Homework 10

Mathematics in Computer Science

1. What are the Boolean functions

   (a) $(x \to y) \to y$?

   (b) $(x \to y) \to x$?

2. There are 16 Boolean functions of two variables. A subset of the 16 Boolean functions is said to be universal if the other functions can be expressed in terms of functions in the subset. A subset is minimal if deleting any element of the subset results in a non universal set. AND and NOT is a minimal universal set as is OR and NOT. Given three other such sets. Demonstrate that your sets are indeed universal.

3. How many Boolean functions of n variables?

4. There are two types of people in a certain group. Liars who always lie and truth tellers who always say the truth. In each of the following can you say anything about the type of person.

   (a) A says one of us is a liar. B says nothing.
   A is truth teller and B is liar.

   (b) A says both of us are truth tellers and B says A is a liar.
   A is liar and B is a truth teller.

   (c) A and B both say we are truth tellers.
   Either A and B are both truth tellers or both liars.

5. Prove that $x \to y$ is equivalent to $\overline{y} \to \overline{x}$

6. Aces and eights - 8 cards 4 aces 4 eights, three players, six cards dealt, two to each player. Other two face down. Each layer show his or her hand to the other two players. In turn each player tries to determine his or her hand. If player does not know his or her hand he/she must say so.

   (a) Alice two aces, Bob two eights, you
   Alice don't know, Bob don't know. What are you holding? two aces, ? two eights? or one of each? I must have one ace and one eight otherwise one of Bob or Alice would know what they are holding.

   (b) you, Alice two eights, Bob ace+eight
   you don't know, Alice don't know, Bob don't know.
   I have either two aces or ace and eight. If I had two aces Bob would have known by (a) that he had an ace+eight. Thus I must have ace+eight.