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19CSE337 Social Networking Security

Lecture 11

Topics to Discuss

- PageRank
- Katz Centrality

PageRank

- PageRank (PR) is an algorithm used by Google Search to rank websites in their search engine results.
- PageRank was named after Larry Page, one of the founders of Google.
- PageRank is a way of measuring the importance of website pages.
- According to Google: PageRank works by counting the number and quality of links to a page to determine a rough estimate of how important the website is. The underlying assumption is that more important websites are likely to receive more links from other websites.

PageRank

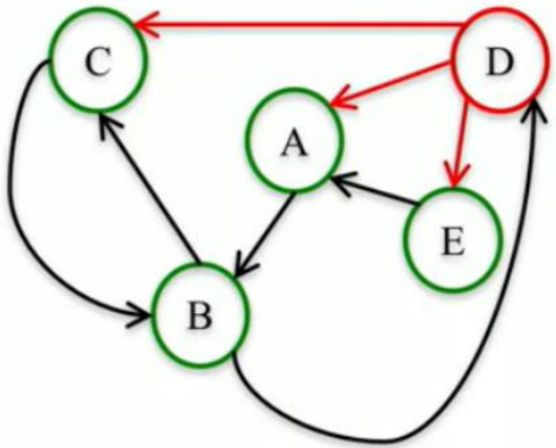
- PageRank is a variant of Eigenvector centrality.
- The difference from eigenvector is that PageRank takes link direction and weight into account to calculate centrality.
- So, the links can pass influence only in one direction and pass different amounts of influence.
- Each node in a network is assigned a score based on its number of incoming links (its indegree) or in other words based on their connections, and their connections' connections.
- These links are also weighted depending on the relative score of its originating node.
- The nodes with many incoming links are influential, and nodes to which they are connected share some of that influence.

PageRank

- **What does PageRank centrality tell me?**
  - Like Eigenvector Centrality, PageRank can help uncover influential or important nodes whose reach extends beyond just their direct connections. It’s especially useful in scenarios where link direction is important.
  - Understanding citations (e.g; patent citations, academic citations).
  - Visualizing IT network activity.
  - Modeling the impact of SEO and link building activity.

PageRank Computation

- Calculate the page rank of the following graph.

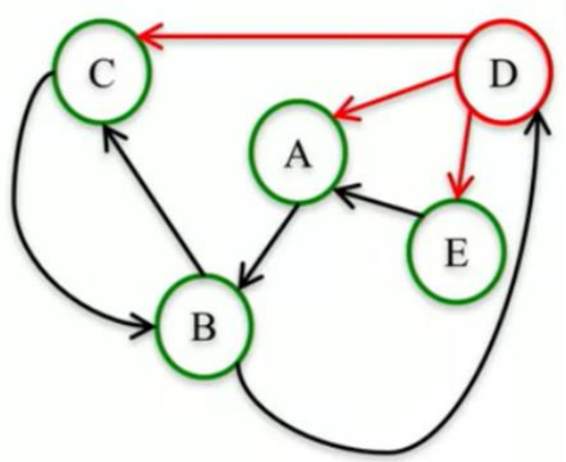


PageRank Computation

- Assume each node has a page rank value of (1/n), where n is the total number of nodes in the graph.
- So, in our example, initially each node has a page rank value of (1/5).
- By applying basic page rank updating rules K times, we will arrive at final page rank value of every node.

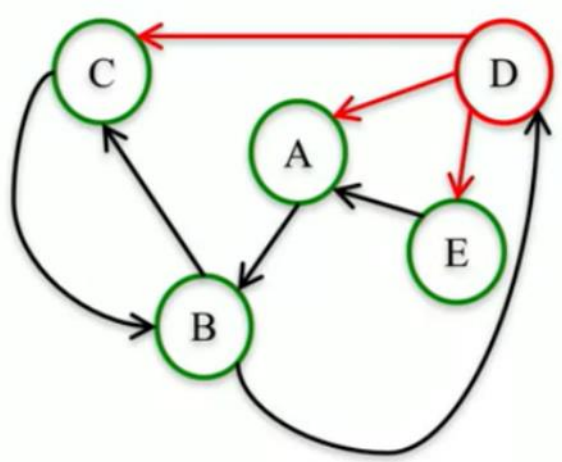
PageRank Computation

- Let's start with node A.
- Nodes D and E point to node A, so A is going to get PageRank from D and E.
- Now let's think about how much **PageRank A** is going to receive from each one of those two nodes.
- If we look at **D**, D has three edges, that points to three different nodes, C, A, and E.
- Therefore, A is going to receive 1/3 of the current page rank that D has.
- D currently has 1/ 5 PageRank, and so A is going to get 1/3 of that 1/5 PageRank that D has.



