Program Reasoning - 12

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① $\boxed{\sim Gp = F\sim p \rightarrow \text{True}}$

L.H.S:

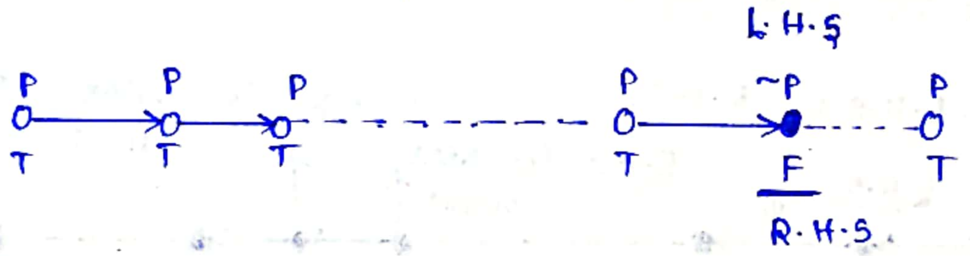
$\sim Gp \rightarrow$ 'P' is not globally true.

$\rightarrow$ It is not satisfied by all state in the future.

R.H.S:

$F\sim p \rightarrow$ '$\sim$p' is true atleast in one state in future.

$\rightarrow$ 'P' is false atleast in one future state.



② $\boxed{\sim Fp = G\sim p \rightarrow \text{True}}$

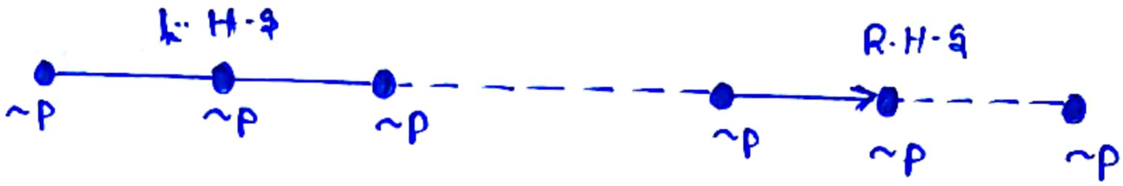
L.H.S:

$\sim Fp \rightarrow$ In future 'p' doesnot become true in any state.

R.H.S:

$G\sim p \rightarrow$ Globally 'p' is false always (or) '$\sim$p' is true always in future.

→ Both are equivalent and hence L.H.S = R.H.S.



(3) $\boxed{\sim Xp = X\sim p \rightarrow \text{True}}$

L.H.S:

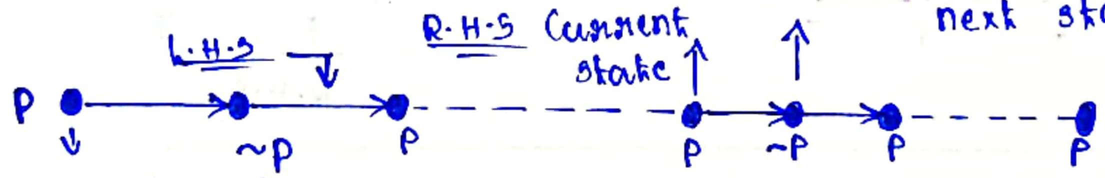
$\sim Xp \rightarrow$ p is not true in the next state.

R.H.S:

$X\sim p \rightarrow \sim p$ is true in the next state.

→ Both are equivalent and conveys the same idea L.H.S = R.H.S.

~p is True in the next state.



Current state → Next state where '~p' is true which implies that p is false in next state.

