

$$2.2.2 \quad P(X=k) = \frac{k}{15}, \quad k=1, 2, 3, 4, 5$$

$$P\left(\frac{1}{5} < X < \frac{5}{2}\right) = P(X=1) + P(X=2) = \frac{1}{5}$$

$$P(1 \leq X \leq 3) = P(X=1) + P(X=2) + P(X=3) = \frac{2}{5}$$

$$P(X > 3) = P(X=4) + P(X=5) = \frac{2}{5}$$

2.2.5 记 X 为汽车首次停下已通过的信号数。

$$P(X=0) = \frac{1}{5}, \quad P(X=1) = \frac{1}{4}, \quad P(X=2) = \frac{1}{8}, \quad P(X=3) = \frac{1}{8}$$

$$2.4.3 \quad F(x) = \begin{cases} 0, & x \leq 0 \\ A + Be^{-2x}, & x > 0 \end{cases} \quad \begin{matrix} F(+\infty) = \lim_{x \rightarrow +\infty} (A + Be^{-2x}) = 1 \\ \text{得 } A=1 \end{matrix}$$

$$\text{由 } F(x) \text{ 连续得 } F(0) = \lim_{x \rightarrow 0^+} F(x), \quad 1+B=0, \quad B=-1$$

$$\text{则 } A=1, \quad B=-1, \quad F(x) = -e^{-2x} + 1, \quad x > 0$$

$$P(-1 < X < 1) = F(1) - F(-1) = 1 - e^{-2}$$

$$f(x) = F'(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

$$2.4.5 \quad P(X \geq 180) = \int_{180}^{+\infty} f(x) dx = \frac{2}{3}, \quad P(\text{3个电子管180h不更换}) \\ = P^3(X \geq 180) = \frac{8}{27}$$

$$2.4.15 \quad P(X \geq 200) = \int_{200}^{+\infty} f(x) dx = e^{-\frac{1}{5}} \\ P(\text{至少一件损坏}) = 1 - P^3(X < 200) = 1 - e^{-1}$$

$$2.5.2 \quad Y = \sin \frac{\pi}{2} X, \quad P(X=k) = \frac{1}{2^k}, \quad k=1, 2, \dots \text{ 则 } Y = -1, 0, 1$$

$$\cancel{P(Y=-1) = P(X=4k-1) = \frac{2}{15}}, \quad \cancel{P(Y=0) = \frac{2}{15}}$$

$$P(Y=\frac{1}{2}) = \sum_{n=0}^{+\infty} P(X=4n+1) = \frac{8}{15} \quad P(Y=-1) = \sum_{n=1}^{+\infty} P(X=4n-1) = \frac{2}{15}$$

$$P(Y=0) = \sum_{n=0}^{+\infty} P(X=2n) = \frac{1}{3}$$

$$2.5.6 \quad Y = \frac{1}{X}, \quad y > 0 \text{ 时}, \quad F_Y(y) = P(\frac{1}{X} < y) + P(0 < \frac{1}{X} \leq y) = P(X < 0) + P(X \geq \frac{1}{y})$$

$$= F(0) + 1 - F(\frac{1}{y}), \quad f_Y(y) = F'_Y(y) = \frac{1}{y^2} f(\frac{1}{y}), \quad y < 0 \text{ 时},$$

$$F_Y(y) = P(\frac{1}{X} \leq y) = P(\frac{1}{y} \leq X < 0) = F(0) - F(\frac{1}{y}),$$

$$f_Y = F'_Y(y) = \frac{1}{y^2} f(\frac{1}{y}), \quad \text{则 } f_Y(y) = \begin{cases} \frac{1}{y^2} f(\frac{1}{y}), & y \neq 0 \\ 0, & y = 0 \end{cases}$$

$$Y = |X|, \quad y \geq 0 \text{ 时}, \quad F_Y(y) = \cancel{P(-y \leq X \leq y)} P(-y \leq X \leq y) = F(y) - F(-y)$$

$$\cancel{f_Y(y)} f_Y(y) = F'_Y(y) = f(y) + \cancel{f(-y)} f(-y), \quad y < 0 \text{ 时},$$

$$F_Y(y) = 0, \quad \text{则 } f_Y(y) = \begin{cases} f(y) + f(-y), & y \geq 0 \\ 0, & y < 0 \end{cases}$$