Assignment (II)

1. (a) Show that the set of all functions form $\mathbb{N}$ to $\mathbb{N}$ is not denumerable. (b) Show that the set of all non-computable total functions from $\mathbb{N}$ to $\mathbb{N}$ is not denumerable.

2. Show by diagonalization that the function f defined by

$$f(x) \simeq \begin{cases} 1 & \phi_x \text{ is total and } \phi_x(y) \leq \phi_x(y+1) \text{ for all } y \\ \uparrow & \text{otherwise} \end{cases}$$

   is not computable.

3. Show that there is a number n s.t. $W_n = E_n = n\mathbb{N}$.

4. Show that there exist total computable functions f and g such that for all x, $E_{f(x)} = W_x$ and $W_{g(x)} = E_x$.

5. Show that for each m there is a total $(m+1)$-ary computable function s^m such that for all n

$$\phi_e^{(m+n)}(\tilde{x}, \tilde{y}) \simeq \phi_{s^m(e,\tilde{x})}(\tilde{y}).$$

   Show that further there is such function s^m that is primitive recursive.

6. Show that for any total computable function f, there is an increasing recursive function $n(t)$ such that for every t, $\phi_{n(t)} = \phi_{f(n(t))}$. [Hint: use padding technique.]