STAT525 HW7

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1

Consider the one-dimensional Ising model that we discussed in class. Let $x = (x_1, x_2, ...x_d)$, where x_i is either +1, or -1. The target distribution is

$$\pi(x) \propto exp\{\mu \sum_{i=1}^{d-1} x_i x_{i+1}\}$$

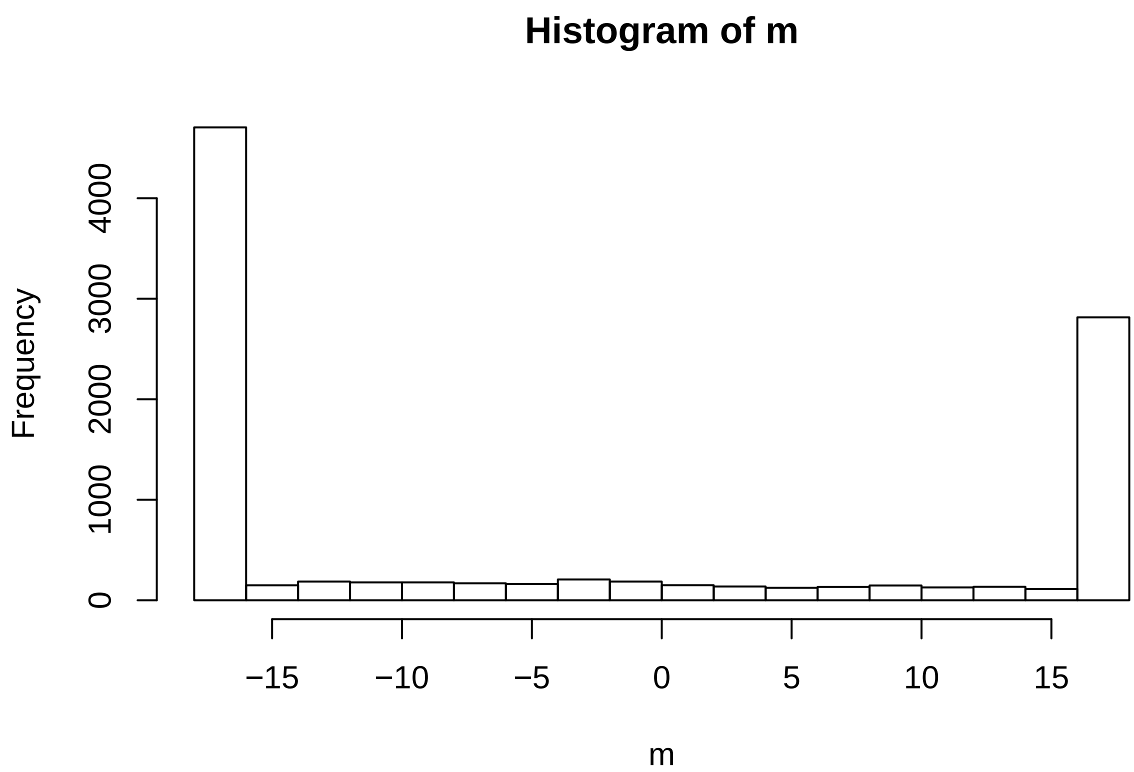
Let $\mu = 2, d = 18$

$$\pi(x_i | x_1^{(t+1)}, ..., x_{i-1}^{(t+1)}, x_{i+1}^{(t)}, ..., x_d^{(t)}) \propto exp\{\mu(x_{i-1}^{(t+1)} + x_{i+1}^{(t)})x_i\}$$

```
mu = 2
d = 18
n = 10000
x = matrix(0, nrow = n, ncol = d)
x1 = runif(d)
x[1, ] = ifelse(x1>0.5,1,-1)
for (i in 2:n) {
  for (j in 1:d) {
    if (j == 1) {
      x[i, ]= x[i-1,]
      p = exp(mu*1*x[i,2])/(exp(mu*1*x[i,2]) + exp(-mu*1*x[i,2]))
    } else if (j == d) {
      p = exp(mu*x[i,d-1])/(exp(mu*1*x[i,d-1]) + exp(-mu*1*x[i,d-1]))

    } else {
      p = exp(mu*x[i,j-1]+mu*x[i,j+1])/(exp(mu*x[i,j-1]+mu*x[i,j+1])+ exp(-mu*x[i,j-1]-mu*x[i,j+1]))
    }
    x[i,j] = ifelse(rbinom(1,size=1, p)==1, 1, -1)
  }

}
m = apply(x, 1, sum)
hist(m)
```



2

Design a Gibbs sampling algorithm to generate samples approximately from the target distribution $p(\mu, \tau|y_1, .., y6)$. Attach your code and results.

$$p(\mu|\tau, y_1, .., y6) \propto N(\frac{\tau \sum y_i}{n\tau + w}, \frac{1}{n\tau + w})$$

$$p(\tau|\mu, y_1, .., y6) \propto Gamma(n/2 + a, \frac{1}{\sum(y_i - \mu)^2/2 + 1/b})$$

```
y = c(1.8, 3.3, 0.4, 2.5, 2.6, 2.3)
w = 0.04
a = 2
b = 0.5
m = 10000
n = 6
para = matrix(0, nrow = m, ncol = 2)
para[1,1] = rnorm(1, mean = 0, sd = sqrt(1/w))
para[1,2] = rgamma(1, shape = a, scale = b)
# first column is mu, second column is tau
for (i in 2:m) {
  para[i, ] = para[i-1, ]
  for (j in 1:2) {
    if (j == 1) {
      tau = para[i,2]
      para[i,1] = rnorm(1,mean = tau*sum(y)/(w+tau*n),sd =sqrt(1/(w+tau*n)))
    } else {
      mu = para[i,1]
```

```r
      para[i,2] = rgamma(1, shape = n/2+a, scale = 1/(sum((y-mu)^2)/2+1/b))
    }
  }
}
# posterior mean os mu and tau
apply(para, 2, mean)
```

```
## [1] 2.139529 1.020202
```