Average

Definition

Simple Average (or Mean) is defined as the ratio of sum of the quantities to the number of quantities.

By Definition, Average $= \dfrac{\text{Sum of all quantities}}{\text{no. of quantities}}$

Putting in symbols, $\overline{X} = \dfrac{x_1 + x_2 + x_3 + \text{- - - - - - - - - - -} + x_n}{N}$

Here x_1, x_2, x_3, ---------- x_n represent the n values of quantity under consideration & $\overline{x}$ is the mean. Average or mean is said to be a measure of central tendency.

Let us take a very simple example of the first five natural numbers 1, 2, 3, 4 & 5.

Their Average $= \dfrac{\text{Sum}}{5} = \dfrac{15}{5} = 3$.

Now let's add 2 more 3's to these 5 numbers.

Now, Average $= \dfrac{15 + 6}{5 + 2} = \dfrac{21}{7} = 3$.

Ex.1 If a person with age 45 joins a group of 5 persons with an average age of 39 years. What will be the new average age of the group?

Sol. Total age will be $45 + 5 \times 39 = 240$. And there will be 6 persons now.
So the average will be 240/6 = 40.

(or)

Since 45 is 6 more than 39, by joining the new person, the total will increase by 6 and so the average will increase by 1.
So, the average is 39 + 1 = 40.

Ex.2 Two students with marks 50 and 54 leave class VIII A and move to class VIII B. As a result the average marks of the class VIII A fall from 48 to 46. How many students were there initially in the class VIII A?

Sol. The average of all the students of class VIII A is 46, excluding these two students.
They have 4 and 8 marks more than 46. So with the addition of these two students, 12 marks are adding more, and hence the average is increasing 2. There should be 6 students in that class including these two. This is the initial number of students,

Ex.3 The average of x successive natural numbers is N. If the next natural number is included in the group, the average increases by:-

(1) Depends on x **(2) Depends on the starting number of the series**

(3) Both (1) and (2) **(4) $\dfrac{1}{2}$**

(5) None of these

Sol. The average of consecutive numbers is the middle number. If one more number is added to the list, the middle number moves 0.5 towards right. So the **answer is (4).**

Weighted Mean

If some body asks you to calculate the combined average marks of both the sections of class X A and X B, when both sections have 60% and 70% average marks respectively? Then your answer will be 65% but this is wrong as you do not know the total number of students in each sections. So to calculate weighted average we have to know the number of students in both the sections. Let $N_1, N_2, N_3, ... N_n$ be the weights attached to variable values $X_1, X_2, X_3, ... X_n$ respectively.

Then the weighted arithmetic mean, usually denoted by

$$\overline{X} = \frac{N_1X_1 + N_2X_2 + N_3X_3 + ... + N_nX_n}{N_1 + N_2 + N_3 + ... + N_n}$$

for any two different quantities taken in different ratios. The weighted average is just like a see-saw. More the ratio of a quantity more will be the inclination of the average from mid value towards the value with more ratios.

Ex.4 *The average marks of 30 students in a section of class X are 20 while that of 20 students of second section is 30. Find the average marks for the entire class X?*

Sol. We can do the question by using both the Simple average & weighted average method.

Simple average = $\dfrac{\text{Sum of marks of all students}}{\text{Total number of students}}$

$= \dfrac{20 \times 30 + 30 \times 20}{30 + 20} = 24.$

By the weighted mean method, Average = $\dfrac{3}{5} \times 20 + \dfrac{2}{5} \times 30$

$= 12 + 12 = 24.$

Real Facts about average

1. If each number is increased / decreased by a certain quantity n, then the mean also increases or decreases by the same quantity.
2. If each number is multiplied/ divided by a certain quantity n, then the mean also gets multiplied or divided by the same quantity.
3. If the same value is added to half of the quantities and same value is subtracted from other half quantities then there will not be any change in the final value of the average.

