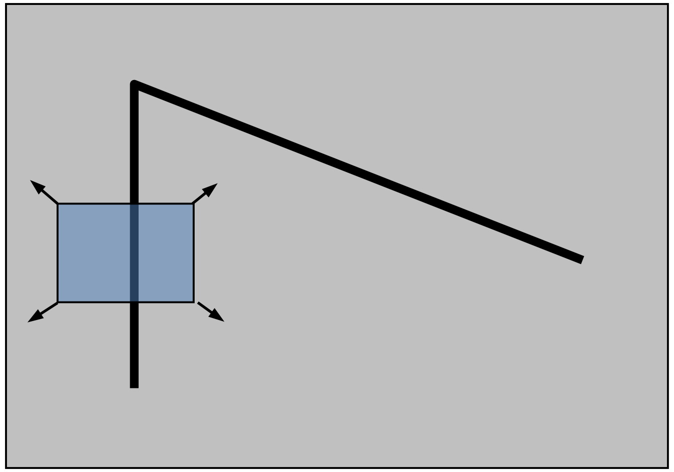
Credit: S. Seitz, D. Frolova, D. Simakov

Local measures of uniqueness

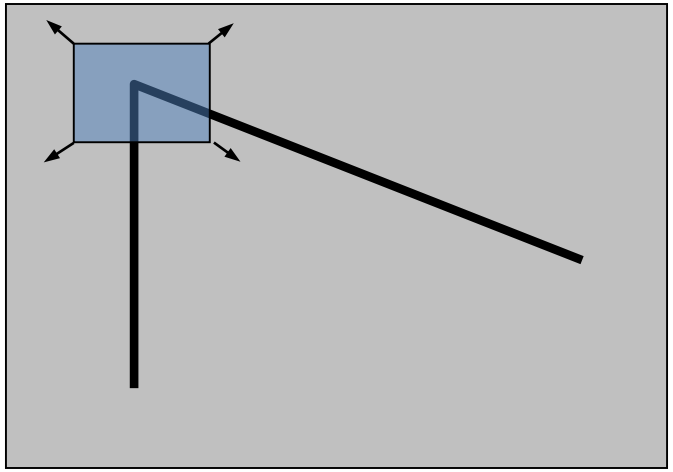
- Suppose we only consider a small window of pixels.
- How does the window change when you shift it?



"flat" region:
no change in all directions



"edge":
no change along the edge direction



"corner":
significant change in all directions

Harris corner detection

- Taylor Series expansion of I:

$$I(x+u, y+v) = I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

- If the motion (u, v) is small, then first order approximation is good

$$I(x+u, y+v) \approx I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

- Plug it into the SSD error term:

$$\begin{aligned} E(u,v) &= \sum_{(x,y)\in W} \left[I(x+u, y+v) - I(x,y)\right]^2 \\ &\approx \sum_{(x,y)\in W} \left[I(x,y) + I_x u + I_y v - I(x,y)\right]^2 \\ &\approx \sum_{(x,y)\in W} \left[I_x u + I_y v\right]^2 \end{aligned}$$

Harris corner detection

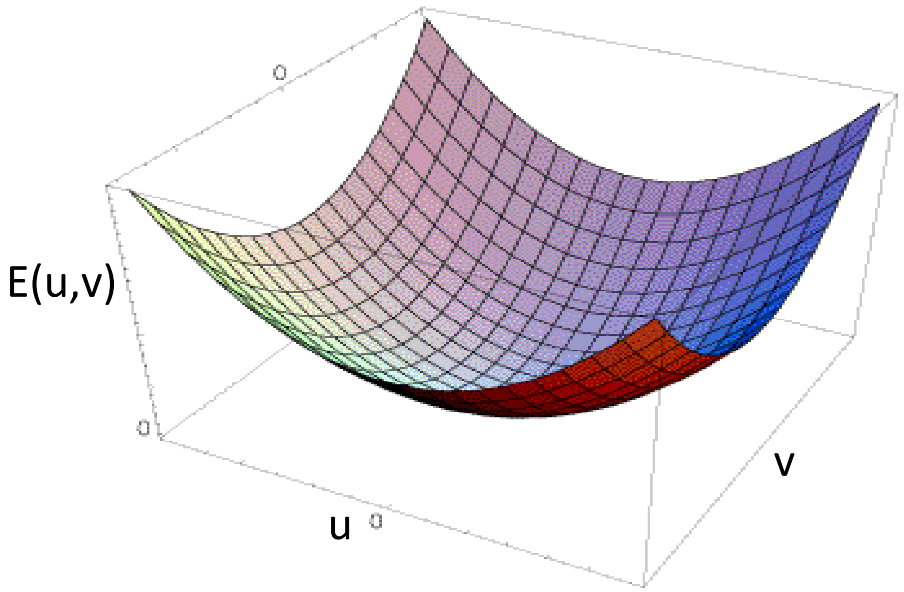
$$
\begin{aligned}
E(u,v) &\approx \sum_{(x,y)\in W} [I_x u + I_y v]^2 \\
&\approx Au^2 + 2Buv + Cv^2 \\
&\approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}
\end{aligned}
$$

$$A = \sum_{(x,y)\in W} I_x^2$$

$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$

Also called **second-moments matrix**



Harris corner detection

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

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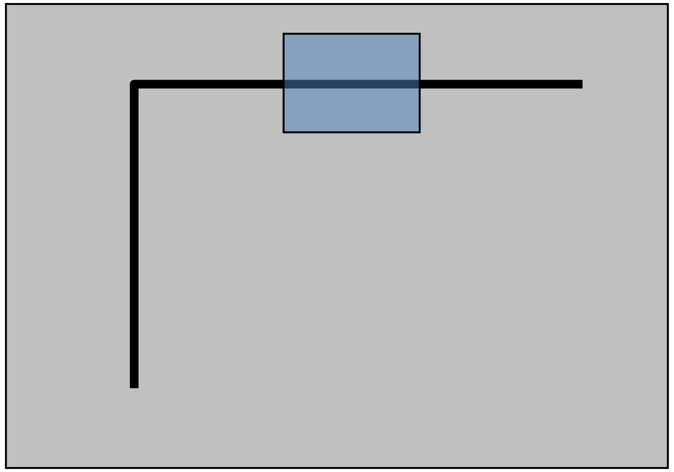
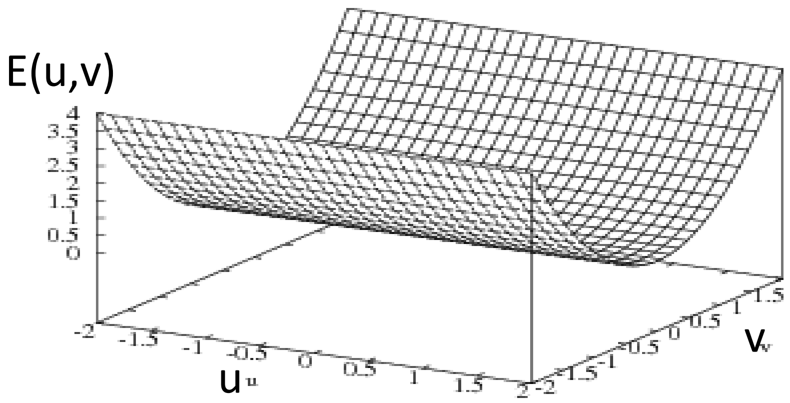
$$C = \sum_{(x,y)\in W} I_y^2$$



Horizontal edge: $I_x = 0$

$$H = \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix}$$



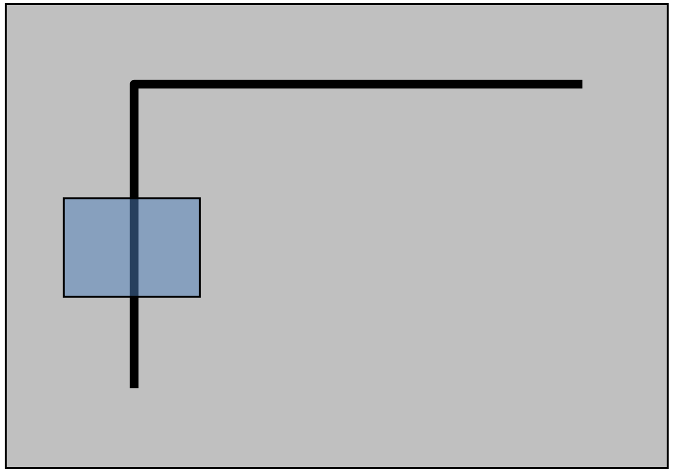
Harris corner detection

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

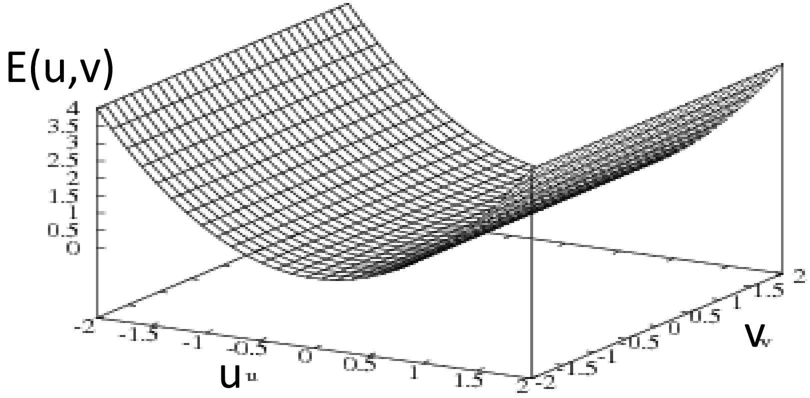
$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



Vertical edge: $I_y = 0$

$$H = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$$



Harris corner detection: probabilistic interpretation

- A real symmetric matrix has an eigendecomposition of:

$$Av = \lambda v$$
$$AQ = Q\Lambda$$
$$A = Q\Lambda Q^{-1}$$
$$\xrightarrow{\text{A is real symmetric matrix}}$$
$$A = Q\Lambda Q^T$$
$$A = \begin{pmatrix} e_1 & e_2 \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} e_1^T \\ e_2^T \end{pmatrix}$$

  – Bonus: eigenvectors are orthonormal if A is real and symmetric.