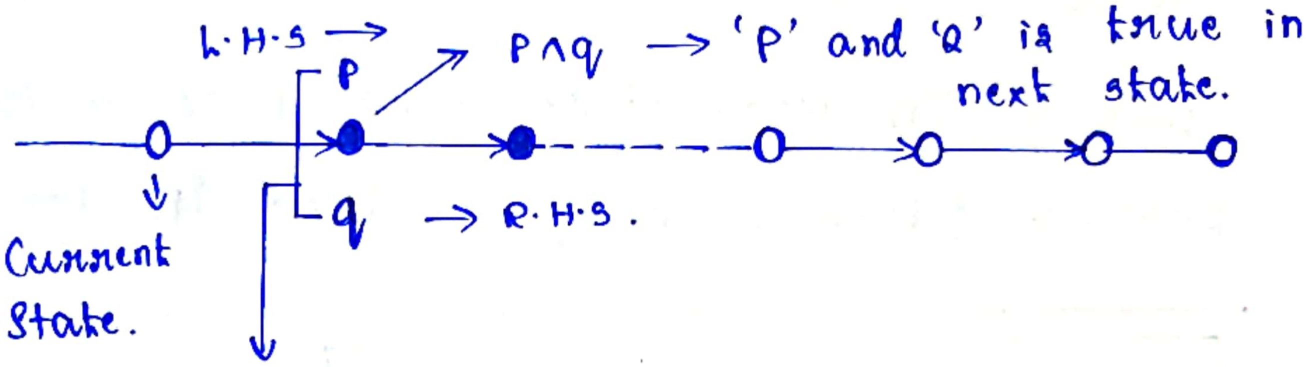
(4) $\boxed{X(p \wedge q) = Xp \wedge Xq \rightarrow \text{True}}$

L.H.S:

$X(p \wedge q) \rightarrow$ In the next state both 'p' and 'q' are True.

R.H.S:

→ $Xp \wedge Xq$ → 'p' is true in the next state and 'q' is also true in the next state.



→ Both P and q are true in next state.

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⑤ $\boxed{X(P \vee q) = Xp \vee Xq \longrightarrow \text{True}}$

L.H.S:

$X(P \vee q)$ → Either 'p' is true (or) 'q' is true in the next state.

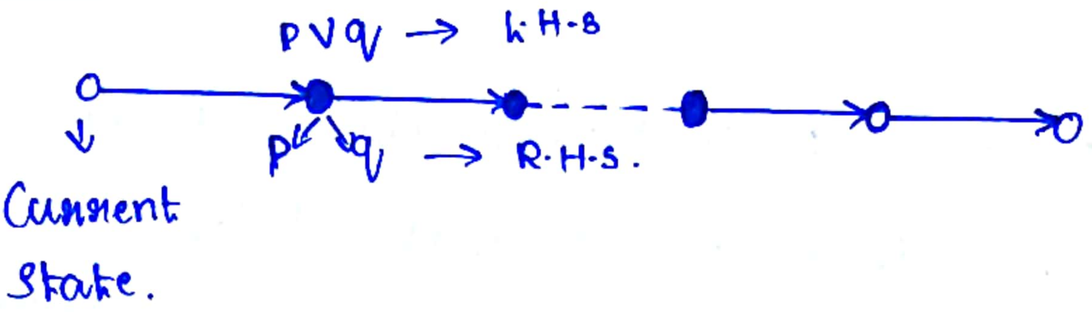
R.H.S:

Xp → P is true in next State.

Xq → Q is true in next state.

$Xp \vee Xq$ → Either 'p' is true in next state (or) q is true in next state.

→ Both the statements are equivalent and hence L.H.S = R.H.S.



L.H.S → In Next state either p or q is true

R.H.S → Either p or q is true in next state.

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(6) $\boxed{X(p \cup q) = Xp \cup Xq \rightarrow \text{True.}}$

L.H.S:

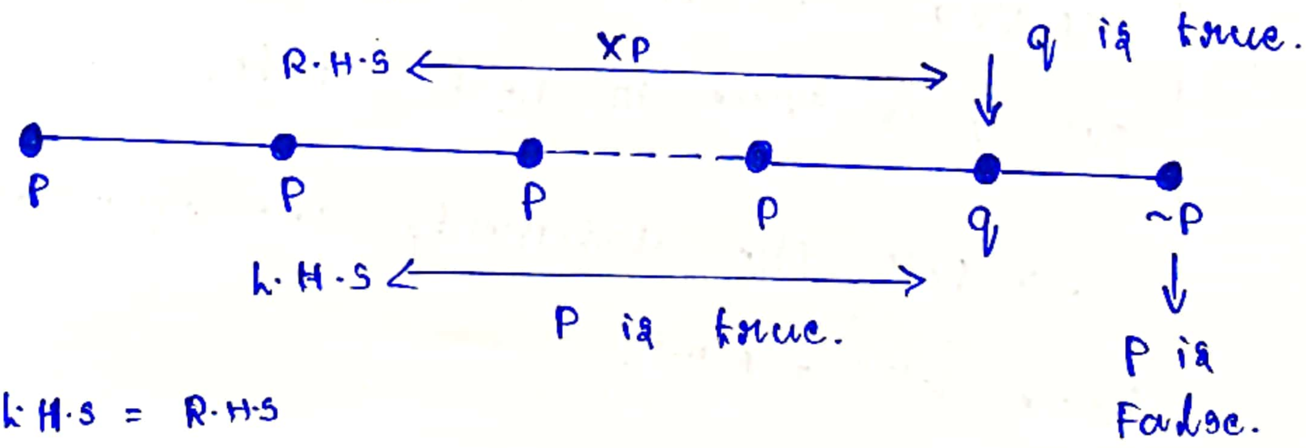
$X(p \cup q)$ → 'p' is true from the next state until q become true.

R.H.S:

Xp → In the next state 'p' is true.

Xq → In the next state 'q' is true.

$Xp \cup Xq$ → The next state 'p' is true until the next state 'q' becomes true.



→ L.H.S = R.H.S

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(7) $\boxed{F(p \vee q) = Fp \vee Fq \rightarrow \text{True}}$

L.H.S:

$F(p \vee q)$ → In future either 'p' (or) 'q' will become true.

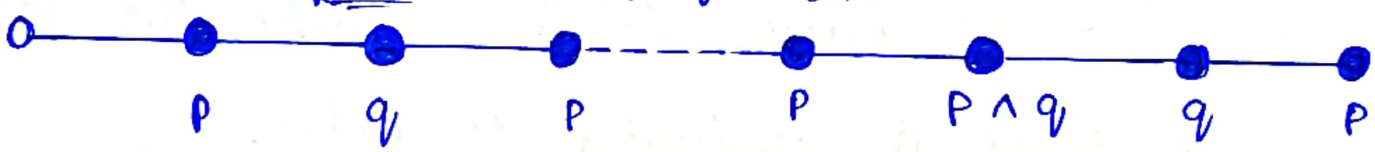
R.H.S:

Fp → In future 'p' holds.

Fq → In future 'q' holds.

$Fq \vee Fq$ → Either 'p' (or) 'q' holds in the future

L.H.S → $F(p \vee q)$ either p (or) q (or) both.



R.H.S → $Fp \vee Fq$.

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(8) $\boxed{F(p \wedge q) = Fp \wedge Fq \rightarrow \text{False}}$

L.H.S:

$F(p \wedge q)$ → In future both 'p' and 'q' are true in same state.

R.H.S:

Fp → In future 'p' is true.

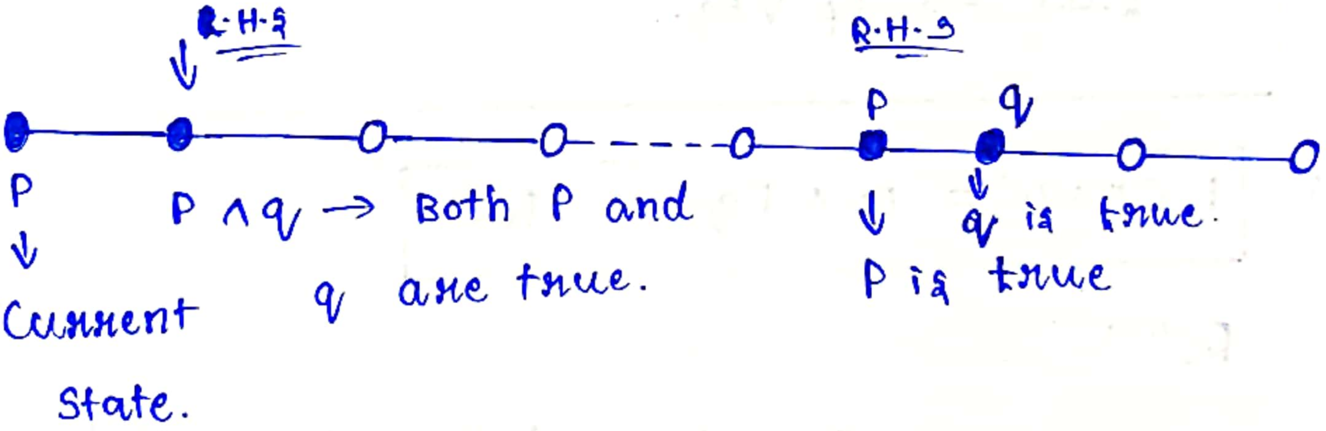
$Fq \rightarrow$ 'q' is true in future.

