

# Chapter 11

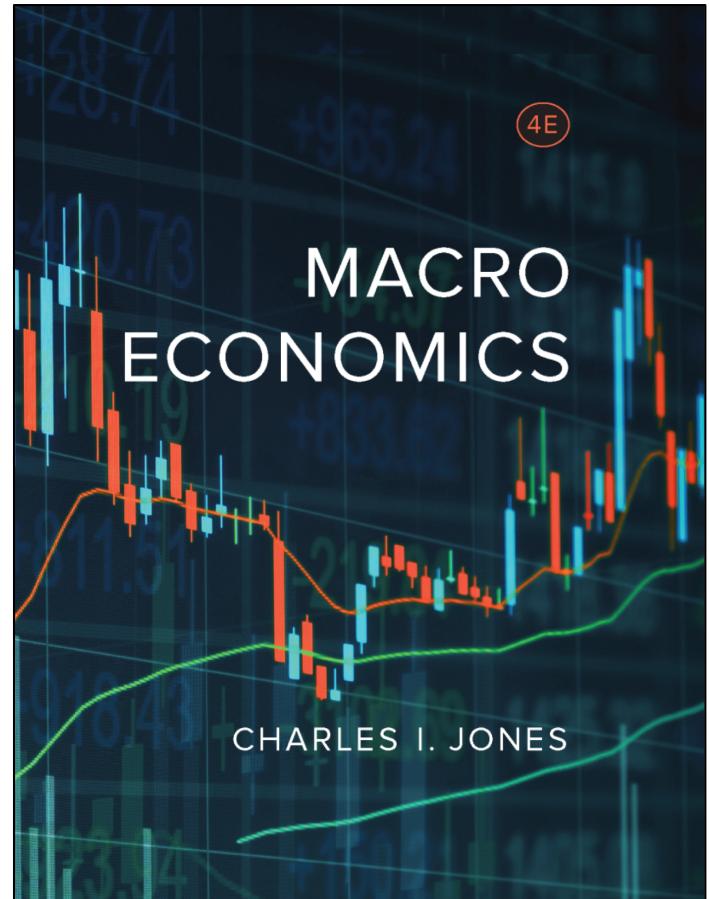
## The IS Curve

IS - LM

IS - LP

Hicks  
Keynes

Emily Marshall, Dickinson College  
Revised, Expanded, and Updated by Simeon Alder  
U of Wisconsin - Madison



# 11.1 Introduction

---

- In this chapter, we learn:

- Foundation of the short-run model: **the IS curve** illustrates the **inverse relationship between the real interest rate and short-run output**
  - Aggregate demand shocks (i.e., consumption, investment, government purchases, or net exports) shift the IS curve.
  - Life-cycle/permanent-income hypothesis describes consumption behavior (next lecture).
  - Changes in real interest rates affect short-run output primarily through **investment**.

# Introduction

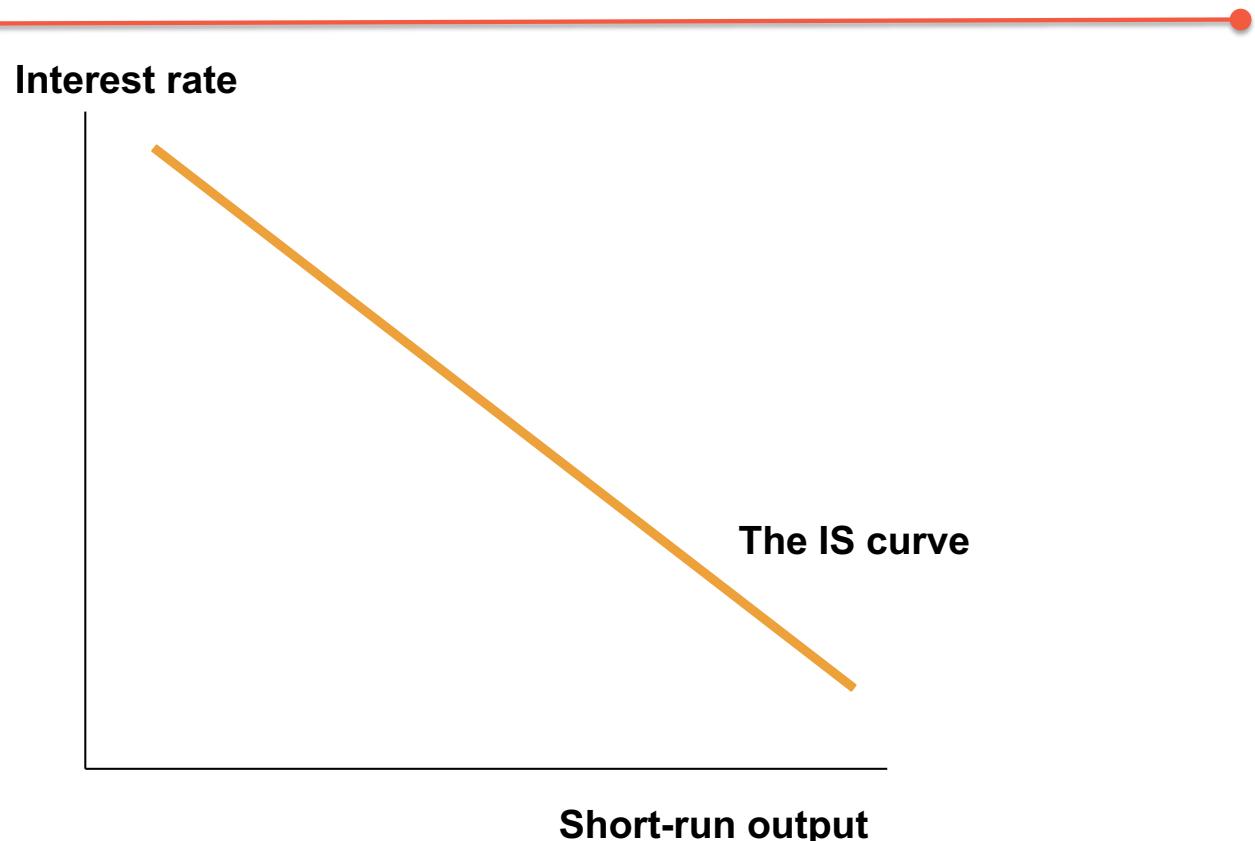
- The Federal Reserve influences the level of economic activity in the short run.
  - The Fed targets the federal funds rate.
  - The federal funds rate is highly correlated with the short-term nominal interest rate at which people borrow and lend in financial markets.
  - In-depth look at inner workings of Fed in chapter 12
- The basic story is as follows:

