

## Week 8

### Assignments:

h7 available \_\_\_\_\_ and due before 10pm on after spring break

p3b available and due before 10pm on 3/28

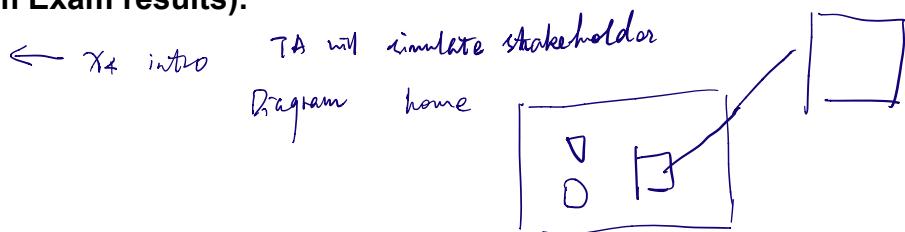
{ x4 available and due before 10pm on 4/1

x5 available and due before 10pm on 4/1

**Read:** Week 8 (start on week and 9 for week after Spring Break)

### THIS WEEK (in addition to the Midterm Exam results):

- Graphs
  - edge representations
  - traversals
  - Spanning Trees
  - topological ordering
  -
- Exam Results (Tuesday or Wednesday in class)



### NEXT WEEK (SPRING BREAK – No classes, lab or office hours)

### Next WEEK (after Spring Break)

- Topological Ordering
- Dijkstra's Shortest Path
- Spanning Trees

## Compute degree of a Vertex

→ Write the code to be added to a Graph class that computes the degree of a given node in an undirected graph.

*Adjacency list:*

```

public int degree( Graphnode<T> n) {
    O(1)    return n.getAdj().size();
}

```

*\* degree:  
 "out" degree for  
 direction.*

*Adjacency matrix:*

```

public int degree( Graphnode<T> n) {
    O(1)    int degree = 0;
    O(1)    int index = n.getAmIndex();
    O(V)    for (int col = 0; col < AM[index].length; col++) {
    O(V)        if (AM[index][col])
    O(V)            degree++;
    }
    return degree;
}

```

$O(V)$   $\uparrow$  # of vertices

## Comparison of Edge Representations

*Ease of Implementation*

\* All are easy

**Space (memory)**  $V \# \text{vertices}, E \# \text{edges}$

AM  $O(V^2)$

AL  $O(V+E)$

**Time (complexity of ops)**

Depend on implementation

node's degree?

AM  $O(V)$  linear

AL  $O(1)$  constant.

edge exists between two given nodes?

AM  $O(1)$  constant  $\text{AM}[\text{IndexOf } \text{From}][\text{IndexOf } \text{To}]$ .

AL  $O(N)$  linear ← must iterate through AL.  
at adjacent nodes for vertex.

## Searches and Traversals

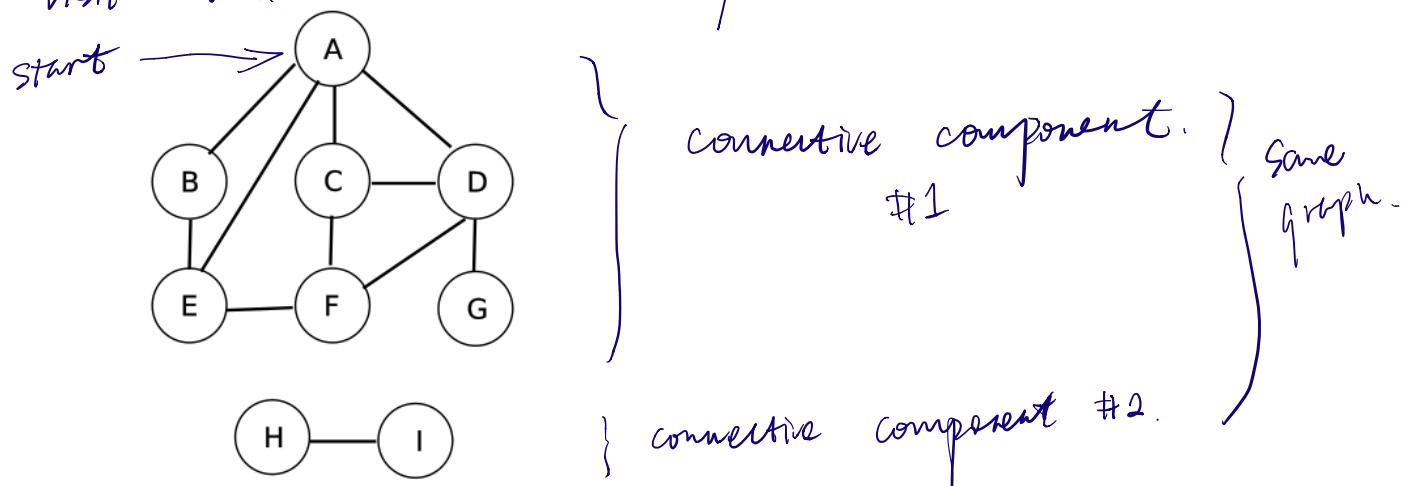
**Search** Given a "start vertex"  
look through collection to find something  
stop when found or reach end of selection

