Graph

Anoop S Babu

Faculty Associate

Dept. of Computer Science & Engineering

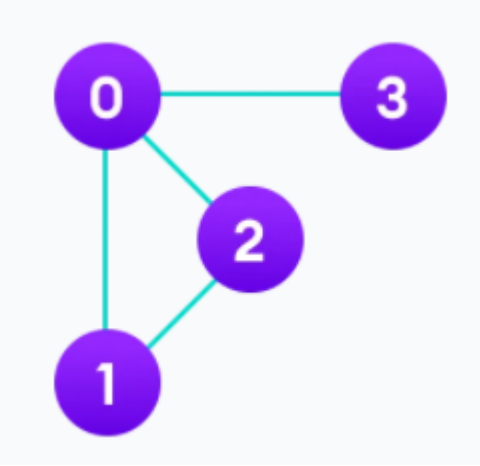
bsanoop@am.amrita.edu

Graph Data Structure

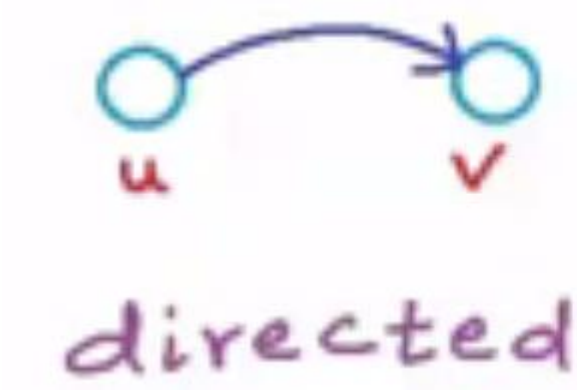
- A data structure that consists of a **set of nodes** (vertices) and a **set of edges** that relate the nodes to each other.

Formal (Mathematical) Definition

- A Graph ***G*** is an ordered pair ***G = (V, E)***
  - ***V*** – set of vertices
  - ***E*** – set of edges
- G = (V, E)
  - V = {0, 1, 2, 3}
  - E = {{0,1}, {0,2}, {0,3}, {1,2}}



Directed and Undirected Edges

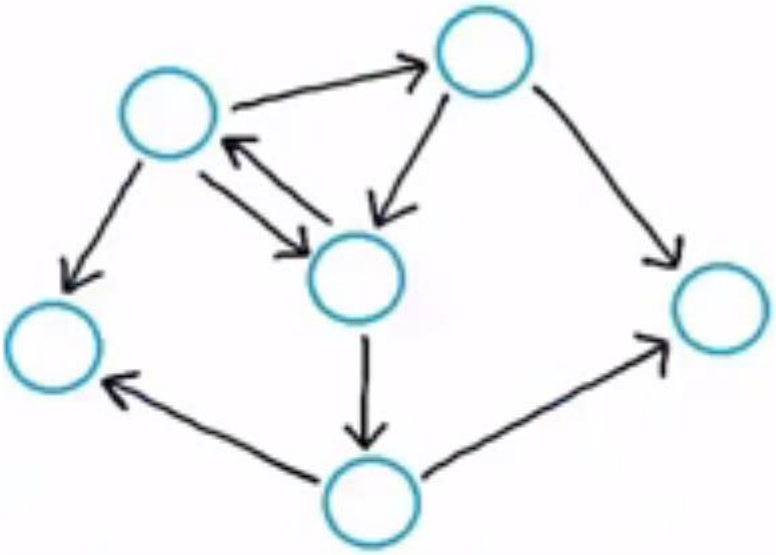


(u, v)

