
PART I

CHAPTER 1

GEOMETRY

INTRODUCTION

Geometry is a branch of mathematics that is concerned with the study of zero, one, two and three dimensional figures and the relationships among them. It focuses on the study of properties and relationships of size, shape, location, direction and orientation of figures like points, lines, planes, angles, polygons, and solids.

It is an important topic for any aptitude test. Geometry can be used to model and represent many mathematical and real life situations; hence, it is used as a tool to test spatial reasoning abilities.

LINES AND ANGLES

Basic concepts

A **point** is the smallest unit in a plane. It is characterized only by its position. It does not have length, width, or thickness. A mark of a sharp pencil tip on a piece of paper closely resembles the concept of a point. A point is denoted by a capital letter. In the figure below, A, B, C, D, and E are all points.

A **line** is a set of all the points in one dimension. It can be extended in both the directions; hence its length is infinite. A line does not have either width or thickness.

A **line segment** is a part of a line. It has a fixed length, with a defined starting point and an end point. Like a line, it too does not have width or thickness.

A **ray** is a part of a line that has a defined starting point (called end point) and extends upto infinity in one direction. A ray has no fixed length. It does not have any width or thickness.

A **plane** is the set of all the points in two dimensions. It does not have any thickness but is indefinitely extended in all directions.

When two rays emerge from a common point, they form an **angle**. The common point is known as the **vertex**. Angles are measured in degrees and radians.

An angle that measures exactly 90° is known as a **right** angle. The angle shown in the figure below is a right angle.

If the measures of two angles add up to 180° , then the angles form a pair of **supplementary** angles.

If the measures of two angles add up to 90° , then both the angles form a pair of **complementary** angles.

When two lines intersect each other, we get four angles. Two alternate, opposite angles form a pair of **vertically opposite angles**. Two such pairs are formed at the intersection of two lines. The angles in each pair of vertically opposite angles are always equal.

When two angles share a common side and a common vertex, we get two **adjacent angles**. For two angles to be adjacent, no angle should be inside the other.

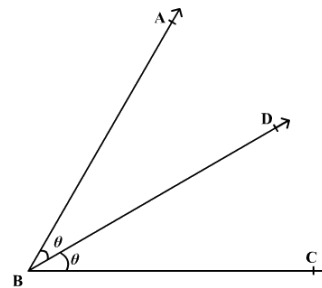
Two lines intersecting each other at 90° are said to be **perpendicular** to each other.

Two lines in the same plane, which never intersect each other, are called **parallel** lines.

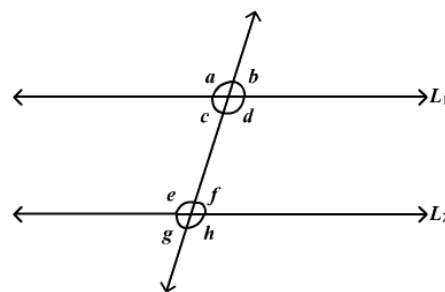
A line segment bisector which makes an angle of 90° with the given segment is

known as the **perpendicular bisector** for the given segment.

A line or a ray (ray BD, in the given figure) which divides the given angle ($\angle ABC$) into two equal parts ($\angle ABD$ and $\angle DBC$) is known as an **angle bisector**.



A line that cuts two or more lines at more than one point is known as a **transversal**. There are many important properties related to two parallel lines and a transversal. To understand these, refer to the following figure.



Here, **Vertically opposite angles** are equal: $a = d$, $b = c$, $e = h$ and $f = g$

Alternate interior angles are equal: $c = f$ and $d = e$

Alternate exterior angles are equal: $a = h$
and $b = g$

Corresponding angles are equal: $a = e$, $b = f$, $c = g$ and $d = h$

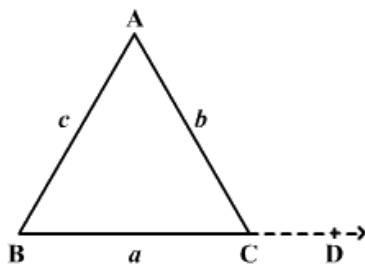
Interior angles on the same side of the transversal are supplementary: $c + e = 180^\circ$
and $d + f = 180^\circ$

Exterior angles on the same side of the transversal are supplementary: $a + g = 180^\circ$
and $b + h = 180^\circ$

TRIANGLE

Basic concepts

A **triangle** is a closed figure bound by three non-parallel coplanar straight lines. It has three non-collinear vertices (A, B and C for example) which are the intersection points of these three lines (AB, BC and CA) known as the **sides** of the triangle.



Properties of triangles:

- Sum of the three interior angles is 180°
- Sum of the lengths of any two sides is more than that of the third side.

- Difference of the lengths of any two sides is less than the third side.
- Side opposite to the greatest angle is the longest, and side opposite to the smallest angle is the shortest.

A line joining the midpoint of a side with the opposite vertex is known as **median** for that side.

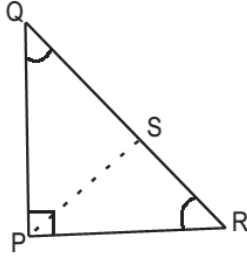
A perpendicular drawn from a vertex to the opposite side is known as **altitude** for that side.

A triangle has three medians. All the three medians intersect each other at a common point. This point of intersection is known as the **centroid**, which divides the three medians in the ratio of 2:1 (2 towards the vertex and 1 towards the side).

A triangle has three altitudes. All the three altitudes intersect each other at a common point. This point of intersection is known as the **orthocentre**.

A triangle, in which all the three angles are acute, is known as an **acute angled** triangle or simply **acute** triangle.

A triangle, in which one angle is a right angle, is known as a **right angled** triangle or simply **right** triangle.



In a right triangle, ΔPQR , $QR^2 = PQ^2 + PR^2$, where QR is hypotenuse. (This is known as the **Pythagoras Theorem**)

A triangle, in which one angle is obtuse, is known as an **obtuse angled** triangle or simply **obtuse** triangle.

A triangle, in which all the three sides are equal in length, is known as an **equilateral** triangle. All the three angles of an equilateral triangle are equal to 60° .

A triangle, in which all the three sides are of different length, is known as a **scalene** triangle. In a scalene triangle, all the three angles are of different measure.

Perimeter of a triangle

The sum of the lengths of the three sides of a triangle is known as the perimeter. So for a triangle with sides a , b and c ;

$$\text{Perimeter (P)} = a + b + c$$

Area of a triangle

Case 1: Lengths of the sides (a , b and c) are given.

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \text{ Where } s \text{ is the semi perimeter and } s = \frac{a+b+c}{2}$$

Case 2: Length of the base and altitude is given.

$$\text{Area} = \frac{1}{2} \times b \times h, \text{ Where } b = \text{base and } h = \text{height or altitude to the base.}$$

Case 3: Lengths of two sides (a and b) and the included angle (θ) is given.

$$\text{Area} = \frac{1}{2} \times a \times b \sin \theta$$

Equilateral triangle

$$\text{Area} = \frac{\sqrt{3}}{4} \times a^2$$

Where a = length of any side.

Similarity and congruency of triangles

Two triangles are said to be **similar** if their corresponding angles are equal. The notation used for similarity is " \sim ".

A-A-A test

If in two triangles all the corresponding angles are equal, then the two triangles are similar.

S-A-S test

If an angle of a triangle is equal to an angle of the other triangle and the corresponding sides including this angle are in the same proportion, then the triangles are similar.

S-S-S test

If all the corresponding sides of two triangles are in the same proportion, then the triangles are similar.

- Also, ratio of the areas of similar triangles will be equal to the square of ratio of the linear measurement.

Two triangles are said to be **congruent** if their corresponding sides are equal. The notation for congruency is " \cong ".

S-S-S test

If three sides of one triangle are equal to three corresponding sides of another triangle, then the two triangles are congruent.

S-A-S test

If two sides and their included angle of one triangle are equal to the two sides and

included angle of another triangle, then the two triangles are congruent.

A-S-A test

If two angles and side included by them of one triangle are equal to the two angles and side included by them of another triangle, then the two triangles are congruent.

S-A-A test

If a side and two angles of one triangle are equal to the side and two corresponding angles of another triangle, then the two triangles are congruent.

Hypotenuse-Side test (for right triangles)

In case of two right triangles, if one side and hypotenuse of one triangle is equal to one side and hypotenuse of another triangle, then the two triangles are congruent.

In any triangle all the three perpendicular bisectors meet at a common point, which is at an equal distance from all the three vertices. Considering this meeting point of perpendicular bisectors as a centre and the distance from this centre to the vertex as radius we can draw a circle. This circle passes through all the three vertices and circumscribes the triangle and hence is known as the **circumcircle**. The centre of

this circle is known as the **circumcentre** and the radius is known as the **circumradius**.

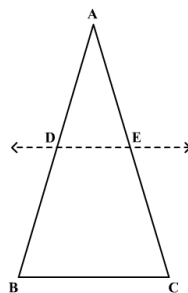
The **circumradius** R is given by the formula, $R = \frac{abc}{4A}$, where a , b and c are the lengths of the sides and A is the area of the triangle.

In any triangle all the three angle bisectors meet at a common point, which is at an equal distance from all the three sides. Considering this meeting point of angle bisectors as the centre and the perpendicular distance from this centre to the sides as the radius, we can draw a circle. This circle will touch all the three sides from inside and hence is known as the **incircle**. The centre of the circle is known as the **incentre** and the radius is known as the **inradius**.

The **inradius** r is given by the formula, $r = \frac{A}{s}$, where s is the semiperimeter and A is the area of the triangle.

Basic proportionality theorem (BPT)

In $\triangle ABC$, if line DE is parallel to side BC then it divides the other two sides AB and AC in the same proportion.



$$\text{Hence } \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

30°-60° - 90° triangle

If the angles of a triangle are of measure 30°, 60° and 90°, then

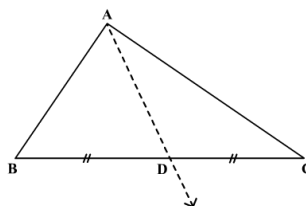
Side opposite to 30° = Half of the hypotenuse

Side opposite to 60° = $\frac{\sqrt{3}}{2}$ times the hypotenuse.

45°-45°-90° triangle

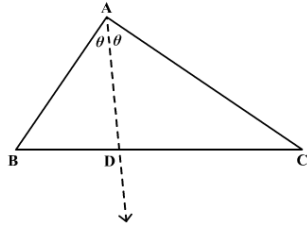
If the angles of a triangle are of measure 45°, 45° and 90°, then

Side opposite to 45° = $\frac{1}{\sqrt{2}}$ times the hypotenuse.

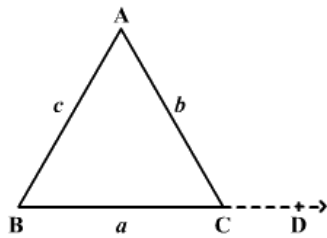


- In $\triangle ABC$, AD is median, then $AB^2 + AC^2 = 2(AD^2 + BD^2)$, this is known as

Apollonius theorem.



- In $\triangle ABC$, if AD is the **angle bisector** for angle A, then $\frac{AB}{BD} = \frac{AC}{CD}$

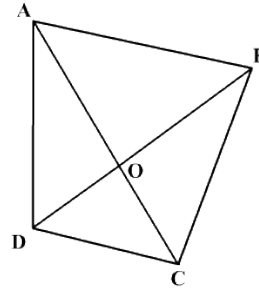


- In $\triangle ABC$, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$
where R is circumradius of the triangle.

QUADRILATERAL AND OTHER POLYGONS

Basic concepts

A **Quadrilateral** is a closed figure bounded by four straight lines (see figure below). It has four vertices (A, B, C and D) such that no three of them are collinear. These vertices are the intersection points of four lines (AB, BC, CD and DA) known as the sides of the quadrilateral.

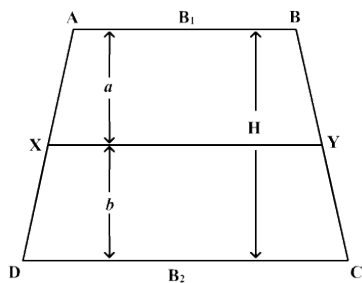


- Sum of the four interior angles is 360°
- Sum of the four exterior angles is 360°

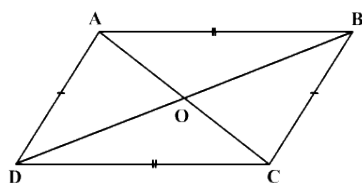
A quadrilateral is said to be a **cyclic quadrilateral** if it is possible to draw a circle passing through all the four vertices of the quadrilateral.

- In a cyclic quadrilateral, opposite angles are supplementary and an external angle is equal to the interior opposite angle.

A quadrilateral in which at least one set of opposite sides is parallel is known as **trapezium** or **trapezoid**. The non-parallel sides if any (AD and BC in the given figure) are known as **oblique sides**. If length of oblique sides is equal, it is known as **isosceles trapezium**. Parallel sides (AB and DC) are generally known as bases and the perpendicular distance between bases is known as height.



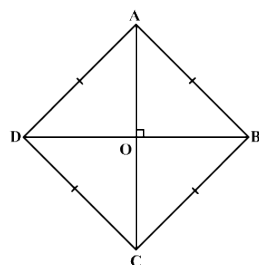
A quadrilateral in which both the sets of opposite sides are parallel to each other is known as a **parallelogram**.



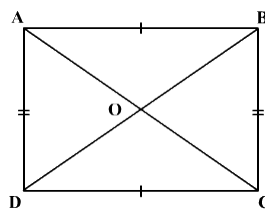
Area (A) = $b \times h$; where b , h represent the base and height of the parallelogram respectively.

- Opposite angles are equal.
- Adjacent angles are supplementary.
- Opposite sides are equal.
- Any of the diagonals (AC or BD) divides the parallelogram into two congruent triangles.
- Both the diagonals bisect each other.
- If we take a point anywhere inside the parallelogram and join it with all the four vertices, we get four triangles such that the sum of the area of the set of vertically opposite triangles is equal.

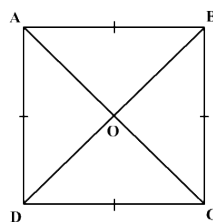
- A parallelogram in which all the sides are of equal length is called a **rhombus**. In a rhombus, the diagonals bisect each other at right angles. Every rhombus is a parallelogram but the converse is not true.



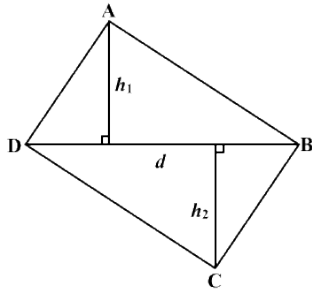
A quadrilateral in which each of the angles is a right angle is called a **rectangle**. In a rectangle, both the diagonals are equal.



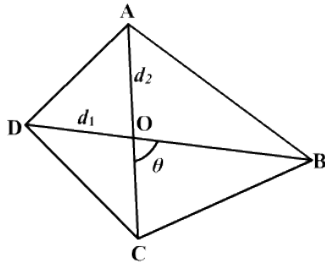
A square is a parallelogram in which all the sides are equal and all the angles are right angles. A square possesses properties of both rhombus and rectangle. Both the diagonals are also equal and they bisect each other perpendicularly.



Area of a quadrilateral

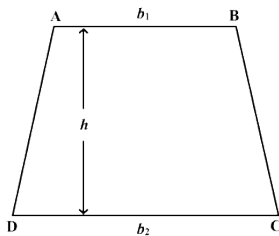


Area (A) = $\frac{1}{2}d(h_1 + h_2)$, where d the diagonal and h_1 and h_2 are the offsets, i.e. perpendiculars drawn to the given diagonal from the other two vertices.



Area (A) = $\frac{1}{2}d_1d_2\sin\theta$, where d_1 and d_2 are the diagonals and θ is the angle between these diagonals.

Area of a trapezium



Area (A) = $\frac{1}{2}(b_1 + b_2)h$, where b_1 and b_2 are the bases and h is the distance between these bases (known as height).

Area of rectangle

Area (A) = lb

Where l is the length and b is the breadth.

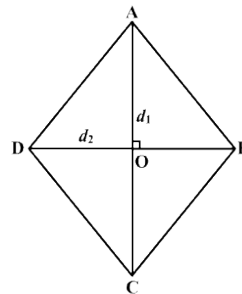
Length of the diagonal, $d = \sqrt{l^2 + b^2}$

Area of square

Area (A) = a^2 , where a is the side

Length of the diagonal, $d = a\sqrt{2}$

Area of rhombus



Area (A) = $\frac{1}{2}d_1d_2$, where d_1 and d_2 are the diagonals.

POLYGON

Basic concepts

A polygon is a closed figure bounded by three or more straight lines, known as sides.

It has as many vertices as the number of sides, with no three of them collinear.

A line joining two non adjacent vertices is known as the **diagonal**.

Properties of polygons

- Number of diagonals in an n sided polygon is given by

$$N_d = \frac{n(n-3)}{2}$$

- The sum total of all the interior angles of any polygon $= (n - 2)\pi$
- The sum total of all the exterior angles of any polygon $= 360^\circ$.

A polygon, in which all the interior angles are less than 180° , is known as a **convex polygon**.

A polygon, in which at least one of the interior angles is more than 180° , is known as a **concave polygon**.

If all the sides and hence all the angles of a polygon are equal, it is known as a **regular polygon**.

A **regular hexagon** with side a can be divided into six equilateral triangles with

side a . The area of each equilateral triangle will be equal to $\frac{\sqrt{3}}{4}a^2$.

Hence, total area will be equal to $6 \times \frac{\sqrt{3}}{4}a^2$

CIRCLE

Basic concepts

A **circle** is a set of all those points in a plane which are equidistant from a given point, called the **centre** of the circle, in the same plane. The distance of any point on the circle from the centre is known as the **radius** of the circle.

A line segment joining two distinct points on the circle is known as a **chord**.

The longest chord that can be drawn in a circle is known as the **diameter**, generally denoted by ' d '. It always passes through the centre of the circle.

A line which lies in the plane of a circle and touches the circle at only one point is known as a **tangent**.

The point at which the tangent touches the circle is called the **point of contact** or **point of tangency**.

A line which passes through two distinct points on a circle is known as a **secant**.

A secant (or a chord) divides the circle in two parts. Each of the two parts along with the two points common with the secant (or chord) is known as an **arc**. The smaller one is the **minor arc** and the larger one is the **major arc**.

An angle made by two points on the circle at the centre is known as a **central angle**.

Two circles in the same plane are known as **concentric circles** when they have the same centre.

Circles having equal radii are called **congruent circles**.

The distance around the curved line which forms the circle is known as the **circumference** of the circle. Circumference is denoted by 'C'

The **circumference of a circle** is given by,
 $C = 2\pi r$

Length of arc, $l = \frac{\theta}{360} \times 2\pi r$, where r is the radius of a circle and θ is the angular measure of the arc in degrees.

The space enclosed by a circle is known as the **area of the circle**. For a circle with radius r , the area is given by, $A = \pi r^2$

The part of a circle that is enclosed by an arc and the two radii joining the end points of that arc to the centre of the circle is known as a **sector**. If the arc of the sector is a minor arc, the sector is known as a **minor sector**. If the arc of the sector is a major arc, the sector is known as a **major sector**.

If r is the radius of a circle and θ is the angular measure of the arc in degrees, then **area of the sector** $A_s = \frac{\theta}{360} \times \pi r^2$

If l is the length of the arc of a circle and r is the radius of the circle, then **area of the sector** $A_s = \frac{lr}{2}$

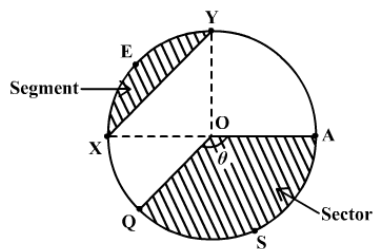
Perimeter of a sector is given by,

$$P_s = l + 2r$$

A chord divides the circular region in two parts, each part known as a **segment**.

Minor segment: The segment of a circle corresponding to a minor arc.

Major segment: The segment of a circle corresponding to a major arc.



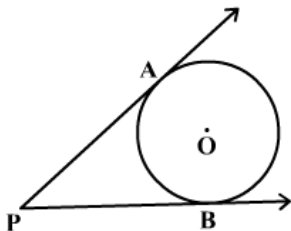
Area of the minor segment (i.e. the segment corresponding to minor arc XEY)
 = area of sector (O-XEY) - area of $\triangle XOY$

$$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin \theta$$

- A tangent at any point on a circle is perpendicular to the radius at the point of contact

- From a point outside a circle, exactly two tangents can be drawn to the circle.

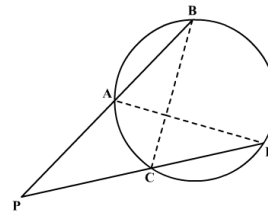
- For example, in the figure below, PA and PB are tangent segments to the circle drawn from the point in the exterior of the circle. Also, $PA = PB$



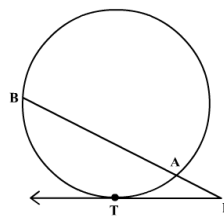
- A perpendicular drawn from the centre of a circle to a chord bisects the chord. Conversely, the segment joining the centre of a circle and midpoint of a chord is perpendicular to the chord.

- Equal chords are equidistant from the centre. Conversely, chords equidistant from the centre are equal in length

- If two secants of a circle passing through point P outside the circle intersect the circle at points A, B, C and D, then $PA \times PB = PC \times PD$ and, $m\angle DPB = \frac{1}{2}\{m(\text{arc } BD) - m(\text{arc } AC)\}$



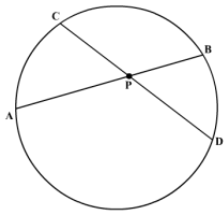
- If PT is a tangent to a circle and a secant passing through P intersects the circle at A and B then $PT^2 = PA \times PB$



- If we draw two chords AB and CD intersecting at a common point P inside the circle, then $PA \times PB = PC \times PD$

$$m\angle DPB = \frac{1}{2}\{m(\text{arc } BD) + m(\text{arc } AC)\}$$

$$m\angle DPA = \frac{1}{2}\{m(\text{arc } AD) + m(\text{arc } BC)\}$$



- Angles subtended by an arc or a chord at distinct points on the circumference, in the same segment, are equal.
- The angle subtended by an arc or a chord, at any point on the circumference, is half of the angle subtended by the same arc or chord at the centre of the circle.

1.1 Quiz

Directions for Q1 and Q2: ABC forms an equilateral triangle in which B is 2 km from A. A person starts walking from B in a direction parallel to AC and stops when he reaches a point D directly east of C. He, then, reverses direction and walks till he reaches a point E directly south of C.

Q1. Then D is

- 3 km east and 1 km North of A
- 3 km east and $\sqrt{3}$ km North of A
- $\sqrt{3}$ km east and 1 km South of A
- $\sqrt{3}$ km west and 3 km North of A

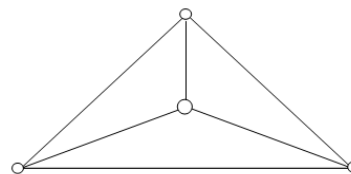
Q2. The total distance walked by the person is

- 3 km
- 4 km
- $2\sqrt{3}$ km
- 6 km

Q3. Consider the five points comprising of the vertices of a square and the intersection point of its diagonals. How many triangles can be formed using these points?

- 4
- 6
- 8
- 10

Q4. Four cities are connected by a road network as shown in the figure. In how many ways can you start from any city and come back to it without travelling on the same road more than once?

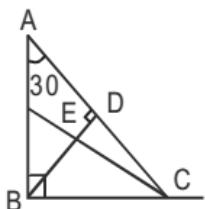


- 8
- 12
- 16
- 20

Q5. The line AB is 6 metres in length and is tangent to the inner one of the two concentric circles at point C. It is known that the radii of the two circles are integers. The radius of the outer circle is

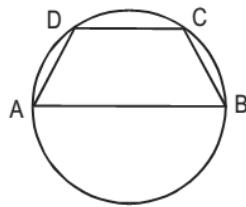
- a. 5 metres
- b. 4 metres
- c. 6 metres
- d. 3 metres

Q6. $AB \perp BC$, $BD \perp AC$ and CE bisects $\angle C$, $\angle A = 30^\circ$. Then what is $\angle CED$?



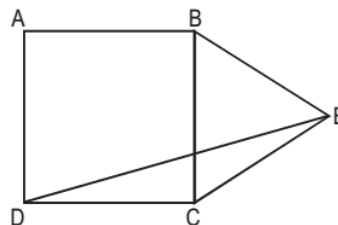
- a. 30°
- b. 60°
- c. 45°
- d. 65°

Q7. In the given figure, AB is diameter of the circle and points C and D are on the circumference such that $\angle CAD = 30^\circ$ and $\angle CBA = 70^\circ$. What is the measure of $\angle ACD$?



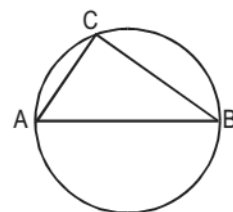
- a. 40°
- b. 50°
- c. 30°
- d. 90°

Q8. If ABCD is a square and BCE is an equilateral triangle, what is the measure of $\angle DEC$?



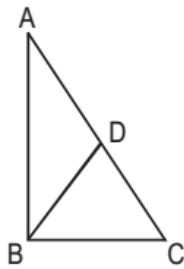
- a. 15°
- b. 30°
- c. 20°
- d. 45°

Q9. The figure shows a circle of diameter AB and radius 6.5 cm. If chord CA is 5 cm long, find the area of $\triangle ABC$.



- a. 60 sq. cm
- b. 30 sq. cm
- c. 40 sq. cm
- d. 52 sq. cm

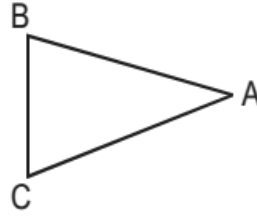
Q10. In $\triangle ABC$, $\angle B$ is a right angle, $AC=6$ cm, and D is the mid-point of AC . The length of BD is



- a. 4 cm
- b. $\sqrt{6}$ cm
- c. 3 cm
- d. 3.5 cm

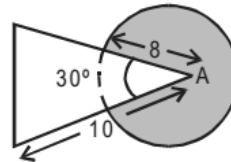
1.2 Worked out Examples

Q1. A cow is tethered at point A by a rope. Neither the rope nor the cow is allowed to enter the triangle ABC. $\angle BAC = 30^\circ$ $AB = AC = 10$ m.



What is the area that can be grazed by the cow if the length of the rope is 8 m?

Solution



Since the length of the rope is more than that of the side AB and AC hence required area will be more than the area calculated as under (area of circle with radius 8) – (area of the sector of the same circle with angle 30°)

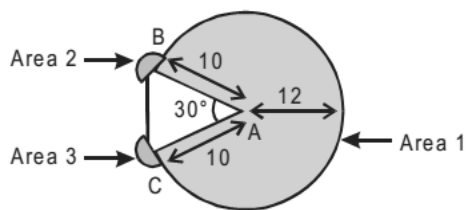
$$\Rightarrow \pi(8)^2 - \frac{30}{360} \pi(8)^2$$

$$= \frac{176\pi}{3}$$

Q2. What is the area that can be grazed by the cow if the length of the rope is 12m?

- a. $133\frac{1}{6}\pi$ sq m
- b. 121π sq m
- c. 132π sq m
- d. $\frac{176}{3}\pi$ sq m

Solution



If the length of the rope is 12 m, then the total area that can be grazed by the cow is as depicted in the diagram. Area 1 is (the area of the circle with radius 12) – (Area of the sector of the same circle with angle 30°)

$$\Rightarrow \pi(12)^2 - \frac{30}{360}\pi(12)^2 = 132\pi$$

Since the length of the rope is higher than the sides of the triangle (viz. AB and AC), if the cow reaches point B or C, there would still be a part of the rope $(12 - 10) = 2$ m in length. With this extra length available the cow can further graze an area equivalent to some part of the circle with radius = 2 m from both points, i.e. B and C. This is depicted as area 2 and area 3 in the diagram. Hence, the actual area grazed will be slightly more than 132π . The only answer choice that supports this is (a).

Q3. Euclid has a triangle in mind. Its longest side was length 20 and another of its sides has length 10. Its area is 80. What is the exact length of its third side?

Solution

Let the 3rd side be x .

Then by using area of triangle formula

Area =

$$\sqrt{s(s-20)(s-10)(s-x)} = 80 \text{ --- (1),}$$

$$\text{where } s = \frac{20+10+x}{2}$$

Solving (1), we get $x = \sqrt{260}$

Q4. Neeraj has agreed to mow the farm lawn, which is a 20 m by 40 m rectangle. The mower mows a 1 m wide strip. If Neeraj starts at one corner and mows around the lawn towards the centre, about how many times would he go round before he has mowed half the lawn?

Solution

Area of the plot = $20 \times 40 = 800$ sq. m

Area left after moving 2 rounds = $(20-4) \times (40-4) = 576$ sq. m

Similarly area left after moving 4 rounds = $(20-6) \times (40-6) = 384$ sq. m

Hence, area should be slightly less than 4 rounds.

Q5. One angle of pentagon is 140° . If the remaining angles are in the ratio 1:2:3:4. The size of the greatest angle is

Solution

One angle of the pentagon is 140° . Since the remaining angles are in the ratio 1:2:3:4. Let the remaining angles be x° , $2x^\circ$, $3x^\circ$ and $4x^\circ$.

Sum of interior angles of a pentagon =

$$(5 - 2)\pi = 540^\circ.$$

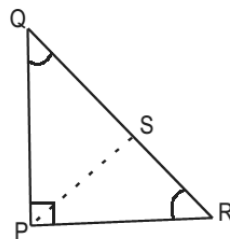
$$\therefore x^\circ + 2x^\circ + 3x^\circ + 4x^\circ + 140^\circ = 540^\circ$$

$$\therefore x = 40$$

\therefore The angles of the pentagon are

$140^\circ, 40^\circ, 80^\circ, 120^\circ$ and 160° .

Hence the size of the greatest angle = 160°



I. Triangle PQS and Triangle RPS are similar.

II Triangle PSQ and Triangle RSP are congruent.

III Triangle PSQ and Triangle RPQ are similar.

Mark the correct option

- a. I and II are correct
- b. I and III are incorrect
- c. Only III is correct
- d. All three are correct

Q3. A wire, if bent into a square, enclose an area of 484 cm^2 . This wire is cut into two pieces; with the bigger piece having a length three-fourth of the original wire's length. Now, if a circle and a square are formed with the bigger and the smaller piece respectively, what should be the area enclosed by the two pieces?

- a. 464 cm^2
- b. 544.25 cm^2
- c. 376.75 cm^2
- d. 424.25 cm^2

1.3 Class Work Problems

Q1. The area of an isosceles triangle is 12 sq. cm . If one of the equal sides is 5 cm long, mark all the options which can give the length of the base.

- a. 4 cm
- b. 6 cm
- c. 8 cm
- d. 9 cm

Q2. In the right-angled triangle QPR given below, PS is the altitude to the hypotenuse. The figure is followed by three possible inferences.

Q4. A spiral is made up of 13 successive semicircles, with center alternately at A and B, starting with center at A. The radii of semicircles, thus developed, are 0.5 cm, 1.0 cm, 1.5 cm, and 2.0 cm and so on. The total length of the spiral is:

- a. 144 cm
- b. 143 cm
- c. 147 cm
- d. None of the above

Q5. The interior angles of a polygon are in Arithmetic Progression. If the smallest angle is 120° and common difference is 5° , then number of sides in the polygon is:

- a. 7
- b. 8
- c. 9
- d. None of the above

Q6. A ladder 25 metres long is placed against a wall with its foot 7 metres away from the foot of the wall. How far should the foot be drawn out so that the top of the ladder may come down by half the distance of the total distance if the foot is drawn out?

- a. 6 metres
- b. 8 metres

- c. 8.75 metres
- d. None of the above

Q7. A pole has to be erected on the boundary of a circular park of diameter 13 metres in such a way that the difference of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. The distance of the pole from one of the gates is:

- a. 8 metres
- b. 8.25 metres
- c. 5 metres
- d. None of the above

Q8. If D is the midpoint of side BC of a triangle ABC and AD is the perpendicular to AC then:

- a. $3AC^2 = BC^2 - AB^2$
- b. $3BC^2 = AC^2 - 3AB^2$
- c. $BC^2 + AC^2 = 5AB^2$
- d. None of the above

Q9. Mohan was playing with a square cardboard of side 2 metres. While playing, he sliced off the corners of the cardboard in such a manner that a figure having all its

sides equal was generated. The area of this eight sided figure is:

- a. $\frac{4\sqrt{2}}{\sqrt{2}+1}$
- b. $\frac{4}{\sqrt{2}+1}$
- c. $\frac{2\sqrt{2}}{\sqrt{2}+1}$
- d. $\frac{8}{\sqrt{2}+1}$

Q10. Find the ratio of shaded area to unshaded area.



- a. $\frac{1}{5}(\sqrt{21} - 2)$
- b. $\frac{1}{5}(3\sqrt{7} - 2)$
- c. $\frac{1}{5}(3\sqrt{7} - 2\sqrt{3})$
- d. None of the above

Q11. An arc AB of a circle subtends an angle 'x' radian at the center O of the circle. If the area of the sector AOB is equal to the square of the length of the arc AB, then x is:

- a. 0.5
- b. 1.0
- c. 0.75
- d. None of the above

Q12. If there is threefold increase in all the side of a cyclic quadrilateral, then the percentage increase in its area will be:

- a. 81%
- b. 9%
- c. 900%
- d. None of the above

Q13. In a square of side 2 metres, isocoles triangles of equal area are cut from the corners to form a regular octagon. Find the perimeter and area of the regular octagon.

- a. $\frac{16}{2+\sqrt{2}}; \frac{4(1+\sqrt{2})}{3+2\sqrt{2}}$
- b. $\frac{8}{2+\sqrt{2}}; \frac{2(1+\sqrt{2})}{3+2\sqrt{2}}$
- c. $\frac{16}{1+\sqrt{2}}; \frac{3(1+\sqrt{2})}{3+2\sqrt{2}}$
- d. Non e of these

Q14. In a triangle ABC the length of side BC is 295. If the length of side AB is a perfect square, then the length of side AC is a power of 2, and the length of side AC is twice the length of side AB. Determine the perimeter of the triangle.

- a. 343
- b. 487
- c. 1063

d. None of these

Q15. In a circular field, there is a rectangular tank of length 130 m and breadth 110 m. If the area of the land portion of the field is 20350m^2 then the radius of the field is

- a. 85 m
- b. 95 m
- c. 105 m
- d. 115 m

Q16. Eight points lie on the circumference of a circle. The difference between the number of triangles and the number of quadrilaterals that can be formed by connecting these points is

- a. 7
- b. 14
- c. 32
- d. 84

Q17. The cost of levelling and turfing a square field at Rs. 160 per hectare is Rs. 2624.40. The cost of surrounding it with a railing costing 25 paise per metre is:

- a. Rs. 350
- b. Rs. 375

c. Rs. 400

d. Rs. 405

Q18. The radius of a circle is so increased that its circumference increased by 5%. The area of the circle then increases by

- a. 12.5%
- b. 10.25%
- c. 10.5%
- d. 11.25%

Q19: The diagonal of a square is $4\sqrt{2}\text{cm}$. The diagonal of another square whose area is double that of the first square is

- a. 8 cm
- b. $8\sqrt{2}\text{ cm}$
- c. $4\sqrt{2}\text{ cm}$
- d. 16 cm

Q20. If C_1 and C_2 are the circumferences of the outer and inner circles respectively, what is $C_1:C_2$?

- (I) The two circles are concentric
 - (II) The area of the ring is $\frac{2}{3}$ the area of greater circle
- a. If statement I alone is sufficient to answer the question.

b. If statement II alone is sufficient to answer the question.

c. If both statement I and II together are necessary to answer the question.

d. If both statements I and II together are not sufficient to answer the question.

Q21. A rectangular park 60 m long and 40 m wide has two concrete crossroads running in the middle of the park and rest of the park has been used as a lawn. If the area of the lawn is 2109 sq m, then what is the width of the road?

a. 2.91 m

b. 3 m

c. 5.82 m

d. None of these

Q22. One side of an equilateral triangle is 24cm. The midpoints of its sides are joined to form another triangle whose midpoints are in turn joined to form still another triangle. This process continues indefinitely. Find the sum of the perimeters of all the triangles.

a. 144 cm

b. 72 cm

c. 536 cm

d. 676 cm

Q23. The length of a height 3m and the stool is kept 9m away from the wall it is leaning against. If lower end of the ladder is kept on a stool of height 3m and the stool is kept 9m away from the wall, the upper end of the ladder coincides with the top of the wall. Then, the height of the wall is :

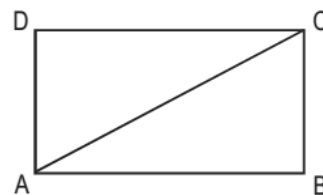
a. 12 m

b. 15 m

c. 18 m

d. 11 m

Q24. In the adjoining figure, $AC + AB = 5AD$ and $AC - AD = 8$. Then, the area of the rectangle ABCD is :



a. 36

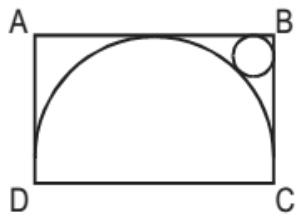
b. 50

c. 60

d. cannot be answered

Q25. The figure shows the rectangle ABCD with a semicircle and a circle inscribed

inside in it as shown. What is the ratio of the area of the circle to that of the semicircle?

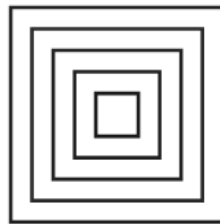


- a. $(\sqrt{2} - 1)^2 : 1$
- b. $2(\sqrt{2} - 1)^2 : 1$
- c. $(\sqrt{2} - 1)^2 : 2$
- d. None of these

Q26. From a circular sheet of paper with a radius 20 cm, four circles are cut out. What is the ratio of the uncut to the cut portion?

- a. 1:3
- b. 4:1
- c. 3:1
- d. 4:3

Q27. The adjoining figures shows a set of concentric squares. If the diagonal of the innermost square is 2 units, and if the distance between the corresponding corners of any two successive squares is 1 unit, find the difference between the areas of the eight and the seventh squares. Counting from the innermost square.



- a. $10\sqrt{2}$ sq unit
- b. 30 sq units
- c. $35\sqrt{2}$ sq units
- d. None of these

Q28. The sum of the areas of two circles, which touch other externally, is 153π . If the sum of their radii is 15, find the ratio of the larger to the smaller radius.

- a. 4
- b. 2
- c. 3
- d. None of these

Q29. Consider a circle with unit radius. There are seven adjacent sectors, $S_1, S_2, S_3, \dots, S_7$, in the circle such that their total area is $1/8$ of the area of the circle. Further, the area of the j^{th} sector is twice that of the $(j-1)^{\text{th}}$ sector, for $j = 2, \dots, 7$. What is the angle, in radians, subtended by the arc of S_1 , at the centre of the circle?

- a. $\frac{\pi}{508}$
- b. $\frac{\pi}{2040}$

- c. $\frac{\pi}{1016}$
 d. $\frac{\pi}{1524}$

Q30. A rectangular pool 20 metres wide and 60 metres long is surrounded by a walkway of uniform width. If the total area of the walkway is 516 square metres, how wide, in metres, is the walkway?

- a. 43
 b. 43
 c. 3
 d. 3.5

1.3 Exercise

Q1. What is the number of distinct triangles with integral valued sides and perimeter 14?

- a. 6
 b. 5
 c. 4
 d. 3

Q2. A ladder leans against a vertical wall. The top of the ladder is 8 m above the ground. When the bottom of the ladder is moved 2 m farther away from the wall, the top of the ladder rests against the foot of the wall. What is the length of the ladder?

- a. 10 m
 b. 15 m
 c. 20 m
 d. 17 m

Q3. Consider obtuse-angled triangles with sides 8 cm, 15 cm and x cm. If x is an integer, then how many such triangles exist?

- a. 5
 b. 21
 c. 10
 d. 15
 e. 14

Q4. How many differently shaped triangles exist in which no two sides are of the same length, each side is of integral unit length and the perimeter of the triangle is less than 14 units?

- a. 3
 b. 4
 c. 5
 d. 6
 e. None of the above.

Q5. If a, b, c are the sides of a triangle, and $a^2 + b^2 + c^2 = bc + ca + ab$, then the triangle is

- a. equilateral
- b. isosceles
- c. right angled
- d. obtuse angled

Q6. Euclid has a triangle in mind, Its longest side has length 20 and another of its sides has length 10. Its area is 80. What is the exact length of its third side?

