Due: 2013/11/15

Homework 6

Problem 1. *We know that when $n \geq 6$, any Y-B colouring of K_n contains a monochromatic triangle. However, we cannot say the triangle can always be found in the major colour.*
(a) Show that for any n, there is a Y-B colouring of K_n such that Y is used more than B yet there are no yellow triangles.
(b) Show that for any n, there is a Y-B colouring of K_n such that Y is used more than 99 percent of times (on the $\binom{n}{2}$ edges) yet there are not yellow K_{2013}'s.

Problem 2. *(R. Graham) Given two graphs G and H, where $V(G)$ and $V(H)$ are disjoint, their* join *is defined to be the graph with a copy of G, a copy of H and additional edges joining all the pairs between G and H. Formally,*

$$G \vee H = (V(G) \cup V(H), E(G) \cup E(H) \cup \{uv : u \in V(G), v \in V(H)\}).$$

Clearly, $K_3 \vee C_5$ does not contain a copy of K_6. Prove that, if its edges are coloured Y and B, there is always a monochromatic K_3.

Problem 3. *(V Chvátal)*
(a) Prove that, for any n, one can colour the edges of the cube Q_n with Y and B such that there are no monochromatic copies of Q_2 (or call it C_4 if you like).
(b) Let T_1 and T_2 be two trees, discuss when it is true that, no matter how we colour the edges of T_1 with Y and B, there is always a monochromatic copy of T_2.

To be more precise, describe a simple (polynomial time) algorithm, given T_1 and T_2, decide if the above property holds. Prove the correctness of your algorithm.