Assignment (III)

Due: 8:00 AM Tuesday, 5 Nov

Problem 1

Suppose that f is a total computable function, A a recursive set and B an r.e. set. Prove that $f^{-1}(A)$ is recursive and that $f(A)$, $f(B)$ and $f^{-1}(B)$ are r.e. but not necessary recursive. What extra information about these sets can be obtained if f is a bijection?

Problem 2

Suppose that A is an r.e. set. Show that the sets $\bigcup_{x \in A} W_x$ and $\bigcup_{x \in A} E_x$ are both r.e. Show that $\bigcap_{x \in A} W_x$ is not necessary r.e.

Problem 3

Prove that if $A \leq_1 B$ and A and B are r.e. and A is infinite then $A \leq_1 B$ via some f such that $f(A) = B$.

Problem 4

Prove that $Inf \equiv_1 Tot \equiv_1 Con$.

Problem 5

We say a set A is an index set if $\forall x \in A \wedge \phi_x \simeq \phi_y \implies y \in A$, A is a *cylinder* if $(\forall B)[B \leq_m A \implies B \leq_1 A]$. Prove that any index set is a *cylinder*.

Problem 6

1. Disjoint sets A and B are *recursively imseparable* if there is no recursive set C such that $A \subseteq C$ and $C \cap B = \emptyset$. Show that there exist disjoint r.e. sets which are recursively inseparable.

2. Prove that if two disjoint r.e. sets A and B are recursively inseparable, then $K \equiv_1 A \equiv_1 B$.