Arithmetic, Logical and Matrix operations in R

In this lecture

- Arithmetic
- Logical
- Matrix operations

Arithmetic operations in R

Arithmetic operations in R

Symbols	Operation
=, < −*	***Assignment***
+	***Addition***
-	***Subtraction***
*	***Multiplication***
/	***Division***
^,**	***Exponent***
%%	***Remainder***
%/%	***Integer division***

**In R only ‘< −' is valid for assignment operation where as in R Studio both = and < −' will work*

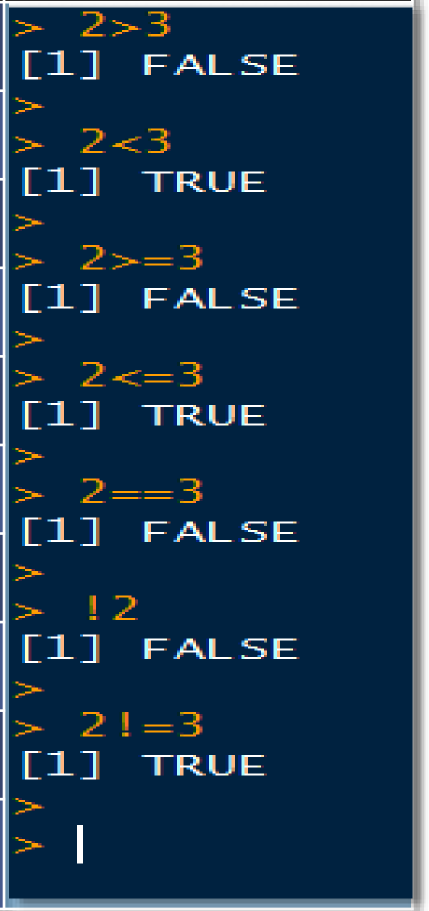
Hierarchy of operations

$$A = 7 - 2\,x\,\frac{27}{3^2} + 4$$

Order of Precedence	Operation
Bracket	( )
Exponent	^,**
Division	/
Multiplication	*
Addition and subtraction	+,-

Logical operations in R

Logical operations in R

Symbols	Operation	Examples
<	*Less than*	
<=	*Less than equal to*	
>	*Greater than*	
>=	*Greater than equal to*	
==	*Exactly equal to*	
!=	*Not equal to*	
!	*Not*	
|	*Or*	
&	*And*	
isTRUE	*Test if variable is TRUE*	

```
> 2>3
[1] FALSE
>
> 2<3
[1] TRUE
>
> 2>=3
[1] FALSE
>
> 2<=3
[1] TRUE
>
> 2==3
[1] FALSE
>
> !2
[1] FALSE
>
> 2!=3
[1] TRUE
>
> 
```

Matrix operations in R

Matrices

A matrix is a rectangular arrangement of numbers in rows and columns

Rows run horizontally and columns run vertically

$$\begin{pmatrix} 1 & 5 & 3 \\ 4 & 9 & 2 \\ 5 & 6 & 7 \end{pmatrix} \qquad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 4 & 5 \end{bmatrix}$$

Creating matrices

Follow these steps to create a matrix

1. Open a curve bracket,

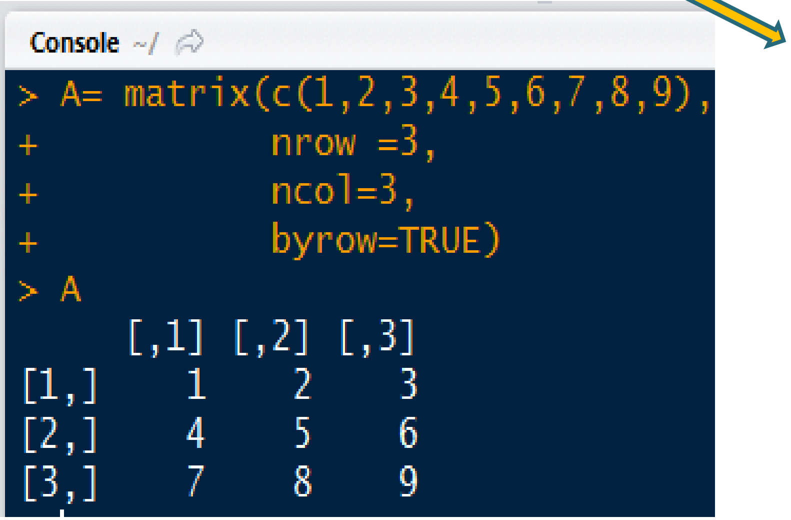
A = matrix()

2. Enter the sequence of elements,

A= matrix(c(1,2,3,4,5,6,7,8,9))

3. Specify the parameters nrow, ncol, byrow

A= matrix(c(1,2,3,4,5,6,7,8,9), nrow =3, ncol=3, byrow=TRUE)



```
> A= matrix(c(1,2,3,4,5,6,7,8,9),
+           nrow =3,
+           ncol=3,
+           byrow=TRUE)
> A
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6
[3,]    7    8    9
```

