

Due: 2012/11/22

Homework 7

Problem 1. Determine the Schur number $S(2, 2)$, justify your answer.

Problem 2. Prove the stronger version of Schur's theorem: For any positive integers c and m , there exists $S^*(c, m)$ such that no matter how we colour $[S^*(c, m)]$ by c colours, there are distinct $x_1, x_2, \dots, x_m, y \in [S(c)]$ with the same colour such that $\sum_{i=1}^m x_i = y$.

Problem 3. Prove that for any positive integer c , there is a number $N = N(c)$ such that for any c -colouring of all subsets of $[N]$, $f : 2^{[N]} \rightarrow [c]$, there exists non-empty disjoint sets $X, Y \subseteq [N]$ such that $f(X)$, $f(Y)$ and $f(X \cup Y)$ are the same.