① $T(n) = 3T(n/2) + n^2.$

$\boxed{a=3}$ $\boxed{b=2}$ $\boxed{k=2}$ $\boxed{p=0}$

$b^k = 2^2 = 4$

$a < b^4 \rightarrow$ case ③.

$\boxed{p=0} \rightarrow T(n) = O(n^k \log^p n)$

$\rightarrow O(n^2 \log^0 n)$

$\rightarrow \boxed{O(n^2)}$

---

② $T(n) = 7T(n/2) + n^2.$

$\boxed{a=7}$ $\boxed{b=2}$ $\boxed{k=2}$ $\boxed{p=0}$

$b^k = 2^2 = 4$

$a > b^k \rightarrow 7 > 4.$

$a > b^k \rightarrow$ case 1

$T(n) = \Theta(n^{\log_b a})$

$T(n) = \Theta(n^{\log_2 7})$

$= \Theta(n^{2.801})$

$\boxed{T(n) = \Theta(n^3)}$

---

3 $T(n) = 4T(n/2) + n^2$

$\boxed{a=4}$ $\boxed{b=2}$ $\boxed{k=2}$ $\boxed{p=0}$

$b^k = 2^2 = 4.$

$a = b^4 \longrightarrow$ case : 2

$p = 0 \longrightarrow T(n) = \Theta(n^{\log_b a} \cdot \log^{p+1} n)$

$T(n) = \Theta(n^{\log_2 4} \cdot \log^1 n)$

$T(n) = \Theta(n^2 \cdot \log^1 n) \rightarrow \boxed{\Theta(n^2 \log n)}$

---

(4) $T(n) = 3T(n/4) + n\log n.$

$\boxed{a=3}$ $\boxed{b=4}$ $\boxed{k=1}$ $\boxed{p=1}$

$a < b^k \longrightarrow$ case : (3)

$\boxed{p=1} \rightarrow T(n) = \theta(n^k \log^p n)$

$T(n) = \theta(n^1 \log^1 n)$

$\boxed{T(n) = \theta(n\log n)}$

---

(5) $T(n) = 4T(n/2) + \log n.$

$\boxed{a=4}$ $\boxed{b=2}$ $\boxed{k=1}$ $\boxed{p=1}$

$a > b^k \longrightarrow$ case (1)

$T(n) = \theta(n^{\log_b a})$

$T(n) = \theta(n^{\log_2 4}) = \boxed{\theta(n^2)}$

---

6 $T(n) = T(n-1) + n$

Here $\boxed{a=1}$ $\boxed{b=1}$ $\boxed{k=1}$ $\boxed{p=0}$

$\boxed{b>1}$ → For masters theorem.

→ So it doesn't applicable for masters theorem.

---

7 $T(n) = 4T(n/2) + n^2 \log n$.

$\boxed{a=4}$ $\boxed{b=2}$ $\boxed{k=2}$ $\boxed{p=1}$

$b^k = 2^2 = 4$

$\boxed{a = b^k}$ → case 2.

$T(n) = \theta(n^{\log_b a} \cdot \log^{p+1} n)$

$= \theta(n^{\log_2 4} \cdot \log^2 n)$

$$\boxed{T(n) = \theta(n^2 \cdot \log^2 n)}$$

(8) $5T(n/2) + n^2 \log n$

$\boxed{a=5}$ $\boxed{b=2}$ $\boxed{k=2}$ $\boxed{p=1}$

$2^2 \leq 5 \rightarrow$ case (1). $\rightarrow a > b^k$

$\boxed{p=1} \rightarrow T(n) = O(n^{k} \log_b^{p} a)$

$= O(n^{2} \log_2^{1} 5)$

$$\boxed{T(n) = O(n \log_2^{5})}$$

---

(9) $T(n) = 3T(n/3) + n/\log n$

$\boxed{a=3}$ $\boxed{b=3}$

→ Here $f(n) = n/\log n$ is not polynomially smaller than $n^{\log_3 3-\epsilon}$ for any $\epsilon > 0$.