This parameter decides how values in the vector would be assigned i.e. “by row” or not

Creating special matrices

Different ways of creating matrices:

a. **Matrix where all rows and columns are filled by a single constant 'k'.**

  - ➢ **For k=3, with 'm' rows & 'n' columns**

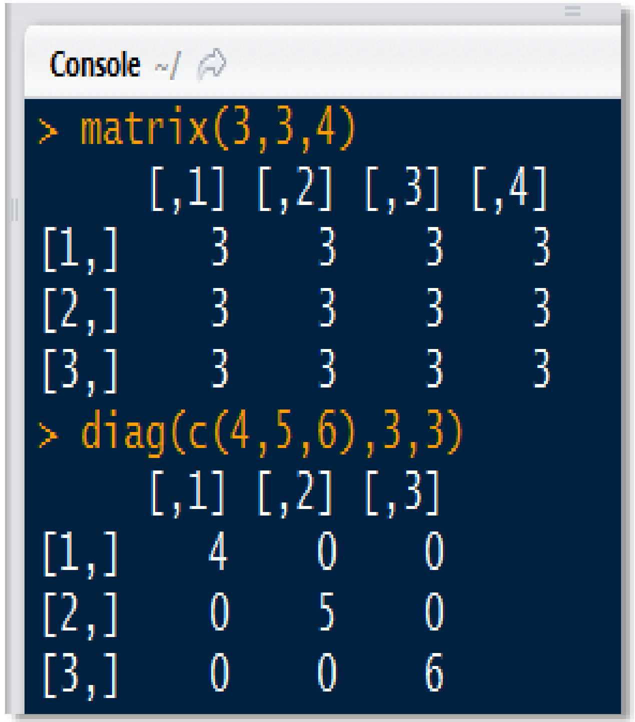
    Command :matrix(3,m,n)

b. **Diagonal matrix:**

  - ➢ **Values in diagonal, similar to 'matrix()'.**
  - ➢ **Mention 'k' as constant/array in first parameter.**
  - ➢ **Command: diag(k,m,n)**

c. **Identity matrix:**

  - ➢ **Use 'diag()' command with k=1**



```
> matrix(3,3,4)
     [,1] [,2] [,3] [,4]
[1,]    3    3    3    3
[2,]    3    3    3    3
[3,]    3    3    3    3
> diag(c(4,5,6),3,3)
     [,1] [,2] [,3]
[1,]    4    0    0
[2,]    0    5    0
[3,]    0    0    6
```

```
>diag(1,3,3)
[,1] [,2] [,3]
[1,] 1 0 0
[2,] 0 1 0
[3,] 0 0 1
```

Exercise: Creating matrices

Create the following matrices in R

$$\begin{bmatrix} 3 & 5 \\ -2 & 0 \end{bmatrix} \quad , \quad \begin{bmatrix} 1 & 10 \\ 3 & -1 \\ 7 & 5 \end{bmatrix}$$

and

$$\begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 2 \\ -1 & -2 & -3 \\ 5 & 4 & 3 \end{bmatrix}$$

Matrix metrics

# create a matrix A

A= matrix(c(1,2,3,4,5,6,7,8,9), nrow =3, ncol=3, byrow=TRUE)

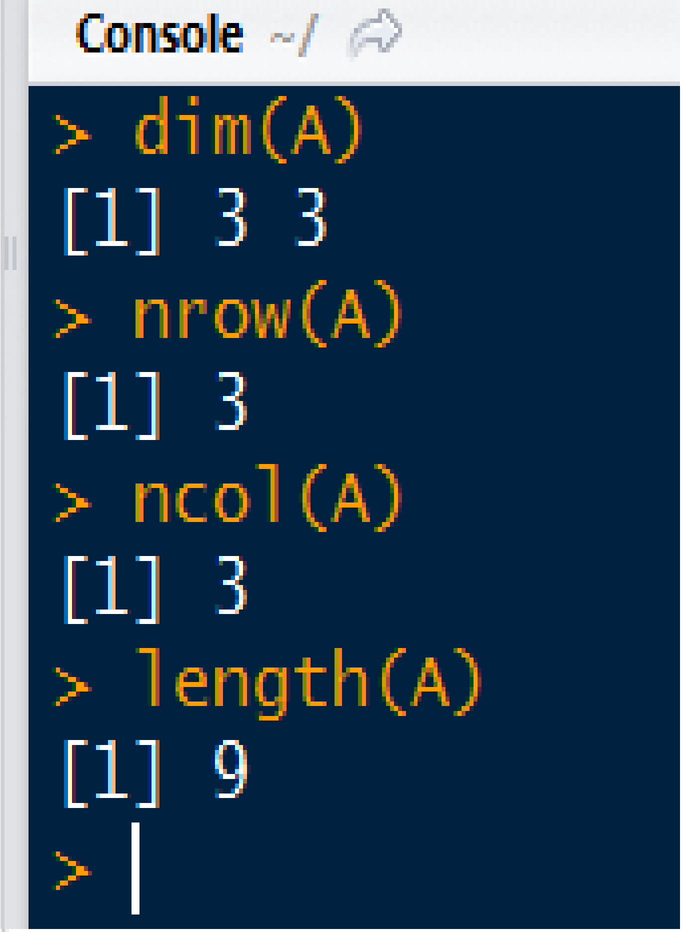
Finding the size of the matrix, A :

