

## Week 2

### ASSIGNMENTS

h0 available and due before 10pm on Monday 1/28

h1 available and due before 10pm on Monday 2/4

p1 available and due before 10pm on Thursday 2/7

**TA Lab Consulting** - See schedule (cs400 home pages)

**Peer Mentoring available** - Friday 8am-12pm, 12:15-1:30pm in 1289CS

**TAs and Peer Mentors will focus on helping students get p1 JUnit tests running in Lab.  
We can not guarantee that we can get your personal computers configured.**

*Module: Week 2 (start on week 3 before next week)*

### THIS WEEK

- Ready Set Program 1!
  - Read Assignment - there are getting started instructions there
  - Create and configure project for JUnit5, compile and run **TestDS\_MyPWT** tests in **DataStructureADTTest** (not the sub class).
- Testing: JUnit5
- Java: inner classes
- Determining Height of a Tree (Recursion Review)
- Binary Search Trees (BST) (Review?)
  - operations
  - implementing
  - complexities
- Classifying Binary Trees
- Balanced Search Trees
- George Adelson-Velsky and Evgenii Landis

### NEXT WEEK

- X-team Exercise x1 (in-class exercise with your assigned teams)
- Watch for instructions to find your team number and how to meet

## Writing JUnit5 Tests

### What is JUnit?

- a testing framework for Java class
- has test runners that runs the test you write

### How do I use it?

1. Add JUnit to the build path
2. Add JUnit test case (class)
3. Add units test to test class
4. Run the test class

### What does a `@Test` method look like?

- code to try things
- code to detect problems → `if (bad conditions) fail()`  
`assertEquals(expr1, expr2)`

### Details:

#### 1. Import JUnit classes

```
import static org.junit.jupiter.api.Assertions.*;
import org.junit.jupiter.api.AfterAll;
import org.junit.jupiter.api.AfterEach;
import org.junit.jupiter.api.BeforeAll;
import org.junit.jupiter.api.BeforeEach;
import org.junit.jupiter.api.Test;
```

Eclipse imports these

```
public class TestClass {
    @Test ~ annotations
    void test123_try_someting_check_results() {
        // try something
        // if (cond_expression) fail("descriptive failure message")
        // assertEquals( expr1, expr2 )
        // fail if something unexpected occurs or if something expected does not occur.
    }
}
```

## Java's Inner Classes

<https://docs.oracle.com/javase/tutorial/java/javaOO/innerclasses.html>

What is an inner class?

- a class defined in another class
- make the inner class private, make members of inner class private

search  
↙

Define a generic Tree class using an inner class for the individual node type of the tree.

```

public class Tree<K extends Comparable<K>> {
    private class Treenode<K> {
        private K data;
        private ListADT<Treenode<K>> children;
        //private constructors and methods
        ...
    }
    private Treenode<K> root;
    private int size;
    public Tree() { ... }           (constructor)
    add();
    remove();
}

}

```

t<sup>0</sup>

height = 0

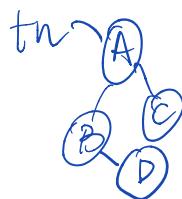
/ = levels.

CS 400 – Programming III

### Determining Height of a Tree (or sub-tree)

t<sup>1</sup>

h<sup>1</sup> = 1  
height of a tree



height = 3.

We define height of a tree as the # nodes on path from the root to the deepest leaf.

Write a recursive definition for the height of a general tree.

height(tree) t)

h=0 , if t is null      base cases

h=1 , if root is leaf

h= max height of children recursion if not a leaf

recursive cases

Complete the recursive height method based on the recursive definition.

Assume the method is added to a Tree class having a root instance variable.

```

public int height() { if (root == null) return 0; }
                     else return height(root)
  
