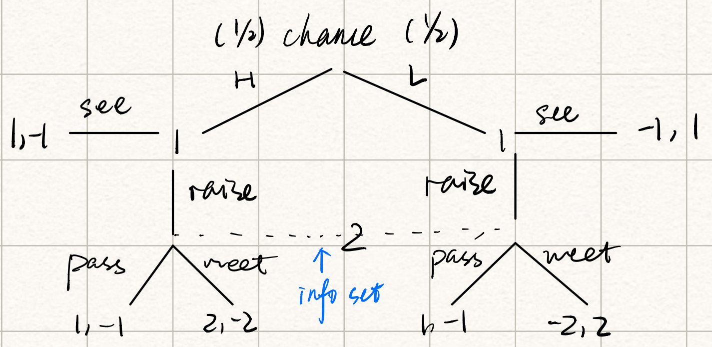
Imperfect information

- players
- terminal histories
- player function
- chance
- preference
- Info set / partitions

Card Game



	P	M
SS	0,0	0,0
SR	1,-1	$-\frac{1}{2},\frac{1}{2}$
RS	0,0	$\frac{1}{2},-\frac{1}{2}$
RR	1,-1	0,0

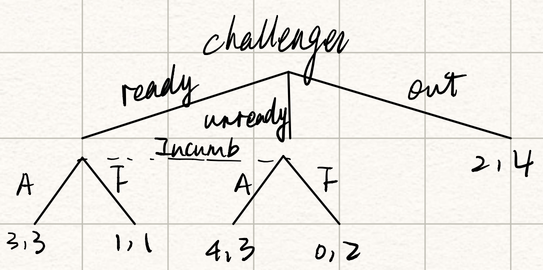
No NE

Strategies

1: SS, SR, RS, RR

2: P, M (do not know what p1 plays)

Entry Game



	A	F
R	3,3	1,1
U	4,3	0,2
O	2,4	2,4

NE (U, A)

(O, F)

Only have the whole game as a the only subgame.

↳ SPE not applicable.

Weak Sequential Equilibrium

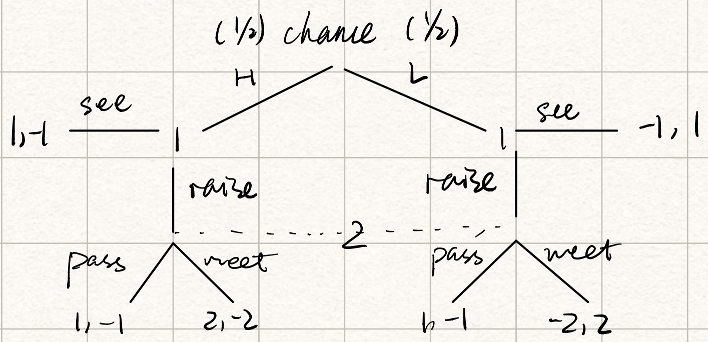
Assessment: Belief (where in the game history takes) μ

Behavioral Strategies β

Requirement: sequentially rationality given beliefs, everyone's strategies. NE: can do anything

weak consistency of beliefs with strategies.

Card Game



belief

chance is H when R is played.

$\mu_H = P(H|R) = \frac{\frac{1}{2}P_H}{\frac{1}{2}P_H + \frac{1}{2}P_L} = \frac{P_H}{P_H+P_L}$

Bayes Rule

$P_H = P(\text{play } R | H)$

$P_L = P(\text{play } R | L)$

$q_P = P(\text{play } P | R)$

2: pay P if $-1 \geq -2\mu_H + 2(1-\mu_H)$

$\Rightarrow -1 \geq 2 - 4\mu_H \Rightarrow \mu_H \geq 3/4$ (prob to get L to be < 1/4)

play M if $\mu_H \leq 3/4$

Case 1: $P_H = P_L = 0$ no eq.

Case 2: $\mu_H > 3/4$ $q_P = 1$ $P_L = 1$, $1 \geq P_H \geq 0$ $\mu_H = \frac{P_H}{P_H+P_L}$. no eq.

