19CSE337 Social Networking Security

Assignment – 1

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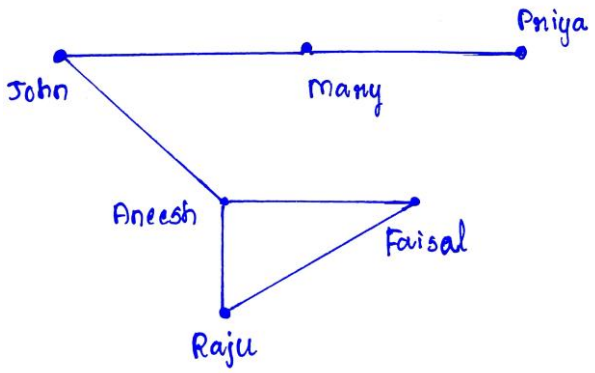
1 - Model the following friendship relation as an undirected graph.

- **John and Mary are friends.**
- **John is also a friend of Aneesh.**
- **Aneesh and Faisal are friends.**
- **Mary and Priya are close friends.**
- **Raju is a mutual friend of Aneesh and Faisal.**



- **A - Give the adjacency matrix and adjacency list notation of the relationship.**

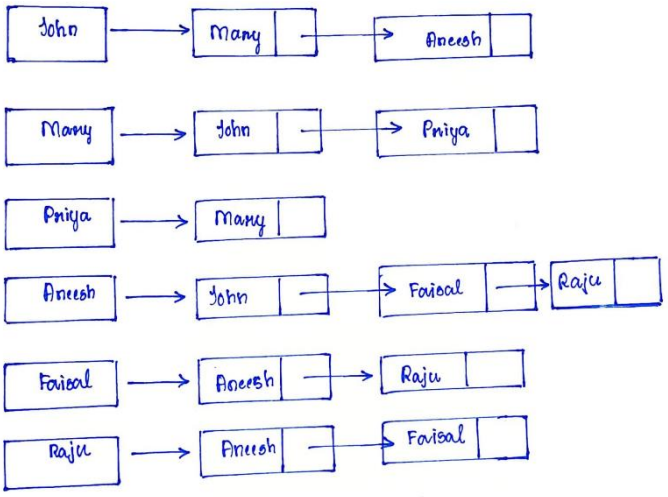
Adjacency Matrix

	John	Mary	Priya	Aneesh	Faisal	Raju
John	0	1	0	1	0	0
Mary	1	0	1	0	0	0
Priya	0	1	0	0	0	0
Aneesh	1	0	0	0	1	1
Faisal	0	0	0	1	0	1
Raju	0	0	0	1	1	0

```
1  print(nx.adjacency_matrix(Graph).todense())
```

```
[[0 1 0 1 0 0]
 [1 0 1 0 0 1]
 [0 1 0 0 0 1]
 [1 0 0 0 1 0]
 [0 0 0 1 0 0]
 [0 1 1 0 0 0]]
```

Adjacency List




```
1  Graph.adj
```

```
AdjacencyView({'John': {'Aneesh': {}, 'Mary': {}}, 'Aneesh': {'John': {}, 'Faisal': {},
```