$Fp \wedge Fq \rightarrow$ In future 'p' can be true and 'q' can be true but it is not necessary to be true in same future state.

→ 'p' can be true in any of the future state and 'q' can be true in any of the other future state.

→ But the L.H.S denotes that both 'p' and 'q' should be true is same future.state.

→ So R.H.S doesn't satisfy the condition and hence it evaluates to false.



→ Both 'p' and 'q' are not true in same state in R.H.S.

L.H.S ≠ R.H.S

(9) $\boxed{G(p \wedge q) = Gp \wedge Gq \rightarrow \text{True}}$

L.H.S:

$G(p \wedge q) \rightarrow$ Globally p and q are true.

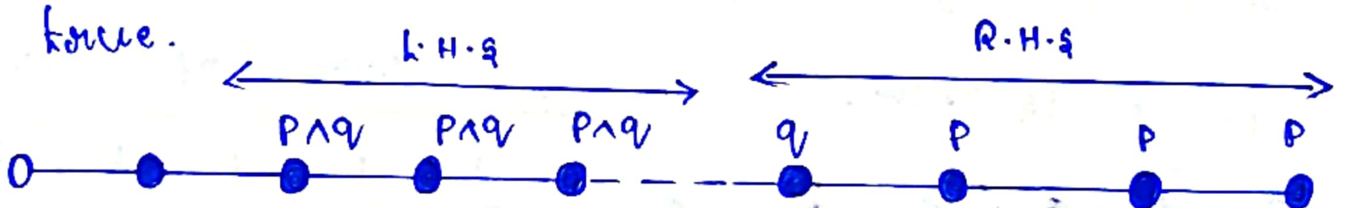
R.H.S:

$Gp \rightarrow$ Globally 'p' is true.

$Gq \rightarrow$ Globally 'q' is true.

$Gp \wedge Gq \rightarrow$ Both 'p' and 'q' are true globally

$\rightarrow$ Wherever 'p' is true 'q' is also true.



$\rightarrow$ $p \wedge q \rightarrow$ True.
R.H.S

$\rightarrow$ Both are true and hence L.H.S = R.H.S

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(10) $\boxed{G(p \vee q) = Gp \vee Gq \rightarrow \text{False}}$

L.H.S:

$G(p \vee q) \rightarrow$ Globally either 'p' can be true (or) 'q' can be true.

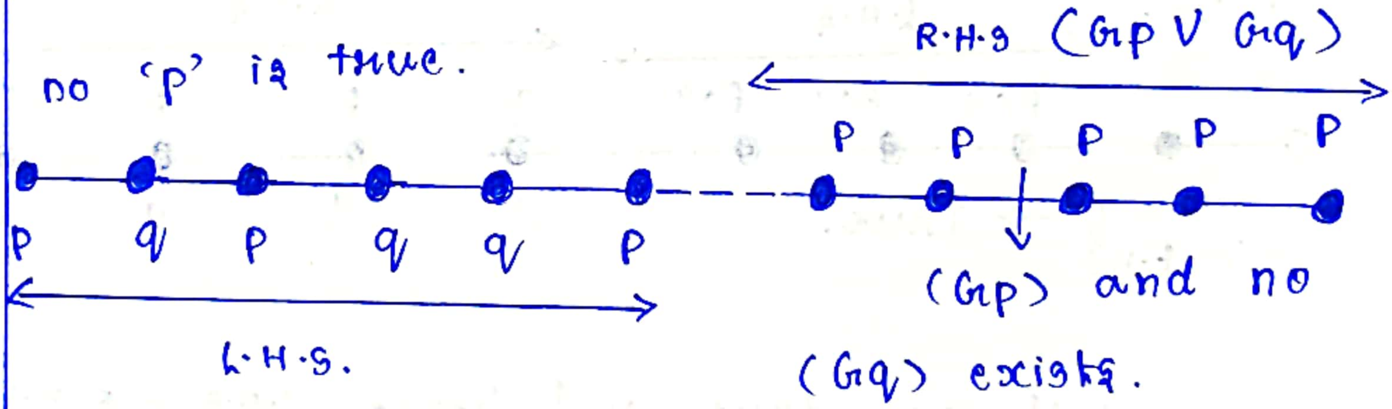
$\rightarrow$ One of them can be true in one of the state.

