Basics of Fractions OR Chain Rule

(1) When a fraction has its numerator greater than the denominator, its value is greater than one. Let us call it ***greater fraction***. Whenever a number (say x) is multiplied by a ***greater fraction***, it gives a value greater than itself.

(2) When a fraction has its numerator less than the denominator, its value is less than one. Let us call it ***less fraction***. Whenever a number (say x) is multiplied by a ***less fraction***, it gives a value less than itself.

Rule of Fractions OR Chain Rule

Let us try to understand the concept with the help of an example.

Ex.1 ***If 6 men can do a piece of work in 30 days of 9 hours each, how many men will it take to do 10 times the amount of work if they work for 25 days of 8 hours?***

Sol. Three points arise:

(1) Less days, so more men required.

(2) Less working hours, so more men required.

(3) More work, more men.

By rule of fraction or Chain rule:

Step I: We look for our required unit. It is the number of men. So, we write down the number of men given in the question. It is 6.

Step II: The number of days gets reduced from 30 to 25, so it will need **more men** (Reasoning: Less days, more men). It simply means that 6 should be multiplied by a **greater fraction** because we need a value greater than 6. So, we have: $6 \times \frac{30}{25}$

Step III: Following in the same way, we see that the above figure should be multiplied by a '**greater fraction**', i.e., by $\frac{9}{8}$. So, we have: $6 \times \frac{30}{25} \times \frac{9}{8}$

Step IV: Following in the same way, we see that the above figure should be multiplied by a '**greater fraction**' i.e. by $\frac{10}{1}$. So, we have: $6 \times \frac{30}{25} \times \frac{9}{8} \times \frac{10}{1}$ = 81 men.

Mixtures & Alligation

As the dictionary meaning of Alligation (mixing), we will deal with problems related to mixing of different compounds or quantities. The concept of alligation and weighted average are the same.

When two or more quantities are mixed together in different ratios to form a mixture, then ratio of the quantities of the two constituents is given by the following formulae:

$$\frac{Q_c}{Q_d} = \frac{d - m}{m - c}$$

Toolkit

Q_c	$\rightarrow$	Cheaper quantity
Q_d	$\rightarrow$	Dearer quantity
c	$\rightarrow$	C.P. of unit qty of 1st constituent.
d	$\rightarrow$	C.P. of unit qty of 2nd constituent.
m	$\rightarrow$	Mean Cost price of unit qty of mixture

$$\begin{array}{ccc} \mathbf{c} & & \mathbf{d} \\ & \mathbf{m} & \\ \mathbf{d-m} & & \mathbf{m-c} \end{array}$$

$$\frac{\mathbf{Q_c}}{\mathbf{Q_d}} = \frac{\mathbf{d-m}}{\mathbf{m-c}}$$

Gives us the ratio of quantities in which the two ingredients should be mixed to get the mixture.

Ex.2 ***A sum of Rs 39 was divided among 45 boys and girls. Each girl gets 50 paise, whereas a boy gets one rupee. Find the number of boys and girls.***

Sol. Average amount of money received by each = $\frac{39}{45}$ = Rs $\frac{13}{15}$

Amount received by each girl = 50 paise = Rs $\frac{1}{2}$

Amount received by each boy = Re. 1

By alligation rule:

$$\frac{\text{Number of boys}}{\text{Number of girls}} = \frac{\frac{13}{15} - \frac{1}{2}}{1 - \frac{13}{15}} = \frac{11}{4}$$

$$\begin{array}{ccc} \textbf{Boys} & & \textbf{Girls} \\ \mathbf{1} & & \mathbf{\frac{1}{2}} \\ & \mathbf{\frac{13}{15}} & \\ \left[\mathbf{\frac{13}{15} - \frac{1}{2}}\right] & & \left[\mathbf{1 - \frac{13}{15}}\right] \end{array}$$

$\therefore$ Number of boys = $\frac{11}{11+4} \times 45 = 33$

Number of girls = 45 – 33 = 12.

Important Funda

Always identify the ingredients as cheaper & dearer to apply the *alligation rule*.

In the alligation rule, the variables c, d & m may be expressed in terms of **percentages** (e.g. A 20% mixture of salt in water), **fractions** (e.g. two-fifth of the solution contains salt) or **proportions** (e.g. A solution of milk and water is such that **milk : Water = 2 : 3**) . The important point is to remember is that c & d may represent pure ingredients or mixtures.

(Mixing a Pure component to a solution)

Ex.3 ***A jar contains a mixture of two liquids A and B in the ratio 4 : 1. When 10 litres of the liquid B is poured into the jar, the ratio becomes 2: 3. How many litres of liquid A were contained in the jar?***

Sol. **Method 1: (weighted average or equation method)**

Let the quantities of A & B in the original mixture be 4x and x litres.

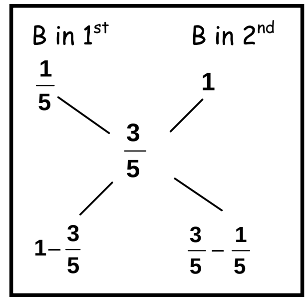
According to the question $\frac{4x}{x+10} = \frac{2}{3}$.