AdjacencyView({

'John': {'Mary': {}, 'Aneesh': {}},

'Mary': {'John': {}, 'Priya': {}},

'Aneesh': {'John': {}, 'Faisal': {}, 'Raju': {}},

'Faisal': {'Aneesh': {}, 'Raju': {}},

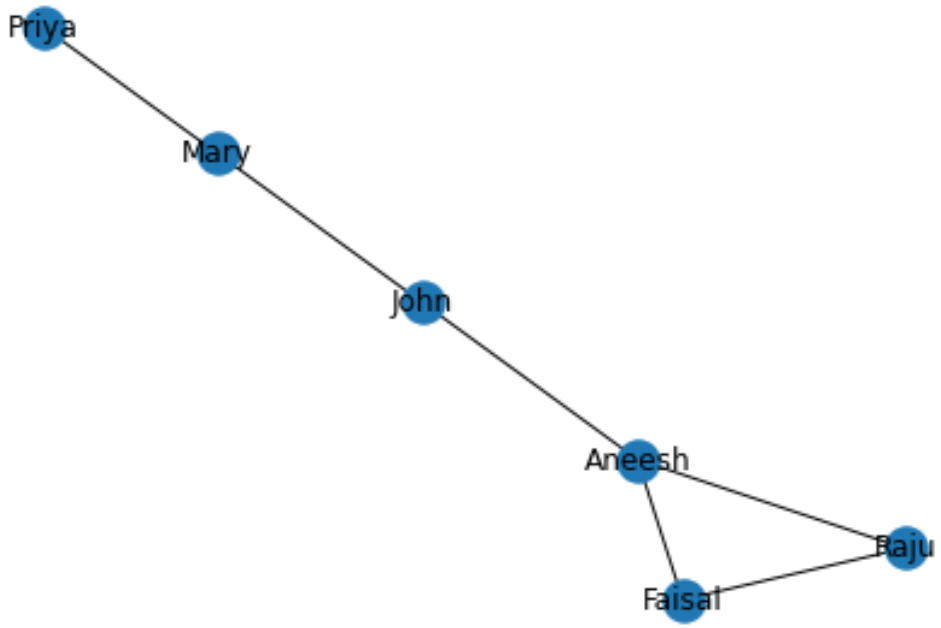
'Priya': {'Mary': {}},

'Raju': {'Faisal': {}, 'Aneesh': {}}

})

- B - Model the graph using NetworkX. Write the code and include a screenshot of the output in the assignment.

```
import networkx as
 nx;

Graph = nx.Graph()

Graph.add_nodes_from(['John', 'Aneesh', 'Faisal', 'Mary', 'Priya', 'Raju'])

Graph.add_edges_from([

        ('John', 'Aneesh'), ('John', 'Mary'),

        ('Mary', 'John'), ('Mary', 'Priya'),

        ('Priya', 'Mary'),

        ('Aneesh', 'John'), ('Aneesh', 'Faisal'),

        ('Faisal', 'Aneesh'), ('Faisal', 'Raju'),

        ('Raju', 'Aneesh'), ('Raju', 'Faisal')

        ])

nx.draw(Graph, with_labels = True)
```



- **C - Compute the Degree Centrality, Betweenness Centrality, Closeness Centrality, and eigenvalue centrality manually.**

Degree Centrality

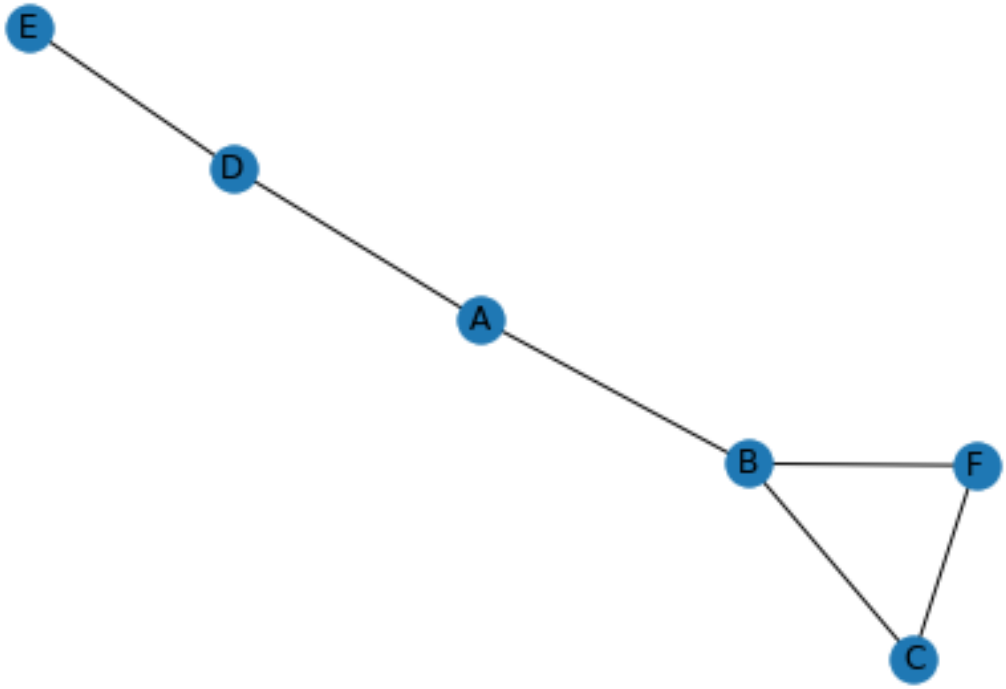
Node	Score	Standardized Score
John	2	2/5 = 0.4
Aneesh	3	3/5 = 0.6
Faisal	2	2/5 = 0.4
Mary	2	2/5 = 0.4
Priya	1	1/5 = 0.2
Raju	2	2/5 = 0.4

