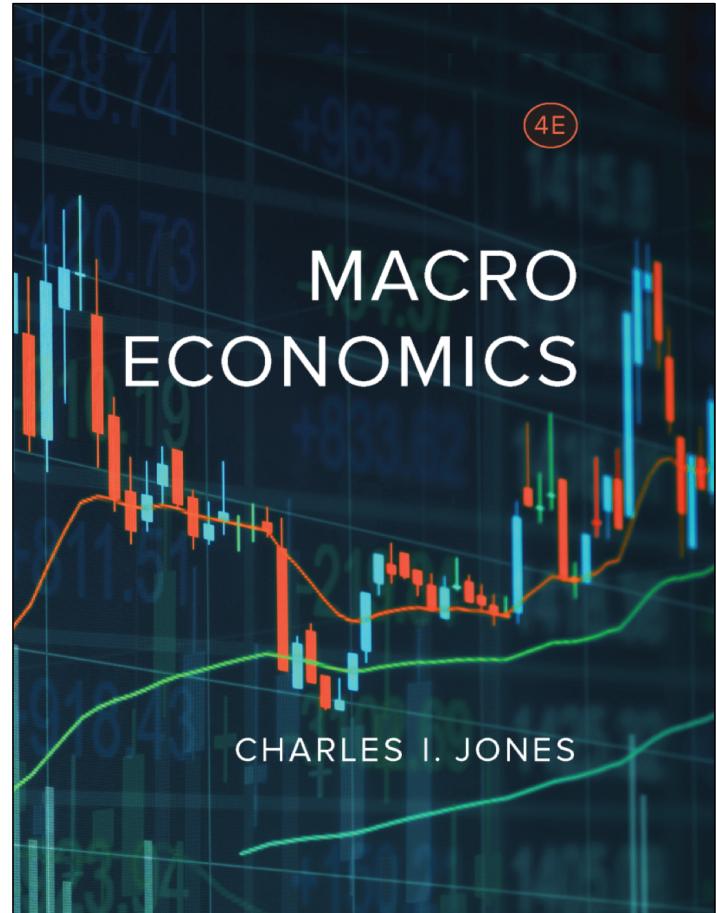


# Chapter 3

## An Overview of Long-Run Economic Growth

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Revised, Expanded, and Updated by Simeon Alder  
U of Wisconsin - Madison



# 3.1 Introduction

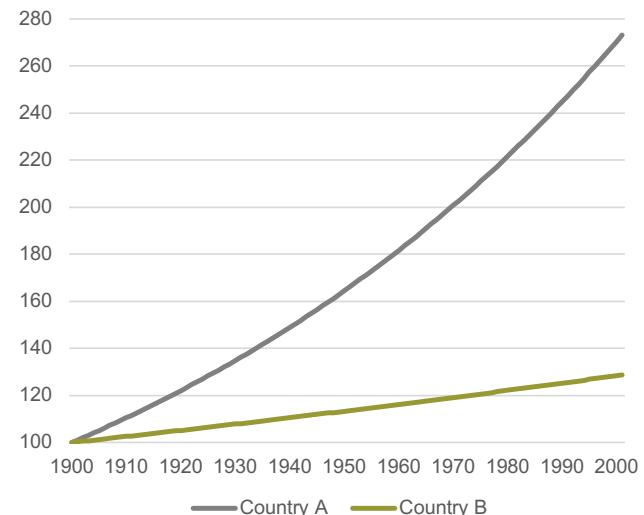
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- In this chapter, we learn:
  - some **facts** related to economic growth that later chapters will seek to explain
  - how economic growth has dramatically improved welfare around the world
  - growth is actually a relatively **recent** phenomenon.
  - some tools used to study economic growth, including how to calculate growth rates
  - why a “ratio scale” makes plots of per capita GDP easier to understand

# Motivation: The Big Effect of Small Growth Rates

- A 1 percent change in annual growth appears small
- But it may lead to large differences in level of output over time
- Consider a 100-year period:
  - Country A and B begin at the same level of GDP (\$100 billion)
  - Country A grows at 1 percent each year *28% growth*
  - Country B grows at 0.25 percent each year *25% growth*

Hypothetical GDP Values  
(in billions of dollars)



