Due: 2013/10/25

Homework 4

Problem 1. *Prove that, if G is a k-regular ($k \geq 1$) bipartite graph with bipartition A and B, then $|A| = |B|$.*

Problem 2. *For any positive integer n, define a graph $G = (V, E)$, where V consists of points in the plane $(i, 0)$ for $i = 0, 1, 2, ..., n+1$, $(i, 1)$ and $(i, -1)$ for $i = 1, 2, ..., n$. Two vertices are adjacent if (1) their distance in the plane is 1; and (2) one of the vertices is on the x-axis. Find the size of $Aut(G)$.*

Problem 3. *For each $n > 5$, find a graph G of order n for which $|Aut(G)| = 1$. (In addition, you may check that any graph G on $[2]$, $[3]$, or $[4]$ has $|Aut(G)| > 1$.)*

Problem 4. *For which n does there exist a graph G on $[n]$ such that $G \cong \overline{G}$? Prove your answer.*