→ So M.T not applicable.

(10) $T(n) = 2T(n/4) + C$

$\boxed{a=2}$ $\boxed{b=4}$ $\boxed{k=0}$

$b^k = 4^0 = 1$ $\quad 1 < 2$ $\quad b^k < a$ (or) $a > b^k$

$a > b^k \rightarrow$ case (1)

$T(n) = \theta(n^{\log_b a})$

$= \theta(n^{\log_4 2})$

$= \theta(n^{\log_4 2}) \rightarrow \theta(n^{0.5})$

$$\boxed{T(n) = \theta(n^{1/2})}$$

---

(11) $T(n) = T(n/4) + \log n.$

$\boxed{a=1}$ $\boxed{b=4}$ $\boxed{k=0}$ $\boxed{p=1}$

$b^k = 4^0 = 1$ $\quad \boxed{a=1}$ $\quad \boxed{b^k=1}$

$a = b^k \rightarrow$ case : 2

$\boxed{p = 1} \rightarrow T(n) = \theta(n^k \log^p n)$

$T(n) = \theta(n^{\log_b a} \cdot \log^{p+1} n)$

$= \theta(n^{\log_4 1} \log^2 n)$

$= \theta(n^0 \log^2 n)$

$$\boxed{T(n) = \theta(\log^2 n)}$$

---

(12) $T(n) = T(n/2) + T(n/4) + n^2$

→ Master theorem doesn't apply.

→ Recursion Tree $T(n) = O(n^2)$.

---

(13) $2T(n/4) + \log n$.

$\boxed{a=2}$ $\boxed{b=4}$ $\boxed{k=0}$ $\boxed{p=1}$

$b^k = (4)^0 = 1$

$b^k < a \Leftarrow a > b^k \longrightarrow$ case ①.

$T(n) = O(n^{\log_b a})$

$T(n) = O(n^{\log_4 2})$

$\boxed{T(n) = O(n^{\frac{1}{2}})}$

---

(14) $T(n) = 3T(n/3) + n\log n$.

$\boxed{a=3}$ $\boxed{b=3}$ $\boxed{k=1}$ $\boxed{p=1}$

$b^k = 3^1 = 3$

$\boxed{a = b^k} \rightarrow$ case ②.

$\boxed{p=1} \rightarrow T(n) = O(n^{\log_b a} . \log^{p+1} n)$

$T(n) = O(n^{\log_3 3} . \log^2 n)$.

$T(n) = O(n^1 \cdot \log^2 n)$

$$\boxed{T(n) = O(n \cdot \log^2 n)}$$

---

(15) $T(n) = 8T((n - \sqrt{n})/4) + n^2$

$\boxed{a=8}$ $\boxed{b=4}$ $\boxed{k=0}$ $\boxed{p=2}$

Master theorem doesn't application.

---

(16) $T(n) = 2T(n/4) + \sqrt{n}$

$\boxed{a=2}$ $\boxed{b=4}$ $\boxed{k=0}$ $\boxed{p=\frac{1}{2}}$

$a = 2 \rightarrow b^k = 4^0 = 1$

$a > b^k \longrightarrow O(n \log_b a) \rightarrow$ case (1)

$T(n) = O(n \log_4 2)$

$= O(n^{0.5})$

$$\boxed{T(n) = O(n)}$$

17) $T(n) = 2T(n/4) + n^{0.51}$

$\boxed{a=2}$ $\boxed{b=4}$ $\boxed{k=0.51}$ $\boxed{p=0}$

$a = 2$

$b^k = 4^{0.51} \longrightarrow 2.02791$

$b^k > a$

$a < b^k \Longleftrightarrow$ case 3

$\boxed{p=0} \rightarrow T(n) = O(n^k \log^p n).$

$T(n) = O(n^{0.51} \log^0 n)$

$$\boxed{T(n) = O(n^{0.51})}$$

---

18) $T(n) = 16T(n/4) + n!$

$\boxed{a=16}$ $\boxed{b=4}$ $\boxed{f(n) = n!}$

$\log_b a = \log_4 16 = 2.$

$n^{\log_b a} = n^2$

Third case of theorem:

