Assignment (IV)

Due: 8:00 PM Sunday, 2013-11-17

Problem 1

Prove that if B is r.e. and $A \cap B$ is productive, then A is productive.

Problem 2

1. Let $\mathcal{A}$ be a set of unary computable functions, and suppose that $g \in \mathcal{A}$ implies for all finite $\theta \subseteq g$, $\theta \notin \mathcal{A}$. Prove that the set $A = \{x \mid \phi_x \in \mathcal{A}\}$ is productive.

2. Prove that $\{x \mid \phi_x$ is a polynomial function $\}$ is productive.

Problem 3

Let $A_x = \{y \mid \phi_y = \phi_x\}$. Show that there is no totoal computable function $f(x, y)$ such that $\forall x.(y \in \bar{K} \iff f(x, y) \in A_x)$.

Problem 4

Suppose that A is r.e. and $A \neq \mathbb{N}$. Prove that A is creative if and only if

$$(\forall \text{ r.e. } B)[A \cap B = \emptyset \implies A \equiv A \cup B].$$

Problem 5

A set A is simple if (i) A is r.e., (ii) $\bar{A}$ is infinite, and (iii) $\bar{A}$ contains on infinite r.e. subset. Prove that

1. A simple set is neither recursive nor creative.

2. Define a function $f(x)$ as follows. Given input x, parallel compute $\phi_x(0), \phi_x(1) \ldots$ in a diagonal way; stop if and only if a number z is found such that $\phi_x(z) > 2x$; in that case we put $f(x) = \phi_x(z)$. Prove that $A = Ran(f)$ is simple.

Problem 6

Let C be a compleixty measure. We will say that a totoal computable function $f(x)$ is *C-constructible* if there is a program number e s.t. $C_e(x) = f(x)$ for all x. Prove or disprove that every computable fucntion is C-constructible.