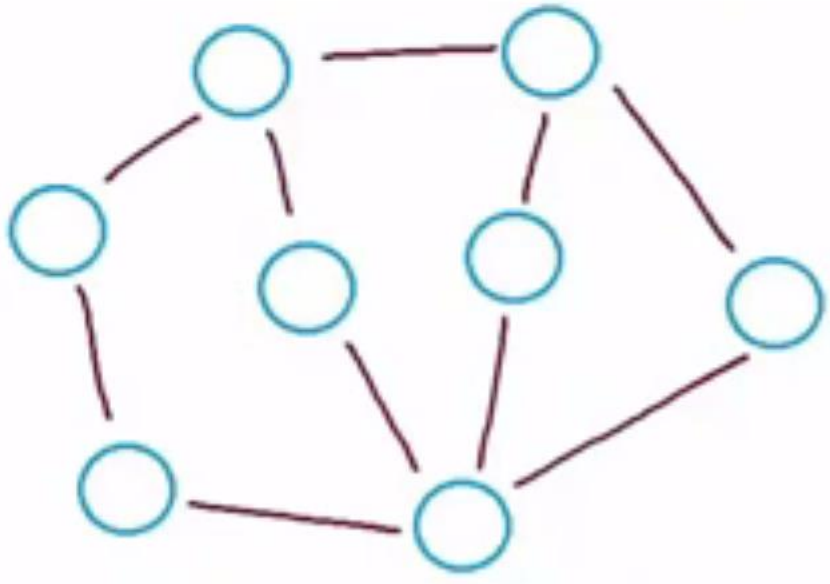
{u, v}

Directed and Undirected Graphs

- A graph with **all directed edges** is called directed graph (or *digraph*)
- A graph with **all undirected edges** is called undirected graph.

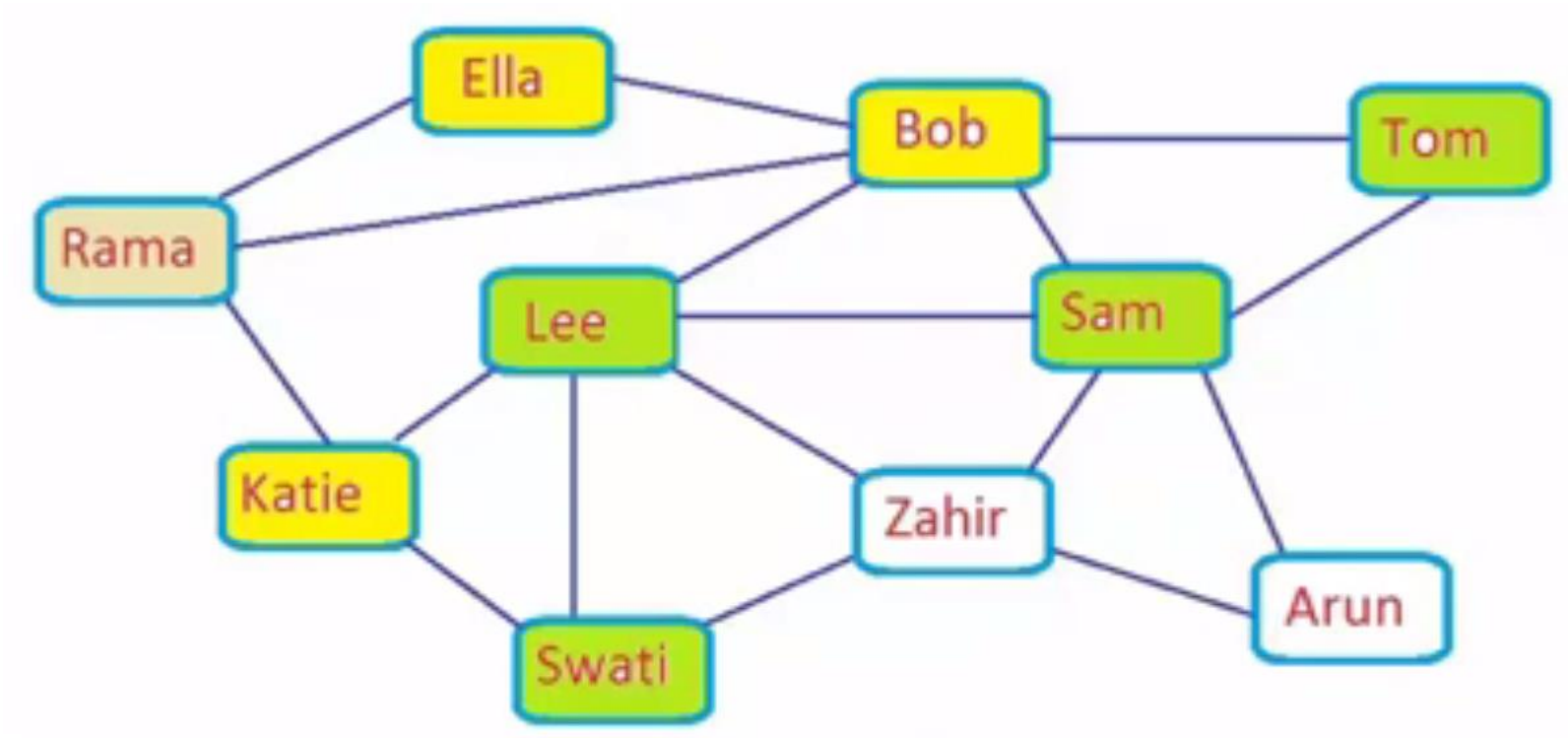


Directed Graph (Digraph)