$\uparrow$  interest rate  $\Rightarrow$   $\downarrow$  investment  $\Rightarrow$   $\downarrow$  output

$\uparrow$  cost of

- The IS curve:
  - illustrates the **negative** relationship between interest rates and short-run output.

# Introducing the IS Curve



## 11.2 Setting Up the Economy

- The national income accounting identity
  - Implies that the total resources available to the economy equal total uses
  - One equation and six unknown variables

$$Y_t = C_t + I_t + G_t + EX_t - IM_t$$

- where

$Y_t$	Output	$I_t$	Investment
$IM_t$	Imports	$G_t$	Government Purchases
$C_t$	Consumption	$EX_t$	Exports

## 11.2 Setting Up the Economy

- The national income accounting identity
  - Implies that the total resources available to the economy equal total uses
  - One equation and six unknown variables

$$\underbrace{Y_t + IM_t}_{\text{total resources}} = C_t + I_t + G_t + EX_t$$

■ where

$Y_t$	Output	$I_t$	<u>Investment</u>
$IM_t$	Imports	$G_t$	Government Purchases
$C_t$	Consumption	$EX_t$	Exports

# Setting Up the Economy

- Five additional equations to solve the model:

$$C_t = \bar{a}_c \bar{Y}_t$$

$$G_t = \bar{a}_g \bar{Y}_t$$

$$EX_t = \bar{a}_{ex} \bar{Y}_t$$

$$IM_t = \bar{a}_{im} \bar{Y}_t$$

$$\frac{I_t}{\bar{Y}_t} = \bar{a}_i - b(R_t - \bar{r}) \Rightarrow I_t = \bar{a}_i \bar{Y}_t$$

(long run)

*(long run)*

*more sensitive to the gap*

*real interest rate =  $r$*

*marginal product of capital.*

*endogenous*

where a bar denotes an exogenous variable

$$\text{(only mn: } \bar{a}_c + \bar{a}_g + \bar{a}_{ex} + \bar{a}_i - \bar{a}_{im} = 1)$$

# The Model—1

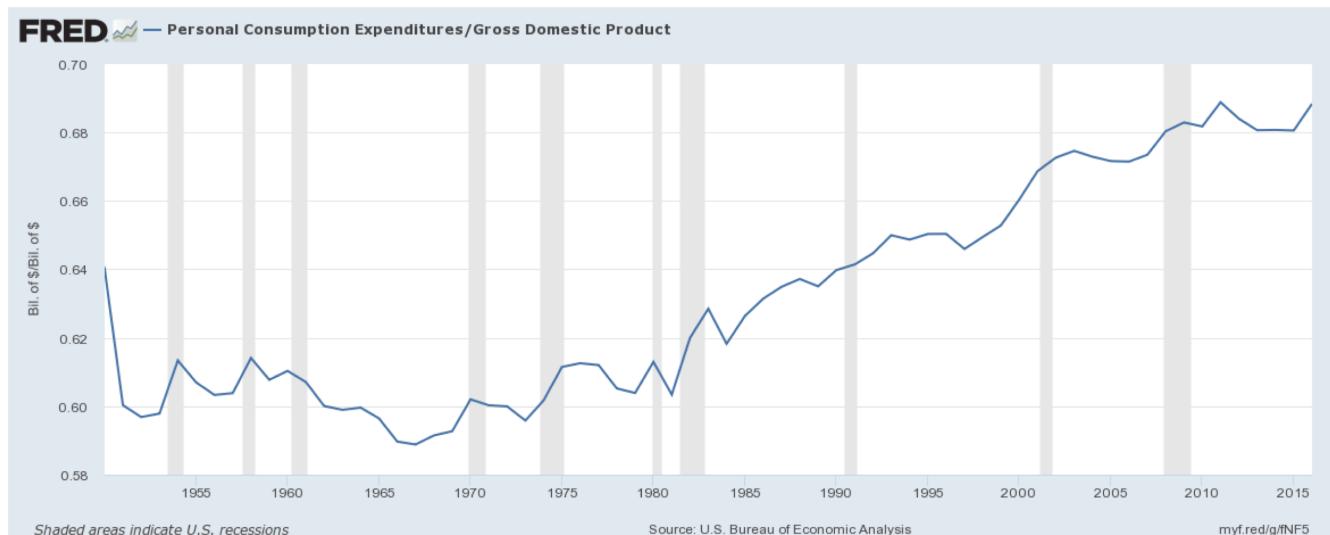
- Assume  $C_t$ ,  $G_t$ ,  $IM_t$ , and  $EX_t$  are given exogenously (i.e. they are functions of *potential output*).
- Example:  $C_t$  is a constant fraction of potential output given by  $\bar{a}_c$ 
  - $\bar{a}_c$  is empirically estimated to be approximately 2/3



