Due: 2013/10/18

Homework 3

Problem 1. *Pick two numbers a and b from $[100]$ uniform randomly and independently, what is the probability that $\mu(a,b) = -1$?*

Problem 2. *Prove that the Stirling number of the 1st kind $s(n,k)$ is an even number whenever $2k < n$ and $n > 0$.*

Problem 3. *Let T be the Stirling number of the 1st kind $T = s(100, 50)$. Compute $T \mod 3$. Justify your answer.*

Problem 4. *Count the number of permutations $x_1, x_2, \cdots, x_{2n}$ of $[2n]$ such that $x_i + x_{i+1} \neq 2n+1$ for all $1 \leq i \leq 2n-1$.*

Problem 5. *Give a combinatorial proof that the Stirling number of the 2nd kind $S(n,k)$ equals the coefficient of x^{n-k} in*

$$\prod_{t=0}^{k}(1 + tx + t^2x^2 + t^3x^3 + ... + t^nx^n).$$