- a. $\sqrt{260}$
- b. $\sqrt{250}$
- c. $\sqrt{240}$
- d. $\sqrt{270}$

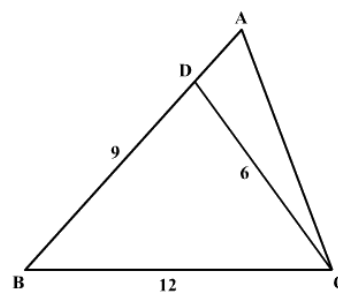
Q7. The internal bisector of an angle A in a triangle ABC meets the side BC at point D . $AB = 4$, $AC = 3$ and $\angle A = 60^\circ$. Then what is the length of the bisector AD ?

- a. $\frac{12\sqrt{3}}{7}$
- b. $\frac{12\sqrt{13}}{7}$
- c. $\frac{4\sqrt{13}}{7}$
- d. $\frac{4\sqrt{3}}{7}$

Q8. A piece of paper is in the shape of a right angled triangle and is cut along a line that is parallel to the hypotenuse, leaving a smaller triangle. There was a 35% reduction in the length of the hypotenuse of the triangle. If the area of the original triangle was 34 square inches before the cut, what is the area (in square inches) of the smaller triangle?

- a. 16.665
- b. 16.565
- c. 15.465
- d. 14.365

Q9. Consider the triangle ABC shown in the following figure where $BC = 12$ cm, $DB = 9$ cm, $CD = 6$ cm and $\angle BCD = \angle BAC$. What is the ratio of the perimeter of the triangle ADC to that of the triangle BDC ?



- a. $7/9$
- b. $8/9$
- c. $6/9$
- d. $5/9$

Q10. In a triangle ABC, the lengths of the sides AB and AC equal 17.5 cm and 9 cm respectively. Let D be a point on the line segment BC such that AD is perpendicular to BC. If $AD = 3$ cm, then what is the radius (in cm) of the circle circumscribing the triangle ABC?

- a. 17.05
- b. 27.85
- c. 22.45
- d. 32.25
- e. 26.25

Q11. Triangle ABD is right-angled at B. On AD there is a point C for which $AC = CD$ and $AB = BC$. The magnitude of angle DAB, in degrees, is :

- a. $67\frac{1}{2}$
- b. 60
- c. 45
- d. 30

Q12. ABCD is a square with sides of length 10 units. OCD is an isosceles triangle with base CD. OC cuts AB at point Q and OD cuts AB at point P. The area of trapezoid

PQCD is 80 square units. The altitude from O of the triangle OPQ is:

- a. 12
- b. 13
- c. 14
- d. 15
- e. None of the above.

Q13. The diagonal of a square is $4\sqrt{2}$ cm. The diagonal of another square whose area is double that of the first square is

- a. 8 cm
- b. $8\sqrt{2}$ cm
- c. $4\sqrt{2}$ cm
- d. 16 cm

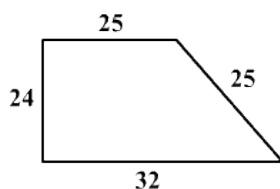
Q14. A square, whose side is 2 metres, has its corners cut away so as to form an octagon with all sides equal. Then the length of each side of the octagon, in metres is

- a. $\frac{\sqrt{2}}{\sqrt{2}+1}$
- b. $\frac{2}{\sqrt{2}+1}$
- c. $\frac{\sqrt{2}}{\sqrt{2}-1}$
- d. $\frac{2}{\sqrt{2}-1}$

Q15. Any five points are taken inside or on a square of side 1. Let a be the smallest possible number with the property that it is always possible to select one pair of points from these five such that the distance between them is equal to or less than a . Then a is:

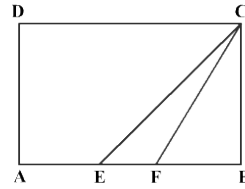
- a. $\frac{\sqrt{3}}{3}$
- b. $\frac{\sqrt{2}}{2}$
- c. $\frac{2\sqrt{2}}{3}$
- d. 1

Q16. Two sides of a plot measure 32 metres and 24 metres and the angle between them is a perfect right angle. The other two sides measure 25 metres each and the other three are not right angles.



What is the area of the plot?

- a. 768
- b. 534
- c. 696.5
- d. 684



Q17. In the above diagram, ABCD is a rectangle with $AE = EF = FB$. What is the ratio of the area of the triangle CEF and that of the rectangle?

- a. $1/6$
- b. $1/8$
- c. $1/9$
- d. None of these

Q18. Consider a square S which is inside a circle A such that the four corner points of the square touch the circumference of the circle. A second circle B is inside the square S so that its four sides touches the circumference of B . Then, the ratio of the areas of the circles $A : B$ equals:

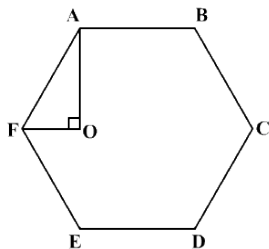
- a. $\sqrt{2} : 1$
- b. $2 : 1$
- c. $\sqrt{3} : 1$
- d. $\pi : 1$

Q19. Each side of a given polygon is parallel to either the X or the Y axis. A corner of such a polygon is said to be convex if the

internal angle is 90° or concave if the internal angle is 270° . If the number of convex corners in such a polygon is 25, the number of concave corners must be

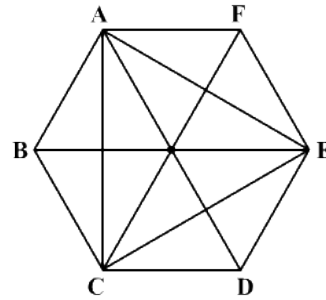
- a. 20
- b. 0
- c. 21
- d. 22

Q20. In the figure below, ABCDEF is a regular hexagon and $\angle AOF = 90^\circ$. FO is parallel to ED. What is the ratio of the area of the triangle AOF to that of the hexagon ABCDEF?



- a. $1/12$
- b. $1/6$
- c. $1/24$
- d. $1/18$

Q21. Let ABCDEF be a regular hexagon. What is the ratio of the area of the triangle ACE to that of the hexagon ABCDEF?



- a. $1/3$
- b. $1/2$
- c. $2/3$
- d. $5/6$

Q22. Four points A, B, C and D lie on a straight line in the X-Y plane, such that $AB = BC = CD$, and the length of AB is 1 metre. An ant at A wants to reach a sugar particle at D. But there are insect repellents kept at points B and C. The ant would not go within one metre of any insect repellent. The minimum distance in metres the ant must traverse to reach the sugar particle is

- a. $3\sqrt{2}$
- b. $1 + \pi$
- c. $4\pi/3$
- d. 5

Q23. Two identical circles intersect so that their centres, and the points at which they intersect, form a square of side 1 cm. The

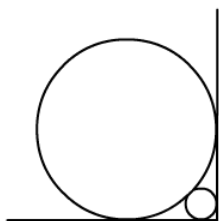
area in sq. cm of the portion that is common to the two circles is

- a. $\pi/4$
- b. $\pi/2 - 1$
- c. $\pi/5$
- d. $\sqrt{2} - 1$

Q24. Two tangents are drawn to a circle from an exterior point A; they touch the circle at points B and C, respectively. A third tangent intersects segment AB in P and AC in R, and touches the circle at Q. If $AB = 20$, then the perimeter of $\triangle APR$ is:

- a. 42
- b. 40.5
- c. 40
- d. Not determined by the given information

Q25. A circle with radius 2 is placed against a right angle. Another smaller circle is also placed as shown in the adjoining figure. What is the radius of the smaller circle?



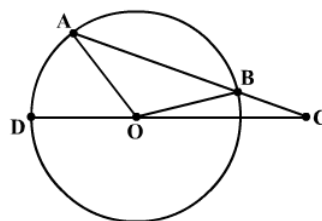
- a. $3 - 2\sqrt{2}$
- b. $4 - 2\sqrt{2}$

- c. $7 - 4\sqrt{2}$
- d. $6 - 4\sqrt{2}$

Q26. What is the distance in cm between two parallel chords of lengths 32 cm and 24 cm in a circle of radius 20 cm?

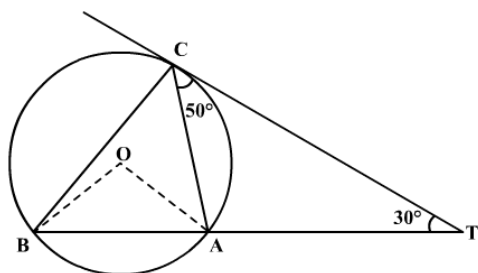
- a. 1 or 7
- 1 or 14
- 2 or 21
- 3 or 28

Q27. In the figure given below, AB is the chord of a circle with centre O. AB is extended to C such that $BC = OB$. The straight line CO is produced to meet the circle at D. If $\angle ACD = y^\circ$ and $\angle AOD = x^\circ$ such that $x = ky$, then the value of k is



- a. 3
- b. 2
- c. 1
- d. None of the above

Q28. In the figure given below (not drawn to scale), A, B and C are three points on a circle with centre O. The chord BA is extended to a point T such that CT becomes a tangent to the circle at point C. If $\angle ATC = 30^\circ$ and $\angle ACT = 50^\circ$, then the angle $\angle BOA$ is:

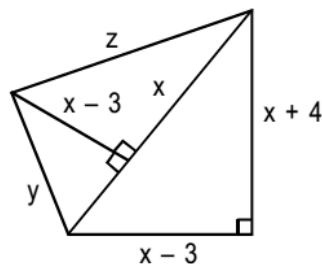


- a. 100°
- b. 150°
- c. 80°
- d. Cannot be determined

Q29. There is a common chord of 2 circles with radius 15 and 20. The distance between the two centres is 25. The length of the chord is

- a. 48
- b. 24
- c. 36
- d. 28

Q30. Based on the figure below, what is the value of x, if $y = 10$.



- a. 10
- b. 11
- c. 12
- d. None of these.

Q31. A piece of paper is in the shape of a right angled triangle and is cut along a line that is parallel to the hypotenuse leaving a smaller triangle. There was a 35% reduction in the length of the hypotenuse of the triangle. If the angled of the original triangle was 34 square inches before the cut, what is the area (in square inches) of the smaller triangle?

- a. 16.665
- b. 16.565
- c. 15.465
- d. 14.365

Q32. Four friends start from four towns, which are at the four corners of an imaginary rectangle. They meet at a point which falls inside the rectangle, after traveling the distances of 40m, 50 m and

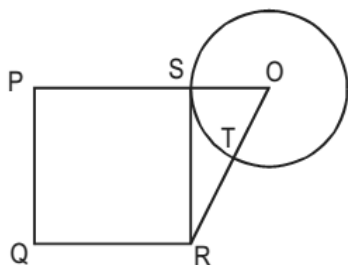
60m. The maximum distance that the fourth could have traveled is approximately :

- a. 67m
- b. 52 m
- c. 22.5 m
- d. Cannot be determined

Q33. The sides of a triangle are 5, 12 and 13 units. A rectangle is constructed, which is equal in area to the triangle, has a width of 10 units. Then, the perimeter of the rectangle is :

- a. 30 units
- b. 36 units
- c. 13 units
- d. None of these

Q34. PQRS is a square. SR is a tangent (at point S) to the circle with centre O and $TR = OS$. Then, the ratio of area of the circle to the area of the square is :



- a. $\pi/3$
- b. $11/7$
- c. $3/\pi$

d. $7/11$.

Q35. Which one of the following cannot be the ratio of angles in a right- angled triangle?

- a. 1 : 2 : 3
- b. 1 : 1 : 2
- c. 1 : 3 : 6
- d. None of these

Q36. In a rectangle, the difference between the sum of the adjacent sides and the diagonal is half the length of longer side. What is the ratio of the shorter to the longer side?

- a. $\sqrt{3} : 2$
- b. $1 : \sqrt{3}$
- c. 2 : 5
- d. 3 : 4

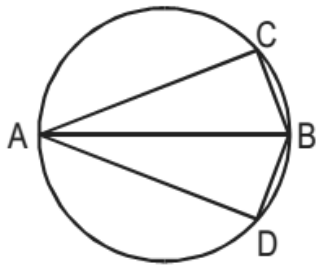
Q37. In ΔABC , points P, Q and R are the mid-points of sides AB, BC and CA respectively. If area ΔABC , is 20 sq. units, find the area of ΔPQR .

- a. 10 sq units
- b. $5\sqrt{3}$ sq units
- c. 5 sq units
- d. None of these

Q38. The value of each of a set of coins varies as the square of its diameter, if its thickness remains constant, and varies as the thickness, if the diameter remain constant. If the diameter of two coins are in the ratio 4:3, what should be the ratio of their thickness be if the value of the first is four times that of the second?

- a. 16:9
- b. 9:4
- c. 9:16
- d. 4:9

Q39. AB is the diameter of the given circle, while points C and D lie on the circumferences as shown. If AB is 15 cm. AC is 12 cm and BD is 9 cm, find the area of quadrilateral ACBD.



- a. 54π sq cm
- b. 216π sq cm
- c. 162π sq cm
- d. None of these

Q40. Four identical coins are placed in a square. For each coin, the ratio of area to circumference is same as the ratio of circumference to area. Then, find the area of the square that is not covered by the coins.

- a. $16(\pi - 1)$
- b. $16(8 - \pi)$
- c. $16(4 - \pi)$
- d. $16\left(4 - \frac{\pi}{2}\right)$

Answers - Geometry

1.1 Quiz

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q10	

1.3 Class Work Problems

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12
Q13	Q14	Q15
Q16	Q17	Q18
Q19	Q20	Q21
Q22	Q23	Q24
Q25	Q26	Q27
Q28	Q29	Q30

1.4 Exercise

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12

Q13	Q14	Q15
Q16	Q17	Q18
Q19	Q20	Q21
Q22	Q23	Q24
Q25	Q26	Q27
Q28	Q29	Q30
Q31	Q32	Q33
Q34	Q35	Q36
Q37	Q38	Q39
Q40		

CHAPTER 2

SIMPLE INTEREST & COMPOUND INTEREST

INTRODUCTION

Money borrowed today is repaid with a higher amount tomorrow. This gives rise to the concept of interest. The amount of money which the creditor lends initially is known as the **Principal (P)** or **Capital** and the time frame for which he lends the money is known as **Time** or **Period (T or n)**.

The difference between the amount of money which the debtor borrows today (i.e. the principal) and the amount of money which he needs to repay at the end of the time period is called the **Interest (I)** over the Principal amount. Also, the total money which he repays at the end is termed as the **Amount (A)**.

In other words, $\text{Amount} = \text{Principal} + \text{Interest}$.

The Interest is calculated based on the **Rate of Interest (R)**, which is specified in terms of percent per annum (p. c. p. a) unless specified otherwise.

SIMPLE INTEREST (SI)

The interest calculated for the given time duration only on the original principal, is called Simple interest.

$$\text{Simple Interest} = \frac{P \times R \times T}{100}$$

$$\text{Amount} = P + I = P + \frac{P \times R \times T}{100}$$

COMPOUND INTEREST (CI)

When money is lent at compound interest, at the end of a fixed period, the interest for that fixed period is added to the principal, and this amount is considered to be the principal for the next year or period. This is repeated until the amount for the last period has been calculated. The difference between the final amount and the original principal is the Compound Interest (CI). This amount can be calculated using the following formula:

$$\text{Amount} = A = P \left(1 + \frac{r}{100}\right)^n$$

- The frequency of compounding can vary. It can be done half yearly (semi-annually), quarterly, monthly etc. When compounding is done more than once a year, the rate of interest for that time period will be less than the effective rate of interest for the entire year. For example, if the annual rate of

interest is 12%, then the rate of interest when the amount is compounded half-yearly will be 6%. Thus, as the time period of compounding doubles, the corresponding rate of interest is halved.

- For half yearly rate, $A = P \left(1 + \frac{r/2}{100}\right)^{2n}$
- For quartely rate, $A = P \left(1 + \frac{r/4}{100}\right)^{4n}$
- For monthly rate, $A = P \left(1 + \frac{r/12}{100}\right)^{12n}$
- If the interest is calculated k times in a year, $A = P \left(1 + \frac{r/k}{100}\right)^{kn}$

DEPRECIATION OF VALUE

The value of any asset decreases with time due to any of a number of factors including wear and tear, outdated technology, usage etc. This decrease is called its depreciation. If P is the original value and r is the rate of depreciation per year, then the final value (F) after n number of years is given by the formula, $F = P \times \left(1 - \frac{r}{100}\right)^n$

2.1 Quiz

Q1. Mr. Amir has Rs.25,000 with him. He invested this amount in two banks A and B at simple interest rate of 8% per annum and

at compound interest rate of 10% per annum respectively. If the total amount of interest earned by him from two banks in 2 years is Rs. 4,750, then what was the amount invested by him in bank A?

- Rs.10,000
- Rs.9,000
- Rs.10,500
- Rs.15,000
- Rs.16,500

Q2. A sum of money lent at compound interest amounts to Rs. 578.40 in 2 years and to Rs. 614.55 in

3 years. If at the same interest rate, the person lends Rs. 16,000 for 3 years at simple interest, then approximately how much interest will he receive?

- Rs.2,500
- Rs.3,000
- Rs.4,000
- Rs.3,300
- None of these

Q3. A sum of money lent out at compound interest increases in value by 50% in 5 years. A person wants to lend three different sums of money x , y and z for 10, 15 and 20 years respectively at the above rate in such a

way that he gets back equal sums of money at the end of their respective periods. What is the ratio of x , y and z ?

- a. $8 : 5 : 3$
- b. $7 : 2 : 1$
- c. $9 : 6 : 4$
- d. $3 : 2 : 1$
- e. None of these

Q4. An amount becomes 3 times in 6 years with fixed rate of simple interest. In how many years it will become 24 times?

- a. 72 years
- b. 69 years
- c. 50 years
- d. 70 years
- e. 48 years

Q5. What is the value of the rate of interest if the difference between the compound interests of first and second year is 4 times that of the principal?

- a. 10%
- b. 200%
- c. 300%
- d. 30%
- e. 100%

Q6. The difference between the compound interest and the simple interest on a certain sum at 10% p.a. For two years is Rs.90. What is the value of this sum?

- a. Rs.1200
- b. Rs.6000
- c. Rs.6900
- d. Rs.8500
- e. Rs.9,000

2.2 Worked out Examples

Q1. Calculate the amount on Rs. 4480 at 8% per annum for 3 years.

Solution

$$\text{Simple Interest} = \frac{P \times N \times R}{100} = \frac{4480 \times 3 \times 8}{100} = \text{Rs. } 1075.20$$

$$\therefore \text{Amount} = \text{Rs. } (4480 + 1075.20) = \text{Rs. } 5555.20$$

Q2. A certain sum of money at simple interest amounts to Rs. 1260 in 2 years and to Rs.1350 in 5 years. What is the rate percent per annum?

Solution

$$\text{S.I. for 3 years} = \text{Rs. } (1350 - 1260) = \text{Rs. } 90$$

$$\therefore \text{S.I. for 2 years} = \text{Rs. } 90 \times \frac{2}{3} = \text{Rs. } 60$$

$$\therefore \text{Principal} = \text{Rs. } (1260 - 60) = \text{Rs. } 1200$$

$$R = \frac{I \times 100}{P \times N} = \frac{100 \times 60}{1200 \times 2} = 2.5 \%$$

Q3. If the compound interest on a certain sum for 3 years at 20% per annum is Rs. 728, find the simple interest.

Solution

$$\text{Given, Rs. 728} = P \left[\left(1 + \frac{20}{100} \right)^3 - 1 \right]$$

$$\therefore \text{Principal (P)} = \text{Rs. 1000}$$

$$\therefore \text{Simple Interest} = \frac{1000 \times 3 \times 20}{100} = \text{Rs. 600}$$

Q4. A sum was put at simple interest at a certain rate for 2 years. Had it been put at 1% higher rate, it would have fetched Rs. 24 more. Find the sum.

Solution

Let the sum be Rs. x and rate be $R\%$ and $(R + 1)\%$. Then, we have

$$\frac{x \times 2 \times (R + 1)}{100} - \frac{x \times 2 \times R}{100} = \text{Rs. 24}$$

$$\text{Solving, } x = \text{Rs. 1200}$$

Q5. The difference between the compound interest and the simple interest on a certain sum at 10% per annum for two years is Rs. 60. Find the sum.

Solution

Let the sum be Rs. x

$$\text{Simple Interest} = \frac{x \times 2 \times 10}{100} = \text{Rs. } \frac{x}{5}$$

Compound Interest =

$$\text{Rs. } x \left[\left(1 + \frac{10}{100} \right)^2 - 1 \right] = \text{Rs. } \frac{21 \times x}{100}$$

$$\therefore \text{Rs. } \left[\frac{21 \times x}{100} - \frac{x}{5} \right] = \text{Rs. 60.}$$

$$\therefore x = \text{Rs. 6000}$$

2.3 Class Work Problems

Q1. The rate of inflation was 1000%. Then, what will be the cost of an article, which costs 6 units of currency now, 2 years from now?

a. 666

b. 660

c. 720

d. 726

Q2. A man invests Rs. 3000 at the rate of 5% per annum. How much more should he invest at the rate of 8%, so that he can earn a total of 6% per annum?

a. Rs. 1200

b. Rs. 1300

c. Rs. 1500

d. Rs. 2000

Q3. Ram purchased a flat at Rs. 1 lakh and Prem purchased a plot of land worth Rs. 1.1 lakh. The respective annual rates at which

the prices of the flat and the plot increased were 10% and 5%. After two years, they exchanged their belongings and one paid the other, the difference. Then:

- a. Ram paid Rs. 275 to Prem*
- b. Ram paid Rs. 475 to Prem*
- c. Ram paid Rs. 375 to Prem*
- d. Prem paid Rs. 475 to Ram.*

Q4. A yearly payment to the servant is Rs. 90 plus one turban. The servant leaves the job after 9 months and receives Rs. 65 and a turban. Then find the price of the turban?

- a. Rs. 10*
- b. Rs. 15*
- c. Rs. 7.50*
- d. Cannot be determined*

Q5. Mr. Jeevan wanted to give some amount of money to his two children, so that although today they may not be using it, in the future the money would be of use to them. He divides a sum of Rs. 18,750/- between his two sons of age 10 years and 13 years respectively in such a way that each would receive the same amount at 3% p.a. Compound interest when he attains the age of 30 years. What would be the original share of the younger son?

- a. 8959.80*
- b. 8559.80*
- c. 8969.80*
- d. 8995.80*

Q6. In 4 years, Rs. 6000 amounts to Rs. 8000. In what time at the same rate will Rs. 525 amount to Rs. 700?

- a. 2 years*
- b. 3 years*
- c. 4 years*
- d. 5 years*

Q7. Mungeri Lal has two investment plans- A and B, to choose from. Plan A offers interest of 10% compounded annually while plan B offers simple interest of 12% per annum. Till how many years is plan B a better investment?

- a. 3*
- b. 4*
- c. 5*
- d. 6*
- e. 7*

Q8. Bennett distribution company, a subsidiary of a major cosmetics manufacturer Bavlton, is forecasting the zonal sales for the next year. Zone I with

current yearly sales of Rs.193.8 lakh is expected to achieve a sales growth of 7.25%; Zone II with current sales of Rs.79.3 lakh is expected to grow by 8.2%; and Zone III with sales of Rs.57.5 lakh is expected to increase sales by 7.15%. What is the Bennett's expected sales growth for the next year?

- a.7.46%
- b.7.53%
- c.7.88%
- d.7.41%

Q9. ICICI bank offers a 1-year loan to a company at an interest rate of 20 % payable at maturity, while Citibank offers on a discount basis at a 19% interest rate for the same period. How much should the ICICI Bank decrease/increase the interest rate to match up the effective interest rate of Citibank?

- a.Increase by 3.5%
- b.Decrease by 1.8%
- c.Increase by 1%
- d.Decrease by 1.4%

Q10. Mr. Mehra is planning for higher education expenses of his two sons aged 15 and 12. He plans to divide Rs 15 lakhs in two equal parts and invest in two

different plans such that his sons may have access to Rs. 21 lakhs each when they reach the age of 21. He is looking for plan that will give him a simple interest per annum. The rates of interest of the plans for his younger son and elder son should be

- a.5% and 7.5% respectively
- b.8% and 12% respectively
- c.10% and 15% respectively
- d.15% and 22.5% respectively
- e.20% and 30% respectively

Q11. Mr. Mishra invested Rs.25,000 in two fixed deposits X and Y offering compound interest @ 6% per annum and 8% per annum respectively. If the total amount of interest accrued in two years through both fixed deposits is Rs.3518, the amount invested in Scheme X is

- a.Rs. 12,000
- b.Rs. 13,500
- c.Rs. 15,000
- d.Cannot be determined

Q12. A man received Rs. 12000 as Puja Bonus. He invested a part of it at 5% per annum and the remaining at 6% per annum, simple interest being allowed in

each case. The total interest earned by him in 4 years is Rs. 2580. The sum invested at 5% per annum is

- a. Rs. 4500
- b. Rs. 4000
- c. Rs. 7500
- d. Rs. 8000

Q13. A finance company declares that, at a certain compound interest rate, a sum of money deposited by anyone will become 8 times in three years. If the same amount is deposited at the same compound rate of interest, then in how many year will it become 16 times ?

- a. 5 years
- b. 4 years
- c. 6 years
- d. 7 years

Q14. Seema invested an amount of Rs. 16,000 for two years on compound interest and received an amount of Rs. 17,640 on maturity. What is the rate of interest ?

- a. 5 pcpa
- b. 8 pcpa
- c. 4 pcpa
- d. Data inadequate

Q15. At $x\%$ rate Aman receives a Simple Interest of Rs. 320 in 2 years over an investment of Rs. 800. What will be the interest earned when he invests another Rs. 1000 at compound interest rate of $x\%$?

- a. Rs. 540
- b. Rs. 440
- c. Rs. 360
- d. Rs. 420
- e. Rs. 460

Q16. A man earns $x\%$ on the first Rs. 2000 and $y\%$ on the rest. If he earns Rs. 700 from Rs. 4000 and Rs. 900 from Rs. 5000, find x .

- a. 20%
- b. 15%
- c. 25%
- d. None of these

Q17. What annual payment will discharge a debt of Rs. 770 due after 5 years, the rate of interest being 5% per annum?

- a. Rs. 140
- b. Rs. 150
- c. Rs. 180
- d. Rs. 200

Q18. Divide Rs. 2379 into three parts so that their amounts after 2, 3 and 4 years

respectively may be equal, the rate of interest being 5% per annum.

- a. Rs. 828, Rs. 792, Rs. 759
- b. Rs. 820, Rs. 800, Rs. 759
- c. Rs. 834, Rs. 786, Rs. 759
- d. Rs. 828, Rs. 799, Rs. 752

Q19. Mr. Arun lends equal sum of money at the same rate of interest to A and B. The money lent to A become twice of the original amount in just four years at simple interest. While Arun had lent to B for the first two years at compound interest and for the rest two years at simple interest. If the difference between the amount of A and B after 4 years is Rs. 2750. What is the amount of money that Arun lends to each one?

- a. Rs. 40000
- b. Rs. 6000
- c. Rs. 8000
- d. Rs. 80000

Q20. Mr. Ram deposited a total amount of Rs. 65000 in three different schemes A, B and C with rate of interest 12% p.a, 16% p.a, and 18% p.a respectively and earned a total interest of Rs. 10,180 in one year. If the amount invested in Scheme A was 72% of

the amount invested in Scheme C, then what was the amount invested in Scheme B?

- a. Rs. 25000
- b. Rs. 22000
- c. Rs. 18000
- d. Cannot be determined

2.4 Exercise

Q1. What sum will yield an interest of Rs. 306 in six years at 5% p.a. simple interest?

- a. Rs. 840
- b. Rs. 765
- c. Rs. 1,210
- d. Rs. 1,020

Q2. A sum amounts to Rs. 12,480 in five years and to Rs. 13,728 in eight years at simple interest. What is the sum?

- a. Rs. 9,800
- b. Rs. 10,400
- c. Rs. 10,200
- d. Rs. 9,600

Q3. A sum triples itself in three years at simple interest. In how many years will the same sum become nine times itself at the same rate?

a.18

b.9

c.12

d.27

Q4. Find the interest on a sum of Rs.88,000 at $9\frac{1}{11}\%$ p.a. rate of simple interest for 45 months.

a.Rs.40,000

b.Rs.30,000

c.Rs.36,000

d.Rs.28,000

Q5. A certain sum is lent at 4% p.a. for the first four years, 8% p.a. for the next three years and 5% p.a. for the remaining period. If for a period of nine years, the simple interest obtained is Rs. 8500, find the sum.

a.Rs.25,500

b.Rs.18,000

c.Rs.12,750

d.Rs.17,000

Q6. The difference between the simple interest received from two banks on Rs.1250 for 4 years, is Rs.125. What is the difference between their annual rates?

a.4%

b.3.5%

c.3%

d.2.5%

Q7. In what time does a sum double itself at $12\frac{1}{2}\%$ p.a. simple interest?

a.16 years

b.10 years

c.8 years

d.20 years

Q8. A sum is invested for three years at compound interest of 8%, 10% and 20% respectively. Find the sum if at the end of three years, it becomes Rs.14,256.

a.Rs.10,000

b.Rs.9000

c.Rs.8000

d.Rs.12,000

Q9. A certain sum is lent at compound interest for three years at rate of interests of 5%, 10% and 12% respectively for three consecutive years. Find the sum if the total interest for the three years is Rs.5,284.80.

a.Rs.12,000

b.Rs.20,000

c.Rs.18,000

d.Rs.16,000

Q10. How much more does a man get by investing Rs.40,000 at 40% p.a compound interest, compounded half yearly, than at 40% p.a compound interest, compounded yearly per one year?

- a. Rs.2,564
- b. Rs.964
- c. Rs.1,078
- d. Rs.1,600

Q11. The compound interest for 2 years and simple interest for 1 year on a certain sum at certain rate of interest are Rs.7040 and Rs.3200 respectively. Find the principal.

- a. Rs.15000
- b. Rs.16000
- c. Rs.18000
- d. Rs.20000

Q12. A sum invested at compound interest, with interest being compounded annually amounts to Rs.62,720 in two years and Rs.70,246.40 in three years. Find the sum.

- a. Rs.50000
- b. Rs.48,000
- c. Rs.52,000
- d. Rs.54,000

Q13. Amar borrows Rs.30000 at 10% compound interest. He repays Rs.20000 after 1 year. What sum should be repaid at the end of the second year such that the debt is discharged?

- a. Rs.14,300
- b. Rs.12,500
- c. Rs.13,750
- d. Rs.15,400

Q14. Find the present value of Rs.29,282 payable after 3 years at 10% p.a. compound interest, interest being compounded annually.

- a. Rs.22,000
- b. Rs.20,000
- c. Rs.24,000
- d. Rs.18,000

Q15. A sum of money becomes four times itself in eight years at compound interest. In how many years will the same sum become sixteen times itself?

- a. 64
- b. 32
- c. 44
- d. 16

Q16. The compound interest and the simple interest on a certain sum for two years are Rs.1,653.60 and Rs.1,560 respectively. Find the sum and the rate of interest.

- a. Rs.7,800, 10% p.a
- b. Rs.7,800, 15% p.a
- c. Rs.6,500, 10% p.a
- d. Rs.6,500, 12% p.a

Q17. The compound interest on a certain sum for 2 years at 20 % p.a. is Rs.528. Find the simple interest for the same amount for the same period, at the same rate of interest.

- a. Rs.560
- b. Rs.420
- c. Rs.480
- d. Rs.580

Q18. A sum was lent at 15% p.a. simple interest for 3 years. If it was lent at 15% p.a. compound interest, interest being compounded annually, it would have fetched Rs.1134 more as interest. Find the sum.

- a. Rs.15000
- b. Rs.16000
- c. Rs.20000
- d. Rs.18000

Q19. A sum lent at compound interest amounts to 72.8% more than itself in 3 years, interest being compounded annually. If it was lent at the same rate of simple interest, it would fetch interest of Rs.2100 at the end of the first 6 years. Find the sum.

- a. Rs.2250
- b. Rs.1750
- c. Rs.1500
- d. Rs.2500

Q20. What annual installment under simple interest will discharge a debt of Rs.66,000 due in 3 years at 10% p.a. if installments are paid at the end of the year?

- a. Rs.26,000
- b. Rs.23,000
- c. Rs.20,000
- d. Rs.24,000

Q21. A sum of Rs. 725 is lent in the beginning of a year at a certain rate of interest. After 8 months, a sum of Rs. 362.50 more is lent but at the rate twice the former. At the end of the year, Rs. 33.50 is earned as interest from both the loans. What was the original rate of interest?

- a. 3.6%
- b. 4.5%

c.5%

d.None of these

Q22. A sum was put at simple interest at a certain rate for 2 years. Had it been put at 3% higher rate, it would have fetched Rs. 72 more. The sum is :

a.Rs. 1200

b.Rs. 1500

c.Rs. 1600

d.Rs. 1800

Q23. On a sum of money, the simple interest for 2 years is Rs. 660, while the compound interest is Rs. 696.30, the rate of interest being the same in both the cases.

The rate of interest is :

a.10%

b.10.5%

c.12%

d.11%

Q24. The effective annual rate of interest corresponding to a nominal rate of 6% per annum payable half-yearly is :

a.6.06%

b.6.07%

c.6.08%

d.6.09%

Q25. Mr. Dua invested money in two schemes A and B offering compound interest at 8 p.c.p.a. and 9 p.c.p.a. respectively. If the total amount of interest accrued through two schemes together in two years was Rs. 4818.30 and the total amount invested was Rs. 27,000, what was the amount invested in Scheme A ?

a.Rs. 12,000

b.Rs. 13,500

c.Rs. 15,000

d. None of these

Answers - Simple Interest & Compound Interest

2.2 Quiz

Q1	Q2	Q3
Q4	Q5	Q6

2.3 Class Work Problems

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12
Q13	Q14	Q15
Q16	Q17	Q18
Q19	Q20	

2.4 Exercise

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12
Q13	Q14	Q15
Q16	Q17	Q18
Q19	Q20	Q21
Q22	Q23	Q24
Q25		

CHAPTER 3

PERMUTATIONS & COMBINATIONS

INTRODUCTION

The chapter focuses on **counting problems** which come under the branch of Mathematics called **combinatorics**.

Fundamental Principle of Counting

Multiplication rule

If an operation can be performed in m different ways and following which a second operation can be performed in n different ways, then the two operations in succession can be performed in $m \times n$ different ways.

Addition Rule

If an operation can be performed in m different ways and a second independent operation can be performed in n different ways, either of the two operations can be performed in $(m+n)$ ways.

Permutation

Permutations are the different arrangements of a given number of things in a particular order by taking some or all at a time

Permutations of n distinct things taken ' r ' at a time (repetition of things not allowed) is denoted by nPr or $P(n,r)$ and is given by

$${}^n P_r = \frac{n!}{(n-r)!} = n(n-1)(n-2)\dots(n-r+1)$$

where $0 \leq r \leq n$.

- ${}^n P_0 = 1$

Permutations of n distinct things taken all at a time (repetition of things not allowed) is denoted by ${}^n P_n$ and is given by ${}^n P_n = n!$

- Normally repetition is not allowed unless mentioned specifically.

Permutation under various constraints

i) The number of permutations of n different things taking ' r ' at a time when each thing may be repeated any number of times in any permutations is given by $(n \times n \times n \dots r \text{ times})$ i.e. **n^r ways**.

ii) The number of arrangements when things are not all different such as arrangement of n things taken all at a time, when p of them are of one kind, q of another kind, r is still of another kind and so on, the total number of permutations is given by

$$\frac{n!}{(p! q! r! \dots)}$$

iii) The total number of arrangements of n things taken ' r ' at a time, in which a particular thing always occurs = ${}^{(n-1)}P_{(r-1)}$

iv) The total number of permutations of n different things taken ' r ' at a time in which a particular thing never occurs = ${}^{(n-1)}P_r$

Circular Permutations

In linear permutation, we fill first place by n ways and next in $(n - 1)$ ways and so on, but in circular arrangement we don't have any first place. So fix any object as a first place and arrange the rest $(n - 1)$ objects around it. Hence, we have to arrange 1 less than the total number of things.

Number of circular permutations of n things all taken at a time = $(n - 1)!$

Number of circular permutations of n different things taking ' r ' at a time = $\frac{nPr}{r}$

If there is no difference between clockwise and anticlockwise arrangements, the total number of circular permutations of n things taking all at a time is $\frac{(n-1)!}{2}$ & the total number of circular permutations n when taking ' r ' at a time all will be $\frac{nPr}{2r}$

Combinations

Number of combinations or selections of n dissimilar things taken ' r ' at a time is denoted by nCr or $C(n, r)$ & is given by

$${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}$$

where $0 \leq r \leq n$

- $n_{C_0} = 1$
- $n_{C_r} = n_{C_{n-r}}$

Combinations under constraints

• Number of combinations of n different things taken ' r ' at a time in which ' p ' particular things will always occur is

$${}^{n-p}C_{r-p}$$

• Number of combinations of n dissimilar things taken ' r ' at a time in which ' p ' particular things will never occur is ${}^{n-p}C_r$

• The total number of ways of dividing n identical items among r persons, each of whom can receive 0, 1, 2, or more items ($\leq n$) is ${}^{(n+r-1)}C_{(r-1)}$.

• Number of diagonals in n sided polygon = ${}^nC_2 - n$

• Sum of all possible combinations of n distinct things

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$$

3.1 Quiz

Q1. In how many ways can 10 prizes be distributed to 4 students if each student has to receive at least one prize?

Q2. How many palindromes are there between 10 and 100000?

Q3. In how many different ways can the six faces of a cube be painted with six different colours?

Q4. How many ways one white rook and one black rook can be placed on a chess board so that they are never in an attacking position?

Q5. Five couples go for a dance party. How many ways can we make dancing couples so that no two spouses are together?

3.2 Worked Out Examples

Q1. Door numbers of flats in an apartment complex consists of a natural number less than 100, followed by one of the letters A, B,

C, D, E and F. How many different door numbers are possible?

Solution

The number can be any one of the natural numbers from 1 to 99. There are 99 choices for the number. The letter can be chosen in 6 ways. Number of possible bus routes are $99 \times 6 = 594$

Q2. If the digits are not to be repeated, in how many possible ways can you make a 3 digit number using digits 0,1,2,3 and 4?

Solution

Three digit numbers will have unit's, ten's and hundred's place.

Out of 5 given digits except zero any one can take the hundred's place. This can be done in 4 ways

Any of the four remaining digits including zero can take the ten's place. This can be done in 4 ways

After filling in ten's place, unit's place can be filled from any of the three remaining digits.

This can be done in 3 ways

By counting principle, the number of 3 digit numbers $= 4 \times 4 \times 3 = 48$

Q3. If repetition of letters is not allowed, how many 3-letter words with or without meaning, can be formed out of the letters of the word, 'ALGORITHMS'?

Solution

The word 'ALGORITHMS' has 10 different letters.

Hence, the number of 3-letter words(with or without meaning) formed by using these letters = ${}^{10}P_3 = 10 \times 9 \times 8 = 720$

Q4. In a school on a working day there are 6 periods. How many ways 5 subjects can be organized such that each subject is allowed at least one period?

Solution

In 6 periods, 5 can be organized in 6P_5 ways. Remaining 1 period can be organized in 5P_1 ways.

Total number of arrangements
= ${}^6P_5 \times {}^5P_1 = 720 \times 5 = 3600$

Q5. How many words of 3 consonants and 2 vowels can be formed using , 8 consonants and all the vowels?

Solution

Number of ways of selecting 3 consonants out of 8 = 8C_3

Number of ways of selecting 2 vowels out of

all vowels = 5C_2

Number of ways of selecting 3 consonants out of 8 and 2 vowels out of 5 = ${}^8C_3 \times {}^5C_2$

Number of ways of arranging 5 letters among themselves = $5!$

Required number of ways = ${}^8C_3 \times {}^5C_2 \times 5! = 67200$

Q6. 5 girls and 5 boys have to be arranged in a circle. How many different arrangements are possible such that the boys and the girls occupy the alternate positions?

Solution

In a circle, 5 boys can be arranged in $4!$ Ways.

Given that the boys and the girls alternate. Hence there are 5 places for girls which can be arranged in $5!$ ways

Total number of ways = $4! \times 5! = 2880$

Q7. A pizza hut has 8 different toppings. If you are given the choice of ordering any number of toppings, how many different ways can you opt for a topping?

Solution

Sum of all possible combinations- either no topping or 1 topping or 2 or 3 etc or all 8 toppings, ${}^8C_0 + {}^8C_1 + {}^8C_2 + \dots + {}^8C_8 = 2^8$
= 256 selections are possible

3.3 Class Work Problems

Q1. There are 8 black balls and 8 white balls. In how many ways can these balls be arranged in a row so that balls of different colours are alternate?

