Minimum Spanning Tree

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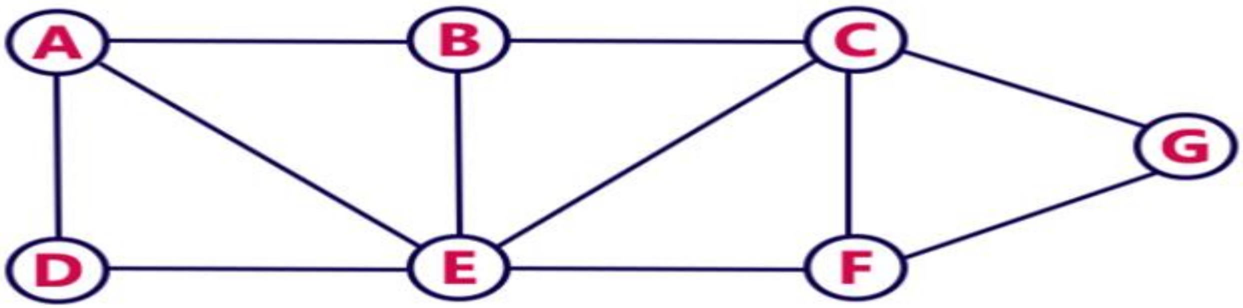
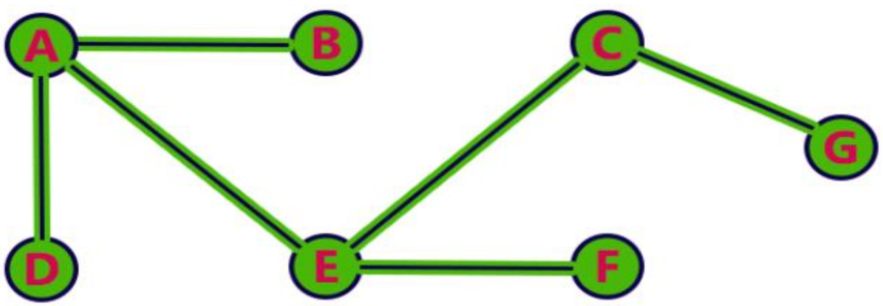
BSF & DFS

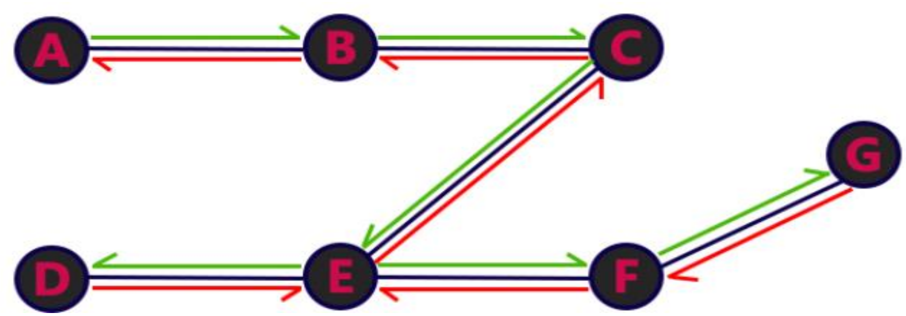


BFS Order: A D E B C F G



Time Complexity O(V + E)

DFS Order: A B C E D F G



Time Complexity O(V + E)

Final result of BFS and DFS traversal is a spanning tree

Spanning Tree

- A spanning tree is a **sub-graph** of an undirected connected graph, which includes **all the vertices** of the graph with **a minimum possible number of edges**.

