



AMRITA
VISHWA VIDYAPEETHAM
DEEMED TO BE UNIVERSITY

19CSE337 Social Networking Security

Lecture 3

A vertical sidebar on the left side of the slide, featuring a dark blue background with a grid of various white and light blue icons. These icons represent different concepts such as technology (television, camera, smartphone), communication (lightbulb, ear, speech bubble), social media (Twitter bird, 't' logo), and general concepts (lock, shopping cart, magnifying glass).

Topics to Discuss

- Introduction to Graphs
- Fundamentals of Graph Theory



Representation of Networks

- We know network is a collection of nodes joined by links.
- How can we represent a network?
 - Some mathematical notations!
- To represent a network, we use Graphs in mathematical literature.
- In mathematics, graph theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects.

A decorative header featuring a grid of various icons in shades of blue and white. The icons include a dollar sign, a wrench and screwdriver, a car, a sun, a shopping cart, a briefcase, a smartphone, a family of three, a lightbulb, a headset, a hand holding a pen, a briefcase, a bar chart, a radio tower, and a book. The background is a gradient of blue and teal.

What is Graph?

- A **graph** is a set of points, called nodes or vertices, which are interconnected by a set of lines called edges.



Graph Definition

- A graph $G=(V,E)$ is a set of vertices $V=\{v_1,v_2,v_3...v_n\}$ and edges $E=\{e_1,e_2,e_3....e_n\}$, such that each edge e_k is identified with a pair of vertices (v_i,v_j) .

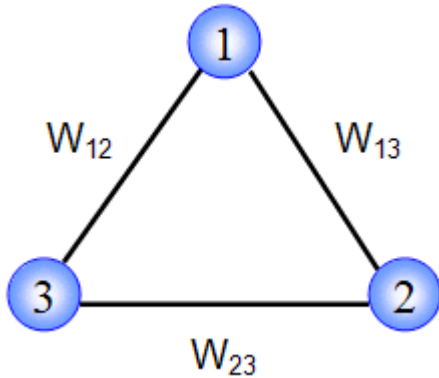
The header features a blue background with a grid of circular icons. These icons include a dollar sign, a wrench and screwdriver, a car, a sun, a briefcase, a smartphone, a family of three, a lightbulb, a handshake, a shopping cart, a bar chart, a radio tower, and a pair of headphones. The title 'Networks in graph perspective' is written in orange text across the middle of this header.

Networks in graph perspective

- We try to redefine networks in the perspective of graphs.
- A network is a collection of vertices and edges.
- In other words, A network is a graph $G=(V,E)$, where V is a set of vertices representing nodes and E is a set of edges representing relationships between the nodes.

Network Representation using Graph

$G=(V,E)$, where $V=\{1,2,3\}$, $E=\{w_{13},w_{12},w_{23}\}$



The edges can also be represented as
 $E=\{(1,2),((1,3),(2,3))\}$

This notation is called edge list.

Fundamentals of Graph Theory

- **Point**

A point is a particular position in a one-dimensional, two-dimensional, or three-dimensional space.



Fundamentals of Graph Theory

- **Line**

A **Line** is a connection between two points.



Fundamentals of Graph Theory

- **Vertex**

- A vertex is a point where multiple lines meet.
- It is also called a **node**.
- Like points, a vertex is also denoted by an alphabet.





Fundamentals of Graph Theory

- **Edge**

- An edge is the mathematical term for a line that connects two vertices.
- Many edges can be formed from a single vertex.
- Without a vertex, an edge cannot be formed.
- There must be a starting vertex and an ending vertex for an edge.

Fundamentals of Graph Theory

- **Loop**

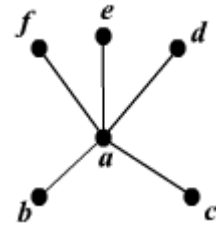
- In a graph, if an edge is drawn from vertex to itself, it is called a loop.



