# Time and Distance

1. The fundamental relationship between Distance (s), time (t) and speed (v) is given by: **s = v x t.**
2. Let $v_1$ and $v_2$ be the velocity of the two vehicles and let $v_1 > v_2$
   If both the vehicles are moving in the same direction then their
   Relative Velocity = R.V. = $v_1 - v_2$
   If both the vehicles are moving in the opposite direction then their
   Relative velocity = R.V. = $v_1 + v_2$

**TIP**

**Conversion:**

1 km/h = $\frac{5}{18}$ m/s and

1 m/s = $\frac{18}{5}$ km/h

3. Average velocity = $\frac{\text{Total distance covered}}{\text{Total time taken}}$

   If $x_1$ & $x_2$ are the distances covered at velocities $v_1$ & $v_2$ respectively then the average velocity over the entire distance $(x_1 + x_2)$ is given by $\frac{x_1 + x_2}{\frac{x_1}{v_1} + \frac{x_2}{v_2}}$.
4. A man travels first half of the distance at a velocity $v_1$, second half of the distance at a velocity $v_2$ then, Average velocity = $\frac{2v_1v_2}{v_1 + v_2}$
5. If the distance is covered in three equal parts with different speeds $v_1$, $v_2$ and $v_3$ then, Average velocity = $\frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$
6. For the same distance, the time is inversely proportion to the speed of the object. These types of problems can be solved as the problems of percentage.

**Do you Know?**

*If the speed of the bodies is changed in the ratio x : y, then ratio of the time taken changes in the ratio y : x.*

**Ex.1** ***If I decrease my speed by 20% of original speed, I reach office 7 minute late. What is my usual time and new time of reaching office?***

**Sol**. Since speed is decreased by 20% i.e. $\frac{1}{5}$ of the original. New speed will become $\frac{4}{5}$ of the original speed. For the same distance, the time will become $\frac{5}{4}$ of original time. Therefore new time increase by $\frac{1}{4}$ of the original. This is given equal to 7 minutes.
So Usual time = $7 \times 4$ = 28 minutes and New time = 28 + 7 = 35 minutes.

7. When time is constant the ratio of speeds of the object is equal to the ratio of the distance covered by them i.e. $\frac{v_1}{v_2} = \frac{d_1}{d_2}$

**Ex.2** ***A train leaves Calcutta for Mumbai, a distance of 1600 kms at the same time a train leaves Mumbai to Calcutta. The trains meet at Nagpur which is at a distance of 700 kms from Mumbai. What is the ratio of the speeds of the trains?***

**Sol.** *From the problem, it is clear that when the first train travels a distance of 1600 – 700 = 900 km, the second train travels a distance equal to 900. So, the ratio of their speeds is 9 : 7.*

## *Trains*

(i) When a train approaches a stationary object (a tree, a stationary man, a lamp-post; we assume the length of the object to be infinitely small; provided its length isn't mentioned)

Time taken by the train to cross Pole = $\frac{\text{length of the train}}{\text{Speed of the train.}}$

(ii) However, when a train approaches a platform, the time taken by the train to cross the platform is same as the time taken by the train to cross a distance equal to its own length plus the length of the platform at its own velocity.

$\therefore$ Time taken to cross the platform = $\frac{\text{Length of Train} + \text{Platform}}{\text{Speed of train}}$

(iii) For two trains having lengths $l_1$ & $l_2$ and traveling in the same direction with speeds $v_1$ & $v_2$ respectively ($v_1 > v_2$).

Time taken to cross each other completely

$= \frac{\text{total distance}}{\text{relative speed}}. = \frac{l_1 + l_2}{v_1 - v_2}$

(iv) Similarly, for two trains traveling in the opposite direction:

Time taken to cross each other completely = total distance /relative velocity = $\frac{l_1 + l_2}{v_1 + v_2}$.

(v) If two trains/Object start at the same time from two points X & Y and move towards each other and after crossing they take a & b hrs respectively to reach opposite points Y and X, then

$$\frac{x's\ \text{speed}}{y's\ \text{speed}} = \sqrt{\frac{b}{a}}.$$

**Ex.3** ***A train 110 m long travels at 60 kmph. How long does it take?***

(a) To pass a telegraph post by the side of the track?