Consumption

- About 2 out of every 3 dollars of GDP is attributed to consumption

# The Model—2



# The Model—3

- Now, we use our national income accounting identity:

$$Y_t + IM_t = C_t + I_t + G_t + EX_t$$

- and substitute in the four equations that contain only parameters:

$$Y_t + \boxed{IM_t} = \boxed{C_t} + \boxed{I_t} + \boxed{G_t} + \boxed{EX_t}$$
$$\boxed{\bar{a}_{im}\bar{Y}_t} \quad \boxed{\bar{a}_c\bar{Y}_t} \quad \boxed{\bar{a}_g\bar{Y}_t} \quad \boxed{\bar{a}_{ex}\bar{Y}_t}$$

- such that

$$Y_t + \bar{a}_{im}\bar{Y}_t = \bar{a}_c\bar{Y}_t + I_t + \bar{a}_g\bar{Y}_t + \bar{a}_{ex}\bar{Y}_t$$

# The Investment Equation

---

- Investment is given by:

$$\frac{I_t}{\bar{Y}_t} = \bar{a}_i - \bar{b}(R_t - \bar{r})$$

$\bar{a}_i$	Share of potential output that goes to investment
$\bar{b}$	Parameter weighting the difference between the real interest rate and the MPK
$R_t$	Real interest rate
$\bar{r}$	Marginal product of capital (MPK)

# Marginal Product of Capital (MPK)–1

---

- Amount of additional output the firm can produce by investing in one more unit of capital
- In the long run,  $\text{MPK} = \bar{r}$ , and  $\bar{r}$  is:
  - Exogenous
  - Time invariant
- Recall the equation for investment:

$$\frac{I_t}{\bar{Y}_t} = \bar{a}_i - \bar{b}(R_t - \bar{r})$$

- Multiply each side by  $\bar{Y}_t$ :

$$I_t = \bar{a}_i \bar{Y}_t - \bar{b} \bar{Y}_t R_t + \bar{b} \bar{Y}_t \bar{r}$$

# Marginal Product of Capital (MPK)–2

---

- Now, we can use the following equation for investment to understand how the gap between MPK and the real interest rate helps determine investment:

$$I_t = \bar{a}_i \bar{Y}_t - \bar{b} \bar{Y}_t R_t + \bar{b} \bar{Y}_t \bar{r}$$

- In the short run,  $\text{MPK} = \bar{r}$  and  $R_t$  can be different
- If  $\text{MPK} = \bar{r} < R_t$ 
  - Firms should **save** and **not invest** in capital
  - Investment will decline
- If  $\text{MPK} = \bar{r} > R_t$ 
  - Firms should **borrow** and **invest** in capital
  - Investment will increase

# The Setup of the Economy for the IS Curve

## The Setup of the Economy for the IS Curve

Endogenous variables:  $Y_t, C_t, I_t, G_t, EX_t, IM_t$

National income identity:  $Y_t = C_t + I_t + G_t + EX_t - IM_t$

Consumption:  $C_t = \bar{a}_c \bar{Y}_t$

Government purchases:  $G_t = \bar{a}_g \bar{Y}_t$

Exports:  $EX_t = \bar{a}_{ex} \bar{Y}_t$

Imports:  $IM_t = \bar{a}_{im} \bar{Y}_t$

Investment:  $\frac{I_t}{\bar{Y}_t} = \bar{a}_i - \bar{b}(R_t - \bar{r})$

Exogenous variables/parameters:  $Y_t, \bar{r}, \bar{a}_c, \bar{a}_i, \bar{a}_g, \bar{a}_{ex}, \bar{a}_{im}, \bar{b}$

Exogenous for now (until next chapter):  $R_t$

# 11.3 Deriving the IS Curve

- Begin with the national income accounting identity:

$$Y_t + IM_t = C_t + I_t + G_t + EX_t$$

- Divide both sides by potential output:

$$\frac{Y_t}{\bar{Y}_t} = \frac{C_t}{\bar{Y}_t} + \frac{I_t}{\bar{Y}_t} + \frac{G_t}{\bar{Y}_t} + \frac{EX_t}{\bar{Y}_t} - \frac{IM_t}{\bar{Y}_t}$$

- Substitute the five remaining equations into the equation above:

$$\frac{Y_t}{\bar{Y}_t} = \frac{\bar{a}_c \bar{Y}_t}{\bar{Y}_t} + \frac{\bar{a}_i \bar{Y}_t - \bar{b} \bar{Y}_t R_t + \bar{b} \bar{Y}_t \bar{r}}{\bar{Y}_t} + \frac{\bar{a}_g \bar{Y}_t}{\bar{Y}_t} + \frac{\bar{a}_{ex} \bar{Y}_t}{\bar{Y}_t} - \frac{\bar{a}_{im} \bar{Y}_t}{\bar{Y}_t}$$

- Simplifying yields:  $\frac{\text{ratio}}{\text{actual}}$   $\frac{\text{ratio}}{\text{potential}}$

$$\frac{Y_t}{\bar{Y}_t} = \bar{a}_c + \bar{a}_i - \bar{b}(R_t - \bar{r}) + \bar{a}_g + \bar{a}_{ex} - \bar{a}_{im}$$

# Deriving the IS Curve – 1

- Recall the definition of short-run output:

$$\tilde{Y}_t \equiv \frac{Y_t - \bar{Y}_t}{\bar{Y}_t} = \frac{Y_t}{\bar{Y}_t} - 1$$

- Subtract 1 from both sides of the equation:

$$\frac{\bar{Y}_t}{\bar{Y}_t} - 1 = \underbrace{\bar{a}_c + \bar{a}_i + \bar{a}_g + \bar{a}_{ex} - \bar{a}_{im}}_{\tilde{\gamma}_t} - 1 - \bar{b}(R_t - \bar{r})$$

