L9 02.20

Mixed Strategy Game

strategy: α_i = probability distribution.

$\alpha_i(a_i)$ = probability of playing a_i

e.g. head or tail

$\alpha_i(H) = 0.5 \quad \alpha_i(T) = 0.5 \quad \Rightarrow$ add to 1.

Pure strategy: all prob on one action.

	H	T
H	10, -10	-1, 1
T	-1, 1	1, -1

less likely to play H for player 2 due to -10 payoff.

Preferences over lotteries. (von Neumann - Morgenstern)

Expected utility.

Mixed strategy NE (MSNE)

For every play i, $u_i(\underset{\text{list of MS}}{\alpha^*}) \geq u_i(\alpha_i, \alpha^*_{-i}) \quad \forall \alpha_i$

L10 02.25

Matching Pennies

if matched ① get 1 ② get -1 ; not, ① get -1, ② get 1

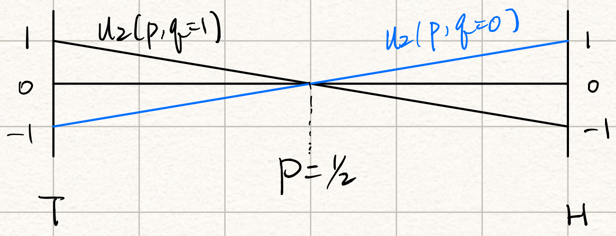
Want to find the prob p, q for NE.

		② prob q: H	(1-q): T
①	prob: p H	1, -1	-1, 1
	(1-p) T	-1, 1	1, -1

payoff for 2: H: $-1 \cdot p + 1 \cdot (1-p) = 1-2p$. $(q=1)$

T: $1 \cdot p + (-1)(1-p) = 2p-1$ $(q=0)$

Player 2 has a mixed strategy if payoff of H = payoff of T



$\Leftarrow 1-2p = 2p-1 \Rightarrow p = 0.5.$

$$BR_2 = \begin{cases} H & q=1 & \text{if } p<1/2 \\ T & q=0. & \text{if } p>1/2 \\ \text{mixed.} & q\in[0,1]. & \text{if } p=1/2 \end{cases}$$

For player 1: $u_1^H = q-(1-q) = 2q-1$

$u_1^T = -q + (1-q) = 1-2q.$

$u_1^T = u_1^H \Rightarrow q = 1/2.$

NE: $(p=1/2,\ q=1/2)$ same payoff of H or T for both players

Bach or Stravinsky

			②: q	$(1-q)$
			B	S
①	p	B	2, 1	0, 0
	$(1-p)$	S	0, 0	1, 2

$u_2^B = p + 0(1-p) = p$

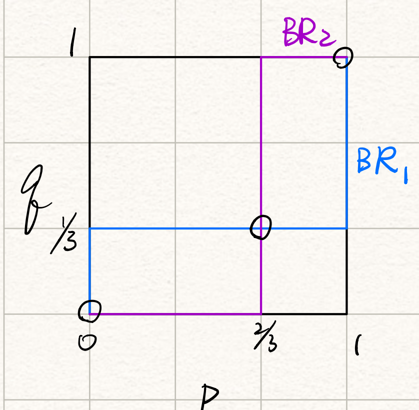
$u_2^S = 0\cdot p + 2(1-p) = 2-2p$

$$BR_2 = \begin{cases} B & q=1 & \text{if } p>2/3 \\ S & q=0 & \text{if } p<2/3 \\ \text{B or S} & q\in[0,1] & \text{if } p=2/3. \end{cases}$$

$u_1^B = 2q + 0(1-p) = 2q$

$u_1^S = 0q + (1-q) = 1-q.$

$$BR_1 = \begin{cases} B & p=1 & \text{if } q>1/3 \\ S & p=0 & \text{if } q<1/3 \\ \text{B or S} & p\in[0,1] & \text{if } q=1/3 \end{cases}$$



NE: $(0,0) \Leftarrow (S,S)$

$(1,1) \Leftarrow (B,B)$

$(2/3, 1/3)$

Pure strategies is included in the mixed strategies.

