Due: 2013/09/27 before class

# Homework 1

**Problem 1.** *Calculate the following:*
*(a)* $\binom{10}{8}$*;*
*(b) The inverse of* $1, 2, 3, 4, 5, 6$ *in* $\mathbf{Z}_7$*, i.e. for each* $1 \leq i \leq 6$*, find* $j = i^{-1}$ *such that* $ij \equiv 1 \mod 7$*;*
*(c) The rightmost digit in the decimal* $\binom{449}{137}$*, i.e.* $\binom{449}{137} \mod 10$*.*

**Problem 2.** *Find the number of ordered pairs* $(A, B)$ *such that* $A, B \subseteq [n]$ *and* $A \cap B = \emptyset$*.*

**Problem 3.** *Consider a 2-d array of* $9 \times 9$ *unit squares. If each unit square is filled with a distinct number from* $[81]$*, prove that there are always two neighboring squares (vertically or horizontally adjacent) with difference at least 9.*

**Problem 4.** *n people are in a big party. There are many triples (a group of 3 among the n) called* fun*. All we know is that, among any 4 people, the number of fun triples is even (that is, 0, 2, or 4). For any two persons a and b, the club* $\overline{ab}$ *is the set of people including a, b, and anyone who forms a fun triple with a and b.*
*Prove that, either there is a club of size n, or there are at least n different clubs.*