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Ratio & Proportion

Concept

If a and b (b ≠ 0) are two quantities of the same kind, then Ratio is the relation which one quantity bears to another of the same kind in magnitude.

Now in two quantities a and b the fraction $\dfrac{a}{b}$ is called the ratio of a to b. It is usually expressed as a : b, a and b are said to be the terms of the ratio. The former (numerator) 'a' is called the **Antecedent** of the ratio and latter (denominator) 'b' is called **consequent**.

Basics of Ratio

(i)   As ratio is a relation between two quantities so ratio is independent of the concrete units employed in the quantities compared.

(ii)   Ratio exists only between two quantities; both the quantities must be in the same units.

Composition of Ratio

I.   Compounded Ratio

When two or more ratios are multiplied term wise, the ratio thus, obtained is called their compounded ratio.

Compounded ratio of (**a : b**, **c : d** and **e : f**) $\dfrac{a}{b} \times \dfrac{c}{d} \times \dfrac{e}{f} = \dfrac{ace}{bdf}$

II.   Reciprocal Ratio

For any ratio a : b, the reciprocal ratio will be $\dfrac{1}{a} : \dfrac{1}{b}$.

The application of it is in calculating the ratio of the wages distributed among the workers, which is equal to the reciprocal ratio of number of days taken by them to complete the work.

Proportion

When two ratios are equal, the four terms involved, taken in order are called proportional, and they are said to be in proportion.

1.   The ratio of a to b is equal to the ratio of c to d i.e. if $\dfrac{a}{b} = \dfrac{c}{d}$, we write a : b : : c : d

2.   If a : b : : c : d Then we have **ad = bc**

i.e. the terms a and d are called Extremes and, the terms b and c are called the Means. The term a, b, c and d are known by the name 1st, 2nd, 3rd and the 4th proportion respectively.

Continued Proportion:

Three quantities are said to be in continued proportion, if the ratio of the first to the second is same as the ratio of the second to the third.

Thus a, b and c are in continued proportion if a : b = b : c

$$\dfrac{a}{b} = \dfrac{b}{c}$$

Mean Proportion:

If a, b, c are in continued proportion then second quantity 'b' is called the mean proportional between 'a' and 'c' and a, b and c are known as 1st, 2nd and 3rd proportion respectively.

$\dfrac{a}{b} = \dfrac{b}{c}$ then $b^2 = ac$

or $\boxed{b = \sqrt{ac}}$

Ex.1 *What is the mean proportional to* $\dfrac{5}{36}, \dfrac{3.2}{25}$ *?*

(1) $\dfrac{2}{25}$ (2) $\dfrac{15}{2}$ (3) $\dfrac{2}{15}$ (4) $\dfrac{4}{25}$

Sol. Let the mean proportion of $\dfrac{5}{36}$ and $\dfrac{3.2}{25}$ be x

$\therefore \dfrac{5}{36} : x = x : \dfrac{3.2}{25} \Rightarrow \dfrac{5}{36} \times \dfrac{1}{x} = x \times \dfrac{25}{3.2}$

$x^2 = \dfrac{5}{36} \times \dfrac{3.2}{25} = \dfrac{16}{900}$ $x = \sqrt{\dfrac{16}{900}} = \dfrac{4}{30} = \dfrac{2}{15} \Rightarrow x = \dfrac{2}{15}$.

Ex.2 *What should be added to each of the numbers 19, 26, 37, 50 so that the resulting number should be in proportion?*

(1) 2 (2) 3 (3) – 2 (4) – 5

Sol. Let the required number be x.

According to question,

19 + x, 26 + x, 37 + x and 50 + x are in proportion

$19 + x : 26 + x = 37 + x : 50 + x \Rightarrow \dfrac{19+x}{26+x} = \dfrac{37+x}{50+x}$

$(19 + x)(50 + x) = (37 + x)(26 + x) \Rightarrow x^2 + 69x + 950 = x^2 + 63x + 962$

$x^2 - x^2 + 69x - 63x = 962 - 950$

Hence the resulting number = 2.