Undirected Graph

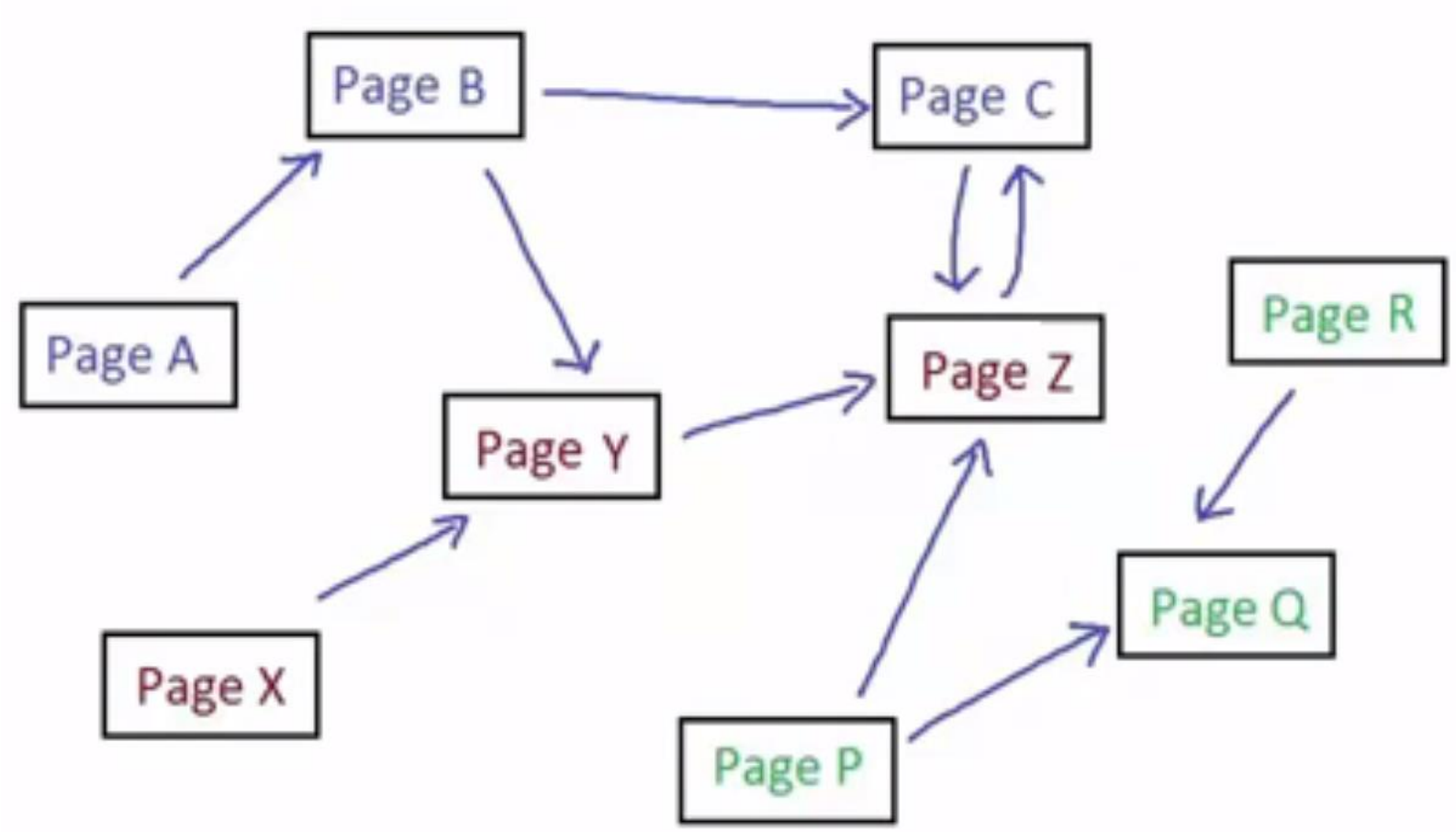
Graph Representation in Real World



Representation of Social Network (Facebook)