(b) To pass a man running at 6 kmph in the same direction as the train?

(c) To pass a man running at 6 kmph in the opposite direction?

(d) To pass a station platform 240 m long?

(e) To pass another train 170 m long, running at 40 kmph in the same direction?

(f) To pass another train 170 m long, running at 60 kmph in the opposite direction?

**Sol.** (a) Speed of train = $60 \times \frac{5}{18}$ m/s = $\frac{50}{3}$ m/s

$\therefore$ Time taken to cross the telegraph post = $\frac{110}{\frac{50}{3}}$ = 6.6 seconds.

(b) Speed of man = $6 \times \frac{5}{18}$ m/s = $\frac{5}{3}$ m/s

$\therefore$ Time taken to pass the man = Length of the train/ Relative velocity

$= \frac{110}{16\frac{2}{3} - 1\frac{2}{3}} = \frac{110}{15} = 7\frac{1}{3}$ seconds.

(c) Time = Length of the train/ Relative velocity = $\frac{110}{16\frac{2}{3}+1\frac{2}{3}}$

$= \frac{110}{18\frac{1}{3}} = 110 \times \frac{3}{55} = 6$ seconds.

(d) Time = (Length of the train + Length of platform) / Relative velocity = $\frac{110+240}{\frac{50}{3}} = 350 \times \frac{3}{50} =$ 21 seconds.

(e) Speed of the second train = $40 \times \frac{5}{18}$ mps = $\frac{100}{9}$ mps.

$\therefore$ Time = Sum of the length of the two trains/ Relative velocity

$= \frac{110+170}{\frac{50}{3}-\frac{100}{9}} = \frac{280 \times 9}{50} = 50.4$ seconds.

(f) Time = Sum of the length of the two trains/Relative velocity

$= \frac{110+170}{\frac{50}{3}+\frac{50}{3}} = \frac{280 \times 3}{100} = 8.4$ seconds.

## *Boats & Streams*

**Let** Speed of boat in still water = b km/hr

Speed of stream = w km/hr

Speed of boat with stream (Down Stream), D = b + w

Speed of boat against stream (Up stream), U = b – w

Speed of boat in still water, b = $\frac{1}{2}$(D + U)

Speed of stream, w = $\frac{1}{2}$(D – U)

**Ex.4** ***A man can row 4.5 km/hr in still water. It takes him twice as long to row upstream as to row downstream. What is the rate of the current?***

**Sol.** Speed of boat in still water (b) = 4.5 km/hr.

It is given upstream time is twice to that of down stream.

$\Rightarrow$ Downstream speed is twice to that of upstream.

So b + u = 2(b – u)

$\Rightarrow u = \frac{b}{3} = 1.5$ km/hr.

## *Circular Motion*

**Circular Motion with two people**

**Ex.5** ***Sachin and Saurav, as a warm-up exercise, are jogging on a circular track. Saurav is a better athlete and jogs at 18km/hr while Sachin jogs at 9 km/hr. The circumference of the track is 500 m (i.e. ½ km). They start from the same point at the same time and in the same direction. When will they be together again for the first time?***

**Sol.** **Method 1:** Since Saurav is faster than Sachin, he will take a lead and as they keep running, the gap between them will keep widening. Unlike on a straight track, they would meet again even if Saurav is faster than Sachin.

The same problem could be rephrased as "In what time would Saurav take a lead of 500 m over Sachin"?

Every second Saurav is taking a lead of $\left[18\times\frac{5}{18}-9\times\frac{5}{18}\right]$m = 2.5 m over Sachin in Therefore, they would meet for the first time after 200 sec.

In general, the first meeting if both are moving in the same direction and after both have started simultaneously occurs after

Time of first meeting = $\frac{\text{Circumference of the circle}}{\text{Relative speed}}$.

**Fact**

1. The faster runner will meet slower only when he will have a lead equal to the length of the track when running in the same direction.

2. When running in opposite direction they will meet when they together covers distance equal to length of the track

**Method 2:** For every round that Sachin makes, Saurav would have made 2 rounds because the ratio of their speeds is 1 : 2. Hence, when Sachin has made 1 full round, Saurav would have taken a lead of 1 round. Therefore, they would meet after $\frac{500}{2.5}$ sec.

