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19CSE337 Social Networking Security

Lecture 5

Topics to Discuss

- Other Fundamentals Concepts in Graph Theory

Other fundamental concepts

- **Clique**: A subset S of the vertices of a graph such that all pairs of vertices in S are adjacent.
- **Coclique**: A subset S of the vertices of a graph such that no two vertices in S are adjacent.
- **Diameter**: The diameter of a graph is the maximal distance between any two points on the graph. If the graph is not connected, its diameter is infinite.
- **Distance**: The distance between two vertices v,v' (often denoted ``d(v,v')'') is the length of the shortest path connecting them.

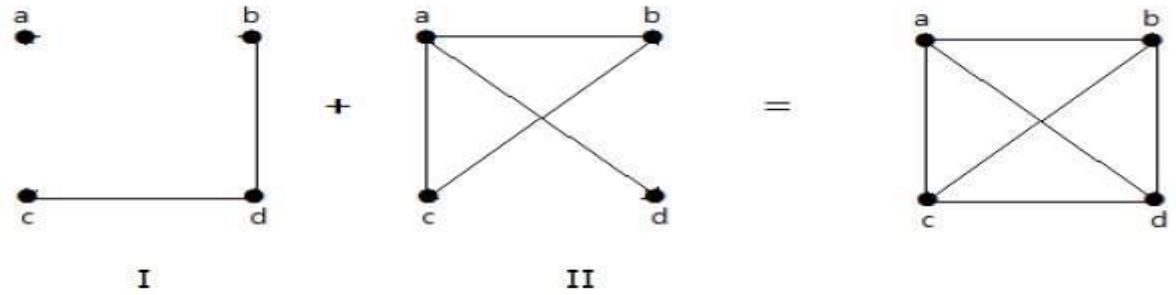
Other fundamental concepts

- **Girth**: The girth of a graph is the length of the shortest cycle(s) in the graph. If the graph has no cycles, its girth is infinite.
- **Tree**: A connected graph with no cycles.
- **Cut set**: In a connected graph G, a cut set is a set of edges which when removed from G leaves G disconnected, provided there is no proper subset of these edges disconnects G.

Other fundamental concepts

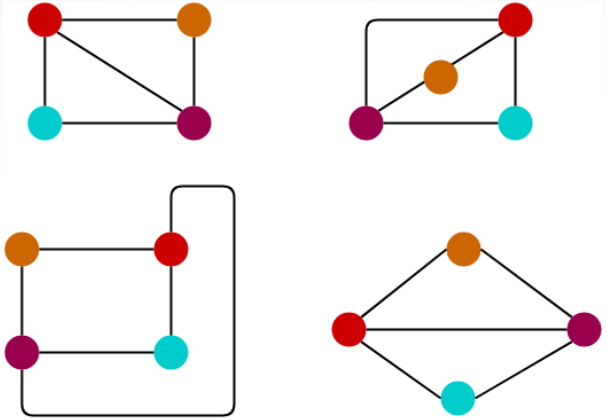
Complement of a graph

- For any graph G, the complementary graph of G is the graph with the same vertices as that of G but with different edges.
- If G' is this complementary graph, then G is in turn the complement of G'.
- When G and G’ merged, it will form a complete graph.



Other fundamental concepts

- **Isomorphism of Graphs**
  - Graph Isomorphism is a phenomenon of existing the same graph in more than one forms.
  - Two graphs G1 and G2 are isomorphic if there exists a matching between their vertices so that two vertices are connected by an edge in G1 if and only if corresponding vertices are connected by an edge in G2.



Graph Isomorphism Example

Other fundamental concepts

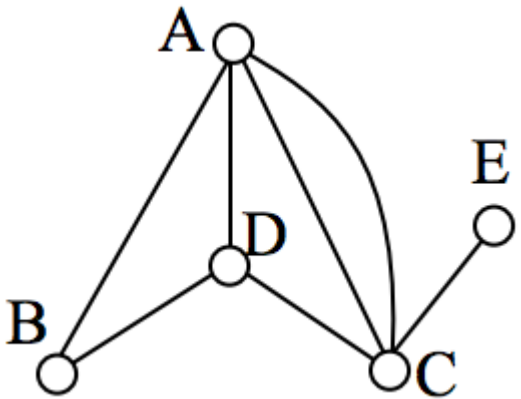
- To check isomorphism of graph, ask the following questions:
  - Are the number of vertices in both graphs the same?
  - Are the number of edges in both graphs the same?
  - Is the degree sequence in both graphs the same?
  - If the vertices in one graph can form a cycle of length k, can we find the same cycle length in the other graph?
- And if we can answer yes to all four of the above questions, then the graphs are isomorphic. In other words, they are the equivalent graphs just in different forms.

Other fundamental concepts

- **Euler Path:** An Euler path or Eulerian path in a graph G is a path that traverses each edge of G exactly once and each vertex at least once. Euler path starts and ends at different vertices.
- **Euler Circuit:** If the initial and final vertices are the same then the path is an Euler(ian) circuit.
- **Euler's Path and Euler's Circuit Theorem**
  - A graph will contain an Euler path if it contains at most two vertices of odd degree.
  - A graph will contain an Euler circuit if all vertices have even degree

Euler's Theorem Example

- In the graph below, vertices A and C have degree 4, since there are 4 edges leading into each vertex. B is degree 2, D is degree 3, and E is degree 1. This graph contains two vertices with odd degree (D and E) and three vertices with even degree (A, B, and C), so Euler's theorems tell us this graph has an Euler path, but not an Euler circuit.