$\overline{\alpha}$

long run  $\bar{a} = 0$

# Deriving the IS Curve—2

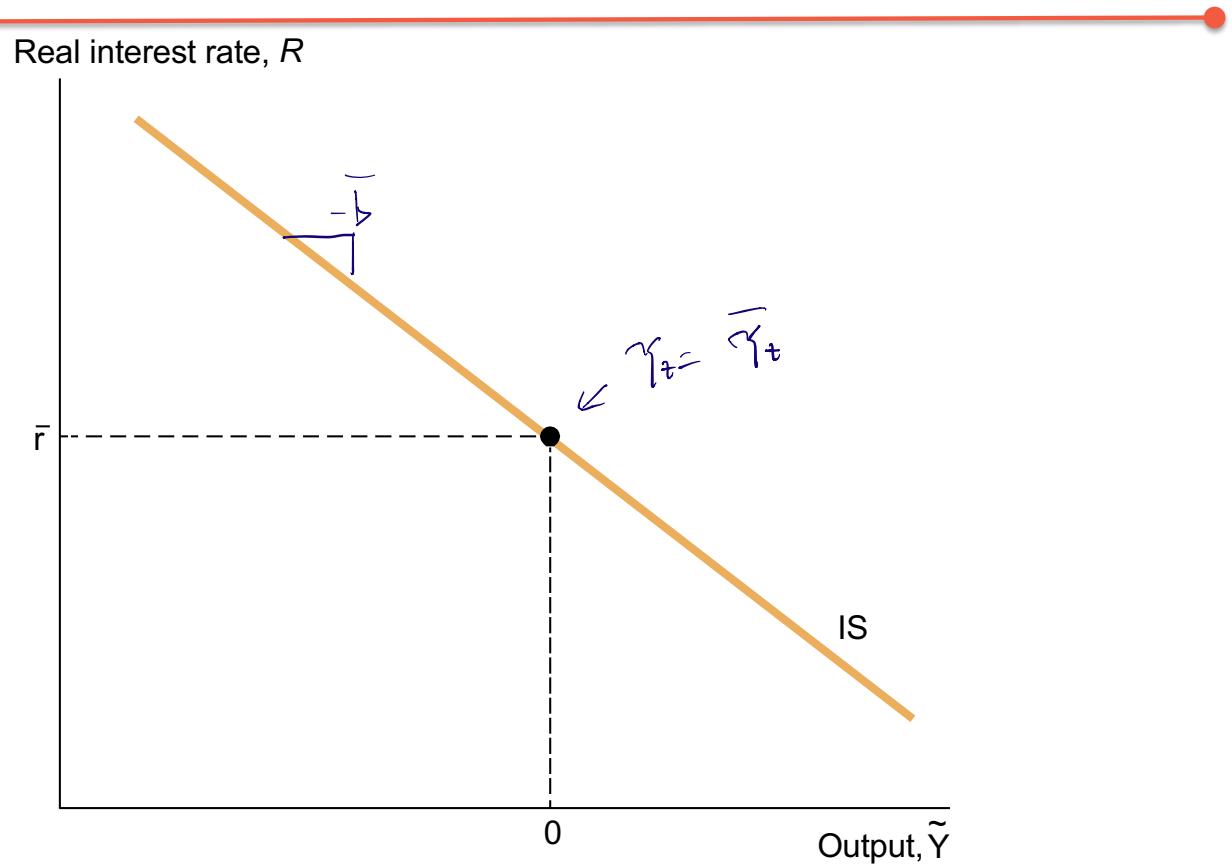
---

- After simplifying:

$$\tilde{Y}_t = \bar{a} - \bar{b}(R_t - \bar{r})$$

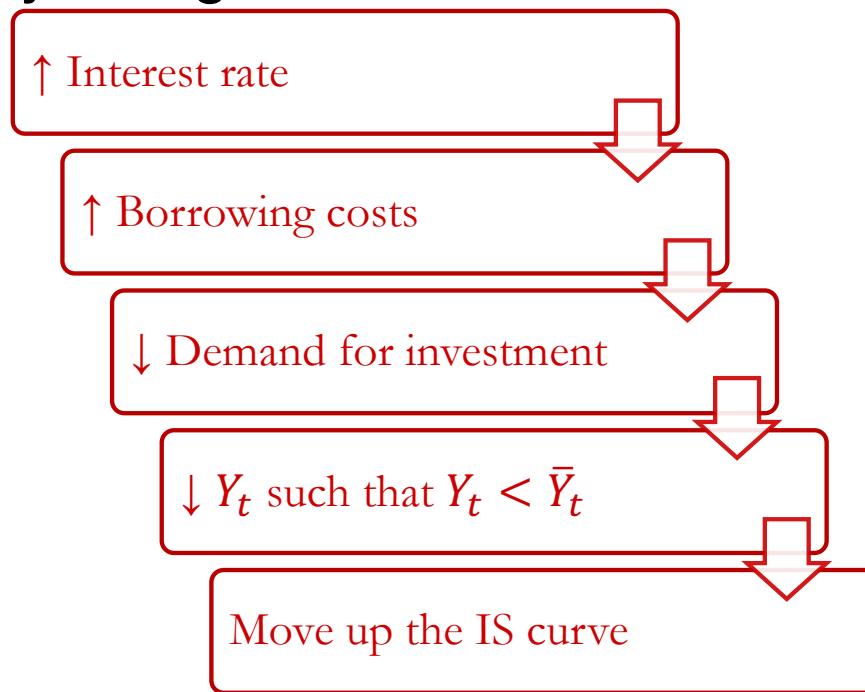
- The gap between the real interest rate and the MPK is what determines output fluctuations
- The parameter  $\bar{a}$  is called the aggregate demand shock *a recession,  $\bar{a}$  shift is curve.*
- Note: when  $Y_t = \bar{Y}_t$  then  $\bar{a} = 0$  and  $R_t = \bar{r}$

