Chair for Algorithms
and Data Structures
Prof. Dr. Hannah Bast
Claudius Korzen

Information Retrieval
WS 2017/2018

http://ad-wiki.informatik.uni-freiburg.de/teaching

Exercise Sheet 10

Submit until Tuesday, January 16 at **12:00pm (noon)**

Exercise 1 (5 points)

Let A be an $m \times n$ matrix with rank r, and let $A = U \cdot S \cdot V$ be the singular value decomposition, where U is an $m \times r$ column-orthonormal matrix, S is a diagonal $r \times r$ matrix, and V is an $r \times n$ row-orthonormal matrix.

Prove that $V = S^{-1} \cdot U^T \cdot A$, that is, V can be easily computed from A, S, and U.

Exercise 2 (10 points)

Let A be the following 3×5 matrix (you can think of it as a term-document matrix, but that is not important for this exercise):

$$A = \begin{pmatrix} 13 & 5 & 5 & 0 & 13 \\ 9 & 15 & 15 & 0 & 9 \\ 0 & 0 & 0 & 20 & 0 \end{pmatrix}$$

Compute the singular value decomposition (SVD) $A = U \cdot S \cdot V$ via the following steps. Like for ES9, you should carry out this computation using pencil and paper, without using a program (but it is ok to use a simple calculator, e.g., to compute $388 \cdot 612 - 384^2 = 90.000$). Like for ES9, pay attention that you don't make yourself more work than necessary. Endless calculations are a clear sign that you are doing something wrong or too complicated.

1. Compute the symmetric 3×3 matrix $A \cdot A^T$. (1 point)

2. Compute the Eigenvector decomposition (EVD) of $A \cdot A^T$. It is ok to *guess* the Eigenvectors from looking at $A \cdot A^T$ (it is simple enough for this example matrix), but then of course you have to verify that they are indeed Eigenvectors. (4 points)

3. From this EVD, determine the U and the S of the SVD of A. Verify that U is indeed a 3×3 column-orthonormal matrix. (2 points)

4. From A, U, and S compute the V of the SVD of A. Verify that V is indeed a 3×5 row-orthonormal matrix. (2 points)

5. Verify that the product $U \cdot S \cdot V$ is indeed A (1 point)

[turn over with a suitable eigenvector]

Exercise 3 (5 points)

Compute the optimal rank-2 approximation A' of A using your SVD from Exercise 2. (1 point)

Let $q = (4, 1, 2)$ be a query vector. Compute the ranking of the 5 documents (columns) of A with respect to q in the original term-space and in the reduced 2-dimensional space. Compute the ranking in the reduced space using all three variants discussed in the lecture. (4 points)

Add your submission *as a single PDF* to a new sub-directory *sheet-10* of your folder in the course SVN, and commit it. Like for ES9, you may upload a handwritten solution, but only if it is neatly written and properly scanned (sorry CS students). In all other cases, you should use $\LaTeX$ (other typesetting programs are neither encouraged nor allowed). Also commit the usual *experiences.txt* with your brief and concise feedback. Confirm that linear algebra is one of the most beautiful things in the universe and imagine how your life would look like without Eigenvectors.