STAT 525 — Spring 2020 — Section A1

Homework 4 Solution

1. Using the following R code, the estimate of he mean squared extension of self-avoiding walks with length 11 is 30.6623. The standard error of the estimate is 0.5000.

   R code:

```
d = 11; n = 2000
directions = matrix(c(1,0,-1,0,0,1,0,-1),4,2)
sample_saw <- function(d){
        x = c(d+1,d+1)
        visited = matrix(0,2*d+1,2*d+1); visited[rbind(x)] = 1;
        w = 1
        for(t in 1:d){
                x_new = sweep(directions,MARGIN=2,STATS=-x,FUN='-')
                tmp = visited[x_new]
                n_valid = 4-sum(tmp)
                if(n_valid==0){
                        return(c(0,0)); break;
                }
                else{
                        x = x_new[which(tmp==0)[sample(1:n_valid)[1]],]
                        w = w*n_valid
                        visited[rbind(x)] = 1
                }
        }
        return(c(sum((x-d-1)^2),w))
}
result = replicate(n,sample_saw(d))
mu = sum(result[1,]*result[2,])/sum(result[2,])
se = sqrt((var(result[1,]*result[2,]) + mu^2*var(result[2,]) - 2*mu*cov(result
```

2. Using the following R code, we found that the number of self-avoiding walks with length 11 is 122681. The standard error of the estimate is 1476.

```
result = replicate(n,sample_saw(d))
sum((result[1,]>0)*result[2,])/n
sd((result[1,]>0)*result[2,])/sqrt(n)
```

3. Given row sums and column sums, the degree of freedom of a 3×3 contingency table is 4. Therefore, we just need to calculate the probabilities of observing the 4 cells on the topleft. The Fréchet bounds for 2-way contingency tables are given by

$$
\begin{aligned}
\max(0, 13-7-4) \leq & t_{11} \leq \min(9, 13), \\
\max(0, 13-6-4) \leq & t_{21} \leq \min(7, 13), \\
\max(0, 1) \leq & t_{12} \leq \min(3, 5), \\
\max(0, 1) \leq & t_{22} \leq \min(2, 3).
\end{aligned}
$$

The probability of generating the given table is $\frac{1}{8} \times \frac{1}{5} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{240}$.