To start with, let's take an example. If 2 men take 10 days to build a wall, 1 man will take 20 days (not 5 days) to build the same wall. The question arises Why?

The reason is simple that man and time are inversely proportional to each other. When men will decrease the number of days required to complete the work will increase. Here men become half, so time will double and will complete the work in 20 days.

$M \propto \frac{1}{D}$ Where M & D are number of men and number of days respectively.

If we further break number of days to hours, then total hours = DH, where H are number of hours per day. Now, our formula becomes

$$M \propto \frac{1}{DH} \quad \text{……………………….. (1)}$$

To elaborate it further, let's say M men take D days to build a room. Now if work is doubled (they have to build two rooms of same size) in same D no. of days, obviously they have to double their strength Or we can say that no. of men are directly proportional to the work to be finished. In mathematics, we can write it as

$M \propto W$, where W is the work to be finished ……………….. (2)

By combining (1) and (2), we get

$M \alpha \frac{1}{DH} \times W$

or $\frac{MDH}{W} = K$ Where K is constant of proportionality

or $\frac{M_1D_1H_1}{W_1} = \frac{M_2D_2H_2}{W_2}$

> **TIP**
>
> If a worker takes x days to finish a work (working alone) and second one takes y days to finish the same work (working alone), then together they will take $\frac{xy}{x+y}$ days to finish the same job.

This is our general formula to solve time & work problems. It is also known as **Work Equivalence Method.**

Majority of time & work questions can be divided into two types.

Type I: Where **efficiency** of individual's is not mentioned (as in the above example): These are the cases where the man-days concept is utilized. Here the Rule of Fractions or the proportion (direct or indirect) concept can also be used.

Type II: Where the **efficiency** is mentioned. Here the case of unit day's or one day's concept is utilized.

Concept of unit time

If A does a work in 10 days, then what is the one-day work done by A?

The answer is one-tenth part of the total work. We can say that

Unit time work = $\frac{1}{\text{Total time taken}}$

Work done in unit time is known as efficiency of the worker or we can say, that if a worker takes less number of days (than second worker) to finish a work, he will be more efficient.

LCM Approach

In this type of approach the total work is considered as the LCM of the time taken by the individuals and then unit time work is calculated. Here the work done is not 1(unity) but the LCM of the time taken by number of persons. Let us understand this with the help of an example.

Ex.1 ***If Manni and Gopi finish a work in 10 and 15 days respectively, what will be number of days taken by both of them to complete the work when both work together?***

Sol. Let the total work is equal to the LCM of number of days of Manni & Gopi taken to do the work respectively.

Work = LCM of 10 & 15 = 30 units

One day work of Manni = $\frac{30}{10}$ = 3 units

One day work of Gopi = $\frac{30}{15}$ = 2 units

One day work of both = 5 units

So number of days taken = $\frac{\text{Work}}{\text{One day work}} = \frac{30}{5}$ = 6 days.

Simultaneous Working Problems

It is always a good way if we try to solve such type of questions with the help of LCM approach. Let us understand the concept with the help of an example.

Ex.2 ***A, B and C can finish a work independently in 10, 12 and 15 days. C starts the work and after 1 day, B joins him. After 1 day of B, A also joins them but leave 3 days before completion of the work, while B left two days before completion of work. What will be the total number of days taken to complete the work?***

Sol. Let total work is LCM (10, 12, 15) = 60 units

I day work: - A = $\frac{60}{10} = 6$ units , B = $\frac{60}{12}$ = 5 units , C = $\frac{60}{15}$ = 4 units

Let total days taken = N

Days of A = (N – 5), Days of B = (N – 3), Days of C = N

According to the question we have,

6(N – 5) + 5(N – 3) + 4N = 60

Solving we have N = 7 days

Alternate Days concept

In these, type of problems, we see the work done by two or more men on alternate days or hours.

Ex.3 ***A can build a wall in 20 days while B can build the same wall in 30 days. If they work on alternate days in how many days, will the wall be completed if A start the job?***

Sol. Let total work = 60 units

1 day work of A = 3 units and 1 day work of B = 2 units

2 days work = 3 + 2 = 5 units

To do 60 units, days required = $\frac{60 \times 2}{5}$ =24 days

Negative Work Concept

These types of problems revel about the problems of pipe and cistern in which one is inlet and other is outlet. Let us solve a puzzle "A monkey wants to climb a pole which is 24 m tall. In 1 minute, he can climb up by 3 m and in the next minute, he slips by 2 m. In how many minutes will the monkey reach the top of the pole?

Ex.4 ***If A build the wall in 20 days and B can destroy that wall in 30 days and work on alternate days. What will be the number of days required to build the wall for the first time?***

Sol. A can build 1/20 th of the wall in 1 day. Where as B will destroy 1/30 th of the wall in 1 day and since they are working on alternate days, So in 2 days, 1/20-1/30 = 1/60 th of the wall will be constructed.