Fundamentals of Graph Theory

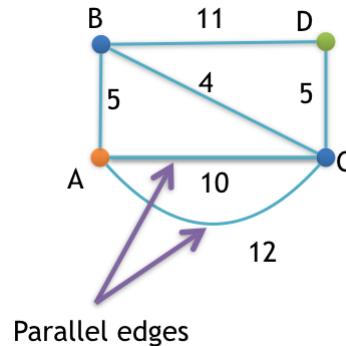
- **Degree of a vertex**

- It is the number of edges connecting to a vertex.
- In the example below, degree of vertex a is 5 and the rest have degree 1.
- It is represented as $\deg(a)=5$.



Fundamentals of Graph Theory

- **Parallel Edges**
 - If a pair of vertices is connected by more than one edge, then those edges are called parallel edges.

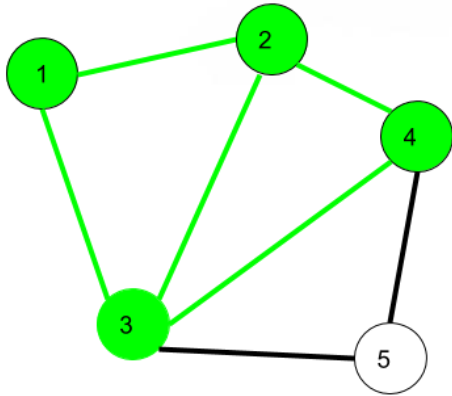




Fundamentals of Graph Theory

- **Neighbour or adjacent**
 - Node j is neighbour of node i , if and only if node i is connected to node j .

Fundamentals of Graph Theory



- **Walk**

- A list of edges that are sequentially connected to form a continuous route on a network.

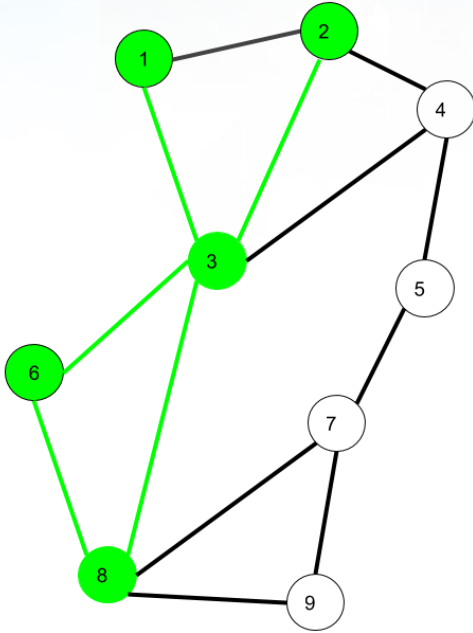
- **Open walk-** A walk is said to be an open walk if the starting and ending vertices are different i.e. the origin vertex and terminal vertex are different.

- $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 3$ is an open walk.

- **Closed walk-** A walk is said to be a closed walk if the starting and ending vertices are identical i.e. if a walk starts and ends at the same vertex, then it is said to be a closed walk.

- $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 3 \rightarrow 1$ is a closed walk.

