

Clocks

The dial of a clock is a circle whose circumference is divided into 12 parts, called hour spaces. Each hour space is further divided into 5 parts, called minute's spaces. This way, the whole circumference is divided into $12 \times 5 = 60$ minute spaces.

If we consider clock as a circular track and the two hands of clock minute hand and hour hand are just like two players running in the same direction. Let total length of the track is 360° and minute hand completes one full round in 1 hour while hour hand cover full round in 12 hours.

$$\text{Speed of hour hand} = \frac{360^\circ}{60} = 6^\circ \text{ per second}$$

$$\text{Speed of minute hand} = \frac{360^\circ}{60 \times 12} = \frac{1^\circ}{2} \text{ per second}$$

TIP

- a. Minute hand and hour hand meet 11 times in 12 hours
- b. Hands of clock will be 22 times at right angle in 12 hours.

Since they are moving in the same direction, so the relative speed of both the hands with respect to each

$$\text{other} = \left(6 - \frac{1}{2}\right)^\circ = \left(\frac{11}{2}\right)^\circ \text{ per second.}$$

Time taken by minute hand to overtake hour hand =

$$\frac{\text{Distance}}{\text{Relative speed}} = \frac{360^\circ}{\left(\frac{11}{2}\right)^\circ} = 65\frac{5}{11} \text{ minutes}$$

There are 4 types of problems on clocks:

1. To calculate the angle between the two hands when time is given.
2. To calculate the time when both the hands will be at some angle.
3. Concept of slow and fast clocks.
4. Overall gain/loss

Calculating the angle

The angle between the two hands is given by the following formula

$$\text{Formula for the angle between the hands} = \left| 30H - \frac{11}{2}M \right|^\circ$$

Where H → Hour reading & M → Minute reading

Ex.1 Calculate the angle between the two hands of clock when the clock shows 5 : 25 p.m.

Sol. Given time = 5 : 25 p.m. Hence H = 5 and M = 25

We can apply the following direct formula to find the angle between the hands

$$= \left| 30H - \frac{11}{2}M \right|^\circ$$

$$\begin{aligned} \text{Required angle} &= \left| 30 \times 5 - \frac{11}{2} \times 25 \right|^\circ = (150 - 137.5)^\circ \\ &= 12.5^\circ. \end{aligned}$$

Alternative Method:

Since at 5 : 25 the minute hand will be at 5 and the angle between them will be same as the distance covered in degree by the hour hand in 25 minutes

$$\text{Required angle} = \text{distance of hour hand} = \text{speed} \times \text{time} = \left(\frac{1}{2} \right)^\circ \times 25 = 12.5^\circ$$

Calculating the time

To calculate the time when both the hands will be at some angle

$$\text{In one minute the net gain of minute hand over hour hand} = \left(6 - \frac{1}{2} \right)^\circ = 5\frac{1}{2}^\circ = \frac{11}{2}^\circ$$

If the gain is $\frac{11}{2}^\circ$ then the time is 1 min.

If the gain is 1° then the time is $\frac{2}{11}$ min

If the gain is x° then the time is $\frac{2}{11} \times x$ min.

If between H and (H + 1) o'clock, the two hands are together at an angle θ then required time = $\frac{2}{11} [H \times 30^\circ \pm \theta]$ minutes, Where H is reading of hour.

Ex.2 At what time between 4 and 5 o'clock are the hands of the clock together?

Sol. Method 1: At 4 o'clock, the hour hand is at 4 and the minute hand is at 12. It means that they are 20 min spaces apart. To be together, the minute hand must gain 20 minutes over the hour hand. Now, we know that 55 min. are gained in 60 min.

$$\therefore 20 \text{ min are gained in } \frac{60}{55} \times 20 = \frac{240}{11} = 21\frac{9}{11} \text{ min.}$$

Therefore, the hands will be together at $21\frac{9}{11}$ min past 4.

Alternate method: Using the formula:

$$\text{Required time} = \frac{2}{11} [H \times 30^\circ \pm \theta]$$

Here $\theta = 0^\circ$ (Hands of clock are together) and $H = 4$

$$\therefore \text{Required time} = \frac{2}{11} [4 \times 30^\circ \pm 0] = \frac{240}{11} = 21 \frac{9}{11} \text{ min.}$$

Therefore, the hands will be together at $21 \frac{9}{11}$ min past 4.