Harris corner detection

- An ellipse can have a matrix form of:

$$x^T \left(e_1 e_2 \right) \begin{pmatrix} \lambda_1 & \\ & \lambda_2 \end{pmatrix} \begin{pmatrix} e_1^T \\ e_2^T \end{pmatrix} x = 1$$

$$\lambda_1 x^T e_1 e_1^T x + \lambda_2 x^T e_2 e_2^T x = 1$$

$$\frac{\left(e_1^T x\right)^2}{\left(\frac{1}{\sqrt{\lambda_1}}\right)^2} + \frac{\left(e_2^T x\right)^2}{\left(\frac{1}{\sqrt{\lambda_2}}\right)^2} = 1$$

- Which is exactly as a rotated ellipse with a center of $(0{,}0)$:

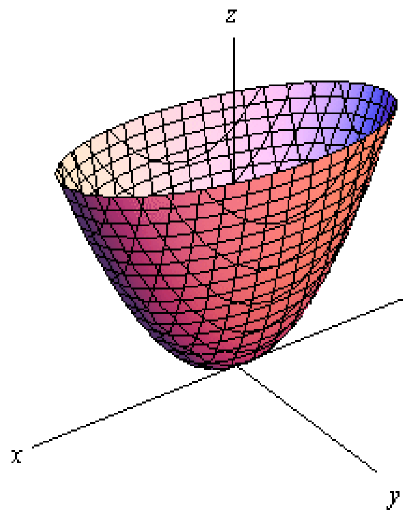
$$\frac{(x\cos(\theta) + y\sin(\theta))^2}{a^2} + \frac{(x\sin(\theta) - y\cos(\theta))^2}{b^2} = 1$$

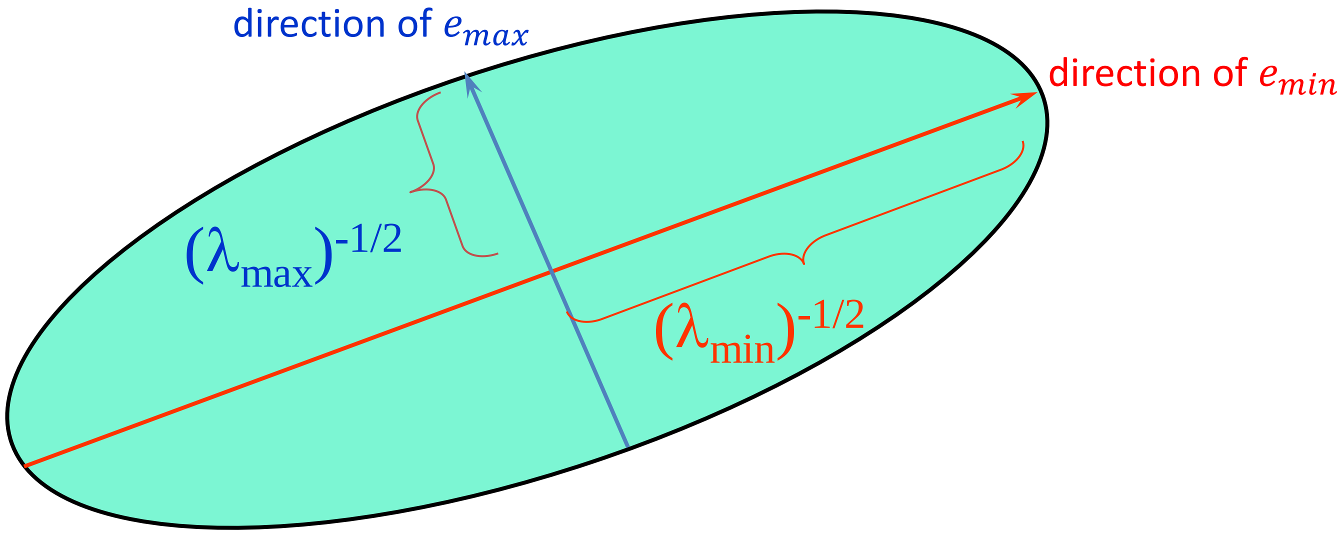
Harris corner detection

- Combining the two equations seen before we can conclude that when taking a cross-section from the error function, we can get an ellipsoid.

$$-\,[u \quad v]H\begin{bmatrix}u\\v\end{bmatrix} = \text{const}$$

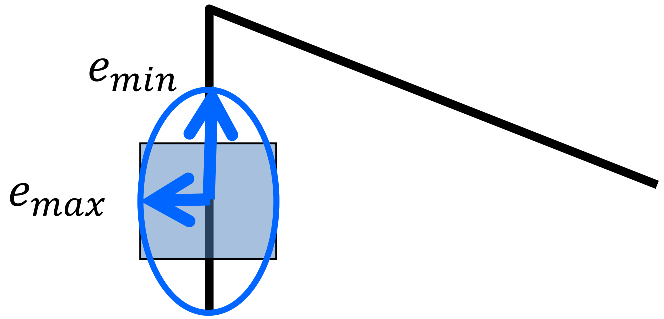
- Assume $\lambda_1 > \lambda_2$



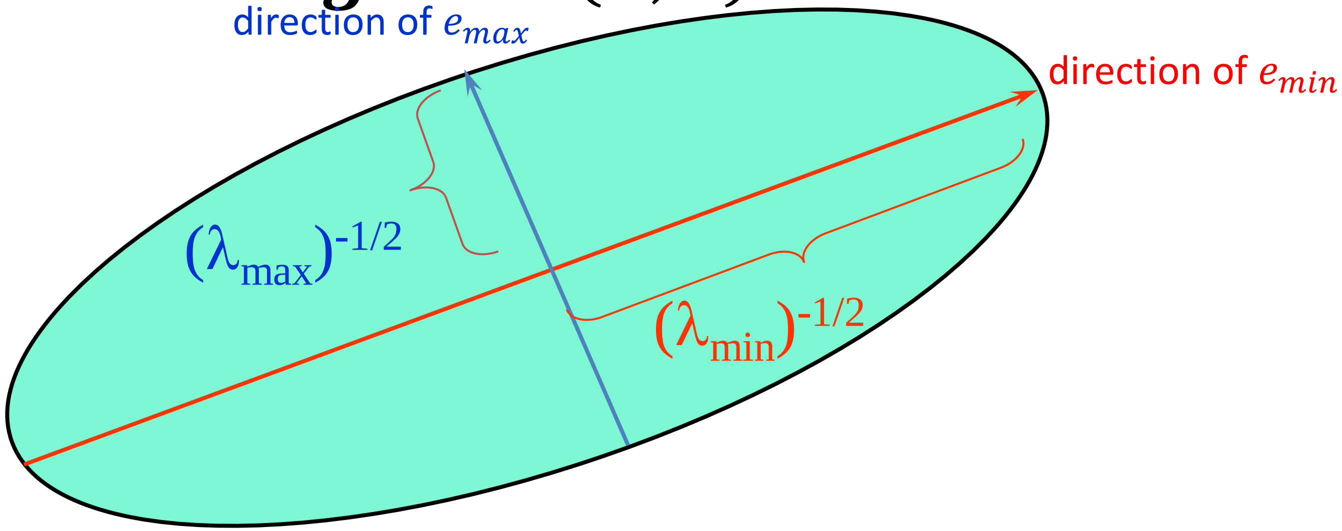


Harris corner detection

- Eigenvalues and eigenvectors of H
  - e_1 = direction of largest increase in E
  - λ_1 = relative increase in direction e_1
  - e_2 = direction of smallest increase in E
  - λ_2 = relative increase in direction e_2



- $\boldsymbol{\lambda\ larger \leftrightarrow \lambda^{-\frac{1}{2}}\ smaller \leftrightarrow faster\ change\ in\ E(u,v)\ in\ e\ direction}$

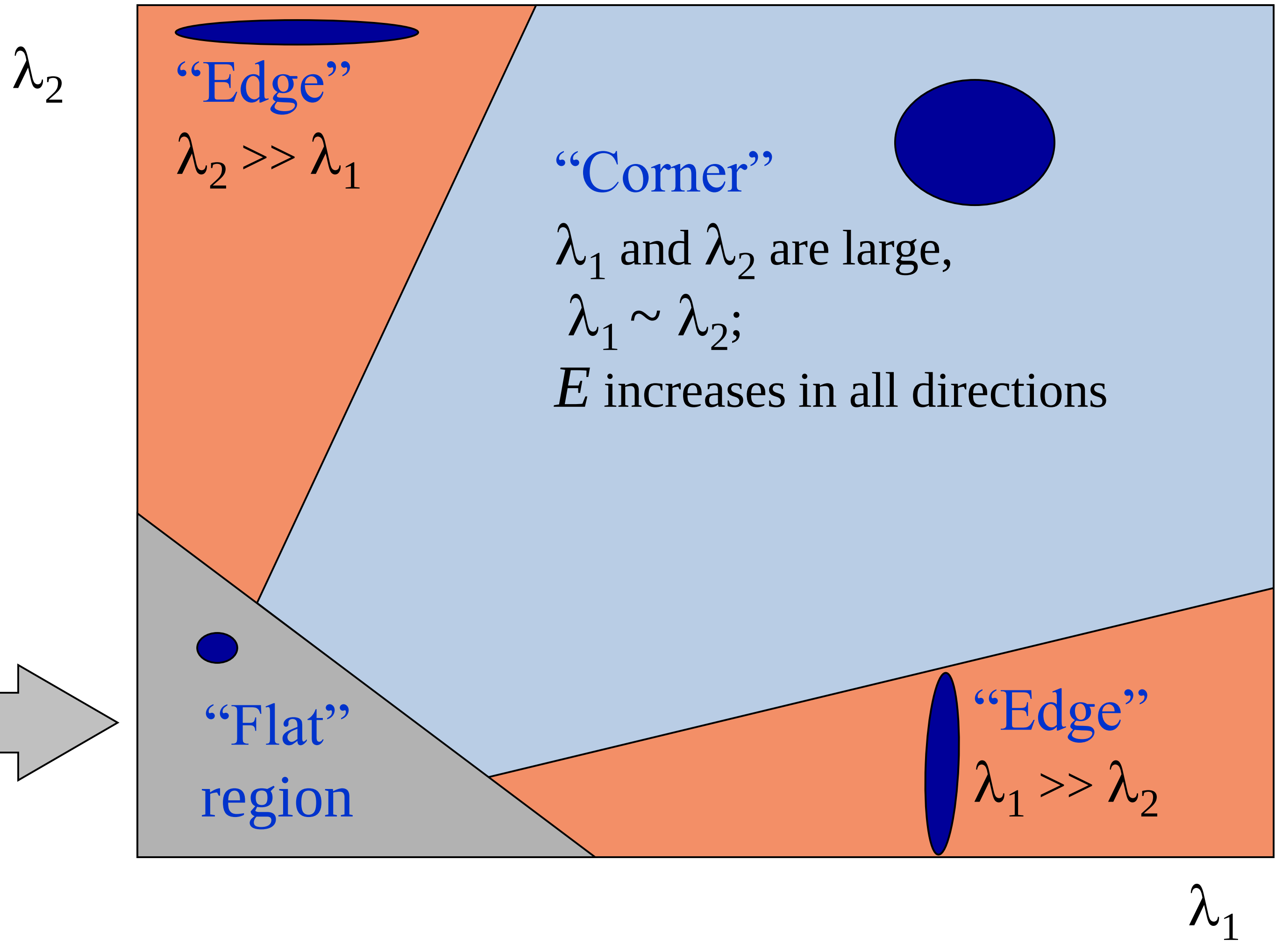


Interpreting the eigenvalues

- A “good” corner will have a large $R = \lambda_{min}$, which means big change of E in both axis.
- Getting the eigenvectors and eigenvalues is computationally inefficient.
- Instead, use two tricks:
  - $\prod_i \lambda_i = \det(A)$
  - $\sum_i \lambda_i = trace(A)$
- Then we can more easily compute $R \approx \lambda_{min}$:
  - $R = \det(A) - \kappa * trace(A)^2 \quad (\kappa \in [0.04, 0.06])$
  - $R = \frac{\det(A)}{trace(A)} = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$

