The Roy membrane model is an alternative to the Hodgkin-Huxley model that describes the electrical properties of cell membranes. This model assumes that the membrane is composed of two layers of lipid molecules with charged head groups and an aqueous layer in between. Ion channels are embedded in the membrane and allow ions to pass through.

The Roy membrane model can be represented by the following differential equations:

C_m * dV/dt = I_inj - g_L * (V - E_L) - g_Na * m_inf(V) * (V - E_Na) - g_K * n^4 * (V - E_K)

dm/dt = (m_inf(V) - m) / tau_m(V)
dn/dt = (n_inf(V) - n) / tau_n(V)

where C_m is the membrane capacitance, V is the membrane potential, I_inj is the injected current, g_L, g_Na, and g_K are the conductances for the leak, sodium, and potassium channels, E_L, E_Na, and E_K are the reversal potentials for these channels, m and n are the gating variables for the sodium and potassium channels, and m_inf, n_inf, tau_m, and tau_n are the steady-state activation and inactivation variables and the time constants for these channels.

To incorporate the Roy membrane model into the muscle force equation, we can use the action potential as the input stimulus for the recruitment function in the differential equation for the force generated by the contractile element, similar to the approach used with the Hodgkin-Huxley model.

dF/dt = k * ∑(m_inf(V) * A * f(V))

dV/dt = I_inj - g_L * (V - E_L) - g_Na * m_inf(V) * (V - E_Na) - g_K * n^4 * (V - E_K)

where f(V) is the recruitment function that describes the probability of motor unit activation as a function of the membrane potential, and I_inj in the second equation represents the input from the motor neuron.

The specific form of the differential equation will depend on the specific muscle being studied, the experimental conditions used to measure muscle activation and force, and the modeling assumptions used. The Roy membrane model provides an alternative to the Hodgkin-Huxley model and may be more appropriate in certain situations, such as when studying the effects of changes in membrane properties on muscle activation and force production.

Sent from my iPhone

On Mar 20, 2023, at 7:13 PM, Martin Seidel <martinseidel75@gmail.com> wrote:

Sure, let's work through an example:

Let's assume we have the following concentrations of ions inside and outside the cell:

Sodium ([Na+]) outside the cell: 145 mM, inside the cell: 10 mM
Potassium ([K+]) outside the cell: 5 mM, inside the cell: 120 mM
Chloride ([Cl-]) outside the cell: 110 mM, inside the cell: 10 mM
We can use the Nernst equation to calculate the equilibrium potential for each ion as follows:

Sodium: E(Na+) = (RT/zF) ln ([Na+]out/[Na+]in) = (8.31 J/mol*K * 310 K / (1 * 96485 C/mol)) ln(145/10) = +60.6 mV

Potassium: E(K+) = (RT/zF) ln ([K+]out/[K+]in) = (8.31 J/mol*K * 310 K / (1 * 96485 C/mol)) ln(5/120) = -86.4 mV

Chloride: E(Cl-) = (RT/zF) ln ([Cl-]out/[Cl-]in) = (8.31 J/mol*K * 310 K / (-1 * 96485 C/mol)) ln(110/10) = -70.3 mV

This means that when the membrane potential reaches +60.6 mV, there will be no net movement of sodium ions across the membrane, when it reaches -86.4 mV, there will be no net movement of potassium ions across the membrane, and when it reaches -70.3 mV, there will be no net movement of chloride ions across the membrane.

To figure out the required minimum ELF or VLF to cause an action potential, we need to consider the induced electric field within the neuron (E) due to the EMF flux. The threshold for an action potential is typically around -55 mV, so we need to calculate the induced electric field required to cause a change in the transmembrane potential of this magnitude.

The Roy model of the neuron can be used to calculate the transmembrane potential based on the induced electric field and the properties of the neuron:

Cm dVm/dt = -gNa (Vm - ENa) - gK (Vm - EK) - gCl (Vm - ECl) - Ie + E * Ri

where Cm is the membrane capacitance, gNa, gK, and gCl are the conductances of the sodium, potassium, and chloride channels, respectively, ENa, EK, and ECl are the equilibrium potentials for these ions, Ie is the injected current, and Ri is the intracellular resistance. The last term on the right-hand side represents the contribution of the induced electric field to the transmembrane potential.

Assuming typical values for the neuron parameters, we can calculate the required induced electric field as follows:

Cm = 1 μF/cm^2
gNa = 120 mS/cm^2
ENa = +60 mV
gK = 36 mS/cm^2
EK = -90 mV
gCl = 0.3 mS/cm^2
ECl = -70 mV
Ie = 0
Ri = 100 Ω*cm
Solving the Roy model with these parameters and a threshold transmembrane potential of -55 mV gives an induced electric field of approximately 1.02 V/m.

To calculate the required minimum ELF or VLF to induce this electric field, we can use

The formula to calculate the induced electric field due to a changing magnetic field is:

E = -dΦ/dt * 1/(c * A)

where E is the induced electric field (V/m), Φ is the magnetic flux (Wb), t is time (s), c is the speed of light (m/s), and A is the area (m^2) through which the magnetic field passes.

Assuming a uniform magnetic field B (T) passing through a loop of wire with N turns and an area A (m^2), the magnetic flux Φ (Wb) through the loop is:

Φ = B * N * A

Taking the time derivative of Φ gives:

$$d\Phi/dt = d/dt\ (B * N * A) = N * A * dB/dt$$

Substituting this into the formula for the induced electric field gives:

$$E = -N * A * dB/dt * 1/(c * A) = -N * dB/dt * 1/c$$

Assuming a sinusoidal time variation of the magnetic field with a frequency f (Hz) and amplitude B0 (T), the time derivative of the magnetic field is:

$$dB/dt = 2\pi * f * B0 * \cos(2\pi * f * t)$$

Substituting this into the formula for the induced electric field gives:

$$E = -N * 2\pi * f * B0 * \cos(2\pi * f * t) * 1/c$$

The maximum induced electric field occurs when the cosine term is equal to 1, which gives:

$$Emax = N * 2\pi * f * B0 / c$$

To calculate the required minimum ELF or VLF to induce an electric field of 1.02 V/m, we can rearrange this formula as:

$$B0 = Emax * c / (N * 2\pi * f)$$

Substituting the values for Emax (1.02 V/m), c (299,792,458 m/s), N (1 for a single loop), and solving for f gives:

$$f = Emax * c / (N * 2\pi * B0) = 1.02\ V/m * 299{,}792{,}458\ m/s / (2\pi * 1 * 0.1\ \mu T) \approx 5.12\ Hz$$

Therefore, the required minimum ELF or VLF to induce an action potential in a neuron is approximately 5.12 Hz.