AMRITA VISHWA VIDYAPEETHAM

DEEMED TO BE UNIVERSITY

19CSE337 Social Networking Security

Lecture 10

Topics to Discuss

- Eigenvector Centrality

Eigenvector Centrality

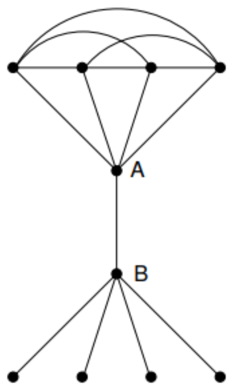
- A limitation of the degree measure is that it gives the same weight to all the neighbors of a node when computing its importance.
- However, it may make more sense to give a larger weight to nodes that are themselves important.
- In a social network, for example, one node may be important because it has social ties with few but important nodes (instead of just participating in many ties).
- Eigenvector centrality is a measure of influence that takes into account the number of links each node has and the number of links their connections have, and so on throughout the network.

Eigenvector Centrality

- Like degree centrality, EigenCentrality measures a node's influence based on the number of links it has to other nodes in the network.
- EigenCentrality then goes a step further by also taking into account how well connected a node is, and how many links their connections have, and so on through the network.
- By calculating the extended connections of a node, EigenCentrality can identify nodes with influence over the whole network, not just those directly connected to it.
- EigenCentrality is a good 'all-round' SNA score, handy for understanding human social networks, but also for understanding networks like malware propagation.

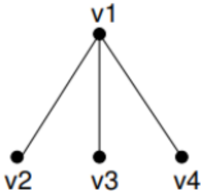
Eigenvector Computation

- Nodes A and B both have degree 5.
- The four nodes (other than A) to which B is adjacent may be unimportant (since they don’t have any interactions among themselves).
- So, A seems more central than B.
- Eigenvector centrality was proposed to capture this.



Eigenvector Computation

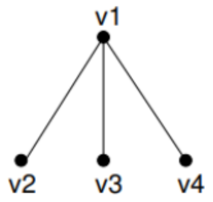
- Consider the following graph and its adjacency matrix.
- We need to find the centrality of each node as function of centrality value of its neighbors.
- The simplest function is the sum of the centrality values.
- The formula to calculate eigenvector centrality of node i is $x_i=1/\lambda\sum x_j$.



0	1	1	1
1	0	0	0
1	0	0	0
1	0	0	0

Eigenvector Computation

- Let x_i denote the centrality of node v_i, i=1,2,3,4.
- To find x1,x2,x3,x4 solve the following equations
  - x1=1/λ(x2+x3+x4)
  - x2=1/λ(x1)
  - x3=1/λ(x1)
  - x4=1/λ(x1)
- Must avoid trivial solution x1=x2=x3=x4=0
- So xi>0, for at least one i∈{1,2,3,4}



$$\begin{matrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{matrix}$$

Eigenvector Computation

- Rewriting above equations
  - $\lambda x1=(x2+x3+x4)$
  - $\lambda x2=(x1)$
  - $\lambda x3=(x1)$
  - $\lambda x4=(x1)$

Matrix version:

$$\lambda \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

- ie; $\lambda x=Ax$, λ is the eigen value of matrix A and x is the corresponding eigen vector (centrality of nodes!).

Eigenvector Computation

- Perron-Frobenius Theorem

  If a matrix A has non-negative entries and is symmetric, then all the values in the eigenvector corresponding to the principal eigen value of A are positive.

Eigenvector Computation

Algorithm: Eigenvector centrality

Input: Adjacency matrix of A representing undirected graph G=(V,E).

Output: The eigenvector centrality of each node of G.

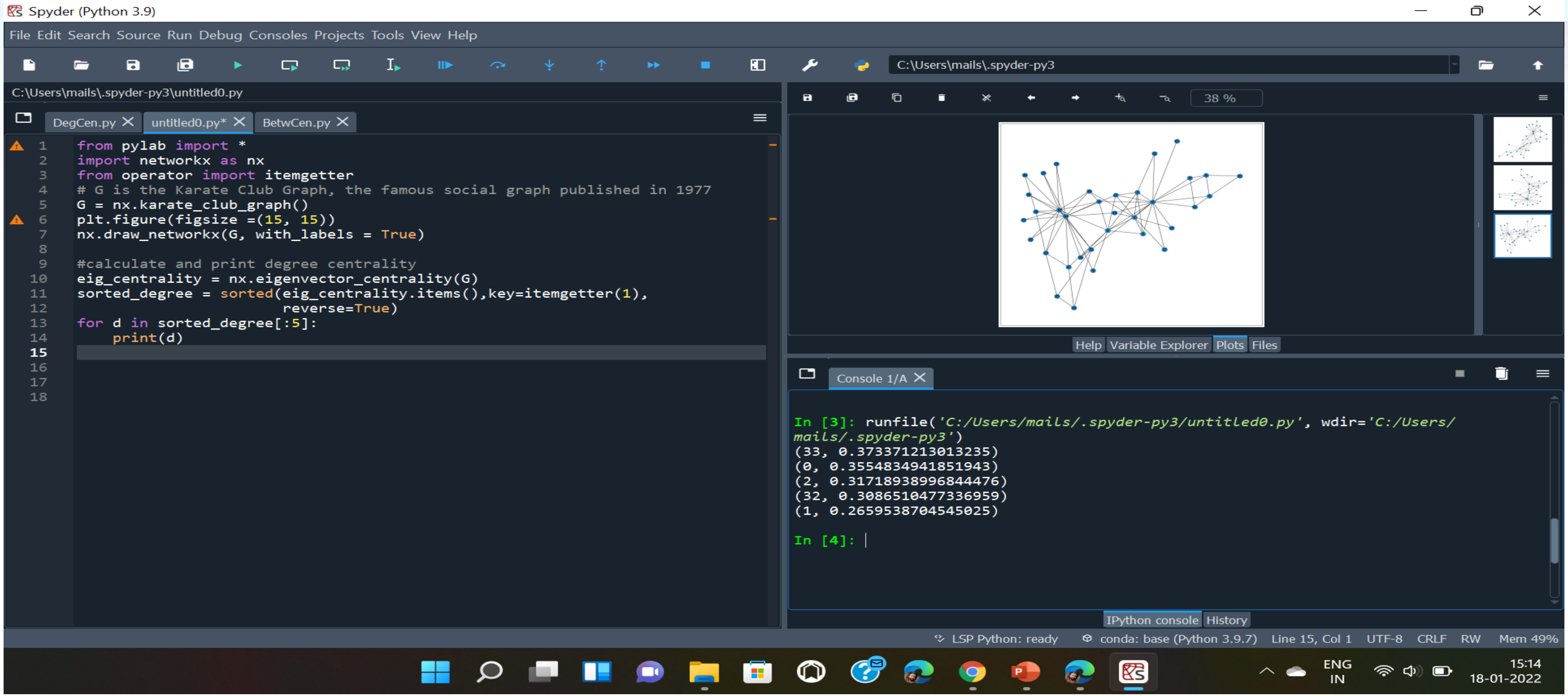
Steps:

- Compute principal eigen value λ of A.
- Compute eigenvector corresponding to λ.
- Each x value gives eigenvector centrality of corresponding node of G.

Eigenvector Computation

- When we solve characteristic equation of matrix A, we get $\lambda=-\sqrt{3},0,0,\sqrt{3}$.
- Principal eigen value is $\sqrt{3}$.
- Eigenvector corresponding to principal eigen value is $[0.707,0.408,0.408,0.408]^T$
- Node v1 has higher centrality.

Eigenvector Computation using NetworkX



Thanks...........