[Here, 9 × $\frac{5}{18}$ m/s = 2.5 m/s is Sachin's speed.]

**Ex.6** ***Suppose in the earlier problem when would the two meet for the first time if they are moving in the opposite directions?***

**Sol.** If the two are moving in the opposite directions, then

Relative speed = 2.5 + 5 = 7.5 m/s.

[Hence, time for the first meeting = Circumference / Relative speed

$= \left(\frac{500}{7.5}\right) = \left(\frac{200}{3}\right)$ sec.

**Ex.7** ***If the speeds of Saurav and Sachin were 8 km/hr and 5 km/hr, then after what time will the two meet for the first time at the starting point if they start simultaneously?***

**Sol.** Let us first calculate the time Saurav and Sachin take to make one full circle.

Time taken by Saurav = $\frac{500}{8\times\frac{5}{18}} = \frac{1800}{8}$ = 225 sec.

Time take by Sachin = $\frac{500}{\left(5\times\frac{5}{18}\right)}$ = 360 sec.

***Do youKnow?***

Problems in circular motion make use of both the relative speed and the LCM concepts.

Hence, after every 225 sec, Saurav would be at the starting point and after every 360 sec Sachin would be at starting point. The time when they will be together again at the starting point simultaneously for the first time, would be the smallest multiple of both 225 and 360 which is the LCM of 225 and 360.
Hence, they would both be together at the starting point for the first time after LCM (225, 360) = 1800 sec = 0.5 hr. Thus, every half an hour, they would meet at the starting point.
**Note:** From the solution you could realise that it is immaterial whether they move in the same direction or in the opposite.

**Circular motion with three people:**

If three persons A, B, and C are running along a circular track of length d meters with speeds $V_a, V_b, V_c$ respectively. To find the time when all the three will meet for the first time, we have to calculate the relative time of the meeting of any one (A or B or C) among the three with other two runners and then calculate the LCM of these two timings. This will be the time when all the three runners will meet for the first time.

**Time of meeting for the first time** $= \text{LCM}\left[\left|\dfrac{d}{V_a - V_b}\right| \text{and} \left|\dfrac{d}{V_a - V_c}\right|\right]$

And to calculate when they all meet for the first time at the starting point, we have to take the LCM of the timings taken by all the runners separately to cover one full circular motion.

**Time when they meet first time at starting point** $= \text{LCM}\left[\dfrac{d}{V_a}, \dfrac{d}{V_b}, \dfrac{d}{V_c}\right]$

**Ex.8 Let us now discuss the cases of circular motion with three people:**
***Laxman joins Saurav and Sachin, and all of them run in the same direction from the same point simultaneously in a track of length 500 m. Laxman moves at 3 km/hr, Sachin at 5 km/hr and Saurav at 8 km/hr. When will all of them be together again?***
***a. for the first time?***
***b. for the first time at the starting point?***

**Sol. (a)** Break the problem into two separate cases.
**In the first case,** Saurav moves at the relative speed of (8 – 5) = 3 km/hr with respect to Sachin.

At a relative speed of 3 km/hr, he would meet Sachin after every $\dfrac{500}{\left(3 \times \dfrac{5}{18}\right)}$ = 600 sec = 10 min.

**In the second case,** Saurav moves at the speed of (8 – 3) km/hr = 5 km/hr with respect to Laxman.
At a relative speed of 5 km/hr, he would meet Laxman after every $\dfrac{500}{\left(5 \times \dfrac{5}{18}\right)}$ = 360 sec = 6 min.

$\therefore$ If all the three have to meet, they would meet after every [LCM (10, 6)] min = 30 min or ½ hour.
Hence, they would all meet for the first time after 30 min.

**(b)** If we need to find the time after which all of them would be at the starting point simultaneously for the first time, we shall use the same method as in the case involving two people.

At a speed of 8 km/hr, Saurav takes 225 sec. to complete one circle.

At a speed of 5 km/hr, Sachin takes 360 sec. to complete one circle.

At a speed of 3 km/hr, Laxman would take 600 sec. to complete one circle.

Hence, they would meet for the first time at the starting point after

LCM (225, 360, 600) sec. = 1800 sec.

