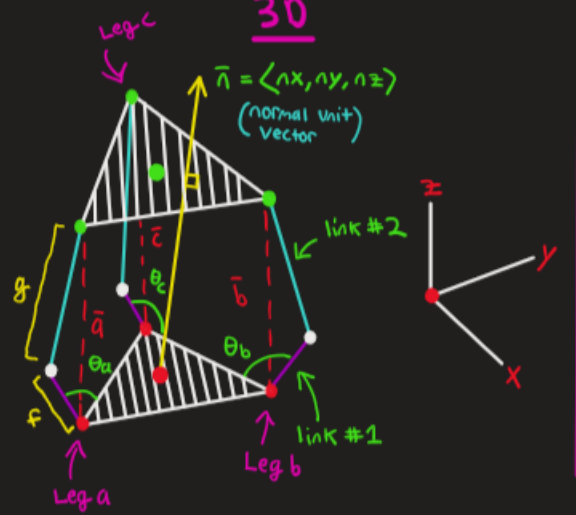
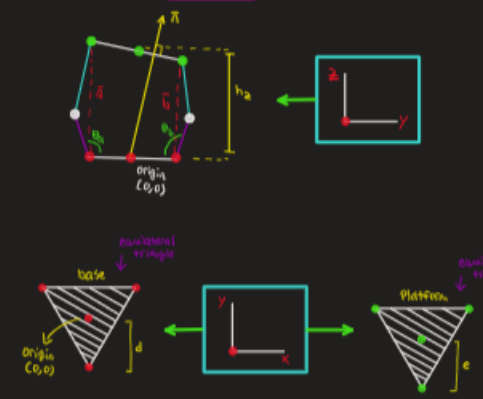
3RPS Parallel Manipulator Inverse Kinematics

3D



2D



Constants

- d → distance from the center of the base to any of its corners
- e → distance from the center of the platform to any of its corners
- f → length of link #1
- g → length of link #2

Parameters

- h → platform height
- $\bar{n} = \langle nx, ny, nz \rangle$ → unit vector normal to the platform plane

Leg a

$$a_y = d + \left(\frac{e}{2}\right)\left(1 - \frac{nx^2 + 3ny^2 + 3nz}{nz + 1 - nx^2} + \frac{nx^4 - 3nx^2ny^2}{(nz+1)(nz+1-nx^2)}\right)$$

$$a_z = h_z + e \cdot ny$$

$$a_m = \sqrt{ay^2 + a_z^2}$$

$$\theta_a = \cos^{-1}\left(\frac{a_y}{a_m}\right) + \cos^{-1}\left(\frac{a_m^2 + f^2 - g^2}{2 a_m f}\right)$$

Leg b

$$b_x = \frac{\sqrt{3}}{2}\left(e\left(1 - \frac{nx^2 + \sqrt{3}\,nx\,ny}{nz+1}\right) - d\right)$$

$$b_y = \frac{b_x}{\sqrt{3}}$$

$$b_z = h_z - \frac{e}{2}(\sqrt{3}\,nx + ny)$$

$$b_m = \sqrt{b_x^2 + b_y^2 + b_z^2}$$

$$\theta_b = \cos^{-1}\left(\frac{\sqrt{3}\,b_x + b_y}{-2 b_m}\right) + \cos^{-1}\left(\frac{b_m^2 + f^2 - g^2}{2 b_m f}\right)$$

Leg c

$$c_x = \frac{\sqrt{3}}{2}\left(d - e\left(1 - \frac{nx^2 - \sqrt{3}\,nx\,ny}{nz+1}\right)\right)$$

$$c_y = -\frac{c_x}{\sqrt{3}}$$

$$c_z = h_z + \frac{e}{2}(\sqrt{3}\,nx - ny)$$

$$c_m = \sqrt{c_x^2 + c_y^2 + c_z^2}$$

$$\theta_c = \cos^{-1}\left(\frac{\sqrt{3}\,c_x - c_y}{2 c_m}\right) + \cos^{-1}\left(\frac{c_m^2 + f^2 - g^2}{2 c_m f}\right)$$