## 3.2 Growth Over the Very Long Run

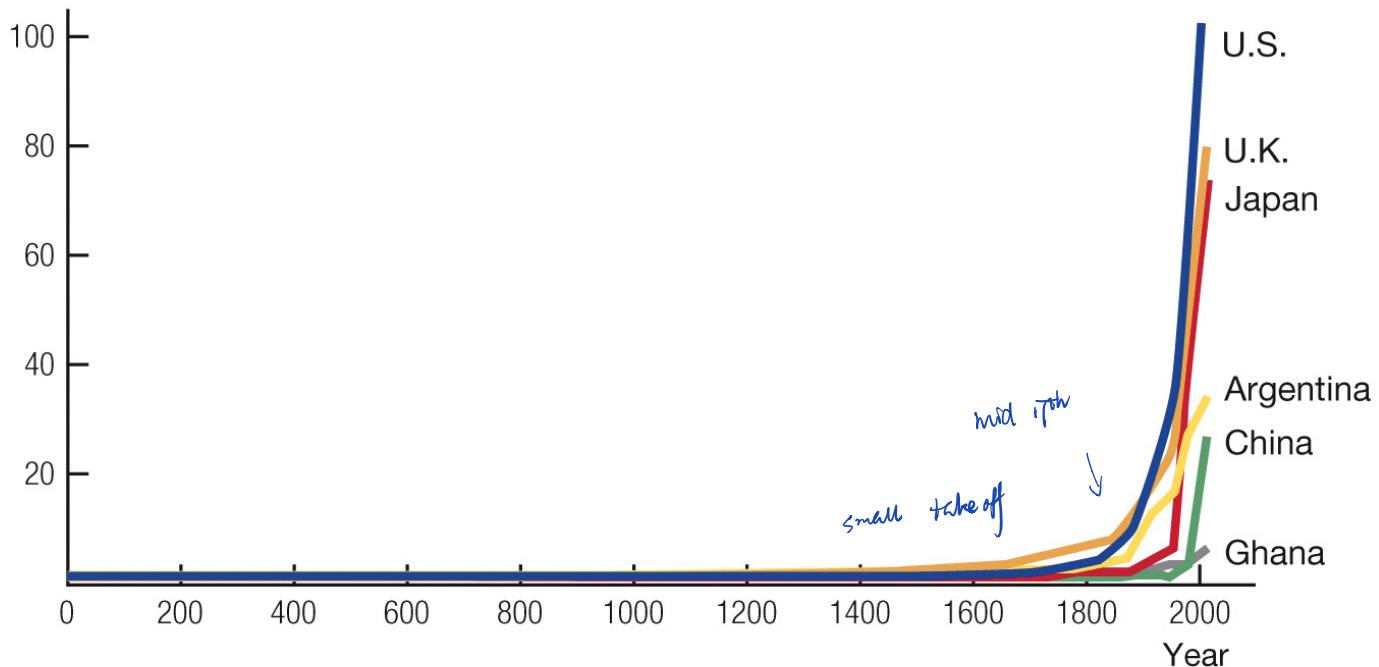
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- Sustained increases in standards of living are a recent phenomenon.
- Economic growth emerges in different places at different times.
- As a result, today's per capita GDPs differ remarkably around the world.

# Growth Over the Very Long Run

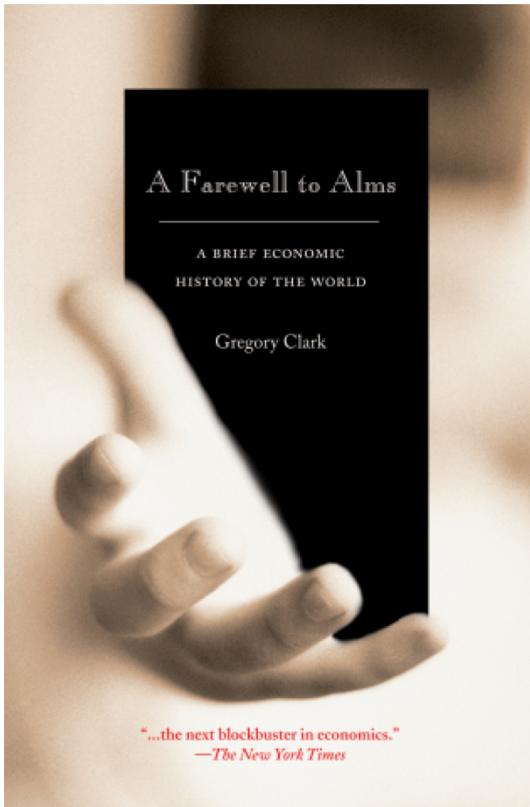
## Economic Growth over the Very Long Run in Six Countries

Per capita GDP  
(multiple of 300 dollars)



# Data on Growth and Incomes a Long Time Ago

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Angus Maddison (1926-2010)

