|             |             |                |               |                             |   |  |  |     |                              |                             | ı     |               |             |
|-------------|-------------|----------------|---------------|-----------------------------|---|--|--|-----|------------------------------|-----------------------------|-------|---------------|-------------|
|             | $a_1$       | $a_2$          | $a_3$         | $a_4$                       | $a_6$   | r                                      | T                                      | s   | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$ | Kodaira       | Isogenies   |
| 31a         |             |                |               |                             | $31a = (3 - 5\varphi) = 31a$ (1 isoger                | ny class)                              |  |     |                              |                             |       |               | 31a         |
| a1          | 1           | $-1-\varphi$   | $\varphi$     | 0                           | 0   | 0                                      | 8                                      | -+  | 1                            | 1                           | 1     | $I_1$         |             |
| a2          | $1+\varphi$ | $-1-\varphi$   | $\varphi$     | $-27+15\varphi$             | $64-40\varphi$  | 0                                      | 8                                      | ++  | 2                            | 2                           | 2     | $I_2$         |             |
| a3          | $\varphi$   | -1             | $\varphi$     | $-2891 + 1786\varphi$       | $71196 - 44002\varphi$                                | 0                                      | 4                                      | +-  | 1                            | 1                           | 1     | $I_1$         |             |
| a4          | 1           | $-1-\varphi$   | $\varphi$     | $-30-40\varphi$             | $-76-130\varphi$                                      | 0                                      | 4                                      | ++  | 4                            | 4                           | 2     | $I_4$         |             |
| a5          |             | $-1-\varphi$   | $\varphi$     | $-45-30\varphi$             | $-117-111\varphi$                                     | 0                                      | 2                                      | +-  | 8                            | 8                           | 2     | $I_8$         |             |
| a6          | $1+\varphi$ | $1+\varphi$    | $1+\varphi$   | $-19898 - 32196\varphi$     | $-2083814 - 3371682\varphi$                           | 0                                      | 2                                      | -+  | 2                            | 2                           | 2     | $I_2$         |             |
| 31b         |             |                |               |                             | $31b = (2 - 5\varphi) = 31b$ (1 isogen                | y class)                               |  |     |                              |                             |       |               | 31b         |
| a1          | 1           | $1+\varphi$    | $\varphi$     | $\varphi$                   | 0   | 0                                      | 8                                      | + - | 1                            | 1                           | 1     | $I_1$         |             |
| a2          | $\varphi$   | -1             | $1+\varphi$   | $-11-17\varphi$             | $24 + 39\varphi$                                      | 0                                      | 8                                      | ++  | 2                            | 2                           | 2     | $I_2$         |             |
| a3          | $1+\varphi$ | •              | $1 + \varphi$ | $-1105 - 1788\varphi$       | $27194 + 44001\varphi$                                | 0                                      | 4                                      | -+  | 1                            | 1                           | 1     | $I_1$         |             |
| a4          | 1           | $1 + \varphi$  | $\varphi$     | $-70 + 41\varphi$           | $-276 + 170\varphi$                                   | 0                                      | 4                                      | ++  | 4                            | 4                           | 2     | $I_4$         |             |
| a5          | 1           | $1 + \varphi$  | $\varphi$     | $-75 + 31\varphi$           | $-303 + 141\varphi$                                   | 0                                      | 2                                      | -+  | 8                            | 8                           | 2     | $I_8$         |             |
| a6          | $\varphi$   | $\varphi$      | $1+\varphi$   | $-52096 + 32197\varphi$     | $-5371204 + 3319586\varphi$                           | 0                                      | 2                                      | + - | 2                            | 2                           | 2     | $I_2$         |             |
| 36a         |             |                |               |                             | $36a = (6) = 2 \cdot 3 \qquad (1 \text{ isogeny})$    | class)                                 |  |     |                              |                             |       |               | 36a         |
| a1          | $1+\varphi$ | $\varphi$      | $\varphi$     | 0                           | 0   | 0                                      | 10                                     |     | 2, 1                         | 2, 1                        | 2, 1  | $I_2, I_1$    |             |
| a2          | $1+\varphi$ | $\varphi$      | $\varphi$     | $-10-10\varphi$             | $10 + 10\varphi$                                      | 0                                      | 10                                     | ++  | 1, 2                         | 1, 2                        | 1, 2  | $I_1, I_2$    |             |
| a3          | $1+\varphi$ | $\varphi$      | $\varphi$     | $-165-165\varphi$           | $-1221 - 1683\varphi$                                 | 0                                      | 2                                      | ++  | 5, 10                        | 5, 10                       | 1, 2  | $I_5, I_{10}$ |             |
| a4          | $1+\varphi$ | $\varphi$      | $\varphi$     | $-5-5\varphi$               | $-37-51\varphi$                                       | 0                                      | 2                                      |     | 10, 5                        | 10,5                        | 2,1   | $I_{10}, I_5$ |             |
| 41a         |             |                |               |                             | $41a = (7 - \varphi) = 41a$ (1 isogen                 | y class)                               |  |     |                              |                             |       |               | 41a         |
| a1          |             | $-1 + \varphi$ |               | 0                           | -arphi  | 0                                      | 7                                      |     | 1                            | 1                           | 1     | $I_1$         |             |
| a2          | 0           | $-1+\varphi$   | $1+\varphi$   | $-30-10\varphi$             | $-82-32\varphi$                                       | 0                                      | 1                                      |     | 7                            | 7                           | 1     | $I_7$         |             |
| <b>41</b> b |             |                |               |                             | $41b = (6 + \varphi) = 41b \qquad (1 \text{ isogen})$ | y class)                               |  |     |                              |                             |       |               | <b>41</b> b |
| a1          | 0           | $-\varphi$     | $\varphi$     | 0                           | 0   | 0                                      | 7                                      |     | 1                            | 1                           | 1     | $I_1$         |             |
| a2          | 0           | $-\varphi$     | $\varphi$     | $-40 + 10\varphi$           | $-113 + 31\varphi$                                    | 0                                      | 1                                      |     | 7                            | 7                           | 1     | $I_7$         |             |
| <b>45</b> a |             |                |               |                             | $45a = (3 - 6\varphi) = 5a \cdot 3$ (1 isoge          | ny class)                              |  |     |                              |                             |       |               | 45a         |
| a1          | 1           | 1              | 1             | -80                         | 242   | 0                                      | 8                                      | ++  | 1, 2                         | 1, 2                        | 1, 2  | $I_1, I_2$    |             |
| a2          | 1           | 1              | 1             | -5                          | 2   | 0                                      | 16                                     | ++  | 2, 4                         | 2,4                         | 2, 4  | $I_2, I_4$    |             |
| a3          | 1           | 1              | 1             | 0                           | 0   | 0                                      | 8                                      |     | 1, 2                         | 1,2                         | 1, 2  | $I_1, I_2$    |             |
| a4          | 1           | 1              | 1             | -10                         | -10   | 0                                      | 16                                     | ++  | 4, 8                         | 4,8                         | 4, 8  | $I_4, I_8$    |             |
| a5          | 1           | 1              | 1             | -135                        | -660  | 0                                      | 8                                      | ++  | 8, 4                         | 8,4                         | 8,4   | $I_8, I_4$    |             |
| a6          | 1           | 1              | 1             | 35                          | -28   | 0                                      | 8                                      |     | 2, 16                        | 2, 16                       | 2, 16 | $I_2, I_{16}$ |             |
| a7          | 1           | 1              | 1             | -2160                       | -39540  | 0                                      | 4                                      | ++  | 4, 2                         | 4,2                         | 4,2   | $I_4, I_2$    |             |
| a8          | 1           | 1              | 1             | -110                        | -880  | 0                                      | 4                                      |     | 16, 2                        | 16, 2                       | 16, 2 | $I_{16}, I_2$ |             |
| a9          | 1           | $1-\varphi$    | $\varphi$     | $-8052529 + 4976732\varphi$ | $-10344409915 + 6393196917\varphi$                    | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\begin{vmatrix} 2 \\ 2 \end{vmatrix}$ | + - | 2, 1                         | 2,1                         | 2, 1  | $I_2, I_1$    |             |
| a10         | 1           | $\varphi$      | $1+\varphi$   | $-3075797 - 4976733\varphi$ | $-3951212998 - 6393196918\varphi$                     | 0                                      | 2                                      | -+  | 2, 1                         | 2,1                         | 2,1   | $I_2, I_1$    |             |

|              | $a_1$       | $a_2$                         | $a_3$         | $a_4$                               | $a_6$  | r                                      | T  | s            | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$            | $c_p$  | Kodaira   | Isogenies    |
|--------------|-------------|-------------------------------|---------------|-------------------------------------|--|--|--|--------------|------------------------------|--|--|---|--------------|
|              |             |                               |               |                                     |  |  |  |              |                              | 1                                      |  |   |              |
| 49a          |             |                               |               |                                     | 40 - (7) 7 (1:   |  |  |              |                              |  |  |   | 49a          |
|              |             |                               |               |                                     | . , ,  | ogeny class)                           |  |              |                              |  |  |   | 49a          |
| a1           | 0           | $1-\varphi$                   | 1             | 1                                   | 0  | 0                                      | 5  |              | 1                            | 1                                      | 1  | $I_1$   |              |
| a2           | 0           | $1-\varphi$                   | 1             | $-29 - 30\varphi$                   | $-84 - 102\varphi$                                       | 0                                      | 1  |              | 5                            | 5                                      | 1  | $I_5$   |              |
|              |             |                               |               |                                     |  |  |  |              |                              |  |  |   |              |
| 55a          |             |                               |               |                                     | $55a = (8 - \varphi) = 5a \cdot 11b$                     | (1 isogeny class)                      |  |              |                              |  |  |   | 55a          |
| a1           | 1           | $\varphi$                     | 1             | $-1+\varphi$                        | 0  | 0                                      | 6  | + -          | 1, 1                         | 1,1                                    | 1,1  | $I_1, I_1$  |              |
| a2           | 1           | $\varphi$                     | 1             | $-11+6\varphi$                      | $16-10\varphi$   | 0                                      | 12   | ++           | 2, 2                         | 2,2                                    | 2, 2   | $I_2, I_2$  |              |
| a3           | $1+\varphi$ | -1                            | 1             | $-1131 + 698\varphi$                | $17565 - 10856\varphi$                                   | 0                                      | 6  | ++           | 1, 4                         | 1,4                                    | 1,2  | $I_1, I_4$  |              |
| a4           | $\varphi$   | $1-\varphi$                   | $\varphi$     | $-60-96\varphi$                     | $333 + 537\varphi$                                       | 0                                      | 6  | -+           | 4, 1                         | 4, 1                                   | 2, 1   | $I_4, I_1$  |              |
| a5           | 1           | $\varphi$                     | 1             | $-46+21\varphi$                     | $-112 + 54\varphi$                                       | 0                                      | 4  | ++           | 6, 6                         | 6, 6                                   | 2, 2   | $I_6, I_6$  |              |
| a6           | 1           | arphi                         | 1             | $-41+26\varphi$                     | $-114 + 70\varphi$                                       | 0                                      | 2  | + -          | 3, 3                         | 3, 3                                   | 1,1  | $I_3, I_3$  |              |
| a7           | 1           | $\varphi$                     | 1             | $54-54\varphi$                      | $-572 + 374\varphi$                                      | 0                                      | 2  | -+           | 12, 3                        | 12, 3                                  | 2, 1   | $I_{12}, I_3$   |              |
| a8           | $\varphi$   | $1-\varphi$                   | $\varphi$     | $-405 - 601\varphi$                 | $-5400 - 8817\varphi$                                    | 0                                      | 2  | ++           | 3, 12                        | 3, 12                                  | 1,2  | $I_{3}, I_{12}$   |              |
|              |             |                               |               |                                     |  |  |  |              |                              |  |  |   |              |
| 55b          |             |                               |               |                                     | $55b = (7 + \varphi) = 5a \cdot 11a$                     | (1 isogeny class)                      |  |              |                              |  |  |   | 55b          |
|              |             | 1 ,0                          | 1             | /2                                  | $\frac{360 - (1 + \varphi) - 3\alpha \cdot 11\alpha}{0}$ |  | 6  | 1            | 1 1                          | 1 1                                    | 1 1  | тт  |              |
| a1<br>a2     | 1 1         | $1 - \varphi \\ 1 - \varphi$  | $1 \\ 1$      | $-arphi \ -5-6arphi$                | $6+10\varphi$  | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$ | - +<br>  + + | 1, 1 $2, 2$                  | $1, 1 \\ 2, 2$                         | $\begin{array}{ c c } 1,1\\2,2\end{array}$   | $\begin{matrix} I_1, I_1 \\ I_2, I_2 \end{matrix}$      |              |
| a2<br>a3     | $\varphi$   | $-\varphi$                    | 1             | $-3 - 6\varphi$ $-432 - 699\varphi$ | $6709 + 10856\varphi$                                    |  | 6  | ++           | 1, 4                         | 1, 4                                   | 1, 2   | $I_1, I_4$  |              |
| $a_4$        | $1+\varphi$ |                               | $1+\varphi$   | $-156 + 94\varphi$                  | $870 - 538\varphi$                                       | 0                                      | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$ | + -          | 4, 1                         | 4,1                                    | $\begin{bmatrix} 1, 2 \\ 2, 1 \end{bmatrix}$ | $I_1, I_4$ $I_4, I_1$                                   |              |
| a5           | 1           | $1-\varphi$                   | 1             | $-25-21\varphi$                     | $-58-54\varphi$  | 0                                      | 4  | ++           | 6, 6                         | 6,6                                    | 2, 2   | $I_6, I_6$  |              |
| a6           | 1           | $1-\varphi$                   | 1             | $-15-26\varphi$                     | $-44-70\varphi$  | 0                                      | 2  | - +          | 3, 3                         | 3, 3                                   | 1, 1   | $I_3, I_3$  |              |
| a7           | 1           | $1-\varphi$                   | 1             | 54arphi                             | $-198 - 374\varphi$                                      | 0                                      | 2  | + -          | 12, 3                        | 12, 3                                  | 2,1  | $I_{12}, I_{3}$   |              |
| a8           | $1+\varphi$ |                               | $1 + \varphi$ | $-1006 + 599\varphi$                | $-14217 + 8816\varphi$                                   | 0                                      | 2  | ++           | 3, 12                        | 3, 12                                  | 1,2  | $I_3, I_{12}$   |              |
|              |             |                               | <u>.</u>      | <u> </u>                            | •  |  | 1  |              |                              | 1                                      | 1  |   |              |
| 64a          |             |                               |               |                                     | C4 (0) 93 (1:  | 1 )                                    |  |              |                              |  |  |   | 64a          |
|              |             | 1 .                           |               |                                     | *                  | ogeny class)                           |  |              | 4                            |  | 0  | ттт   | U4a          |
| a1           |             | $-1+\varphi$                  | 0             | $-\varphi$                          | 0 17.  | 0                                      | 8  | ++           | 4                            | 0                                      | 2  | III<br>*  |              |
| a2<br>a3     | 0           | $-\varphi$                    | $0 \\ 0$      | $-16 + 11\varphi$ $-5 - 11\varphi$  | $27 - 17\varphi$   | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 8 8  | + -          | 8<br>8                       | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 4  | т*  |              |
| a5<br>a4     | 0           | $-1 + \varphi$ $-1 + \varphi$ | 0             | $-5-11\varphi \ -5-6\varphi$        | $10 + 17\varphi$ $-7 - 11\varphi$                        |  | $\begin{vmatrix} 6 \\ 4 \end{vmatrix}$                 | - +<br>  + + | 8                            | 0                                      | $\begin{vmatrix} 4\\2 \end{vmatrix}$         | $\begin{bmatrix} I_1^* \\ I_1^* \\ I_1^* \end{bmatrix}$ |              |
| a5           | 0           | $-1 + \varphi$ $-\varphi$     | 0             | $-3 - 6\varphi$ $-171 + 106\varphi$ | $-7 - 11\varphi$ $-1050 + 647\varphi$                    |  | 2  | + -          | 10                           | 0                                      | $\frac{2}{2}$                                | $_{\mathrm{III}^{*}}^{\mathrm{I}_{1}}$                  |              |
| a6           |             | $-1+\varphi$                  | 0             | $-65-106\varphi$                    | $-403 - 647\varphi$                                      | 0                                      | 2  | - +          | 10                           | 0                                      | 2  | III*  |              |
|              |             | - · r                         |               |                                     | 200 02.7   |  |  | '            |                              |  |  |   |              |
| <b>7</b> 1 - |             |                               |               |                                     |  |  |  |              |                              |  |  |   | <b>₽</b> 1 - |
| 71a          |             |                               |               |                                     | . , ,  | isogeny class)                         |  |              |                              | T                                      |  | T _   | 71a          |
| a1           |             | $-1+\varphi$                  | 1             | 0                                   | 0  | 0                                      | 6  | + -          | 1                            | 1                                      | 1  | $I_1$   |              |
| a2           |             | $-1+\varphi$                  | 1             | $-5\varphi$                         | $2\varphi$   | 0                                      | $\frac{6}{2}$  | -+           | 2                            | 2                                      | 2  | $I_2$   |              |
| a3           |             | $-1+\varphi$                  | 1             | $-20+15\varphi$                     | $-34 + 22\varphi$  | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 2  | +-           | 3                            | 3                                      | $\begin{vmatrix} 1 \\ 2 \end{vmatrix}$       | $I_3$   |              |
| a4           | $1+\varphi$ | $-1+\varphi$                  | 1             | $-25 + 5\varphi$                    | $-46 + 12\varphi$  | 0                                      | 2  | -+           | 6                            | 6                                      | 2  | $I_6$   |              |

|  |                 |                                |                 |                                  |   |  | Leni          | 1            | 1/4)                         | 1 (1)                                   |                                      |  |           |
|--|-----------------|--------------------------------|-----------------|----------------------------------|---|--|---------------|--------------|------------------------------|---|--------------------------------------|--|-----------|
|  | $a_1$           | $a_2$                          | $a_3$           | $a_4$                            | $a_6$                                       | r                                      | T             | s            | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$             | $c_p$                                | Kodaira                                | Isogenies |
|  |                 |                                |                 |                                  |   |  |               |              |                              |   |                                      |  |           |
| 71b  |                 |                                |                 |                                  | 711 (0 + 12) 711 (1 :00)                    | manus alagg)                           |               |              |                              |   |                                      |  | 71b       |
|  |                 | 1                              |                 |                                  | · · · · · · · · · · · · · · · · · · ·       | geny class)                            | - C           |              | 1                            | 1                                       | 1                                    | т                                      | 110       |
| a1<br>a2                                   | $\varphi$       | $1 + \varphi \\ 1 + \varphi$   | $\varphi$       | $rac{arphi}{-5+6arphi}$         | $0 \ 7-2arphi$                              | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 6             | - +<br>  + - | $\frac{1}{2}$                | $\begin{array}{c c} 1 \\ 2 \end{array}$ | $\begin{vmatrix} 1\\2 \end{vmatrix}$ | $egin{array}{c} I_1 \ I_2 \end{array}$ |           |
| $\begin{vmatrix} a_2 \\ a_3 \end{vmatrix}$ | $arphi \ arphi$ | $1+\varphi$<br>$1+\varphi$     | $\varphi$       | $-5 + 6\varphi$ $-5 - 14\varphi$ | $7-2\varphi$ $-27-42\varphi$                |  | 2             | -+           | 3                            | 3                                       | 1                                    | $I_3$                                  |           |
| a4   | $\varphi$       | $1 + \varphi$<br>$1 + \varphi$ | $arphi \ arphi$ | $-20-4\varphi$                   | $-39 - 37\varphi$                           |  | $\frac{2}{2}$ | + -          | 6                            | 6                                       | $\frac{1}{2}$                        | $I_6$                                  |           |
|  | τ               | - 1 7                          | τ               |                                  | 30 317                                      |  |               | '            |                              |   | _                                    | -0                                     |           |
|  |                 |                                |                 |                                  |   |  |               |              |                              |   |                                      |  |           |
| 76a  |                 |                                |                 | 7                                | $76a = (2 - 8\varphi) = 2 \cdot 19a$ (2 iso | ogeny classes                          | )             |              |                              |   |                                      |  | 76a       |
| a1   | 1               |                                | $1 + \varphi$   | $-1-2\varphi$                    | 0   | 0                                      | 5             |              | 1, 1                         | 1,1                                     | 1, 1                                 | $I_1, I_1$                             |           |
| a2   | $1+\varphi$     | $-\varphi$                     | $1+\varphi$     | $-214 + 132\varphi$              | $-1464 + 902\varphi$                        | 0                                      | 1             |              | 5, 5                         | 5, 5                                    | 1,1                                  | $I_5, I_5$                             |           |
| b1   | $\varphi$       | $1-\varphi$                    | 1               | -1                               | 0   | 0                                      | 9             |              | 1,3                          | 1,3                                     | 1,3                                  | $I_1, I_3$                             |           |
| b2   | $\varphi$       | $1-\varphi$                    | 1               | $9-5\varphi$                     | $6-8\varphi$                                | 0                                      | 9             |              | 3,9                          | 3, 9                                    | 3, 9                                 | $I_3, I_9$                             |           |
| b3   | $\varphi$       | $1-\varphi$                    | 1               | $-151-45\varphi$                 | $-858 - 264\varphi$                         | 0                                      | 3             |              | 9,3                          | 9, 3                                    | 9, 3                                 | $I_9, I_3$                             |           |
| b4   | $1+\varphi$     | $-\varphi$                     | 1               | $-35336 - 54686\varphi$          | $-4653177 - 7490886\varphi$                 | 0                                      | 1             |              | 3, 1                         | 3, 1                                    | 3, 1                                 | $I_3, I_1$                             |           |
|  |                 |                                |                 |                                  |   |  |               |              |                              |   |                                      |  |           |
|  |                 |                                |                 |                                  |   |  |               |              |                              |   |                                      |  |           |
| <b>76</b> b                                |                 |                                |                 | •                                | $76b = (6 - 8\varphi) = 2 \cdot 19b$ (2 iso | geny classes)                          | )             |              |                              |   |                                      |  | 76b       |
| a1   | 1               | 0                              | $\varphi$       | $-2+\varphi$                     | $1-\varphi$                                 | 0                                      | 5             |              | 1, 1                         | 1, 1                                    | 1,1                                  | $I_1, I_1$                             |           |
| a2   | $\varphi$       | 0                              | $\varphi$       | $-80 - 134\varphi$               | $-561 - 903\varphi$                         | 0                                      | 1             | <u> </u>     | 5, 5                         | 5, 5                                    | 1,1                                  | $I_5, I_5$                             | L         |
|  | $1 + \varphi$   | 0                              | 1               | $-1-\varphi$                     | 0   | 0                                      | 9             |              | 1, 3                         | 1,3                                     | 1,3                                  | $I_1, I_3$                             |           |
|  | $1+\varphi$     | 0                              | 1               | $4+4\varphi$                     | $-2+8\varphi$                               | 0                                      | 9             |              | 3,9                          | 3, 9                                    | 3,9                                  | $I_3, I_9$                             |           |
|  | $1 + \varphi$   | 0                              | 1               | $-196 + 44\varphi$               | $-1122 + 264\varphi$                        | 0                                      | 3             |              | 9,3                          | 9, 3                                    | 9, 3                                 | $I_9, I_3$                             |           |
| b4   | $\varphi$       | 0                              | 1               | $-90021 + 54685\varphi$          | $-12144063 + 7490886\varphi$                | 0                                      | 1             |              | 3,1                          | 3, 1                                    | 3,1                                  | $I_3, I_1$                             |           |
|  |                 |                                |                 |                                  |   |  |               |              |                              |   |                                      |  |           |
| 79a  |                 |                                |                 |                                  | $79a = (3 - 8\varphi) = 79a$ (1 iso         | geny class)                            |               |              |                              |   |                                      |  | 79a       |
| al   | φ               | $1+\varphi$                    | 0               | $1+\varphi$                      | 0   | 0                                      | 4             | -+           | 1                            | 1                                       | 1                                    | $I_1$                                  |           |
| a2   | $\varphi$       | $1+\varphi$ $1+\varphi$        | 0               | $-4-4\varphi$                    | $-10-15\varphi$                             | 0                                      | 4             | ++           | 2                            | 2                                       | 2                                    | $I_2$                                  |           |
|  | $1+\varphi$     | -1                             | $\varphi$       | $-427-689\varphi$                | $-6529 - 10566\varphi$                      | 0                                      | 2             | ++           | 1                            | 1                                       | 1                                    | $I_1$                                  |           |
| a4   | $\varphi$       | $1 + \varphi$                  | 0               | $-19+6\varphi$                   | $-43+4\varphi$                              | 0                                      | 2             | + -          | 4                            | 4                                       | 2                                    | $I_4$                                  |           |
|  | · · ·           | · '                            |                 | · · ·                            |   |  |               |              |                              |   |                                      |  |           |
|  |                 |                                |                 |                                  |   |  |               |              |                              |   |                                      |  |           |
| <b>7</b> 9b                                |                 |                                |                 |                                  | $79b = (5 - 8\varphi) = 79b$ (1 iso         | geny class)                            |               |              |                              |   |                                      |  | 79b       |
| a1   | $1+\varphi$     | $-1+\varphi$                   | $\varphi$       | 0                                | 0   | 0                                      | 4             | + -          | 1                            | 1                                       | 1                                    | $I_1$                                  |           |
|  | $1+\varphi$     | $-1+\varphi$                   | $\varphi$       | $-10+5\varphi$                   | $-10 + 5\varphi$                            | 0                                      | 4             | ++           | 2                            | 2                                       | 2                                    | $I_2$                                  |           |
| a3   | $\varphi$       | $-\varphi$                     | $1+\varphi$     | $-1115 + 687\varphi$             | $-17095 + 10565\varphi$                     | 0                                      | 2             | ++           | 1                            | 1                                       | 1                                    | $I_1$                                  |           |
| a4   | $1 + \varphi$   | $-1+\varphi$                   | $\varphi$       | $-15-5\varphi$                   | $-29-19\varphi$                             | 0                                      | 2             | -+           | 4                            | 4                                       | 2                                    | $I_4$                                  |           |
|  |                 |                                |                 |                                  |   |  |               |              |                              |   |                                      |  |           |

|  | $a_1$  | $a_2$  | $a_3$  | $a_4$   | $a_6$  | r  | T  | s  | $\operatorname{ord}(\Delta)$  | $\operatorname{ord}_{-}(j)$  | $c_p$   | Kodaira  | Isogenies  |
|--|--|--|--|---|--|--|--|--|---|--|---|--|------------|
|  |  |  |  |   |  | <u> </u>   |  |  |   |  |   |  |            |
| 80a  |  |  |  |   | $80a = (4 - 8\varphi) = 2^2 \cdot 5a \tag{1 is}$   | sogeny class                                       |  |  |   |  |   |  | 80a        |
| a1   | 0  | 1  | 0  | -1  | 0  | 0  | 12   | ++   | 4, 2  | 2  | 3, 2  | $IV, I_2$  |            |
| a2   | 0  | 1  | 0  | $-11-5\varphi$  | $-1+17\varphi$   | 0  | 6  | ++   | 8, 1  | 1  | 3, 1  | $IV^*, I_1$  |            |
| a3   | 0  | 1  | 0  | $-16+5\varphi$  | $16-17\varphi$   | 0  | 6  | ++   | 8, 1  | 1  | 3, 1  | $IV^*, I_1$  |            |
| a4   | 0  | 1  | 0  | 4   | 4  | 0  | 6  |  | 8, 4  | 4  | 3, 2  | $IV^*, I_4$  |            |
| a5   | 0  | 1  | 0  | -41   | -116   | 0  | 4  | ++   | 4,6   | 6  | 3, 2  | $IV, I_6$  |            |
| a6   |  | $-1-\varphi$   | 0  | $-3281 + 2025\varphi$   | $-84572 + 52269\varphi$  | 0  | 2  | ++   | 8, 3  | 3  | 3, 1  | $IV^*, I_3$  |            |
| a7   | 0  | $1+\varphi$  |  | $-1257 - 2023\varphi$   | $-33560 - 54293\varphi$  | 0  | 2  | ++   | 8,3   | 3  | 3, 1  | $IV^*, I_3$  |            |
| a8   | 0  | 1  | 0  | -36   | -140   | 0  | 2  |  | 8, 12   | 12   | 3, 2  | $IV^*, I_{12}$   |            |
| 81a  |  |  |  |   | $81a = (9) = 3^2$ (1 isoger  | ny class)  |  |  |   |  |   |  | 81a        |
| a1   | 1  | -1   | $\varphi$  | $-2\varphi$   | $\varphi$  | 0  | 6  |  | 3   | 0  | 2   | III  |            |
| a2   | 1  |  | $1+\varphi$  | $-2+\varphi$  | 1-2arphi   | 0  | 6  |  | 3   | 0  | 2   | III  |            |
| a3   | $\varphi$  | $-1-\varphi$   | $\varphi$  | $-80 - 128\varphi$  | $569 + 921\varphi$   | 0  | 6  | ++   | 3   | 0  | 2   | III  |            |
| a4   | $1+\varphi$  | 1  | 0  | $-207 + 129\varphi$   | $1283 - 793\varphi$  | 0  | 6  | ++   | 3   | 0  | 2   | III  |            |
| a5   | $\varphi$  | $-1-\varphi$   | $1+\varphi$  | $-719 - 1153\varphi$  | $-13501 - 21854\varphi$  | 0  | 2  | ++   | 9   | 0  | 2   | $III^*$  |            |
| a6   | $1+\varphi$  | 1  | 1  | $-1871 + 1153\varphi$   | $-37225 + 23006\varphi$  | 0  | 2  | ++   | 9   | 0  | 2   | $III^*$  |            |
| a7   | 1  | -1   | $\varphi$  | $-15+13\varphi$   | $-26+20\varphi$  | 0  | 2  |  | 9   | 0  | 2   | $III^*$  |            |
| a8   | 1  | -1   | $1 + \varphi$  | $-2-14\varphi$  | $-6-21\varphi$   | 0  | 2  |  | 9   | 0  | 2   | $III^*$  |            |
|  |  |  |  |   |  |  |  |  |   |  |   |  |            |
| 89a  |  |  |  |   | $89a = (10 - \varphi) = 89a$ (1 iso  | ogeny class)                                       |  |  |   |  |   |  | 89a        |
| 89a  | $1+\varphi$  | -1   | 1  | $-1-\varphi$  | $89a = (10 - \varphi) = 89a$ (1 iso  | ogeny class)                                       | 6  | ++   | 1   | 1  | 1   | $I_1$  | 89a        |
|  |  | -1<br>-1   | 1<br>1   | $-1 - \varphi$ $-6 - 11\varphi$   |  |  | 6<br>6   | ++   | 1<br>2  | 1<br>2   | 1<br>2  | $\begin{array}{ c c c }\hline I_1\\I_2\\ \end{array}$  | 89a        |
| a1   | $1+\varphi$  |  |  |   | 0  | 0  |  |  |   | 1<br>2<br>3  | 1<br>2<br>1   |  | 89a        |
| a1<br>a2   | $\begin{array}{c} 1+\varphi \\ 1+\varphi \end{array}$  | -1   | 1  | $-6-11\varphi$  | $0 \\ 10 + 16\varphi$  | 0  | 6  | -+   | 2   |  |   | $I_2$  | 89a        |
| a1<br>a2<br>a3   | $\begin{aligned} 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \end{aligned}$   | $-1 \\ -1$   | 1<br>1   | $-6 - 11\varphi$ $-16 - 11\varphi$  | $   \begin{array}{c}     0 \\     10 + 16\varphi \\     -28 - 32\varphi \\     -24 - 12\varphi   \end{array} $   | 0 0 0  | $\frac{6}{2}$  | - +<br>+ +   | 2<br>3  | 3  | 1   | $egin{array}{c} I_2 \ I_3 \end{array}$   | 89a<br>89b |
| a1<br>a2<br>a3<br>a4<br><b>89</b> b  | $   \begin{array}{c}     1 + \varphi \\     1 + \varphi \\     1 + \varphi \\     1 + \varphi   \end{array} $  | -1<br>-1<br>-1   | 1<br>1   | $-6 - 11\varphi$ $-16 - 11\varphi$  | $   \begin{array}{c}     0 \\     10 + 16\varphi \\     -28 - 32\varphi \\     -24 - 12\varphi   \end{array} $   | 0<br>0<br>0<br>0                                   | 6<br>2<br>2  | -+<br>++<br>-+                                     | 2<br>3  | 3  | 1   | I <sub>2</sub> I <sub>3</sub> I <sub>6</sub>   |            |
| a1<br>a2<br>a3<br>a4<br><b>89b</b>   | $ \begin{array}{c} 1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\end{array} $   | $ \begin{array}{c} -1 \\ -1 \\ -1 \end{array} $ $ -\varphi$  | 1 1 1  | $ \begin{array}{c} -6 - 11\varphi \\ -16 - 11\varphi \\ -16 - 16\varphi \end{array} $   | $     \begin{array}{c}       0 \\       10 + 16\varphi \\       -28 - 32\varphi \\       -24 - 12\varphi   \end{array} $ $     89b = (9 + \varphi) = 89b \qquad (1 \text{ iso}) $  | 0<br>0<br>0<br>0<br>0<br>geny class)               | 6 2 2  | - +<br>+ +<br>- +                                  | 2<br>3<br>6   | 3 6  | 1 2   | I <sub>2</sub> I <sub>3</sub> I <sub>6</sub>   |            |
| a1<br>a2<br>a3<br>a4<br><b>89b</b><br>a1<br>a2   | $ \begin{array}{c} 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \end{array} $  | $ \begin{array}{c} -1 \\ -1 \\ -1 \end{array} $ $ -\varphi$ $ -\varphi$  | 1<br>1<br>1  | $ \begin{array}{c} -6 - 11\varphi \\ -16 - 11\varphi \\ -16 - 16\varphi \end{array} $ $ -16 - 16\varphi $   | $0$ $10 + 16\varphi$ $-28 - 32\varphi$ $-24 - 12\varphi$ $89b = (9 + \varphi) = 89b \qquad (1 \text{ iso}$ $0$ $26 - 16\varphi$  | 0 0 0 0 geny class)                                | 6<br>2<br>2<br>2                                     | -+<br>++<br>-+<br>++<br>+-                         | 2<br>3<br>6<br>1<br>2   | 3<br>6<br>1<br>2   | 1   | $\begin{array}{c c} I_2 \\ I_3 \\ I_6 \\ \hline \\ I_1 \\ I_2 \\ \end{array}$  |            |
| a1<br>a2<br>a3<br>a4<br><b>89b</b>   | $ \begin{array}{c} 1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\end{array} $   | $ \begin{array}{c} -1 \\ -1 \\ -1 \end{array} $ $ -\varphi$  | 1<br>1<br>1  | $ \begin{array}{r} -6 - 11\varphi \\ -16 - 11\varphi \\ -16 - 16\varphi \end{array} $   | $     \begin{array}{c}       0 \\       10 + 16\varphi \\       -28 - 32\varphi \\       -24 - 12\varphi   \end{array} $ $     89b = (9 + \varphi) = 89b \qquad (1 \text{ iso}) $  | 0 0 0 0 geny class) 0 0 0                          | 6 2 2  | - +<br>+ +<br>- +                                  | 2<br>3<br>6   | 3 6  | 1<br>2<br>1<br>2  | I <sub>2</sub> I <sub>3</sub> I <sub>6</sub>   |            |
| a1<br>a2<br>a3<br>a4<br><b>89b</b><br>a1<br>a2<br>a3<br>a4                                     | $ \begin{array}{c} 1+\varphi \\ 1+\varphi \\ 1+\varphi \\ 1+\varphi \\ \end{array} $   | $ \begin{array}{c} -1 \\ -1 \\ -1 \end{array} $ $ -\varphi$ $ -\varphi$ $ -\varphi$  | 1<br>1<br>1<br>1<br>1<br>1                               | $ \begin{array}{c} -6 - 11\varphi \\ -16 - 11\varphi \\ -16 - 16\varphi \end{array} $ $ \begin{array}{c} -1 \\ -16 + 10\varphi \\ -26 + 10\varphi \\ -31 + 15\varphi \end{array} $  | $   \begin{array}{c}     0 \\     10 + 16\varphi \\     -28 - 32\varphi \\     -24 - 12\varphi   \end{array} $ $   \begin{array}{c}     89b = (9 + \varphi) = 89b & (1 \text{ iso}) \\     0 \\     26 - 16\varphi \\     -60 + 32\varphi \\     -36 + 12\varphi   \end{array} $   | 0<br>  0<br>  0<br>  0<br>  0<br>  0<br>  0<br>  0 | 6<br>2<br>2<br>6<br>6<br>6<br>2<br>2                 | -+<br>++<br>-+<br>++<br>+-<br>++                   | 2<br>3<br>6<br>1<br>2<br>3  | 3<br>6<br>1<br>2<br>3  | 1<br>2<br>1<br>2<br>1   | $\begin{array}{c c} I_2 \\ I_3 \\ I_6 \\ \hline \\ I_1 \\ I_2 \\ I_3 \\ \end{array}$   | 89b        |
| a1 a2 a3 a4  89b a1 a2 a3 a4  95a  | $ \begin{array}{c} 1+\varphi \\ 1+\varphi \\ 1+\varphi \\ 1+\varphi \\ \end{array} $   | $ \begin{array}{c} -1 \\ -1 \\ -1 \end{array} $ $ -\varphi$ $ -\varphi$ $ -\varphi$ $ -\varphi$  | 1<br>1<br>1<br>1<br>1<br>1<br>1                          | $ \begin{array}{r} -6 - 11\varphi \\ -16 - 11\varphi \\ -16 - 16\varphi \end{array} $ $ \begin{array}{r} -1 \\ -16 + 10\varphi \\ -26 + 10\varphi \\ -31 + 15\varphi \end{array} $  | $     \begin{array}{c}       0 \\       10 + 16\varphi \\       -28 - 32\varphi \\       -24 - 12\varphi   \end{array} $ $     \begin{array}{c}       89b = (9 + \varphi) = 89b & (1 \text{ iso}) \\       0 \\       26 - 16\varphi \\       -60 + 32\varphi \\       -36 + 12\varphi   \end{array} $ $     \begin{array}{c}       95a = (9 + 2\varphi) = 5a \cdot 19a & (1 \text{ iso})   \end{array} $                                    | 0   0   0   0                                      | 6<br>2<br>2<br>6<br>6<br>6<br>2<br>2                 | -+<br>++<br>-+<br>+-<br>++<br>+-                   | 2<br>3<br>6<br>1<br>2<br>3<br>6   | 3<br>6<br>1<br>2<br>3<br>6   | 1<br>2<br>1<br>2<br>1<br>2  | I <sub>2</sub> I <sub>3</sub> I <sub>6</sub> I <sub>1</sub> I <sub>2</sub> I <sub>3</sub> I <sub>6</sub>   |            |
| a1 a2 a3 a4  89b a1 a2 a3 a4  95a a1   | $ \begin{array}{c} 1+\varphi \\ 1+\varphi \\ 1+\varphi \\ 1+\varphi \\ 1+\varphi \\ \end{array} $ $ \begin{array}{c} \varphi \\ \varphi \\ \varphi \\ \end{array} $ $ \begin{array}{c} \varphi \\ \varphi \\ \varphi \\ \end{array} $              | $ \begin{array}{c} -1 \\ -1 \\ -1 \end{array} $ $ -\varphi$ $ -\varphi$ $ -\varphi$ $ -\varphi$  | 1<br>1<br>1<br>1<br>1<br>1<br>1                          | $   \begin{array}{r}     -6 - 11\varphi \\     -16 - 11\varphi \\     -16 - 16\varphi   \end{array} $ $   \begin{array}{r}     -1 \\     -16 + 10\varphi \\     -26 + 10\varphi \\     -31 + 15\varphi   \end{array} $                                | $     \begin{array}{c}       0 \\       10 + 16\varphi \\       -28 - 32\varphi \\       -24 - 12\varphi   \end{array} $ $     \begin{array}{c}       89b = (9 + \varphi) = 89b & (1 \text{ iso}) \\       0 \\       26 - 16\varphi \\       -60 + 32\varphi \\       -36 + 12\varphi   \end{array} $ $     \begin{array}{c}       95a = (9 + 2\varphi) = 5a \cdot 19a & (1 \text{ iso}) \\       1 \\     \end{array} $                    | 0   0   0   0   0   0   0   0   0   0              | 6<br>2<br>2<br>2<br>6<br>6<br>2<br>2<br>2            | -+<br>++<br>-+<br>+-<br>++<br>+-                   | 2<br>3<br>6<br>1<br>2<br>3<br>6   | 3<br>6<br>1<br>2<br>3<br>6   | 1<br>2<br>1<br>2<br>1<br>2  | I <sub>2</sub> I <sub>3</sub> I <sub>6</sub> I <sub>1</sub> I <sub>2</sub> I <sub>3</sub> I <sub>6</sub> I <sub>1</sub> I <sub>2</sub> I <sub>3</sub> I <sub>6</sub> | 89b        |
| a1 a2 a3 a4  89b a1 a2 a3 a4  95a a1 a2  | $ \begin{array}{c} 1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\ \end{array} $ $ \begin{array}{c} \varphi\\\varphi\\\varphi\\\varphi\\1+\varphi\\ \end{array} $   | $ \begin{array}{c} -1 \\ -1 \\ -1 \end{array} $ $ -\varphi$ $ -\varphi$ $ -\varphi$ $ -\varphi$ $ -1 + \varphi$ $ -1 + \varphi$  | 1<br>1<br>1<br>1<br>1<br>1<br>1                          | $ \begin{array}{c} -6 - 11\varphi \\ -16 - 11\varphi \\ -16 - 16\varphi \end{array} $ $ \begin{array}{c} -1 \\ -16 + 10\varphi \\ -26 + 10\varphi \\ -31 + 15\varphi \end{array} $ $ \begin{array}{c} -4 - \varphi \\ 1 - \varphi \end{array} $       | $     \begin{array}{r}       0 \\       10 + 16\varphi \\       -28 - 32\varphi \\       -24 - 12\varphi   \end{array} $ $     \begin{array}{r}       89b = (9 + \varphi) = 89b & (1 \text{ iso}) \\       0 \\       26 - 16\varphi \\       -60 + 32\varphi \\       -36 + 12\varphi   \end{array} $ $     \begin{array}{r}       95a = (9 + 2\varphi) = 5a \cdot 19a & (1 \\       1 \\       -1   \end{array} $                          | 0   0   0   0   0   0   0   0   0   0              | 6<br>2<br>2<br>6<br>6<br>6<br>2<br>2<br>2            | -+<br>++<br>-+<br>+-<br>++<br>+-                   | 2<br>3<br>6<br>1<br>2<br>3<br>6<br>2,6<br>1,3                               | 3<br>6<br>1<br>2<br>3<br>6   | 1<br>2<br>1<br>2<br>1<br>2<br>2<br>2,6<br>1,3   | $\begin{array}{c c} I_2 \\ I_3 \\ I_6 \\ \hline \\ I_1 \\ I_2 \\ I_3 \\ I_6 \\ \hline \\ I_2, I_6 \\ I_1, I_3 \\ \hline \end{array}$                                 | 89b        |
| a1 a2 a3 a4  89b a1 a2 a3 a4  95a a1 a2 a3 a4  | $ \begin{array}{c} 1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\ \end{array} $ $ \begin{array}{c} \varphi\\\varphi\\\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\\end{array} $   | $-1$ $-1$ $-\varphi$ $-\varphi$ $-\varphi$ $-\varphi$ $-1+\varphi$ $-1+\varphi$  | 1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1                | $-6 - 11\varphi$ $-16 - 11\varphi$ $-16 - 16\varphi$ $-1$ $-16 + 10\varphi$ $-26 + 10\varphi$ $-31 + 15\varphi$ $-4 - \varphi$ $1 - \varphi$ $-54 - 26\varphi$  | $     \begin{array}{c}       0 \\       10 + 16\varphi \\       -28 - 32\varphi \\       -24 - 12\varphi   \end{array} $ $     \begin{array}{c}       89b = (9 + \varphi) = 89b & (1 \text{ iso}) \\       0 \\       26 - 16\varphi \\       -60 + 32\varphi \\       -36 + 12\varphi   \end{array} $ $     \begin{array}{c}       95a = (9 + 2\varphi) = 5a \cdot 19a & (1 \\       1 \\       -1 \\       161 + 80\varphi   \end{array} $ | 0   0   0   0   0   0   0   0   0   0              | 6<br>2<br>2<br>6<br>6<br>2<br>2<br>2<br>)            | -+<br>++<br>-+<br>+-<br>++<br>+-                   | 2<br>3<br>6<br>1<br>2<br>3<br>6<br>2,6<br>1,3<br>4,3                        | 3<br>6<br>1<br>2<br>3<br>6<br>2,6<br>1,3<br>4,3                        | 1<br>2<br>1<br>2<br>1<br>2<br>2<br>2,6<br>1,3<br>4,3  | $\begin{array}{c c} I_2 \\ I_3 \\ I_6 \\ \hline \\ I_1 \\ I_2 \\ I_3 \\ I_6 \\ \hline \\ I_2, I_6 \\ I_1, I_3 \\ I_4, I_3 \\ \hline \end{array}$                     | 89b        |
| a1<br>a2<br>a3<br>a4<br>89b<br>a1<br>a2<br>a3<br>a4<br>95a<br>a1<br>a2<br>a3<br>a4             | $ \begin{array}{c} 1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\\end{array} $ $ \begin{array}{c} \varphi\\\varphi\\\varphi\\\\ 1+\varphi\\1+\varphi\\1+\varphi\\\end{array} $  | $-1$ $-1$ $-\varphi$ $-\varphi$ $-\varphi$ $-\varphi$ $-1+\varphi$ $-1+\varphi$ $-1+\varphi$   | 1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1                | $-6 - 11\varphi$ $-16 - 11\varphi$ $-16 - 16\varphi$ $-16 - 16\varphi$ $-16 + 10\varphi$ $-26 + 10\varphi$ $-31 + 15\varphi$ $-4 - \varphi$ $1 - \varphi$ $-54 - 26\varphi$ $-34 + 24\varphi$   | $   \begin{array}{c}     0 \\     10 + 16\varphi \\     -28 - 32\varphi \\     -24 - 12\varphi   \end{array} $ $   \begin{array}{c}     89b = (9 + \varphi) = 89b & (1 \text{ iso}) \\     0 \\     26 - 16\varphi \\     -60 + 32\varphi \\     -36 + 12\varphi   \end{array} $ $   \begin{array}{c}     1 \\     1 \\     -1 \\     161 + 80\varphi \\     -71 + 60\varphi   \end{array} $   | 0  | 6<br>2<br>2<br>2<br>6<br>6<br>12<br>6                | -+<br>++<br>-+<br>+-<br>++<br>+-                   | 2<br>3<br>6<br>1<br>2<br>3<br>6<br>2,6<br>1,3<br>4,3<br>1,12                | 3<br>6<br>1<br>2<br>3<br>6<br>2,6<br>1,3<br>4,3<br>1,12                | 1<br>2<br>1<br>2<br>1<br>2<br>2<br>2,6<br>1,3<br>4,3<br>1,12  | $\begin{array}{c c} I_2\\I_3\\I_6\\ \hline\\ I_1\\I_2\\I_3\\I_6\\ \hline\\ I_2,I_6\\I_1,I_3\\I_4,I_3\\I_4,I_3\\I_1,I_{12}\\ \end{array}$                             | 89b        |
| a1<br>a2<br>a3<br>a4<br>89b<br>a1<br>a2<br>a3<br>a4<br>a1<br>a2<br>a3<br>a4<br>a5              | $ \begin{array}{c} 1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\ \end{array} $ $ \begin{array}{c} \varphi\\\varphi\\\varphi\\ \end{array} $ $ \begin{array}{c} \varphi\\\varphi\\\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\\end{array} $ | $-1$ $-1$ $-\varphi$ $-\varphi$ $-\varphi$ $-\varphi$ $-1+\varphi$ $-1+\varphi$ $-1+\varphi$ $-1+\varphi$  | 1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1      | $-6 - 11\varphi$ $-16 - 11\varphi$ $-16 - 16\varphi$ $-16 - 16\varphi$ $-16 + 10\varphi$ $-26 + 10\varphi$ $-31 + 15\varphi$ $-4 - \varphi$ $1 - \varphi$ $-54 - 26\varphi$ $-34 + 24\varphi$ $-219 + 54\varphi$                                      | $ \begin{array}{c} 0 \\ 10 + 16\varphi \\ -28 - 32\varphi \\ -24 - 12\varphi \end{array} $ $ 89b = (9 + \varphi) = 89b \qquad (1 \text{ iso}) $ $ \begin{array}{c} 0 \\ 26 - 16\varphi \\ -60 + 32\varphi \\ -36 + 12\varphi \end{array} $ $ 95a = (9 + 2\varphi) = 5a \cdot 19a \qquad (1 \text{ iso}) $ $ \begin{array}{c} 1 \\ -1 \\ 161 + 80\varphi \\ -71 + 60\varphi \\ -991 + 594\varphi \end{array} $                                | geny class)    0                                   | 6<br>2<br>2<br>6<br>6<br>2<br>2<br>9<br>12<br>6<br>4 | -+<br>++<br>-+<br>+-<br>++<br>+-<br>++<br>++       | 2<br>3<br>6<br>1<br>2<br>3<br>6<br>1,3<br>4,3<br>1,12<br>12,1               | 3<br>6<br>1<br>2<br>3<br>6<br>2,6<br>1,3<br>4,3<br>1,12<br>12,1        | 1<br>2<br>1<br>2<br>1<br>2<br>2<br>2,6<br>1,3<br>4,3<br>1,12<br>12,1  | $\begin{array}{c c} I_2\\I_3\\I_6\\ \hline\\ I_1\\I_2\\I_3\\I_6\\ \hline\\ I_2,I_6\\I_1,I_3\\I_4,I_3\\I_1,I_{12}\\I_{12},I_1\\ \end{array}$                          | 89b        |
| a1<br>a2<br>a3<br>a4<br>89b<br>a1<br>a2<br>a3<br>a4<br>95a<br>a1<br>a2<br>a3<br>a4<br>a5<br>a6 | $ \begin{array}{c} 1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\ \end{array} $ $ \begin{array}{c} \varphi\\\varphi\\\varphi\\ \end{array} $ $ \begin{array}{c} \varphi\\\varphi\\\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\\end{array} $ | $     \begin{array}{c}         -1 \\         -1 \\         -1 \\     \end{array} $ $     \begin{array}{c}         -\varphi \\         -\varphi \\         -\varphi \\     \end{array} $ $     \begin{array}{c}         -1 + \varphi \\         -1 + \varphi \\         -1 + \varphi \\         -1 + \varphi \\         -1 + \varphi \\     \end{array} $ | 1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1 | $-6 - 11\varphi$ $-16 - 11\varphi$ $-16 - 16\varphi$ $-16 - 16\varphi$ $-16 + 10\varphi$ $-26 + 10\varphi$ $-26 + 10\varphi$ $-31 + 15\varphi$ $-4 - \varphi$ $1 - \varphi$ $-54 - 26\varphi$ $-34 + 24\varphi$ $-219 + 54\varphi$ $-204 + 74\varphi$ | $ \begin{array}{c} 0 \\ 10 + 16\varphi \\ -28 - 32\varphi \\ -24 - 12\varphi \end{array} $ $ 89b = (9 + \varphi) = 89b \qquad (1 \text{ iso}) $ $ \begin{array}{c} 0 \\ 26 - 16\varphi \\ -60 + 32\varphi \\ -36 + 12\varphi \end{array} $ $ 95a = (9 + 2\varphi) = 5a \cdot 19a \qquad (1) $ $ \begin{array}{c} 1 \\ -1 \\ 161 + 80\varphi \\ -71 + 60\varphi \\ -991 + 594\varphi \\ -1109 + 460\varphi \end{array} $                      | 0   0   0   0                                      | 6<br>2<br>2<br>2<br>6<br>6<br>12<br>6<br>4<br>4      | ++<br>++<br>+-<br>++<br>+-<br>++<br>++<br>++<br>++ | 2<br>3<br>6<br>1<br>2<br>3<br>6<br>2,6<br>1,3<br>4,3<br>1,12<br>12,1<br>6,2 | 3<br>6<br>1<br>2<br>3<br>6<br>2,6<br>1,3<br>4,3<br>1,12<br>12,1<br>6,2 | $\begin{array}{c} 1\\ 2\\ 1\\ 2\\ 1\\ 2\\ \end{array}$ $\begin{array}{c} 2,6\\ 1,3\\ 4,3\\ 1,12\\ 12,1\\ 6,2\\ \end{array}$ | $\begin{array}{c c} I_2\\I_3\\I_6\\ \hline\\ I_1\\I_2\\I_3\\I_6\\ \hline\\ I_2,I_6\\I_1,I_3\\I_4,I_3\\I_1,I_{12}\\I_{12},I_1\\I_6,I_2\\ \end{array}$                 | 89b        |
| a1<br>a2<br>a3<br>a4<br>89b<br>a1<br>a2<br>a3<br>a4<br>a1<br>a2<br>a3<br>a4<br>a5              | $ \begin{array}{c} 1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\ \end{array} $ $ \begin{array}{c} \varphi\\\varphi\\\varphi\\ \end{array} $ $ \begin{array}{c} \varphi\\\varphi\\\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\\end{array} $ | $     \begin{array}{c}         -1 \\         -1 \\         -1 \\     \end{array} $ $     \begin{array}{c}         -\varphi \\         -\varphi \\         -\varphi \\     \end{array} $ $     \begin{array}{c}         -1 + \varphi \\         -1 + \varphi \\         -1 + \varphi \\         -1 + \varphi \\         -1 + \varphi \\     \end{array} $ | 1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1      | $-6 - 11\varphi$ $-16 - 11\varphi$ $-16 - 16\varphi$ $-16 - 16\varphi$ $-16 + 10\varphi$ $-26 + 10\varphi$ $-31 + 15\varphi$ $-4 - \varphi$ $1 - \varphi$ $-54 - 26\varphi$ $-34 + 24\varphi$ $-219 + 54\varphi$                                      | $ \begin{array}{c} 0 \\ 10 + 16\varphi \\ -28 - 32\varphi \\ -24 - 12\varphi \end{array} $ $ 89b = (9 + \varphi) = 89b \qquad (1 \text{ iso}) $ $ \begin{array}{c} 0 \\ 26 - 16\varphi \\ -60 + 32\varphi \\ -36 + 12\varphi \end{array} $ $ 95a = (9 + 2\varphi) = 5a \cdot 19a \qquad (1 \text{ iso}) $ $ \begin{array}{c} 1 \\ -1 \\ 161 + 80\varphi \\ -71 + 60\varphi \\ -991 + 594\varphi \end{array} $                                | geny class)    0                                   | 6<br>2<br>2<br>6<br>6<br>2<br>2<br>9<br>12<br>6<br>4 | -+<br>++<br>-+<br>+-<br>++<br>+-<br>++<br>++       | 2<br>3<br>6<br>1<br>2<br>3<br>6<br>1,3<br>4,3<br>1,12<br>12,1               | 3<br>6<br>1<br>2<br>3<br>6<br>2,6<br>1,3<br>4,3<br>1,12<br>12,1        | 1<br>2<br>1<br>2<br>1<br>2<br>2<br>2,6<br>1,3<br>4,3<br>1,12<br>12,1  | $\begin{array}{c c} I_2\\I_3\\I_6\\ \hline\\ I_1\\I_2\\I_3\\I_6\\ \hline\\ I_2,I_6\\I_1,I_3\\I_4,I_3\\I_1,I_{12}\\I_{12},I_1\\ \end{array}$                          | 89b        |

|     | $a_1$                   | $a_2$                          | $a_3$                               | $a_4$                         | $a_6$  | r               | T                                      | s        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$            | Kodaira                  | Isogenies   |
|-----|-------------------------|--------------------------------|-------------------------------------|-------------------------------|--|-----------------|--|----------|------------------------------|-----------------------------|------------------|--------------------------|-------------|
| 95b |                         |                                |                                     |                               | $95b = (11 - 2\varphi) = 5a \cdot 19b $        | 1 isogeny clas  | s)                                     |          |                              |                             |                  |                          | 95h         |
| al  |                         | $1+\varphi$                    | $\varphi$                           | $-5+2\varphi$                 | $\frac{2-4\varphi}{}$                          | 0               | 12                                     | ++       | 2,6                          | 2,6                         | 2,6              | $I_{2}, I_{6}$           |             |
| a2  |                         | $1 + \varphi$<br>$1 + \varphi$ | $\varphi$                           | $\frac{3+2\varphi}{2\varphi}$ | $\varphi$                                      |                 | 6                                      | + -      | 1, 3                         | 1,3                         | 1, 3             | $I_{1}, I_{3}$           |             |
| a3  |                         | $1+\varphi$<br>$1+\varphi$     | $\varphi$                           | $-80 + 27\varphi$             | $267-134\varphi$                               |                 | 12                                     |          | 4, 3                         | 4,3                         | $\frac{1}{4}, 3$ | $I_{4}, I_{3}$           |             |
| a4  | $\varphi$               | $1+\varphi$ $1+\varphi$        | $\varphi$                           | $-10-23\varphi$               | $-35-94\varphi$                                |                 | 6                                      | - +      | 1, 12                        | 1,12                        | 1,12             | $I_1, I_{12}$            |             |
| a5  | ,                       | $1+\varphi$                    | $\varphi$                           | $-165-53\varphi$              | $-451 - 813\varphi$                            |                 | 4                                      | ++       | 12, 1                        | 12, 1                       | 12, 1            | $I_{12}, I_{1}$          |             |
| a6  |                         | $1+\varphi$                    | $\varphi$                           | $-130 - 73\varphi$            | $-723-664\varphi$                              |                 | 4                                      | + +      | 6, 2                         | 6, 2                        | 6, 2             | $I_6, I_2$               |             |
| a7  | 1                       | 1                              | $\varphi$                           | $-420 + 258\varphi$           | $-4069 + 2513\varphi$                          |                 | 2                                      | + -      | 3, 1                         | 3,1                         | 3, 1             | $I_3, I_1$               |             |
| a8  | 1                       | $\varphi$                      | 0                                   | $-52017 - 84164\varphi$       | $-8717980 - 14105985\varphi$                   | 0               | 2                                      | -+       | 3, 4                         | 3,4                         | 3, 4             | $I_3, I_4$               |             |
| 99a |                         |                                |                                     |                               | $99a = (3 - 9\varphi) = 3 \cdot 11a \tag{1}$   | isogeny class   | )                                      |          |                              |                             |                  |                          | 99a         |
| al  | $1+\varphi$             | 0                              | 0                                   | -4                            | $\frac{-2-3\varphi}{}$                         | 0               | 8                                      | ++       | 4, 2                         | 4,2                         | 4, 2             | $I_4, I_2$               |             |
| a2  | $1+\varphi$ $1+\varphi$ | 0                              | 0                                   | 1                             | 0  |                 | 4                                      |          | 2, 1                         | 2, 1                        | 2, 1             | $I_{2}, I_{1}$           |             |
| a3  | $1+\varphi$ $1+\varphi$ | 0                              | 0                                   | $-34+15\varphi$               | $79-48\varphi$                                 |                 | 8                                      | ++       | 8, 1                         | 8,1                         | 8,1              | $I_{8}, I_{1}$           |             |
| a4  | $1 + \varphi$           | _                              | $\varphi$                           | $-154-241\varphi$             | $-1336 - 2145\varphi$                          |                 | 4                                      | + +      | 2, 4                         | 2,4                         | 2,4              | $I_{2}, I_{4}$           |             |
| a5  | $\varphi$               | $-\varphi$                     | $\varphi$                           | $-16465 - 26640\varphi$       | $-1541390 - 2494020\varphi$                    |                 | 2                                      | - +      | 1, 2                         | 1, 2                        | 1, 2             | $I_{1}, I_{2}$           |             |
| a6  | $\varphi$               | 0                              | $\overset{\boldsymbol{\varphi}}{0}$ | $-3550 + 2187\varphi$         | $-95697 + 59133\varphi$                        |                 | $\frac{1}{2}$                          | + -      | 1, 2 $1, 8$                  | 1,8                         | 1,8              | $I_1, I_2 \\ I_1, I_8$   |             |
| 0.0 | <u>r</u>                |                                |                                     | 3333 + 2237 7                 | 3000.   00100.                                 |                 |  | <u> </u> |                              |                             | _, ~             | -1, -0                   |             |
| 99b |                         |                                |                                     |                               | $99b = (6 - 9\varphi) = 3 \cdot 11b \tag{1}$   | isogeny class)  | )                                      |          |                              |                             |                  |                          | <b>99</b> k |
| a1  | $\varphi$               | $1-\varphi$                    | 0                                   | -4                            | $-5+3\varphi$                                  | 0               | 8                                      | ++       | 4, 2                         | 4, 2                        | 4, 2             | $I_4, I_2$               |             |
| a2  | $\varphi$               | $1-\varphi$                    | 0                                   | 1                             | 0  | 0               | 4                                      |          | 2, 1                         | 2, 1                        | 2, 1             | $I_2, I_1$               |             |
| a3  | $\varphi$               | $1-\varphi$                    | 0                                   | $-19-15\varphi$               | $31 + 48\varphi$                               | 0               | 8                                      | ++       | 8, 1                         | 8, 1                        | 8, 1             | $I_8, I_1$               |             |
| a4  | 1                       | $\varphi$                      | $1+\varphi$                         | $-395 + 240\varphi$           | $-3481 + 2144\varphi$                          | 0               | 4                                      | ++       | 2, 4                         | 2,4                         | 2, 4             | $I_2, I_4$               |             |
| a5  | $1 + \varphi$           | $-\varphi$                     | 0                                   | $-1363 - 2187\varphi$         | $-36564 - 59133\varphi$                        | 0               | 2                                      | - +      | 1, 8                         | 1,8                         | 1,8              | $I_1, I_8$               |             |
| a6  | $1+\varphi$             | -1                             | $1+\varphi$                         | $-43105 + 26638\varphi$       | $-4035410 + 2494019\varphi$                    | 0               | 2                                      | +-       | 1, 2                         | 1,2                         | 1,2              | $I_1, I_2$               |             |
| 100 | a                       |                                |                                     |                               | $100a = (10) = 2 \cdot 5a^2$ (2 is             | ogeny classes)  |  |          |                              |                             |                  |                          | 100a        |
| a1  | $1+\varphi$             | 9                              | $1+\varphi$                         | $-25-25\varphi$               | $46 + 53\varphi$                               | 0               | 5                                      | T        | 3, 2                         | 3                           | 1, 1             | $I_3, II$                |             |
| a2  | $1+\varphi$             |                                | $1+\varphi$                         | 0                             | 0  | 0               | 5                                      |          | 1, 2                         | 1                           | 1,1              | $I_1$ , II               |             |
| a3  | $1+\varphi$             |                                | $1+\varphi$                         | $-15-15\varphi$               | $-48-69\varphi$                                | 0               | 1                                      |          | 5, 10                        | 5                           | 1,1              | $I_5, II^*$              |             |
| a4  | $1+\varphi$             |                                | $1+\varphi$                         | $110 + 110\varphi$            | $352 + 506\varphi$                             | 0               | 1                                      |          | 15, 10                       | 15                          | 1, 1             | $I_{15}, II^*$           |             |
| b1  |                         | · <del>′</del> -               | '- '<br>1                           | -3                            | 1  | 0               | 15                                     | -        | 5,4                          | <u>-</u><br>  5             | 5,3              | $I_5, IV$                | <u>-</u>    |
| b2  | 1                       | 0                              | 1                                   | -1                            | -2   |                 | 3                                      |          | 1, 8                         | 1                           | 1, 3             | $I_5, IV$<br>$I_1, IV^*$ |             |
| b3  | 1                       | 1                              | 1                                   | $\frac{1}{22}$                | _9   |                 | 5                                      |          | 15, 4                        | 15                          | 15, 1            | $I_{15}$ , IV            |             |
| b4  | 1                       | 0                              | 1                                   | -126                          | -552   | 0               | $\begin{vmatrix} 0 \\ 1 \end{vmatrix}$ |          | 3,8                          | 3                           | 3,1              | $I_3, IV^*$              |             |
| 116 | a                       |                                |                                     |                               | $116a = (12 - 2\varphi) = 2 \cdot 29a \tag{2}$ | l isogeny class | es)                                    |          |                              |                             |                  |                          | 116a        |
| al  | 1                       | -1                             | $1+\varphi$                         | -1                            | $\frac{1100}{-\varphi}$                        | 0               | 5                                      | T        | 1,1                          | 1,1                         | 1,1              | $I_1, I_1$               |             |
| a2  | 1                       |                                | $1+\varphi$                         | $-6-10\varphi$                | $-32-2\varphi$                                 |                 | 1                                      |          | 5, 5                         | 5,5                         | 1,1              | $I_5, I_5$               |             |