Betweenness Centrality

$$B(v) = \sum \left[\frac{\sigma_{st}(v)}{\sigma_{st}} \right]$$

Consider,

Node	Notation
John	A
Aneesh	B
Faisal	C
Mary	D
Priya	E
Raju	F



Betweenness Centrality of A - John

Pair	Shortest Distance?	Passes through A?	β(v)
BC	1	0	0
BD	1	1	1
BE	1	1	1
BF	1	0	0
CD	1	1	1
CE	1	1	1
CF	1	0	0
DE	1	0	0
DF	1	1	1
EF	1	1	1

- β(v) = 6
- **[β(v)/((N-1)(N-2)/2)]** = 6/((5)(4)/2) = 6/10 = 0.6

Betweenness Centrality of B - Aneesh

Pair	Shortest Distance?	Passes through B?	β(v)
AC	1	1	1
AD	1	0	0
AE	1	0	0
AF	1	1	1
CD	1	1	1

CE	1	1	1
CF	1	0	0
DE	1	0	0
DF	1	1	1
EF	1	1	1

- $\beta(v) = 6$
- **$[\beta(v)/((N-1)(N-2)/2)]$** $= 6/((5)(4)/2) = 6/10 = 0.6$

Betweenness Centrality of C - Faisal

Pair	Shortest Distance?	Passes through C?	β(v)
AB	1	0	0
AD	1	0	0
AE	1	0	0
AF	1	0	0
BD	1	0	0
BE	1	0	0
BF	1	0	0
DE	1	0	0
DF	1	0	0
EF	1	0	0

- $\beta(v) = 0$
- **$[\beta(v)/((N-1)(N-2)/2)]$** $= 0/((5)(4)/2) = 0/10 = 0$

Betweenness Centrality of D - Mary

Pair	Shortest Distance?	Passes through D?	β(v)
AB	1	0	0
AC	1	0	0
AE	1	1	1
AF	1	0	0
BC	1	0	0
BE	1	1	1
BF	1	0	0
CE	1	1	1
CF	1	0	0
EF	1	1	1

- $\beta(v) = 4$
- **$[\beta(v)/((N-1)(N-2)/2)]$** $= 4/((5)(4)/2) = 4/10 = 0.4$

Betweenness Centrality of E - Priya

Pair	Shortest Distance?	Passes through E?	β(v)
AB	1	0	0
AC	1	0	0
AD	1	0	0
AF	1	0	0
BC	1	0	0
BD	1	0	0
BF	1	0	0
CD	1	0	0
CF	1	0	0
DF	1	0	0

- $\beta(v) = 0$
- $\mathbf{[\beta(v)/((N-1)(N-2)/2)]} = 0/((5)(4)/2) = 0/10 = 0$

Betweenness Centrality of F - Raju

Pair	Shortest Distance?	Passes through F?	β(v)
AB	1	0	0
AC	1	0	0
AD	1	0	0
AE	1	0	0

- β(v) = 0

BC	1	0	0
BD	1	0	0
BE	1	0	0
CD	1	0	0
CE	1	0	0
DE	1	0	0

- β(v) = 0
- **[β(v)/((N-1)(N-2)/2)]** = 0/((5)(4)/2) = 0/10 = 0

Notation	Node	Betweenness Centrality
A	John	0.6
B	Aneesh	0.6
C	Faisal	0
D	Mary	0.4
E	Priya	0
F	Raju	0