**Traversal**

visit each vertex exactly one

specify a "start" vertex

visit vertices that are reachable from start vertex.



→ Which connected component in the graph above can produce the longest path?

Problem: How do we avoid cycles?

Solution: traverse, mark visited vertices

and when reach a marked vertex, that's a cycle

**\*\*CS400 Convention:** pick next unvisited vertex in increasing alphanumeric order

### Depth-First Search

Assume: all vertices are unvisited at start.

Relies on: stack, use call stack + recursion recursive

Algorithm:  $\text{DFS}(v)$

mark  $v$  as visited  
for each unvisited successor ( $s$ ) of  $v$   
 $\text{DFS}(s)$

Equivalent to: pre order tree traversal.

### Breadth-First Search

Assume: all vertices are unvisited at start.

Relies on: queue; need to create queue x recursive

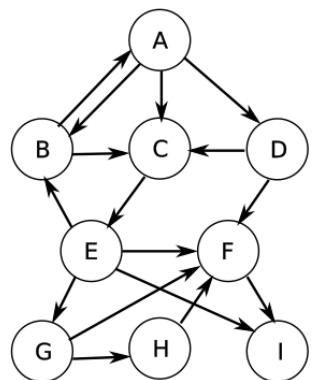
Algorithm:

```
BFS(v)
q = new Queue()
mark v as visited first.
q.enqueue(v) // add to queue
while (!queue.isEmpty())
    c = q.dequeue() // remove
    for each unvisited successor (s) of c;
        mark s as visited
        q.enqueue(s)
```

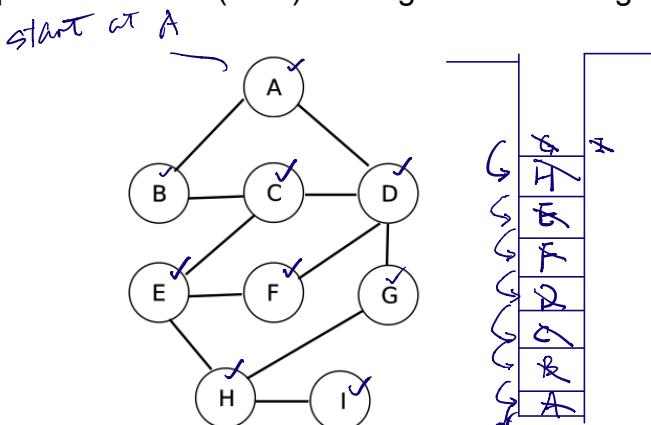
Equivalent to: level - order traversal

### DFS Practice

Give the order that vertexes are visited for depth-first search (DFS) starting at A for each graph



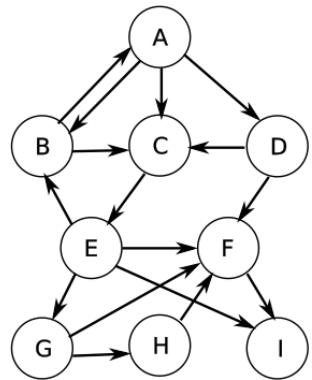
Graph 1:



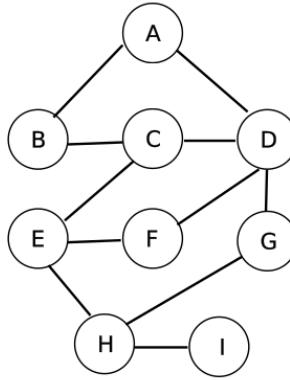
Graph 2:

A B C D F E H G I

Give the order that vertexes are visited for depth-first search (DFS) starting at C for each graph



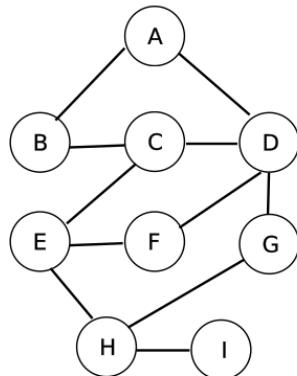
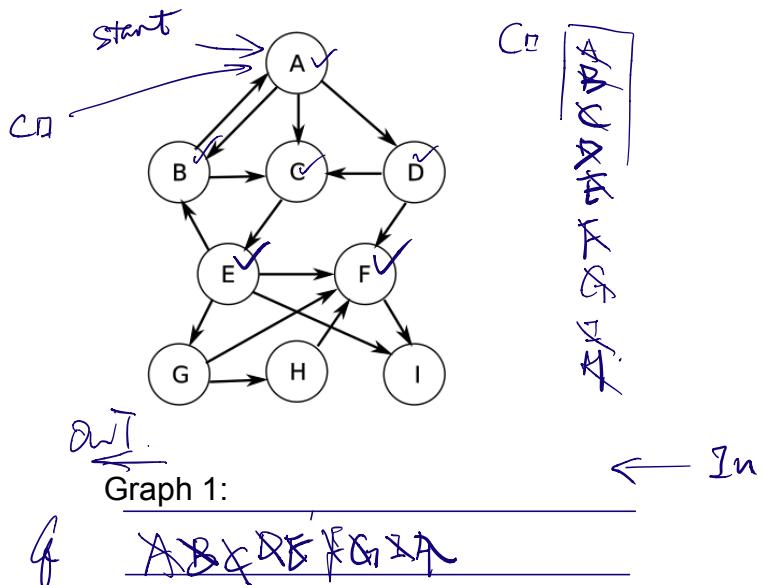
Graph 1:



Graph 2:

### BFS Practice

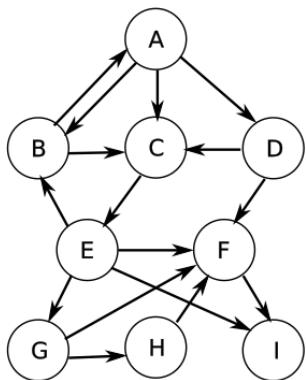
Give the order that vertexes are visited for breadth-first search (BFS) starting at A for each graph



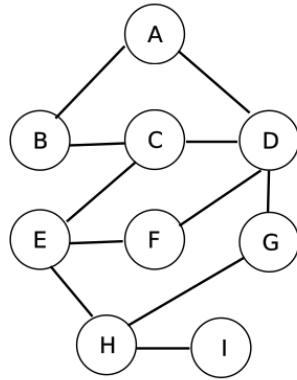
Graph 2:

Output / traversal order  
A B C D E F G I H

Give the order that vertexes are visited for breadth-first search (BFS) starting at C for each graph



Graph 1:



Graph 2:

traversal works.

**Applications of DFS & BFS**

- Is the graph connected → one connected component.
- what vertices are reachable from start?

## Path Detection

Is there a path from  $S \xrightarrow{\text{start}}$   $T \xrightarrow{\text{target}}$ ?

what is the path. (DFS)

what is the shortest path? (fewer # edges)

lowest "cost".

work if graph is unweighted.  
use bfs

## Cycle Detection

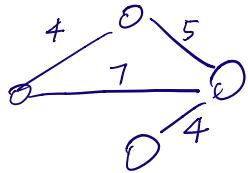
Is there a cycle = path from start to start.

cycle: have 3 or more vertices

Are there cycles that don't include start vertex

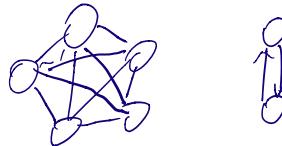
## More Graph Terminology

**Weighted graph:** assign a "cost" to each edge

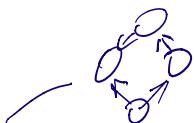


**Network:** a weighted directed graph-edge, weights are non-negative.

**Complete graph:** an edge exists between every vertex pair



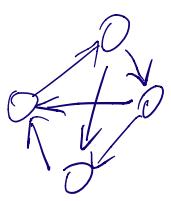
**Connected graph (undirected):** a path exist between every vertex pair



not has to be complete

**Connected graph (directed):**

**weakly connected**: a path exists between every vertex pair  
if direction ignored



**strongly connected**

$\rightarrow$  (strongly  $\dashv$  is weakly connected)  $\dashv$  not ignored.