|  |                            |                               |           |          |                                       |                                       |                                |  |  |            |                              |                             |  | T   |           |
|--|----------------------------|-------------------------------|-----------|----------|---------------------------------------|---------------------------------------|--------------------------------|--|--|------------|------------------------------|-----------------------------|--|---|-----------|
|  | $a_1$                      | $a_2$                         | 2 0       | 3        | $a_4$                                 |                                       | $a_6$                          | r                                      | T                                      | s          | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$  | Kodaira                                       | Isogenies |
| 116  | a                          |                               |           |          | 110                                   | $6a = (12 - 2\varphi) = 2 \cdot 29a$  | (2 isogeny                     | y classes                              | )                                      |            |                              |                             |  |   | 116a      |
| b1   | 1                          | $-\varphi$                    | Ç         | )        | $2+\varphi$                           |                                       | $-\varphi$                     | 0                                      | 7                                      |            | 7, 1                         | 7, 1                        | 7, 1   | $I_7, I_1$                                    |           |
| b2   | 1                          | $-\varphi$                    | (         | )        | $-588 - 489\varphi$                   | -6684 -                               | $7663\varphi$                  | 0                                      | 1                                      |            | 1,7                          | 1,7                         | 1,7  | $I_1, I_7$                                    |           |
| 116  | b                          |                               |           |          | 11                                    | $6b = (10 + 2\varphi) = 2 \cdot 29b$  | (2 isogeny                     | y classes)                             | )                                      |            |                              |                             |  |   | 116b      |
| a1   | 1                          |                               |           | )        | $-\varphi$                            |                                       | 0                              | 0                                      | 5                                      |            | 1, 1                         | 1, 1                        | 1,1  | $I_1, I_1$                                    |           |
| a2   | 1                          |                               | '         |          | $-15 + 9\varphi$                      |                                       | $33 + \varphi$                 | 0                                      | 1                                      |            | 5,5                          | 5,5                         | 1,1  | $I_5, I_5$                                    |           |
| b1   |                            | $-1+\varphi$                  |           |          | $3-2\varphi$                          | 1 49 47                               | -1                             | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 7                                      |            | 7, 1                         | 7,1                         | 7,1  | $I_7, I_1$                                    |           |
| b2   | 1                          | $-1+\varphi$                  | 1+4       | 2        | $-1077 + 488\varphi$                  | -14347 +                              | $-7662\varphi$                 | 0                                      | 1                                      |            | 1,7                          | 1,7                         | 1,7  | $I_1, I_7$                                    |           |
| 121  | a                          |                               |           |          |                                       | $121a = (11) = 11a \cdot 11b$         | (1 isogeny                     | class)                                 |  |            |                              |                             |  |   | 121a      |
| a1   | 0                          |                               |           |          | 0                                     |                                       | 0                              | 0                                      | 5                                      |            | 1, 1                         | 1, 1                        | 1, 1   | $I_1, I_1$                                    |           |
| a2   | 0                          |                               |           |          | -10                                   |                                       | -20                            | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 5                                      |            | 5, 5                         | 5, 5                        | 5, 5   | $I_5, I_5$                                    |           |
| a3   | 0                          | -1                            |           | L        | -7820                                 | _                                     | 263580                         | 0                                      | 1                                      |            | 1,1                          | 1,1                         | 1,1  | $I_1, I_1$                                    |           |
| 124  | a                          |                               |           |          | 12                                    | $24a = (6 - 10\varphi) = 2 \cdot 31a$ | (1 isoger                      | ny class)                              |  |            |                              |                             |  |   | 124a      |
| a1   | $1+\varphi$                |                               | 1 + 9     | )        | $-54 - 87\varphi$                     | 269 -                                 | $+435\varphi$                  | 0                                      | 6                                      | ++         | 1, 2                         | 1, 2                        | 1, 2   | $I_1, I_2$                                    |           |
| a2   | $\varphi$                  | •                             |           |          | 1                                     |                                       | 0                              | 0                                      | 6                                      | - +        | 2, 1                         | 2, 1                        | 2,1  | $I_2, I_1$                                    |           |
| a3   | $\varphi$                  | •                             |           |          | $-89 + 35\varphi$                     |                                       | $+204\varphi$                  | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 6                                      | ++         | 3, 6                         | 3,6                         | 3,6  | $I_3, I_6$                                    |           |
| a4<br>a5                                   | $\varphi$                  |                               |           | l        | $-9 - 5\varphi$ $-3099 - 4882\varphi$ | -22 $-123561 - 19$                    | $2-20\varphi$                  | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{bmatrix} 6 \\ 2 \end{bmatrix}$ | -+         | 6, 3<br>2, 1                 | 6, 3                        | $\begin{bmatrix} 6, 3 \\ 2 \end{bmatrix}$    | $I_6, I_3$                                    |           |
| a6   | $1+\varphi$<br>$1+\varphi$ | -1<br>$-1+\varphi$            | $1+\zeta$ |          | $-305684 + 188901\varphi$             | -125301 - 13 $-76307979 + 4716$       | ,                              | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\begin{bmatrix} 2\\2 \end{bmatrix}$   | - +<br>+ + | $\frac{2}{1}, \frac{1}{2}$   | 2, 1 $1, 2$                 | $\begin{bmatrix} 2, 1 \\ 1, 2 \end{bmatrix}$ | $\begin{matrix} I_2,I_1\\I_1,I_2\end{matrix}$ |           |
| ao   | ΙΙΥ                        | Ι   Ψ                         | ± 1 \     |          | σοσοσί   100σοίφ                      | 10001313   11110                      | σουσφ                          | 0                                      |  | 1 1        | 1,2                          | 1,2                         | 1,2  | 11,12   |           |
| 124  | b                          |                               |           |          |                                       | $24b = (4 - 10\varphi) = 2 \cdot 31b$ | (1 isogen                      | ny class)                              |  |            |                              | T                           | ı  | ı   | 124b      |
| a1   | $\varphi$                  | •                             |           | )        | $-139 + 85\varphi$                    | 705                                   | $-436\varphi$                  | 0                                      | 6                                      | ++         | 1, 2                         | 1, 2                        | 1,2  | $I_1, I_2$                                    |           |
| a2   | •                          | $-1+\varphi$                  |           |          | -1                                    | 115                                   | $1-\varphi$                    | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 6                                      | + -        | 2,1                          | 2,1                         | $\begin{bmatrix} 2,1\\ 2,c \end{bmatrix}$    | $I_2, I_1$                                    |           |
| a3<br>a4                                   |                            | $-1 + \varphi$ $-1 + \varphi$ |           |          | $-56 - 35\varphi$ $-16 + 5\varphi$    |                                       | $-260\varphi$<br>$21+4\varphi$ | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 6                                      | ++-        | $3, 6 \\ 6, 3$               | $3, 6 \\ 6, 3$              | $\begin{bmatrix} 3, 6 \\ 6, 3 \end{bmatrix}$ | $I_3, I_6 \\ I_6, I_3$                        |           |
| $\begin{vmatrix} a_4 \\ a_5 \end{vmatrix}$ | $\varphi$                  | •                             |           | 9        | $-7979 + 4880\varphi$                 | -323066 + 19                          | •                              |  | $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ | + -        | 0, 3<br>2, 1                 | 0, 3 $2, 1$                 | $\begin{bmatrix} 0, 3 \\ 2, 1 \end{bmatrix}$ | $I_{6}, I_{3}$<br>$I_{2}, I_{1}$              |           |
| a6   | φ                          |                               |           | ĺ        | $-116781 - 188901\varphi$             | -29336020 - 4740                      | ,                              |  | $\frac{2}{2}$                          | ++         | 1, 2                         | 1, 2                        | 1, 2   | $I_1, I_2$                                    |           |
| 144  |                            | . ,                           |           |          |                                       |                                       |                                | logg)                                  | l                                      |            | ,                            | ,                           | ,  | 1) 2  | 144a      |
|  |                            | 1                             |           | <u> </u> | 0 0                                   | $144a = (12) = 2^2 \cdot 3$           | (1 isogeny c                   |  | 4                                      | 1 1        | 4.0                          | 0                           | 1.0  | TV/ T   | 1444      |
| a1<br>a2                                   | 0                          | r                             |           | )<br>)   | $-2 - 3\varphi$ $-32 - 48\varphi$     |                                       | $-2 - 2\varphi$ $-188\varphi$  | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ | +++        | $4, 2 \\ 8, 1$               | 2 1                         | $\begin{array}{ c c } 1,2\\1,1\end{array}$   | $IV, I_2$<br>$IV^*, I_1$                      |           |
| a2<br>a3                                   | 0                          |                               |           | )        | $-32 - 48\varphi$ $-80 + 48\varphi$   |                                       | $^{-188}\varphi$               |  | $\begin{bmatrix} 2\\2 \end{bmatrix}$   | + +        | 8, 1                         | 1                           | $\begin{bmatrix} 1,1\\1,1 \end{bmatrix}$     | $IV^*, I_1$<br>$IV^*, I_1$                    |           |
| a4   | 0                          | ,                             |           | )        | $3+2\varphi$                          |                                       | $1 - 13\varphi$                | 0                                      | 2                                      |            | 8,4                          | 4                           | 1, 2   | $IV^*, I_4$                                   |           |
| 145  | a                          | ·                             |           |          | ·                                     | $5a = (3 - 11\varphi) = 5a \cdot 29b$ | (3 isogen                      | v classes                              | s)                                     |            |                              | ı                           | 1  | ı   | 145a      |
| a1   | 0                          | 0                             | 1+0       | 2        | $\frac{-16-27\varphi}{}$              | . , ,                                 | $3+79\varphi$                  | 0                                      | 7                                      |            | 1,7                          | 1, 7                        | 1,7  | $I_1, I_7$                                    |           |
| a2   | 0                          |                               | 1 + 6     |          | $-48607 + 30045\varphi$               | -4849210 + 299                        |                                |  | $\stackrel{\cdot}{1}$                  |            | 7, 1                         | 7, 1                        | 7, 1   | $I_7, I_1$                                    |           |
|  |                            |                               |           |          | ,                                     |                                       | •                              |  |  |            |                              | <u> </u>                    | · ·  |   |           |

| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   |     | $a_1$       | $a_2$        | $a_3$       | $a_4$                     | $a_6$   | r             | T        | s                | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$ | Kodaira                     | Isogenies       |
|--|-----|-------------|--------------|-------------|---------------------------|---|---------------|----------|------------------|------------------------------|-----------------------------|-------|-----------------------------|-----------------|
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   |     | <u>ω1</u>   | <u> </u>     |             | <i>w</i> <sub>4</sub>     |   | '             | -        |                  | Ora(\(\Delta\)               | 0142(J)                     |       | Hodaira                     | Isogemes        |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 145 | a           |              |             | 14                        | $5a = (3 - 11\varphi) = 5a \cdot 29b \tag{3}$ | isogeny clas  | ses)     |                  |                              |                             |       |                             | 145a            |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | b1  | $\varphi$   | $-1-\varphi$ | 1           | $-421 + 256\varphi$       | $3929 - 2421\varphi$                          | 0             | 4        | ++               | 1, 1                         | 1,1                         | 1, 1  | $I_1, I_1$                  |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | b2  | $1+\varphi$ | 1            | $1+\varphi$ | 0                         | 0   | 0             | 4        |                  | 1, 1                         | 1, 1                        | 1, 1  | $I_1, I_1$                  |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | b3  | $\varphi$   | $-1-\varphi$ | 1           | $-26+16\varphi$           | $63 - 39\varphi$                              | 0             | 8        | ++               | 2, 2                         | 2,2                         | 2, 2  | $I_2, I_2$                  |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | b4  | $1+\varphi$ | 1            | $1+\varphi$ | $-15-15\varphi$           | $-34-57\varphi$                               | 0             | 8        | ++               | 4, 4                         | 4,4                         | 4, 4  | $\mathrm{I}_4,\mathrm{I}_4$ |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | b5  | $1+\varphi$ | 1            | $1+\varphi$ | $-200-255\varphi$         | $-1849 - 2798\varphi$                         | 0             | 4        | ++               | 2,8                          | 2,8                         | 2, 8  | $I_2, I_8$                  |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | b6  | $1+\varphi$ | 1            | $1+\varphi$ | $10-15\varphi$            | $-39 - 112\varphi$                            | 0             | 4        |                  | 8, 2                         | 8, 2                        | 8, 2  | $I_8, I_2$                  |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | b7  |             |              | 0           | $-127067 - 205599\varphi$ | $-33078478 - 53522100\varphi$                 | 0             | 2        | -+               | 1, 4                         | 1,4                         | 1, 4  | ${ m I}_1, { m I}_4$        |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | b8  | $\varphi$   | $-1-\varphi$ | 1           | $-3391 + 2041\varphi$     | $-88145 + 54311\varphi$                       | 0             | 2        | + -              | 1, 16                        | 1, 16                       | 1, 16 | $I_{1}, I_{16}$             |                 |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$  | c1  |             |              | $1+\varphi$ |                           | $-1-\varphi$                                  | 0             | 3        |                  | 1,1                          | 1,1                         | 1,1   | $I_1, I_1$                  |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | c2  |             |              |             | •                         | •   |               |          |                  | ,                            |                             | 1 '   |                             |                 |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   |     |             | ·            |             | ,                         | ·   | l             |          | -                |                              |                             |       |                             |                 |
| $ \begin{array}{c c c c c c c c c c c c c c c c c c c $  | 145 | b           |              |             | 14                        | $5b = (8 - 11\varphi) = 5a \cdot 29a \tag{3}$ | isogeny class | ses)     |                  |                              |                             |       |                             | $145\mathrm{b}$ |
| $ \begin{array}{c c c c c c c c c c c c c c c c c c c $  | a1  | 0           | 0            | $\varphi$   | $-43 + 27\varphi$         | $128 - 80\varphi$                             | 0             | 7        |                  | 1,7                          | 1,7                         | 1,7   | $I_1, I_7$                  |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | a2  | 0           | 0            |             | $-18562 - 30045\varphi$   | $-1852249 - 2996961\varphi$                   |               | 1        |                  |                              |                             |       |                             |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | b1  | · · · · · · | $-1-\omega$  |             |                           |   |               | <u> </u> | :<br>            | 1.1                          |                             | 1.1   |                             | <u>  </u>       |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |     | •           | 1            | -           | $-165 - 256\varphi$       | <u> </u>                                      | _             |          | + +              |                              |                             |       |                             |                 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |     |             | 1            |             | •                         |   |               |          |                  |                              |                             |       |                             |                 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |     | •           | $-1-\varphi$ |             | •                         | ·   |               |          |                  | ,                            |                             |       |                             |                 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |     | ,           |              |             | •                         |   |               |          | I                |                              |                             |       |                             |                 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |     | ,           |              |             | •                         |   |               |          |                  |                              |                             |       |                             |                 |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | b7  | ,           | ,            | $1+\varphi$ | •                         | •   |               |          | + -              |                              |                             |       |                             |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   |     |             | _            | •           | •                         |   |               |          |                  | *                            |                             |       |                             |                 |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$  |     |             |              |             |                           |   | <del>-</del>  |          | ' <sup>·</sup> - |                              | .'                          |       |                             | <u>-</u>        |
|  |     | -           | •            |             | •                         | · ·   |               |          |                  |                              |                             | 1 '   |                             |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 02  |             | Τ   Υ        | Ψ           |                           | 20 019  |               | -        |                  |                              | 3,3                         | 1,1   | 13,13                       |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 155 | a           |              |             | 1                         | $55a = (13 - \varphi) = 5a \cdot 31a \tag{1}$ | isogeny clas  | s)       |                  |                              |                             |       |                             | 155a            |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | a1  | 1           | $1-\varphi$  | $1+\varphi$ |                           |   |               | 1        | T - +            | 1.4                          | 1.4                         | 1.2   | $I_1, I_4$                  |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   |     | 1           | •            |             |                           | •   |               |          |                  |                              |                             |       |                             |                 |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   |     | 1           |              |             |                           |   |               |          | I                |                              |                             |       |                             |                 |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$  |     |             |              |             | •                         | •   |               |          |                  |                              |                             |       |                             |                 |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$  |     |             |              |             |                           |   | '             |          |                  |                              | •                           |       |                             |                 |
| $ \begin{vmatrix} a2 \\ a3 \end{vmatrix}  \begin{vmatrix} 1 \\ 0 \end{vmatrix}  \varphi  \varphi \qquad \qquad \begin{vmatrix} -18+3\varphi \\ -98+48\varphi \end{vmatrix} \qquad \begin{vmatrix} 11-24\varphi \\ -382+230\varphi \end{vmatrix}  \begin{vmatrix} 0 \\ 2 \\ 0 \end{vmatrix}  \begin{vmatrix} 4 \\ 2 \\ 0 \end{vmatrix}  + + \begin{vmatrix} 2,8 \\ 1,16 \end{vmatrix}  \begin{vmatrix} 2,2 \\ 1,1_{1} \\ 1,16 \end{vmatrix}  \begin{vmatrix} 1_{1},1_{1}_{1} \\ 1_{1},1_{1}_{1} \end{vmatrix} $ | 155 | b           |              |             | 1                         | $55b = (12 + \varphi) = 5a \cdot 31b \tag{1}$ | isogeny clas  | s)       |                  |                              |                             |       |                             | 155b            |
| $ \begin{vmatrix} a2 \\ a3 \end{vmatrix}  \begin{vmatrix} 1 \\ 0 \end{vmatrix}  \varphi  \varphi \qquad \qquad \begin{vmatrix} -18+3\varphi \\ -98+48\varphi \end{vmatrix} \qquad \begin{vmatrix} 11-24\varphi \\ -382+230\varphi \end{vmatrix}  \begin{vmatrix} 0 \\ 2 \\ 0 \end{vmatrix}  \begin{vmatrix} 4 \\ 2 \\ 0 \end{vmatrix}  + + \begin{vmatrix} 2,8 \\ 1,16 \end{vmatrix}  \begin{vmatrix} 2,2 \\ 1,1_{1} \\ 1,16 \end{vmatrix}  \begin{vmatrix} 1_{1},1_{16} \\ 1_{1} \end{vmatrix} $              | a1  | 1           | $\varphi$    | $\varphi$   | $-13 + 8\varphi$          | $23-15\varphi$                                | 0             | 4        | + -              | 1,4                          | 1,4                         | 1, 2  | $I_1, I_4$                  |                 |
| a3   1 $\varphi$ $\varphi$   |     | 1           |              |             | •                         | •   | 0             | 4        | ++               |                              |                             |       |                             |                 |
|  |     |             |              |             |                           | •   |               | 2        | 1                |                              |                             |       |                             |                 |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$   | a4  | $\varphi$   | $1-\varphi$  | 1           | $-402-666\varphi$         | $-6045 - 9803\varphi$                         | 0             | 2        | -+               | 4, 4                         | 4, 4                        | 2, 2  | $I_4, I_4$                  |                 |

|              |               |              |             |                         |   |  |   | 1   |                              |                             | 1     | T           |           |
|--------------|---------------|--------------|-------------|-------------------------|---|--|---|-----|------------------------------|-----------------------------|-------|-------------|-----------|
|              | $a_1$         | $a_2$        | $a_3$       | $a_4$                   | $a_6$   | r                                      | T | s   | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$ | Kodaira     | Isogenies |
| 164          | a             |              |             | 1                       | $64a = (14 - 2\varphi) = 2 \cdot 41a \tag{1}$ | isogeny class)                         | ) |     |                              |                             |       |             | 164a      |
| a1           | $1+\varphi$   | $-1-\varphi$ | 0           | $-2+\varphi$            | $-2-\varphi$                                  | 0                                      | 2 |     | 6, 1                         | 6, 1                        | 2, 1  | $I_6, I_1$  |           |
| a2           | $\varphi$     | -1           | 0           | $-213 + 128\varphi$     | $-1330 + 819\varphi$                          | 0                                      | 2 | ++  | 3, 2                         | 3, 2                        | 1,2   | $I_3, I_2$  |           |
| 164          | b             |              |             | 1                       | $64b = (12 + 2\varphi) = 2 \cdot 41b \tag{1}$ | isogeny class)                         |   |     |                              |                             |       |             | 164b      |
| a1           | $\varphi$     | -1           | 0           | $-1-\varphi$            | $-3+\varphi$                                  | 0                                      | 2 |     | -, -                         | 6, 1                        | 2, 1  | $I_6, I_1$  |           |
| a2           | $1+\varphi$   | $-1-\varphi$ | 0           | $-85 - 128\varphi$      | $-511 - 819\varphi$                           | 0                                      | 2 | ++  | 3, 2                         | 3, 2                        | 1,2   | $I_3, I_2$  |           |
| 171          | a             |              |             | 1                       | $71a = (3 - 12\varphi) = 3 \cdot 19a \tag{1}$ | isogeny class)                         | ) |     |                              |                             |       |             | 171a      |
| a1           | $1+\varphi$   | -1           | 1           | $-21-16\varphi$         | $-23-66\varphi$                               | 0                                      | 6 | ++  |                              | 1,6                         | 1,6   | $I_1, I_6$  |           |
| a2           | $1+\varphi$   | -1           | 1           | $-1-\varphi$            | $-1-2\varphi$                                 | 0                                      | 6 |     | , -                          | 2,3                         | 2,3   | $I_2, I_3$  |           |
| a3           | $1+\varphi$   | -1           | 1           | $-61 - 106\varphi$      | $-397-626\varphi$                             | 0                                      | 2 |     | ٠, ـ                         | 6, 1                        | 6, 1  | $I_6, I_1$  |           |
| a4           | 1             | φ            | 1           | $-7018 - 11344\varphi$  | $-434352 - 702788\varphi$                     | 0                                      | 2 | ++  | 3, 2                         | 3, 2                        | 3, 2  | $I_3, I_2$  |           |
| <b>171</b> ] | b             |              |             | 1                       |   | isogeny class)                         |   |     |                              |                             |       |             | 171b      |
| a1           | $\varphi$     | $-\varphi$   | 1           | $-36+15\varphi$         | $-89+66\varphi$                               | 0                                      | 6 | ++  |                              | 1,6                         | 1,6   | $I_1, I_6$  |           |
| a2           | $\varphi$     | $-\varphi$   | 1           | -1                      | $-3+2\varphi$                                 | 0                                      | 6 |     | -, -                         | 2,3                         | 2,3   | $I_2, I_3$  |           |
| a3           | $\varphi$     | $-\varphi$   | 1           | $-166 + 105\varphi$     | $-1023 + 626\varphi$                          | 0                                      | 2 |     | 6, 1                         | 6, 1                        | 6,1   | $I_6, I_1$  |           |
| a4           | 1             | $1-\varphi$  | 1           | $-18362 + 11344\varphi$ | $-1137140 + 702788\varphi$                    | 0                                      | 2 | ++  | 3, 2                         | 3, 2                        | 3, 2  | $I_3, I_2$  |           |
| 176          | a             |              |             |                         | $76a = (4 - 12\varphi) = 2^2 \cdot 11a $      | 1 isogeny class                        | ) |     |                              |                             |       |             | 176a      |
| a1           | 0             | -1           | 0           | $-1+\varphi$            | 0   | 0                                      | 6 | + - | ,                            | 1                           | 1,3   | $I_1, IV$   |           |
| a2           | 0             | -1           | 0           | $4-4\varphi$            | $-4+4\varphi$                                 | 0                                      | 6 | -+  |                              | 2                           | 2,3   | $I_2, IV^*$ |           |
| a3           | 0             | -1           | 0           | $-101 + 61\varphi$      | $-424 + 260\varphi$                           | 0                                      | 2 | + - | 3,4                          | 3<br>6                      | 1,3   | $I_3, IV$   |           |
| a4           | 0             | -1           | 0           | $-116 + 36\varphi$      | $-468 + 196\varphi$                           | 0                                      | 2 | -+  | 6,8                          | 0                           | 2,3   | $I_6, IV^*$ |           |
| 176          | b             |              |             | 1'                      | $76b = (8 - 12\varphi) = 2^2 \cdot 11b $      | l isogeny class                        | ) |     |                              |                             |       |             | 176b      |
| a1           | 0             | -1           | 0           | -arphi                  | 0   | 0                                      | 6 | -+  | ,                            | 1                           | 1,3   | $I_1, IV$   |           |
| a2           | 0             | -1           | 0           | 4arphi                  | $-4\varphi$                                   | 0                                      | 6 | + - | ,                            | 2                           | 2,3   | $I_2, IV^*$ |           |
| a3           | 0             | -1           | 0           | $-40-61\varphi$         | $-164 - 260\varphi$                           | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 2 | -+  | ,                            | 3                           | 1,3   | $I_3, IV$   |           |
| a4           | 0             | -1           | 0           | $-80 - 36\varphi$       | $-272 - 196\varphi$                           | 0                                      | 2 | + - | 6,8                          | 6                           | 2,3   | $I_6, IV^*$ |           |
| 179          | a             |              |             | 1                       | $79a = (5 - 12\varphi) = 179a \qquad (2 i)$   | sogeny classes                         | ) |     |                              |                             |       |             | 179a      |
| a1           | 1             |              | $1+\varphi$ | $1-\varphi$             | $-\varphi$                                    | 0                                      | 3 |     | 1                            | 1                           | 1     | $I_1$       |           |
| a2           | $\varphi$     | $1+\varphi$  | $1+\varphi$ | $-14 - 19\varphi$       | $-49 - 76\varphi$                             | 0                                      | 1 |     | 3                            | 3                           | 1     | $I_3$       |           |
| b1           | φ             | $-1-\varphi$ | 1           | $-26+15\varphi$         | $-57 + 35\varphi$                             | 0                                      | 1 |     | 1                            | 1                           | 1     | $I_1$       |           |
| 179          | b             |              |             | 1                       | $79b = (7 - 12\varphi) = 179b \qquad (2 i)$   | sogeny classes)                        |   |     |                              |                             |       |             | 179b      |
| a1           | 1             | 1            | $\varphi$   | 1                       | 0   | 0                                      | 3 |     | 1                            | 1                           | 1     | $I_1$       |           |
| a2           | $1 + \varphi$ | $-1+\varphi$ | 0           | $-33 + 20\varphi$       | $-71 + 42\varphi$                             | 0                                      | 1 |     | 3                            | 3                           | 1     | $I_3$       |           |
|              |               |              |             |                         |   |  |   |     |                              | 1                           | -     |             |           |

|   | $a_1$   | $a_2$   | $a_3$   | $a_4$  | $a_6$   | r   | T                                    | s                                     | $\operatorname{ord}(\Delta)$                  | $\operatorname{ord}_{-}(j)$                         | $c_p$   | Kodaira   | Isogenies    |
|---|---|---|---|--|---|---|--------------------------------------|---------------------------------------|---|---|---|---|--------------|
| 179   | b   |   |   |  | $179b = (7 - 12\varphi) = 179b$   | (2 isogeny cla  | asses)                               | )                                     |   |   |   |   | 179b         |
| b1  | $1+\varphi$   | 1   | $\varphi$   | $-11-15\varphi$  | $-33-50\varphi$   | 0   | 1                                    |                                       | 1   | 1   | 1   | $I_1$   |              |
|   |   |   |   |  |   | '   |                                      |                                       |   | 1   | I   | 1   |              |
| 180   | a   |   |   |  | $180a = (6 - 12\varphi) = 2 \cdot 5a \cdot 3$   | (1 isogeny  | class                                | s)                                    |   |   |   |   | 180a         |
| a1  | 1   |   | 1   | -289   | 1862  | 0   | 6                                    |                                       | 1, 3, 8                                       | 1, 3, 8   | 1, 3, 2   | $I_1, I_3, I_8$   |              |
| a2  | 1   |   | 1   | -19  | 26  | 0   | 12                                   |                                       | 2, 6, 4                                       | 2, 6, 4   | 2, 6, 2   | $I_2, I_6, I_4$   |              |
| a3  | 1   |   | 1   | 1  | 2   | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$                        | 12                                   |                                       | 4, 3, 2                                       | 4, 3, 2   | 4, 3, 2   | $I_4, I_3, I_2$   |              |
| a4  | 1   |   | 1   | -69  | -194  | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$                        | 6                                    |                                       | 1, 12, 2                                      | 1,12,2  | 1, 12, 2  | $I_1, I_{12}, I_2$  |              |
| a5<br>a6                                      | 1 1   |   | 1<br>1  | $-454 \\ -334$   | -544 $-2368$  | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$                        | $\begin{vmatrix} 2\\4 \end{vmatrix}$ |                                       | 3, 1, 24                                      | 3, 1, 24  | 3, 1, 2   | $I_3, I_1, I_{24}$  |              |
| a7  | 1   |   | 1   | -334 $-14$   | -2508 $-64$   |   | $\begin{vmatrix} 4\\4 \end{vmatrix}$ |                                       | 6, 2, 12 $12, 1, 6$                           | 6, 2, 12 $12, 1, 6$                                 | 6, 2, 2<br>12, 1, 2                                 | $I_6, I_2, I_{12}$  |              |
| a8  | 1   |   | 1   | -14 $-5334$  | -04 $-150368$   |   | $\begin{vmatrix} 4\\2 \end{vmatrix}$ |                                       | 3, 4, 6                                       | $\begin{bmatrix} 12, 1, 0 \\ 3, 4, 6 \end{bmatrix}$ | $\begin{bmatrix} 12, 1, 2 \\ 3, 4, 2 \end{bmatrix}$ | $\begin{array}{ c c c c c }\hline I_{12}, I_1, I_6 \\ I_3, I_4, I_6 \\ \hline \end{array}$  |              |
| ao  | 1   | 0   |   | -0004  | -130300   | 0   |                                      | + +                                   | 3, 4, 0                                       | 3, 4, 0   | 3, 4, 2   | 13, 14, 16  |              |
| 191   | a   |   |   |  | $191a = (13 + 2\varphi) = 191a$   | (1 isogeny c  | lass)                                |                                       |   |   |   |   | 191a         |
| a1  | $1+\varphi$   | $\varphi$   | 0   | $1+\varphi$  | 0   | 0   | 4                                    | -+                                    | 1   | 1   | 1   | $I_1$   |              |
| a2  | $1+\varphi$   | $\varphi$   |   | $-4-4\varphi$  | $-9-16\varphi$  | 0   | 4                                    | ++                                    | 2   | 2   | 2   | $I_2$   |              |
| a3  | $1+\varphi$   | $\varphi$   | 0   | $-79-89\varphi$  | $-396-722\varphi$   | 0   | 2                                    | ++                                    | 1   | 1   | 1   | $I_1$   |              |
| a4  | $1+\varphi$   | $\varphi$   | 0   | $-9+\varphi$   | $-18 - 14\varphi$   | 0   | 2                                    | +-                                    | 4   | 4   | 2   | $I_4$   |              |
| 191   | b   |   |   |  | $191b = (15 - 2\varphi) = 191b$   | (1 isogeny c  | lass)                                |                                       |   |   |   |   | 191b         |
| a1  |   |   |   |  |   | (   | , ,                                  |                                       |   |   |   |   |              |
| CL  | 1 (/)   | -1 + 0  | - 1   | 0  | 0   | 0   | 4                                    | + -                                   | 1   | 1   | 1   | I <sub>1</sub>  |              |
| a2  |   | $-1 + \varphi$<br>$-1 + \varphi$  |   | $0 \\ -10 + 5\varphi$  | $0 \\ -10 + 6\varphi$   | 0 0   | 4 4                                  | + -                                   | $\frac{1}{2}$                                 | $\begin{array}{c c} 1 \\ 2 \end{array}$             | $\frac{1}{2}$                                       | $I_1$ $I_2$   |              |
| a2<br>a3                                      | φ   | $-1+\varphi$  | 1   | $-10 + 5\varphi$   | $-10+6\varphi$  | 0   | 4                                    | ++                                    | 2   | 2   | 2   | $I_2$   |              |
| a2<br>a3<br>a4                                | $\varphi$   | $-1 + \varphi$ $-1 + \varphi$   | 1<br>1  |  | $-10 + 6\varphi$ $-858 + 552\varphi$  |   |                                      | 1                                     |   |   |   | $egin{array}{c} I_2 \ I_1 \end{array}$  |              |
| a3  | $\varphi$   | $-1+\varphi$  | 1<br>1  | $-10 + 5\varphi$ $-170 + 90\varphi$  | $-10+6\varphi$  | 0 0   | $\begin{vmatrix} 4\\2 \end{vmatrix}$ | ++++                                  | 2<br>1  | 2<br>1  | 2<br>1  | $I_2$   |              |
| a3  | φ<br>φ<br>φ   | $-1 + \varphi$ $-1 + \varphi$   | 1<br>1  | $-10 + 5\varphi$ $-170 + 90\varphi$  | $     \begin{array}{r}       -10 + 6\varphi \\       -858 + 552\varphi \\       -22 + 4\varphi   \end{array} $  | 0 0   | 4<br>2<br>2                          | ++++                                  | 2<br>1  | 2<br>1  | 2<br>1  | $egin{array}{c} I_2 \ I_1 \end{array}$  | 196a         |
| a3<br>a4                                      | φ<br>φ<br>φ   | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ -1 + \varphi \end{array} $   | 1<br>1  | $-10 + 5\varphi$ $-170 + 90\varphi$  | $     \begin{array}{r}       -10 + 6\varphi \\       -858 + 552\varphi \\       -22 + 4\varphi   \end{array} $  | 0 0 0   | 4<br>2<br>2                          | ++++                                  | 2<br>1  | 2<br>1  | 2<br>1  | $egin{array}{c} I_2 \ I_1 \end{array}$  | 196a         |
| a3<br>a4<br>196                               | $\mathbf{a}$  | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ -1 + \varphi \end{array} $   | 1<br>1<br>1   | $   \begin{array}{r}     -10 + 5\varphi \\     -170 + 90\varphi \\     -10   \end{array} $   |   | 0<br>0<br>0   | 4 2 2 2 s)                           | ++++-+                                | 2<br>1<br>4                                   | 2<br>1<br>4   | 2<br>1<br>2   | $\begin{bmatrix} I_2 \\ I_1 \\ I_4 \end{bmatrix}$   | 196a         |
| a3 a4 196 a1                                  | $\mathbf{a}$  | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ -1 + \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $  | 1 1 1   | $-10 + 5\varphi$ $-170 + 90\varphi$ $-10$ $-11$  |   | 0 0 0 0 1 isogeny class 0                                     | s) 6 6 6 6                           | +++-+                                 | 1, 2<br>2, 1<br>3, 6                          | 1, 2<br>2, 1<br>3, 6                                | 1, 2<br>2, 1<br>3, 6                                | $\begin{array}{c c} I_2 \\ I_1 \\ I_4 \\ \end{array}$ $\begin{array}{c c} I_1, I_2 \\ I_2, I_1 \\ I_3, I_6 \\ \end{array}$                                  | 196a         |
| a3<br>a4<br>1966<br>a1<br>a2<br>a3<br>a4      | $\varphi$ $\varphi$ $\varphi$ $\varphi$ $\varphi$ $\varphi$ $\varphi$   | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ -1 + \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array} $   | 1<br>1<br>1<br>1<br>1<br>1                          | $ \begin{array}{r} -10 + 5\varphi \\ -170 + 90\varphi \\ -10 \end{array} $ $ \begin{array}{r} -11 \\ -1 \\ -36 \\ 4 \end{array} $  |   | 1 isogeny class 0 0 0 0 0 0 0 0 0 0 0                         | s) 6 6 6 6 6                         | +++                                   | 1, 2<br>2, 1<br>3, 6<br>6, 3                  | 1, 2<br>2, 1<br>3, 6<br>6, 3                        | 1, 2<br>2, 1<br>3, 6<br>6, 3                        | $\begin{array}{c c} I_2 \\ I_1 \\ I_4 \\ \end{array}$ $\begin{array}{c c} I_1, I_2 \\ I_2, I_1 \\ I_3, I_6 \\ I_6, I_3 \\ \end{array}$                      | 196a         |
| a3<br>a4<br>196<br>a1<br>a2<br>a3<br>a4<br>a5 | $\varphi$ $\varphi$ $\varphi$ $\varphi$ $\varphi$ $\varphi$ $\varphi$   | $-1 + \varphi$ $-1 + \varphi$ $-1 + \varphi$ $0$ $0$ $0$ $0$ $0$ $0$  | 1<br>1<br>1<br>1<br>1<br>1<br>1                     | $ \begin{array}{r} -10 + 5\varphi \\ -170 + 90\varphi \\ -10 \end{array} $ $ \begin{array}{r} -11 \\ -1 \\ -36 \\ 4 \\ -2731 \end{array} $   | $-10 + 6\varphi$ $-858 + 552\varphi$ $-22 + 4\varphi$ $196a = (14) = 2 \cdot 7 \qquad ($ $12$ $0$ $-70$ $-6$ $-55146$   | 1 isogeny class 0 0 0 0 0 0 0 0 0 0 0 0 0                     | s) 6 6 6 6 2                         | +++-+                                 | 1, 2<br>2, 1<br>3, 6<br>6, 3<br>9, 2          | 1, 2<br>2, 1<br>3, 6<br>6, 3<br>9, 2                | 1,2<br>2,1<br>3,6<br>6,3<br>9,2                     | $\begin{array}{c c} I_2 \\ I_1 \\ I_4 \\ \hline \\ I_1, I_2 \\ I_2, I_1 \\ I_3, I_6 \\ I_6, I_3 \\ I_9, I_2 \\ \hline \end{array}$                          | 196a         |
| a3<br>a4<br>1966<br>a1<br>a2<br>a3<br>a4      | $\varphi$ $\varphi$ $\varphi$ $\varphi$ $\varphi$ $\varphi$ $\varphi$   | $-1 + \varphi$ $-1 + \varphi$ $-1 + \varphi$ $0$ $0$ $0$ $0$ $0$ $0$  | 1<br>1<br>1<br>1<br>1<br>1                          | $ \begin{array}{r} -10 + 5\varphi \\ -170 + 90\varphi \\ -10 \end{array} $ $ \begin{array}{r} -11 \\ -1 \\ -36 \\ 4 \end{array} $  |   | 1 isogeny class 0 0 0 0 0 0 0 0 0 0 0                         | s) 6 6 6 6 6                         | +++                                   | 1, 2<br>2, 1<br>3, 6<br>6, 3                  | 1, 2<br>2, 1<br>3, 6<br>6, 3                        | 1, 2<br>2, 1<br>3, 6<br>6, 3                        | $\begin{array}{c c} I_2 \\ I_1 \\ I_4 \\ \end{array}$ $\begin{array}{c c} I_1, I_2 \\ I_2, I_1 \\ I_3, I_6 \\ I_6, I_3 \\ \end{array}$                      | 196a         |
| a3<br>a4<br>196<br>a1<br>a2<br>a3<br>a4<br>a5 | $\varphi$   | $-1 + \varphi$ $-1 + \varphi$ $-1 + \varphi$ $0$ $0$ $0$ $0$ $0$ $0$  | 1<br>1<br>1<br>1<br>1<br>1<br>1                     | $ \begin{array}{r} -10 + 5\varphi \\ -170 + 90\varphi \\ -10 \end{array} $ $ \begin{array}{r} -11 \\ -1 \\ -36 \\ 4 \\ -2731 \end{array} $   | $-10 + 6\varphi$ $-858 + 552\varphi$ $-22 + 4\varphi$ $196a = (14) = 2 \cdot 7 \qquad ($ $12$ $0$ $-70$ $-6$ $-55146$   | 1 isogeny class 0 0 0 0 0 0 0 0 0 0 0 0 0                     | s) 6 6 6 6 2 2                       | +++++                                 | 1, 2<br>2, 1<br>3, 6<br>6, 3<br>9, 2          | 1, 2<br>2, 1<br>3, 6<br>6, 3<br>9, 2                | 1,2<br>2,1<br>3,6<br>6,3<br>9,2                     | $\begin{array}{c c} I_2 \\ I_1 \\ I_4 \\ \hline \\ I_1, I_2 \\ I_2, I_1 \\ I_3, I_6 \\ I_6, I_3 \\ I_9, I_2 \\ \hline \end{array}$                          |              |
| a3 a4  196 a1 a2 a3 a4 a5 a6  199             | $egin{array}{c} arphi \ ar$ | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ -1 + \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $  | 1<br>1<br>1<br>1<br>1<br>1<br>1<br>1                | $ \begin{array}{r} -10 + 5\varphi \\ -170 + 90\varphi \\ -10 \end{array} $ $ \begin{array}{r} -11 \\ -1 \\ -36 \\ 4 \\ -2731 \\ -171 \end{array} $   | $-10 + 6\varphi$ $-858 + 552\varphi$ $-22 + 4\varphi$ $196a = (14) = 2 \cdot 7 \qquad ($ $12$ $0$ $-70$ $-6$ $-55146$ $-874$  | 0 0 0 0 1 isogeny class 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   | 4 2 2 2 s) 6 6 6 6 2 2 2 asses       | ++++                                  | 1, 2<br>2, 1<br>3, 6<br>6, 3<br>9, 2          | 1, 2<br>2, 1<br>3, 6<br>6, 3<br>9, 2                | 1, 2<br>2, 1<br>3, 6<br>6, 3<br>9, 2<br>18, 1       | $\begin{matrix} I_2\\I_1\\I_4\end{matrix}$  |              |
| a3 a4  196. a1 a2 a3 a4 a5 a6  199. a1        | $egin{array}{c} arphi \ arphi \ arphi \ arphi \ \end{array}$  | $-1+\varphi\\-1+\varphi\\-1+\varphi$ $0\\0\\0\\0\\0\\-1-\varphi$  | 1<br>1<br>1<br>1<br>1<br>1<br>1<br>1                | $ \begin{array}{c} -10 + 5\varphi \\ -170 + 90\varphi \\ -10 \end{array} $ $ \begin{array}{c} -11 \\ -1 \\ -36 \\ 4 \\ -2731 \\ -171 \end{array} $   | $-10 + 6\varphi$ $-858 + 552\varphi$ $-22 + 4\varphi$ $196a = (14) = 2 \cdot 7 \qquad ($ $12$ $0$ $-70$ $-6$ $-55146$ $-874$ $199a = (13 + 3\varphi) = 199a$ $0$  | 0 0 0 0 1 isogeny class 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 | 8) 6 6 6 6 2 2 2 asses 3             | +++                                   | 1, 2<br>2, 1<br>3, 6<br>6, 3<br>9, 2<br>18, 1 | 1,2<br>2,1<br>3,6<br>6,3<br>9,2<br>18,1             | 1, 2<br>2, 1<br>3, 6<br>6, 3<br>9, 2<br>18, 1       | $\begin{array}{c c} I_2 \\ I_1 \\ I_4 \\ \hline \\ I_1, I_2 \\ I_2, I_1 \\ I_3, I_6 \\ I_6, I_3 \\ I_9, I_2 \\ I_{18}, I_1 \\ \hline \\ I_1 \\ \end{array}$ |              |
| a3 a4  196 a1 a2 a3 a4 a5 a6  199 a1 a2       | $egin{array}{c} arphi \ arphi \ arphi \ arphi \ \end{array}$  | $-1+\varphi\\-1+\varphi\\-1+\varphi$ $0\\0\\0\\0\\0\\-1-\varphi$  | 1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1      | $ \begin{array}{c} -10 + 5\varphi \\ -170 + 90\varphi \\ -10 \end{array} $ $ \begin{array}{c} -11 \\ -1 \\ -36 \\ 4 \\ -2731 \\ -171 \end{array} $ $ \begin{array}{c} \varphi \\ -10 + 11\varphi \end{array} $ | $-10 + 6\varphi$ $-858 + 552\varphi$ $-22 + 4\varphi$ $196a = (14) = 2 \cdot 7 \qquad ($ $12$ $0$ $-70$ $-6$ $-55146$ $-874$ $199a = (13 + 3\varphi) = 199a$  | 0 0 0 0 1 isogeny class 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   | 4 2 2 2 s) 6 6 6 6 2 2 2 asses       | +++                                   | 1, 2<br>2, 1<br>3, 6<br>6, 3<br>9, 2<br>18, 1 | 1,2<br>2,1<br>3,6<br>6,3<br>9,2<br>18,1             | 1, 2<br>2, 1<br>3, 6<br>6, 3<br>9, 2<br>18, 1       | $\begin{array}{c c} I_2\\ I_1\\ I_4 \\ \\ I_4 \\ \\ I_1, I_2\\ I_2, I_1\\ I_3, I_6\\ I_6, I_3\\ I_9, I_2\\ I_{18}, I_1 \\ \\ \\ I_{13} \\ \end{array}$      |              |
| a3 a4  196 a1 a2 a3 a4 a5 a6  199 a1 a2 a3 a3 | $egin{array}{c} arphi \ arphi \ arphi \ arphi \ \end{array}$  | $   \begin{array}{c}     -1 + \varphi \\     -1 + \varphi \\     -1 + \varphi   \end{array} $ $   \begin{array}{c}     0 \\     0 \\     0 \\     0 \\     0   \end{array} $ $   \begin{array}{c}     0 \\     0 \\     0 \\     0 \\     1 + \varphi   \end{array} $ | 1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1 | $-10 + 5\varphi$ $-170 + 90\varphi$ $-10$ $-11$ $-1$ $-36$ $4$ $-2731$ $-171$ $-10 + 11\varphi$ $-46382 + 28665\varphi$  | $-10 + 6\varphi$ $-858 + 552\varphi$ $-22 + 4\varphi$ $196a = (14) = 2 \cdot 7 \qquad ($ $12$ $0$ $-70$ $-6$ $-55146$ $-874$ $199a = (13 + 3\varphi) = 199a$ $0$ $-21 + 6\varphi$ $-4525688 + 2797026\varphi$ | 0 0 0 0 1 isogeny class 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   | s) 6 6 6 6 2 2 asses 3 3 1           | + + + + + + + + + + + + + + + + + + + | 1, 2<br>2, 1<br>3, 6<br>6, 3<br>9, 2<br>18, 1 | 1,2<br>2,1<br>3,6<br>6,3<br>9,2<br>18,1             | 1,2<br>2,1<br>3,6<br>6,3<br>9,2<br>18,1             | $\begin{array}{c c} I_2 \\ I_1 \\ I_4 \\ \end{array}$   |              |
| a3 a4  196 a1 a2 a3 a4 a5 a6  199 a1 a2       | $ \begin{array}{c c} \varphi \\ \varphi \\ \varphi \end{array} $  | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ -1 + \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} -1 - \varphi \\ -1 - \varphi \\ 1 + \varphi \end{array} $   | 1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1<br>1      | $ \begin{array}{c} -10 + 5\varphi \\ -170 + 90\varphi \\ -10 \end{array} $ $ \begin{array}{c} -11 \\ -1 \\ -36 \\ 4 \\ -2731 \\ -171 \end{array} $ $ \begin{array}{c} \varphi \\ -10 + 11\varphi \end{array} $ | $-10 + 6\varphi$ $-858 + 552\varphi$ $-22 + 4\varphi$ $196a = (14) = 2 \cdot 7 \qquad ($ $12$ $0$ $-70$ $-6$ $-55146$ $-874$ $199a = (13 + 3\varphi) = 199a$ $0$ $-21 + 6\varphi$                             | 0 0 0 0 1 isogeny class 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   | s) 6 6 6 6 2 2 asses 3 3             | +++                                   | 1,2<br>2,1<br>3,6<br>6,3<br>9,2<br>18,1       | 1,2<br>2,1<br>3,6<br>6,3<br>9,2<br>18,1             | 1,2<br>2,1<br>3,6<br>6,3<br>9,2<br>18,1             | $\begin{array}{c c} I_2\\ I_1\\ I_4 \\ \\ I_4 \\ \\ I_1, I_2\\ I_2, I_1\\ I_3, I_6\\ I_6, I_3\\ I_9, I_2\\ I_{18}, I_1 \\ \\ \\ I_{13} \\ \end{array}$      | 196a<br>199a |

|              |                |                |                |             |                         |  |  | Imi                                   |          | 1(A)                         | 1 (:)                               |                        | Kodaira       | T            |
|--------------|----------------|----------------|----------------|-------------|-------------------------|--|--|---------------------------------------|----------|------------------------------|-------------------------------------|------------------------|---------------|--------------|
|              |                | $a_1$          | $a_2$          | $a_3$       | $a_4$                   | $a_6$  | r                                      | T                                     | s        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$         | $c_p$                  | Kodaira       | Isogenies    |
| 199a         | a              |                |                |             | 1                       | $99a = (13 + 3\varphi) = 199a \qquad (3 i)$  | sogeny classes                         | )                                     |          |                              |                                     |                        |               | 199a         |
| c1           |                | 1              | -1             | φ           | -2                      | -1   | 0                                      | 2                                     | + -      | 1                            | 1                                   | 1                      | $I_1$         |              |
| c2           | (              | $\varphi$ –    | $1-\varphi$    | φ           | $-43-69\varphi$         | $-177 - 286\varphi$                          | 0                                      | 2                                     | -+       | 2                            | 2                                   | 2                      | $I_2$         |              |
| 199          | b              |                |                |             | 1                       | $.99b = (16 - 3\varphi) = 199b \qquad (3 is$ | sogeny classes)                        |                                       |          |                              |                                     |                        |               | <b>199</b> b |
| a1           |                | 0              | $1+\varphi$    | 1           | φ                       | 0  | 1                                      | 3                                     | -+       |                              | 1                                   | 1                      | $I_1$         |              |
| a2           |                |                | $1+\varphi$    | 1           | $-9\varphi$             | $-15-16\varphi$                              | 1                                      | 3                                     | -+       | 3                            | 3                                   | 3                      | $I_3$         |              |
| a3           |                | 0 –            | $1-\varphi$    | 1           | $-17718 - 28663\varphi$ | $-1710944 - 2768362\varphi$                  | 1                                      | 1                                     | - +      | 1                            | 1                                   | 1                      | $I_1$         |              |
| b1           |                | 0              | 0              | 1           | $-9+4\varphi$           | $11-6\varphi$                                | 0                                      | 5                                     | - +      | 1                            | 1                                   | 1                      | $I_1$         |              |
| b2           |                | 0              | 0              | 1           | $-186 - 317\varphi$     | $-1966 - 3180\varphi$                        | 0                                      | 1                                     | -+       | 5                            | 5                                   | 1                      | $I_5$         |              |
| c1           |                | 1              | -1             | $1+\varphi$ | $-2-\varphi$            | -1-arphi                                     | 0                                      | $\begin{vmatrix} 1 & 2 \end{vmatrix}$ | - +      | 1                            | 1                                   | 1                      | $I_1$         |              |
| c2           | 1+             | $\varphi$      | 1              | 0           | $-111 + 70\varphi$      | $-574 + 355\varphi$                          | 0                                      | 2                                     | + -      | 2                            | 2                                   | 2                      | $I_2$         |              |
| <b>205</b> a | a              |                |                |             | 20                      | $5a = (9 - 13\varphi) = 5a \cdot 41a$ (3)    | isogeny classe                         | s)                                    |          |                              |                                     |                        |               | <b>205</b> a |
| a1           | (              | $\varphi$      | $1-\varphi$    | $1+\varphi$ | $-37 + 20\varphi$       | $79-49\varphi$                               | 0                                      | 6                                     | ++       | 2, 1                         | 2,1                                 | 2, 1                   | $I_2, I_1$    |              |
| a2           | (              |                | $1-\varphi$    |             | -2                      | -arphi                                       | 0                                      | 6                                     |          | 1, 2                         | 1, 2                                | 1, 2                   | $I_1, I_2$    |              |
| a3           | 1 + 6          | $\varphi$      | $-\varphi$     | $1+\varphi$ | $-76-106\varphi$        | $-408-657\varphi$                            | 0                                      | 2                                     | ++       | 6,3                          | 6,3                                 | 2, 1                   | $I_6, I_3$    |              |
| a4           | (              | $\varphi$      | $1-\varphi$    | $1+\varphi$ | $13-10\varphi$          | 7-7arphi                                     | 0                                      | 2                                     |          | 3,6                          | 3,6                                 | 1,2                    | $I_3, I_6$    |              |
| b1           |                | 1 –            | $1+\varphi$    | $\varphi$   | $-144 - 230\varphi$     | $1206 + 1947\varphi$                         | 0                                      | 8                                     | + +      | 1,8                          | 1,8                                 | 1,8                    | $I_1, I_8$    |              |
| b2           | 1 + 6          |                | $\varphi$      | 1           | $-4+\varphi$            | $-2+\varphi$                                 | 0                                      | 8                                     | ++       | 2, 4                         | 2, 4                                | 2,4                    | $I_2, I_4$    |              |
| b3           | 1 + 6          | $\varphi$      | $\varphi$      | 1           | $1+\varphi$             | arphi  | 0                                      | 4                                     |          | 1, 2                         | 1, 2                                | 1, 2                   | $I_1, I_2$    |              |
| b4           | 1 + 0          | $\varphi$      | $\varphi$      | 1           | $-54 + 26\varphi$       | $-162 + 81\varphi$                           | 0                                      | 4                                     | ++       | 4, 2                         | 4, 2                                | 4, 2                   | $I_4, I_2$    |              |
| b5           |                |                | $1+\varphi$    | 1           | $-5562 + 3418\varphi$   | $-184493 + 113973\varphi$                    | 0                                      | 2                                     | ++       | ,                            | 2, 1                                | 2, 1                   | $I_2, I_1$    |              |
| b6           | $1 + \epsilon$ | $\varphi$      | $\varphi$      | 1           | $-39 + 46\varphi$       | $-184 + 75\varphi$                           | 0                                      | 2                                     |          | 8, 1                         | 8, 1                                | 8,1                    | $I_8, I_1$    |              |
| c1           |                | $\varphi$      | $\varphi$      | 1           | $-266-133\varphi$       | $352 + 2075\varphi$                          | 0                                      | 2                                     | + +      | 2,5                          | 2,5                                 | [2, 1]                 | $I_2, I_5$    |              |
| c2           | (              | φ              | $\varphi$      | 1           | $-16-8\varphi$          | $2+25\varphi$                                | 0                                      | 2                                     |          | 1, 10                        | 1,10                                | 1,2                    | $I_1, I_{10}$ |              |
| 205          | b              |                |                |             | 20                      | $5b = (4 - 13\varphi) = 5a \cdot 41b$ (3)    | isogeny classe                         | s)                                    |          |                              |                                     |                        |               | <b>205</b> b |
| a1           | 1 + 6          | $\varphi$      | 0              | φ           | $-16-22\varphi$         | $31 + 48\varphi$                             | 0                                      | 6                                     | ++       | 2, 1                         | 2, 1                                | 2, 1                   | $I_2, I_1$    |              |
| a2           | 1 + 6          | $\varphi$      | 0              | $\varphi$   | $-1-2\varphi$           | 0  | 0                                      | 6                                     |          | ,                            | 1, 2                                | 1,2                    | $I_1, I_2$    |              |
| a3           | (              | $\varphi$      | 0              | $\varphi$   | $-180 + 104\varphi$     | $-1064 + 656\varphi$                         | 0                                      | 2                                     | ++       |                              | 6,3                                 | 2, 1                   | $I_6, I_3$    |              |
| a4           | 1 + 6          | $\varphi_{\_}$ | 0              | $\varphi$   | $4+8\varphi$            | $1+6\varphi$                                 | 0                                      | 2                                     | <u> </u> | 3, 6                         | 3,6                                 | 1,2                    | $I_3, I_6$    |              |
| b1           |                | 1              | $-\varphi^{-}$ | $1+\varphi$ | $-374 + 229\varphi$     | $3153 - 1948\varphi$                         | 0                                      | 8                                     | + +      | 1,8                          | 1,8                                 | 1,8                    | $I_1, I_8$    |              |
| b2           |                |                | $1 + \varphi$  | 0           | -4                      | $3-5\varphi$                                 | 0                                      | 8                                     | ++       | ,                            | 2, 4                                | 2, 4                   | $I_2, I_4$    |              |
| b3           |                |                | $1+\varphi$    | 0           | 1                       | 0  | 0                                      | 4                                     |          | 1, 2                         | 1, 2                                | 1, 2                   | $I_1, I_2$    |              |
| b4           |                | •              | $1+\varphi$    | 0           | $-29-25\varphi$         | $-77 - 110\varphi$                           | 0                                      | 4                                     | ++       | ,                            | 4, 2                                | 4,2                    | $I_4, I_2$    |              |
| b5           |                |                | $1+\varphi$    | 0           | $6-45\varphi$           | $-160-69\varphi$                             | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 2                                     |          | -, -                         | 8,1                                 | 8, 1                   | $I_8, I_1$    |              |
| b6           | 1+             |                | $\varphi$      | 0           | $-2143 - 3417\varphi$   | $-73938 - 119535\varphi$                     | 0                                      | 2                                     | + +      |                              | $\begin{vmatrix} 2,1 \end{vmatrix}$ | $\lfloor 2, 1 \rfloor$ | $I_2, I_1$    |              |
| c1           | 1 + 6          |                | $1 + \varphi$  | 0           | $-398 + 136\varphi$     | $2561 - 2340\varphi$                         | 0                                      | 2                                     | ++       |                              | 2,5                                 | 2,1                    | $I_2, I_5$    |              |
| c2           | 1 + 6          | $\varphi$      | $1+\varphi$    | 0           | $-23 + 11\varphi$       | $36-40\varphi$                               | 0                                      | 2                                     |          | 1, 10                        | 1, 10                               | 1,2                    | $I_1, I_{10}$ |              |