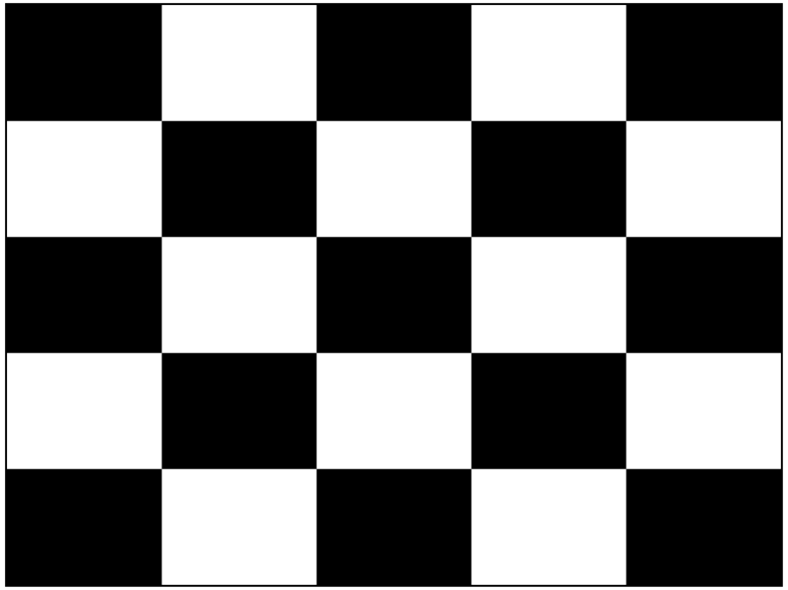
Interpreting the eigenvalues

λ_1 and λ_2 are small;
E is almost constant in all directions

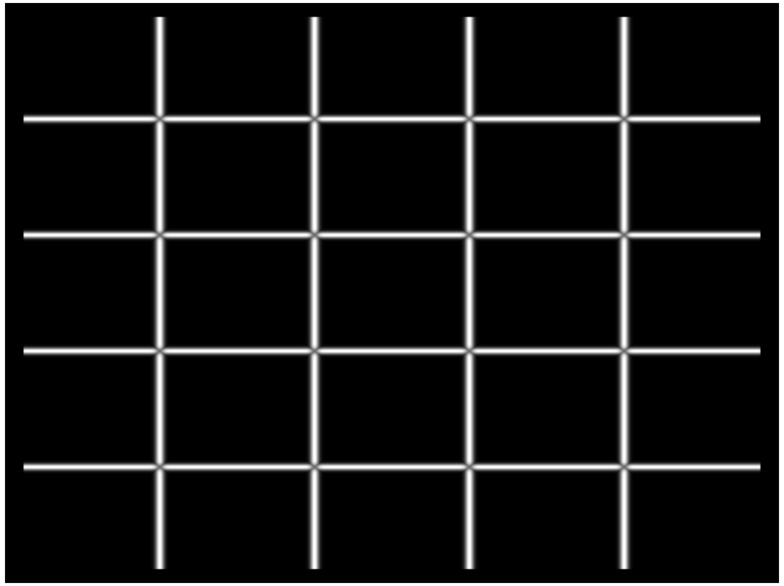


Interpreting the eigenvalues

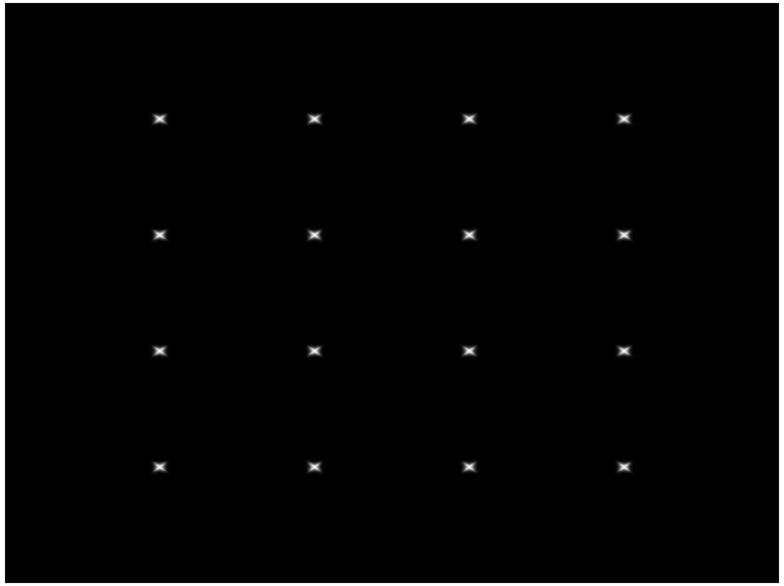
- A binary threshold of pixels above λ_{max} and λ_{min}



I



$\lambda_{\max}$



$\lambda_{\min}$

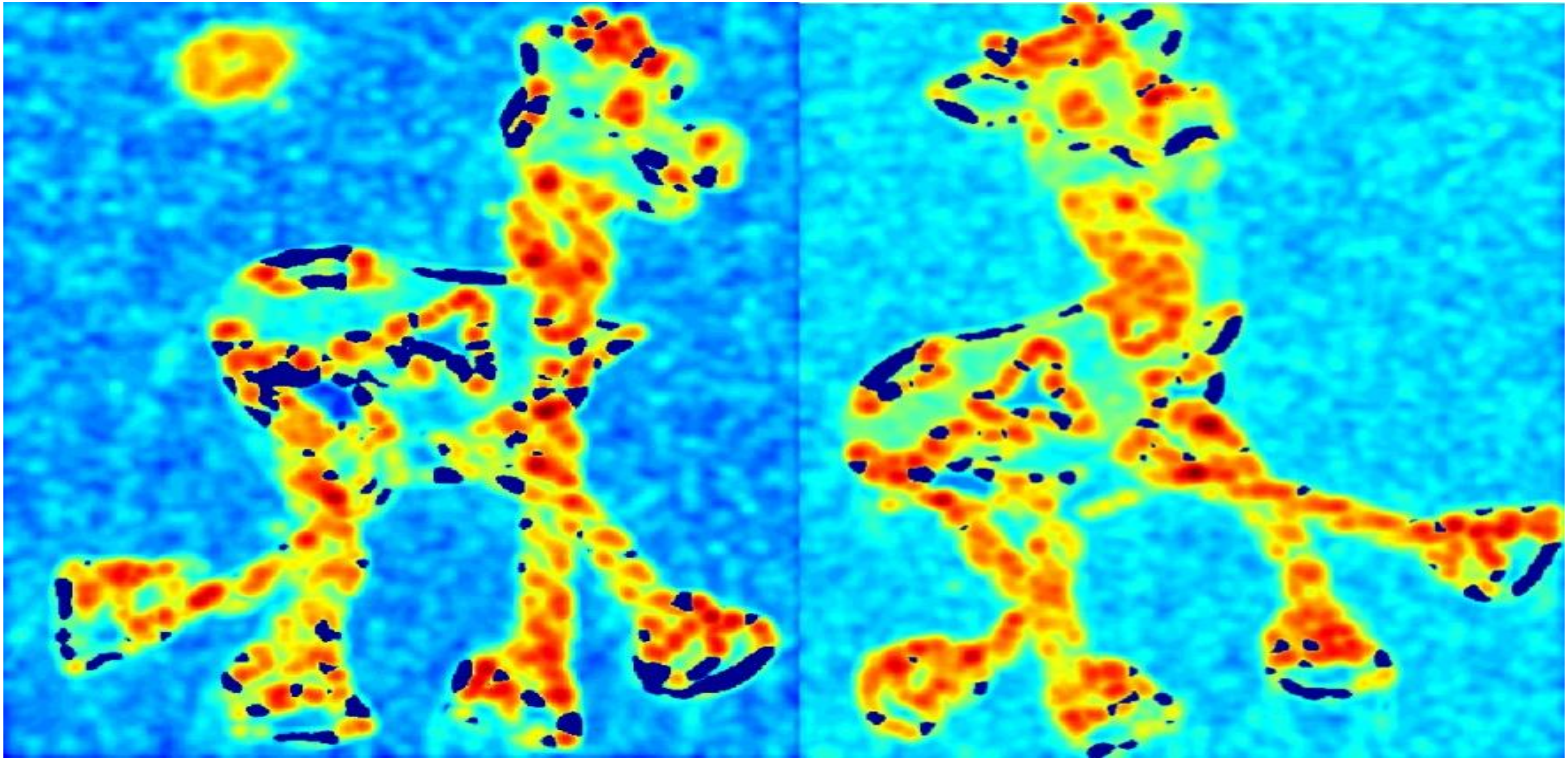
Harris corner detection

- Compute gradients of patch around each pixel.
- ~~Subtract the mean from each patch gradient.~~
- Compute the second-moment matrix.
- Compute eigendecomposition of covariance matrix.
- Use eigenvalues to find corners.

