Qsys Mathematics

Evolution

The quantum system will evolve according to the Schrodinger equation in the absence of any user input:

$$\frac{\partial}{\partial t}\psi(x,t) = \frac{-i}{\hbar}\hat{H}(t)\psi(x,t) \tag{1}$$

Without explicit time dependence in the Hamiltonian, we can immediately find the general solution as:

$$\hat{U}(t) = exp\left(\frac{-i\hat{H}t}{\hbar}\right) \tag{2}$$

For the sake of computational simplicity, we partition the simulation time finely and evaluate the hamiltonian at each time step:

$$[t_0, t_f] \to t_0, t_1, t_2, ..., t_f$$

$$\hat{H}_0 := \hat{H}(t_0) \qquad U_0 := exp\left(\frac{-i\hat{H}_0\delta t}{\hbar}\right)$$

$$\hat{H}_1 := \hat{H}(t_1) \qquad U_1 := exp\left(\frac{-i\hat{H}_1\delta t}{\hbar}\right)$$

$$\hat{H}_2 := \hat{H}(t_2) \qquad U_2 := exp\left(\frac{-i\hat{H}_2\delta t}{\hbar}\right)$$

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where $\delta t = t_{n+1} - t_n$ is the width of the time partition. Note that U_n does not have explicit time dependence but it does keep a factor of δt. This is because we are shifting all of the time dependence into the evaluation of the Hamiltonian. Each of these propogators should only advance a single time step before the next propogator is used.

Measurements occur when user input is detected. The measurement procedure takes several steps:

- Wavefunction Sampling
- Conditional Probabilities Sampling
- Bayesian Update

Step 1: Wavefunction Sampling:
The wavefunction vector is multiplied by its complex conjugate with a dot product to give a probability distribution:

$$P(x) = \psi(x) \cdot \psi^*(x) \tag{3}$$

This probability distribution is sampled to give the "hidden state" of the system, λ.
Step 2: Conditional Probabilities Sampling:
The user input, E_ϕ, and λ together index an $n \times n \times n$ collection of conditional probabilities (preset and arbitrary, you can think of this as a list of n, $n \times n$ matrices.) These probabilities are of the form

$$P(o|\lambda, E_\phi) \tag{4}$$

Selecting a λ and a E_ϕ will yield a conditioned probability distribution for the output pitches, o. This distribution is sampled to give the result of the measurement and thus the pitch that is played. **Step 3: Bayesian Update:**

A matrix D is constructed by putting the elements of the distribution $P(o|\lambda, E_\phi)$ for all output pitches o along the diagonal:

$$M = \begin{pmatrix} P(A|\lambda, E_\phi) & 0 & 0 & 0 & ... \\ 0 & P(B|\lambda, E_\phi) & 0 & 0 & ... \\ 0 & 0 & P(C|\lambda, E_\phi) & 0 & ... \\ . & & & & \\ . & & & & \\ . & & & & \end{pmatrix} \tag{5}$$

and then the state is updated to:

$$\psi(x)_{new} = \frac{M\psi(x)_{old}M}{\psi^*(x)_{old}M\psi(x)_{old}} \tag{6}$$

where $\psi(x)_{old}$ was the state immediately before the measurement occurred.

- Wavefunction
  - Mathematical Type: complex-valued probability amplitude
  - Computational Type: complex array (used here as a column vector)
  - Parameters:
    - Dimension - 2-12 complex dimensions, 4-24 real
    - Time Dependent
- Symbol: $\psi(x, t)$
- Hamiltonian
  - Mathematical Type: complex-valued Hermetian matrix
  - Computational Type: complex array (always square, symmetric under conjugate transpose)
  - Parameters:
    - Dimension - 2-12 cplx, 4-24 real
    - Time Dependent
- Symbol: $\hat{H}(t)$