Another formula: Between H and $(H + 1)$ o'clock, the two hands will be together at

$$5 \times H \times \left(\frac{12}{11} \right) \text{ min past } H.$$

$$\text{In this case; } 5 \times 4 \times \left(\frac{12}{11} \right) = 21 \frac{9}{11} \text{ min past } 4$$

Ex.3 At what time between 4 and 5 o'clock will the hand of clock be at right angle?

Sol. At 4 o'clock there are 20 min. spaces between hour and minute hands. To be at right angle, they should be 15 min spaces apart.

So, there are two cases:

Case I: When the minute hand is 15 min spaces behind the hour hand.

To be in this position, the min hand should have to gain $20 - 15 = 5$ min spaces.

Now, we know that 55 min spaces are gained in 60 min.

$$\therefore 5 \text{ min spaces are gained in } \frac{60}{55} \times 5 = 5 \frac{5}{11} \text{ min}$$

$$\therefore \text{They are at right angle at } 5 \frac{5}{11} \text{ min past } 4.$$

Case II: When the minute hand is 15 min spaces ahead of the hour hand.

To be in this position, the min hand should have to gain $20 + 15 = 35$ min spaces.

Now, we know that 55 min spaces are gained in 60 min

$$\therefore 35 \text{ min spaces will be gained in } \frac{60}{55} \times 35 = 38 \frac{2}{11} \text{ min}$$

$$\therefore \text{They are at right angle at } 38 \frac{2}{11} \text{ min past } 4.$$

Alternate method:

As the hands of the clock are at right angle therefore $\theta = 90^\circ$

Also time is between 4 and 5 o'clock, no. of hours = 4

$$\begin{aligned} \text{Required time} &= \frac{2}{11} [(\text{no. of hours}) \times 30^\circ \pm \theta] \\ &= \frac{2}{11} [4 \times 30^\circ \pm 90^\circ] = \frac{420}{11} \text{ or } \frac{60}{11} = 38 \frac{2}{11} \text{ min or } 5 \frac{5}{11} \text{ min} \end{aligned}$$

∴ They are at right angle at $5\frac{5}{11}$ min or $38\frac{2}{11}$ min past 4.

Another Formula: Between x and (x + 1) o'clock the two hands are at angle at

$$(5x \pm 15) \times \frac{12}{11} \text{ min past } x$$

In the case; they will be at right angle at

$$(5 \times 4 - 15) \times \frac{12}{11} \text{ and } (5 \times 4 + 15) \times \frac{12}{11} \text{ min past } 4$$

$$\text{or } 5\frac{5}{11} \text{ min and } 38\frac{2}{11} \text{ min past } 4.$$

Concept of slow and fast clocks

Too Fast and Too Slow:

If a watch indicates 9.20 when the correct time is 9.10, it is said to be 10 minutes too fast. And if it indicates 9.00 when the correct time is 9.10, it is said to be 10 minutes too slow.

Ex.4 Two clocks are set at 1 p.m. Fast clock gains 1 min for every hour. Find the time when the fast clock shows 6 p.m.

Sol. For every 60 min of true clock, the fast clock will show 61 min.

For 61 minutes of fast clock, true time = 60 minutes

$$\text{For 300 minutes (5 hrs) of fast clock, true time} = \frac{60}{61} \times 300$$

$$= \frac{18000}{61} = 295\frac{5}{61} \text{ min.} = 240 + 55\frac{5}{61} \text{ min}$$

$$\text{Actual time in the true clock} = 5 : 55\frac{5}{61} \text{ hrs.}$$

Overall gain/loss

After every $65\frac{5}{11}$ min. = $\frac{720}{11}$ min. the two hands will coincide. If the hands of a clock coincide every 'x' min,

then

Gain/loss per day by a watch, is given by

$$\left(\frac{720}{11} - x \right) \times \frac{60 \times 24}{x}$$

[If answer is (+) then there will be gain and if (–) then there will be loss.]

Calendar

There are 12 months in 1 year.

Jan –31 (days), Feb –28; 29 (for a leap year), Mar –31, Apr – 30, May –31,

June – 30; July – 31; Aug – 31; Sept –30; Oct – 31; Nov –30; Dec – 31.

