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DEEMED TO BE UNIVERSITY

19CSE337 Social Networking Security

Lecture 3

Topics to Discuss

- Introduction to Graphs
- Fundamentals of Graph Theory

Representation of Networks

- We know network is a collection of nodes joined by links.
- How can we represent a network?
  - Some mathematical notations!
- To represent a network, we use Graphs in mathematical literature.
- In mathematics, graph theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects.

What is Graph?

- A **graph** is a set of points, called nodes or vertices, which are interconnected by a set of lines called edges.

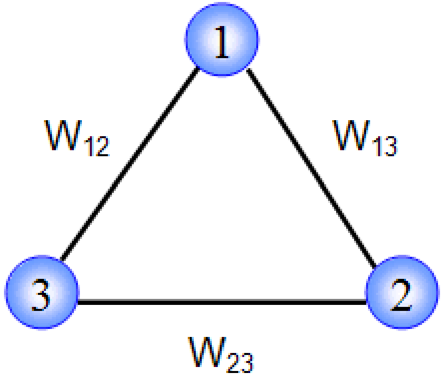
Graph Definition

- A graph $G=(V,E)$ is a set of vertices $V=\{v_1,v_2,v_3...v_n\}$ and edges $E=\{e_1,e_2,e_3....e_n\}$, such that each edge e_k is identified with a pair of vertices (v_i,v_j).

Networks in graph perspective

- We try to redefine networks in the perspective of graphs.
- A network is a collection of vertices and edges.
- In other words, A network is a graph G=(V,E), where V is a set of vertices representing nodes and E is a set of edges representing relationships between the nodes.

Network Representation using Graph



G=(V,E), where V={1,2,3}, E={w_{13},w_{12},w_{23}}

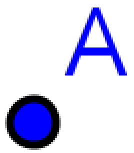
The edges can also be represented as
E={(1,2),((1,3),(2,3))}

This notation is called edge list.

Fundamentals of Graph Theory

- **Point**

  A point is a particular position in a one-dimensional, two-dimensional, or three-dimensional space.



Fundamentals of Graph Theory

- **Line**

  A **Line** is a connection between two points.

Fundamentals of Graph Theory

- **Vertex**
  - A vertex is a point where multiple lines meet.
  - It is also called a **node**.
  - Like points, a vertex is also denoted by an alphabet.

Fundamentals of Graph Theory

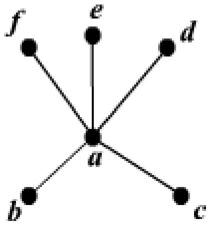
- **Edge**
  - An edge is the mathematical term for a line that connects two vertices.
  - Many edges can be formed from a single vertex.
  - Without a vertex, an edge cannot be formed.
  - There must be a starting vertex and an ending vertex for an edge.

Fundamentals of Graph Theory

- **Loop**
  - In a graph, if an edge is drawn from vertex to itself, it is called a loop.

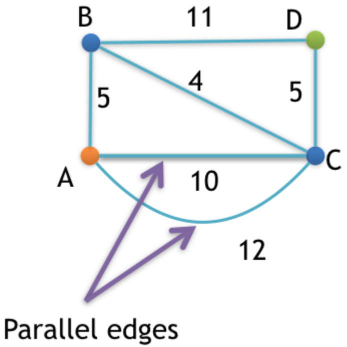
Fundamentals of Graph Theory

- **Degree of a vertex**
  - It is the number of edges connecting to a vertex.
  - In the example below, degree of vertex a is 5 and the rest have degree 1.
  - It is represented as deg(a)=5.



Fundamentals of Graph Theory

- **Parallel Edges**
  - If a pair of vertices is connected by more than one edge, then those edges are called parallel edges.

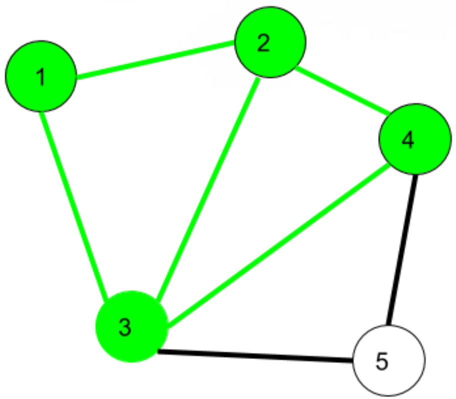


Fundamentals of Graph Theory

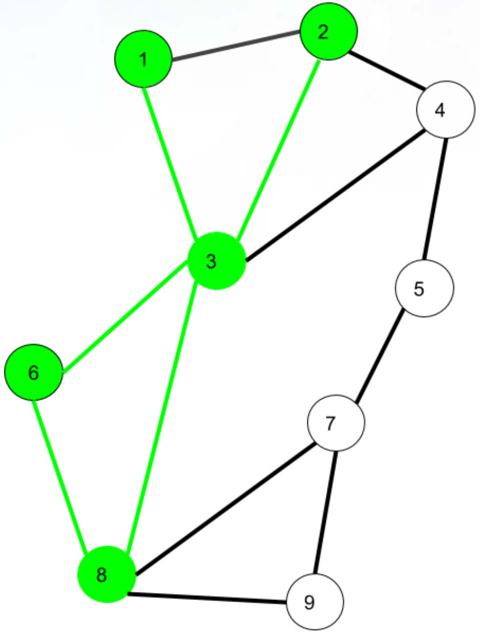
- **Neighbour or adjacent**
  - Node j is neighbour of node i, if and only if node i is connected to node j.