|  | $a_1$  | $a_2$ $a_3$   | $a_4$   | $a_6$   | $\mid r \mid$   | T   | s                               | $\operatorname{ord}(\Delta)$  | $\operatorname{ord}_{-}(j)$  | $c_p$   | Kodaira  | Isogenies |
|--|--|---|---|---|---|---|---------------------------------|---|--|---|--|-----------|
| 09a  | a  |   | 209a =  | $(15 - \varphi) = 11a \cdot 19b \qquad (3 \text{ isog})$  | eny classes   | ;)  |                                 |   |  |   |  | 209       |
| a1   | 0  | $1-\varphi$ $\varphi$   | $-112 + 69\varphi$  | $-601 + 371\varphi$   | 0   | 1   |                                 | 1, 1  | 1,1  | 1, 1  | $I_1, I_1$   |           |
| $1 \stackrel{!}{\mid}$                     |  | $-1-\varphi$ $1+\varphi$  | $-416 + 251\varphi$   | $3823 - 2351\varphi$  |   | <del> </del>  | ++                              | 1,1   | [ 1, 1]  | 1, 1  | $ \tilde{I}_1, \tilde{I}_1 $   |           |
| $\frac{1}{2}$                              | $\varphi$  | $1 \qquad \varphi$  | 0   | 0   |   | 4   |                                 | 1, 1  | 1, 1   | 1, 1  | $I_1, I_1$   |           |
| 3  | $\varphi$  | $1 \qquad \stackrel{'}{arphi}$  | -5  | $2-\varphi$   | 0   | 8   | ++                              | 2, 2  | 2,2  | 2, 2  | $I_2, I_2$   |           |
| 4  | arphi  | $1 \qquad \stackrel{\cdot}{arphi}$  | -10-5arphi  | $-11-15\overset{\cdot}{arphi}$  | 0   | 4   | ++                              | 4, 4  | 4, 4   | 2, 2  | $I_4, I_4$   |           |
| 5  | $\varphi$  | $1 \qquad \varphi$  | $-55-135\varphi$  | -509-852 arphi  | 0   | 2   | -+                              | 8, 2  | 8, 2   | 2, 2  | $I_8, I_2$   |           |
| 6  | $\varphi$  | $1 \qquad \varphi$  | $-45+45\varphi$   | $-225 + 66\varphi$  | 0   | 2   | + -                             | 2, 8  | 2,8  | 2,2   | $I_2, I_8$   |           |
| 1  | 0  | $1+\varphi$ $\varphi$   | -arphi  | $-2-3\varphi$   | 0   | 3   |                                 | 3, 1  | 3,1  | 3,1   | $I_3, I_1$   |           |
| :2   | 0  | $1+\varphi$ $\varphi$   | $-100 - 141\varphi$   | $-733 - 1154\varphi$  | 0   | 1   |                                 | 1,3   | 1,3  | 1, 3  | $I_1, I_3$   |           |
| 09ł  | b  |   | 209b =  | $(14 + \varphi) = 11b \cdot 19a \qquad (3 \text{ isog})$  | eny classes   | 1)  |                                 |   |  |   |  | 209       |
| a1   | 0  | $\varphi$ 1+ $\varphi$  | $\frac{2600 - 43 - 69\varphi}{-43 - 69\varphi}$   | $\frac{(11+\varphi)-110^{-100}}{-230-372\varphi}$   |   | 1   |                                 | 1,1   | 1,1  | 1, 1  | $I_1, I_1$   |           |
| <u>-</u><br>51                             | 1  | $1+\varphi$ $1+\varphi$   | $-165-250\varphi$   | $1308 + 2099\varphi$  |   | ¦   | ++                              | 1,1   | $egin{array}{cccccccccccccccccccccccccccccccccccc$                                   | 1,1   | $ \tilde{I}_1, \tilde{I}_1 $   |           |
| $\frac{1}{2}$                              | $1+\varphi$  | $1 + \varphi + 1 + \varphi$<br>$1 - \varphi + 1 + \varphi$  | $-2\varphi$   | •   |   | $\stackrel{\mathbf{a}}{4}$  |                                 | 1, 1 $1, 1$   | 1,1  | 1, 1 $1, 1$   | $I_1, I_1$ $I_1, I_1$  |           |
|  | $1 + \varphi$<br>$1 + \varphi$   | $1 - \varphi  1 + \varphi$ $1 - \varphi  1 + \varphi$   | $-5-2\varphi$   | -arphi 1  |   | 8   | + +                             | 2, 2  | 2, 2   | 2, 2  | $I_1, I_1$ $I_2, I_2$  |           |
| $_{\mathrm{b4}}^{\mathrm{b5}}$             | •  | $1 - \varphi$ $1 + \varphi$ $1 - \varphi$   | $-15+3\varphi$  | $-26+14\varphi$   |   | $\frac{\circ}{4}$   | ++                              | $\frac{2}{4}, \frac{2}{4}$  | $\frac{2}{4}, \frac{2}{4}$   | 2, 2  | $I_{2}, I_{2}$ $I_{4}, I_{4}$  |           |
| $_{05}$                                    | $1+\varphi$ $1+\varphi$  | $1 - \varphi  1 + \varphi$ $1 - \varphi  1 + \varphi$   | $-47\varphi$  | $-159-67\varphi$  |   | $\frac{1}{2}$   | - +                             | 2, 8  | 2,8  | 2, 2  | $I_2, I_8$   |           |
|  | •  | $1 - \varphi$ $1 + \varphi$ $1 + \varphi$   | $-190 + 133\varphi$   | $-1361 + 851\varphi$  |   | 2   | + -                             | 8, 2  | 8,2  | 2, 2  | $I_8, I_2$   |           |
| c1   | <del>-</del>   | $-1-\varphi$ $1+\varphi$  | $-2+3\varphi$   | -3  | 0   | 3   | - <u>·</u>                      | $\frac{1}{3}$ , $\frac{1}{1}$   | 3,1  | $\frac{1}{3}, \frac{1}{1}$  | $\overline{\mathrm{I}}_{3},\overline{\mathrm{I}}_{1}$  |           |
| c2   |  | $-1-\varphi$ $1+\varphi$  | $-242 + 143\varphi$   | $-1645 + 1011\varphi$   | 0   | 1   |                                 | 1,3   | 1,3  | 1, 3  | $I_1, I_3$   |           |
|  |  |   |   |   |   |   |                                 |   |  |   |  |           |
| 090  | $\mathbf{c}$   |   |   | $(8 - 13\varphi) = 11a \cdot 19a \qquad (3 \text{ isog})$   | geny classe   |   |                                 |   | 1  |   | T  | 209       |
| 4  | $\varphi$  | $\varphi$ 1+ $\varphi$  | -1  | $-\varphi$  | 1   | 2   | ++                              | 1, 1  | 1,1  | 1, 1  | $\mathrm{I}_1,\mathrm{I}_1$  |           |
|  |  | $\varphi 1 + \varphi$   | $-1-5\varphi$   | 2   | 1   | 2   | -+                              | $\frac{2}{2}$   | 2, 2   | 2, 2  | $I_2, I_2$   |           |
|  | $\varphi$  |   |   |   |   |   |                                 | 3, 1  | 3,1  | 3, 1  | $I_3, I_1$   |           |
| a2  <br>o1                                 | $\frac{1}{1}$  | $\frac{1}{0}$ $\varphi$   | $-3-3\varphi$   | $1+3\varphi$  | 0   | 6   | + +                             |   | 1 2  |   | 0, 1   |           |
| a2  <br>o1  <br>o2                         | 1<br>1   |   | $-3-8\varphi$   | $-6-5\varphi$   | 0   | 6   | + +                             | 6, 2  | 6, 2   | 6, 2  | $I_6, I_2$   |           |
| 12   01   02   03                          | 1  | $0 \qquad \varphi$  | $-3 - 8\varphi$ $-583 + 353\varphi$   | $-6-5\varphi \\ -6449+3978\varphi$  | $\left  egin{array}{c} 0 \\ 0 \end{array} \right $      | 6 2   |                                 | $6, 2 \\ 1, 3$  | 6, 2 $1, 3$  | 6, 2 $1, 3$   | $\begin{matrix} I_6,I_2\\I_1,I_3\end{matrix}$  |           |
| b1   b2   b3                               | 1<br>1   | $ \begin{array}{ccc} 0 & \varphi \\ 0 & \varphi \end{array} $   | $-3 - 8\varphi \\ -583 + 353\varphi \\ -378 - 423\varphi$   | $-6-5\varphi$   | $\left \begin{array}{c} 0 \\ 0 \\ 0 \end{array}\right $ | $\begin{bmatrix} 6 \\ 2 \\ 2 \end{bmatrix}$   | -+                              | 6, 2  | 6, 2   | 6, 2  | $\begin{bmatrix} I_6, I_2 \\ I_1, I_3 \\ I_2, I_6 \end{bmatrix}$   |           |
| b4  <br>c1                                 | $ \begin{array}{c} 1\\ 1\\ 1+\varphi\\ \end{array} $                                 | $ \begin{array}{ccc} 0 & \varphi \\ 0 & \varphi \\ -\varphi & \varphi \end{array} $   | $   \begin{array}{r}     -3 - 8\varphi \\     -583 + 353\varphi \\     -378 - 423\varphi \\     -2 - \varphi   \end{array} $                        | $     \begin{array}{r}       -6 - 5\varphi \\       -6449 + 3978\varphi \\       -4064 - 5772\varphi   \end{array} $  | $\left  egin{array}{c} 0 \\ 0 \end{array} \right $      | $\begin{bmatrix} 6 \\ 2 \\ 2 \end{bmatrix}$   | - +<br>+ +<br>- +<br>+ +        | 6, 2 $1, 3$ $2, 6$ $1, 1$   | 6, 2<br>1, 3<br>2, 6<br>1, 1   | 6, 2 $1, 3$ $2, 6$ $1, 1$   | $\begin{array}{c c} I_6, I_2 \\ I_1, I_3 \\ I_2, I_6 \\ \hline I_1, I_1 \end{array}$                         |           |
| a2   b1   b2   b3   b4   c1   c2           | $ \begin{array}{c} 1\\ 1\\ 1+\varphi\\ 1 \end{array} $                               | $ \begin{array}{ccc} 0 & \varphi \\ 0 & \varphi \\ -\varphi & \varphi \\ 0 & \varphi \\ \hline 0 & 1 \\ -\varphi & 1 \end{array} $  | $   \begin{array}{r}     -3 - 8\varphi \\     -583 + 353\varphi \\     -378 - 423\varphi \\     -2 - \varphi \\     -70 + 42\varphi   \end{array} $ | $     \begin{array}{r}       -6 - 5\varphi \\       -6449 + 3978\varphi \\       -4064 - 5772\varphi \\       \hline       \varphi \\       -277 + 171\varphi   \end{array} $ |   | $\begin{bmatrix} 6 \\ 2 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 4 \\ 4 \end{bmatrix}$      | - +<br>+ +<br>- +<br>+ +<br>+ + | $   \begin{array}{c}     6, 2 \\     1, 3 \\     \hline     2, 6 \\     \hline     1, 1 \\     2, 2   \end{array} $       | $ \begin{array}{c c} 6, 2 \\ 1, 3 \\ 2, 6 \\ \hline 1, 1 \\ 2, 2 \end{array} $       | $   \begin{array}{c}     6,2 \\     1,3 \\     2,6 \\     \hline     1,1 \\     2,2   \end{array} $ | $\begin{array}{c c} I_6, I_2 \\ I_1, I_3 \\ I_2, I_6 \\ \hline I_1, I_1 \\ I_2, I_2 \end{array}$             |           |
| a2   b1   b2   b3   b4   c1   c2           | $ \begin{array}{c} 1\\ 1\\ 1+\varphi\\ \end{array} $                                 | $ \begin{array}{ccc} 0 & \varphi \\ 0 & \varphi \\ -\varphi & \varphi \\ \hline 0 & \frac{\varphi}{1} \end{array} $   | $-3 - 8\varphi$ $-583 + 353\varphi$ $-378 - 423\varphi$ $-2 - \varphi$ $-70 + 42\varphi$ $-7610 + 4703\varphi$                                      | $-6 - 5\varphi$ $-6449 + 3978\varphi$ $-4064 - 5772\varphi$ $-277 + 171\varphi$ $-304778 + 188363\varphi$   |   | $\begin{bmatrix} 6 \\ 2 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix}$ | - +<br>+ +<br>- +<br>- +        | 6, 2 $1, 3$ $2, 6$ $1, 1$   | $\begin{array}{c c} 6, 2 \\ 1, 3 \\ 2, 6 \\ \hline 1, 1 \\ 2, 2 \\ 1, 1 \end{array}$ | $6, 2 \\ 1, 3 \\ 2, 6 \\ 1, 1 \\ 2, 2 \\ 1, 1$  | $\begin{array}{c c} I_6, I_2 \\ I_1, I_3 \\ I_2, I_6 \\ \hline I_1, I_1 \\ I_2, I_2 \\ I_1, I_1 \end{array}$ |           |
| a2   b1   b2   b3   b4   c1   c2   c3      | $ \begin{array}{c} 1\\ 1\\ 1+\varphi\\ 1\\ 1+\varphi \end{array} $                   | $ \begin{array}{ccc} 0 & \varphi \\ 0 & \varphi \\ -\varphi & \varphi \\ 0 & \varphi \\ \hline 0 & 1 \\ -\varphi & 1 \end{array} $  | $   \begin{array}{r}     -3 - 8\varphi \\     -583 + 353\varphi \\     -378 - 423\varphi \\     -2 - \varphi \\     -70 + 42\varphi   \end{array} $ | $     \begin{array}{r}       -6 - 5\varphi \\       -6449 + 3978\varphi \\       -4064 - 5772\varphi \\       \hline       \varphi \\       -277 + 171\varphi   \end{array} $ |   | $\begin{bmatrix} 6 \\ 2 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 4 \\ 4 \end{bmatrix}$      | - +<br>+ +<br>- +<br>+ +<br>+ + | $   \begin{array}{c}     6, 2 \\     1, 3 \\     \hline     2, 6 \\     \hline     1, 1 \\     2, 2   \end{array} $       | $ \begin{array}{c c} 6, 2 \\ 1, 3 \\ 2, 6 \\ \hline 1, 1 \\ 2, 2 \end{array} $       | $   \begin{array}{c}     6,2 \\     1,3 \\     2,6 \\     \hline     1,1 \\     2,2   \end{array} $ | $\begin{array}{c c} I_6, I_2 \\ I_1, I_3 \\ I_2, I_6 \\ \hline I_1, I_1 \\ I_2, I_2 \end{array}$             |           |
| b1   b2   b3   b4   c1                     | $ \begin{array}{c} 1\\ 1\\ 1+\varphi\\ 1\\ 1\\ 1+\varphi\\ \varphi\\ 1 \end{array} $ | $\begin{array}{cccc} 0 & \varphi \\ 0 & \varphi \\ -\varphi & \varphi \\ \hline 0 & \varphi \\ \hline -\varphi & \varphi \\ \hline 0 & 1 \\ -\varphi & 1 \\ 1-\varphi & 1 \\ \end{array}$ | $-3 - 8\varphi$ $-583 + 353\varphi$ $-378 - 423\varphi$ $-2 - \varphi$ $-70 + 42\varphi$ $-7610 + 4703\varphi$ $-27 - 21\varphi$                    | $-6 - 5\varphi$ $-6449 + 3978\varphi$ $-4064 - 5772\varphi$ $-\varphi$ $-277 + 171\varphi$ $-304778 + 188363\varphi$ $-68 - 73\varphi$  |   | $ \begin{array}{c c} 6 \\ 2 \\ 2 \\ 4 \\ 4 \\ 2 \\ 2 \end{array} $                      | - +<br>+ +<br>- +<br>+ +<br>+ + | $   \begin{array}{c}     6,2 \\     1,3 \\     \hline     2,6 \\     \hline     1,1 \\     2,2 \\     1,1   \end{array} $ | $\begin{array}{c c} 6, 2 \\ 1, 3 \\ 2, 6 \\ \hline 1, 1 \\ 2, 2 \\ 1, 1 \end{array}$ | $6, 2 \\ 1, 3 \\ 2, 6 \\ 1, 1 \\ 2, 2 \\ 1, 1$  | $\begin{array}{c c} I_6, I_2 \\ I_1, I_3 \\ I_2, I_6 \\ \hline I_1, I_1 \\ I_2, I_2 \\ I_1, I_1 \end{array}$ | 209       |
| a2   b1   b2   b3   b4   c1   c2   c3   c4 | $ \begin{array}{c} 1\\ 1\\ 1+\varphi\\ 1\\ 1\\ 1+\varphi\\ \varphi\\ 1 \end{array} $ | $\begin{array}{cccc} 0 & \varphi \\ 0 & \varphi \\ -\varphi & \varphi \\ \hline 0 & \varphi \\ \hline -\varphi & \varphi \\ \hline 0 & 1 \\ -\varphi & 1 \\ 1-\varphi & 1 \\ \end{array}$ | $-3 - 8\varphi$ $-583 + 353\varphi$ $-378 - 423\varphi$ $-2 - \varphi$ $-70 + 42\varphi$ $-7610 + 4703\varphi$ $-27 - 21\varphi$                    | $-6 - 5\varphi$ $-6449 + 3978\varphi$ $-4064 - 5772\varphi$ $-\varphi$ $-277 + 171\varphi$ $-304778 + 188363\varphi$ $-68 - 73\varphi$  |   | $ \begin{array}{c c} 6 \\ 2 \\ 2 \\ 4 \\ 4 \\ 2 \\ 2 \end{array} $                      | - +<br>+ +<br>- +<br>+ +<br>+ + | $   \begin{array}{c}     6,2 \\     1,3 \\     \hline     2,6 \\     \hline     1,1 \\     2,2 \\     1,1   \end{array} $ | $\begin{array}{c c} 6, 2 \\ 1, 3 \\ 2, 6 \\ \hline 1, 1 \\ 2, 2 \\ 1, 1 \end{array}$ | $6, 2 \\ 1, 3 \\ 2, 6 \\ 1, 1 \\ 2, 2 \\ 1, 1$  | $\begin{array}{c c} I_6, I_2 \\ I_1, I_3 \\ I_2, I_6 \\ \hline I_1, I_1 \\ I_2, I_2 \\ I_1, I_1 \end{array}$ | 209       |

|     | $a_1$   | $a_2$        | $a_3$                          | $a_4$                                   | $a_6$   | r                                      | T                                      | S  | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$             | $c_p$          | Kodaira                               | Isogenies    |
|-----|---|--------------|--------------------------------|---|---|--|--|--|------------------------------|---|----------------|---------------------------------------|--------------|
| 200 | J   |              |                                |   | 00 1 (7 10 ) 11 101                           | (0.                                    | ,                                      | ,  |                              |   |                |                                       | 2004         |
| 209 |   |              |                                |   | . , ,   | (3 isogeny                             | 1                                      |  |                              | 1                                       | T              |                                       | 209d         |
| b1  | 1   |              | $1+\varphi$                    | $-6+2\varphi$                           | $4-4\varphi$                                  | 0                                      | 6                                      | ++   | 3, 1                         | 3, 1                                    | 3, 1           | $I_3, I_1$                            |              |
| b2  | 1   |              | $1+\varphi$                    | $-11+7\varphi$                          | $-11+4\varphi$                                | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 6                                      | +-   | 6, 2                         | 6, 2                                    | 6, 2           | $I_6, I_2$                            |              |
| b3  | $\varphi$                                     |              | $1+\varphi$                    | $-229 - 355\varphi$                     | $-2471 - 3979\varphi$                         | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 2                                      | ++   | 1,3                          | 1,3                                     | 1,3            | $I_1, I_3$                            |              |
| b4  | 1   |              | $1+\varphi$                    | $-801 + 422\varphi$                     | $-9836 + 5771\varphi$                         |  | 2                                      | + -  | 2,6                          | 2,6                                     | 2,6            | $I_2, I_6$                            |              |
| c1  | 1   | 0            | 1                              | $-3+\varphi$                            | $1-\varphi$                                   | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 4                                      | ++   | 1, 1                         | 1,1                                     | 1,1            | $I_1, I_1$                            |              |
| c2  | $\varphi$                                     | 0            | 1                              | $-27 - 43\varphi$                       | $-106 - 171\varphi$                           | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\frac{4}{2}$                          | ++   | 2, 2                         | 2,2                                     | 2, 2           | $I_2, I_2$                            |              |
| c3  | $\begin{vmatrix} 1+\varphi\\ 1 \end{vmatrix}$ | $0 \\ 0$     | $1 \\ 1$                       | $-2907 - 4704\varphi$ $-48 + 21\varphi$ | $-116415 - 188363\varphi$                     | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\begin{vmatrix} 2\\2 \end{vmatrix}$   | -+   | 1, 1                         | 1, 1<br>4, 4                            | $1, 1 \\ 2, 2$ | $I_1, I_1$                            |              |
| c4  | 1   | 0            |                                | $-48 + 21\varphi$                       | $-141 + 73\varphi$                            | 0                                      |  | + -  | 4,4                          | 4,4                                     | 2, 2           | $I_4, I_4$                            |              |
|     |   |              |                                |   |   |  |  |  |                              |   |                |                                       |              |
| 220 | a   |              |                                | 22                                      | $0a = (16 - 2\varphi) = 2 \cdot 5a \cdot 11b$ | (3 isogeny                             | z class                                | ses)   |                              |   |                |                                       | 220a         |
| a1  | 1   | 1            | $1+\varphi$                    | $\frac{-5+3\varphi}{}$                  | $\frac{6-4\varphi}{}$                         | 0                                      | 10                                     | <del>-                                    </del> | 1, 2, 5                      | 1, 2, 5                                 | 1, 2, 5        | $I_1, I_2, I_5$                       |              |
| a2  | 1   |              | $1 + \varphi$<br>$1 + \varphi$ | $-5-7\varphi$                           | $20+2\varphi$                                 |  | 10                                     | _ +  | 2, 1, 10                     | 2, 1, 10                                | 2, 1, 10       | $I_1, I_2, I_3$<br>$I_2, I_1, I_{10}$ |              |
| a3  | 1   |              | $1+\varphi$                    | $-350 + 143\varphi$                     | $-2608 + 1319\varphi$                         |  | 2                                      | + -  | 5, 10, 1                     | 5, 10, 1                                | 5, 2, 1        | $I_5, I_{10}, I_1$                    |              |
| a4  | $\varphi$                                     | $-1-\varphi$ | ,                              | $-3629 - 5772\varphi$                   | $-154896 - 250296\varphi$                     | 0                                      | $\frac{1}{2}$                          | 1 '  | 10, 5, 2                     | 10, 5, 2                                | 10, 1, 2       | $I_{10}, I_5, I_2$                    |              |
| b1  | $\begin{vmatrix} 1 - \varphi \end{vmatrix}$   |              | 0                              | $-2+2\varphi$                           | - <u> </u>                                    |  | 6                                      | <u> </u>   | 3, 2, 1                      | $\frac{1}{3}, \frac{1}{2}, \frac{1}{1}$ | 3, 2, 1        | $  I_3, I_2, I_1  $                   |              |
| b2  | $1+\varphi$                                   | 0            | 0                              | $8-8\varphi$                            | $-2-4\varphi$                                 |  | 6                                      | - +  | 6, 1, 2                      | 6, 1, 2                                 | 6, 1, 2        | $I_6, I_1, I_2$                       |              |
| b3  | $1+\varphi$                                   | 0            | 0                              | $-137 + 67\varphi$                      | $-687 + 376\varphi$                           | 0                                      | 2                                      | + -  | 1, 6, 3                      | 1, 6, 3                                 | 1, 6, 1        | $I_1, I_6, I_3$                       |              |
| b4  | 1   | $1-\varphi$  | $\varphi$                      | $-914 - 1440\varphi$                    | $-19589 - 31584\varphi$                       | 0                                      | 2                                      | - +  | 2, 3, 6                      | 2, 3, 6                                 | 2, 3, 2        | $I_2, I_3, I_6$                       |              |
| c1  | $\varphi$                                     | $1+\varphi$  |                                | $-3+2\varphi$                           | 2-arphi                                       |  | 12                                     |  | 1, 4, 3                      | 1,4,3                                   | 1,4,3          | $  I_1, I_4, I_3  $                   | <u> </u>     |
| c2  | $\varphi$                                     | $1+\varphi$  | 0                              | $-23-18\varphi$                         | $18+51\varphi$                                | 0                                      | 12                                     |  | 2, 2, 6                      | 2, 2, 6                                 | 2, 2, 6        | $I_2, I_2, I_6$                       |              |
| c3  | $1+\varphi$                                   | -1           | $\varphi$                      | $-1707 - 2721\varphi$                   | $50348 + 81533\varphi$                        | 0                                      | 6                                      |  | 1, 1, 3                      | 1, 1, 3                                 | 1, 1, 3        | $I_1, I_1, I_3$                       |              |
| c4  | $\varphi$                                     | $1+\varphi$  | 0                              | 7-38arphi                               | $-28 + 69\varphi$                             | 0                                      | 6                                      | -+   | 4, 1, 12                     | 4, 1, 12                                | 2, 1, 12       | $I_4, I_1, I_{12}$                    |              |
| c5  | $\varphi$                                     | $1+\varphi$  | 0                              | $-33 + 37\varphi$                       | $-119 + 81\varphi$                            | 0                                      | 4                                      | + -  | 3, 12, 1                     | 3, 12, 1                                | 1, 12, 1       | $I_3, I_{12}, I_1$                    |              |
| c6  | 1   | 1            | 0                              | $-4436 + 2733\varphi$                   | $-135379 + 83661\varphi$                      | 0                                      | 4                                      | ++   | 6, 6, 2                      | 6, 6, 2                                 | 2, 6, 2        | $I_6,I_6,I_2$                         |              |
| c7  | $1+\varphi$                                   | $-1+\varphi$ | $\varphi$                      | $-486101 + 300415\varphi$               | $-153092515 + 94616388\varphi$                | 0                                      | 2                                      | ++   | 3, 3, 1                      | 3, 3, 1                                 | 1, 3, 1        | $I_3, I_3, I_1$                       |              |
| c8  | $\varphi$                                     | $1+\varphi$  | 0                              | $-993 - 163\varphi$                     | $-12703 - 3631\varphi$                        | 0                                      | 2                                      | -+   | 12, 3, 4                     | 12, 3, 4                                | 2, 3, 4        | $I_{12}, I_3, I_4$                    |              |
|     |   |              |                                |   |   |  |  |  |                              |   |                |                                       |              |
| വസ  | L   |              |                                |   |   | (0.1                                   | ,                                      | `  |                              |   |                |                                       | 220L         |
| 220 | ນ   |              |                                |   | $0b = (14 + 2\varphi) = 2 \cdot 5a \cdot 11a$ | (3 isogeny                             |  |  |                              | 1                                       | ı              |                                       | <b>220</b> b |
| a1  | 1   | -1           | $\varphi$                      | $-1-4\varphi$                           | $3+3\varphi$                                  | 0                                      | 10                                     | 1  | 1, 2, 5                      | 1, 2, 5                                 | 1, 2, 5        | $I_1, I_2, I_5$                       |              |
| a2  | 1   | -1           | $\varphi$                      | $-11+6\varphi$                          | $23-3\varphi$                                 | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 10                                     | 1  | 2, 1, 10                     | 2, 1, 10                                | 2, 1, 10       | $I_2, I_1, I_{10}$                    |              |
| a3  | 1   | -1           | $\varphi$                      | $-206 - 144\varphi$                     | $-1288 - 1320\varphi$                         | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\begin{vmatrix} 2 \\ 2 \end{vmatrix}$ |  | 5, 10, 1                     | 5, 10, 1                                | 5, 2, 1        | $I_5, I_{10}, I_1$                    |              |
| a4  | $1+\varphi$                                   | 1            |                                | $-9400 + 5772\varphi$                   | $-414591 + 256067\varphi$                     | 0                                      | $\frac{1}{2}$                          | + -  | $\frac{10, 5, 2}{2}$         | 10, 5, 2                                | 10, 1, 2       | $  I_{10}, I_5, I_2  $                |              |
| b1  | $\varphi$                                     | $1-\varphi$  | 0                              | $-2\varphi$                             | 0   | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\frac{6}{c}$                          | -+   | 3, 2, 1                      | 3, 2, 1                                 | 3, 2, 1        | $I_3, I_2, I_1$                       |              |
| b2  | φ   | $1-\varphi$  | 0                              | $8\varphi$                              | $-6+4\varphi$                                 | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\frac{6}{2}$                          | +-   | 6, 1, 2                      | 6, 1, 2                                 | 6, 1, 2        | $I_6, I_1, I_2$                       |              |
| b3  | $\varphi$                                     | $1-\varphi$  | 0                              | $-70 - 67\varphi$                       | $-311 - 376\varphi$                           | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\begin{vmatrix} 2 \\ 2 \end{vmatrix}$ | -+   | 1, 6, 3                      | 1, 6, 3                                 | 1, 6, 1        | $I_1, I_6, I_3$                       |              |
| b4  | 1   | φ            | $1+\varphi$                    | $-2354 + 1439\varphi$                   | $-51173 + 31583\varphi$                       | 0                                      | 2                                      | +-   | 2, 3, 6                      | 2, 3, 6                                 | 2, 3, 2        | $I_2, I_3, I_6$                       |              |

|              | $a_1$         | $a_2$        | $a_3$         | $a_4$                         | $a_6$   | r                                      | T             | s        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$           | $c_p$    | Kodaira                   | Isogenies    |
|--------------|---------------|--------------|---------------|-------------------------------|---|--|---------------|----------|------------------------------|---------------------------------------|----------|---------------------------|--------------|
|              |               |              |               |                               |   |  | •             |          |                              |                                       |          |                           |              |
| 220l         | )             |              |               | 220                           | $b = (14 + 2\varphi) = 2 \cdot 5a \cdot 11a$ (3 is      | ogeny                                  | class         | es)      |                              |                                       |          |                           | 220b         |
| c1           | $1 + \varphi$ | $-1+\varphi$ | $\varphi$     | $-3-\varphi$                  | $3-2\varphi$  | 0                                      | 12            | -+       | 1, 4, 3                      | 1, 4, 3                               | 1, 4, 3  | $I_1, I_4, I_3$           |              |
| c2           | $1+\varphi$   | $-1+\varphi$ | $\varphi$     | $-43 + 19\varphi$             | $131 - 94\varphi$                                       | 0                                      | 12            | ++       |                              | 2, 2, 6                               | 2, 2, 6  | $I_2, I_2, I_6$           |              |
| c3           | $\varphi$     | $-\varphi$   | $1+\varphi$   | $-4427 + 2719\varphi$         | $131881 - 81534\varphi$                                 | 0                                      | 6             | ++       |                              | 1, 1, 3                               | 1, 1, 3  | $I_1, I_1, I_3$           |              |
| c4           | ,             | $-1+\varphi$ | $\varphi$     | $-33+39\varphi$               | $113-102\varphi$  | 0                                      | 6             | +-       | 4, 1, 12                     | 4, 1, 12                              | 2, 1, 12 | $I_4, I_1, I_{12}$        |              |
| c5           | ,             | -            | $\varphi$     | $2-36\varphi$                 | $-76-79\varphi$   | 0                                      | 4             | -+       | , ,                          | 3, 12, 1                              | 1, 12, 1 | $I_3, I_{12}, I_1$        |              |
| c6           | 1             | 1            | 0             | $-1703 - 2733\varphi$         | $-51718 - 83661\varphi$                                 | 0                                      | $\frac{4}{2}$ | ++       | , ,                          | 6, 6, 2                               | 2, 6, 2  | $I_6, I_6, I_2$           |              |
| c7<br>c8     | $\varphi$     | $1+\varphi$  | 0             | $-185685 - 300414\varphi$     | $-58776542 - 95102489\varphi$                           | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\frac{2}{2}$ | ++       | , ,                          | 3, 3, 1                               | 1, 3, 1  | $I_3, I_3, I_1$           |              |
| 68           | $1+\varphi$   | $-1+\varphi$ | φ             | $-1158 + 164\varphi$          | $-15012 + 2473\varphi$                                  | 0                                      | 2             | +-       | 12, 3, 4                     | 12, 3, 4                              | 2, 3, 4  | $I_{12}, I_3, I_4$        |              |
| 225a         | ì             |              |               |                               | $225a = (15) = 5a^2 \cdot 3$ (3 isogen                  | y clas                                 | ses)          |          |                              |                                       |          |                           | 225a         |
| a1           | 0             | $1-\varphi$  | 1             | $-1-2\varphi$                 | $1+2\varphi$  | 0                                      | 5             |          | 1, 2                         | 1                                     | 1,1      | $I_1, II$                 |              |
| a2           | 0             | $1-\varphi$  | 1             | $9+8\varphi$                  | -55-76arphi   | 0                                      | 1             |          | 5, 10                        | 5                                     | 1, 1     | $I_5, II^*$               |              |
| b1           | 0             | 1            | 1             | 2                             | 4   |  | 5             | <u> </u> | 5, 4                         | 5                                     | 5,1      | $I_5, IV$                 |              |
| b2           | 0             | -1           | 1             | -8                            | -7  | 0                                      | 1             |          | 1,8                          | 1                                     | 1, 1     | $I_1, IV^*$               |              |
| c1           | $1+\varphi$   | φ            | $1+\varphi$   | $-25-25\varphi$               | $-83-119\varphi$  | - 0                                    | 4             | <u> </u> | 2,10                         | [2, 4]                                | 2, 4     | $I_2, I_4^*$              | <u> </u>     |
| c2           | $1+\varphi$   | -            | $1+\varphi$   | Ó                             | $-3-4\varphi$   | 0                                      | 2             |          | 1,8                          | 1,2                                   | 1, 2     | $I_1, I_2^*$              |              |
| c3           | $1+\varphi$   | $\varphi$    | $1+\varphi$   | $-10800 - 10800\varphi$       | $571497 + 758396\varphi$                                | 0                                      | 4             | ++       | 4, 8                         | 4, 2                                  | 2, 4     | $I_4, I_2^{\overline{*}}$ |              |
| c4           | $1 + \varphi$ | $\varphi$    | $1 + \varphi$ | $-675-675\varphi$             | $8547 + 11171\varphi$                                   | 0                                      | 4             | ++       | 8, 10                        | 8,4                                   | 2, 4     | $I_8, I_4^*$              |              |
| c5           | $1+\varphi$   | $\varphi$    | $1+\varphi$   | $-50-50\varphi$               | $47 + 46\varphi$  | 0                                      | 4             | ++       | 4, 14                        | 4,8                                   | 2, 4     | $I_4, I_8^*$              |              |
| c6           | $1+\varphi$   | $1+\varphi$  | 1             | $-15378983 + 9504683\varphi$  | $27286831393 - 16864189127\varphi$                      | 0                                      | 2             | + -      | 2,7                          | 2,1                                   | 2, 2     | $I_2, I_1^*$              |              |
| c7           | $1+\varphi$   | $1-\varphi$  | 1             | $-40262649 - 65146315\varphi$ | $187026724347 + 302615596781\varphi$                    | 0                                      | 2             | -+       | 2,7                          | 2,1                                   | 2, 2     | $I_2, I_1^*$              |              |
| c8           | $1+\varphi$   | -            | •             | $-550 - 550\varphi$           | $12097 + 15946\varphi$                                  | 0                                      | 2             |          | 16, 8                        | 16, 2                                 | 2, 4     | $I_{16}, I_2^*$           |              |
| c9           | $1+\varphi$   |              | $1+\varphi$   | $-400 - 400\varphi$           | $-4433 - 6044\varphi$                                   | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\frac{2}{2}$ | ++       | 1,8                          | 1,2                                   | 1, 2     | $I_1, I_2^*$              |              |
| c10          | $1+\varphi$   | $\varphi$    | $1+\varphi$   | $175 + 175\varphi$            | $767 + 1081\varphi$                                     | 0                                      | 2             |          | 2,22                         | 2,16                                  | 2,4      | $I_2, I_{16}^*$           |              |
| 229a         | ı             |              |               |                               | $229a = (17 - 3\varphi) = 229a \qquad (1 \text{ isog})$ | geny o                                 | class)        |          |                              |                                       |          |                           | <b>229</b> a |
| a1           | $\varphi$     | $-1+\varphi$ | 1             | $-\varphi$                    | 0   | 1                                      | 1             |          | 1                            | 1                                     | 1        | $I_1$                     |              |
| 2291         | `             |              |               |                               | 2201 (14 + 2) 2201 (1:                                  |  | 1)            |          |                              |                                       |          |                           | 229b         |
|              |               |              |               |                               | $229b = (14 + 3\varphi) = 229b$ (1 isog                 | geny c                                 | <del> </del>  | 1        |                              |                                       |          |                           | <b>449</b> D |
| a1           | $1+\varphi$   | φ            | 0             | $2\varphi$                    | 1   | 1                                      | 1             |          | 1                            | 1                                     | 1        | $I_1$                     |              |
| <b>239</b> a | ì             |              |               |                               | $239a = (16 - \varphi) = 239a$ (3 isoge                 | ny cla                                 | asses)        |          |                              |                                       |          |                           | 239a         |
| a1           | $1+\varphi$   | -1           | φ             | $-1-2\varphi$                 | 0   | 1                                      | 1             | -+       | 1                            | 1                                     | 1        | $I_1$                     |              |
| b1           |               | $-1+\varphi$ | •             | -1                            | -arphi  | 0                                      | 4             | + -      | 1                            | 1                                     | 1        | $I_1$                     |              |
| b2           |               | $-1+\varphi$ |               | $-11+5\varphi$                | $-11+6\varphi$  | 0                                      | 4             | ++       | 2                            | 2                                     | 2        | $I_2$                     |              |
| b3           |               | $-1+\varphi$ |               | $-176 + 90\varphi$            | $-894 + 584\varphi$                                     | 0                                      | 2             | ++       | 1                            | 1                                     | 1        | $I_1$                     |              |
| b4           |               | $-1+\varphi$ |               | -6                            | $-32 + 16\varphi$                                       | 0                                      | 2             | <u> </u> | <u>.</u> 4                   | 4                                     | 2        | $I_4$                     |              |
| c1           | 1             | $1+\varphi$  | $1+\varphi$   | $-67 - 103\varphi$            | $-448 - 720\varphi$                                     | 0                                      | 1             | -+       | 1                            | 1                                     | 1        | $I_1$                     |              |
| ·            | ·             |              | ·             |                               |   |  |               |          |                              | · · · · · · · · · · · · · · · · · · · |          |                           |              |

|              |               |                             |               |  |   |  |  |                |                              |  |  | 1   |              |
|--------------|---------------|-----------------------------|---------------|--|---|--|--|----------------|------------------------------|--|--|---|--------------|
|              | $a_1$         | $a_2$                       | $a_3$         | $a_4$                                  | $a_6$   | r                                      | T                                      | s              | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$            | $c_p$  | Kodaira   | Isogenies    |
| 2391         | )             |                             |               |  | $239b = (15 + \varphi) = 239b \qquad (3 i)$   | sogeny classes)                        |  |                |                              |  |  |   | <b>23</b> 9b |
| a1           | $\varphi$     | $-\varphi$                  | $1+\varphi$   | -2                                     | $-\varphi$                                    | 1                                      | 1                                      | + -            | 1                            | 1                                      | 1  | $I_1$   |              |
| b1           | 1             | $-\varphi$                  | $\varphi$     | $-\varphi$                             | 0   | 0                                      | 4                                      | - +            | 1                            | 1                                      | 1  | $I_1$   |              |
| b2           | 1             | $-\varphi$                  | $\varphi$     | $-5-6\varphi$                          | $-4-7\varphi$                                 | 0                                      | 4                                      | ++             | 2                            | 2                                      | 2  | $I_2$   |              |
| b3           | 1             | $-\varphi$                  | $\varphi$     | $-85-91\varphi$                        | $-309 - 585\varphi$                           | 0                                      | 2                                      | ++             | 1                            | 1                                      | 1  | $I_1$   |              |
| b4           | 1             | $-\varphi$                  | $\varphi$     | $-5-\varphi$                           | $-15 - 17\varphi$                             | 0                                      | 2                                      | + -            | 4                            | 4                                      | 2  | $I_4$   |              |
| c1           | 1             | $-1-\varphi$                | $1+\varphi$   | $-171 + 104\varphi$                    | $-997 + 615\varphi$                           | 0                                      | 1                                      | + -            | 1                            | 1                                      | 1  | $I_1$   |              |
| <b>244</b> a | ì             |                             |               | 2-                                     | $44a = (8 - 14\varphi) = 2 \cdot 61a \tag{2}$ | 2 isogeny classes                      | s)                                     |                |                              |  |  |   | 244a         |
| a1           | 1             | $1+\varphi$                 | 1             | $3-16\varphi$                          | $-25-29\varphi$                               | 0                                      | 1                                      |                | 11,1                         | 11,1                                   | 1,1  | $I_{11},I_1$  |              |
| b1           | $1+\varphi$   | 0                           | 1             | 0                                      | 0   | 0                                      | 3                                      |                | 1,1                          | 1,1                                    | 1,1  | $\overline{\mathrm{I}}_{1},\overline{\mathrm{I}}_{1}$         | [ ]          |
|              | $1+\varphi$   |                             | 1             | $-5-5\varphi$                          | $-12-8\varphi$                                | 0                                      | 3                                      |                | 3, 3                         | 3, 3                                   | 3, 3   | $I_3, I_3$  |              |
| b3           | 1             | $1-\varphi$                 | $1 + \varphi$ | $-1994 - 3150\varphi$                  | $-63379 - 102293\varphi$                      | 0                                      | 1                                      |                | 1,1                          | 1,1                                    | 1,1  | $I_1, I_1$  |              |
| <b>244</b> l | )             |                             |               | 2                                      | $44b = (6 - 14\varphi) = 2 \cdot 61b \tag{2}$ | isogeny classes                        | 4)                                     |                |                              |  |  |   | 244b         |
| a1           | 1             | $-1-\varphi$                | 0             | $-14+18\varphi$                        | $-40 + 12\varphi$                             | 0                                      | 1                                      |                | 11,1                         | 11, 1                                  | 1,1  | $I_{11}, I_{1}$   |              |
| b1           | <br>φ         |                             | 1             | $1-\varphi$                            | 0   |  |  | <u> </u>       | 1, 1                         | 1,1                                    | 1,1  | $\overline{\mathrm{I}}_{1},\overline{\mathrm{I}}_{1}$         | <u>-</u>     |
| b2           | $\varphi$     | $1-\varphi$                 | 1             | $-9+4\varphi$                          | $-20+8\varphi$                                |  | 3                                      |                | 3, 3                         | 3,3                                    | 3,3  | $I_3, I_3$  |              |
| b3           | 1             | φ                           | $\varphi$     | $-5143 + 3149\varphi$                  | $-165671 + 102292\varphi$                     | 0                                      | 1                                      |                | 1,1                          | 1,1                                    | 1,1  | $I_1, I_1$  |              |
| <b>245</b> a | ì             |                             |               |  | $245a = (7 - 14\varphi) = 5a \cdot 7 \tag{1}$ | l isogeny class)                       |  |                |                              |  |  |   | 245a         |
| a1           | 0             | 1                           | 1             | -1                                     | 0   | 1                                      | 3                                      |                | 2,1                          | 2,1                                    | 2, 1   | $I_2, I_1$  |              |
| a2           | 0             | 1                           | 1             | 9                                      | 1   | 1                                      | 3                                      |                | 6, 3                         | 6, 3                                   | 2,3  | $I_6, I_3$  |              |
| a3           | 0             | 1                           | 1             | -131                                   | -650  | 1                                      | 1                                      |                | 18, 1                        | 18,1                                   | 2,1  | $I_{18}, I_1$   |              |
| 251a         | ì             |                             |               |  | $251a = (15 + 2\varphi) = 251a \tag{1}$       | isogeny class)                         |  |                |                              |  |  |   | 251a         |
| a1           | $1 + \varphi$ | 1                           | $\varphi$     | 0                                      | 0   | 1                                      | 1                                      |                | 1                            | 1                                      | 1  | $I_1$   |              |
| <b>251</b> l | )             |                             |               |  | $251b = (17 - 2\varphi) = 251b \tag{1}$       | isogeny class)                         |  |                |                              |  |  |   | 251b         |
| a1           |               | $-1-\varphi$                | 1             | 0                                      | 0   | 1                                      | 1                                      |                | 1                            | 1                                      | 1  | $I_1$   |              |
| 070          |               |                             |               |  |   |  | L                                      | 1              |                              | 1                                      | I  | 1   | 070          |
| 256a         |               |                             |               |  | . , ,   | eny classes)                           |  | ı              |                              |  |  |   | 256a         |
| a1           | 0             | ,                           | 0             | $-5-6\varphi$                          | $7+11\varphi$                                 | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 8                                      | ++             | 8                            | 0                                      | 4  | $I_0^*$   |              |
| a2           | 0             | $1-\varphi$                 | 0             | $-\varphi$                             | $0$ $403 + 647\varphi$                        | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 4                                      | ++             | 4<br>10                      | 0                                      | $\begin{array}{ c c }\hline 1\\ 2 \end{array}$ | II<br>*   |              |
| a3<br>a4     | $0 \\ 0$      | $1-\varphi$                 | $0 \\ 0$      | $-65 - 106\varphi$ $-171 + 106\varphi$ | $403 + 647\varphi$ $1050 - 647\varphi$        | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\begin{vmatrix} 4 \\ 4 \end{vmatrix}$ | - +<br>  + -   | 10                           | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\frac{2}{2}$                                  | 1 <sub>2</sub><br>I*  |              |
| a4<br>a5     | 0             | $\frac{\varphi}{1-\varphi}$ | 0             | $-171 + 100\varphi$ $-5 - 11\varphi$   | $-10 - 047\varphi$ $-10 - 17\varphi$          |  | $\frac{4}{2}$                          | <del>+</del> - | 8                            | 0                                      | 1  | I*  |              |
| a6           | 0             | $\varphi$                   | 0             | $-16 + 11\varphi$                      | $-27 + 17\varphi$                             |  | $\frac{2}{2}$                          | + -            | 8                            | 0                                      | 1  | $egin{array}{c} I_2^* \\ I_2^* \\ I_0^* \\ I_0^* \end{array}$ |              |
|              |               | 7                           |               | 10   119                               | Ξ. ; Ξ.γ                                      |  |  | <u>'</u>       |                              |  |  | -0  |              |

|              | $a_1$         | $a_2$         | $a_3$       | $a_4$               | a                                       | r                   | T | s   | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$                             | Kodaira  | Isogenies    |
|--------------|---------------|---------------|-------------|---------------------|---|---------------------|---|-----|------------------------------|-----------------------------|-----------------------------------|--|--------------|
|              |               |               |             |                     |   |                     |   |     |                              |                             |                                   |  |              |
| <b>256</b> a | ı             |               |             |                     | $256a = (16) = 2^4 		(3 is$             | sogeny classes)     |   |     |                              |                             |                                   |  | <b>256</b> a |
| b1           | 0             | $-1-\varphi$  | 0           | $\varphi$           | (                                       | 0                   | 4 | ++  | 4                            | 0                           | 1                                 | II   |              |
| b2           |               | $-1-\varphi$  | 0           | $-4\varphi$         | $4+8\varphi$                            |                     | 4 | - + |                              | 0                           | 2                                 | $\mathrm{I}_0^*$   |              |
| b3           |               | $-1-\varphi$  | 0           | $-5+\varphi$        | $4-3\varphi$                            | l l                 | 4 | ++  |                              | 0                           | 2                                 | $\overset{\overset{\ast}{I_0^*}}{I_0^*}$   |              |
| b4           | 0             | $-1-\varphi$  | 0           | $-5+6\varphi$       | $-7+3\varphi$                           | l l                 | 2 | +-  |                              | 0                           | 1                                 | $\mathrm{I}_0^*$   |              |
| b5           | 0             | -1            | 0           | $-448 + 277\varphi$ | $4444 - 2747\varphi$                    | l l                 | 4 | + - |                              | 0                           | 4                                 | $\mathrm{I}_2^*$   |              |
| b6           | 0             | 1             | 0           | $-171 - 277\varphi$ | $-1697 - 2747\varphi$                   | 0                   | 2 | - + | 10                           | 0                           | $\begin{vmatrix} 2 \end{vmatrix}$ | $egin{array}{c} oxed{\mathrm{I}_2^*} \ oxed{\mathrm{I}_2^*} \ & \dots \end{array}$ |              |
| c1           | 0             | $1 + \varphi$ | 0           | arphi               | (                                       | 0                   | 4 | ++  | 4                            | 0                           | 1                                 | II   |              |
| c2           | 0             | $1 + \varphi$ | 0           | $-5+6\varphi$       | $7-3\varphi$                            | l l                 | 4 | + - |                              | 0                           | 2                                 | $I_0^*$  |              |
| c3           | 0             | $1 + \varphi$ | 0           | $-5+\varphi$        | $-4+3\varphi$                           | l l                 | 4 | ++  |                              | 0                           | 2                                 | $\mathrm{I}_0^*$   |              |
| c4           | 0             | -1            | 0           | $-171-277\varphi$   | $1697 + 2747\varphi$                    | l l                 | 4 | - + |                              | 0                           | 4                                 | $egin{array}{c} 	ilde{I}_2^* \ 	ilde{I}_0^* \end{array}$                           |              |
| c5           | 0             | $1+\varphi$   | 0           | $-4\varphi$         | $-4-8\varphi$                           |                     | 2 | -+  |                              | 0                           | 1                                 | $\mathrm{I}_0^*$   |              |
| c6           | 0             | 1             | 0           | $-448 + 277\varphi$ | $-4444 + 2747\varphi$                   | 0                   | 2 | +-  | 10                           | 0                           | 2                                 | $\mathrm{I}_2^*$   |              |
|              |               |               |             |                     |   |                     |   |     |                              |                             |                                   |  |              |
| <b>269</b> a | ì             |               |             | 2                   | $69a = (4 - 15\varphi) = 269a$          | (2 isogeny classes) |   |     |                              |                             |                                   |  | <b>269</b> a |
| al           |               | $-1+\varphi$  |             |                     | • | , , ,               | 1 |     | 1                            | 1                           | 1                                 | Т.   |              |
|              |               |               |             |                     | $1-\varphi$                             |                     | 1 | + - |                              | 1                           | 1                                 | $I_1$  |              |
| b1           | 1             | $\varphi$     | 0           | $-12+8\varphi$      | $21-13\varphi$                          |                     | 3 | + - |                              | 1                           | 1                                 | $I_1$  |              |
| b2           | $\varphi$     | $1-\varphi$   | $1+\varphi$ | $-57 - 86\varphi$   | $-292 - 465\varphi$                     | 0                   | 1 | +-  | 3                            | 3                           | 1                                 | $I_3$  |              |
|              |               |               |             |                     |   |                     |   |     |                              |                             |                                   |  |              |
| <b>269</b> b | )             |               |             | 20                  | $69b = (11 - 15\varphi) = 269b$         | (2 isogeny classes) |   |     |                              |                             |                                   |  | 269b         |
| a1           | 1             | $-\varphi$    | $1+\varphi$ | $-1-\varphi$        | (                                       | ) 1                 | 1 | -+  | 1                            | 1                           | 1                                 | $I_1$  |              |
| b1           | 1             | $1-\varphi$   |             | $-4-8\varphi$       | $8+13\varphi$                           | 0                   | 3 | - + | 1                            | 1                           | 1                                 | $I_1$  | [            |
| b2           | $1 + \varphi$ | 0             | $\varphi$   | $-142 + 84\varphi$  | $-756 + 464\varphi$                     | 0                   | 1 | -+  | 3                            | 3                           | 1                                 | $I_3$  |              |
|              |               |               |             |                     |   | <u>'</u>            |   |     |                              |                             |                                   |  |              |
| <b>271</b> a | ı             |               |             |                     | $271a = (17 - \varphi) = 271a$          | (1 isogeny class)   |   |     |                              |                             |                                   |  | 271a         |
| a1           | 1             | $\varphi$     | $\varphi$   | 0                   | (                                       | 0                   | 4 | +-  | 1                            | 1                           | 1                                 | $I_1$  |              |
| a2           | 1             | arphi         | arphi       | $-5-5\varphi$       | $2+5\varphi$                            | 0                   | 4 | ++  | 2                            | 2                           | 2                                 | $ m I_2$   |              |
| a3           | 1             | $\varphi$     | $\varphi$   | $-85-80\varphi$     | $241 + 517\varphi$                      |                     | 2 | ++  | 1                            | 1                           | 1                                 | $I_1$  |              |
| a4           | 1             | $\varphi$     | $\varphi$   | $-5-10\varphi$      | $-9-7\varphi$                           | 0                   | 2 | -+  | 4                            | 4                           | 2                                 | ${ m I}_4$   |              |
|              |               |               |             |                     |   |                     |   |     |                              |                             |                                   |  |              |
| <b>271</b> k | )             |               |             |                     | $271b = (16 + \varphi) = 271b$          | (1 isogeny class)   |   |     |                              |                             |                                   |  | 271b         |
| a1           | 1             | $1-\varphi$   | $1+\varphi$ | $-\varphi$          | $-\varphi$                              | 0                   | 4 | -+  | 1                            | 1                           | 1                                 | $I_1$  |              |
| a2           | 1             | $1-\varphi$   |             | $-10+4\varphi$      | $7-6\varphi$                            |                     | 4 | ++  |                              | 2                           | 2                                 | $ m I_2$   |              |
| a3           | 1             | $1-\varphi$   |             | $-165 + 79\varphi$  | 758 - 5184                              |                     | 2 | ++  |                              | 1                           | 1                                 | $\overline{\mathrm{I}_{1}}$  |              |
| a4           | 1             | $1-\varphi$   |             | $-15+9\varphi$      | $-16+6\varphi$                          |                     | 2 | +-  |                              | 4                           | 2                                 | $ m I_4$   |              |
|              |               | •             | •           | ,                   | •                                       | ı                   |   |     |                              | 1                           | 1                                 | 1  |              |

|  |                                |                             |     |                         |                                   |  |         |             | Imi            | l <u>.</u>     | J(A)                         | 1 (:)                                   |  | I/ - J - :                    | T            |
|--|--------------------------------|-----------------------------|-----|-------------------------|-----------------------------------|--|---------|-------------|----------------|----------------|------------------------------|---|--|-------------------------------|--------------|
|  | $a_1$                          | $a_2$                       |     | $a_3$                   | $a_4$                             | $a_6$  |         | r           | T              | s              | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$             | $c_p$  | Kodaira                       | Isogenies    |
| 275  | a                              |                             |     |                         | 27                                | $5a = (5 - 15\varphi) = 5a^2 \cdot 11a$        | (1 iso  | geny class  | s)             |                |                              |   |  |                               | <b>275</b> a |
| a1   | φ                              | 1                           |     | φ                       | $-1-\varphi$                      | -2   |         | 0           | 4              | T - +          | 1,7                          | 1,1                                     | 1,4  | $I_1, I_1^*$                  |              |
| a2   | $1+\varphi$                    | $1+\varphi$                 |     | $\varphi$               | $-203 - 330\varphi$               | $1853 + 2994\varphi$                           |         | 0           | 4              | -+             | 3,9                          | 3,3                                     | 1,4  | $I_3, I_3^*$                  |              |
| a3   | arphi                          | 1                           |     | $\varphi$               | $-151 + 24\varphi$                | $623-175\varphi$                               |         | 0           | 4              | ++             | 6, 12                        | 6, 6                                    | 2, 4   | $I_6, I_6^*$                  |              |
| a4   | $\varphi$                      | 1                           |     | $\varphi$               | $-26-\varphi$                     | $-62-5\varphi$                                 |         | 0           | 4              | ++             | 2,8                          | 2, 2                                    | 2,4  | $\mathrm{I}_2,\mathrm{I}_2^*$ |              |
| a5   | $\varphi$                      | 1                           |     | $\varphi$               | $-2026 + 974\varphi$              | $35863 - 24330\varphi$                         |         | 0           | 2              | ++             | 3, 18                        | 3, 12                                   | 1, 4   | $I_3, I_{12}^*$               |              |
| a6   | $1+\varphi$                    | $1+\varphi$                 |     | $\varphi$               | $-828 - 1330\varphi$              | $-18532 - 29986\varphi$                        |         | 0           | 2              | ++             | ,                            | 1,4                                     | 1,4  | $\mathrm{I}_1,\mathrm{I}_4^*$ |              |
| a7   | $\varphi$                      | 1                           |     | $\varphi$               | $-276 + 274\varphi$               | $-1377 + 2200\varphi$                          |         | 0           | 2              | + -            | 12, 9                        | 12, 3                                   | 2,4  | $I_{12}, I_3^*$               |              |
| a8   | 1                              | $-1-\varphi$                | 1+  | $\varphi$               | $-2028 + 1252\varphi$             | $-41213 + 25468\varphi$                        |         | 0           | 2              | + -            | 4,7                          | 4,1                                     | 2,4  | $I_4, I_1^*$                  |              |
| 275  | b                              |                             |     |                         | 275                               | $5b = (10 - 15\varphi) = 5a^2 \cdot 11b$       | (1 iso  | ogeny clas  | s)             |                |                              |   |  |                               | <b>275</b> b |
| a1   | $\varphi$                      | $\varphi$                   |     | $\varphi$               | $-536 + 332\varphi$               | $\phantom{00000000000000000000000000000000000$ | `       | 0           | 4              | + -            | 3,9                          | 3, 3                                    | 1,4  | $I_{3}, I_{3}^{*}$            |              |
| a2   | $1+\varphi$                    | $1-\varphi$                 |     |                         | $-2-\varphi$                      | $-2-\varphi$                                   |         |             | 4              | + -            |                              | 1,1                                     | 1,4  | $I_1, I_1^*$                  |              |
| a3   | $1+\varphi$                    | $1-\varphi$                 |     | •                       | $-127-26\varphi$                  | $448 + 174\varphi$                             |         | 0           | 4              | ++             | ,                            | 6,6                                     | 2, 4   | $I_6, I_6^*$                  |              |
| a4   | $1+\varphi$                    | $1-\varphi$                 | 1+  | $\varphi$               | $-27-\varphi$                     | $-67+4\varphi$                                 |         | 0           | 4              | ++             | ,                            | 2,2                                     | 2, 4   | $I_2, I_2^*$                  |              |
| a5   | arphi                          | arphi                       |     | $\varphi$               | $-2161 + 1332\varphi$             | $-45027 + 27826\varphi$                        |         | 0           | 2              | ++             |                              | 1,4                                     | 1, 4   | $I_1, I_4^*$                  |              |
| a6   | $1+\varphi$                    | $1-\varphi$                 | 1 + | $\varphi$               | $-1052 - 976\varphi$              | $11533 + 24329\varphi$                         |         | 0           | 2              | ++             | 3, 18                        | 3, 12                                   | 1,4  | $I_3, I_{12}^*$               |              |
| a7   | $1+\varphi$                    | $1-\varphi$                 | 1 + | $\varphi$               | $-2-276\varphi$                   | $823-2201\varphi$                              |         | 0           | 2              | -+             | 12, 9                        | 12, 3                                   | 2, 4   | $I_{12}, I_3^*$               |              |
| a8   | 1                              | $1+\varphi$                 | 1+  | $\varphi$               | $-776 - 1251\varphi$              | $-16520 - 26721\varphi$                        |         | 0           | 2              | -+             | 4,7                          | 4,1                                     | 2,4  | $I_4, I_1^*$                  |              |
| 279  | a                              |                             |     |                         | 27                                | $9a = (9 - 15\varphi) = 3 \cdot 31a$           | 3 isoge | eny classes | ;)             |                |                              |   |  |                               | 279a         |
| a1   | $1 + \omega$                   | $-1+\varphi$                |     | 1                       | $-2+\varphi$                      | -1   |         | 0           | 2              | <u> </u>       | 1, 2                         | 1, 2                                    | 1, 2   | $I_1, I_2$                    |              |
|  |                                | $-1+\varphi$                |     | 1                       | $-42+16\varphi$                   | $-97 + 36\varphi$                              |         | 0           | $\overline{2}$ | ++             |                              | 2, 1                                    | 2, 1   | $I_2, I_1$                    |              |
| b1   |                                | $-1-\varphi$                |     |                         | $-1-2\varphi$                     | —1   |         | 0           | 4              | <u> </u>       |                              | $\frac{1}{2}, \frac{1}{1}$              | $\frac{1}{2}, \frac{1}{1}$                   | $I_2, I_1$                    |              |
| b2   |                                | $-1-\varphi$                |     |                         | $-11+3\varphi$                    | $-12+6\varphi$                                 |         |             | 4              | + +            | ,                            | $\frac{2}{4}, \frac{1}{2}$              | 2, 1 $2, 2$                                  | $I_4, I_2$                    |              |
| b3   |                                | $-1-\varphi$                |     |                         | $-87-133\varphi$                  | $-493 - 802\varphi$                            |         |             | 2              | + +            | ,                            | 8, 1                                    | 2, 1   | $I_8, I_1$                    |              |
| b4   |                                | $-1-\varphi$                |     |                         | $-146 + 93\varphi$                | $-795 + 483\varphi$                            |         | 0           | 2              | + -            | *                            | 2, 4                                    | 2,2  | $I_2, I_4$                    |              |
| c1   |                                | $1+\varphi$                 |     | ,<br>0                  | $-99+62\varphi$                   | $-464 + 283\varphi$                            |         |             | 2              | <u>-</u>       |                              | $\frac{1}{6}, \frac{1}{1}$              | $\frac{1}{2}, \frac{1}{1}$                   | $I_6, I_1$                    |              |
| c2   | $1 + \varphi$<br>$1 + \varphi$ | $1+\varphi$ $1+\varphi$     |     | 0                       | $-4+7\varphi$                     | $-7 + 7\varphi$                                |         | 0           | 2              |                |                              | 3, 2                                    | 1, 2   | $I_3, I_2$                    |              |
| 279  | h                              |                             |     |                         | 27                                | $9b = (6 - 15\varphi) = 3 \cdot 31b$ (         | 3 isoge | eny classes | )              |                |                              |   |  |                               | <b>279</b> b |
| a1   |                                | 1 ± /0                      |     | (0                      |                                   | $\frac{-2-2\varphi}{}$                         | 2 10080 | 0           | 2              | T              | 1, 2                         | 1,2                                     | 1, 2   | $I_1, I_2$                    |              |
| a1<br>a2   | $\varphi$                      | $1 + \varphi$ $1 + \varphi$ |     | arphi                   | $-26-15\varphi$                   | $-2 - 2\varphi$ $-77 - 78\varphi$              |         |             | 2              | ++             |                              | 2, 1                                    | $\begin{bmatrix} 1,2\\2,1 \end{bmatrix}$     | $I_1, I_2 \\ I_2, I_1$        |              |
| b1   | $\varphi$                      | -1                          |     | $\varphi^{r} = \varphi$ | -1                                | $-\varphi$                                     |         | 0           | 4              | <u> </u>       |                              | $\frac{1}{2}, \frac{1}{2}, \frac{1}{1}$ | $\begin{bmatrix} 2, 1 \\ 2, 1 \end{bmatrix}$ | $ \tilde{I}_2, \tilde{I}_1 $  |              |
| b2   | $\varphi$                      | -1                          |     | arphi                   | $-6-5\varphi$                     | $-5-7\varphi$                                  |         |             | 4              |                |                              | $\frac{2}{4}, \frac{1}{2}$              | $2, 1 \\ 2, 2$                               | $I_{2}, I_{1}$ $I_{4}, I_{2}$ |              |
| b3   | $\frac{\varphi}{1}$            | $1+\varphi$                 | 1+  | •                       | $-220 + 134\varphi$               | $-1514 + 934\varphi$                           |         |             | 2              |                |                              | 8, 1                                    | 2, 2 $2, 1$                                  | $I_{8}, I_{1}$                |              |
| b4   | $\varphi$                      | -1                          |     | $\varphi$               | $-51-95\varphi$                   | $-311 - 484\varphi$                            |         |             | 2              | - +            | *                            | 2, 4                                    | 2, 1 $2, 2$                                  | $I_{2}, I_{4}$                |              |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$ |                                |                             |     | <u> </u>                | $-41 - 59\varphi$                 | $-201 - 323\varphi$                            |         | 0           | 2              | <u>'</u>       |                              | $\frac{1}{6}, \frac{2}{1}$              | $\begin{bmatrix} 2, 2 \\ 2, 1 \end{bmatrix}$ | $ I_6, I_1 $                  |              |
| $\begin{vmatrix} c_1 \\ c_2 \end{vmatrix}$             | arphi $arphi$                  | $\varphi$                   |     | 1                       | $-41 - 39\varphi$ $-1 - 4\varphi$ | $-201 - 323\varphi$ $-5 - 7\varphi$            |         |             | 2              | <del>+ +</del> |                              | 3, 2                                    | 1, 2   | $I_6, I_1 \\ I_3, I_2$        |              |
| 02   | Ψ                              | $\varphi$                   |     | 1                       | $-1-4\varphi$                     | $-3-1\varphi$                                  |         | 0           |                |                | 5, 4                         | 5, 4                                    | 1, 4   | 13, 12                        |              |