**Ex.9** ***A thief is spotted by a policeman from a distance of 200 m. When the policeman starts a chase, the thief starts running. Speed of thief is 10 Kmph and that of policeman is 12 kmph. After how many hours will the policeman catch the thief?***

**Sol.** $t = \frac{s}{R.V.} = \frac{200}{1000 \times (12-10)} = \frac{200}{1000 \times 2} = \frac{1}{10}$ hr = 6 min.

**Ex.10** ***A man steals a car at 1 : 30 pm & drives at 40 kmph. At 2 pm the owner starts chasing his car at 50 kmph. At what time does he catch the man?***

**Sol.** Distance covered by the thief in 1h = 40 km

Distance covered in $\frac{1}{2}$h = 20 km

Now, time taken to catch the thief = $\frac{\text{Distance}}{\text{Relative velocity}}$

Relative velocity = 50 – 40 = 10 kmph ($\because$ Both are moving in same direction)

$\therefore$ $t = \frac{20}{10}$ = 2 hrs.

Time = 4 p.m. ($\because$ 2 p.m. + 2)

**Ex.11 A and B started moving simultaneously from P towards Q and their respective speeds are 36 kmph and 15 m/s respectively. What is the distance between them after moving for 2 minutes after starting from P?**

**Sol.** Speed of A = 36 kmph = $\frac{5}{18} \times 36$ = 10 m/s.

Speed of B = 15 m/s

So in 1 sec, B covers 5 m extra than A and so, the distance between them will be 5 m.

So in 2 mins = 120 sec, the distance between will be 120 $\times$ 5 m = 600 m.

**Ex.12 Two trains are moving in opposite directions with the respective speeds of 36 kmph and 45 kmph. They will cross each other in 20 seconds. If they are moving in the same direction at the same speed, how much time will they take to cross each other?**

**Sol.** Speeds = 36 kmph, 45 kmph = 10 m/s, 12.5 m/s

If they are moving in opposite directions, relative velocity = 10 + 12.5 = 22.5m/s.

And they take 20 seconds to cross each other.

If they move in the same direction, relative velocity = 12.5 – 10 = 2.5

Since the velocity in the second case is $\frac{1}{9}$ times the velocity in the first case.

So, they will take 9 times more time to cross each other. i.e., 20 $\times$ 9 = 180 sec = 3 min.

**Ex.13 Two trains are moving in the same direction. The speed of the faster train is twice the speed of the slower train. The faster train takes 60 sec to overtake the slower train. If they move in opposite directions, how much time will they take to cross each other?**

**Sol.** Assume 'v' is the speed of the slower train, so 2v is the speed of the faster train.

If they move in the same direction, relative speed = 2v – v = v.

If they move in opposite directions, relative velocity = 2v + v = 3v.

Since, the velocity is three times, the time required is $\frac{1}{3}$rd. i.e., $\frac{60}{3}$ = 20 sec.

**Ex.14 Train A starts from city P to city Q with a velocity of 40 kmph. Train B starts at city P towards city Q, 1 hour after train A with a speed of 60 kmph. If both the trains reach station Q simultaneously, what is the distance between cities, P and Q?**

**Sol.** Assume, the distance as d.

$\therefore \frac{d}{40} - 1 = \frac{d}{60}$ (since train B takes 1 hour less than that of train A)

$\therefore$ d = 120 km.

**Ex.15 Two friends A and B are moving towards Q and P respectively from P and Q respectively. The distance between P and Q is 600 m and the speeds of the friends, A and B are 6 m/s and 8 m/s respectively. How much time after A starts for P, does B have to start for Q so that they meet at the exact midpoint of P and Q?**

**Sol.** The midpoint means exactly 300 m from both the sides.

A can walk this 300 m at a speed 6 m/s speed in 50 sec.

Whereas B can cover this distance in $\frac{300}{8}$ = 37.5 sec.

