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Lecture - 18

Opinion formation.

What is opinion?

→ An opinion is a belief or attitude about something that is not necessarily based on facts.

→ An opinion is a belief or judgement or belief based on experience and on certain facts but not amounting to sure knowledge.

→ information diffusion makes Opinion formation

→ in social networks users will form opinions based on the information and contributions available in social networks.

→ Everyone will not make opinions.

→ active users only.

→ Opinions may be +ve, -ve or neutral.

- Opinions are influenced by their social networks.
- Three key components.
  - users → share information
    → forms opinion.
  - network → group of connected people.
  - message → information shared
    it's types, polarity etc.

- Opinions are highly time variant.
  i.e; evolving.
- user forms opinions from information shared in the network & their own ideas or beliefs.
- There are various models representing opinion formation.
  eg: DeGroot model, value-expectancy model
- How an individual will combine his/her own opinion and information they receive?

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- There are two prominent approaches to model this
  (i) Bayesian model
  (ii) Non-Bayesian model.
- Non Bayesian approach is more popular.
- DeGroot model is an example.

DeGroot Model.

- Simple and earliest model introduced by Morris H. DeGroot in 1974
- A model based on repeated communication, where people communicate with each other and take weighted averages of information.
- The people start with their initial beliefs, and successive repeated communication will update their belief using weighted average.
- existing beliefs may be changed.

- The agents in this model are boundedly rational. means, they fail to adjust repetitions and dependencies in information.
- This will lead to duplication of information which is a shortcoming of the DeGroot model.
- Analysis of convergence is very easy in DeGroot model.
- Consensus belief is a weighted average of the agents initial beliefs and these beliefs provide a measure of agent's influence or social importance.

- Consider an example.
Someone may ask / post a question is social network.
for eg: Is Tajmahal beautiful?
- Opinion may be yes / no / neutral.
- The consensus can be yes.
- we have to produce a weighted arg. corresponding to yes.

- Consider a group of agents.
  $V = \{1, 2, 3 \cdots n\}$
- At time $t = 0$, each agent $i \in V$ has his/her own initial belief (opinion) $x_i(t) \in [0, 1]$ concerning some topic (Taymahal is beautiful).
- This initial weight will be updated at each time $t \in \{1, 2, \cdots n\}$.
- The weight updation may lead to a yes/NO. [0 or 1].
- Agents exchange information about their beliefs with their neighbours.
- this interaction may be represented using an $n \times n$ non negative matrix.
  $W = [w_{ij}]$, $w_{ij} > 0$ means i listens to j
- W is strongly connected means every two individual can communicate with each other.

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- At any time $t \geq 1$, agent i update his/her belief according to

$$x_i(t) = \sum_j w_{ij}\, x_i(t-1)$$

(Agent i updates their belief by taking weighted average of the current beliefs of agent j, forming his/her own belief for the next period).

$x(t)$ → is a column vector of beliefs.

- initial beliefs at $t=0$

$$X(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \\ \vdots \\ x_n(0) \end{bmatrix}$$

weight matrix

$$W = \begin{bmatrix} w_{11} & w_{12} & \cdots \\ w_{21} & \cdots & \\ \vdots & & \\ w_{n1} & \cdots & w_{nn} \end{bmatrix}$$

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- This process will repeat till it converges to a limiting belief. ($t \to \infty$)

- The necessary and sufficient Condition for convergence is that every nodes should be strongly connected and closed is aperiodic.

- Example:

$$W = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 1/4 & 3/4 \end{bmatrix}$$

- Agent 1 listens to everybody equally.

- Agent 2 don't believe agent 3's opinion.

- Agent 3 don't listen to agent 1. and trust on his own belief.

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Suppose the initial belief of 3 agents on certain issue (is Taj mahal beautiful?) is as follows.

- initial vector

$$X(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Agent 1 yes
Agent 2 } no
Agent 3 }

updating (may change opinion when they talk each other)

- $x_i(t) = \sum_j w_{ij}\, x_i(t-1)$

$$X(1) = W\,X(0)$$

$$= \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 1/4 & 3/4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/2 \\ 0 \end{bmatrix}$$

$$X(2) = W\,X(1)$$

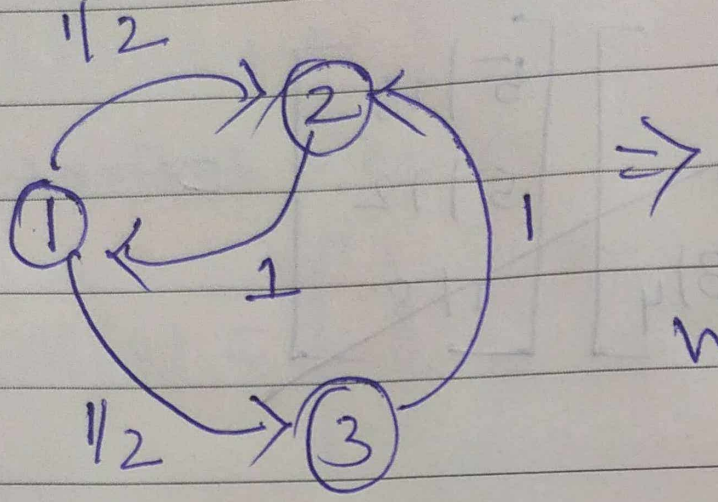
$$= \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 1/4 & 3/4 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/2 \\ 0 \end{bmatrix} = \begin{bmatrix} 5/18 \\ 5/12 \\ 1/8 \end{bmatrix}$$

- At some point in time it will converge at

$$x = \begin{bmatrix} 3/11 \\ 3/11 \\ 3/11 \end{bmatrix}$$

when all the agents agrees on a common opinion

Example: 2



$$\Rightarrow W = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$X(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$x(1) = Wx(0) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$x(2) = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$$

$$X(3) = \begin{bmatrix} 3/4 \\ 1/2 \\ 0 \end{bmatrix}$$

Converges at:

$$\cdots X = \begin{pmatrix} 2/5 \\ 2/5 \\ 2/5 \end{pmatrix}$$