|          | $a_1$            |   | $a_2$   | $a_3$       | $a_4$  | $a_6$                                     | r                                      | T             | s       | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$            | $c_p$  | Kodaira  | Isogenies |
|----------|------------------|---|---|-------------|--|---|--|---------------|---------|------------------------------|--|--|--|-----------|
| 284      | a                |   |   |             |  | $284a = (18 - 2\varphi) = 2 \cdot 71a$    | (1 isogeny class)                      |               |         |                              |  |  |  | 284a      |
| a1       | $\varphi$        |   | φ   | 0           | $-7-2\varphi$  | $-10-9\varphi$                            | 0                                      | 2             | + -     | 2, 1                         | 2, 1                                   | 2,1  | $I_2, I_1$   |           |
| a2       | $1+\varphi$      | - | $1-\varphi$                                   | 1           | $-363 - 587\varphi$  | $-5188 - 8393\varphi$                     | 0                                      | 2             | ++      | 1, 2                         | 1,2                                    | 1,2  | $I_1, I_2$   |           |
| 284      | b                |   |   |             |  | $284b = (16 + 2\varphi) = 2 \cdot 71b$    | (1 isogeny class)                      |               |         |                              |  |  |  | 284b      |
| a1       | $1 + \varphi$    |   | $1+\varphi$                                   | 1           | $-9+5\varphi$  | $-16 + 3\varphi$                          | 0                                      | 2             | -+      | 2, 1                         | 2, 1                                   | 2,1  | $I_2, I_1$   |           |
| a2       | $\varphi$        |   | 1   | 1           | $-949 + 586\varphi$  | $-13581 + 8393\varphi$                    | 0                                      | 2             | ++      | 1, 2                         | 1, 2                                   | 1,2  | $I_1, I_2$   |           |
| 289      | a                |   |   |             |  | $289a = (17) = 17 \qquad (13)$            | isogeny class)                         |               |         |                              |  |  |  | 289a      |
| a1       | 1                |   | -1  | 1           | -1   | 0   | 1                                      | 4             | ++      | 1                            | 1                                      | 1  | $I_1$  |           |
| a2       | 1                |   | -1  | 1           | -6   | -4  | 1                                      | 4             | ++      | 2                            | 2                                      | 2  | $I_2$  |           |
| a3<br>a4 | 1<br>1           |   | $-1 \\ -1$                                    | 1<br>1      | $     \begin{array}{r}       -91 \\       -1   \end{array} $     | $-310 \\ -14$                             | 1                                      | 2<br>4        | ++      | $\frac{1}{4}$                | $\begin{vmatrix} 1 \\ 4 \end{vmatrix}$ | $\begin{vmatrix} 1 \\ 4 \end{vmatrix}$       | $egin{array}{c} I_1 \ I_4 \end{array}$   |           |
| ач       | 1                |   | 1   | 1           | 1  | 14  | 1                                      | -1            |         |                              | 4                                      | 4  | 14   |           |
| 304      |                  |   |   |             |  | $304a = (4 - 16\varphi) = 2^2 \cdot 19a$  | (1 isogeny class                       |               |         |                              | 1                                      | ı  |  | 304a      |
| a1       | $0 \\ 0$         |   | $-1 \\ -1$                                    | 0           | $-2+\varphi$   | -1  | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\frac{2}{2}$ | + -     | $1, 4 \\ 2, 8$               | $\frac{1}{2}$                          | $\begin{array}{ c c } 1,1\\2,1\end{array}$   | $I_1, IV$  |           |
| a2       | 0                |   | -1  | 0           | $-7-9\varphi$  | $-11-13\varphi$                           | 0                                      |               | -+      | 2,8                          | 2                                      | 2, 1   | $I_2, IV^*$  |           |
| 304      | b                |   |   |             |  | $304b = (12 - 16\varphi) = 2^2 \cdot 19b$ | (1 isogeny class                       |               |         |                              |  |  |  | 304b      |
| a1       | 0                |   | -1  | 0           | $-1-\varphi$   | -1  | 0                                      | 2             | -+      | 1,4                          | 1                                      | 1,1  | $I_1, IV$  |           |
| a2       | 0                |   | -1  | 0           | $-16 + 9\varphi$   | $-24 + 13\varphi$                         | 0                                      | 2             | + -     | 2,8                          | 2                                      | 2, 1   | $I_2, IV^*$  |           |
| 305      | a                |   |   |             |  | $305a = (18 - \varphi) = 5a \cdot 61a$    | (1 isogeny class)                      |               |         |                              |  |  |  | 305a      |
| a1       |                  |   | $1-\varphi$                                   |             | $-2-\varphi$   | -arphi                                    | 0                                      | 4             | ++      | 1, 1                         | 1, 1                                   | 1,1  | $I_1, I_1$   |           |
| a2       |                  |   | $egin{array}{l} -arphi \ 1-arphi \end{array}$ |             | $-7-\varphi$   | -5  | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 4             | ++      | 2, 2                         | 2, 2                                   | $\begin{bmatrix} 2,2\\ 4,4 \end{bmatrix}$    | $I_2, I_2$   |           |
| a3<br>a4 |                  |   | $1-\varphi$<br>$1-\varphi$                    |             | $\begin{array}{c} 3 - 6\varphi \\ -176 - 261\varphi \end{array}$ | $-22 + 8\varphi$ $-1444 - 2303\varphi$    | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\frac{4}{2}$ | <br>+ + | $4, 4 \\ 1, 1$               | 4, 4<br>1, 1                           | $\begin{bmatrix} 4, 4 \\ 1, 1 \end{bmatrix}$ | $egin{array}{c} \mathrm{I}_4,\mathrm{I}_4 \ \mathrm{I}_1,\mathrm{I}_1 \end{array}$ |           |
|          |                  |   | - 7   | - 1 7       | 110 2019   | 1111 20007                                | 0                                      |               | - 1     | -,-                          | -,-                                    | -,-  | -1,-1  |           |
| 305      | b                |   |   |             |  | $305b = (17 + \varphi) = 5a \cdot 61b$    | (1 isogeny class)                      |               |         |                              |  |  |  | 305b      |
| a1       | $\varphi$        |   | -1  | $\varphi$   | $-1-\varphi$   | 0   | 0                                      | 4             | ++      | 1, 1                         | 1, 1                                   | 1,1  | $I_1, I_1$   |           |
| a2       | $\varphi$        |   | -1  | $\varphi$   | $-6-\varphi$   | $-4-\varphi$                              | 0                                      | 4             | ++      | 2, 2                         | 2, 2                                   | 2, 2   | $I_2, I_2$   |           |
| a3       | $\varphi$ 1      |   | -1<br>1   (2                                  | $\varphi$   | $-1 + 4\varphi$ $-437 + 262\varphi$                              | $-13 - 9\varphi$ $-4183 + 2563\varphi$    | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\frac{4}{2}$ |         | $4, 4 \\ 1, 1$               | 4, 4 $1, 1$                            | $\begin{bmatrix} 4, 4 \\ 1, 1 \end{bmatrix}$ | $I_4, I_4$   |           |
| a4       | 1                | - | $1+\varphi$                                   | $1+\varphi$ | $-437 + 202\varphi$  | $-4163 + 2503\varphi$                     | 0                                      |               | ++      | 1, 1                         | 1, 1                                   | 1,1  | $I_1, I_1$   |           |
| 311      | a                |   |   |             |  | $311a = (11 - 16\varphi) = 311a$          | (1 isogeny class)                      |               |         |                              |  | 1  | ı  | 311a      |
| a1       | 0                |   | -1  | $1+\varphi$ | 0  | $-\varphi$                                | 1                                      | 1             | -+      | 1                            | 1                                      | 1  | $I_1$  |           |
| 311      | $\mathbf{b}^{-}$ |   |   |             |  | $311b = (5 - 16\varphi) = 311b$           | (1 isogeny class)                      |               |         |                              |  |  |  | 311b      |
| a1       | 0                |   | -1  | $\varphi$   | 0  | 0   |  | 1             | + -     | 1                            | 1                                      | 1  | $I_1$  |           |
|          |                  |   |   | ,           |  |   |  |               | •       |                              |  | 1  | _  |           |

|   | $a_1$   | $a_2$   | $a_3$  | $a_4$   | $a_6$   | r                                      | T   | s                                       | $\operatorname{ord}(\Delta)$  | $\operatorname{ord}_{-}(j)$                               | $c_p$  | Kodaira   | Isogenies    |
|---|---|---|--|---|---|--|---|---|---|---|--|---|--------------|
| 210   |   |   |  |   |   | ·                                      |   |   |   |   |  |   | 210          |
| 316   |   |   |  |   |   | sogeny class)                          | )   |   |   |   |  |   | 316a         |
| a1  | $1 + \varphi$   | -1  | $\varphi$  | $-2-4\varphi$   | $1+2\varphi$  | 0                                      | 3   |   | 1, 1  | 1,1   | 1,1  | $I_1, I_1$  |              |
| a2  | $\varphi$   | $1+\varphi$   | 0  | $-69 + 46\varphi$   | $-278 + 175\varphi$   | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 3   |   | 3, 3  | 3, 3  | 3, 3   | $I_3, I_3$  |              |
| a3  | $1+\varphi$   | $-1+\varphi$  | φ  | $-268494 + 165938\varphi$   | $-62809056 + 38818131\varphi$   | 0                                      | 1   |   | 1,1   | 1,1   | 1,1  | $I_1, I_1$  |              |
| 316   | b   |   |  | 31  | $6b = (10 - 16\varphi) = 2 \cdot 79b \tag{1 i}$   | sogeny class                           | )   |   |   |   |  |   | 316b         |
| a1  | $\varphi$   | $-\varphi$  | $1+\varphi$  | $-5+2\varphi$   | $3-3\varphi$  | 0                                      | 3   |   | 1, 1  | 1,1   | 1, 1   | $I_1, I_1$  |              |
| a2  | $1+\varphi$   | $-1+\varphi$  | $\varphi$  | $-25-45\varphi$   | $-123-200\varphi$   | 0                                      | 3   |   | 3, 3  | 3,3   | 3, 3   | $I_3, I_3$  |              |
| a3  | $\varphi$   | $1+\varphi$   | 0  | $-102555 - 165937\varphi$   | $-24156863 - 39086625\varphi$   | 0                                      | 1   |   | 1, 1  | 1,1   | 1,1  | $I_1, I_1$  |              |
| 319   | a   |   |  | 31:   | $\theta a = (7 - 16\varphi) = 11b \cdot 29b \tag{1}$  | isogeny class                          | s)  |   |   |   |  |   | 319a         |
| a1  | 1   | $-\varphi$  | $1+\varphi$  | -1  | -arphi  | 1                                      | 2   | -+                                      | 1, 1  | 1,1   | 1, 1   | $I_1, I_1$  |              |
| a2  | 1   | $-\varphi$  | $1+\varphi$  | $-16 + 10\varphi$   | $24-16\varphi$  | 1                                      | 2   | + -                                     | 2, 2  | 2,2   | 2, 2   | $I_2, I_2$  |              |
| 319   | b   |   |  | 319   | $\theta b = (9 - 16\varphi) = 11a \cdot 29a \tag{1}$  | isogeny class                          | s)  |   |   |   |  |   | 319b         |
| a1  | 1   | $-1+\varphi$  | $\varphi$  | $-\varphi$  | 0   | 1                                      | 2   | + -                                     | 1, 1  | 1,1   | 1,1  | $I_1, I_1$  |              |
| a2  | 1   | $-1+\varphi$  | $\varphi$  | $-5-11\varphi$  | $9+15\varphi$   | 1                                      | 2   | -+                                      | 2, 2  | 2,2   | 2,2  | $I_2, I_2$  |              |
|   |   |   |  |   |   |  |   |   |   |   |  |   |              |
| 319   | $\mathbf{c}$  |   |  | 31:   | $\theta c = (17 + 2\varphi) = 11b \cdot 29a \tag{1}$  | isogeny class                          | s)  |   |   |   |  |   | 319c         |
| a1  | $\mathbf{c}$ $\varphi$  | $1-\varphi$   | 1  | 319   | 0   | isogeny class                          | s) 4  | -+                                      | 1, 1  | 1, 1  | 1, 1   | $I_1, I_1$  | 319c         |
| a1<br>a2  | $\varphi$ $\varphi$   | $\begin{array}{c} 1 - \varphi \\ 1 - \varphi \end{array}$   | 1<br>1   | $-\varphi \\ -10 + 4\varphi$  | $\begin{matrix} 0 \\ 8-6\varphi \end{matrix}$   | 1 1                                    | 4 4   | ++                                      | 2, 2  | 2, 2  | 2, 2   | $I_2, I_2$  | 319c         |
| a1<br>a2<br>a3  | $\varphi$ $\varphi$ 1   | $1-\varphi$ $\varphi$   | $\frac{1}{\varphi}$  | $-\varphi$ $-10 + 4\varphi$ $-1093 + 672\varphi$  | $\begin{matrix} 0\\ 8-6\varphi\\ 16452-10174\varphi \end{matrix}$   | 1<br>1<br>1                            | 4 4 2   | ++++                                    | $2, 2 \\ 1, 1$  | $2, 2 \\ 1, 1$  | $2, 2 \\ 1, 1$   | $\begin{matrix} I_2,I_2\\I_1,I_1\end{matrix}$   | 319c         |
| a1<br>a2  | $\varphi$ $\varphi$   | $1-\varphi$   | 1  | $-\varphi \\ -10 + 4\varphi$  | $\begin{matrix} 0 \\ 8-6\varphi \end{matrix}$   | 1 1                                    | 4 4   | ++                                      | 2, 2  | 2, 2  | 2, 2   | $I_2, I_2$  | 319c         |
| a1<br>a2<br>a3  | $\varphi$ $\varphi$ 1 $\varphi$   | $1-\varphi$ $\varphi$   | $\frac{1}{\varphi}$  | $     \begin{array}{r}       -\varphi \\       -10 + 4\varphi \\       -1093 + 672\varphi \\       -10 + 9\varphi   \end{array} $   | $   \begin{array}{r}     0 \\     8 - 6\varphi \\     16452 - 10174\varphi \\     -2 - 4\varphi   \end{array} $   | 1<br>1<br>1                            | 4<br>4<br>2<br>2  | ++++                                    | $2, 2 \\ 1, 1$  | $2, 2 \\ 1, 1$  | $2, 2 \\ 1, 1$   | $\begin{matrix} I_2,I_2\\I_1,I_1\end{matrix}$   | 319c<br>319d |
| a1<br>a2<br>a3<br>a4<br>319                           | $\frac{\varphi}{\varphi}$ $\frac{1}{\varphi}$ $\frac{\mathbf{d}}{1+\varphi}$  | $ \begin{array}{c} 1 - \varphi \\ \varphi \\ 1 - \varphi \end{array} $  | $\begin{matrix} 1 \\ \varphi \\ 1 \end{matrix}$  |   | $0 \\ 8 - 6\varphi \\ 16452 - 10174\varphi \\ -2 - 4\varphi$ $0d = (19 - 2\varphi) = 11a \cdot 29b \qquad (1)$ $0$  | 1 1 1 1 1 1 sisogeny class             | 4 4 2 2 2 S) 4  | +++++-                                  | 2, 2<br>1, 1<br>4, 4  | 2, 2<br>1, 1<br>4, 4                                      | 2, 2<br>1, 1<br>2, 4   | $I_{2}, I_{2}$ $I_{1}, I_{1}$ $I_{4}, I_{4}$ $I_{1}, I_{1}$   |              |
| a1<br>a2<br>a3<br>a4<br>319<br>a1<br>a2               | $\frac{\varphi}{\varphi}$ $\frac{1}{\varphi}$ $\frac{\mathbf{d}}{1+\varphi}$ $\frac{1+\varphi}{1+\varphi}$  | $ \begin{array}{c} 1 - \varphi \\ \varphi \\ 1 - \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \end{array} $  | $\begin{array}{c} 1\\ \varphi\\ 1\\ \end{array}$   | $     \begin{array}{r}                                     $  | $0$ $8 - 6\varphi$ $16452 - 10174\varphi$ $-2 - 4\varphi$ $0d = (19 - 2\varphi) = 11a \cdot 29b \qquad (1)$ $0$ $2 + 6\varphi$  | 1                                      | 3 4 4 4 5 5 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6   | +++++++++++++++++++++++++++++++++++++++ | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2  | 2, 2<br>1, 1<br>4, 4                                      | 2, 2<br>1, 1<br>2, 4<br>1, 1<br>2, 2   | $\begin{array}{c} I_2,I_2\\ I_1,I_1\\ I_4,I_4 \end{array}$ $\begin{array}{c} I_1,I_1\\ I_2,I_2 \end{array}$   |              |
| a1<br>a2<br>a3<br>a4<br>3196<br>a1<br>a2<br>a3        | $ \frac{\varphi}{\varphi} $ $ \frac{1}{\varphi} $ $ \frac{\mathbf{d}}{1+\varphi} $ $ \frac{1+\varphi}{1+\varphi} $ $ \frac{1}{1+\varphi} $  | $ \begin{array}{c} 1 - \varphi \\ \varphi \\ 1 - \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 1 - \varphi \end{array} $   | $ \begin{array}{c} 1\\ \varphi\\ 1\\ \hline 1\\ 1+\varphi \end{array} $  | $     \begin{array}{r}                                     $  | $0 \\ 8 - 6\varphi \\ 16452 - 10174\varphi \\ -2 - 4\varphi$ $0d = (19 - 2\varphi) = 11a \cdot 29b \qquad (1)$ $0 \\ 2 + 6\varphi \\ 6278 + 10173\varphi$   | 1                                      | $ \begin{array}{ c c c } \hline 4 & 4 & 4 \\ 2 & 2 & 2 \\ \hline 8 & 4 & 4 \\ 2 & 2 & 4 \\ \hline \end{array} $   | +++++++++++++++++++++++++++++++++++++++ | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1  | 2, 2<br>1, 1<br>4, 4                                      | 2, 2<br>1, 1<br>2, 4<br>1, 1<br>2, 2<br>1, 1   | $\begin{array}{c} I_2,I_2\\ I_1,I_1\\ I_4,I_4 \end{array}$ $\begin{array}{c} I_1,I_1\\ I_2,I_2\\ I_1,I_1 \end{array}$   |              |
| a1<br>a2<br>a3<br>a4<br>319<br>a1<br>a2               | $\frac{\varphi}{\varphi}$ $\frac{1}{\varphi}$ $\frac{\mathbf{d}}{1+\varphi}$ $\frac{1+\varphi}{1+\varphi}$  | $ \begin{array}{c} 1 - \varphi \\ \varphi \\ 1 - \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \end{array} $  | $\begin{array}{c} 1\\ \varphi\\ 1\\ \end{array}$   | $     \begin{array}{r}                                     $  | $0$ $8 - 6\varphi$ $16452 - 10174\varphi$ $-2 - 4\varphi$ $0d = (19 - 2\varphi) = 11a \cdot 29b \qquad (1)$ $0$ $2 + 6\varphi$  | 1                                      | 3 4 4 4 5 5 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6   | +++++++++++++++++++++++++++++++++++++++ | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2  | 2, 2<br>1, 1<br>4, 4                                      | 2, 2<br>1, 1<br>2, 4<br>1, 1<br>2, 2   | $\begin{array}{c} I_2,I_2\\ I_1,I_1\\ I_4,I_4 \end{array}$ $\begin{array}{c} I_1,I_1\\ I_2,I_2 \end{array}$   |              |
| a1<br>a2<br>a3<br>a4<br>3196<br>a1<br>a2<br>a3        | $ \frac{\varphi}{\varphi} $ $ \frac{1}{\varphi} $ $ \frac{\mathbf{d}}{1+\varphi} $ $ \frac{1}{1+\varphi} $ $ \frac{1}{1+\varphi} $  | $ \begin{array}{c} 1 - \varphi \\ \varphi \\ 1 - \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 1 - \varphi \end{array} $   | $ \begin{array}{c} 1\\ \varphi\\ 1\\ \hline 1\\ 1+\varphi \end{array} $  | $     \begin{array}{r}                                     $  | $0 \\ 8 - 6\varphi \\ 16452 - 10174\varphi \\ -2 - 4\varphi$ $0d = (19 - 2\varphi) = 11a \cdot 29b \qquad (1)$ $0 \\ 2 + 6\varphi \\ 6278 + 10173\varphi$   | 1                                      | $\begin{bmatrix} 4 & 4 & 4 & 2 & 2 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3$   | +++++++++++++++++++++++++++++++++++++++ | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1  | 2, 2<br>1, 1<br>4, 4                                      | 2, 2<br>1, 1<br>2, 4<br>1, 1<br>2, 2<br>1, 1   | $\begin{array}{c} I_2,I_2\\ I_1,I_1\\ I_4,I_4 \end{array}$ $\begin{array}{c} I_1,I_1\\ I_2,I_2\\ I_1,I_1 \end{array}$   | 319d         |
| a1 a2 a3 a4 320a3 a4                                  | $ \frac{\varphi}{\varphi} $ $ \frac{1}{\varphi} $ $ \frac{\mathbf{d}}{1+\varphi} $ $ \frac{1}{1+\varphi} $ $ \frac{1}{1+\varphi} $ $ \frac{1}{0} $                                    | $ \begin{array}{c} 1 - \varphi \\ \varphi \\ 1 - \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 1 - \varphi \\ 0 \end{array} $  | $ \begin{array}{c} 1\\ \varphi\\ 1\\ \end{array} $ $ \begin{array}{c} 1\\ 1\\ 1+\varphi\\ 1\\ \end{array} $                                      | $     \begin{array}{r}                                     $  | $0 \\ 8 - 6\varphi \\ 16452 - 10174\varphi \\ -2 - 4\varphi$ $0d = (19 - 2\varphi) = 11a \cdot 29b \qquad (1$ $0$ $2 + 6\varphi \\ 6278 + 10173\varphi \\ -6 + 4\varphi$ $0a = (8 - 16\varphi) = 2^3 \cdot 5a \qquad (3 \text{ iso})$   | 1                                      | $\begin{bmatrix} 4 & 4 & 4 & 2 & 2 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3$   | +++++++++++++++++++++++++++++++++++++++ | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1<br>4, 4  | 2, 2<br>1, 1<br>4, 4                                      | 2, 2<br>1, 1<br>2, 4<br>1, 1<br>2, 2<br>1, 1<br>2, 4                                 | $I_{2}, I_{2} \\ I_{1}, I_{1} \\ I_{4}, I_{4}$ $I_{1}, I_{1} \\ I_{2}, I_{2} \\ I_{1}, I_{1} \\ I_{4}, I_{4}$ $III, I_{2}$  | 319d         |
| a1 a2 a3 a4 320 a1 a2 a1 a2                           | $ \frac{\varphi}{\varphi} $ $ \frac{1}{\varphi} $ $ \frac{\mathbf{d}}{1+\varphi} $ $ \frac{1}{1+\varphi} $ $ \frac{1}{1+\varphi} $ $ \frac{1}{0} $ $ 0 $                              | $ \begin{array}{c} 1 - \varphi \\ \varphi \\ 1 - \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 1 - \varphi \\ 0 \end{array} $  | $ \begin{array}{c} 1\\ \varphi\\ 1\\ \end{array} $ $ \begin{array}{c} 1\\ 1\\ 1\\ \end{array} $ $ \begin{array}{c} 0\\ 0\\ \end{array} $         | $     \begin{array}{r}                                     $  | $0 \\ 8 - 6\varphi \\ 16452 - 10174\varphi \\ -2 - 4\varphi$ $0d = (19 - 2\varphi) = 11a \cdot 29b \qquad (1)$ $0 \\ 2 + 6\varphi \\ 6278 + 10173\varphi \\ -6 + 4\varphi$ $0a = (8 - 16\varphi) = 2^3 \cdot 5a \qquad (3 \text{ iso})$ $1 \\ 334 + 540\varphi$                                       | 1                                      | $ \begin{array}{ c c } \hline 4 & 4 \\ 4 & 2 \\ 2 & 2 \end{array} $ S) $ \begin{array}{ c c } \hline 4 & 4 \\ 4 & 2 \\ 2 & 2 \end{array} $ S) $ \begin{array}{ c c } \hline 8 & 4 \\ 4 & $  | +++++++++++++++++++++++++++++++++++++++ | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1<br>4, 4<br>4, 2<br>8, 1  | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1<br>4, 4      | 2, 2<br>1, 1<br>2, 4<br>1, 1<br>2, 2<br>1, 1<br>2, 4<br>2, 2<br>2, 1                 | $I_{2}, I_{2}$ $I_{1}, I_{1}$ $I_{4}, I_{4}$ $I_{1}, I_{1}$ $I_{2}, I_{2}$ $I_{1}, I_{1}$ $I_{4}, I_{4}$ $III, I_{2}$ $I_{1}^{*}, I_{1}$  | 319d         |
| a1 a2 a3 a4  319 a1 a2 a3 a4  320 a1 a2 a3 a4         | $ \frac{\varphi}{\varphi} $ $ \frac{1}{\varphi} $ $ \frac{1}{1+\varphi} $ $ \frac{1}{1+\varphi} $ $ \frac{1}{1+\varphi} $ $ \frac{0}{0} $ $0$   | $ \begin{array}{c} 1 - \varphi \\ \varphi \\ 1 - \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 1 - \varphi \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array} $      | $ \begin{array}{c} 1\\ \varphi\\ 1\\ \end{array} $ $ \begin{array}{c} 1\\ 1\\ 1\\ 1\\ \end{array} $ $ \begin{array}{c} 0\\ 0\\ 0\\ \end{array} $ |   | $0 \\ 8 - 6\varphi \\ 16452 - 10174\varphi \\ -2 - 4\varphi$ $0d = (19 - 2\varphi) = 11a \cdot 29b \qquad (1)$ $0 \\ 2 + 6\varphi \\ 6278 + 10173\varphi \\ -6 + 4\varphi$ $0a = (8 - 16\varphi) = 2^3 \cdot 5a \qquad (3 \text{ is}$ $1 \\ 334 + 540\varphi \\ 50 - 28\varphi$                       | 1                                      | $ \begin{array}{ c c c } \hline 4 & 4 & 4 \\ 2 & 2 & 2 \\ \hline 8 & 4 & 4 \\ 4 & 4 & 4 \\ \hline 8 & 4 & 4 \\ 4 & 4 & 4 \\ \hline 8 & 4 & 4 \\ 4 & 4 & 4 \\ \hline 9 & 2 & 2 \\ \hline 8 & 4 & 4 \\ \hline 4 & 4 & 4 \\ \hline 9 & 2 & 2 \\ \hline 8 & 4 & 4 \\ \hline 4 & 4 & 4 \\ \hline 9 & 2 & 2 \\ \hline 9 & 3 & 4 \\ \hline 9 & 4 & 4 \\ \hline 9 & 2 & 2 \\ \hline 9 & 3 & 4 \\ \hline 9 & 4 & 4 \\ \hline 9 & 9 & 8 \\ \hline 9 & 1 & 1 \\ \hline 9 & 1 & $ | +++++++++++++++++++++++++++++++++++++++ | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1<br>4, 4<br>4, 2<br>8, 1<br>8, 1                                    | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1<br>4, 4      | 2, 2<br>1, 1<br>2, 4<br>1, 1<br>2, 2<br>1, 1<br>2, 4<br>2, 2<br>2, 1<br>2, 1         | $I_{2}, I_{2} \\ I_{1}, I_{1} \\ I_{4}, I_{4}$ $I_{1}, I_{1} \\ I_{2}, I_{2} \\ I_{1}, I_{1} \\ I_{4}, I_{4}$ $III, I_{2} \\ I_{1}^{*}, I_{1} \\ I_{1}^{*}, I_{1}$  | 319d         |
| a1 a2 a3 a4 320 a1 a2 a3 a4                           | $ \frac{\varphi}{\varphi} $ $ \frac{1}{\varphi} $ $ \frac{1}{\varphi} $ $ \frac{1+\varphi}{1+\varphi} $ $ \frac{1}{1+\varphi} $ $ \frac{a}{0} $ $ 0 $ $ 0 $                           | $ \begin{array}{c} 1 - \varphi \\ \varphi \\ 1 - \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 1 - \varphi \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array} $      | $ \begin{array}{c} 1 \\ \varphi \\ 1 \end{array} $ $ \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $                         |   | $0 \\ 8 - 6\varphi \\ 16452 - 10174\varphi \\ -2 - 4\varphi$ $0d = (19 - 2\varphi) = 11a \cdot 29b \qquad (1)$ $0 \\ 2 + 6\varphi \\ 6278 + 10173\varphi \\ -6 + 4\varphi$ $0a = (8 - 16\varphi) = 2^3 \cdot 5a \qquad (3 \text{ iso}$ $1 \\ 334 + 540\varphi \\ 50 - 28\varphi \\ -6$                | 1                                      | s) 4 4 4 2 2 2 3 8) 8 4 4 4 8   | +++++++++++++++++++++++++++++++++++++++ | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1<br>4, 4<br>4, 2<br>8, 1<br>8, 1<br>8, 4                            | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1<br>4, 4      | 2, 2<br>1, 1<br>2, 4<br>1, 1<br>2, 2<br>1, 1<br>2, 4<br>2, 2<br>2, 1<br>2, 1<br>4, 4 | $I_{1}, I_{2}$ $I_{1}, I_{1}$ $I_{4}, I_{4}$ $I_{1}, I_{1}$ $I_{2}, I_{2}$ $I_{1}, I_{1}$ $I_{4}, I_{4}$ $III, I_{2}$ $I_{1}^{*}, I_{1}$ $I_{1}^{*}, I_{1}$ $I_{1}^{*}, I_{4}$  | 319d         |
| a1 a2 a3 a4 320 a1 a2 a3 a4 a5                        | $ \frac{\varphi}{\varphi} $ $ \frac{1}{\varphi} $ $ \frac{1+\varphi}{1+\varphi} $ $ \frac{1+\varphi}{0} $ $ 0 $ $ 0 $ $ 0 $ $ 0 $ $ 0 $   | $ \begin{array}{c} 1 - \varphi \\ \varphi \\ 1 - \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 1 - \varphi \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $ | $ \begin{array}{c} 1 \\ \varphi \\ 1 \end{array} $ $ \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $                    | $     \begin{array}{r}       -\varphi \\       -10 + 4\varphi \\       -1093 + 672\varphi \\       -10 + 9\varphi   \end{array} $ $     \begin{array}{r}       319 \\       -1 \\       -6 - 5\varphi \\       -421 - 673\varphi \\       -1 - 10\varphi   \end{array} $ $     \begin{array}{r}       320 \\       -2 \\       -59 - 96\varphi \\       -22 + 15\varphi \\       -7 \\       -107   \end{array} $             | $0 \\ 8 - 6\varphi \\ 16452 - 10174\varphi \\ -2 - 4\varphi$ $0d = (19 - 2\varphi) = 11a \cdot 29b \qquad (1)$ $0 \\ 2 + 6\varphi \\ 6278 + 10173\varphi \\ -6 + 4\varphi$ $0a = (8 - 16\varphi) = 2^3 \cdot 5a \qquad (3 \text{ iso}$ $1 \\ 334 + 540\varphi \\ 50 - 28\varphi \\ -6 \\ -426$        | 1                                      | S) 4 4 4 2 2 2 Ss) 8 4 4 4 8 4 4  | +++++++++++++++++++++++++++++++++++++++ | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1<br>4, 4<br>4, 2<br>8, 1<br>8, 1<br>8, 4<br>10, 2                   | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1<br>4, 4<br>2 | 2, 2<br>1, 1<br>2, 4<br>1, 1<br>2, 2<br>1, 1<br>2, 4<br>2, 1<br>4, 4<br>2, 2         | $\begin{array}{c} I_2,I_2\\ I_1,I_1\\ I_4,I_4 \end{array}$ $\begin{array}{c} I_1,I_1\\ I_2,I_2\\ I_1,I_1\\ I_4,I_4 \end{array}$ $\begin{array}{c} III,I_2\\ I_1^*,I_1\\ I_1^*,I_1\\ I_1^*,I_1\\ I_1^*,I_1\\ I_1^*,I_2\\ I_1^*,I_1\\ I_1^*,I_2\\ I_1^*,I_1\\ I_1^*,I_2\\ I_1^*,I_3\\ I_1^*,I_3\\ I_1^*,I_4\\ I_1^*,I_5\\ I_1^*,I_5\\ I_1^*,I_6\\ I_1^$ | 319d         |
| a1 a2 a3 a4  3196 a1 a2 a3 a4  3206 a1 a2 a3 a4 a5 a6 | $ \begin{array}{c} \varphi \\ \varphi \\ 1 \\ \varphi \end{array} $ $ \frac{1}{\varphi}$ $ \frac{1}{1+\varphi}$ $ \frac{1}{1+\varphi}$ $ \frac{1}{0}$ $0$ $0$ $0$ $0$ $0$ $0$ $0$ $0$ | $ \begin{array}{c} 1 - \varphi \\ \varphi \\ 1 - \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 1 - \varphi \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $ | $ \begin{array}{c} 1 \\ \varphi \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $   | $     \begin{array}{r}       -\varphi \\       -10 + 4\varphi \\       -1093 + 672\varphi \\       -10 + 9\varphi   \end{array} $ $     \begin{array}{r}       319 \\       -1 \\       -6 - 5\varphi \\       -421 - 673\varphi \\       -1 - 10\varphi   \end{array} $ $     \begin{array}{r}       320 \\       -2 \\       -59 - 96\varphi \\       -22 + 15\varphi \\       -7 \\       -107 \\       13   \end{array} $ | $0 \\ 8 - 6\varphi \\ 16452 - 10174\varphi \\ -2 - 4\varphi$ $0d = (19 - 2\varphi) = 11a \cdot 29b \qquad (1)$ $0 \\ 2 + 6\varphi \\ 6278 + 10173\varphi \\ -6 + 4\varphi$ $0a = (8 - 16\varphi) = 2^3 \cdot 5a \qquad (3 \text{ iso}$ $1 \\ 334 + 540\varphi \\ 50 - 28\varphi \\ -6 \\ -426 \\ -34$ | 1                                      | S) 4 4 4 2 2 2 Ss) 8 4 4 4 8 8 4 4 4  | +++++++++++++++++++++++++++++++++++++++ | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1<br>4, 4<br>4, 2<br>8, 1<br>8, 1<br>8, 4<br>10, 2<br>10, 8          | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1<br>4, 4      | 2, 2<br>1, 1<br>2, 4<br>1, 1<br>2, 2<br>1, 1<br>2, 4<br>2, 1<br>4, 4<br>2, 2<br>2, 8 | $I_{1}, I_{2}$ $I_{1}, I_{1}$ $I_{4}, I_{4}$ $I_{1}, I_{1}$ $I_{2}, I_{2}$ $I_{1}, I_{1}$ $I_{4}, I_{4}$ $III, I_{2}$ $I_{1}^{*}, I_{1}$ $I_{1}^{*}, I_{1}$ $I_{1}^{*}, I_{4}$ $III^{*}, I_{2}$ $III^{*}, I_{8}$  |              |
| a1 a2 a3 a4 a5 a1 a2 a3 a4 a5                         | $ \frac{\varphi}{\varphi} $ $ \frac{1}{\varphi} $ $ \frac{1+\varphi}{1+\varphi} $ $ \frac{1+\varphi}{0} $ $ 0 $ $ 0 $ $ 0 $ $ 0 $ $ 0 $   | $ \begin{array}{c} 1 - \varphi \\ \varphi \\ 1 - \varphi \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 1 - \varphi \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $ | $ \begin{array}{c} 1 \\ \varphi \\ 1 \end{array} $ $ \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $                    | $     \begin{array}{r}       -\varphi \\       -10 + 4\varphi \\       -1093 + 672\varphi \\       -10 + 9\varphi   \end{array} $ $     \begin{array}{r}       319 \\       -1 \\       -6 - 5\varphi \\       -421 - 673\varphi \\       -1 - 10\varphi   \end{array} $ $     \begin{array}{r}       320 \\       -2 \\       -59 - 96\varphi \\       -22 + 15\varphi \\       -7 \\       -107   \end{array} $             | $0 \\ 8 - 6\varphi \\ 16452 - 10174\varphi \\ -2 - 4\varphi$ $0d = (19 - 2\varphi) = 11a \cdot 29b \qquad (1)$ $0 \\ 2 + 6\varphi \\ 6278 + 10173\varphi \\ -6 + 4\varphi$ $0a = (8 - 16\varphi) = 2^3 \cdot 5a \qquad (3 \text{ iso}$ $1 \\ 334 + 540\varphi \\ 50 - 28\varphi \\ -6 \\ -426$        | 1                                      | S) 4 4 4 2 2 2 Ss) 8 4 4 4 8 4 4  | +++++++++++++++++++++++++++++++++++++++ | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1<br>4, 4<br>4, 2<br>8, 1<br>8, 1<br>8, 4<br>10, 2<br>10, 8<br>11, 1 | 2, 2<br>1, 1<br>4, 4<br>1, 1<br>2, 2<br>1, 1<br>4, 4<br>2 | 2, 2<br>1, 1<br>2, 4<br>1, 1<br>2, 2<br>1, 1<br>2, 4<br>2, 1<br>4, 4<br>2, 2         | $\begin{array}{c} I_2,I_2\\ I_1,I_1\\ I_4,I_4 \end{array}$ $\begin{array}{c} I_1,I_1\\ I_2,I_2\\ I_1,I_1\\ I_4,I_4 \end{array}$ $\begin{array}{c} III,I_2\\ I_1^*,I_1\\ I_1^*,I_1\\ I_1^*,I_1\\ I_1^*,I_1\\ I_1^*,I_2\\ I_1^*,I_1\\ I_1^*,I_2\\ I_1^*,I_1\\ I_1^*,I_2\\ I_1^*,I_3\\ I_1^*,I_3\\ I_1^*,I_4\\ I_1^*,I_5\\ I_1^*,I_5\\ I_1^*,I_6\\ I_1^$ | 319d         |

|                        | $a_1$                                     | $a_2$        | $a_3$                             | $a_4$  | $a_6$  | $\mid r \mid$                 | T     | s     | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$          | Kodaira  | Isogenies    |
|------------------------|---|--------------|-----------------------------------|--|--|-------------------------------|-------|-------|------------------------------|-----------------------------|----------------|--|--------------|
| 320                    | a   |              |                                   | 320a   | $= (8 - 16\varphi) = 2^3 \cdot 5a \qquad (3 \text{ isogen})$   | ny classes)                   | )     |       |                              |                             |                |  | 320a         |
| b1                     | 0   | $-\varphi$   | 0                                 | -4   | 4arphi   | 0                             | 8     | ++    | 8, 2                         | 2                           | 4, 2           | $I_1^*, I_2$   |              |
| b2                     | 0   | $-\varphi$   | 0                                 | 1  | 0  | 0                             | 4     |       | 4, 1                         | 1                           | 2, 1           | $\mathrm{III},\mathrm{I}_1$  |              |
| b3                     | 0   | $-1+\varphi$ | 0                                 | $-149-235\varphi$  | $1270 + 2045\varphi$   | 0                             | 4     | + +   | 10, 1                        | 1                           | 2, 1           | $\mathrm{III}^*, \mathrm{I}_1$   |              |
| b4                     | 0   | $-\varphi$   | 0                                 | $-44 + 20\varphi$  | $-128 + 76\varphi$   |                               | 4     | + +   | 10, 4                        | 4                           | 2,4            | $\mathrm{III}^*,\mathrm{I}_4$  |              |
| b5                     | 0   | $-\varphi$   | 0                                 | $-84-60\varphi$  | $-400-276\varphi$  |                               | 2     | - +   | 11, 8                        | 8                           | 1,8            | $\mathrm{II}^*,\mathrm{I}_8$   |              |
| b6                     | 0   | $1-\varphi$  | 0                                 | $-4480 + 2772\varphi$  | $-138192 + 85404\varphi$   | 0                             | 2     | + -   | 11,2                         | 2                           | 1,2            | $II^*, I_2$  |              |
| c1                     |   | $-1+\varphi$ | 0                                 | -4   | 4-4arphi   | 0                             | 8     | ++    | 8, 2                         | 2                           | 4, 2           | $\mathrm{I}_1^*,\mathrm{I}_2$  |              |
| c2                     | 0   | $-1+\varphi$ | 0                                 | 1  | 0  | 0                             | 4     |       | 4, 1                         | 1                           | 2, 1           | $III, I_1$   |              |
| c3                     | 0   | $-\varphi$   | 0                                 | $-384 + 235\varphi$  | $3315 - 2045\varphi$   | 0                             | 4     | + +   | 10, 1                        | 1                           | 2, 1           | $\mathrm{III}^*, \mathrm{I}_1$   |              |
| c4                     |   | $-1+\varphi$ | 0                                 | $-24-20\varphi$  | $-52-76\varphi$  | 0                             | 4     | + +   | 10, 4                        | 4                           | 2, 4           | $\mathrm{III}^*, \mathrm{I}_4$   |              |
| c5                     | 0   | $-1+\varphi$ | 0                                 | $-144 + 60\varphi$   | $-676 + 276\varphi$  | 0                             | 2     | + -   | 11, 8                        | 8                           | 1, 8           | $\mathrm{II}^*,\mathrm{I}_8$   |              |
| c6                     | 0   | $\varphi$    | 0                                 | $-1708 - 2772\varphi$  | $-52788 - 85404\varphi$  | 0                             | 2     | -+    | 11, 2                        | 2                           | 1,2            | $\mathrm{II}^*,\mathrm{I}_2$   |              |
| <b>324</b>             | a   |              |                                   | ;  | $324a = (18) = 2 \cdot 3^2$ (1 isogeny of  | class)                        |       |       |                              |                             |                |  | 324a         |
| a1                     | $1+\varphi$                               | 1            | 1                                 | $-1490 - 1490\varphi$  | $28499 + 37999\varphi$   | 0                             | 2     | ++    | 5, 16                        | 5, 10                       | 1,4            | $I_5, I_{10}^*$  |              |
| a2                     | $1+\varphi$                               | 1            | 1                                 | $-95-95\varphi$  | -553-737arphi  | 0                             | 2     | ++    | 1,8                          | 1, 2                        | 1, 4           | $\mathrm{I}_1,\mathrm{I}_2^*$  |              |
| a3                     | $1+\varphi$                               | 1            | 1                                 | $-50-50\varphi$  | $851 + 1135\varphi$  | 0                             | 2     |       | 10, 11                       | 10, 5                       | 2, 2           | $I_{10}, I_5^*$  |              |
| a4                     | $1+\varphi$                               | 1            | 1                                 | $-5-5\varphi$  | $-13-17\varphi$  | 0                             | 2     |       | 2,7                          | 2,1                         | 2, 2           | $\mathrm{I}_2,\mathrm{I}_1^*$  |              |
| 341                    | $\mathbf{c}$                              |              |                                   | 341c   | $= (17 + 4\varphi) = 11a \cdot 31b$ (1 isoge   | eny class)                    | )     |       |                              |                             |                |  | <b>341</b> c |
| a1                     | $1+\varphi$                               | $-1-\varphi$ | 0                                 | $\varphi$  | 0  | 1                             | 2     | + -   | 2, 1                         | 2, 1                        | 2, 1           | $I_2, I_1$   |              |
| a2                     | $1+\varphi$                               | $-1-\varphi$ | 0                                 | $-4\varphi$  | $1+3\varphi$   | 1                             | 2     | -+    | 4, 2                         | 4, 2                        | 4, 2           | $I_4, I_2$   |              |
| 341                    | d   |              |                                   | 341d   | $= (21 - 4\varphi) = 11b \cdot 31a$ (1 isoge   | geny class)                   | )     |       |                              |                             |                |  | 341d         |
| a1                     | $\varphi$                                 | -1           | 0                                 | $1-\varphi$  | 0  | 1                             | 2     | -+    | 2, 1                         | 2, 1                        | 2, 1           | $\mathrm{I}_2,\mathrm{I}_1$  |              |
| a2                     | $\varphi$                                 | -1           | 0                                 | $-4+4\varphi$  | $4-3\varphi$   | 1                             | 2     | + -   | 4, 2                         | 4, 2                        | 4, 2           | $I_4, I_2$   |              |
| 256                    |   |              |                                   | 3566   | $a = (20 - 2\varphi) = 2 \cdot 89a \qquad (1 \text{ isoge})$   | 1)                            |       |       |                              |                             |                |  | 356a         |
| 356                    | a   |              |                                   | 333.   | $x = (20 - 2\varphi) = 2 \cdot 69a$ (1 isoge.  | eny ciass)                    |       |       |                              |                             |                |  |              |
| a1                     | ${f a}$                                   | 0            | $1+\varphi$                       | -2   | $\frac{1 - (20 - 2\varphi) - 2 \cdot 69a}{-\varphi}$   | eny class)                    | 2     | ++    | 2, 1                         | 2,1                         | 2, 1           | $I_2, I_1$   |              |
|                        |   |              | $1 + \varphi \\ 1 + \varphi$      |  |  | <del>, , , ,</del>            | 2 2   | ++++  | 2, 1<br>1, 2                 | $2, 1 \\ 1, 2$              | $2, 1 \\ 1, 2$ | $\begin{matrix} \mathrm{I}_2,\mathrm{I}_1 \\ \mathrm{I}_1,\mathrm{I}_2 \end{matrix}$ |              |
| a1                     | 1<br>1                                    |              |                                   | $-2 \\ -22 + 10\varphi$  | -arphi   |                               |       |       |                              |                             |                |  | 356b         |
| a1<br>a2               | 1<br>1                                    |              |                                   | $-2 \\ -22 + 10\varphi$  | $ \begin{array}{c} -\varphi \\ 38 - 27\varphi \end{array} $  |                               |       |       |                              |                             |                |  | 356b         |
| a1<br>a2<br><b>356</b> | $egin{array}{c} rac{1}{1} \ \end{array}$ | 0            | $1+\varphi$                       | $ \begin{array}{r} -2\\ -22 + 10\varphi \end{array} $  | $-\varphi$ $38 - 27\varphi$ $b = (18 + 2\varphi) = 2 \cdot 89b \qquad (1 \text{ isogen}$   | 1 1 ny class)                 | 2     | ++    | 1,2                          | 1,2                         | 1,2            | $I_1, I_2$   | 356b         |
| a1 a2 356 a1           | 1<br>1<br><b>b</b>                        | 0            | $\frac{1+\varphi}{\varphi}$       | $-2$ $-22 + 10\varphi$ $356$ $-1 - \varphi$ $-11 - 11\varphi$  | $-\varphi$ $38 - 27\varphi$ $b = (18 + 2\varphi) = 2 \cdot 89b \qquad (1 \text{ isogen}$ $0$   | 1   1   1   my class)   1   1 | 2     | ++    | 2,1                          | 2,1                         | 2,1            | $I_1,I_2$ $I_2,I_1$  | 356b<br>359a |
| a1 a2 356 a1 a2        | 1<br>1<br><b>b</b> 1<br>1                 | 0 0          | $1 + \varphi$ $\varphi$ $\varphi$ | $ \begin{array}{r} -2\\ -22 + 10\varphi \end{array} $ $ 356$ $ \begin{array}{r} -1 - \varphi\\ -11 - 11\varphi \end{array} $ $ 3596$ | $-\varphi$ $38 - 27\varphi$ $b = (18 + 2\varphi) = 2 \cdot 89b \qquad (1 \text{ isoger})$ $0$ $12 + 26\varphi$ $a = (7 - 17\varphi) = 359a \qquad (2 \text{ isogeny})$                                 | 1   1   1   my class)   1   1 | 2     | +++++ | 2,1                          | 2,1                         | 2,1            | $I_1, I_2$ $I_2, I_1$ $I_1, I_2$   |              |
| 356 a1 a2 359 a1       | 1<br>1<br><b>b</b><br>1<br>1<br>2         | 0 0          | $1 + \varphi$ $\varphi$ $\varphi$ | $-2$ $-22 + 10\varphi$ $356$ $-1 - \varphi$ $-11 - 11\varphi$ $359\alpha$ $-2 + \varphi$   | $ \frac{-\varphi}{38 - 27\varphi} $ $ b = (18 + 2\varphi) = 2 \cdot 89b \qquad (1 \text{ isoger}) $ $ 0 $ $ 12 + 26\varphi $ $ a = (7 - 17\varphi) = 359a \qquad (2 \text{ isogeny}) $ $ 1 - \varphi $ | 1                             | 2 2 2 | +++++ | 2,1                          | 2,1                         | 2,1            | $I_1, I_2$ $I_2, I_1$ $I_1, I_2$ $I_1$   |              |
| a1 a2 356 a1 a2 359    | 1<br>1<br><b>b</b> 1<br>1                 | 0 0          | $1 + \varphi$ $\varphi$ $\varphi$ | $ \begin{array}{r} -2\\ -22 + 10\varphi \end{array} $ $ 356$ $ \begin{array}{r} -1 - \varphi\\ -11 - 11\varphi \end{array} $ $ 3596$ | $-\varphi$ $38 - 27\varphi$ $b = (18 + 2\varphi) = 2 \cdot 89b \qquad (1 \text{ isoger})$ $0$ $12 + 26\varphi$ $a = (7 - 17\varphi) = 359a \qquad (2 \text{ isogeny})$                                 | ny class)  1 1 1 vy classes)  | 2     | +++++ | 2,1                          | 2,1                         | 2,1            | $I_1, I_2$ $I_2, I_1$ $I_1, I_2$   |              |

|     |              |           |                    |                |             |       |                |       |  |       |               |                |              |                    |                              | T                           | _      | 1                           | I                |
|-----|--------------|-----------|--------------------|----------------|-------------|-------|----------------|-------|--|-------|---------------|----------------|--------------|--------------------|------------------------------|-----------------------------|--------|-----------------------------|------------------|
|     |              | $a_1$     |                    | $a_2$          | $a_{i}$     | 3     | a              | 4     | $a_6$  | i     | r             | $ \Gamma $     | '            | s                  | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$  | Kodaira                     | Isogenies        |
| 359 | b            |           |                    |                |             |       |                | 359   | $9b = (10 - 17\varphi) = 359b \tag{2}$               | 2 iso | ogeny classes | )              |              |                    |                              |                             |        |                             | <b>359</b> b     |
| a1  |              | 0         |                    | 0              | $1+\varphi$ | ,     | $-1-\varphi$   | ρ     | 0  |       | 1             | 1              |              | + -                | 1                            | 1                           | 1      | $I_1$                       |                  |
| b1  |              | 0         |                    | 1              | $1+\varphi$ | )     | -3 + 24        | <br>0 | $1-2\varphi$   |       | 0             | 3              | - <u>†</u> - | + -                | 1                            | 1                           | 1      | $I_1$                       | <u> </u>         |
| b2  |              | 0         |                    |                | $1+\varphi$ |       | -23 - 184      |       | $-57-67\varphi$                                      |       | 0             | 1              |              | + -                | 3                            | 3                           | 1      | $I_3$                       |                  |
| 361 | a            |           |                    |                |             |       |                | 3     | $61a = (19) = 19a \cdot 19b \tag{2}$                 | isog  | geny classes) |                |              |                    |                              |                             |        |                             | 361a             |
| a1  |              | 0         |                    | 1              | 1           |       |                | 1     | 0  |       | 0             | 3              |              |                    | 1, 1                         | 1, 1                        | 1,1    | $I_1, I_1$                  |                  |
| a2  |              | 0         |                    | 1              | 1           |       | -9             |       | -15  |       | 0             | 3              |              |                    | 3, 3                         | 3,3                         | 3,3    | $I_3, I_3$                  |                  |
| a3  |              | 0         |                    | 1              | 1           |       | -769           | 9     | -8470  |       | 0             | 1              | <u>.</u>     |                    | 1,1                          | 1,1                         | 1,1    | $I_1, I_1$                  |                  |
| b1  | 1 +          | •         |                    | 1              | C           | )     | $1+\varphi$    |       | 0  |       | 0             | 2              |              |                    | 1, 1                         | 1, 1                        | 1,1    | $I_1, I_1$                  |                  |
| b2  | 1 +          |           |                    | 1              | C           | )     | $-4-4\varphi$  |       | $-9-12\varphi$                                       |       | 0             | 4              |              | + +                | 2, 2                         | 2, 2                        | 2,2    | $I_2, I_2$                  |                  |
| b3  |              | r         | -1 -               | •              | φ           |       | -150 + 904     |       | $-805 + 500\varphi$                                  |       | 0             | 2              |              | + +                | 1, 4                         | 1,4                         | 1,2    | $I_1, I_4$                  |                  |
| b4  | 1 +          | - φ       |                    | 1              | C           | )     | -59 - 894      | ρ     | $-364 - 590\varphi$                                  |       | 0             | 2              |              | ++                 | 4, 1                         | 4,1                         | 2,1    | $I_4, I_1$                  |                  |
| 361 | b            |           |                    |                |             |       |                | 36    | $31b = (9 - 17\varphi) = 19a^2 \tag{2}$              | iso   | geny classes) |                |              |                    |                              |                             |        |                             | 361b             |
| a1  |              | 0         | 1 –                | $\varphi$      | $\varphi$   | )     | -3             |       | 0  |       | 1             | 1              |              | + +                | 2                            | 0                           | 1      | II                          |                  |
| a2  |              | 0         |                    | $\varphi$      | $\varphi$   | )<br> | -172 - 2734    |       | $1710 + 2772\varphi$                                 |       | 1             | _ 1            | _            | + +                | 2                            | 0                           | 1      | II                          |                  |
| b1  |              |           | -1 -               | $\varphi$      | $\varphi$   | )     | -25 - 44       |       | $19 - 38\varphi$                                     |       | 0             | 3              |              | + +                |                              | 0                           | 3      | $IV^*$                      |                  |
| b2  |              | 0         |                    | 1              | φ           | )     | -5116 - 82124  | ρ     | $-267380 - 432445\varphi$                            |       | 0             | 1              |              | ++                 | 8                            | 0                           | 3      | IV*                         |                  |
| 361 | $\mathbf{c}$ |           |                    |                |             |       |                | 36    | $31c = (8 - 17\varphi) = 19b^2 \tag{2}$              | iso   | geny classes) |                |              |                    |                              |                             |        |                             | 361c             |
| a1  |              | 0         |                    | $\varphi$ :    | $1+\varphi$ | )     | -:             | 1     | $-\varphi$   |       | 1             | 1              |              | + +                | 2                            | 0                           | 1      | II                          |                  |
| a2  |              | 0         | -1 +               | $\varphi$      | $1+\varphi$ | )     | -445 + 2734    | ρ     | $4482 - 2773\varphi$                                 |       | 1             | 1              |              | + +                | 2                            | 0                           | 1      | II                          |                  |
| b1  |              | 0         | 1+                 | $\varphi^{-1}$ | $1+\varphi$ | ,     | -30 + 64       | ρ -   | $-49 + 42\varphi$                                    |       | 0             | 3              | - ī          | + +                | 8                            | 0                           | 3      | IV*                         |                  |
| b2  |              | 0         |                    | 1              | $1+\varphi$ | )     | -13328 + 82124 | ρ     | $-699825 + 432444\varphi$                            |       | 0             | 1              |              | + +                | 8                            | 0                           | 3      | IV*                         |                  |
| 369 | a            |           |                    |                |             |       |                | 369   | $\partial a = (21 - 3\varphi) = 3 \cdot 41a \tag{3}$ | 3 is  | ogeny classes | )              |              |                    |                              |                             |        |                             | 369a             |
| a1  |              | 0         | -1 +               | $\varphi$ :    | $1+\varphi$ | )     | $-2\varphi$    |       | 0  |       | 1             | 1              |              |                    | 2, 1                         | 2, 1                        | 2, 1   | $I_2, I_1$                  |                  |
| b1  |              |           |                    | <u>-</u>       | '- '-<br>φ  |       | $-3+\varphi$   |       | $1-\varphi$  |       |               | <br>4          | - ¦ -        | . <u></u> .<br>+ + | 1, 1                         | 1, 1                        | 1,1    | $I_1, I_1$                  |                  |
| b2  | 1 +          | ,         | -1 +               |                |             |       | -11 - 174      |       | $-32-52\varphi$                                      |       |               | 4              |              | + +                | 2, 2                         | 2, 2                        | 2,2    | $I_2, I_2$                  |                  |
| b3  |              |           | $-1 + \frac{1}{1}$ |                |             |       | -176 - 2579    |       | $-1667 - 2665\varphi$                                |       | 0             | 2              |              | + +                |                              | 1,1                         | 1,1    | $I_1, I_1$                  |                  |
| b4  |              | $\varphi$ |                    | -φ             | φ           |       | 17 - 144       |       | $-28 + 14\varphi$                                    |       | 0             | 4              |              |                    | 4, 4                         | 4, 4                        | 4,4    | $\mathrm{I}_4,\mathrm{I}_4$ |                  |
| c1  |              | 1         | 1+                 | $\varphi$      | φ           |       | -16 - 36       |       | $-1 - 31\varphi$                                     |       | 0             | - <sub>-</sub> | - i -        | + +                | 6, 2                         | 6, 2                        | [2, 2] | $I_6, I_2$                  | [                |
| c2  |              | 1         | 1 +                |                | φ           |       | -6 - 84        |       | $-16-25\varphi$                                      |       | 0             | 2              |              | + +                |                              | 3, 1                        | 1,1    | $I_3, I_1$                  |                  |
| c3  |              | 1         | 1 +                |                | $\varphi$   |       | -41 - 584      |       | $161 + 186\varphi$                                   |       | 0             | 2              |              | - +                | 12, 1                        | 12, 1                       | 2, 1   | $I_{12},I_1$                |                  |
| c4  | 1 +          | - φ       | 1 –                | $\varphi$      | φ           |       | -1153 + 7174   | ρ     | $17138 - 10597\varphi$                               |       | 0             | 2              |              | + -                | 3, 4                         | 3, 4                        | 1,2    | $I_3, I_4$                  |                  |
| 369 | b            |           |                    |                |             |       |                | 369   | $9b = (18 + 3\varphi) = 3 \cdot 41b \tag{3}$         | 3 is  | ogeny classes | )              |              |                    |                              |                             |        |                             | $369 \mathrm{b}$ |
| a1  |              | 0         |                    | -φ             | φ           |       | -2 + 2q        |       | $1-\varphi$  |       | 1             | 1              |              |                    | 2,1                          | 2, 1                        | 2,1    | $I_2, I_1$                  |                  |
|     |              |           |                    | 7              | Υ           |       | 2   29         | -     | τ Ψ  |       | 1             |                |              |                    | -, +                         |                             |        | -2,-1                       |                  |