BR in pure strategies are also BR in mixed strategies.

Equality of Payoff Theorem

α^* is a NE iff, for each play i, if $\alpha_i^*(a_i) > 0$, $u_i(a_i, \alpha_{-i}^*) = E_i^*$.

↳ For any action has positive action, utility is the same.

If $\alpha_i^*(a_i) = 0$, $u_i(a_i, \alpha_{-i}^*) \leq E_i^*$.

↳ some action chosen not to play, cannot have better utility with this.

⇒ Find NE and check whether this holds.

E.g.

as P(L)=0 not matter

			② 0	1/3	2/3
			L	C	R
①	0.75	T	–, 2	3, 3	1, 1
	0	M	–, –	0, –	2, –
	0.25	B	–, 4	5, 1	0, 7

For player 1: T: $\frac{1}{3} \cdot 3 + \frac{2}{3} \cdot 1 = \frac{5}{3} = E_2^*$

M: $\frac{1}{3} \cdot 0 + \frac{2}{3} \cdot 2 = \frac{4}{3} \leq E_2^*$

B: $\frac{1}{3} \cdot 5 + \frac{2}{3} \cdot 0 = \frac{5}{3} = E_2^*$

For player 2: L: $\frac{3}{4} \cdot 2 + \frac{1}{4} \cdot 4 = \frac{10}{4} \leq E_1^*$

C: $\frac{3}{4} \cdot 3 + \frac{1}{4} \cdot 1 = \frac{10}{4}$

B: $\frac{3}{4} \cdot 1 + \frac{1}{4} \cdot 7 = \frac{10}{4}$

$\Big\}\ E_1^*$

$((\frac{3}{4}, 0, \frac{1}{4}), (0, \frac{1}{3}, \frac{2}{3}))$ is NE because satisfies EPT.

L11 02.27

Midterm 1. mean 80.7, median 83 sd. 13.1

90~100 ×17 80~90 ×23 70~92 ×12 <70×15.

Game with finitely many action for each player always has a mixed-strategy NE.

Dominated strategies in mixed strategies.

E.g.1 Payoffs for player 1

① \ ②	L	R
T	1	1
M	4	0
B	0	3
$\frac{1}{2}M+\frac{1}{2}B$	2	1.5

$\frac{1}{2}M+\frac{1}{2}B$ strictly dominate T

← whether player 2 plays mixed strategy does not matter for strictly dominated strategies.

payoffs for T always 1, for $\frac{1}{2}(M+B) \in [1.5, 2]$.

E.g.2 Matching Pennies (modified)

		q	$(1-q)$
	① \ ②	H	T
p	H	1,-1	-1,1
$1-p$	T	-1,1	-1,-1

t is weakly dominated in pure strategy.

2 willing to play tails if $p-(1-p) \geq -1+1-p$, i.e. $p \geq \frac{1}{2}$.

NE $(p \geq \frac{1}{2}, q=0)$

Every Finite game has a NE in which no one plays a weakly dominated strategy.

Symmetric Game in mixed strategies

same set of actions, symmetric preferences.

symmetric equilibrium. if $\alpha_1^* = \alpha_2^* = \cdots = \alpha_n^*$ and α^* is NE.

Eg. 1

		②	q N	$1-q$ S
①				
p	N		0, 0	1, 1
$1-p$	S		1, 1	0, 0

For ② $1-p=p \Rightarrow p=\frac{1}{2}$

For ① $1-q=q \Rightarrow q=\frac{1}{2}$.

$(\frac{1}{2}, \frac{1}{2}) \leftarrow$ symmetric NE.