R.H.S:

$Gp \vee Gq$ → Globally either 'p' is true in all state (or) 'q' is true in all state.

→ The alternate 'p' and 'q' is not possible because if we consider only 'Gp' then in all states only 'p' is true and the (or) condition doesn't allow the 'q' to be true.

We can consider this viceversa also where 'q' has completely occupied the 'true' place and no 'p' is true.



$G(p \vee q)$

L.H.S ≠ R.H.S

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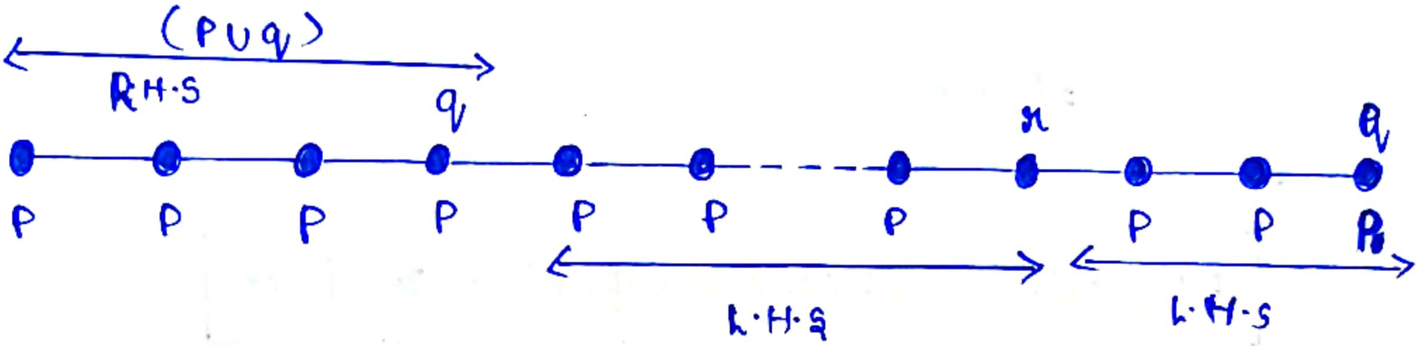
(11) $\boxed{p\,U\,(q \vee r) = (p\,U\,q) \vee (p\,U\,r) \rightarrow \text{True.}}$

L.H.S:

$p\,U\,(q \vee r)$ → 'p' is true until 'q' (or) 'r' becomes true.

R.H.S:

$(PUq) \vee (PUr) \rightarrow$ 'P' is true until q is true (or) 'p' is true until 'r' becomes true.



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(12) $\boxed{PU(q \wedge r) = (PUq) \wedge (PUr) \rightarrow \text{False}}$

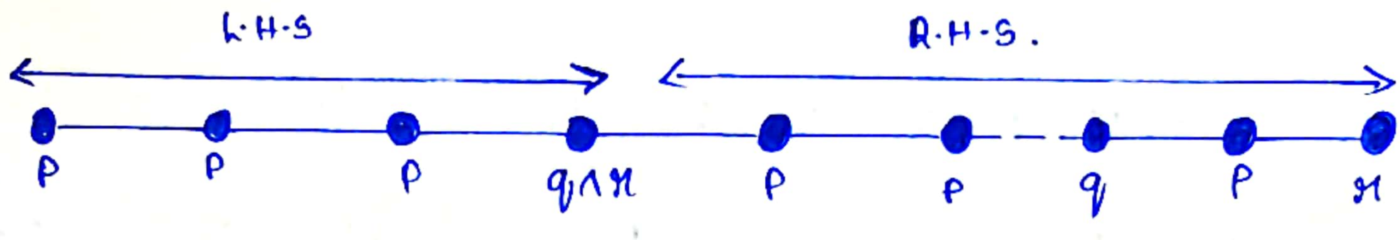
L.H.S:

$PU(q \wedge r) \rightarrow$ P is true until q and r are true in same state.