```

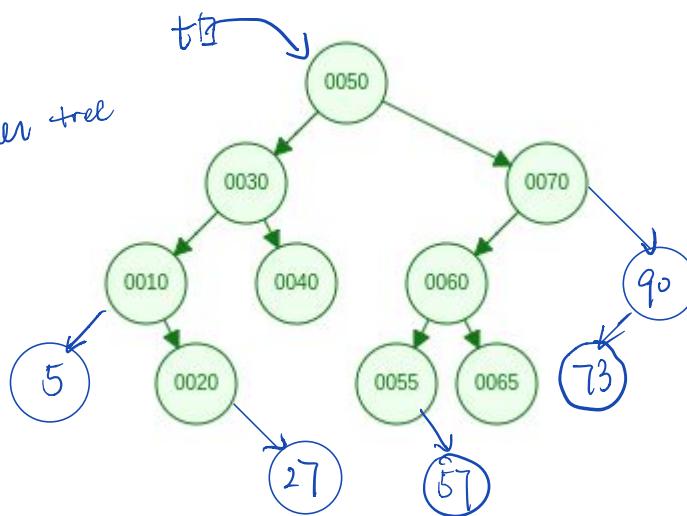
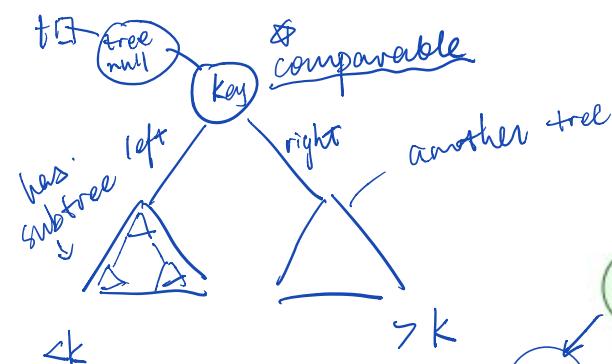
private int height (Treenoode<T> node) {

}

X Duplicate.

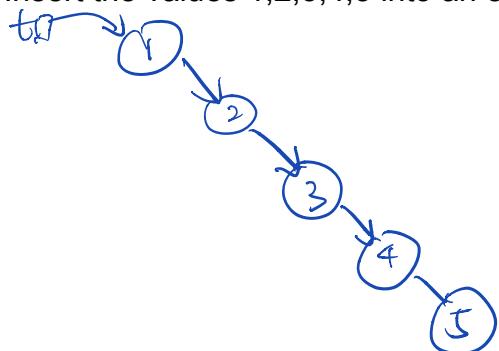
## Binary Search Tree (BST) Review

Image created at: <https://www.cs.usfca.edu/~galles/visualization/BST.html>



Insert 5, 27, 90, 73, 57 into the above BST tree (recall binary search algorithm)

Insert the values 1,2,3,4,5 into an empty BST      sorted number → linear structure



What can you conclude about the shape of a BST when the values are inserted in sorted order?

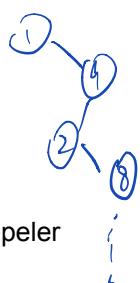
It's linear       $O(N)$

FATAL FLAW  
↳ Shape depends upon insert order

Will you only get this shape if inserted in sorted order?

X      1, 9, 2, 8, 3, 7, 4, 6, 5

↳ complexity depends upon shape

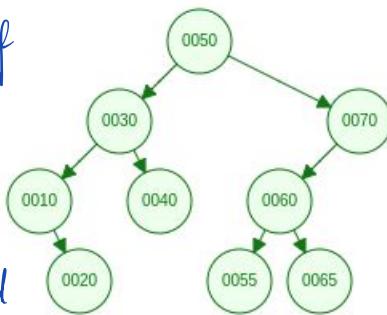


## Practice Deleting from a BST

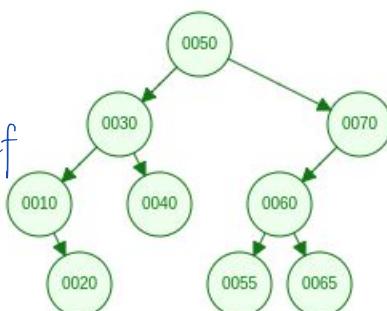
Delete 90 from this tree. *No 90,*

Delete 40 and then 65 from this tree.

① Unlink 40

by setting right child of  
30 to null② Unlink 65,  
set right child of 60 to null

Delete 10 and then 70 from this tree.

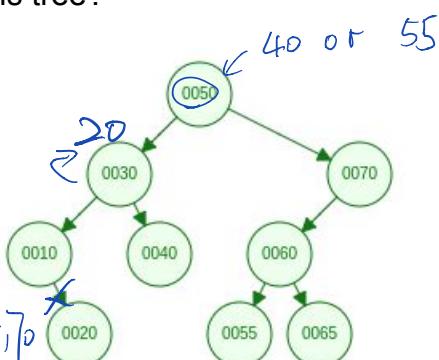
10: set left child  
of 30  
to 20 (right child of  
10)70: set right child  
of 50 (root)  
to 60 (left child of  
70)

How do you delete 30 or 50 from this tree?