Eg 2. Reporting a Crime

n players call or not

payoff of i
$$\begin{cases} v & \text{if some one calls not } i \\ v-c & \text{if } i \text{ call} \\ 0 & \text{if no one call} \end{cases}$$

pure strategies NE: exactly 1 person call

MSNE. Let p = probability that one person calls.

$$v-c = v \cdot p(\text{someone else calls})$$

$$v-c = v \cdot (1-p(\text{no one calls}))$$

$$c = v \cdot p(\text{no one else calls}) = v(1-p)^{n-1}$$

$$p = 1-\left(\frac{c}{v}\right)^{\frac{1}{n-1}}$$

If $n\uparrow \rightarrow \frac{1}{n-1}\downarrow \rightarrow \left(\frac{c}{v}\right)^{\frac{1}{n-1}}\uparrow \rightarrow 1-\left(\frac{c}{v}\right)^{\frac{1}{n-1}}\downarrow \rightarrow p\downarrow$. $(0<c<v)$

$P(\text{no one calls}) = (1-p)^n$ is also increasing in n $(n\uparrow \Rightarrow p\downarrow \rightarrow (1-p)\uparrow \rightarrow (1-p)^{\overset{\uparrow}{\underset{\Downarrow}{n}}}\uparrow)$

Find MSNE:

1. eliminate strictly dominated strategies
2. For each player i
   - for each different subset S_i of i's strategies A_i.
     check for α such that S_i are the only action assigned positive prob. and EPT satisfied.

Eg. Bach or Stravinsky (modified)

		q_B	q_S	q_X
	②	B	S	X
① p	B	<u>4</u>,<u>2</u>	0,0	0,1
$1-p$	S	0,0	<u>2</u>,<u>4</u>	<u>1</u>,3

Case 1: P2: B & S: $q_B, q_S \geq 0$, $q_X = 0$.

$2p = 4(1-p) \Rightarrow p = \frac{2}{3}$

P_1: $4q_B = 2(1-q_B) \Rightarrow q_B = \frac{1}{3}$.

$u_2^B = \frac{4}{3} = u_2^S < u_2^X = \frac{2}{3} + 3 \cdot \frac{1}{3} = \frac{5}{3}$

not a NE. → in this situation P_2 will play x.

L12 03.03

Short Cut: graphs (if one has only 2 actions)

For P_2: payoff for B $2p$

S $4-4p$

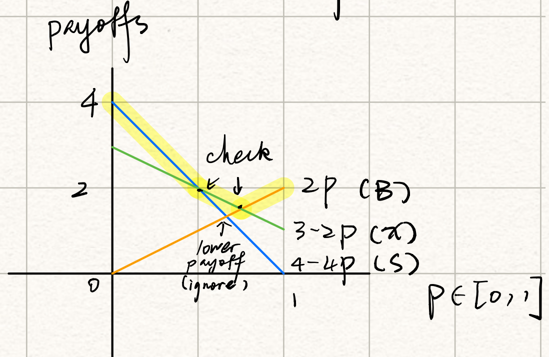
X $p + 3(1-p) = 3-2p$.

Best Responses: S if $p < \frac{1}{2}$

x if $\frac{1}{2} < p < \frac{3}{4}$

B if $p \geq \frac{3}{4}$.

↓



S and X: $4-4p = 3-2p$ $p = \frac{1}{2}$

need $u_1^B = 0q_S + 0(1-q_S) = u_1^S = 2q_S + (1-q_S)$ $1 + q_S = 0 \Rightarrow$ no NE

B and X: $2p = 3-2p$ $p = \frac{3}{4}$

$u_2^S = 4-4p = 1 < 2p = 3-2p = \frac{6}{4}$

$u_1^B = 4q_B + 0(1-q_B) = 0q_B + (1-q_B) = u_1^S$ → $q_B = \frac{1}{5}$.

NE: ($p = \frac{3}{4}$, $q_B = \frac{1}{5}$, $q_S = 0$, $q_X = \frac{4}{5}$)