Basics of Proportion

(i) Let us take four quantities a, b, c & d such that they are in proportion i.e. **a : b : : c : d** then,

$\dfrac{a+b}{a-b} = \dfrac{c+d}{c-d}$

This operation is called **componendo and Dividendo**.

(ii) If $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f}$ then

$\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} = \dfrac{a+c+e}{b+d+f} = \dfrac{a+c-e}{b+d-f} = \dfrac{k_1a + k_2c + k_3e}{k_1b + k_2d + k_3f}$

Partnership & Share

<table>
<tr><td colspan="2" align="center">**Partnership & Share**</td></tr>
<tr><td colspan="2">If there is profit in the business run by two partners A and B then,</td></tr>
<tr><td align="center">$\dfrac{\text{Amount of A's investment} \times \text{No. of months invested by A}}{\text{Amount of B's investment} \times \text{No. of months invested by B}}$</td><td align="center">$= \dfrac{\text{Profit of A}}{\text{Profit of B}}$</td></tr>
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Ex.3 *Saman begins business with a capital of Rs. 50,000 and after 3 months takes Manu into partnership with a capital of Rs. 75000. Three months later Amandeep joins the firm with a capital of Rs. 1, 25,000. At the end of the year the firm makes a profit of Rs. 99,495. How much of this sum should Amandeep receive?*

Sol. Money invested by Saman for 12 months = Rs. 50,000

Money invested by Manu for 9 months = Rs. 75000

Money invested by Amandeep for 6 months = Rs. 1,25,000

Share of Saman: Manu: Amandeep

$= 50,000 \times 12 : 75,000 \times 9 ; 1,25,000 \times 6$

$= 6,00,000 : 6,75,000 : 7,50,000 = 600 : 675 : 750 = 8 : 9 : 10$

Total profit = Rs. 99,495.

Profit of Amandeep $= \left(\dfrac{10}{8+9+10}\right) \times 99495$

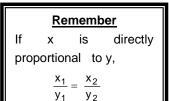$= \dfrac{10}{27} \times 99,495$ = Rs. 36,850.

PROPORTIONS

Direct Proportion

If A is directly proportional to B, then as A increases B also increases proportionally or in other words the proportional change occurs in the same direction. In general when A is directly proportional to B, then

$\dfrac{\mathbf{A}}{\mathbf{B}}$ **= Constant**.

Examples of Direct proportion are Time & Distance problems; time taken to travel a distance is directly proportional to the distance traveled when the speed is constant. This means the distance-traveled doubles if the time taken doubles provided speed remains constant.

Remember
If x is directly proportional to y, $\dfrac{x_1}{y_1} = \dfrac{x_2}{y_2}$

Direct Relation

In these types of cases an increase in one causes increase in the other but the increase is not proportional as in the case of direct proportion.

For example:

With simple interest the amount increases with increase of the number of years but not proportionally while on the other hand interest doubles or triples after 2nd and 3rd years. So the increment in amount is in direct relation while increment in interest is direct proportion.

Remember
If x is in direct relation to y then, $x = K_1 + yK_2$

Ex.4 *If 6 men can lay 8 bricks in one day, then how many men are required to lay 60 bricks in the same time?*

(1) 45 men   *(2) 40 men*   *(3) 60 men*   *(4) 50 men*

Sol. Since the time is same so to do more work we need more persons. Hence this is the problem of direct proportion,

i.e. $\dfrac{M_1}{B_1} = \dfrac{M_2}{B_2} \Rightarrow M_2 = \dfrac{6 \times 60}{8}$ = 45 men

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Ex.5 *The cost of New Year party organized in TCY is directly related to the number of persons attending that party. If 10 persons attend the party the cost per head is Rs 250 and if 15 people attend, the cost per head is Rs. 200. What will be the total cost of the party if 20 persons attend it?*