dim(A) will return the size of the matrix

nrow(A) will return the number of rows

ncol(A) will return the number of columns

prod(dim(A)) or length(A) will return the number of elements



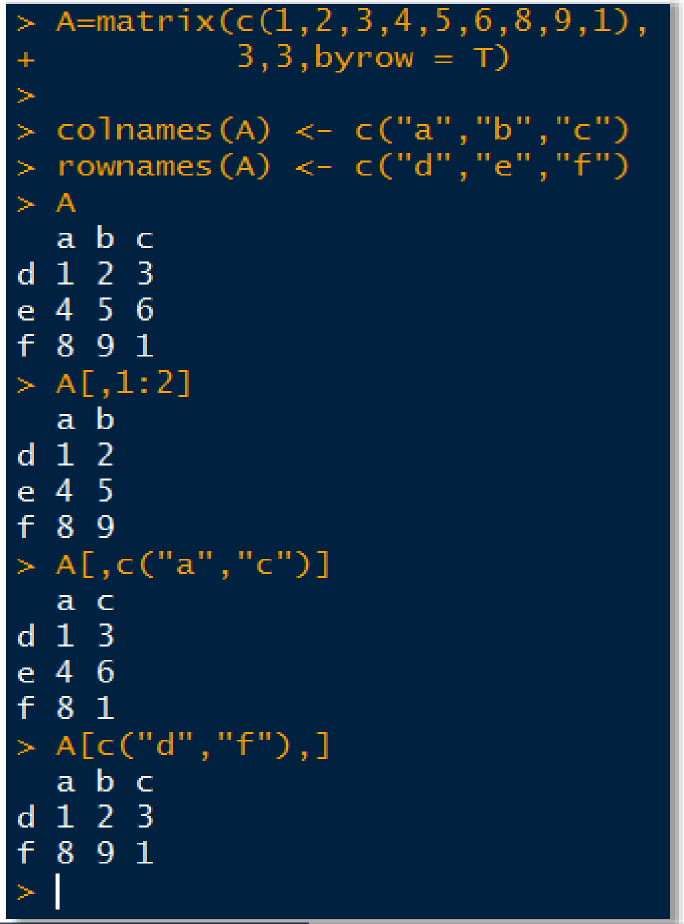
```
> dim(A)
[1] 3 3
> nrow(A)
[1] 3
> ncol(A)
[1] 3
> length(A)
[1] 9
> |
```

Accessing, editing, deleting in elements in matrices

They follow the same convention as dataframes such as

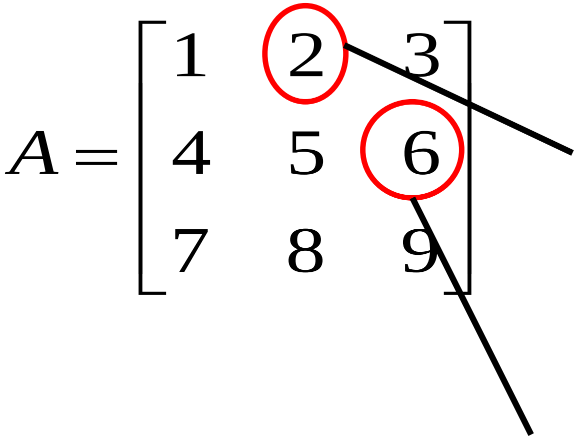
- Array/value before “,” for accessing rows
- Array/value before “,” for accessing columns
- use of ‘-’ for removing rows/columns
- Strings can be assigned as names of rows and columns using:
  - rownames() and Colnames()

$$A=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 8 & 9 & 1\end{bmatrix}_{3\times 3}$$



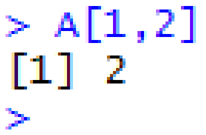
```
> A=matrix(c(1,2,3,4,5,6,8,9,1),
+          3,3,byrow = T)
>
> colnames(A) <- c("a","b","c")
> rownames(A) <- c("d","e","f")
> A
  a b c
d 1 2 3
e 4 5 6
f 8 9 1
> A[,1:2]
  a b
d 1 2
e 4 5
f 8 9
> A[,c("a","c")]
  a c
d 1 3
e 4 6
f 8 1
> A[c("d","f"),]
  a b c
d 1 2 3
f 8 9 1
> |
```