## 11.4 Using the IS Curve

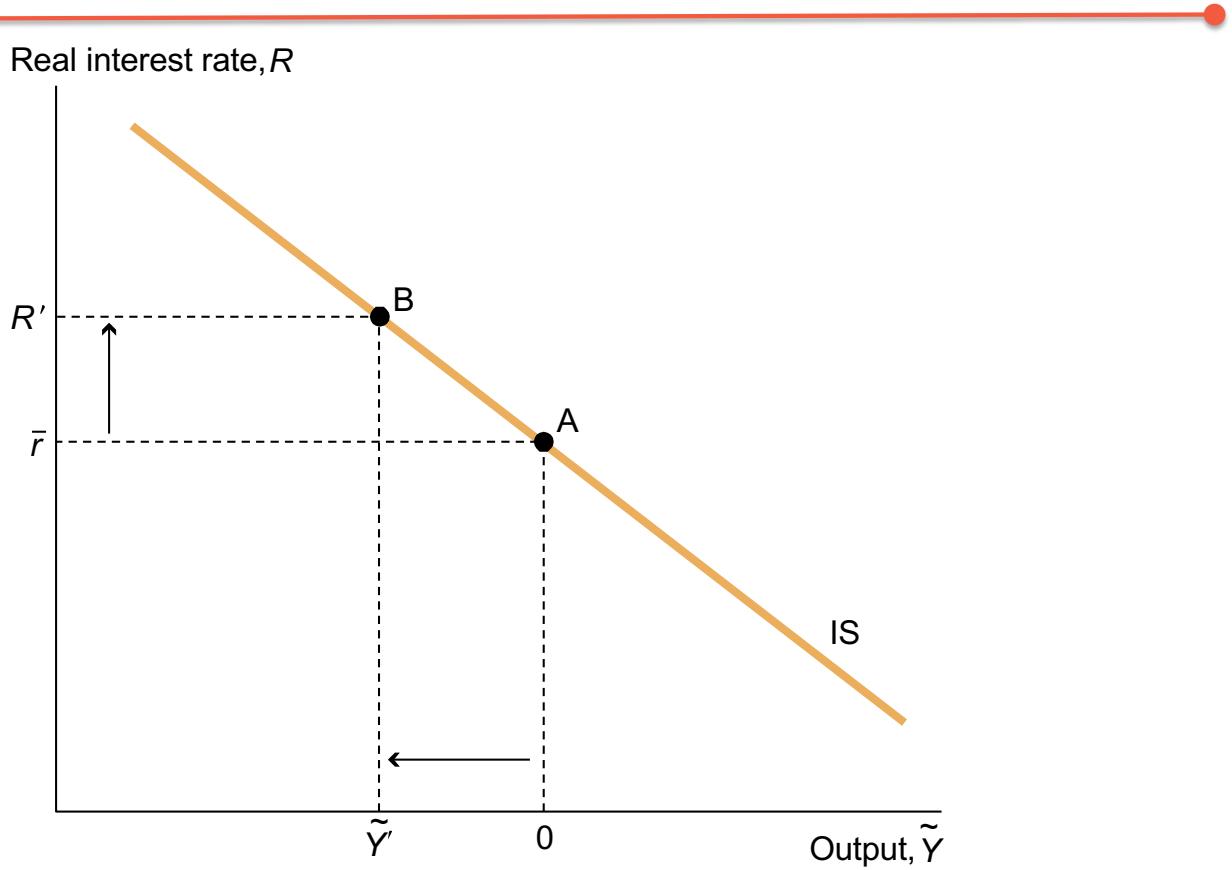


# A Change in the Interest Rate—1

- A change in the *real* interest rate moves the economy along the IS Curve



# An Increase in the Real Interest Rate to $R'$



# A Change in the Interest Rate—2

---

- If the sensitivity to the interest rate ( $\bar{b}$ ) were **higher**:
  - The IS curve would be **flatter**
  - A change in the interest rate would be associated with larger changes in output

# A Change in the Interest Rate—3

- Rearranging the IS equation  $\tilde{Y}_t = \bar{a} - \bar{b}(R_t - \bar{r})$  and solving for  $R_t$ :
  - $\tilde{Y}_t - \bar{a} + \bar{b}\bar{r} = -\bar{b}R_t$
  - $R_t = \frac{\tilde{Y}_t - \bar{a} + \bar{b}\bar{r}}{-\bar{b}}$
  - $R_t = \frac{\bar{a}}{\bar{b}} + \bar{r} - \left(\frac{1}{\bar{b}}\right)\tilde{Y}_t$
- The **slope** of the IS curve is *negative*
- As  **$\bar{b}$  increases**, the **slope decreases**, and the **IS curve becomes flatter**

# Shift of the IS Curve— Demand Shocks [1]

---

- Exogenous change in  $R_t \Rightarrow$  **move along** IS curve
- Exogenous change in  $\frac{I_t}{Y_t}$  for **given**  $R_t \Rightarrow$  **shift of** the IS curve
- Recall the equation for investment:

$$\frac{I_t}{\bar{Y}_t} = \bar{a}_i - \bar{b}(R_t - \bar{r})$$

# Shift of the IS Curve— Demand Shocks [2]

- IS curve in this chapter is very stylized
  - Important link to Solow model in chapter 5 is left out:
    - $\bar{a}_i$  is long-run investment share in short-run model
    - $\bar{s}$  is investment/saving rate in Solow model
    - What happens to MPK in long run if  $\bar{s}$  rises in Solow model?
    - What happens to MPK if  $\bar{a}_i$  rises in short-run model?
- $$K^* = \left( \frac{\bar{s}A}{\delta} \right)^{\frac{1}{1-\alpha}} L$$
- $$Y^* = \bar{A} K^* L^{1-\alpha} = \bar{A}^{\frac{1}{1-\alpha}} \left( \frac{\bar{s}}{\delta} \right)^{\frac{\alpha}{1-\alpha}} L$$
- $$\frac{\partial Y}{\partial K} = \frac{\alpha \bar{A}^{\frac{1}{1-\alpha}} L^{1-\alpha}}{\delta} = \frac{\alpha \bar{s}}{\delta}$$