## 3.3 Modern Economic Growth

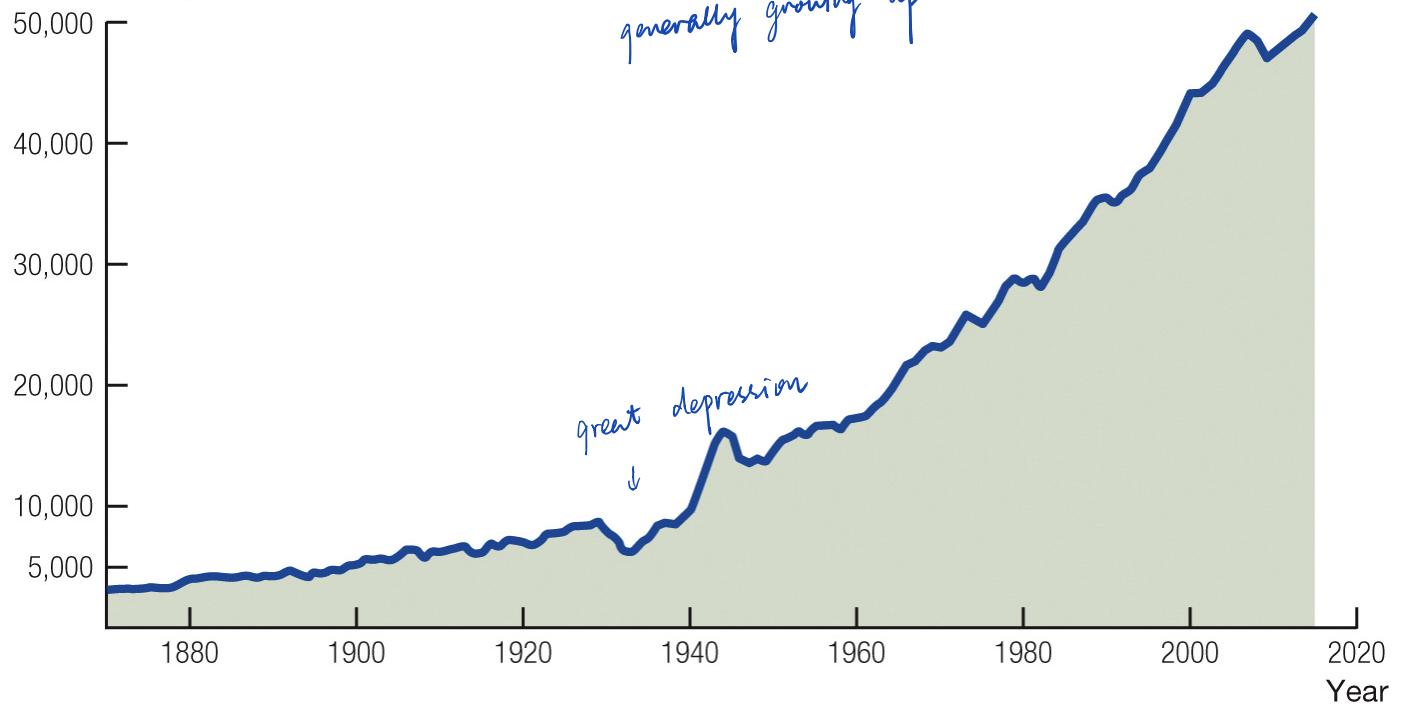
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- Timeline: from 1870 to 2000, United States real per capita GDP rose by a factor of 15.
  - A typical college student today will earn a lifetime income about twice that of his or her parents.
- one generation is  
twice better than  
last one*
- ↙

# Modern Economic Growth

Per Capita GDP in the United States

Per capita GDP  
(2009 dollars)



# Definition of Economic Growth

- Growth of per capita GDP
  - The exact rate of change of per capita GDP
- A percentage change
  - The change between two periods divided by the value of the variable in the initial period
- Percentage change in GDP between period  $t$  and  $t + 1$ :

$1+g$  growth factor

$$\bar{g} = \frac{y_{t+1} - y_t}{y_t}$$

$$y_{t+1} = y_t(1 + \bar{g})$$

# Population Growth—1

- Population growth evolves the same way:

$$(1) \quad L_{t+1} = L_t(1 + \bar{n})$$

↑ growth factor  
↑ growth rate

- Intuitively, tomorrow's population in time period  $t + 1$  depends on today's population in period  $t$ .
- If equation (1) is true in time  $t$ , it also applies to time  $t+1$ . It follows that:

$$(2) \quad L_{t+2} = L_{t+1}(1 + \bar{n})$$

# Population Growth—2

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- But, equation (1) gives us the value for  $L_{t+1}$ .  
So, we can plug (1) into (2)

$$(3) \quad L_{t+2} = L_t(1 + \bar{n})(1 + \bar{n})$$

$$(4) \quad L_{t+2} = L_t(1 + \bar{n})^2$$

- And we could continue this process for any number of time periods.
- Until we recognize the pattern:

$$(5) \quad L_t = L_0(1 + \bar{n})^t$$

# The Constant Growth Rate Rule

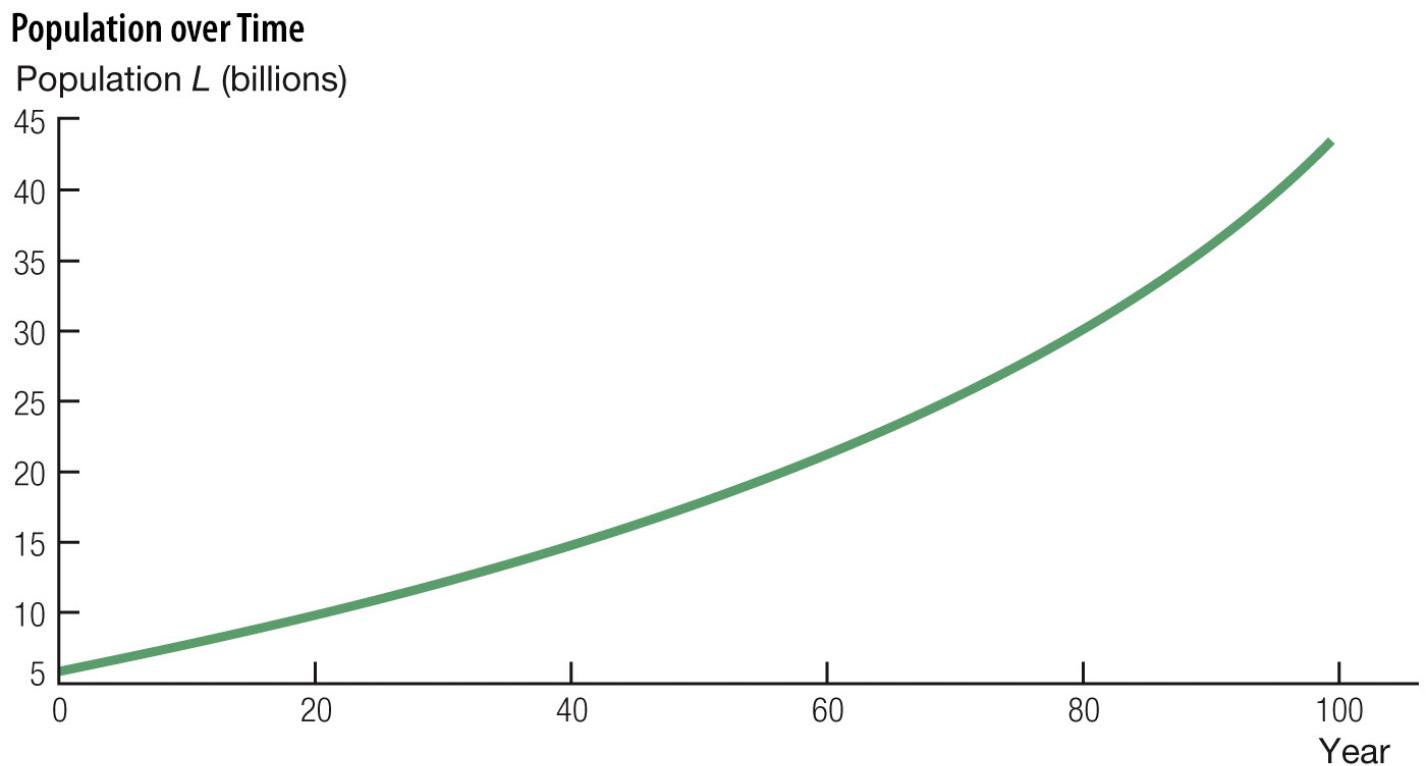
- The constant growth rate rule states that:

$$(6) \quad y_t = y_0(1 + \bar{g})^t$$

where

- $t$  is the time period
- $y_t$  is the value of variable  $y$  in time  $t$
- $y_0$  is the initial value of variable  $y$  in period 0
- $\bar{g}$  is the constant growth rate

# Population over Time



# The Rule of 70

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- The Rule of 70
  - If  $y$  grows at a rate of  $g$  percent per year, then the number of years it takes  $y$  to double is approximately equal to  $70/g$ .  
*↳ growth rate 2 →  $\frac{70}{2} = 35$  years  
↓  
a generation*
- Notes:
  - Small differences in growth rates result in large differences over time.
  - The time it takes to double only depends on the growth rate and not the initial value.

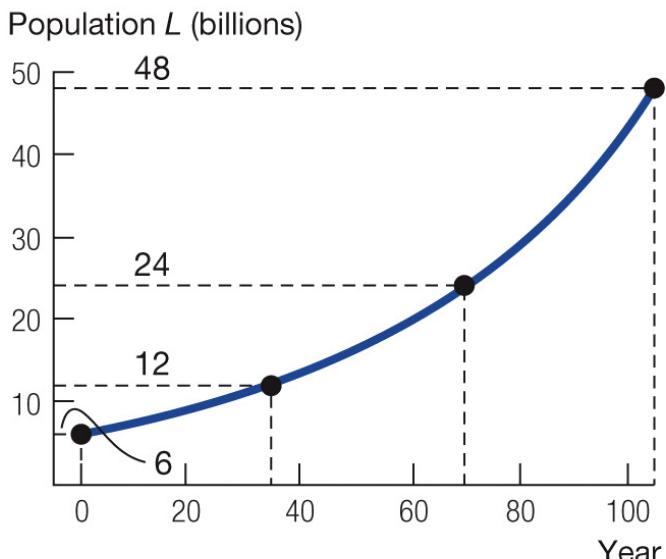
# The Ratio Scale

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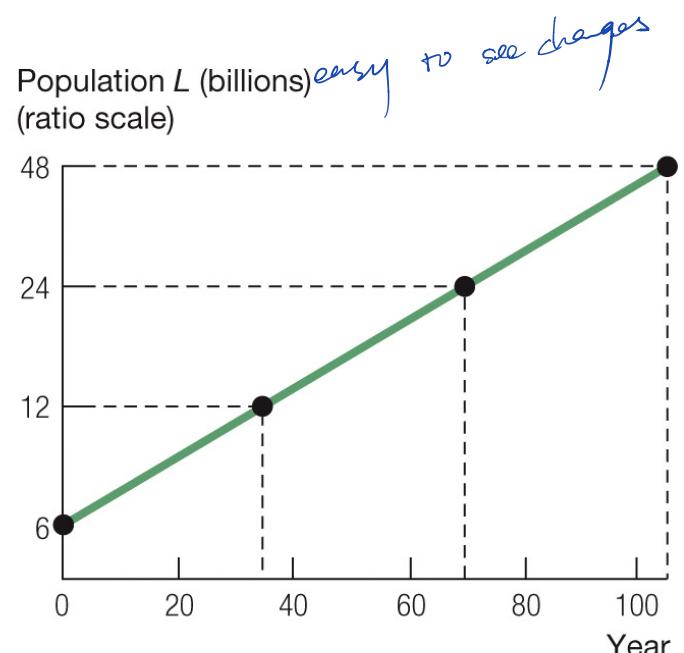
- Plot where equally spaced tick marks on the vertical axis are labeled consecutively with numbers that exhibit a **constant ratio**.
- When plotted on a ratio scale, a variable that grows at a **constant rate** will be a **straight line**.

# Population over Time, Revisited

## Population over Time, Revisited



(a) On a standard scale...