Fundamentals of Graph Theory

- **Walk**
  - A list of edges that are sequentially connected to form a continuous route on a network.
- **Open walk-** A walk is said to be an open walk if the starting and ending vertices are different i.e. the origin vertex and terminal vertex are different.
  - 1->2->3->4->5->3 is an open walk.
- **Closed walk-** A walk is said to be a closed walk if the starting and ending vertices are identical i.e. if a walk starts and ends at the same vertex, then it is said to be a closed walk.
  - 1->2->3->4->5->3->1 is a closed walk.

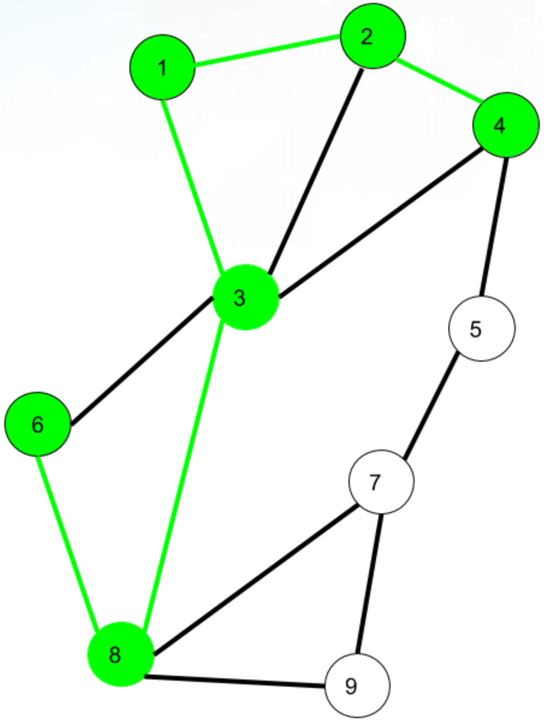


Fundamentals of Graph Theory



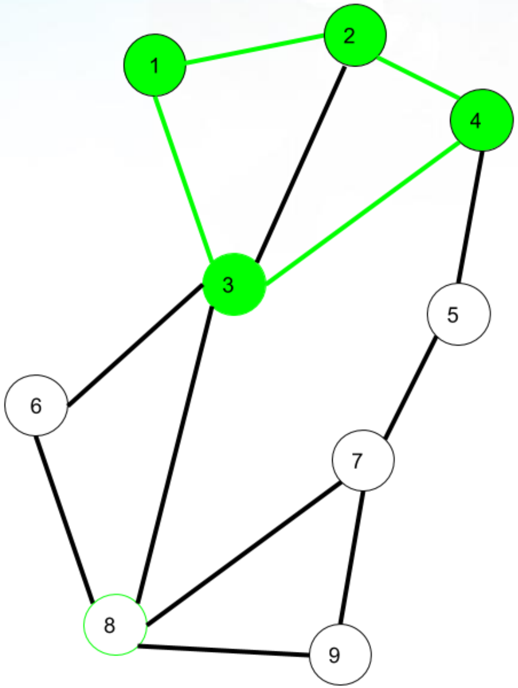
- **Trail**
  - A walk that doesn’t go through any edge more than once.
  - Trail is an open walk in which no edge is repeated.
  - Here 1->3->8->6->3->2 is trail Also 1->3->8->6->3->2->1 will be a closed trail

Fundamentals of Graph Theory



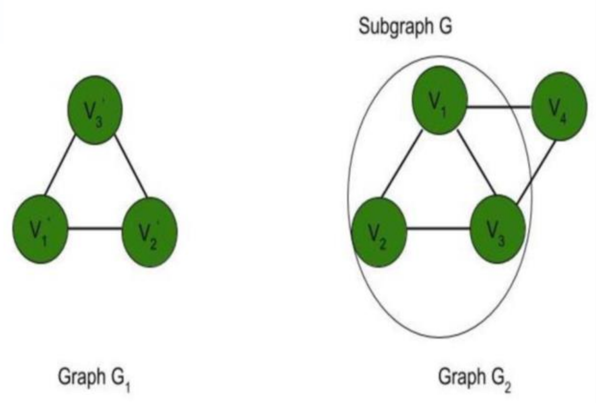
- **Path**
  - A walk that doesn't go through any node (and therefore any edge, too) more than once.
  - It is a trail in which neither vertices nor edges are repeated i.e. if we traverse a graph such that we do not repeat a vertex and nor we repeat an edge.
  - As path is also a trail, thus it is also an open walk.
  - Vertex not repeated, Edge not repeated.
  - Here 6->8->3->1->2->4 is a Path

Fundamentals of Graph Theory



- **Cycle**
  - A walk that starts and ends at the same node without going through any node more than once on its way.
  - Traversing a graph such that we do not repeat a vertex nor we repeat a edge but the starting and ending vertex must be same i.e. we can repeat starting and ending vertex only then we get a cycle.
  - Vertex not repeated. Edge not repeated.
  - Here 1->2->4->3->1 is a cycle.

Fundamentals of Graph Theory



- **Subgraph**
  - Part of the graph.
  - **Definition:** A graph whose vertices and edges are subsets of another graph.
  - **Formal Definition:** A graph G'=(V', E') is a subgraph of another graph G=(V, E) if
    - $V' \subseteq V$, and
    - $E' \subseteq E \wedge ((v_1, v_2) \in E' \rightarrow v_1, v_2 \in V')$.

Thanks...........