R.H.S:

$(PUq) \wedge (PUr) \rightarrow$ P is true until and unless 'q' becomes true and 'p' remains to be true until 'r' also becomes true.

→ This doesn't necessarily happen in same state because 'q' can be true any where in future and 'r' can be true any where in some other future.



$P \cup (q \wedge r)$ $\quad$ $(P \cup q) \wedge (P \cup r)$

L.H.S $\neq$ R.H.S

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(13) $\boxed{(p \wedge q) \cup r = (p \wedge r) \cup (q \wedge r) \longrightarrow \text{True}}$

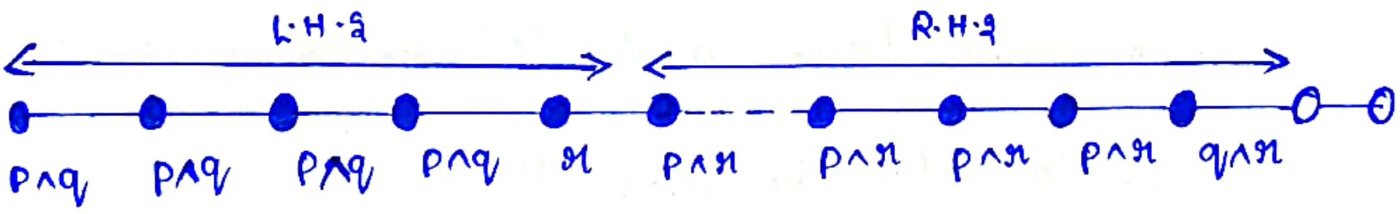
L.H.S:

$(p \wedge q) \cup r$ → 'p' and 'q' is true until 'r' becomes true.

R.H.S:

$(p \wedge r) \cup (q \wedge r)$ → 'p' and 'r' remains true until 'q' and 'r' remains true.

$(p \wedge q) \cup r$ $\quad$ $(p \wedge r) \cup (q \wedge r)$



→ L.H.S = R.H.S.

→ 'p' and 'r' remains true until 'q' also remains true with 'r' as true.

(14) $\boxed{(p \vee q)\, U\, r = (p \vee r)\, U\, (q \vee r) \rightarrow \text{False}}$

<u>L.H.S:</u>

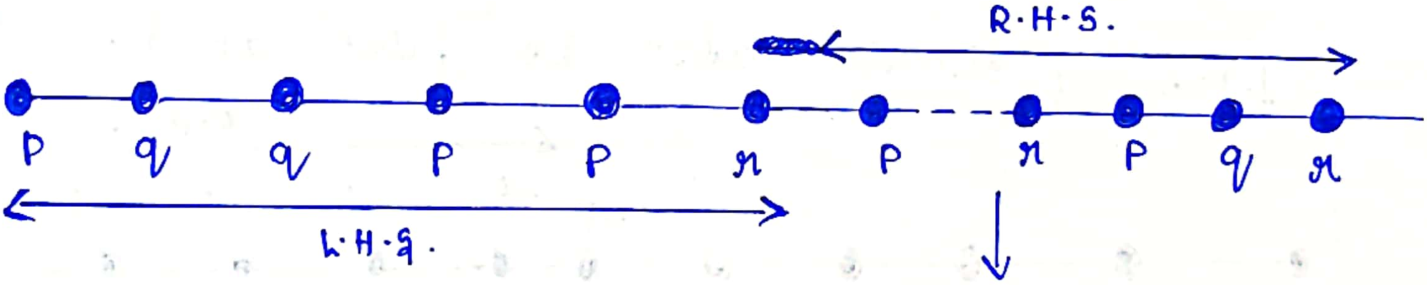
$(p \vee q)\, U\, r \rightarrow$ Either p (or) q is true 'r' becomes true.