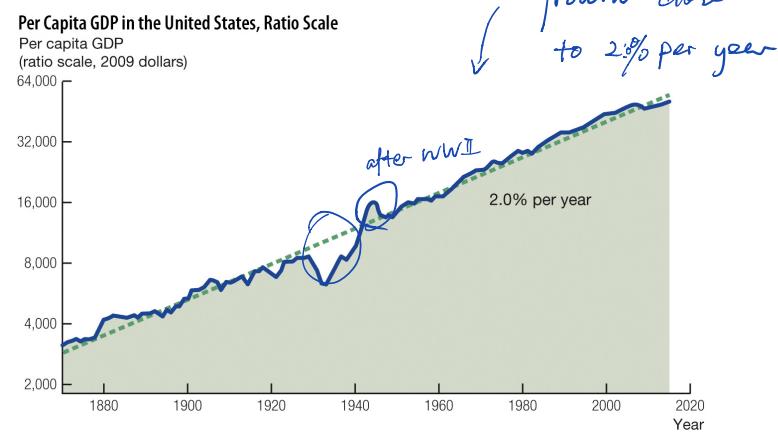
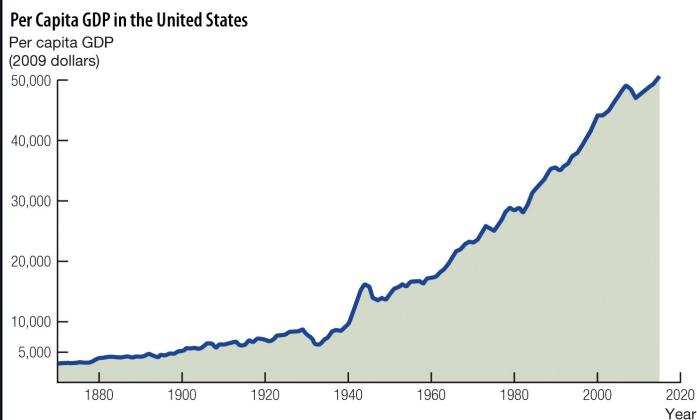
(b) ...and a ratio scale.

# U.S. GDP on a Ratio Scale

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- If growth rates are constant, the slope will be constant.
- If growth rates are rising, the slope will be increasing.
- Per capita GDP in the United States has grown at approximately 2 percent per year over the last 130 years.
  - Easy to see with a ratio scale
  - Approximately linear

# Per Capita GDP in the United States, Ratio Scale



# Calculating Growth Rates

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- Begin with the constant growth rate rule:

$$y_t = y_0(1 + \bar{g})^t$$

- Use data for  $y$  to solve for  $\bar{g}$
- Divide both sides by  $y_0$ :

$$\frac{y_t}{y_0} = (1 + \bar{g})^t$$

- Raise both sides to the  $1/t$  power:

$$\left(\frac{y_t}{y_0}\right)^{1/t} = 1 + \bar{g}$$

- Subtract 1 from both sides:

$$\bar{g} = \left(\frac{y_t}{y_0}\right)^{1/t} - 1$$

## 3.4 Modern Growth around the World

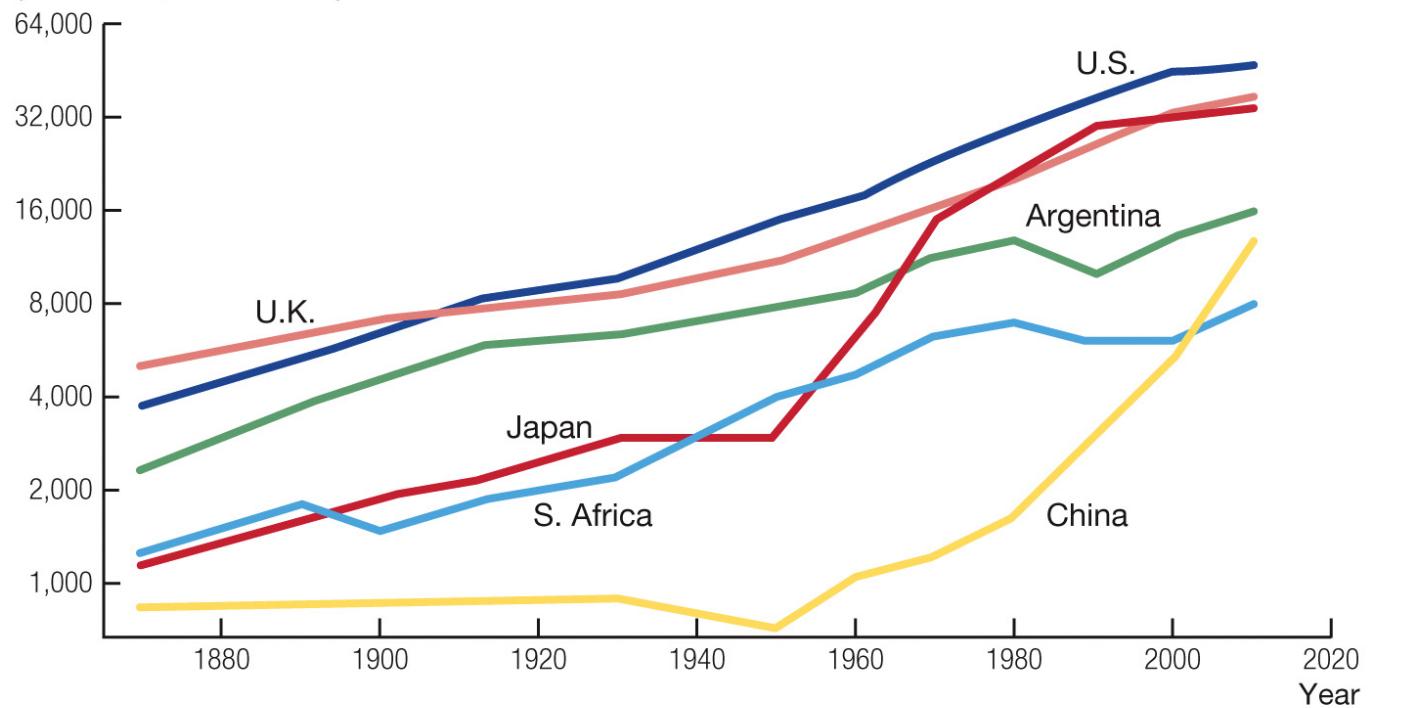
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- Convergence (definition):  
Occurs when relatively poor countries "catch up" to richer countries
- After World War II, growth in Germany and Japan accelerated.
- Other convergence / divergence patterns:
  - Latin America converged in the first half of the 19<sup>th</sup> century, followed by divergence
  - India / China first diverged, then caught up.