So B has to wait for (50 – 37.5) = 12.5 sec, if they want to reach the midpoint simultaneously

**Ex.16 A river flows at a speed of 1.5 kmph and a boatman who can row his boat at a speed of 2.5 kmph in still water, takes $7\frac{1}{2}$ hours to go a certain distance up stream and return to the starting point. What is the total distance covered by the boatman?**

**Sol.** The speeds of the boatman upstream and downstream are 1 kmph and 4 kmph respectively.

If the distance covered each way = x km, then total time taken to go upstream and downstream

$= \frac{x}{1} + \frac{x}{4} = 7\frac{1}{2}$,

i.e. $\frac{5}{4}x = \frac{15}{2} \Rightarrow x = \frac{15}{2} \times \frac{4}{5}$ = 6 km

The distance covered both ways is 12 km.

## *Races & Games of Skill*

**Races:** Any contest of speed in running, riding, driving, sailing or rowing is called a ***race***. The path on which the contests are held is called a ***race course***. The point from which the race begins is called the ***starting point***. The point where the race ends is called the ***winning post*** or ***the goal***. The person who first reaches the winning post is called the ***winner***. If all the contestants reach the goal at the same time then the race is called a ***dead heat race***.

There are two types of races

**(a)** **Handicap Race:** In this type of races 1st runner gives 2nd runner benefit of running for some time before 1st start the race.

e.g. *A can give B a handicap of 3 sec. mans they started from the same point but A start 3 sec after.*

**(b)** **Headway/Start race:** This means that runner A has started from the "K" distance behind runner B but at the same time.

e.g. A gives B a start of 10 m means before starting A is on starting point and B is 10 meter forward from A and they start the race at the same time.

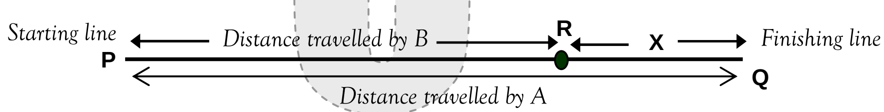
In a contest with 2 participants, one is the winner and the other is the loser.

(i) The winner can give/allow the loser a start of t seconds or x metres, i.e. start distance = x metres and start time = t seconds.

(ii) The winner can beat the loser by t seconds or x metres, i.e. beat distance = x metres and beat time = t seconds

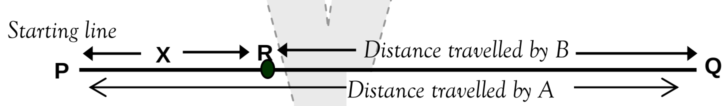
Interpretation from the statements given in the problems concerned with race

**(1)** **A beats B by X meter:** This means that the winner of the race is A and B is X meters behind A when he crosses the finishing line.




**(2)** **A and B start together at P:** This means both the runners have started from the same starting point. The ratio of the speed of the runners is the ratio of the distance covered by them in the time, in which winner reaches the finishing line.

**(3)** **A gives B a start of X metre:** This means that A stands at starting point and B is X meter forward of the starting line at the starting of the race.




**(4)** **A beats B by t seconds**

This states that after coming of A at the finishing point B will take more t seconds to cover the left distance. If we know that distance and time taken we can cover the speed of the looser?

Winner's (A) time = Loser's (B) time – t

A and B starts together at P, A finishes at Q, but t seconds before B finishes.

From the above figures, following formulae have been derived for a race (contest) of two participants, *Toolkit*

| | |
|---|---|
| (a) | Winner's distance = Length of race |
| (b) | Loser's distance = Winner's distance – (beat distance + start distance) |
| (c) | Winner's time = Loser's time – (beat time + start time) |
| (d) | $\frac{\text{Winner's time}}{\text{Loser's distance}} = \frac{\text{Loser's time}}{\text{Winner's distance}}$ $= \frac{\text{beat times} + \text{start time}}{\text{beat distance} + \text{start distance}}$ |
| (e) | If a race ends in a dead lock, i.e. both reach the winning post together then beat time = 0 and beat distance = 0. |

**Ex.17** ***A can run 330 metres in 41 seconds and B in 44 seconds. By how many seconds will B win if he has 30 metres start?***

**Sol.** B runs 330 metres in 44 seconds.

$\therefore$ B runs (330 – 30) metres in $\frac{44}{330} \times 300$ sec = 40 sec.

But A runs 330 metres in 41 seconds

So, B wins by (41 – 40) seconds, i.e. 1 second.