Delete 30: ?

Δ in-order traversal

10, 20, 30, 40, 50, 55, 60, 65, 70

LVR

calling recursively

Δ Pre order

VLR 50; 30, 10, 20, 40, 70, 60, 55, 65

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Δ post order

LDR 30, 10, 40, 30, 55, 65, 60, 70, 50

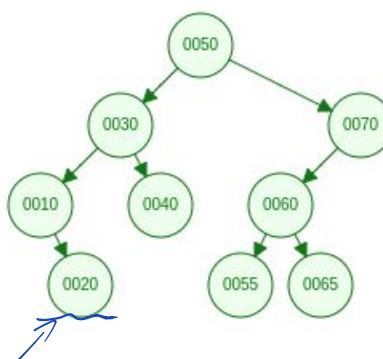
A level order

50, 30, 70, 10, 40, 60, 20, 55, 65

Delete 30 from this tree using the in-order predecessor

① replace 30 with value of  
in-order predecessor

② delete 20 from  
left of 30 subtree

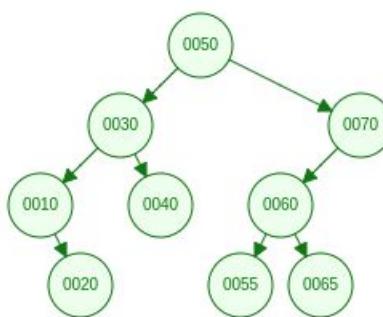


in-order predecessor of 30

Delete 50 from this tree using the in-order successor

① replace 50 with value of  
in-order successor

② delete 55  
from right subtree



How do we find in-order predecessor or in-order successor?

further to right of  
left-subtree

greatest value in left-subtree

go left, then right  
as far as possible

-----  
left  
right-subtree

lowest value in right-subtree

go right, then left -----

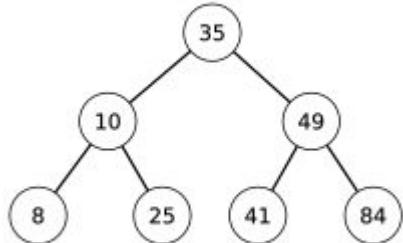
```
public boolean lookup(K key) { return lookup(root, key); }
```

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private helper method. Implementing BST Operations

assume key is not null

```
^ boolean lookup(BSTnode<K> n, K key) {
```



```
    if (key == null) throw IllegalArgumentException;  
    if (key.compareTo(n.getKey()) == 0)  
        return true;  
    if (key.compareTo(n.getKey()) < 0)  
        return lookup(n.getLeft(), key);  
    return lookup(n.getRight(), key);
```

```
void { root = insert(root, key); }  
public insert(K key) { insert(root, key); }
```

/\* pre-cond: key is NOT null \*/

```
private BSTnode<K> insert(BSTnode<K> n, K key)  
throws DuplicateKeyException {
```

```
    if (key == null) throw IllegalArgumentException("null key");  
    if (n == null) create a new node {  
        BSTnode<K> newNode = new BSTnode<K>(key, null, null)  
        return newNode;  
    }  
    if (n.key.equals(key)) throw DuplicateException;  
    if (n.key.compareTo(key) < 0)  
        n.left = insert(n.left, key);
```

else  
 n.right = insert(n.right, key)

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public void delete (K key) {  
 root = *delete (root, key);*}  
Implementing BST Operations

delete

/\* pre-cond: key is not null \*/

private BSTnode<K> delete(BSTnode<K> n, K key) {

if (n == null) throw exception ("key not found");  
return null;

: if (key.compareTo(n.getKey()) == 0) // find match, delete this

CASE #1: n has no children

if (n.left == null && n.right == null)  
return null;

CASE #2: n has one child

// return n's other child to parent

else if (n.left == null) return n.right;

else if (n.right == null) return n.left;

CASE #3: n has two children

n.key = inOrderPredecessor(n.left) // return key not node

n.left = delete(n.left, n.key);

// 1. Find a replacement key

// 2. Replace n's key with replacement

// 3. Delete replacement key from a prop Subtree

// Key Concept: Find replacement

the minimize changes to structure

return n.