Closeness Centrality

$$C_c(s) = \frac{n-1}{\sum_{t \in V} d(s,t)}$$

Node	A	B	C	D	E	F	Total Distance	Score	Standardized Score
A	0	1	2	1	2	2	8	1/8	5/8 = 0.625
B	1	0	1	2	3	1	8	1/8	5/8 = 0.625
C	2	1	0	3	4	1	11	1/11	5/11 = 0.4545
D	1	2	3	0	1	3	10	1/10	5/10 = 0.5
E	2	3	4	1	0	4	14	1/14	5/14 = 0.3571
F	2	1	1	3	4	0	11	1/11	5/11 = 0.4545

Eigenvector Centrality

$Av = \lambda v$

$Av - \lambda v = (A - \lambda I) \cdot v = 0$ Equation has a non zero solution if and only if $\det(A - \lambda I) = 0$

$$\det(A - \lambda I) = \begin{vmatrix} 0-\lambda & 1 & 0 & 1 & 0 & 0 \\ 1 & 0-\lambda & 0 & 0 & 0 & 0 \\ 0 & 1 & 0-\lambda & 0 & 0 & 0 \\ 1 & 0 & 0 & 0-\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0-\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0-\lambda \end{vmatrix}$$

$$= \lambda^6 - 6\lambda^4 - 2\lambda^3 + 8\lambda^2 + 4\lambda - 1$$

$$= (\lambda+1) \cdot (\lambda^5 - \lambda^4 - 5\lambda^3 + 3\lambda^2 + 5\lambda - 1)$$

$$= (\lambda+1)(\lambda+1)(\lambda^4 - 2\lambda^3 - 3\lambda^2 + 6\lambda - 1)$$

$$= (\lambda+1)(\lambda+1)(\lambda+1.775) \cdot (\lambda - 0.186)(\lambda - 1.360)$$

$$(\lambda - 2.228) = 0$$

① $\lambda_1 = -1$

② $\lambda_2 = -1.775$

③ $\lambda_3 = 0.186$

④ $\lambda_4 = 1.360$

⑤ $\lambda_5 = 2.228$

Principle Eigen value is 2.228

Eigen vector for $\lambda = 2.228$

$$A - \lambda_5 I = \begin{bmatrix} -2.228 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2.228 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2.228 & 0 & 0 & 0 \\ 1 & 0 & 0 & -2.228 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2.228 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2.228 \end{bmatrix}$$

Using gaussian elimination, solve the linear equations.

① $R_1/(-2.228) \rightarrow R_1$ ② $R_2 - 1 \cdot R_1 \rightarrow R_2$ ③ $R_4 - 1 \cdot R_1 \rightarrow R_4$

After several steps, we will get the final matrix.

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & -0.737 & 0 \\ 0 & 1 & 0 & 0 & 0 & -0.414 & 0 \\ 0 & 0 & 1 & 0 & 0 & -0.186 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1.228 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 0.737 x_6$

