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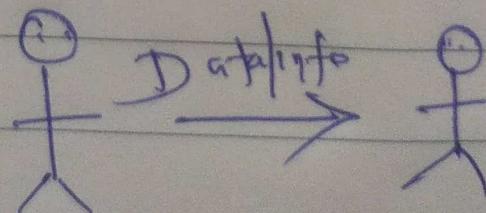
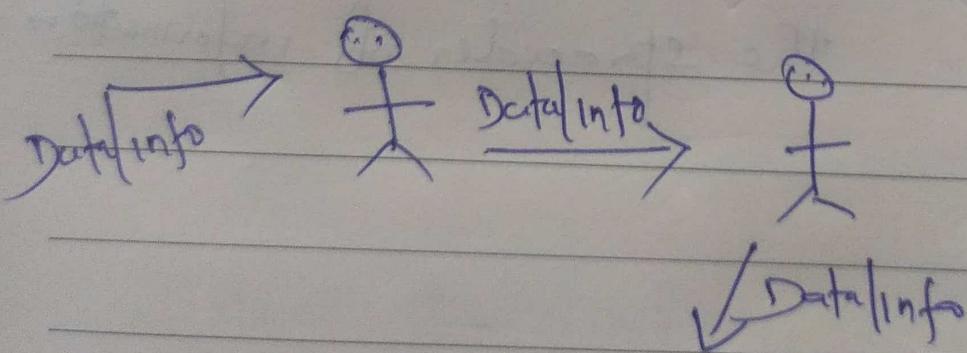
Lecture - 16

Contagion

Meaning: Spreading of diseases
in social networks:

Information spreading from
person to person.

Otherwise called diffusion/
information diffusion.



- Can be used to model infection, economic ~~spread~~

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- social networks are prominent tool for information diffusion in society.
- modelling and predicting information diffusion is very important.
- useful in predicting, marketing, advertising, political campaigns etc.
- tie strength is very important in information diffusion.

Social Contagion:

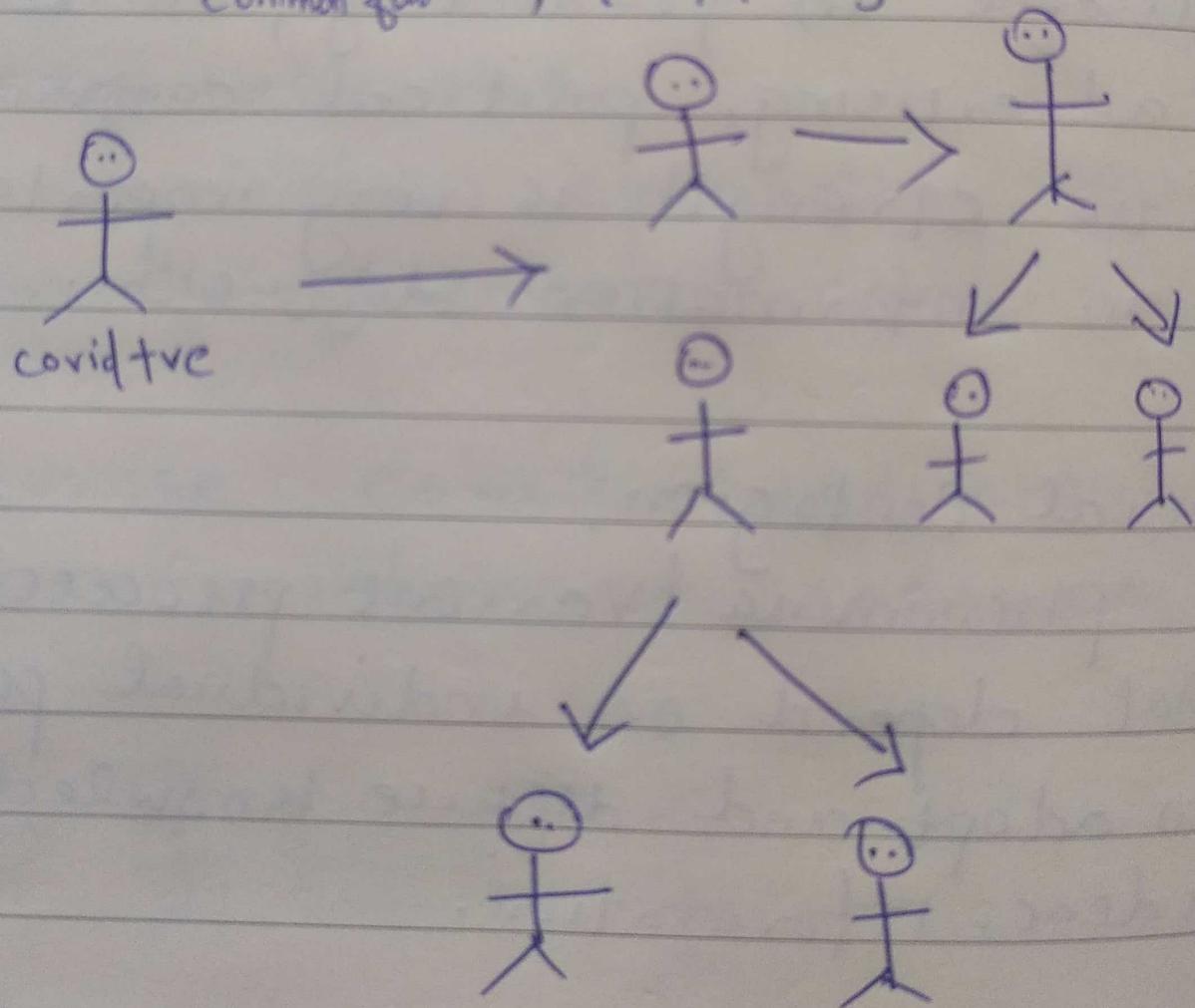
phenomena | various processes that depend on individual propensity to adopt and diffuse knowledge, ideas, information.

