

Given $\Delta^n y$ is the n^{th} forward difference of sequence y

$$\Delta f(x) = f(x+1) - f(x) \quad (1)$$

And x^n is the falling sequential product (falling factorial, factorial power)

$$x^n = \overbrace{x(x-1)\dots(x-n+1)}^{n \text{ factors}} \quad \text{for integer } n \geq 0 \quad (2)$$

And C_n is the quotient of the 0^{th} element of $\Delta^n y$ over $n!$

$$C_n = \frac{\Delta^n y[0]}{n!} \quad (3)$$

Where $y = \langle 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, \dots \rangle$

$$\begin{aligned} \Delta^0 y &= \langle 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, \dots \rangle \\ \Delta^1 y &= \langle 1, 3, 5, 7, 9, 11, 13, 15, 17, \dots \rangle \\ \Delta^2 y &= \langle 2, 2, 2, 2, 2, 2, 2, 2, \dots \rangle \\ \Delta^3 y &= \langle 0, 0, 0, 0, 0, 0, 0, \dots \rangle \end{aligned} \quad (4)$$

$$\begin{aligned} C &= \left\langle \frac{0}{0!}, \frac{1}{1!}, \frac{2}{2!} \right\rangle \\ &= \left\langle \frac{0}{1}, \frac{1}{1}, \frac{2}{2} \right\rangle \\ &= \langle 0, 1, 1 \rangle \end{aligned} \quad (5)$$

$$\begin{aligned} x^0 &= 0 \\ x^1 &= x \\ x^2 &= x(x-1) = x^2 - x \end{aligned} \quad (6)$$

$$\begin{aligned} G &= \sum C_n x^n \\ &= (0 \cdot 0) + (1 \cdot x) + (1 \cdot (x^2 - x)) \\ &= x + x^2 - x \\ &= x^2 \end{aligned} \quad (7)$$