Accessing an entry of a matrix



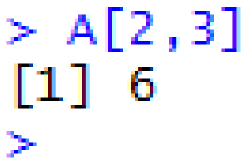
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

First row, second column



```
> A[1,2]
[1] 2
>
```

Second row, third column



```
> A[2,3]
[1] 6
>
```

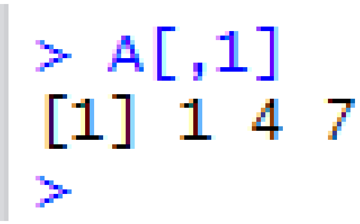
The part before the comma should be the row number
The part after the comma should be the column number

Accessing a column

- Specify the column index
- Leave the rows index unspecified
- This means accessing all row elements of the given column index

$$A = \begin{bmatrix} \boxed{\begin{matrix} 1 \\ 4 \\ 7 \end{matrix}} & \begin{matrix} 2 \\ 5 \\ 8 \end{matrix} & \begin{matrix} 3 \\ 6 \\ 9 \end{matrix} \end{bmatrix}$$

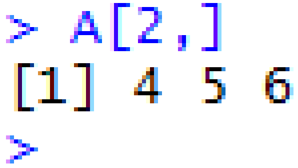
All rows in first column



```
> A[,1]
[1] 1 4 7
>
```

Accessing a row

$$A = \left[\begin{array}{ccc} 1 & 2 & 3 \\ \multicolumn{3}{c}{\boxed{4 \quad 5 \quad 6}} \\ 7 & 8 & 9 \end{array}\right]$$



```
> A[2,]
[1] 4 5 6
>
```

Leaving the column index empty means choose all the columns

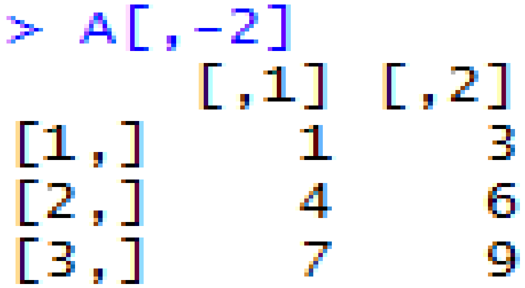
How do you access the last row ?

A[nrow(A),]

Accessing everything but one column

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

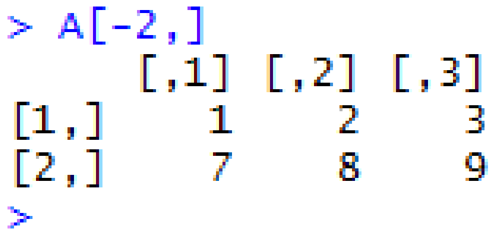
- Access the column that has to be avoided and then put a '-' sign in front of it
  - For example: A[,-2]
  - This will fetch all the columns except the 2nd column



```
> A[,-2]
     [,1] [,2]
[1,]    1    3
[2,]    4    6
[3,]    7    9
```

Accessing everything but one row

$$A = \begin{bmatrix} 1 & 2 & 3 \\ \multicolumn{3}{c}{\boxed{4 \quad 5 \quad 6}} \\ 7 & 8 & 9 \end{bmatrix}$$



```
> A[-2,]
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    7    8    9
>
```

- Access the row that has to be avoided and then put a ‘-’ sign in front of it
  - For example: A[-2,]
  - This will fetch all the row except the 2nd row

Exercise: Accessing elements of a matrix

Do the following in R

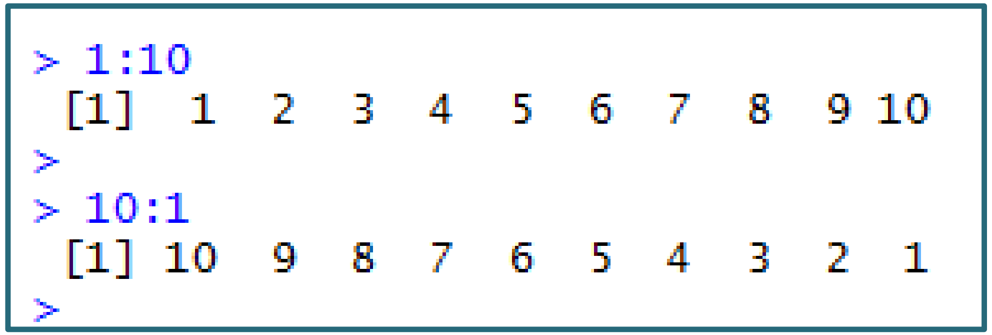
Assign the following matrix

$$A = \begin{bmatrix} 1 & 7 & 3 \\ 4 & 4 & 6 \\ 4 & 7 & 12 \end{bmatrix}$$

- Change the element 12 to 13
- Access the second row and the third column
- List all the elements in the second column and third row