- The spread of behaviours, attitudes and affect through crowds and other types of social aggregates from one member to others.

- Simple Contagion

- Single exposure sufficient for transmission.

eg: disease, information etc
covid
common flu, traffic jam



- Complex contagion.

- if the behaviour is risky or technology requires coordination

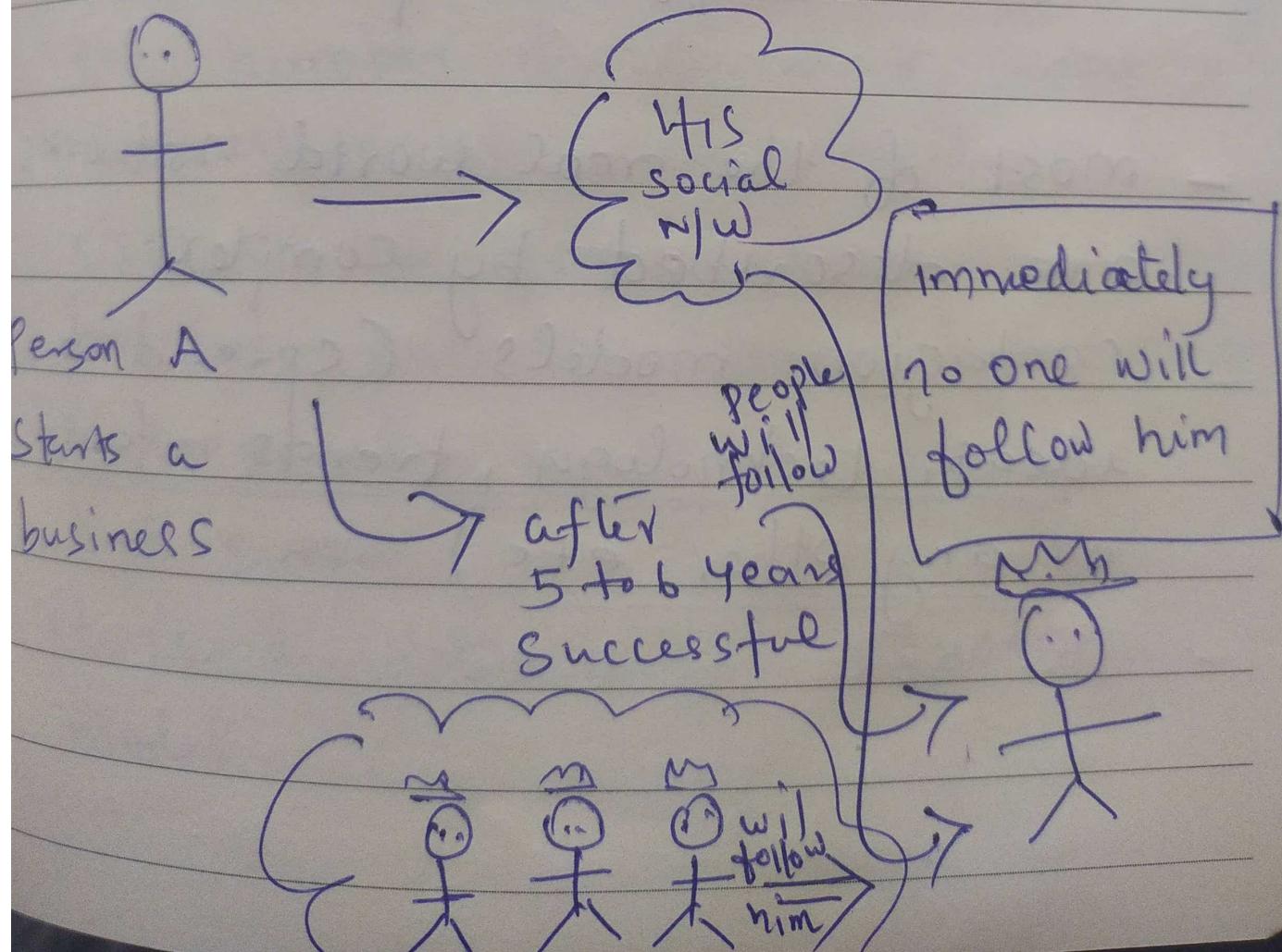
then

single exposure is not enough
ie; simple contagion not sufficient

There comes

- Complex contagion

- Repeated exposure is needed for transmission





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To analyze contagion we need to use some mathematical models.

- Simple contagion mainly inspired by disease spreading.
- Various epidemic models such as SIR, SIS are examples.
- Complex contagion can be modelled using threshold model, generalized epidemic models, diffusion percolation etc.
- Most of the real world networks are described by complex contagion models (spread of ideas, technology, trends in fabric or any other etc.)

simple epidemic model | simple Contagion model.

S-I-R model.

S - Susceptible

(people/nodes
capable of
spreading
a disease/info)

I - Infected

(people infected/
or done
with what they can)

R - Recovered

(People recovered
from an affect/
disease)

- SIR model capable of predicting the number of S, I, R. people (illness or information impact etc)
- SIR is a compartmental model means can be separated into groups.
- Introduced in 1927, popular and simple.
- It is a differential equation model.



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- There will be dependent variables and independent variables to model a disease situation
- Dependent variables.
 - people in each group
 - total population
(fraction of population in each category)
- Independent Variable.
 - Time t . (measured)
in days/hrs/etc

Other parameters.

- Susceptible individuals may ~~not~~ become infected.
- This measures rate of transfer from susceptible to infected.
- The rate of transfer depends on number of individuals in each compartments. and frequent contacts.

- The rate of contact which converts susceptible to infected is β .
- This β value may be reduced through quarantines, lockdowns etc.
- Next value, rate of recovery(γ)
(mortality)
(infected to recovered status)
- $R_0 \rightarrow$ Basic reproduction no.
the mean number of infections caused by a single infected individual over the course of their illness.
- R_0 is the ratio between β and γ
- $R_0 \rightarrow$ Contacts per infection
- a decrease in β reduces R_0 .
- If $R_0 > 1$, spread will be increased.
 $R_0 < 1$ spread will be reduced.