## Complexities of BST Operations

Problem size:  $N = \# \text{ of data items}$

print:  $\Theta(N)$  ~ must traverse all nodes

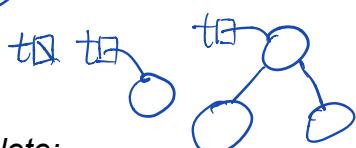
lookup:  $O(\log N) \rightarrow O(H)$   $H$ : height of the tree ( $\log N \leq H \leq N$ )

insert:  $\Theta(H)$

delete:  $\Theta(H)$

## k-ary Tree Classification Terms

**Perfect** (referred to as full in some readings)  $(2^n - 1)$  nodes



all interior trees have  $k$  children  
all leaves at same level

Complete:

a heap is complete

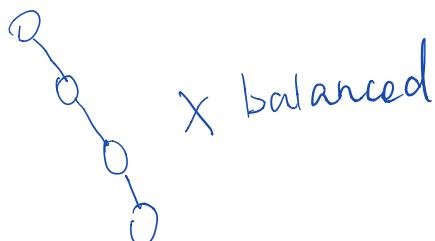
- interior nodes have  $k$  children
- perfect to 2nd last level, last level filled left to right with leaves

Height-Balanced:

Every node has subtree height that differ by at most one.

Balanced

a tree is balanced if its insert operation is  $O(\log_k N)$



Binary	at most 2 children
Triary	3
K-ary	$k$ .

## Balanced Search Trees

Goal: keep height at  $O(\log_2 N)$   
 $\hookrightarrow \# \text{ of nodes}$

Idea:  
have insert + delete maintain balance

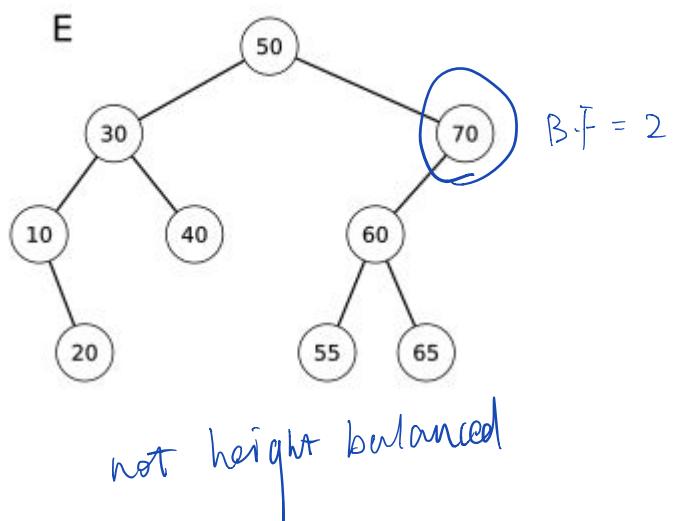
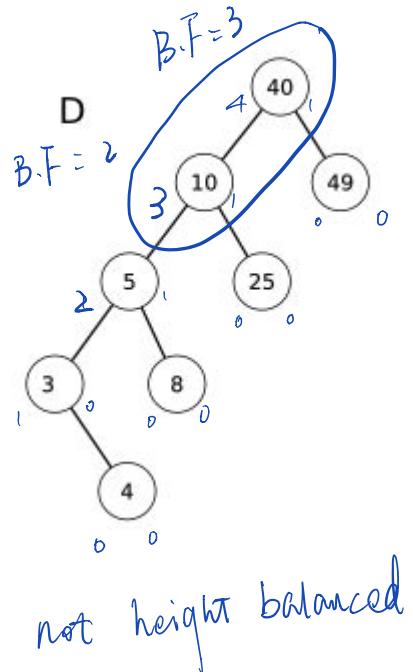
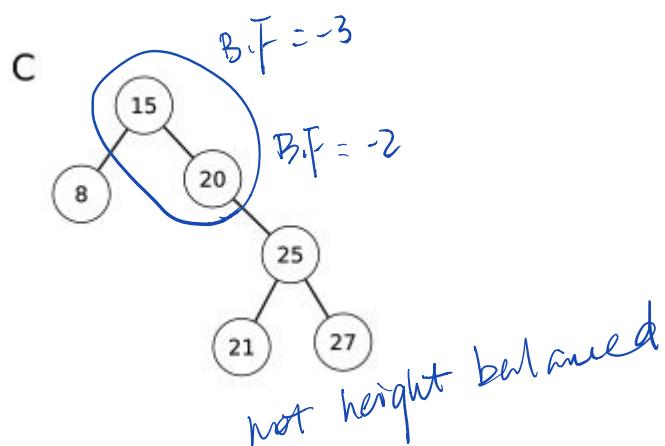
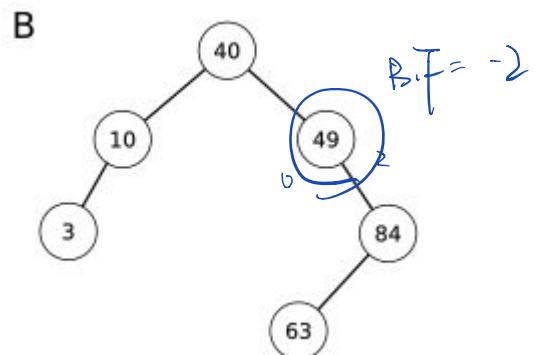
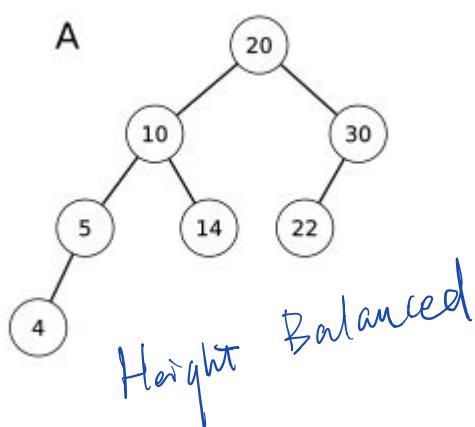
How do we detect and fix?