# Per Capita GDP since 1870

## Per Capita GDP since 1870

Per capita GDP  
(ratio scale, 2009 dollars)



# Broader Sample of Countries

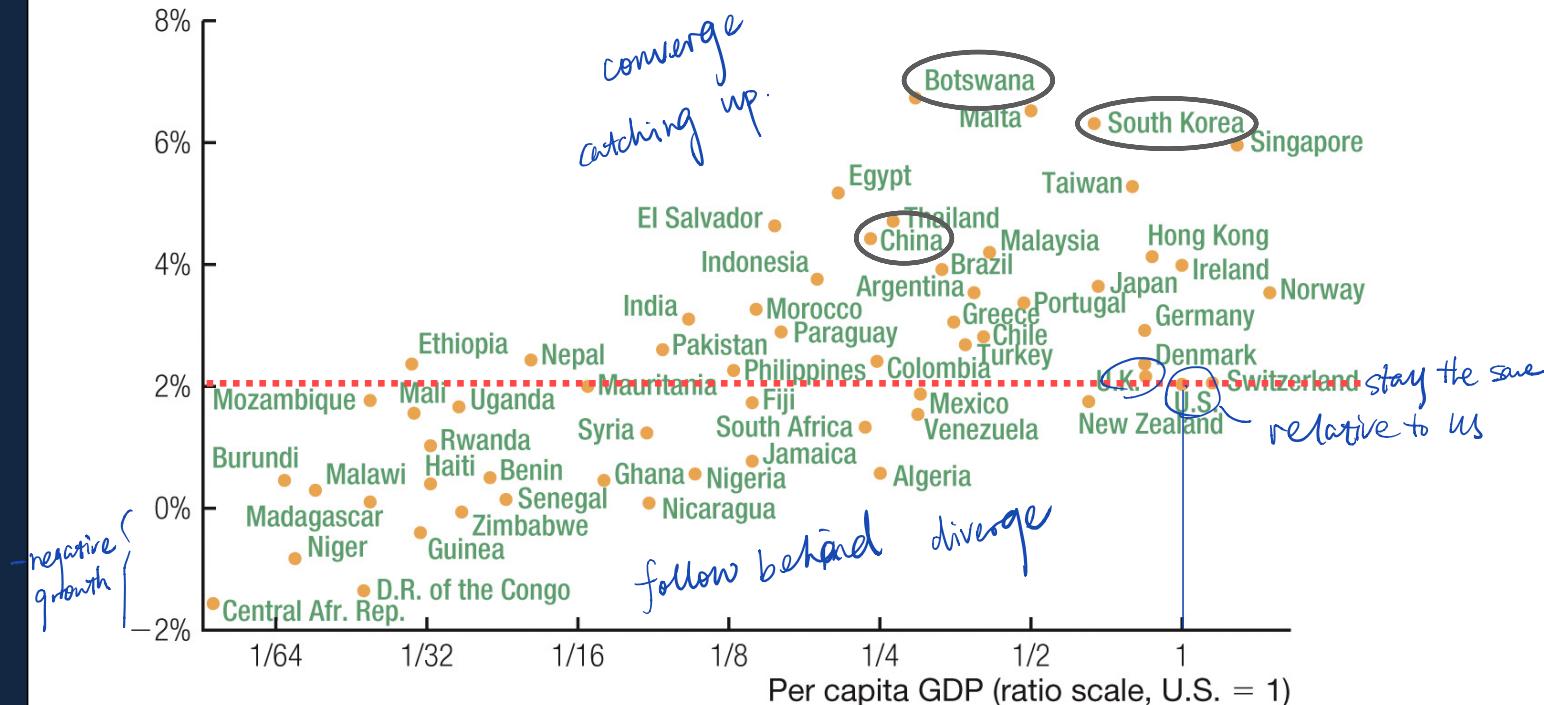
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- Over the period 1960–2007
  - Some countries have exhibited a negative growth rate.
  - Other countries have sustained nearly 7 percent growth. *a country in africa*
- Small differences in growth rates result in large differences in standards of living.