**Ex.18** ***In one kilometer race, A beats B by 36 metres or 9 seconds. Find A's time over the course.***

**Sol.** Here A is the winner and B is the loser.

The time taken by B to cover the distance of 36 metres = 9 seconds.

Time taken by B to cover 1 kilometer = (1000 × 9)/36 = 250 sec.

Therefore time taken by A to cover 1km = 250 – 9 = 241 seconds.

**Alternative method:**

Using formula: $\frac{\text{Winners time}}{\text{Loser's distance}} = \frac{\text{beat time} + \text{start time}}{\text{beat distance} + \text{start distance}}$

$\Rightarrow \frac{\text{A's time}}{1000 - 36} = \frac{9 + 0}{36 + 0} \Rightarrow \text{A's time} = \frac{9}{36} \times 964 = 241 \text{ sec}.$

## Escalator

Let us understand this concept with the help of an example.

**Ex.19** ***In an escalator Rajesh covers 4 steps with 3 steps of Suresh in the same time in a static escalator. When the escalator is moving Rajesh takes 24 steps while Suresh takes 21 steps to reach the top of the escalator. What are the total numbers of steps in the escalator?***

**Sol.** Assume that S be the number of steps of escalator that helped Rajesh and Suresh for the same time. Then,

For every 4 steps of Rajesh, Suresh will take 3 steps.

So for 24 steps Rajesh take, Suresh will take 18 steps.

But Suresh has taken 21 steps which are $\frac{7}{6}$ of the original steps.

These extra steps taken are the result of extra time consumed by him and helped by the escalator. So, the equation will become

$$24 + S = 21 + \frac{7}{6}S \Rightarrow S = 18$$

Total number of steps in the escalator = 24 + S = 24 + 18 = 42 steps.

**Alternate Method**

| Rajesh's Steps | Suresh's Steps | Escalator's Steps |
|---|---|---|
| 24 | 18 | S |
| | 21 | $\frac{7}{6}S$ |

So, according to the table we will have

$$24 + S = 21 + \frac{7}{6}S \Rightarrow \text{Solving } S = 18$$

Total number of steps = 24 + 18 = 42 steps.

**Ex.20 In a 100m race, A can beat B by 10m, and C by 20m. If B and C run with the same velocity, By how many meters can B beat C in a 900 m race?**

**Sol.** From the first line of the question we can understand that when A runs 100m, B runs 90 m and C runs 80 m. So B can beat C by 10m in a 90 m race. So in a 900m race, B can beat C by **100m.**

**Ex.21 A can beat B by 44 metres in a 1760 meter race, while in a 1320 metres race, B can beat C by 30 metres. By what distance (in meters) will A beat C in a 880 meter race?**

**Sol.** When A runs 1760 metres, B runs 1716 metres.

When B runs 1320 metres, C runs 1290 metres.

When B runs 1716 metres, C runs 1290 (1719/1320) = 1677 metres.

| A | B | C |
|---|---|---|
| 1760 | 1716 | 1677 |

When A runs 880 metres, C runs 1677 (880/1760) = 838.5 metres.

A beats C by (880 – 838.5) = 41.5 metres.

**Ex.22 Both A and B run a 2 km race. A gives B a start of 100 m and still beats him by 20 seconds. If A runs at a speed of 20 km per hour, find B's speed in kilometres per hour.**

**(1) 17 (2) 18 (3) 19 (4) 19.5**

**Sol.** A covers a distance of 2 km. in (2/20) hour i.e. 360 sec.

B covers the distance of (2000 – 100) i.e. 1900 m in 360 + 20 i.e. 380 sec.

B's speed = (1900/380) = 5 m/s = $5 \times \frac{18}{5}$ = 18 kmph.

**Ex.23 J is $1\frac{3}{8}$ times as fast as K. If J gives K a start of 150 m, how far must be the winning post so that the race ends in a dead heat?**

**(1) 100 m (2) 440 m (3) 550 m (4) 200 m**

**Sol.** Race ends in a dead heat, i.e. times taken by J and K are the same.

$\therefore$ use t = $\frac{\text{J's distance}}{\text{J's speed}} = \frac{\text{K'distance}}{\text{K's speed}}$

$\therefore \frac{x}{\frac{11}{8}k} = \frac{x-150}{k}$

$\therefore$x = Length of race = 550 metres.