- a. $8! \times 7!$
- b. $2 \times 8! \times 7!$
- c. $2 \times (8!)^2$
- d. $(8!)^2$

Q2. In how many ways can 5 boys and 4 girls be arranged in a line so that there will be a boy at the beginning and at the end?

- a. $\left(\frac{3!}{5!}\right)7!$
- b. $\left(\frac{5!}{3!}\right)7!$
- c. $\left(\frac{5!}{6!}\right)7!$
- d. $\left(\frac{5!}{7!}\right)7!$

Q3. How many different ways can the letters of the word 'JUDGE' be rearranged such that the vowels always come together?

- a. 48
- b. 32
- c. 64
- d. None of these

Q4. In a queue for a movie ticket there are 12 people standing. If out of them there are three friends who will always stand together, how many total arrangements of the line are possible?

- a. $3! \times 2! \times 9!$
- b. $3! \times 10!$
- c. $12 P_3 \times 9!$
- d. $12C_3 \times 9!$

Q5. How many 4 digit numbers divisible by 5 can be formed with the digits 0, 1, 2, 3, 4, 5, 6 and 6?

- a. 220
- b. 249
- c. 216
- d. 288

Q6. A questionnaire asks students what activities they enjoy. They were given the options of video games, trekking, Drawing, Gardening, Cooking, Singing, Acting. A student may select no activity, one activity, or more than one activity. How many different ways a student could fill out the questionnaire?

- a. 21
- b. 42

- c. 128
- d. 5040

Q7. There are 20 people among whom 2 are sisters. Find the number of ways in which we can arrange them around a circle so that there is exactly one person between the 2 sisters.

- a. $18!$
- b. $2! \times 19!$
- c. $19!$
- d. $2! \times 18!$

Q8. How many different 9 digit numbers can be formed from the number 223355888 by rearranging the digits so that the odd digits occupy even positions?

- a. 120
- b. $\frac{9!}{(2!)^3} 3!$
- c. $\frac{4!}{(2!)^3} 3!$
- d. 60

Q9. In how many ways can the letters of the English alphabet be arranged so that there are 7 letters between the letters A and B?

- a. $31! 2!$
- b. $24P_7 \times 18! \times 2$

- c. $24! 36$
- d. $26P_7 \times 20! \times 2$

Q10. Four married couples and one single woman want to play tennis while vacationing in a resort. How many different mixed doubles tennis match can be organized if no man is willing to play with his wife in the same match? [In Mixed Doubles tennis match, each team consists of one male and another female player].

- a. 36
- b. 12
- c. 18
- d. 54

Q11. If you jumble and arrange the word LABOUR in all possible ways and arrange all the words so formed as in a dictionary. What will be the rank of the word LABOUR?

- a. 275
- b. 251
- c. 240
- d. 242

Q12. How many ways can a group of 4 men and 3 women be selected from a group

containing 7 men and 6 women so that Miss .X refuses to be in the same group as Mr. Y?

- a. 500
- b. 375
- c. 350
- d. 275

Q13. In a game some people were standing on a circle at distinct points. Each possible pair of persons, who are not adjacent, sings a 3 minute song, one pair after another. The total time taken by all the pairs to sing is 1 hour. Find the number of people on the circle?

- a. 6
- b. 7
- c. 8
- d. 9

Q14. In how many ways can you pack 5 different gifts in 3 identical boxes such that no box is empty, if any of the boxes may hold all of the gifts?

- a. 20
- b. 25
- c. 30
- d. 600

Q15. In an exam the question paper had two sections, each section containing 5 questions. The students are required to answer 6 out of 10 questions but not permitted to attempt more than 4 from each group. In how many ways can a student make the choice?

- a. 210
- b. 150
- c. 100
- d. 200

3.4 Exercise

Q1. How many factors of $3^5 \times 5^6 \times 13^7$ are perfect squares?

Q2. How many diagonals a decagon has?

Q3. Assume $P(x, y)$ denote the number of ways of distributing "x" pens to "y" students so that each student receives at the most 2 toys then find $P(4, 3)$?

Q4. A security code was formed with 2 odd digits followed by 4 even digits. For the generation of more codes, it was decided to form the code with 2 even digits followed by

any three digits. If repetition of digits is allowed find the increase in number of codes generated.

Q5. Three points each are marked on each sides AB, BC and CA of a given triangle ABC. How many triangles can be formed by joining these points?

Q6. How many three letter computer passwords can be formed (no repetition allowed) with at least one symmetric letter?

Q7. At a business meeting, every person shakes each other's hands once. How many people were present for the meeting if there was a total of 91 handshakes?

- a. 14
- b. 15
- c. 24
- d. 46

Q8. The number of ways 3 tickets can be pulled out of a bag containing 20 tickets is same as pulling "k" tickets out of 20 tickets. Find the value of k .

Q9. In how many ways can the letters of the word MANAGEMENT be rearranged so that the two A's do not appear together?

Q10. In how many ways can 5 members forming a committee out of 10 be selected so that

(a) Two particular members must be included.

(b) Two particular numbers must not be included

Q11. There are 100 balls numbered $n_1, n_2, n_3, \dots, n_{100}$. They are arranged in all possible ways. How many arrangements should be there in which n_{28} ball will always be before n_{29} ball and the two of them will be adjacent to each other.

Q12. In how many ways can a group of 4 women and 5 men be selected from 10 couples such that a particular couple is included?

Q13. Find the number of numbers greater than 20000 by arranging all the digits of the number 242302.

Q14. There are 6 tasks and 6 people. Task 1 cannot be assigned either to person 1 or to person 2, task 2 must be assigned either to person 3 or person 4 and every person must be assigned one task. How many ways of task allotments are there?

Q15. How many words, with or without meaning, can be made from the letters of the word TRIANGLE without repetition such that words begin with 't' and ends with 'e'?

Q16. How many different albums can be formed using 5 rock songs, 6 carnatic songs & 3 pop songs ,if the albums should contain at least one rock song and 1 carnatic song?

Q17. How many ways can you put 5 balls in 3 boxes if

A. The boxes are similar and the balls are similar

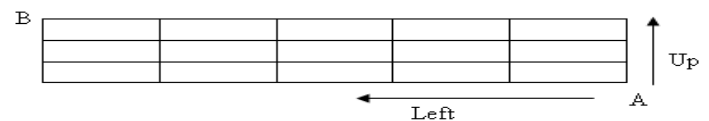
B. The boxes are different but the balls are similar

C. The boxes are similar but the balls are different.

D. The boxes are different and the balls are different

Q18. Let a, b, c , and d be non-negative integers. How many possible values of a, b, c, d exist such that $a + b + c + d = 11$?

Q19. In the figure given below, the lines represent one way road allowing travel from any point, only to up or to the left. Along how many distinct routes can a vehicle reach a point B from Point A?(CAT)



- a. 15
- b. 56
- c. 120
- d. 336

Q20. There are 10 different fruits and 5 different vegetables. A grocer has to choose 3 fruits and 2 vegetables. In all the possible selections he can make, what is the number of selections in which a particular fruit and a particular vegetable are always there?

- a. 120
- b. 144
- c. 160
- d. 80

Q21. In how many ways is it possible to choose a white square and a black square on a chess board so that the squares must not lie in the same row or column?

- a. 56
- b. 896
- c. 60
- d. 768

Q22. A man has nine friends-4 boys and 5 girls. In how many ways can he invite them, if there have to be exactly 3 girls in the invitees?

- a. 320
- b. 160
- c. 80
- d. 200

Q23. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then the number of words which have at least one letter repeated is

Q24. In a chess competition involving some boys and girls of a school, every student had to play exactly one game with every other student. It was found that in 45 games both the players were girls, and in 190 games

both were boys. The number of games in which one player was a boy and the other was a girl is

- a. 200
- b. 216
- c. 235
- d. 256

Q25. A new flag is to be designed with six vertical stripes using some or all of the colours yellow, green, blue and red. Then, the number of ways this can be done such that no two adjacent stripes have the same colour is

- a. 12×81
- b. 16×192
- c. 20×125
- d. 24×216

Q26. In a chess competition in 45 games, both the players were women and in 190 games, both players were men. How many games would have been played in which one person was a man and other person was a woman (every player play exactly one game with every other player)?

- a. 40
- b. 200
- c. 180

d. 120

Q27. Different words with six letters are formed from 10 different letters of alphabet. How many words can be formed which have at least one letter repeated?

a. $^{10}P_6$

b. 10^6

c. $^{10}C_6$

d. $10^6 - ^{10}P_6$

Q28. In how many ways can 8 distinguishable cards be put into 5 distinguishable covers if no box can contain more than one ball?

a. 0

b. 8^5

c. 8P_5

d. 5^8

Q29. Find the number of ways in which the letters of the word 'EQUATION' be written such that no two consonants are together?

a. 126000

b. 36000

c. 10000

d. 14400

Q30. In how many ways can 16 identical fruits be distributed among three identical boxes in such a way that no two boxes get the same number of fruits and each box gets at most half the total number of fruits and at least one fruit?

a. 11

b. 12

c. 8

d. 9

e. 5

**Answers - Permutations &
Combinations**

3.1 Quiz

Q1 Q2 Q3

Q4 Q5

3.3 Class Work Problems

Q1	Q2	Q3
----	----	----

Q4	Q5	Q6
----	----	----

Q7	Q8	Q9
----	----	----

Q10	Q11	Q12
-----	-----	-----

Q13	Q14	Q15
-----	-----	-----

3.4 Exercise

Q1	Q2	Q3
----	----	----

Q4	Q5	Q6
----	----	----

Q7	Q8	Q9
----	----	----

Q10	Q11	Q12
-----	-----	-----

Q13	Q14	Q15
-----	-----	-----

Q16	Q17	Q18
-----	-----	-----

Q19	Q20	Q21
-----	-----	-----

Q22	Q23	Q24
-----	-----	-----

Q25	Q26	Q27
-----	-----	-----

Q28	Q29	Q30
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CHAPTER 4

PROBABILITY

INTRODUCTION

In this chapter we study probability, which is the mathematical measure of “chance.”

Probability is used to indicate a possibility of an event to occur.

- *In any experiment if the result of an experiment is unique or certain, then the experiment is said to be deterministic in nature.*
- *If the result of the experiment is not unique and can be one of the several possible outcomes then the experiment is said to be probabilistic in nature.*

Various Terms Used in Defining Probability

*(i) **Random Experiment:** Whenever an experiment is conducted any number of times under identical conditions and if the result is not certain and is any one of the several possible outcomes, the experiment is called a trial or a random experiment, the outcomes are known as events. The sample space of an experiment is the set of all possible outcomes.*

eg, When a die is thrown is a trial, getting a

number 1 or 2 or 3 or 4 or 5 or 6 is an event and $\{1,2,3,4,5,6\}$ is the sample space

*(ii) **Equally Likely Events:** Events are said to be equally likely when there is no reason to expect any one of them rather than any one of the others.*

eg, When a die is thrown any number 1 or 2 or 3 or 4 or 5 or 6 may occur. In this trial, the six events are equally likely.

*(iii) **Mutually Exclusive Events:** If the occurrence of any one of the events in a trial prevents the occurrence of any one of the others, then the events are said to be mutually exclusive events. eg, When a die is thrown the event of getting faces numbered 1 to 6 are mutually exclusive.*

If E and F are mutually exclusive events in a sample space S, then the probability of E or F is $P(E \cup F) = P(E) + P(F)$

*(iv) **Independent Events:** When the occurrence of one event does not affect the probability of the other event, the events are said to be independent.*

For instance, if a balanced coin is tossed, the probability of showing heads on the second toss is

1/2, irrespective of the outcome of the first toss. So any two tosses of the coin are independent.

If E and F are independent events in a sample space S , then the probability of E and F is $P(E \cap F) = P(E) \cdot P(F)$

Classical Definition of Probability

If in a random experiment, there are n mutually exclusive and equally likely events in which n events are favourable to a particular event E , then the probability of the event E is defined as $P(E)$

$$P(E) = \frac{\text{Favourable Events}}{\text{Total number of Events}} = \frac{n(E)}{n(S)}$$

- If the probability of occurrence of an event E is p and the probability of non-occurrence is q then, $p + q = 1$, the sum of the probabilities of success and failure is 1.

Non occurrence of event E is called the complementary event of E usually denoted as \bar{E}

- The probability of an event must satisfy $0 \leq P(E) \leq 1$. If $P(E) = 1$, the event E is called a certain event and if $P(E) = 0$, the event E is called an impossible event.

- The probability of either A or B occurring (or both) is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- If E is an event, then the odds in favour of E are defined as $\frac{P(E)}{P(\bar{E})}$

- The odds against E are $\frac{P(\bar{E})}{P(E)}$

Conditional Probability

Often it is required to compute the probability of an event given that another event has occurred.

The probability the event B occurs, given that event A has happened, is represented as $P(B | A)$

This is read as “the probability of B given A ”

If Events A and B are not independent, then

$$P(B | A) = \frac{P(A \cap B)}{P(A)}$$

Some Facts

- When a die is rolled six events occur. They are $\{1, 2, 3, 4, 5 \text{ and } 6\}$

- When two dice are rolled 36 events occur. They are $[(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)]$

- When a coin is tossed 2 events occur. They are $\{H, T\}$

- When two coins are tossed 4 events occur. They are {HH, HT, TH, TT}
- When three coins are tossed 8 events occur. They are {HHH HHT, HTH, HT T, T HH, THT, T TH, T T T}
- In a standard deck or pack of 52 cards there are 26 red cards and 26 black cards. The 26 red cards are divided into 13 heart cards and 13 diamond cards. The 26 black cards are divided into 13 club cards and 13 spade card. In each of the colours, hearts, diamonds, clubs and spades is called a suit. In a suit, we have 13 cards (ie, A, K, Q, J, 10, 9, 8, 7, 6, 5, 4, 3 and 2)

4.1 Quiz

Q1. What is the probability that in a class of 35 students, at least two have the same birthday?

Q2. In your drawer you have 10 pairs of socks, 6 of which are white. If you reach in and randomly grab two pairs of socks, what is the probability that both are white?

Q3. A card is pulled a deck of cards and noted. The card is then replaced, the deck is

shuffled, and a second card is removed and noted. What is the probability that both cards are Aces?

Q4. At some random moment, you look at your clock and note the minutes reading. What is probability the minutes reading is 15?

Q5. A monkey is trained to arrange wooden blocks in a straight line. He is then given 11 blocks showing the letters A, B, B, I, I, L, O, P, R, T, Y. What is the probability that the monkey will arrange the blocks to spell the word PROBABILITY?

Q6. Eight horses are entered in a race. You randomly predict a particular order for the horses to complete the race. What is the probability that your prediction is correct?

4.2 Worked Out Examples

Q1. If you pull a random card from a deck of playing cards, what is the probability it is not a heart?

Solution

There are 13 hearts in the deck, so

$$P(\text{heart}) = \frac{13}{52} = \frac{1}{4}$$

The probability of not drawing a heart

$$= 1 - P(\text{heart}) = 1 - \frac{1}{4} = \frac{3}{4}$$

Q2. Suppose we flipped a coin and rolled a die, what is the probability of getting a head on the coin and a 6 on the die?

Solution

We could list all possible outcomes: {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}.

Notice there are $2 \cdot 6 = 12$ total outcomes.

Out of these, only 1 is the desired outcome, so the probability is $\frac{1}{12}$.

Q3. Suppose we draw one card from a standard deck. What is the probability that we get a red card or a King?

Solution

Half the cards are red, so $P(\text{red}) = \frac{26}{52}$

There are four kings, so $P(\text{King}) = \frac{4}{52}$

There are two red kings, so

$$P(\text{King and Red}) = \frac{2}{52}$$

We can then calculate

$$P(\text{Red or king}) =$$

$$P(\text{red}) + P(\text{king}) - P(\text{red \& king}) = \frac{28}{52}$$

Q4. If you pull 2 cards out of a deck, what is the probability that both are spades?

Solution

The probability that the first card is a spade is $\frac{13}{52}$

The probability that the second card is a spade, given the first was a spade, is $\frac{12}{51}$

The probability that both cards are spades is

$$\frac{13}{52} \times \frac{12}{51} = \frac{3}{51}$$

Q5. In a single throw of a fair die what is the probability that the number on the top is more than 2?

Solution

In a die there are 6 faces numbered 1, 2, 3, 4, 5, and 6. So, the total number of possible events = 1, 2, 3, 4, 5 and 6 = 6 and, the total number of favourable events = 3, 4, 5 and 6 = 4

So the required Probability is $\frac{4}{6} = \frac{2}{3}$.

6. If out of 20 numbers from 1 to 20

Mr. X selects a number at random. What is the probability that this number will be a multiple of 4 or 7?

Solution

Total number of possible events = 1, 2, ...20
= 20 in number

Number divisible by 4 = 4, 8, 12, 16, 20 = 5
in number

Number divisible by 7 = 7 and 14 = 2 in
number

Since from 1 to 20 there is not any number
which is divisible by both 4 and 7. It is a
case of mutually exclusive events.

So number of possible outcomes = $5 + 2 = 7$

So the required probability is $= \frac{7}{20}$.

4.3 Class Work Problems

Q1. When 2 dice are drawn simultaneously,
what is the probability that

(i) One die shows up 2 and the other shows
up 5?

(ii) 1st die shows up 6 and 2nd die does not
show up 6?

(iii) The sum of the scores on the dice is less
than or equal to 4?

(iv) Sum obtained is less than 9?

Q2. If 2 cards are drawn simultaneously
from a well shuffled pack of cards, then find
the probability of both being

(i) Kings

(ii) Blacks

(iii) Numbered card

(iv) Diamonds

(v) black honours

Q3. A bag contains 3 white balls and 2
black balls. Another bag contains 2 white
and 4 black balls. A bag and a ball are picked
random. The probability that the ball will be
white is:

a. $\frac{7}{11}$

b. $\frac{7}{30}$

c. $\frac{7}{15}$

d. $\frac{5}{11}$

Q4. What is the probability that the sum of
two different single-digit prime numbers
will not be prime?

a. 0

b. $\frac{1}{3}$

c. $\frac{2}{3}$

d. $\frac{1}{2}$

Q5. What is the probability that a word
formed by randomly rearranging the letters

of the word ALGAE is the word ALGAE itself?

- a. $\frac{1}{120}$
- b. $\frac{1}{60}$
- c. $\frac{2}{7}$
- d. $\frac{1}{30}$

Q6. I forgot the last digit of a 7-digit telephone number. If I randomly dial the final 3 digits after correctly dialing the first four, then what is the chance of dialing the correct number?

- a. $\frac{1}{1001}$
- b. $\frac{1}{1000}$
- c. $\frac{1}{999}$
- d. $\frac{1}{900}$

Q7. A bag contains 6 white and 4 red balls. Three balls are drawn one by one with replacement. What is the probability that all 3 balls are red?

- a. $\frac{8}{125}$
- b. $\frac{1}{20}$
- c. $\frac{1}{30}$
- d. $\frac{1}{120}$

Q8. In a class of 60 boys, there are 45 boys who play cards and 30 boys play carom. Find how many boys play both games (assuming that every boy play either cards or carom or both)

- a. 15
- b. 17
- c. 20
- d. 21

Q9. What is the probability that a 2 digit number selected at random will be a multiple of 3 and not a multiple of 5?

- a. $\frac{2}{15}$
- b. $\frac{4}{15}$
- c. $\frac{1}{15}$
- d. $\frac{4}{90}$

Q10. A man bets on number 16 on a roulette wheel 14 times and losses each time. On the 15th span he does a quick calculation and finds out that the number 12 had appeared twice in the 14 spans and is therefore, unable to decide if to bet on 16 or 12 in the 15th span. Which will give him the best chance and what are the odds against winning on the bet that he takes?

- a. 16 ; 22 : 14

- b. 12; 72 : 1
- c. 12; 7 : 1
- d. either; 35 : 1

Q11. Two squares are chosen at random on a chess board. What is the probability that they have a side in common?

- a. $\frac{1}{18}$
- b. $\frac{64}{4032}$
- c. $\frac{63}{164}$
- d. $\frac{1}{9}$

Q12. When two dice are thrown simultaneously, what is the probability that the sum of the two numbers that turn up is less than 11?

- a. $\frac{5}{6}$
- b. $\frac{11}{12}$
- c. $\frac{1}{6}$
- d. $\frac{1}{12}$

Q13. When 4 dice are thrown, what is the probability that the same number appears on each of them?

- a. $\frac{1}{36}$
- b. $\frac{1}{18}$

- c. $\frac{1}{216}$
- d. $\frac{1}{5}$

Q14. An experiment succeeds twice often as it fails. What is the probability that in the next 5 trials there will be 4 successes?

- a. 0
- b. $(\frac{2}{3})^4$
- c. $5x(\frac{1}{3})x(\frac{2}{3})^4$
- d. $(\frac{1}{3})x(\frac{2}{3})^4$

Q15. An anti aircraft gun can fire 4 shots at a time. If the probabilities of the 1st, 2nd, 3rd and the last shot hitting the enemy aircraft are 0.7, 0.6, 0.5 and 0.4, what is the probability that 4 shots aimed at an enemy aircraft will bring the aircraft down?

- a. 0.084
- b. 0.916
- c. 0.036
- d. 0.964

Q16. A number is selected at random from 1st thirty natural numbers. What is the chance that it is a multiple of either 3 or 13?

- a. $\frac{17}{30}$
- b. $\frac{2}{5}$

c. $\frac{11}{30}$

d. $\frac{4}{15}$

Q17. A man can hit a target once in 4 shots. If he fires 4 shots in succession, what is the probability that he will hit his target?

a. 1

b. $\frac{1}{256}$

c. $\frac{81}{256}$

d. $\frac{175}{256}$

Q18. There are 2 bags-one containing 3 one rupee coins, 6 five rupee coins and the other containing 2 one rupee coins, 7 five rupee coins. One bag is chosen at random and from that one coin is drawn at random. What is the probability that it is a 5 rupee coin?

19. A box has 10 bulbs in which 3 are defective. 4 bulbs are drawn at random. What is probability that atleast one bulb is good?

20. A speaks truth in 70% cases and B in 85% cases. What is the probability that they contradict each other in stating the same fact?

4.4 Exercise

Q1. In a game there are 70 people in which 40 are boys and 30 are girls, out of which 10 people are selected at random. One from the total group, thus selected is selected as a leader at random. What is the probability that the person, chosen as the leader is a boy?

a. $\frac{4}{7}$

b. $\frac{4}{9}$

c. $\frac{5}{7}$

d. $\frac{2}{7}$

Q2. Two urns contain 5 white and 7 black balls and 3 white and 9 black balls respectively. One ball is transferred to the second urn and then one ball is drawn from the second urn. Find the probability that the first ball transferred is black, given that the ball drawn is black?

a. $\frac{13}{23}$

b. $\frac{11}{23}$

c. $\frac{14}{23}$

d. $\frac{7}{23}$

Q3. There are three similar boxes, containing

(i). 6 black and 4 white balls

(ii). 3 black and 7 white balls

(iii). 5 black and 5 white balls, respectively.

If you choose one of the three boxes at random and from that particular box picks up a ball at random, and find that to be black, what is the probability that the ball picked up from the second box?

a. $\frac{14}{30}$

b. $\frac{3}{14}$

c. $\frac{7}{30}$

d. $\frac{1}{14}$

Q4. A can solve 80% of the problems given in an exam and B can solve 70%. What is the probability that at least one of them will solve a problem selected at random from the exam?

a. $\frac{21}{49}$

b. $\frac{12}{49}$

c. $\frac{2}{49}$

d. $\frac{37}{49}$

Q5. There are 2 positive integers x and y . What is the probability that $x + y$ is odd?

a. $\frac{1}{3}$

b. $\frac{1}{2}$

c. $\frac{1}{4}$

d. $\frac{1}{5}$

Q6. A person has 9 pairs of dark blue socks and 9 pairs of black socks. He keeps them all in the same bag. If he picks out three socks at random, then what is the probability that he will get a matching pair?

a. $\frac{(2 \times 9 C_3 \times 9 C_1)}{18 C_3}$

b. $\frac{(9 C_3 \times 9 C_1)}{18 C_3}$

c. 1

d. none of these

Q7. The probability that an arrow fired from a point will hit the target is $\frac{1}{4}$. Three such arrows are fired simultaneously towards the target from that very point. What is the probability that the target will be hit?

a. $\frac{19}{64}$

b. $\frac{23}{64}$

c. $\frac{23}{67}$

d. $\frac{37}{64}$

Q8. A and B play a game where each is asked to select a number from 1 to 5. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial is:

- a. $\frac{1}{25}$
- b. $\frac{24}{25}$
- c. $\frac{23}{23}$
- d. $\frac{2}{25}$

Q9. There are three events A, B and C, one of which must and only can happen. If the odds are 8:3 against A, 5:2 against B, the odds against C must be:

- a. 13 : 7
- b. : 2
- c. 43 : 34
- d. 43 : 77

Q10. If the probability that X will live 15 year is $\frac{7}{8}$ and that Y will live 15 years is $\frac{9}{10}$, then what is the probability that both will live after 15 years?

- a. $\frac{1}{5}$
- b. $\frac{63}{80}$
- c. $\frac{2}{49}$
- d. none of these

Q11. Four boys and three girls stand in queue for an interview. The probability that they stand in alternate positions is:

- a. $\frac{1}{35}$
- b. $\frac{1}{34}$
- c. $\frac{1}{17}$
- d. $\frac{1}{68}$

Q12. There are 6 positive and 8 negative numbers. Four numbers are chosen at random and multiplied. The probability that the product is a positive number is:

- a. $\frac{500}{1001}$
- b. $\frac{503}{1001}$
- c. $\frac{505}{1001}$
- d. $\frac{101}{1001}$

Q13. A bag contains 2 red, 3 green and 2 blue balls are to be drawn randomly. What is the probability that the balls drawn contain no blue ball?

- a. $\frac{10}{21}$
- b. $\frac{11}{21}$
- c. $\frac{2}{7}$
- d. $\frac{5}{7}$

Q14. Two people agree to meet on January 9, 2005 between 6.00 P.M. to 7.00 P.M., with the understanding that each will wait no longer than 20 minutes for the other. What is the probability that they will meet?

- a. $\frac{4}{9}$
- b. $\frac{5}{9}$
- c. $\frac{7}{9}$
- d. $\frac{2}{9}$

Q15. The game 'Chunk' is played at carnivals in some parts of Europe. Its rules are as follows: If you pick a number from 1 to 6 and the operator rolls three dice. If the number you picked comes up on all three dice, the operator pays you Rs 3 ; If it comes up on two dice, you are paid Rs 2 ; and it comes up on just one dice, you are paid Rs 1. Only if the number you picked does not come up at all, you pay the operator Rs 1. The probability that you will win money playing in this game is

- a. 0.52
- b. 0.753
- c. 0.42
- d. None of the above

Q16. There are three similar boxes, containing

(i) 6 black and 4 white balls (ii) 3 black and 7 white balls (iii) 5 black and 5 white balls, respectively. If you choose one of the three boxes at random and from that particular box picks up a ball at random, and find that to be black, what is the probability that the ball picked up from the second box?

- a. $\frac{3}{14}$
- b. $\frac{14}{30}$
- c. $\frac{7}{30}$
- d. $\frac{7}{14}$

Q17. Each of X and Y take not more than 10 pens at random from a box containing unlimited number of pens. What is the probability that the difference in the number of pens they take is not more than 5?

- a. $\frac{91}{121}$
- b. $\frac{1}{2}$
- c. $\frac{93}{121}$
- d. $\frac{95}{121}$

Q18. Four different objects 1,2,3,4 are distributed at random in four places marked 1,2,3,4. What is the probability that none of

the objects occupy the place corresponding to its number?

a. $\frac{17}{24}$

b. $\frac{3}{8}$

c. $\frac{1}{2}$

d. $\frac{5}{8}$

b. $\frac{18}{25}$

c. $\frac{4}{5}$

d. $\frac{1}{6}$

Q19. Find the probability that in a random arrangement of the letters of the word 'UNIVERSITY' the two I's come together.

a. $\frac{1}{7}$

b. $\frac{3}{5}$

c. $\frac{5}{11}$

d. $\frac{1}{5}$

Q20. A bag contains 10 balls numbered from 0 to 9. The balls are such that the person picking a ball out of the bag is equally likely to pick anyone of them. A person picked a ball and replaced it in the bag after noting its number. He repeated this process 2 more times. What is the probability that the ball picked first is numbered higher than the ball picked second and the ball picked second is numbered higher than the ball picked third?

a. $\frac{3}{25}$

Answers - Probability

4.1 Quiz

Q1 Q2 Q3

Q4 Q5 Q6
--

4.3 Class Work Problems

Q1 Q2 Q3
--

Q4 Q5 Q6
--

Q7 Q8 Q9

Q10 Q11 Q12

Q13 Q14 Q15

Q16 Q17 Q18

Q19 Q20

4.4 Exercise

Q1 Q2 Q3
--

Q4 Q5 Q6
--

Q7 Q8 Q9

Q10 Q11 Q12

Q13 Q14 Q15

Q16 Q17 Q18

Q19 Q20

CHAPTER 5

DATA SUFFICIENCY

Data Sufficiency has become another important area in many of the Campus recruitment tests as well as management entrance exams. The questions asked in this section are mostly based on Mathematics. A firm grasp over the basics of all topics being covered under the Quantitative Ability section is an essential prerequisite for doing well in the Data Sufficiency section. There will also be questions on Data Sufficiency which are based on simple reasoning or logic.

Data Sufficiency, as the name suggests, tests your ability to identify whether the data given are sufficient to answer the question or not. Hence, in all the questions in Data Sufficiency, you must stop at the stage of determining the sufficiency of the data and need not solve the problem completely. For certain questions, you are not required to do any calculation but can answer the question by mere observation. However, in some cases, it is difficult to say whether the data are sufficient or not unless complete calculations are done.

Structure of Data Sufficiency Questions

In Data Sufficiency questions typically, there will be a question followed by some data. The data are given in the form of two or three statements. (Statements I, II and III or statements A, B and C). You have to decide the answer choice for the question depending on whether the data given in the statements are sufficient to answer the questions or not. The directions are provided to you as to how to arrive at the correct answer choice. While, as mentioned above, there can be three statements, giving data, it is questions with two statements that are very common. So, for the purpose of this discussion, we will take questions with data given in two statements only.

The directions for questions on Data Sufficiency as they appear in different exams can be of five types. We will look at all these five types – designated Set – I, Set – II Set – III Set – IV and Set – V. You must read the directions carefully before you start answering the questions.

DIRECTIONS - SET I

"Each problem contains a question and two statements, A and B, giving certain data.

You have to select the correct answer from (1) to (4) depending on the sufficiency of data given in the statements to answer the question.

Choose 1 if statement A alone is sufficient and statement B alone is not sufficient to answer the question.

Choose 2 if statement B alone is sufficient and statement A alone is not sufficient to answer the question.

Choose 3 if statements A and B together are sufficient but neither statement alone is sufficient to answer the question.

Choose 4 if both statements A and B together are not sufficient to answer the question and additional data specific to the problem are needed."

DIRECTIONS - SET II

"Each problem contains a question and two statements, A and B, giving certain data. You have to select the correct, answer from (1) to (5) depending on the sufficiency of data given in the statements to answer the question.

Choose 1 if statement A alone is sufficient and statement B alone is not sufficient to answer the question.

Choose 2 if statement B alone is sufficient and statement A alone is not sufficient to answer the question.

Choose 3 if statements A and B together are sufficient but neither statement alone is sufficient to answer the question.

Choose 4 if each statement alone is sufficient to answer the question.

Choose 5 if both statements A and B together are not sufficient to answer the question and additional data specific to the problem are needed."

DIRECTIONS - SET III

"Each question is followed by two statements giving certain data. You have to decide whether the information provided in the statements is sufficient for answering the question.

Choose 1 if the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.

Choose 2 if the question can be answered by using either statement alone.

Choose 3 if the question can be answered by using both statements together, but cannot be answered by using either statement alone.

Choose 4 if the question cannot be answered even by using both the statements together"

Note that choices (1) and (2) of Set – II are both taken care of by choice (1) in set III, while choice (4) of Set II is equivalent to choice (2) of Set III.

DIRECTIONS - SET IV

"Each question is followed by two statements, A and B giving certain data. Answer each question using the following instructions:

Choose 1 if the question can be answered by using statement A alone but not by using B alone.

Choose 2 if the question can be answered by using statement B alone but not by using A alone.

Choose 3 if the question can be answered by using either statement alone.

Choose 4 if the question can be answered by using both the statements together but not by either statement alone.

This set of directions does not allow for the "cannot be answered even by using both the statements" case.

DIRECTIONS - SET V

Each problem contains a question and two statements, A and B, giving certain data. You have to select the correct answer from (1) to (5) depending on the sufficiency of data given in the statements to answer the question.

Choose 1 if statement A alone is sufficient and statement B alone is not sufficient to answer the question.

Choose 2 if statement B alone is sufficient and statement A alone is not sufficient to answer the question.

Choose 3 if each statement alone is sufficient to answer the question.

Choose 4 if statements A and B together are sufficient but neither statement alone is sufficient to answer the question.

Choose 5 if both statements A and B together are not sufficient to answer the question and additional data specific to the problem are needed."

Rules for solving Data Sufficiency questions

- 1. Only a unique answer from a single statement is acceptable.*
- 2. Two different answers from two different statements are acceptable.*
- 3. 'NO' is an answer in Data Sufficiency*

4. Always try to see whether the statements taken independently can solve the question. Combine them only when it is certain that statements taken independently cannot answer the question.

5.1 WORKED OUT EXAMPLES

Q1 What is the value of $2x + 7$, where x is an integer?

Statements:

A. $x > 3$

B. $x < 5$

Solution

We need to find the value of $2x + 7$. Clearly, to find this value we need x . The question only states that x is an integer.

Step I:

Take only statement A and try to solve the question. Here, $x > 3$

So x can take the values 4, 5, 6, 7...

$\Rightarrow 2x + 7$ can be 15, 17, 19, 21...

Thus, we cannot find a unique value of $2x + 7$ using statement A alone.

Step II:

Take only statement B and try to solve the question. Here, $x < 5$

So x can take values 4, 3, 2, 1, 0, 1, 2...

$\Rightarrow 2x + 7$ can be 15, 13, 11, 9...

Thus, we cannot find a unique value of $2x + 7$ using statement B alone.

Step III:

Since step I and step II have failed to give an answer, let's consider statements A and B

together. Now, we have $x > 3$ and $x < 5$ i.e. $3 < x < 5$. This gives $x = 4$

$\Rightarrow 2x + 7 = 15$

We get a unique value of $2x + 7$ by considering both the statements together.

Hence, the correct answer is option 3.

If we were unable to get the answer even after using both the statements A and B together, then the data would have been insufficient to solve the question.

Q2. What is the value of $x + y$?

Statements:

A. $x + 2y = 7$

B. $2x + 4y = 10$

Solution

It is very obvious that, using statement A alone or using statement B alone, we cannot find the value of $x + y$.

But when we use both the statements A and B together, we get two equations which are inconsistent. Thus, we cannot find out the

values of x and y and thus, the value of $(x + y)$ cannot be found out even by combining statements A and B together.

So, the question can be answered using neither statement A alone nor statement B alone and also cannot be answered using both the statements together.

Q3. What is the LCM of 3 and x ?

Statements:

A. HCF of 3 and x is 1.

B. x is a positive composite number less than 5.

Solution

Using statement A alone:

HCF of 3 and x is 1. This means that 3 and x do not have any common divisors other than 1 and hence, 3 and x are relatively prime numbers. But, this does not give us a unique value of x . So, we cannot find the LCM of 3 and x . Statement A alone is not sufficient to answer the question.

Using statement B alone:

The only composite number less than 5 is 4.

So we get the unique value of x as 4. LCM of 3 and 4 is 12. Statement B alone is sufficient to answer the question.

Q4. What is the value of x ?

Statements:

A. $x + 2 = 5$

B. $x + 2 = 6$

Solution

These two statements will give two different answers. Using statement A alone we get $x = 3$, but using statement B alone we get $x = 4$. The two answers are different, but we are not concerned about the answers. What is important is whether the data is sufficient to solve the question or not. In this case both the statements A and B are individually sufficient to solve the question.

Q5. Is x a prime number?

Statements:

A. $x + y = 5$

B. $x + 12 = 18$

Solution

Using statement A alone, we do not get any unique answer as y is not given.

Using statement B alone, we get $x = 6$ which is not a prime number, hence the answer to the question "Is x a prime number?" is "No".

Although, the answer is “No”, we still have an answer and the data is sufficient to solve the question.

5.2 Class Work Problems

Each problem contains a question and two statements, A and B, giving certain data. You have to select the correct, answer from (1) to (5) depending on the sufficiency of data given in the statements to answer the question.

Choose 1 if statement A alone is sufficient and statement B alone is not sufficient to answer the question.

Choose 2 if statement B alone is sufficient and statement A alone is not sufficient to answer the question.

Choose 3 if statements A and B together are sufficient but neither statement alone is sufficient to answer the question.

Choose 4 if each statement alone is sufficient to answer the question.

Choose 5 if both statements A and B together are not sufficient to answer the question and additional data specific to the problem are needed."

Q1. What is the pass percentage for an exam conducted by ABC University?

A. A candidate scoring 25% of the total marks fails by 30 marks.

B. A candidate scoring 50% of the total marks gets 20 marks more than pass marks.

Q2. A scooter was parked in a parking lot for 10 days. For the first seven days, what was the average cost per day for parking?

A. The total cost for parking for 10 days was Rs. 30.50, which included the rate of Rs. 2 per day for each day after the first seven days.

B. The average cost per day for parking, all 10 days was Rs. 3.05.

Q3. The number of soldiers in a parade is less than 250. How many soldiers are there?

A. The soldiers can arrange themselves in rows of 3, 5 and 7.

B. The number of soldiers is an even number.

Q4. For a company xyz, were the profits of 1981 greater than those 1982?

A. Company's sales, in 1982 were greater than the sales in 1981.

B. Profits of 1982 were 1.25 times the average of the profits of 1981 and 1982 together.

Q5. What is the rate of compound interest given by a bank?

A. A deposit of Rs. 3000 becomes Rs. 3300 in three years.

B. The interest received on Rs. 8000 is Rs. 560.

Q6. What is the percentage of students who are not intelligent?

A. The ratio of number of boys and girls is 3: 2.

B. 20% of the boys and 20% of the girls are intelligent.

Q7. Is the sum of the costs of a TV and a sofa more than the sum of the costs of a VCR and a table?

A. 30% of the cost of the TV and 20% of a cost of the sofa is more than 40% of a cost of the VCR and 60% of the cost of table

B. 20% of the cost of the TV and 30% of the cost of the sofa is less than 10% of the cost of the VCR and 15% of the cost of the table.

Q8. What is the Greatest Common Divisor of the integers p and q ?

A. The Greatest Common Divisor of $2p$ and $2q$ is 10.

B. Both p and q are odd.

Q9. When integer p is divided by 2, the remainder is 1. What is the remainder when p is divided by 4?

A. When p is divided by 8, the remainder is 3.

B. p is a multiple of 5.

Q10. A and B are solutions of water and glycerine. If solution C consists of 2 parts of A and 3 parts of B, then what percent of solution C will glycerine form?

A. Solution A contained 10% glycerine and solution B contained 3% glycerine

B. There was a total of 10 litres of the solution C

Q11. What is the compound interest earned on the sum of Rs. 10,000 for 2 years?

A. If the rate of interest had been 2 percentage points more, an additional interest of Rs. 444 would have been earned.

B. At the same rate of simple interest, the interest earned in 2 years would have been Rs. 2000.

Q12. Given that $p^q = r^q$ where q is a whole number. Is $p = r$?

A. q is divisible by 3.

B. q is odd.

Q13. What is the total income of Anand, Bhargav and Charan?

A. Income of Anand is Rs. 300.

B. 25% of Anand's income is equal to 15% of Bhargav's income, which in turn is equal to 20% of Charan's income.

Q14. When a table and a chair are purchased, 11% of the price of the table is given as a discount on the price of the chair while no discount is offered on the price of table. What is the total amount actually paid for the two items purchased after the discount?

A. Before the discount, total price of the chair and table is Rs. 500 and the price of the table is Rs. 200 more than that of the chair.

B. After discount, the price of the chair is reduced by Rs. 38.50 from Rs. 150

Q15. If n is an integer and $10 < 3^n < 300$, then what is the value of n ?

A. n is the square of an integer.

B. 3^n is the square of an integer.

Q16. Find $x + y$ where x and y are non-negative integers.

A. $2^x + 3^y = 17$.

B. $9^x + 8^y = 145$.

Q17. What is the profit percentage when two varieties of rice priced at Rs. 6 per kg and Rs. 8 per kg respectively are mixed and sold at Rs. 9 per kg?

A. The total quantity of mixture sold was 10 kg and the total cost of the mixture was Rs. 68.

B. Two varieties of rice costing Rs. 6 per kg and Rs. 8 per kg are mixed in the ratio of 3 : 2

Q18. How long will it take for a deposit to become, four times itself at simple interest?