$x_2 = 0.414 x_6$

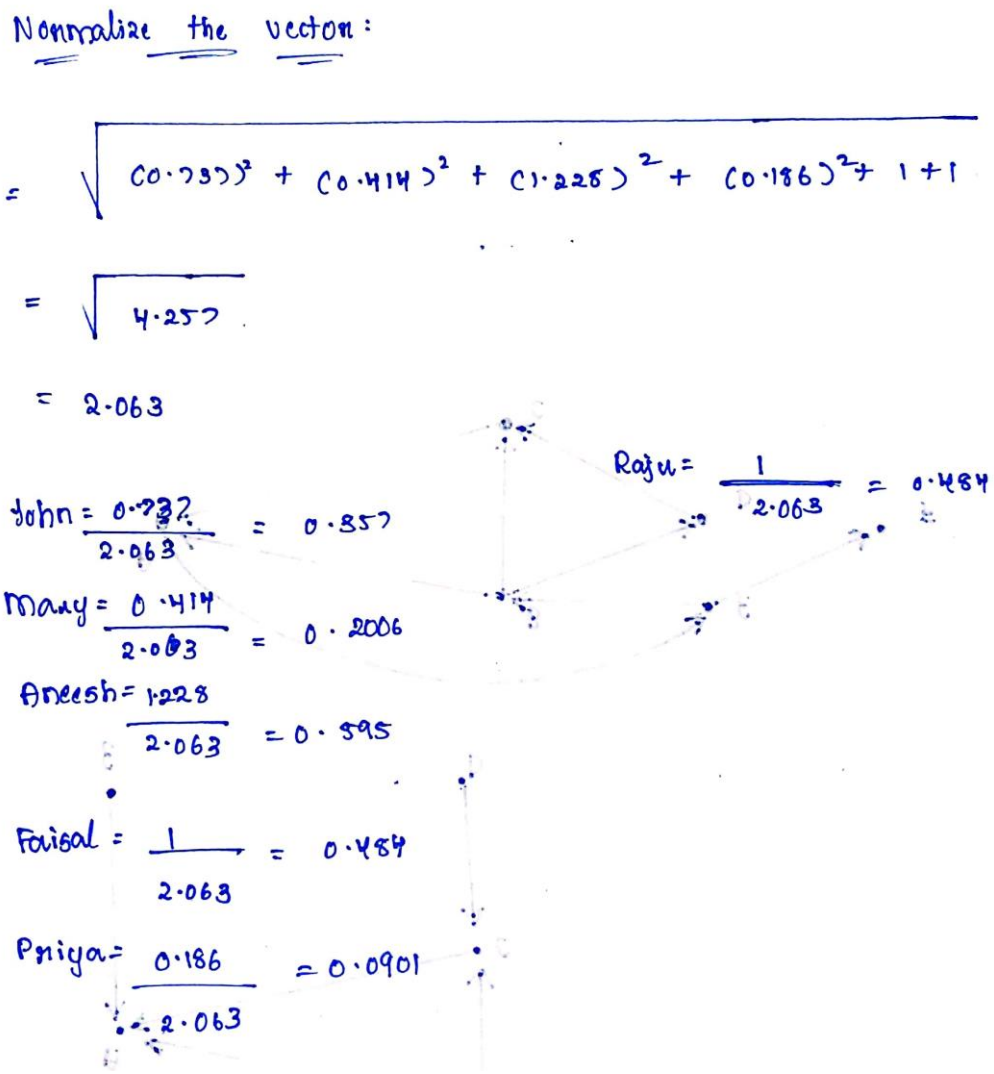
$x_3 = 0.186 x_6$

$x_4 = 1.228 x_6$

$x_5 = x_6$

$x_6 = x_6$

let $x_6 = 1 \rightarrow$ $V_6 = \begin{pmatrix} 0.737 \\ 0.414 \\ 0.186 \\ 1.228 \\ 1 \\ 1 \end{pmatrix}$

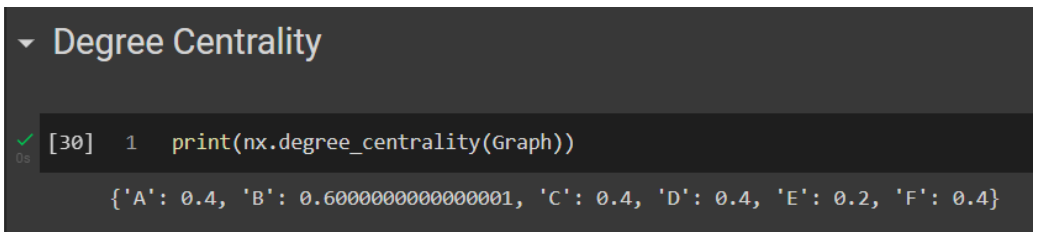


- **D - Compute question c using NetworkX and compare the values.**

  Note: Include code and a screenshot of the output obtained.