# An Aggregate Demand Shock—1

- Suppose that information technology improvements create an investment boom:

↑ Business optimism

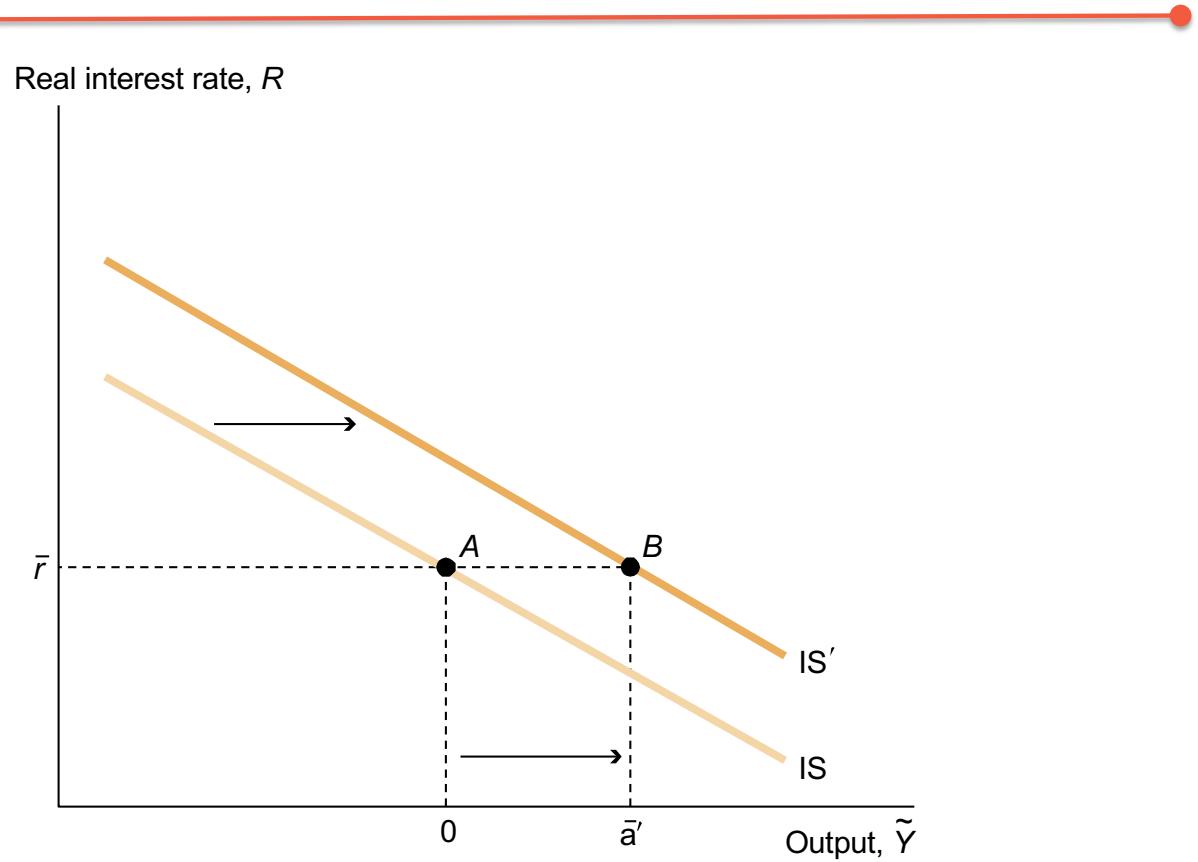
↑ Demand for capital

↑ Investment at a given  $R_t$

↑  $\bar{a}_i$ , which is part of  $\bar{a}$

IS curve shifts right

# An Aggregate Demand Shock—2



# A Shock to Potential Output

- Suppose there is a discovery of a new technology:

↑ Potential output (why?)

↑ Actual output

Short-run output  
unchanged

- Recall the definition of short-run output:  $\tilde{Y}_t = \frac{Y_t}{\bar{Y}_t} - 1$
- If  $Y_t$  and  $\bar{Y}_t$  increase by the same amount, the ratio  $\frac{Y_t}{\bar{Y}_t}$  will be equal to 1 and  $\tilde{Y}_t = 0$