A. The fixed deposit is Rs. 50000.

B. The rate of interest is 8% p.a.

Q19. What percentage of the employees of company X are assistants?

A. Exactly 40% of the men and 55% of the women are assistants.

B. The ratio of the number of assistants to the number of non-assistants is 9 : 11.

Q20. A bike can be purchased by paying a down payment plus five (interest free) installments. What is the price of the bike?

A. Each installment is Rs. 600.

B. Down payment is 40% of the total price of the bike.

Q21. If p , q and d are positive integers and d is odd, are both p and q divisible by d ?

A. $p + q$ is divisible by d .

B. $p - q$ is divisible by d .

Q22. If x , y and z are positive integers, is the sum of $(2x + 4y)$ and $(x - y + z)$ divisible by 3?

A. $(x + y)$ is divisible by 3.

B. z is divisible by 3.

Q23. If the arithmetic mean of six numbers is 30, then how many numbers are greater than 30?

A. Three of the six numbers are equal to 40.

B. Three of the six numbers are equal to 20.

Q24. The difference between the compound interest and the simple interest earned on a sum of money at same rate of interest for 3 years is Rs. 76.25. What is the sum?

A. The simple interest earned in 3 years is Rs. 1500.

B. Rate of interest per annum is 5%.

Q25. A piece of copper wire is bent in the form of an arc of a circle. What is the length of the copper wire?

A. The arc of the circle subtends an angle of 60° at the centre of the circle.

B. Radius of the circle is 5 cm.

Q26. Is the speed of the passenger train more than 36 kmph?

A. The train takes 40 seconds to cross a platform 400 m long.

B. The train takes one minute to cross a 360 m long bridge.

Q27. From a tank full of petrol, the petrol is discharged into a tub through a pipe. What is the quantity of petrol discharged from the tank in a minute?

A. The tank was emptied in 30 minutes.

B. The dimensions of the tub are

$$3m \times 2m \times 2m$$

Q28. An arithmetic progression has 81 terms. Find the sum of its terms.

A. Twice its twenty first term exceeds its first term by 63.

B. Its second term is 5

Q29. Is quadrilateral PQRS a rhombus?

A. One of the angles of the quadrilateral is obtuse.

B. The opposite sides are parallel and equal.

Q30. If Ramesh walked from his home to his office at a kmph, he would be a minutes early. Instead had he walked at b kmph, he would be b minutes early, where a and b are distinct. Find the usual time he takes to reach his office.

A. $a+b=10$

B. $a-b=2$

Q31. Amar and Babu completed a job working on alternate days. Find the time taken by them to complete it.

A. Had they worked together they would have completed the job in 20 days.

B. Times taken by Amar and Babu working individually to complete a

similar job are 30 days and 60 days respectively

Q32. A geometric progression G has $3N$ terms. Find the sum of its terms.

A. The sum of the first N terms of a geometric progression having the same first term as G and a common ratio which is the cube of the common ratio of G is $2(3^9 - 1)$.

B. The first term and the common ratio of G are in the ratio 4 : 3.

Q33. What is the ratio of the volume of cube X to that of sphere Y ?

A. If the cube is melted, the metal can be used to make 2 spheres, one of size equal to that of sphere Y and the other with a diameter half that of Y .

B. The surface areas of the cube and the sphere are not equal.

Q34. What is the volume of the sphere?

A. The surface area of the sphere is 120 sq. cm.

B. The sphere can be reformed into a cuboid of a total surface area of 120 sq. cm. by melting.

Q35. Rakesh rowed his boat from a point A in a river 24 km. downstream and then returned to A. Find the speed of his boat in still water if his upstream journey took p hours.

A. The time he took for the downstream journey was 3 hours less than that for the upstream journey and $p = 6$.

B. His upstream and downstream speeds were 4 kmph and 8 kmph respectively.

Q36. Amal, Bhuvan and Chittan can complete a job in 6 days, a days and b days respectively. They started it and worked for 2 days, then Amal left. Bhuvan left after another day. Chittan completed the remaining part of the job in another day. Who completed the maximum part of the job?

A. $a > 9$

B. $b > 12$

Q37. Four non-zero integers are in geometric progression. Find the square of the common ratio.

A. The sum of the middle numbers is more than the first number by five times the first number.

B. The product of the second and last numbers exceeds the product of the first and third numbers by 12 times the square of the first number.

Q38. R, S and T ran a race with distinct speeds. Who won the race?

A. The average of the speeds of R and T when doubled would equal the sum of the average of the speeds of R and S and the average of the speeds of S and T.

B. Twice the speed of R was equal to thrice the speed of T.

Q39. Pipes P, Q and R can be used for filling or emptying with the same capacity. If P and Q are used for filling and R is used for emptying, a tank would be filled in 6 hours. Find the time taken by R to fill it.

A. If all the pipes are used for filling, the tank would be filled in 2 hours.

B. If Q and R are used for filling and P is used for emptying, the tank would be filled in 6 hours.

Q40. What is the third number in an arithmetic progression consisting of nine numbers?

A. Sum of the first and the last numbers is 24.

B. First number is 4 and the last number is 20

Q41. Between A and B, one always tells the truth (truth teller) and the other always lies (liar). Who is the truth teller?

A. A said, "We both are liars".

B. B said, "We both are truth tellers"

Q42. Six person A, B, C, D, E and F, among whom there are two couples and two bachelors, are sitting around a circular table. Each married person is neither opposite nor adjacent to his/her spouse Is F a bachelor?

A. D and E are sitting together and none of them is a bachelor. A and B are opposite each other

B. C, who is one of the married persons. is to the immediate right of A. B is a bachelor.

Q43. Did David write a letter to Paddy today?

A. David wrote a letter to Molly yesterday.

B. David writes a letter to Paddy, only if he has written a letter to Molly the previous day.

Q44. What is the area of the triangle?

A. One of the sides of the triangle is on the y-axis.

B. Two of the sides lie on $x + y = 1$ and $x - y = 1$.

Q45. Each of A, B and C make two statements either both true or both false. They are from different cities Mumbai, Delhi and Kolkata. A said "Exactly two of us are truth tellers and C is from Delhi". Who is from Kolkata?

A. B said, "Exactly two of us are liars. C is from Mumbai".

B. C said, "A is from Delhi. B is from Kolkata".

Q46. Find the total distance between two stations A and B. Given the A is to the west of B

A. There are four stations between A and B with the distance between the two neighboring middle stations being 160 km.

B. For any station, the distance when compared to the immediate next station towards east is twice that towards west

Q47. In a colony, 100 people read "The Hindu". 30 among them also read "The Express". How many read "The Express"?

A. 40% of the people who read "The Express" do not read "The Hindu"

B. 20% of the people read neither of the newspapers

Q48. What is the area of the square ABCD?

A. The midpoints of AB and CD are (1, 4) and (1, 8) respectively.

B. The point of intersection of the diagonals is (2, 10) and one of the vertices of the square is (4, 6).

Q49. A team of four persons is to be selected from six persons A, B, C, D, E and F, such that at most one of C and E can be selected. Which pair of persons will always be selected?

A. If C is selected, then D must not be selected.

B. If D is selected, then A must not be selected.

Q50. Akash is to the east of Bala and to the south of Dhruv. In which direction is Akash located with respect to Chandru?

A. Bala is to the north of Eric, which is to the west of Chandru.

B. Fahad is to the east of Dhruv and to the north of Chandru.

Q51. Some jugs are mugs. All mugs are rugs. All rugs are hugs. Is Albert a hug?

A. Albert is a jug.

B. Albert is a mug.

Q52. How many letters are there in the word? (There is no repetition of the letters in the word)

A. The number of ways of choosing 3 letters such that two of them are consonants and one is a vowel is 24.

B. The number of ways of choosing 4 letters containing at least 3 vowels is 16.

Q53. Is Pradip the tallest person in his office?

A. There is nobody in the office who is taller than Pradip.

B. Not all the employees in the office are shorter than Pradip.

Q54. Five professors P, Q, R, S and T have to give seminars on five different days of a week., Monday through Friday, such that P

and Q cannot give seminars on consecutive days. On which day does S give the seminar?'

A. R gives the seminar on the day immediately following the day on which S gave the seminar.

B. P gave the seminar on the third day following the day on which T gave the seminar

Q55. A cube is painted with red, blue and green such that each colour is painted on two faces. The cube is cut into 64 small and identical cubes. How many small cubes have exactly two colours on them?

A. Both faces painted in blue are adjacent to at least one face with red paint.

B. Both faces painted in red are adjacent to exactly one face with green paint.

Q56. There are some ladies and some gentlemen in a group. How many persons are in the group?

A. The probability of selecting 3 persons from the group such that at least 1 lady is included is $\frac{31}{35}$.

B. The probability of selecting 3 ladies in the group is $\frac{1}{35}$.

Q57. When does a wall clock show the correct time?

A. The clock previously showed the correct time at 7 O'clock on Wednesday

B. The clock is 12 minutes fast on Sunday at 10:00 a.m. and is 12 minutes slow on the same day at 10:00 p.m.

Q58. If the first day of a month is Monday, what will be the first day of the next year?

A. It is not a leap year.

B. The first day of the given month, which is in the second half of the year, coincides with the first day of the first month of that year.

Q59. If P is the father of Q, then how is M related to P?

A. Q's only cousin's, only cousin's only aunt is M.

B. M is the only daughter of N, who is the grandfather of Q

Q60. How much time does a frog, which is at the bottom of the well, take to come out of the well?

A. Every hour the frog moves up and slips down. In this process, it changes its position by one meter upward for every hour.

B. The well is seven metres deep.

5.3 Exercise

Each question Q is followed by two statements A and B . Indicate your responses based on the following directives:

Mark (1) if Q can be answered from A alone but not from B alone.

Mark (2) if Q can be answered from B alone but not from A alone.

Mark (3) if Q can be answered from A alone as well as from B alone.

Mark (4) if Q can be answered A and B together but not from any of them alone.

Mark (5) if Q cannot be answered even from A and B together.

In a single elimination tournament, any player is eliminated with a single loss. The tournament is played in multiple rounds subject to the following rules:

(a) If the number of players, say n , in any round is even, the players are grouped into $n/2$ pairs. The players in each pair play a match against each other and the winner moves on to the next round.

(b) If the number of players, say n in any round is odd, then one of them is given a bye, that is, he automatically moves on to

the next round. The remaining $(n - 1)$ players are grouped into $(n - 1)/2$ pairs. The players in each pair play a match against each other and the winner moves on to the next round. No player gets more than one bye in the entire tournament.

Thus, if n is even, then $n/2$ players move on to the next round while if n is odd, then $(n + 1)/2$ players move on to the next round. The process is continued till the final round- which obviously is played between two players. The winner in the final round is the champion of the tournament.

Q1. What is the number of matches played by the champion?

A. The entry list for the tournament consists of 83 players

B. The champion received one bye

Q2. If the number of players, say n , in the first round was between 65 and 128, then what is the exact value of n ?

A. Exactly one player received a bye in the entire tournament

B. One player received a bye while moving on to the fourth round from the third round.

Each question is followed by two statements A and B. Indicate your responses based on the following directives:

Mark (1) if the question can be answered using A alone but not using B alone

Mark (2) If the question can be answered using B alone but not using A alone

Mark (3) if the question can be answered using A and B together, but not using either A or B alone

Mark (4) if the question cannot be answered even using A and B together.

Q3. The average weight of a class of 100 students is 45 kg. The class consists of two sections- I and II, each with 50 Students. The average weight, W_i , of Section I is smaller than the average weight, W_{ii} , of Section II. If the heaviest student, say Deepak, of Section II is moved to Section I, and the lightest student, say Poonam, of Section I is moved to Section II, then the average weights of the two sections are switched, i.e., the average weight of Section I becomes W_{ii} and that of Section II becomes W_i . What is the weight of Poonam?

A. $W_{ii} - W_i = 1.0$

B. Moving Deepak from Section II to I (without any move from I to II) makes the average weights of the two sections equal.

Q4. ABC Corporation is required to maintain at least 400 Kilolitres of water at all times in its factory, in order to meet safety and regulatory requirements.

ABC is considering the suitability of a spherical tank with uniform wall thickness for the purpose. The outer diameter of the tank is 10 meters. Is the tank capacity adequate to meet ABC' requirements?

A. The Inner diameter of the tank is at least 8 metres.

B. The tank weight 30,000 kg when empty, and is made of a material with density of 3 gm/cc

Q5. Consider integers x , y and z .. What is the minimum possible value of $x^2 + y^2 + z^2$?

A. $x + y + z = 89$

B. Among x , y , z two are equal.

Q6. Rahim plans to draw a square JKLM with a point O on the side JK but is not successful. Why is Rahim unable to draw the square?

A. The length of OM is twice that of OL

B. The length OM is 4

Q7. What is the 57th number in a series of numbers?

A. Each number in the series is three more than the preceding number.

B. The tenth number in the series is 29.

Mark (1) if the question can be answered using A alone but not using B alone

Mark (2) If the question can be answered using B alone but not using A alone

Mark (3) if the question can be answered using A and B together, but not using either A or B alone

Mark (4) if the question cannot be answered even using A and B together.

Questions are followed by two statements labeled as I and II. Decide if these statements are sufficient to conclusively answer the question. Choose the appropriate answer from the options given below.

Mark (1) if the question can be answered using one statement alone but not using the other statement alone

Mark (2) If the question can be answered either statement alone

Mark (3) if the question can be answered using both the statements together, but not using either statements alone

Mark (4) if the question cannot be answered even using both statements together.

Q8. A sequence of positive integer is defined as $A_{n+1} = A_n^2 + 1$ for each $n \geq 0$. What is the value of Greatest Common Divisor of A_{900} and A_{1000} ?

A. $A_0 = 1$

B. $A_1 = 2$

Answers - Data Sufficiency

5.2 Class Work Problems

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12
Q13	Q14	Q15
Q16	Q17	Q18
Q19	Q20	Q21
Q22	Q23	Q24
Q25	Q26	Q27
Q28	Q29	Q30
Q31	Q32	Q33
Q34	Q35	Q36
Q37	Q38	Q39
Q40	Q41	Q42
Q43	Q44	Q45
Q46	Q47	Q48
Q49	Q50	Q51
Q52	Q53	Q54
Q55	Q56	Q57
Q58	Q59	Q60

5.3 Exercise

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	

Chapter 6

PROFIT, LOSS & DISCOUNT

INTRODUCTION

Profit & Loss

Profit and Loss are part and parcel of everyday transactions. The terms 'cost price' and 'selling price' are used at every stage of goods exchanging hands.

The price at which a person buys (or produces) a product is the **Cost Price** (CP) of the product w.r.t. to that person and the price at which a person sells a product is the sales price or the **Selling Price** (SP) of the product, again w.r.t. that person. At each stage, the cost price for one person becomes the selling price for another.

For example, if A buys Apples at Rs. 75 per kg from a wholesaler and sells them to B at Rs. 80 per kg, then for one kg of apples that exchanged hands between the wholesaler, A and B.

The wholesaler's Selling Price = Rs. 75 = A's Cost Price and As Selling Price = Rs. 80 = B's Cost Price

When a person is able to sell a product at a price higher than its cost price for him, then we say he has earned a **Profit** (P).

Profit = Selling price - Cost Price

$$P = SP - CP$$

Similarly, if a person sells an item for a price lower than its cost price for him, we say a **Loss** (L) has been incurred.

Loss = Cost price - Selling Price

$$L = CP - SP$$

Percentage Profit or Loss

We often need to compare the gains or losses of two business transactions. The actual gains or losses are not comparable by themselves as the investment or the capital of the two businesses may differ. In such cases, the comparison of gains and losses can be made by converting them into percentages.

Formulae

1. Gain = SP-CP
2. Loss = CP-SP
3. Loss or gain is always reckoned on CP.
4. Gain % = $\frac{SP-CP}{CP} \times 100\%$.
5. Loss % = $\frac{CP-SP}{CP} \times 100\%$.
6. When a person sells two similar items, one at a gain of say x%, and the other at a loss of x%, then the seller always incurs a loss given by:

$$\text{Loss \%} = \left(\frac{\text{Common Loss and Gain \%}}{10} \right)^2 = \left(\frac{x}{10} \right)^2$$

7. If a trader professes to sell his goods at cost price, but uses false weights, then

$$\text{Gain \%} = \left[\frac{\text{Error}}{\text{True Value} - \text{Error}} \right] \times 100 \%$$

Marked Price & Discount

The difference between the Selling Price of a good and its Cost Price is known as **markup**. Manufacturers add a markup to the Cost Price of an item in order to make profits. The price that is printed on an article or written on the label attached to it is the sum of the Cost Price and the markup, and is called the **Marked Price** (MP) or **List Price** of the item.

A shopkeeper or retailer buys goods from the manufacturer or wholesaler. The retailer then marks up the cost price (the increased price forming his selling price) so as to have a good profit. Note that the term “markup” is used to denote the amount of increase in the Cost Price, while “Marked Price” is used to denote the increased price as it appears on the product.

i.e. $\text{Cost Price} + \text{Markup} = \text{Marked Price}$

Markup can either be expressed as an amount (as shown above) or as a percentage

of the Cost Price. So, another expression for Marked Price would be,

$$\begin{aligned} &\text{Cost Price} + \\ &\frac{\text{Markup (as a \%)}}{100} \times \text{Cost Price} \\ &= \text{Marked Price.} \end{aligned}$$

Generally, $\text{MP} = \text{SP}$. However, sometimes, in order to increase sales or to sell-off the old stock, retailers reduce the marked price of the article by a certain amount called **Discount**. In this case, the Selling Price will be the reduced price (i.e. price after deducting the discount). i.e. $\text{Selling Price} = \text{Marked Price} - \text{Discount}$

Similar to markup, discount can also be represented both as an amount (shown above) and as a percentage. So, another expression could be,

$$\begin{aligned} &\text{Marked Price} - \\ &\frac{\text{Discount (as a \%)}}{100} \times \text{Marked Price} \\ &= \text{Selling Price} \end{aligned}$$

More Formulae

$$\text{Discount Percentage} = \frac{\text{Discount}}{\text{Marked Price}} \times 100$$

$$\frac{\text{SP}}{\text{MP}} = 1 - \frac{\text{Discount Percentage}}{100}$$

Important: You must always determine Percentage Profit on the Cost Price of an item, unless it is stated otherwise in the question.

6.1 Quiz

Q1. A man sells two flats at the rate of Rs. 1.995 lakhs each. On one he gains 5% and on the other, he loses 5%. His gain or loss percent in the whole transaction is :

- a. 0.25% loss
- b. 2.5% loss
- c. 0.25% gain
- d. 25% loss

Q2. The percentage profit earned by selling an article for Rs. 1920 is equal to the percentage loss incurred by selling the same article for Rs. 1280. At what price should the article be sold to make 25% profit?

- a. Rs. 2000
- b. Rs. 2200
- c. Rs. 2400
- d. Data inadequate

Q3. When a plot is sold for Rs. 18,700, the owner loses 15%. At what price must that plot be sold in order to gain 15%?

- a. Rs. 21,000
- b. Rs. 22,500
- c. Rs. 25,300
- d. Rs. 25,800

Q4. The difference between the cost price and sale price of an article is Rs. 240. If the profit is 20%, the selling price is :

- a. Rs. 1240
- b. Rs. 1440
- c. Rs. 1400
- d. None of these

Q5. 10% loss on selling price is what percent loss on the cost price ?

- a. $9\frac{1}{11}\%$
- b. 10%
- c. $9\frac{2}{11}\%$
- d. 11%

Q6. A shopkeeper sold an article for Rs.2090.42. Approximately, what will be the percentage profit if he sold that article for Rs.2602.58?

- a. 15%

- b. 25%
- c. 20%
- d. 30%

Q7. A man purchased a box full of pencils at the rate of 7 for Rs. 9 and sold all of them at the rate of 8 for Rs. 11. In this transaction, he gained Rs. 10. How many pencils did the box contain?

- a. 100
- b. 114
- c. 112
- d. 115

Q8. If books bought at prices ranging from Rs. 200 to Rs. 350 are sold at prices ranging from Rs. 300 to Rs. 425, what is the greatest possible profit that might be made in selling eight books?

- a. Rs. 400
- b. Cannot be determined
- c. Rs. 600
- d. None of these

Q9. A man purchased sugar worth Rs. 400. he sold $\frac{3}{4}$ th at a loss of 10% and the remainder at a gain of 10%. On the whole, he gets:

- a. a loss of 5%

- b. a loss of $5\frac{1}{19}\%$
- c. a gain of $5\frac{1}{2}\%$
- d. a loss of $5\frac{5}{19}\%$

Q10. A shopkeeper sells one transistor for Rs. 840 at a gain of 20% and another for Rs. 960 at a loss of 4%. His total gain or loss percent is :

- a. $5\frac{15}{17}\%$ loss
- b. $6\frac{2}{3}\%$ gain
- c. $5\frac{15}{17}\%$ gain
- d. None of these

Q11. Saransh purchased 120 reams of paper at Rs. 80 per ream. He spent Rs. 280 on transportation, paid octroi at the rate of 40 paise per ream and paid Rs. 72 to the coolie. If he wants to have a gain of 8%, what must be the selling price per ream?

- a. Rs. 86
- b. Rs. 89
- c. Rs. 87.48
- d. Rs. 90

Q12. A house worth Rs. 1, 50,000 is sold by X to Y at 5% profit. Y sells the house back to X at 2% loss. Then, in the entire transaction

- a. X loses Rs. 1350
- b. X loses Rs. 4350
- c. X gains Rs. 3150
- d. X gains Rs. 4350

6.2 Worked out Problems

Q1. A man buys a cycle for Rs. 1400 and sells it at a loss of 15%. What is the selling price of the cycle?

- a. Rs. 1090
- b. Rs. 1160
- c. Rs. 1190
- d. Rs. 1202

Solution

$$\begin{aligned} \text{S.P.} &= 85\% \text{ of Rs. } 1400 \\ &= \text{Rs. } \left(\frac{85}{100} \times 1400 \right) = \text{Rs. } 1190 \end{aligned}$$

Q2. On selling 17 balls at Rs. 720, there is a loss equal to the cost price of 5 balls. The cost price of a ball is:

- a. Rs. 45
- b. Rs. 50
- c. Rs. 55
- d. Rs. 60

Solution

$$\begin{aligned} &(\text{C. P. of 17 balls}) - (\text{S.P. of 17 balls}) \\ &= (\text{C. P. of 5 balls}) \end{aligned}$$

$$\text{C. P. of 12 balls} = \text{S.P. of 17 balls} = \text{Rs. } 720.$$

$$\text{C. P. of 1 ball} = \text{Rs. } \left(\frac{720}{12} \right) = \text{Rs. } 60.$$

Q3. The cost price of 20 articles is the same as the selling price of x articles. If the profit is 25%, then the value of x is:

- a. 15
- b. 16
- c. 18
- d. 25

Solution

Let C.P. of each article be Re. 1
C.P. of x articles = Rs. x .

$$\text{S.P. of } x \text{ articles} = \text{Rs. } 20.$$

$$\text{Profit} = \text{Rs. } (20 - x).$$

$$\frac{20-x}{x} \times 100 = 25$$

$$2000 - 100x = 25x$$

$$125x = 2000$$

$$x = 16.$$

6.3 Class Work Problems

Q1. A man buys an article for 10% less than its value and sells it for 10% more than its value. His gain or loss percent is:

- a. Neither profit nor loss
- b. Less than 20% profit

- c. 20% profit
- d. More than 20% profit

Q2. Jacob bought a scooter for a certain sum of money. He spent 10% of the cost on repairs and sold the scooter for a profit of Rs. 1100. How much did he spend on repairs if he made a profit of 20%?

- a. Rs. 400
- b. Rs. 500
- c. Rs. 440
- d. Rs. 550

Q3. Some articles were bought at 6 for Rs. 5 and sold at 5 for Rs. 6. Gain percent is :

- a. 30%
- b. 35%
- c. $33\frac{1}{3}\%$
- d. 44%

Q4. By selling a pen for Rs. 15, a man loses one-sixteenth of what it costs him. The cost price of the pen is :

- a. Rs. 16
- b. Rs. 20
- c. Rs. 18
- d. Rs. 21

Q5. Karthik sells two horses at the same price. On one he makes a profit of 20% and on the other he suffered a loss of 20%. Which one of the following statement is true?

- a. He makes neither profit nor loss
- b. He makes a profit of 4%
- c. He suffers a loss of 4%
- d. He suffers a loss of 20%

Q6. A mechanic buys an old bike for Rs. 3600. He spent Rs. 500 for its repairs. For what sum should he sell it to gain 14%?

- a. Rs. 4560
- b. Rs. 4750
- c. Rs. 4674
- d. Rs. 5550

Q7. A reduction of 10% in price would enable a man to get 5 kilogram more sugar for Rs. 50. The reduced price per kilogram is

- a. Rs.10
- b. Rs. 9
- c. Rs. 45
- d. Rs. 1

Q8. The Marked Price of an article is Rs. 160. A discount of $12\frac{1}{2}\%$ is allowed for cash payment. What is the selling Price?

- a. Rs. 140
- b. Rs. 145
- c. Rs. 150
- d. Rs. 160

Q9. An article costs Rs. 6. A man marked it is Rs. 10 and allowed a discount of 25% on the cost price for cash payment. Find his loss or gain percent.

- a. 20% gain
- b. 25% gain
- c. 20% loss
- d. Loses 5%

Q10. A dishonest dealer professes to sell his goods at cost price, but he uses a false weight and thus gains $6\frac{18}{47}\%$. For a kg, he uses a weight of:

- a. 940 gms
- b. 953 gms
- c. 947 gms
- d. 960 gms

Q11. A shopkeeper expects a gain of $22\frac{1}{2}\%$ on his cost price. If in a week, his sale was of Rs. 392, what was his profit?

- a. Rs. 18.20
- b. Rs. 72

- c. Rs. 70
- d. Rs. 88.25

Q12. An article when sold at a gain of 5% yields Rs. 15 more than when sold at a loss of 5%. Its cost price would be:

- a. Rs. 150
- b. Rs. 250
- c. Rs. 200
- d. Rs. 300

Q13. A fruit seller sells mangoes at the rate of Rs. 9 per kg and thereby loses 20%. At what price per kg, he should have sold them to make a profit of 5%?

- a. Rs. 11.81
- b. Rs. 12.25
- c. Rs. 12
- d. Rs. 12.31

Q14. A fruit seller purchases fruits at the rate of 6 fruits for a rupee and sells them at the rate of Rs. 2.50 per dozen. Find his loss or gain percent.

- a. 25% gain
- b. 25% loss
- c. 20% gain
- d. 20% loss

Q15. If on selling 12 notebooks, a seller makes a profit equal to the selling price of 4 notebooks, what is his percent profit?

- a. $16\frac{2}{3}$
- b. 50
- c. 25
- d. Data Inadequate
- e. None of these

d. 10% gain

Q3. An article when sold at a gain of 5% yields Rs. 15 more than when sold at a loss of 5%. Its cost price would be:

- a. Rs. 150
- b. Rs. 250
- c. Rs. 200
- d. Rs. 300

6.4 Exercise

Q1. Sunil purchased equal number of two varieties of oranges at 4 and 6 per rupee respectively. He sold both varieties at 10 for Rs. 2. Then he.

- a. Gains 4%
- b. Gains 5%
- c. Loses 4%
- d. Loses 5%

Q2. A cloth merchant sold half of his cloth at 20% profit, half of the remaining at 20% loss and the rest was sold at the cost price. In the total transaction, his gain or loss will be

- a. Neither loss nor gain
- b. 5% gain
- c. 5% loss

Q4. A grocer sells rice at a profit of 10% and uses weights which are 20% less than the market weight. The total gain earned by him will be :

- a. 30%
- b. 37.5%
- c. 35%
- d. None of these

Q5. Profit earned by selling an article for Rs. 1060 is 20% more than the loss incurred by selling the article for Rs. 950. At what price should the article be sold to earn 20% profit?

- a. Rs. 980
- b. Rs. 1800
- c. Rs. 1080
- d. None of these

Q6. The cash difference between the selling prices of an article at a profit of 4% and 6% is Rs. 3. The ratio of the two selling prices is

- a. 51 : 52
- b. 51 : 53
- c. 52 : 53
- d. 52 : 55

Q7. A shopkeeper purchased 70 kg of potatoes for Rs. 420 and sold the whole lot at the rate of Rs. 6.50 per kg. What will be his gain percentage?

- a. $4\frac{1}{6}\%$
- b. $8\frac{1}{3}\%$
- c. $6\frac{1}{4}\%$
- d. 20%

Q8. A house and a shop were sold for 1 lakh each. In this transaction, the house sale resulted into 20% loss whereas the shop sale resulted into 20% profit. The entire transaction resulted in

- a. no loss, no gain
- b. loss of Rs. $\frac{1}{18}$ lakh
- c. loss of Rs. $\frac{1}{12}$ lakh
- d. gain of Rs. $\frac{1}{24}$ lakh

Q9. By selling a pen for Rs. 15, a man loses one-sixteenth of what it costs him. The cost price of the pen is

- a. Rs. 16
- b. Rs. 20
- c. Rs. 18
- d. Rs. 21

Q10. 100 oranges are bought at the rate of Rs. 350 and sold at the rate of Rs. 48 per dozen. The percentage of profit or loss is

- a. $14\frac{2}{7}\%$ gain
- b. $14\frac{2}{7}\%$ loss
- c. 15% gain
- d. 15% loss

Answers - Profit, Loss & Discounts

6.1 Quiz

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12

6.3 Class Work Problems

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12
Q13	Q14	Q15

6.4 Exercise

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10		

PART II

CHAPTER 7

MENSURATION

INTRODUCTION

Mensuration is a branch of geometry where we study measurements of the perimeter, area and volume of geometrical figures and solid objects.

A solid is a portion of space bounded by surfaces, known as faces. The intersection of the surfaces gives lines known as edges. The intersection of the edges gives points known as vertices.

Lateral Surface Area (LSA): The sum of the areas of all the vertical/lateral surfaces (excluding the top and bottom) is known as the lateral surface area of the solid

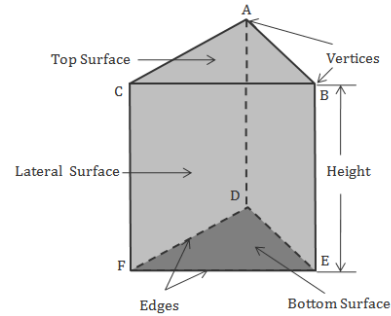
Total Surface Area (TSA): The sum of the areas of all surfaces (including the top and bottom) is known as the total surface area of the solid

$$TSA = LSA + \text{Area of top and bottom surfaces}$$

RIGHT PRISM

A solid in which the top and bottom surfaces are identical polygons and the lateral (vertical) surfaces are rectangular in shape

and perpendicular to the top and bottom surfaces is known as a right prism.



In general, the relations applicable to all kinds of right prism are given by:

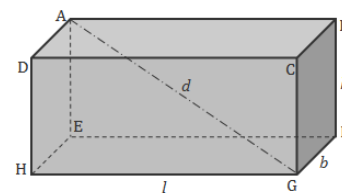
$$LSA = \text{Perimeter of base} \times \text{height}$$

$$TSA = LSA + 2 \times \text{Area of base}$$

$$\text{Volume (V)} = \text{Area of base} \times \text{height}$$

CUBOID

A right prism in which all the six surfaces are rectangular in shape is known as a cuboid or a rectangular parallelepiped.



For a cuboid with base length l , base breadth b and height h :

$$LSA = 2(lh + bh)$$

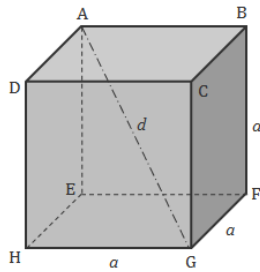
$$TSA = 2(lh + bh + lb)$$

$$\text{Volume (V)} = lbh$$

$$\text{Body diagonal} = \sqrt{l^2 + b^2 + h^2}$$

CUBE

A right prism in which all the six surfaces are square in shape is known as a cube. It is a special case of the cuboid.



For a cube with side a :

$$LSA = 4a^2$$

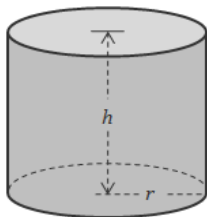
$$TSA = 6a^2$$

$$\text{Volume (V)} = a^3$$

$$\text{Body diagonal} = a\sqrt{3}$$

CYLINDER

A right circular cylinder is a special case of the right prism. The top and bottom surfaces are identical polygons with infinite number of sides, or are circles. Its lateral surfaces are curved; hence the concept of LSA is replaced by the concept of CSA (curved surface area).



For a cylinder with height h and radius of base circle r :

$$CSA = 2\pi rh$$

$$TSA = 2\pi rh + 2\pi r^2$$

$$\text{Volume (V)} = \pi r^2 h$$

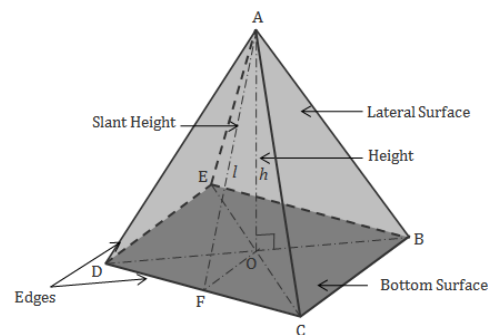
RIGHT PYRAMID

A right pyramid is a solid in which:

The bottom surface (i.e. base) is a polygon, each vertex of which is joined with the help of slanted edges to a single vertex at the top.

The lateral (slanted-vertical) surfaces are triangular in shape.

The line joining the top vertex to the centre of the polygon which forms the bottom surface (also known as the height) is perpendicular to the bottom surface.



In general, the relations applicable to all the right pyramids are given by:

$$LSA = \quad \quad \quad =$$

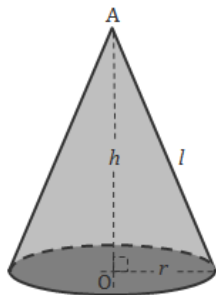
$$\frac{1}{2} \times \text{Perimeter of Base} \times \text{Slant Height}$$

$$TSA = LSA + \text{Area of base}$$

$$\text{Volume (V)} = \frac{1}{3} \times \text{Area of Base} \times \text{Height}$$

CONE

A Right Circular Cone is a special case of the right pyramid, in which the bottom surface is a polygon with an infinite number of sides; a circle. Its lateral surfaces are curved; hence the LSA generalizes to the concept of a CSA (curved surface area).



For a cone with height h , slant height l and radius of the base circle r :

$$CSA = \pi r l$$

$$TSA = \pi r l + \pi r^2$$

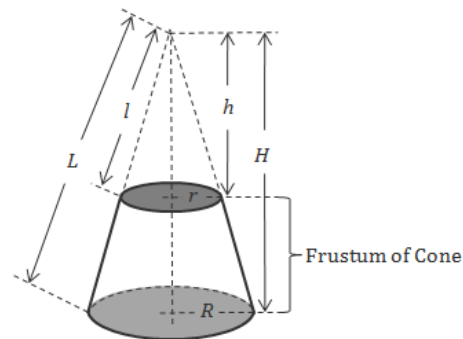
$$\text{Volume (V)} = \frac{1}{3} \pi r^2 h$$

$$\text{Slant Height (l)} = \sqrt{r^2 + h^2}$$

FRUSTUM OF A CONE

If the upper part of the cone (the part containing the top vertex) is cut off by a plane parallel to the base, the remaining part of the original cone is known as the frustum

of the cone. The upper part so removed will also be a cone, similar to the original cone.



$$CSA = \pi (r+R) l_f$$

$$TSA = \pi (r+R) l_f + \pi r^2 + \pi R^2$$

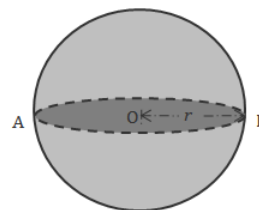
$$\text{Volume (V)} = \frac{1}{3} \pi (r^2 + R^2 + rR) H_f$$

$$\text{Slant Height (l}_f\text{)} = \sqrt{(R - r)^2 + H_f^2}$$

$$\text{Where } H_f = H - h \text{ and } l_f = L - l$$

SPHERE

A sphere is a solid for which every point on the surface is at the same distance from its centre. The distance from the centre to any point on the surface is known as its radius.



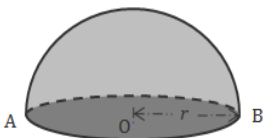
For a sphere with radius r :

$$CSA = TSA = 4 \pi r^2$$

$$\text{Volume (V)} = \frac{4}{3} \pi r^3$$

HEMISPHERE

If a sphere is cut into two halves symmetrically, either of the parts thus formed is called a hemisphere.



For a hemisphere with radius r :

$$\text{CSA} = 2 \pi r^2$$

$$\text{TSA} = 3 \pi r^2$$

$$\text{Volume (V)} = \frac{2}{3} \pi r^3$$

7.1 Quiz

Q1. A right circular cone is enveloping a right circular cylinder such that the base of the cylinder rests on the base of the cone. If the radius and the height of the cone is 4 cm and 10 cm respectively, then the largest possible curved surface area of the cylinder of radius r is:

Q2. Radius of a spherical balloon, of radii 30 cm, increases at the rate of 2 cm per second. Then its curved surface area increases by:

Q3. A hemispherical bowl is filled with hot water to the brim. The contents of the bowl are transferred into a cylindrical vessel whose radius is 50% more than its height. If diameter of the bowl is the same as that of the vessel, the volume of the hot water in the cylindrical vessel is

Q4. The length, breadth and height of a room are in the ratio 3:2:1. If the breadth and height are halved while the length is doubled, then the total area of the four walls of the room will

7.2 Worked out Examples

Q1. Find the lateral surface area, total surface area, volume and body diagonals of a cuboid with dimensions of length, breadth and height of 10 m, 8 m and 6 m respectively.

Solution:

$$\text{LSA} = 2(lh + bh) = 2(10 \times 6 + 8 \times 6) = 216 \text{ m}^2$$

$$\text{TSA} = \text{LSA} + 2lb = 216 + 2 \times 10 \times 8 = 376 \text{ m}^2$$

$$\text{Volume (V)} = lbh = 10 \times 8 \times 6 = 480 \text{ m}^3$$

$$\text{Body diagonal} = \sqrt{10^2 + 8^2 + 6^2} = 10\sqrt{2} \text{ m}$$

Q2. Three metal cubes of body diagonals $6\sqrt{3}, 8\sqrt{3}, 10\sqrt{3}$ are melted and recast into a bigger cube of body diagonal $x\sqrt{3}$ without any loss of metal. Find x .

Solution:

Let the sides of the three smaller cubes be a , b and c . Then the body diagonals of these cubes will be $a\sqrt{3}, b\sqrt{3}, c\sqrt{3}$ which are given as $6\sqrt{3}, 8\sqrt{3}, 10\sqrt{3}$. Hence $a = 6$, $b = 8$ and $c = 10$.

Since the cubes are recast without any loss of metal, the volume of the bigger cube would be same as the total of the volumes of the individual cubes.

Now, since the body diagonal of the bigger cube is $x\sqrt{3}$, its side will be x and its volume $= x^3$

$$\therefore x^3 = a^3 + b^3 + c^3$$

$$= 6^3 + 8^3 + 10^3 = 216 + 512 + 1000 = 1728$$

$$\therefore x = 12$$

Q3. Find the area of canvas required to make a conical tent of height 3 m and base radius 4 m.

Solution:

$$\text{Slant height } l = \sqrt{r^2 + h^2} = \sqrt{4^2 + 3^2} = 5 \text{ m}$$

$$\begin{aligned} \text{Area of canvas required} &= \text{CSA of the above cone} \\ &= \pi r l = \pi \times 4 \times 5 = 20\pi \text{ m}^2 \end{aligned}$$

Q4. Eight small solid spherical metal balls are melted, and this material is cast into a bigger spherical ball without any loss of metal. Find the percentage change in the total surface area.

Solution:

Let the radius and volume of the smaller ball be r and v and those of the bigger ball be R and V respectively.

Since no metal is wasted, the total volume of the 8 smaller balls must equal the volume of the bigger ball.

$$\text{Thus, } V = 8v$$

$$\therefore 4\pi R^3/3 = 8 \times 4\pi r^3/3$$

$$\therefore R^3 = 8 \times r^3 \text{ i.e. } R = 2r$$

Q5. A sphere is cut into two hemispheres; find the change in the total surface area.

Solution:

$$\begin{aligned} \text{Total surface area of the original sphere} &= 4\pi r^2 \end{aligned}$$

$$\begin{aligned} \text{Total surface area of the two hemispheres} &= 2 \times (3\pi r^2) = 6\pi r^2 \end{aligned}$$

The new TSA is 1.5 times that of the original, hence the percentage change is equal to 50% (increase).