|          | $a_1$                      | $a_2$                 | $a_3$                          | $a_4$                               | $a_6$   | r                                      | T                | s              | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                                       | $c_p$  | Kodaira   | Isogenies       |
|----------|----------------------------|-----------------------|--------------------------------|-------------------------------------|---|--|------------------|----------------|------------------------------|---|--|---|-----------------|
|          |                            |                       |                                |                                     |   |  |                  |                |                              |   |  |   |                 |
| 369      | b                          |                       |                                |                                     | $369b = (18 + 3\varphi) = 3 \cdot 41b $                                 | 3 isogeny cla                          | asses)           |                |                              |   |  |   | 369b            |
| b1       | $1+\varphi$                | -1                    | $1+\varphi$                    | $-2-3\varphi$                       | 0   | 0                                      | 4                | ++             | 1,1                          | 1,1   | 1, 1   | $I_1, I_1$  |                 |
| b2       | φ                          | $1+\varphi$           | 1                              | $-26 + 17\varphi$                   | $-67 + 41\varphi$   | 0                                      | 4                | ++             | 2,2                          | 2, 2  | 2,2  | $I_2, I_2$  |                 |
| b3       | $\varphi$                  | $1+\varphi$           | 1                              | $-431 + 257\varphi$                 | $-4075 + 2489\varphi$   | 0                                      | 2                | ++             | 1, 1                         | 1, 1  | 1,1  | $I_1, I_1$  |                 |
| b4       | $1+\varphi$                | -1                    | $1 + \varphi$                  | $3+12\varphi$                       | $-14-15\varphi$   | 0                                      | 4                |                | 4, 4                         | 4, 4  | 4, 4   | $I_4, I_4$  |                 |
| c1       | 1                          | $-1-\varphi$          | $\varphi$                      | $-20+4\varphi$                      | $-12+27\varphi$   | 0                                      | $\overline{4}$   | + +            | 6, 2                         | $\frac{1}{6}, 2$  | [-2, 2]  | $I_6, I_2$  |                 |
| c2       | 1                          | $-1-\varphi$          | arphi                          | $-15+9\varphi$                      | $-26+16\varphi$   | 0                                      | 2                | ++             | 3, 1                         | 3,1   | 1,1  | $I_3, I_1$  |                 |
| c3       | 1                          | $-1-\varphi$          | $\varphi$                      | $-100 + 59\varphi$                  | $447-245\varphi$  | 0                                      | 2                | +-             | 12, 1                        | 12, 1   | 2, 1   | $I_{12},I_1$  |                 |
| c4       | $\varphi$                  | 1                     | $1 + \varphi$                  | $-435 - 719\varphi$                 | $6541 + 10596\varphi$   | 0                                      | 2                | -+             | 3, 4                         | 3,4   | 1, 2   | $I_3, I_4$  |                 |
|          |                            |                       |                                |                                     |   |  |                  |                |                              |   |  |   |                 |
| 380      | ล                          |                       |                                | 3⊗                                  | $0a = (18 + 4\varphi) = 2 \cdot 5a \cdot 19a$                           | (3 isogeny                             | classe           | ·e)            |                              |   |  |   | 380a            |
| a1       | 1                          | -/0                   | 1                              | $-36-61\varphi$                     | $\frac{6a - (16 + 4\varphi) - 2 \cdot 5a \cdot 15a}{-145 - 245\varphi}$ | 0                                      | 2                | - +            | 1, 2, 7                      | 1, 2, 7   | 1, 2, 1  | $I_1, I_2, I_7$   |                 |
| a2       | 1                          | $-\varphi$ $-\varphi$ | 1                              | $-30 - 61\varphi$ $-66 - 41\varphi$ | $-143 - 243\varphi$ $-97 - 289\varphi$                                  |  | 2                |                | 2, 1, 14                     | $\begin{bmatrix} 1, 2, 7 \\ 2, 1, 14 \end{bmatrix}$               | $\begin{bmatrix} 1, 2, 1 \\ 2, 1, 2 \end{bmatrix}$ | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$                      |                 |
|          |                            |                       |                                | $-3-\varphi$                        | $2-\varphi$   | : :                                    | $\frac{1}{6}$    |                |                              |   |  |   |                 |
| b1<br>b2 | $1+\varphi$                | -1                    | $rac{arphi}{0}$               | $-3 - \varphi$ $-223 + 140\varphi$  | $ \begin{array}{c} 2 - \varphi \\ 1482 - 915\varphi \end{array} $       | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 6                | + -            | 1, 2, 1<br>2, 1, 2           | $\begin{vmatrix} 1, 2, 1 \\ 2, 1, 2 \end{vmatrix}$                | $\begin{vmatrix} 1, 2, 1 \\ 2, 1, 2 \end{vmatrix}$ | $\begin{array}{ c c c c c } & I_1, I_2, I_1 \\ & I_2, I_1, I_2 \end{array}$ |                 |
| b3       | $\varphi$ $1+\varphi$      | $1+\varphi$ $-1$      |                                | $-223 + 140\varphi$ $2 - 16\varphi$ | $-9 - 22\varphi$  |  | 6                | <del>-</del> + |                              | $\begin{bmatrix} 2, 1, 2 \\ 3, 6, 3 \end{bmatrix}$                | $\begin{bmatrix} 2, 1, 2 \\ 3, 6, 1 \end{bmatrix}$ | $I_2, I_1, I_2  I_3, I_6, I_3$  |                 |
| b4       | $1+\varphi$<br>$1+\varphi$ | -1<br>-1              | $\varphi$                      | $-38 + 24\varphi$                   | $ \begin{array}{c} -3 - 22\varphi \\ 31 - 94\varphi \end{array} $       |  | 6                | + -            | 6, 3, 6                      | $\begin{bmatrix} 3, 0, 3 \\ 6, 3, 6 \end{bmatrix}$                | $\begin{bmatrix} 5, 0, 1 \\ 6, 3, 2 \end{bmatrix}$ | $I_6, I_3, I_6$   |                 |
| b5       | $1 \cdot \varphi$          | $\varphi$             | arphi                          | $-3360 - 5408\varphi$               | $-143894 - 232769\varphi$   |  | 2                |                |                              | 1, 2, 9   | $\begin{bmatrix} 0, 3, 2 \\ 1, 2, 1 \end{bmatrix}$ | $I_1, I_2, I_9$   |                 |
| b6       | $1+\varphi$                | -1                    | arphi                          | $-2018 + 154\varphi$                | $-34589 + 3448\varphi$  |  | 2                | + -            | 2, 1, 18                     | $\begin{bmatrix} 1, 2, 3 \\ 2, 1, 18 \end{bmatrix}$               | $\begin{bmatrix} 1, 2, 1 \\ 2, 1, 2 \end{bmatrix}$ | $I_1, I_2, I_9$<br>$I_2, I_1, I_{18}$                                       |                 |
| c1       | 1                          |                       | $1+\varphi$                    | $-1-2\varphi$                       | $-1-\varphi$  | - 0                                    | - <del>-</del> - | '<br>          | 1, 4, 1                      | $\begin{array}{c c} 1 & 2, 2, 13 \\ \hline & 1, 4, 1 \end{array}$ | 1, 4, 1  | $  I_1, I_4, I_1  $   | <u> </u>        |
| c2       | 1                          |                       | $1 + \varphi$<br>$1 + \varphi$ | $-21-22\varphi$                     | $-45-57\varphi$   |  | 4                | + +            | 2, 2, 2                      | 2, 2, 2   | $\begin{bmatrix} 1, 4, 1 \\ 2, 2, 2 \end{bmatrix}$ | $I_1, I_4, I_1 $<br>$I_2, I_2, I_2$   |                 |
| c3       | $1+\varphi$                | $\varphi$             | 0                              | $-11615 - 18793\varphi$             | $-929455 - 1503890\varphi$  |  | 2                |                | 1, 1, 1                      | $\begin{bmatrix} 2, 2, 2 \\ 1, 1, 1 \end{bmatrix}$                | $\begin{bmatrix} 2, 2, 2 \\ 1, 1, 1 \end{bmatrix}$ | $I_2, I_2, I_2$<br>$I_1, I_1, I_1$  |                 |
| c4       | $1+\varphi$                | $1+\varphi$           | $\varphi$                      | $-588 + 361\varphi$                 | $-6523 + 4020\varphi$   |  | 2                |                | 4, 1, 4                      | 4, 1, 4   | $\begin{bmatrix} 1, 1, 1 \\ 2, 1, 4 \end{bmatrix}$ | $I_{4}, I_{1}, I_{4}$   |                 |
| 0.1      | - 1 7                      | - 1 7                 | τ                              | 300   3019                          | 0023   10209  |  | _                | _ '            | -, -, -                      | 1 -, -, -   | _, _, _,   | 14, 11, 14  |                 |
|          | _                          |                       |                                |                                     |   |  |                  |                |                              |   |  |   | _               |
| 380      | b                          |                       |                                | 38                                  | $0b = (22 - 4\varphi) = 2 \cdot 5a \cdot 19b$                           | (3 isogeny                             | classe           | s)             |                              |   |  |   | $380\mathrm{b}$ |
| a1       |                            | $-1+\varphi$          | 1                              | $-97 + 61\varphi$                   | $-390 + 245\varphi$   | 0                                      | 2                | +-             | 1, 2, 7                      | 1, 2, 7   | 1, 2, 1  | $I_1, I_2, I_7$   |                 |
| a2       | 1                          | $-1+\varphi$          | 1                              | $-107 + 41\varphi$                  | $-386 + 289\varphi$   | 0                                      | 2                | -+             | 2, 1, 14                     | 2, 1, 14  | [2, 1, 2]  | $I_2, I_1, I_{14}$  |                 |
| b1       | $\varphi$                  | $-\varphi$            | $1+\varphi$                    | $-3-\varphi$                        | 1   | 0                                      | 6                | + -            | 1, 2, 1                      | [1, 2, 1]   | [1, 2, 1]  | $  I_1, I_2, I_1  $   |                 |
| b2       | $1+\varphi$                | $-1+\varphi$          | arphi                          | $-85-139\varphi$                    | $513 + 830\varphi$  | 0                                      | 6                | 1              | 2, 1, 2                      | 2, 1, 2   | 2, 1, 2  | $I_2, I_1, I_2$   |                 |
| b3       | $\varphi$                  | -arphi                | $1+\varphi$                    | $-13 + 14\varphi$                   | $-31+21\varphi$   | 0                                      | 6                | I              | 3, 6, 3                      | 3, 6, 3   | 3, 6, 1  | $I_{3}, I_{6}, I_{3}$   |                 |
| b4       | $\varphi$                  |                       | $1+\varphi$                    | $-13-26\varphi$                     | $-63 + 93\varphi$   | 0                                      | 6                | 1              | 6, 3, 6                      | 6, 3, 6   | 6, 3, 2  | $I_{6}, I_{3}, I_{6}$   |                 |
| b5       | 1                          | $1-\varphi$           | $1 + \varphi$                  | $-8768 + 5407\varphi$               | $-376663 + 232768\varphi$   | 0                                      | 2                |                | 1, 2, 9                      | 1, 2, 9   | 1, 2, 1  | $I_1, I_2, I_9$   |                 |
| b6       | $\varphi$                  | $-\varphi$            | $1+\varphi$                    | $-1863 - 156\varphi$                | $-31141 - 3449\varphi$  | 0                                      | 2                | -+             | 2, 1, 18                     | 2, 1, 18  | [2, 1, 2]  | $I_2, I_1, I_{18}$  |                 |
| c1       | 1                          | $-1+\varphi$          | $\varphi$                      | $-2+\varphi$                        | -1  | 0                                      | 4                | + -            | 1, 4, 1                      | 1,4,1   | [1, 4, 1]  | $  I_1, I_4, I_1  $   |                 |
| c2       |                            | $-1+\varphi$          | arphi                          | $-42 + 21\varphi$                   | $-101 + 56\varphi$  | 0                                      | 4                | ++             | 2, 2, 2                      | 2, 2, 2   | 2, 2, 2  | $I_2, I_2, I_2$   |                 |
| c3       | $\varphi$                  | $-1+\varphi$          | 1                              | $-30410 + 18794\varphi$             | $-2384141 + 1473480\varphi$   | 0                                      | 2                | ++             | 1, 1, 1                      | 1, 1, 1   | 1, 1, 1  | $I_1, I_1, I_1$   |                 |
| c4       | $\varphi$                  | $\varphi$             | $\varphi$                      | $-230-359\varphi$                   | $-2634 - 4249\varphi$   | 0                                      | 2                | -+             | 4, 1, 4                      | 4, 1, 4   | 2, 1, 4  | $I_4, I_1, I_4$   |                 |

|     |               |               |               |                           |   |              |               |     |                              |                             |   | T                         |           |
|-----|---------------|---------------|---------------|---------------------------|---|--------------|---------------|-----|------------------------------|-----------------------------|---|---------------------------|-----------|
|     | $a_1$         | $a_2$         | $a_3$         | $a_4$                     | $a_6$   | r            | T             | s   | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$                                   | Kodaira                   | Isogenies |
| 395 | a             |               |               | 39                        | $5a = (19 + 2\varphi) = 5a \cdot 79a \tag{2}$ | 2 isogeny cl | lasses        | s)  |                              |                             |   |                           | 395a      |
| a1  | $1+\varphi$   | 0             | 1             | $-4+2\varphi$             | $3-2\varphi$                                  | 1            | 2             | -+  | 2,1                          | 2,1                         | 2, 1                                    | $I_2, I_1$                |           |
| a2  | $1+\varphi$   | 0             | 1             | $-59 + 37\varphi$         | $205 - 126\varphi$                            | 1            | 2             | + - | 1,2                          | 1,2                         | 1, 2                                    | $I_1, I_2$                |           |
| b1  | $1+\varphi$   | $-\varphi$    | 0             | $-656 + 405\varphi$       | $-7744 + 4786\varphi$                         | 0            | $\frac{1}{2}$ | + + | 1, 2                         | 1,2                         | 1,2                                     | $ $ $I_1, I_2$            |           |
| b2  | 1             | 0             | 0             | $-7+2\varphi$             | $-8+3\varphi$                                 | 0            | 2             | -+  | 2,1                          | 2,1                         | 2, 1                                    | $I_2, I_1$                |           |
| 395 | b             |               |               | 39                        | $5b = (21 - 2\varphi) = 5a \cdot 79b \tag{2}$ | 2 isogeny cl | asses         | )   |                              |                             |   |                           | 395b      |
| a1  | $\varphi$     | $1-\varphi$   | 1             | $-1-3\varphi$             | $1+2\varphi$                                  | 1            | 2             | +-  | 2,1                          | 2,1                         | 2, 1                                    | $I_2, I_1$                |           |
| a2  | $\varphi$     | $1-\varphi$   | 1             | $-21-38\varphi$           | $79 + 126\varphi$                             | 1            | 2             | -+  | 1, 2                         | 1, 2                        | 1, 2                                    | $I_1, I_2$                |           |
| b1  | $\varphi$     | 0             | 0             | $-251 - 405\varphi$       | $-2958 - 4786\varphi$                         | 0            | $\frac{1}{2}$ | + + | 1,2                          | 1, 2                        | [1, 2]                                  | $ $ $I_1, I_2$            |           |
| b2  | 1             | 0             | 0             | $-5-2\varphi$             | $-5-3\varphi$                                 | 0            | 2             | +-  | 2, 1                         | 2, 1                        | 2, 1                                    | $I_2, I_1$                |           |
| 396 | a             |               |               | 396                       | $a = (6 - 18\varphi) = 2 \cdot 3 \cdot 11a$   | (5 isogeny o | classe        | s)  |                              |                             |   |                           | 396a      |
| a1  | $1+\varphi$   | -1            | 0             | $1+5\varphi$              | $5+4\varphi$                                  | 0            | 5             | T   | 1, 3, 5                      | 1, 3, 5                     | 1, 1, 5                                 | $I_1, I_3, I_5$           |           |
| a2  | $1+\varphi$   | -1            | 0             | $-224 - 340\varphi$       | $-2464 - 3728\varphi$                         | 0            | 1             |     | 5, 15, 1                     | 5, 15, 1                    | $\begin{bmatrix} 5, 1, 1 \end{bmatrix}$ | $I_5, I_{15}, I_1$        |           |
| b1  | $\varphi$     | -1            | 0             | $-4+\varphi$              | $-7-2\varphi$                                 | 0            | 1             |     | 7, 1, 1                      | 7,1,1                       | [1, 1, 1]                               | $I_7, I_1, I_1$           |           |
| c1  | 1             | $\varphi$     | $1+\varphi$   | $-426 + 257\varphi$       | $3985 - 2473\varphi$                          | 0            | 8             | + + | 4, 4, 2                      | 4, 4, 2                     | [2,4,2]                                 | $  I_4, I_4, I_2  $       |           |
| c2  | 1             |               | $1+\varphi$   | $-26+17\varphi$           | $65-41\varphi$                                | 0            | 8             |     | 2, 8, 1                      | 2, 8, 1                     | 2, 8, 1                                 | $I_2, I_8, I_1$           |           |
| c3  | $\varphi$     | $1 + \varphi$ | 1             | $-318015 + 196545\varphi$ | $81095854 - 50119994\varphi$                  | 0            | 4             | ++  | 2, 2, 1                      | 2, 2, 1                     | 2, 2, 1                                 | $I_2, I_2, I_1$           |           |
| c4  | arphi         | $1-\varphi$   | 0             | $-340-413\varphi$         | $-3227 - 5628\varphi$                         | 0            | 4             | ++  | 8, 2, 4                      | 8, 2, 4                     | 2, 2, 4                                 | $I_8, I_2, I_4$           |           |
| c5  | $\varphi$     | $1-\varphi$   | 0             | $-250-413\varphi$         | $-3389 - 6942\varphi$                         | 0            | 2             |     | ,-,-                         | 16, 1, 2                    | 2, 1, 2                                 | $I_{16}, I_1, I_2$        |           |
| c6  | $1+\varphi$   | $-\varphi$    | 0             | $-30579 - 49435\varphi$   | $-3912205 - 6330026\varphi$                   | 0            | 2             | + + | 4,1,8                        | 4, 1, 8                     | [2, 1, 8]                               | $\mid I_4, I_1, I_8$      |           |
| d1  | $\varphi$     | -1            | $\varphi$     | $2-2\varphi$              | $-2+\varphi$                                  | 0            | 5             |     | -, -, -                      | 1, 5, 1                     | 1, 5, 1                                 | $I_1, I_5, I_1$           |           |
| d2  | $\varphi$     | -1            | $\varphi$     | $-398 + 228\varphi$       | $-3384 + 2055\varphi$                         |              | 1             |     | 5, 1, 5                      | 5, 1, 5                     | $\begin{bmatrix} 5, 1, 1 \end{bmatrix}$ | $I_5, I_1, I_5$           |           |
| e1  | 1             | 0             | 1             | $-19-21\varphi$           | $35 + 66\varphi$                              | 0            | 3             |     | 1, 1, 0                      | 1, 1, 3                     | 1, 1, 3                                 | $I_1,I_1,I_3$             |           |
| e2  | $\varphi$     | $1-\varphi$   | 1             | $-15308 + 9460\varphi$    | $-865777 + 535080\varphi$                     | 0            | 1             |     | 3, 3, 1                      | 3, 3, 1                     | 1, 3, 1                                 | $I_3, I_3, I_1$           |           |
| 396 | b             |               |               | 396                       | $b = (12 - 18\varphi) = 2 \cdot 3 \cdot 11b$  | (5 isogeny   | classe        | es) |                              |                             |   |                           | 396b      |
| a1  | $\varphi$     | $-\varphi$    | 0             | $6-5\varphi$              | $9-4\varphi$                                  | 0            | 5             |     | 1, 3, 5                      | 1, 3, 5                     | 1, 1, 5                                 | $I_1, I_3, I_5$           |           |
| a2  | $\varphi$     | $-\varphi$    | 0             | $-564 + 340\varphi$       | $-6192 + 3728\varphi$                         | 0            | 1             |     | 5, 15, 1                     | 5, 15, 1                    | 5, 1, 1                                 | $I_5, I_{15}, I_1$        |           |
| b1  | $1+\varphi$   | $-1-\varphi$  | 0             | $-3-\varphi$              | $-9+2\varphi$                                 | 0            |               |     | 7, 1, 1                      | 7,1,1                       | $\begin{bmatrix} 1,1,1 \end{bmatrix}$   | $\mid I_7, I_1, I_1 \mid$ |           |
| c1  | 1             | $1-\varphi$   | $\varphi$     | $-168 - 258\varphi$       | $1513 + 2472\varphi$                          | 0            | 1 8           |     | $\bar{4}, \bar{4}, \bar{2}$  | 4,4,2                       | [2,4,2]                                 | $ $ $I_4, I_4, I_2$       | <u>-</u>  |
| c2  | 1             | $1-\varphi$   | arphi         | $-8-18\varphi$            | $25+40\varphi$                                | 0            | 8             |     | 2, 8, 1                      | 2, 8, 1                     | 2, 8, 1                                 | $I_2, I_8, I_1$           |           |
| c3  |               |               | $1 + \varphi$ | $-121472 - 196545\varphi$ | $30900787 + 49998522\varphi$                  | 0            | 4             |     | 2, 2, 1                      | 2, 2, 1                     | 2, 2, 1                                 | $I_2, I_2, I_1$           |           |
| c4  | $1 + \varphi$ | 0             | 0             | $-753 + 413\varphi$       | $-8855 + 5628\varphi$                         | 0            | 4             |     | 8, 2, 4                      | 8, 2, 4                     | 2, 2, 4                                 | $I_8, I_2, I_4$           |           |
| c5  | $1 + \varphi$ | 0             | 0             | $-663 + 413\varphi$       | $-10331 + 6942\varphi$                        | 0            | 2             | 1   | 16, 1, 2                     | 16, 1, 2                    | 2, 1, 2                                 | $I_{16}, I_1, I_2$        |           |
| c6  | $\varphi$     | 0             | 0             | $-80014 + 49435\varphi$   | $-10242231 + 6330026\varphi$                  | 0            | 2             | + + | 4,1,8                        | 4,1,8                       | [2, 1, 8]                               | $\mid I_4, I_1, I_8$      |           |
| d1  |               | $-1-\varphi$  |               | 0                         | $-1-2\varphi$                                 | 0            | 5             | 1   | 1, 5, 1                      | 1,5,1                       | [1, 5, 1]                               | $I_1, I_5, I_1$           |           |
| d2  | $1 + \varphi$ | $-1-\varphi$  | $1+\varphi$   | $-170 - 230\varphi$       | $-1329 - 2056\varphi$                         | 0            | 1             |     | 5, 1, 5                      | 5, 1, 5                     | 5, 1, 1                                 | $I_5, I_1, I_5$           |           |

|   |   |  |   |  |   |   |   |     |                                       |  | 1  |   |              |
|---|---|--|---|--|---|---|---|-----|---------------------------------------|--|--|---|--------------|
|   | $a_1$   | $a_2$  | $a_3$   | $a_4$  | $a_6$   | r   | T   | s   | $\operatorname{ord}(\Delta)$          | $\operatorname{ord}_{-}(j)$  | $c_p$  | Kodaira   | Isogenies    |
| 396   | b   |  |   | 396b =   | $(12 - 18\varphi) = 2 \cdot 3 \cdot 11b$ (5 is  | ogeny cl  | asses)  | ı   |                                       |  |  |   | <b>396</b> b |
| e1  | 1   | 0  | 1   | $-40 + 21\varphi$  | $101-66\varphi$   | 0   | 3   |     | 1, 1, 3                               | 1, 1, 3  | 1, 1, 3  | $I_1, I_1, I_3$   |              |
| e2  | $1+\varphi$   | 0  | 1   | $-5848 - 9461\varphi$  | $-330697 - 535080\varphi$   | 0   | 1   |     | 3, 3, 1                               | 3, 3, 1  | 1, 3, 1  | $I_3, I_3, I_1$   |              |
| 400   | ล   |  |   | 40   | $0a = (20) = 2^2 \cdot 5a^2$ (1 isoge   | eny class   | ١   |     |                                       |  |  |   | 400a         |
| a1  | 0   | $1-\varphi$  | 0   | $-6-7\varphi$  | $\frac{6a - (20) - 2}{-6 - 6\varphi}$   |   | 4   | ++  | 4,8                                   | 2  | 1,4  | $IV, I_2^*$   | 1000         |
| a2  | 0   | $1-\varphi$  | 0   | $-206-207\varphi$  | $1534 + 2114\varphi$  | 0   | 4   | + + | 4, 12                                 | 6  | 1, 4   | $IV, I_6^*$   |              |
| a3  | 0   | $-\varphi$   | 0   | $-16408 - 26528\varphi$  | $1546332 + 2502004\varphi$  | 0   | 2   | ++  | 8,9                                   | 3  | 1, 2   | $IV^*, I_3^*$   |              |
| a4  | 0   | $-1+\varphi$   | 0   | $-42936 + 26528\varphi$  | $4048336 - 2502004\varphi$  | 0   | 2   | ++  | 8, 9                                  | 3  | 1, 2   | $IV^*, I_3^*$   |              |
| a5  | 0   | $1-\varphi$  | 0   | $-81 - 107\varphi$   | $-406 - 681 \varphi$  | 0   | 2   | ++  | 8, 7                                  | 1  | 1, 2   | $IV^*, I_1^*$   |              |
| a6  | 0   | arphi  | 0   | $-188 + 107\varphi$  | $-1087 + 681\varphi$  | 0   | 2   | ++  | 8, 7                                  | 1  | 1, 2   | $IV^*, I_1^*$   |              |
| a7  | 0   | $1-\varphi$  | 0   | $-181-182\varphi$  | $1919 + 2619\varphi$  | 0   | 2   |     | 8, 18                                 | 12   | 1, 4   | $IV^*, I_{12}^*$  |              |
| a8  | 0   | $1-\varphi$  | 0   | $19 + 18\varphi$   | $-41 - 61\varphi$   | 0   | 2   |     | 8, 10                                 | 4  | 1,4  | $IV^*, I_4^*$   |              |
|   |   |  |   | 40.4   | (11 10 ) 401 (1:  | 1   | aa)   |     |                                       |  |  |   | 401a         |
| 401   | ล   |  |   | 401a   | a = (11 - 1802) = 401a (1.180)  | geny cla  |   |     |                                       |  |  |   |              |
| 401   |   | $-1+\varphi$   | 1   | 0  | $a = (11 - 18\varphi) = 401a$ (1 iso,   | geny cla<br>1   | 1   |     | 1                                     | 1  | 1  | $I_1$   | 1016         |
| a1  | 1   | $-1 + \varphi$ $-\varphi$  | 1   | 0  | 0   |   | 1   |     | 1                                     | 1  | 1  | I <sub>1</sub>  |              |
| 401 a1  | 1<br><b>b</b>   | · ·  |   | 0<br>401<br>0  | $b = (7 - 18\varphi) = 401b \qquad (1 \text{ isog} $  | geny clas   | s) 1  |     |                                       |  |  |   | 401h         |
| 401 a1  | 1<br><b>b</b>   | · ·  |   | 0<br>401<br>0  | $b = (7 - 18\varphi) = 401b \qquad (1 \text{ isog} $  | geny clas   | s) 1  |     |                                       |  |  |   | 401b         |
| 401 a1 404  | 1<br><b>b</b> 1   | $-\varphi$   | 1   | 0 $401$ $0$ $404a =$   | $b = (7 - 18\varphi) = 401b \qquad (1 \text{ isog}$ $0$ $= (10 - 18\varphi) = 2 \cdot 101a \qquad (4 \text{ iso}$   | geny clas   | s) 1  |     | 1                                     | 1  | 1  | $I_1$   | 401b         |
| a1 401 a1 404 a1  | 1 <b>b</b> 1 <b>a</b>   | $-\varphi$ $1+\varphi$   | 1   | $0$ $401$ $0$ $404a = -3 + 3\varphi$   | $b = (7 - 18\varphi) = 401b \qquad (1 \text{ isog}$ $0$ $= (10 - 18\varphi) = 2 \cdot 101a \qquad (4 \text{ iso}$ $2 - \varphi$   | geny clas  1  ogeny cla  1                                  | 1 s) 1 sses) 1 1 - 1 - 1                          |     | 1,2                                   | 1,2  | 1,2  | $I_1$ $I_1, I_2$  | 401h         |
| a1 401 a1 a1 b1 b1 b1   | $\mathbf{b}$ $\mathbf{a}$ $1$ $1+\varphi$   | $ \begin{array}{c} -\varphi \\ 1+\varphi \\ \hline 0 \end{array} $                                       | $\frac{1}{\varphi}$   | $0$ $401$ $0$ $404a = $ $-3 + 3\varphi$ $-60 + 38\varphi$ $-11 - 17\varphi$  | $b = (7 - 18\varphi) = 401b \qquad (1 \text{ isog}$ $0$ $= (10 - 18\varphi) = 2 \cdot 101a \qquad (4 \text{ iso}$ $\frac{2 - \varphi}{-219 + 135\varphi}$   | geny clas  1  ogeny cla  1  1  ogeny cla  1                 | s) 1 asses) 1 1 1 1                               |     | 1,2,1,7                               | 1 1,2 - 1,7  | 1,2,1,1  | $I_1$ $I_1, I_2$ $I_1, I_7$ $I_1, I_1$  | 401k         |
| a1 401 a1 404 a1 b1 c1  | $\mathbf{b}$ $\mathbf{a}$ $1$ $1+\varphi$   | $ \begin{array}{c c} -\varphi \\ \hline 1+\varphi \\ \hline 0 \\ \hline 1 \end{array} $                  | 1   | $0$ $401$ $0$ $404a = $ $-3 + 3\varphi$ $-60 + 38\varphi$  | $b = (7 - 18\varphi) = 401b \qquad (1 \text{ isog}$ $0$ $= (10 - 18\varphi) = 2 \cdot 101a \qquad (4 \text{ iso}$ $\frac{2 - \varphi}{-219 + 135\varphi}$ $-36 - 57\varphi$   | geny clas  1  ogeny clas  1  ogeny cla  1  0                | s) 1 sses) 1 1 1 1 1 1                            |     | 1,2,1,7,1,1                           | 1<br>1,2<br>1,7<br>1,1   | 1<br>1,2<br>1,1<br>1,1   | $I_1$ $I_1, I_2$ $I_1, I_7$   | 4011         |
| a1 401 a1 404 a1 b1 c1 d1 d2  | $\mathbf{a}$ $\mathbf{a}$ $\frac{1}{1+\varphi}$ $\frac{1}{\varphi}$ $\frac{1}{1+\varphi}$ | $ \begin{array}{c} -\varphi \\ 1+\varphi \\ 0 \\ 1 \\ -1\varphi \end{array} $                            | 1   | $0$ $401a = \frac{404a = \frac{-3 + 3\varphi}{-60 + 38\varphi} - \frac{-11 - 17\varphi}{-\varphi} - \frac{-\varphi}{-4919 + 3019\varphi}$                        | $b = (7 - 18\varphi) = 401b \qquad (1 \text{ isog}$ $0$ $= (10 - 18\varphi) = 2 \cdot 101a \qquad (4 \text{ iso}$ $\frac{2 - \varphi}{-219 + 135\varphi}$ $-36 - 57\varphi$ $1 + 6\varphi$ $-154440 + 95396\varphi$   | 1   1   1   1   1   1   1   1   1   1                       | s) 1 sses) 1 1 1 1 1 1 5 1                        |     | 1<br>1,2<br>1,7<br>1,1<br>1,10        | $ \begin{array}{ c c c c c } \hline 1, 2 \\ \hline 1, 7 \\ \hline 1, 1 \\ \hline 1, 10 \end{array} $   | $ \begin{array}{c c} 1 \\ \hline 1,2 \\ \hline 1,1 \\ \hline 1,1 \\ \hline 1,10 \end{array} $                    | $I_{1}$ $I_{1}, I_{2}$ $I_{1}, I_{7}$ $I_{1}, I_{1}$ $I_{1}, I_{10}$                | 4018         |
| a1 401 a1 404 a1 b1 c1 d1 d2 404                                    | $\mathbf{b}$ $\mathbf{a}$ $1$ $1$ $1$ $1 + \varphi$ $\varphi$ $1$ $\mathbf{b}$            | $ \begin{array}{c c} -\varphi \\ \hline 1+\varphi \\ \hline -1-\varphi \\ -1-\varphi \\ -1 \end{array} $ | 1   | $0$ $401a = -3 + 3\varphi$ $-60 + 38\varphi$ $-11 - 17\varphi$ $-\varphi$ $-4919 + 3019\varphi$ $404b = -2$  | $b = (7 - 18\varphi) = 401b \qquad (1 \text{ isog} $ $0$ $= (10 - 18\varphi) = 2 \cdot 101a \qquad (4 \text{ iso} $ $2 - \varphi$ $-219 + 135\varphi$ $-36 - 57\varphi$ $1 + 6\varphi$ $-154440 + 95396\varphi$ $= (8 - 18\varphi) = 2 \cdot 101b \qquad (4 \text{ isog} $                                    | geny clas    1     0     0     0     0     0     geny class | s) 1 sses) 1 1 1 1 1 1 5 1                        |     | 1<br>1,2<br>1,7<br>1,1<br>1,10<br>5,2 | $ \begin{array}{ c c c } \hline 1,2\\ \hline 1,7\\ \hline 1,1\\ \hline 1,10\\ 5,2\\ \end{array} $  | $ \begin{array}{c c} 1,2\\ 1,1\\ 1,1\\ 1,10\\ 5,2 \end{array} $  | $I_1 \\ I_1, I_2 \\ I_1, I_7 \\ I_1, I_1 \\ I_1, I_{10} \\ I_5, I_2$                | 401b         |
| a1 401 a1 404 a1 b1 d1 d2 404                                       | $\mathbf{b}$ $\mathbf{a}$ $1$ $1$ $1$ $1 + \varphi$ $\varphi$ $1$ $\mathbf{b}$            | $ \begin{array}{c} -\varphi \\ 1+\varphi \\ 0 \\ -1 \\ -1-\varphi \\ -1 \end{array} $                    | $ \begin{array}{c} 1 \\ \hline                                  $ | $0$ $401a = \frac{401a}{0}$ $404a = \frac{-3 + 3\varphi}{-60 + 38\varphi}$ $-11 - 17\varphi$ $-\varphi$ $-4919 + 3019\varphi$ $404b = \frac{404b}{-1 - \varphi}$ | $b = (7 - 18\varphi) = 401b \qquad (1 \text{ isog}$ $0$ $= (10 - 18\varphi) = 2 \cdot 101a \qquad (4 \text{ iso}$ $\frac{2 - \varphi}{-219 + 135\varphi}$ $-36 - 57\varphi$ $1 + 6\varphi$ $-154440 + 95396\varphi$ $= (8 - 18\varphi) = 2 \cdot 101b \qquad (4 \text{ isog}$ $2 + 3\varphi$                  | geny clas  geny clas  ogeny cla  1  0  0  0  geny clas      | s) 1 sses) 1 1 1 1 1 1 5 1                        |     | 1, 2<br>1, 7<br>1, 1<br>1, 10<br>5, 2 | $ \begin{array}{c c} 1, 2 \\ \hline 1, 7 \\ \hline 1, 1 \\ \hline 1, 10 \\ 5, 2 \end{array} $  | $ \begin{array}{c c} 1 & 1,2 \\ \hline 1,1 & 1,1 \\ \hline 1,10 & 5,2 \end{array} $                              | $I_{1}$ $I_{1}, I_{2}$ $I_{1}, I_{1}$ $I_{1}, I_{10}$ $I_{5}, I_{2}$ $I_{1}, I_{2}$ | 4011         |
| a1       401       a1       404       a1       b1       d1       d2 | $\mathbf{b}$ $\mathbf{a}$ $1$ $1 + \varphi$ $1$ $\mathbf{b}$ $1$                          | $ \begin{array}{c} -\varphi \\ 1+\varphi \\ 0 \\ -1 \\ -1-\varphi \\ -1 \end{array} $                    | 1   | $0$ $401$ $0$ $404a = -3 + 3\varphi$ $-60 + 38\varphi$ $-11 - 17\varphi$ $-\varphi$ $-4919 + 3019\varphi$ $404b = -1 - \varphi$ $-22 - 39\varphi$                | $b = (7 - 18\varphi) = 401b \qquad (1 \text{ isog} $ $0$ $= (10 - 18\varphi) = 2 \cdot 101a \qquad (4 \text{ isog} $ $2 - \varphi$ $-219 + 135\varphi$ $-36 - 57\varphi$ $1 + 6\varphi$ $-154440 + 95396\varphi$ $= (8 - 18\varphi) = 2 \cdot 101b \qquad (4 \text{ isog} $ $2 + 3\varphi$ $-84 - 136\varphi$ | 1   1   1   1   1   1   1   1   1   1                       | s) 1 usses) 1 1 1 1 1 5 1 1 1 1 1 1 1 1 1 1 1 1 1 |     | 1,2<br>1,7<br>1,1<br>1,10<br>5,2      | $ \begin{array}{ c c c } \hline 1, 2 \\ \hline 1, 7 \\ \hline 1, 1 \\ \hline 1, 10 \\ 5, 2 \\ \hline 1, 2 \\ \hline 1, 7 \\ \hline \end{array} $ | $ \begin{array}{c c} 1,2\\ 1,1\\ 1,1\\ 1,10\\ 5,2\\ \end{array} $ $ \begin{array}{c c} 1,2\\ 1,1\\ \end{array} $ | $I_{1}$ $I_{1}, I_{2}$ $I_{1}, I_{7}$ $I_{1}, I_{10}$ $I_{5}, I_{2}$ $I_{1}, I_{7}$ | 401b         |
| a1 401 a1 404 a1 b1 d1 d2 404                                       | $\mathbf{b}$ $\mathbf{a}$ $1$ $1 + \varphi$ $1$ $\mathbf{b}$ $1$                          | $ \begin{array}{c} -\varphi \\ 1+\varphi \\ 0 \\ -1 \\ -1-\varphi \\ -1 \end{array} $                    | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$             | $0$ $401a = \frac{401a}{0}$ $404a = \frac{-3 + 3\varphi}{-60 + 38\varphi}$ $-11 - 17\varphi$ $-\varphi$ $-4919 + 3019\varphi$ $404b = \frac{404b}{-1 - \varphi}$ | $b = (7 - 18\varphi) = 401b \qquad (1 \text{ isog}$ $0$ $= (10 - 18\varphi) = 2 \cdot 101a \qquad (4 \text{ iso}$ $\frac{2 - \varphi}{-219 + 135\varphi}$ $-36 - 57\varphi$ $1 + 6\varphi$ $-154440 + 95396\varphi$ $= (8 - 18\varphi) = 2 \cdot 101b \qquad (4 \text{ isog}$ $2 + 3\varphi$                  | geny clas  geny clas  ogeny cla  1  0  0  0  geny clas      | s) 1 usses) 1 1 1 1 1 5 1 1 1 1 1 1 1 1 1 1 1 1 1 |     | 1, 2<br>1, 7<br>1, 1<br>1, 10<br>5, 2 | $ \begin{array}{c c} 1, 2 \\ \hline 1, 7 \\ \hline 1, 1 \\ \hline 1, 10 \\ 5, 2 \end{array} $  | $ \begin{array}{c c} 1 & 1,2 \\ \hline 1,1 & 1,1 \\ \hline 1,10 & 5,2 \end{array} $                              | $I_{1}$ $I_{1}, I_{2}$ $I_{1}, I_{1}$ $I_{1}, I_{10}$ $I_{5}, I_{2}$ $I_{1}, I_{2}$ | 401b         |

|              | $a_1$         | $a_2$                                 | $a_3$         | $a_4$                                 |   | $a_6$        | r                                      | T | s   | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$             | Kodaira                                       | Isogenies |
|--------------|---------------|---------------------------------------|---------------|---------------------------------------|---|--------------|--|---|-----|------------------------------|-----------------------------|-------------------|---|-----------|
| 405a         | 1             |                                       |               | 405a                                  | $= (9 - 18\varphi) = 5a \cdot 3^2 $ (1 i          | sogeny cla   | )<br>266)                              |   |     |                              |                             |                   |   | 405a      |
| a1           | 1             | -1                                    | 0             | -19440                                | $\frac{-(3-10\varphi)-5u\cdot 3}{1048}$           |              | $\frac{0}{0}$                          | 4 | ++  | 10, 2                        | 4, 2                        | 4, 2              | $I_4^*, I_2$                                  |           |
| a1<br>a2     | 1             | -1 $-1$                               | 0             | -19440 $-1215$                        |   | I .          | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 4 | + + | 10, 2 $14, 4$                | 8,4                         | $\frac{4,2}{4,2}$ | $I_{4}^{1}, I_{2}^{1}$ $I_{8}^{*}, I_{4}^{1}$ |           |
| a3           | 1             | -1                                    | 0             | -90                                   |   |              | 0                                      | 4 | + + | 10, 8                        | 4,8                         | 4, 2              | $I_{8}^{*}, I_{4}$ $I_{4}^{*}, I_{8}$         |           |
| a4           | 1             | -1                                    | 0             | -45                                   |   |              | $\stackrel{\circ}{0}$                  | 4 | ++  | 8,4                          | 2, 4                        | 4, 2              | $I_2^*, I_4$                                  |           |
| a5           | 1             | -1                                    | 0             | 0                                     |   |              | 0                                      | 4 |     | 7, 2                         | 1, 2                        | 4, 2              | $I_1^*, I_2$                                  |           |
| a6           | 1             | -1                                    | 1             | $-27682172 - 44790597\varphi$         | 106637960337 + 17254384400                        | I .          | 0                                      | 2 | -+  | 8, 1                         | 2, 1                        | 4, 1              | $I_2^*, I_1$                                  |           |
| a7           | 1             | -1                                    | 1             | $-72472769 + 44790597\varphi$         | 279181804337 - 17254384400                        |              | 0                                      | 2 | + - | 8, 1                         | 2, 1                        | 4, 1              | $I_2^*, I_1$                                  |           |
| a8           | 1             | -1                                    | 0             | -990                                  | 22  | 765          | 0                                      | 2 |     | 22, 2                        | 16, 2                       | 4, 2              | $\overline{\mathrm{I}_{16}^*}, \mathrm{I}_2$  |           |
| a9           | 1             | -1                                    | 0             | -720                                  |   |              | 0                                      | 2 | + + | 7, 2                         | 1, 2                        | 4, 2              | $I_1^*, I_2$                                  |           |
| a10          | 1             | -1                                    | 0             | 315                                   | 1   | 066          | 0                                      | 2 |     | 8, 16                        | 2, 16                       | 4, 2              | $I_2^*, I_{16}$                               |           |
| 409a         | l             |                                       |               | 409a                                  | $a = (19 + 3\varphi) = 409a$ (1 is                | ogeny cla    | ss)                                    |   |     |                              |                             |                   |   | 409a      |
| a1           | 1             | $-1-\varphi$                          | $1 + \varphi$ | 0                                     |   | $-\varphi$   | 1                                      | 1 | -+  | 1                            | 1                           | 1                 | $I_1$   |           |
| 409h         | )             |                                       |               | 409                                   | $b = (22 - 3\varphi) = 409b$ (1 is                | ogeny clas   | ss)                                    |   |     |                              |                             |                   |   | 409b      |
| a1           | 1             | $1+\varphi$                           | $1 + \varphi$ | arphi                                 |   | 0            | 1                                      | 1 | + - | 1                            | 1                           | 1                 | $I_1$   |           |
| <b>419</b> a | L             |                                       |               | 419a                                  | $a = (21 - \varphi) = 419a$ (2 isog               | geny class   | ses)                                   |   |     |                              |                             |                   |   | 419a      |
| a1           | $\varphi$     | 1                                     | $1+\varphi$   | $-4+2\varphi$                         | 2 —   | $2\varphi$   | 1                                      | 1 |     | 1                            | 1                           | 1                 | $I_1$   |           |
| b1           | 1             | $-1+\varphi$                          | 1             | $-\varphi$                            |   | 0            | 1                                      | 1 |     | 1                            | 1                           | 1                 | $I_1$   |           |
| 419h         | )             |                                       |               | 419b                                  | $\phi = (20 + \varphi) = 419b$ (2 isog            | eny class    | es)                                    |   |     |                              |                             |                   |   | 419b      |
| a1           | $1 + \varphi$ | $1-\varphi$                           | $\varphi$     | $-1-4\varphi$                         | 1 -   | <b>+</b> φ   | 1                                      | 1 |     | 1                            | 1                           | 1                 | $I_1$   |           |
| b1           | 1             | $-\varphi$                            | 1             | $-1+\varphi$                          |   | 0            | 1                                      | 1 |     | 1                            | 1                           | 1                 | $I_1$   |           |
| 431a         | L             |                                       |               | 431a                                  | $=(5-19\varphi)=431a$ (4 iso                      | geny clas    | ses)                                   |   |     |                              |                             |                   |   | 431a      |
| a1           | 0             | $\varphi$                             | $1 + \varphi$ | $-1+\varphi$                          |   | $-\varphi$   | 1                                      | 1 | + - | 1                            | 1                           | 1                 | $I_1$   |           |
| b1           | $1+\varphi$   | 0                                     | $\varphi$     | 0                                     |   | 0            | 0                                      | 3 |     | 1                            | 1                           | 1                 | $I_1$   | [         |
| b2           | $1+\varphi$   | 0                                     | arphi         | $-5-10\varphi$                        | -15 - 5   | $20\varphi$  | 0                                      | 1 |     | 3                            | 3                           | 1                 | $I_3$   |           |
| c1           | $\varphi$     | 1                                     | $1+\varphi$   | -2                                    |   | $-\varphi$   | 0                                      | 4 | + - | 1                            | 1                           | 1                 | $I_1$   | [         |
| c2           | $\varphi$     | 1                                     | $1+\varphi$   | -7-5arphi                             | 1+  | , ,          | 0                                      | 4 | ++  | 2                            | 2                           | 2                 | $I_2$   |           |
| c3           | $1+\varphi$   | $1 + \varphi$                         |               | $-426-676\varphi$                     | 5706 + 924  | $ 9\varphi $ | 0                                      | 2 | ++  | 1                            | 1                           | 1                 | $I_1$   |           |
| c4           | $\varphi$     |                                       | $1+\varphi$   | $8-15\varphi$                         |   | $4\varphi$   | 0                                      | 2 | -+  | 4                            | 4                           | 2                 | $I_4$   |           |
| d1           | 0             | 1                                     | $1+\varphi$   | $-22-32\varphi$                       | 59 + 9  | $9\varphi$   | 0                                      | 3 | + - | 1                            | 1                           | 1                 | $I_1$   |           |
| d2           | 0             | $-1-\varphi$                          | $1+\varphi$   | $-985 + 604\varphi$                   | -13812 + 858                                      | $60\varphi$  | 0                                      | 1 | + - | 3                            | 3                           | 1                 | $I_3$   |           |
| <b>431</b> k | )             |                                       |               | 431b                                  | $= (14 - 19\varphi) = 431b \qquad (4 \text{ iso}$ | geny clas    | ses)                                   | ) |     |                              |                             |                   |   | 431b      |
| a1           | 0             | $1-\varphi$                           | $\varphi$     | $-\varphi$                            |   | 0            | 1                                      | 1 | -+  | 1                            | 1                           | 1                 | $I_1$   |           |
|              |               | · · · · · · · · · · · · · · · · · · · | •             | · · · · · · · · · · · · · · · · · · · |   |              |  |   |     |                              | 1                           |                   | 1   |           |

|     | $a_1$       |                    |                 | $a_3$      | $a_4$                   |   | 6 r                | r             | T                            | s   | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                   | $c_p$  | Kodaira  | Isogenies       |
|-----|-------------|--------------------|-----------------|------------|-------------------------|---|--------------------|---------------|------------------------------|-----|------------------------------|---|--|--|-----------------|
|     | α1          | a                  | -2              |            |                         |   | 6 /                |               | -                            |     |                              | ord_(J)                                       |  | Rodalia  | isogemes        |
| 431 | b           |                    |                 |            | 4                       | $31b = (14 - 19\varphi) = 431b \tag{4}$ | isogeny classe     | $\mathbf{s})$ |                              |     |                              |   |  |  | 431b            |
| b1  | $\varphi$   | $1-\varphi$        | $\rho$ 1        | $+\varphi$ | $1-2\varphi$            | —(                                      | 0                  |               | 3                            |     | 1                            | 1   | 1  | $I_1$  |                 |
| b2  | $\varphi$   | $1-\zeta$          | $\rho$ 1        | $+\varphi$ | $-14 + 8\varphi$        | -35 + 196                               | $\circ$ 0          |               | 1                            |     | 3                            | 3   | 1  | $I_3$  |                 |
| c1  | $1+\varphi$ | 1 - 9              | ρ               | $\varphi$  | $-1-2\varphi$           |   | 0   0              | -             | 4                            | - + | 1                            | 1   | 1  | $\bar{\mathrm{I}}_1$                           | [               |
| c2  | $1+\varphi$ | $1-\zeta$          | ρ               | $\varphi$  | $-11+3\varphi$          | 8 - 76                                  | $\circ$ 0          |               | 4                            | ++  | 2                            | 2   | 2  | $I_2$  |                 |
| c3  | $\varphi$   | 4                  | $\rho$ 1        | $+\varphi$ | $-1104 + 677\varphi$    | 16735 - 103536                          | 0                  |               | 2                            | + + | 1                            | 1   | 1  | $I_1$  |                 |
| c4  | $1+\varphi$ | $1-\varphi$        | ρ               | $\varphi$  | $-6+13\varphi$          | 5 - 50                                  | $o \mid 0$         |               | 2                            | + - | 4                            | 4   | 2  | $I_4$  |                 |
| d1  | 0           |                    | 1               | $\varphi$  | $-54+32\varphi$         | 159 - 1000                              | $\rho$ $\mid 0$    |               | 3                            | -+  | 1                            | 1   | 1  | $I_1$  | <u> </u>        |
| d2  | 0           | $1+\varsigma$      | ρ               | φ          | $-382 - 602\varphi$     | -5643 - 91546                           |                    |               | 1                            | -+  | 3                            | 3   | 1  | $I_3$  |                 |
| 441 | a           |                    |                 |            |                         | $441a = (21) = 3 \cdot 7$ (3 iso        | geny classes)      |               |                              |     |                              |   |  |  | 441a            |
| a1  | 1           |                    | 0               | 0          | -39                     | 9                                       |                    | Τ.            | 8                            | ++  | 8,1                          | 8, 1  | 8, 1   | $I_8, I_1$                                     |                 |
| a2  | 1           |                    | 0               | 0          | -4                      | _                                       |                    | - 1           | 8                            | + + | 4, 2                         | 4, 2  | 4, 2   | $I_4, I_2$                                     |                 |
| a3  | 1           |                    | 0               | 0          | 1                       |   | 0                  |               | 4                            |     | 2, 1                         | 2, 1  | 2,1  | $I_2, I_1$                                     |                 |
| a4  | 1           |                    | 0               | 0          | -49                     | -13                                     | 0                  |               | 4                            | ++  | 2, 4                         | 2, 4  | 2, 4   | $I_2, I_4$                                     |                 |
| a5  | 1           |                    | 0               | 0          | -34                     | -21                                     | 7 0                |               | 2                            |     | 1,8                          | 1,8   | 1,8  | $I_1, I_8$                                     |                 |
| a6  | 1           | (                  | 0               | 0          | -784                    | -851                                    | 0                  |               | 2                            | ++  | 1, 2                         | 1, 2  | 1, 2   | $I_1, I_2$                                     |                 |
| b1  | $1+\varphi$ | (                  | 0               |            | $-26096 + 16128\varphi$ | 1907142 - 11786796                      | $o \mid 0$         | - i           | $2^{-1}$                     | ++  | 1, 1                         | 1,1   | 1,1  | $\overline{I}_1,\overline{I}_1$                | <u>  </u>       |
| b2  | 1           | $1-\varphi$        | ρ               | $\varphi$  | $-239 + 143\varphi$     | 1511 - 9386                             |                    |               | 4                            | ++  | 2, 2                         | 2, 2  | 2,2  | $I_2, I_2$                                     |                 |
| b3  | $\varphi$   | — <i>'</i>         |                 | $\varphi$  | -3                      | -2c                                     | 0                  |               | 4                            |     | 4, 1                         | 4, 1  | 4, 1   | $I_4, I_1$                                     |                 |
| b4  | $1+\varphi$ | -1 + 9             | $\rho$ 1        | $+\varphi$ | $-1429 - 2302\varphi$   | -40441 - 654526                         | $\circ$ 0          |               | 2                            | ++  | 1, 4                         | 1,4   | 1,4  | $I_1, I_4$                                     |                 |
| c1  | $1+\varphi$ |                    | - <b>-</b><br>ρ | $\varphi$  | $-3-3\varphi$           | -6 - 90                                 | $\circ$ $\mid$ $0$ | - i           | $\frac{1}{2}$                | ++  | 3,1                          | 3,1   | 1,1  | $I_3, I_1$                                     | <u>  </u>       |
| c2  | $1+\varphi$ |                    | ρ               | φ          | $2+2\varphi$            | -11 - 146                               |                    |               | 2                            |     | 6, 2                         | 6, 2  | 2,2  | $I_6, I_2$                                     |                 |
| 445 | а           |                    |                 |            | 44:                     | $5a = (12 - 19\varphi) = 5a \cdot 89a$  | (2 isogeny class   | ses)          | ١                            |     |                              |   |  |  | 445a            |
| a1  | φ           | <u> </u>           |                 | (0         | -2                      |   | 0 1                | T             | 4                            | ++  | 2,1                          | 2,1   | 2, 1   | $I_2, I_1$                                     |                 |
| a2  | $\varphi$   | -γ                 |                 | arphi      | $-12+5\varphi$          | -19 + 110                               |                    |               | 4                            | ++  | $\frac{2}{4}, \frac{1}{2}$   | $\frac{2}{4}, \frac{1}{2}$                    | $\begin{bmatrix} 2, 1 \\ 2, 2 \end{bmatrix}$ | $I_2, I_1$ $I_4, I_2$                          |                 |
| a3  | $\varphi$   | — <u>Y</u>         |                 | $\varphi$  | $-172 + 110\varphi$     | -1080 + 6600                            |                    |               | $\frac{1}{2}$                | + - | 8, 1                         | 8, 1  | 2, 2   | $I_{8}, I_{1}$                                 |                 |
| a4  | $1+\varphi$ | $-1 + \frac{7}{9}$ |                 |            | $-82-134\varphi$        | -612 - 9840                             |                    |               | $\frac{1}{2}$                | - + | 2, 4                         | 2, 4  | 2, 4   | $I_2, I_4$                                     |                 |
| b1  | $1+\varphi$ | 1 – 4              |                 | $\varphi$  | $-38 + 21\varphi$       | 83 - 530                                |                    | -:            | $\frac{1}{4}^{-\frac{1}{4}}$ | ++  | $\frac{1}{2}, \frac{1}{1}$   | $\frac{1}{2}, \frac{1}{1}$                    | $\frac{1}{2}, \frac{1}{1}$                   | $I_2, I_1$                                     | <u>-</u>        |
| b2  | 1           | $1+\varsigma$      |                 | $\varphi$  | $-74-108\varphi$        | -484 - 7936                             |                    |               | $\frac{1}{4}$                | ++  | 4, 2                         | 4, 2  | $\frac{2}{4}, \frac{1}{2}$                   | $I_{2}, I_{1}$ $I_{4}, I_{2}$                  |                 |
| b3  | 1           |                    | 0               | $\varphi$  | $-64-113\varphi$        | -471 - 8156                             |                    |               | $\stackrel{1}{4}$            |     | 8, 4                         | 8,4   | 8,4  | $I_{8}, I_{4}$                                 |                 |
| b4  |             | $-1-\zeta$         |                 |            | $-52081 - 84268\varphi$ | -8685673 - 140537146                    |                    |               | 2                            | ++  | 2, 1                         | 2, 1  | 2, 1   | $I_2, I_1$                                     |                 |
| 445 | b           |                    |                 |            | 44                      | $5b = (7 - 19\varphi) = 5a \cdot 89b$ ( | 2 isogeny class    | es)           |                              |     |                              |   |  |  | $445\mathrm{b}$ |
| al  | $1+\varphi$ |                    | 1 1             | + 10       | $-2-2\varphi$           | -(· 104)                                |                    | Ť             | 4                            | ++  | 2, 1                         | 2,1   | 2, 1   | $I_2, I_1$                                     |                 |
| a2  | $1+\varphi$ |                    | 1 1             |            | $-7-7\varphi$           | -8 - 126                                |                    |               | 4                            | ++  | $\frac{2}{4}, \frac{1}{2}$   | $\frac{2}{4}, \frac{1}{2}$                    | $\begin{bmatrix} 2, 1 \\ 2, 2 \end{bmatrix}$ | $egin{array}{c} I_2,I_1 \ I_4,I_2 \end{array}$ |                 |
| a3  | $\varphi$   | $1+\varsigma$      |                 | 1  1       | $-214+134\varphi$       | -1462 + 9026                            |                    |               | 2                            | + - | $\frac{4}{2}, \frac{2}{4}$   | 2, 4  | $\begin{bmatrix} 2, 2 \\ 2, 4 \end{bmatrix}$ | $I_{2}, I_{4}$                                 |                 |
| a4  | $1+\varphi$ |                    | 1 1             |            | $-62 - 112\varphi$      | -420 - 6616                             |                    |               | $\frac{2}{2}$                | _ + | 8,1                          | 8,1   | $\begin{bmatrix} 2, 4 \\ 2, 1 \end{bmatrix}$ | $I_{8}, I_{1}$                                 |                 |
| ат  | - 1 Ψ       | •                  |                 | ' "        | υς 112ψ                 | 420 0010                                | _   1              |               | -                            | - 1 | 0, 1                         | , <u>, , , , , , , , , , , , , , , , , , </u> |  | -8,-1  |                 |

|              |               |              |               |                          |  |  | ı    | T        |                              | 1                           | ı     | I   |           |
|--------------|---------------|--------------|---------------|--------------------------|--|--|------|----------|------------------------------|-----------------------------|-------|---|-----------|
|              | $a_1$         | $a_2$        | $a_3$         | $a_4$                    | $a_6$  | r                                      | T    | s        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$ | Kodaira   | Isogenies |
| 445          | )             |              |               | 44                       | $5b = (7 - 19\varphi) = 5a \cdot 89b$ (2 i     | sogeny classe                          | s)   |          |                              |                             |       |   | 445b      |
| b1           | $\varphi$     |              | $1+\varphi$   | $-16-23\varphi$          | $30 + 52\varphi$                               | 0                                      | 4    | ++       | 2, 1                         | 2, 1                        | 2,1   | $I_2, I_1$  |           |
| b2           | 1             | r            | $\varphi$     | $-183 + 109\varphi$      | $-1094 + 684\varphi$                           | 0                                      | 4    | ++       | 4, 2                         | 4, 2                        | 4, 2  | $I_4, I_2$  |           |
| b3           | 1             | - r          | $\varphi$     | $-178 + 114\varphi$      | $-1108 + 701\varphi$                           | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 4    |          | 8,4                          | 8,4                         | 8,4   | $I_8, I_4$  |           |
| b4           | φ             | -1           | φ             | $-136347 + 84266\varphi$ | $-22739386 + 14053713\varphi$                  | 0                                      | 2    | ++       | 2,1                          | 2,1                         | 2,1   | $I_2, I_1$  |           |
| 449a         | a.            |              |               | 44                       | $49a = (11 - 19\varphi) = 449a$ (2 is          | sogeny classes                         |      |          |                              |                             |       |   | 449a      |
| a1           | 0             | $-\varphi$   | $1 + \varphi$ | $-1+\varphi$             | $-\varphi$                                     | 1                                      | 1    |          | 1                            | 1                           | 1     | $I_1$   |           |
| b1           | 0             |              | $\varphi$     | -arphi                   | 0  | 0                                      | 3    |          | 1                            | 1                           | 1     | $I_1$   |           |
| b2           | 0             | $-1-\varphi$ | φ             | $-28 + 20\varphi$        | $-70 + 41\varphi$                              | 0                                      | 1    |          | 3                            | 3                           | 1     | $I_3$   |           |
| <b>449</b> ł | )             |              |               | 4                        | $49b = (8 - 19\varphi) = 449b$ (2 iso          | ogeny classes)                         |      |          |                              |                             |       |   | 449b      |
| a1           | 0             | $-1+\varphi$ | $\varphi$     | $-\varphi$               | 0  | 1                                      | 1    |          | 1                            | 1                           | 1     | $I_1$   |           |
| b1           | 0             | 1            | $1+\varphi$   | $-1+\varphi$             | -arphi   | 0                                      | 3    |          | 1                            | 1                           | 1     | $I_1$   |           |
| b2           | 0             | $1+\varphi$  | $1 + \varphi$ | $-9-18\varphi$           | $-38-61\varphi$                                | 0                                      | 1    |          | 3                            | 3                           | 1     | $I_3$   |           |
| 451a         | ì             |              |               | 4516                     | $a = (10 - 19\varphi) = 11a \cdot 41a \tag{3}$ | isogeny class                          | ses) |          |                              |                             |       |   | 451a      |
| a1           | 0             | $1-\varphi$  | $1+\varphi$   | $2-\varphi$              | $-\varphi$                                     | 1                                      | 1    |          | 2, 1                         | 2, 1                        | 2, 1  | $I_2, I_1$  |           |
| b1           | $\varphi$     | $-\varphi$   |               | $-\varphi$               | arphi  | 0                                      | 3    | <u> </u> | 1, 1                         | 1,1                         | 1,1   | $I_1, I_1$  |           |
| b2           | $\varphi$     | $-\varphi$   | 0             | $-35 + 24\varphi$        | $-106 + 68\varphi$                             | 0                                      | 1    |          | 3, 3                         | 3, 3                        | 1,1   | $I_3, I_3$  |           |
| c1           | $1+\varphi$   | $-1+\varphi$ | 0             | $-127 - 193\varphi$      | $-1070 - 1754\varphi$                          | 0                                      | 1    |          | 5, 1                         | 5,1                         | 1,1   | $I_5, I_1$  |           |
| 451k         | )             |              |               | 451                      | $b = (9 - 19\varphi) = 11b \cdot 41b \tag{3}$  | isogeny classe                         | es)  |          |                              |                             |       |   | 451b      |
| a1           | 0             | $\varphi$    | $\varphi$     | $1+\varphi$              | 0  | 1                                      | 1    |          | 2, 1                         | 2,1                         | 2,1   | $I_2, I_1$  |           |
| b1           | $1+\varphi$   | -1           |               | $-1+\varphi$             | 1-arphi  |  | 3    | <u> </u> | 1,1                          | 1,1                         | 1,1   | $\overline{\mathrm{I}}_{1},\overline{\mathrm{I}}_{1}$ | <u>  </u> |
| b2           | $1 + \varphi$ | -1           | 0             | $-11-24\varphi$          | $-38-68\varphi$                                | 0                                      | 1    |          | 3, 3                         | 3,3                         | 1,1   | $I_3, I_3$  |           |
| c1           | $\varphi$     | $1+\varphi$  | $1+\varphi$   | $-321 + 194\varphi$      | $-2631 + 1626\varphi$                          | 0                                      | 1    |          | 5, 1                         | 5,1                         | 1,1   | $I_5, I_1$  |           |
| 4510         | :             |              |               | 45                       | $1c = (23 - 3\varphi) = 11a \cdot 41b \tag{1}$ | isogeny class                          | s)   |          |                              | 1                           |       | ı   | 451c      |
| a1           | 1             | $\varphi$    | 1             | -1                       | $-1-\varphi$                                   | 0                                      | 2    |          | 2, 1                         | 2, 1                        | 2,1   | $I_2, I_1$  |           |
| a2           | $1 + \varphi$ |              | 1             | $-48 + 26\varphi$        | $-130 + 77\varphi$                             | 0                                      | 2    | ++       | 1, 2                         | 1, 2                        | 1,2   |   |           |
| 4510         | i             |              |               | 45                       | $1d = (20 + 3\varphi) = 11b \cdot 41a \tag{1}$ | isogeny class                          | s)   |          |                              |                             |       |   | 451d      |
| a1           | 1             | $1-\varphi$  | 1             | -1                       | $-2+\varphi$                                   | 0                                      | 2    |          | ,                            | 2, 1                        | 2, 1  | $I_2, I_1$  |           |
| a2           | $\varphi$     | $-\varphi$   | 1             | $-21-27\varphi$          | $-53 - 77\varphi$                              | 0                                      | 2    | ++       | 1,2                          | 1, 2                        | 1,2   | $I_1, I_2$  |           |
| 4648         |               |              |               |                          | $4a = (24 - 4\varphi) = 2^2 \cdot 29a \tag{3}$ | isogeny classe                         | s)   |          |                              |                             |       |   | 464a      |
| a1           | 0             |              | 0             | $-2+\varphi$             | 10.7   | 1                                      | 2    | ++       |                              | 1                           | 3,1   | $IV, I_1$   |           |
| a2           | 0             | $-\varphi$   | 0             | $-7+6\varphi$            | $-10 + 7\varphi$                               | 1                                      | 2    | + -      | 8, 2                         | 2                           | 3, 2  | $IV^*, I_2$   |           |