PageRank Computation

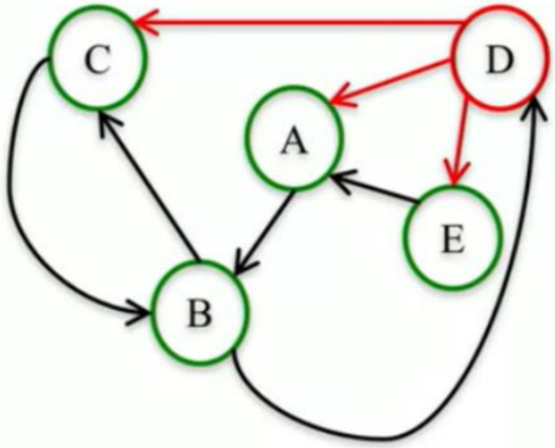
- Now A is also going to get PageRank from node **E**.
- Since E only points to A, then it's going to give all of its PageRank to node A.
- So A is going to get 1/5 PageRank from node E.
- Therefore, in total, A is going to get 4/15 PageRank from those two nodes.
- Hence, the new value PageRank of node A is 4/15.
- Repeat the above procedure for every nodes.
- Repeat this process for K times until



PageRank Computation

K=1					
	A	B	C	D	E
Old PR Value	1/5	1/5	1/5	1/5	1/5
Updated PR	4/15	2/5	1/6	1/10	1/15
Value	0.267	0.4	0.16	0.1	0.067

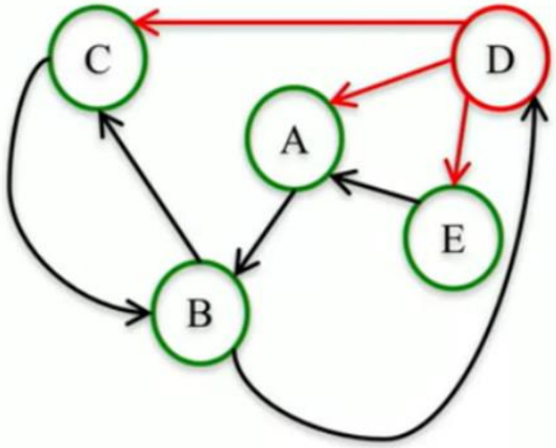
Calculation	
A	(1/3)*(1/5)+(1/1)*(1/5)=(1/15)+(1/5)=(4/15)
B	(1/1/)*(1/5)+(1/1)*(1/5)=(2/5)
C	(1/3)*(1/5)+(1/2)*(1/5)=(5/30)=(1/6)
D	(1/2)*(1/5)=(1/10)
E	(1/3)*(1/5)=(1/15)



PageRank Computation

K=2					
	A	B	C	D	E
Old PR Value	4/15	2/5	1/6	1/10	1/15
Updated PR	1/10	13/30	7/30	1/5	1/30
Value	0.1	0.43	0.23	0.2	0.033