Graph Problem: Finding all the nodes having length of shortest path from Rama equal to 2

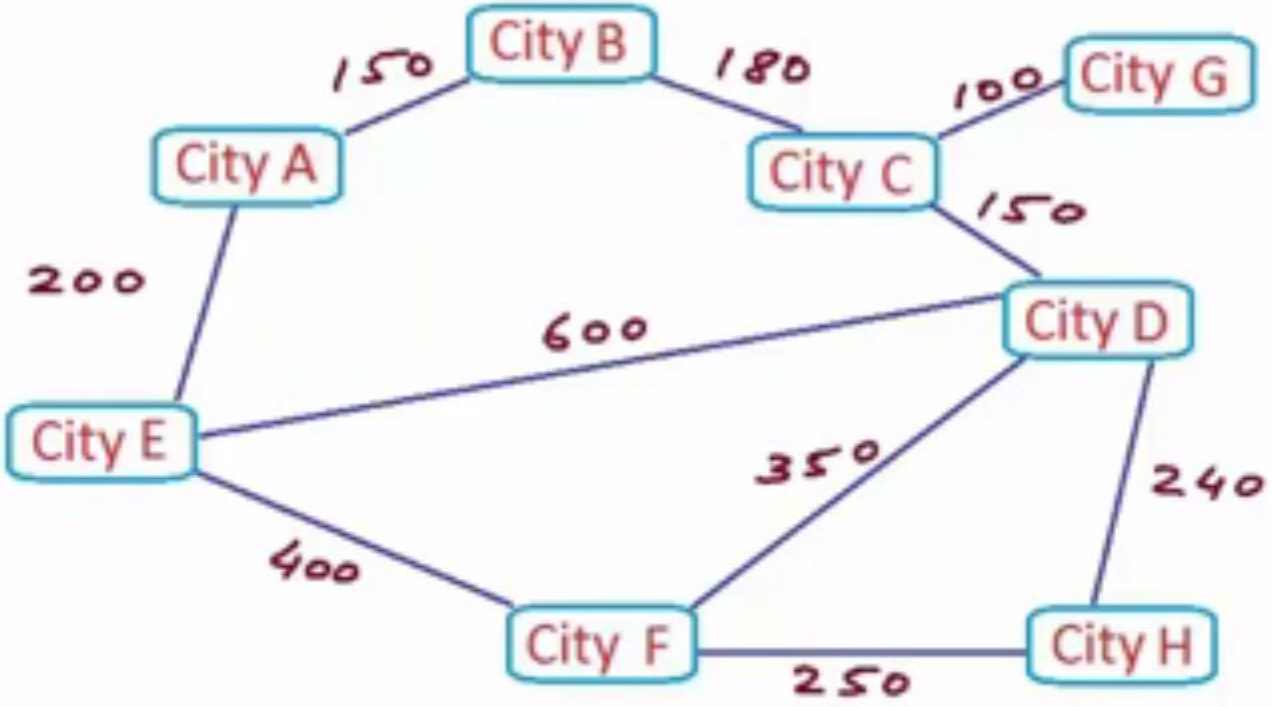
Graph Representation in Real World



Interlinked webpages in www

Weighted and Unweighted Graphs

- In a weighted graph, each edge is assigned with some weight or cost.



- **Unweighted Graph**: A weighted graph with all the edges having same weight. (ie., weight = 1 unit)

Graph Terminology

- **Self-loop**: A edge is called self-loop or self edge if both the end points of an edge is same.

- Two undirected edges to have the same end vertices, and for two directed edges to have the same origin and the same destination. Such edges are called **parallel edges** or **multi edges.**

Simple Graph: If a graph doesn't contain any self-loops or multi-edges then its called a simple graph

Graph Terminology

- **Adjacent Nodes**: If two nodes u and v are connected via an edge e, then the nodes u and v are called as neighbours or adjacent nodes.

- **Walk:** A sequence of vertices where each adjacent pair is connected by an edge.

