STAT525 HW5

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###1

```
library('Rlab')
```

```
## Rlab 2.15.1 attached.
```

```
##
## Attaching package: 'Rlab'
```

```
## The following objects are masked from 'package:stats':
##
##     dexp, dgamma, dweibull, pexp, pgamma, pweibull, qexp, qgamma,
##     qweibull, rexp, rgamma, rweibull
```

```
## The following object is masked from 'package:datasets':
##
##     precip
```

```
y=c(-0.4, -1.4, 1.1, -0.3, 0.7, -0.1, 1.3, 0.3, 0.1, 1.2, 2.0, 0.9, -0.4, 0.9, 1.0)
w0 = 1
m = 10000
mu0 = 0
mu = matrix(0, nrow = m, ncol = 15)
weight = matrix(0, nrow = m, ncol = 15)
for (i in 1:m) {
  for (j in 1:15) {
    if (rbinom(1, 1, 0.9) == 1){
      if (j == 1) {
        mu[i,j] = 0
      } else {
        mu[i,j] = mu[i,j-1]
      }
    } else {
      mu[i,j] = rnorm(1, mean = 0, sd = 1)
    }

    if (j == 1){
      weight[i,j] = dnorm(y[j], mean = mu[i,j], sd=1)
    } else {
      weight[i,j] = weight[i,j-1]* dnorm(y[j], mean = mu[i,j], sd=1)
    }

  }
}
w_sum = apply(weight, 2, sum)
mean = apply(mu*weight,2, sum)/w_sum
mean
```

```
##  [1] -0.014930554 -0.125358793  0.045899355 -0.008107693  0.060059247
##  [6]  0.024317083  0.170818606  0.146169623  0.111131602  0.255542358
```

```
## [11]  0.616849497  0.645813034  0.358809108  0.460380067  0.547045229
```

###2 The variance of $X(b^*)$ is smaller than the variance I obtained earlier for estimating $E(\frac{1}{1+X^3})$ with Naive Monte Carlo method.

```
x = runif(1000)
b = cov(1/(1+x^3),(1+x^3))/var(1+x^3)
xb = 1/(1+x^3)-b*(1+x^3-5/4)
# MEAN OF THE OF Xb
mean(xb)
```

```
## [1] 0.8353137
```

```
# VARIANCE OF Xb
var(xb) / 1000
```

```
## [1] 6.981546e-07
```

```
# Variance OF ESTIMATE BY NAIVE MONTE CARLO
var(1/(1+x^3)) / 1000
```

```
## [1] 2.486547e-05
```

3

$$f(x) = 160x^{-6}, 2 \leq x \leq \infty$$

The inverse CDF is given by

$$g(u) = F^{-1}(u) = 2(1-u)^{-1/5}, 0 \leq x \leq 1$$

g is monotonic function, $X = g(U), X' = g(1-U)$ and $U1, U2$ are independent uniform random variables, we have

$$E(\{g(U1) - g(U2)\}\{g(1-U1) - g(1-U2)\}) = 2Cov(X, X') \leq 0$$

Thus, $Var((X + X')/2) \leq Var(X)/2$

```
u = runif(1000)
x = 2*(1-u)^(-1/5)
x2 = 2*(u)^(-1/5)
mean(x)
```

```
## [1] 2.536989
```

```
var(x) / 1000
```

```
## [1] 0.0005583147
```

```
mean((x+x2)/2)
```

```
## [1] 2.511285
```

```
var((x+x2)/2) / 1000
```

```
## [1] 0.0001890023
```