Clocks and Calendar

Clocks

The dial of a clock is a circle whose circumference is divided into 12 parts, called hour spaces. Each hour space is further divided into 5 parts, called minute's spaces. This way, the whole circumference is divided into $12 \times 5 = 60$ minute spaces.

If we consider clock as a circular track and the two hands of clock minute hand and hour hand are just like two players running in the same direction. Let total length of the track is 360^o and minute hand completes one full round in 1 hour while hour hand cover full round in 12 hours.

Speed of hour hand = $\frac{360^o}{60} = 6^o$ per second

Speed of minute hand = $\frac{360^0}{60x12} = \frac{1}{2}^o$ per second

TIP

a. Minute hand and hour hand meet 11 times in 12 hours

b. Hands of clock will be 22 times at right angle in 12 hours.

Since they are moving in the same direction, so the relative speed of both the hands with respect to each other = $\left(6 - \frac{1}{2}\right)^o = \left(\frac{11}{2}\right)^o$ per second.

Time taken by minute hand to overtake hour hand =

$$\frac{\text{Distance}}{\text{Relative speed}} = \frac{360^o}{\left(\frac{11}{2}\right)^o} = 65\frac{5}{11}\text{minutes}$$

There are 4 types of problems on clocks:

1. To calculate the angle between the two hands when time is given.
2. To calculate the time when both the hands will be at some angle.
3. Concept of slow and fast clocks.
4. Overall gain/loss

Calculating the angle

The angle between the two hands is given by the following formula

Formula for the angle between the hands = $\left|30H - \frac{11}{2}M\right|^0$

Where H $\rightarrow$ Hour reading & M $\rightarrow$ Minute reading

Ex.1 ***Calculate the angle between the two hands of clock when the clock shows 5 : 25 p.m.***

Sol. Given time = 5 : 25 p.m. Hence H = 5 and M = 25

We can apply the following direct formula to find the angle between the hands

$$= \left|30H - \frac{11}{2}M\right|^0$$

Required angle = $\left|30 \times 5 - \frac{11}{2} \times 25\right|^0$ = $(150 - 137.5)^o$

$= 12.5^o$.

Alternative Method:

Since at 5 : 25 the minute hand will be at 5 and the angle between them will be same as the distance covered in degree by the hour hand in 25 minutes

Required angle = distance of hour hand = speed $\times$ time = $\left(\frac{1}{2}\right)^o \times 25 = 12.5^o$

Calculating the time

To calculate the time when both the hands will be at some angle

In one minute the net gain of minute hand over hour hand = $\left(6 - \frac{1}{2}\right)^o = 5\frac{1}{2}^o = \frac{11}{2}^o$

If the gain is $\frac{11}{2}^o$ then the time is 1 min.

If the gain is 1^o then the time is $\frac{2}{11}$ min

If the gain is x^o then the time is $\frac{2}{11} \times x$ min.

> If between H and (H +1) o'clock, the two hands are together at an angle θ then required time = $\frac{2}{11}$ [H$\times$ $30^o \pm \theta$] minutes, Where H is reading of hour.

Ex.2 ***At what time between 4 and 5 o'clock are the hands of the clock together?***

Sol. **Method 1:** At 4 o'clock, the hour hand is at 4 and the minute hand is at 12. It means that they are 20 min spaces apart. To be together, the minute hand must gain 20 minutes over the hour hand. Now, we know that 55 min. are gained in 60 min.

$\therefore$ 20 min are gained in $\frac{60}{55} \times 20 = \frac{240}{11} = 21\frac{9}{11}$ min.

Therefore, the hands will be together at $21\frac{9}{11}$ min past 4.

<u>Alternate method:</u> Using the formula:

Required time = $\frac{2}{11}$ [H × 30° ± θ]

Here θ = 0° (Hands of clock are together) and H = 4

$\therefore$ Required time = $\frac{2}{11}$ [4 × 30° ± 0] = $\frac{240}{11} = 21\frac{9}{11}$ min.

Therefore, the hands will be together at $21\frac{9}{11}$ min past 4.

Another formula: Between H and (H + 1) o'clock, the two hands will be together at

$5 \times H \times \left(\frac{12}{11}\right)$ min past H.

In this case; $5 \times 4 \times \left(\frac{12}{11}\right) = 21\frac{9}{11}$ min past 4

Ex.3 ***At what time between 4 and 5 o'clock will the hand of clock be at right angle?***

Sol. At 4o'clock there are 20 min. spaces between hour and minute hands. To be at right angle, they should be 15 min spaces apart.

So, there are two cases:

Case I: When the minute hand is 15 min spaces behind the hour hand.

To be in this position, the min hand should have to gain 20 – 15 = 5 min spaces.

Now, we know that 55 min spaces are gained in 60 min.

$\therefore$ 5 min spaces are gained in $\frac{60}{55} \times 5 = 5\frac{5}{11}$ min

$\therefore$ They are at right angle at $5\frac{5}{11}$ min past 4.

Case II: When the minute hand is 15 min spaces ahead of the hour hand.

To be in this position, the min hand should have to gain 20 + 15 = 35 min spaces.

Now, we know that 55 min spaces are gained in 60 min

$\therefore$ 35 min spaces will be gained in $\frac{60}{55} \times 35 = 38\frac{2}{11}$ min

$\therefore$ They are at right angle at $38\frac{2}{11}$ min past 4.

<u>Alternate method:</u>

As the hands of the clock are at right angle therefore θ = 90^0

Also time is between 4 and 5 o'clock, no of hours = 4

Required time = $\frac{2}{11}$ [(no. of hours) × 30° ± θ]

= $\frac{2}{11}$ [4 × 30° ± 90°] = $\frac{420}{11}$ or $\frac{60}{11}$ = $38\frac{2}{11}$ min or $5\frac{5}{11}$ min

$\therefore$ They are at right angle at $5\frac{5}{11}$min or $38\frac{2}{11}$ min past 4.

Another Formula: Between x and (x + 1) o'clock the two hands are rt angle at

$(5x \pm 15) \times \frac{12}{11}$ min past x

In the case; they will be at right angle at

$(5 \times 4 - 15) \times \frac{12}{11}$ and $(5 \times 4 + 15) \times \frac{12}{11}$ min past 4

or $5\frac{5}{11}$ min and $38\frac{2}{11}$ min past 4.

Concept of slow and fast clocks

Too Fast and Too Slow:

If a watch indicates 9.20 when the correct time is 9.10, it is said to be 10 minutes too fast. And if it indicates 9.00 when the correct time is 9.10, it is said to be 10 minute too slow.

Ex.4 ***Two clocks are set at 1 p.m. Fast clock gains 1 min for every hour. Find the time when the fast clock shows 6 p.m.***

Sol. For every 60 min of true clock, the fast clock will show 61 min.

For 61 minutes of fast clock, true time = 60 minutes

For 300 minutes (5 hrs) of fast clock, true time = $\frac{60}{61} \times 300$

$= \frac{18000}{61} = 295\frac{5}{61}$ min. $= 240 + 55\frac{5}{61}$ min

Actual time in the true clock = 5 : $55\frac{5}{61}$ hrs.

Overall gain/loss

After every $65\frac{5}{11}$ min. = $\frac{720}{11}$ min. the two hands will coincide. If the hands of a clock coincide every 'x' min, then

Gain/loss per day by a watch, is given by

$$\left(\frac{720}{11} - x\right) \times \frac{60 \times 24}{x}$$

[If answer is (+) then there will be gain and if (–) then there will be loss.]