Case 3: $\mu_H < 3/4$ $q_P = 0$ $P_H = 1$, $P_L = 0$ $\mu_H = \frac{P_H}{P_H+P_L} = 1$ no eq.

Case 4: $\mu_H = 3/4 = \frac{P_H}{P_H+P_L}$ $3P_H + 3P_L = 4P_H \Rightarrow P_H = 3P_L$

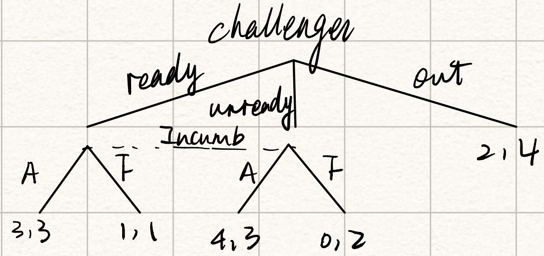
P1 be indifferent between S/R at L ($P_L \neq 0$, $P_L \neq 1$)

L: $-1 = q_P - 2(1-q_P)$ $q_P = 1/3$.

$P_H = 1$ $P_L = 1/3$. $\mu_H = 3/4$.

WSE

Entry Game

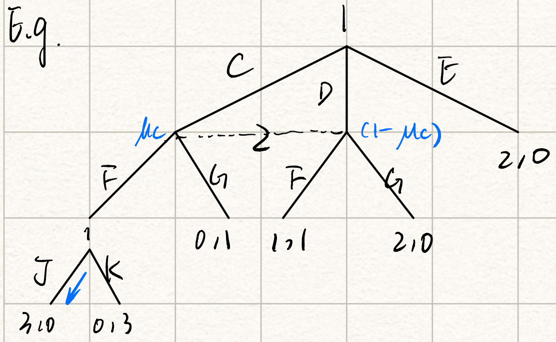


↑ Always prefer accomodate.

NE: (U, A), (O, F)

WSE: $(U, A$, belief of $u)$

E.g.



Player 2: F if $0 \cdot \mu_c + 1 \cdot (1-\mu_c) \geq 1 \cdot \mu_c + 0 \cdot (1-\mu_c)$

$\mu_c \leq 1/2$

backward induction.

Case 1: $P_E = 1$ (payoff for $E >$ payoff for C, D) $P_C, P_D = 0$.

$(E > C)$ $2 > 3q_F + 0 \cdot (1-q_F)$ $q_F \leq 2/3$

$(E > D)$ $2 > q_F + 2 \cdot (1-q_F) = 2 - 2q_F$ $q_F > 0$.

$\Big\}$ 2 choose $q_F \leq 2/3$.

↓

$\mu_c \geq 1/2$.

WSE: $(EJ,\ q_F \leq 2/3,\ \mu_c = 1/2)$

$(EJ,\ q_F = 0,\ \mu_c > 1/2)$

↖ belief.

Case 2: $P_C > 0$ or $P_D > 0$ or both.

$$\mu_c = \frac{P_C}{P_C + P_D}$$

F if $P_C < P_D$

G if $P_C > P_D$.

F or G if $P_C = P_D$.

a) 2 chooses F : $P_C < P_D$. $\Rightarrow$ $P_C = 1$ (payoff for $C > D$) $\Rightarrow$ no eq.

b) . . . G : $P_C > P_D$ $\Rightarrow$ P_1 never choose C, indifferent between D or E $\Rightarrow P_C = 0$ $\Rightarrow$ no eq.

c) F or G if $P_C = P_D$. $\Rightarrow$ $P_D = 0$ (payoff $<$ E) $\Rightarrow$ no eq.

Signaling.

reference: P291. figure 8.5.

Sender: has hidden info

e.g. of lemons "adverse selection"

Receiver: doesn't have info

making decision

bad cars draw good cars out of the market.

Hiring

		Finance	other
p	A	160	125
1-p	C	60	30.