# 11.5 Micro-Foundations of the IS Curve

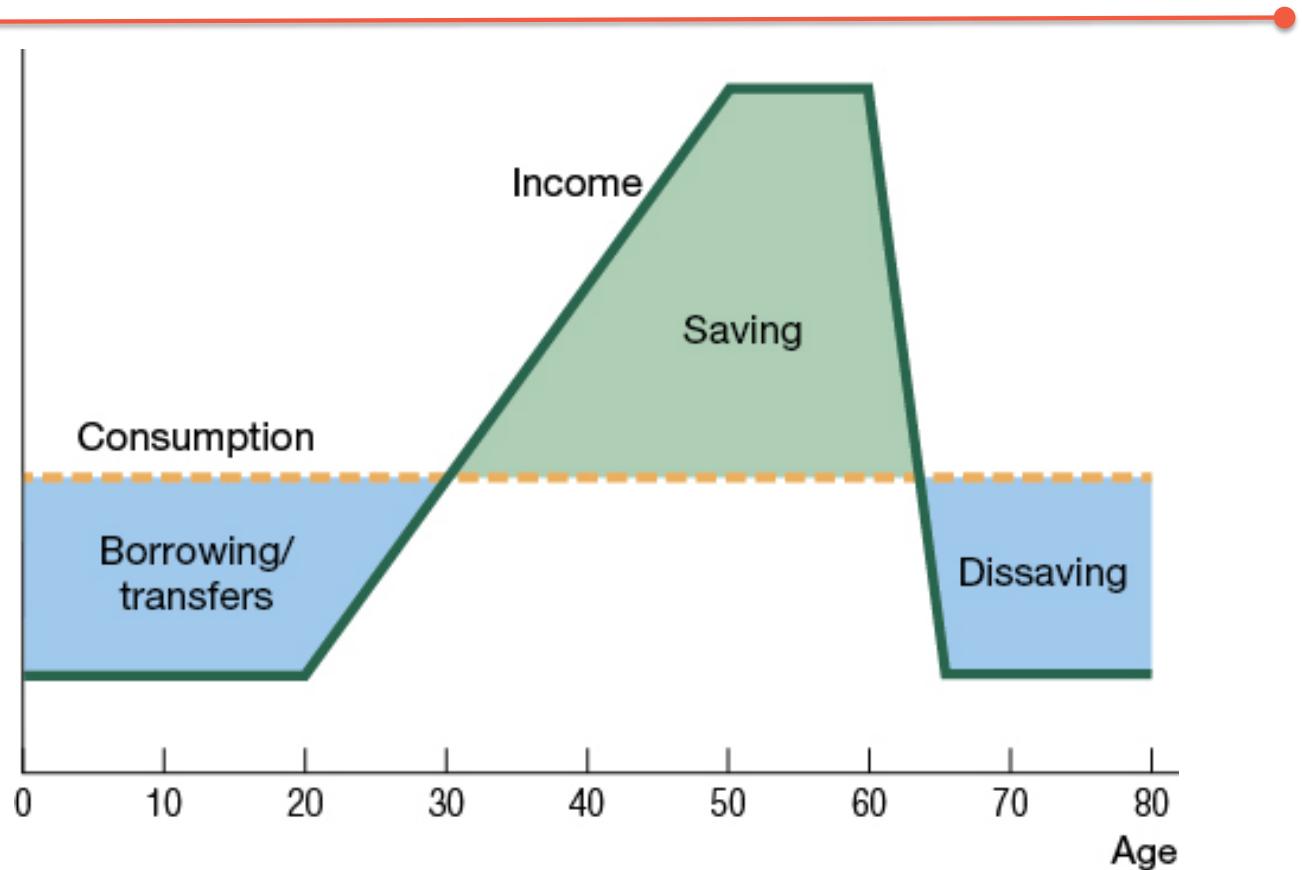
- Microfoundations:
  - Explain the **microeconomic** behavior that establishes the demands for  $C, I, G, EX$  and  $IM$
- Key theory of **consumption** behavior:  
Individuals prefer to smooth their consumption spending over time.
  - The **permanent-income hypothesis**: People will base their consumption on an average of their income over time rather than on their current income
  - The **life-cycle model of consumption**: Suggests that consumption is based on average lifetime income rather than on income at any given age

*not react to income  
steadily.*

# Consumption

- Consider an example: You are given the choice between Option A and Option B below.
- Option A:
  - Consumption: one piece of cake (\$3) every day Monday through Friday
  - Income: \$15 on Friday, but you may borrow \$3 a day (at no interest) to consume during the week.
- Option B:
  - Consumption: five pieces of cake (\$15) on Friday
  - Income: \$15 on Friday
- The permanent income hypothesis predicts that people will choose Option A.

# The Life-Cycle Model of Consumption



# Empirical Evidence—1

---

- Is there any evidence to support the two hypotheses?
- Case study: Alaska – How do consumers respond to two **different** types of income shocks?
  - Annual payment from the State of Alaska's Permanent Fund (from oil revenues)
  - Federal tax refunds
  - What distinguishes the annual payment from tax refunds?

# Empirical Evidence—2

---

- The LC/PI hypothesis predicts that consumption should not change when the Permanent Fund check is received
  - Hsieh finds that this is the case for the Permanent Fund
  - But, the same is not true in response to a tax refund
  - For every \$1 of a tax refund, consumers spend 0.30 cents
- Hsieh concludes that the LC/PI hypothesis applies to large and anticipated changes, but not necessarily to unanticipated changes.

# Empirical Evidence—3

---

- Additional evidence from temporary tax changes in the United States:
  - 1992 change in federal tax withholding schedule (but no change in tax liability itself) under H.W. Bush
  - Economic Growth and Tax Relief Reconciliation Act of 2001 rebates of \$300 / \$500 / \$600, depending on filing status (study by Shapiro and Slemrod)

# Multiplier Effects—1

- Suppose we conclude that consumption also depends on temporary changes in income  $\Rightarrow$  multiplier effect
- Consumption equals

$$\frac{C_t}{\bar{Y}_t} = \bar{a}_c + \bar{x} \tilde{Y}_t$$

$\bar{x}$  between 0 and 1

where  $\bar{x}$  is a parameter that determines how much consumption rises when the economy expands.

- We assume  $\bar{x}$  is between 0 and 1

# Multiplier Effects—2

- Solving for the IS curve assuming that consumption also depends on temporary changes in income:
  - We begin with the national income accounting identity and divide both sides by potential GDP
  - Substituting the new consumption equation yields:

$$\frac{Y_t}{\bar{Y}_t} = \frac{\bar{a}_c \bar{Y}_t + \bar{x} \bar{Y}_t \bar{Y}_t}{\bar{Y}_t} + \frac{\bar{a}_i \bar{Y}_t - \bar{b} \bar{Y}_t R_t + \bar{b} \bar{Y}_t \bar{r}}{\bar{Y}_t} + \frac{\bar{a}_g \bar{Y}_t}{\bar{Y}_t} + \frac{\bar{a}_{ex} \bar{Y}_t}{\bar{Y}_t} - \frac{\bar{a}_{im} \bar{Y}_t}{\bar{Y}_t}$$