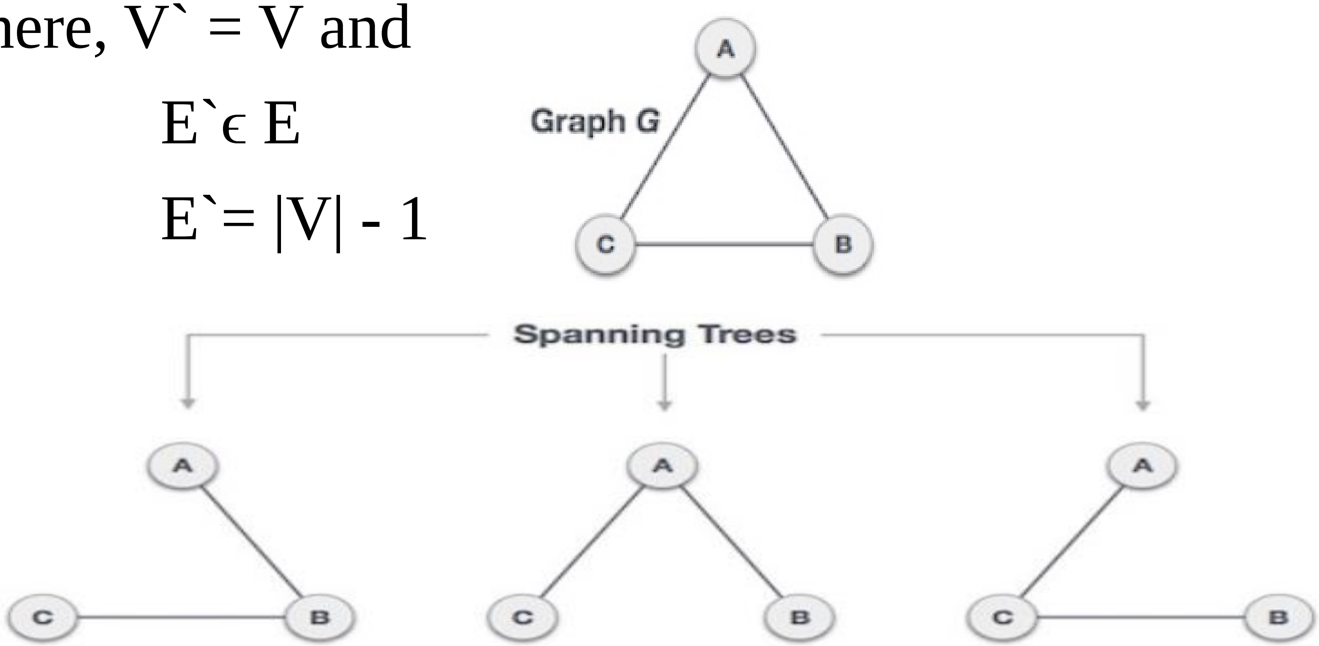
Let the graph **G(V, E)**

Then we can represent spanning tree as **G\`(V\`, E\`)**

where, V\` = V and

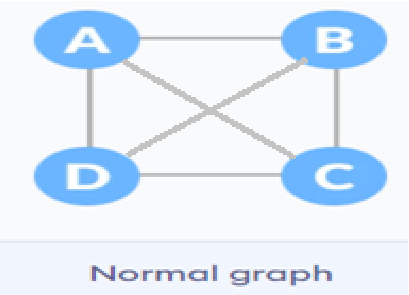
E\`ϵ E

E\`= |V| - 1

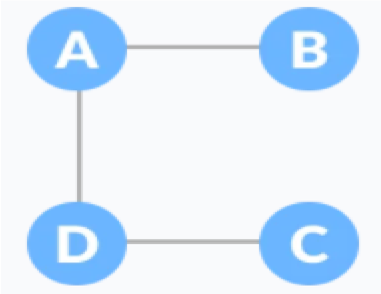


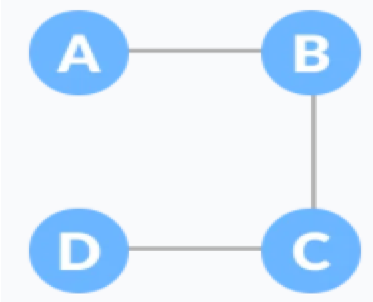
Example of a Spanning Tree

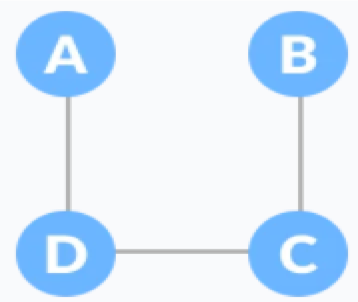
Let the original graph be

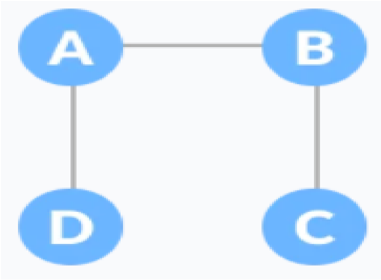


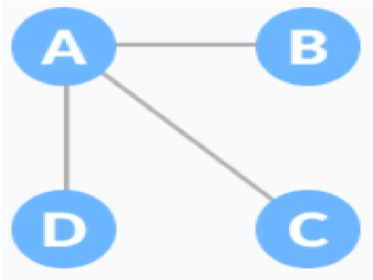
Some of the possible spanning trees

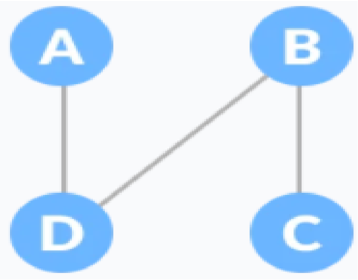












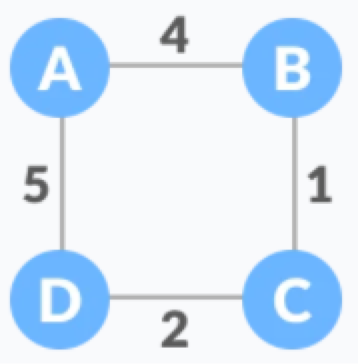
The total number of spanning trees with n vertices that can be created from a **complete graph** is equal to $\mathbf{n^{(n-2)}}$. Here, n = 4 ie, 16 spanning trees.

General Properties of Spanning Tree