# Levels and Growth Rates of Per Capita GDP

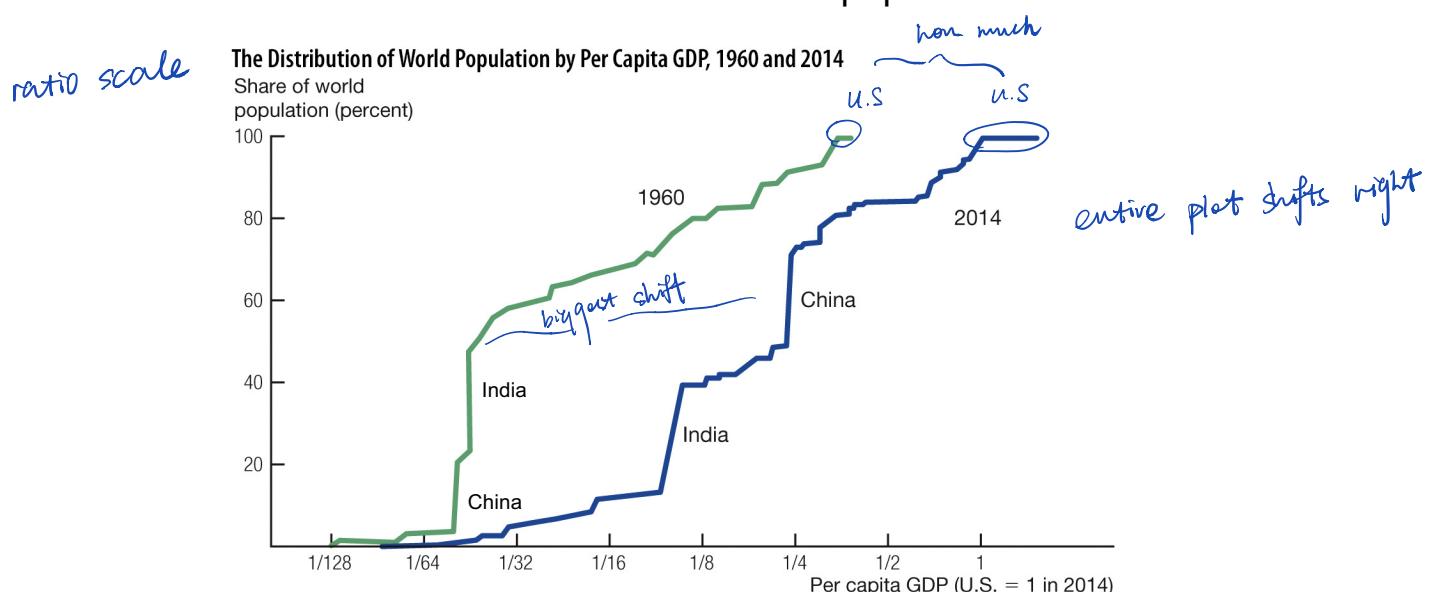
Levels and Growth Rates of Per Capita GDP

Per capita GDP growth  
(1960–2014)

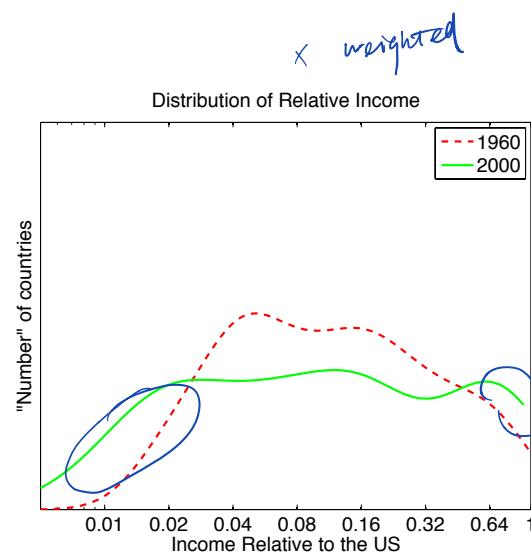


# Case Study: People versus Countries

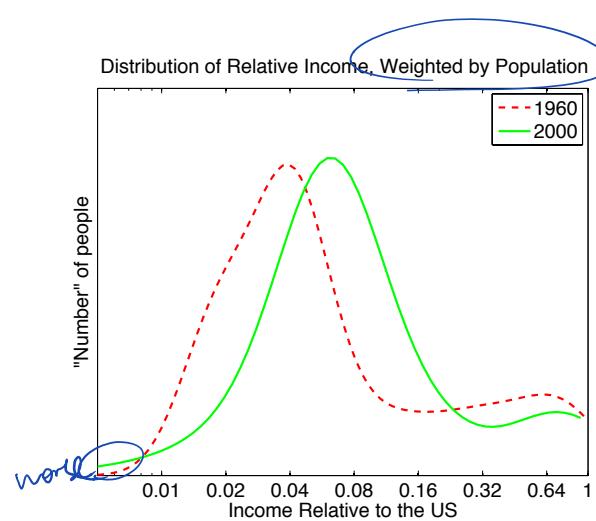
- Since 1960:
    - The bulk of the world's population is substantially richer.
    - The fraction of people living in poverty has fallen.
  - A major reason for changes:
    - Economic growth in China and India
    - These two countries account for 40 percent of the world population.