$f(n) = \Omega(n^{\log_b a + \epsilon}) \rightarrow c > 0$

$T(n) = \theta(f(n))$

$n! = \Omega(n^{2+\epsilon})$

$16 f(n/4) \leq c f(n)$

$16 (n/4)! \leq cn!$

$\boxed{c = 0.5}$ $\rightarrow$ Then the condition satisfies

So case 3 $\rightarrow$ $T(n) = \theta(n!)$

---

(19) $T(n) = 3T(n/2) + n.$

$\boxed{a = 3}$ $\boxed{b = 2}$ $\boxed{k = 0}$ $\boxed{p = 1}$

$b^k = 2^0 = 1$

$a > b^k \rightarrow$ case (1)

$T(n) = O(n^{\log_b a})$

$T(n) = O(n^{\log_2 3})$

$$\boxed{T(n) = O(n^{\log_2 3})}$$

---

(20) $T(n) = 4T(n/2) + cn.$

$\boxed{a=4}$ $\boxed{b=2}$ $\boxed{p=1}$ $\boxed{k=0}$

$b^k = 2^0 = 1$

$a > b^k \rightarrow$ case (1)

$T(n) = O(n^{\log_b a})$

$= O(n^{\log_2 4})$

$$\boxed{T(n) = O(n^2)}$$

(21) $T(n) = 3T(n/3) + n/2$

$\boxed{a=3}$ $\boxed{b=3}$ $\boxed{k=1}$ $\boxed{p=0}$

$b^k = 3^1 = 3$

$\boxed{a = b^k} \Rightarrow$ case (2)

$\boxed{p=0} \rightarrow T(n) = \theta(n^{\log_b a} \cdot \log^{p+1} n)$

$T(n) = \theta(n^{\log_3 3} \cdot \log^1 n)$

$= \theta(n^1 \log n)$

$\boxed{T(n) = \theta(n \log n)}$

---

(22) $T(n) = 4T(n/2) + n/\log n$

$\boxed{a=4}$ $\boxed{b=2}$ $\boxed{k=1}$ $\boxed{p=0}$

$b^k = 2^1 = 2.$

$a > b^4 \longrightarrow$ case 1

$\rightarrow \Theta(n^{\log_b a})$

$\rightarrow \Theta(n^{\log_2 4})$

$\rightarrow \Theta(n^2)$

$\boxed{T(n) = \Theta(n^2)}$

---

23 $T(n) = 7T(n/3) + n^2$

$\boxed{a=7}$ $\boxed{b=3}$ $\boxed{k=2}$ $\boxed{p=0}$

$a = 7.\quad b^k = 3^2 = 9.$

$a < b^k \rightarrow$ case : 3

$\boxed{p=0} \longrightarrow \Theta(n^k \log^p n)$

$T(n) = \Theta(n^k \cdot \log^p n)$

$T(n) = \Theta(n^2 \cdot \log^0 2) \rightarrow \boxed{\Theta(n^2)}$

(24) $T(n) = 8T(n/3) + 2^n$.

$\boxed{a=8}$ $\boxed{b=3}$ $f(n) = 2^n$.

$8f(n/3) <= c(2^n)$

$8(2^{n/3}) <= c(2^n)$

$\boxed{c=0.5}$

$8(2^{n/3}) <= 2^n$ $\boxed{\text{case - } 3}$

$\boxed{T(n) = \Theta(2^n)}$

(25) $T(n) = 16T(n/4) + n$

$\boxed{a=16}$ $\boxed{b=4}$ $\boxed{k=1}$ $\boxed{p=0}$

$b^k = 4^1 = 4$

$\rightarrow a > b^k = \Theta(n^{\log_b a}) \rightarrow$ case ①

$\rightarrow \Theta(n^{\log_4 16})$

$\rightarrow \boxed{T(n) = \Theta(n^2)}$