Graph Representation

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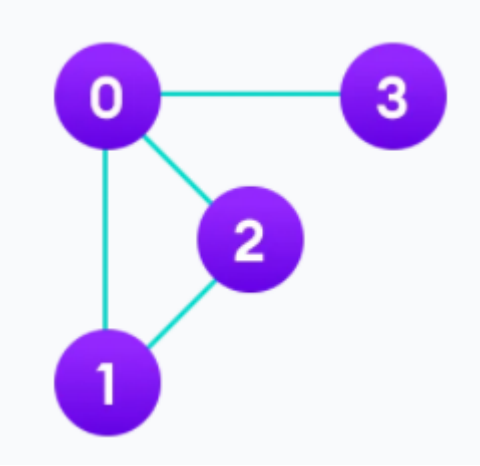
Graph Data Structure

- A data structure that consists of a **set of nodes** (vertices) and a **set of edges** that relate the nodes to each other.

Formal (Mathematical) Definition

- A Graph ***G*** is an ordered pair ***G = (V, E)***
  - ***V*** – set of vertices
  - ***E*** – set of edges
- G = (V, E)
  - V = {0, 1, 2, 3}
  - E = {{0,1}, {0,2}, {0,3}, {1,2}}

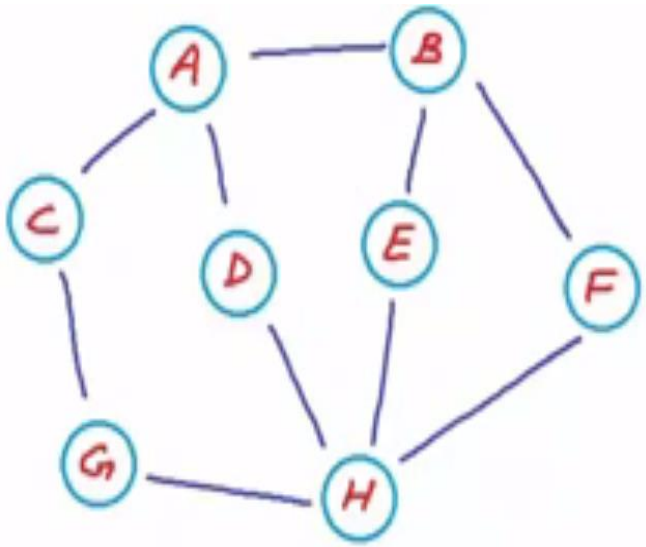


Graph Representation

- Graphs are commonly represented in 3 ways:
  - Edge List
  - Adjacency Matrix
  - Adjacency List

Edge List

- The ***edge list*** structure is the simplest, though not the most efficient, representation of a graph *G*.
  - All vertex objects are stored in an unordered list *V*,
  - All edge objects are stored in an unordered list *E*.



Vertex List

A
B
C
D
E
F
G
H

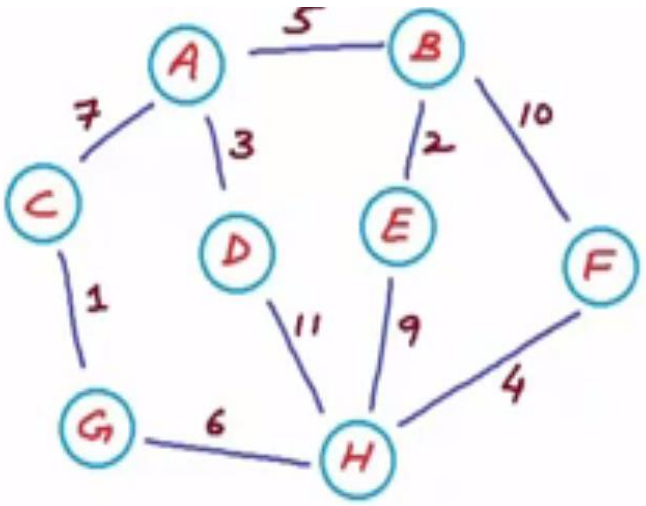
```
class Edge:
    def __init__(self, start, end):
        self.startVertex = start
        self.endVertex = end
```

Edge List

A	B
A	C
A	D
B	E
B	F
C	G
D	H
E	H
F	H
G	H

Edge List

```
class Edge:
    def __init__(self, start, end, weight):
        self.startVertex = start
        self.endVertex = end
        self.weight = weight
```