# Case Study: People versus Countries



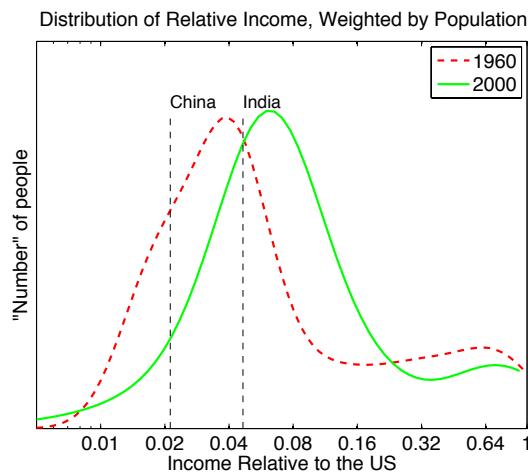
# Case Study: People versus Countries



China / India factor

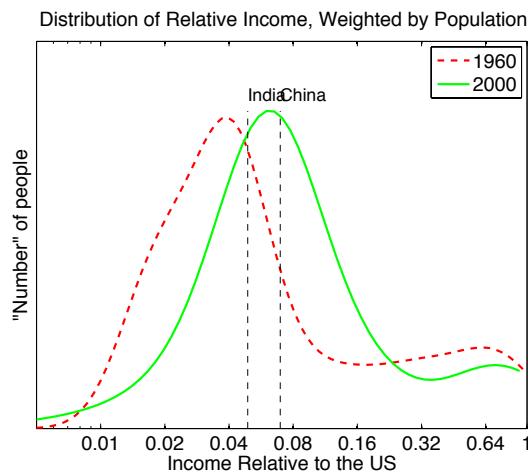
# Case Study: People versus Countries

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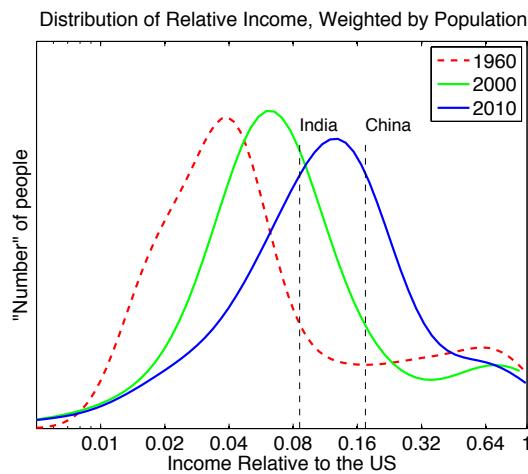
# Case Study: People versus Countries

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# Case Study: People versus Countries

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## 3.5 Some Useful Properties of Growth Rates

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- Growth rates of ratios, products, and powers follow several simple rules.
- Growth rates obey mathematical operations that are a level simpler than the operation on the original variable.
  - Variables divided → growth rates subtracted
  - Variables multiplied → growth rates added
  - Variable taken to a power number → growth rate multiplied by that number

# Some Useful Properties of Growth Rates

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- Suppose two variables  $x$  and  $y$  have average annual growth rates of  $g_x$  and  $g_y$ , respectively.
- Assume also that  $g_z$  is the average annual growth rate of  $z$ .
- Then the following rules apply:

1. If  $z = x/y$  then  $g_z = g_x - g_y$

2. If  $z = x \times y$  then  $g_z = g_x + g_y$

3. If  $z = x^a$  then  $g_z = a \times g_x$

# Examples of Growth Rate Calculations

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TABLE 3.1

## Examples of Growth Rate Calculations

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Suppose  $x$  grows at rate  $g_x = 0.10$  and  $y$  grows at rate  $g_y = 0.03$ .  
What is the growth rate of  $z$  in the following cases?

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$$z = x \times y \quad \Rightarrow \quad g_z = g_x + g_y = 0.13$$

$$z = x/y \quad \Rightarrow \quad g_z = g_x - g_y = 0.07$$

$$z = y/x \quad \Rightarrow \quad g_z = g_y - g_x = -0.07$$

$$z = x^2 \quad \Rightarrow \quad g_z = 2 \times g_x = 0.20$$

$$z = y^{1/2} \quad \Rightarrow \quad g_z = 0.5 \times g_y = 0.015$$

$$z = x^{1/2}y^{-1/4} \quad \Rightarrow \quad g_z = 0.5 \times g_x - 0.25 \times g_y = 0.1425$$

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# Growth Rules with Cobb-Douglas Production

- Applying rules of growth rates to a Cobb Douglas production function:
- Original output equation:

$$Y_t = A_t K_t^{1/3} L_t^{2/3}$$

- Use multiplication rule to get:

$$g(Y_t) = g(A_t) + g\left(K_t^{\frac{1}{3}}\right) + g\left(L_t^{\frac{2}{3}}\right)$$

Labor income share

- Use exponent rule to get:

$$g(Y_t) = g(A_t) + \frac{1}{3}g(K_t) + \frac{2}{3}g(L_t)$$

## 3.6 The Benefits and Costs of Economic Growth

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- The benefits of economic growth
  - Improvements in health
  - Higher incomes
  - Increase in the variety of goods and services
- Costs of economic growth include:
  - Environmental problems
  - Income inequality across and within countries (?)
  - Loss of certain types of jobs
- Economists generally have a consensus that the benefits of economic growth outweigh the costs.

## 3.7 A Long-Run Roadmap

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- The next few chapters examine the following:
  - Chapter 4: How can we measure differences in income levels across countries?
  - Chapter 5: Develops the Solow growth model
  - Chapter 6: Is human capital a driver of economic growth?
- Also part of the long-run discussion is:
  - Chapter 7: The labor market, wages, and unemployment in the long run
  - Chapter 8: Determinants of long-run inflation