|  | $a_1$   | $a_2$   | $a_3$  | $a_4$  | $a_6$   | r   | T                              | s   | $\operatorname{ord}(\Delta)$   | $\operatorname{ord}_{-}(j)$   | $c_p$   | Kodaira  | Isogenies      |
|--|---|---|--|--|---|---|--------------------------------|---|--|---|---|--|----------------|
|  |   |   |  |  |   |   |                                |   |  |   |   |  |                |
|  |   |   |  |  |   |   |                                |   |  |   |   |  |                |
| <b>64</b> a  | l   |   |  | 464a   | $= (24 - 4\varphi) = 2^2 \cdot 29a $ (3 is  | ogeny classe  | s)                             |   |  |   |   |  | <b>46</b> 4    |
| b1   | 0   | -1  | 0  | $-4+2\varphi$  | 5-3arphi  | 0   | 6                              | +++   | 4, 1   | 1   | 3, 1  | $IV, I_1$  |                |
| $^{\circ 2}$   | 0   | -1  | 0  | $-4-3\varphi$  | $12 + \varphi$  | 0   | 6                              | - +   | 8, 2   | 2   | 3, 2  | $IV^*, I_2$  |                |
| 03   | 0   | $-1-\varphi$  | 0  | $-142-222\varphi$  | $-1083 - 1747\varphi$   | 0   | 2                              | ++  | 4, 3   | 3   | 3, 1  | $IV, I_3$  |                |
| 04   | 0   | -1  | 0  | $-4-43\varphi$   | $-268 + 33\varphi$  | 0   | 2                              | -+  | 8,6  | 6   | 3, 2  | $IV^*, I_6$  |                |
| 1  | 0   | 0   | 0  | $-29 + 17\varphi$  | $-69 + 43\varphi$   | 0   | 2                              | ++  | 4, 1   | 1   | 1,1   | $IV, I_1$  |                |
| 2  | 0   | 0   | 0  | $-47-67 \varphi$   | $202 + 336\varphi$  | 0   | 2                              | -+  | 8, 2   | 2   | 1, 2  | $IV^*, I_2$  |                |
|  |   |   |  |  |   |   |                                |   |  |   |   |  |                |
|  |   |   |  |  |   |   |                                |   |  |   |   |  |                |
| 64l  | )   |   |  | 464b   | $= (20 + 4\varphi) = 2^2 \cdot 29b $ (3 iso   | ogeny classes   | s)                             |   |  |   |   |  | 46             |
| 1  | 0   | $-1+\varphi$  | 0  | $-1-\varphi$   | 1   | 1   | 2                              | ++  | 4, 1   | 1   | 3, 1  | $IV, I_1$  |                |
| 2  |   | $-1+\varphi$  | 0  | $-1-6\varphi$  | -3-7arphi   | 1   | $\frac{1}{2}$                  | - +   | 8, 2   | 2   | 3, 2  | $IV^*, I_2$  |                |
| $1 \stackrel{!}{\mid}$   |   | <del>'</del><br>-1  | 0  | $-2-2\varphi$  | $2+3\varphi$  | 0   | ' ·<br>  6                     | <br>  | $\frac{1}{4}, \frac{1}{1}$   | '<br>  1  | $\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$  | $  IV, I_1  $  | <u>-</u> :<br> |
| $\frac{1}{2}$  | 0   | -1  | 0  | $-7+3\varphi$  | $13 - \varphi$  |   | 6                              | + -   | 8, 2   | 2   | 3, 2  | $IV^*, I_2$  |                |
| $\frac{1}{3}$  | 0   | $1+\varphi$   | 0  | $-365+224\varphi$  | $-3195 + 1970\varphi$   | 0   | $\frac{3}{2}$                  | ++  | 4, 3   | 3   | 3, 1  | $IV, I_3$  |                |
| 4  | 0   | -1  | 0  | $-47 + 43\varphi$  | $-235-33\varphi$  | 0   | $\frac{1}{2}$                  | + -   | 8,6  | 6   | 3, 2  | $IV^*, I_6$  |                |
|  |   |   |  | $-12-17\varphi$  | $\begin{array}{cccccccccccccccccccccccccccccccccccc$  |   | 2                              | <br>  | $\frac{1}{4}, \frac{1}{1}$   | <u>-</u> ' 1  | 1,1   | $ \tilde{IV}, \tilde{I_1} $  | <u>-</u>       |
| 1  | ()  | ()  | · · · · · ·  |  |   | ()  |                                |   |  |   |   |  |                |
|  | $0 \\ 0$  | $0 \\ 0$  | $0 \\ 0$   |  | •   | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$  | $\frac{2}{2}$                  | + -   |  | $\frac{1}{2}$   | $1, 1 \\ 1, 2$  |  |                |
|  |   | •   | -  | $-12 - 17\varphi$ $-114 + 67\varphi$   | $-26 - 43\varphi$ $538 - 336\varphi$  |   |                                |   | 8, 2   |   |   | $IV^*, I_2$  |                |
|  |   | •   | -  |  | •   |   |                                |   |  |   |   |  |                |
| c2   | 0   | •   | -  | $-114 + 67\varphi$   | $538 - 336\varphi$  | 0   | 2                              |   |  |   |   |  | 47:            |
| 7 <b>5</b> a   | 0<br><b>1</b>   | 0   | 0  | $-114 + 67\varphi$ $475a =$  | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^2 \cdot 19a \qquad (3 \text{ is}$  | osogeny classe  | 2<br>es)                       | +-  | 8, 2   | 2   | 1,2   | $IV^*, I_2$  | 47             |
| 7 <b>5</b> 2   | 0<br><b>1</b> φ   | $\frac{0}{0}$   | $\varphi$  | $-114 + 67\varphi$ $475a =$ $-1 - \varphi$   | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^2 \cdot 19a \qquad (3 \text{ is} $   | sogeny classe   | es) 2                          | -+  | 1,3  | 2   | 1,2   | $IV^*, I_2$ $I_1, III$   | 47             |
| 75a  | $\frac{1}{\varphi}$   | $\begin{array}{c} 0 \\ -\varphi \\ -\varphi \end{array}$  | $\varphi$ $\varphi$  | $-114 + 67\varphi$ $475a =$ $-1 - \varphi$ $-11 - 6\varphi$  | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^2 \cdot 19a \qquad (3 \text{ is} $ $0$ $15 + 15\varphi$  | sogeny classed 1 1  | es) 2 2                        | + -<br>- +<br>+ +   | 1, 3<br>2, 3   | 1 2   | 1,2   | $IV^*, I_2$ $I_1, III$ $I_2, III$  | 47             |
| 75a  | $\frac{\mathbf{v}}{\varphi}$  | $ \begin{array}{c}                                     $  | $\frac{\varphi}{1}$  | $-114 + 67\varphi$ $475a =$ $-1 - \varphi$ $-11 - 6\varphi$ $-327 - 483\varphi$  | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^{2} \cdot 19a \qquad (3 \text{ is}$ $0$ $15 + 15\varphi$ $3852 + 6105\varphi$  | sogeny classo $ \begin{array}{c c}  & 1 \\ \hline  & 1 \\ \hline  & 0 \end{array} $ | es) 2 2 4                      | + -<br>  - +<br>  + +<br>  - +                                | 1,3<br>2,3<br>3,7  | 1<br>2<br>3,1   | 1,2<br>2,2<br>1,4   |  | 47             |
| 7 <b>5</b> a 1 2 1 2 1 2 1 2   | $ \begin{array}{c} 0 \\ \hline 1 \\ \varphi \\ \varphi \end{array} $  | $ \begin{array}{c}                                     $  | $\begin{matrix} \varphi \\ \varphi \\ \vdots \\ 1 \end{matrix}$  | $-114 + 67\varphi$ $475a =$ $-1 - \varphi$ $-11 - 6\varphi$ $-327 - 483\varphi$ $-2 - 8\varphi$  | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^{2} \cdot 19a \qquad (3 \text{ is}$ $0$ $15 + 15\varphi$ $3852 + 6105\varphi$ $2 + 5\varphi$   | sogeny classe   1   | es) 2 2 4 4                    | + -<br>  - +<br>  + +<br>  - +                                | $ \begin{array}{c} 1,3 \\ -2,3 \\ -3,7 \\ 1,9 \end{array} $  | $ \begin{array}{c c}  & 1 \\  & 2 \\ \hline  & 3, 1 \\  & 1, 3 \end{array} $  | 1, 2<br>2, 2<br>1, 4<br>1, 4  | $IV^*, I_2$ $I_1, III$ $I_2, III$ $I_3, I_1^*$ $I_1, I_3^*$  | 47             |
| 75a 1   2   01   02   03   | $ \begin{array}{c} 0 \\ \hline 1 \\ \varphi \\ \varphi \end{array} $  | $ \begin{array}{c}                                     $  | $\begin{array}{c} \varphi \\ \varphi \\ \varphi \\ 1 \\ 1 \\ 1 \end{array}$  | $-114 + 67\varphi$ $475a = \frac{-1 - \varphi}{-11 - 6\varphi}$ $-327 - 483\varphi$ $-2 - 8\varphi$ $-27 - 33\varphi$  | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^{2} \cdot 19a \qquad (3 \text{ is}$ $0$ $15 + 15\varphi$ $3852 + 6105\varphi$ $2 + 5\varphi$ $-78 - 135\varphi$  | sogeny classe    1  | es) 2 2 4 4 4                  | -+<br>++<br>-+<br>-+<br>++                                    | 1,3<br>2,3<br>3,7<br>1,9<br>2,12   | $ \begin{array}{c c}  & 1 \\  & 2 \\ \hline  & 3, 1 \\  & 1, 3 \\  & 2, 6 \end{array} $   | 1, 2<br>2, 2<br>1, 4<br>1, 4<br>2, 4  | $IV^*, I_2$ $I_1, III$ $I_2, III$ $I_3, I_1^*$ $I_1, I_3^*$ $I_2, I_6^*$   | 47             |
| 7 <b>5</b> a 1 2 1 2 3 4   | $ \begin{array}{c} 0 \\ \hline 1 \\ \varphi \\ \varphi \end{array} $  | $ \begin{array}{c}                                     $  | $ \begin{array}{c} \varphi \\ \varphi \\ -\frac{\varphi}{1} \\ 1 \\ 1 \\ \varphi \end{array} $   | $-114 + 67\varphi$ $-174 + 67\varphi$ $-17 - \varphi$ $-11 - 6\varphi$ $-17 - 483\varphi$ $-27 - 483\varphi$ $-27 - 33\varphi$ $-2416 + 1393\varphi$   | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^{2} \cdot 19a \qquad (3 \text{ is})$ $0$ $15 + 15\varphi$ $3852 + 6105\varphi$ $2 + 5\varphi$ $-78 - 135\varphi$ $51811 - 31488\varphi$  | 0<br>  sogeny classe<br>  1<br>  1<br>  1<br>  0<br>  0<br>  0<br>  0               | 2 2 2 4 4 4 4 4                | -+<br>++<br>-+<br>++<br>++                                    | 1,3<br>2,3<br>3,7<br>1,9<br>2,12<br>6,8  | 1<br>2<br>3,1<br>1,3<br>2,6<br>6,2  | $ \begin{array}{ c c c } \hline 1,2\\2,2\\\hline 1,4\\1,4\\2,4\\2,4\\2,4\end{array} $                   |  | 47             |
| 75a 1 2 1 2 3 4 5  | $ \begin{array}{c}                                     $  | $ \begin{array}{c}                                     $  | $\begin{matrix} \varphi \\ \varphi \\ \vdots \\ \varphi \\ \varphi \end{matrix}$   | $-114 + 67\varphi$ $-17 - \varphi$ $-11 - 6\varphi$ $-17 - 483\varphi$ $-27 - 483\varphi$ $-27 - 33\varphi$ $-2416 + 1393\varphi$ $-466 + 293\varphi$  | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^{2} \cdot 19a \qquad (3 \text{ is})$ $0$ $15 + 15\varphi$ $3852 + 6105\varphi$ $2 + 5\varphi$ $-78 - 135\varphi$ $51811 - 31488\varphi$ $4431 - 2778\varphi$   | 0<br>  sogeny classe<br>  1<br>  1<br>  1<br>  0<br>  0<br>  0<br>  0<br>  0<br>  0 | 2 2 2 4 4 4 4 2 2              | -+-<br>++<br>-+<br>++<br>++                                   | 1,3<br>2,3<br>3,7<br>1,9<br>2,12<br>6,8<br>1,18  | 1<br>2<br>3,1<br>1,3<br>2,6<br>6,2<br>1,12  | 1,2<br>2,2<br>1,4<br>1,4<br>2,4<br>2,4<br>1,4   |  | 47             |
| 75a 1  | $ \begin{array}{c} 0 \\ \hline  & \varphi \\ \hline  & \varphi \\  & \varphi \\  & \varphi \\  & 1 \\ 1 \\ 1 + \varphi \end{array} $  | $ \begin{array}{c}                                     $  | $ \begin{array}{c} \varphi \\ \varphi \\ -\frac{\varphi}{1} \\ 1 \\ 1 \\ \varphi \end{array} $   | $-114 + 67\varphi$ $475a = \frac{-1 - \varphi}{-11 - 6\varphi}$ $-327 - 483\varphi$ $-2 - 8\varphi$ $-27 - 33\varphi$ $-2416 + 1393\varphi$ $-466 + 293\varphi$ $-260086 + 160737\varphi$  | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^2 \cdot 19a \qquad (3 \text{ is} $ $0$ $15 + 15\varphi$ $3852 + 6105\varphi$ $2 + 5\varphi$ $-78 - 135\varphi$ $51811 - 31488\varphi$ $4431 - 2778\varphi$ $59979689 - 37069439\varphi$  | 0<br>  sogeny classe<br>  1   | 2 2 2 4 4 4 4 2 2 2            | - + -<br>+ + +<br>- +<br>+ +<br>+ +<br>+ -<br>+ -             | 1,3<br>2,3<br>3,7<br>1,9<br>2,12<br>6,8<br>1,18<br>3,10  | $\begin{array}{ c c c }\hline & 1 \\ & 2 \\ \hline & 3,1 \\ & 1,3 \\ & 2,6 \\ & 6,2 \\ & 1,12 \\ & 3,4 \\ \end{array}$                  | $\begin{array}{ c c c }\hline 1,2\\ 2,2\\ \hline 1,4\\ 1,4\\ 2,4\\ 2,4\\ 1,4\\ 1,4\\ 1,4\\ \end{array}$ | $IV^*, I_2$ $I_1, III$ $I_2, III$ $I_3, I_1^*$ $I_1, I_3^*$ $I_2, I_6^*$ $I_6, I_2^*$ $I_1, I_{12}^*$ $I_3, I_4^*$   | 47             |
| 75a<br>1   22   33   44   55   66   7  | $ \begin{array}{c}                                     $  | $ \begin{array}{c} -\varphi \\ -\varphi \\ -1+\varphi \\ -1+\varphi \\ -1+\varphi \\ -\varphi \\ -\varphi \\ 1+\varphi \\ 1+\varphi \end{array} $                             | $ \begin{array}{c} \varphi \\ \varphi \\ 1 \\ 1 \\ \varphi \\ \varphi \\ 1 + \varphi \\ 1 \end{array} $  | $-114 + 67\varphi$ $475a = -1 - \varphi$ $-11 - 6\varphi$ $-327 - 483\varphi$ $-2 - 8\varphi$ $-27 - 33\varphi$ $-2416 + 1393\varphi$ $-2466 + 293\varphi$ $-260086 + 160737\varphi$ $-402 - 533\varphi$   | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^2 \cdot 19a \qquad (3 \text{ is} $ $0$ $15 + 15\varphi$ $3852 + 6105\varphi$ $2 + 5\varphi$ $-78 - 135\varphi$ $51811 - 31488\varphi$ $4431 - 2778\varphi$ $59979689 - 37069439\varphi$ $-5078 - 7885\varphi$  | 0<br>  sogeny classe<br>  1   | es)  2 2 4 4 4 4 2 2 2         | -+<br>++<br>-+<br>++<br>++<br>++<br>+-<br>++                  | 1,3<br>2,3<br>3,7<br>1,9<br>2,12<br>6,8<br>1,18<br>3,10<br>4,9   | $\begin{array}{ c c c }\hline 2\\ \hline & 1\\ & 2\\ \hline & 3,1\\ & 1,3\\ & 2,6\\ & 6,2\\ & 1,12\\ & 3,4\\ & 4,3\\ \\ \end{array}$    | 1, 2<br>2, 2<br>1, 4<br>1, 4<br>2, 4<br>2, 4<br>1, 4<br>2, 4<br>1, 4<br>2, 4                            | $IV^*, I_2$ $I_1, III$ $I_2, III$ $I_3, I_1^*$ $I_1, I_3^*$ $I_2, I_6^*$ $I_6, I_2^*$ $I_1, I_{12}^*$ $I_3, I_4^*$ $I_4, I_3^*$  | 47             |
| 75a 1   2   3   3   4   5   5   66   67   68   3   6   6   6   7   6   8   5   6   6   6   7   6   8   5   6   6   6   7   6   8   5   6   6   6   6   6   7   6   6   6   6   | $\begin{matrix} \varphi \\ \varphi \\ \varphi \\ \varphi \\ \varphi \\ 1 \\ 1 \\ 1 + \varphi \\ \varphi \\ 1 \end{matrix}$  | $ \begin{array}{c} -\varphi \\ -\varphi \\ -1+\varphi \\ -1+\varphi \\ -1+\varphi \\ -\varphi \\ -\varphi \\ 1+\varphi \\ -1+\varphi \\ -\varphi \\ -\varphi \end{array} $    | $ \begin{array}{c} \varphi \\ \varphi \\ 1 \\ 1 \\ 1 \\ \varphi \\ \varphi \\ 1 + \varphi \\ 1 \\ \varphi \end{array} $  | $-114 + 67\varphi$ $475a = \frac{-1 - \varphi}{-11 - 6\varphi}$ $-327 - 483\varphi$ $-2 - 8\varphi$ $-27 - 33\varphi$ $-2416 + 1393\varphi$ $-466 + 293\varphi$ $-260086 + 160737\varphi$ $-402 - 533\varphi$ $-2466 + 1368\varphi$  | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^2 \cdot 19a \qquad (3 \text{ is})$ $0$ $15 + 15\varphi$ $3852 + 6105\varphi$ $2 + 5\varphi$ $-78 - 135\varphi$ $51811 - 31488\varphi$ $4431 - 2778\varphi$ $59979689 - 37069439\varphi$ $-5078 - 7885\varphi$ $50386 - 31188\varphi$                                       | 0<br>  sogeny classe<br>  1   | es) 2 2 4 4 4 2 2 2 2          | -+<br>++<br>-+<br>++<br>++<br>+-<br>+-<br>++                  | 1,3<br>2,3<br>3,7<br>1,9<br>2,12<br>6,8<br>1,18<br>3,10<br>4,9<br>12,7   | 1<br>2<br>3,1<br>1,3<br>2,6<br>6,2<br>1,12<br>3,4<br>4,3<br>12,1  | 1,2<br>2,2<br>1,4<br>1,4<br>2,4<br>2,4<br>1,4<br>1,4<br>2,4<br>2,4                                      | $ \begin{array}{c c} IV^*, I_2 \\ \hline I_1, III \\ I_2, III \\ \hline I_3, I_1^* \\ I_1, I_3^* \\ I_2, I_6^* \\ I_6, I_2^* \\ I_1, I_{12}^* \\ I_3, I_4^* \\ I_4, I_3^* \\ I_{12}, I_1^* \\ \hline \end{array} $                             | 47             |
| 75a 1   2   3   4   5   5   66   77   68   1   1   1   | $\begin{matrix} 0 \\ \\ \mathbf{v} \\ \varphi \\ \varphi \\ \varphi \\ 1 \\ 1 \\ 1 + \varphi \\ \varphi \\ 1 \\ 1 \\ 1 + \varphi \end{matrix}$  | $ \begin{array}{c} -\varphi \\ -\varphi \\ -1+\varphi \\ -1+\varphi \\ -1+\varphi \\ -\varphi \\ -\varphi \\ 1+\varphi \\ 1+\varphi \\ 1+\varphi \end{array} $                | $ \begin{array}{c} \varphi \\ \varphi \\ 1 \\ 1 \\ 1 \\ \varphi \\ \varphi \\ 1 + \varphi \\ 1 \\ \varphi \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0$ | $-114 + 67\varphi$ $-17 - \varphi$ $-11 - 6\varphi$ $-11 - 6\varphi$ $-327 - 483\varphi$ $-2 - 8\varphi$ $-27 - 33\varphi$ $-2416 + 1393\varphi$ $-466 + 293\varphi$ $-260086 + 160737\varphi$ $-402 - 533\varphi$ $-2466 + 1368\varphi$ $-3 + 3\varphi$                             | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^{2} \cdot 19a \qquad (3 \text{ is})$ $0$ $15 + 15\varphi$ $3852 + 6105\varphi$ $2 + 5\varphi$ $-78 - 135\varphi$ $51811 - 31488\varphi$ $4431 - 2778\varphi$ $59979689 - 37069439\varphi$ $-5078 - 7885\varphi$ $50386 - 31188\varphi$ $-7 + 3\varphi$                     | sogeny classe    1  | es)  2 2 4 4 4 2 2 2 2 2       | -+-<br>++<br>-+<br>++<br>++<br>+-<br>+-<br>++<br>++           | $\begin{array}{c} 8,2 \\ \hline \\ 1,3 \\ 2,3 \\ \hline \\ 3,7 \\ 1,9 \\ 2,12 \\ 6,8 \\ 1,18 \\ 3,10 \\ 4,9 \\ 12,7 \\ \hline \\ 1,9 \\ \end{array}$ | $\begin{array}{ c c c }\hline 2\\\hline &1\\&2\\\hline &3,1\\&1,3\\&2,6\\&6,2\\&1,12\\&3,4\\&4,3\\&12,1\\\hline &1\\\hline \end{array}$ | 1,2<br>2,2<br>1,4<br>1,4<br>2,4<br>2,4<br>1,4<br>2,4<br>2,4<br>1,2                                      | $ \begin{array}{ c c c c }\hline IV^*,I_2\\\hline &I_1,III\\\hline &I_2,III\\\hline &I_3,I_1^*\\\hline &I_1,I_3^*\\&I_2,I_6^*\\&I_6,I_2^*\\&I_1,I_{12}^*\\&I_3,I_{4}^*\\&I_4,I_{3}^*\\&I_{12},I_{1}^*\\\hline &I_1,III^*\\\hline \end{array} $ | 47             |
| 75a 1   2   2   2   2   2   2   2   2   2  | $\begin{matrix} \varphi \\ \varphi \\ \varphi \\ \varphi \\ \varphi \\ 1 \\ 1 \\ 1 + \varphi \\ \varphi \\ 1 \end{matrix}$  | $ \begin{array}{c} -\varphi \\ -\varphi \\ -1+\varphi \\ -1+\varphi \\ -1+\varphi \\ -\varphi \\ -\varphi \\ 1+\varphi \\ -1+\varphi \\ -\varphi \\ -\varphi \end{array} $    | $ \begin{array}{c} \varphi \\ \varphi \\ 1 \\ 1 \\ 1 \\ \varphi \\ \varphi \\ 1 + \varphi \\ 1 \\ \varphi \end{array} $  | $-114 + 67\varphi$ $475a = \frac{-1 - \varphi}{-11 - 6\varphi}$ $-327 - 483\varphi$ $-2 - 8\varphi$ $-27 - 33\varphi$ $-2416 + 1393\varphi$ $-466 + 293\varphi$ $-260086 + 160737\varphi$ $-402 - 533\varphi$ $-2466 + 1368\varphi$  | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^2 \cdot 19a \qquad (3 \text{ is})$ $0$ $15 + 15\varphi$ $3852 + 6105\varphi$ $2 + 5\varphi$ $-78 - 135\varphi$ $51811 - 31488\varphi$ $4431 - 2778\varphi$ $59979689 - 37069439\varphi$ $-5078 - 7885\varphi$ $50386 - 31188\varphi$                                       | 0<br>  sogeny classe<br>  1   | es) 2 2 4 4 4 2 2 2 2          | -+<br>++<br>-+<br>++<br>++<br>+-<br>+-<br>++                  | 1,3<br>2,3<br>3,7<br>1,9<br>2,12<br>6,8<br>1,18<br>3,10<br>4,9<br>12,7   | 1<br>2<br>3,1<br>1,3<br>2,6<br>6,2<br>1,12<br>3,4<br>4,3<br>12,1  | 1,2<br>2,2<br>1,4<br>1,4<br>2,4<br>2,4<br>1,4<br>1,4<br>2,4<br>2,4                                      | $ \begin{array}{c c} IV^*, I_2 \\ \hline I_1, III \\ I_2, III \\ \hline I_3, I_1^* \\ I_1, I_3^* \\ I_2, I_6^* \\ I_6, I_2^* \\ I_1, I_{12}^* \\ I_3, I_4^* \\ I_4, I_3^* \\ I_{12}, I_1^* \\ \hline \end{array} $                             | 47             |
| 75a 1   22   275a 1   22   275a 2   275 | $\begin{matrix} 0 \\ \\ \mathbf{v} \\ \varphi \\ \varphi \\ \varphi \\ 1 \\ 1 \\ 1 + \varphi \\ \varphi \\ 1 \\ 1 \\ 1 + \varphi \end{matrix}$  | $ \begin{array}{c} -\varphi \\ -\varphi \\ -1+\varphi \\ -1+\varphi \\ -1+\varphi \\ -\varphi \\ -\varphi \\ 1+\varphi \\ 1+\varphi \\ 1+\varphi \end{array} $                | $ \begin{array}{c} \varphi \\ \varphi \\ 1 \\ 1 \\ 1 \\ \varphi \\ \varphi \\ 1 + \varphi \\ 1 \\ \varphi \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0$ | $-114 + 67\varphi$ $-17 - \varphi$ $-11 - 6\varphi$ $-11 - 6\varphi$ $-327 - 483\varphi$ $-2 - 8\varphi$ $-27 - 33\varphi$ $-2416 + 1393\varphi$ $-466 + 293\varphi$ $-260086 + 160737\varphi$ $-402 - 533\varphi$ $-2466 + 1368\varphi$ $-3 + 3\varphi$                             | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^{2} \cdot 19a \qquad (3 \text{ is})$ $0$ $15 + 15\varphi$ $3852 + 6105\varphi$ $2 + 5\varphi$ $-78 - 135\varphi$ $51811 - 31488\varphi$ $4431 - 2778\varphi$ $59979689 - 37069439\varphi$ $-5078 - 7885\varphi$ $50386 - 31188\varphi$ $-7 + 3\varphi$                     | sogeny classe    1  | es)  2 2 4 4 4 2 2 2 2 2       | -+-<br>++<br>-+<br>++<br>++<br>+-<br>+-<br>++<br>++           | $\begin{array}{c} 8,2 \\ \hline \\ 1,3 \\ 2,3 \\ \hline \\ 3,7 \\ 1,9 \\ 2,12 \\ 6,8 \\ 1,18 \\ 3,10 \\ 4,9 \\ 12,7 \\ \hline \\ 1,9 \\ \end{array}$ | $\begin{array}{ c c c }\hline 2\\\hline &1\\&2\\\hline &3,1\\&1,3\\&2,6\\&6,2\\&1,12\\&3,4\\&4,3\\&12,1\\\hline &1\\\hline \end{array}$ | 1,2<br>2,2<br>1,4<br>1,4<br>2,4<br>2,4<br>1,4<br>2,4<br>2,4<br>1,2                                      | $ \begin{array}{ c c c c }\hline IV^*,I_2\\\hline &I_1,III\\\hline &I_2,III\\\hline &I_3,I_1^*\\\hline &I_1,I_3^*\\&I_2,I_6^*\\&I_6,I_2^*\\&I_1,I_{12}^*\\&I_3,I_{4}^*\\&I_4,I_{3}^*\\&I_{12},I_{1}^*\\\hline &I_1,III^*\\\hline \end{array} $ | 47             |
| 75a  1   | $\begin{matrix} \varphi \\ \varphi \\ \varphi \\ \varphi \\ \varphi \\ \varphi \\ 1 \\ 1 \\ 1 + \varphi \\ \varphi \\ 1 \\ 1 + \varphi \\ 1 \\ 1 + \varphi \\ 1 + \varphi \end{matrix}$ | $ \begin{array}{c} -\varphi \\ -\varphi \\ -1+\varphi \\ -1+\varphi \\ -1+\varphi \\ -\varphi \\ -\varphi \\ 1+\varphi \\ 1+\varphi \\ 1+\varphi \end{array} $                | $ \begin{array}{c} \varphi \\ \varphi \\ 1 \\ 1 \\ 1 \\ \varphi \\ \varphi \\ 1 + \varphi \\ 1 \\ \varphi \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0$ | $-114 + 67\varphi$ $-17 - \varphi$ $-11 - 6\varphi$ $-17 - 8\varphi$ $-27 - 483\varphi$ $-27 - 33\varphi$ $-2416 + 1393\varphi$ $-466 + 293\varphi$ $-260086 + 160737\varphi$ $-402 - 533\varphi$ $-2466 + 1368\varphi$ $-3 + 3\varphi$ $-78 + 28\varphi$                            | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^{2} \cdot 19a \qquad (3 \text{ is})$ $0$ $15 + 15\varphi$ $3852 + 6105\varphi$ $2 + 5\varphi$ $-78 - 135\varphi$ $51811 - 31488\varphi$ $4431 - 2778\varphi$ $59979689 - 37069439\varphi$ $-5078 - 7885\varphi$ $50386 - 31188\varphi$ $-7 + 3\varphi$ $-307 + 103\varphi$ | 0     0   | es)  2 2 4 4 4 2 2 2 2 2 2     | -+-<br>++<br>-+<br>++<br>++<br>+-<br>+-<br>++<br>++           | $\begin{array}{c} 8,2 \\ \hline \\ 1,3 \\ 2,3 \\ \hline \\ 3,7 \\ 1,9 \\ 2,12 \\ 6,8 \\ 1,18 \\ 3,10 \\ 4,9 \\ 12,7 \\ \hline \\ 1,9 \\ \end{array}$ | $\begin{array}{ c c c }\hline 2\\\hline &1\\&2\\\hline &3,1\\&1,3\\&2,6\\&6,2\\&1,12\\&3,4\\&4,3\\&12,1\\\hline &1\\\hline \end{array}$ | 1,2<br>2,2<br>1,4<br>1,4<br>2,4<br>2,4<br>1,4<br>2,4<br>2,4<br>1,2                                      | $ \begin{array}{ c c c c }\hline IV^*,I_2\\\hline &I_1,III\\\hline &I_2,III\\\hline &I_3,I_1^*\\\hline &I_1,I_3^*\\&I_2,I_6^*\\&I_6,I_2^*\\&I_1,I_{12}^*\\&I_3,I_{4}^*\\&I_4,I_{3}^*\\&I_{12},I_{1}^*\\\hline &I_1,III^*\\\hline \end{array} $ |                |
| 75a a1 a2 b1 b2 b3 b4 b5 b6 b7 b8 c1 c2 75k  | $ \begin{array}{c}                                     $  | $ \begin{array}{c} -\varphi \\ -\varphi \\ -1+\varphi \\ -1+\varphi \\ -1+\varphi \\ -\varphi \\ -1+\varphi \\ 1+\varphi \\ 1+\varphi \\ 1+\varphi \\ 1+\varphi \end{array} $ | $ \begin{array}{c} \varphi \\ \varphi \\ 1 \\ 1 \\ \varphi \\ \varphi \\ 1 \\ \varphi \\ 1 \\ 1 \end{array} $  | $-114 + 67\varphi$ $475a = -1 - \varphi$ $-11 - 6\varphi$ $-327 - 483\varphi$ $-2 - 8\varphi$ $-27 - 33\varphi$ $-2416 + 1393\varphi$ $-466 + 293\varphi$ $-260086 + 160737\varphi$ $-402 - 533\varphi$ $-2466 + 1368\varphi$ $-3 + 3\varphi$ $-78 + 28\varphi$ $475b = -126\varphi$ | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^{2} \cdot 19a \qquad (3 \text{ is} $ $0$ $15 + 15\varphi$ $3852 + 6105\varphi$ $2 + 5\varphi$ $-78 - 135\varphi$ $51811 - 31488\varphi$ $4431 - 2778\varphi$ $59979689 - 37069439\varphi$ $-5078 - 7885\varphi$ $50386 - 31188\varphi$ $-7 + 3\varphi$ $-307 + 103\varphi$ | sogeny classe    1  | es)  2 2 4 4 4 2 2 2 2 2 2 2 2 | - + -<br>+ +<br>- +<br>+ +<br>+ -<br>+ -<br>+ +<br>+ +<br>+ + | 1,3<br>2,3<br>3,7<br>1,9<br>2,12<br>6,8<br>1,18<br>3,10<br>4,9<br>12,7<br>1,9<br>2,9   | $\begin{array}{ c c c }\hline 2\\\hline &1\\&2\\\hline &3,1\\&1,3\\&2,6\\&6,2\\&1,12\\&3,4\\&4,3\\&12,1\\\hline &1\\\hline \end{array}$ | 1,2<br>2,2<br>1,4<br>1,4<br>2,4<br>2,4<br>1,4<br>2,4<br>2,4<br>2,4<br>2,2                               | $IV^*, I_2$ $I_1, III$ $I_2, III$ $I_3, I_1^*$ $I_1, I_3^*$ $I_2, I_6^*$ $I_6, I_2^*$ $I_1, I_{12}^*$ $I_3, I_4^*$ $I_4, I_3^*$ $I_{12}, I_1^*$ $I_1, III^*$ $I_2, III^*$  |                |
| b7   b8   c1   c2   <b>75k</b> a1  | $\begin{matrix} \varphi \\ \varphi \\ \varphi \\ \varphi \\ \varphi \\ \varphi \\ 1 \\ 1 \\ 1 + \varphi \\ \varphi \\ 1 \\ 1 + \varphi \\ 1 \\ 1 + \varphi \\ 1 + \varphi \end{matrix}$ | $ \begin{array}{c} -\varphi \\ -\varphi \\ -1+\varphi \\ -1+\varphi \\ -1+\varphi \\ -\varphi \\ -\varphi \\ 1+\varphi \\ 1+\varphi \\ 1+\varphi \end{array} $                | $ \begin{array}{c} \varphi \\ \varphi \\ 1 \\ 1 \\ 1 \\ \varphi \\ \varphi \\ 1 + \varphi \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} $   | $-114 + 67\varphi$ $-17 - \varphi$ $-11 - 6\varphi$ $-17 - 8\varphi$ $-27 - 483\varphi$ $-27 - 33\varphi$ $-2416 + 1393\varphi$ $-466 + 293\varphi$ $-260086 + 160737\varphi$ $-402 - 533\varphi$ $-2466 + 1368\varphi$ $-3 + 3\varphi$ $-78 + 28\varphi$                            | $538 - 336\varphi$ $= (5 - 20\varphi) = 5a^{2} \cdot 19a \qquad (3 \text{ is})$ $0$ $15 + 15\varphi$ $3852 + 6105\varphi$ $2 + 5\varphi$ $-78 - 135\varphi$ $51811 - 31488\varphi$ $4431 - 2778\varphi$ $59979689 - 37069439\varphi$ $-5078 - 7885\varphi$ $50386 - 31188\varphi$ $-7 + 3\varphi$ $-307 + 103\varphi$ | 0     0   | es)  2 2 4 4 4 2 2 2 2 2 2     | -+-<br>++<br>-+<br>++<br>++<br>+-<br>+-<br>++<br>++           | $\begin{array}{c} 8,2 \\ \hline \\ 1,3 \\ 2,3 \\ \hline \\ 3,7 \\ 1,9 \\ 2,12 \\ 6,8 \\ 1,18 \\ 3,10 \\ 4,9 \\ 12,7 \\ \hline \\ 1,9 \\ \end{array}$ | $\begin{array}{ c c c }\hline 2\\\hline &1\\&2\\\hline &3,1\\&1,3\\&2,6\\&6,2\\&1,12\\&3,4\\&4,3\\&12,1\\\hline &1\\\hline \end{array}$ | 1,2<br>2,2<br>1,4<br>1,4<br>2,4<br>2,4<br>1,4<br>2,4<br>2,4<br>1,2                                      | $ \begin{array}{ c c c c }\hline IV^*,I_2\\\hline &I_1,III\\\hline &I_2,III\\\hline &I_3,I_1^*\\\hline &I_1,I_3^*\\&I_2,I_6^*\\&I_6,I_2^*\\&I_1,I_{12}^*\\&I_3,I_{4}^*\\&I_4,I_{3}^*\\&I_{12},I_{1}^*\\\hline &I_1,III^*\\\hline \end{array} $ | 475            |

|          |  |   |     |                     |   |  |          |  |        | ı     |                              |                             | T  |   |                 |
|----------|--|---|-----|---------------------|---|--|----------|--|--------|-------|------------------------------|-----------------------------|--|---|-----------------|
|          | $a_1$                                  | $a_2$   |     | $a_3$               | $a_4$   | $a_6$  |          | r                                      | T      | s     | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$  | Kodaira   | Isogenies       |
| 475      | b                                      |   |     |                     | 47  | $5b = (15 - 20\varphi) = 5a^2 \cdot 19b$           | (3 iso   | geny c                                 | lasses | s)    |                              |                             |  |   | $475\mathrm{b}$ |
| b1       | $1+\varphi$                            | $\varphi$   |     | 0                   | $-809 + 484\varphi$                                   | $10440 - 6432\varphi$                              |          | 0                                      | 4      | + -   | 3,7                          | 3,1                         | 1,4  | $I_3, I_1^*$  |                 |
| b2       | $1+\varphi$                            | $\varphi$   |     | 0                   | $-9+9\varphi$   | $15-7\varphi$                                      |          | 0                                      | 4      | +-    | 1, 9                         | 1,3                         | 1,4  | $I_1, I_3^*$  |                 |
| b3       | $1+\varphi$                            | $\varphi$   |     | 0                   | $-59 + 34\varphi$                                     | $-180 + 108\varphi$                                |          | 0                                      | 4      | ++    | 2,12                         | 2,6                         | 2,4  | $I_2, I_6^*$  |                 |
| b4       | 1                                      | $-1+\varphi$  |     |                     | $-1023 - 1394\varphi$                                 | $20323 + 31487\varphi$                             |          | 0                                      | 4      | ++    | 6, 8                         | 6, 2                        | 2,4  | $I_6, I_2^*$  |                 |
| b5       | 1                                      | $-1+\varphi$  |     | •                   | $-173 - 294\varphi$                                   | $1653 + 2777\varphi$                               |          | 0                                      | 2      | -+    | 1,18                         | 1,12                        | 1,4  | $I_1, I_{12}^*$   |                 |
| b6       | $\varphi$                              | $\varphi$   | 1 + | $\varphi$           | $-99351 - 160736\varphi$                              | $22848864 + 36970088\varphi$                       |          | 0                                      | 2      | -+    | 3, 10                        | 3,4                         | 1,4  | $I_3, I_4^*$  |                 |
| b7       | $1+\varphi$                            | $\varphi$   |     | 0                   | $-934 + 534\varphi$                                   | $-12430 + 7483\varphi$                             |          | 0                                      | 2      | ++    | 4,9                          | 4,3                         | 2,4  | $I_4, I_3^*$  |                 |
| b8       | 1                                      | $-1+\varphi$  | 1 + | $\varphi$           | $-1098 - 1369\varphi$                                 | $19198 + 31187\varphi$                             |          | 0                                      | 2      | ++    | 12, 7                        | 12, 1                       | 2,4  | $I_{12},I_1^*$  |                 |
| c1       | $\varphi$                              | $\varphi$   |     | 0                   | -3  | $-3-5\varphi$                                      |          | 0                                      | 2      | + -   | 1,9                          | 1                           | 1,2  | $   I_1, III^* $  |                 |
| c2       | $\varphi$                              | $\varphi$   |     | 0                   | $-53 - 25\varphi$                                     | $-178 - 155\varphi$                                |          | 0                                      | 2      | ++    | 2,9                          | 2                           | 2,2  | $I_2, III^*$  |                 |
| 479      | a                                      |   |     |                     |   | $479a = (21 + 2\varphi) = 479a$                    | (1 isoge | eny cla                                | uss)   |       |                              |                             |  |   | 479a            |
| a1       | 1                                      | $1+\varphi$   |     | $\varphi$           | 0   | $-1-\varphi$                                       |          | 0                                      | 2      | -+    | 1                            | 1                           | 1  | $I_1$   |                 |
| a2       | 1                                      | $1+\varphi$   |     | $\varphi$           | $-5+5\varphi$   | $-9+4\varphi$                                      |          | 0                                      | 2      | +-    | 2                            | 2                           | 2  | $I_2$   |                 |
| 479      |  |   |     |                     |   | . , ,  | 1 isoge  | eny cla                                | ss)    |       |                              |                             |  |   | 479b            |
| a1       |  | $-1-\varphi$  |     | $\varphi$           | $-1+\varphi$  | -1   |          | 0                                      | 2      | + -   | 1                            | 1                           | 1  | $I_1$   |                 |
| a2       | 1                                      | $-1-\varphi$  |     | $\varphi$           | $-1-4\varphi$   | -4   |          | 0                                      | 2      | -+    | 2                            | 2                           | 2  | $I_2$   |                 |
| 484      | a                                      |   |     |                     | 4   | $184a = (22) = 2 \cdot 11a \cdot 11b $             | 4 i soge | eny cla                                | sses)  |       |                              |                             |  |   | 484a            |
| a1       |  | $-1-\varphi$  |     | 1                   | $-6-6\varphi$   | $8+11\varphi$                                      |          | 0                                      | 5      |       | 1, 1, 1                      | 1,1,1                       | 1,1,1  | $I_1, I_1, I_1$   |                 |
| a2       | $1+\varphi$                            | $-1-\varphi$  |     | 1                   | $39 + 39\varphi$                                      | $-82 - 109\varphi$                                 |          | 0                                      | 5      |       | 5, 5, 5                      | 5, 5, 5                     | 5, 5, 1                                      | $I_5, I_5, I_5$   |                 |
| a3       | $1+\varphi$                            | $-1-\varphi$  |     | 1                   | $-6066-6066\varphi$                                   | $-242434 - 323245\varphi$                          |          | 0                                      | 1      |       | 1, 1, 25                     | 1, 1, 25                    | 1, 1, 1                                      | $I_1, I_1, I_{25}$  |                 |
| b1       | $1+\varphi$                            | $\varphi$   | 1 + | $\varphi$           | -1-arphi  | $-2-3\varphi$                                      |          | 0                                      | 2      | Ī — — | 1, 1, 4                      | 1,1,4                       | [1, 1, 2]                                    | $ $ $I_1, I_1, I_4$   |                 |
| b2       | $1+\varphi$                            | $\varphi$   | 1 + | $\varphi$           | $-21-21\varphi$                                       | $-66-95\varphi$                                    |          | 0                                      | 4      | ++    | 2, 2, 2                      | 2, 2, 2                     | 2, 2, 2                                      | $I_2, I_2, I_2$   |                 |
| b3       | 1                                      | $-\varphi$  |     | 0                   | $-4384 + 2709\varphi$                                 | $-132609 + 81957\varphi$                           |          | 0                                      | 2      | ++    | 4, 1, 1                      | 4, 1, 1                     | 4, 1, 1                                      | $I_4, I_1, I_1$   |                 |
| b4       | 1                                      | $-1+\varphi$  |     | 0                   | $-1675 - 2709\varphi$                                 | $-50652 - 81957\varphi$                            |          | 0                                      | 2      | ++    | 1, 4, 1                      | 1, 4, 1                     | 1, 4, 1                                      | $I_1, I_4, I_1$   |                 |
| c1       | $1+\varphi$                            | 1   |     | 0                   | $-2-2\varphi$   | $-12-16\varphi$                                    |          | 0                                      | 1      |       | 1, 1, 11                     | 1,1,11                      | [1, 1, 1]                                    | $  I_1, I_1, I_{11}$  |                 |
| d1       | $1+\varphi$                            | -1  | 1+  | $\varphi$           | $-1-3\varphi$   | -1-2arphi  |          | 0                                      | 3      |       | 3, 1, 1                      | 3,1,1                       | [3, 1, 1]                                    | $  I_3, I_1, I_1  $   |                 |
| d2       | 1                                      |   | 1+  |                     | $-420-678\varphi$                                     | $-6547 - 10592\varphi$                             |          | 0                                      | 1      |       | 1, 3, 3                      | 1, 3, 3                     | 1, 1, 3                                      | $I_1, I_3, I_3$   |                 |
| 101      | h                                      |   |     |                     | 46  | (14 00 ) 0 11 2                                    | (0.:     | 1                                      |        | `     |                              |                             |  | ,   | 484b            |
| 484      |  | 1   |     |                     |   | $\frac{34b = (14 - 20\varphi) = 2 \cdot 11a^2}{2}$ | (2 1808  | geny cl                                |        | Ī     | 0.0                          | 0                           | 1.0  | TT T  | 4040            |
| a1<br>a2 | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | 1<br>1  |     | $\varphi$ $\varphi$ | -arphi $-35-61arphi$                                  | $0$ $147 + 241\varphi$                             |          | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | 1<br>1 |       | $2, 2 \\ 2, 6$               | 6                           | $ \begin{array}{c c} 1,2\\ 1,2 \end{array} $ | $\begin{array}{c c} & \text{II}, I_2 \\ & \text{II}, I_6 \end{array}$ |                 |
| b1       | <del>-</del> -<br>  1                  |   |     |                     | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | $33-24\varphi$                                     |          |  |        | '_    | 8,2                          | 1 2                         | $\begin{bmatrix} 1, 2 \\ 3, 2 \end{bmatrix}$ | $  IV^*, I_2  $   |                 |
| b2       | $\varphi$                              | $\begin{array}{c} \varphi \\ 1-\varphi \end{array}$ |     | $\varphi$ 1         | $-536 - 894\varphi$                                   | $-9431 - 15172\varphi$                             |          | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 1      |       | 8, 6                         | 6                           | 1,6  | $IV^*, I_6$   |                 |
|          | •                                      | *   |     |                     | · · · · · · · · · · · · · · · · · · ·                 | · · · · · · · · · · · · · · · · · · ·              |          |  |        |       |                              | 1                           | 1  |   | 1               |

|     | <i>a</i> .                                       | <i>a a</i>  | a.                        | <i>a</i> .                                      |  |   | s         | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                   | C  | Kodaira                         | Isogenies |
|-----|--|---|---------------------------|---|--|---|-----------|------------------------------|---|--|---------------------------------|-----------|
|     | $a_1$  | $a_2$ $a_3$   | $a_4$                     | $a_6$   | /                                      | 1   | 3         | σια(Δ)                       | ord_(J)                                       | $c_p$  | Rodalia                         | Isogemes  |
|     |  |   |                           |   |  |   |           |                              |   |  |                                 |           |
| 484 | $\mathbf{c}$                                     |   | 48                        | $34c = (6 - 20\varphi) = 2 \cdot 11b^2 \tag{2}$ | 2 isogeny cl                           | asses)                                    |           |                              |   |  |                                 | 484c      |
| a1  | 1  | $1  1 + \varphi$  | -1                        | $-\varphi$                                      | 1                                      | 1   |           | 2,2                          | 2   | 1, 2   | $II, I_2$                       |           |
| a2  | 1  | $1  1 + \varphi$  | $-96+60\varphi$           | $388 - 242\varphi$                              | 1                                      | 1   |           | 2, 6                         | 6   | 1,2  | $II, I_6$                       |           |
| b1  | 1  | $1-\varphi$ $1+\varphi$   | -5-5arphi                 | $9+23\varphi$                                   | 0                                      | 3   |           | 8,2                          | 2   | [-3, 2]  | $ $ $IV^*, I_2$                 |           |
| b2  | $1+\varphi$                                      | 0 1   | $-1430 + 893\varphi$      | $-24603 + 15172\varphi$                         | 0                                      | 1   |           | 8,6                          | 6   | 1,6  | $IV^*, I_6$                     |           |
|     |  |   |                           |   |  |   |           |                              |   |  |                                 |           |
| 495 | a  |   | 495                       | $a = (24 - 3\varphi) = 5a \cdot 3 \cdot 11b$    | (3 isogeny                             | classe                                    | s)        |                              |   |  |                                 | 495a      |
| a1  | $1+\varphi$                                      | $1+\varphi$ $\varphi$   | $1+\varphi$               | 0   | 1                                      | 2   | - +       | 1, 1, 1                      | 1, 1, 1                                       | 1, 1, 1  | $I_1,I_1,I_1$                   |           |
| a2  | $1+\varphi$                                      | $ \begin{array}{ccc} 1 + \varphi & \varphi \\ 1 + \varphi & \varphi \end{array} $ | $-4-4\varphi$             | $1+\varphi$                                     |  | $\frac{2}{4}$                             | + +       | 2, 2, 2                      | 2, 2, 2                                       | $\begin{bmatrix} 1, 1, 1 \\ 2, 2, 2 \end{bmatrix}$             | $I_1, I_1, I_1$ $I_2, I_2, I_2$ |           |
| a3  | $1+\varphi$                                      | $1+\varphi \qquad \varphi$ $1+\varphi \qquad \varphi$                             | $-69-99\varphi$           | $280 + 468\varphi$                              | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | 2   |           | 1, 4, 1                      | 1, 4, 1                                       | 1, 2, 1  | $I_1, I_4, I_1$                 |           |
| a4  | $\begin{vmatrix} - & r \\ \varphi \end{vmatrix}$ | $1 \qquad \varphi$  | $-129 + 79\varphi$        | $615 - 379\varphi$                              | 1                                      | 2   | + -       | 4, 1, 4                      | 4, 1, 4                                       | 4, 1, 2  | $I_4, I_1, I_4$                 |           |
| b1  | : <del>:</del> -<br>  1                          | $\varphi$ 1+ $\varphi$  | $-3-\varphi$              | 1-arphi   | 0                                      | $\frac{1}{4} - \frac{7}{4} - \frac{1}{4}$ | <br>  - + |                              | $\frac{1}{1}$ , $\frac{1}{1}$ , $\frac{1}{1}$ | $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ | $  I_1, I_1, I_1  $             | <u>  </u> |
| b2  | 1  | $\varphi  1 + \varphi$  | $-28+14\varphi$           | $64-41\varphi$                                  | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 8   | + +       | 2, 2, 2                      | 2, 2, 2                                       | 2, 2, 2  | $I_2, I_2, I_2$                 |           |
| b3  | $\varphi$  | $1+\varphi$ 1   | $-19855 + 12273\varphi$   | $1262856 - 780487\varphi$                       | 0                                      | 4   | + -       | 1, 1, 1                      | 1, 1, 1                                       | 1, 1, 1  | $I_1, I_1, I_1$                 |           |
| b4  | 1  | $\varphi$ 1+ $\varphi$  | $-38-\varphi$             | $29-92\varphi$                                  | 0                                      | 8   | ++        | 4, 4, 4                      | 4, 4, 4                                       | 2, 4, 4  | $I_4, I_4, I_4$                 |           |
| b5  | $\varphi$  | $1-\varphi$ 0   | $-1203 - 1919\varphi$     | $-30120 - 48680\varphi$                         | 0                                      | 4   | ++        | 8, 2, 2                      | 8, 2, 2                                       | 2, 2, 2  | $I_8, I_2, I_2$                 |           |
| b6  | 1  | $\varphi$ 1+ $\varphi$  | $27-16\varphi$            | $184-286\varphi$                                | 0                                      | 8   |           | 2, 8, 8                      | 2, 8, 8                                       | 2,8,8  | $I_2, I_8, I_8$                 |           |
| b7  | 1  | $\varphi$ 1 + $\varphi$   | $7-391\varphi$            | $-3655 - 1769\varphi$                           | 0                                      | 2   |           | 16, 1, 1                     | 16, 1, 1                                      | 2, 1, 1  | $I_{16}, I_1, I_1$              |           |
| b8  | $1+\varphi$                                      | $-\varphi$ 0  | $-130608 - 211264\varphi$ | $-34550086 - 55903043\varphi$                   | 0                                      | 2   | ++        | 4, 1, 1                      | 4, 1, 1                                       | 2, 1, 1  | $I_4, I_1, I_1$                 |           |
| c1  | 1  | $-\varphi$ 0  | $-28-41\varphi$           | $-88-139\varphi$                                | 0                                      | $\overline{4}$                            | + +       | 2, 4, 2                      | [-2,4,2]                                      | [2, 2, 2]  | $  I_2, I_4, I_2  $             |           |
| c2  | $1+\varphi$                                      | $1+\varphi$ 1   | $-12+10\varphi$           | $-23+15\varphi$                                 | 0                                      | 2   | + -       | 1, 2, 1                      | 1, 2, 1                                       | 1, 2, 1  | $I_1, I_2, I_1$                 |           |
| c3  | $\varphi$  | $-1+\varphi$ $1+\varphi$  | $-2880 - 4657\varphi$     | $-114271 - 184899\varphi$                       | 0                                      | 2   | ++        | 1, 2, 4                      | 1, 2, 4                                       | 1, 2, 2  | $I_1, I_2, I_4$                 |           |
| c4  | $1+\varphi$                                      | $1+\varphi$ 1   | $38-30\varphi$            | $-117 + 51\varphi$                              | 0                                      | 2   | -+        | 4, 8, 1                      | 4, 8, 1                                       | 2, 2, 1  | $I_4, I_8, I_1$                 |           |
|     |  |   |                           |   |  |   |           |                              |   |  |                                 |           |
| 495 | b  |   | 495                       | $b = (21 + 3\varphi) = 5a \cdot 3 \cdot 11a$    | (3 isogeny                             | classe                                    | s)        |                              |   |  |                                 | 495b      |
| a1  | $\varphi$  | $\varphi$ $\varphi$   | $-1+\varphi$              | 0   |  | 2   | <u> </u>  | 1, 1, 1                      | 1, 1, 1                                       | 1, 1, 1  | $I_1, I_1, I_1$                 |           |
| a2  | $\varphi$  | $\varphi$ $\varphi$   | $-11+6\varphi$            | 17-11arphi                                      | 1                                      | 4   | + +       | 2, 2, 2                      | 2, 2, 2                                       | 2, 2, 2  | $I_2, I_2, I_2$                 |           |
| a3  | $\varphi$  | $\varphi$ $\varphi$   | $-171 + 101\varphi$       | $1018 - 638\varphi$                             | 1                                      | 2   | + +       | 1, 4, 1                      | 1, 4, 1                                       | 1, 2, 1  | $I_1, I_4, I_1$                 |           |
| a4  | $1+\varphi$                                      | $1-\varphi$ $1+\varphi$   | $-50-81\varphi$           | $236 + 378\varphi$                              | 1                                      | 2   | -+        | 4, 1, 4                      | 4, 1, 4                                       | 4, 1, 2  | $I_4, I_1, I_4$                 |           |
| b1  | : <del>:</del> -<br>  1                          | $1-\varphi$ $\varphi$   | -3                        |   | 0                                      | $\frac{1}{1} - \frac{1}{4} - \frac{1}{4}$ | (         | 1,1,1                        | $\frac{1}{1}$ , $\frac{1}{1}$ , $\frac{1}{1}$ | $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$         | $  I_1, I_1, I_1  $             | <u></u>   |
| b2  | $\begin{vmatrix} & -1 \\ 1 & \end{vmatrix}$      | $1-\varphi$ $\varphi$   | $-13-15\varphi$           | $24 + 40\varphi$                                | 0                                      | 8   | + +       |                              | 2, 2, 2                                       | 2, 2, 2  | $I_2, I_2, I_2$                 |           |
| b3  | $1+\varphi$                                      | $-1+\varphi$ $1+\varphi$  | $-7584 - 12273\varphi$    | $477680 + 772903\varphi$                        | 0                                      | 4   |           | 1, 1, 1                      | 1, 1, 1                                       | 1, 1, 1  | $I_1, I_1, I_1$                 |           |
| b4  | 1  | $1-\varphi$ $\varphi$   | -38                       | $-62 + 91\varphi$                               | 0                                      | 8   |           | 4, 4, 4                      | 4, 4, 4                                       | 2,4,4  | $I_4, I_4, I_4$                 |           |
| b5  | $1+\varphi$                                      | 0 0   | $-3122 + 1919\varphi$     | $-78800 + 48680\varphi$                         | 0                                      | 4   | 1         | 8, 2, 2                      | 8, 2, 2                                       | 2, 2, 2  | $I_8, I_2, I_2$                 |           |
| b6  | 1  | $1-\varphi$ $\varphi$   | $12 + 15\varphi$          | $-101 + 285\varphi$                             | 0                                      | 8   | 1         | 2, 8, 8                      | 2, 8, 8                                       | 2, 8, 8  | $I_2, I_8, I_8$                 |           |
| b7  | 1  | $1-\varphi$ $\varphi$   | $-383 + 390\varphi$       | $-5423 + 1768\varphi$                           | 0                                      | 2   | 1         | 16, 1, 1                     | 16, 1, 1                                      | 2, 1, 1  | $I_{16}, I_1, I_1$              |           |
| b8  | $\varphi$  | 0 0   | $-341872 + 211264\varphi$ | $-90453129 + 55903043\varphi$                   | 0                                      | 2   | ++        | 4, 1, 1                      | 4, 1, 1                                       | 2, 1, 1  | $I_4, I_1, I_1$                 |           |

|            |               |               |               |                       |  |                  | T              |       |                              |                                     | T       | T               |           |
|------------|---------------|---------------|---------------|-----------------------|--|------------------|----------------|-------|------------------------------|-------------------------------------|---------|-----------------|-----------|
|            | $a_1$         | $a_2$         | $a_3$         | $a_4$                 | $a_6$  | r                | T              | s     | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$         | $c_p$   | Kodaira         | Isogenies |
| 495        | b             |               |               | 49                    | $95b = (21 + 3\varphi) = 5a \cdot 3 \cdot 11a$ | (3 isogeny cl    | asses          | )     |                              |                                     |         |                 | 495b      |
| c1         | 1             | $-1+\varphi$  | 0             | $-69 + 41\varphi$     | $-227 + 139\varphi$                            | 0                | 4              | ++    | 2, 4, 2                      | 2,4,2                               | 2, 2, 2 | $I_2, I_4, I_2$ |           |
| c2         | $\varphi$     | $\varphi$     | 0             | $-5-7\varphi$         | $-12-19\varphi$                                | 0                | 2              | I     | 1, 2, 1                      | 1, 2, 1                             | 1, 2, 1 | $I_1, I_2, I_1$ |           |
| c3         | $1 + \varphi$ | $\varphi$     |               | $-7536 + 4656\varphi$ | $-294513 + 182018\varphi$                      | 0                | 2              | ++    | 1, 2, 4                      | 1, 2, 4                             | 1, 2, 2 | $I_1, I_2, I_4$ |           |
| c4         | φ             | φ             | 0             | $5 + 33\varphi$       | $-40 - 45\varphi$                              | 0                | 2              | +-    | 4, 8, 1                      | 4, 8, 1                             | 2, 2, 1 | $I_4, I_8, I_1$ |           |
| 505        | a             |               |               |                       | $505a = (23 - \varphi) = 5a \cdot 101a$        | (3 isogeny cla   | sses)          |       |                              |                                     |         |                 | 505a      |
| a1         | 0             | $1 + \varphi$ | 1             | $-211 + 132\varphi$   | $-1421 + 879\varphi$                           | 0                | 1              |       | 1, 1                         | 1,1                                 | 1, 1    | $I_1, I_1$      |           |
| b1         | 0             | $-\varphi$    | 1             | $2\varphi$            | $-1-2\varphi$                                  | 0                | 5              |       | 1,5                          | 1,5                                 | 1,5     | $I_1, I_5$      |           |
| b2         | 0             | $-\varphi$    | 1             | $-210-328\varphi$     | $-2089 - 3266\varphi$                          | 0                | 1              |       | 5, 1                         | 5,1                                 | 5, 1    | $I_5, I_1$      |           |
| c1         | 1             | $\varphi$     | 0             | -4                    | $-1-4\varphi$                                  | 0                | $\frac{1}{4}$  | + +   | 2, 2                         | $\begin{vmatrix} 2,2 \end{vmatrix}$ | 2,2     | $I_2, I_2$      |           |
| c2         | 1             | $\varphi$     | 0             | 1                     | 0  | 0                | 2              |       | 1, 1                         | 1, 1                                | 1,1     | $I_1, I_1$      |           |
| c3         | 1             | $\varphi$     | 0             | $-39 + 20\varphi$     | $106-68\varphi$                                | 0                | 4              | ++    | 4, 1                         | 4, 1                                | 4, 1    | $I_4, I_1$      |           |
| c4         | φ             | $1-\varphi$   | $1+\varphi$   | $-160 - 252\varphi$   | $-1439 - 2316\varphi$                          | 0                | 2              | ++    | 1,4                          | 1,4                                 | 1,4     | $I_1, I_4$      |           |
| 505        | b             |               |               |                       | $505b = (22 + \varphi) = 5a \cdot 101b$        | (3 isogeny clas  | sses)          |       |                              |                                     |         |                 | 505b      |
| a1         | 0             | $-1-\varphi$  | 1             | $-80-130\varphi$      | $-462 - 748\varphi$                            | 0                | 1              |       | 1, 1                         | 1,1                                 | 1,1     | $I_1, I_1$      |           |
| b1         | 0             | $-1+\varphi$  | 1             | $2-2\varphi$          | $-3+2\varphi$                                  | 0                | 5              |       | 1, 5                         | 1,5                                 | 1,5     | $I_1, I_5$      | Ī         |
| b2         | 0             | $-1+\varphi$  | 1             | $-538 + 328\varphi$   | $-5355 + 3266\varphi$                          | 0                | 1              |       | 5, 1                         | 5,1                                 | 5, 1    | $I_5, I_1$      |           |
| c1         | 1             | $1-\varphi$   | 0             | -4                    | $-5+4\varphi$                                  | 0                | $\overline{4}$ | + +   | 2, 2                         | $\begin{vmatrix} 2,2 \end{vmatrix}$ | 2,2     | $I_2, I_2$      |           |
| c2         | 1             | $1-\varphi$   | 0             | 1                     | 0  | 0                | 2              |       | 1, 1                         | 1,1                                 | 1,1     | $I_1, I_1$      |           |
| c3         | 1             | $1-\varphi$   | 0             | $-19-20\varphi$       | $38 + 68\varphi$                               | 0                | 4              | ++    | 4, 1                         | 4,1                                 | 4, 1    | $I_4, I_1$      |           |
| c4         | $1+\varphi$   | 0             | $\varphi$     | $-411 + 250\varphi$   | $-3754 + 2315\varphi$                          | 0                | 2              | ++    | 1,4                          | 1,4                                 | 1,4     | $I_1, I_4$      |           |
| 509        | a             |               |               |                       | $509a = (21 + 4\varphi) = 509a$                | (2 isogeny class | ses)           |       |                              |                                     |         |                 | 509a      |
| a1         | $1 + \varphi$ | $-1+\varphi$  | $1 + \varphi$ | $-1-\varphi$          | -arphi   | 1                | 1              | +-    | 1                            | 1                                   | 1       | $I_1$           |           |
| b1         | $1+\varphi$   | 0             | 1             | -5                    | $-5-\varphi$                                   | 0                | 1              |       | 1                            | 1                                   | 1       | $I_1$           |           |
| 509        | b             |               |               |                       | $509b = (25 - 4\varphi) = 509b $               | 2 isogeny class  | es)            |       |                              |                                     | •       |                 | 509b      |
| a1         | $\varphi$     | $1+\varphi$   | 1             | $\varphi$             | 0  | 1                | 1              | -+    | 1                            | 1                                   | 1       | $I_1$           |           |
| b1         | $\varphi$     | $1-\varphi$   | 1             | $-4-\varphi$          | $-6+\varphi$                                   | 0                | 1              | ;<br> | 1                            | 1                                   | 1       | $I_1$           | <u>  </u> |
| <b>521</b> |               |               |               |                       | $521a = (16 - 21\varphi) = 521a$               | (1 isogeny cla   | ss)            |       |                              |                                     |         |                 | 521a      |
| a1         | $\varphi$     | 1             | 0             | $-1+\varphi$          | 0  | 1                | 2              | ++    | 1                            | 1                                   | 1       | $I_1$           |           |
| a2         | $1+\varphi$   | $1+\varphi$   | 0             | $-4-8\varphi$         | $-20-33\varphi$                                | 1                | 2              | -+    | 2                            | 2                                   | 2       | $I_2$           |           |
| <b>521</b> | b             |               |               |                       | $521b = (5 - 21\varphi) = 521b$                | (1 isogeny clas  | s)             |       |                              |                                     |         |                 | 521b      |
| a1         | $1+\varphi$   | $1-\varphi$   | 0             | $-\varphi$            | 0  | 1                | 2              | ++    | 1                            | 1                                   | 1       | $I_1$           |           |
| a2         | $\varphi$     | φ             | 1             | $-16 + 11\varphi$     | $-28 + 18\varphi$                              | 1                | 2              | +-    | 2                            | 2                                   | 2       | $I_2$           |           |
|            |               |               |               |                       |  |                  |                |       |                              |                                     |         |                 |           |

