

What is the meaning of percent?

The terms **percent** means “for every hundred”. A fraction whose denominator is 100 is called **percentage** and the numerator of the fraction is called the **rate percent**. Thus, when we say a man made a profit of 20 percent we mean to say that he gained Rs. 20 for every hundred rupees he invested in the business, i.e. 20/100 rupees for each Rupee.

The abbreviation of percent is p.c. and it is generally denoted by %.

**Ex.1** 84% of a particular total is 630 marks. What is 90% equal to?

(1) 750

(2) 675

(3) 450

(4) 550

(5) None of these

**Sol.** The required answer is  $\frac{90}{84} \times 630 = 675$ . **Answer: (2)**

**Ex.2** Two numbers are greater than the third number by 25% and 20% respectively. What percent of first number is the second number?

(1) 92%

(2) 94 %

(3) 96 %

(4) 98 %

(5) None of these

**Sol.** Assume the third number is 100. So the first number is 125 and the second number is 120.

So the required answer is  $\frac{120}{125} \times 100 = 96\%$ . **Answer: (3)**

Basics**(a) Percentage and Fraction Equivalents:**

If some one asks you to represent 50% in fractions then what will you do?

Certainly, you will come with the answer  $\frac{1}{2}$ . What this value actually represents? This is nothing but the

fractional equivalent of the given percentage. From CAT point of view it is very important for us to mug up fractional equivalent of the percentages. Some fractional equivalent are given as under:-

$$\frac{1}{1} = 100\%$$

$$\frac{1}{2} = 50\%$$

$$\frac{1}{3} = 33.33\%$$

$$\frac{1}{4} = 25\%$$

$$\frac{1}{5} = 20\%$$

$$\frac{1}{6} = 16.67\%$$

$$\frac{1}{7} = 14.28\%$$

$$\frac{1}{8} = 12.5\%$$

$$\frac{1}{9} = 11.11\%$$

$$\frac{1}{10} = 10\%$$

$$\frac{1}{11} = 9.09\%$$

$$\frac{1}{12} = 8.33\%$$

$$\frac{1}{13} = 7.69\%$$

$$\frac{1}{14} = 7.14\%$$

$$\frac{1}{15} = 6.66\%$$

$$\frac{1}{16} = 6.25\%$$

$$\frac{1}{17} = 5.88\%$$

$$\frac{1}{18} = 5.55\%$$

$$\frac{1}{19} = 5.29\%$$

$$\frac{1}{20} = 5.00\%$$

**IMPORTANT**

It will be very useful if you see percentage in the form of fraction. Also important for D.I.

**37.5%**

$$= 3 \times 12.5\% = \frac{3}{8}$$

**10.58%**

$$= 2 \times 5.29\% = \frac{2}{19}$$

**Ex.3** A is earning 20% more than B, who earns 20% less than C. By what percent A earns more or less than C?

**Sol.**  $A = B + \frac{B}{5} = \frac{6}{5}B$  ..... (1)

$$B = C - \frac{C}{5} = \frac{4}{5}C \dots\dots\dots (2)$$

From (1) & (2) we have

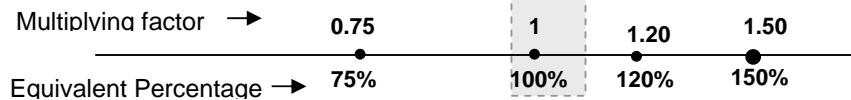
$$A = \frac{6}{5} \times \frac{4}{5}C = \frac{24}{25}C = C - \frac{C}{25}$$

i.e. A is 4% less than C. **(OR)**

Assume that 'C' earns 100.. Thus B earns 80 and A earn 96. So 'A' earns 4% less than C.

### (b) Multiplying Factor

While dealing with %age increase or decrease picture the following scale in your mind **with reference as 100% (= 1) in the center.**



So we can say that multiplying factor (M.F) of 10% increase is 1.1 and that of 15% decrease is 0.85. An increase by x% implies the value lies on the right hand side of 100% & vice versa. Let's start with a number X

1. X increased by 10% would become  $X + 0.1X = 1.1X$
2. X increased by 1% would become  $X + 0.01X = 1.01X$
3. X increased by 0.1 % would become  $X + 0.001X = 1.001X$
4. X decreased by 10% would become  $X - 0.1X = 0.9X$
5. X decreased by 1% would become  $X - 0.01X = 0.99X$
6. X decreased by 0.1% would become  $X - 0.001X = 0.999X$
7. X increased by 200% would become  $X + 2X = 3X$
8. X decreased by 300% would become  $X - 3X = -2X$

**Ex.4 Coconut oil is now being sold at Rs. 27 per kg. During last month its cost was Rs. 24 per kg. Find by how much percent a family should reduce its consumption, to keep the expenditure the same.**

**Sol.** Assume the consumption last year is 1 kg, then it cost Rs. 24.

But now for Rs. 24, only  $\frac{24}{27}$  kg of oil will come.

$$\text{So the \% reduction in consumption} = \frac{1 - \frac{24}{27}}{1} \times 100 = 11 \frac{1}{9} \%$$

### (c) Successive Percentage change:

The population of a city increases by 10% in one year and again increases by 10% in the next year, then what is the net increase in the population in two years. The very common answer is 20% which is wrong. Why? Let us see

If Original population = P

$$\text{After 1}^{\text{st}} \text{ year} = \left(1 + \frac{1}{10}\right)P = \frac{11}{10}P$$

$$\text{After 2}^{\text{nd}} \text{ year} = \left(1 + \frac{1}{10}\right) \frac{11}{10}P = \frac{121}{100}P = \left(1 + \frac{21}{100}\right)P \text{ i.e. increases by 21\% of the original value.}$$

This successive change in the percentage can be calculated in the shortcut way as explained below:

Let us consider a product of two quantities  $A = a \times b$ .