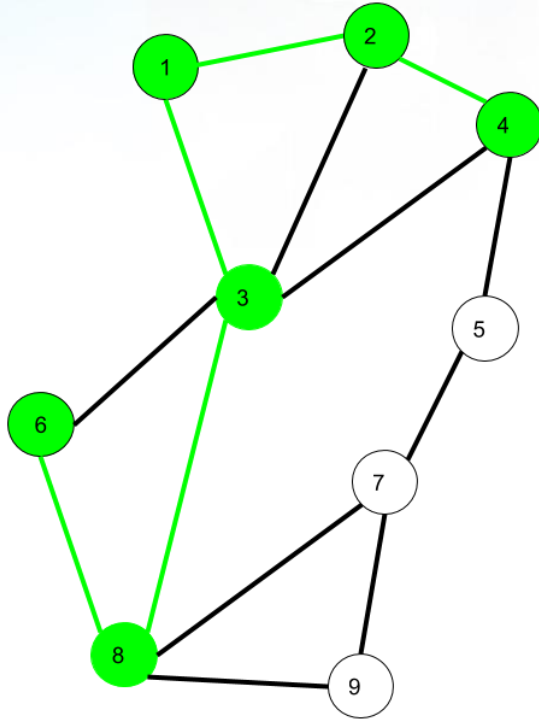
Fundamentals of Graph Theory



- **Trail**

- A walk that doesn't go through any edge more than once.
- Trail is an open walk in which no edge is repeated.
- Here $1 \rightarrow 3 \rightarrow 8 \rightarrow 6 \rightarrow 3 \rightarrow 2$ is trail
Also $1 \rightarrow 3 \rightarrow 8 \rightarrow 6 \rightarrow 3 \rightarrow 2 \rightarrow 1$ will be a closed trail

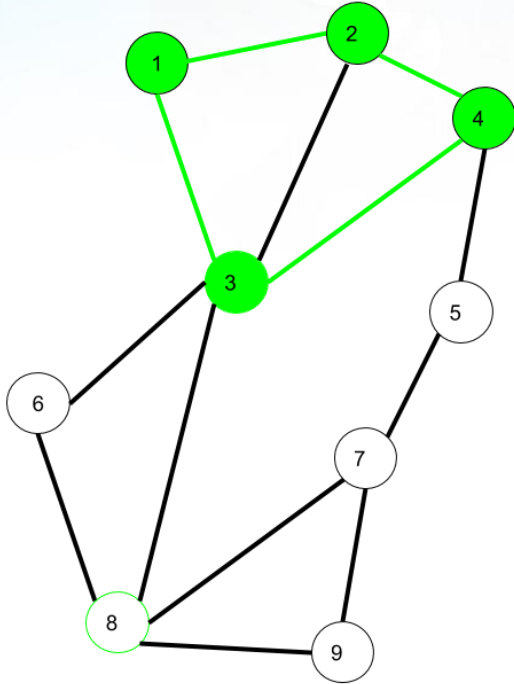
Fundamentals of Graph Theory



- **Path**

- A walk that doesn't go through any node (and therefore any edge, too) more than once.
- It is a trail in which neither vertices nor edges are repeated i.e. if we traverse a graph such that we do not repeat a vertex and nor we repeat an edge.
- As path is also a trail, thus it is also an open walk.
- Vertex not repeated, Edge not repeated.
- Here $6 \rightarrow 8 \rightarrow 3 \rightarrow 1 \rightarrow 2 \rightarrow 4$ is a Path

Fundamentals of Graph Theory



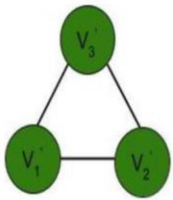
- **Cycle**

- A walk that starts and ends at the same node without going through any node more than once on its way.
- Traversing a graph such that we do not repeat a vertex nor we repeat a edge but the starting and ending vertex must be same i.e. we can repeat starting and ending vertex only then we get a cycle.
- Vertex not repeated. Edge not repeated.
- Here $1 \rightarrow 2 \rightarrow 4 \rightarrow 3 \rightarrow 1$ is a cycle.

Fundamentals of Graph Theory

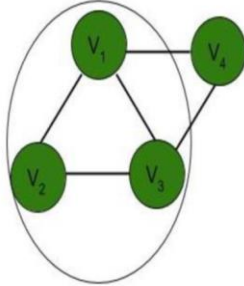
- **Subgraph**

- Part of the graph.
- **Definition:** A graph whose vertices and edges are subsets of another graph.
- **Formal Definition:** A graph $G'=(V', E')$ is a subgraph of another graph $G=(V, E)$ if
 - $V' \subseteq V$, and
 - $E' \subseteq E \wedge ((v_1, v_2) \in E' \rightarrow v_1, v_2 \in V')$.



Graph G_1

Subgraph G



Graph G_2



Thanks.....