**Length of a path:**

# edges on the path

**Sub-graph:**  $T$  is a sub-graph of  $G$

if all vertices and edges in  $T$  exists in  $G$ .

P.S. two graph, same, both sub-graph  
of the other -

"cost" of the path: sum of weights of edges on the path.

## Spanning Trees

### Definition

*Tree*

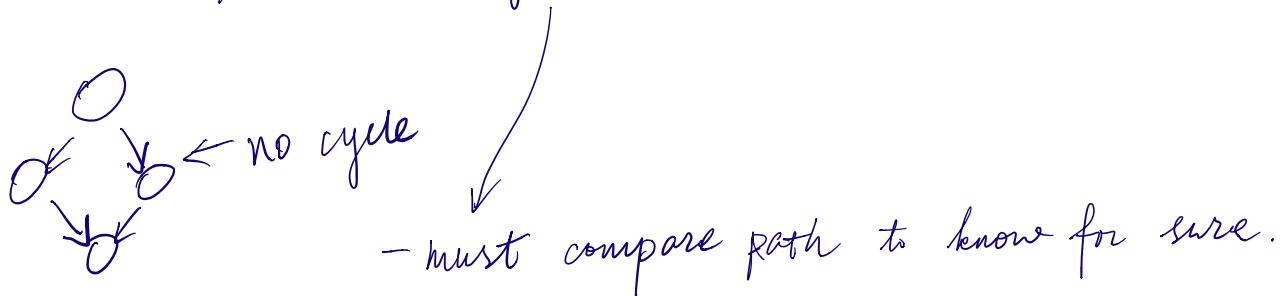
- if  $T$  is a sub-graph of  $G$ ,  
(must pick a root)
- and contains all vertices of  $G$ , it is a spanning Tree.
- All spanning trees are sub-graphs, have root
- they are tree - have a root, no cycles
- have exactly  $N-1$  edges,  $N$ : # vertices (order of the graph)

Easy Spanning Tree: remove edges until there is no cycles.  
and pick a root.

### Cycle detection

$\text{DFS}(s)$  – recognize cycle when we visit an already visited node  
successor

Can  $\text{BFS}(s)$  detect cycle?



## DFS and BFS Spanning Trees

*DFS algorithm*

```
add vertex V to T
mark vertex V as visited
push V to stack
while stack is not empty
    C is vertex on top of stack (not popped)
    if no unvisited successor of C
        pop C from the stack
    else
        S is next unvisited successor of C
        add S to T
        add edge (C,S) to T
        mark S as visited
        push S to stack
```

*BFS algorithm*

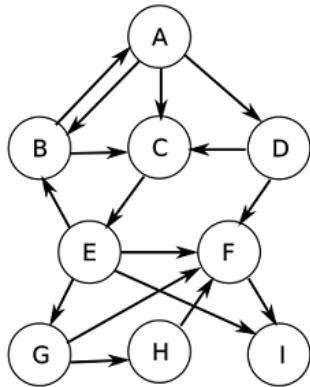
Follow the ~~DFS~~ or BFS algorithms as for the traversal

**Add V to T**

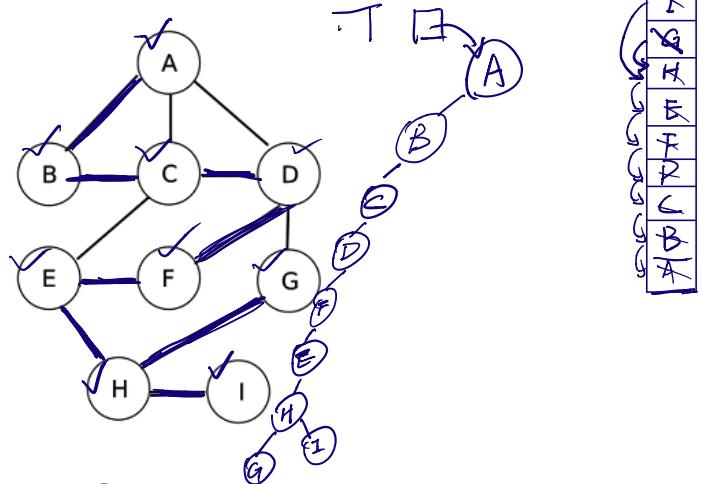
```
Mark V as visited
Add V to queue
while queue is not empty:
    C = queue.dequeue()
    for each unvisited successor (S) of C
        add S to T
        add edge (C,S) to T
        mark S as visited
        add S to queue
```