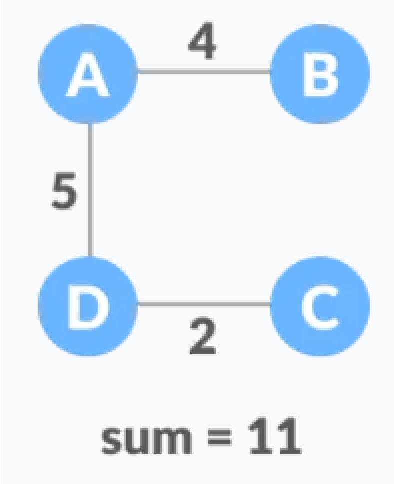
- A connected graph G can have more than one spanning tree.
- All possible spanning trees of graph G, have the same number of edges and vertices.
- The spanning tree does not have any cycle (loops).
- Removing one edge from the spanning tree will make the graph disconnected, i.e. the spanning tree is **minimally connected**.
- Adding one edge to the spanning tree will create a circuit or loop, i.e. the spanning tree is **maximally acyclic**.
- Spanning tree has **n-1** edges, where **n** is the number of nodes (vertices) of the graph.
- From a complete graph, by removing maximum **e - n + 1** edges, we can construct a spanning tree.
- A complete graph can have maximum $\mathbf{n^{n-2}}$ number of spanning trees.
- Every connected and undirected graph has atleast 1 spanning tree.