|  |  |  |  |   |   |  |   | ı   |  | 1  |  | 1   |              |
|--|--|--|--|---|---|--|---|---|--|--|--|---|--------------|
|  | a  | $a_1$ $a_2$  | $a_3$  | $a_4$   | $a_6$   | r  | T   | s   | $\operatorname{ord}(\Delta)$   | $\operatorname{ord}_{-}(j)$  | $c_p$  | Kodaira   | Isogenies    |
| <b>524</b>   | a.   |  |  | Ę   | $524a = (24 - 2\varphi) = 2 \cdot 131a$   | (1 isogeny class   | )   |   |  |  |  |   | <b>524</b> a |
| a1   | $1+\varphi$  | 0  | $1 + \varphi$  | $-2-\varphi$  | $-\varphi$  | 1  | 1   |   | 1, 2   | 1,2  | 1,2  | $I_1, I_2$  |              |
| <b>524</b>   | .b   |  |  |   | $524b = (22 + 2\varphi) = 2 \cdot 131b$   | (1 isogeny class   | )   |   |  |  |  |   | 524b         |
| a1   | 4  | $\rho$ 1 – $\varphi$   | $\varphi$  | $-1-\varphi$  | 0   | 1  | 1   |   | 1, 2   | 1, 2   | 1,2  | $I_1, I_2$  |              |
| <b>52</b> 9  | a  |  |  |   | $529a = (23) = 23 \qquad (1)$   | isogeny class)   |   |   |  |  |  |   | <b>529</b> a |
| a1   | (  | $0 - 1 + \varphi$  | 1  | $-3-4\varphi$   | $-4-6\varphi$   | 0  | 1   |   | 1  | 1  | 1  | $I_1$   |              |
| 531  | a  |  |  | 5:  | $31a = (15 - 21\varphi) = 3 \cdot 59a$  | (2 isogeny classe  | s)  |   |  |  |  |   | 531a         |
| a1   | 1  | $1 - \varphi$  | $1+\varphi$  | $1-\varphi$   | $-\varphi$  | 1  | 1   |   | 1, 1   | 1,1  | 1,1  | $I_1, I_1$  |              |
| b1   | $1+\varphi$  | ρ φ  | $\varphi$  | -2  | $1-\varphi$   | 1  | 1   | Ī — —   | 1, 1   | 1,1  | 1,1  | $I_1, I_1$  |              |
| 531  | . <b>b</b>   |  |  | Ę   | $531b = (6 - 21\varphi) = 3 \cdot 59b$  | (2 isogeny classes                                       | )   |   |  |  |  |   | <b>53</b> 1b |
| a1   | 1  | $1 \qquad \varphi$   | $\varphi$  | 1   | 0   | 1  | 1   |   | 1, 1   | 1, 1   | 1,1  | $I_1, I_1$  |              |
| b1   | 4  | $\varphi$ $-1+\varphi$   | $\varphi$  | -3  | $3-2\varphi$  | 1  | 1   |   | 1, 1   | 1,1  | 1,1  | $I_1, I_1$  |              |
|  |  |  |  |   |   |  |   |   |  |  |  |   |              |
| 539  | a  |  |  | 5   | $39a = (7 - 21\varphi) = 11a \cdot 7$   | (5 isogeny classes                                       | s)  |   |  |  |  |   | 539a         |
| 539<br>a1  | $\frac{\partial \mathbf{a}}{ 1+\varphi }$  | $\varphi$ 1 + $\varphi$  | $\varphi$  | $-8 - 11\varphi$  | $39a = (7 - 21\varphi) = 11a \cdot 7$ $6 + 10\varphi$   | (5 isogeny classes                                       | s)<br>1   | +-  | 3, 1   | 3,1  | 3,1  | $I_3, I_1$  | 539a         |
| a1<br>b1   | $\frac{1+\varphi}{1}$  | $1 - 1 + \varphi$  | <del>'</del> - <del>'</del> - <del>'</del> φ   | $-8 - 11\varphi$ $-378 - 566\varphi$  | $\frac{6+10\varphi}{-4998-8206\varphi}$   |  | 1 2   | + -   | 3, 2   | 3,2  | 1,2  | $I_3, I_2$  | 539a         |
| a1<br>b1<br>b2                                       | $\begin{array}{ c c c }\hline 1+\varphi\\ \hline & 1\\ \hline & 1+\varphi\\ \hline \end{array}$  | $ \begin{array}{cccc} 1 & -1 + \varphi \\ \varphi & & \varphi \end{array} $  | $\varphi$ $1$  | $-8 - 11\varphi$ $-378 - 566\varphi$ $-11 + \varphi$  | $   \begin{array}{r}       6 + 10\varphi \\       -4998 - 8206\varphi \\       8 - 22\varphi   \end{array} $  |  | $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$                             | ++  | 3, 2<br>6, 1   | 3, 2<br>6, 1   | 1, 2 2, 1  | $I_3, I_2 \\ I_6, I_1$  | 539a         |
| a1<br>b1<br>b2<br>c1                                 | $\begin{array}{ c c c }\hline 1+\varphi\\ \hline & 1\\ 1+\varphi\\ \hline & 1\end{array}$  | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $\frac{1}{\varphi}$  | $-8 - 11\varphi$ $-378 - 566\varphi$ $-11 + \varphi$ $-2 + 2\varphi$  | $6 + 10\varphi$ $-4998 - 8206\varphi$ $8 - 22\varphi$ $2 - \varphi$   |  | $\begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}$                        | + +<br> <br>  + -   | $ \begin{array}{c} 3,2\\ 6,1\\ 1,1 \end{array} $   | $ \begin{array}{c c} 3,2 \\ 6,1 \\ \hline 1,1 \end{array} $  | $\begin{bmatrix} 1, 2 \\ 2, 1 \\ 1, 1 \end{bmatrix}$   | $egin{array}{c c} I_3, I_2 \\ I_6, I_1 \\ I_1, I_1 \end{array}$   | 539a         |
| a1<br>b1<br>b2<br>c1<br>c2                           | $ \begin{array}{ c c c } \hline 1 + \varphi \\ \hline 1 + \varphi \\ \hline 1 + \varphi \\ \hline 1 \\ 1 \\ \hline 1 \end{array} $   | $ \begin{array}{ccc} 1 & -1 + \varphi \\ \varphi & & \varphi \\ 1 & & \varphi \\ 1 & & \varphi \end{array} $   | $ \begin{array}{cccc}  & & & & & \\  & & & & \\  & & & & \\  & & & &$  | $-8 - 11\varphi$ $-378 - 566\varphi$ $-11 + \varphi$ $-2 + 2\varphi$ $-27 + 17\varphi$  | $\begin{array}{c} 6 + 10\varphi \\ -4998 - 8206\varphi \\ 8 - 22\varphi \\ \hline 2 - \varphi \\ -61 + 36\varphi \end{array}$   | $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$              | $\begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \\ 1 \end{bmatrix}$                   | + +<br> <br>  + -<br>  + -  | $   \begin{array}{c}     3, 2 \\     6, 1 \\     \hline     1, 1 \\     3, 3   \end{array} $   | $ \begin{array}{c c} 3,2\\ 6,1\\ \hline 1,1\\ 3,3 \end{array} $  | $ \begin{array}{ c c c c } \hline 1,2\\2,1\\1,1\\1,3\\\end{array} $  | $\begin{array}{ c c c c }\hline I_3, I_2 \\ I_6, I_1 \\\hline I_1, I_1 \\ I_3, I_3 \\\hline \end{array}$  | 539a         |
| a1<br>b1<br>b2<br>c1<br>c2<br>d1                     | $\begin{array}{ c c c }\hline 1+\varphi\\ \hline & 1\\ 1+\varphi\\ \hline & 1\\ \hline & 1\\ \hline & \varphi\\ \end{array}$   | $ \begin{array}{ccc} 1 & -1 + \varphi \\ \varphi & \varphi \\ 1 & \varphi \\ 1 & \varphi \\ \varphi & -1 \end{array} $   | $ \begin{array}{ccc}  & & & & \\  & & & & \\  & & & & \\  & & & &$   | $-8 - 11\varphi$ $-378 - 566\varphi$ $-11 + \varphi$ $-2 + 2\varphi$ $-27 + 17\varphi$ $-48 + 26\varphi$  | $   \begin{array}{r}                                     $  | $\begin{array}{c c} & 0 \\ \hline 0 \\ 0 \\ \end{array}$ | $\begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \\ 1 \\ 2 \end{bmatrix}$              | + +<br> <br>  + -<br>  + -  | $ \begin{array}{c} 3,2\\ 6,1\\ 1,1\\ 3,3\\ 1,2 \end{array} $   | $ \begin{array}{c c} 3, 2 \\ 6, 1 \\ \hline 1, 1 \\ 3, 3 \\ \hline 1, 2 \end{array} $  | $ \begin{array}{ c c c c } \hline 1,2\\ 2,1\\ \hline 1,1\\ 1,3\\ \hline 1,2\\ \end{array} $  | $\begin{array}{c c} & I_3, I_2 \\ & I_6, I_1 \\ \hline & I_1, I_1 \\ & I_3, I_3 \\ \hline & I_1, I_2 \\ \end{array}$  | 539a         |
| a1<br>b1<br>b2<br>c1<br>c2<br>d1<br>d2               | $ \begin{array}{c c} 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \\ 1 \\ 1 \\                            $   | $ \begin{array}{ccc} 1 & -1 + \varphi \\ \varphi & \varphi \\ 1 & \varphi \\ 1 & \varphi \\ \varphi & -1 \\ \varphi & -1 \end{array} $   | $ \begin{array}{c} \varphi \\ 1 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 + \varphi \end{array} $  | $-8 - 11\varphi$ $-378 - 566\varphi$ $-11 + \varphi$ $-2 + 2\varphi$ $-27 + 17\varphi$ $-48 + 26\varphi$ $-3 + \varphi$   | $   \begin{array}{r}                                     $  |  | 1 2 2 2 3 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2                                 | + +<br> <br>  + -<br>  + -<br>  | $   \begin{array}{c}     3, 2 \\     6, 1 \\     \hline     1, 1 \\     3, 3 \\     \hline     1, 2 \\     2, 1   \end{array} $                  | $ \begin{array}{c c} 3, 2 \\ 6, 1 \\ \hline 1, 1 \\ 3, 3 \\ \hline 1, 2 \\ 2, 1 \end{array} $  | $ \begin{array}{c c}  & 1, 2 \\  & 2, 1 \\ \hline  & 1, 1 \\  & 1, 3 \\ \hline  & 1, 2 \\  & 2, 1 \\ \hline \end{array} $  | $ \begin{vmatrix} I_3, I_2 \\ I_6, I_1 \end{vmatrix} $ $ \begin{vmatrix} I_1, I_1 \\ I_3, I_3 \end{vmatrix} $ $ \begin{vmatrix} I_1, I_2 \\ I_2, I_1 \end{vmatrix} $  | 539a         |
| a1<br>b1<br>b2<br>c1<br>c2<br>d1                     | $\begin{array}{ c c c }\hline 1+\varphi\\ \hline & 1\\ 1+\varphi\\ \hline & 1\\ \hline & 1\\ \hline & \varphi\\ \end{array}$   | $ \begin{array}{ccc} 1 & -1 + \varphi \\ \varphi & \varphi \\ 1 & \varphi \\ 1 & \varphi \\ \varphi & -1 \\ \varphi & -1 \end{array} $   | $ \begin{array}{ccc}  & & & & \\  & & & & \\  & & & & \\  & & & &$   | $-8 - 11\varphi$ $-378 - 566\varphi$ $-11 + \varphi$ $-2 + 2\varphi$ $-27 + 17\varphi$ $-48 + 26\varphi$  | $   \begin{array}{r}                                     $  | $\begin{array}{c c} & 0 \\ \hline 0 \\ 0 \\ \end{array}$ | $\begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \\ 1 \\ 2 \end{bmatrix}$              | + +<br> <br>  + -<br>  + -  | $ \begin{array}{c} 3,2\\ 6,1\\ 1,1\\ 3,3\\ 1,2 \end{array} $   | $ \begin{array}{c c} 3, 2 \\ 6, 1 \\ \hline 1, 1 \\ 3, 3 \\ \hline 1, 2 \end{array} $  | $ \begin{array}{ c c c c } \hline 1,2\\ 2,1\\ \hline 1,1\\ 1,3\\ \hline 1,2\\ \end{array} $  | $\begin{array}{c c} & I_3, I_2 \\ & I_6, I_1 \\ \hline & I_1, I_1 \\ & I_3, I_3 \\ \hline & I_1, I_2 \\ \end{array}$  | 539a         |
| a1 b1 b2 c1 c2 d1 d2 e1                              | $ \begin{array}{c c} 1 + \varphi \\ \hline  & 1 \\ 1 + \varphi \\ \hline  & 1 \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & \varphi \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & \varphi \\ \hline  & 1 \\ \hline  & \varphi \\  & \varphi$ | $ \begin{array}{ccc} 1 & -1 + \varphi \\ \varphi & \varphi \\ 1 & \varphi \\ 1 & \varphi \\ \varphi & -1 \\ \varphi & -1 \end{array} $   | $ \begin{array}{c} \varphi \\ 1 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 + \varphi \end{array} $  | $-8 - 11\varphi$ $-378 - 566\varphi$ $-11 + \varphi$ $-2 + 2\varphi$ $-27 + 17\varphi$ $-48 + 26\varphi$ $-3 + \varphi$ $-367 + 228\varphi$   |   | 0 0 0 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0                  | 1 2 2 3 1 2 2 2 1 1 ss)   | + +<br> <br>  + -<br>  + -<br>  | $ \begin{array}{c} 3,2 \\ 6,1 \\ 1,1 \\ 3,3 \\ 1,2 \\ 2,1 \\ 3,1 \end{array} $   | $ \begin{array}{c c} 3, 2 \\ 6, 1 \\ \hline 1, 1 \\ 3, 3 \\ \hline 1, 2 \\ 2, 1 \\ \hline 3, 1 \end{array} $   | 1,2<br>2,1<br>1,1<br>1,3<br>1,2<br>2,1<br>1,1  | $ \begin{array}{c c} I_3, I_2 \\ I_6, I_1 \\ I_1, I_1 \\ I_3, I_3 \\ I_1, I_2 \\ I_2, I_1 \\ I_3, I_1 \end{array} $   | 539a         |
| a1<br>b1<br>b2<br>c1<br>c2<br>d1<br>d2<br>e1         | $ \begin{array}{c c} 1 + \varphi \\ \hline  & 1 \\ 1 + \varphi \\ \hline  & 1 \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & \varphi \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & \varphi \\ \hline  & 1 \\ \hline  & \varphi \\  & \varphi$ | $ \begin{array}{ccc} 1 & -1 + \varphi \\ \varphi & \varphi \\ 1 & \varphi \\ 1 & \varphi \\ \varphi & -1 \\ \varphi & -1 \end{array} $   | $ \begin{array}{c} \varphi \\ -1 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \end{array} $  | $-8 - 11\varphi$ $-378 - 566\varphi$ $-11 + \varphi$ $-2 + 2\varphi$ $-27 + 17\varphi$ $-48 + 26\varphi$ $-3 + \varphi$ $-367 + 228\varphi$   |   |  | 1 2 2 3 1 2 2 2 1 1 ss)   | + +<br> <br>  + -<br>  + -<br>  | $   \begin{array}{c}     3, 2 \\     6, 1 \\     \hline     1, 1 \\     3, 3 \\     \hline     1, 2 \\     2, 1   \end{array} $                  | $ \begin{array}{c c} 3, 2 \\ 6, 1 \\ \hline 1, 1 \\ 3, 3 \\ \hline 1, 2 \\ 2, 1 \\ \hline 3, 1 \end{array} $   | 1,2<br>2,1<br>1,1<br>1,3<br>1,2<br>2,1<br>1,1  | $ \begin{vmatrix} I_3, I_2 \\ I_6, I_1 \end{vmatrix} $ $ \begin{vmatrix} I_1, I_1 \\ I_3, I_3 \end{vmatrix} $ $ \begin{vmatrix} I_1, I_2 \\ I_2, I_1 \end{vmatrix} $  |              |
| a1 b1 b2 c1 c2 d1 d2 e1  539                         | $ \begin{array}{c c} 1 + \varphi \\ \hline  & 1 \\ 1 + \varphi \\ \hline  & 1 \\ \hline  & 1 \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & \varphi \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & \varphi \\ \hline  & 1 \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & \varphi \\ \hline  & 1 \\ \hline  & 2 \\ \hline  & 2 \\ \hline  & 2 \\ \hline  & 3 \\ \hline  & 4 \\ \hline  &$  | $ \begin{array}{cccc} 1 & -1 + \varphi \\ \varphi & & \varphi \\ 1 & & \varphi \\ 1 & & \varphi \\ \varphi & & -1 \\ \varphi & & -1 \\ \varphi & & & \varphi \end{array} $                             | $ \begin{array}{c} \varphi \\ -1 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \end{array} $  | $-8 - 11\varphi$ $-378 - 566\varphi$ $-11 + \varphi$ $-2 + 2\varphi$ $-27 + 17\varphi$ $-48 + 26\varphi$ $-3 + \varphi$ $-367 + 228\varphi$ $5$ $-22 + 13\varphi$ $-944 + 565\varphi$   | $6 + 10\varphi$ $-4998 - 8206\varphi$ $8 - 22\varphi$ $2 - \varphi$ $-61 + 36\varphi$ $-131 + 79\varphi$ $-2$ $-3117 + 1927\varphi$ $39b = (14 - 21\varphi) = 11b \cdot 7$ $49 - 31\varphi$ $-13204 + 8205\varphi$  | 0  | 1 2 2 2 3 1 2 2 1 1 Ss) 1 2 2   | + +<br> <br>  + -<br>  + -<br>  | $ \begin{array}{c} 3,2 \\ 6,1 \\ \hline 1,1 \\ 3,3 \\ \hline 1,2 \\ \hline 2,1 \\ \hline 3,1 \\ \hline 3,2 \\ \end{array} $                      | $ \begin{array}{c c} 3, 2 \\ 6, 1 \\ \hline 1, 1 \\ 3, 3 \\ \hline 1, 2 \\ 2, 1 \\ \hline 3, 1 \end{array} $   | 1,2<br>2,1<br>1,1<br>1,3<br>1,2<br>2,1<br>1,1<br>3,1   | $\begin{array}{c c} I_3, I_2 \\ I_6, I_1 \\ \hline I_1, I_1 \\ I_3, I_3 \\ \hline I_1, I_2 \\ I_2, I_1 \\ \hline I_3, I_1 \\ \hline I_3, I_1 \\ \hline I_3, I_2 \\ \hline \end{array}$  |              |
| a1 b1 b2 c1 c2 d1 d2 e1                              | $ \begin{array}{c c} 1 + \varphi \\ \hline  & 1 \\ 1 + \varphi \\ \hline  & 1 \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & 1 + \varphi \end{array} $   | $ \begin{array}{cccc} 1 & -1 + \varphi \\ \varphi & \varphi \\ 1 & \varphi \\ 1 & \varphi \\ \varphi & -1 \\ \varphi & -1 \\ \varphi & \varphi \\ 1 & -\varphi \\ \varphi & -1 + \varphi \end{array} $ | $ \begin{array}{c} \varphi \\ 1 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \end{array} $   | $-8 - 11\varphi$ $-378 - 566\varphi$ $-11 + \varphi$ $-2 + 2\varphi$ $-27 + 17\varphi$ $-48 + 26\varphi$ $-3 + \varphi$ $-367 + 228\varphi$ $5$ $-22 + 13\varphi$ $-944 + 565\varphi$ $-11$   | $6 + 10\varphi$ $-4998 - 8206\varphi$ $8 - 22\varphi$ $2 - \varphi$ $-61 + 36\varphi$ $-131 + 79\varphi$ $-2$ $-3117 + 1927\varphi$ $39b = (14 - 21\varphi) = 11b \cdot 7$ $49 - 31\varphi$ $-13204 + 8205\varphi$ $-3 + 11\varphi$   | 0  | 1 2 2 3 1 2 2 2 1 1 ss)   | ++-   | $ \begin{array}{c} 3,2\\ 6,1\\ 1,1\\ 3,3\\ 1,2\\ 2,1\\ 3,1\\ \hline 3,1\\ 3,2\\ 6,1\\ \end{array} $  | $ \begin{array}{c c} 3, 2 \\ 6, 1 \\ \hline 1, 1 \\ 3, 3 \\ \hline 1, 2 \\ 2, 1 \\ \hline 3, 1 \\ \hline 3, 1 \\ \hline 3, 2 \\ 6, 1 \end{array} $   | $ \begin{array}{c c} 1,2 \\ 2,1 \\ \hline 1,1 \\ 1,3 \\ \hline 1,2 \\ 2,1 \\ \hline 1,1 \\ \hline 1,2 \\ 2,1 \\ \hline \end{array} $                             | $\begin{array}{c c} I_3, I_2 \\ I_6, I_1 \\ \hline I_1, I_1 \\ I_3, I_3 \\ \hline I_1, I_2 \\ I_2, I_1 \\ \hline I_3, I_1 \\ \hline I_3, I_1 \\ \hline I_3, I_2 \\ \hline I_4, I_2 \\ \hline I_5, I_1 \\ \hline I_6, I_1 \\ \hline I_6, I_1 \\ \hline \end{array}$  |              |
| a1 b1 b2 c1 c2 d1 d2 e1  539 a1 b1 b2 c1             | $ \begin{array}{c c} 1 + \varphi \\ \hline  & 1 \\ 1 + \varphi \\ \hline  & 1 \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & 1 + \varphi \end{array} $ $ \begin{array}{c c} b \\ \hline  & \varphi \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & 1 \\ \hline  & 1 \\ \hline  & 2 \\ \hline  & 2 \\ \hline  & 3 \\ \hline  & 4 \\ \hline  & 5 \\ \hline  & 5 \\ \hline  & 6 \\ \hline  & 7 \\ \hline  $  | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $ \begin{array}{c} \varphi \\ 1 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \end{array} $ $ \begin{array}{c} \varphi \\ 1 + \varphi \\ 0 \\ 0 \end{array} $                       | $-8 - 11\varphi$ $-378 - 566\varphi$ $-11 + \varphi$ $-2 + 2\varphi$ $-27 + 17\varphi$ $-48 + 26\varphi$ $-3 + \varphi$ $-367 + 228\varphi$ $5$ $-22 + 13\varphi$ $-944 + 565\varphi$ $-11$ $-2\varphi$                                     | $6 + 10\varphi$ $-4998 - 8206\varphi$ $8 - 22\varphi$ $2 - \varphi$ $-61 + 36\varphi$ $-131 + 79\varphi$ $-2$ $-3117 + 1927\varphi$ $39b = (14 - 21\varphi) = 11b \cdot 7$ $49 - 31\varphi$ $-13204 + 8205\varphi$ $-3 + 11\varphi$ $1 + \varphi$                                     | 0  | 1 2 2 3 1 2 2 2 1 1 S) 1 2 2 2 3 3 3 3 1 2 3 3 3 3 3 3 3 3 3 3          | ++-   | $ \begin{array}{c} 3,2\\ 6,1\\ 1,1\\ 3,3\\ 1,2\\ 2,1\\ 3,1\\ 3,1\\ 3,2\\ 6,1\\ 1,1\\ \end{array} $   | $ \begin{array}{c c} 3, 2 \\ 6, 1 \\ \hline 1, 1 \\ 3, 3 \\ \hline 1, 2 \\ 2, 1 \\ \hline 3, 1 \end{array} $ $ \begin{array}{c c} 3, 1 \\ \hline 3, 2 \\ 6, 1 \\ \hline 1, 1 \end{array} $         | $ \begin{array}{c c} 1,2\\2,1\\\hline 1,1\\1,3\\\hline 1,2\\2,1\\\hline 1,1\\\hline 1,2\\2,1\\\hline 1,1\\\hline 1,2\\2,1\\\hline 1,1\\\hline \end{array} $      | $\begin{array}{c c} I_3, I_2 \\ I_6, I_1 \\ \hline I_1, I_1 \\ I_3, I_3 \\ \hline I_1, I_2 \\ I_2, I_1 \\ \hline I_3, I_1 \\ \hline I_3, I_1 \\ \hline I_3, I_2 \\ \hline I_6, I_1 \\ \hline I_1, I_1 \\ \hline \end{array}$  |              |
| a1 b1 b2 c1 c2 d1 d2 e1  539 a1 b1 b2 c1 c2          | $ \begin{array}{c c} 1 + \varphi \\ \hline  & 1 \\ 1 + \varphi \\ \hline  & 1 \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & \varphi \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & 1 \\ \hline  & 2 \\ \hline  & 2 \\ \hline  & 3 \\ \hline  & 4 \\ \hline  & 4 \\ \hline  & 5 \\ \hline  & 6 \\ \hline  & 7 \\ \hline  & 1 \\ \hline  & 1 \\ \hline  & 1 \\ \hline  & 2 \\ \hline  & 2 \\ \hline  & 3 \\ \hline  & 4 \\ \hline  & 4 \\ \hline  & 2 \\ \hline  & 3 \\ \hline  & 4 \\ \hline  & 4 \\ \hline  & 5 \\ \hline  & 6 \\ \hline  & 7 \\ \hline  & 1 \\  & 1 \\ \hline  & 1 \\  & 1 \\ \hline  & 1 \\  & 1 \\ \hline  & 1 \\ \hline$   | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $\begin{array}{c} \varphi \\ -\frac{1}{0} \\ 0 \\ 0 \\ 1+\varphi \\ 1+\varphi \\ 1+\varphi \\ \end{array}$ $\begin{array}{c} \varphi \\ 1+\varphi \\ 0 \\ \end{array}$                       | $-8 - 11\varphi$ $-378 - 566\varphi$ $-11 + \varphi$ $-2 + 2\varphi$ $-27 + 17\varphi$ $-48 + 26\varphi$ $-3 + \varphi$ $-367 + 228\varphi$ $5$ $-22 + 13\varphi$ $-944 + 565\varphi$ $-11$ $-2\varphi$ $-10 - 17\varphi$                   | $6 + 10\varphi$ $-4998 - 8206\varphi$ $8 - 22\varphi$ $2 - \varphi$ $-61 + 36\varphi$ $-131 + 79\varphi$ $-2$ $-3117 + 1927\varphi$ $39b = (14 - 21\varphi) = 11b \cdot 7$ $49 - 31\varphi$ $-13204 + 8205\varphi$ $-3 + 11\varphi$ $1 + \varphi$ $-25 - 36\varphi$                   |  | $ \begin{array}{c c} 1 \\ 2 \\ 2 \\ 3 \\ 1 \\ 2 \\ 2 \\ 1 \end{array} $ | + + +       + + +     - +       + +       - +   | $ \begin{array}{c} 3,2\\ 6,1\\ 1,1\\ 3,3\\ 1,2\\ 2,1\\ 3,1\\ 3,1\\ 3,2\\ 6,1\\ 1,1\\ 3,3\\ 3,3\\ 3,3\\ 3,3\\ 3,3\\ 3,3\\ 3$                      | $ \begin{array}{c c} 3, 2 \\ 6, 1 \\ \hline 1, 1 \\ 3, 3 \\ \hline 1, 2 \\ 2, 1 \\ \hline 3, 1 \end{array} $ $ \begin{array}{c c} 3, 1 \\ \hline 3, 2 \\ 6, 1 \\ \hline 1, 1 \\ 3, 3 \end{array} $ | 1,2<br>2,1<br>1,1<br>1,3<br>1,2<br>2,1<br>1,1<br>3,1<br>1,2<br>2,1<br>1,1<br>1,3   | $\begin{array}{c c} I_3, I_2 \\ I_6, I_1 \\ \hline I_1, I_1 \\ I_3, I_3 \\ \hline I_1, I_2 \\ I_2, I_1 \\ \hline I_3, I_1 \\ \hline I_3, I_1 \\ \hline I_3, I_1 \\ \hline I_4, I_1 \\ \hline I_5, I_1 \\ \hline I_6, I_1 \\ \hline I_6, I_1 \\ \hline I_7, I_1 \\ \hline I_8, I_3 \\ \hline I_9, I_1 \\ \hline I_$ |              |
| a1 b1 b2 c1 c2 d1 d2 e1  539 a1 b1 b2 c1 c2 d1 c2 d1 | $ \begin{array}{c c} 1 + \varphi \\ \hline  & 1 \\ 1 + \varphi \\ \hline  & 1 \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & 1 \\ \hline  & \varphi \\ \hline  & \varphi \\ \hline  & 1 \\ \hline  &$  | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $\begin{array}{c} \varphi \\ 1 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \\ \hline 1 + \varphi \\ 0 \\ 0 \\ \varphi \end{array}$  | $-8 - 11\varphi$ $-378 - 566\varphi$ $-11 + \varphi$ $-2 + 2\varphi$ $-27 + 17\varphi$ $-48 + 26\varphi$ $-3 + \varphi$ $-367 + 228\varphi$ $5$ $-22 + 13\varphi$ $-944 + 565\varphi$ $-11$ $-2\varphi$ $-10 - 17\varphi$ $-21 - 28\varphi$ | $6 + 10\varphi$ $-4998 - 8206\varphi$ $8 - 22\varphi$ $2 - \varphi$ $-61 + 36\varphi$ $-131 + 79\varphi$ $-2$ $-3117 + 1927\varphi$ $39b = (14 - 21\varphi) = 11b \cdot 7$ $49 - 31\varphi$ $-13204 + 8205\varphi$ $-3 + 11\varphi$ $1 + \varphi$ $-25 - 36\varphi$ $-51 - 80\varphi$ | (5 isogeny classes 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$                   | + + +     + +     + +     + +     + +     + +     + +     + +     + +     + +       + + | $ \begin{array}{c} 3,2\\ 6,1\\ \hline 1,1\\ 3,3\\ \hline 1,2\\ 2,1\\ \hline 3,1\\ \hline 3,2\\ 6,1\\ \hline 1,1\\ 3,3\\ \hline 1,2 \end{array} $ | $\begin{array}{c c} 3,2\\ 6,1\\ \hline 1,1\\ 3,3\\ \hline 1,2\\ 2,1\\ \hline 3,1\\ \hline 3,2\\ 6,1\\ \hline 1,1\\ 3,3\\ \hline 1,2\\ \end{array}$   | $ \begin{array}{c c} 1,2\\2,1\\\hline 1,1\\1,3\\\hline 1,2\\2,1\\\hline 1,1\\\hline 1,2\\2,1\\\hline 1,1\\\hline 1,3\\\hline 1,2\\1,3\\\hline 1,2\\\end{array} $ | $\begin{array}{c c} I_3, I_2 \\ I_6, I_1 \\ \hline I_1, I_1 \\ I_3, I_3 \\ \hline I_1, I_2 \\ I_2, I_1 \\ \hline I_3, I_1 \\ \hline I_3, I_1 \\ \hline I_3, I_1 \\ \hline I_3, I_2 \\ I_6, I_1 \\ \hline I_3, I_3 \\ \hline I_1, I_2 \\ \hline I_1, I_2 \\ \hline I_2, I_1 \\ \hline I_3, I_2 \\ \hline I_3, I_2 \\ \hline I_3, I_2 \\ \hline I_4, I_1 \\ \hline I_5, I_1 \\ \hline I_5, I_1 \\ \hline I_5, I_2 \\ \hline I_6, I_1 \\ \hline I_7, I_1 \\ \hline I_8, I_2 \\ \hline I_9, I_1 \\ \hline I_9,$      |              |
| a1 b1 b2 c1 c2 d1 d2 e1  539 a1 b1 b2 c1 c2          | $\begin{vmatrix} 1+\varphi \\ 1 \\ 1+\varphi \end{vmatrix}$  | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $ \begin{array}{c} \varphi \\ 1 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \end{array} $ $ \begin{array}{c} \varphi \\ 1 + \varphi \\ 0 \\ 0 \\ \varphi \\ \varphi \end{array} $ | $-8 - 11\varphi$ $-378 - 566\varphi$ $-11 + \varphi$ $-2 + 2\varphi$ $-27 + 17\varphi$ $-48 + 26\varphi$ $-3 + \varphi$ $-367 + 228\varphi$ $5$ $-22 + 13\varphi$ $-944 + 565\varphi$ $-11$ $-2\varphi$ $-10 - 17\varphi$                   | $6 + 10\varphi$ $-4998 - 8206\varphi$ $8 - 22\varphi$ $2 - \varphi$ $-61 + 36\varphi$ $-131 + 79\varphi$ $-2$ $-3117 + 1927\varphi$ $39b = (14 - 21\varphi) = 11b \cdot 7$ $49 - 31\varphi$ $-13204 + 8205\varphi$ $-3 + 11\varphi$ $1 + \varphi$ $-25 - 36\varphi$                   |  | $ \begin{array}{c c} 1 \\ 2 \\ 2 \\ 3 \\ 1 \\ 2 \\ 2 \\ 1 \end{array} $ | + + +       + + +     - +       + +       - +   | 3, 2<br>6, 1<br>1, 1<br>3, 3<br>1, 2<br>2, 1<br>3, 1<br>3, 2<br>6, 1<br>1, 1<br>3, 3<br>1, 2<br>2, 1<br>1, 1<br>3, 3<br>2, 1<br>2, 1<br>1, 1     | $ \begin{array}{c c} 3, 2 \\ 6, 1 \\ \hline 1, 1 \\ 3, 3 \\ \hline 1, 2 \\ 2, 1 \\ \hline 3, 1 \end{array} $ $ \begin{array}{c c} 3, 1 \\ \hline 3, 2 \\ 6, 1 \\ \hline 1, 1 \\ 3, 3 \end{array} $ | 1,2<br>2,1<br>1,1<br>1,3<br>1,2<br>2,1<br>1,1<br>3,1<br>1,2<br>2,1<br>1,1<br>1,3   | $\begin{array}{c c} I_3, I_2 \\ I_6, I_1 \\ \hline I_1, I_1 \\ I_3, I_3 \\ \hline I_1, I_2 \\ I_2, I_1 \\ \hline I_3, I_1 \\ \hline I_3, I_1 \\ \hline I_3, I_1 \\ \hline I_4, I_1 \\ \hline I_5, I_1 \\ \hline I_6, I_1 \\ \hline I_6, I_1 \\ \hline I_7, I_1 \\ \hline I_8, I_3 \\ \hline I_9, I_1 \\ \hline I_$ |              |

|  |  |   |  |   |   |  |  | _  |  | 1  |   | 1   |           |
|--|--|---|--|---|---|--|--|--|--|--|---|---|-----------|
|  | $a_1$  | $a_2$   | $a_3$  | $a_4$   | $a_6$   | r                                      | T  | s  | $\operatorname{ord}(\Delta)$   | $\operatorname{ord}_{-}(j)$  | $c_p$   | Kodaira   | Isogenies |
| 541  | a  |   |  |   | $541a = (22 + 3\varphi) = 541a$   | (1 isogeny class)                      |  |  |  |  |   |   | 541a      |
| a1   | $1+\varphi$  | $-\varphi$  | $\varphi$  | $-\varphi$  | 0   | 1                                      | 3  |  | 1  | 1  | 1   | $I_1$   |           |
| a2   | $1+\varphi$  | $-\varphi$  | $\varphi$  | $-5-\varphi$  | $-9-6\varphi$   | 1                                      | 1  |  | 3  | 3  | 3   | $I_3$   |           |
| 541  | b  |   |  |   | $541b = (25 - 3\varphi) = 541b$   | (1 isogeny class)                      |  |  |  |  |   |   | 541b      |
| a1   | $\varphi$  |   | $1+\varphi$  | -arphi  | $-\varphi$  | 1                                      | 3  |  | 1  | 1  | 1   | $I_1$   |           |
| a2   | $\varphi$  | 0 ]   | $1+\varphi$  | $-5-\varphi$  | $-15 + 5\varphi$  | 1                                      | 1  |  | 3  | 3  | 3   | $I_3$   |           |
| 545  | a  |   |  | 545   | $5a = (8 - 21\varphi) = 5a \cdot 109b$  | (3 isogeny classe                      | es)  |  |  |  |   |   | 545a      |
| a1   | $\varphi$  | $\varphi$   | $\varphi$  | -2  | $-2\varphi$   | 0                                      | 4  | ++   | 1,1  | 1,1  | 1,1   | $I_1, I_1$  |           |
| a2   | $\varphi$  | $\varphi$   | $\varphi$  | $-12-15\varphi$   | $-31-51\varphi$   | 0                                      | 4  | ++   | 2, 2   | 2, 2   | 2,2   | $I_2, I_2$  |           |
| a3   | $\varphi$  | $\varphi$   | $\varphi$  | $-2-20\varphi$  | $-34-54\varphi$   | 0                                      | 4  |  | 4, 4   | 4, 4   | 4,4   | $I_4, I_4$  |           |
| a4   | $1+\varphi$  | $1-\varphi$ 1   | $1+\varphi$  | $-1115 - 1799\varphi$   | $-27554 - 44575\varphi$   | 0                                      | 2  | ++   | 1,1  | 1,1  | 1,1   | $I_1, I_1$  | <u> </u>  |
| b1   | 1  | $\varphi$   | 0  | $-2+\varphi$  | 0   | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 6  | +++  | 1,3  | 1,3  | 1,3   | $I_1, I_3$  |           |
| b2   | 1  | arphi   | 0  | $8-4\varphi$  | $-2+3\varphi$   | 0                                      | 6  |  | 2,6  | 2,6  | 2,6   | $I_2, I_6$  |           |
| b3   | 1  | $\varphi$   | 0  | $-122 + 66\varphi$  | $-574 + 333\varphi$   | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 2  | ++   | 3, 1   | 3, 1   | $\begin{bmatrix} 3,1\\c \end{matrix}$   | $I_3, I_1$  |           |
| b4   | 1  | <del></del>   | 0  | $-117 + 71\varphi$  | $-597 + 328\varphi$   | 0                                      | 2  | <u> </u>                                     | 6,2  | 6,2  | 6,2   | $I_6, I_2$  |           |
| c1   | $\varphi$  | 1   | $\varphi$  | $-34 + 19\varphi$   | $-99 + 61\varphi$   | 0                                      | $\frac{2}{2}$  | +++  | 1,5  | 1,5  | 1,1   | $I_1, I_5$  |           |
| c2   | φ  | 1   | φ  | $-29 + 19\varphi$   | $-115 + 76\varphi$  | 0                                      | 2  |  | 2, 10  | 2, 10  | 2,2   | $I_2, I_{10}$   |           |
| 545  | •  |   |  |   |   |  |  |  |  |  |   |   |           |
| 343  | b  |   |  |   | $b = (13 - 21\varphi) = 5a \cdot 109a$  | (3 isogeny class                       | ses)   | ,  |  |  |   |   | 545b      |
| a1   | $1+\varphi$  | $1 + \varphi$   | φ  | $-1+2\varphi$   | $-1+\varphi$  | 0                                      | 4  | ++   | 1,1  | 1, 1   | 1,1   | $I_1, I_1$  | 545b      |
| a1<br>a2   | $\begin{array}{c} 1+\varphi \\ 1+\varphi \end{array}$  | $1+\varphi$   | $\varphi$  | $-1 + 2\varphi$ $-26 + 17\varphi$   | $ \begin{array}{c} -1+\varphi\\ -66+40\varphi \end{array} $   | 0 0                                    | 4 4  | ++   | 2, 2   | 2, 2   | 2,2   | $\mathrm{I}_2,\mathrm{I}_2$   | 545b      |
| a1<br>a2<br>a3   | $\begin{array}{c} 1+\varphi\\ 1+\varphi\\ \varphi \end{array}$   | $1 + \varphi$ $1$   |  | $-1 + 2\varphi \\ -26 + 17\varphi \\ -2912 + 1797\varphi$   | $-1 + \varphi \\ -66 + 40\varphi \\ -72128 + 44574\varphi$  | 0 0 0                                  | 4 4 2  | ++++   | $2, 2 \\ 1, 1$   | $2, 2 \\ 1, 1$   | $ \begin{array}{c c} 2, 2 \\ 1, 1 \end{array} $   | $\begin{matrix} I_2,I_2\\I_1,I_1\end{matrix}$   | 545b      |
| a1<br>a2<br>a3<br>a4   | $\begin{array}{c} 1+\varphi \\ 1+\varphi \end{array}$  | $1 + \varphi$ $1$ $1 + \varphi$   | $\varphi$ $\varphi$  | $-1 + 2\varphi$ $-26 + 17\varphi$ $-2912 + 1797\varphi$ $-21 + 22\varphi$   | $   \begin{array}{r}     -1 + \varphi \\     -66 + 40\varphi \\     -72128 + 44574\varphi \\     -67 + 53\varphi   \end{array} $  | 0<br>0<br>0<br>0                       | 4 4 2 4  | ++   | 2, 2<br>1, 1<br>4, 4   | 2, 2<br>1, 1<br>4, 4   | $ \begin{array}{ c c c } 2,2 \\ 1,1 \\ 4,4 \\ \hline \end{array} $  | $\begin{bmatrix} I_2, I_2 \\ I_1, I_1 \\ I_4, I_4 \end{bmatrix}$  | 545b      |
| a1<br>a2<br>a3<br>a4<br>b1   | $ \begin{array}{c} 1 + \varphi \\ 1 + \varphi \\ \varphi \\ 1 + \varphi \end{array} $  | $1 + \varphi$ $1$ $\frac{1 + \varphi}{1 - \varphi}$   | $\varphi$ $\varphi$ $\varphi$ $\varphi$ $0$  | $ \begin{array}{r} -1 + 2\varphi \\ -26 + 17\varphi \\ -2912 + 1797\varphi \\ -21 + 22\varphi \\ -1 - \varphi \end{array} $   | $   \begin{array}{r}     -1 + \varphi \\     -66 + 40\varphi \\     -72128 + 44574\varphi \\     -67 + 53\varphi \\     \hline     0   \end{array} $  | 0 0 0 0 0 0 0 0                        | 4 4 2 4 5 6  | + +<br>  + +<br> <br>  + +                   | $ \begin{array}{c} 2, 2 \\ 1, 1 \\ 4, 4 \\ \hline 1, 3 \end{array} $   | $ \begin{array}{ c c c c } 2,2 \\ 1,1 \\ 4,4 \\ \hline 1,3 \end{array} $   | $ \begin{array}{ c c c } 2,2 \\ 1,1 \\ 4,4 \\ \hline 1,3 \end{array} $  | $\begin{array}{ c c c c c }\hline I_2, I_2 \\ I_1, I_1 \\ I_4, I_4 \\ \hline I_1, I_3 \\ \hline \end{array}$  | 545b      |
| a1<br>a2<br>a3<br>a4<br>b1<br>b2   | $\begin{array}{c} 1+\varphi\\ 1+\varphi\\ \varphi \end{array}$   | $1 + \varphi$ $1$ $1 + \varphi$ $1 - \varphi$ $1 - \varphi$   | $\varphi \\ \varphi \\ -\frac{\varphi}{0} \\ 0$  | $-1 + 2\varphi$ $-26 + 17\varphi$ $-2912 + 1797\varphi$ $-21 + 22\varphi$ $-1 - \varphi$ $4 + 4\varphi$   | $   \begin{array}{r}     -1 + \varphi \\     -66 + 40\varphi \\     -72128 + 44574\varphi \\     -67 + 53\varphi \\     \hline     0 \\     1 - 3\varphi   \end{array} $  | 0<br>0<br>0<br>0<br>0                  | $ \begin{array}{ c c c } \hline  & 4 & \\  & 4 & \\  & 2 & \\  & 4 & \\  & & 6 & \\ \hline  & 6 & \\ \hline  & 6 & \\ \hline \end{array} $ | + +<br>  + +<br> <br>  + +<br>               | $ \begin{array}{c} 2, 2 \\ 1, 1 \\ 4, 4 \\ \hline 1, 3 \\ 2, 6 \end{array} $   | $ \begin{array}{c cccc} 2, 2 \\ 1, 1 \\ 4, 4 \\ \hline 1, 3 \\ 2, 6 \end{array} $  | $ \begin{array}{ c c c } 2,2 \\ 1,1 \\ 4,4 \\ \hline 1,3 \\ 2,6 \end{array} $   | $\begin{array}{ c c c c }\hline I_2, I_2 \\ I_1, I_1 \\ \hline I_4, I_4 \\ \hline \hline I_1, I_3 \\ I_2, I_6 \\ \end{array}$   | 545b      |
| a1<br>a2<br>a3<br>a4<br>b1<br>b2<br>b3   | $ \begin{array}{c} 1 + \varphi \\ 1 + \varphi \\ \varphi \\ 1 + \varphi \end{array} $  | $1 + \varphi$ $1$ $1 + \varphi$ $1 - \varphi$ $1 - \varphi$ $1 - \varphi$   | $ \begin{array}{c} \varphi \\ \varphi \\ \varphi \\ \hline 0 \\ 0 \\ 0 \end{array} $                                       | $-1 + 2\varphi$ $-26 + 17\varphi$ $-2912 + 1797\varphi$ $-21 + 22\varphi$ $-1 - \varphi$ $4 + 4\varphi$ $-56 - 66\varphi$   | $   \begin{array}{r}     -1 + \varphi \\     -66 + 40\varphi \\     -72128 + 44574\varphi \\     -67 + 53\varphi \\     \hline     0 \\     1 - 3\varphi \\     -241 - 333\varphi $   | 0<br>0<br>0<br>0<br>0                  | $ \begin{array}{ c c c c } \hline  & 4 & 4 & \\  & 4 & 2 & \\  & 4 & \\  & & 6 & \\  & 6 & & 2 & \\ \hline \end{array} $                   | +++++  | $ \begin{array}{c} 2, 2 \\ 1, 1 \\ 4, 4 \\ \hline 1, 3 \\ 2, 6 \\ 3, 1 \end{array} $   | $\begin{array}{c c} 2,2 \\ 1,1 \\ 4,4 \\ \hline \\ 1,3 \\ 2,6 \\ 3,1 \end{array}$  | $ \begin{array}{ c c c } 2,2 \\ 1,1 \\ 4,4 \\ \hline 1,3 \\ 2,6 \\ 3,1 \end{array} $  | $\begin{array}{c c} I_2, I_2 \\ I_1, I_1 \\ I_4, I_4 \\ \hline -I_1, I_3 \\ I_2, I_6 \\ I_3, I_1 \end{array}$   | 545b      |
| a1<br>a2<br>a3<br>a4<br>b1<br>b2<br>b3<br>b4   | $ \begin{array}{c} 1+\varphi \\ 1+\varphi \\ \varphi \\ 1+\varphi \\ 1\\ 1\\ 1\\ 1 \end{array} $   | $1 + \varphi$ $1$ $1 + \varphi$ $1 - \varphi$ $1 - \varphi$ $1 - \varphi$ $1 - \varphi$   | $ \varphi \\ \varphi \\ \varphi \\ -\overline{0} \\ 0 \\ 0 \\ 0 $  | $-1 + 2\varphi$ $-26 + 17\varphi$ $-2912 + 1797\varphi$ $-21 + 22\varphi$ $-1 - \varphi$ $4 + 4\varphi$ $-56 - 66\varphi$ $-46 - 71\varphi$   | $-1 + \varphi$ $-66 + 40\varphi$ $-72128 + 44574\varphi$ $-67 + 53\varphi$ $0$ $1 - 3\varphi$ $-241 - 333\varphi$ $-269 - 328\varphi$   | 0<br>0<br>0<br>0<br>0                  | $ \begin{array}{ c c c } \hline 4 & 4 \\ 2 & 4 \\ \hline 6 & 6 \\ 2 & 2 \end{array} $  | +++  | $ \begin{array}{c} 2,2 \\ 1,1 \\ 4,4 \\ \hline 1,3 \\ 2,6 \\ 3,1 \\ 6,2 \end{array} $  | $\begin{array}{ c c c }\hline 2,2\\ 1,1\\ 4,4\\\hline &1,3\\ 2,6\\ 3,1\\ 6,2\\\hline \end{array}$  | $ \begin{array}{ c c c } 2,2 \\ 1,1 \\ 4,4 \\ \hline 1,3 \\ 2,6 \\ 3,1 \\ 6,2 \\ \end{array} $  | $\begin{array}{ c c c }\hline I_2, I_2\\ I_1, I_1\\ I_4, I_4\\\hline\hline I_1, I_3\\ I_2, I_6\\ I_3, I_1\\ I_6, I_2\\\hline\end{array}$  | 545b      |
| a1<br>a2<br>a3<br>a4<br>b1<br>b2<br>b3<br>b4<br>c1   | $ \begin{array}{c} 1+\varphi \\ 1+\varphi \\ \varphi \\ 1+\varphi \\ 1\\ 1\\ 1\\ 1+\varphi \\ 1+\varphi$  | $1 + \varphi$ $1$ $1 + \varphi$ $1 - \varphi$   | $ \varphi \\ \varphi \\ \varphi \\ 0 \\ 0 \\ 0 \\ 1 + \varphi $  | $-1 + 2\varphi$ $-26 + 17\varphi$ $-2912 + 1797\varphi$ $-21 + 22\varphi$ $-1 - \varphi$ $4 + 4\varphi$ $-56 - 66\varphi$ $-46 - 71\varphi$ $-15 - 21\varphi$   | $-1 + \varphi$ $-66 + 40\varphi$ $-72128 + 44574\varphi$ $-67 + 53\varphi$ $0$ $1 - 3\varphi$ $-241 - 333\varphi$ $-269 - 328\varphi$ $-38 - 62\varphi$   |  | $ \begin{array}{ c c c } \hline 4 & 4 & \\ 2 & 4 & \\ \hline 6 & 6 & \\ 2 & 2 & \\ \hline 2 & \\ \hline \end{array} $                      | + +<br>  + +<br> <br>  + +<br> <br>  + +     | $ \begin{array}{c} 2,2 \\ 1,1 \\ 4,4 \\ \hline 1,3 \\ 2,6 \\ 3,1 \\ 6,2 \\ \hline 1,5 \end{array} $  | $ \begin{array}{c c} 2,2\\ 1,1\\ 4,4 \end{array} $ $ \begin{array}{c c} 1,3\\2,6\\3,1\\6,2\\ \end{array} $ $ \begin{array}{c c} 1,5 \end{array} $  | $ \begin{array}{ c c c } 2,2\\ 1,1\\ 4,4\\ \hline 1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline 1,1\\ \end{array} $  | $\begin{array}{ c c c c }\hline & I_2, I_2 \\ & I_1, I_1 \\ & I_4, I_4 \\ \hline & I_1, I_3 \\ & I_2, I_6 \\ & I_3, I_1 \\ & I_6, I_2 \\ \hline & I_1, I_5 \\ \hline \end{array}$   | 545b      |
| a1<br>a2<br>a3<br>a4<br>b1<br>b2<br>b3<br>b4   | $ \begin{array}{c} 1+\varphi \\ 1+\varphi \\ \varphi \\ 1+\varphi \\ 1\\ 1\\ 1\\ 1 \end{array} $   | $1 + \varphi$ $1$ $1 + \varphi$ $1 - \varphi$ $1 - \varphi$ $1 - \varphi$ $1 - \varphi$   | $ \varphi \\ \varphi \\ \varphi \\ 0 \\ 0 \\ 0 \\ 1 + \varphi $  | $-1 + 2\varphi$ $-26 + 17\varphi$ $-2912 + 1797\varphi$ $-21 + 22\varphi$ $-1 - \varphi$ $4 + 4\varphi$ $-56 - 66\varphi$ $-46 - 71\varphi$   | $-1 + \varphi$ $-66 + 40\varphi$ $-72128 + 44574\varphi$ $-67 + 53\varphi$ $0$ $1 - 3\varphi$ $-241 - 333\varphi$ $-269 - 328\varphi$   | 0<br>0<br>0<br>0<br>0                  | $ \begin{array}{ c c c } \hline 4 & 4 \\ 2 & 4 \\ \hline 6 & 6 \\ 2 & 2 \end{array} $  | +++  | $ \begin{array}{c} 2,2 \\ 1,1 \\ 4,4 \\ \hline 1,3 \\ 2,6 \\ 3,1 \\ 6,2 \end{array} $  | $\begin{array}{ c c c }\hline 2,2\\ 1,1\\ 4,4\\\hline &1,3\\ 2,6\\ 3,1\\ 6,2\\\hline \end{array}$  | $ \begin{array}{ c c c } 2,2 \\ 1,1 \\ 4,4 \\ \hline 1,3 \\ 2,6 \\ 3,1 \\ 6,2 \\ \end{array} $  | $\begin{array}{ c c c }\hline I_2, I_2\\ I_1, I_1\\ I_4, I_4\\\hline\hline I_1, I_3\\ I_2, I_6\\ I_3, I_1\\ I_6, I_2\\\hline\end{array}$  | 545b      |
| a1<br>a2<br>a3<br>a4<br>b1<br>b2<br>b3<br>b4<br>c1   | $ \begin{array}{c} 1+\varphi \\ 1+\varphi \\ \varphi \\ 1+\varphi \\ 1\\ 1\\ 1\\ 1+\varphi \\ 1+\varphi \\ 1+\varphi \\ \end{array} $  | $1 + \varphi$ $1$ $1 + \varphi$ $1 - \varphi$   | $ \varphi \\ \varphi \\ \varphi \\ 0 \\ 0 \\ 0 \\ 1 + \varphi $  | $-1 + 2\varphi$ $-26 + 17\varphi$ $-2912 + 1797\varphi$ $-21 + 22\varphi$ $-1 - \varphi$ $4 + 4\varphi$ $-56 - 66\varphi$ $-46 - 71\varphi$ $-15 - 21\varphi$ $-10 - 21\varphi$   | $   \begin{array}{r}     -1 + \varphi \\     -66 + 40\varphi \\     -72128 + 44574\varphi \\     -67 + 53\varphi \\     \hline     0 \\     1 - 3\varphi \\     -241 - 333\varphi \\     -269 - 328\varphi \\     \hline     -38 - 62\varphi \\     -39 - 77\varphi   \end{array} $ |  | $ \begin{array}{ c c c } \hline 4 & 4 & \\ 2 & 4 & \\ \hline 6 & 6 & \\ 2 & 2 & \\ \hline 2 & 2 & \\ \hline \end{array} $                  | + +<br>  + +<br> <br>  + +<br> <br>  + +     | $ \begin{array}{c} 2,2 \\ 1,1 \\ 4,4 \\ \hline 1,3 \\ 2,6 \\ 3,1 \\ 6,2 \\ \hline 1,5 \end{array} $  | $ \begin{array}{c c} 2,2\\ 1,1\\ 4,4 \end{array} $ $ \begin{array}{c c} 1,3\\2,6\\3,1\\6,2\\ \end{array} $ $ \begin{array}{c c} 1,5 \end{array} $  | $ \begin{array}{ c c c } 2,2\\ 1,1\\ 4,4\\ \hline 1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline 1,1\\ \end{array} $  | $\begin{array}{ c c c c }\hline & I_2, I_2 \\ & I_1, I_1 \\ & I_4, I_4 \\ \hline & I_1, I_3 \\ & I_2, I_6 \\ & I_3, I_1 \\ & I_6, I_2 \\ \hline & I_1, I_5 \\ \hline \end{array}$   | 545b      |
| a1<br>a2<br>a3<br>a4<br>b1<br>b2<br>b3<br>b4<br>c1<br>c2   | $ \begin{array}{c} 1+\varphi \\ 1+\varphi \\ \varphi \\ 1+\varphi \\ 1 \\ 1 \\ 1 \\ 1+\varphi \\ 1+\varphi \\ \mathbf{a} \end{array} $   | $1 + \varphi$ $1$ $1 + \varphi$ $1 - \varphi$   | $ \varphi \\ \varphi \\ \varphi \\ 0 \\ 0 \\ 0 \\ 1 + \varphi $  | $-1 + 2\varphi$ $-26 + 17\varphi$ $-2912 + 1797\varphi$ $-21 + 22\varphi$ $-1 - \varphi$ $4 + 4\varphi$ $-56 - 66\varphi$ $-46 - 71\varphi$ $-15 - 21\varphi$ $-10 - 21\varphi$   | $   \begin{array}{r}     -1 + \varphi \\     -66 + 40\varphi \\     -72128 + 44574\varphi \\     -67 + 53\varphi \\     \hline     0 \\     1 - 3\varphi \\     -241 - 333\varphi \\     -269 - 328\varphi \\     \hline     -38 - 62\varphi \\     -39 - 77\varphi   \end{array} $ | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0   | 4 4 2 4 4 6 6 6 2 2 2 2 es)  | + +<br>  + +<br> <br>  + +<br> <br>  + +     | $ \begin{array}{c} 2,2 \\ 1,1 \\ 4,4 \\ \hline 1,3 \\ 2,6 \\ 3,1 \\ 6,2 \\ \hline 1,5 \end{array} $  | $ \begin{array}{c c} 2,2\\ 1,1\\ 4,4 \end{array} $ $ \begin{array}{c c} 1,3\\2,6\\3,1\\6,2\\ \end{array} $ $ \begin{array}{c c} 1,5 \end{array} $  | $ \begin{array}{ c c c } 2,2\\ 1,1\\ 4,4\\ \hline 1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline 1,1\\ \end{array} $  | $\begin{array}{ c c c c }\hline & I_2, I_2 \\ & I_1, I_1 \\ & I_4, I_4 \\ \hline & I_1, I_3 \\ & I_2, I_6 \\ & I_3, I_1 \\ & I_6, I_2 \\ \hline & I_1, I_5 \\ \hline \end{array}$   |           |
| a1 a2 a3 a4 b1 b2 b3 b4 c1 c2 5493   | $ \begin{array}{c} 1+\varphi\\1+\varphi\\\varphi\\1+\varphi\\1\\1\\1\\1+\varphi\\1+\varphi\end{array} $ $ \begin{array}{c} 1\\1\\1\\1\\1+\varphi\\1+\varphi\end{array} $   | $1 + \varphi$ $1$ $1 + \varphi$ $1 - \varphi$   | $ \varphi \\ \varphi \\ \varphi \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 + \varphi $  | $-1 + 2\varphi$ $-26 + 17\varphi$ $-2912 + 1797\varphi$ $-21 + 22\varphi$ $-1 - \varphi$ $4 + 4\varphi$ $-56 - 66\varphi$ $-46 - 71\varphi$ $-15 - 21\varphi$ $-10 - 21\varphi$ $54$  | $-1 + \varphi$ $-66 + 40\varphi$ $-72128 + 44574\varphi$ $-67 + 53\varphi$ $0$ $1 - 3\varphi$ $-241 - 333\varphi$ $-269 - 328\varphi$ $-38 - 62\varphi$ $-39 - 77\varphi$ $9a = (12 - 21\varphi) = 3 \cdot 61a$   | 0                                      | 4 4 4 2 4 1 6 6 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2  | + +<br>  + +<br> <br>  + +<br> <br>  + +<br> | $ \begin{array}{c} 2, 2 \\ 1, 1 \\ 4, 4 \\ \hline 1, 3 \\ 2, 6 \\ 3, 1 \\ 6, 2 \\ \hline 1, 5 \\ 2, 10 \end{array} $   | $\begin{array}{ c c c } 2,2\\ 1,1\\ 4,4\\ \hline 1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline 1,5\\ 2,10\\ \end{array}$  | $ \begin{array}{c c} 2,2\\ 1,1\\ 4,4\\ \hline 1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline 1,1\\ 2,2\\ \end{array} $  | $\begin{array}{c c} I_2, I_2 \\ I_1, I_1 \\ I_4, I_4 \\ \hline & I_1, I_3 \\ \hline & I_2, I_6 \\ I_3, I_1 \\ \hline & I_6, I_2 \\ \hline & I_1, I_5 \\ \hline & I_2, I_{10} \\ \end{array}$  |           |
| a1 a2 a3 a4 b1 b2 b3 b4 c1 c2 5493   | $ \begin{array}{c} 1+\varphi\\1+\varphi\\\varphi\\1+\varphi\\1\\1\\1\\1+\varphi\\1+\varphi\end{array} $ $ \begin{array}{c} 1\\1\\1\\1\\1+\varphi\\1+\varphi\end{array} $   | $1 + \varphi$ $1$ $1 + \varphi$ $1 - \varphi$   | $ \varphi \\ \varphi \\ \varphi \\ 0 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 + \varphi $   | $-1 + 2\varphi$ $-26 + 17\varphi$ $-2912 + 1797\varphi$ $-21 + 22\varphi$ $-1 - \varphi$ $4 + 4\varphi$ $-56 - 66\varphi$ $-46 - 71\varphi$ $-15 - 21\varphi$ $-10 - 21\varphi$ $54$  |   | 0                                      | 4 4 4 2 4 1 6 6 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2  | + +<br>  + +<br> <br>  + +<br> <br>  + +<br> | $ \begin{array}{c} 2, 2 \\ 1, 1 \\ 4, 4 \\ \hline 1, 3 \\ 2, 6 \\ 3, 1 \\ 6, 2 \\ \hline 1, 5 \\ 2, 10 \end{array} $   | $ \begin{array}{c c} 2,2\\ 1,1\\ 4,4\\ \hline & 1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline & 1,5\\ 2,10\\ \hline & 1,1\\ \hline & 5,1\\ \hline \end{array} $   | $ \begin{array}{c c} 2,2\\ 1,1\\ 4,4\\ \hline \\ 1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline \\ 1,1\\ 2,2\\ \hline \end{array} $   | $ \begin{array}{c c} I_2, I_2 \\ I_1, I_1 \\ I_4, I_4 \\ \hline & I_1, I_3 \\ I_2, I_6 \\ I_3, I_1 \\ I_6, I_2 \\ \hline & I_1, I_5 \\ I_2, I_{10} \\ \hline \\ & I_1, I_1 \\ \hline & I_5, I_1 \\ \hline \end{array} $   |           |
| a1 a2 a3 a4 b1 b2 b3 b4 c1 c2 549  | $ \begin{array}{c} 1+\varphi\\1+\varphi\\\varphi\\1+\varphi\\1\\1\\1\\1\\1+\varphi\\1+\varphi\end{array} $ $ \begin{array}{c} 1\\1\\1\\1\\1\\1\\1\\1\\1\end{array} $   | $1 + \varphi$ $1$ $1 + \varphi$ $1 - \varphi$   | $ \begin{array}{cccc} \varphi \\ \varphi \\ -\frac{\varphi}{0} \\ 0 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 + \varphi \end{array} $  | $-1 + 2\varphi$ $-26 + 17\varphi$ $-2912 + 1797\varphi$ $-21 + 22\varphi$ $-1 - \varphi$ $4 + 4\varphi$ $-56 - 66\varphi$ $-46 - 71\varphi$ $-15 - 21\varphi$ $-10 - 21\varphi$ $54$ $-1 - \varphi$ $-4 + 6\varphi$ $-48 + 16\varphi$   |   | 0                                      | 4   4   2   4   6   6   2   2   2   2   2   1   1   1   1   1  | + +<br>  + +<br> <br>  + +<br> <br>  + +<br> | $ \begin{array}{c} 2,2 \\ 1,1 \\ 4,4 \\ \hline 1,3 \\ 2,6 \\ 3,1 \\ 6,2 \\ \hline 1,5 \\ 2,10 \end{array} $  | $ \begin{array}{c cccc} 2, 2 \\ 1, 1 \\ 4, 4 \end{array} $ $ \begin{array}{c cccc} 1, 3 \\ 2, 6 \\ 3, 1 \\ 6, 2 \end{array} $ $ \begin{array}{c cccc} 1, 5 \\ 2, 10 \end{array} $        | 2,2<br>1,1<br>4,4<br>1,3<br>2,6<br>3,1<br>6,2<br>1,1<br>2,2   | $\begin{array}{ c c c c }\hline I_2,I_2\\I_1,I_1\\I_4,I_4\\\hline\hline I_1,I_3\\I_2,I_6\\I_3,I_1\\I_6,I_2\\\hline\hline I_1,I_5\\I_2,I_{10}\\\hline\hline I_1,I_5\\I_2,I_{10}\\\hline\hline I_1,I_1\\\hline\hline I_1,I_1\\\hline I_1,I_1\\\hline\hline I_1,I_1\\\hline I_1,I_1\\\hline\hline I_1,I_1\\\hline I$ |           |
| a1 a2 a3 a4 b1 b2 b3 b4 c1 c2 5498   | $ \begin{array}{c} 1+\varphi\\1+\varphi\\\varphi\\1+\varphi\\1\\1\\1\\1\\1+\varphi\\1+\varphi\end{array} $ $ \begin{array}{c} 1\\1\\\varphi\\\varphi\\\varphi \end{array} $  | $1 + \varphi$ $1$ $1 + \varphi$ $1 - \varphi$   | $ \begin{array}{c} \varphi \\ \varphi \\ -\frac{\varphi}{0} \\ 0 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 + \varphi \end{array} $     | $ \begin{array}{r} -1 + 2\varphi \\ -26 + 17\varphi \\ -2912 + 1797\varphi \\ -21 + 22\varphi \\ -1 - \varphi \\ 4 + 4\varphi \\ -56 - 66\varphi \\ -46 - 71\varphi \\ -15 - 21\varphi \\ -10 - 21\varphi \end{array} $ $ \begin{array}{r} 54 \\ -1 - \varphi \\ -4 + 6\varphi \\ -48 + 16\varphi \\ 7 - 4\varphi \end{array} $ |   | 0   0   0   0   0   0   0   0   0   0  | $ \begin{array}{ c c c c c } \hline  & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & $   | + +<br>  + +<br> <br>  + +<br> <br>  + +<br> | $ \begin{array}{c} 2,2\\ 1,1\\ 4,4\\ \hline 1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline 1,5\\ 2,10 \end{array} $ $ \begin{array}{c} 1,1\\ 5,1\\ \hline 10,1\\ 5,2\\ \end{array} $   | $\begin{array}{ c c c }\hline 2,2\\ 1,1\\ 4,4\\ \hline &1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline &1,5\\ 2,10\\ \hline &1,1\\ \hline &5,1\\ \hline &10,1\\ 5,2\\ \hline \end{array}$                | $ \begin{array}{c c} 2,2\\ 1,1\\ 4,4\\ \hline \end{array} $ $ \begin{array}{c c} 1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline \end{array} $ $ \begin{array}{c c} 1,1\\ 2,2\\ \hline \end{array} $ $ \begin{array}{c c} 1,1\\ \hline \end{array} $ $ \begin{array}{c c} 5,1\\ \hline \end{array} $ $ \begin{array}{c c} 2,1\\ 1,2\\ \hline \end{array} $ | $\begin{array}{c c} I_2, I_2 \\ I_1, I_1 \\ I_4, I_4 \\ \hline I_1, I_3 \\ I_2, I_6 \\ I_3, I_1 \\ \hline I_6, I_2 \\ \hline I_1, I_5 \\ I_2, I_{10} \\ \hline \end{array}$   |           |
| a1 a2 a3 a4 b1 b2 b3 b4 c1 c2 549 a1 c1 c2 d1 c1 c2 c1 d1 c2 c1 d1 c2 c1 d1 c2 c1 d1 c2 c1 c1 c2 c1 d1 c1 d1 d1 c2 c1 d1 d1 c1 d1 | $ \begin{array}{c} 1+\varphi\\1+\varphi\\\varphi\\1+\varphi\\1\\1\\1\\1\\1+\varphi\\1+\varphi\end{array} $ $ \begin{array}{c} 1\\1\\\varphi\\\varphi\\\varphi \end{array} $  | $1 + \varphi$ $1$ $1 + \varphi$ $1 - \varphi$   | $ \begin{array}{cccc} \varphi \\ \varphi \\ \varphi \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 + \varphi \end{array} $ | $-1 + 2\varphi$ $-26 + 17\varphi$ $-2912 + 1797\varphi$ $-21 + 22\varphi$ $-1 - \varphi$ $4 + 4\varphi$ $-56 - 66\varphi$ $-46 - 71\varphi$ $-15 - 21\varphi$ $-10 - 21\varphi$ $54$ $-1 - \varphi$ $-4 + 6\varphi$ $-48 + 16\varphi$ $7 - 4\varphi$ $-65 + 36\varphi$  | $     \begin{array}{r}                                     $  | 0                                      | 4   4   2   4   6   6   2   2   2   2   1   1   2   2   1   1  | + +<br>  + +<br> <br>  + +<br> <br>  + +<br> | $ \begin{array}{c} 2,2\\ 1,1\\ 4,4\\ \hline 1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline 1,5\\ 2,10\\ \hline \\ 1,1\\ \hline 5,1\\ \hline 10,1\\ 5,2\\ \hline 11,1\\ \end{array} $   | $\begin{array}{ c c c }\hline 2,2\\ 1,1\\ 4,4\\ \hline &1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline &1,5\\ 2,10\\ \hline &1,1\\ \hline &5,1\\ \hline &10,1\\ 5,2\\ \hline &11,1\\ \hline \end{array}$ | $ \begin{array}{c c} 2,2\\ 1,1\\ 4,4\\ \hline \\ 1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline \\ 1,1\\ 2,2\\ \hline \end{array} $   | $\begin{array}{ c c c }\hline I_2,I_2\\I_1,I_1\\I_4,I_4\\\hline\hline I_1,I_3\\I_2,I_6\\I_3,I_1\\I_6,I_2\\\hline\hline I_1,I_5\\I_2,I_{10}\\\hline\hline I_1,I_5\\I_2,I_{10}\\\hline\hline I_1,I_1\\\hline\hline I_5,I_1\\\hline\hline I_{10},I_1\\I_5,I_2\\\hline\hline I_{11},I_1\\I_5,I_2\\\hline\hline I_{11},I_1\\\hline\hline I_{11},I_1\\\hline\hline I_{11},I_1\\\hline\hline I_{12},I_{12}\\\hline\hline I_{11},I_1\\\hline\hline I_{11},I_1\\\hline\hline I_{12},I_{13}\\\hline\hline I_{11},I_{14}\\\hline\hline I_{12},I_{14}\\\hline\hline I_{11},I_{14}\\\hline\hline I_{12},I_{14}\\\hline\hline I_{11},I_{14}\\\hline\hline I_{12},I_{14}\\\hline\hline I_{11},I_{14}\\\hline\hline I_{12},I_{14}\\\hline\hline I_{12},I_{14}\\\hline\hline I_{11},I_{14}\\\hline\hline I_{12},I_{14}\\\hline\hline I_{12},I_{14}\\\hline\hline I_{12},I_{14}\\\hline\hline I_{12},I_{14}\\\hline\hline I_{12},I_{14}\\\hline\hline I_{13},I_{14}\\\hline\hline I_{14},I_{15}\\\hline\hline I_{14},I_{15}\\\hline\hline I_{14},I_{15}\\\hline\hline I_{14},I_{15}\\\hline\hline I_{14},I_{15}\\\hline\hline I_{15},I_{15}\\\hline\hline I_{15},I_{15}\\\hline I_{15},I_{15}\\\hline\hline I_{15}$   |           |
| a1 a2 a3 a4 b1 b2 b3 b4 c1 c2 549:   | $ \begin{array}{c} 1+\varphi \\ 1+\varphi \\ \varphi \\ 1+\varphi \\ 1 \\ 1 \\ 1 \\ 1+\varphi \\ 1+\varphi \\ \mathbf{a} \end{array} $ $ \begin{array}{c} 1+\varphi \\ 1+$ | $1 + \varphi$ $1$ $1 + \varphi$ $1 - \varphi$ | $ \begin{array}{c} \varphi \\ \varphi \\ -\frac{\varphi}{0} \\ 0 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 + \varphi \end{array} $     | $ \begin{array}{r} -1 + 2\varphi \\ -26 + 17\varphi \\ -2912 + 1797\varphi \\ -21 + 22\varphi \\ -1 - \varphi \\ 4 + 4\varphi \\ -56 - 66\varphi \\ -46 - 71\varphi \\ -15 - 21\varphi \\ -10 - 21\varphi \end{array} $ $ \begin{array}{r} 54 \\ -1 - \varphi \\ -4 + 6\varphi \\ -48 + 16\varphi \\ 7 - 4\varphi \end{array} $ |   | 0                                      | $ \begin{array}{ c c c c c } \hline  & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & $   | + +<br>  + +<br> <br>  + +<br> <br>  + +<br> | $ \begin{array}{c} 2,2 \\ 1,1 \\ 4,4 \\ \hline 1,3 \\ 2,6 \\ 3,1 \\ 6,2 \\ \hline 1,5 \\ 2,10 \end{array} $ $ \begin{array}{c} 1,1 \\ \hline 5,1 \\ \hline 10,1 \\ 5,2 \\ \hline 11,1 \\ \hline 1,2 \\ \end{array} $ | $\begin{array}{ c c c }\hline 2,2\\ 1,1\\ 4,4\\ \hline &1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline &1,5\\ 2,10\\ \hline &1,1\\ \hline &5,1\\ \hline &10,1\\ 5,2\\ \hline \end{array}$                | $ \begin{array}{c c} 2,2\\ 1,1\\ 4,4\\ \hline \end{array} $ $ \begin{array}{c c} 1,3\\ 2,6\\ 3,1\\ 6,2\\ \hline \end{array} $ $ \begin{array}{c c} 1,1\\ 2,2\\ \hline \end{array} $ $ \begin{array}{c c} 1,1\\ \hline \end{array} $ $ \begin{array}{c c} 5,1\\ \hline \end{array} $ $ \begin{array}{c c} 2,1\\ 1,2\\ \hline \end{array} $ | $\begin{array}{c c} I_2, I_2 \\ I_1, I_1 \\ I_4, I_4 \\ \hline I_1, I_3 \\ I_2, I_6 \\ I_3, I_1 \\ \hline I_6, I_2 \\ \hline I_1, I_5 \\ I_2, I_{10} \\ \hline \end{array}$   |           |

