

Homework 4

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1. (a) The characteristic equation is $x^2 - 5x + 6 = 0$. $x_1 = 2, x_2 = 3$.

$$\therefore f(n) = ax_1^n + bx_2^n$$

$$\therefore f(0) = a + b = 0, f(1) = 2a + 3b = 1$$

$$\therefore a = -1, b = 1$$

$$\therefore f(n) = -2^n + 3^n$$

$$f(0) = -2^0 + 3^0 = 0$$

$$f(1) = -2^1 + 3^1 = 1$$

$$f(2) = -2^2 + 3^2 = 5 = 5f(1) - 6f(0)$$

$$f(3) = -2^3 + 3^3 = 19 = 5f(1) - 6f(0)$$

- (b) The characteristic equation is $x^3 - 6x^2 + 12x - 8 = 0, x_1 = x_2 = x_3 = 2$

$$\therefore f(n) = ax_1^n + bnx_1^n + cn^2x_1^n$$

$$\therefore f(0) = a = 1, f(1) = 2a + 2b + 2c = 6, f(2) = 4a + 8b + 16c = 32$$

$$\therefore a = 1, b = \frac{1}{2}, c = \frac{3}{2}$$

$$\therefore f(n) = \left(1 + \frac{1}{2}n + \frac{3}{2}n^2\right)2^n$$

$$f(0) = \left(1 + \frac{1}{2} \times 0 + \frac{3}{2} \times 0^2\right)2^0 = 1$$

$$f(1) = \left(1 + \frac{1}{2} \times 1 + \frac{3}{2} \times 1^2\right)2^1 = 6$$

$$f(2) = \left(1 + \frac{1}{2} \times 2 + \frac{3}{2} \times 2^2\right)2^2 = 32$$

$$f(3) = \left(1 + \frac{1}{2} \times 3 + \frac{3}{2} \times 3^2\right)2^3 = 128 = 6f(2) - 12f(1) + 8f(0)$$

2. (a) $x^2 + 4 = 0$

(b) $x_1 = 2i, x_2 = -2i$

(c) $f(n) = a(2i)^n + b(-2i)^n$

$$(d) \because f(0) = a + b, f(1) = (2a - 2b)i$$

$$\therefore a = \frac{f(0)}{2} + \frac{f(1)}{4i}, b = \frac{f(0)}{2} - \frac{f(1)}{4i}$$

$$\therefore f(n) = \left[\frac{f(0)}{2} + \frac{f(1)}{4i} \right] (2i)^n + \left[\frac{f(0)}{2} - \frac{f(1)}{4i} \right] (-2i)^n$$

$$(e) f(2) = -4f(0), f(3) = -4f(1), f(4) = 16f(0), f(5) = 16f(1)$$

3. The characteristic equation is $x^3 - 5x^2 + 8x - 4 = 0$. $x_1 = x_2 = 2, x_3 = 1$.

$$\therefore f(n) = (a + bn)2^n + c$$