7.3 Class Work Problems

Q1. A regular hexagonal prism has perimeter of its base as 1200 cm and height equal to 150 cm. How many liters of petrol can it hold? Find the weight of petrol if density is 1.2 gm/cc.

Q2. A solid metallic right circular cone of height 36 cm and radius 12 cm is melted and two solid of cylinders of height 15 cm are prepared. If the volume of one is 8 times that of the other, find the radius of smaller one.

Q3. An iron ball of diameter 10 inches is dropped into a cylindrical vessel of diameter of 20 inches filled with water. Find the rise in water level.

Q4. A rectangular sheet of paper of length 12 cm and breath 44 cm is rolled end to end to form a right circular cylinder of height 12 cm. Find the volume of the cylinder.

Q5. Find the volume, CSA, and TSA of a right circular cone of height and radius 8 cm and 6 cm respectively.

Q6. The radius of sphere is 9 cm and that of hemisphere is 6 cm. Find the ratio of their volumes and ratio of their TSA's.

Q7. A regular hexagon has a perimeter of 36 cm. What is its area?

Q8. A square has a diagonal measuring 16 cm. When its area is expressed as 2^x square centimeters, what is x ?

Q9. A circle is inscribed inside a square. The square is inscribed inside another circle. If the area of the small circle is $4\pi \text{ cm}^2$, what is the area of the large circle, in square centimeters?

Q10. The radius of a cylinder is increased by 33.33%. By what percent should the height of the cylinder be reduced to maintain the volume of the cylinder?

Q11. A sphere is carved out of a cone with height 16 cm and radius of base circle 12cm. What is the maximum volume of the sphere?

Q12. The volume of the solid generated by the revolution of an isosceles right angled triangle about its hypotenuse of length 6 cm is:

Q13. A solid metallic cylinder of base radius 6 cm and height 15 cm is melted to make 'n' solid cones of height 5 cm and base radius 2 cm. Find the value of 'n'.

Q14. Three cubes of volumes, 27 cm^3 , 64 cm^3 and 125 cm^3 are melted to form a new cube. What is the diagonal of the new cube?

Q15. If the height of a cylinder is decreased by 15% and the radius of the cylinder is increased by 25%, then the volume of the cylinder increases by _____ %

Q16. A sphere is inscribed in a cone whose radius and height are 3 and 4 units, respectively. Then, the volume of the sphere is

Q17. For two cubes S_1 and S_2 , the sum of their volumes is numerically equal to the sum of the lengths of their edges. Then the ratio of the lengths of the edges of S_1 and S_2 is

Q18. If the curved surface area of a cone is 4 times that of another cone and slant height of the second cone is 9 times that of the first, find the ratio of the area of their bases.

Q19. A right circular cone of height 12 cm is divided into three parts by cutting the cone by two planes parallel to the base at a height of 3 cm & 7 cm from the base respectively. Find the ratio of $V_1:V_2:V_3 = ?$

Q20. Sum of the radius of the base and the height of a solid cylinder is 14 cm. If the total surface area of the cylinder is $112\pi \text{ sq.cm}$, then find the height of the cylinder

7.4 Exercise

Q1. Three identical cones with base radius r are placed on their bases so that each is touching the other two. The radius of the circle drawn through their vertices is

- a. smaller than r .
- b. equal to r .
- c. larger than r .
- d. depends on the height of the cones.

Q2. The diameter of a hollow cone is equal to the diameter of a spherical ball. If the ball is placed at the base of the cone, what portion of the ball will be outside the cone?

- a. 50%
- b. less than 50%
- c. more than 50%
- d. 100%

Q3. A slab of ice 8 inches in length, 11 inches in breadth, and 2 inches thick was melted and resolidified into the form of a rod of 8 inches diameter. The length of such a rod, in inches, is nearest to

- a. 3
- b. 3.5
- c. 4
- d. 4.5

Q4. A cube of side 12 cm is painted red on all the faces and then cut into smaller cubes, each of side 3 cm. What is the total number of smaller cubes having none of their faces painted?

- a. 16
- b. 8
- c. 12
- d. 24

Q5. A wooden box (open at the top) of thickness 0.5 cm, length 21 cm, width 11 cm and height 6 cm is painted on the inside. The expenses of painting are Rs. 70. What is the rate of painting per square centimetres?

- a. Re 0.7
- b. Re 0.5
- c. Re 0.1
- d. Re 0.2

Q6. The value of each of a set of coins varies as the square of its diameter, if its thickness remains constant, and it varies as the thickness, if the diameter remains constant. If the diameter of two coins are in the ratio 4 : 3, what should be the ratio of their thickness if the value of the first is four times that of the second?

- a. 16 : 9
- b. 9 : 4
- c. 9 : 16
- d. 4 : 9

Q7. A farmer has decided to build a wire fence along one straight side of his property. For this, he planned to place several fence-posts at 6 m intervals, with posts fixed at both ends of the side. After he bought the posts and wire, he found that the number of

posts he had bought was 5 less than required. However, he discovered that the number of posts he had bought would be just sufficient if he spaced them 8 m apart. What is the length of the side of his property and how many posts did he buy?

- a. 100 m, 15
- b. 100 m, 16
- c. 120 m, 15
- d. 120 m, 16

Q8. Four horses are tethered at four corners of a square plot of side 14 m so that the adjacent horses can just reach one another. There is a small circular pond of area 20 m^2 at the centre. Find the ungrazed area.

- a. 22 m^2
- b. 42 m^2
- c. 84 m^2
- d. 168 m^2

Q9. A piece of paper is in the shape of a right-angled triangle and is cut along a line that is parallel to the hypotenuse, leaving a smaller triangle. There was 35% reduction in the length of the hypotenuse of the triangle. If the area of the original triangle was 34 square inches before the cut, what is

the area (in square inches) of the smaller triangle?

- a. 16.665
- b. 16.565
- c. 15.465
- d. 14.365

Q10. The water from a roof, 9 sq. metres in area, flows down to a cylindrical container of 900 cm^2 base. To what height will the water rise in cylinder if there is a rainfall of 0.1 mm?

- a. 0.1 cm
- b. 0.1 metre
- c. 0.11 cm
- d. 1 cm

Q11. A regular pyramid has a square base with side 10 cm and a vertical height of 20 cm. If the height increases by 10% of its original value and the volume is constant, the percentage change in the side of the square base with respect to its original value is approximately:

- a. +5%
- b. +10%
- c. -5%
- d. -10%

Q12. What is the area of the copper sheet required to prepare a cone of base radius 30 cm with the height 40 cm?

- a. 7543 cm^2
- b. 5146 cm^2
- c. 5432 cm^2
- d. 7246 cm^2

Q13. A child consumed an ice cream of inverted right-circular conical shape from the top and left only 12.5% of the cone for her mother. If the height of the ice cream-cone was 8 cm, what was the height of the remaining ice cream-cone?

- a. 2.5 cm
- b. 3.0 cm
- c. 3.5 cm
- d. 4.0 cm

Q14. A cylinder, a Hemi-sphere and a cone stand on the same base and have the same heights. The ratio of the areas of their curved surface is:

- a. $2 : 2 : 1$
- b. $2 : \sqrt{2} : 1$
- c. $\sqrt{2} : 3 : 1$
- d. None of the above

Q15. An iron cube of size 10 cm is hammered into a rectangular sheet of thickness 0.5 cm. If the sides of the sheet be in the ratio 1 : 5, then the sides are

- a. 20 cm, 100 cm
- b. 10 cm, 50 cm
- c. 40 cm, 200 cm
- d. None of these

Q16. Three pipes are made of different shapes. The cross-sections of the pipes are an equilateral triangle, a hexagon and a circle. The perimeter of each of these cross-sections is equal. The flow through the pipes is proportional to the area of cross section. If it takes 8 minutes for the triangular pipe to fill up the tank, what will be the difference in the time taken by the hexagonal and circular pipes?

- a. 45 seconds
- b. 1 minute
- c. 0.5 minutes
- d. 7.9 minutes

Q17. The radius of a cylindrical cistern is 10 metres and its height is 15 metres. Initially the cistern is empty, we start filling the cistern with water through a pipe whose diameter is 50 cm, water is coming out of

the pipe with a velocity of 5 m/s. How many minutes will it take in filling the cistern with water?

a. 20

b. 40

c. 60

d. 80

Q18. A carpenter constructed a rectangular sandbox with a capacity of 10 cubic feet. If the carpenter were to make a similar sandbox twice as long, twice as wide, and twice as high as the first sandbox, what would be the capacity, in cubic feet, of the second sandbox?

a. 20

b. 40

c. 60

d. 80

e. 100

Q19. If the area of a square region having sides of length 6 centimeters is equal to the area of a rectangular region having width 2.5 centimeters, then the length of the rectangle, in centimeters, is

a. 8.5

b. 9.5

c. 9.6

d. 10.5

e. 14.4

Q20. A glucose solution contains 15 grams of glucose per 100 cubic centimeters of solution. If 45 cubic centimeters of the solution were poured into an empty container, how many grams of glucose would be in the container?

a. 3.00

b. 5.00

c. 5.50

d. 6.50

e. 6.75

Answers- Mensuration

7.1 Quiz

Q1 Q2 Q3

Q4

7.3 Class Work Problems

Q1 Q2 Q3

Q4 Q5 Q6

Q7 Q8 Q9

Q10 Q11 Q12

Q13 Q14 Q15

Q16 Q17 Q18

Q19 Q20

7.4 Exercise

Q1 Q2 Q3

Q4 Q5 Q6

Q7 Q8 Q9

Q10 Q11 Q12

Q13 Q14 Q15

Q16 Q17 Q18

Q19 Q20

CHAPTER 8

CLOCKS & CALENDERS

INTRODUCTION

CLOCKS

A clock dial has 60 divisions called minute spaces. It is the distance covered by the minute hand in one minute. So, the minute hand covers 60 minute spaces in an hour. The hour hand covers only 5 minute spaces in an hour.

Hence, the minute hand moves faster than the hour hand. In 1 hour, the minute hand moves 55 minute spaces more than the hour hand.

Hence, the time taken by the minute hand to gain x minute spaces over the hour hand =

$$\frac{60}{55}x \text{ i.e. } \frac{12}{11}x \text{ minutes}$$

Remember

In a 12-hour period, the clock will only make 22 right angles, NOT 24.

A straight line is said to be formed when an angle of 0° or 180° is formed between the hour and minute hands.

In a 12-hour period, the hands make an angle of 180° with each other 11 times.

Every hour, the minute hand and hour hand coincide or overlap each other once. But, between 11 a.m. to 1 p.m. and between 11 p.m. to 1 a.m., the hands overlap only once, that is, at 12 p.m. and 12 a.m. respectively. Hence, the hour and minute hands overlap 22 times in 24 hours. In one minute, Minute hand covers 6° and Hour hand covers 0.5° . Therefore, Relative Velocity = 5.5° .

Alternatively, in 1 hour, the minute hand covers 60 minute divisions where as the hour hand covers 5 minute divisions. Therefore, Relative speed = 55 minutes per hour.

To calculate the angle ϕ between the hands of a clock, we use the following formula where m – minutes and h – hours.

$$\phi = \frac{11}{2}m - 30h \left(\text{when } \frac{11}{2}m > 30h \right)$$

$$\phi = 30h - \frac{11}{2}m \left(\text{when } \frac{11}{2}m < 30h \right)$$

CALENDERS

Odd Days

To find the day of the week on a given date, we use the concept of 'odd days'.

In a given period, the number of days more than the complete weeks are called odd days.

Leap Year

- Every year divisible by 4 is a leap year, if it is not a century.
- Every 4th century is a leap year and no other century is a leap year.
- Each of the years 1940, 2012, 1696 etc. is a leap year.
- Each of the years 400, 800, 1200, 1600, 2000 etc. is a leap year.
- None of the years 2001, 2002, 2003, 2005, 1800, 2100 is a leap year.

Note: A leap year has 366 days.

Ordinary Year

The year which is not a leap year is called an ordinary year. An ordinary year has 365 days.

Counting of Odd Days

1 ordinary year = 365 days = (52 weeks + 1 day.)

Ordinary year has 1 odd day.

1 leap year = 366 days = (52 weeks + 2 days)

• **1 leap year has 2 odd days.**

100 years = 76 ordinary years + 24 leap years

= (76 x 1 + 24 x 2) odd days = 124 odd days.

= (17 weeks + days) 5 odd days.

- Number of odd days in 100 years = 5
- Number of odd days in 200 years = 3 odd days. (i.e 5 x 2)
- Number of odd days in 300 years = 1 odd day. (i.e 5 x 3)
- Number of odd days in 400 years = 0 odd day. (i.e 5 x 4 + 1)
- Similarly, each one of 800 years, 1200 years, 1600 years, 2000 years etc. has 0 odd days.

Day of the Week Related to Odd Days

Sunday	0
Monday	1
Tuesday	2
Wednesday	3
Thursday	4
Friday	5
Saturday	6

8.1 Quiz

Q1. What is the angle between the two hands of a clock when time is 20 minutes past 5?

Q2. At what time between 8 and 9 o'clock, will the minute hand and the hour hand of the clock coincide?

Q3. The minute hand of a clock overtakes the hour hand after every 68 minutes of correct time. How much time does the clock lose or gain in a day of normal time?

Q4. The first Republic day was celebrated on 26th Jan 1950. It was a _____ day.

Q5. If 17th September 1993 was a Friday, then which day of the week was 25th June 1972?

8.2 Worked out Examples

Q1. It was Sunday on Jan 1, 2006. What was the day of the week Jan 1, 2010?

Solution

On 31st December, 2005 it was Saturday.
Number of odd days from the year 2006 to the year 2009 = $(1 + 1 + 2 + 1) = 5$ days.

On 31st December 2009, it was Thursday.

Thus, on 1st Jan, 2010 it is Friday.

Q2. What was the day of the week on 28th May, 2006?

Solution

28 May, 2006 = (2005 years + Period from 1.1.2006 to 28.5.2006)

Odd days in 2000 years = 0

5 years = (4 ordinary years + 1 leap year) = $(4 \times 1 + 1 \times 2) = 6$ odd days

Jan + Feb + March + April + May
 $(31 + 28 + 31 + 30 + 28) = 148$ days

148 days = (21 weeks + 1 day) 1 odd day.

Total number of odd days = $(0 + 0 + 6 + 1) = 7 = 0 = \text{odd day.}$

Given day is Sunday.

Q3. Today is Monday. After 61 days, it will be:

Solution

Each day of the week is repeated after 7 days.

So, after 63 days, it will be Monday.

After 61 days, it will be Saturday

Q4. What is the angle between the minute hand and the hour hand of a clock at 5 hours 40 minutes?

Solution

The angle between the hands can be calculated by $\phi = \left| \frac{11}{2}m - 30h \right|$, where m is minutes and h is hours. Here $m=40$ and $h=5$

$$\therefore \phi = \left| \frac{11}{2} \times 40 - 30 \times 5 \right| = 70^\circ$$

The angle between the two hands is 70° .

Q5. Find the time between 3 and 4 O'clock at which the minute hand and the hour hand make an angle 60° with each other.

Solution

In the formula $\phi = \left| \frac{11}{2}m - 30h \right|$, $\phi = 60^\circ$ and $h = 3$

$$\therefore 60 = \left| \frac{11}{2}m - 30 \times 3 \right|$$

$$60 = \frac{11}{2}m - 30 \times 3$$

$$\frac{11}{2}m = 150$$

$$m = \frac{300}{11} = 27\frac{3}{11} \text{ min past 3}$$

Or

$$60 = 30 \times 3 - \frac{11}{2}m$$

$$m = \frac{60}{11} = 5\frac{5}{11} \text{ min past 3}$$

Therefore, the angle between the hour hand and the minute hand is 60° at $27\frac{3}{11}$ minutes past 3 O'clock and at $5\frac{5}{11}$ minutes past 3 O'clock.

8.3 Class Work Problems

Q1. At what time between 5 o'clock and 6 o'clock do the minute and hour hands of a clock coincide?

Q2. When are the minute and hour hand at 180° (with respect to each other) between 1 p.m. and 2 p.m.?

Q3. A clock is currently showing the right time as 12 p.m. If, after this, it starts gaining 10 minutes every hour, what time will it show at 10 p.m.?

Q4. A clock is showing the right time at 12 p.m. After this it started gaining 15 minutes per hour. So, when this clock shows 5 pm, what is the actual time?

Q5. At what time between 3 p.m. and 4 p.m. will the minute hand coincide with the hour hand?

Q6. At what time between 5 o'clock and 6 o'clock will the hands of a watch be at right angles?

Q7. Imagine that your watch was correct at noon, but then it began to lose 30 minutes each hour. It now shows 4 p.m., but it stopped 5 hours ago. What is the correct time now?

- | | |
|------------|------------|
| a. 9.30 pm | b. 11 pm |
| c. 1 am | d. 1.30 am |

Q8. Assuming that three of the following four dates in the year 2004 are right, which one would be wrong?

- a. Sunday, 17th of January
- b. Sunday, 21st of February
- c. Sunday, 19th of March
- d. Sunday, 10th of April

Q9. A mechanical grandfather clock is at present showing 7 hrs 40 min 6 sec. Assuming that it loses 4 sec in every hour, what time will it show after exactly $6\frac{1}{2}$ hours?

- a. 2 hr 9 min 40 sec
- b. 2 hr 10 min 6 sec
- c. 14 hr 9 min 34 sec
- d. 14 hr 10 min 32 sec

8.4 Exercise

Q1. It takes the pendulum of a clock 7 seconds to strike 4 o'clock. How much time will it take to strike 11 o'clock?

- a. 18 seconds
- b. 20 seconds
- c. 19.25 seconds
- d. 23.33 seconds

Q2. If 09/12/2001(DD/MM/YYYY) happens to be Sunday, then 09/12/1971 would have been a

- a. Wednesday
- b. Tuesday
- c. Saturday
- d. Thursday

Q3. In a month of 31 days, the third Wednesday falls on the 15th. What will be the last day of that month?

- a. Fifth Thursday
- b. Fifth Wednesday
- c. Fourth Sunday
- d. Fifth Friday

Q4. If a clock is kept on the table in such a way that at 3:10 pm the hour hand points south, after how much time will the minute hand point east?

- a. 20 minutes
- b. 35 minutes
- c. 50 minutes
- d. 90 minutes

Q5. What is the closest time between 7 and 8 when the hands of your watch are exactly opposite to each other?

- a. 7 Hr - 5 Min
- b. 7 Hr - 5.5 Min
- c. 7 Hr - 6 Min
- d. 7 Hr - 6.5 Min

Q6. What year comes next in the sequence 1973, 1979, 1987, 1993, 1997, 1999..... ?

- a. 2001
- b. 2003
- c. 2005
- d. 2007

Q7. A clock loses 12 minutes every 24 hours. It is set right at 7:25 p.m. on Monday. What will be the time when the clock shows 1:45 p.m. the following day?

- a. 1:20:35 p.m.
- b. 1:35:50 p.m.
- c. 1:25:35 p.m.
- d. None of these

Q8. A clock strikes once at 1 o'clock, twice at 2 o'clock and so on. If it takes 6 seconds to strike at 3 o'clock, how much time will it take to strike at 9 o'clock?

- a. 24 seconds
- b. 18 seconds
- c. 20 seconds

d. None of these

Q9. A man on his way to dinner shortly after 6:00 p.m. observes that the hands of his watch form an angle of 110° . Returning before 7:00 p.m. he notices that again the hands of his watch form an angle of 110° . The number of minutes that he has been away is:

- a. $36\frac{2}{3}$
- b. 40
- c. 42
- d. 42.4

Q10. Sangeeta and Swati bought two wristwatches from Jamshedpur Electronics at 11:40 a.m. IST. After purchasing they found that when 60 minutes elapses on a correct clock (IST), Sangeeta's wristwatch registers 62 minutes whereas Swati's wristwatch registers 56 minutes. Later in the day Sangeeta's wristwatch reads 10 p.m., then the time on Swati's wristwatch is:

- a. 8:40 p.m.
- b. 9:00 p.m.
- c. 9:20 p.m.
- d. 9:40 p.m.
- e. Cannot be calculated

Q11. How much does a watch gain per day, if its hands coincide every 64 minutes?

- a. 96 min
- b. 90 min
- c. $36\frac{5}{11}$ min
- d. $32\frac{8}{11}$ min

Q12. If March 1, 2006 was Wednesday, which day was it on March 1, 2002?

- a. Wednesday
- b. Thursday
- c. Friday
- d. Saturday

Answers - Clocks & Calenders

8.1 Quiz

Q1 Q2 Q3

Q4 Q5

8.3 Class Work Problems

Q1 Q2 Q3

Q4 Q5 Q6

Q7 Q8 Q9

8.4 Exercise

Q1 Q2 Q3

Q4 Q5 Q6

Q7 Q8 Q9

Q10 Q11 Q12

CHAPTER 9

CUBES

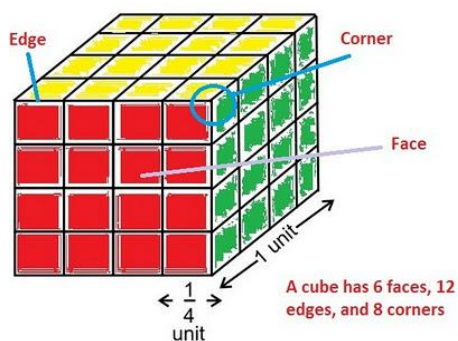
INTRODUCTION

A cube is a 3-dimensional diagram with all sides equal.

In a cube length, breadth and height are same while in cuboid these are different.

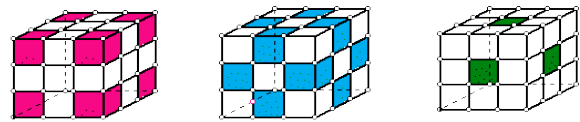
In a cube the number of unit cubes = (side)³.

Shown below is a cube which is painted on all the sides and the cut into $(1/4)^{\text{th}}$ of its original side.

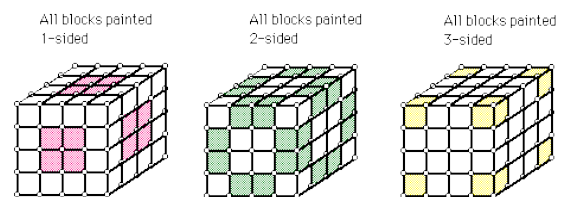
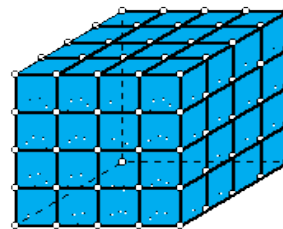


Some Observations

A cube has 6 faces, 12 edges and 8 corners. These 3 cubes all represent $3 \times 3 \times 3$ cubes, the first one has the blocks shaded (pink) that will be painted 3 sides, the second cube has the blocks shaded (blue) that will have 2 sides painted, and the third cube has blocks shaded (green) that will have 1 side painted.



let us see an example of a $4 \times 4 \times 4$ cube, then below is three $4 \times 4 \times 4$ cubes with the cubes that will be painted 1, 2, and 3 sides.



If a cube is divided into the size $1/n^{\text{th}}$ of its original side after get painted all the sides, Then

Total number of cubes = n^3

Cubes with 3 sides painting = 8

Cubes with 2 sides painting = $12 \times (n-2)$

Cubes with 1 side painting = $6 \times (n-2)^2$

Cubes with no painting = $(n-2)^3$

9.1 Quiz

Directions for Questions Q1-Q4: A cube is cut into 125 small cubes and placed in one corner of a room. The bottom of the cube is touching the floor and the top of the cube is touching with the roof. Then the cube is painted with Black color.

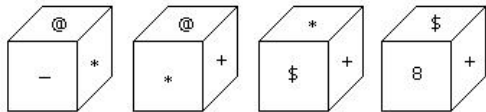
Q1. How many cubes have three faces painted?

Q2. How many cubes have two faces painted?

Q3. How many cubes have one face painted?

Q4. How many cubes have no face painted?

Q5. Which symbol will be on the face opposite to the face with symbol * ?

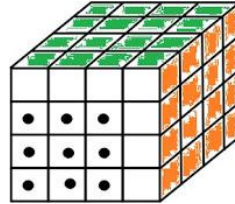


9.2 Worked out Examples

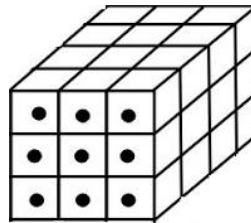
Q1. A cube whose two adjacent faces are coloured is cut into 64 identical small cubes. How many of these small cubes are not coloured at all?

Solution

Assume the top face of the cube and its right side are colored green and orange respectively.



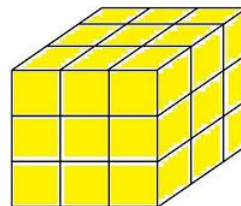
Now if we remove the colored faces, we left with a cuboid, whose front face is indicated with dots.



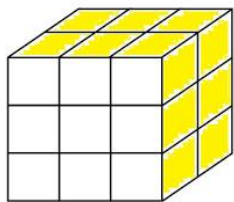
So on the front face there are 9 cubes, and behind it lies 4 stacks. So total $9 \times 4 = 36$

Q2. A cube, painted yellow on all-faces is cut into 27 small cubes of equal size. How many small cubes have no face painted?

Solution



Assume we have taken out the front 9 cubes. Then the cube looks like the one below.



Now the cube which is in the middle has not got any painting. The cubes on the Top row, bottom row, left column and right column all got painting on atleast one face.

Alternative method:

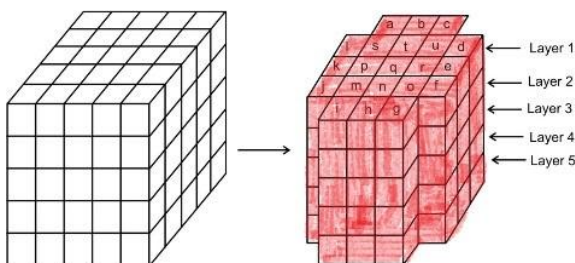
Use formula: $(n-2)^3$ Here $n = 3$

So $(3-2)^3 = 1$

Directions for Questions Q3-Q5: One hundred and twenty-five cubes of the same size are arranged in the form of a cube on a table. Then a column of five cubes is removed from each of the four corners. All the exposed faces of the rest of the solid (except the face touching the table) are colored green. Now, answer these questions based on the above statement:

Q3. How many small cubes are there in the solid after the removal of the columns?

Solution:



Since out of 125 total number of cubes, we removed 4 columns of 5 cubes each, the remaining number of cubes =

$$125 - (4 \times 5) = 125 - 20 = 105.$$

Q4. How many cubes do not have any colored face?

Solution

Cubes with no painting lie in the middle. So cubes which are below the cubes named as s, t, u, p, q, r, m, n, o got no painting. Since there are 4 row below the top layer, total cubes with no painting are $(9 \times 4) = 36$.

Q5. How many cubes have only one green face each?

Solution

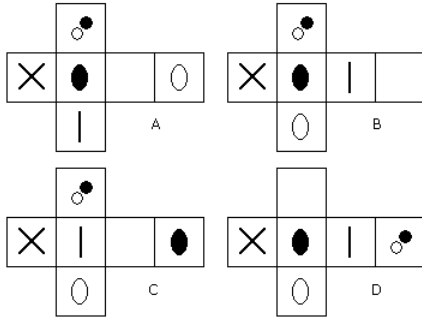
There are 9 cubes named as m, n, o, p, q, r, s, t and u in layer 1, and 4 cubes (in columns b, e, h and k) in each of the layers 2, 3, 4 and 5 got one green face. Thus, there are $9 + (4 \times 4) = 25$ Cubes

Q6. Which of the patterns when folded will make the cube shown?

Completed Cube



Patterns



Solution: (B)

9.3 Class Work Problems

Directions for Questions Q1-Q3: There are two cubes. Each has to be cut into 8 small cubes. After cutting, one of the corners of the first cube is removed then the second is inserted in the corner. Now two cubes have to be painted so that parallel sides have the same colour with red, green and yellow.

Q1. How many small cubes have three faces painted?

Q2. How many small cubes have two faces painted?

Q3. How many small cubes have one face painted?

Directions for Questions Q4-Q8: A cube is cut into 64 small cubes. From each face the cubes in the edges are left intact and the rest are removed. Then the cube has to be painted with Red, Blue and green such that opposite faces have same color.

Q4. How many cubes have three faces painted?

Q5. How many cubes have one face painted?

Q6. How many cubes have four faces painted?

Q7. How many cubes have painted red in two faces and green in two faces?

Q8. How many cubes have painted red in two faces and white in two faces?

Directions for Questions Q9-Q13: A cube has to be cut into 64 small cubes. Then cubes in second and third row of each face is painted with brown. The remaining cubes in each face have to be painted with different colors such as Red, Green and Yellow provided the opposite faces have same color.

Q9. How many cubes have painted Brown on two faces?

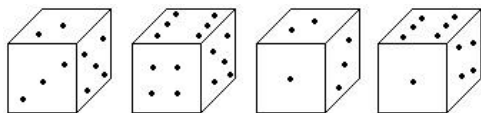
Q10. How many cubes have painted brown in one face and different colors in other face?

Q11. How many cubes have painted red in one face and Green in other face?

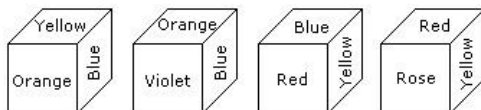
Q12. How many cubes have painted green in one face and yellow in other face?

Q13. How many cubes have one face painted?

Q14. How many points will be on the face opposite to the face which contains 2 points?



Q15. From the four positions of a dice given below, find the color which is opposite to yellow?



9.4 Exercise

Q1. Cube is made into a number of small cubes by dividing each edge into four equal parts. What is the minimum number of additional small cubes required to create a bigger cube that will completely enclose the original cube?

- a. 152
- b. 216
- c. 96
- d. 148

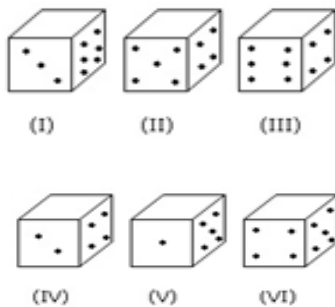
Q2. A cube to be coloured in such a way as to avoid the same color on adjacent surfaces. What is the minimum number of colors you will require?

- a. Four
- b. Three
- c. Six
- d. Nine

Q3. A rectangular floor is fully covered with square tiles of identical size. The tiles on the edges are white and the tiles in the interior are red. The number of white tiles is the same as the number of red tiles. A possible value of the number of tiles along one edge of the floor is

- a. 10
- b. 12
- c. 14
- d. 16

Directions for questions Q4 & Q5 : Six dice with upper faces erased are as shows.



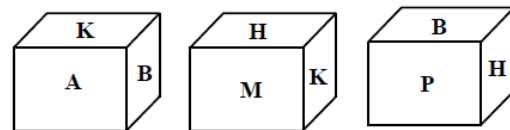
Q4. If even numbered dice have even number of dots on their top faces, then what would be the total number of dots on the top faces of their dice?

- a.12
- b.14
- c.18
- d.24

Q5. If the odd numbered dice have even number of dots on their top faces, then what would be the total number of dots on the top faces of their dice?

- a.8
- b.10
- c.12
- d.14

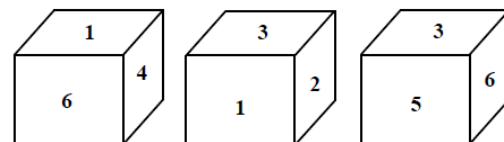
Q6. Three views of a cube following a particular motion are given below: [CSAT]



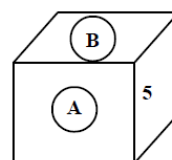
What is the letter opposite to A?

- a. H
- b. P
- c. B
- d. M

Q7. A cube has six numbers marked 1, 2, 3, 4, 5 and 6 on its faces. Three views of the cube are shown below:



What possible numbers can exist on the two faces marked "A" and "B" respectively on the cube?



a. 2 and 3

b. 6 and 1

c. 1 and 4

d. 3 and 1

Answers - Cubes

9.2 Quiz

Q1 Q2 Q3

Q4 Q5

9.3 Class Work Problems

Q1 Q2 Q3
--

Q4 Q5 Q6
--

Q7 Q8 Q9
--

Q10 Q11 Q12

Q13 Q14 Q15

9.4 Exercise

Q1 Q2 Q3
--

Q4 Q5 Q6
--

Q7

CHAPTER 10

CODING AND DECODING

INTRODUCTION

A CODE is a 'system of signals'. Therefore, Coding is a method of transmitting a message between the sender and the receiver without a third person knowing it.

Before transmitting, the data is encoded and at receiver side encode data is decoded in order to obtain original data by determining common key in encoded data.

The Coding and Decoding is classified into six types.

Type1: Letter Coding.

Type2: Number Coding.

Type3: Matrix Coding.

Type4: Substitution.

Type5: Mixed Letter Coding.

Type6: Mixed Number Coding.

Before we proceed to discuss the various types of questions related to coding, it is better to have an idea regarding the general types of coding. Some of the major types of coding are:

- Constant addition in the position of alphabets.

- Constant subtraction in the position of alphabets.
- Denoting the position of alphabets in the whole alphabetical order.
- Addition of the positions of all the alphabets to make code for the word.
- Constant addition and subtraction respectively in the position of all the alphabets.
- Squaring the number of letters in the word.
- Arranging the letters in the alphabetic order.
- Arranging the letters given in the main word, in the reverse order.
- Interchanging each pair of the letters, in the whole word.
- Constant addition and then reversing the letters to make the final code.

10.1 Quiz

Q1. In a certain code QUESTION is written as PTDRSHNM, how is NUMBER written in that code?

Q2. In a certain language GROUP is coded as HQPTQ, which word will be coded as COUNTRY?

Q3. In a certain code PERSONAL is written as GNUPPMNY, how is COMPUTER written in that code?

Q4. In a certain code MORNING is written as USXVSZU, how is HISTORY written in that code?

Q5. In a certain code MOVE is written as OSDU, how is STORY written in that code?

10.2 Worked out Examples

TYPE1: Letter Coding

In this type the real alphabets in a word are replaced by certain other alphabets according to a specific rule to form its code. The candidate is required to detect the common rule and answer the questions accordingly.

Q1. If PROGRAM is coded as QSPHSBN. How is SYSTEM coded in that code?

Solution

We are given that PROGRAM is coded as QSPHSBN. In which the word PROGRAM is moved one step forward to obtain the

corresponding letter of the code that is P is coded as Q; R is coded as S and so on. Therefore,

P	R	O	G	R	A	M
↓	↓	↓	↓	↓	↓	↓
Q	S	P	H	S	B	N

Implies

S	Y	S	T	E	M
↓	↓	↓	↓	↓	↓
T	Z	T	U	F	N

Likewise code may be moved one step backward also.

Q2. If BOUNDRY is coded as OVPCXQZC, then how is YOUR SELF coded?

Solution

BOUN	DARY
------	------

Reverse

NUOB	YRAD
+1+1+1+1	-1-1-1-1

OVPC	XQZC
YOUR	SELF
↓	↓

Reverse

RUOY	FLES
+1+1+1+1	-1-1-1-1
SVPZ	EKDR

Q3. In a certain code, MENTION is written as LNEITNO. How is PATTERN written in that code?

Solution

Clearly, to obtain the code, the first letter of the word MENTION is moved one step backward and the remaining letters are reversed in order, taking two at a time.

So, in PATTERN, P will be coded as O, and the sequence of the remaining letter in the cod would be TAETNR. Thus the code becomes OTAETNR. Hence, the answer is OTAETNR.

Q4. If in a certain language CARROM is coded as BZQQNL, how is HOUSE written the code?

Solution

Each letter of the word is one step ahead of the corresponding letter of the code

C	A	R	R	O	M	H	O	U	S	E
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
B	Z	Q	Q	N	L	G	N	T	R	D

So, H is coded as G, O as N, U as T, S as R and E as D. HOUSE is coded as GNTRD.

TYPE 2: Number Coding

In these questions, either numerical code values are assigned to a word or Alphabetical code letters are assigned to the numbers. The candidate is required to analyze the code as per the directions.

Q5. If in a certain language A is coded as 1, B is coded as 2, and so on, how is BLUE is coded in that code?

Solution: As given the letters are coded as

A	B	C	D	E	F	G
1	2	3	4	5	6	7
H	I	J	K	L	M	N
8	9	10	11	12	13	14
O	P	Q	R	S	T	
15	16	17	18	19	20	
U	V	W	X	Y	Z	
21	22	23	24	25	26	

So in BLUE, B is coded as 2, L as 12, U as 21 and E as 5. Thus, BLUE is coded as 212215

Q6. In a certain code, DEER is written as 96. How is SONG written in that code?

Solution

Clearly, in the given code, the alphabets are coded as follows

D E E R S O N G

4 5 5 18 19 15 14 7

So, DEER----->4+5+5+18=32*3=96

SONG---->19+15+14+7=55*3=155

Q7. In a certain code, 2 is coded as P, 3 as N, 9 as Q, 5 as R, 4 as A and 6 as B. How is 599423 coded in that code?

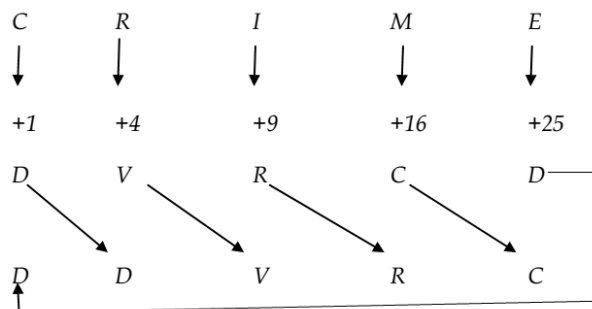
Solution

Clearly as given 5 is coded as R, 9 as Q, 4 as A, 2 as P, 3 as N. So, 599423 is coded as RQQAPN.

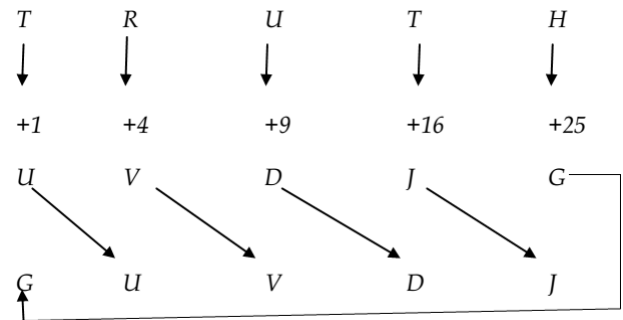
Q8. If the word CRIME be written as DDVRC in coded form, how can TRUTH be written the following the same doing?

- a. UVDDJ
- b. GUVDJ
- c. GUDJG
- d. UGVJD

Solution



From the above we can conclude, To the first letter 1² is added, To the second letter 2² is added..... to the fifth letter 5² is added . Then the obtain result DVRCD is shifted by 1 unit in a cycle order.



So the correct answer is choice b.

TYPE 3: Matrix Coding

In this type of questions, a word is represented by only one set of numbers as given in any one of the alternatives. The set of alternatives are represented by two classes of alphabets as in the given matrices.

Examples:

MATRIX 1

MATRIX 2

	0	1	2	3	4		5	6	7	8	9
0	D	K	A	E	C	5	P	L	O	T	N
1	C	D	K	A	E	6	T	P	N	L	O
2	K	C	E	A	D	7	P	N	T	O	L
3	K	C	D	E	A	8	O	N	T	P	L
4	E	D	A	K	C	9	L	O	P	N	T

Q8. Find COLD?

a. 44,96,95,22

b. 31,99,77,22

c. 30,66,86,43

d. 10,85,79,24

Solution

From matrix 1, C can be coded as 04,10,21,31 or 44.

From matrix 2, O can be coded as 57,69,78,85 or 96.

From matrix 2, L can be coded as 56,68,79,89 or 95.

From matrix 1, D can be coded as 00,11,24,32 or 41.

Clearly, only (d) contains all correct codes.

TYPE 4: Substitution

In this type of questions, some particular objects are assigned code names. Then a question is asked that is to be answered in the code language.

Q9. 'Reds' are 'blues', 'blues' are 'whites', 'whites' are 'yellows', 'yellows' are 'oranges', 'oranges' are 'pinks', then what is the colour of the sky?

Solution

We need to find the colour of the sky and we know that the colour of sky is blue but we

are given that blues are whites. So the colour of sky is white.

Q10. 'man' is coded as 'woman', 'woman' is coded as 'girl', 'girl' is coded as 'boy', 'boy' is coded as 'worker' then 6 years female is known as?

Solution

We know that 6 years female is girl, but we are given that 'girl' is coded as 'boy'. So the 6 year girl will be called as 'boy'.