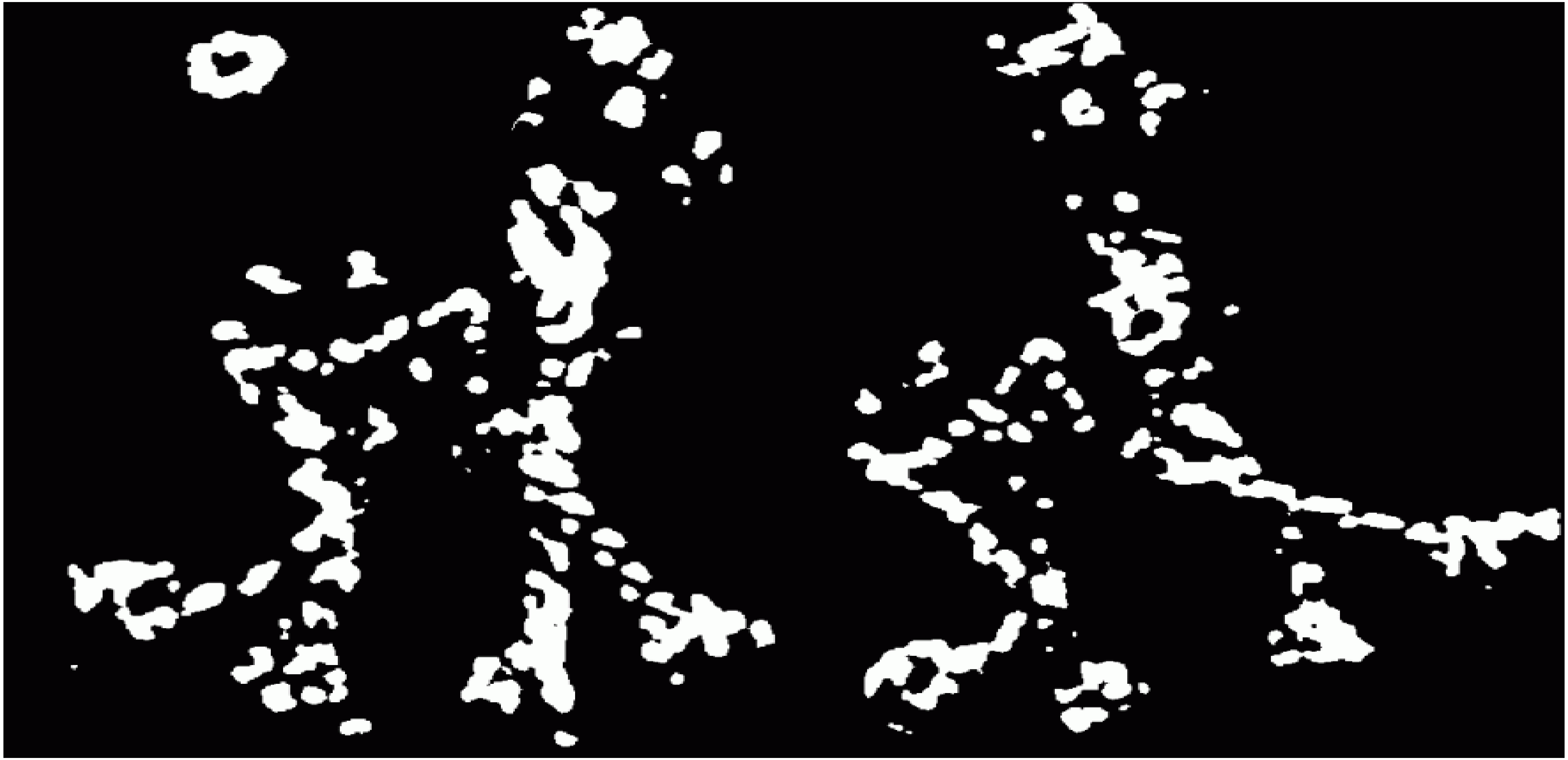
This is PCA (principal component analysis). Out of scope but really interesting!

Harris detector example

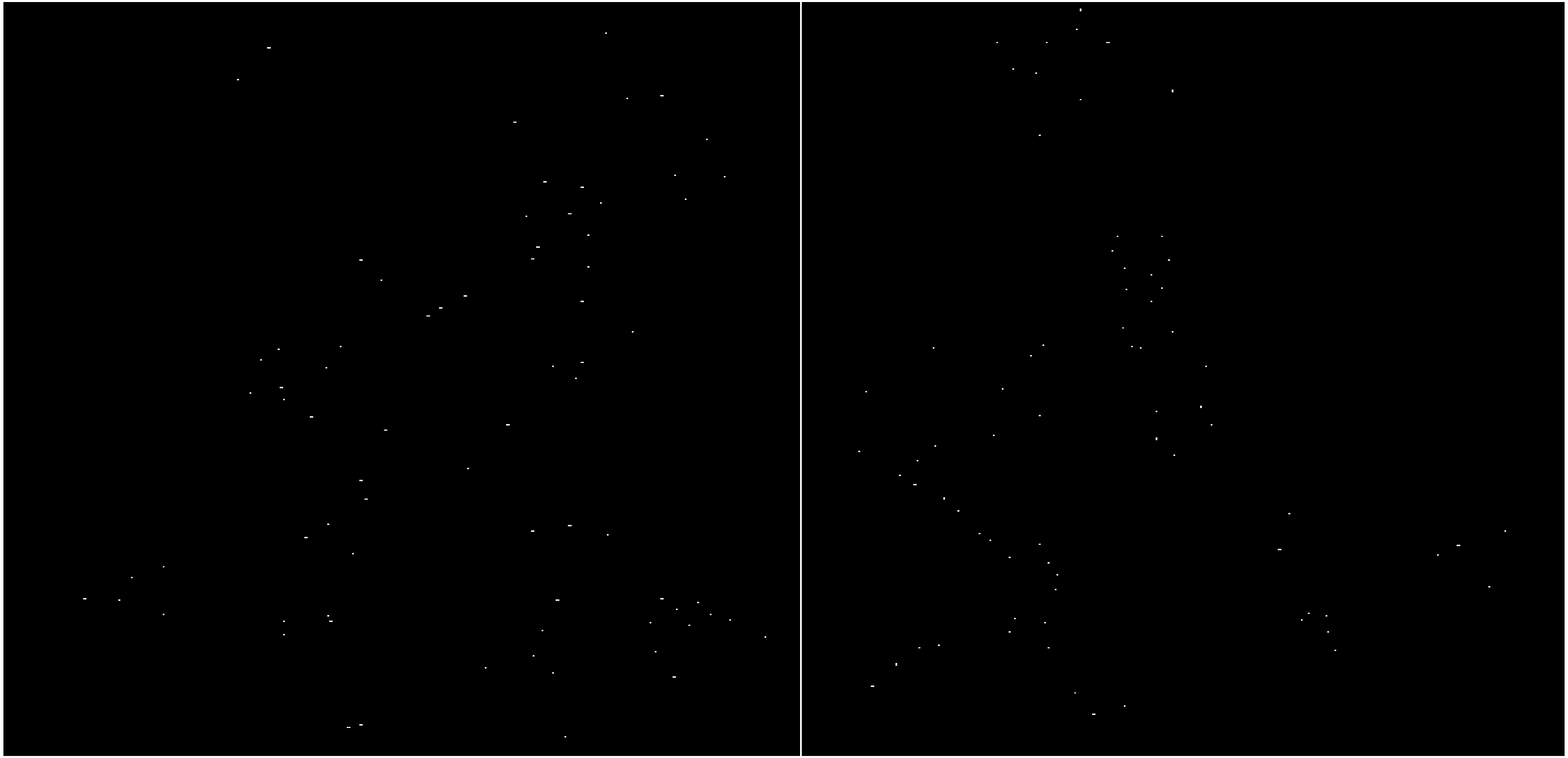
f value (red high, blue low)



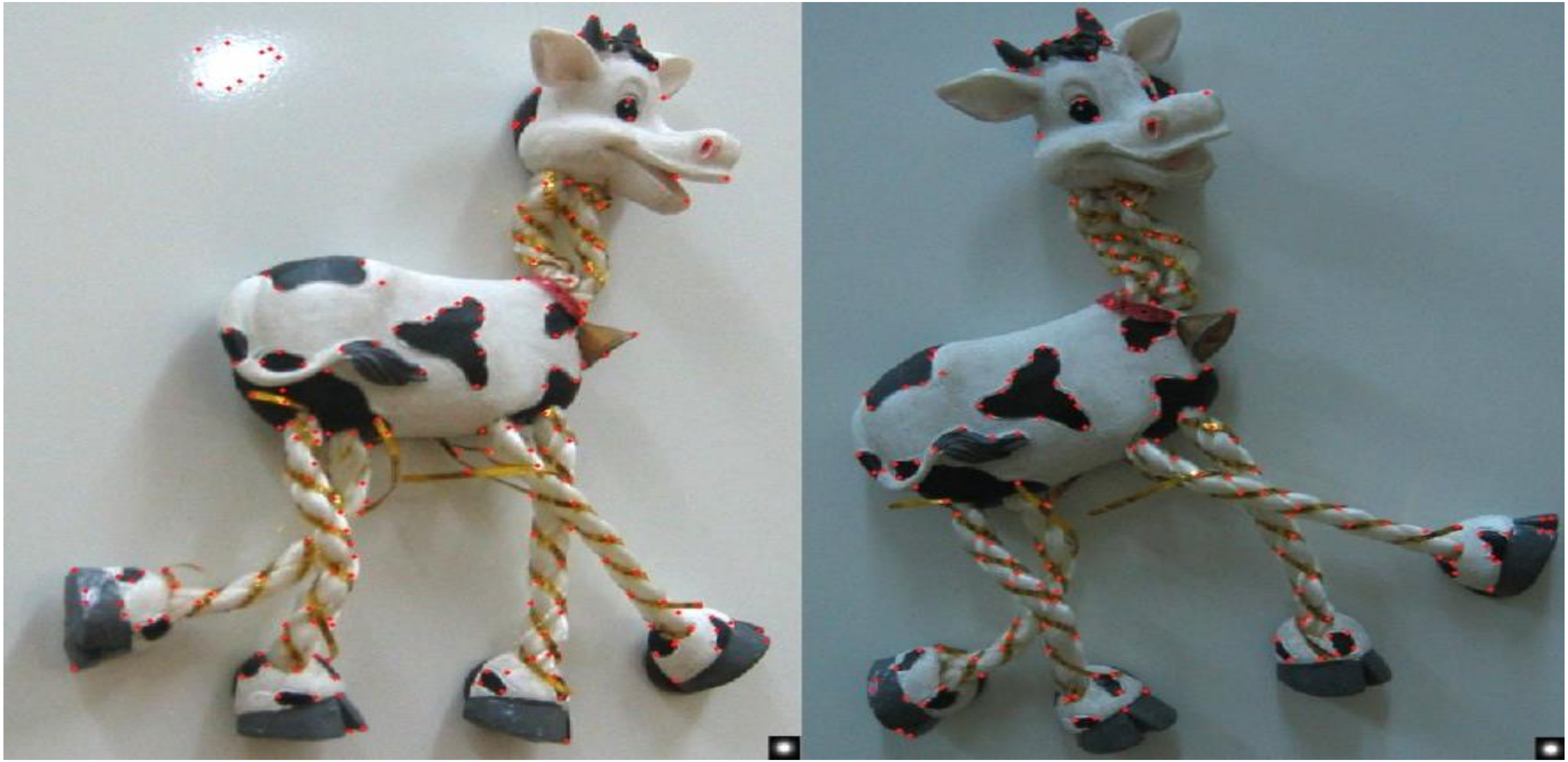
Threshold (f > value)