## DFS Spanning Tree Practice

Which edges are not in the DFS spanning tree starting at A for each graph

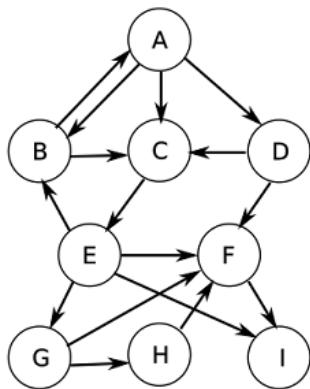


Graph 1:

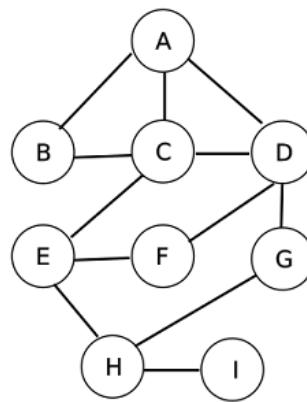


Graph 2:

Which edges are not in the BFS spanning tree starting at E for each graph



Graph 1:



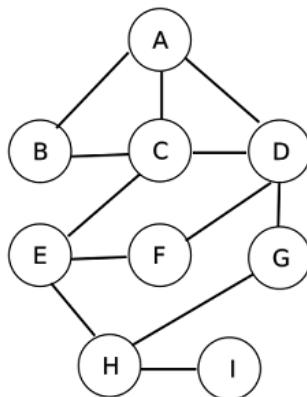
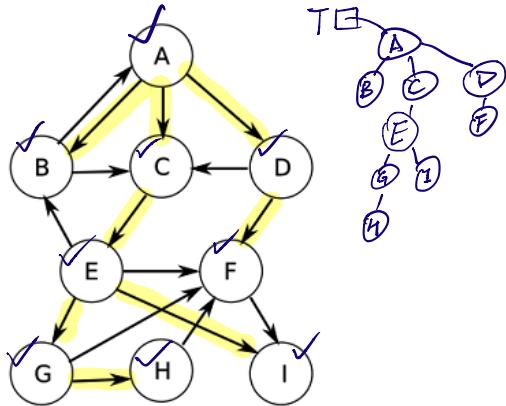
Graph 2:

**B->A, B->C, D->C  
E->B, E->F,  
F->I  
G->F  
H->F**

**B-C, C-D, E-F, G-H**

## BFS Spanning Tree Practice

Which edges are not in the BFS spanning tree starting at A for each graph



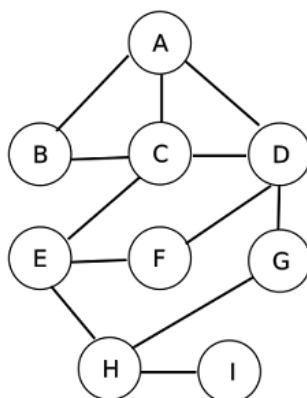
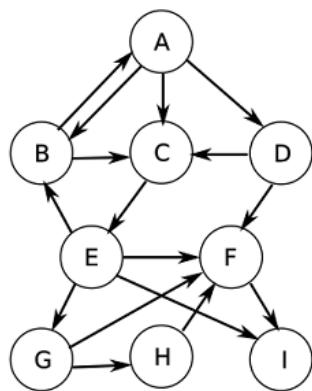
Graph 1:

Out      Queue      In

A B & D E F G & H

Graph 2:

Which edges are not in the BFS spanning tree starting at D for each graph



Graph 1:

Graph 2:

Exam.

- ⑧ If the same element is inserted in to an AVL and Red Black tree,  
the height of two tree will differ by most one level  
False

14. Consider full resize.

Load Factor  $\geq$  LoadFactorThreshold

28. copy file from remote repo to local repo. push

我搞錯的是 git push 問題.