TYPE 5: Mixed Letter Coding

In this type of questions, three or four complete messages are given in the coded language and the code for a particular word is asked. To analyze such codes, any two messages bearing a common word are picked up. The common code word will mean that word. Proceeding similarly by picking up all possible combinations of two, the entire message can be analyzed.

Q11. If 'nso ptr kli chn' stands for 'sharma gets marriage gift', 'ptr lnm wop chn' stands for 'wife gives marriage gift', 'tti wop nhi' stands for 'he gives nothing', what would mean 'gives'?

a.chn

b.nhi

c.ptr

d.wop

Solution

In the second and third statements the common word is 'gives' and the common code word is 'wop'. So 'wop' means 'gives'.

TYPE 6: Mixed Number Coding

In this type of questions, a few groups of numbers codes a certain short message not in the order, are given. Through a comparison of the given coded messages, taking two at a time, the candidate is required to find the number code for each word and then formulate the code for the message given.

Q12. In a certain code, '786' means 'study very hard', '958' means 'hard work pays' and '645' means 'study and work'. Which of the following is the code for 'very'?

Solution

In the first and second statements, the common word is 'hard' and the common code digit is '8'. So, '8' means 'hard'.

In the first and third statements, the common word is 'study' and the common code digit is '6'. So, '6' means 'study'. Thus, in the first statement '7' means 'very'.

10.3 Class Work Problems

Q1. If the word DISCUSS be written as JFXGXYZ in coded form, how can EXAMPLE be written following the same coding?

- a. XEPMAEL
- b. XEPMFKS
- c. YGSQFKS
- d. YGSMAEL

Q2. If the word BENEFIT be written as AQQOKIC in coded form, how can PRESENT be written following the same coding?

- a. QTHWTKA
- b. QTHTKWQ
- c. AHTTKTA
- d. AHTTKWQ

Q3. If the word PHYSICAL be written as SKHBXROZ in coded form, how can RESPONSE be written following the same coding?

- a. VIKHMLEH
- b. IVHKLMEH
- c. VKHILHME
- d. HIVKMLEH

Q4. If the word BACHELOR be written as SXZYILOV in coded form, how can HYDROGEN be written following the same coding?

- a. RDGOYHNE
- b. IWBSMVTL
- c. RDGOMVLT
- d. IWBOHYNE

Q5. If the word DIRECTOR be written as FGRGATQP in coded form, how can POSITION be written following the same coding?

- a. NQSKRICQ
- b. RMJKRJQL
- c. RMUKRIMP
- d. MRKSRIQL

Q6. If the word STATE be written as HATTV in coded form, how can TRADE be written following the same coding?

- a. UDAVR
- b. GADRV
- c. UADRV
- d. UDARF

Q7. If the word THEORY be written as UFJASQ in coded form, how can CHARGE be written following the same coding?

- a. DBITIG
- b. DBJGHT
- c. DBJTHG
- d. DBITHT

Q8. In a certain code, RIPPLE is written as 613382 and LIFE is written as 8192. How is PILLER written in that code?

- a. 318826
- b. 318286
- c. 618826
- d. 338816

Q9. In a certain code BOARD is written as 54#12 and MORE is written as 941\$. How is DREAM written on that code?

- a. 21\$#9
- b. 2\$1#9
- c. 51\$#9
- d. 25\$#9

Q10. In a certain language, 'sun shines brightly' is written as 'sa re ka', 'houses are brightly lit' as 'ma pa tha sa' and 'light comes from sun' as 'ni saa re rega'. What

code-words are written for 'sun' and 'brightly'

a. sa, ka

b. ka, re

c. re, sa

d. sa, re

Directions for Questions Q11&Q12: Read the information given below to answer the questions that follow:

In a certain code language,

(i) 'van ma ra' means 'you are welcome';

(ii) 'ma si xa ya' means 'they are very good';

(iii) 'ku dai ya' means 'who is good';

(iv) 'su si van ya' means 'they welcome good people'.

Q11. Which of the following means 'people' in that code language?

a. su

b. ya

c. si

d. van

e. Data inadequate

Q12. Which of the following means 'very' in that code language?

a. xa

b. ma

c. dai

d. Data inadequate

e. None of these

10.4 Exercise

Q1. If SAND is called AIR, Air is called PLATEAU, PLATEAU is called WELL, WELL is called ISLAND and Island is called SKY, then from where will a woman draw water?

a. WELL

b. ISLAND

c. SKY

d. AIR

Q2. If CLOUD is called WHITE, WHITE is called RAIN, RAIN is called GREEN, GREEN is called AIR, AIR is called BLUE and BLUE is called WATER, where will the birds fly?

a. BLUE

b. SKY

c. YELLOW

d. WATER

Q3. In a certain code, 'nee tim see' means 'how are you', 'ble nee see' means 'where are you', what is code for 'where'?

- a. nee
- b. tim
- c. see
- d. ble

Q4. If 'gnr tag zog qmp' stands for 'Seoul Olympic Organising committee'. 'hyto gnr emf' stands for 'summer Olympic games' and 'esm sdr hyto' stands for 'modern games history', what would be the code for 'summer' ?

- a. hyto
- b. gnr
- c. emf
- d. zog

Q5. In certain code language, 'Tom Kun Sud' means "Dogs are barking"; 'Kun Jo Mop' means 'Dogs and Horses' and 'Mut Tom Ko' means 'Donkeys are mad'. Which word in that language means 'barking'?

- a. Sud
- b. Kun
- c. Jo
- d. Tom

e. ko

Q6. In a certain code '256' means 'you are good', '637' means 'we are bad' and '358' means 'good and bad'. Which of the following represents 'and' in that code?

- a. 2
- b. .5
- c. 8
- d. 3

Q7. In certain code, '123' means 'hot filtered coffee'; '356' means 'very hot day' and '589' means 'day and night'. Which digit in that code means 'very'?

- a. 9
- b. 5
- c. 8
- d. 8
- e. 6

Q8. In certain code language, '851' means 'good sweet fruit'; '783' means 'good red rose' and '341' means 'rose and fruit'. Which of the following digit stands for 'sweet' in that language?

- a. 8
- b. 5

c.1

d.3

Q9. In certain code language, '479' means 'fruit is sweet'; '248' means 'very sweet voice' and '637' means 'eat fruit daily'. Which of the following digit stands for 'is' in that language?

a.7

b.9

c.4

d.6

Q10. If JAPAN is coded KCSES, then the code for CASTLE will be

a. DIJOB T

b. DJKRDP

c. DKMSG R

d. DCVXQ K

Q11. If CAUTIOUS is BBTUHPTT, then NEGLIGENT is

a. MFFMHFOS

b. MFHMHHD MU

c. MFFMHHD OS

d. MFFMHJD OS

Q12. In a certain language, 'cul max dir' means 'nice little boxes', 'sut med bix' means 'well arranged row', 'bix fac dir' means 'row of boxes'. Which of the following stands for 'of' in that language?

a. max

b. bix

c. fac

d. sut

Q13. In a certain code language, 'pit nae tom' means 'apple is green', 'nae to tap' means 'green and white', and 'ho tom ka' means 'shirt is white'. Which of the following represents 'apple' in that language?

a. nae

b. tom

c. pit

d. ho

Q14. The letter of TEACHING may be rearranged as CHEATING; if you follow the same code, what will happen to GRADIENT?

a. DIRAGENT

b. RADIGENT

c. DIERATIN

d. RATINGDI

Q15. What is the value of A, B, C and D in the following matrix?

A	17	14	11
B	15	16	21
C	12	19	22
23	D	13	8

- a. A =20, B =10, C =18, D = 9
- b. A =10, B =18, C = 9, D = 10
- c. A =20, B =10, C = 9, D = 18
- d. A = 10, B =20, C =18, D = 9

Q16. In a certain code language, 'kew xas huma deko' means 'she is eating apples'. 'kew tepo qua' means 'she sells toys' and 'sut time deko' means 'I like apples'. Which words in the language means 'she' and 'apples'?

- a. 'xas' and 'deko'
- b. 'deko' and 'tepo'
- c. 'kew' and 'deko'
- d. 'xas' and 'kew'

Answers - Coding & Decoding

10.1 Quiz

Q1 Q2 Q3

Q4 Q5

10.3 Class Work Problems

Q1 Q2 Q3

Q4 Q5 Q6

Q7 Q8 Q9

Q10 Q11 Q12

10.4 Exercise

Q1 Q2 Q3

Q4 Q5 Q6

Q7 Q8 Q9

Q10 Q11 Q12

Q13 Q14 Q15

Q16

CHAPTER 11

LOGICAL CONNECTIVITY & BINARY LOGIC

INTRODUCTION

LOGICAL CONNECTIVES

Connective

Two or more propositions can be combined together to make compound repositions with the help of logical connectives.

Example:

Above two propositions can be used to make a compound proposition using any of the logical connectives.

- 2 is an odd number AND 4 is a perfect square.
- 2 is an odd number OR 4 is a perfect square.

Their truth values are false and true respectively. For the first compound proposition to be true both the propositions have to be true as the connective is AND and as OR is the connective for the second one if either of the propositions is true the truth value of the compound proposition is true.

The following are the logical connectives used commonly:

Conjunction

The logical “conjunction” is understood in the same way as commonly used AND the compound proposition truth-value is true iff all the constituent propositions hold true. It is represented as " \wedge ". Truth table for two individual propositions p and q with conjunction is given below

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

The logical “disjunction” is understood in the same way as commonly used OR the compound proposition truth-value is true iff at least one of the constituent propositions hold true. It is represented as " \vee ". Truth table for two individual propositions p and q with conjunction is given below

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation

This is the logical "negation" and it is expressed by \sim as $\sim p$ for "not p ". Truth table is given below

p	$\sim p$
T	F
F	T

Conditional

It is define as "a proposition holds true if another proposition is true" i.e. $p \rightarrow q$ is read as "if p , then q ". Truth table is given below

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

$p \rightarrow q$ is also expressed in a number of different (but equivalent) ways in English.

The following table represents different forms of

IF - THEN statement and its equivalent statements:

Statement	Representation Using \rightarrow	Equivalent Statements
If p , then q	$p \rightarrow q$	$\sim q \rightarrow \sim p$
q , if p	$p \rightarrow q$	$\sim q \rightarrow \sim p$
When p , then q	$p \rightarrow q$	$\sim q \rightarrow \sim p$
Whenever p , then q	$p \rightarrow q$	$\sim q \rightarrow \sim p$
q , when p	$p \rightarrow q$	$\sim q \rightarrow \sim p$
q , whenever p	$p \rightarrow q$	$\sim q \rightarrow \sim p$
Everytime p, q	$p \rightarrow q$	$\sim q \rightarrow \sim p$
q , Everytime p	$p \rightarrow q$	$\sim q \rightarrow \sim p$
q , only if p	$q \rightarrow p$	$\sim p \rightarrow \sim q$
Unless p, q	$\sim p \rightarrow q$	$\sim q \rightarrow p$
q , unless p	$\sim p \rightarrow q$	$\sim q \rightarrow p$
P , otherwise q	$\sim p \rightarrow q$	$\sim q \rightarrow p$

Bi Conditional

A proposition $(p \rightarrow q) \wedge (q \rightarrow p)$ can be abbreviated using bi conditional conjunction \leftrightarrow as $p \leftrightarrow q$ and is read as "if p then q , and if q then p ".

Tautology

A compound proposition, which is true in every case. $p \vee \sim p$

Contradiction

This is the opposite of tautology, which is false in every case. $p \wedge \sim p$

Logical Implication and Equivalence

If the value of $p \rightarrow q$ is true in every case, then p is said to logically imply q . It is represented as $p \Rightarrow q$. If p and q have the same truth-value in every case then they are said to be logically equivalent and it is represented as $p \leftrightarrow q$.

Following are some of the useful identities and implications from propositional logic.

Identities

- $\sim (p \vee q) \leftrightarrow (\sim p \wedge \sim q)$ [demorgan's law]
- $\sim (p \wedge q) \leftrightarrow (\sim p \vee \sim q)$ [demorgan's law]
- $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$ [implication]
- $[(p \wedge q) \rightarrow r] \leftrightarrow [p \rightarrow (q \rightarrow r)]$ [exportation]
- $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ [contra positive]

Binary Logic

In these problems, you find people answer questions in two or three different statements and some of them are true and

some are false. Based on the clues given, we have to figure out the actual category of persons.

Category of persons:

- **Truth-Tellers**
- **Liars**
- **Alternators** (If first statement of a person is true, then next is false or vice versa)

11.1 Quiz

Directions for Q1: The following question consists of a main statement followed by **four/five** answer options. From these options, select the one that logically follows the main statement.

Q1. If I can read, I can clear the exam.

- a. I can read hence I can clear the exam
- b. I cannot clear the exam implies I cannot read.
- c. I can read hence I cannot clear the exam
- d. Both a and b.

Q2. Among the three persons X, Y and Z - one is a truth-teller, one is a liar and other is an alternator. Each of them made the

following two statements in reply to the question asked about them.

X: I am not the liar. Z is the liar.

Y: I am the liar. X is the truth-teller.

Z: I am the alternator. X is the liar.

Find out who the truth-teller is, who the liar is and who the alternator is.

11.2 Worked out Examples

Directions for Q1 – Q3: The following question consists of a main statement followed by four answer options. From these options, select the one that logically follows the main statement.

Q1.If you're in the army, you've to wear uniform

- a. You wear uniform means you are in the army
- b. You are in the army hence you don't wear uniform
- c. You don't wear uniform, though you are in the army
- d. You don't wear uniform implies that you are not in the army

Solution

Given "If you're in army (p) then you have to wear uniform (q)", in such situation the only valid inference for if $p \rightarrow q$ is $\sim q \rightarrow \sim p$

(i.e) "you don't wear uniform implies you are not in army".

Q2. He scores a century, only if the match is fixed.

- a. The match is not fixed implies he doesn't score century
- b. He didn't scores century hence the match is not fixed
- c. He scores a century means the match is fixed
- d. The match is fixed hence he scores a century
- e. Both a and c

Solution

Given "He scores a century (q), only if the match is fixed (p)". The valid inference for q, only if p is $q \rightarrow p$ and $\text{Neg } p \rightarrow \text{Neg } q$ ($\sim p \rightarrow \sim q$). Both a and c valid. So answer is (e).

Q3. If you don't bribe the minister, you'll not get the 2G license.

- a. you bribe the minister , so you will get the 2G license
- b. you will get 2G license means you bribe the minister
- c. You don't bribe the minister implies you will not get 2G license

d. Both b and c

Solution

Given "Unless you bribe the minister (p), you will not get the 2G license (q)". The valid inference for unless p, q is $\text{Neg } p \rightarrow q$ and $\text{Neg } q \rightarrow p$ ($\sim q \rightarrow p$). So answer is d (Both b and c).

Q4. Consider the following statements:

Ram: Raj did it.

Raj: Sam did it.

Deva: I did not do it.

Sam: Raj lied when he said that I did it.

(a) If only one out of all above statements is true, who did it?

(b) If only one out of all above statements is false, who did it?

Solution

We solve this question by assuming that Ram is thief. Then Raj, then after Sam. and put it in a table

Thief	Ram	Raj	Deva
Ram	F	T	F
Raj	F	F	F
Deva	T	T	F
Sam	T	T	T
		Raj did it	Deva did it

From the table, it is clear that only one statement is false when we assume Raj is thief. So answer for (b) is Raj. And only one statement became true, when we assume Deva is thief. So answer for (a) is Deva

Q5. The Public caught up Ragu, Ramesh and Venkat yesterday because one of them was suspected of having robbed the Jewellery shop. The three suspects made the following statements under intensive questioning.

Ragu: I'm innocent.

Ramesh: I'm innocent.

Venkat: Ramesh is the guilty one.

If only one of the statements turned out to be true, who robbed the Jewellery shop?

Solution

Assume Ragu is the thief. Now Except Ramesh statement, remaining two statements became false which is given in the question. So Ragu is the thief.

Directions for Q6 & Q7: Three robbers were arrested for shop lifting. However, when interrogated only one told the truth in both his statements, while the other two each told one true statement and one lie. The statement were

KHAN:

- (a) Dim passed the goods.
- (b) Dhoni created the diversion.

DHONI:

- (a) Khan passed the goods.
- (b) I created the diversion.

DIM:

- (a) I took the goods out of the shop.
- (b) Dhoni passed the goods.

Q6. Who created the diversion?

- a. Khan
- b. Dim
- c. Dhoni
- d. either (a) or (c)
- e. either b or c

Q7. Which of these statements is correct?

- a. Dim created the diversion.
- b. Khan took the goods out of the shop.
- c. Dim passed the goods.
- d. Khan created the diversion.
- e. Khan passed the goods.

Solution

Let 'T' represents true statement and 'F' represents false statement. We have to check possibilities and contradictions by assuming one person speaking truth and others will

say truth or lie alternatively. Assuming Dhoni to speak truth

Khan	F	T
Dhoni	T	T
Dim	T	F

Above mentioned possibility satisfies the conditions as others give contradictions. So, Khan passed the goods. Dhoni created diversion. Dim took goods out of shop.

Directions for question Q8: On an Island there live three types of families Cabrera, Hawa and Nana. Cabrerases always tell the truth, Hawas always lie and Nanases tell the truth and lie (alternators). Three persons (of different tribes) from this Island give these statements.

GOOD:

UGLY is of Cabrera tribe; I am of Nana tribe

BAD:

GOOD is of Hawa tribe; I am of Cabrera Tribe

UGLY:

BAD is of Hawa tribe; I am of Nana tribe.

Q8. GOOD belongs which tribe?

- a. Cabrera

- b. Hawa
- c. Nana
- d. either a or c
- e. cannot say

Solution

If we assume Good is of Cabrera tribe person, His both statements should be true. But one of his statement Ugly is of Cabrera tribe should be wrong as there is only one Cabrera tribe person. Now assume BAD is of Cabrera tribe person. Now his second statement is obviously true and His first statement indicates that Good is of Hawa type which implies that Ugly is of Nana type. Now checking of the truth fullness of the statements of Good and Ugly, we get Good's both the statements are wrong and Ugly's one statements is correct and one is wrong. So Good Belong to Hawa tribe.

11.3 Class Work Problems

Directions for Q1 – Q7: The following question consists of a main statement followed by four answer options. From these options, select the one that logically follows the main statement.

Q1. If I can run fast, I will get the prize.

- a. I can run fast hence I will get the prize.
- b. I wouldn't get the prize implies I cannot run fast
- c. I can run fast hence I don't get the prize
- d. Both A and B

Q2. Ram purchases either a television or air conditioner.

- a. Ram is not purchasing air conditioner implies he is purchasing a television
- b. Ram is purchasing neither a television nor air conditioner.
- c. Ram is not purchasing a television implies he is purchasing air conditioner.
- d. Both b and c

Q3. Vimal will go, only if his friend will come.

- a. His friend will come, hence Vimal will go
- b. Vimal will not go implies that his friend have not come.
- c. Vimal will go though his friend is not come.
- d. Vimal will go implies his friend will come.

Q4. Unless he was very ill, he would be at work.

- a. He was very ill, so cannot be work
- b. He cannot work means he was very ill.
- c. He was not ill implies he would not be at work.
- d. Both B and C

Q5. Kavin eats fruits, whenever he is hungry.

- I. Kavin is not hungry
 - II. Kavin is hungry
 - III. Kavin eats fruits
 - IV. Kavin does not eat fruits
- a. III & II
 - b. II&III
 - c. IV & I
 - d. II, III & IV, I

Q6. If my health is good, then we go to party.

- a. We go to party means my health is good.
- b. My health is good; hence we do not go to party.
- c. We did not go to party, though my health is good.
- d. We did not go to party implies that my health was not good.

Q7. Either several cars or a mini-bus is used to transport the officers to the parade

- a. Several cars used means mini buses will not used to transport the officers to the parade
- b. Mini buses are not used to transport the officers to the parade hence cars used
- c. Several cars are used hence mini buses are used to transport the officers to the parade
- d. Mini buses are not used to transport the officers to the parade implies that several cars are not used.

Q8. Vijay, Anand, Mahesh participated in kick boxing and each won some cash prizes among 5 lakhs, 2 Lakhs and 1 lakh. These things are not necessary in the same order. Among these three members one person always tells truth, one always lies and one speak truth and lie any order. When they were asked about the cash prizes, they replied the following:

Vijay:

I won 5 Lakhs .

Anand won 1 lakh

Anand:

I won 5 Lakhs .

Mahesh won 1 lakh

Mahesh:

I won 1 Lakh

Vijay won 5 lakhs

Who among them won the cash Prize of Rs 2 Lakhs?

a. Vijay

b. Anand

c. Mahesh

d. cannot be determined

Q9. On the way to my room, I met three friends Saro, Senthil and Nathan. One among them always speaks truth, one always lies and other alternates between truth and lie in any order. I know one among them has room key. I asked them about the room key they replied like the following:

Saro:

Nathan has the key

Only one among us has the key

Senthil:

I don't have the key

Saro have the key

Nathan:

Each one among us has the key.

I have the key

Who among have the key?

a. Saro

b. Nathan

c. Senthail

d. cannot be determined

Q10. Three Friends Joy, Happy and Delight are sitting in three seater in the bus. I called them and asked about their sitting positions. Each of them made two statements as follows. In these two statements one is true and another is false statement in any order.

Happy:

I am at the extreme left

Delight is at extreme left

Joy:

Happy is between me and Delight

I am at the extreme right

Delight:

I am at the extreme right

Joy is at the extreme right

What are the actual positions from right to left?

a. Happy-Joy -Delight

b. Delight-Joy-Happy

c. Joy-Delight- Happy

d. None of these

Directions for Questions Q11&Q12: In annual day function I met three teachers Jack, Jill and Mike. One of them is a math teacher, one is physics teacher and the other is a language teacher not necessarily in that order. One among them always speaks truth, one always lies and other alternates between truth and lie in any order. When I asked them about their teaching subjects, each of them made two statements as follows

Jack:

I am math teacher

Jill is not a physics teacher

Jill:

I am a language teacher

Mike is a physics teacher

Mike:

I am a language teacher

Jack is a math teacher.

Q11. Who is math teacher?

a. Jack

b. Jill

c. Mike

d. cannot be determined

Q12. Who among them is a language teacher?

a. Jack

b. Jill

c. Mike

d. cannot be determined

Directions for Questions Q13&Q14: In College I met three members Sana, Jana, Maha. One among them always speaks truth, one always lies and other alternates between truth and lie in any order. They got the first three highest marks in the internal examination. I asked about their marks in the internal examination, they replied like the following.

Sana:

I got more marks than Maha

I got less marks than Jana

Jana:

I got less marks than Maha

I got more marks than Sana

Maha:

I got more marks than Sana and Jana

Sana got more marks than Jana.

Q13. Who got the Second highest marks?

a. Sana

b. Jana

c. Maha

d. cannot be determined

Q14. If Maha is the topper of the class, then who among them is the liar?

- a. Sana*
- b. Jana*
- c. Maha*
- d. cannot be determined*

Directions for questions Q15 & Q16: A, B and C participated in a race and one of them won the race. They belong to three different Countries - India, Pak and China. Indians always speak the truth, Paks always lie and Chinas always tell the truth and lie alternatively. (Each of A, B and C belongs to one Continent). After the race they gave these statements.

A:

- 1. I would have won the race if C had not obstructed me at the last moment.*
- 2. C always speaks the truth.*
- 3. C is the winner.*

B:

- 1. A won the race.*
- 2. C is not a Pak.*

C:

- 1. I hadn't obstructed A at the last moment.*
- 2. B won the race.*

Q15. *C belongs to which country?*

- a. India*
- b. Pak*
- c. China*
- d. either b or c*
- e. cannot say*

Q16. *Who won the race?*

- a. B*
- b. C*
- c. India*
- d. A*
- e. cannot say*

11.4 Exercise

Directions for Q1 & Q2: The following question consists of a main statement followed by four answer options. From these options, select the one that logically follows the main statement.

Q1. If I talk to my professors then I didn't need to take a pill for headache

- A. I talked to my professors*
- B. I did not need to take a pill for headache*
- C. I needed to take a pill for headache*
- D. I did not talk to my professor.*

- a. AB
- b. DC
- c. CD
- d. AB and CD

Q2. Either the train is late or it has derailed

- A. Train is late
- B. Train is not late
- C. Train is derailed
- D. Train is not derailed

- a. AB
- b. DB
- c. CA
- d. BC

Directions for Questions Q3-Q7: In an island, each person belongs to one of the following three categories - truth tellers(who always speak the truth), liars(who always lie), alternators(who alternate among truth and lie in any order). Anil, Binoy, Chakri and Dhanu belong to that island. Each pursuing one of the degrees among B.Sc., B.Com, B.A., and B.Tech not necessarily in that order. At least one person from each category is present among these 4 persons. No 2 persons pursue the same degree. They made the following statements when asked

about their categories and studies.
Anil:

Exactly two of us are truth tellers.

I am pursuing B.Sc.

The person pursuing B.Com is an alternator.

Binoy:

I am an alternator

I am pursuing B.Tech.

Chakri is a liar.

Chakri:

I am a truth-teller.

I am pursuing B.Com

Dhanu is pursuing B.A.

Dhanu:

Exactly two of us are liars.

I am pursuing B.Com

Binoy is pursuing B.Sc

Q3. If Binoy is studying B.Sc. , who must be a liar?

- a. Dhanu
- b. Chakri
- c. Binoy
- d. Anil
- e. More than one of the above.

Q4. If Dhanu is an alternator, then the person pursuing B.Tech is a/an

- a. liar*
- b. truth teller*
- c. alternator*
- d. either liar or alternator*
- e. either liar or truth teller*

- a. only A*
- b. only B*
- c. only C*
- d. only a and b*
- e. only B and C*

Q5. If Chakri is pursuing B.Tech , then who is pursuing B.A.?

- a. Dhanu*
- b. Binoy*
- c. Binoy or Anil*
- d. Anil or Chakri*
- e. Anil*

Q6. If Chakri is not a liar, then who is pursuing B.Sc.

- a. Anil*
- b. Binoy*
- c. Chakri*
- d. Dhanu*
- e. CBD*

Q7. From which of the following conditions, we will get complete information about the studies of all the four?

- A. Binoy is a liar.*
- B. Dhanu is not an alternator.*
- C. Chakri is not an alternator.*

***Answers - LOGICAL CONNECTIVITY
& BINARY LOGIC***

11.1 Quiz

Q1 Q2

11.3 Class Work Problems

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12
Q13	Q14	Q15
Q16		

11.4 Exercise

Q1	Q2	Q3
Q4	Q5	Q6
Q7		

CHAPTER 12

BASICS OF NON VERBAL REASONING

INTRODUCTION

Non verbal reasoning measures ability to recognize patterns, analogies and classifications in diagrams. Non-verbal reasoning is classified into three groups. They are series, analogy, and classification. Most of the questions are from series.

BASIC CONCEPTS

SERIES

Type I

There should be two sets of figures in series section. The first one is question set, which consists of five figures. The second set is answer set. There are five figures and is numbered from 1 to 5. In the question set five figures are formed in a particular series you required. To find the answer figure from answer choices, which should be the sixth figure? At first you should observe whether there is any change in the shape or number of elements in the figures. Sometimes we can see that the elements in the figures move in a particular manner such as addition, deletion of elements, movement (At first watch the

direction of movement of element. The element moves either in clockwise direction or in anticlockwise direction), rotation (Watch the direction of rotation of element while observing the rotation of elements. They move either in clockwise direction or in anticlockwise direction. Then note, how much degree or by what distance they move. There are two types of rotation. In the first kind the element is stable while rotating. In the second is the position of element changes while rotating). Otherwise one of the elements will be replaced by a new one. In some cases both these changes occur at the same time.

Type II

Choosing the Missing Figure in a Series

In this type of questions, two sets of figures are given. The given first set will be the question set and it consists of five figures, labeled A, B, C, D, E. The second set is the answer set. There are five answer choices numbered 1, 2, 3, 4, 5. However the figure at C is missing. The candidate is required to choose this figure from the alternatives 1, 2, 3, 4 and 5. The number of selected figure is the answer. The easiest method for choosing the missing figure is to observe the series from the first and last figure.

ANALOGY

Analogy implies “Corresponding”. In the problems based on analogy, a pair of related figures is provided and a similar relationship is to be established between two other figures, by selecting one or both of them from a set of alternatives. The concepts used in series can be used here. There are various types of problems in Analogy.

Choosing One Element of a Similarly Related Pair

Analogy consists of two sets, Problem set and Answer set. Problem set involves four figures marked A, B, C and D. Answer set involves five figures marked 1, 2, 3, 4 and 5. Compare the figures A and B and find out the relationship between A and B. Establish a similar relationship between figures C and D by choosing a suitable figure for (D) from the answer set.

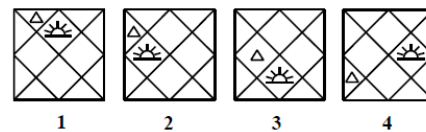
CLASSIFICATION

In this type of problems, we are given a set of figures, such that, all except one have similar characteristics. We are required to select the figure which differs from all other figures in the given set. Out of the five figures (1), (2), (3), (4) and (5) given in each

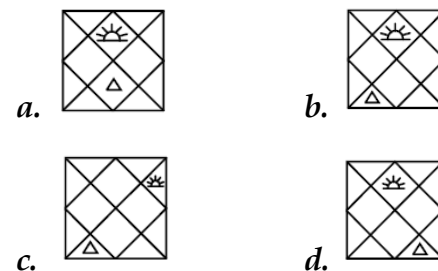
problem, four are similar in a certain way. However, one figure is not like the other four. Choose the figure which is different from the rest.

12.1 Quiz

Q1. Consider the following figures 1, 2, 3 and 4:



In the figures from 1 to 4 above, two symbols are shown to change their position in a regular direction. Following the same sequence, which one of the following will appear at the fifth stage?

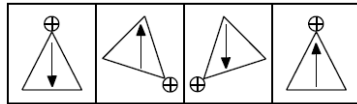


Directions for the following 2 (two) items:
In each item, there are two sets of figures: first four figures named Problem figures and the next four figures named Answer figures indicated as (a), (b), (c) and (d). **The problem figures follow a particular sequence.** In accordance with the same,

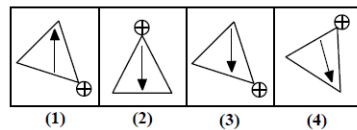
which one of the four answer figures should appear as the fifth figure?

Q2.

Problem figures:

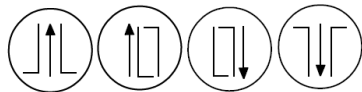


Answer figures:

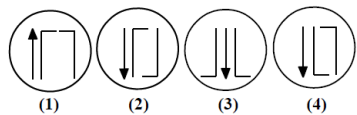


Q3.

Problem figures:

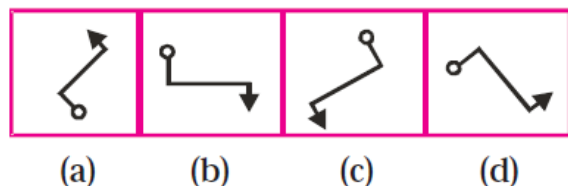


Answer figures:



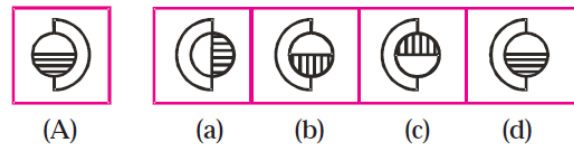
Directions for question Q4: Out of the given four figures, three are similar in a certain way. One figure is not like the other three. That means four figures form a group based on some common characteristics.

Q4. Find out the figure which does not belong to the group i.e., which does not share the common features / characteristics with the other four figures.

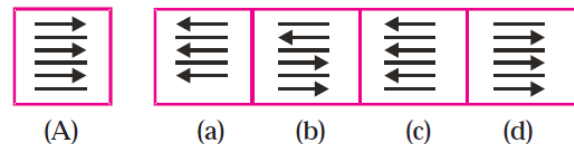


Directions for questions Q5 & Q6: In each of the following questions, choose the correct mirror image from alternatives (a), (b), (c), (d) of the figure (A).

Q5.

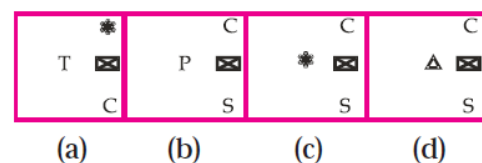
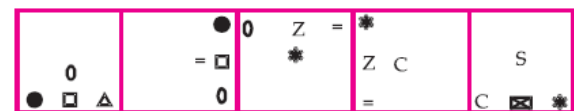


Q6.



12.2 Worked out Examples

Q1. In the following question given below which one of the four answer figures on the right should come after the problem figures on the left, if the sequence were continued?

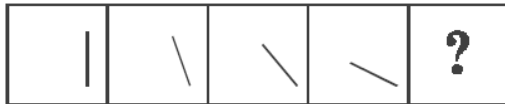


Solution: (b)

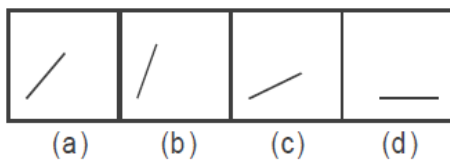
In each step the CW-end element shifts to the opposite corner, the central element becomes CW-end element, the second

element becomes CW-end element, the second from CW end shifts one side ACW and a new element appears at the centre. The elements get in turns replaced by new ones in alternate steps.

Q2. Problem figures



Answer figures



Solution

In the example above, as you go from left to right, you find the line across the Problem Figures falling downward. The question is, if the line continues to fall, what will be its next, i.e. fifth position? The answer would be: it would fall further and be lying horizontal.

Now, look at the Answer Figures. You find that Answer Figure (d) has a line lying horizontal. Therefore, the answer is (d).

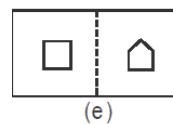
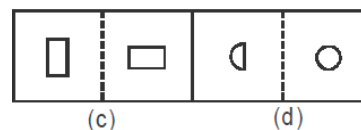
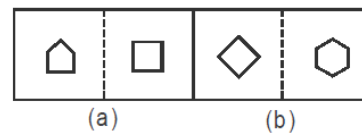
Q3. In the following question, there are two sets of figures. The Problem Figures and the Answer Figures. The Problem Figures are presented in one unit. This unit contains

two figures. You have to find out which of the Answer Figures share same relationship as in the problem figures.

Problem figures



Answer figures

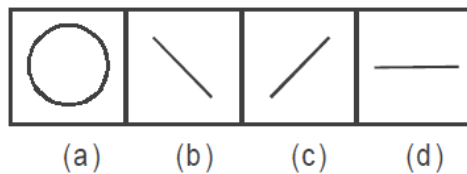


Solution

In the Problem Figure, the right hand element is related to the left hand element in a certain way. The right hand element has one side more than the left hand element. The pair in the Answer Figure No (e) has the **same** relationship, i.e. the right hand element has one side more than the left hand element of the figure. Therefore, (e) is the answer.

Q4. In the given questions, the Problem Figures themselves are also the Answer

Figures. Out of the four figures (a), (b), (c) and (d), three are similar in a certain way. One figure is not like the other three. That means three figures form a group. The question is: which of the figures **does not** belong to this group? Study the following problem based on odd man out.

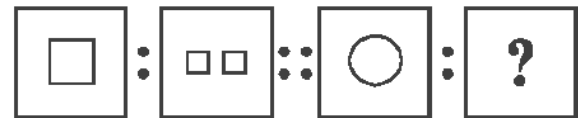


Solution

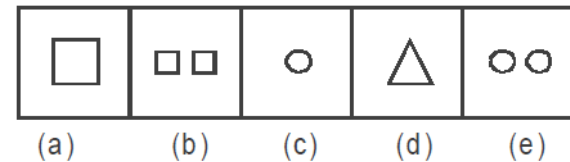
Of the four figures, three are straight-lines whereas one is a circle. The three lines form a group. The circle **does not** belong to this group. Therefore, the answer is the circle, which is figure (a).

Q5. In these questions, there are two sets of figures. The Problem Figures and the Answer Figures. The Problem Figures are presented in two units. The first unit contains two figures and the second unit contains one figure and a question mark. You have to find out which of the Answer Figures should be in place of the question mark.

Problem figures



Answer figures



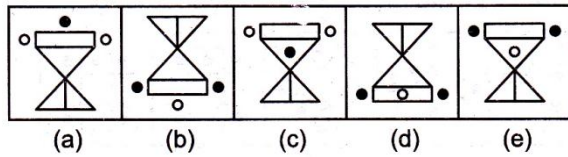
Solution

Look at the first two Problem Figures. The second figure is related to the first figure in a certain way. The elements in the second figure double the elements in the first figure. The first figure contains one square and the second figure contains two squares. The third and fourth figures should have the same relationship as that of the first and the second. The third figure is a circle; so the fourth should be two circles. Therefore, the answer is (e).

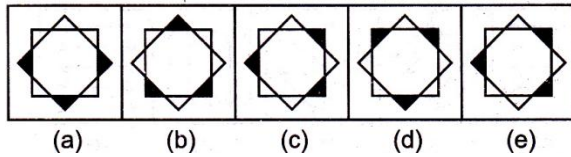
12.3 Class Work Problems

Directions for questions (1 to 15): Out of the five given figures, four are similar in a certain way. Find out the figure which does not belong to the group.

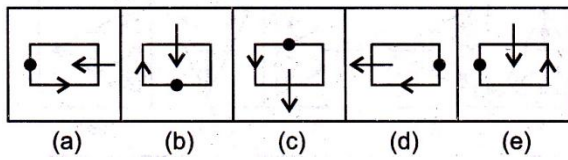
Q1.



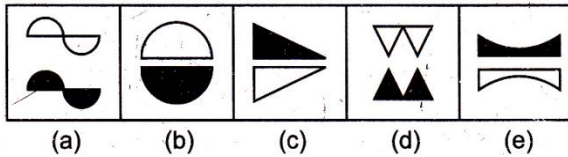
Q2.



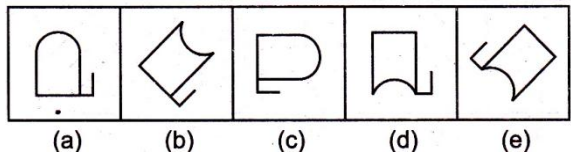
Q3.



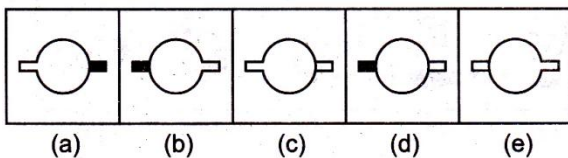
Q4.



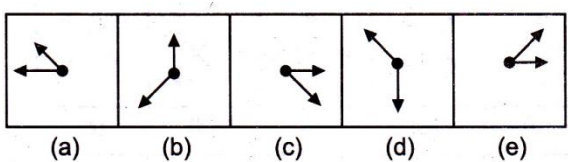
Q5.



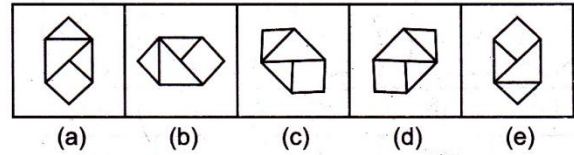
Q6.



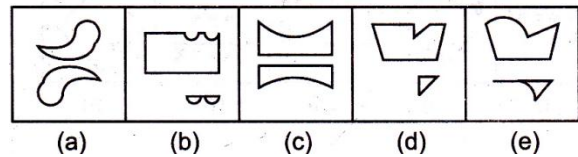
Q7.



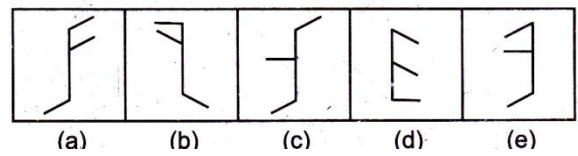
Q8.



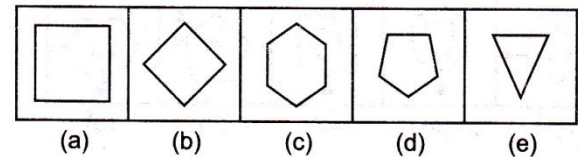
Q9.



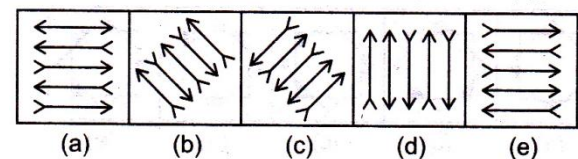
Q10.



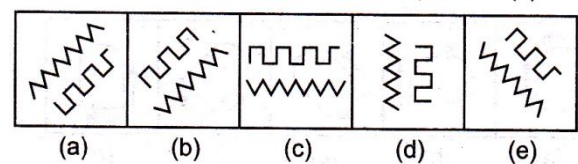
Q11.