A person get in finance or other industry with level A or C.

not have complete info

pooling: wage $= 160p + 60(1-p) = 60 + 100p < 125$

$\Rightarrow p < 0.65$ A drop out

$\Rightarrow$ only Cs left, paid 60

competitive labor market.

Company's Signal: tough classes { take n or more $\Rightarrow$ A ; fewer than n $\Rightarrow$ C } "screening"

Students:

Idea: A takes n classes, C takes 0 classes.

cost: 3/class for A

15/class for C

A: $160 - 3n \geq 60 \Rightarrow n \leq 33.33$

C: $60 \geq 160 - 15n \Rightarrow n \geq 6.67$

$\Rightarrow$ Company will set n in this range.

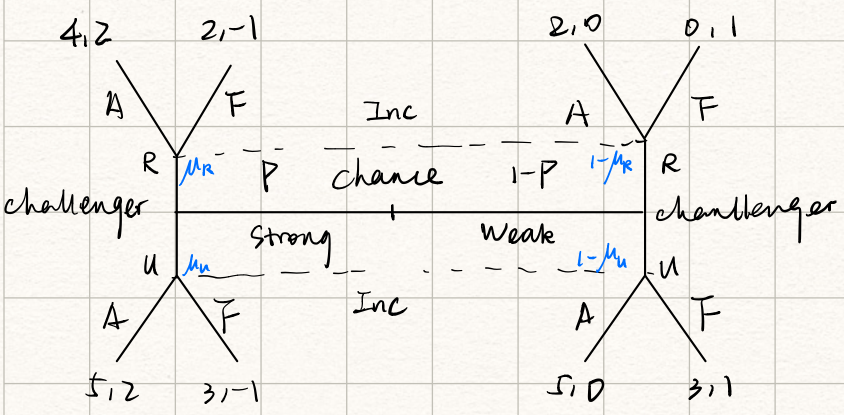
Incentive compatibility constraint

Compared with other options: A: $160 - 3n \geq 125 \Rightarrow n \leq 11.67$ $\longrightarrow$ Combining

C: $60 \geq 30$

$n \in [6.67, 11.67]$.

Entry Game



costly for weak choose ready.
⇑
weak challenger always choose U

Seperating: strong chooses R   equilibrium. ⇐ $\mu_u = 0$, $\mu_R = 1$

Inc [play A if R
play F if U

Pooling. strong chooses U.

$$\mu_u = \frac{p}{p+(1-p)} = p$$

bottom info set – After U: Inc: play A if $2p + 0(1-p) \geq -1p + (1-p)$

$$\Rightarrow p \geq 1/4$$

top info set – After R: Inc play A if $\mu_R \geq 1/4$ (same calculation)

确认I的action 使c play U. – If $p \geq 1/4$. strong challenger's plays U. (Inc can play either) ← payoff 5 > both payoff in R.

if $p < 1/4$ $4q^R_A + 2(1-q^R_A) \leq 3$ ← bottom pay F (payoff 3) $q^R_A \geq 1/2$.

strong challenger's payoff for U

top info set, bottom info set.

WSE: $(RU, AF, \mu_u = 0, \mu_R = 1)$ ⇐ seperating.

top info set not cared. → so μ_R not cared.