- Expressing S, I, & R values
in terms of t.

$$S(t) \rightarrow$$

$$I(t)$$

$$R(t)$$

- we know total population
 n is the sum of $S(t)$, $I(t)$ &
 $R(t)$

$$n = S(t) + I(t) + R(t)$$

To form differential equations.

$$\left. \begin{array}{l} \frac{dS}{dt} \\ \frac{dI}{dt} \\ \frac{dR}{dt} \end{array} \right\} \begin{array}{l} \text{Rate of change} \\ \text{of } S, I \text{ & } R \\ \text{w.r.t time } t \end{array}$$

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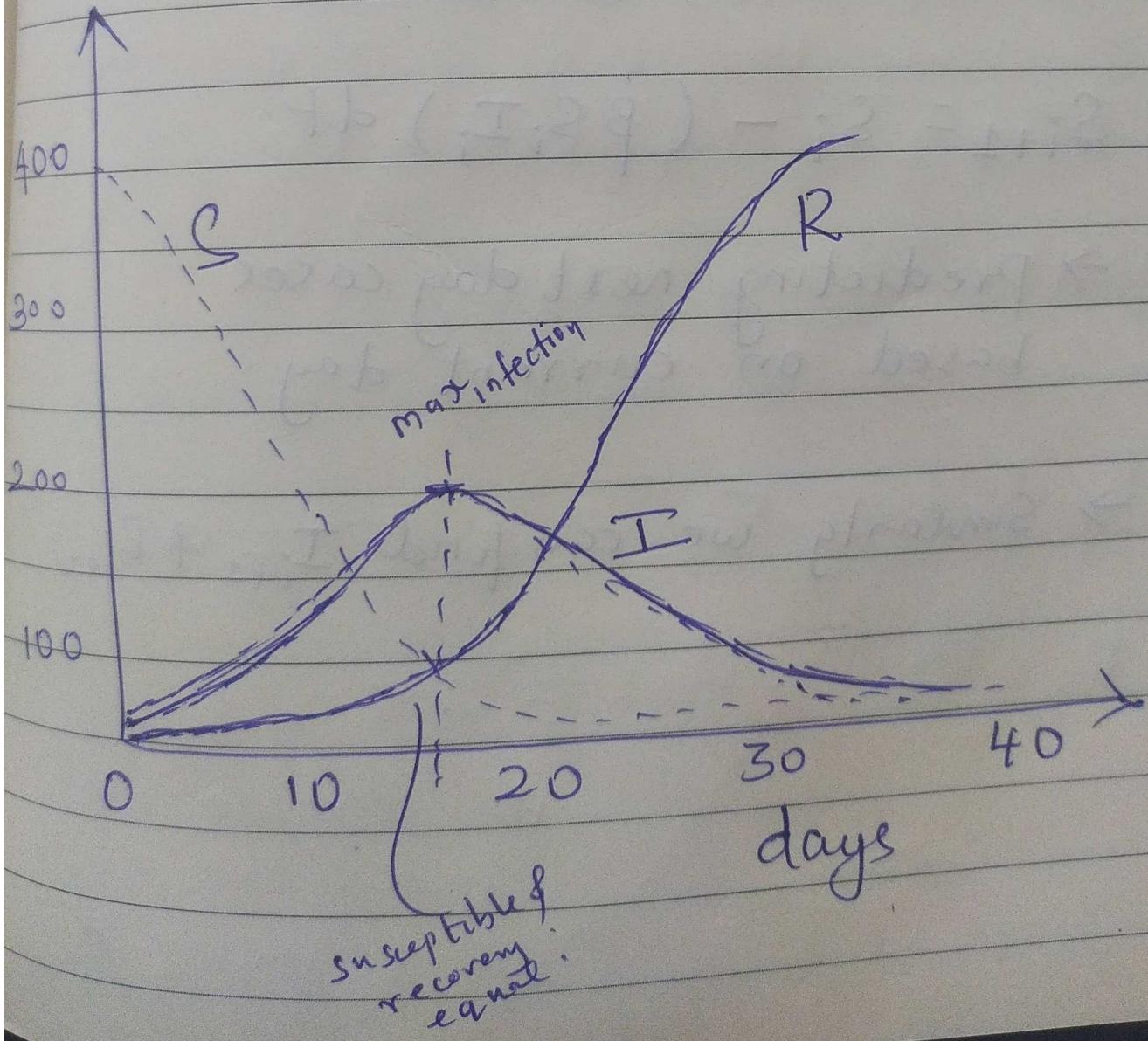
$$\frac{dS}{dt} = -\beta S I$$