Vertex List

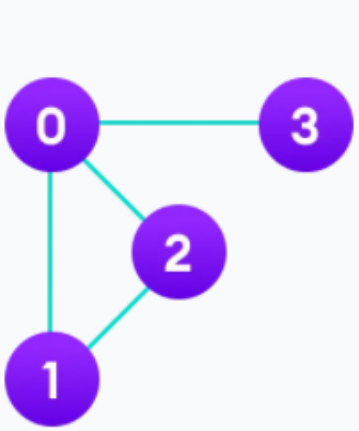
A
B
C
D
E
F
G
H

Edge List

A	B	5
A	C	7
A	D	3
B	E	2
B	F	10
C	G	1
D	H	11
E	H	9
F	H	4
G	H	6

Adjacency Matrix

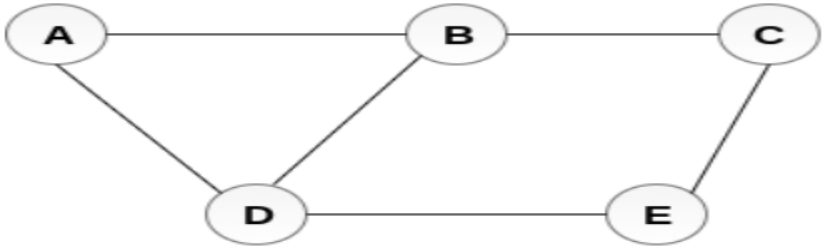
- An adjacency matrix is a 2D array of V x V vertices.
- Each row and column represent a vertex.
- If the value of any element **a[i][j] is 1**, it represents that there is **an edge connecting vertex i and vertex j**.



	0	1	2	3
0	0	1	1	1
1	1	0	1	0
2	1	1	0	0
3	1	0	0	0

Adjacency matrix from a graph

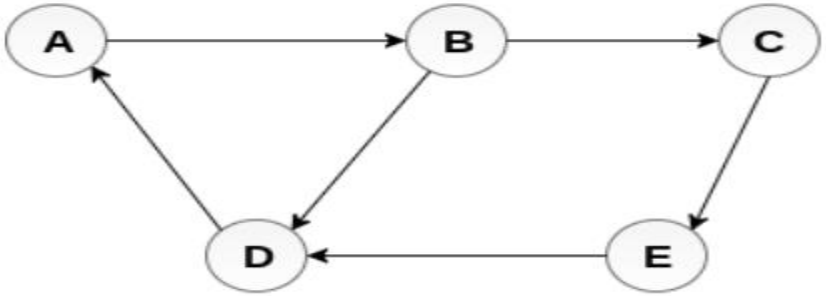
Adjacency Matrix for Directed and Undirected Graphs



Undirected Graph

	A	B	C	D	E
A	0	1	0	1	0
B	1	0	1	1	0
C	0	1	0	0	1
D	1	1	0	0	1
E	0	0	1	1	0

Adjacency Matrix



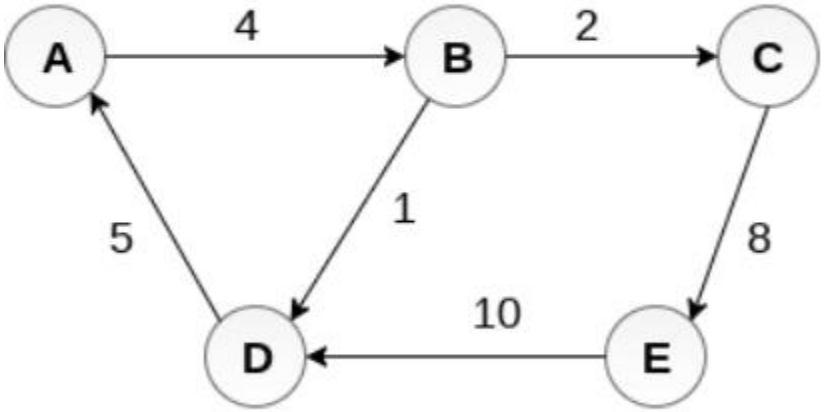
Directed Graph

	A	B	C	D	E
A	0	1	0	0	0
B	0	0	1	1	0
C	0	0	0	0	1
D	1	0	0	0	0
E	0	0	0	1	0

Adjacency Matrix

Adjacency Matrix for Weighted Graphs

- The Non- zero entries of the adjacency matrix are represented by the weight of respective edges.