# Multiplier Effects—3

- After simplifying:

$$\frac{Y_t}{\bar{Y}_t} = \bar{a}_c + \bar{x}\tilde{Y}_t + \bar{a}_i - \bar{b}R_t + \bar{b}\bar{r} + \bar{a}_g + \bar{a}_{ex} - \bar{a}_{im}$$

- Subtract 1 from both sides of the equation:

$$\frac{Y_t}{\bar{Y}_t} - 1 = \bar{x}\tilde{Y}_t + \bar{a}_i - \bar{b}R_t + \bar{b}\bar{r} + \bar{a}_c + \bar{a}_g + \bar{a}_{ex} - \bar{a}_{im} - 1$$

- Simplifying further:

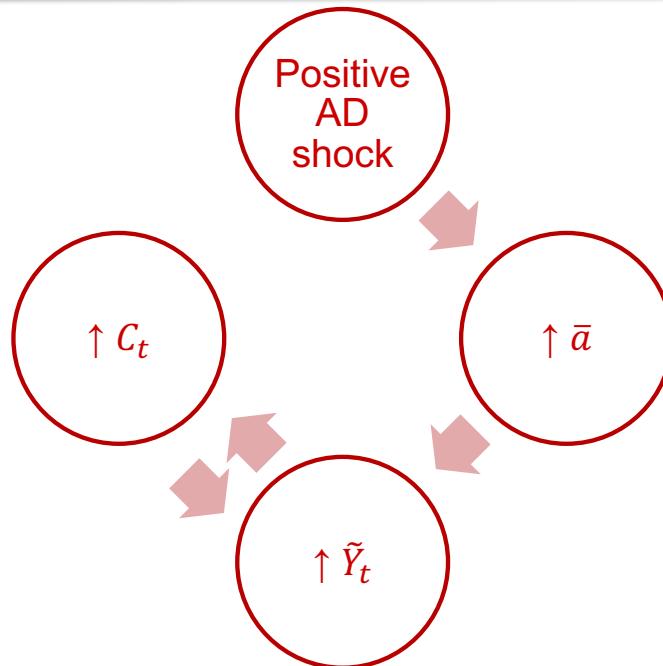
$$\tilde{Y}_t = \frac{1}{1 - \bar{x}} (\bar{a} - \bar{b}(R_t - \bar{r}))$$

# Multiplier Effects—4

- The new IS curve:

$$\tilde{Y} = \underbrace{\frac{1}{1 - \bar{x}}}_{\text{multiplier}} \times \underbrace{[\bar{a} - \bar{b}(R_t - \bar{r})]}_{\text{original IS curve}}$$

# Multiplier Effects—4



Note: In this case,  $\frac{C_t}{\tilde{Y}_t} = \bar{a}_C + \bar{x}\tilde{Y}_t$ , and consumption is affected by changes in short-run output .

# Investment—1

---

- Recall the equation for investment:

$$I_t = \bar{a}_i \bar{Y}_t - \bar{b}(\bar{R}_t - \bar{r}) \bar{Y}_t$$

- There are two main determinants of investment at the firm level:

- The gap between the real interest rate and the MPK
  - Cash flow

moral hazard      adverse selection

# Investment—2

---

- How do we calculate the return to capital?
  - In a simple model, the return on capital = MPK - depreciation
  - A more sophisticated framework would have to account for:
    - Corporate income taxes
    - Investment tax credits
    - Depreciation allowances

# Agency Problems

---

- Investment spending can be financed through:
  - Internal cash flow
  - Borrowing (which tends to be more costly)
- Borrowing introduces agency problems
  - Asymmetric information between individuals involved in a transaction
  - Two main types of agency problems:
    - **Adverse selection** (ex ante) *who shows up → loan*
    - **Moral hazard** (ex post)

# Government Purchases – 1

*assume to be a fix percent.*

- Government purchases of goods and services are an important source of demand
  - In recent years, government purchases are about 20% of GDP
- Government purchases can be:
  - A source of short-run fluctuations
  - An instrument to reduce fluctuations

# Government Purchases—2

- Discretionary fiscal policy

(expressive or not)

- Purchases of new goods or services

- American Recovery and Reinvestment Act of 2009

- Tax rate changes

spend resource  
information

- Investment tax credit of 1961

- Economic Growth and Tax Relief Reconciliation Act of 2001

Reagan tax reforms

# Fiscal Policy

---

- Automatic stabilizers
  - Transfer spending programs (e.g., unemployment insurance, Medicare)  
*→ older*
- The impact of fiscal policy depends on:
  - Timing  
*to slow > x response to*
  - The “no free lunch” principle (based on PDV of government’s budget balance)

(long run bonds)

# Ricardian Equivalence

- Analogous to the permanent-income hypothesis
- According to Ricardian equivalence:
  - The timing of tax changes does not matter for consumer behavior
  - The present value of government tax collection determines behavior
- Consider an example:
  - Suppose Congress decides to hire more teachers, increasing government purchases by \$500 million



# Net Exports—1

- Trade balance = Net exports

$$= EX - IM$$

$$= \bar{a}_{ex} \bar{Y}_t - \bar{a}_{im} \bar{Y}_t$$

$$= \bar{Y}_t (\bar{a}_{ex} - \bar{a}_{im})$$

- If  $EX > IM$

- $NX > 0$

- Trade surplus

- If  $EX < IM$

- $NX < 0$

- Trade deficit

# Net Exports—2

---

- Suppose there is:
  - An increase in demand for U.S. goods from the rest of the world ( $\uparrow \bar{a}_{ex}$ ) → IS shifts right →  $\uparrow$  short-run output
  - An increase in demand for imports from the rest of the world ( $\uparrow \bar{a}_{im}$ ) → IS shifts left →  $\downarrow$  short-run output