L12 03.03

Extensive games (sequential)

- players
- terminal history (all the sequences of actions to get to the outcome)
  - ↳ can represent the structure of the tree.
- player function over subhistories / nodes
- preferences over the (terminal histories) / outcomes

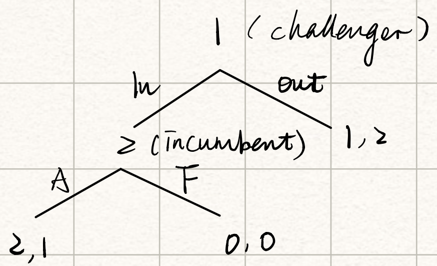
Strategy: complete action plan

NE: $u_i(O(s^*)) \geq u_i(O(s_i, s_{-i}^*)) \quad \forall s_i, i$

E.g.1 Entry Game

A - accomodate

F - fight



Terminal Histories: (Out)

(In, A)

(In, F)

normal / strategic form

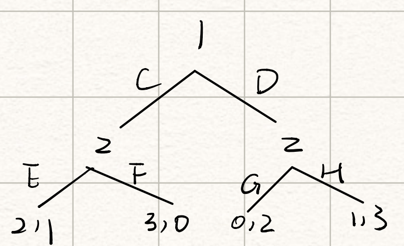
	A	F
In	2,1	0,0
Out	1,2	1,2

NE: (In, A), (Out, F)

non-credible threat

strategy said do the action but do sth. else when at the point.

E.g.2

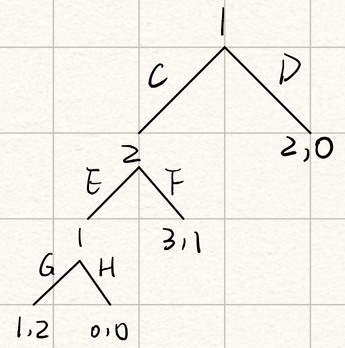


Terminal Histories (C,E), (C,F), (D, G), (D,H).

	EG	EH	FG	FH
C	2,1	2,1	3,0	3,0
D	0,2	1,3	0,2	1,3

NE (C, EG), (C, EH)

E.g. 3



Terminal History: (C, E, G) (C, F)

(C, E, H) (D)

	E	F
CG	1,2	3,1
CH	0,0	3,1
DG	2,0	2,0
DH	2,0	2,0

NE (CH, F) — non-credible threat.

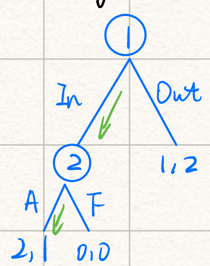
(DG, E)

(DH, E)

don't get to this point, no matter.

L3 03.05

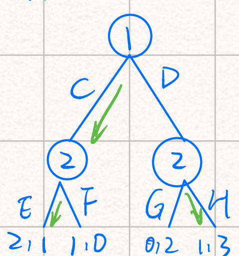
Subgame: game that begins at a particular node.



2 subgames: entire game (at beginning node)

game when player 1 chooses In

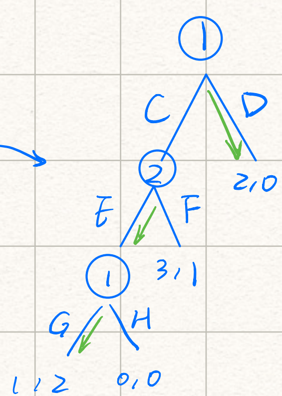
SPE: (In, A)



3 subgames.

SPE: (C, EH)

specify complete action plan



SPE: (DG, E)

Subgame Perfect Equilibrium

S^* is a SPE if $u_i(O_h(S^*)) \geq u_i(O_h(S_i, S^*_{-i}))$ $\forall s_i, i, h$

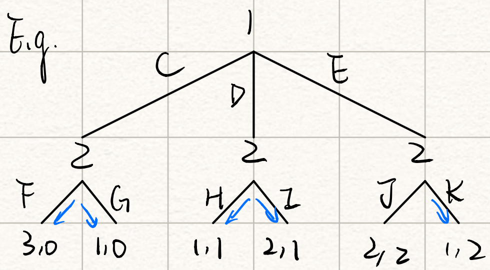
outcome; subgame h; for all players; strategy for i; for each subgame

make choices to maximize utility in all subgame. (→)

roll back equilibrium or backward induction

SPEs are also NE.