Colon operator

Colon operator can be used to create a row matrix



```
> 1:10
 [1]  1  2  3  4  5  6  7  8  9 10
>
> 10:1
 [1] 10  9  8  7  6  5  4  3  2  1
>
```

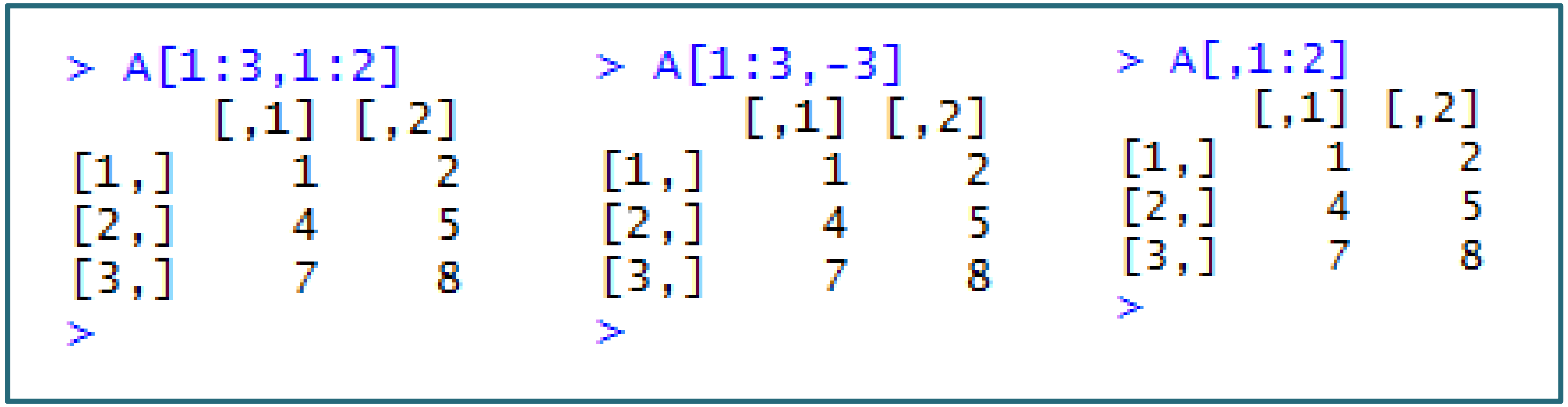
Colon operator: sub matrices selection

The colon notation can also be used to pick sub-matrices

$$A = \begin{bmatrix} \boxed{\begin{matrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{matrix}} & \begin{matrix} 3 \\ 6 \\ 9 \end{matrix} \end{bmatrix}_{3\times 3}$$

Sub-matrix

The sub-matrix occupies the **first three rows** and the **first two columns**



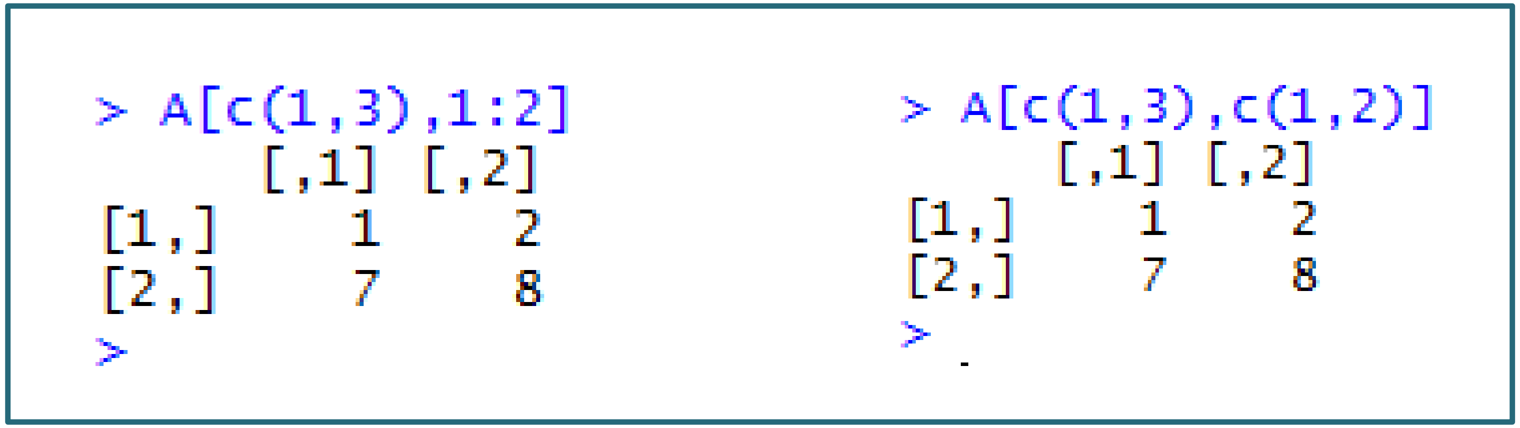
```
> A[1:3,1:2]
     [,1] [,2]
[1,]    1    2
[2,]    4    5
[3,]    7    8
>
```

```
> A[1:3,-3]
     [,1] [,2]
[1,]    1    2
[2,]    4    5
[3,]    7    8
>
```

```
> A[,1:2]
     [,1] [,2]
[1,]    1    2
[2,]    4    5
[3,]    7    8
>
```

