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# 19CSE337 Social Networking and Security

Lecture 23

A vertical sidebar on the left side of the slide, featuring a dark blue background with a grid of various white and light blue icons. These icons represent different social media and technology concepts, including a television, a camera, a lightbulb, a hand, a speech bubble, a padlock, a smartphone, a person, a shopping cart, a Twitter bird, and a lowercase 't' (Tumblr).

## Topics to Discuss

- Evaluating Recommendations.



# Evaluating Recommendations

- Recommendation algorithms can predict missing rates but we need mechanisms to evaluate the accuracy of predictions or recommendations.
- The methods are:
  - Evaluating accuracy of predictions.
  - Relevancy of recommendations.
  - Rankings of recommendations.

# Evaluating accuracy of predictions

- Measure how close predicted ratings are to the true ratings.
- One simplest method Mean Absolute Error (MAE).

$$MAE = \frac{\sum_{ij} |\hat{r}_{ij} - r_{ij}|}{n}$$

Where  $n$  is the number of predicted ratings,  $\hat{r}_{ij}$  is the predicted rating,  $r_{ij}$  is the true rating.

# Evaluating accuracy of predictions

- Normalized MAE (NMAE)

$$NMAE = \frac{MAE}{r_{\max} - r_{\min}}$$

Where  $r_{\max}$  is the max. rate an item can take and  $r_{\min}$  is the min. rate an item can take.

- Root Mean Squared Error (RMSE)

$$RMSE = \sqrt{\frac{1}{n} \sum_{i,j} (\hat{r}_{ij} - r_{ij})^2}$$

# Problem

- Consider the following table with both predicted ratings and true rating of five items. Calculate MAE, NMAE, RMSE?

<i>Item</i>	<i>Predicted Rating</i>	<i>True Rating</i>
1	1	3
2	2	5
3	3	3
4	4	2
5	4	1

# Solution

*The MAE, NMAE, and RMSE values are*

$$MAE = \frac{|1 - 3| + |2 - 5| + |3 - 3| + |4 - 2| + |4 - 1|}{5} = 2.$$

$$NMAE = \frac{MAE}{5 - 1} = 0.5.$$

$$\begin{aligned} RMSE &= \sqrt{\frac{(1 - 3)^2 + (2 - 5)^2 + (3 - 3)^2 + (4 - 2)^2 + (4 - 1)^2}{5}} \\ &= 2.28. \end{aligned}$$





# Evaluating relevancy of recommendations

- When evaluating recommendations based on relevancy, ask users if they find the recommendation relevant or not.
- User will describe relevancy either as relevant or irrelevant.
- Based on selection of items for recommendations and their relevancy, four types are there: selected relevant, selected irrelevant, not-selected relevant, not-selected irrelevant.



# Evaluating relevancy of recommendations

	Selected	Not Selected	Total
Relevant	$N_{rs}$	$N_{rn}$	$N_r$
Irrelevant	$N_{is}$	$N_{in}$	$N_i$
Total	$N_s$	$N_n$	$N$



# Evaluating relevancy of recommendations

- Precision: fraction of relevant items among recommended items.

$$P = \frac{N_{rs}}{N_s}$$

- Recall: probability of selecting a relevant item for recommendation.

$$R = \frac{N_{rs}}{N_r}$$

# Evaluating relevancy of recommendations

- F-measure: Harmonic mean of precision and recall.

$$F = \frac{2PR}{P + R}.$$

# Problem

- Consider the following recommendation relevancy matrix. Calculate precision, recall and F-score.

	<i>Selected</i>	<i>Not Selected</i>	<i>Total</i>
<i>Relevant</i>	9	15	24
<i>Irrelevant</i>	3	13	16
<i>Total</i>	12	28	40



# Solution

$$P = \frac{9}{12} = 0.75.$$

$$R = \frac{9}{24} = 0.375.$$

$$F = \frac{2 \times 0.75 \times 0.375}{0.75 + 0.375} = 0.5.$$



# Evaluating ranking of recommendations

- We predict ratings for multiple products for a user.
- Ranking of products are based on its level of interestingness to the user.
- Rank correlation measures the correlation between predicted ranking and true ranking.

# Evaluating ranking of recommendations

- Spearman's rank correlation.

$$\rho = 1 - \frac{6 \sum_{i=1}^n (x_i - y_i)^2}{n^3 - n},$$

Where  $n$  is the total number of items,  $x_i$ ,  $1 < x_i < n$ ,  
denote the rank predicted for an item  $i$ ,  $y_i$ ,  $1 < y_i < n$   
denote the true rank of item  $i$ .



# Evaluating ranking of recommendations

- Kendall's tau

A pair of items  $(i,j)$  are concordant if their ranks  $(x_i, y_i)$  and  $(x_j, y_j)$  are in order:

$$x_i > x_j, \quad y_i > y_j \quad \text{or} \quad x_i < x_j, \quad y_i < y_j$$

Or discordant if their corresponding ranks are not in order.

When  $x_i = x_j$ , or  $y_i = y_j$  the pair is neither concordant nor discordant.

# Evaluating ranking of recommendations

- Let  $c$  denote the total number of concordant item pairs and  $d$  the total number of discordant item pairs.
- Kendall's tau computes the difference between the two normalized by the item pairs.
- Kendall's tau takes the value in the range  $[-1,1]$ .

$$\tau = \frac{c - d}{\binom{n}{2}}.$$



# Evaluating ranking of recommendations

- When the ranks completely agree, all pairs are concordant and Kendall's tau takes value 1 and when the ranks completely disagree, all pairs are discordant and Kendall's tau takes value -1.

# Problem

- Consider the set of four items  $I = \{i_1, i_2, i_3, i_4\}$  for which the predicted and true rankings are as follows. Calculate Kendall's tau for the items.

	<i>Predicted Rank</i>	<i>True Rank</i>
$i_1$	1	1
$i_2$	2	4
$i_3$	3	2
$i_4$	4	3

# Solution

- The possible pairs are  $(i_1, i_2)$ ,  $(i_1, i_3)$ ,  $(i_1, i_4)$ ,  $(i_2, i_3)$ ,  $(i_2, i_4)$ ,  $(i_3, i_4)$ .

$$x_i > x_j, \quad y_i > y_j \quad \text{or} \quad x_i < x_j, \quad y_i < y_j$$

- Check for  $(i_1, i_2)$ .
  - $1 < 2, 1 < 4 \rightarrow$  concordant.
- Check for  $(i_1, i_3)$ 
  - $1 < 3, 1 < 2 \rightarrow$  concordant.

	<i>Predicted Rank</i>	<i>True Rank</i>
$i_1$	1	1
$i_2$	2	4
$i_3$	3	2
$i_4$	4	3

# Solution

*The pair of items and their status {concordant/discordant} are*

$(i_1, i_2)$  : *concordant*

$(i_1, i_3)$  : *concordant*

$(i_1, i_4)$  : *concordant*

$(i_2, i_3)$  : *discordant*

$(i_2, i_4)$  : *discordant*

$(i_3, i_4)$  : *concordant*

*Thus, Kendall's tau for the rankings is*

$$\tau = \frac{4 - 2}{6} = 0.33.$$



Thanks.....