Problem 1

Problem 2

Let us begin by finding the variances of the two variables. First, note that the probability of choosing someone with the trait in the entire population is $P(T) = P(Sub_1, T \cup Sub_2, T) = qp_1 + (1-q)p_2$, where p_i is the probability of finding the trait in the ith population, T is an indicator variable for the trait, and Sub_i is an indicator for whether we are in subpopulation i or not. Because X consists of indicator random variables, we can easily find variance.

$$\begin{aligned} Var(X) &= \frac{1}{n}\sigma^2 \\ &= \frac{1}{n}p(1-p) \\ &= \frac{1}{n}\left\{[qp_1 + (1-q)p_2][1 - qp_1 - (1-q)p_2]\right\} \end{aligned}$$

Now to find $Var(Y)$, let S_1 be the sampled number of people with the trait in population 1 and let S_2 be the sampled number of people with the trait in population 2.

$$\begin{aligned} Var(Y) &= Var(S_1 + S_2) \\ &= \frac{1}{n^2}\sum_{i=1}^{n} Var(Y_i) \\ &= \frac{1}{n^2}\left[qnVar(S_1) + (1-q)nVar(S_2)\right] \\ &= \frac{1}{n}[qVar(S_1) + (1-q)Var(S_2)] \\ &= \frac{1}{n}[qp_1(1-p_1) + (1-q)p_2(1-p_2)] \end{aligned}$$

We can also make the observation that, in the case of the problem at hand, $q = (1-q)$ and $p_1 = (1-p_2)$. Finding $Var(Y)/Var(X)$ is a matter of algebra at this point.

$$\begin{aligned} Var(Y)/Var(X) &= \frac{qp_1(1-p_1) + (1-q)p_2(1-p_2)}{[qp_1 + (1-q)p_2][1 - qp_1 - (1-q)p_2]} \\ &= \frac{qp_1p_2 + qp_2p_1}{[qp_1 + q(1-p_1)][1 - qp_1 + q(1-p_1)]} \\ &= \frac{2qp_1p_2}{q(1-q)} \\ &= \frac{2p_1p_2}{q} \end{aligned}$$

Plugging in our values for p_1, p_2, and q, we get

$$Var(Y)/Var(X) = 0.75$$

Problem 3