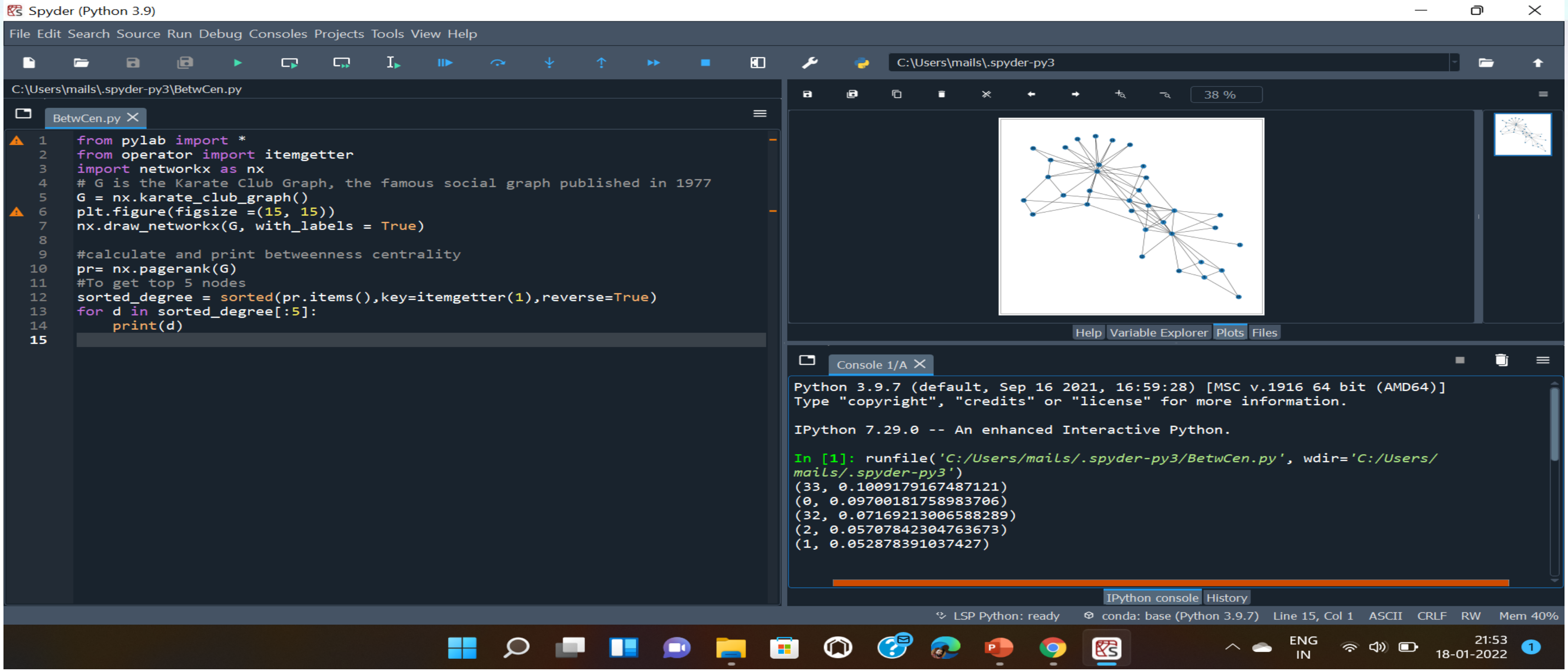
Calculation	
A	(1/3)*(1/10)+(1/1)*(1/15)=(1/30)+(1/15)=(1/10)
B	(1/1/)*(4/15)+(1/1)*(1/6)=(39/90)=(13/30)
C	(1/3)*(1/10)+(1/2)*(2/5)=(1/30)+(2/10)=(7/30)
D	(1/2)*(2/5)=(2/10)=(1/5)
E	(1/3)*(1/10)=(1/30)



PageRank Computation

- From the results obtained after two steps we find that node B has the highest PageRank (0.43), followed by node C, then node D, node A, and E.
- Hence Bis most important in this network.
- **What happens if we continue for another step?** If we continue with more steps, we might notice that the values change a little bit, but they still have the same order and B is still the highest PageRank node.

PageRank using NetworkX



Katz Centrality

- Katz centrality is a measure of centrality in a network.
- It was introduced by Leo Katz in 1950.
- It is a variant of eigenvector centrality which overcomes the problems of eigenvector centrality.
- Katz centrality is mainly used for directed acyclic graphs.
- Katz centrality measures influence by taking into account the total number of walks between a pair of nodes.

Katz Centrality

- The adjacency matrix will be constructed based on the direction of links. The matrix will be asymmetric in nature.
- In directed networks, if a node is not pointed by any other nodes, its centrality will be zero and so its neighbours too!
- Katz centrality assigns a minimum score to all nodes and include this in calculation.

Katz Centrality

- Katz centrality of node i is $x_i = 1/\lambda \sum x_j + \beta$
- First part in the above formula is normal eigenvector centrality and second part is bias or free centrality score.
- Usually, we set $\beta = 1$ for the computation.

Thanks...........