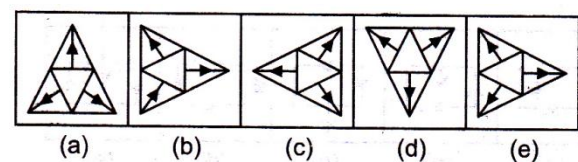
Q12.



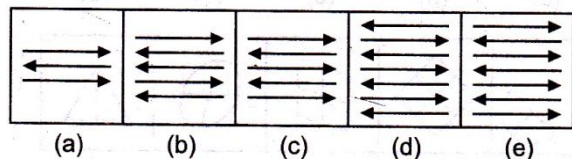
Q13.



Q14.



Q15.



Q16. The elements of the problem figures given below are changing with a certain rule as we observe them from left to right:

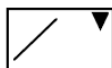


According to this rule, which of the following would be the next figure if the changes were continued with the same rule?

a.



b.



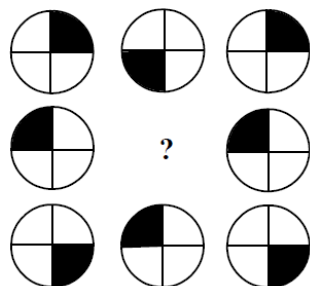
c.



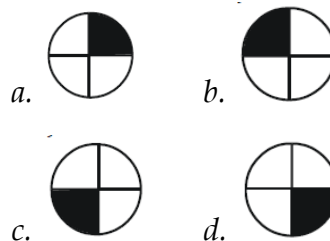
d.



Q17.



Which one of the figure shown below occupies the blank space (?) in the matrix given above?

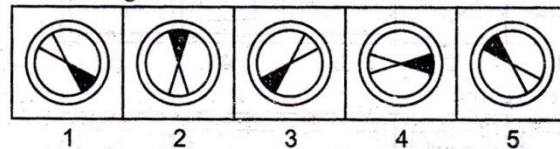


12.4 EXERCISE

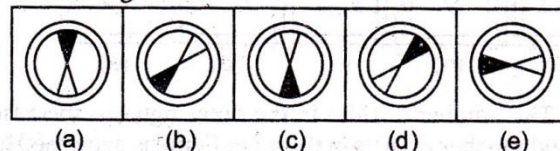
Directions for questions (1 to 15) : There are two sets of figures namely problem figure containing five figures namely 1,2,3,4 and 5. There are five answer figures named as (a), (b), (c), (d) and (e). Select one figure from the answer figure set which will continue the same series as given in problem set figures.

Q1.

Problem figures

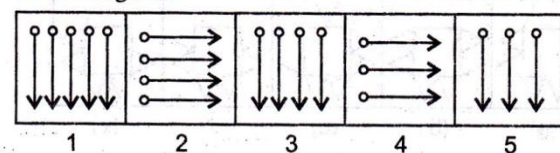


Answer Figures

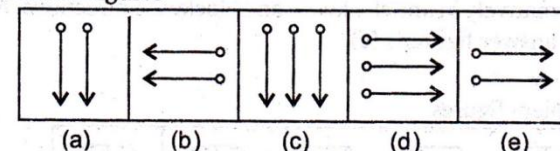


Q2.

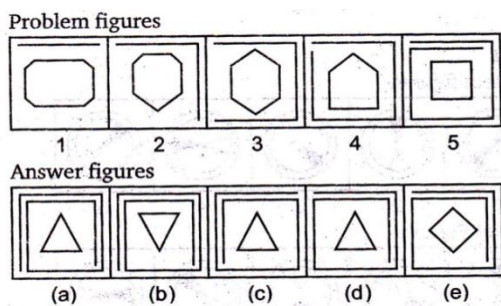
Problem figures



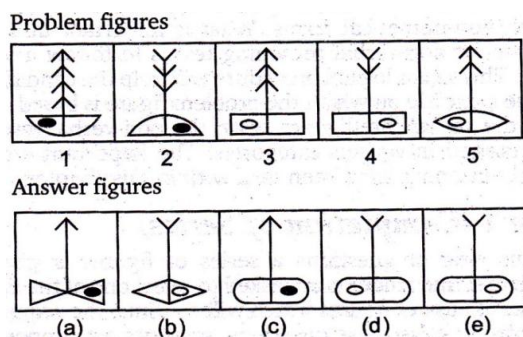
Answer figures



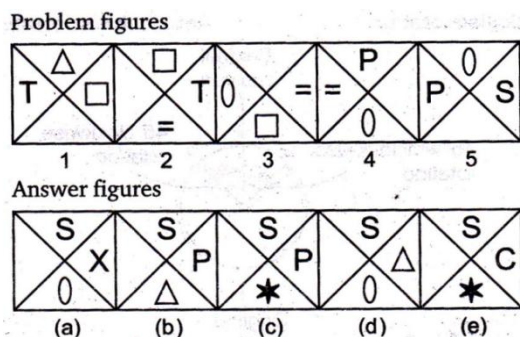
Q3.



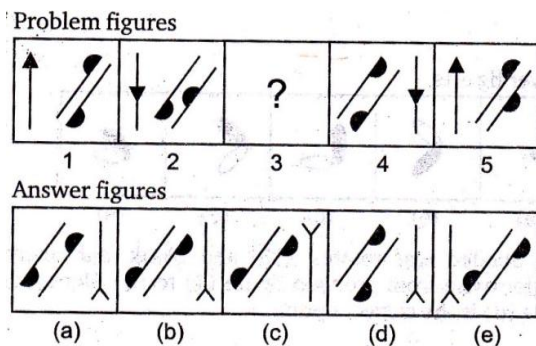
Q4.



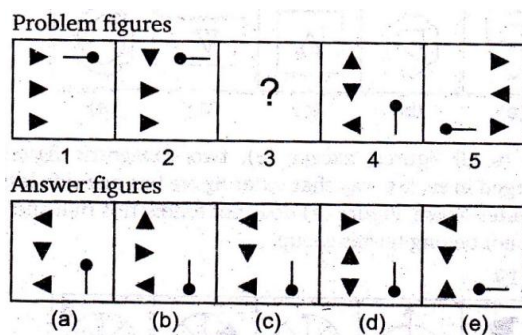
Q5.



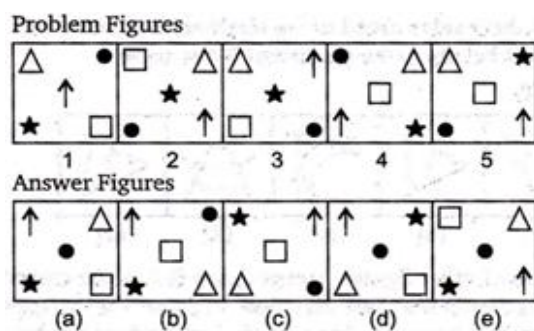
Q6.



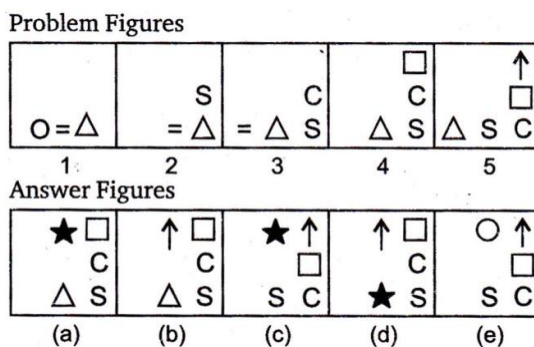
Q7.



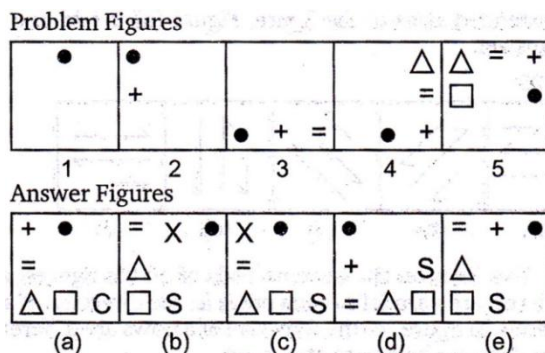
Q8.



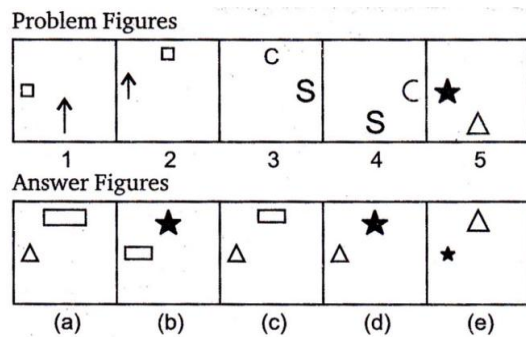
Q9.



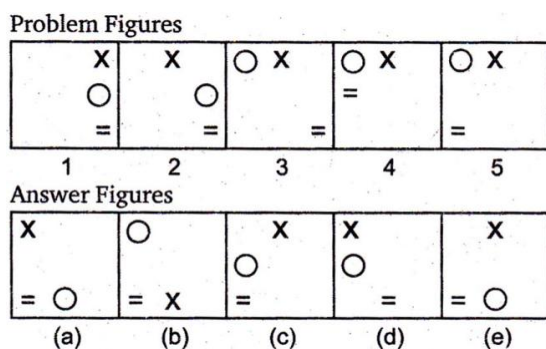
Q10.



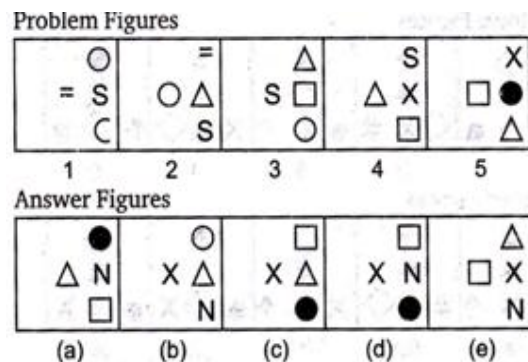
Q11.



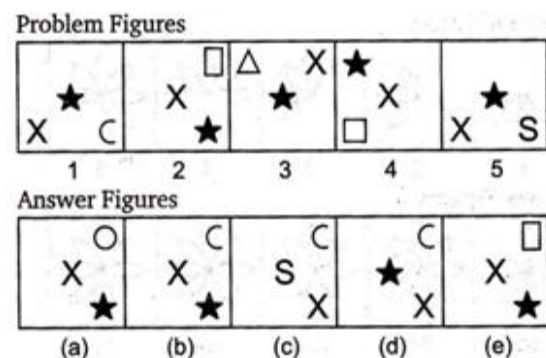
Q12.



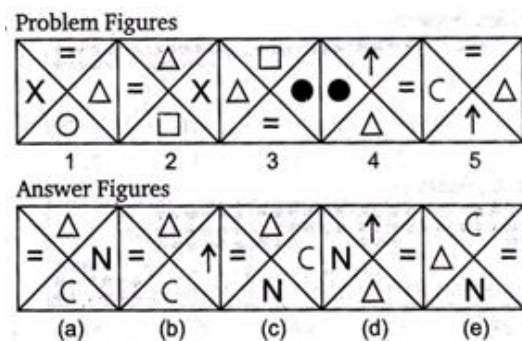
Q13.



Q14.

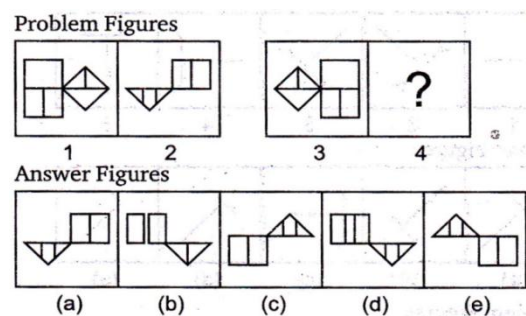


Q15.

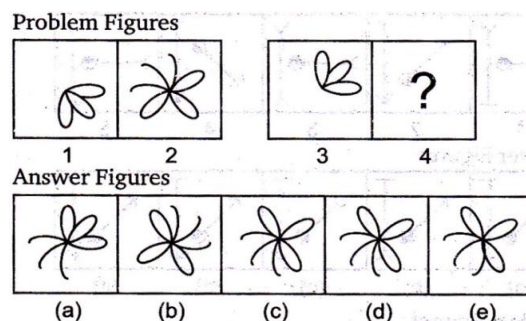


Directions for questions (16 to 30): The second figure in the first part of the problem figures bears a certain relationship to the first figure. Similarly, one of the figures in the answer figure bears the same relationship to the first figure in the second part. Select the figure from the set of answer figures which would replace the question mark.

Q16.

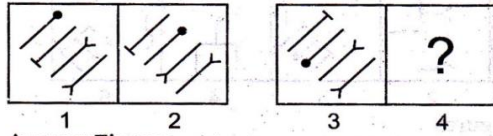


Q17.

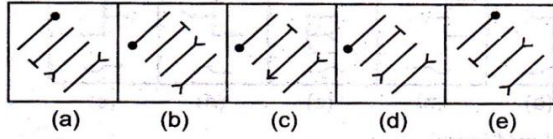


Q18.

Problem Figures

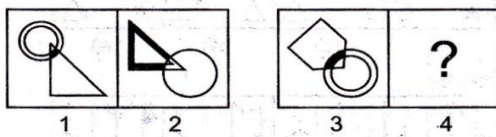


Answer Figures

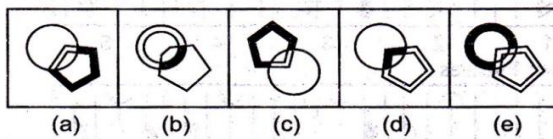


Q19.

Problem Figures

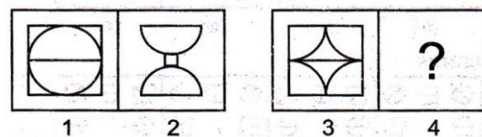


Answer Figures

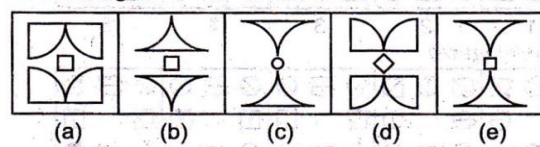


Q20.

Problem Figures

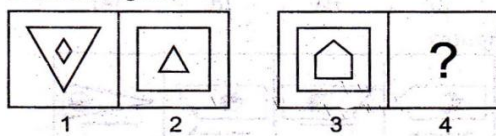


Answer Figures

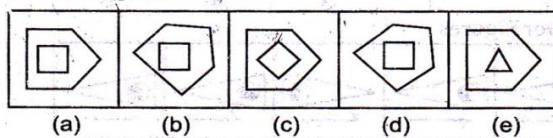


Q21.

Problem Figures

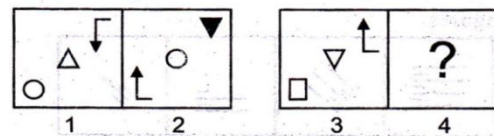


Answer Figures

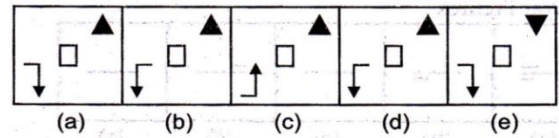


Q22.

Problem Figures

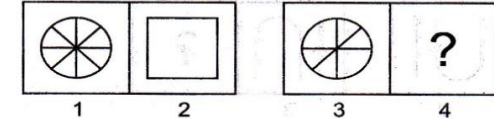


Answer Figures

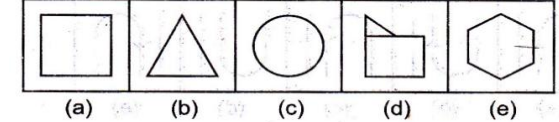


Q23.

Problem Figures

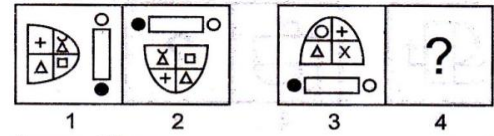


Answer Figures

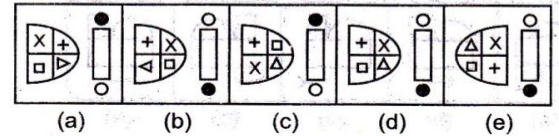


Q24.

Problem Figures

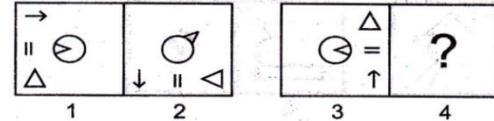


Answer Figures

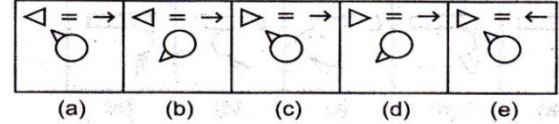


Q25.

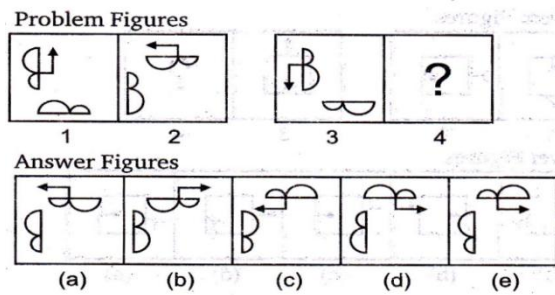
Problem Figures



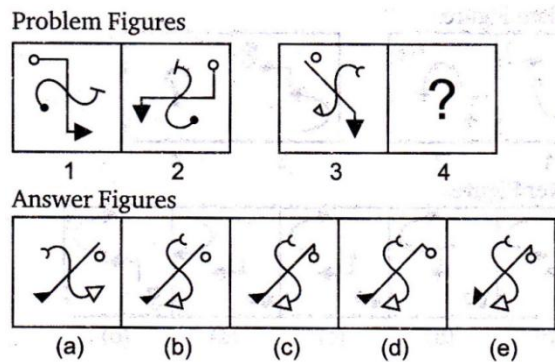
Answer Figures



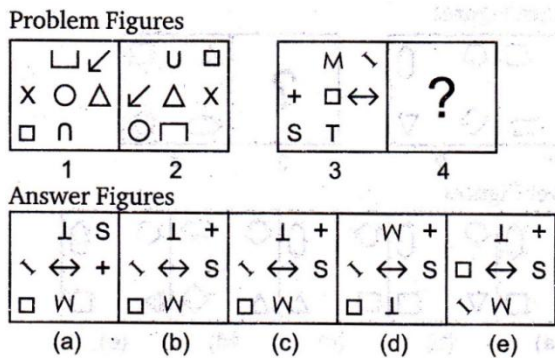
Q26.



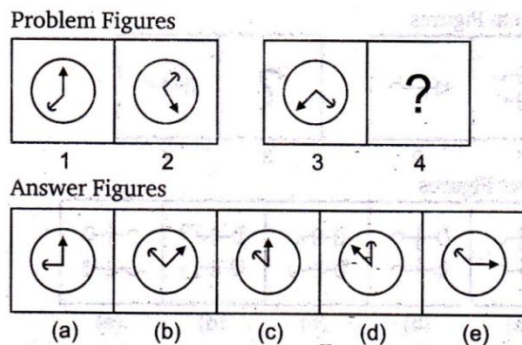
Q27.



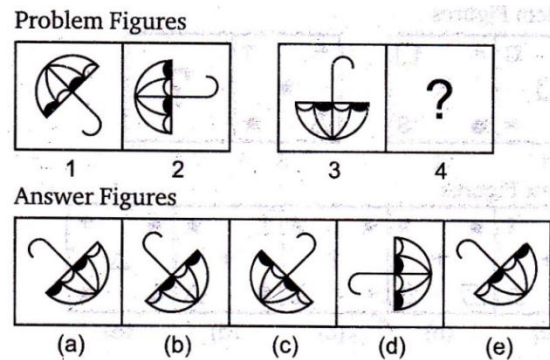
Q28.



Q29.



Q30.



***Answers - BASICS OF NON VERBAL
REASONING***

12.1 Quiz

Q1	Q2	Q3
Q4	Q5	Q6

12.3 Class Work Problems

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12
Q13	Q14	Q15
Q16	Q17	

12.4 Exercise

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12
Q13	Q14	Q15
Q16	Q17	Q18
Q19	Q20	Q21
Q22	Q23	Q24
Q25	Q26	Q27
Q28	Q29	Q30

CHAPTER 13

CAMPUS RECRUITMENT PAPERS

INTRODUCTION

In this chapter we have collected questions from various placement papers under 5 exercises. Every exercise deals with different difficulty level, and types of questions (including puzzles asked in written test as well as in the interview).

13.1 Exercise

Q1. Four horses are tethered at the four corners of a square of side 14m such that two horses along the same side can just reach each other. They were able to graze the area in 11 days. How many days will they take in order to graze the left out area?

- a.3
- b.7
- c.10
- d.4

Q2. Let A and B be two solid spheres such that the surface area of B is 300% higher than the surface area of A. The volume of A is found to be k% lower than the volume of B. The value of k must be

- a. 85.5
- b. 92.5
- c. 90.5
- d. 87.5

Q3. If the ratio of work done by $(x-1)$ men in $(x+1)$ days to the work done by $(x+2)$ men in $(x-1)$ days is 9:10, then x is equal to

- a. 5
- b. 6
- c. 7
- d. 8

Q4. The average age of 8 persons in a committee is increased by 2 years when two men aged 35 years and 45 years are substituted by two women. The average age of these two women is:

- a. 52 years
- b. 56 years
- c. 48 years
- d. 44 years

Q5. After a get-together every person present shakes the hand of every other person. If there were 105 hands-shakes in all, how many persons were present in the party?

- a. 15
- b. 14
- c. 13
- d. 16

Q6. A class consists of 100 students, 25 of them are girls and 75 boys; 20 of them are rich and remaining poor; 40 of them are fair complexioned. The probability of selecting a fair complexioned rich girl is:

- a. 0.05
- b. 0.04
- c. 0.02
- d. 0.08

Q7. In town of 500 people, 285 read Hindu and 212 read Indian express and 127 read Times of India, 20 read only Hindu and Times of India and 29 read only Hindu and Indian express and 35 read only Times of India and Indian express. 50 read no news paper. Then how many read all the three papers?

- a. 90
- b. 40
- c. 45
- d. 70

Q8. There are three departments having students 64, 58 and 24. In an exam they have to be seated in rooms such that each room has equal number of students and each room has students of one type only (No mixing of departments). Find the minimum number rooms required?

- a. 146
- b. 73
- c. 49
- d. 36

Q9. Argentina had football team of 22 players of which captain is from Brazilian team and goalkeeper is from European team. For remaining players, they have picked 6 from Argentinean and 14 from European. Now for a team of 11 they must have goalkeeper and captain so out of 9 now they plan to select 3 from Argentinean and 6 from European. Find out number of methods available for it.

- a. 60060
- b. 10010
- c. 16060
- d. 59060

Directions for Questions Q10-Q12: Convert the given binary numbers.

Q10. $(1110\ 0111)_2 = ()_{16}$

- a. $(F7)_{16}$
- b. $(E7)_{16}$
- c. $(D7)_{16}$
- d. $(D6)_{16}$

Q11. $(01011010)_2 = ()_8$

- a. $(132)_8$
- b. $(128)_8$
- c. $(256)_8$
- d. $(268)_8$

Q12. $(11110000)_2 = ()_{10}$

- a. $(120)_{10}$
- b. $(240)_{10}$
- c. $(250)_{10}$
- d. $(496)_{10}$

Q13. One boy can eat 100 chocolates in half a minute, and another can eat half as many in twice the length of time. How many chocolates can both boys eat in 15 seconds?

- a. 150
- b. 62.5
- c. 57.5
- d. 50

Q14. A merchant sells an item at a 20 percent discount. But still makes a gross profit of 20 percent of the cost. What percent of cost would be gross profit on the item have been if it had been sold without the discount?

- a. 20%
- b. 40%
- c. 50%
- d. 60%
- e. 66.6%

Q15. How many integers n greater than 10 and less than 100 are there such that, if the digits of n are reversed, the resulting integer is $n+9$?

- a. 5
- b. 6
- c. 7
- d. 8
- e. 9

Q16. Working independently, Tina can do a certain job in 12 hours. Working independently, Anna can do the same job in 9 hours. If Tina works independently at the job for 8 hours and then Anna works independently, how many hours will it take

Anna to complete the remainder of the job?

- a. $\frac{2}{3}$*
- b. $\frac{3}{4}$*
- c. 1*
- d. 2*
- e. 3*

Q17. *Worker A produces n units in 5 hours. Workers A and B can work together and produce n units in 2 hours. How long would it take B alone to produce n units?*

- a. 1 hr 26 min*
- b. 1 hr 53 min*
- c. 2 hr 30 min*
- d. 3 hr 30 min*
- e. 3 hr 20 min*

Q18. *A 5 litre jug contains 4 litres of salt water solution that is 15 percent salt. If 1.5 litres of the solution spills out of the jug, and the jug is then filled to capacity with water, approximately what percent of the resulting solution in the jug is salt?*

- a. 7.5%*
- b. 9.5%*
- c. 10.5%*
- d. 12%*
- e. 15%*

Q19. *A plane travelled K miles in the first 96 minutes of flight time. If it completed the remaining 300 miles of the trip in 1 minute, what was its average speed in miles per hour for the entire trip?*

- a. $(300)/K+96$*
- b. $(300+k)/97 * 60$*
- c. $(300+k)/96 * 60$*
- d. $(300+k)/97$*

Q20. *A sink has 12 litres of water. Some quantity of water is taken out. If the remaining water is 6 litres less than the water taken out then quantity of water taken out is.*

- a. 3*
- b. 6*
- c. 9*
- d. 1*

Q21. *Which is the 4 digit number whose second digit is thrice the first digit and 3rd digit is sum of 1st and 2nd and last digit is twice the second digit.*

- a. 2674*
- b. 1349*
- c. 3343*
- d. 3678*

Q22. A warehouse had a square floor with area 10,000 sq metres. A rectangular addition was built along one entire side of the warehouse that increased the floor by one-half as much as the original floor. How many metres did the addition extend beyond the original buildings?

- a. 10
- b. 20
- c. 50
- d. 200
- e. 500

Q23. A digital wristwatch was set accurately at 8.30 a.m and then lost 2 seconds every 5 minutes. What time was indicated on the watch at 6.30 p.m of the same day if the watch operated continuously that time?

- a. 5:56
- b. 5:58
- c. 6.00
- d. 6.23
- e. 6.26

Q24. There are 6561 balls out of them 1 is heavy. Find the minimum number of times the balls have to be weighed for finding out the heavy ball.

- a. 7
- b. 8
- c. 9
- d. 10

Q25. Karan and Arjun run a 100 metre race, where Karan beats Arjun by 10 metres. To do a favour to Arjun, Karan starts 10 metres behind the starting line in a second 100 metre race. They both run at their earlier speeds. Which of the following is true in connection with the second race?

- a. Karan and Arjun reach the finishing line simultaneously.
- b. Arjun beats Karan by 1 metre.
- c. Arjun beats Karan by 11 metres.
- d. Karan beats Arjun by 1 metre.

Q26. A coffee seller has two types of coffee Brand A costing 5 bits per pound and Brand B costing 3 bits per pound. He mixes two brands to get a 40 pound mixture. He sold this at 6 bits per pound. The seller gets a profit of $33\frac{1}{2}$ percent. How much he has used Brand A in the mixture?

- a. 15 pounds
- b. 20 pounds
- c. 25 pounds

d. 30 pounds

Q27. A ship went on a voyage. After it had travelled 180 miles a plane started with 10 times the speed of that ship. Find the distance when they meet from starting point.

a.250

b.300

c.200

d.220

e.240

Q28. A man sold two cows for Rs. 210 at a total profit of 5 %. He sold one cow at a loss of 10% and another at a profit of 10%. What is the cost price of each cow?

a. Rs. 150 and Rs. 50

b. Rs 160 and Rs 40

c. Rs 100 and Rs 100

d. Rs 120 and Rs 80

e. Rs 110 and Rs 90

Q29. At 6'o clock the clock ticks 6 times. The time between first and last tick was 30sec. How much time it takes at 12'o clock?

a.72

b.66

c.60

d.90

e. None of these

Q30. A man reaches his office just on time every day. On a particular day, he wants to reach his office 15 minutes earlier than usual, after taking care of some work on the way for 10 minutes. Accordingly, he starts early from home. He takes care of his work on the way. But a little latter, he gets stuck for 20 minutes in a traffic jam. His watch is running for 15 minutes slow, which he doesn't know. Which of the following statements is true?

a. He reaches office 10 minutes earlier than the regular office time.

b. He reaches office 20 minutes later than the regular office time.

c. He reaches office 25 minutes later than the regular office time.

d. He reaches office at his regular office time.

Q31. Two twins have certain peculiar characteristics. One of them always lies on Monday, Wednesday, and Friday. The other always lies on Tuesdays, Thursdays and

Saturdays. On the other days they tell the truth. You are given a conversation.

Person A: Today is Sunday my name is Anil

Person B: Today is Tuesday, my name is Bill. What day is today?

- a. Sunday
- b. Tuesday
- c. cannot be determined
- d. None of these

Q32. A Roman was born the first day of the 35th year before Christ and died the first day of the 35th year after Christ. How many years did he live?

- a. 70
- b. 69
- c. 71
- d. 72

Q33. A horse starts to chase a dog that has left the stable two hours earlier. The horse runs at an average speed of 22km/hr. It crosses a 10 metre road, two small ponds 3 meters deep, and finally runs along two small streets of 200 meters long. After traveling 6 hrs, 2hrs after sunset, it catches

the dog. Compute the speed of the dog in Km/hr?

- a. 20
- b. 22
- c. 16.5
- d. 18.5

Q34. Adam sat with his friends in the Chinnaswamy stadium at Madurai to watch the 100 metres running race organized by the Asian Athletics Association. Five rounds were run. After every round half the teams were eliminated. Finally, one team wins the game. How many teams participated in the race?

- a. 30
- b. 32
- c. 41
- d. 54

Q35. A garrison of 3300 men has provisions for 32 days when supplied at the rate of 850 g per head. At the end of 7 days, reinforcement arrives, and it is found that the provisions can last for 17 days more when supplied at the rate of 825 g per head. What is the strength of the reinforcement?

- a. 1700
- b. 1000
- c. 3000
- d. 2700

Q36. Two unemployed young men decided to start a business together. They pooled in their savings, which came to Rs. 2,000. They were both lucky, their business prospered and they were able to increase their capital by 50 percent every three years. How much did they have in all at the end of eighteen years?

- a. Rs. 22,781.25
- b. Rs. 24,150.25
- c. Rs. 28,140.50
- d. Rs. 18,000

Q37. A train starts with certain number of passengers. At the first station, it drops one-third of the passengers and takes 280 more. At the second station, it drops one-half of the new total and takes 12 more. On arriving at the third station, it is found to have 248 passengers. Find the number of passengers in the beginning.

- a. 240
- b. 248

- c. 280
- d. 288

Q38. A manufacturer undertakes to supply 2000 pieces of a particular component at Rs.25 per piece. According to his estimates, even if 5% fail to pass the quality tests, then he will make a profit of 25%. However, as it turned out, 50% of the components were rejected. What is the loss to the manufacturer?

- a. Rs.12000
- b. Rs.13000
- c. Rs.14000
- d. Rs.15000

Q39. 6 persons standing in queue with different age group, after two years their average age will be 43 and seventh person joined with them. Hence the current average age has become 45. Find the age of seventh person?

- a. 69
- b. 70
- c. 40
- d. 45

Q40. Which is the smallest number that divides 2880 and gives a perfect square?

- a. 1
- b. 2
- c. 5
- d. 6

Q41. 23 people are there, they are shaking hands together, how many handshakes possible, if they are in pair of cyclic sequence.

- a. 22
- b. 23
- c. 44
- d. 46

Q42. There are two water tanks A and B, A is much smaller than B. While water fills at the rate of one liter every hour in A, it gets filled up like 10lt, 20lt, 40lt, 80lt, 160lt...in tank B. (At the end of first hour, B has 10 liters, second hour it has 20, and so on). If tank B is $\frac{1}{32}$ filled after 21 hours, what is the total duration required to fill it completely?

- a. 26 hrs
- b. 25 hrs
- c. 5 hrs

d. 27 hrs

Q43. A man jogs at 6 mph over a certain journey and walks over the same route at 4 mph. What is his average speed for the journey?

- a. 2.4 mph
- b. 4 mph
- c. 4.8 mph
- d. 5 mph

Q44. A man, a woman, and a child can do a piece of work in 6 days. Man only can do it in 24 days. Woman can do it in 16 days and in how many days child can do the same work?

- a. 8
- b. 14
- c. 16
- d. 18

Q45. Two trains move in the same direction at 50 kmph and 32 kmph respectively. A man in the slower train observes the 15 seconds elapse before the faster train completely passes by him. What is the length of faster train?

- a. 100m
- b. 75m
- c. 120m
- d. 50m

Directions for Q46-Q55: Use the following answer choices for the questions below.

- A.** Statement 1 alone is sufficient but statement 2 alone is not sufficient to answer the question asked.
- B.** Statement 2 alone is sufficient but statement 1 alone is not sufficient to answer the question asked.
- C.** Both statements 1 and 2 together are sufficient to answer the question but neither statement alone is sufficient.
- D.** Each statement alone is sufficient to answer the question.
- E.** Statements 1 and 2 are not sufficient to answer the question asked and additional data is needed to answer the statements.

Q46. Is the product of x and y greater than 60?

- 1. The sum of x and y is greater than 60.
- 2. Each of the variables is greater than 2.

- a. A
- b. B
- c. C
- d. D
- e. E

Q47. Is $x > 0$?

- 1. $-2x < 0$
- 2. $x^3 > 0$

- a. A
- b. B
- c. C
- d. D
- e. E

Q48. A certain straight corridor has four doors, A, B, C and D (in that order) leading off from the same side. How far apart are doors B and C?

- 1. The distance between doors B and D is 10 metres.
- 2. The distance between A and C is 12 metres.

- a. A
- b. B
- c. C
- d. D
- e. E

Q49. Given that x and y are real numbers, what is the value of $x + y$?

1. $(x^2 - y^2) / (x - y) = 7$

2. $(x + y)^2 = 49$

a. A

b. B

c. C

d. D

e. E

Q50. Two socks are to be picked at random from a drawer containing only black and white socks. What is the probability that both are white?

1. The probability of the first sock being black is $1/3$.

2. There are 24 white socks in the drawer.

a. A

b. B

c. C

d. D

e. E

Q51. A bucket was placed under a dripping tap which was dripping at a uniform rate. At what time was the bucket full?

1. The bucket was put in place at 2pm.

2. The bucket was half full at 6pm and three-quarters full at 8pm.

a. A

b. B

c. C

d. D

e. E

Q52. One number, n , is selected at random from a set of 10 integers. What is the probability that $\frac{1}{2}n + 13 = 0$?

1. The largest integer in the set is 13.

2. The arithmetic mean of the set is zero.

a. A

b. B

c. C

d. D

e. E

Q53. What is the volume of a cubical box in cubic centimeters?

1. One face of the box has an area of 49 sq cm

2. The longest diagonal of the box is 12.12 cm

a. A

b. B

c. C

- d. D
- e. E

Q54. If a ground is rectangular, what is its width?

1. The ratio of its length to its breadth is 7:2

2. Perimeter of the playground is 396 mts.

- a. A
- b. B
- c. C
- d. D
- e. E

Q55. How many brothers and sisters are there in the family of seven children?

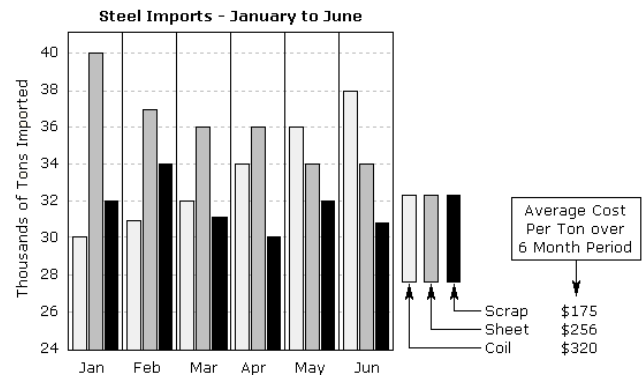
1. Each boy in the family has as many sisters as brothers

2. Each of the girl in the family has twice as many brothers as sisters

- a. A
- b. B
- c. C
- d. D
- e. E

Directions for Questions Q56-Q60: The figure below shows imports for three types of

steel over a six month period. Use this information to answer the following questions



Q56. Which month showed the largest decrease in total imports over the previous month?

- a. February
- b. March
- c. April
- d. May
- e. June

Q57. What was the total value of sheet steel (in \$) imported over the 6 month period?

- a. 56,750
- b. 75,300
- c. 55,550
- d. 42,370
- e. 44,750

Q58. What was the percentage of scrap steel imported in the 6 month period?

- a. 37.5
- b. 35.2
- c. 36.1
- d. 31.2
- e. 38.3

Q59. What was the difference (in thousands of tons) between coil steel and sheet steel imports in the first 3 months of the year?

- a.10
- b.16
- c.18
- d.19
- e. 20

Q60. What was the approximate ratio of sheet steel and coil steel imports in the first 3 months of the year?

- a.11:9
- b.8:9
- c. 7:11
- d. 3:8
- e.7:4

13.2 Exercise

Directions for Questions Q1 - Q3:

Answer the questions on the basis of the information given below. A, B, C, D, E, and F are a group of friends. There are two housewives, one professor, one engineer, one accountant and one lawyer in the group. There are only two married couples in the group. The lawyer is married to D, who is a housewife. No woman in the group is either an engineer or an accountant. C, the accountant, is married to F, who is a professor. A is married to a housewife. E is not a housewife.

Q1. Which of the following is one of the married couples?

- a. A & B
- b. B & E
- c. D & E
- d. A & D

Q2. What is E's profession?

- a. Engineer
- b. Lawyer
- c. Professor
- d. Accountant

Q3. How many members of the group are males?

- a. 2
- b. 3
- c. 4
- d. Cannot be determined.

Q6. How many cubes have at least two faces colored?

- a. 19
- b. 20
- c. 21
- d. 23

Directions for questions Q4- Q8: A cube is colored orange on one face, pink on the opposite face, brown on one face and silver on a face adjacent to the brown face. The other two faces are left uncolored. It is then cut into 125 smaller cubes of equal size.

Now answer the following questions based on the above statements:

Q4. How many cubes have at least one face colored pink?

- a. 1
- b. 9
- c. 16
- d. 25

Q7. How many cubes are colored orange on one face and have the remaining faces uncolored?

- a. 8
- b. 12
- c. 14
- d. 16

Q5. How many cubes have all the faces uncolored?

- a. 24
- b. 36
- c. 48
- d. 64

Q8. How many cubes have one colored silver on one face, orange or pink on another face and have four uncolored faces?

- a. 8
- b. 10
- c. 12
- d. 16

Directions for Questions Q9&Q10:
Study the following arrangement and answer the questions given below:
R 4 T M 7 W % J 9 5 I # 1 P B 2 T A 3 D \$
6 E N F 8 U H @

Q9. How many such vowels are there in the above arrangement, each of which is immediately preceded by a number and not immediately followed by a consonant?

- a. None
- b. One
- c. Two
- d. Three
- e. Four

Q10. What should come in place of the question mark (?) in the following series based on the above arrangement?

TM% 951 B23?

- a. \$EF
- b. \$6F
- c. D\$N
- d. \$E8
- e. None of these

Directions for Questions Q11&Q12:

In the following questions, the symbols @,?, %, ^, and \$ are used with the following meanings illustrated. 'P % Q' means 'P is either greater than or equal to Q'. 'P? Q' means 'P is neither greater than nor smaller than Q'. 'P \$ Q' means 'P is smaller than Q'. 'P @ Q' means 'P is either smaller than or equal to Q'. 'P ^ Q' means 'P is greater

than Q'. In each of the following questions assuming the given statements to be true, find out which of the three conclusions I, II and III given below them is/are definitely true and mark your answer accordingly.

Q11. Statements: M \$ T, T@ K, K ? D

Conclusions:

I. D % T

II. K ^ M

III. D ^ M

- a. Only I and II are true
- b. Only I and III are true
- c. Only II and III are true
- d. All are true
- e. None of these

Q12. Statements: B^ H, H % A, A? K

Conclusions:

I. B % K

II. K@ H

III. A \$ B

- a. Only I and II are true
- b. Only I and III are true
- c. only II and III are true
- d. only II is true.
- e. None of these

Q13. In a murder case there are four suspects P,Q,R,S. Each of them makes a statement. They are

P: "I had gone to the theatre with S at the time of the murder".

Q: "I was playing cards with P at the time of the murder".

R: "Q didn't commit the murder".

S: "R is not the murderer".

Assuming the only one of the above statement is false and that one of them is the murderer, who is the murderer?