1. check height of each subtree (compute balanceFactors)
2. If  $|B.F.| > 1$  - must rebalance

Balance Factor:

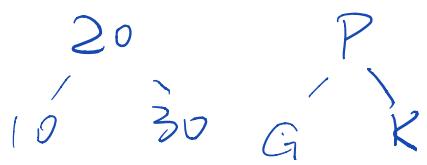
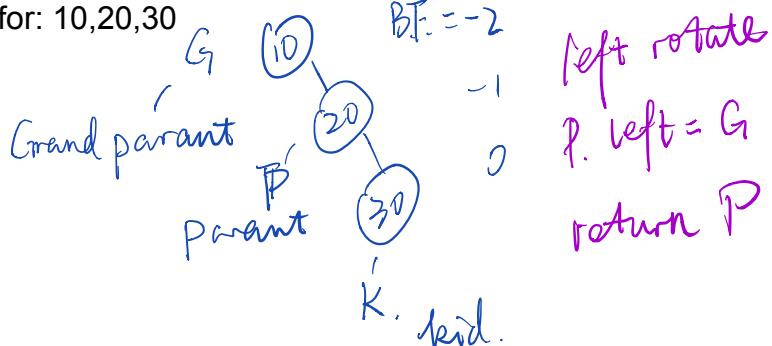
$BF = \text{height of left subtree} - \text{height of right subtree}$

## Height-Balanced Practice

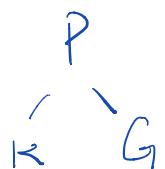
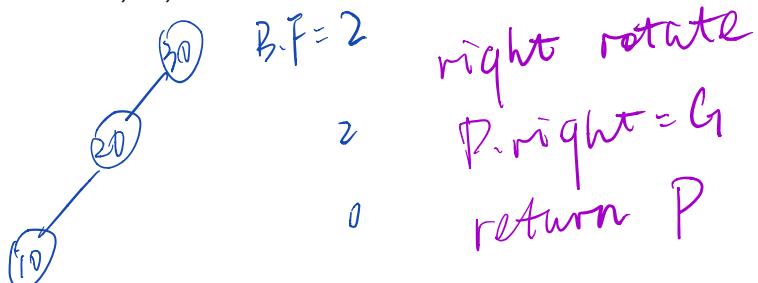


