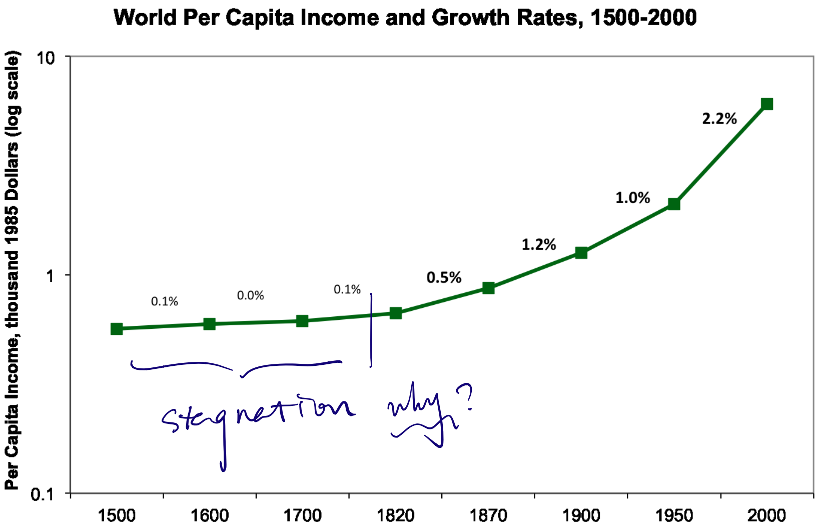
Malthusian Stagnation and Fertility

Understanding Malthusian Stagnation



Source: Maddison (2010)

Understanding Malthusian Stagnation

The facts to be explained are:

1. low incomes until 1800 (no growth)
2. small income differences across countries
3. productivity differences reflected in population density rather than income per capita.

↑ urban, tech

↓

↑ population

The Malthus Model with Endogenous Fertility

- Agricultural production function:

exponent of land

$$Y_t = (A_t X)^{\alpha} N_t^{1-\alpha},$$

where A_t is productivity, X is fixed amount of land, and N_t is population.

- Income per capita is given by:

efficiency unit of land.

$y_t = \frac{Y_t}{N_t}$ $\quad y_t = \left(\frac{A_t X}{N_t}\right)^{\alpha}$ $0<\alpha<1$ decreasing return to scale.

- People have utility function over consumption c_t and number of children n_t:

want diminishing marginal income. natural log does this,

$$u(c_t, n_t) = \ln(c_t) + \ln(n_t).$$

Fertility

which they maximize subject to the budget constraint:

$$c_t + p n_t = y_t.$$

Optimal Fertility Choice

$\max(U(n_t)) = \ln(y_t - pn_t) + \ln(n_t)$

first order condition
i.e. $f'(x)$

$U'(n_t) = \frac{-p}{y_t - pn_t} + \frac{1}{n_t} = 0$

- The solution to the utility maximization problem is:

$$n_t = \frac{y_t}{2p}$$

- Law of motion for population (assumes that people live for one period/generation):

$$N_{t+1} = N_t n_t = N_t \frac{y_t}{2p}.$$

Results for Constant Productivity Level

$$y_t = \left(\frac{A\ X}{N_t}\right)^{\alpha}$$
$$\Uparrow$$
$$N_{t+1} = N_t$$
$$\Uparrow$$
$$n_t = 1$$

- Economy converges to steady state with constant income per capita, which depends only on cost of children:

$$\bar{y} = 2p$$

- Steady state $\bar{N}$:

$$2p = \left(\frac{AX}{\bar{N}}\right)^{\alpha} \qquad \bar{N} = \left(\frac{1}{2p}\right)^{\frac{1}{\alpha}} AX$$

- Population density (people per unit of land):

$$\frac{\bar{N}}{X} = \left(\frac{1}{2p}\right)^{\frac{1}{\alpha}} A$$

- Thus, more productive locations have higher population density, but not higher incomes.

Introducing Productivity Growth

- So far, we have focused on the case of constant productivity:

$$A_t = A$$

  in all periods.
- However, in reality productivity increased over time even before industrialization, e.g.:
  - transition from hunter - gatherer to farming
  - selective breeding, crop selection $\Rightarrow$ make land $\uparrow$ productive.
  - crop rotation, plow technology
- Let us therefore consider the case of positive productivity growth:

$$A_{t+1} = (1+g)A_t$$

  with $g > 0$.