E.g.



SPE: (C, FHK) (C, GHK)
(D, FIK) (D, GHK)
(D, GIK) (E, GHK)

Stackelberg.

$\Rightarrow$ Cournot with Sequential Moves.

i.e. 1 chooses q_1, then 2 chooses q_2.

SPE: 2 maximizes $q_2 P_d(q_1+q_2) - C_2(q_2)$ to get $b_2(q_1)$

versus demand curve — best response.

1 maximizes $q_1 P_d(q_1 + b_2(q_1)) - C_1(q_1)$

e.g. $P_d(Q) = \begin{cases} \alpha - Q & \text{if } Q \le \alpha \\ 0 & \text{otherwise} \end{cases}$

$C_i(q_i) = c\,q_i$

For player 2: $\max_{q_2} q_2(\alpha - q_1 - q_2 - c)$

$$\frac{\partial u_2}{\partial q_2} = \alpha - q_1 - 2q_2 - c \overset{set}{=} 0.$$

$$b_2(q_1) = \frac{\alpha - q_1 - c}{2}$$ $\Leftarrow$ SPE best response in the subgame. instead of $\frac{\alpha-c}{4}$

For player 1: $\max_{q_1} q_1\left(\alpha - q_1 - \frac{\alpha - q_1 - c}{c} - c\right)$

$$= \max_{q_1} \tfrac{1}{2} q_1(\alpha - q_1 - c)$$

$$\frac{\partial u_1}{\partial q_1} = \frac{1}{2}(\alpha - 2q_1 - c) \overset{set}{=} 0$$

$$q_1 = \frac{\alpha - c}{2}$$

SPE: $\left(q_1 = \frac{\alpha-c}{2},\ q_2 = \begin{cases} \frac{\alpha - q_1 - c}{2} & \text{if } \alpha > q_1 + c \\ 0 & \text{otherwise} \end{cases}\right)$

Cournot: $\left(\frac{\alpha-c}{3}, \frac{\alpha-c}{3}\right)$

E.g. 21 Flags. 2 players (f, s).

each take 1, 2, or 3 flags

players take the last flag wins.

1:	f(1)	5	f(1)	9	f(1)	13	f(1)	17	f(1)	21:	f
2	f(2)	6	f(2)	10	f(2)	14	f(2)	18	f(2)		
3	f(3)	7	f(3)	11	f(3)	15	f(3)	19	f(3)		
4	s	8	s	12	s	16	s	20	s		

L14 03.10

E.g. Pirate Game (Oldest propose, if not agreed on, die; else, split as proposed)

age ↓

	A	B	C	D	E
E					100
D				100	0
C			99	0	1
B		99	0	1	0
A	98	0	1	0	1

Ties: goes to proposer.

D needs 1 votes (Tie effect)

C needs 2 votes (include himself)

B needs 2 votes (Tie effect)

A needs 3 votes

Because this is backward induction, this is SPE

The extension: wiki discussed

Important: identify as sequential game

E.g. Ultimation Game

2 players split prize with value C

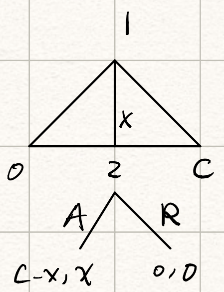
player 1 makes offer to give player 2 an amount x

2 accept or reject → nobody gets anything. ⇒ continuous game.

↳ discussion occurs

tree structure not applicable

"Synthetic" tree structure



SPE Backward Induction

Start with P2 : A if $x>0$

A or R if $x=0$

P1: want to keep as much by himself while still making this offer acceptable.

(i.e. choose the smallest x that can be accepted)

Hence: $x=0$, always A.

Note: if player 2 chooses R, no SPE.

(any x, R) is also NE, but not SPE due to incredible threat. ⇐ Lots of NEs.