Find local maxima of f



Harris features (in red)



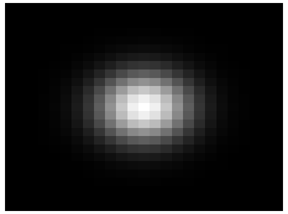
Weighting the derivatives

- In practice, using a simple window *W* doesn't work too well

$$H = \sum_{(x,y)\in W} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

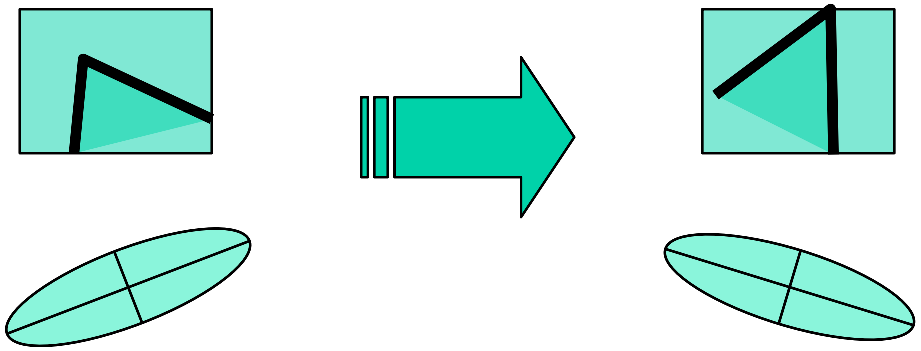
- Instead, we'll *weight* each derivative value based on its distance from the center pixel

$$H = \sum_{(x,y)\in W} w_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



$w_{x,y}$

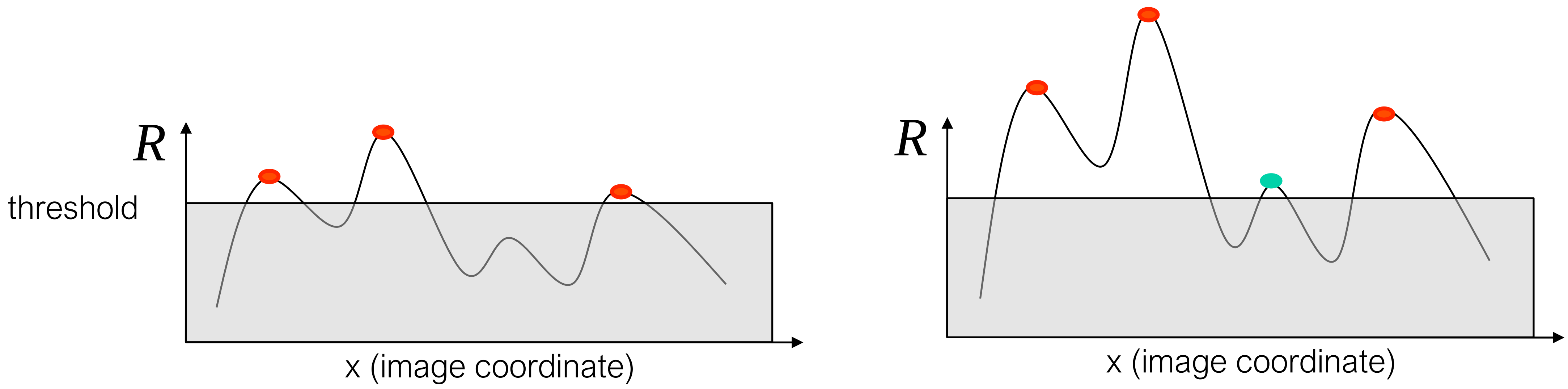
Harris corner detector- rotation and translation



- Ellipse rotates but its shape (**eigenvalues**) remains the same => invariant to rotation!
- The feature is also translation invariant (easy to see).

Harris corner detector- intensity

- Partial invariance to *affine intensity* change
- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$
- Not completely invariance to Intensity scale: $I \rightarrow a \cdot I$



The Harris corner detector is not invariant to scale

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- What and why we need features detection?
- Feature detection
  - Blob detection
  - Harris corner detection
  - **SIFT detector**
- Feature description
  - HOG
  - SIFT descriptor
- SIFT feature matching
- Panoramas

SIFT keypoint detection

- Find blobs using the improved blob detection (across different octave scale).
- Use interpolation to find exact peak of keypoint.
  - The interpolation takes place in **x, y and scale dimension**.
- Eliminate edge response with Harris corner detector variant (called **principal curvature**) around temp keypoints in interpolated space and scale.

- SIFT has the advantages of both previous technics and is invariance to: rotation, translation, scale, illumination shift and partially to 3D change of viewpoint since it is a local keypoint detector.

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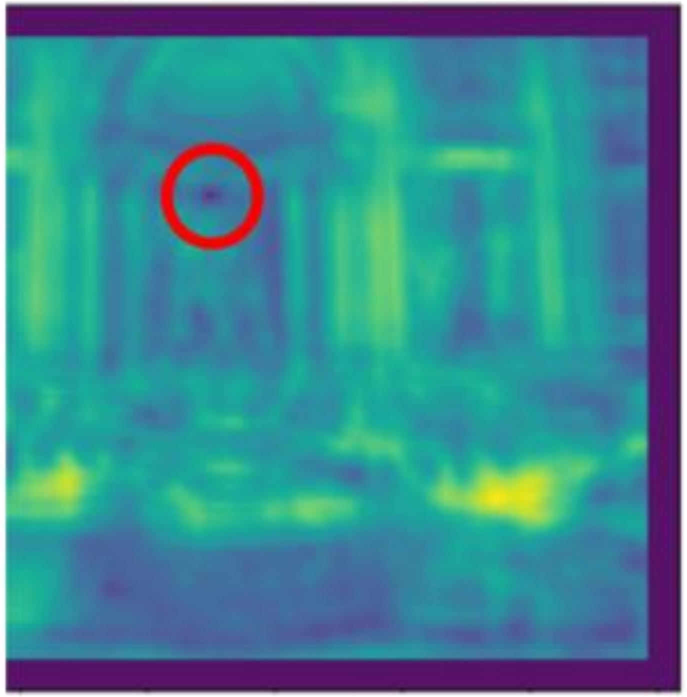
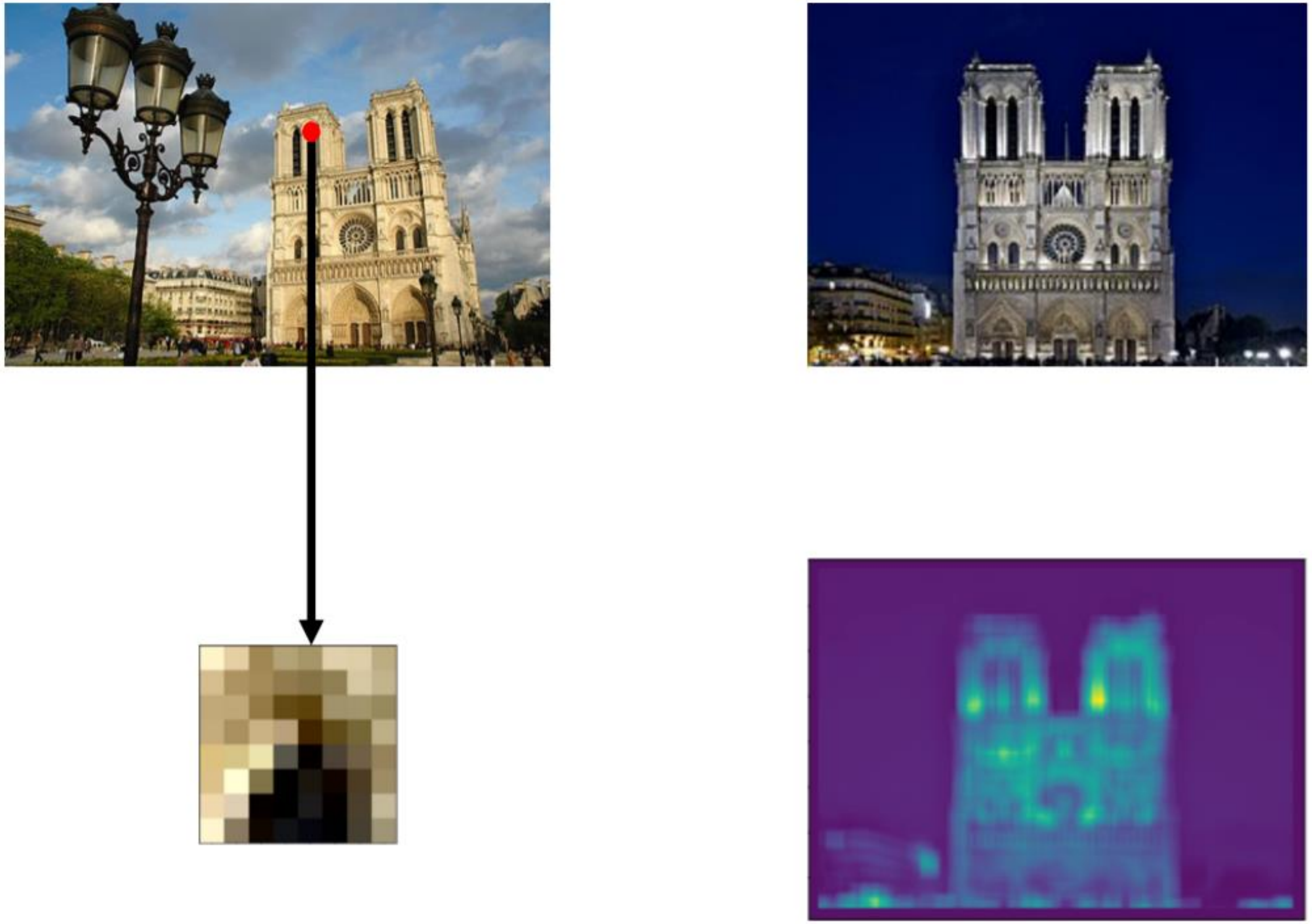
SSD