- a. P
- b. Q
- c. R
- d. S
- e. Cannot be concluded

Q14. Mohan earned twice as much as Deep. Yogesh earned Rs.3 more than half as much as deep. If the amounts earned by Mohan, Deep, Yogesh are M, D, Y respectively, which of the following is the correct ordering of these amounts?

- a. $M < D < Y$
- b. $M < Y < D$
- c. $D < M < Y$
- d. $D < Y < M$

e. cannot be determined

Q15. Given that A,B,C,D,E each represent one of the digits between 1 and 9 and that the following multiplication holds:

A B C D E

X 4

E D C B A

What digit does E represent ?

- a. 4
- b. 6
- c. 8
- d. 7
- e. 2

Directions for Questions Q16-Q20: Six knights - P, Q, R, S, T and U - assemble for a long journey in two traveling parties. For security, each traveling party consists of at least two knights. The two parties travel by separate routes, northern and southern. After one month, the routes of the northern and southern groups converge for a brief time and at that point the knights can, if they wish, rearrange their traveling parties before continuing, again in two parties along separate northern and southern routes. Throughout the entire trip, the composition of traveling parties must be in

accord with the following conditions:
P and R are deadly enemies and, although they may meet briefly, can never travel together. P must travel in the same party with S. Q can't travel by the southern route. U can't change routes

Q16. If one of the two parties of knights consists of P and U and two other knights and travels by the southern route, the other members of this party besides P and U must be

- a. Q and S
- b. Q and T
- c. R and S
- d. R and T
- e. S and T

Q17. If each of the two parties of knights consists of exactly three members, which of the following is not a possible traveling party and route?

- a. P,S,U by the southern route
- b. P,S,T by the northern route
- c. P,S,T by the southern route
- d. Q,U,R by the northern route
- e. Q,R,T by the northern route

Q18. If one of the two parties of knights consists of U and two other knights and travels by the northern route, the other members of this party besides U must be

- a. P and S
- b. P and T
- c. Q and R
- d. Q and T
- e. R and T

Q19. If each of the two parties of knights consists of exactly three members of different parties and R travels by the northern route, then T must travel by the

- a. Southern route with P and S
- b. Southern route with Q and R
- c. Southern route with R and U
- d. Northern route with Q and R
- e. Northern route with R and U

Q20. If, when the two parties of knights encounter one another after a month, exactly one knight changes from one traveling party to the other traveling party, that knight must be

- a. P
- b. Q
- c. R
- d. S

e. T

Directions for Questions Q21 to Q25:

Read the following information carefully and answer the questions given it.

There are six persons A, B, C, D, E and F in a school. Each of the teachers teaches two subjects, one compulsory subject and the other optional subject. D's optional subject was History while three others have it as compulsory subject. E and F have Physics as one of their subjects. F's compulsory subject is Mathematics which is an optional subject of both C and E. History and English are A's subjects but in terms of compulsory and optional subjects, they are just reverse of those of D's. Chemistry is an optional subject of only one of them. The only female teacher in the school has English as her compulsory subject.

Q21. What is C's compulsory subject?

- a. History
- b. Physics
- c. Chemistry
- d. English
- e. Mathematics

Q22. Who is a female member in the group?

- a. A
- b. B
- c. C
- d. D
- e. E

Q23. Which of the following has some compulsory and optional subjects as those of F's ?

- a. D
- b. B
- c. A
- d. C
- e. None of these

Q24. Disregarding which is the compulsory and which is the optional subject, who has the same two subject combination as F?

- a. A
- b. B
- c. E
- d. D
- e. None of these

Q25. Which of the following groups has History as the compulsory subject?

- a. A, C, D
- b. B, C, D
- c. C, D
- d. A, B, C
- e. A, D

Directions for Questions Q26-Q28: In each question below are given three Statements followed by three Conclusions numbered I, II and III. You have to take the given Statements to be true even if they seem to be at variance from commonly known facts. Read all the conclusions and then decide which of the given Conclusions logically follows from the given Statements disregarding commonly known facts.

Q26. Statements:

Some cycles are buses.

All cars are buses.

Some buses are trains.

Conclusions:

- I. All cars are cycles.
- II. Some trains are buses.
- III. Some trains are cars.
- a. None follows
- b. Only I and II follow

- c. Only land III follow
- d. Only II and III follow
- e. None of these

Q27. Statements:

Some birds are animals.

All animals are rivers.

Some rivers are lions.

Conclusions:

- I. Some lions are animals
- II. Some rivers are birds
- III. No animal is lion
- a. Only II follows
- b. Only either I or III follows
- c. I and II follows
- d. Only either II or III follow
- e. None of these

Q28. Statements:

All boxes are pans.

Some boxes are jugs .

Some jugs are glasses.

Conclusions:

- I. Some glasses are boxes
- II. No glass is box
- III. Some jugs are pans
- IV. No jug is pan

- a. Only I and II follows
- b. Either I or II and III follows
- c. Only III follows
- d. Either I or II, and either III or IV follow
- e. None of these

Q29. Sunita goes 30 km towards North from a fixed point, then after turning to her right she goes 15 km. After this she goes 30 km after turning to her right. How far and in what direction is she from her starting point?

- a. 45 km, East
- b. 15 km, East
- c. 45 km, West
- d. 45 Km, North
- e. None of these

Q30. P, Q, R and S are playing a game of carom. P, R, and S, Q are partners. S is to the right of R who is facing west. Then Q is facing?

- a. North
- b. South
- c. East
- d. West
- e. None of these

13.3 Exercise

Q1. $(1024 - 263 - 233) \div (986 - 764 - 156)$
=?

- a. 9
- b. 6
- c. 7
- d. 8
- e. None of these

Q2. $(42)^2 \div 6.3 \times 26$ =?

- a. 7182
- b. 7269
- c. 7260
- d. 7240
- e. None of these

Q3. The compound interest on a certain amount for 2 years at the rate of 8 p.c.p.a. is Rs.312. What will be the simple interest on the same amount and at the same rate and same time?

- a. Rs. 349.92
- b. Rs. 300
- c. Rs. 358.92
- d. Rs. 400
- e. None of these

Q4. The length of a rectangle exceeds its breadth by 7cm. If the length is decreased by 4 cm. and the breadth is increased by 3cm, then the area of the new rectangle will be the same as the area of the original rectangle. What will be the perimeter of the original rectangle?

- a. 45cm
- b. 40cm
- c. 50cm
- d. 55cm
- e. None of these

Q5. In a fraction, twice the numerator is two more than the denominator. If 3 is added to the numerator and the denominator each, then the resultant fraction will be $\frac{2}{3}$. What was the original fraction?

- a. $\frac{5}{18}$
- b. $\frac{6}{13}$
- c. $\frac{13}{6}$
- d. $\frac{7}{12}$
- e. None of these

Q6. A shopkeeper purchased 200 bulbs for Rs. 10 each. However, 5 bulbs were fused and had to be thrown away. The remaining

were sold at Rs.12 each. What will be the percentage profit?

- a. 25
- b. 15
- c. 13
- d. 17
- e. None of these

Q7. Girish started a business investing Rs. 45,000. After 3 months, Vijay joined him with a capital of Rs. 60,000. After another 6 months, Ankush joined them with a capital of Rs. 90,000. At the end of the year, they made a profit of Rs. 16,500. What is Girish's share of profit?

- a. Rs. 5,500
- b. Rs. 6,000
- c. Rs. 6,600
- d. As. 5,900
- e. None of these

Q8. A and B are two taps which can fill a tank individually in 10 minutes and 20 minutes respectively. However, there is a leakage at the bottom which can empty a filled tank in 40 minutes. If the tank is empty initially, how much time will both the

taps take to fill the tank (leakage is still there)?

- a. 8 minutes
- b. 7 minutes
- c. 10 minutes
- d. 15 minutes
- e. None of these

Q9. An urn contains 9 blue, 7 white and 4 black balls. If 2 balls are drawn at random, then what is the probability that only one ball is white?

- a. 71/190
- b. 121/190
- c. 91/190
- d. 93/190
- e. None of these

Q10. A train B speeding with 120 kmph crosses another train C running in the same direction, in 2 minutes. If the lengths of the trains B and C be 100 m and 200 m respectively, what is the speed (in kmph) of the train C?

- a. 111
- b. 123
- c. 127
- d. 129

Q11. A merchant has 1000 kg of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity (in kg.) sold at 18% profit is:

- a. 560
- b. 600
- c. 400
- d. 640

Q12. A sum of money becomes Rs. 13, 380 after 3 years and Rs. 20, 070 after 6 years on compound interest. The sum (in Rupees) is:

- a. 8800
- b. 8890
- c. 8920
- d. 9040

Q13. A rectangular carpet has an area of 120 sq meters and a perimeter of 46 meters. The length of its diagonal (in meters) is:

- a. 11
- b. 13
- c. 15
- d. 17

Q14. Three numbers are in the ratio of 3:4:5 respectively. If the sum of the first and third numbers is more than the second number by 52, then which will be the largest number?

- a. 65
- b. 52
- c. 79
- d. 63
- e. None of these

Q15. The compound interest on a certain amount for 2 years at the rate of 8% p.a. is Rs.312. What will be the simple interest on the same amount and at the same rate and same time?

- a. Rs. 349.92
- b. Rs. 300
- c. Rs. 358.92
- d. Rs. 400
- e. None of these

Q16. The length of a rectangle exceeds its breadth by 7cm. If the length and breadth is decreased by 4cm and 3cm, then the area of the new rectangle is 44sq.cm less than that of the original. What will be the perimeter of the original rectangle?

- a. 45 cm

- b. 40 cm
- c. 50 cm
- d. 60 cm
- e. None of these

Q17. In a fraction, the numerator is three fourth of the denominator. If 2 is added to the numerator and the denominator each, then the resultant fraction will be $\frac{14}{11}$. What was the original fraction?

- a. $\frac{5}{18}$
- b. $\frac{6}{13}$
- c. $\frac{13}{6}$
- d. $\frac{7}{12}$
- e. None of these

Q18. Father is aged three times more than his son Ronit. After 8 years, he would be two and a half times of Ronit's age. After further 8 years how many times would he be of Ronit's age?

- a. $2\frac{1}{5}$ times
- b. $2\frac{1}{2}$ times
- c. $2\frac{1}{3}$ times
- d. 3 times

Q19. If a man walk at 3 kmph, he miss the bus by 2 minutes. If, however, he walks at 4 kmph, then reach the station 2 minutes before the arrival of the bus. How far does he walk to reach the station?

- a. $\frac{3}{4}$ km
- b. $\frac{7}{8}$ km
- c. $\frac{4}{9}$ km
- d. $\frac{4}{5}$ km

Q20. Two taps can fill a tank in 18 and 24 min respectively. When both the taps are opened find when the first tap be turned off so that the tank may be filled in 12 min.

- a. 6 min
- b. 7min
- c. 8 min
- d. 9min

Q21. A plant reproduces at the rate of 25% every 12 min. In approximately what time will it triple itself?

- a. 90 min
- b. 75min
- c. 60 min
- d. 40 min

Q22. A square garden has fourteen posts along each side at equal interval. Find how many posts are there in all four sides:

- a. 56
- b. 52
- c. 44
- d. 60

Q23. What is the maximum number of half-pint bottles of cream that can be filled with a 4-gallon can of cream?

(2 pt =1 qt and 4 qt =1 gal)

- a. 16
- b. 24
- c. 30
- d. 64

Q24. The number of times a bucket of capacity 4 litres to be used to fill up a tank is less than the number of times another bucket of capacity 3 litres used for the same purpose by 4. What is the capacity of the tank?

- a. 36 litres
- b. 25 litres
- c. 48 litres
- d. 52 litres

Q25. Last year Mr Basu bought two scooters. This year he sold both of them for Rs 30,000 each. On one, he earned 20% profit, and on the other he made a 20% loss. What was his net profit or loss?

- a. He gained less than Rs 2000
- b. He gained more than Rs 2000
- c. He lost less than Rs 2000
- d. He lost more than Rs 2000

Directions for Questions Q26&Q27:
Study the following arrangement carefully and answer the questions given below:

S M P 6 3 * \$ A 4 L N @ Z 5 # V U 7 E 8 B
% T Y X G 2 I & 9 W

Q26. Which of the following is the third to the left of the fifteenth from the left end?

- a. @
- b. 5
- c. 7
- d. U
- e. None of these

Q27. What should come in place of the question mark (?) in the following series based on the above arrangement?

P M 3 A \$ L Z @ # ?

- a. U V E
- b. V # E
- c. 7 U 8
- d. V 7 #
- e. None of these

Q28. How many such pairs of letters are there in the word RELUCTANCE each of which has many letters between them in the word (in both forward and backward) as they have between them in the English alphabetical series?

- a. 1
- b. 2
- c. 3
- d. 4
- e. None of these

Q29. In a certain code FIGHT is coded as GJFIU and WRITE is coded as XSHUF. Then how will JUDGE be coded in the same code?

- a. KVCHF
- b. HFEKV
- c. KVEHF
- d. VKCFH
- e. KVDHF

Q30. If 2 is subtracted from each even digit and 1 is added to each odd digit in the number 8567284. Which of the following will be the sum of the third digit from the left and second digit from the right of the new number thus formed?

- a. 10
- b. 8
- c. 4
- d. 6
- e. 16

Q31. If it is possible to make only one meaningful word with the Third, Seventh, Eighth and Tenth letters of the word COMPATIBILITY, which of the following would be the last letter of that word? If no such word can be made, give 'X' as your answer and if more than one such word can be formed, give your answer as 'Y'.

- a. I
- b. B
- c. L
- d. X
- e. Y

Q32. Four of the following five are alike in a certain way and so form a group. Which is the one that does not belong to that group?

- a. Stem
- b. Tree
- c. Root
- d. Branch
- e. Leaf

Q33. Each vowel of the word ADJECTIVE is substituted with the next letter of the English alphabetical series and each consonant is substituted with the letter preceding it. How many vowels are present in the new arrangement?

- a. None
- b. One
- c. Two
- d. Three
- e. None of these

Directions for Questions Q34&Q35:
Study the following information carefully and answer the questions carefully

A, B, C, D, E, G and H are sitting in a circle facing at the centre. D is second to the left of F and third to the right of H. A is second to the right of F and an immediate neighbour of H. C is second to the right of B and F is

third to the right of B.G is not an immediate neighbour of F.

Q34. Who is fourth to the right of B?

- a. E
- b. C
- c. A
- d. Data inadequate
- e. None of these

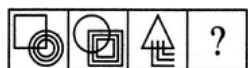
Q35. In which of the following pairs is the first person sitting to the immediate left to the second person?

- a. CD
- b. BG
- c. HA
- d. FC
- e. None of these

Directions for Questions Q36 & Q37:

There is a definite relationship between figures A and B. Establish a similar relationship between figures C and D by selecting a suitable figure

Q36.



(1) (2) (3) (4)



(A) (B) (C) (D) (E)

Q37.



(1) (2) (3) (4)



(A) (B) (C) (D) (E)

Q38. Statements:

- a. Most doctors are engineers.
- b. No engineer is a pilot
- c. All pilots are doctors

Conclusions:

- I. Some engineers are doctors.
- II. All doctors are pilots.
- III. No pilot is an engineer.
- IV. Some pilots are engineers.
- a. Only I follow
- b. Only II and III follow
- c. Only I and III follow
- d. Only either III or IV follows
- e. None of these

Q39. Statements:

- a. All files are folders.
- b. All folders are boxes.
- c. All boxes are drawers.

Conclusions:

- I. All folders are drawers.
- II. All boxes are files.
- III. All files are drawers.

IV. All drawers are folders.

- a. Only I and II follow
- b. Only I and III follow
- c. Only II and III follow
- d. All follow
- e. None of these

Q40. At what time between 3 and 4 O' clock is the angle between the hands of watch one-third of a right angle?

- a. 10 10/11 min past 3
- b. 21 9/11 min past 3
- c. Either (1) or (2)
- d. Both (1) & (2)
- e. None of these

13.4 Exercise

Q1. A wizard named Harry says "I am only three times my son's age. My father is 40 years more than twice my age. Together the three of us are a mere 1240 years old." How old is Harry?

- a. 360 yrs
- b. 340 yrs
- c. 680 yrs
- d. 36 yrs

Q2. There are 3 ants at 3 corners of a triangle; they randomly start moving towards another corner. What is the probability that they don't collide?

- a. 1/6
- b. 1/2
- c. 1/4
- d. 1/8

Q3. If 3 monkeys take 7 minutes to eat 3 bananas, what time will 10 monkeys take to eat 10 bananas?

- a. 8
- b. 9
- c. 10
- d. 7

Q4. How many words can be formed by re-arranging the letters of the word PROBLEMS such that P and S occupy the first and last position respectively?

- a. $8! / 2!$
- b. $8! - 2!$
- c. $6!$
- d. $8! - 2 \times 7!$

Q5. Assume that there are n people in the room. Ignoring leap years, what is the probability that no one else in the room shares your birthday?

a. $\frac{{}^{365}C_n}{{}^{365}P_n}$

b. $\frac{n}{365^n}$

c. $\frac{{}^{365}C_n}{365^n}$

d. $\frac{{}^{365}P_n}{365^n}$

Q6. You meet a stranger on the street, and ask how many children he has. He truthfully says two. You ask "Is the older one a girl?" He truthfully says yes. What is the probability that both children are girls?

a. $\frac{1}{2}$

b. $\frac{2}{3}$

c. $\frac{1}{3}$

d. None of the above

Q7. The letters A, B, C, D, E each stands for 1,2,3,4 and 5 but not necessarily in that order. A is odd, B is neither 4 nor 5, C is 1, D is either 4 or 5 and E is neither 2,3 nor 4. The correct order of the digits according to the correct order of the English alphabet is?

a. 32145

b. 52341

c. 35142

d. None of these

Q8. Complete the series: 0, 7, 26, 63, 124.

a. 215

b. 217

c. 115

d. Cannot be determined

Q9. There are some chicken in poultry. They are fed with corn. One sack of corn will come for 9 days. The farmer decides to sell some chicken and wanted to hold 12 chickens with him. He cuts the feed by 10% and sack of corn comes for 30 days. So initially how many chickens are there?

a. 35

b. 36

c. 37

d. 38

Q10. Buvika walks up a staircase. She finds that if she walks up 25 steps, she requires 11 seconds more to reach the top and if she walks up 40 steps she needs only 8 seconds more to reach the top. Find the number of steps in the stair case.

a. 100

- b. 125
- c. 75
- d. 80

Q11. A rectangle is 14cm long and 10 cm wide. If the length is reduced by x cms and its width is increased by x cms so as to make it a square then its area changes by:

- a. 4
- b. 144
- c. 12
- d. 2

Q12. A train running at the speed of 60 km/hr crosses a pole in 9 seconds. What is the length of the train?

- a. 120 m
- b. 180 m
- c. 324 m
- d. 150 m

Q13. If the radius of a circle is increased by 20 % then the area is increased by:

- a. 44 %
- b. 120 %
- c. 144 %
- d. 40 %

Q14. Three times the first of three consecutive odd integers is 3 more than twice the third. The third integer is:

- a. 9
- b. 11
- c. 13
- d. 15

Q15. There are 200 questions in a 3 hr examination. Among these questions are 50 mathematics problems. It is suggested that twice as much time be spent on each math problem as for each other question. How many minutes should be spent on mathematics problems?

- a. 36
- b. 72
- c. 60
- d. 100

Q16. In a group of 15, 7 have studied Latin, 8 have studied Greek, and 3 have not studied either. How many of these studied both Latin and Greek

- a. 0
- b. 3
- c. 4
- d. 5

Q17. If a and b are positive integers and $(a-b)/3.5 = 4/7$, then

- a. $b < a$
- b. $b > a$
- c. $b = a$
- d. $b \geq a$

Q18. In June a baseball team that played 60 games had won 30% of its game played. After a phenomenal winning streak this team raised its average to 50% .How many games must the team have won in a row to attain this average?

- a. 12
- b. 20
- c. 24
- d. 30

Q19. A company contracts to paint 3 houses. Mr. A can paint a house in 6 days while Mr. B would take 8 days and Mr. C 12 days. After 8 days Mr. A goes on vacation and Mr. B begins to work for a period of 6 days. Then Mr. C joins the works. How many days will it take Mr. C to complete the contract?

- a. 7
- b. 8
- c. 11

d. 12

Q20. Two hours after a freight train leaves Delhi a passenger train leaves the same station travelling in the same direction at an average speed of 16 km/hr. After travelling 4 hrs the passenger train overtakes the freight train. The average speed of the freight train was?

- a. 30
- b. 40
- c. 58
- d. 60

Q21. A man spends half of his salary on house hold expenses, $1/4^{\text{th}}$ for rent, $1/5^{\text{th}}$ for travel expenses, the man deposits the rest in a bank. If his monthly deposits in the bank amount to Rs 50. What is his monthly salary?

- a. Rs 500
- b. Rs 1500
- c. Rs 1000
- d. Rs 900

Q22. The ratio of marks obtained by Vinod and Vasu is 6:5. If the combined average of their percentage is 68.75 and their sum of

the marks is 275, the total marks for which exam was conducted is?

- a. 500*
- b. 200*
- c. 400*
- d. 150*

Q23. *A zookeeper counted heads of the animals in a zoo and found it to be 80. When he counted the legs of the animals he found it to be 260. if the zoo had either pigeons or horses, how many horses are there in the zoo?*

- a. 30*
- b. 40*
- c. 50*
- d. 60*

Q24. *If each child is given 10 sweets there are 3 sweets left over. But if each is given 11, then the number of sweets is 4 less. Find the number of sweets*

- a. 37*
- b. 57*
- c. 75*
- d. 73*

Q25. *A tap can fill the tank in 15 minutes and another can empty it in 8 minutes. If the tank is already half full and both the taps are opened together, the tank will be*

- a. filled in 12 mins*
- b. emptied in 12 mins*
- c. filled in 8 mins*
- d. emptied in 8 mins*

Q26. *Gold is 19 times as heavy as water and copper is 9 times as heavy as water. In what ratio should these be mixed to get an alloy 15 times as heavy as water?*

- a. 1:1*
- b. 2:3*
- c. 1:2*
- d. 3:2*

Q27. *A train 125m long passes a man, running at 5 kmph in the same direction in which the train is going in 10s. The speed of the train is*

- a. 45kmph*
- b. 50kmph*
- c. 54kmph*
- d. 55kmph*

Q28. The difference between the length and breadth of a rectangle is 23m. If its perimeter is 206m, then its area is:

- a. 1520sq.m
- b. 2420sq.m
- c. 2480 sq.m
- d. 2520 sq.m

Q29. A man walked diagonally across a square lot. Approximately, what was the percentage of distance saved by not walking along the edges?

- a. 20
- b. 24
- c. 30
- d. 33

Q30. If n is a positive integer, and $k+2 = 3^n$, which of the following could not be a value of k ?

- a. 1
- b. 4
- c. 7
- d. 25

Q31. Two students appeared at an examination. One of them secured 9 marks more than the other and his marks was 56%

of the sum of their marks. The marks obtained by them are

- a. 39, 30
- b. 41, 32
- c. 42, 33
- d. 43, 34

Q32. If the operation, $\#$ is defined by the equation $x\#y = 2x + y$, what is the value of a in $2\#a = a\#3$

- a. 0
- b. 1
- c. -1
- d. 4

Q33. A particular seafood restaurant serves dinner Tuesday through Sunday. The restaurant is closed on Monday. Five entrees- Squid, Crab, lobster, Salmon and tuna – are served each week according to the following restrictions:

- Crab is served on three days each week, but never on Friday.
- Lobster is served on one day each week.
- Salmon is served on three days each week, but never on consecutive days.
- Crab and squid are both served on Saturday and Sunday.

- Tuna is served five days each week.
- No more than three different entrees are served on any given day.

Which of the following is complete and accurate list of days on which crab and lobster may both be served?

- Tuesday, Thursday
- Tuesday, Wednesday, Thursday
- Monday, Tuesday, Wednesday
- Tuesday, Wednesday, Thursday, Friday

Directions for Questions Q 34 - Q38:

After months of talent searching for an administrative assistant to the president of the college the field of applicants has been narrowed down to 5--A, B, C, D, E .It was announced that the finalist would be chosen after a series of all-day group personal interviews were held. The examining committee agreed upon the following procedure

- The interviews will be held once a week
- 3 candidates will appear at any all-day interview session
- Each candidate will appear at least once
- If it becomes necessary to call applicants for additional interviews, no more 1 such applicant should be asked to appear the next week

V. Because of a detail in the written applications, It was agreed that whenever candidate B appears, A should also be present.

VI. Because of travel difficulties it was agreed that C will appear for only 1 interview.

Q34. At the first interview the following candidates appear A, B, D. Which of the following combinations can be called for the interview to be held next week?

- BCD
- CDE
- ABE
- ABC

Q35. Which of the following is a possible sequence of combinations for interviews in 2 successive weeks

- ABC: BDE
- ABD: ABE
- ADE: ABC
- BDE: ACD

Q36. If A, B and D appear for the interview and D is called for additional interview the following week, which 2 candidates may be asked to appear with D?

I. A

II. B

III. C

IV. E

a. I and II

b. I and III only

c. II and III only

d. III and IV only

Q37. Which of the following correctly state(s) the procedure followed by the search committee

I. After the second interview all applicants have appeared at least once

II. The committee sees each applicant a second time

III. If a third session, it is possible for all applicants to appear at least twice

a. I only

b. II only

c. III only

d. Both I and II

Directions for Questions Q38- Q40: In a certain society, there are two marriage groups, red and brown. No marriage is permitted within a group. On marriage, males become part of their wives groups; women remain in their own group. Children

belong to the same group as their parents. Widowers and divorced males revert to the group of their birth. Marriage to more than one person at the same time and marriage to a direct descendant are forbidden

Q38. A brown female could have had

I. A grandfather born Red

II. A grandmother born Red

III Two grandfathers born Brown

a. I only

b. III only

c. I, II and III

d. I and II only

Q39. A male born into the brown group may have

a. An uncle in either group

b. A brown daughter

c. A brown son

d. A son-in-law born into red group

Q40. If widowers and divorced males retained their group they had upon marrying which of the following would be permissible (Assume that no previous marriage occurred)

a. A woman marrying her dead sister's husband

- b. A woman marrying her divorced daughter's ex-husband
- c. A widower marrying his brother's daughter
- d. A woman marrying her mother's brother who is a widower.

Directions for Questions Q41-Q45: The office staff of XYZ Corporation presently consists of three book keepers--A, B, C and 5 secretaries D, E, F, G, H. The management is planning to open a new office in another city using 2 book keepers and 3 secretaries of the present staff. To do so they plan to separate certain individuals who don't function well together. The following guidelines were established to set up the new office

I. Book keepers A and C are constantly finding fault with one another and should not be sent together to the new office as a team

II. C and E function well alone but not as a team , they should be separated

III. D and G have not been on speaking terms and shouldn't go together

IV Since D and F have been competing for promotion they shouldn't be a team

Q41. A is to be moved as one of the book keepers, which of the following cannot be a possible working unit.

- a. ABDEH
- b. ABDGH
- c. ABEFH
- d. ABEGH

Q42. If C and F are moved to the new office, how many combinations are possible

- a. 1
- b. 2
- c. 3
- d 4

Q43. If C is sent to the new office, which member of the staff cannot go with C

- a. B
- b. D
- c .F
- d. G

Q44. Under the guidelines developed, which of the following must go to the new office

- a. B
- b. D
- c. E
- d. G

Q45. If D goes to the new office, which of the following is/are true

I.C cannot go

II.A cannot go

III.H must also go

a. I only

b .II only

c .I and II only

d .I and III only

Q46. If you had an infinite supply of water and a 5 quart and 3 quart pail, how would you measure exactly 4 quarts?

Q47. There are 4 ants at 4 corners of a square; they randomly start moving towards another corner. What is the probability that they don't collide?

Q48. What is the sum of the numbers from 1 to 1000?

13.5 Exercise

Q1. 6 equidistant vertical lines are drawn on a board. 6 equidistant horizontal lines are also drawn on the board cutting the 6 vertical lines, and the distance between any two consecutive horizontal lines is equal to that between any two consecutive vertical

lines. What is the maximum number of squares thus formed?

a. 34

b. 44

c. 46

d. None of the above

Q2. A person has to completely put each of the three liquids: 403 liters of petrol 465 liters of Diesel and 496 liters of Mobile oil in bottles of equal size without mixing any of the above 3 types of liquids such that each bottle is completely filled. What is the least possible number of bottles required?

a. 34

b. 44

c. 46

d. None of the above

Q3. The average salary of 100 employees in an office is Rs 16000 per month. The management decided to raise the salary of every employee by 5% but stop a transport allowance of Rs 800 per month which was paid earlier to every employee. What will be the new average monthly salary?

a. Rs 16000

b. Rs16500

c. Rs 16800

d. Data insufficient

Q4. *The average speed of a train in the onward journey is 25 % more than that of the return journey. The train halts for 1 hour on reaching the destination. The total time taken for the complete to and fro journey is 17 hours covering a distance of 800kms. The speed of the train in the onward journey is*

- a. 45 km/hr*
- b. 47.06 km/hr*
- c. 50 km/hr*
- d. 56.25 km/hr*

Q5. *Total time taken by a person in going to a place by walking and returning on cycle is 5 hrs 45 mins. He would have gained 2 hrs by cycling both ways. The time taken by him to walk both ways is*

- a. 6hrs 45 mins*
- b. 7 hrs 45 mins*
- c. 8 hrs 15 mins*
- d. 8 hrs 30 mins*

Q6. *A table has 3 drawers. It is known that one of the drawers contain 2 silver coins another contains 2 Gold coins and the 3rd one contains a silver and gold coin. One of*

the drawers is opened at random and a coin is drawn and it is found to be a silver coin. What is the probability that other coin in the drawer is a gold coin?

- a. 0.25*
- b. 1*
- c. 0.50*
- d. 0.60*

Q7. *Ten identical particles are moving randomly inside a closed box. What is the probability that at any given point of time all the 10 particles will be lying in the same half of the box?*

- a. $\frac{1}{2}$*
- b. $\frac{1}{5}$*
- c. $\frac{2}{9}$*
- d. $\frac{2}{11}$*

Q8. *In a tournament one of the participants was to play 1 match against each of the other participants. Three players fell ill after each of them had played 3 matches and had to leave the tournament. What was the total number of participants at the beginning, If the total number of matches played was 75?*

- a. 8*
- b. 10*
- c. 12*

d. 15

Q9. A, B, C, D, E, F & G are members of a family consisting of 4 adults and 3 children, two of whom F and G are girls. A & D are brothers and A is a doctor. E is an engineer married to one of the brothers and has 2 children. B is married to D and G is their child. Who is C?

- a. G's brother
- b. F's father
- c. E's father
- d. A's son

Q10. All the two-digit natural numbers are selected which have their unit's digit greater than their ten's digit. If all these numbers are written one after the other in a series, how many digits are there in the resulting number?

- a. 90
- b. 72
- c. 36
- d. None of these

Q11. A rectangular plot of area 900 sq. m is to be fenced: two adjoining sides with bricks and two others with wooden fence. One metre of the wooden fence costs Rs. 10 and

one metre of the brick fence costs Rs. 25. Sujit, the contractor, has been given Rs. 2,000 to complete the task. Sujit will be in

- a. Profit
- b. Loss
- c. No profit No loss
- d. Data insufficient

Q12. A circle of radius 6.5 cm is circumscribed around a right-angled triangle with the sides a, b and c cm. a, b and c are all natural numbers. What is the perimeter of the triangle?

- a. 30 cm
- b. 26 cm
- c. 28 cm
- d. Data insufficient

Q13. There is a clock hanging on the wall. The reflection of the clock in a mirror shows exactly the same time as the clock. How many times does this happen in 6 days?

- a. 11
- b. 12
- c. 22
- d. 24

Directions for Questions Q14&Q15: In a local petrol bunk, the bunk owner was adulterating the petrol with kerosene. He had two identical tanks – the first was full of pure petrol while the second was empty. First he transferred an arbitrary amount of petrol from the first tank into the second and then replaced the petrol removed from the first tank with kerosene. He then repeated this process one more time but this time he ensured that by the end of the process the second tank was exactly full.

Q14. Which of the following can be the concentration of petrol in the second tank?

- a. 50%
- b. 60%
- c. $200/3$ %
- d. 80%

Q15. If the concentration of petrol in the second tank is 75% and the cost price of kerosene is half that of petrol, then what is his net profit percentage on selling the contents of the second tank given that he claims to sell the petrol at a profit of 25%? a

- a. $300/7$ %
- b. $200/3$ %
- c. $250/3$ %
- d. 100%

Q16. In Mr. Mehta's family, there are one grandfather, one grandmother, two fathers, two mothers, one father – in – law, four children, three grandchildren, one brother, two sisters, two sons, two daughters, and one daughter – in – law. How many members are there in Mr. Mehta's family? Give minimal possible answer.

Q17. Every day Rakesh goes to his office from his suburban house. His driver Bahadur drops him at the office in the morning and picks him up in the evening. Every evening Rakesh reaches the office gate at 6 O'clock. Bahadur also reaches at the same time. One day his office gets over at 5 O'clock and Rakesh instead of waiting for the car, started walking towards home. Bahadur starts at normal time, picks him up on the way and takes him back to house, half an hour early. How much time did Rakesh walk?

- a. 15 minutes
- b. 30 minutes
- c. 45 minutes
- d. Data Insufficient

Q18. An escalator is descending at a constant speed. Ravi found that walking down 26 steps it took him 30 seconds to reach the bottom whereas walking down 34 steps it took him only 18 seconds to get to the bottom. If the time is measured from the moment the top step begins to descend to the time he steps off the last step at the bottom, find the number of steps visible on the escalator, when it is stationary.

- a. 46
- b. 36
- c. 56
- d. Cannot be determined.

Q19. India plays two matches each with Pakistan and Australia. In any match the probabilities of India getting 0, 1 and 2 points are 0.45, 0.05 and 0.50 respectively. Assume that the outcomes are independent, the probability of India getting at least 7 points is

- a. 0.04
- b. 0.0375
- c. 0.0875
- d. 0.0650

Q20. Sam and Mala have a conversation. Sam says I am certainly not over 40. Mala says I am 38 and you are at least 5 years older than me. Now, Sam says you are at least 39. All the statements by the two are false. How old are they really?

- a. Mala 38 yrs; Sam = 41 yrs.
- b. Mala 36 yrs; Sam = 40 yrs.
- c. Mala 38 yrs; Sam = 42 years
- d. Mala 37 yrs; Sam = 41 years

Q21. In a Bus station, there are two buses going. One from Platform 1 & other from platform 2, each is having a frequency of 10 minutes. The Platform 1 service starts at 4 o'clock and the platform 2 starts at 4.03A.M. A man goes to the station every day to catch the first bus that comes. What is the probability of the man catching the first bus?

- a. 0.7
- b. 0.8
- c. 0.75
- d. 0.85

Q22. Ramu and Raju went for a vacation. Unfortunately it rained for 18 days when they were there. But whenever it rained in the mornings, they had clear afternoons and

vice versa. In all they enjoyed 14 mornings and 16 afternoons. How many days did they stay there totally?

- a. 22.*
- b. 23*
- c. 27*
- d. 24*

Q23. *Bird is flying 60 km/hr between B to R which is 180 km distance. Two trains at B to R at 45 kmph. The distance traveled by the bird before it is killed?*

- a. 120 kms*
- b. 165 kms*
- c. 150 kms*
- d. 155 kms*

Q24. *A vender sold two things at same cost 12 Rs with one item at 25% profit and other at 20% loss, by this transaction he made profit or loss by how much?*

- a. Gain 60 paise*
- b. Loss 55 paise*
- c. Loss,60 paise*
- d. Gain 60 paise*

Q25. *A young girl counted in the following way on the fingers of her left hand. She started calling the thumb 1, the index finger 2, middle finger 3, ring finger 4, little finger*

5, then reversed direction, calling the ring finger 6, middle finger 7, index finger 8, thumb 9 and then back to the index finger for 10, middle finger for 11, and so on. She counted up to 1994. She ended on her

- a. Thumb*
- b. Index finger*
- c. Middle finger*
- d. Ring finger*

Directions for Questions Q26 - Q30:
Answer the questions based on the given information

There are one thousand lockers and one thousand students in a school. The principal asks the first student to go to each locker and open it. Then he asks the second student go to every second locker and close it. The third student goes to every third locker, and if it is closed, he opens it, and if it is open, he closes it. The fourth student does it to every fourth locker and so on. The process is completed with all the thousand students.

Q26. *How many lockers are closed at the end of the process?*

- a. 969*
- b. 968*
- c. 32*
- d. None of these*

Q27. How many students can go to only one locker?

- a. 500
- b. 499
- c. 501
- d. None of these

Q28. How many lockers are open after 970 students have done their job?

- a. 31
- b. 61
- c. 64
- d. Can't say

Q29. Which of the following locker is open at the end of the process?

- a. 2
- b. 4
- c. 6
- d. 8

Q30. How many students go to locker no. 840?

- a. 16
- b. 64
- c. 128
- d. 32

Directions for questions Q31&Q32:

Answer the questions based on the given information. In the following table:

A		D
B	G	E
C		F

The letters A, B, C, D, E, F and G represent distinct digits chosen from {0, 1, 2, 3, 4, 5, 6, 7, 8 and 9} such that $A*B*C = B*G*E = D*E*F$, where '*' means multiplication.

Q31. What does the letter G represent?

- a. 6
- b. 2
- c. 1
- d. Cannot be determined

Q32. What does the letter A represent if the letter B represent 4?

- a. 3
- b. 6
- c. 1
- d. Either (a) or (b)

Q33. Which three digits are not represented by any of the seven letters?

- a. 1, 3, 7
- b. 0, 1, 5

c. 0, 5, 7

f. 0, 3, 8

Q34. What is the value of $A*B*C$?

a. 60

b. 72

c. 36

d. Cannot be determined

Q35. What is the value of B if the letter D represents 8?

a. 4

b. 9

c. 6

d. Cannot be determined

Directions for Questions Q36 - Q39:

Two players A and B are playing a game of matchsticks. Any player can pick 1, 2 or 3 sticks in his turn. The player who picks the last stick always loses.

36. If there were 10 matchsticks in all and A starts the game, what should he pick in order to ensure a win always?

a. 1

b. 2

c. 3

d. He can never win

Q37. A starts the game and there are 10 matchsticks. What should he pick in order to ensure win, if the option of picking 3 sticks is not exercisable?

a. 1

b. 2

c. 3

d. He can never win

Q38. In question 37, if in his first chance, A picks up 2, what would B pick in order to win?

a. 1

b. 2

c. 3

d. Any of these

Q39. Which of the following is always true, if there were 16 matchsticks and the rule of the game was that anybody picking the last one wins?

a. The person who starts can never win

b. The person who starts will always pick 2

c. The person who plays second can never win.

d. The person who plays second always picks 1

Q40. Each of the alphabets given below takes different value among 2 to 8. Also, $G + C + E = A + D + G = F + B + D = 15$. Find the value of A?

- a. 5
- b. 6
- c. 7
- d. 8

Answers - Campus Recruitment Papers

13.1 Exercise

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12
Q13	Q14	Q15
Q16	Q17	Q18
Q19	Q20	Q21
Q22	Q23	Q24
Q25	Q26	Q27
Q28	Q29	Q30
Q31	Q32	Q33
Q34	Q35	Q36
Q37	Q38	Q39
Q40	Q41	Q42
Q43	Q44	Q45
Q46	Q47	Q48
Q49	Q50	Q51
Q52	Q53	Q54
Q55	Q56	Q57
Q58	Q59	Q60

13.2 Exercise

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12
Q13	Q14	Q15
Q16	Q17	Q18
Q19	Q20	Q21
Q22	Q23	Q24
Q25	Q26	Q27
Q28	Q29	Q30

13.3 Exercise

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12
Q13	Q14	Q15
Q16	Q17	Q18
Q19	Q20	Q21
Q22	Q23	Q24
Q25	Q26	Q27
Q28	Q29	Q30
Q31	Q32	Q33

Q34	Q35	Q36
Q37	Q38	Q39
Q40		

13.4 Exercise

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12
Q13	Q14	Q15
Q16	Q17	Q18
Q19	Q20	Q21
Q22	Q23	Q24
Q25	Q26	Q27
Q28	Q29	Q30
Q31	Q32	Q33
Q34	Q35	Q36
Q37	Q38	Q39
Q40	Q41	Q42
Q43	Q44	Q45
Q46	Q47	Q48
Q49	Q50	

13.5 Exercise

Q1	Q2	Q3
Q4	Q5	Q6
Q7	Q8	Q9
Q10	Q11	Q12
Q13	Q14	Q15
Q16	Q17	Q18
Q19	Q20	Q21
Q22	Q23	Q24
Q25	Q26	Q27
Q28	Q29	Q30
Q31	Q32	Q33
Q34	Q35	Q36
Q37	Q38	Q39
Q40		