Accessing submatrices: Example 2

$$A = \begin{bmatrix} \boxed{1 \quad 2} & 3 \\ 4 \quad\; 5 & 6 \\ \boxed{7 \quad 8} & 9 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 7 & 8 \end{bmatrix}$$



```
> A[c(1,3),1:2]
     [,1] [,2]
[1,]    1    2
[2,]    7    8
>
```

```
> A[c(1,3),c(1,2)]
     [,1] [,2]
[1,]    1    2
[2,]    7    8
>
```

Exercise: Accessing sub-matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

How do you access this sub-matrix $\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$

Matrix concatenation

- Matrix concatenation refers to merging of a row or column to a matrix
  - Concatenation of a row to a matrix is done using **rbind()**
  - Concatenation of a column to a matrix is done using **cbind()**
  - Consistency of the dimensions between the matrix and the vector should be checked before concatenation

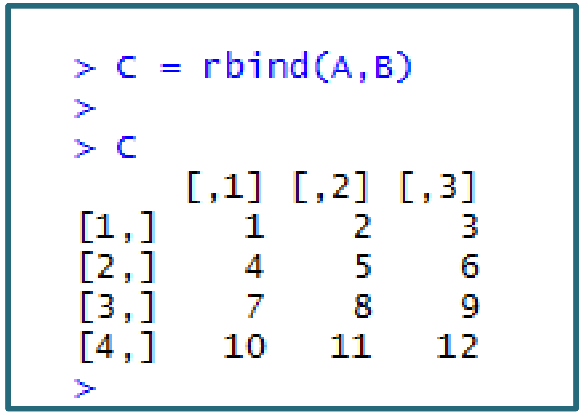
Matrix concatenation – rbind()

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 10 & 11 & 12 \end{bmatrix}$$

Use rbind() to append B row vector to the rows of A

$$C = \begin{bmatrix} A \\ B \end{bmatrix}$$



```
> C = rbind(A,B)
>
> C
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    4    5    6
[3,]    7    8    9
[4,]   10   11   12
>
```

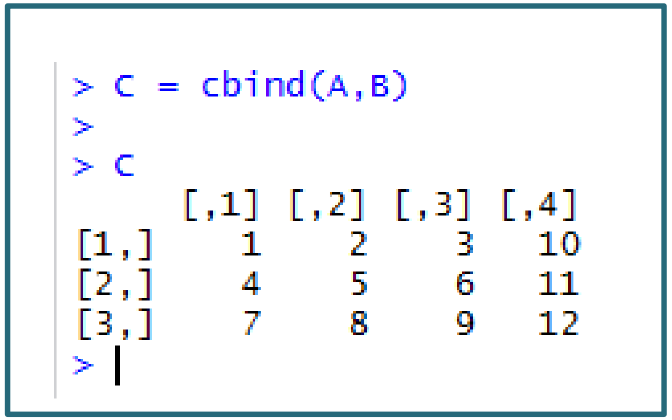
Matrix concatenation – cbind()

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$$

Use cbind() to append B column vector to the columns of A

$$C = \begin{bmatrix} A & B \end{bmatrix}$$



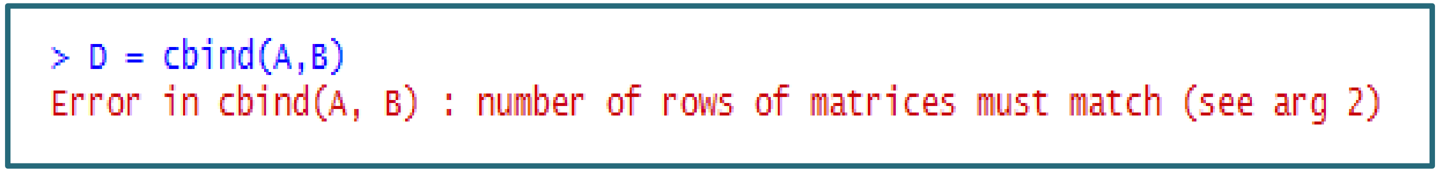
```
> C = cbind(A,B)
>
> C
     [,1] [,2] [,3] [,4]
[1,]    1    2    3   10
[2,]    4    5    6   11
[3,]    7    8    9   12
> |
```

Dimension inconsistency –cbind()

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 10 & 11 & 12 \end{bmatrix}$$