Total 365 (ordinary year); 366 days (for a leap year)

To find the number of weeks in an ordinary year:

$$\frac{365}{7} = 52 \text{ weeks} + 1 \text{ odd day (remainder)}$$

To find the number of weeks in a leap year:

$$\frac{366}{7} = 52 \text{ weeks} + 2 \text{ odd days (remainder)}$$

Leap year: It is so called as it comes after a leap of 3 years from the previous leap year.

Leap Year

Method to find whether a given year is a leap year or an ordinary year.

Every year which is not a century year (i.e. which is not a multiple of 100) is a leap year if and only if it is completely divisible by 4.

Every century year is a leap year if and only if it is completely divisible by 400 or is an integral multiple of 400 (i.e. the remainder ought to be 0).

e.g. 2000 is a leap year.

1900 is not.

1996 is a leap year, 1998 is not.

Odd day's concept

To find the number of odd days in a century

A century, i.e. 100 year has 76 ordinary year and 24 leap year.

$$= [(76 \times 52) \text{ weeks} + 76 \text{ days}] - [(24 \times 52) \text{ weeks} + 24 \times 2 \text{ days}]$$

$$= 5200 \text{ weeks} + 124 \text{ days}$$

$$= 5200 \text{ weeks} + 17 \text{ weeks} + 5 \text{ odd days}$$

$$= 5217 \text{ weeks} + 5 \text{ odd days}$$

Therefore, 100 years contain 5 odd days.

Now, (i) 200 years contain $5 \times 2 = 10$, i.e., 3 odd days.

(ii) 300 years contain $5 \times 3 = 15$ i.e., 1 odd day.

(iii) 400 years contain $5 \times 4 + 1 = 21$, i.e., no odd day.

Similarly, 800, 1200 years etc. contain no odd day.

- Note:** (i) $5 \times 2 = 10$ days = 1 week + 3 days i.e., 3 odd days
(ii) $5 \times 3 = 15$ days = 2 weeks + 1 day i.e. 1 odd day.
(iii) 400th year is a leap year therefore one additional day is added.

Odd days and their numeral values

When we have to calculate the number of days on any particular Extra Days and their numeral Values

- 0 → Sunday
1 → Monday
2 → Tuesday
3 → Wednesday
4 → Thursday
5 → Friday
6 → Saturday

Ex.5 *Father of Nation Mahatma Gandhi died on 30th January 1948. What was the day on which he died?*

Sol. Up to 1600 AD we have 0 odd days, up to 1900 AD we have 1 odd day. Now for in 47 years we have 11 leap years and 36 normal years.

Odd days from 1901 to 1947 = $11 \times 2 + 36 \times 1 = 22 + 36 = 58$ odd days
= 8 weeks + 2 odd days

Total odd days up to 31st December 1947 = $1 + 2 = 3$ odd days

30 days of January contain only 4 weeks + 2 odd days

So 30th January 1948 has total 5 odd days

Day on 30th January 1948 = Friday.

Ex.6 *How does the number of odd days help us in finding the day of a week? (Please take care of this point)*

When a specific day is given:

Suppose a question like: Jan 1, 1992 was Wednesday. What day of the week will it be on Jan 1, 1993? If you recall, 1992 being a leap year it has 2 odd days. So, the above said day will be two days beyond Wednesday, i.e., it will be Friday.

When no specific day is given:

Here, we count days according to number of odd days.

Sunday for 0 odd day, Monday for 1 odd day and so on. (i.e. from 0 to 6; 6 being Saturday)

Suppose someone asks you to find the day of the week on 12th January 1979.

12th Jan, 1979 means 1978 year + 12 days

Now, 1600 years have 0 odd day.

300 years have 1 odd day

78 years have 59 ordinary year + 19 leap years = 6 odd days.

Total no. of odd days = $0 + 1 + 6 + 12 = 19$ or 5 odd days. So, the day was "Friday"

Know Me

An easy way to remember no. of odd days per month are 30 32 32 33 23 23. Where 0 represents the no. of odd days in February if it is not a leap year. If it is a leap year take it as 1.

Points not to be ignored:

1. 400th year is a leap year or a century multiple of 400 is a leap year, rests are not.
2. 100 years has 5 odd days
200 years has 3 odd days
300 years has 1 odd day
400 years has 0 odd days. And so on