Sol. This is the problem of direct relation

Let the total cost of party is

Cost = $K_1 + K_2 N$ (where K_1 & K_2 are fixed and variable costs and N is number of persons)

250 x 10 = $K_1 + 10K_2$ ....(1)

200 x 15 = $K_1 + 15K_2$ ....(2)

Solving them we get $K_1 = 1500$ and $K_2 = 100$

So total cost for 20 persons = 1500 + 20 x 100 = **Rs. 3500**

Indirect/Inverse Proportion

A is in indirect proportion to B if as A increases, B decreases proportionally i.e. the proportional change occurs in the opposite direction. In general if A is in indirect proportion to B, then

$$AB = constant.$$

Examples of Indirect proportion are Price & Quantity (expenditure remaining same), Number of men required & rate of work done (amount of work remaining same), Time & Speed problems for same distance.

> **Remember**
>
> If x is inversely proportional to y, then
>
> $$xy = K$$
>
> $\Rightarrow x_1y_1 = x_2y_2$

Ex.6 *If 6 men can build a wall in 9 days then 60 men can build a similar wall in _____ days?*

Sol. Work = Men x Days

$W = 6 \times 9 = 60 \times D \Rightarrow D = \dfrac{6 \times 9}{60}$ = **0.9 days**

Ex.7 *A can do a piece of work in 12 days, B is 60% more efficient than A. Find the number of days required for B to do the same piece of work.*

Sol. Ratio of the efficiencies is A : B = 100 : 160 = 5 : 8. Since efficiency is inversely proportional to the number of days, the ratio of days taken to complete the job is 8 : 5

So, the number of days taken by B = $\dfrac{5}{8}$ x 12 = $7\dfrac{1}{2}$ days.

Ex.8 *Ten years ago, the ratio of ages of A and B is 3 : 4, now, it is 4 : 5. What is the present age of A?*

Sol. 10 years ago, let their ages be 3k and 4k.

So, present ages are 3k + 10 and 4k + 10.

The ratio is given as 4 : 5.

$\therefore \dfrac{3k + 10}{4k + 10} = \dfrac{4}{5}$

k = 10   Percentage of A = 3k + 10 = 40 years.

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Ex.9 **Three friends A, B, C earn Rs. 2000 together. If they want to distribute this money such that 'A' should get Rs. 300 more than B and Rs. 100 more than C, in what ratio, they have to distribute the money?**

Sol. A = 300 + B = 100 + C

So, A = 300 + B and C = 200 + B

A + B + C = 2000

$\therefore$ B = 500 $\Rightarrow$ A = 800 and C = 700 So, the required ratio = 8 : 5 : 7

Ex.10 **Four friends A, B, C, D share Rs. 10,500 in the ratio $\dfrac{1}{3}:\dfrac{1}{4}:\dfrac{1}{6}:\dfrac{1}{8}$, how much more money A and C together get than B and D together?**

Sol. The given ratio is $\dfrac{1}{3}:\dfrac{1}{4}:\dfrac{1}{6}:\dfrac{1}{8}$,.

Multiply with 24 = 8 : 6 : 4 : 3.

Take A = 8k, B = 6k, C = 4k, D = 3k.

Total = 21k = 10,500

$\Rightarrow$ k = 500

Required answer = (A + C) – (B + D)

$\qquad\qquad$ = (8k + 4k) – (6k + 3k) = 3k = Rs. 1500

Ex.11 **xyz has to distribute Rs. 3500 such that, for every 2 rupees x takes, y will take 3 rupees and for every 4 rupees y takes, z will take 5 rupees. How much money z will get?**

Sol. We can write from the given information that,

x : y = 2 : 3 ....(1)

and y : z = 4 : 5   ....(2)

Equation (1) $\times$ 4 $\Rightarrow$ x : y = 8 : 12

Equation (2) $\times$ 3 $\Rightarrow$ y : z = 12 : 15

$\therefore$ x : y : z = 8 : 12 : 15

$\therefore$ z will get $\dfrac{15}{8+12+15} \times 3500 = $ Rs. 1500.