Discrete case : Player 2: if $x>0$ A

$x=0$ A or R. ← if always A, $p1 \to 0$

SPE: ($x=1$, A if $x>0$, R if $x=0$)

($x=0$, always A)

NE: Lots of NEs.

e.g. ① R if $x<\frac{1}{2}C$ and A if $x\geq\frac{1}{2}C$

$x=\frac{1}{2}C$

② R if even and A if odd

$x=1$

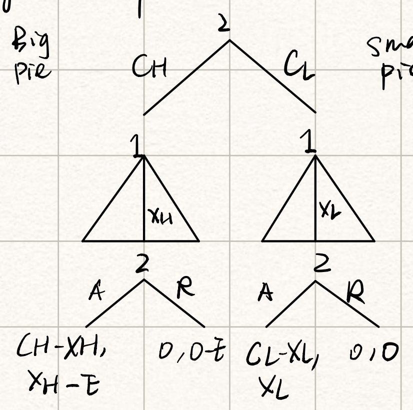
③ R rational x, A irrational x

$x=$ small irrational value

Experimental Exercise: 1 offers $0.35c$ on avg, 20% offers are rejected.

Reason: c being small so worth arguing "fairness" matters for future treatment (negotiable game)

E.g. Holdup Game



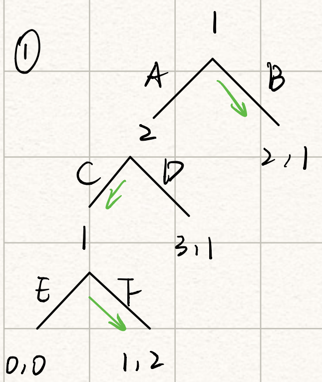
CH>CL

Backward Induction:

SPE: XH=0, XL=0, Always A, ~~0~~

Where SPE not make sense.

①



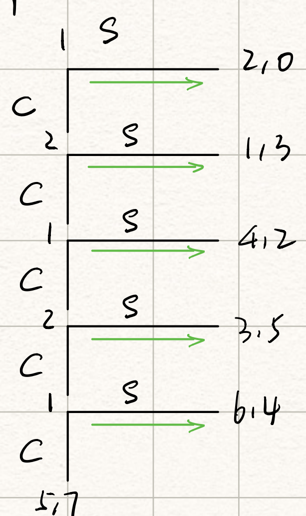
Possible issue: what if sth. unexpected happen?

Continuous plan and think everyone rational

Or distinct opponent's rationality and start on that?

② Chain store example: repeat entry games 20-times against different opponents.

③ Centipede Game



SPE: Always want to stop immediately

But if continuous and stop → get better outcome

Reason: maybe someone making "incredible offer"

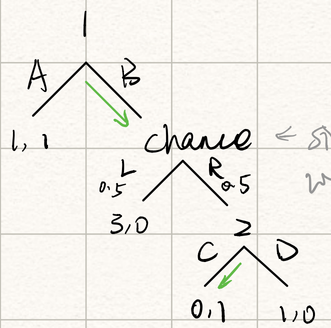
i.e. if 1 continuous, 2 continuous.

Practical: actually continue few rounds

L15 03.12

Exogeneous Uncertainty. (come from out of the game)

"Nature" or "Chance" being a player



< similar to a player with particular mixed strategy. } not in payoff puzzle.

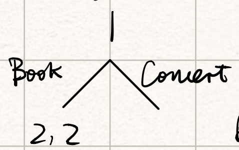
	C	D
A	1, 1	1, 1
B	3/2, 1/2	2, 0

NE: (B, C)

SPE: (B, C)

Simultaneous Moves (Multistage)

can assign multiple players to the same node



	B	S
B	3, 1	0, 0
S	0, 0	1, 3

NE: (B, B)

(S, S)

Find NE

And consider each NE

with backward reduction

→

	B	S
Book, B	2, 2	2, 2
Book, S	2, 2	2, 2
Concert, B	3, 1	0, 0
Concert, S	0, 0	1, 3

NE ((Book, B), S)

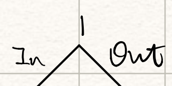
((Book, S), S)

((Concert, B), B).

SPE: ((Concert, B), B)

((Book, S), S)

Entry Game



In with a fixed cost f.

Cournot choose q_1, q_2.