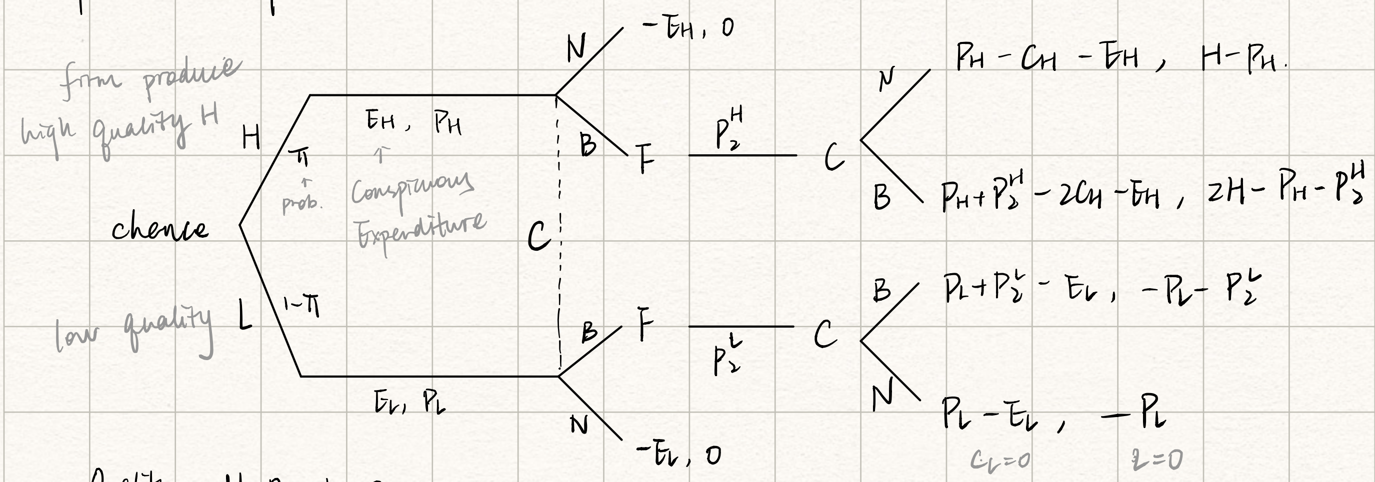
$p \geq 1/4$ $(UU, AA \text{ or } FA, \mu_u = p, \mu_R \in [0,1])$ } pooling.

$p < 1/4$ $(UU, FF, \mu_u = p, \mu_R \leq 1/4)$

$(UU, q^R_A \leq 1/2, F, \mu_u = p, \mu_R = 1/4)$

⇑ mixed strategy.

Conspicuous Expenditure - firm used to advertise, build the brand, etc.



Quality: $H > 0$, $L = 0$

$C_H > C_L = 0$, $H > C_H$

Seperate: 2^{nd}

C $\left[\begin{array}{l} L: \text{ buy only if } P_2^L = 0 \\ H: \text{ buy if } H \geq P_2^H \end{array}\right.$

F $\left[\begin{array}{l} L: \text{ any } P_2^L, \text{ make zero profits.} \\ H: P_2^H = H \end{array}\right.$

C' beliefs: H if $E = E_H$, $P = P_H$

L otherwise.

Eq. F: $P_H + H - 2C_H - E_H \geq 0$ $E_L = 0$, $P_L = 0$ ⇐ High quality firm will not signaling as low quality firm.

High quality firm signaling that they are high quality

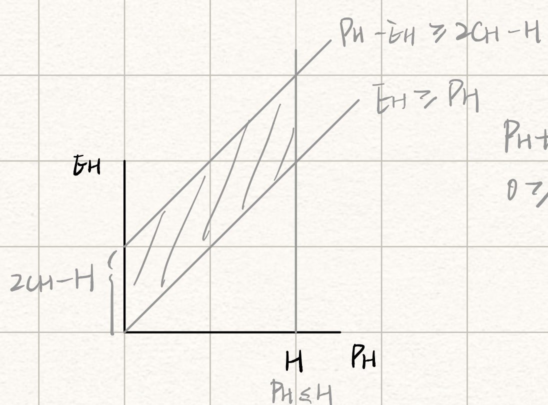
$0 \geq P_H - E_H$ ⇐ Low quality firm will not pretend as high quality firm.

zero profit

C will not go into second round

the payoff for L F to signal as high quality firm.

$H \geq P_H$. ⇐ change the price that C will purchase



$P_H + H - 2C_H - E_H \geq 0$

$0 \geq P_H - E_H \geq 2C_H - H$

Pooling: L, H firms make same E and charge same P in the first round

C: have same E^*, P^*, then play $\pi H + (1-\pi) L = P^*$.
any other combination $E \neq E^*$ or $P \neq P^*$, don't buy.