<u>R.H.S:</u>

$(p \vee r) \rightarrow$ p (or) r.

$(p \vee r)\, U\, (q \vee r) \rightarrow$ 'p' (or) 'r' remains true until 'q' (or) 'r' becomes true.

→ This condition is not strict as L.H.S.



p (or) r remains to be true until when either q (or) r becomes true.

L.H.S $\neq$ R.H.S.

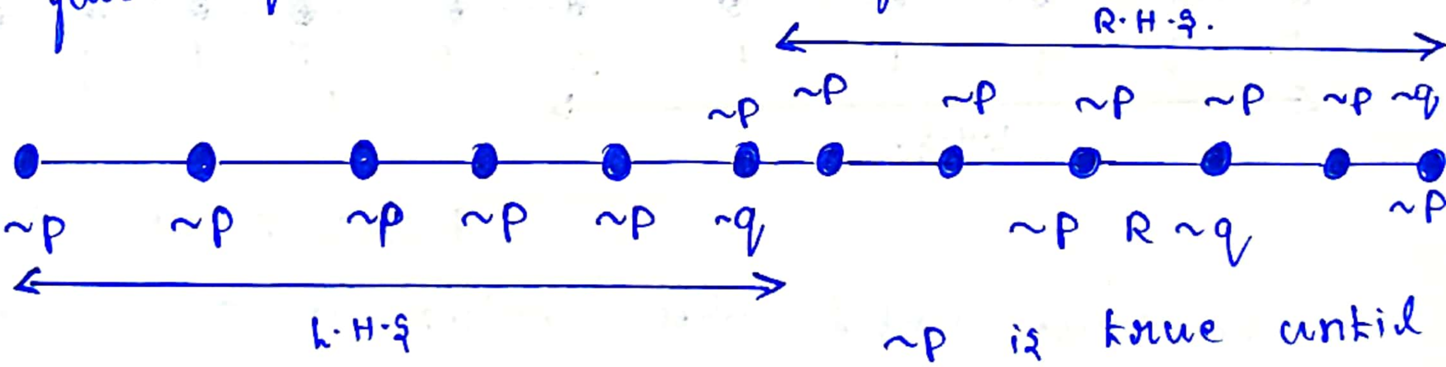
(15) $\boxed{\sim(p\,U\,q) = (\sim p\;R\,\sim q) \rightarrow \text{True.}}$

L.H.S:

$\sim(p\,U\,q)$ → p until q is not true.
→ p is false until q becomes false.

R.H.S:

$(\sim p\;R\,\sim q)$ → p is false until q becomes false and the place where p becomes false q should also be 'false' at that state.



~p is true until ~q is true.

$\sim(p\,U\,q) = \sim p\;U\,\sim q$

L.H.S = R.H.S.

(16) $\boxed{\sim(p \, R \, q) = (\sim p \, U \sim q) \rightarrow \text{True}}$

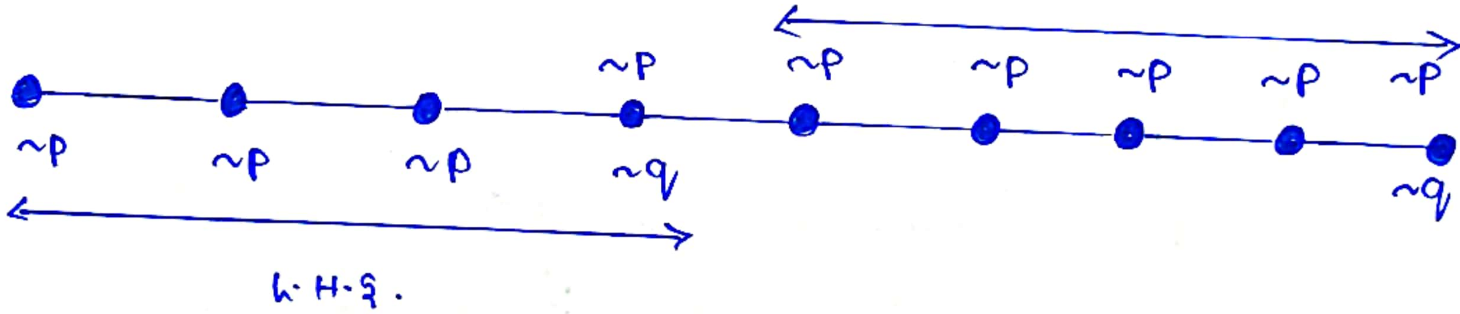
L.H.S:

$\sim(p R q) \rightarrow$ p is false until q is false.

