STAT 525 Homework 7
Spring 2020 Mar 26

Homework 7

Due on Thursday, Apr 2.

1. Consider the one-dimensional Ising model that we discussed in class. Let $\boldsymbol{x} = (x_1, \ldots, x_d)$, where x_i is either $+1$ or -1. The target distribution is

$$\pi(\boldsymbol{x}) \propto \exp\left\{\mu \sum_{i=1}^{d-1} x_i x_{i+1}\right\}.$$

Let $\mu = 2$, $d = 18$.

Design a Gibbs sampling algorithm to generate samples approximately from the target distribution $\pi(\boldsymbol{x})$, and implement your algorithm. Suppose the output of your Gibbs sampling algorithm at step t is $\boldsymbol{x}^{(t)} = (x_1^{(t)}, \ldots, x_d^{(t)})$. Define the total magnetization $M^{(t)} = \sum_{i=1}^{d} x_i^{(t)}$. Plot the histogram of $M^{(t)}$. Attach the code and the figure.

2. Consider a random sample $y_1, \ldots, y_n$ from a normal density with mean μ and variance τ^{-1}. In Bayesian inference, we may assume that μ is subject to a normal prior with mean 0 and variance ω^{-1}, and τ is subject to a gamma prior with shape parameter α and scale parameter β. Given that the two priors are independent, the posterior distribution

$$p(\mu, \tau | y_1, \ldots, y_n) \propto p(y_1, \ldots, y_n | \mu, \tau) p(\mu, \tau) = (2\pi)^{-\frac{n+1}{2}} \tau^{\frac{n}{2}} e^{-\frac{\tau}{2}\sum_{i=1}^{n}(y_i - \mu)^2} \omega^{\frac{1}{2}} e^{-\frac{\omega}{2}\mu^2} \frac{\tau^{\alpha-1}}{\Gamma(\alpha)\beta^{\alpha}} e^{-\frac{\tau}{\beta}}.$$

Let $n = 6$ and $(y_1, \ldots, y_6) = (1.8, 3.3, 0.4, 2.5, 2.6, 2.3)$. Let $\omega = 0.04$, $\alpha = 2$, and $\beta = 0.5$. Design a Gibbs sampling algorithm to generate samples approximately from the target distribution $p(\mu, \tau | y_1, \ldots, y_6)$. Implement your algorithm and estimate the posterior mean of μ and τ. Attach your code and results.