- Sum of squared distances.
- Use the whole patch as a template and find best match in other image.
- $d = \sum ||x_i - y_i||^2$



SSD

- Not good enough for light changes



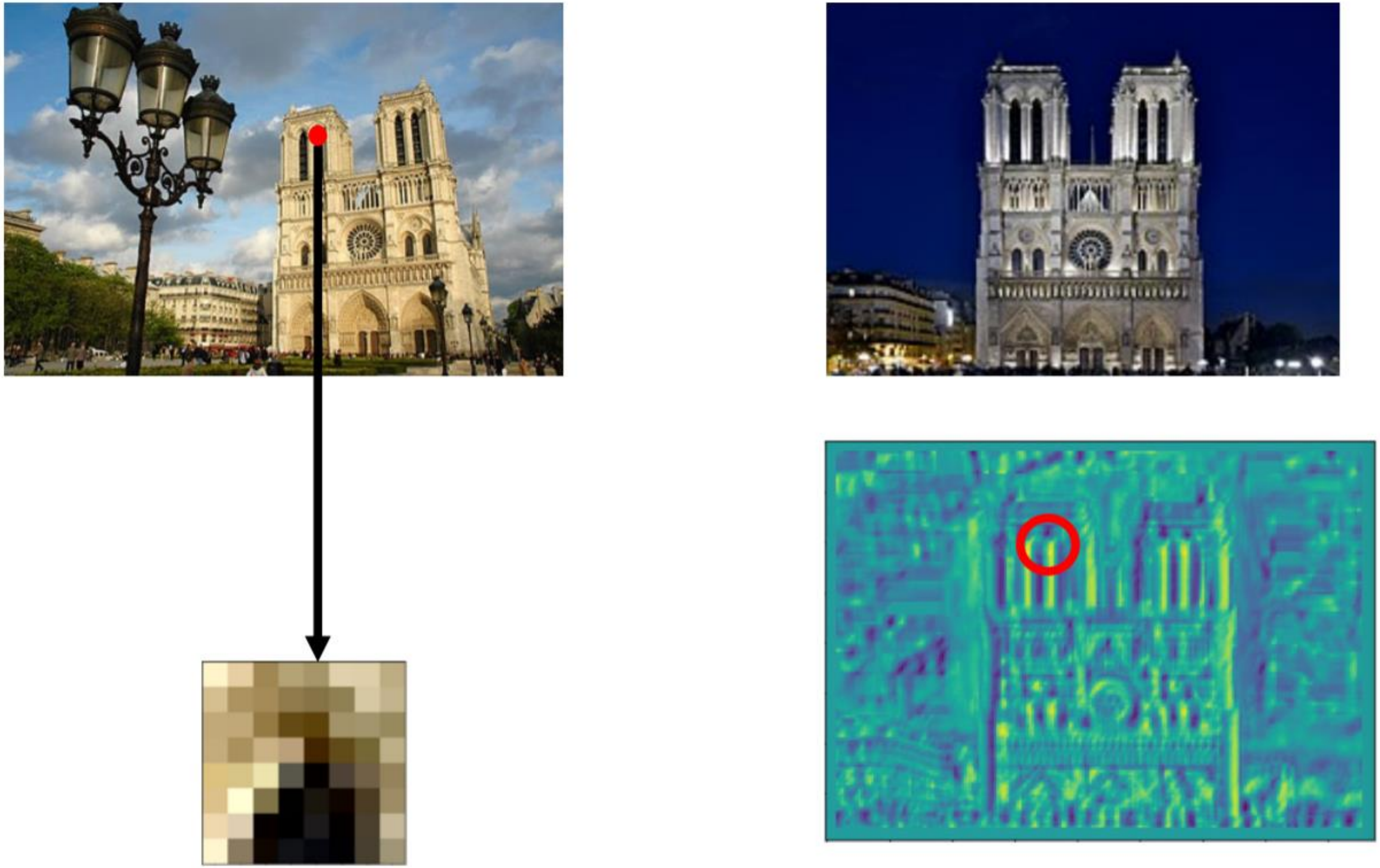
NCC

- **NCC**: Normalized cross correlation. Sometimes called **ZNCC** (zeroed NCC).
- Can handle change in lighting: $I' = \alpha I + \beta$
- Subtract patch mean: invariance to β.
- Divide by STD of template: invariance to α.

$$x' = \frac{x - \bar{x}}{\sigma_x}$$

- Run on each normalized patch of second image $NCC = x' \cdot y'$

NCC



NCC

- Problem: Still can’t handle rotation…

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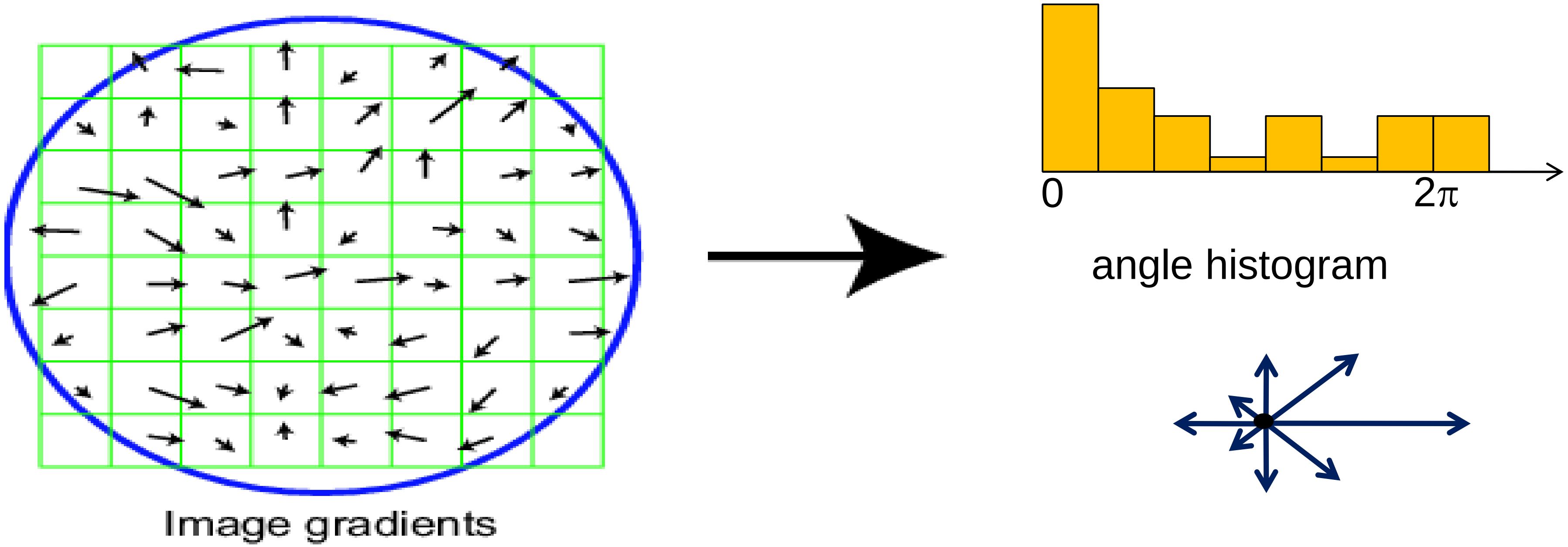
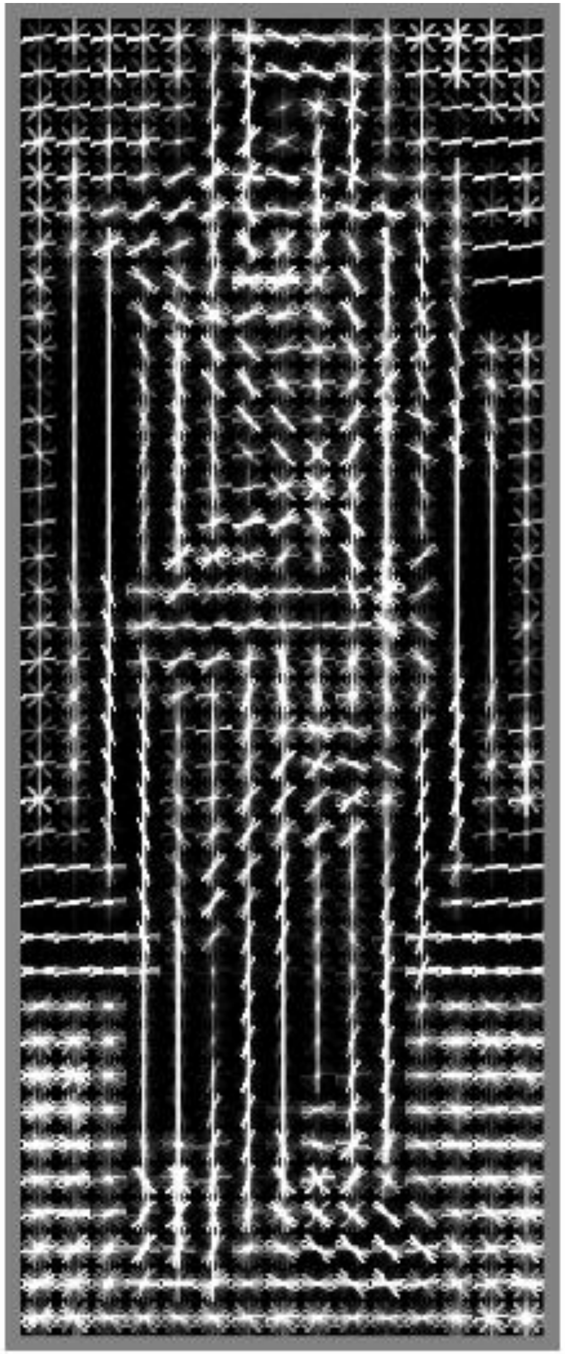
HOG- Histogram of Oriented Gradients

