Due: 2013/10/11 before class

Homework 2

Problem 1. *Consider a poset* $\mathcal{P} = (X, \preccurlyeq)$, *where*

$$X = \{(a_1, a_2, a_3, a_4) : a_i \in [4]\}$$

and $(a_1, a_2, a_3, a_4) \preccurlyeq (b_1, b_2, b_3, b_4)$ *iff* $a_i \leq b_i$ *for all* i. *What is the number of pairs* $x, y \in X$ *such that* $\mu(x, y) < 0$? *Briefly justify your answer.*

Problem 2. *Prove*

$$\sum_{r=0}^{n} r^2 \binom{n}{r} = n(n+1)2^{n-2}.$$

For extra challenge, find a combinatorial proof.

Problem 3. *Give a combinatorial proof for the following equation. For any positive integers* a *and* b,

$$\sum_{i=0}^{a} \binom{a}{i} \binom{b+i}{a} = \sum_{i=0}^{a} \binom{a}{i} \binom{b}{i} 2^i.$$

Problem 4. *Given* $n \geq 1$, *we construct a graph on the set of all permutations of* $[2n]$. *Two permutations* π *and* π' *are adjacent if (1)* π *and* π' *are the same except on two positions* i *and* $i+1$; *and (2)* $|\pi(i) - \pi(i+1)| = n$. *Explain that the number of connected components in this graph is* $\sum_{i=0}^{n} (-1)^i \binom{n}{i} (2n-i)!$.

Problem 5 (Extra)**.** *Give a combinatorial proof for*

$$\sum_{k=0}^{n} \binom{2k}{k} \binom{2n-2k}{n-k} = 4^n.$$