STAT 525 Homework 3
Spring 2020 Feb 6

Homework 3

Due in class on Thursday, Feb 13.

1. Suppose X has a uniform distribution on $[0, 1]$ and we want to estimate $E\left[\sin(\sqrt{X})\right]$. Design an importance sampling algorithm which will give an estimate with smaller standard error than the naive Monte Carlo method based on the same number of samples. Implement the naive Monte Carlo method and your importance sampling algorithm based on 1000 samples. Give your estimate and the standard error your estimate for both algorithms. Attach your code and results.

2. In Problem 3 of Homework 2, how many samples (denote this number by N) do you need to generate from the instrumental distribution in order to have 1000 accepted samples from $\pi(\theta|x_1, \ldots, x_5)$? As we discussed in class, this instrumental distribution can also be used as the proposal distribution for an importance sampling algorithm to estimate the mean of $\pi(\theta|x_1, \ldots, x_5)$. Describe your importance sampling algorithm and implement the importance sampling algorithm based on N samples. Give your estimate of the mean of $\pi(\theta|x_1, \ldots, x_5)$ and the standard error of your estimate. Compare this standard error with the one you obtained in Problem 3 of Homework 2. Attach the code and results. (you may rerun your rejection sampling algorithm if you do not have the results from Homework 2)

3. Use importance sampling to estimate $\sigma^2 = E(X^2)$, where X has the density that is proportional to $e^{-x^8/2}$, $-\infty < x < \infty$. Implement your algorithm. Give your estimate and the standard error of your estimate based on 1000 samples.