(Negative because Susceptible changes to Infected)

$$\frac{dI}{dt} = \beta S I - \gamma I \quad (\text{Balanced})$$

$$\frac{dR}{dt} = \gamma I \quad (\text{Recovered})$$

$1/\gamma$ = days to recover





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$$dS = -(\beta SI) dt$$

$$dI = (\beta SI - \gamma I) dt$$

$$dR = \gamma I dt$$

$$\text{change in } dS = S_{i+1} - S_i$$

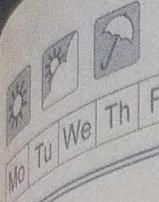
(i-day)

$$S_{i+1} - S_i = -(\beta S_i I_i) dt$$

$$S_{i+1} = S_i - (\beta S_i I_i) dt$$

→ Predicting next day cases
based on current day.

→ Similarly we can find I_{i+1} & R_{i+1}



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Limitations of SIR.

- Not including any other parameters other than S, I, R . (carrier, etc)
vaccinated,
- assumes homogeneous mixing of population ($S \times I$). In reality, human networks are ^{highly} local.



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Consider the statistics of an epidemic in country X.

$$S = 1000, I = 10, R = 0$$

The time t is measured in days.

Each infected person would make infecting contacts every two days.

The average recovery period is 3 days.

Calculate S , I & R on the 2nd day of epidemic.

Solution:

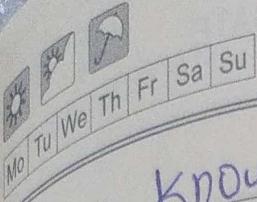
Day 0

$$S_0 = 1000 \quad I_0 = 10 \quad R_0 = 0$$

$$\beta = \frac{1}{2} = 0.5, \gamma = \frac{1}{3} = 0.33$$

$$R_0 = \beta/\gamma = 1.51 > 1$$

\therefore The epidemic will spread



We know that

$$S_{i+1} = S_i - (\beta S_i I_i) dt$$

$$I_{i+1} = I_i + (\beta S_i I_i - \gamma I_i) dt$$

$$R_{i+1} = R_i + \gamma I_i dt$$

Day 1 $dt = 1$ day (change in time)

$$S_1 = S_0 - (\beta S_0 I_0) dt$$

$$= 1000 - (0.5 \times 1000 \times 10)$$

$$= -4000$$

$$I_1 = I_0 + (\beta S_0 I_0 - \gamma I_0) dt$$

$$= 10 + (5000 - 3.3)$$

$$= 5006.7$$



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$$R_1 = R_0 + \gamma I_0 dt$$

$$= 0 + 3.3$$

$$= 3.3$$

Day 2

$$S_2 = S_1 - (\beta S_1 I_1) dt$$

$$= -4000 - (0.5 \times -4000 \times 5006.7)$$

$$= -4000 + 10,013,400$$

$$= 10,009,400$$

$$I_2 = I_1 + (\beta S_1 I_1 - \gamma I_1) dt$$

$$= 5006.7 + (-10,013,400 - 0.33 \times 5006.7)$$

$$= 5006.7 + (-10,013,400 - 1652.21)$$

$$= 5006.7 - 10,015,052.2$$

$$= -10,010,045.5$$

$$R_2 = R_1 + \gamma I_1$$

$$= 3.3 + 0.33 \times 5006.7$$

$$= 1655.511$$

~~1655.511~~