Do not need to hand in

Homework 12

Problem 1. *Let A be the adjacency matrix of the complete bipartite graph $K_{3,3}$, compute the eigenvalues of A.*

Problem 2. *Let A_1, A_2, ..., A_m be m distinct subsets of $[n]$ such that each $|A_i|$ is even and for any $i \neq j$, $|A_i \cap A_j|$ is odd. How big can m be? Prove your answer. (i.e., Prove the bound, and show examples where the bound is reached.)*

Problem 3. *Let t be a fixed integer. Let A_1, A_2, ... A_m be distinct subsets of $[n]$ such that $|A_i \cap A_j| = t$ for any $i \neq j$. Prove that $m \leq n$.*

We will discuss the following cute problems in class next time.

Problem 4. *Let n be a positive integer. Consider*

$$S = \{(x, y) : x, y \in [n] \cup \{0\}, x + y > 0\}$$

as a set of $(n+1)^2 - 1$ points in $\mathbf{R}^3$. Determine the smallest number of lines, the union of which contains S but does not inlcude $(0, 0)$.

Problem 5. *Let n be a positive integer. Consider*

$$S = \{(x, y, z) : x, y, z \in [n] \cup \{0\}, x + y + z > 0\}$$

as a set of $(n + 1)^3 - 1$ points in $\mathbf{R}^3$. Determine the smallest number of planes, the union of which contains S but does not inlcude $(0, 0, 0)$.