$12x = 2x + 20 \quad \Rightarrow 10x = 20 \quad \Rightarrow x = 2$

The quantity of A in the original mixture = $4x = 4 \times 2 = 8$ litres.

Method 2: (Alligation with composition of B)

The average composition of B in the first mixture is 1/5.
The average composition of B in the second mixture = 1
The average composition of B in the resultant mixture = 3/5
Hence applying the rule of Alligation we have
[1 – (3/5)]/[(3/5) – (1/5)] = (2/5)/(2/5) = 1
So, initial quantity of mixture in the jar = 10 litres.
And, quantity of A in the jar = $(10 \times 4)/5$ = 8 litres.

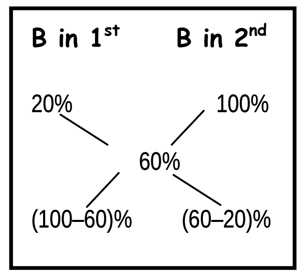


Method 3: (Alligation with percentage of B)

The percentage of B in 1st mixture = 20%
The percentage of B in 2nd mixture = 100%
The percentage of B in Final Mixture = 60%
By rule of allegation we have
Volume 1st : Volume 2nd = (100% - 60%) : (60% - 20%)

$V_1 : V_2 = 1 : 1$

Volume of mixture 1st = 10 litres
Volume of A in mixture 1st = 80% of 10 litres = 8 litres. **Answer**



Removal and Replacement

> If a vessel contains "x" litres of milk and if "y" litres be withdrawn and replaced by water, then if "y" litres of the mixture be withdrawn and replaced by water, and the operation repeated 'n' times in all, then :
>
> $$\frac{\text{Milk left in vessel after nth operation}}{\text{Initial quantity of Milk in vessel}} = \left[\frac{x-y}{x}\right]^n = \left[1-\frac{y}{x}\right]^n$$

Ex.4 ***Nine litres of solution are drawn from a cask containing water. It is replaced with a similar quantity of pure milk. This operation is done twice. The ratio of water to milk in the cask now is 16 : 9. How much does the cask hold?***

Sol. Let there be x litres in the cask
After n operations

$$\frac{\text{Water left in vessel after n operations}}{\text{Original quantity of water in vessel}} = (1 - 9/x)^n$$

$(1 - 9/x)^2 = 16/25$

$\therefore$ x = 45 litres.

Ex.5 ***There are two containers A and B of milk solution. The ratio of milk and water in container A is 3 : 1 and in container B, it is 4 : 1. How many liters of container B solution has to be added to 20 lts of container 'A' solution such that in the resulting solution; the ratio of milk to water should be 19 : 6?***

Sol. In container A, the part of milk = $\frac{3}{3+1} = \frac{3}{4}$.

In container B, it is $\frac{4}{4+1} = \frac{4}{5}$.

Required = $\frac{19}{19+6} = \frac{19}{25}$.

Use allegation:

Container A		**Container B**
$\frac{3}{4}$		$\frac{4}{5}$
	$\frac{19}{25}$	
$\frac{1}{25}$		$\frac{1}{100}$

$\Rightarrow$ 4 : 1

It is given 20 lts of container A is added. So, the quantity of container B should be 5 lts.

Ex.6 ***There are two alloys A and B. Alloy A contains zinc, copper and silver, as 80% 15% and 5% respectively, whereas alloy B also contains the same metals with percentages 70%, 20%, 10% respectively. If these two alloys are mixed such that the resultant will contain 8% silver, what is the ratio of these three metals in the resultant alloy?***

Sol. Since, the resultant alloy contains 8% silver, first we will find, in what ratio these two alloys A and B were mixed to form the resultant.

Alloy A		**Alloy B**
5		10
	8	
2		3

Then the resultant zinc percentage is

$\frac{2 \times 80 + 3 \times 70}{2+3}$ = 74%.

So, copper percentage = 100 – (74 + 8) = 18

$\therefore$ The ratio of these metals = 74 : 18 : 8 = 37 : 9 : 4.

Ex.7 ***The cost of an apple is directly proportional to square of its weight in a fruit bazaar. Two friends A and B went there to purchase apples. A got exactly 5 apples per kg and each apple is of same weight. Where as B got exactly 4 apples per kg each weight is exactly same. If B paid Rs. 10 more than A per kg apples, what is the cost of an apple which weighs 1 kg?***

Sol. It is given cost $\propto$ (weight)^2

$\Rightarrow c = k w^2$.

A got 5 apples per kg and each apple is of same weight. $\Rightarrow$ Each apple is 200 gm. = 1/5 kg.

$\therefore$ His cost per apple = $k(^1/_5)^2$.

Since, he will get 5 apples per kg, so his cost per kg = $5k(^1/_5)^2 = k/5$

Similarly, the cost per B is $4k(^1/_4)^2$. It is given $4k\,(^k/_4)^2 - 5k\,(^k/_5)^2$ = Rs. 10. $\Rightarrow k = 200$

Our required answer is $k\,(1)^2$ = Rs. 200.