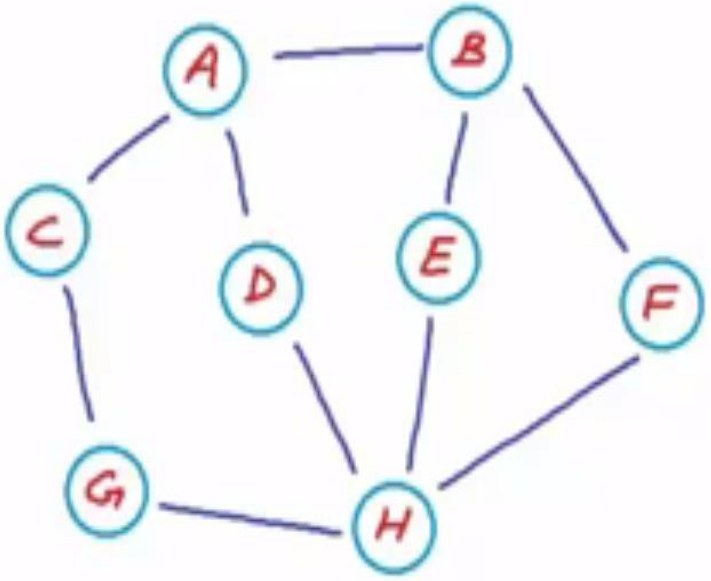
  Eg: <A, B, F, H, E, B, A, D>

- **Path (Simple Path):** A walk in which no vertices (thus no edges) are repeated.

  Eg: <A, B F, H>

- **Trail:** A walk in which no edges are repeated; but vertices can be repeated.

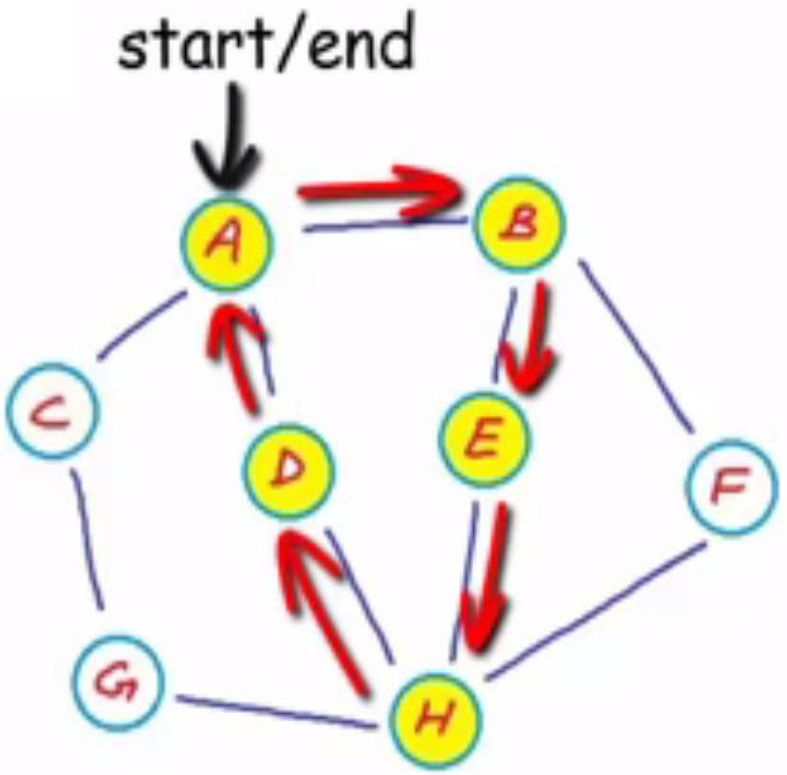
  Eg: <A, B, E, H, D, A, C>



Graph Terminology

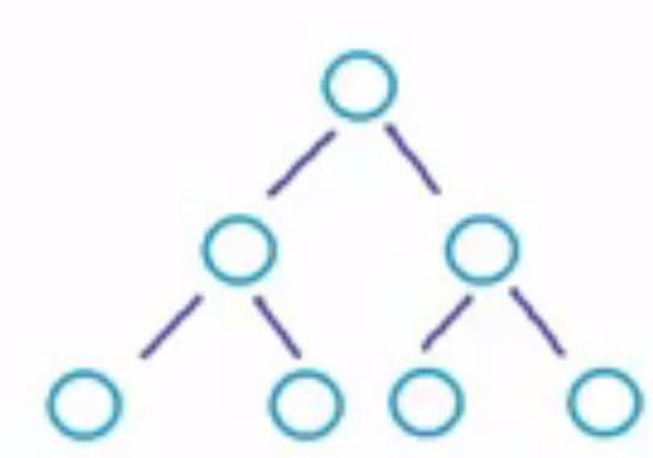
- **Closed Walk:** A walk is called a closed walk if it starts and end at the same vertex. And length of the walk (ie. no of edges in walk) must be greater than 0.