- A dense representation of image as blocks, each with its own histogram of gradient directions, weighted by the gradient magnitude.
- Originally used to detect humans in images.
- Equations for gradient magnitude and orientation:

$$m(x,y) = \sqrt{(L(x+1,y) - L(x-1,y))^2 + (L(x,y+1) - L(x,y-1))^2}$$

$$\theta(x,y) = \tan^{-1}((L(x,y+1) - L(x,y-1))/(L(x+1,y) - L(x-1,y)))$$



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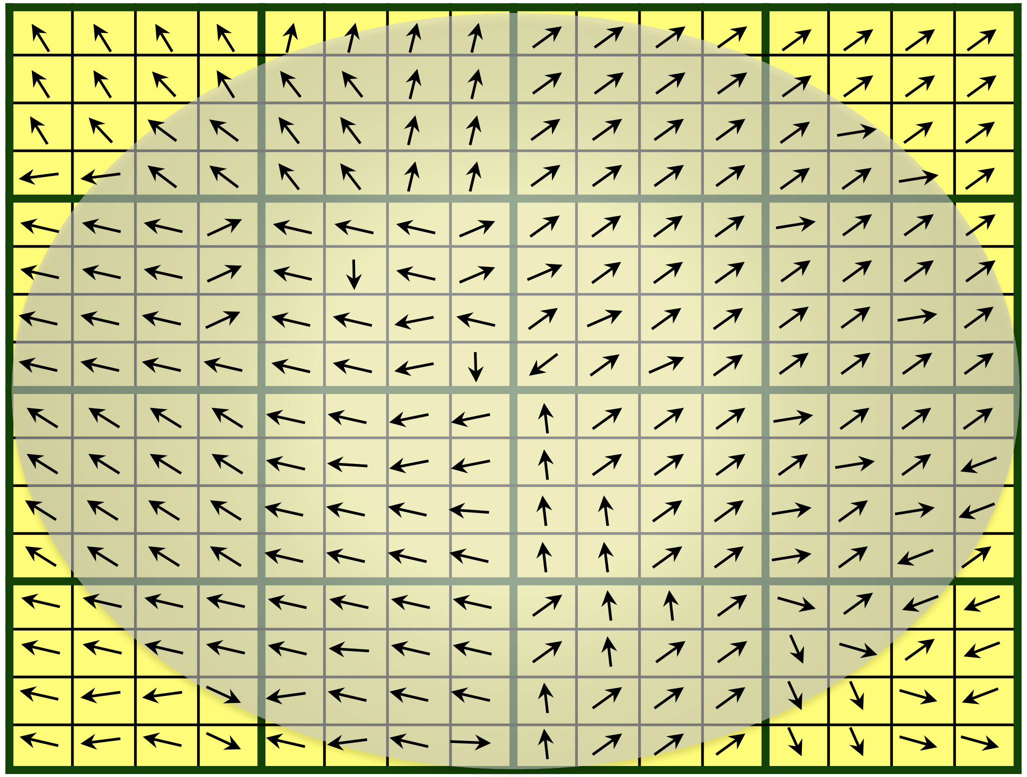
HOG variation as SIFT keypoint descriptor

- First, find the main patch direction using a HOG on all gradients around keypoint at the selected scale, all further calculation is done around this direction (this is how to get a rotation invariance descriptor).
- Take 16x16 patch around detected feature and calculate gradient orientation and magnitude to each pixel.
- Magnitude is also weighted by a Gaussian around the keypoint.
- Build sub-blocks of 4X4 of the patch.
- Create 8-bin histogram of edge orientations (weighted by Gaussian and magnitude of gradient) to each sub-block.
- Take the 4X4X8=128 results of histograms and concatenate them to a single feature vector. Normalize this vector to be invariance to illumination scale.

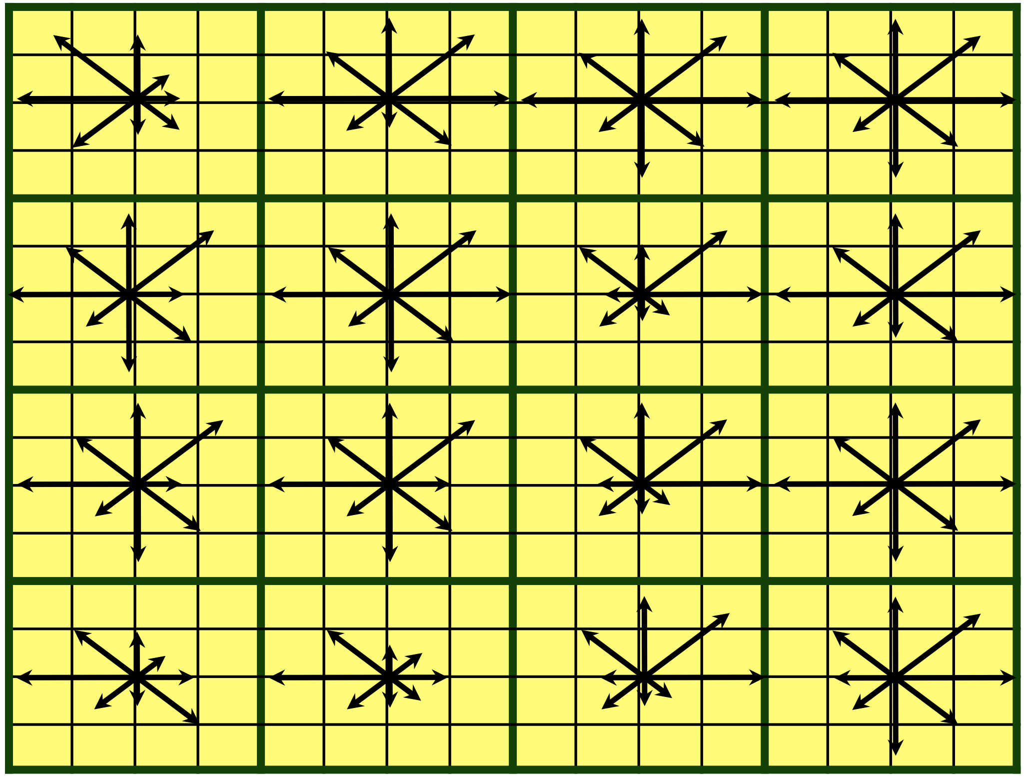
HOG variation as SIFT keypoint descriptor

Image Gradients
(4 x 4 pixel per cell, 4 x 4 cells)



Gaussian weighting

SIFT descriptor
(16 cells x 8 directions = 128 dims)



HOG variation as SIFT keypoint descriptor

HOG variation as SIFT keypoint descriptor

- Invariant to:
  - Rotation (due to shifting of the histograms around the main direction).
  - Translation (easy to see).
  - Scale (build at a specific scale).
  - Illumination shift (only derivatives are used).
  - Illumination scale (normalize the feature vector).
  - 3D change of view (the descriptor is of local keypoints).

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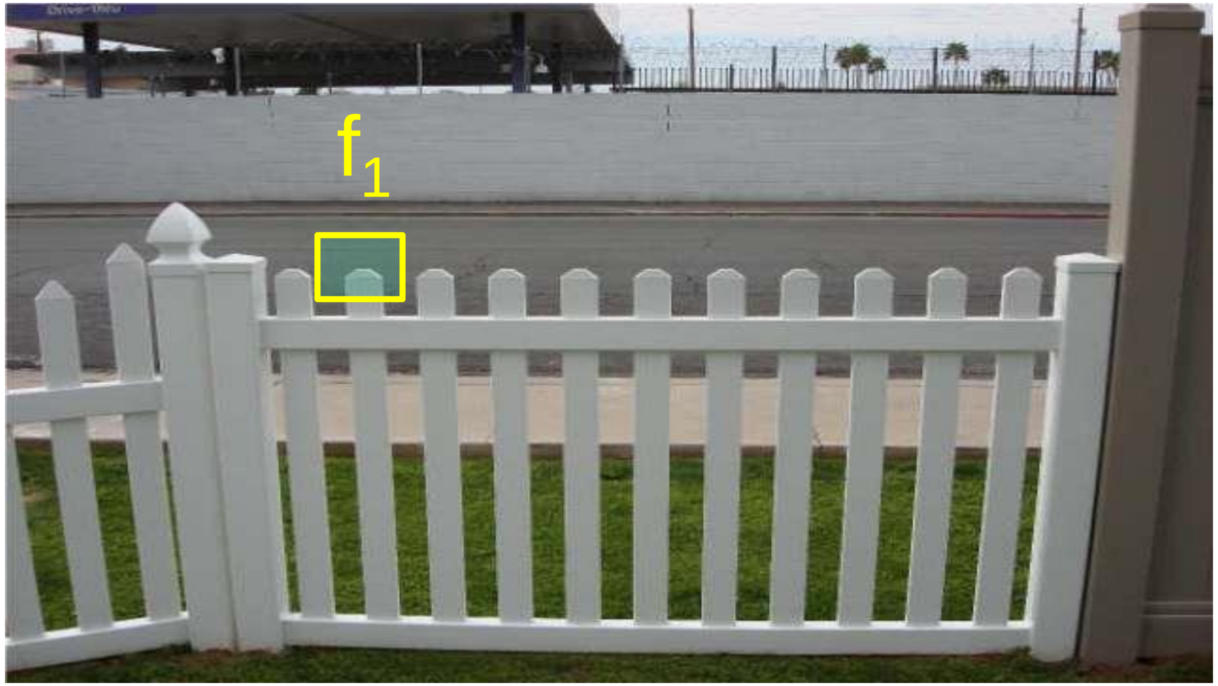
SIFT Feature matching

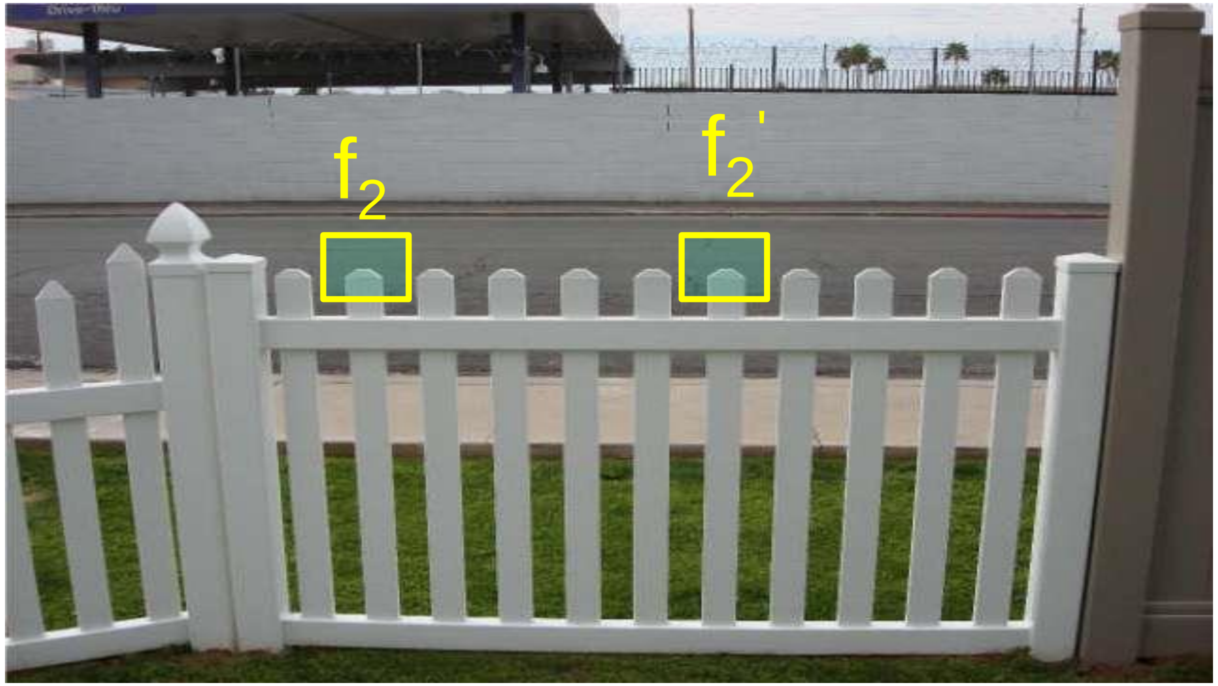
- Given a feature in I_1, how to find the best match in I_2?
  1. Define distance function that compares two descriptors
  2. Test all the features in I_2, find the one with min distance
- What distance function to use?