L participates if $P^* > E^*$ — $P_2^L = 0$ same in seperate.

H participates if $P^* + H - 2C_H - E^* \geq 0$.
(H: second round same in seperating.)

Bayesian Game

Meet (2: Meet), belief $\frac{1}{2}$:

	B	S
B	2, 1	0, 0
S	0, 0	1, 2

Avoid (2: Avoid), belief $\frac{1}{2}$:

	B	S
B	2, 0	0, 2
S	0, 1	1, 0

P_2: Meet / Avoid; 1; $\frac{1}{2}$ — Beliefs

Players, states, Actions,
Signals, Beliefs, Payoffs.

Signal: 1: only 1
2: meet or avoid signal receive.

treat playes in different states as different players

play 1 believe: ½ times P_2 wants meet, ½ times P_2 wants avoid.

1's payoffs.

	BB	BS	SB	SS
B	2	1	1	0
S	0	½	½	1

NE (B, BS)

Cournot Game

C_L Low (2: L), θ | C_H High (2: H), $1-\theta$ | 1

$1: \max\ \theta q_1(\alpha - q_1 - q_L - c) + (1-\theta) q_1(\alpha - q_1 - q_H - c)$

$\quad = q_1(\alpha - q_1 - \theta q_L - (1-\theta) q_H - c)$

$2L: \max\ q_L(\alpha - q_1 - q_L - c_L)$

$2H: \max\ q_H(\alpha - q_1 - q_H - c_H)$

BR. $q_1 = \dfrac{\alpha - c - \theta q_L - (1-\theta) q_H}{2}$

$q_L = \dfrac{\alpha - c_L - q_1}{2}$

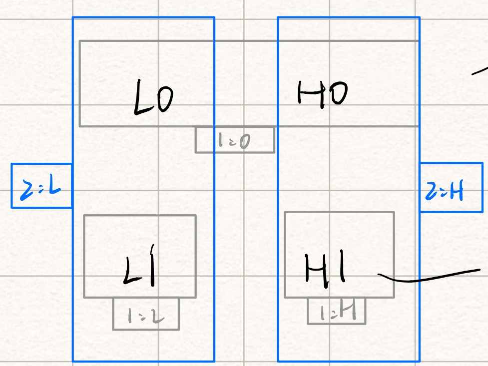
$q_H = \dfrac{\alpha - c_H - q_1}{2}$

Eq: $q_1 = \dfrac{\alpha - 2c + \theta c_L + (1-\theta) c_H}{3}$

$q_L = \dfrac{2\alpha + 2c - (3+\theta) c_L - (1-\theta) c_H}{6}$ $\Leftarrow$ take into account the cost of other type as q_1 consider such.

$q_H = \dfrac{2\alpha + 2c - (4+\theta) c_H - \theta c_L}{6}$

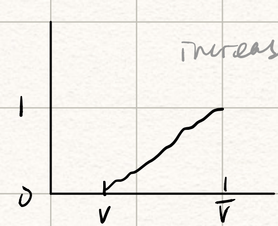
Extension:



P1 does not know the price P2 plays.

P1 knows P2 plays H or L

Public Good with private valuation.



increasing function.

i) valuation with distribution func: $F(v) = P_r(x \leq v)$

cost $c > 0$ of provision.

provide or not provide.

receive own valuation if someone provide the good.

provide if $v_i \geq v^*$.

Assume Symmetric Eq.

prob that any other player provide the good

provide → $v_i - c = v_i \left(1 - F(v^*)^{n-1}\right)$ ← not provide.

Eq: $c = v^* [f(v^*)]^{n-1}$

point of Eq.

代入.

Prob of provision $= 1 - [F(v^*)]^n = 1 - \dfrac{c F(v^*)}{v^*}$

ii) Uniform distribution $v \sim \text{uniform}[0,1]$ $F(v) = v$.

$c = v^* (v^*)^{n-1} = (v^*)^n \Rightarrow v^* = c^{\frac{1}{n}}$

Prob of provision $= 1 - F(v^*)^n = 1 - c$