So, in $57 \times 2 = 114$ days, $\frac{57}{60}$ th of the work will be completed and on the 115th day,

A will come and completes the remaining $\frac{3}{60} = \frac{1}{20}$ th of the work.

So it takes 115 days to construct the wall.

Pipes and Cisterns

Pipes and Cisterns problems are almost the same as those of Time and Work problems. Thus, if a pipe fills a tank in 6 hrs, then the pipe fills $\frac{1}{6}$th of the tank in 1 hour. The only difference with Pipes and Cisterns problems is that there are outlets as well as inlets. Thus, there are agents (the outlets) which perform negative work too. The rest of the process is almost similar.

Inlet: A pipe connected with a tank (or a cistern or a reservoir) is called an **inlet**, if it fills the tank. The work done by this is taken as positive work.

Outlet: A pipe connected with a tank is called an, **outlet**, if it empties the tank and the work done by this is taken as the negative work.

Ex.5 ***Two pipes A and B can fill a tank in 36 hours and 45 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?***

Sol. Part filled by A alone in 1 hour = $\frac{1}{36}$

Part filled by B alone in 1 hour = $\frac{1}{45}$

$\therefore$ Part filled by (A + B) in 1 hour = $\left(\frac{1}{36} + \frac{1}{45}\right) = \frac{9}{180} = \frac{1}{20}$

Hence, both the pipes together will fill the tank in 20 hours.

Ex.6 ***Pipe A can fill a tank in 36 hours and pipe B can empty it in 45hours. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?***

Sol. Total Volume of the tank = 180 units (LCM of 36 and 45)

1 hour work of A = 5 units

1 hour work of B = – 4 units

1 hour work of A + B = 1 units

So, Total time taken = 180 hours

Ex.7 ***A can do a work in 30 days, and B can do in 40 days. If A and B work together for 10 days and A left, then C joined with B and completed the work in 10 days. How many days C alone can complete the work?***

Sol. A work only for 10 days, B work for 20 days and C work for 10 days.

Let us assume C require X days to complete the work.

So, in 1 day, A can do 1/30th of the work. B can do 1/40 th of the work, and c can do 1/X of the work.

So, 10/30 + 20/40 + 10/X = 1. So X = 60 days.

So, C can do the work in **60 days.**

Ex.8 ***Four men can do work in 15 days. If a man left the work after 5 days and again joined after 5 more days, The remaining three work continuously till the end of work. How many more days than the estimated, it takes to complete the work?***

Sol. After 15 days, the work that is left is equal to the amount of work, that can be done by 1 man in 5 days. This work can be done in 5/4 days by the 4 men. So it takes 5/4 more days to complete the work.

Ex.9 ***One man started a work on first day, the second day one more joined with him. The next day one more joined. Everyday one new person joined until the work gets completed. If the work is completed in 15 days, how many days it takes for 10 men to complete the same work, if they work regularly?***

Sol. Let us assume one man can do x amount of work in 1 day.

So the amount of work, that can be completed

On 1st day = x

2nd day = 2x

……………..

..……………

15th day = 15x

Total = $\sum 15\,x$ = 120 x

But 10 men can do 10x work in 1 day.

So, 10 men take 12 days to complete the work.

Ex.10 ***A is 25% more efficient than B. With the help of C, B can complete a work in $\frac{2}{3}$ times that required for A alone. How much percent less efficient is C, compared to A.***

Sol. Assume B can do ‘x’ work in 1 day.

$\Rightarrow$ A will do $\frac{5}{4}$x in 1 day.

Since B and C together take $\frac{2}{3}$rd of the time required for A, (B + C) is $\frac{3}{2}$ times efficient than A.

$\Rightarrow$ B + C will do $\frac{3}{2}\left(\frac{5}{4}x\right)$

So C will do $\frac{15}{8}x - x = \frac{7}{8}x$ in 1 day.

Answer = $\dfrac{\frac{5}{4}x - \frac{7}{8}x}{\frac{5}{4}x} \times 100$ = 30% less.

Ex.11 ***B is twice as efficient as 'A' and 'C' is 50% more efficient than B. If B and C together can complete a work in 10 days, how much time it takes for A and B to complete the work, if they work on alternate days starting with 'A'?***

(1) 28 days **(2) $29\frac{1}{2}$ days** **(3) $30\frac{1}{3}$ days** **(4) $32\frac{1}{4}$ days** **(5) $33\frac{1}{2}$ days**

Sol. If A can do x work in 1 day.

B can do 2x

C can do 3x

B and C together takes 10 days $\Rightarrow$ Total work = 50 x.

If A and B work on alternate days,

In 2 days $\Rightarrow$ x + 2x = 3x work will completed.

In 16 $\times$ 2 = 32 days $\Rightarrow$ 16 $\times$ 3x = 48x will be completed.

33^{rd} day $\Rightarrow$ A will come and do x work. So, 48x + x = 49x will be completed.

34^{th} day $\Rightarrow$ B will come and complete the work in $\frac{1}{2}$ day.

So, answer = $33\frac{1}{2}$ days. **Answer: (5)**