## Balancing I - Practice

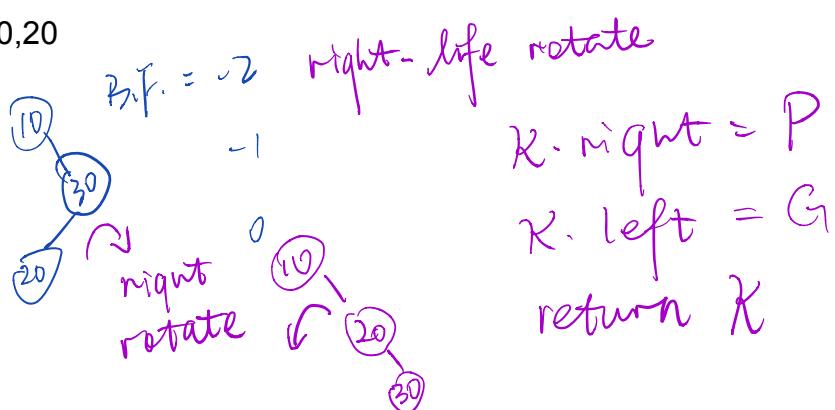
Draw BST for: 10,20,30



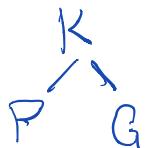
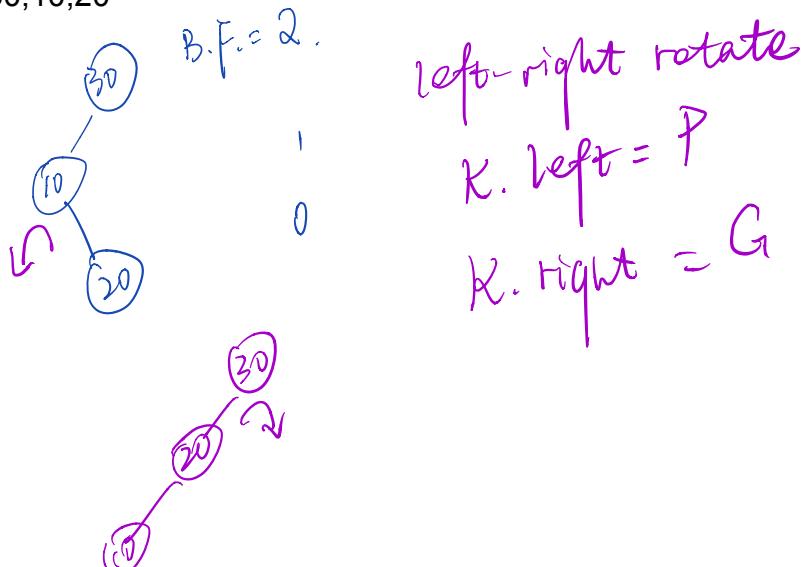
Draw BST for: 30,20,10



Draw BST for 10,30,20



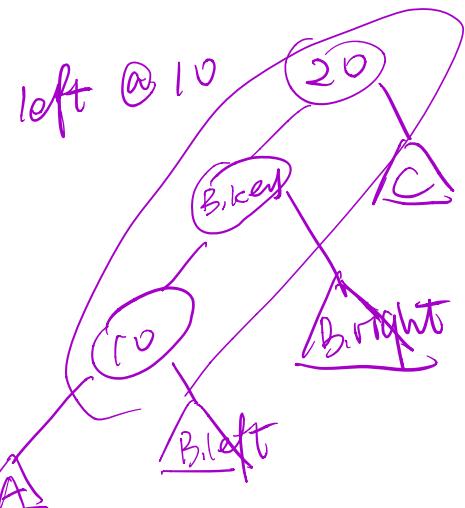
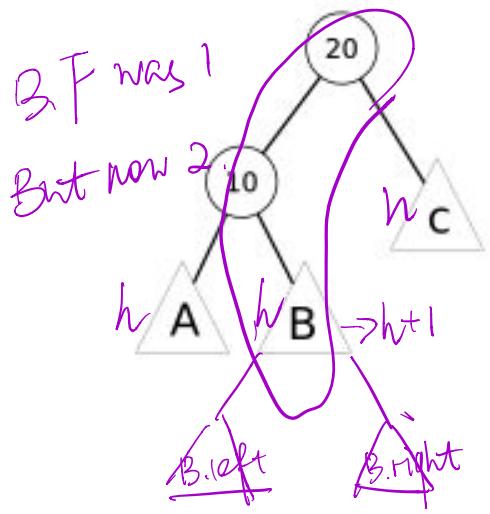
Draw BST for 30,10,20