$u_1 = q_1(\alpha - c - q_1 - q_2) - f$

$u_2 = q_2(\alpha - c - q_1 - q_2)$

monopoly for 2 q_2

$u_2 = q_2(a - c - q_2)$

Subgame: Cournet

$q_1 = q_2 = \frac{\alpha - c}{3}$

$u_1 = \frac{(\alpha - c)^2}{9} - f$

$u_2 = \frac{(\alpha - c)^2}{9}$

Monopoly.

$q_1 = 0, \quad q_2 = \frac{\alpha - c}{2}$

$u_1 = 0 \quad u_2 = \frac{(\alpha - c)^2}{4}$

SPE: 1 chooses In if $\frac{(\alpha - c)^2}{9} \geq f$ when the fixed cost is relatively low.

NE: For example, 2 threaties to drive 1's profit to < 0

so 1 choose out.

Hoteling Election

candidates pick location

voters pick each candidate

SPE: for each set of positions, users chooses ← need NE.

eliminate WD strategies

(voting for least prefered candidates)

Candidates choose positions based on voter's choices

2 candidates:

voter choose closest,

candidates choose median

Lots of SPE, NE.

E.g. all vote for 1 if she chooses x_i, vote for 2 otherwise

1 choose x_i and 2 choose x_j

Committee decision making binary agendas

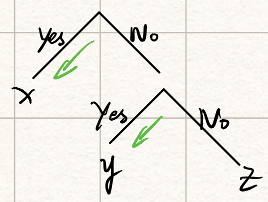
odd numbers of members

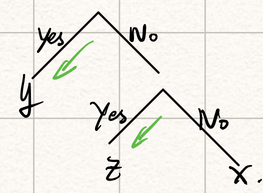
Strategic votes: vote for prefered.

1. $x > y > z$

2. $y > z > x$

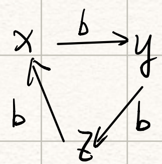
3. $z > x > y$.





⇐ the one voted first is chosen.

Top cycle set: set of alternatives that can win for some agenda.



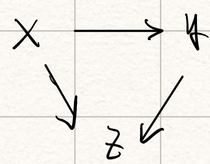
(indirectly beats all other alternatives)

Condorcet winner: only element in top cycle set. ⇐ only winner

$x > y > z$

$y > x > z$

$z > x > y$



Lib 03.24

Mistakes affect equilibrium

e.g.

	A	B
A	1,1	0,0
B	0,0	0,0

NE: (A, A) (B, B)

mistake rate p_1, p_2

	choose A	choose B
choose A	$(1-p_1)(1-p_2)$	$(1-p_1)p_2$
choose B	$p_1(1-p_2)$	p_1p_2.

$(A, A): (1-p_1)(1-p_1) \cdot 1 + (1-p_1)p_2 \cdot 0 + p_1(1-p_2) 0 + p_1 p_2 \cdot 0$.

← 两者相同.

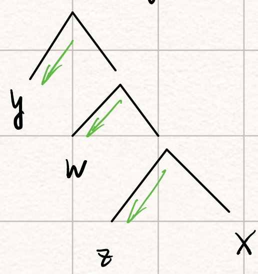
If $p_1 < \frac{1}{2}$ $p_2 < \frac{1}{2}$ NE: (A, A).

Cont: Committee Decision Making. Binary Agenda

$x > y > z > w$

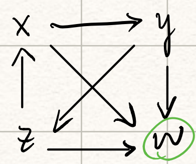
$y > z > x > w$

$w > z > x > y$



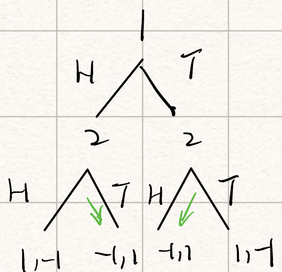
top cycle set:

2/3 prefer → beat.



not in top cycle set.

Simultaneous → Sequential

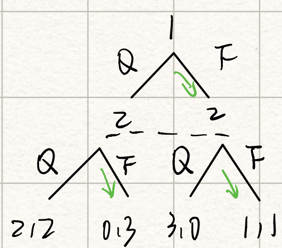


SPE: (H, TH) (T, TH)

	Q	F
Q	2,2	0,3
F	3,0	1,1

NE: (F, F)

→



SPE (F, FF)

Information set ← p2 does not know what p1 does

⇓

= simultaneous game.