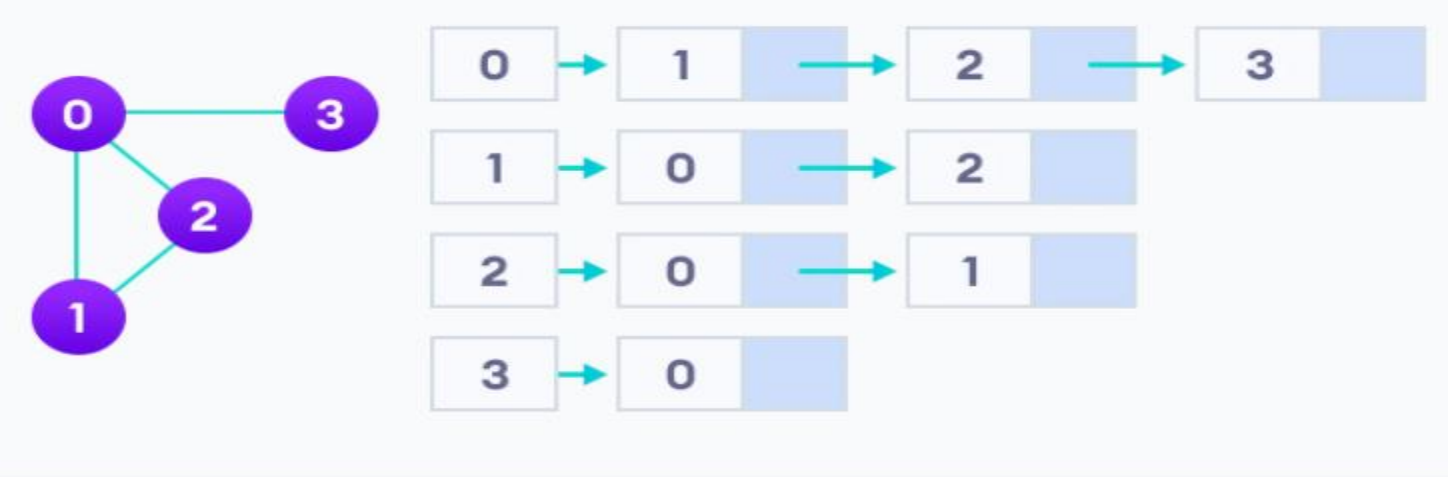
Weighted Directed Graph

	A	B	C	D	E
A	0	4	0	0	0
B	0	0	2	1	0
C	0	0	0	0	8
D	5	0	0	0	0
E	0	0	0	10	0

Adjacency Matrix

Adjacency List

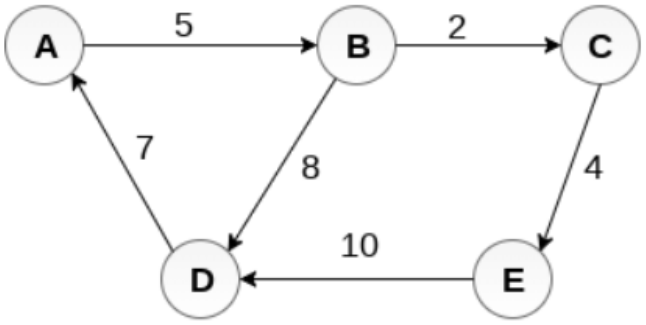
- An adjacency list represents a graph as an array of linked lists.
- The index of the array represents a vertex and each element in its linked list represents the other vertices that form an edge with the vertex.



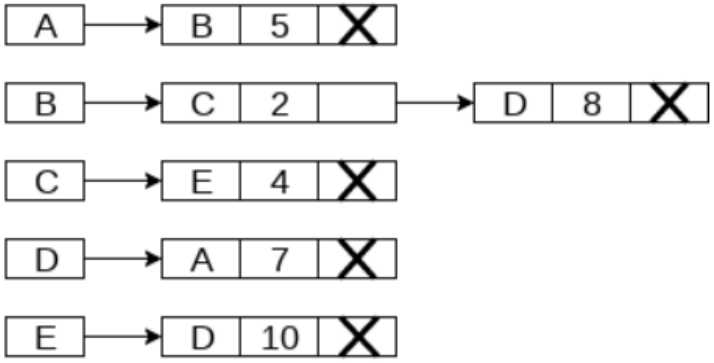
Adjacency List representation

Adjacency List for Weighted Graphs

- In the case of weighted directed graph, each node contains an extra field that is called the weight of the node.



Weighted Directed Graph



Adjacency List