Homework 2

Due in class on Thursday, Feb 6.

1. Design a rejection sampling algorithm for generating samples from a distribution with the following pdf:

$$\pi(x) = \frac{3}{2}x^3 + \frac{11}{8}x^2 + \frac{1}{6}x + \frac{1}{12}, \quad 0 \le x \le 1.$$

Describe your rejection sampling algorithm. Implement your algorithm to generate 1000 samples from this distribution. Based on the 1000 samples you generated, give an estimate of $E(X^2)$, where X follows distribution $\pi(x)$, and the standard error of your estimate. Attach your code and results.

2. Design a rejection sampling algorithm for generating samples from a distribution with the following pdf:

$$\pi(x) \propto \{[\sin(8x)]^2 + 2[\cos(3x)]^4 + 1\}e^{-x}, \quad 0 < x < \infty.$$

Describe your rejection sampling algorithm. Implement your algorithm to generate 1000 samples from this distribution. Based on the 1000 samples you generated, give an estimate of mean of $\pi(x)$ and the standard error of your estimate. Attach your code and results.

3. Let $X_1, \ldots, X_n$ be i.i.d. from $N(\theta, 1)$, where θ is the unknown parameter. In Bayesian inference, we may put a Cauchy prior distribution on θ, and the Bayes estimate of θ is the mean of the posterior distribution

$$\pi(\theta|x_1, \ldots, x_n) \propto \frac{1}{\pi(1+\theta^2)} \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} e^{-(x_i-\theta)^2/2}.$$

Suppose $n = 5$ and the observed values are $x_1 = 1.6, x_2 = 0.6, x_3 = -0.7, x_4 = 1.1, x_5 = 0.8$. Design a rejection sampling algorithm to generate samples from $\pi(\theta|x_1 = 1.6, x_2 = 0.6, x_3 = -0.7, x_4 = 1.1, x_5 = 0.8)$. Describe your algorithm. What is your instrumental distribution? Implement your algorithm to generate 1000 (accepted) samples from $\pi(\theta|x_1 = 1.6, x_2 = 0.6, x_3 = -0.7, x_4 = 1.1, x_5 = 0.8)$. How many samples (denote this number by N) do you need to generate from the instrumental distribution in order to have 1000 accepted samples? What is your estimated acceptance rate (i.e., $1000/N$)? Based on the 1000 samples you generated, give an estimate of the mean of $\pi(\theta|x_1 = 1.6, x_2 = 0.6, x_3 = -0.7, x_4 = 1.1, x_5 = 0.8)$ and the standard error of your estimate. Attach the code and results.