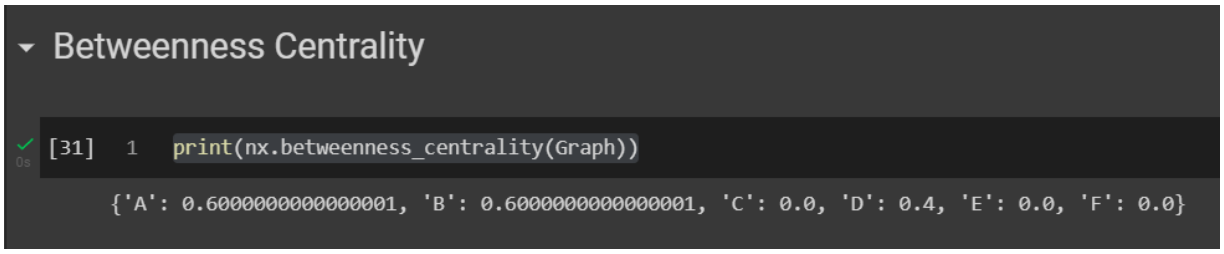
Degree Centrality

print(nx.degree_centrality(Graph))



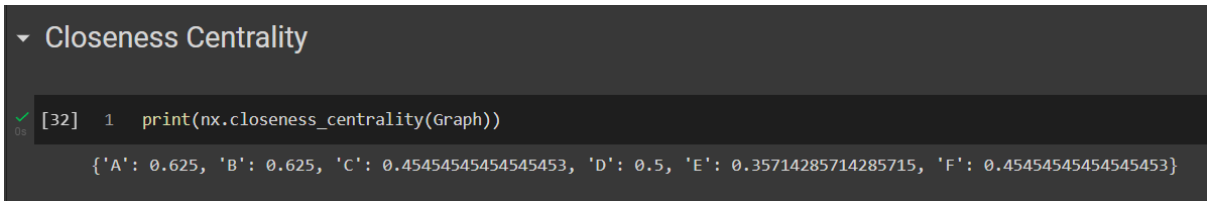
Betweenness Centrality

print(nx.betweenness_centrality(Graph))



Closeness Centrality

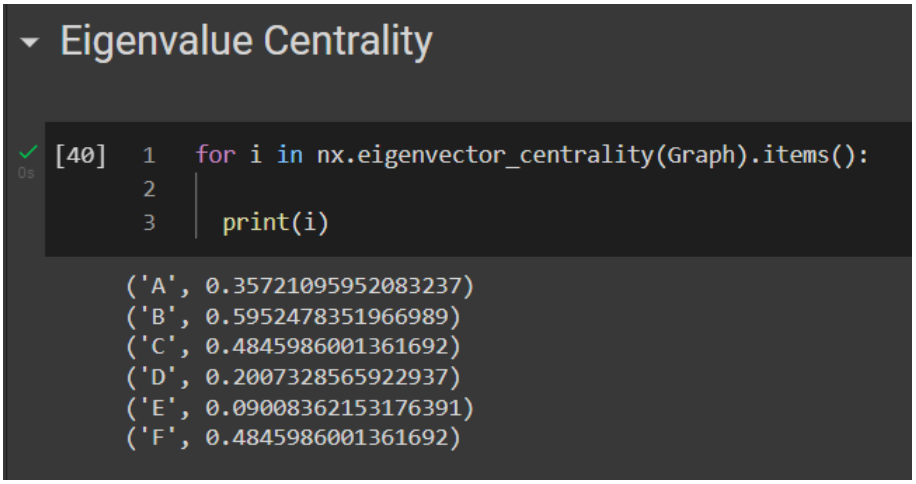
print(nx.closeness_centrality(Graph))



Eigenvalue Centrality

for i in nx.eigenvector_centrality(Graph).items():

  print(i)



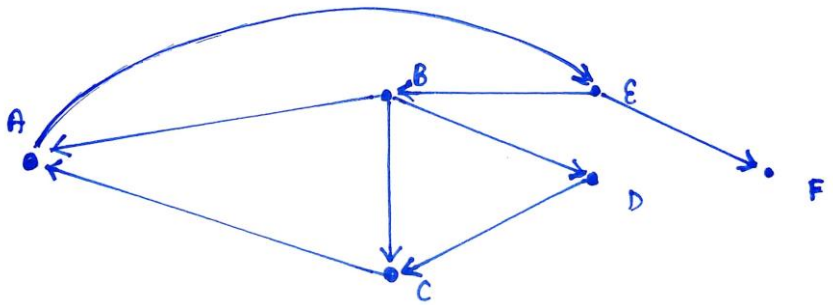
```
[45]  1  from operator import itemgetter
      2
      3  for i in sorted(nx.eigenvector_centrality(Graph).items(), key = itemgetter(1), reverse = True):
      4
      5    print(i)
```

```
('B', 0.5952478351966989)
('C', 0.4845986001361692)
('F', 0.4845986001361692)
('A', 0.35721095952083237)
('D', 0.2007328565922937)
('E', 0.09008362153176391)
```