Can these two matrices be merged to give

$$C = \begin{bmatrix} A & B \end{bmatrix}$$



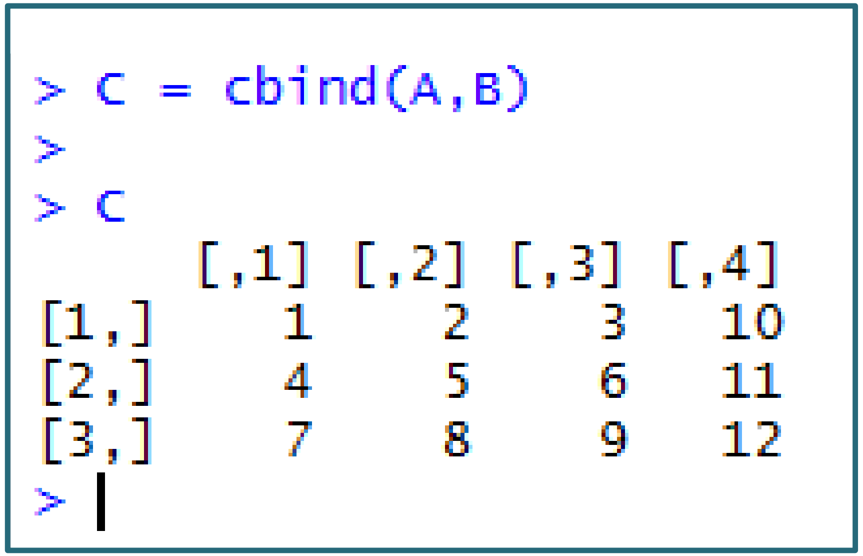
```
> D = cbind(A,B)
Error in cbind(A, B) : number of rows of matrices must match (see arg 2)
```

Fixing the dimension inconsistency

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$$

$$C = \begin{bmatrix} A & B \end{bmatrix}$$



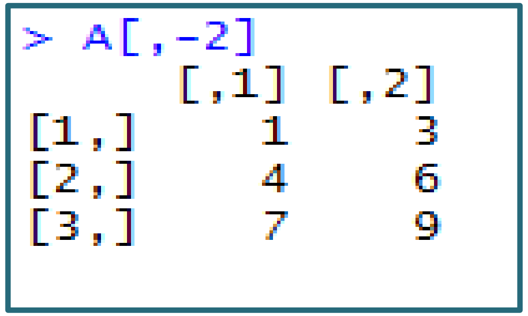
```
> C = cbind(A,B)
>
> C
     [,1] [,2] [,3] [,4]
[1,]    1    2    3   10
[2,]    4    5    6   11
[3,]    7    8    9   12
>
```

Deleting a column

$$A = \left[\begin{matrix}1\\4\\7\end{matrix}\quad\boxed{\begin{matrix}2\\5\\8\end{matrix}}\quad\begin{matrix}3\\6\\9\end{matrix}\right]$$

- Access the column that has to be deleted and then put a ‘-’ sign in front of it
  - For example: A=A[,-2]
  - This will fetch all the columns

    except the 2nd column

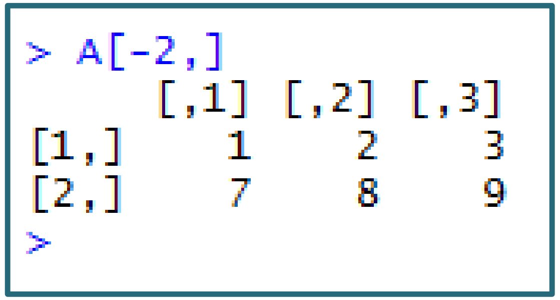


```
> A[,-2]
     [,1] [,2]
[1,]    1    3
[2,]    4    6
[3,]    7    9
```

Deleting a row

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

- Access the row that has to be deleted and then put a ‘-’ sign in front of it
  - For example: A=A[-2,]
  - This will fetch all the rows except the 2nd row



```
> A[-2,]
     [,1] [,2] [,3]
[1,]    1    2    3
[2,]    7    8    9
>
```

