No need to hand in

Homework 5

Problem 1. *Consider the graph Q_n, the n-dimensional cube for $n \geq 1$. Find $v(Q_n)$, $e(Q_n)$, $\delta(Q_n)$, and $\Delta(Q_n)$. What is the distance from $\emptyset$ to $[n]$? And what is the number of shortest paths from $\emptyset$ to $[n]$? What is the size of $Aut(Q_n)$?*

Problem 2. *Let G be a graph on n vertices ($n > 3$) with no vertex of degree $n-1$. Suppose that for any two vertices of G, there is a unique vertex adjacent to both of them.*
(a) If u and v are not adjacent, prove that they have the same degree. (Hint: Construct a bijection between the two sets of neighbors.)
(b) Show that G is k-regular for some k.
(c) Express n in terms of k.

Problem 3. *Let G_n be the graph with $2n$ ($n \geq 3$) vertices v_1, v_2, ..., v_n and u_1, u_2, ..., u_n, where $v_1v_2...v_n$ form a cycle, and u_i is only adjacent to v_i for each i. Compute the chromatic polynomial for G_n.*

Problem 4. *Suppose G is a graph on n vertices and $G \cong \overline{G}$. Prove that (a) G is connected; (b) Either n or $n-1$ is a multiple of 4; (c) If $n = 4k+1$ for some integer k, then there is a vertex v such that $deg(v) = 2k$.*

Problem 5. *Color the edges of K_n by yellow and blue. A cycle is called* monochromatic *if all its edges have the same color. Prove that, if there is a monochromatic cycle of length $2k+1$ for some $k > 2$, then there is also a monochromatic cycle of length $2k$.*

Problem 6. *Prove that there exists a constant $c > 0$ and $N > 0$, such that the number of isomorphic classes of trees on $[n]$ is at least c^n whenever $n > N$.*