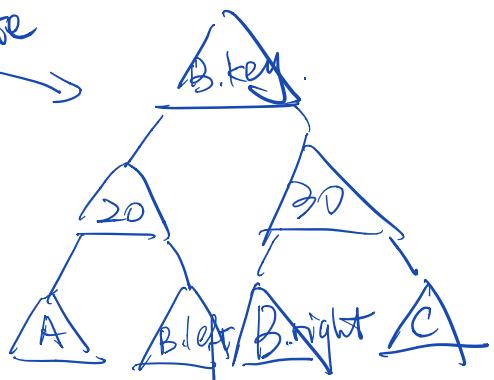
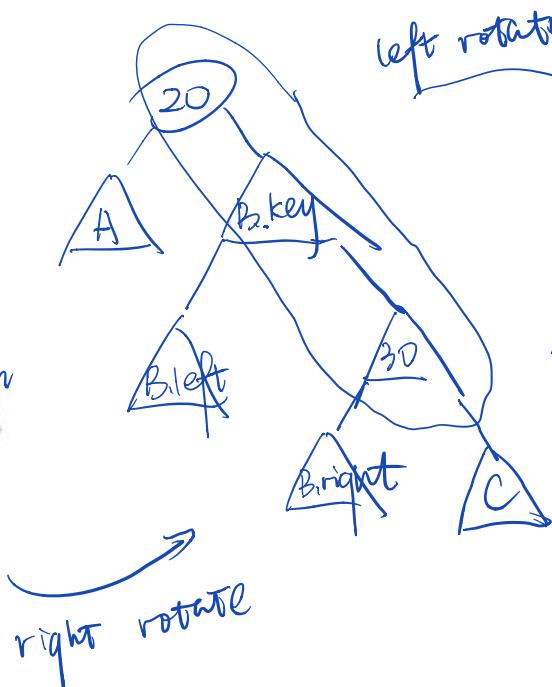
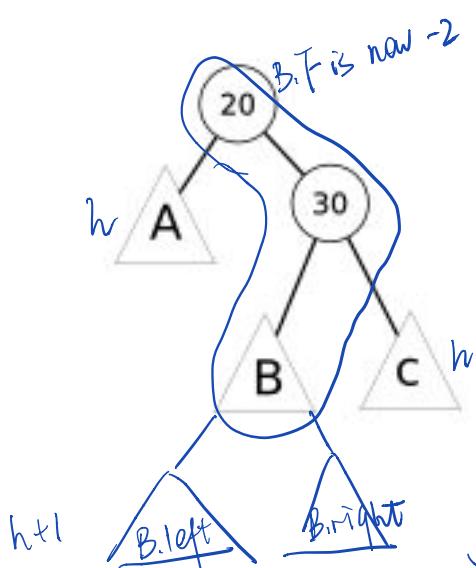
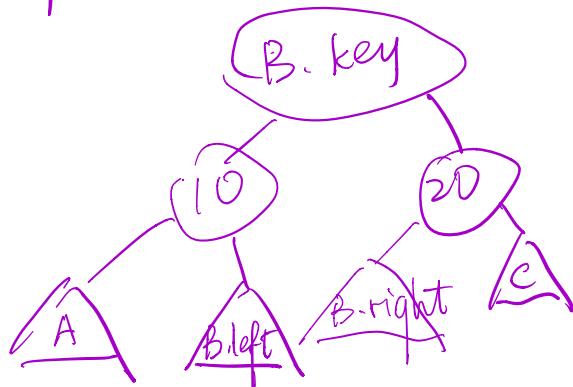
## Balancing II - General Balancing

Assume: a triangle represents sub-tree of unknown height, but that trees shown are *height-balanced*, and show the relative difference (balance factor of -1, 0, 1)