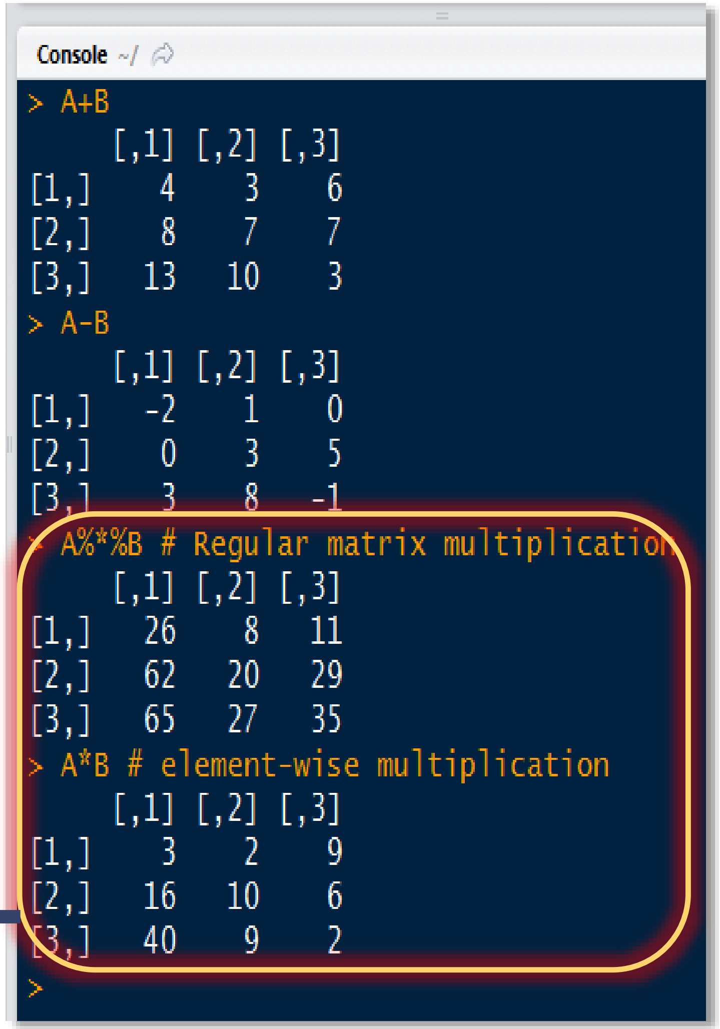
Matrix algebra

- **Addition/subtraction**
- **Multiplication**
- **Matrix Operations in R**
- **Matrix Division**

Matrix addition/subtraction & multiplication

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 9 & 1 \end{bmatrix}_{3\times 3}$$

$$B = \begin{bmatrix} 3 & 1 & 3 \\ 4 & 2 & 1 \\ 5 & 1 & 2 \end{bmatrix}_{3\times 3}$$



```
> A+B
     [,1] [,2] [,3]
[1,]    4    3    6
[2,]    8    7    7
[3,]   13   10    3
> A-B
     [,1] [,2] [,3]
[1,]   -2    1    0
[2,]    0    3    5
[3,]    3    8   -1
> A%*%B # Regular matrix multiplication
     [,1] [,2] [,3]
[1,]   26    8   11
[2,]   62   20   29
[3,]   65   27   35
> A*B # element-wise multiplication
     [,1] [,2] [,3]
[1,]    3    2    9
[2,]   16   10    6
[3,]   40    9    2
>
```

Element-wise multiplication is based on multiplication between corresponding elements of two matrices.

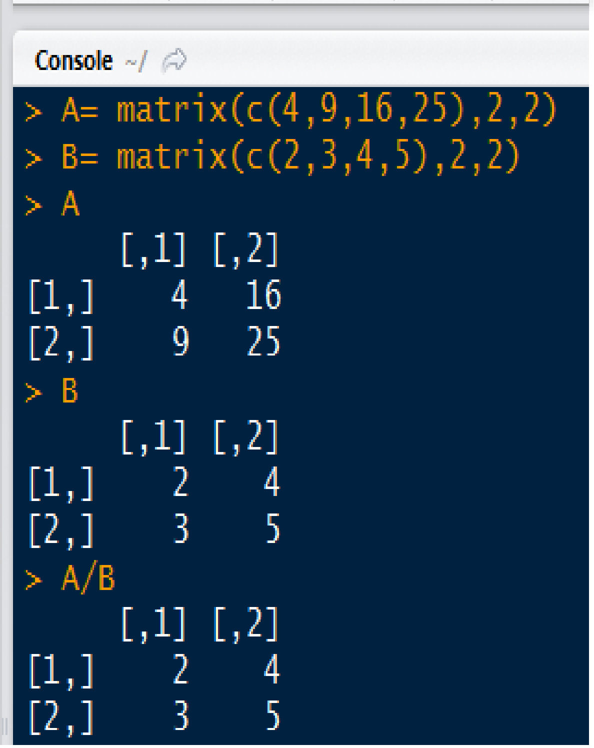
Matrix division

WARNING :

The following operation is not inverse of a matrix but element wise division between matrices A & B.

$$A=\begin{bmatrix}4 & 9\\16 & 25\end{bmatrix}\quad B=\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}$$

$$\boxed{A/B=\frac{a_{ij}}{b_{ij}}}$$



```
> A= matrix(c(4,9,16,25),2,2)
> B= matrix(c(2,3,4,5),2,2)
> A
     [,1] [,2]
[1,]    4   16
[2,]    9   25
> B
     [,1] [,2]
[1,]    2    4
[2,]    3    5
> A/B
     [,1] [,2]
[1,]    2    4
[2,]    3    5
```