Other fundamental concepts

- **Hamiltonian path**: A Hamiltonian path in a graph G is a path that goes through each vertex of G once.
- **Hamiltonian circuit:** If the initial and final vertices are adjacent, then the path can be completed to a Hamiltonian circuit.

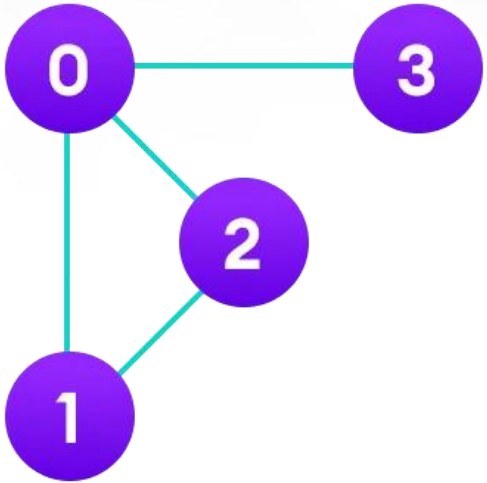
How are Network Graphs represented?

- Graphs can be represented in two ways
  - Adjacency Matrix
  - Adjacency List

Adjacency Matrix

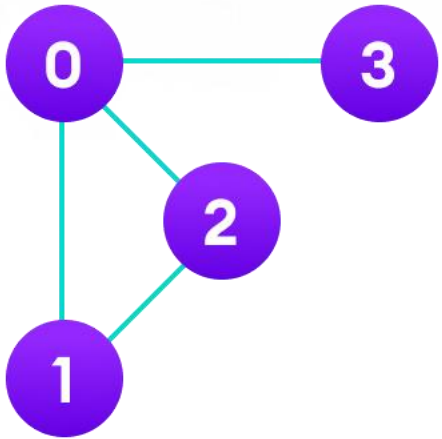
- It is a matrix representation of a graph.
- Each cell in a matrix is represented as a_{ij}, where i and j are vertices.
- a_{ij} ={1, if there is an edge between i and j
- a_{ij} ={0, if there is no edge between i and j
- If it is a weighted directed graph, matrix cells are filled by corresponding weights taking direction into consideration.

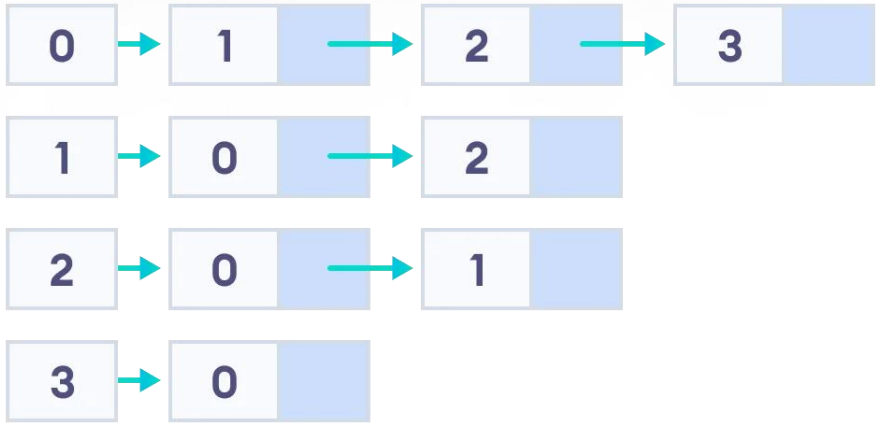
Adjacency Matrix



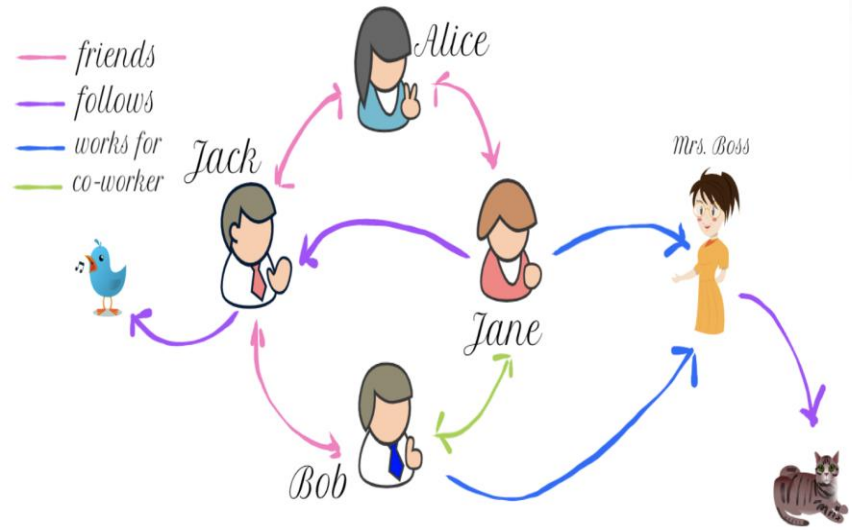
i → / j ↓	0	1	2	3
0	0	1	1	1
1	1	0	1	0
2	1	1	0	0
3	1	0	0	0

Adjacency List





Social Graphs



G=(V,E)

V=(Alice, Jack, Jane, Bob, Mrs.Boss, Cat)

E=(Friends, Follows, Works for, Co-worker)

How Jane related to Bob?

Follow the edge forming the pair (Jane, Bob) → Co-worker.

Thanks……