SIFT Feature matching

- What distance function to use?
  - Simple approach: L_2 distance, $\mathrm{L}_2 = ||f_1 - f_2|| = \sqrt{\sum_i (f_{1_i} - f_{2_i})^2}$
  - Good overall **but** can also give small distances for ambiguous (incorrect) matches.



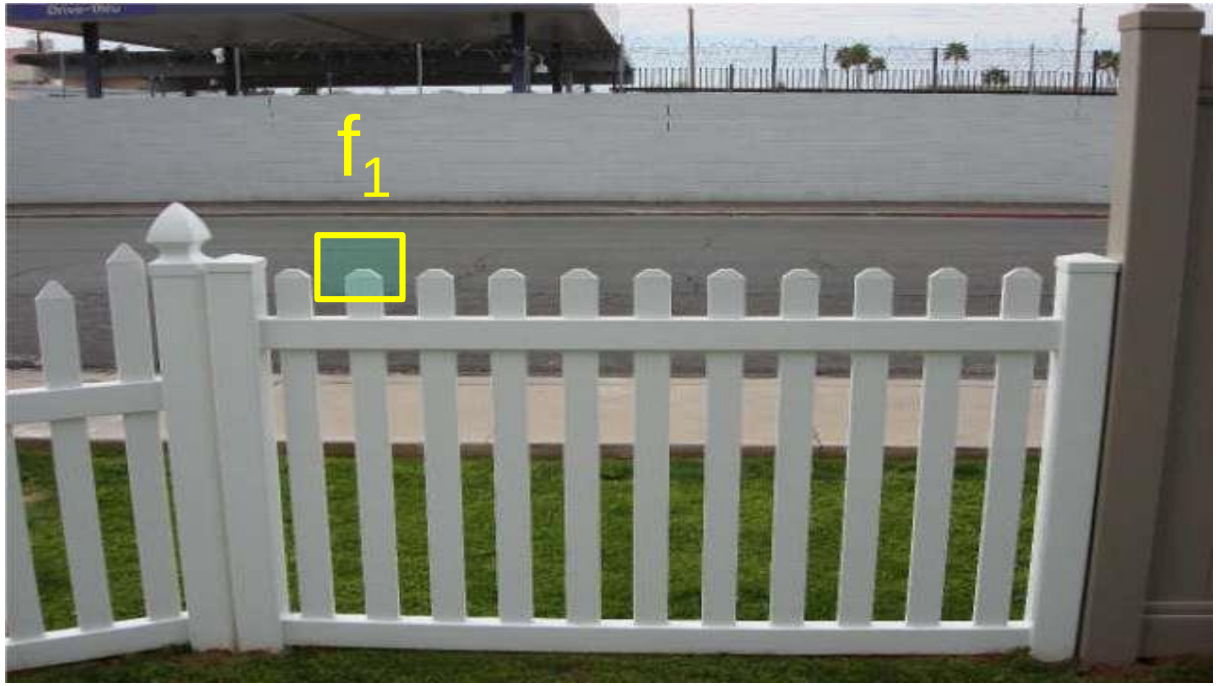
I_1



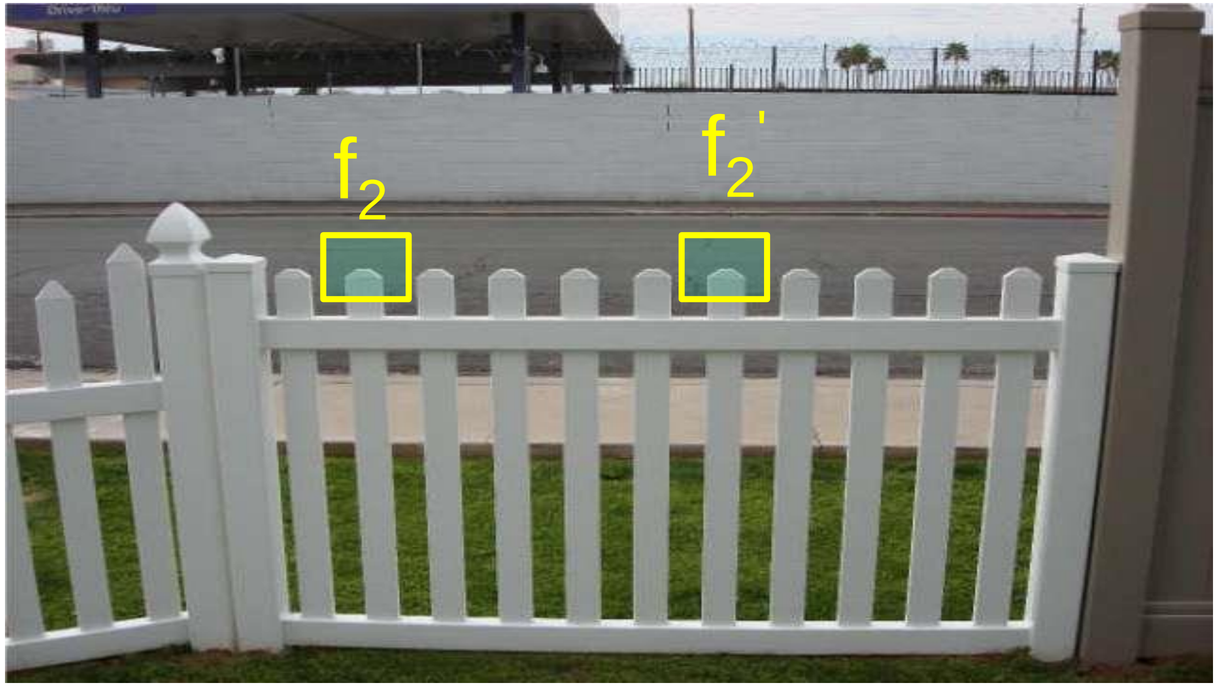
I_2

SIFT Feature matching

- Better approach: ratio distance: $||f_1 - f_2|| \,/\, ||f_1 - f_2'|| < TH$
  - f_2 is best match to f_1 in I_2
  - f_2' is 2^{nd} best match to f_1 in I_2
  - gives larger values for distinct matches.



I_1



I_2

What haven't been covered about SIFT

- Computational fast search of descriptors.
- A lot of minor engineering steps (e.g. thresholding of features).
- And more… this algorithm is 28 pages long article (with 52000 citations!!!)
  - https://www.cs.ubc.ca/~lowe/papers/ijcv04.pdf

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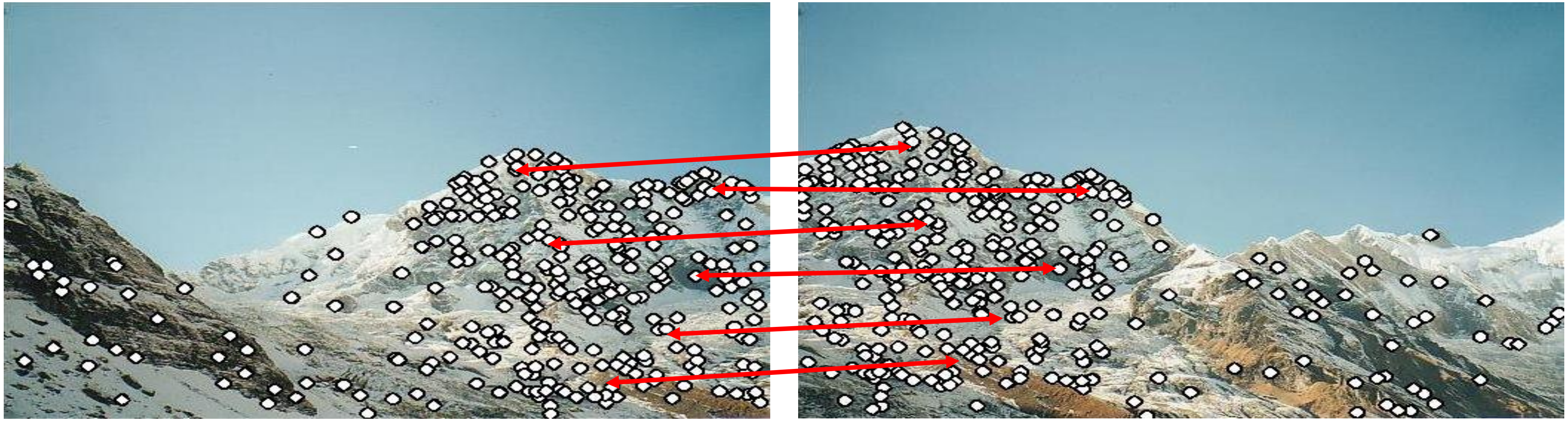
Panoramas

- We have two images – how do we combine them?

Panoramas

- Use SIFT to match descriptors of the two images.
- Between the two images there can be a unknown homographic transformation.
  - How do we align the two images?



Panoramas

- Find the best homographic projection from I_2 to I_1 using RANSAC.
  - Finding this homographic projection is the same as finding the camera calibration matrix that we've seen, only with 3X3 matrix.
  - RANSAC is used to drop wrongly matched points (outliers). Only 3 points needed to be chosen at random to find a RANSAC projection.