$\sim p \, R \sim q \rightarrow$ p is not true until q is not true.

R.H.S:

$\sim p \, U \sim q \rightarrow$ p is not true until q is not true.



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(17) $\boxed{Fp = FFp \rightarrow \text{True}}$

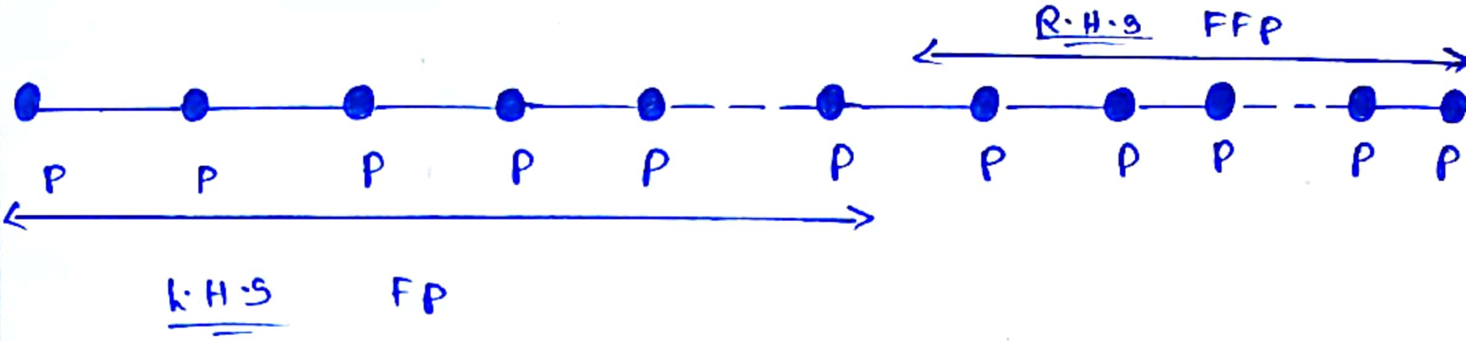
L.H.S:

$Fp \rightarrow$ Globally in future p is true.

$\rightarrow$ p is true eventually.

R.H.S:

$FFp \rightarrow$ In future in future p is true.



→ L.H.S = R.H.S

→ Future P = Future P, where P is true everywhere.

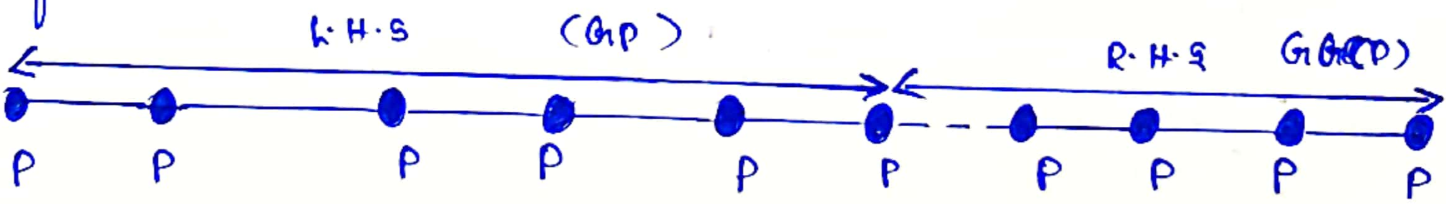
(18) $\boxed{GP = GGP. \rightarrow \text{True}}$

L.H.S :

GP → 'P' is true always in future.

R.H.S :

GGP → Globally of globally 'p' is true which is same as 'p' is always true in future.



→ Both L.H.S and R.H.S denotes that the p remains true always in the future.