Introducing Productivity Growth

- If productivity goes up over time, a steady state with constant population is no longer possible:
  - If the population were constant, a rise in A leads to a rise in income
  - a rise in income $\rightarrow$ a rise in fertility.
- Thus, would like to find a steady state in which population is growing over time.

Introducing Productivity Growth

- What turns out to be constant in the long run is the ratio m_t of population to *effective* land:

$$m_t = \frac{N_t}{A_t X}.$$

- Population per effective land unit m_t is a measure of population density (people per unit of land), but it corrects for the productivity of land at different times by multiplying land by A_t.
- Population N_t satisfies:

$$N_t = m_t A_t X.$$

The Law of Motion for m_t

- The general law of motion for population:

$$N_{t+1} = N_t n_t = m_t A_t X \frac{y_t}{2p} = \frac{N_t}{2p} \left(\frac{A_t X}{N_t} \right)^{\alpha}$$

$$N_{t+1} = (A_t X)^{\alpha} N_t^{1-\alpha} \frac{1}{2p}$$

- Substituting N_{t+1} and N_t gives:

$$A_{t+1} X m_{t+1} = \frac{1}{2p} (A_t X)^{\alpha} (A_t X m_t)^{1-\alpha} = \frac{A_t X}{2p} m_t^{1-\alpha}$$

- Since $A_{t+1} = (1+g)A_t$, this is:

$$(1+g) \cancel{A_t} \cancel{X} m_{t+1} = \frac{\cancel{A_t X}}{2p} m_t^{1-\alpha}$$

- Dividing by $A_t X$ and $1+g$ on both sides gives:

$$m_{t+1} = \frac{1}{2p(1+g)} m_t^{1-\alpha}$$

The Law of Motion for m_t

- Law of motion depends on m_t only, and since we have $1 - \alpha < 1$, m_t converges to steady state $\bar{m}$.
- The steady-state ratio $\bar{m}$ of population to effective land satisfies:

$$\bar{m} = \frac{1}{2p(1+g)} \bar{m}^{1-\alpha}$$

  which gives:

$$\bar{m} = \left(\frac{1}{2p(1+g)}\right)^{\frac{1}{\alpha}}$$

  or:

Population Growth in the Steady State

- We find that in the long run, the ratio:

$$m_t = \frac{N_t}{A_t X}$$

  is constant.
- This implies that population and productivity have to grow at the same rate.
- The steady-state fertility rate $\bar{n}$ therefore must be given by:

$$\bar{n} = 1 + g$$

Income per Capita in the Steady State

- The formula for income per capita is:

$$y_t = \left(\frac{A_t X}{N_t}\right)^{\alpha}.$$

- Term inside brackets is inverse of m_t. We therefore have:

$$y_t = m_t^{-\alpha}.$$

- Given that m_t is constant in the long-run, income per capita is constant as well!
- The steady state income per capita $\bar{y}$ is:

$$\bar{y} = 2p(1+g)$$

$\Rightarrow$ high productivity growth rate $\downarrow$

$\uparrow$ income.

but relatively small

Income per Capita in the Steady State

- We could have gotten the same result by working backwards from population growth.
- We know that steady-state population growth has to satisfy:

$$\bar{n} = 1 + g$$

- From optimal fertility choice, $\bar{n}$ is given by:

$$\bar{n} = \frac{\bar{y}}{2p}.$$

- Thus, $\bar{y}$ satisfies:

$$1 + g = \frac{\bar{y}}{2p}$$

or:

$$\bar{y} = (1 + g)2p.$$

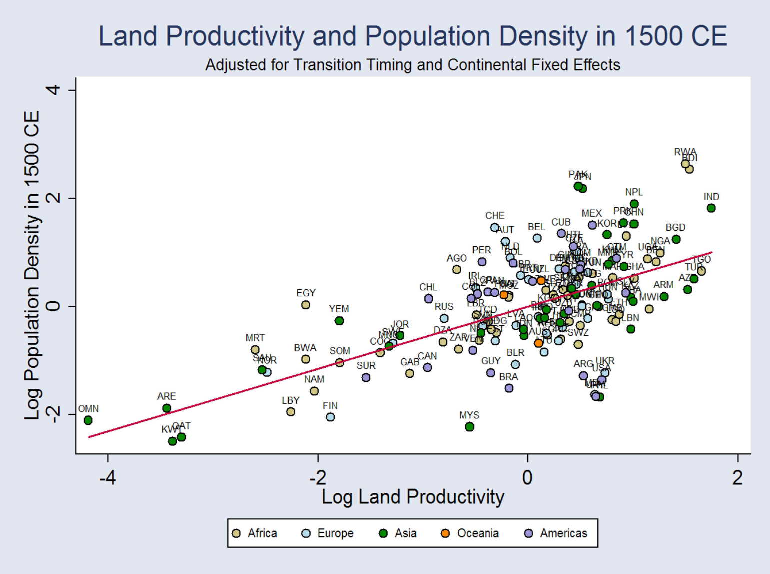
Summary of Results

- Economy converges to steady state with constant income per capita.
- Countries with higher rate of productivity growth have higher level of income per capita, but even they stagnate over time.
- Channel of adjustment (still) is link between income per capita and population growth: income per capita adjusts until population growth matches productivity growth.
- More productive countries are characterized (mostly) by higher population density rather than higher income per capita.

Cross-Country Predictions versus the Data

- Across countries, the main prediction is that more productive countries and regions should have higher population density, but similar income per capita.
- That is exactly what we observed in the pre-industrial era.
  - The areas of highest population density were Western Europe and China.
  - Those were also the technologically most advanced areas.
  - Population density was low in Australia and most of the Americas.
  - Here, technology was much less advanced compared to Eurasia.

Cross-Country Predictions versus the Data



Predictions for Changes

- Consider a country that is currently in the steady state.
- A sudden decline in population (war, famine, or epidemic) will:
  - Lower the level of output in the short run.
  - Increase the level of output per capita in the short run. ← decreasing return to scale
  - Increase the growth rate of population and output in the short run.
  - Decrease the growth rate of output per capita in the short run.
  - Have no effect on the level and growth rate of population, output, and output per capita in the long run.

Predictions for Changes

- Consider a country that is currently in the steady state.
- A sudden increase in land (say, annexation of sparsely populated frontier area) will:
  - Increase the level of output in the short run.
  - Increase the level of output per capita in the short run.
  - Increase the growth rate of population and output in the short run.
  - Lower the growth rate of output per capita in the short run (negative growth).
  - Have no effect on the level and growth rate of output per capita in the long run.
  - Increase the level of population and output in the long run, but have no long-run impact on the growth rate.

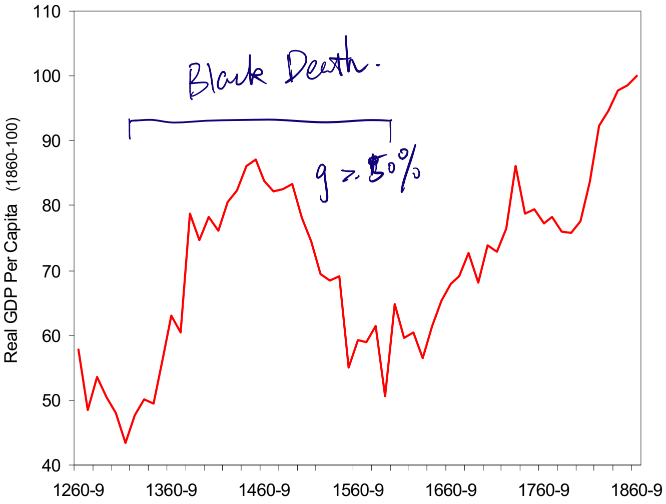
Predictions for Changes

- Consider a country that is currently in the steady state.
- A sudden increase in the growth rate of productivity will:
  - Have no impact on the level of population, output, and output per capita in the short run.
  - Increase the growth rate of population, output, and output per capita in the short run.
  - Increase the level and growth rate of population and output in the long run.
  - Increase the level of output per capita in the long run, but have no effect on the growth rate (still zero).

Predictions for Changes and the Data

- The predictions for changes are in line with the empirical evidence.
- Classic example for reduction in population is the Black Death:
  - Arrival of Plague in 1348 killed about one-third of European population.
  - Subsequently, income per capita was higher, and population growth went up.
  - Over a couple of centuries, population loss was recovered, and income per capita returned to previous level.
- Classic example for increase in land is the American Frontier:
  - North America was sparsely populated before arrival of European settlers.
  - Compared to Europe, income, fertility, and population growth were substantially higher at the American Frontier.
  - Population growth slowed down as population density increased.

The Black Death and Income per Capita in England



also suggest
hard to escape
the growing trend.

Population and Wages in England