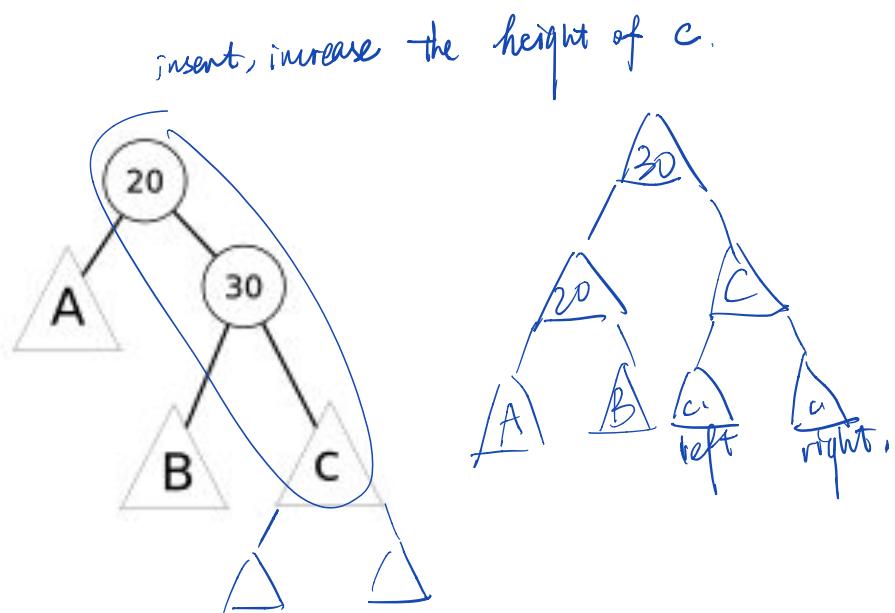
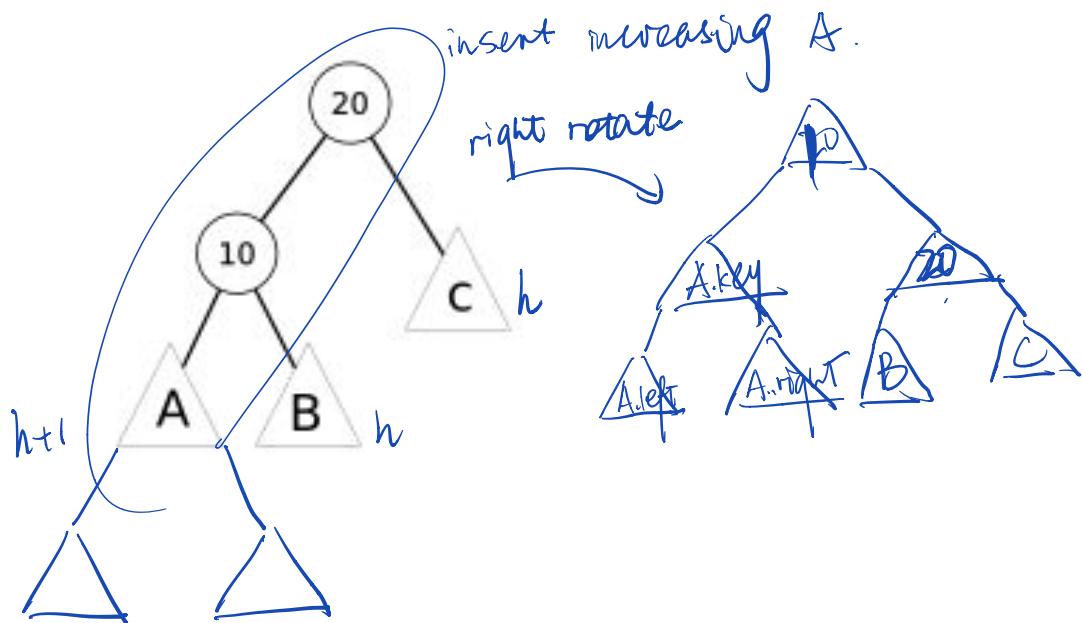
insert increase the height of B.



right rotate



## Balancing III - More General Balancing



Georgy Adelson-Velsky and Evgenii Landis

height balancing  
use one of 4 rotation sequence

*Complexity of operations*

Still  $O(H)$  bad AVL paper "proved"

$$H \approx 1.44 \log_2 N$$

*Implementation*

must know or compute B.F. for each Node  
detect of balance when  $|B.F.| > 1$ .

How can we know height of each sub-tree?