- **Cycle (Simple Cycle):** A closed walk in which has no repeated edges or vertices except start and end vertices.

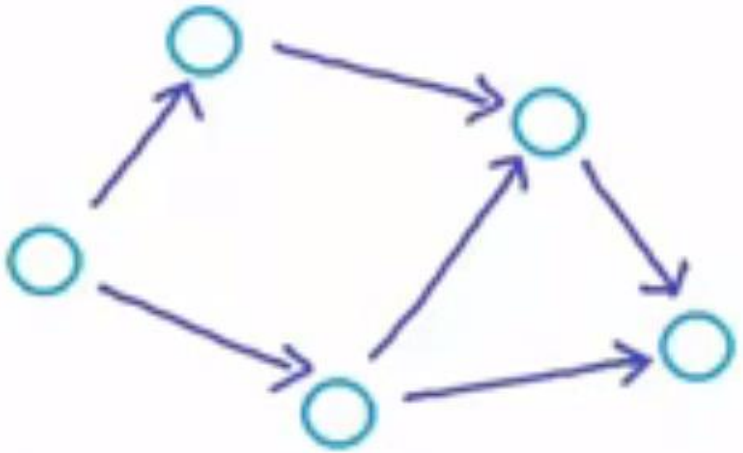


Graph Terminology

- **Acyclic Graph**: A graph with no cycle.



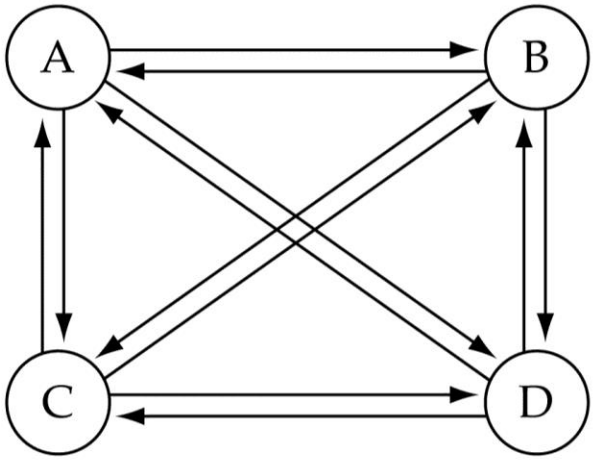
Undirected Acyclic Graph



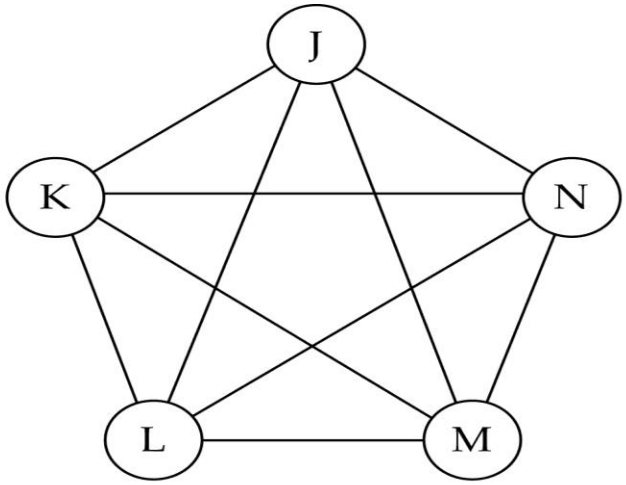
Directed Acyclic Graph (DAG)

Complete Graph

- **Complete Graph**: A graph in which every vertex is directly connected to every other vertex.



(a) Complete directed graph.



(b) Complete undirected graph.

- No of edges in a complete directed graph with N vertices is ***N * (N-1)***
- No of edges in a complete undirected graph with N vertices is ***N * (N-1) / 2***