Average Speed

Average Speed = $\dfrac{\text{Total distance covered}}{\text{Total time taken}}$

If d_1 & d_2 are the distances covered at speeds v_1 & v_2 respectively and the time taken are t_1 & t_2 respectively, then the average speed over the entire distance ($x_1 + x_2$) is given by

$$\frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{d_1 + d_2}{t_1 + t_2} = \frac{d_1 + d_2}{\dfrac{d_1}{v_1} + \dfrac{d_2}{v_2}}$$

If both the distances are equal i.e. $d_1 = d_2 = d$ then,

Average speed = $\dfrac{2v_1v_2}{v_1 + v_2}$ {i.e. Harmonic mean of two velocities}

But if both the time taken are equal i.e. $t_1 = t_2 = t$ the

Average speed = $\dfrac{v_1 + v_2}{2}$ {i.e. Algebraic mean of two velocities}

Ex.5 **The average of 10 consecutive numbers starting from 21 is :**

Sol. The average is simply the middle number, which is the average of 5th & 6th no. i.e, 25 & 26 i.e. **25.5.**

Ex.6 **There are two classes A and B., each has 20 students. The average weight of class A is 38 and that of class B is 40. X and Y are two students of classes A and B respectively. If they interchange their classes, then the average weight of both the classes will be equal. If weight of x is 30 kg, what is the weight of Y**?

Sol. Total weight of class A = 38 × 20, and class B = 40 × 20, if X & Y are interchanged, then the total ages of both the classes are equal.

⇒ 38 × 20 − x + y = 40 × 20 − y + x . 2(y − x) = 2 × 20,

⇒ y = x + 20 = **50** (OR)

Since both the classes have same number of students, after interchange, the average of each class will be 39. Since the average of class 'A' is increasing by 1, the total should increase by 20.

So, x must be replaced by 'y', who must be 20 years elder to 'x'. So, y must be 50 years old..

Ex.7 **The average weight of 10 apples is 0.4 kg. If the heaviest and lightest apples are taken out, the average is 0.41 kg. If the lightest apple weights 0.2 kg, what is the weight of heaviest apple?**

Sol. Total weight of the apples is 0.4 × 10 = 4 kg.

Weight of apples except heaviest & lightest = 0.41 × 8 = 3.28 kg

∴ Heaviest + lightest = 4 − 3.28 = 0.72 kg. It is given lightest = 0.2 kg.

∴ Heaviest is 0.72 − 0.2 = **0.52 kg.**

Ex.8 **While finding the average of '9' consecutive numbers starting from X; a student interchanged the digits of second number by mistake and got the average which is 8 more than the actual. What is X?**

Sol. Since the average is 8 more than the actual, the second no will increase by 72 (9 × 8) by interchanging the digits.

If ab is the second no, then 10a + b + 72 = 10b + a, ⇒ 9(b − a) = 72. ∴ b − a = 8.

The possible number ab is 19. Since the second no is 19.

The first no is 18. ∴ **X = 18**

Ex.9 **There are 30 consecutive numbers. What is the difference between the averages of first and last 10 numbers?**

Sol. The average of first 10 numbers is the average of 5th & 6th no. Where as the average of last 10 numbers is the average of 25th & 26th no. Since all are consecutive numbers, 25th number is 20 more than the 5th number. We can say that the average of last 10 nos is 20 more than the average of first 10 nos. So, the required answer is **20.**

Instructions for next 3 examples:

There are 60 students in a class. These students are divided into three groups A, B, C of 15, 20 & 25 students each. The groups A & C are combined to form group D.

Ex.10 **What is the average weight of the students in group D?**
   1) More than the average weight of A.
   2) More than the average weight of C.
   3) Less than the average weight of C.
   4) Less than the average weight of B.
   5) Cannot be determined.

Sol. We know only the no of students in each class, but we don't know the average weight of any class, we can't find the answer.

Ex.11 **If one student from group A is shifted to group B, which of the following is necessarily true?**
   1) The average weight of both groups increases
   2) The average weight of both groups decreases.
   3) The average weight of class remains the same.
   4) The average weight of group A decreases and that of group B increases.
   5) None of these

Sol. Options (1) & (2) are not possible. Average of both cannot increase or decrease. Option (4) can be eliminated because we are not sure, whether the average of A increases & B decreases or A decreases & B increases or both remains unchanged. It depends on the weight of the student, who shifted from A to B.

Option (3) is always true because even the student shifts from one group to other. The average weight of the whole class does not change. **Answer: (3)**

Ex.12 **If all the students of the class have the same weight, which of the following is false?**
   1) The average weight of all the four groups is same.
   2) Total weight of A & C is twice that of B.
   3) The average weight of D is greater than that of A.
   4) The average weight of class remains same even the students shifts from one group to other.
   5) None of these.

Sol. Since each student has same weight (1) & (4) are right. Since the number of students in group A & C together is 15 + 25 = 40. Where as in B, there are only 20 students. So option (2) is also correct. But (3) is false, because, the average weight of each group is same, since all the students have same weight. **Answer: (3)**