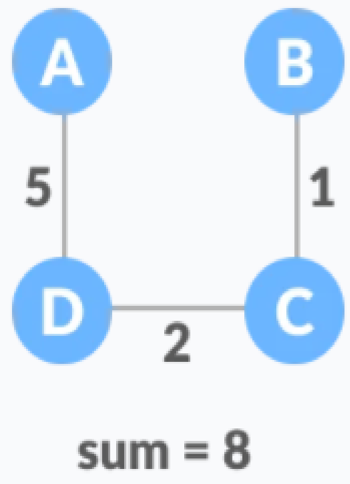
Minimum Spanning Tree (MST)

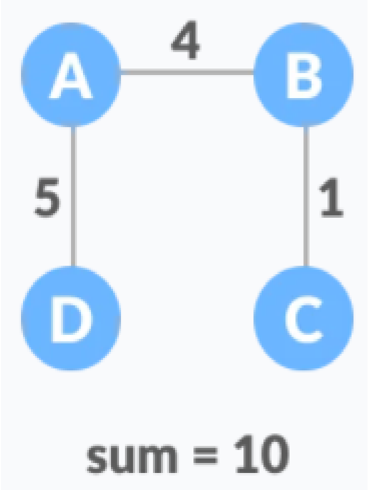
- A minimum spanning tree is a spanning tree in which the **sum of the weight of the edges is as minimum** as possible.

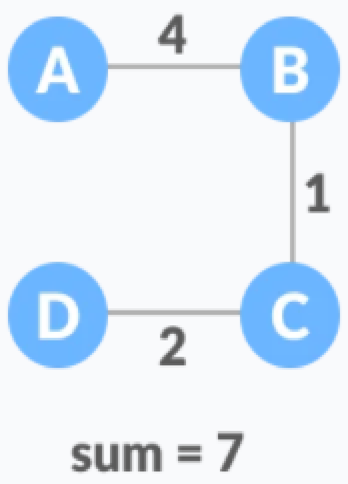


Minimum Spanning Tree



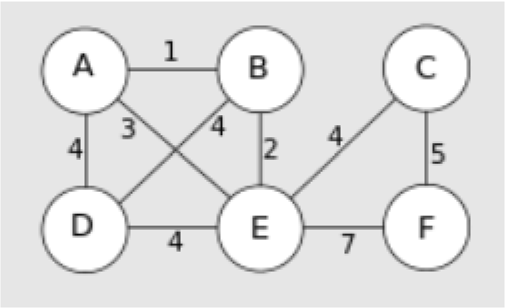


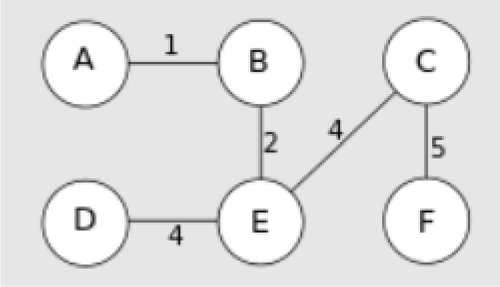




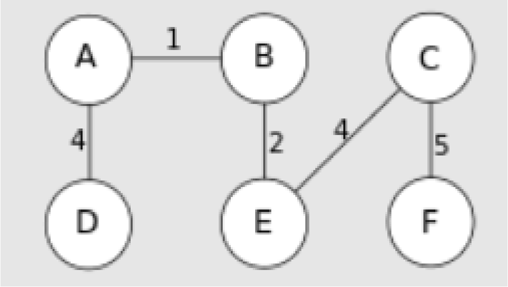
If each edge has distinct weights then there will be only one and unique MST

MST: Example





Sum = 16



Sum = 16

- A graph may can have more than one minimum spanning tree.

Minimum Spanning Tree Algorithms

- The minimum spanning tree from a graph is found using the following algorithms:
  - Prim's Algorithm
  - Kruskal's Algorithm
- These algorithms are falls under a class of greedy algorithms
- Prim's algorithm, treats the **nodes as a single tree and keeps on adding new nodes** to the spanning tree from the given graph.
- Kruskal's algorithm, treats the **graph as a forest** and every node it has as an individual tree and keeps on **adding the edges** with least cost.

Note: A greedy algorithm is an approach for solving a problem by selecting the best option available at the moment, without worrying about the future result it would bring.