2 - Consider the following bank network of lending money. Model it as a directed graph (a Manual model is enough).

- **Bank A lends money from Bank B and Bank C.**
- **Bank C lends money from Bank D and Bank B.**
- **Bank D lends money from Bank B.**
- **Bank B lends money from Bank E.**
- **Bank E lends money from Bank A.**
- **Bank F lends money from Bank E.**

- Compute the PageRank of each node in the graph and report the largest and smallest PageRank.



(A) $A = \frac{1}{6}$

$A = 1/6,\ B = 1/6,\ C = 1/6,\ D = 1/6,\ E = 1/6,\ F = 1/6$

(1) Incoming : B, C.

$B = \frac{1}{3}$

$C = 1$

$A = \frac{1}{3} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{1} = \frac{1}{18} + \frac{1}{6} = \frac{1+3}{18} = \frac{4}{18} = \frac{2}{9}$

(B) $B = \frac{1}{6}$

Incoming : E

$E = \frac{1}{2}$

$B = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

$= 0.083$

(C) $C = \frac{1}{6}$

$= 0.22$

Incoming = B, D

$B = \frac{1}{3}$

$D = 1$

$C = \frac{1}{6} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{1} = \frac{1}{18} + \frac{1}{6}$

$C = \frac{2}{9} = 0.22$

(D) $D = \frac{1}{6}$

Incoming = B

$B = \frac{1}{3}$

$D = \frac{1}{6} \times \frac{1}{3}$

$D = \frac{1}{18} = 0.055$

(E) $E = \frac{1}{6}$

Incoming = A

$A = 1$

$E = \frac{1}{6} \times \frac{1}{1} = \frac{1}{6}$

$E = 0.1667$

(F) $F = \frac{1}{6}$

Incoming = E

$E = \frac{1}{2}$

$F = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12} = 0.083$

② $A = \frac{2}{9}$ $B = \frac{1}{12}$ $C = \frac{2}{9}$ $D = \frac{1}{18}$ $E = \frac{1}{6}$ $F = \frac{1}{12}$

$A = \frac{1}{3} \times \frac{1}{12} + \frac{1}{1} \times \frac{2}{9} = \frac{1}{4}$

$B = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

$C = \frac{1}{3} \times \frac{1}{12} + \frac{1}{1} \times \frac{1}{18} = \frac{3}{36} = \frac{1}{12}$

$D = \frac{1}{3} \times \frac{1}{12} = \frac{1}{36}$

$E = \frac{2}{9} = \frac{2}{9}$

$F = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

③ $A = \frac{1}{4}$ $B = \frac{1}{12}$ $C = \frac{1}{12}$ $D = \frac{1}{36}$ $E = \frac{2}{9}$ $F = \frac{1}{12}$

$A = \frac{1}{3} \times \frac{1}{12} + 1 \times \frac{1}{12} = \frac{4}{3} \times \frac{1}{12} = \frac{1}{9} = 0.111$

$B = \frac{1}{2} \times \frac{2}{9} = \frac{1}{9} = 0.111$

$C = \frac{1}{3} \times \frac{1}{12} + \frac{1}{36} = \frac{2}{36} = \frac{1}{18} = 0.055$

$D = \frac{1}{3} \times \frac{1}{12} \times \frac{1}{36} = 0.0277$

$E = 1 \times \frac{1}{4} = 0.25$

$F = \frac{1}{2} \times \frac{2}{9} = \frac{1}{9} = 0.111$

After 3 iterations, Bank E has the highest Page Rank and the Bank D has the lowest Page Rank.

Thankyou!!