1 Point in AA-Rectangle

Checking if our point P is in rectangle with origin at its top left $(x_0; y_0)$ (assuming everything is axis-aligned) is trivially simple, we just perform simple boundary checks:

$$y_0 \leq P_y \leq y_0 + height \tag{1}$$
$$x_0 \leq P_x \leq x_0 + width \tag{2}$$

If those two conditions are met we can be certain that our point is in a rectangle. Keep in mind the fact that our Y axis grows downwards when working with computer graphics.

2 Point in Any Rectangle

When we want to check if our point is in any kind of rectangle, no matter how oriented, we need to perform a little bit of math and understand something about lines, or more approriately, vectors that make up that rectangle.

For example, let's check if a certain point P is in rectangle. For that we can perform a couple of scalar and vector projections. This little visual aid might help in understanding some of it.

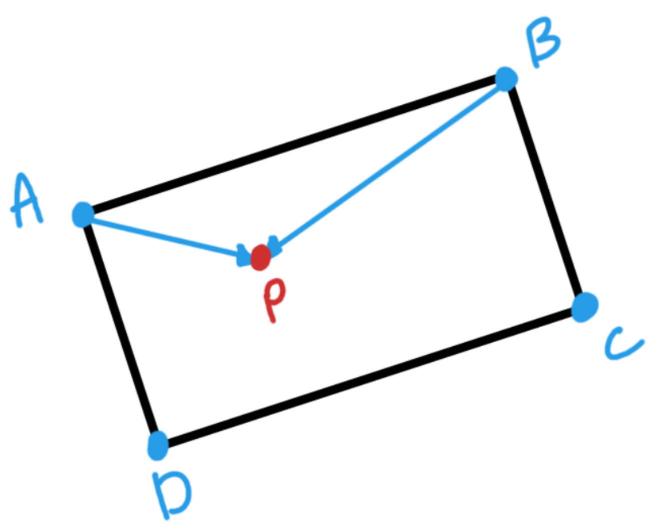

Figure 1: Rectangle with marked vectors

We begin defining a simple scalar projection of $\vec{AP}$ on $\vec{AB}$:

$$S = \frac{\vec{AB} \circ \vec{AP}}{||\vec{AB}||} \tag{3}$$

Now for the things we know: We know that our scalar S cannot be negative, something like that would mean 'stretching' our vector $\vec{AB}$ to the other side (effectively changing its direction). Vector $\vec{AB}$ also cannot stretch more than its actual length $||\vec{AB}||$.

$$0 \leq \frac{\vec{AB} \circ \vec{AP}}{||\vec{AB}||} \leq ||\vec{AB}|| \tag{4}$$

$$0 \leq \vec{AB} \circ \vec{AP} \leq ||\vec{AB}||^2 \tag{5}$$

$$0 \leq \vec{AB} \circ \vec{AP} \leq \vec{AB} \circ \vec{AB} \tag{6}$$

This is only half of the equations we need, these define our border for rectangle's top. If we were to work within the above inequalities, we would be selecting every point above and below the $\vec{AB}$ vector. That's why we need to introduce another one, say $\vec{BC}$ and perform the same operation:

$$0 \leq \frac{\vec{BC} \circ \vec{AP}}{||\vec{BC}||} \leq ||\vec{BC}|| \tag{7}$$

$$0 \leq \vec{BC} \circ \vec{AP} \leq ||\vec{BC}||^2 \tag{8}$$

$$0 \leq \vec{BC} \circ \vec{AP} \leq \vec{BC} \circ \vec{BC} \tag{9}$$

It is <u>important</u> that we select a vector like $\vec{BC}$ or $\vec{AD}$, because it's those two that define our rectangle's border on the sides. Vector $\vec{DC}$ wouldn't work, with it we would still have one degree of freedom available (our point could move freely up and down without any limitation).

When working with more complex shapes it's probably better to perform rasterisation or triangulation. Finding equations for different kinds of shapes would be tiresome and not scalable.