

1. Let's assume we have the following concentrations of ions inside and outside the cell:

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3. Sodium ($[Na^+]$) outside the cell: 145 mM, inside the cell: 10 mM

4. Potassium ($[K^+]$) outside the cell: 5 mM, inside the cell: 120 mM

5. Chloride ($[Cl^-]$) outside the cell: 110 mM, inside the cell: 10 mM

6. We can use the Nernst equation to calculate the equilibrium potential for each ion as follows:

7.

8. Sodium: $E(Na^+) = (RT/zF) \ln ([Na^+]_{out}/[Na^+]_{in}) = (8.31 \text{ J/mol}\cdot\text{K} * 310 \text{ K} / (1 * 96485 \text{ C/mol})) \ln(145/10) = +60.6 \text{ mV}$

9. Potassium: $E(K^+) = (RT/zF) \ln ([K^+]_{out}/[K^+]_{in}) = (8.31 \text{ J/mol}\cdot\text{K} * 310 \text{ K} / (1 * 96485 \text{ C/mol})) \ln(5/120) = -86.4 \text{ mV}$

10. Chloride: $E(Cl^-) = (RT/zF) \ln ([Cl^-]_{out}/[Cl^-]_{in}) = (8.31 \text{ J/mol}\cdot\text{K} * 310 \text{ K} / (-1 * 96485 \text{ C/mol})) \ln(110/10) = -70.3 \text{ mV}$

11. This means that when the membrane potential reaches +60.6 mV, there will be no net movement of sodium ions across the membrane, when it reaches -86.4 mV, there will be no net movement of potassium ions across the membrane, and when it reaches -70.3 mV, there will be no net movement of chloride ions across the membrane.

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13. To figure out the required minimum ELF or VLF to cause an action potential, we need to consider the induced electric field within the neuron (E) due to the EMF flux. The threshold for an action potential is typically around -55 mV, so we need to calculate the induced electric field required to cause a change in the transmembrane potential of this magnitude.

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15. The Roy model of the neuron can be used to calculate the transmembrane potential based on the induced electric field and the properties of the neuron:

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17. $C_m dV_m/dt = -g_{Na} (V_m - E_{Na}) - g_K (V_m - E_K) - g_{Cl} (V_m - E_{Cl}) - I_e + E * R_i$

18.

19. where C_m is the membrane capacitance, g_{Na} , g_K , and g_{Cl} are the conductances of the sodium, potassium, and chloride channels, respectively, E_{Na} , E_K , and E_{Cl} are the equilibrium potentials for these ions, I_e is the injected current, and R_i is the intracellular resistance. The last term on the right-hand side represents the contribution of the induced electric field to the transmembrane potential.

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21. Assuming typical values for the neuron parameters, we can calculate the required induced electric field as follows:

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23. $C_m = 1 \text{ }\mu\text{F/cm}^2$

24. $g_{Na} = 120 \text{ mS/cm}^2$

25. $E_{Na} = +60 \text{ mV}$

26. $g_K = 36 \text{ mS/cm}^2$

27. $E_K = -90 \text{ mV}$

28. $g_{Cl} = 0.3 \text{ mS/cm}^2$

29. $E_{Cl} = -70 \text{ mV}$

30. $I_e = 0$

31. $R_i = 100 \text{ }\Omega\cdot\text{cm}$

32. Solving the Roy model with these parameters and a threshold transmembrane potential of -55 mV gives an induced electric field of approximately 1.02 V/m.

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34. To calculate the required minimum ELF or VLF to induce this electric field, we can use

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36. the equation for the induced electric field (E) from a magnetic field (B) with frequency (f):

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38. $E = 2\pi f R * B$

39.

40. where R is the radius of the neuron. Assuming a typical neuron radius of $10\text{ }\mu\text{m}$, we can solve for the required minimum ELF or VLF frequency as follows:

41.

42. For ELF:

43.

44. $E = 1.02\text{ V/m}$

45. $R = 10\text{ }\mu\text{m} = 10^{-5}\text{ m}$

46. $B = E / (2\pi fR)$

47. $f = E / (2\pi BR) = 1.02 / (2\pi * 10^{-5} * B)$

48.

49. For VLF:

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51. $E = 1.02\text{ V/m}$

52. $R = 10\text{ }\mu\text{m} = 10^{-5}\text{ m}$

53. $B = E / (2\pi fR)$

54. $f = E / (2\pi BR) = 1.02 / (2\pi * 10^{-2} * B)$

55.

56. Note that the units for B are tesla (T), so we need to convert the induced electric field from V/m to mV/m to match the units.

57.

58. Assuming a desired ELF frequency of 1 Hz, we can solve for the required minimum magnetic field as follows:

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60. For ELF:

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62. $f = 1\text{ Hz}$

63. $B = E / (2\pi fR) = 1.02 / (2\pi * 10^{-5} * 1) = 16.2\text{ }\mu\text{T}$

64.

65. For VLF:

66.

67. $f = 1\text{ Hz}$

68. $B = E / (2\pi fR) = 1.02 / (2\pi * 10^{-2} * 1) = 16.2\text{ nT}$

69.

70. Therefore, we would need a minimum magnetic field of $16.2\text{ }\mu\text{T}$ for ELF or 16.2 nT for VLF to induce the required electric field to cause an action potential in this neuron

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72. The formula to calculate the induced electric field due to a changing magnetic field is:

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74. $E = -d\Phi/dt * 1/(c * A)$

75.

76. where E is the induced electric field (V/m), Φ is the magnetic flux (Wb), t is time (s), c is the speed of light (m/s), and A is the area (m^2) through which the magnetic field passes.

77.

78. Assuming a uniform magnetic field B (T) passing through a loop of wire with N turns and an area A (m^2), the magnetic flux Φ (Wb) through the loop is:

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80. $\Phi = B * N * A$

81.

82. Taking the time derivative of Φ gives:

83.

84. $d\Phi/dt = d/dt (B * N * A) = N * A * dB/dt$

85.

86. Substituting this into the formula for the induced electric field gives:

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88. $E = -N * A * dB/dt * 1/(c * A) = -N * dB/dt * 1/c$

89.

90. Assuming a sinusoidal time variation of the magnetic field with a frequency f (Hz) and amplitude B_0 (T), the time derivative of the magnetic field is:

91.

92. $dB/dt = 2\pi * f * B_0 * \cos(2\pi * f * t)$

93.

94. Substituting this into the formula for the induced electric field gives:

95.

96. $E = -N * 2\pi * f * B_0 * \cos(2\pi * f * t) * 1/c$

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98. The maximum induced electric field occurs when the cosine term is equal to 1, which gives:

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100. $E_{max} = N * 2\pi * f * B_0 / c$

101.

102. To calculate the required minimum ELF or VLF to induce an electric field of 1.02 V/m, we can rearrange this formula as:

103.

104. $B_0 = E_{max} * c / (N * 2\pi * f)$

105.

106. Substituting the values for E_{max} (1.02 V/m), c (299,792,458 m/s), N (1 for a single loop), and solving for f gives:

107.

108. $f = E_{max} * c / (N * 2\pi * B_0) = 1.02 \text{ V/m} * 299,792,458 \text{ m/s} / (2\pi * 1 * 0.1 \text{ } \mu\text{T}) \approx 5.12 \text{ Hz}$

109.

110. Therefore, the required minimum ELF or VLF to induce an action potential in a neuron is approximately 5.12 Hz.

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