If  $a$  &  $b$  change (increase or decrease) by a certain percentage say  $x$  &  $y$  respectively, then the overall percentage change in their product is given by the formula:  $\left(x + y + \frac{xy}{100}\right)\%$

**Ex.5 If the volume of a milk and water solution is increased by 25% by pouring only water. By what percentage does the concentration of milk reduce?**

**Sol.** Assume initially, there is 100 lts of solution, out of which  $x$  lts is milk.

So the concentration of milk is  $\frac{x}{100}$ . Now it is  $\frac{x}{125}$ .

$$\text{So percentage decrease} = \frac{\frac{x}{100} - \frac{x}{125}}{\frac{x}{100}} \times 100 = 20\%.$$

This formula also holds true if there are successive changes as in the case of population increase or decrease. But care has to be taken when there are either more than 2 successive changes or there is a product of more than 2 quantities as in the case of volume. **In these cases we have to apply the same formula twice.**

(i) If there is successive increase of  $x\%$  and  $y\%$ , then the net change will be

$$x + y + \frac{xy}{100} \%$$

(ii) If there is successive discount of  $x\%$  and  $y\%$ , then the total discount will be.

$$x + y - \frac{xy}{100} \%$$

(iii) (a) If there is  $x\%$  increase and then  $x\%$  decrease, then the net change

$$- \frac{x^2}{100} \%$$

(b) If the values are different, then net change

$$x - y - \frac{xy}{100}$$

**Ex.6 If A is increased by 20% and B is decreased by 20%, then both the quantities will be equal. What percentage of B is A.**

**Sol.** If A is increased by 20%, it will become 1.2 A. and B is decreased by 20%, it will become 0.8 B.

It is given  $1.2 A = 0.8 B$ ,  $\Rightarrow A = \frac{2}{3} B$ .  $\therefore A = 0.66 B$ .  $\therefore A$  is 66.66% of B.

**Ex.7 If A is 3 times to B then B is what percentage of A.**

**Sol.**  $A = 3B$ .

$$\Rightarrow B = \frac{A}{3} = 0.33 A \quad \therefore B \text{ is } 33\% \text{ of } A.$$

**Ex.8** If 'x' is increased by 20% & 25% successively, then its value increases by 30. What is the value of X?

**Sol.** If X is increased by 20%, it will become  $1.2X$  or  $\frac{6}{5}X$ .

If it is again increased by 25%, its value becomes  $= \frac{5}{4} \times \frac{6}{5}X = \frac{3}{2}X$ .

$$\therefore \text{Change} = \frac{3}{2}X - X = \frac{X}{2}.$$

It is given 30.  $\therefore \frac{X}{2} = 30 \Rightarrow X = 60$ .

(or) If a value is successively increases by X% & Y%, then its value changes by  $X + Y + \frac{XY}{100}$ .

So X increases by  $20 + 25 + \frac{20 \times 25}{100} = 50\%$   $\therefore$  50% of X is given 30.  $\therefore X = 60$ .

**Ex.9** A man spends 30% of his salary for food and 20% of the remaining on rent and 20% of the remaining on other expenses. If he saves Rs. 8960, what is his salary?

**Sol.** Let his salary be 'K'. If he spent 30% on food, he will be left with 70% of K i.e.  $0.7K$  or

$$\frac{70}{100}K \text{ or } \frac{7}{10}K \text{ on this, he spent 20% on rent and 80 left with 80% i.e., } \frac{80}{100} \times \frac{70}{100}K.$$

After spending 20% of this on other expenses he will be left with 80% of this.

$$\text{i.e. } \frac{80}{100} \times \frac{80}{100} \times \frac{70}{100}K = \frac{448}{1000}K. \text{ It is given as Rs. 8960}$$

$$\Rightarrow K = 20000.$$

**Ex.10** Last year an employee used to save 40% of his salary. But now his salary is increased by 50% and expenses also increases by 20%. What is his percentage savings now?

**Sol.** Assume 'X' was the salary of the employee last year. Since his savings were 40%, his expenses were 60%

i.e,  $0.6X$ . At present, his salary is  $1.5X$  and expenses are  $1.2(0.6X) = 0.72X$

$$\therefore \text{Savings} = 1.5X - 0.72X = 0.78X. \therefore \% \text{ Savings} = \frac{0.78X}{1.5X} \times 100 = 52\%$$

**Ex.11** A man earns X% on the first Rs. 2000 and Y% on the rest of income. If he earns Rs. 700 from Rs. 4000 and Rs. 900 from Rs. 5000 income; find X%.

**Sol.** We can form two equations from the above information as

$$2000\left(\frac{X}{100}\right) + (4000 - 2000)\left(\frac{Y}{100}\right) = 700 \dots\dots (1)$$

$$2000\left(\frac{X}{100}\right) + (5000 - 2000)\left(\frac{Y}{100}\right) = 900 \dots\dots (2)$$

$$\text{Equation (1)} \times 3 - \text{Equation (2)} \times 2 \Rightarrow 2000\left(\frac{X}{100}\right) = 300. \Rightarrow X = 15\%. \therefore X\% = 15\%.$$