|              | $a_1$            | $a_2$         | $a_3$      | $a_4$                                   | $a_6$   |  | T   | s     | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$  | Kodaira  | Isogenies |
|--------------|------------------|---------------|------------|---|---|--|---|-------|------------------------------|-----------------------------|--|--|-----------|
| 549          | b                |               |            | Ę                                       | $549b = (9 - 21\varphi) = 3 \cdot 61b$                              | (5 isogeny classes                     | )   |       |                              |                             |  |  | 549b      |
| a1           | $\varphi$        | -1 1          | $+\varphi$ | $-1-\varphi$                            | $-\varphi$  | 1                                      | 1   | -+    | 1, 1                         | 1,1                         | 1,1  | $I_1, I_1$   |           |
| b1           | 1                | $\varphi$     | 0          | $2-6\varphi$                            | $3\varphi$  | 1                                      | 1   | +     | 5,1                          | 5,1                         | 5,1  | $I_5, I_1$   |           |
| c1           | $1+\varphi$      | $1-\varphi$   | 1          | $-32-17\varphi$                         | $-7+72\varphi$  | 0                                      | 2   | ++    | 10,1                         | 10,1                        | [2,1]  | $I_{10}, I_1$                                      |           |
| c2           | $1+\varphi$      | $1-\varphi$   | 1          | $3+3\varphi$                            | $3+8\varphi$  | 0                                      | 2   |       | 5, 2                         | 5,2                         | 1,2  | $I_5, I_2$   |           |
| d1           | $1 + \varphi$    | $\varphi$     | $\varphi$  | $-28-36\varphi$                         | $35 + 90\varphi$  | 0                                      | 1   | -+    | 11, 1                        | 11,1                        | 1, 1   | $\mathrm{I}_{11},\mathrm{I}_{1}$                   |           |
| e1           | 1                | $-1-\varphi$  | 0          | $-1-\varphi$                            | $1-\varphi$   | 0                                      |   | Ī — — | 1, 2                         | 1,2                         | 1,2  | $I_1, I_2$   |           |
| e2           | $\varphi$        | 1             | 1          | $-138 - 219\varphi$                     | $-1237 - 2003\varphi$   | 0                                      | 2   | ++    | 2, 1                         | 2,1                         | 2,1  | $I_2, I_1$   |           |
| <b>551</b> 3 | a                |               |            | 5                                       | $51a = (11 - 21\varphi) = 19b \cdot 29b$                            | (1 isogeny clas                        | s)  |       |                              |                             |  |  | 551a      |
| a1           | $1+\varphi$      | •             | 0          | 1                                       | 0   | 1                                      | 2   | -+    | 1, 1                         | 1,1                         | 1,1  | $I_1, I_1$   |           |
| a2<br>a3     | $1+\varphi$      | •             | 0 $1$      | $-4$ $-379 + 234\varphi$                | $ \begin{array}{c} 2 - 7\varphi \\ 3251 - 2010\varphi \end{array} $ | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | $\begin{array}{ c c }\hline 4 \\ 4 \end{array}$ | +++   | $2, 2 \\ 1, 4$               | $2, 2 \\ 1, 4$              | $\begin{bmatrix} 2,2\\1,4 \end{bmatrix}$     | $\begin{matrix} I_2, I_2 \\ I_1, I_4 \end{matrix}$ |           |
| a4           | $rac{arphi}{1}$ | $-arphi \ 1$  | $\varphi$  | $-379 + 234\varphi$ $-176 - 283\varphi$ | $-1771 - 2863\varphi$   |  | 2   | ++    | 4, 1                         | 4, 1                        | $\begin{bmatrix} 1, 4 \\ 2, 1 \end{bmatrix}$ | $I_{1}, I_{4} \\ I_{4}, I_{1}$                     |           |
| 551          | <b>b</b>         |               |            | 5.                                      | $51b = (10 - 21\varphi) = 19a \cdot 29a$                            | (1 isogeny clas                        | s)  |       |                              | · · ·                       | 1 1  | 1 -7 -   | 551b      |
| a1           | $\varphi$        | $1+\varphi$ 1 | $+\varphi$ | $\varphi$                               | 0   | 1                                      | 2   | + -   | 1, 1                         | 1,1                         | 1,1  | $I_1, I_1$   |           |
| a2           | $\varphi$        | $1+\varphi$ 1 | -          | $-5+\varphi$                            | $-5+2\varphi$   | 1                                      | 4   | ++    | 2, 2                         | 2, 2                        | 2,2  | $I_2, I_2$   |           |
| a3           | $1+\varphi$      | -1            | . 1        | $-145 - 235\varphi$                     | $1241 + 2010\varphi$  | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | 4   | -+    | 1, 4                         | 1,4                         | 1,4  | $I_1, I_4$   |           |
| a4           | 1                | 1 1           | $+\varphi$ | $-459 + 282\varphi$                     | $-4634 + 2862\varphi$   | 1                                      | 2   | ++    | 4,1                          | 4,1                         | 2, 1   | $I_4, I_1$   |           |
| 551          |                  |               |            |   | $551c = (24 - \varphi) = 19a \cdot 29b$                             | (1 isogeny class)                      |   |       |                              |                             |  |  | 551c      |
| a1           |                  | $-1-\varphi$  | 0          | $\varphi$                               | 0   | 1                                      | 2   | +-    | 1, 1                         | 1,1                         | 1,1  | $I_1, I_1$   |           |
| a2           | 1                | $-1-\varphi$  | 0          | $-4\varphi$                             | $4+7\varphi$  | 1                                      | 2   | -+    | 2, 2                         | 2,2                         | 2,2  | $I_2, I_2$   |           |
| 551          | $\mathbf{d}$     |               |            | ļ                                       | $551d = (23 + \varphi) = 19b \cdot 29a$                             | (1 isogeny class                       | )   |       |                              |                             |  |  | 551d      |
| a1           | 1                | $1+\varphi$   | 1          | arphi                                   | 0   | 1                                      | 2   | -+    | 1, 1                         | 1, 1                        | 1,1  | $I_1, I_1$   |           |
| a2           | 1                | $1+\varphi$   | 1          | $-5+6\varphi$                           | $6-2\varphi$  | 1                                      | 2   | + -   | 2, 2                         | 2,2                         | 2,2  | $I_2, I_2$   |           |
| 556          | a                |               |            | Į.                                      | $556a = (22 + 4\varphi) = 2 \cdot 139a$                             | (1 isogeny class                       | )   |       |                              |                             |  |  | 556a      |
| a1           | $\varphi$        | $-1-\varphi$  | $\varphi$  | 0                                       | 0   | 1                                      | 1   |       | 1, 2                         | 1,2                         | 1, 2   | $I_1, I_2$   |           |
| 556          | b                |               |            |   | $556b = (26 - 4\varphi) = 2 \cdot 139b$                             | (1 isogeny class)                      | )   |       |                              |                             |  |  | 556b      |
| a1           | $1 + \varphi$    | 1             | 0          | $1+\varphi$                             | 1   | 1                                      | 1   |       | 1, 2                         | 1, 2                        | 1,2  | $I_1, I_2$   |           |
| 571          | a                |               |            |   | $571a = (23 + 2\varphi) = 571a$                                     | (1 isogeny class)                      |   |       |                              |                             |  |  | 571a      |
| a1           | $1 + \varphi$    | $1-\varphi$   | 1          | $-\varphi$                              | 0   | 1                                      | 1   |       | 1                            | 1                           | 1  | $I_1$  |           |
| 571          | <b>o</b>         |               |            |   | $571b = (25 - 2\varphi) = 571b$                                     | (1 isogeny class)                      |   |       |                              |                             |  |  | 571b      |
| a1           | $\varphi$        | 1             | 1          | 0                                       | 0   | 1                                      | 1   |       | 1                            | 1                           | 1  | $I_1$  |           |
|              |                  |               |            |   |   |  |   | L     |                              | 1                           | İ  | 1  | I.        |

|              | $a_1$                          | $a_2$                 | $a_3$           | $a_4$                 | $a_6$   | r                | T             | s            | $\operatorname{ord}(\Delta)$            | $\operatorname{ord}_{-}(j)$                                      | $c_p$  | Kodaira  | Isogenies       |
|--------------|--------------------------------|-----------------------|-----------------|-----------------------|---|------------------|---------------|--------------|---|--|--|--|-----------------|
| 576a         | ι                              |                       |                 |                       | $576a = (24) = 2^3 \cdot 3 \tag{4}$                               | 4 isogeny classe | s)            |              |   |  |  |  | 576a            |
| a1           | 0                              | -1                    | 0               | -4                    | 4   | 0                | 8             | ++           | 8,2                                     | 2  | 4, 2   | $I_1^*, I_2$                                   |                 |
| a2           | 0                              | -1                    | 0               | 1                     | 0   | 0                | 4             |              | 4, 1                                    | 1  | 2,1  | $III, I_1$                                     |                 |
| a3           | 0                              | -1                    | 0               | -64                   | 220   | 0                | 4             | ++           | 10, 1                                   | 1  | 2, 1   | $III^*, I_1$                                   |                 |
| a4           | 0                              | -1                    | 0               | -24                   | -36   | 0                | 4             | ++           | 10, 4                                   | 4  | 2,4  | $III^*, I_4$                                   |                 |
| a5           | 0                              | -1                    | 0               | -384                  | -2772   | 0                | 2             | ++           | 11, 2                                   | 2  | 1, 2   | $II^*, I_2$                                    |                 |
| a6           | 0                              |                       | 0               | 16                    | -180  | 0                | 2             |              | 11,8                                    | 8  | 1,8  | $II^*, I_8$                                    |                 |
| b1           | 0                              | $1-\varphi$           | 0               | $-40-44\varphi$       | $-112-196\varphi$   | 0                | 2             | ++           | 11, 2                                   | 2  | 1, 2   | $II^*, I_2$                                    |                 |
| b2           | 0                              | $1-\varphi$           | 0               | $-4\varphi$           | $-4\varphi$   | 0                | 2             |              | 10,1                                    | 1  | $\begin{bmatrix} 2,1 \end{bmatrix}$                              | $ $ III $^*$ , $I_1$                           |                 |
| c1           | 0                              | $-1-\varphi$          | 0               | $-1664 + 1024\varphi$ | $30876 - 19072\varphi$  | 0                | 2             | ++           | 11, 1                                   | 1  | 1, 1   | $II^*, I_1$                                    |                 |
| c2           | 0                              | 1                     | 0               | $-20 + \varphi$       | $12-29\varphi$  | 0                | 4             | ++           | 10, 2                                   | 2  | 2,2  | $\mathrm{III}^*,\mathrm{I}_2$                  |                 |
| c3           | 0                              | 1                     | 0               | arphi                 | -arphi  | 0                | 4             |              | 8, 1                                    | 1  | 4, 1   | $I_1^*, I_1$                                   |                 |
| c4           | 0                              | 1                     | 0               | $-100-119\varphi$     | $-556 - 909\varphi$   | 0                | 2             | ++           | 11, 4                                   | 4  | 1,4  | $  II^*, I_4$                                  |                 |
| d1           | 0                              | $1-\varphi$           | 0               | $-33-34\varphi$       | $63 + 95\varphi$  | 0                | $\frac{1}{2}$ | + +          | 11,6                                    | 6  | [1, 2]   | $  II^*, I_6$                                  |                 |
| d2           | 0                              | $1-\varphi$           | 0               | $7+6\varphi$          | $7+7\varphi$  | 0                | 2             |              | 10, 3                                   | 3  | 2,1  | $III^*, I_3$                                   |                 |
| 580a         | ı                              |                       |                 | 580                   | $a = (6 - 22\varphi) = 2 \cdot 5a \cdot 29b$                      | (3 isogeny       | classe        | s)           |   |  |  |  | 580a            |
| a1           | $\varphi$                      | $-1-\varphi$          | $\varphi$       | $-12-23\varphi$       | $-29 - 45\varphi$   | 0                | 1             |              | 9, 1, 1                                 | 9, 1, 1  | 1, 1, 1  | $I_9, I_1, I_1$                                |                 |
| b1           | <del>΄</del> -<br><i>φ</i>     | $1-\frac{1}{\varphi}$ |                 | $-2+7\varphi$         | $-1-8\varphi$   | 0                |               | <u>-</u>     | $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}$ | 1,3,5  | 1, 3, 1  | $  I_1, I_3, I_5  $                            |                 |
| b2           | $\varphi$                      | $1-\varphi$           | 0               | $-52-148\varphi$      | $-504 - 880\varphi$   |                  | 1             |              | 3, 1, 15                                | 3, 1, 15   | $\begin{bmatrix} 1, 0, 1 \\ 3, 1, 1 \end{bmatrix}$               | $I_3, I_1, I_{15}$                             |                 |
|              | $1+\varphi$                    | <u>7</u><br>-1        |                 | $1-2\varphi$          | -1  |                  | 3             | <u> </u>     | 1, 1, 3                                 | $\begin{bmatrix} 1 & 3, 1, 13 \\ 1, 1, 3 \end{bmatrix}$          | $\begin{bmatrix} 1, 1, 1, 3 \end{bmatrix}$                       | $  I_1, I_1, I_3  $                            |                 |
| c2           | $1 + \varphi$                  | $\varphi$             | $arphi \ arphi$ | $-85 - 131\varphi$    | $-599 - 964\varphi$   |                  | 1             |              | 3, 3, 1                                 | $\begin{bmatrix} 1, 1, 3 \\ 3, 3, 1 \end{bmatrix}$               | $\begin{bmatrix} 1, 1, 3 \\ 3, 1, 1 \end{bmatrix}$               | $I_1, I_1, I_3  I_3, I_3, I_1$                 |                 |
| 580b         | <b>1</b>                       | <u> </u>              | <u> </u>        | 5801                  | $\mathbf{v} = (16 - 22\varphi) = 2 \cdot 5a \cdot 29a$            | (3 isogeny       | class         | ng)          |   |  |  |  | $580\mathrm{b}$ |
|              | $\frac{1+\varphi}{1+\varphi}$  | 1                     | 0               | $-34 + 24\varphi$     | $\frac{10 - 22\varphi) - 2 \cdot 3a \cdot 29a}{-108 + 68\varphi}$ | (3 isogeny       | 1             |              | 9, 1, 1                                 | 9, 1, 1  | 1, 1, 1  | $I_9, I_1, I_1$                                |                 |
| <u>-</u> ' - | $1+\varphi$                    | 0                     |                 | $5-7\varphi$          | $-9+8\varphi$   |                  | 3             |              | $\frac{1}{1}, \frac{3}{5}$              | $\begin{array}{c c} 1 & 3, -5, -7 \\ \hline 1, 3, 5 \end{array}$ | $\begin{bmatrix} 1, 3, 1 \end{bmatrix}$                          | $  I_1, I_3, I_5  $                            |                 |
|              | $1 + \varphi$<br>$1 + \varphi$ | 0                     | 0               | $-200 + 148\varphi$   | $-9 + 8\varphi$ $-1384 + 880\varphi$                              |                  | 1             |              | 3, 1, 15                                | $\begin{bmatrix} 1, 3, 3 \\ 3, 1, 15 \end{bmatrix}$              | $\begin{bmatrix} 1, 3, 1 \\ 3, 1, 1 \end{bmatrix}$               | $I_1, I_3, I_5$<br>$I_3, I_1, I_{15}$          |                 |
| c1           | $\varphi^{-}$                  | $-\varphi$ 1          |                 | 0                     | $-1-\varphi$  |                  | 3             | <u> </u>     | $\frac{1}{1}, \frac{1}{1}, \frac{3}{3}$ | $\begin{bmatrix} 1 & 3, 2, 23 \\ 1, 1, 3 \end{bmatrix}$          | $\begin{bmatrix} 1 & 3 & -2 & -2 \\ 1 & 1 & 1 & 3 \end{bmatrix}$ | $  I_1, I_1, I_3  $                            | <u>-</u>        |
| c2           | $\frac{\varphi}{1}$            | $1-\varphi$ 1         |                 | $-216 + 130\varphi$   | $-1563 + 963\varphi$  |                  | 1             |              |   | $\begin{bmatrix} 1, 1, 5 \\ 3, 3, 1 \end{bmatrix}$               | $\begin{bmatrix} 1, 1, 5 \\ 3, 1, 1 \end{bmatrix}$               | $I_{1}, I_{1}, I_{3}$<br>$I_{3}, I_{3}, I_{1}$ |                 |
| 589a         | ı                              |                       |                 | 589                   | $9a = (26 - 3\varphi) = 19b \cdot 31b$                            | (3 isogeny cl    | lasses        | )            |   |  |  |  | 589a            |
| al           |                                | $-1+\varphi$          | 1               | $1-3\varphi$          | $\frac{-1-2\varphi}{}$  | 1                | 1             | <del> </del> | 3,1                                     | 3,1  | 3,1  | $I_3, I_1$                                     |                 |
| b1           | 1                              |                       | 0               | $1-\varphi$           |   | 1                | 1             | <br>  +      | 1,1                                     | 1,1  | 1,1  | $  I_1, I_1  $                                 |                 |
| c1           | $\varphi$                      | $1-\varphi$           | $\varphi$       | $-4+\varphi$          | $-7+4\varphi$   | 0                | 3             | '<br>  - +   | 1,3                                     | 1, 3   | -1,3   | $ I_1,I_3 $                                    |                 |
| c2           | φ                              | $1-\varphi$           | $\varphi$       | $-359 + 191\varphi$   | $-3143 + 1861\varphi$   | 0                | 1             | -+           | 3, 1                                    | 3, 1   | 3, 1   | $I_3, I_1$                                     |                 |
| 589h         | )                              |                       |                 | 589                   | $9b = (23 + 3\varphi) = 19a \cdot 31a$                            | (3 isogeny c     | lasses        | (3)          |   |  |  |  | 589b            |
| a1           | $1+\varphi$                    | $\varphi$             | 0               | $-1+4\varphi$         | $3\varphi$  | 1                | 1             | +-           | 3, 1                                    | 3,1  | 3,1  | $I_3, I_1$                                     |                 |
| b1           | 1                              | $-1+\varphi$          | 0               | $\varphi$             | 1   | 1                | 1             | ·            | 1,1                                     | $\frac{1}{1}$ , $\frac{1}{1}$                                    | $\begin{bmatrix} 1,1 \end{bmatrix}$                              | $  I_1, I_1  $                                 |                 |
|              |                                | · r                   | -               | r                     |   |                  | 1             | <u> </u>     | ,                                       | ,  | L ,  | 1 1  |                 |

|          |                     |                         |                     |                                       |  |   | Imi           | 1            | 1/4)                         | 1 (1)  |   | 77 1 .  | T .       |
|----------|---------------------|-------------------------|---------------------|---------------------------------------|--|---|---------------|--------------|------------------------------|--|---|---|-----------|
|          | $a_1$               | $a_2$                   | $a_3$               | $a_4$                                 | $a_6$  |   | T             | s            | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                  | $c_p$                                       | Kodaira                                       | Isogenies |
| 5891     | )                   |                         |                     | 58                                    | $9b = (23 + 3\varphi) = 19a \cdot 31a$                 | (3 isogeny classe                       | es)           |              |                              |  |   |   | 589b      |
| c1       | $1+\varphi$         |                         | $1+\varphi$         | $-3-3\varphi$                         | $-3-5\varphi$  | 0                                       | 3             | +-           |                              | 1,3  | 1,3   | $I_1, I_3$                                    |           |
|          | $1+\varphi$         | 0                       | $1+\varphi$         | $-168 - 193\varphi$                   | $-1282 - 1862\varphi$                                  | 0                                       | 1             | + -          | 3,1                          | 3,1  | 3,1   | $I_3, I_1$                                    |           |
| 5890     |                     |                         |                     |                                       | $89c = (15 - 22\varphi) = 19a \cdot 31b$               | · • • • • • • • • • • • • • • • • • • • | s)            | 1            |                              |  | ı   | ı   | 589c      |
| a1       | 1                   | $-1-\varphi$            | 0                   | $-5+3\varphi$                         | $-5+3\varphi$  | 0                                       | 1             |              | 3, 1                         | 3, 1   | 1,1   | $I_3, I_1$                                    |           |
| 5890     | d                   |                         |                     | 56                                    | $89d = (7 - 22\varphi) = 19b \cdot 31a$                | (1 isogeny class                        | (             |              |                              |  |   |   | 589d      |
| a1       | 1                   | $1 + \varphi$           | 1                   | $-3-\varphi$                          | $-5-5\varphi$  | 0                                       | 1             |              | 3, 1                         | 3, 1   | 1,1   | $I_3, I_1$                                    |           |
| 596a     | a.                  |                         |                     | 59                                    | $06a = (8 - 22\varphi) = 2 \cdot 149a$                 | (2 isogeny classe                       | s)            |              |                              |  |   |   | 596a      |
| a1       | $1+\varphi$         | $\varphi$               | $\varphi$           | 1                                     | 0  | 1                                       | 1             | -+           | 1, 1                         | 1,1  | 1,1   | $I_1, I_1$                                    |           |
| b1       | $\varphi$           |                         | $1+\varphi$         | -5                                    | $3-\varphi$  |   | 5             | -+           | ,                            | 1,5  | 1,5   | $I_1, I_5$                                    |           |
| b2       | $1+\varphi$         | $-1+\varphi$            | φ                   | $-596 - 970\varphi$                   | $-11196 - 18087\varphi$                                | 1                                       | 1             | -+           | 5,1                          | 5, 1   | 1,1   | $I_5, I_1$                                    |           |
| 596l     | )                   |                         |                     |                                       | $6b = (14 - 22\varphi) = 2 \cdot 149b$                 | (2 isogeny classe                       | s)            |              |                              |  |   |   | 596b      |
| a1       |                     | $-1+\varphi$            | $\varphi$           | 0                                     | 0  | 1                                       | 1             | + -          | <del>-</del>                 | 1,1  | 1,1   | $I_1, I_1$                                    |           |
| b1 b2    | $1+\varphi$         | $-1$ $1+\varphi$        | $arphi \ 0$         | $-4 - 2\varphi$ $-1565 + 971\varphi$  | $3$ $-28313 + 17491\varphi$                            | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$  | 5<br>1        | + -<br>  + - | ,                            | $\begin{bmatrix} 1, 5 \\ 5, 1 \end{bmatrix}$ | $\begin{array}{ c c } 1,5\\1,1 \end{array}$ | $\begin{matrix} I_1,I_5\\I_5,I_1\end{matrix}$ |           |
|          | -                   | 1   Ψ                   |                     | ·                                     | ·  |   |               | 1            | 0,1                          | 0, 1   | 1,1   | 15,11   | <b>-</b>  |
| 599a     |                     |                         |                     |                                       | , , ,  | 2 isogeny classes)                      | 0             |              | 1                            | 1  | -1  | т   | 599a      |
| a1<br>a2 | $\frac{1}{\varphi}$ | $-\varphi$ $-1+\varphi$ | $\frac{1}{\varphi}$ | $-78 + 46\varphi$ $-123 - 175\varphi$ | $-326 + 204\varphi$<br>$809 + 1335\varphi$             | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$  | $\frac{2}{2}$ | +-           |                              | $\begin{array}{c c} 1 \\ 2 \end{array}$      | $\frac{1}{2}$                               | $egin{array}{c} I_1 \ I_2 \end{array}$        |           |
| b1       | $\varphi$           | 1                       | 0                   | $-5-7\varphi$                         | $-11-16\varphi$  |   | 1             | <u> </u>     |                              | 1  | 1   | $I_1$   | <u>  </u> |
| 599l     | <u> </u>            |                         |                     |                                       | $599b = (24 + \varphi) = 599b \tag{2}$                 | 2 isogeny classes)                      |               |              |                              | ı  |   |   | 599b      |
|          |                     | $1-\varphi$             | 0                   | $-12 + 7\varphi$                      | $\frac{3990 - (24 + \varphi) - 3990}{-27 + 16\varphi}$ | 1 sogeny classes)                       | 1             |              | 1                            | 1  | 1   | $I_1$   |           |
| b1       |                     | $-1+\varphi$            | $\frac{\circ}{1}$ - | $-32 - 46\varphi$                     | $-122 - 204\varphi$                                    | ' '                                     |               | <u> </u>     |                              | 1 1  | 1 1   | $\frac{1}{1} - \frac{1}{1}$                   | <u> </u>  |
|          | $1+\varphi$         | $\varphi$               | $\varphi$           | $-297 + 175\varphi$                   | $2319 - 1458\varphi$                                   |   | 2             | + -          |                              | 2  | 2   | $ \stackrel{-1}{\mathrm{I}_{2}} $             |           |
| 601a     | a                   |                         |                     |                                       | $601a = (9 - 22\varphi) = 601a$                        | (1 isogeny class)                       |               |              |                              |  |   |   | 601a      |
| a1       | 0                   | $-1-\varphi$            | $\varphi$           | $1+\varphi$                           | $-\varphi$   | 1                                       | 1             |              | 1                            | 1  | 1   | $I_1$   |           |
| 6011     | )                   |                         |                     |                                       | $601b = (13 - 22\varphi) = 601b$                       | (1 isogeny class)                       |               |              |                              |  |   |   | 601b      |
| a1       | 0                   | $1+\varphi$             | $1+\varphi$         | $1+\varphi$                           | 0  |   | 1             |              | 1                            | 1  | 1   | $I_1$   |           |
| 604a     | a                   |                         |                     | 60-                                   | $4a = (12 - 22\varphi) = 2 \cdot 151a$                 | (2 isogeny classe                       | es)           | •            |                              |  |   | 1   | 604a      |
| a1       | $1+\varphi$         | $\varphi$               | $1+\varphi$         | -1                                    | $-\varphi$   |   | 2             | -+           |                              | 1, 2   | 1, 2  | $I_1, I_2$                                    |           |
| a2       | $\varphi$           | $-1+\varphi$            | $1 + \varphi$       | $-85 + 52\varphi$                     | $400 - 248\varphi$                                     | 1                                       | 2             | +-           | 0.4                          | 2, 1   | 2, 1  | $I_2, I_1$                                    |           |

|          |                     |      |                |                 |   |   |            |        | ı       |                           |                              |                             |           |  |                 |
|----------|---------------------|------|----------------|-----------------|---|---|------------|--------|---------|---------------------------|------------------------------|-----------------------------|-----------|--|-----------------|
|          | $a_1$               |      | $a_2$          | $a_3$           | $a_4$   | $a_6$   |            | r      | T       | s                         | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$     | Kodaira                                    | Isogenies       |
| 604      | 0                   |      |                |                 |   | 04 (10 00 ) 0 171   | (0:        | ,      | \       |                           |                              |                             |           |  | 604a            |
| 604      |                     |      |                |                 |   | $04a = (12 - 22\varphi) = 2 \cdot 151a$                   | (2 isogeny |        |         |                           |                              |                             |           |  | 004a            |
| b1       | 1                   |      | 1              | 1               | $-\varphi$  | 0   |            | 0      | 3       |                           | 1, 1                         | 1,1                         | 1,1       | $I_1, I_1$                                 |                 |
| b2<br>b3 | 1                   | 1 1  | 1              | 1               | $-5 + 9\varphi$   | $-12 + 10\varphi$   |            | 0      | 3       |                           | 3, 3                         | 3,3                         | 3,3       | $I_3, I_3$                                 |                 |
| DO       | $1+\varphi$         | -17  | + φ .          | $1 + \varphi$   | $-4287 + 2637\varphi$   | $-124539 + 76939\varphi$                                  |            | 0      | 1       |                           | 1,9                          | 1,9                         | 1,9       | $I_1, I_9$                                 |                 |
| 604      | b                   |      |                |                 | (   | $604b = (10 - 22\varphi) = 2 \cdot 151b$                  | (2 isogeny | cla    | sses)   |                           |                              |                             |           |  | 604b            |
| a1       | φ                   | -1+  | + φ .          | $1+\varphi$     | $-1-\varphi$  | $-\varphi$  | · · ·      | 1      | 2       | +-                        | 1, 2                         | 1,2                         | 1,2       | $I_1, I_2$                                 |                 |
| a2       | $1+\varphi$         |      | •              | $1+\varphi$     | $-32-53\varphi$   | $100 + 162\varphi$  |            | 1      | 2       | -+                        | 2, 1                         | 2,1                         | 2,1       | $I_2, I_1$                                 |                 |
| b1       | 1                   |      | 1              | 1               | $-1+\varphi$  | 0   |            | 0      | 3       |                           | 1, 1                         | 1,1                         | 1,1       | $\bar{\mathbf{I}}_1, \bar{\mathbf{I}}_1$   |                 |
| b2       | 1                   |      | 1              | 1               | $4-9\varphi$  | $-2-10\varphi$  |            | 0      | 3       |                           | 3, 3                         | 3,3                         | 3,3       | $I_3, I_3$                                 |                 |
| b3       | $\varphi$           | 1 +  | + φ            | 1               | $-1648 - 2637\varphi$   | $-50237 - 81226\varphi$                                   |            | 0      | 1       |                           | 1, 9                         | 1,9                         | 1,9       | $I_1, I_9$                                 |                 |
| 605      | a                   |      |                |                 | 605   | $a = (11 - 22\varphi) = 5a \cdot 11a \cdot 11b$           | (2 isoge   | nv     | classe  | es)                       |                              |                             |           |  | 605a            |
| al       | 1                   |      | -1             | 0               | -59   | 190   | · ·        | 1      | 4       | ++                        | 1,1,8                        | 1,1,8                       | 1, 1, 8   | $I_1, I_1, I_8$                            |                 |
| a2       | 1                   |      | -1             | 0               | -4  | 3   |            | 1      | 8       |                           | 2, 2, 4                      | 2, 2, 4                     | 2, 2, 4   | $I_1, I_1, I_8$<br>$I_2, I_2, I_4$         |                 |
| a3       | 1                   |      | -1             | 0               | 1   | 0   |            | 1      | 4       |                           | 1, 1, 2                      | 1,1,2                       | 1, 1, 2   | $I_1, I_1, I_2$                            |                 |
| a4       | 1                   |      | -1             | 0               | -29   | -52   |            | 1      | 4       | ++                        | 4, 4, 2                      | 4, 4, 2                     | 2, 2, 2   | $I_4, I_4, I_2$                            |                 |
| a5       | $\varphi$           | -1 - | - φ            | 0               | $-878 - 1422\varphi$  | $-18511 - 29934\varphi$                                   |            | 1      | 2       | -+                        | 8, 2, 1                      | 8, 2, 1                     | 2, 2, 1   | $I_8, I_2, I_1$                            |                 |
| a6       | $1+\varphi$         |      |                | $1+\varphi$     | $-2301 + 1422\varphi$   | $-50746 + 31356\varphi$                                   |            | 1      | 2       | + -                       | 2, 8, 1                      | 2, 8, 1                     | 2, 2, 1   | $I_2, I_8, I_1$                            |                 |
| b1       | $1+\varphi$         |      | $\varphi$      | 0               | 1   | 0   |            | 0      | 4       | - +                       | 2, 1, 1                      | [2, 1, 1]                   | [2, 1, 1] | $[ I_2, I_1, I_1 ]$                        | Ī               |
| b2       | $1+\varphi$         |      | $\varphi$      | 0               | -4  | $-2-7\varphi$   |            | 0      | 8       | ++                        | 4, 2, 2                      | 4, 2, 2                     | 4, 2, 2   | $I_4, I_2, I_2$                            |                 |
| b3       | $1+\varphi$         |      | $\varphi$      | 0               | $-34 + 35\varphi$   | $119 - 84\varphi$   |            | 0      | 4       | + -                       | 8, 1, 1                      | 8, 1, 1                     | 8, 1, 1   | $I_8, I_1, I_1$                            |                 |
| b4       | $1+\varphi$         |      | $\varphi$      | 0               | $-54-35\varphi$   | $-211-238\varphi$   |            | 0      | 4       | ++                        | 2, 4, 4                      | 2, 4, 4                     | 2, 2, 4   | $I_2, I_4, I_4$                            |                 |
| b5       | $1+\varphi$         |      | $\varphi$      | 0               | $-29-45\varphi$   | $-327-207\varphi$   |            | 0      | 2       |                           | 1, 8, 8                      | 1, 8, 8                     | 1, 2, 8   | $I_1, I_8, I_8$                            |                 |
| b6       | 1                   | -1+  | + φ .          | $1+\varphi$     | $-3520 - 5574\varphi$   | $-150513 - 243182\varphi$                                 |            | 0      | 2       | ++                        | 1, 2, 2                      | 1, 2, 2                     | 1, 2, 2   | $I_1, I_2, I_2$                            |                 |
| 605      | h                   |      |                |                 | 6   | $505b = (27 - 4\varphi) = 5a \cdot 11a^2$                 | (3 isogeny | ela    | eene)   |                           |                              |                             |           |  | $605\mathrm{b}$ |
| a1       |                     |      | -1             |                 | $-21+4\varphi$  | $\frac{660 - (21 - 4\varphi) - 6a - 11a}{42 - 14\varphi}$ |            |        | 2       | ++                        | 3,6                          | 6                           | 2,6       | $III, I_6$                                 |                 |
| a1<br>a2 | $\varphi$ $\varphi$ |      | -1             | $arphi \ arphi$ | $ \begin{array}{c} -21 + 4\varphi \\ -1 - \varphi \end{array} $ | $42 - 14\varphi$ $1 - \varphi$                            |            | 1<br>1 | 2       | + +                       | 3, 0 $3, 3$                  | 3                           | 2,0 $2,3$ | $III, I_6$ $III, I_3$                      |                 |
| b1       | <u>r</u>            |      | - <del>-</del> | $\frac{r}{0}$ - | -2  | $10 - 7\varphi$   |            | 0      | <br>  4 | ' <del>'</del> -<br>  - + | <del>-</del> ,               | 1,1                         | 4,1       | $\begin{bmatrix} I_1^*, I_1 \end{bmatrix}$ |                 |
| b2       | 1                   |      | 1              | 0               | $-87 + 35\varphi$   | $261-210\varphi$  |            | 0      | 4       | ++                        | 8, 2                         | 2,2                         | 4, 2      | $I_2^{1}, I_2$                             |                 |
| b3       | $\varphi$           |      | + φ            | 0               | $-229-376\varphi$   | $2325 + 3798\varphi$                                      |            | 0      | 4       | -+                        | 9, 3                         | 3,3                         | 4,3       | $I_3^*, I_3$                               |                 |
| b4       | 1                   |      | 1              | 0               | $-1267 + 770\varphi$  | $19717 - 12257\varphi$                                    |            | 0      | 2       | +-                        | 10, 1                        | 4,1                         | 4, 1      | $I_4^*, I_1$                               |                 |
| b5       | 1                   |      | 1              | 0               | $-547 + 260\varphi$   | $-5141 + 3445\varphi$                                     |            | 0      | 4       | ++                        | 12, 6                        | 6,6                         | 4,6       | $I_{6}^{*}, I_{6}$                         |                 |
| b6       | $\varphi$           | 1 +  | $+\varphi$     | 0               | $-954 - 1501\varphi$  | $-21995 - 35563\varphi$                                   |            | 0      | 2       | ++                        | 7, 4                         | 1,4                         | 2,4       | $\mathrm{I}_1^*,\mathrm{I}_4$              |                 |
| b7       | 1                   |      | 1              | 0               | $-1322 + 935\varphi$  | $16554 - 8620\varphi$                                     |            | 0      | 2       | +-                        | 18, 3                        | 12, 3                       | 4, 3      | $I_{12}^*, I_3$                            |                 |
| b8       | 1                   |      | 1              | 0               | $-8252 + 4785\varphi$   | $-335084 + 204530\varphi$                                 |            | 0      | 2       | + +                       | 9,12                         | 3,12                        | 2,12      | $I_3^*, I_{12}$                            |                 |
| c1       | $1+\varphi$         |      |                | 1               | $-7-5\varphi$   | $21-25\varphi$  |            | 0      | 2       | -+                        | 9, 3                         | 3                           | 2,1       | $III^*, I_3$                               |                 |
| c2       | $1+\varphi$         | 1 +  | + φ            | 1               | -192  | $345 - 1041\varphi$                                       |            | 0      | 2       | ++                        | 9,6                          | 6                           | 2,2       | $III^*, I_6$                               |                 |

|  | $a_1$                   | $a_2$                            | $a_3$                  | $a_4$                           | $a_6$   | r                                      | T   | s                    | $\operatorname{ord}(\Delta)$            | $ord_{-}(j)$                                      | $c_p$  | Kodaira   | Isogenies |
|--|-------------------------|----------------------------------|------------------------|---------------------------------|---|--|---|----------------------|---|---|--|---|-----------|
| 605  |                         |                                  |                        | 60                              |   | 3 isogeny cla                          | reede)                                    |                      |   | (67   | P  |   | 605c      |
| a1   |                         | $-1-\varphi$                     | 1   (2                 | $-17-6\varphi$                  | $\frac{36c - (23 + 4\varphi) - 3a + 116}{28 + 13\varphi}$ |  | 2   | 1 , ,                | 3,6                                     | 6   | 2,6  | III, I <sub>6</sub>   |           |
| a1<br>a2                                   |                         | $-1 - \varphi$<br>$-1 - \varphi$ |                        | $-17 - 6\varphi$ $-2 - \varphi$ | $\begin{array}{c} 20 + 13 \varphi \\ 0 \end{array}$       | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | $\frac{2}{2}$                             | ++                   | 3, 0 $3, 3$                             | 3   | $\begin{bmatrix} 2, 0 \\ 2, 3 \end{bmatrix}$ | $III, I_6$<br>$III, I_3$  |           |
| $\begin{vmatrix} a_2 \\ b_1 \end{vmatrix}$ | 1   \psi   \psi       1 | <del>-</del> 1                   |                        | -2                              | $3+7\varphi$  |  | ÷   | ·[                   |   | <u> </u>  |  |   |           |
| $\begin{vmatrix} b_1 \\ b_2 \end{vmatrix}$ | 1                       | 1                                | $0 \\ 0$               | $-2$ $-52 - 35\varphi$          | $5 + i\varphi$ $51 + 210\varphi$                          | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{vmatrix} 4 \\ 4 \end{vmatrix}$    | + - + +              | $7, 1 \\ 8, 2$                          | $\begin{array}{ c c } & 1,1 \\ & 2,2 \end{array}$ | $4, 1 \\ 4, 2$                               | $\begin{array}{ c c c } & I_1^*, I_1 \\ & I_2^*, I_2 \end{array}$ |           |
| b3   | _                       | $-1+\varphi$                     | $\varphi$              | $-607 + 377\varphi$             | $7107 - 4405\varphi$                                      |  | 4   | + -                  | 9, 3                                    | $\begin{bmatrix} 2,2\\3,3 \end{bmatrix}$          | $\frac{4,2}{4,3}$                            | $I_3^{2}, I_3$  |           |
| b4   | $1 \cdot \varphi$       | 1 · Ψ                            | $\overset{\varphi}{0}$ | $-497 - 770\varphi$             | $7460 + 12257\varphi$                                     |  | 2   | _ +                  | 10, 1                                   | 4, 1  | 4,1  | $I_4^{*}, I_1$  |           |
| b5   | 1                       | 1                                | 0                      | $-287 - 260\varphi$             | $-1696 - 3445\varphi$                                     |  | 4   | + +                  | 12, 6                                   | 6,6   | 4,6  | $I_6^*, I_6$  |           |
| b6   | $1+\varphi$             | $-1+\varphi$                     | $\varphi$              | $-2457 + 1502\varphi$           | $-53599 + 33106\varphi$                                   | 0                                      | 2   | + +                  | 7, 4                                    | 1,4   | 2,4  | $I_1^*, I_4$  |           |
| b7   | 1                       | 1                                | 0                      | $-387 - 935\varphi$             | $7934 + 8620\varphi$                                      | 0                                      | 2   | -+                   | 18, 3                                   | 12, 3   | 4,3  | $I_{12}^*, I_3$   |           |
| b8   | 1                       | 1                                | 0                      | $-3467 - 4785\varphi$           | $-130554 - 204530\varphi$                                 | 0                                      | 2   | ++                   | 9,12                                    | 3, 12   | 2,12   | $I_3^*, I_{12}$   |           |
| c1   | $\varphi$               | $\varphi$                        |                        | $-15 + 8\varphi$                | $17 + 11\varphi$  | 0                                      | 2   | + -                  | 9,3                                     | 3   | 2, 1   | $ $ $III^*, I_3$  |           |
| c2   | $\varphi$               | $\varphi$                        | 0                      | $-195 + 3\varphi$               | $-500 + 847\varphi$                                       | 0                                      | 2   | ++                   | 9,6                                     | 6   | 2,2  | $III^*, I_6$  |           |
| 620  | a                       |                                  |                        | 620                             | $0a = (26 - 2\varphi) = 2 \cdot 5a \cdot 31a$             | (3 isogeny cl                          | lasses                                    | )                    |   |   |  |   | 620a      |
| a1   | $\varphi$               | -1                               | 1                      | -2                              | $2-\varphi$   |  | 2   | <u> </u>             | 2, 1, 2                                 | 2, 1, 2   | 2, 1, 2                                      | $I_{2}, I_{1}, I_{2}$   |           |
| a2   | φ                       | -1                               | 1                      | $-32 + 20\varphi$               | $94-57\varphi$  | 1                                      | 2   | + -                  | 1, 2, 4                                 | 1,2,4   | 1, 2, 2                                      | $I_1, I_2, I_4$   |           |
| b1   | 1                       | $\varphi$                        | 1                      | $-7-7\varphi$                   | $-13-19\varphi$   | 0                                      | 1   | .'<br>               | $\frac{1}{3}, \frac{1}{1}, \frac{1}{1}$ | 3, 1, 1   | 1, 1, 1                                      | $I_3, I_1, I_1$   | <u>-</u>  |
|  | $\varphi$               | 0                                | <u>-</u>               | 0                               | 0   |  | 3   | ;;                   | 1,1,1                                   | 1, 1, 1   | 1,1,1  | $\bar{I}_1, \bar{I}_1, \bar{I}_1$                                 | <u> </u>  |
| c2   | $\varphi$               | 0                                | 1                      | -5arphi                         | $-8-2\varphi$   |  | 3   |                      | 3, 3, 3                                 | 3, 3, 3   | 3, 3, 1                                      | $I_3, I_3, I_3$   |           |
| c3   | $1+\varphi$             | 0                                | 1                      | $-1224 - 1933\varphi$           | $-31267 - 50492\varphi$                                   | 0                                      | 1   |                      | 1, 1, 9                                 | 1, 1, 9   | 1, 1, 1                                      | $I_1, I_1, I_9$   |           |
| 620  | b                       |                                  |                        | 620                             | $0b = (24 + 2\varphi) = 2 \cdot 5a \cdot 31b$             | (3 isogeny cl                          | asses                                     | )                    |   |   |  |   | 620b      |
| a1   | $1+\varphi$             | $-1-\varphi$                     | 1                      | $-2-\varphi$                    | $1+\varphi$   |  | 2   | 1 + -                | 2, 1, 2                                 | 2,1,2   | 2, 1, 2                                      | $I_2, I_1, I_2$   |           |
| a2   | ,                       | $-1-\varphi$                     | 1                      | $-12-21\varphi$                 | $37 + 57\varphi$  | 1                                      | 2   | -+                   | 1, 2, 4                                 | 1,2,4   | 1, 2, 2                                      | $I_1, I_2, I_4$   |           |
| b1   | 1                       | $1-\varphi$                      | 1                      | $-14 + 7\varphi$                | $-32 + 19\varphi$   | 0                                      | 1   | ·                    | 3, 1, 1                                 | 3, 1, 1   | 1, 1, 1                                      | $I_3, I_1, I_1$   | <u>-</u>  |
| c1   | $1+\varphi$             | $-\varphi$                       | <u>-</u>               | -arphi                          | 0   | 0                                      | 3   | <u> </u>             | 1,1,1                                   | 1,1,1   | 1,1,1  | $\bar{I}_1, \bar{I}_1, \bar{I}_1$                                 |           |
| c2   | $1+\varphi$             | $-\varphi$                       | 1                      | $-5+4\varphi$                   | $-10 + 2\varphi$  | 0                                      | 3   |                      | 3, 3, 3                                 | 3, 3, 3   | 3, 3, 1                                      | $I_3, I_3, I_3$   |           |
| c3   | $\varphi$               | $1-\varphi$                      | 1                      | $-3156 + 1932\varphi$           | $-81759 + 50492\varphi$                                   | 0                                      | 1   |                      | 1, 1, 9                                 | 1,1,9   | 1, 1, 1                                      | $I_1, I_1, I_9$   |           |
| 639  | a                       |                                  |                        | 6                               | $639a = (27 - 3\varphi) = 3 \cdot 71a \tag{5}$            | isogeny clas                           | sses)                                     |                      |   |   |  |   | 639a      |
| a1   | 0                       | $-1-\varphi$                     | $1+\varphi$            | $\varphi$                       | $-\varphi$  | 1                                      | 1   | -+                   | 1, 1                                    | 1, 1  | 1,1  | $I_1, I_1$  |           |
| b1   | 0                       | <u>-</u><br>1                    | $1 + \varphi$          | $-129 + 76\varphi$              | $610 - 376\varphi$  |  | † - <sub>-</sub> -                        | ;<br>  - +           | 7,1                                     | 7,1   | $\frac{1}{7}, \frac{1}{1}$                   | $\bar{\mathrm{I}}_{7}, \bar{\mathrm{I}}_{1}$                      | <u>-</u>  |
| b2   | 0                       |                                  | $1+\varphi$            | $-49525 - 80172\varphi$         | $-8056864 - 13036101\varphi$                              | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 1   | -+                   | 1, 7                                    | 1,7   | 1,7  | $I_1, I_7$  |           |
| c1   | 9                       | $-1+\varphi$                     | $\varphi$              | $-13-13\varphi$                 | $-19-43\varphi$   | 0                                      | $\frac{1}{1} - \frac{1}{2} - \frac{1}{2}$ | ·<br>  + -           | 4,1                                     | 4,1   | [2,1]  | $\bar{\mathrm{I}}_4, \bar{\mathrm{I}}_1$                          | <u>-</u>  |
| c2   | $1+\varphi$             | $\varphi$                        | $\varphi$              | $-1008 - 1631\varphi$           | $-24565 - 39748\varphi$                                   |  | 2   | - +                  | 2, 2                                    | 2, 2  | 2, 2   | $I_2, I_2$  |           |
| d1   | $1+\varphi$             |                                  | <del>′</del>           | $-6-\varphi$                    | $-3+3\varphi$   | 0                                      | $\frac{1}{1} - \frac{1}{4} - \frac{1}{4}$ | .' <u>·</u><br>  + + | 2, 2                                    | $\frac{1}{2}, \frac{1}{2}$                        | $\frac{1}{2}, \frac{1}{2}$                   | $I_1$ , $I_2$   |           |
| d2   | $1+\varphi$             | 0                                | 1                      | $-36-46\varphi$                 | $105 + 171\varphi$  |  | 4   | ++                   | 1, 4                                    | 1,4   | 1,4  | $I_1, I_4$  |           |
| d3   | $1+\varphi$             | 0                                | 1                      | $-1-\varphi$                    | $-1-\varphi$  | 0                                      | 2   | - +                  | 1, 1                                    | 1,1   | 1,1  | $I_1, I_1$  |           |
| d4   | $\varphi$               | 0                                | 1                      | $-411 + 255\varphi$             | $-3792 + 2345\varphi$                                     | 0                                      | 2   | + -                  | 4, 1                                    | 4, 1  | 4,1  | $I_4, I_1$  |           |

|  | $a_1$                          | $a_2$                     | $a_3$                           | $a_4$                         | $a_6$   | r              | T             | s          | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$  | Kodaira  | Isogenies |
|--|--------------------------------|---------------------------|---------------------------------|-------------------------------|---|----------------|---------------|------------|------------------------------|-----------------------------|--------|--|-----------|
| 639a                                       | a                              |                           |                                 | 63:                           | $9a = (27 - 3\varphi) = 3 \cdot 71a$ (5 is      | sogeny classes | s)            |            |                              |                             |        |  | 639a      |
| e1   | 0                              | $-\varphi$                | $1+\varphi$                     | $-223 + 138\varphi$           | $-1555 + 960\varphi$                            | 0              | 1             | -+         | 1,1                          | 1,1                         | 1,1    | $\mathrm{I}_1,\mathrm{I}_1$                            |           |
| 6391                                       | b                              |                           |                                 | 63                            | $9b = (24 + 3\varphi) = 3 \cdot 71b$ (5 is      | sogeny classes | )             |            |                              |                             |        |  | 639b      |
| a1   | 0                              | $1+\varphi$               | $\varphi$                       | arphi                         | 0   | 1              | 1             | + -        | 1,1                          | 1,1                         | 1,1    | $I_1, I_1$   |           |
| b1   | 0                              | 1                         | $\varphi$                       | $-53-76\varphi$               | $235 + 375\varphi$                              |                | ' ·<br>  7    | :<br>  + - | 7, 1                         | $\frac{1}{7}, \frac{1}{1}$  | 7,1    | $\overline{\mathrm{I}}_{7}, \overline{\mathrm{I}}_{1}$ | <u>-</u>  |
| b2   | 0                              | -1                        | arphi                           | $-129697 + 80172\varphi$      | $-21092964 + 13036100\varphi$                   | 0              | 1             | + -        | 1, 7                         | 1,7                         | 1,7    | $I_1, I_7$   |           |
| c1   | $1+\varphi$                    | $\varphi$                 | $\varphi$                       | $-25+13\varphi$               | $-49 + 30\varphi$                               | 0              | 2             | - +        | 4, 1                         | 4,1                         | [2, 1] | $\overline{\mathrm{I}}_{4},\overline{\mathrm{I}}_{1}$  |           |
| c2   | arphi                          | $-1+\varphi$              | arphi                           | $-2640 + 1631\varphi$         | $-60042 + 37108\varphi$                         | 0              | 2             | + -        | 2, 2                         | 2, 2                        | 2, 2   | $I_2, I_2$   |           |
| d1   | $\varphi$                      | $1-\varphi$               | 1                               | -6                            | $-3\varphi$                                     | 0              | 4             | ++         | 2, 2                         | [2, 2]                      | [2, 2] | $I_2, I_2$   |           |
| d2   | arphi                          | $1-\varphi$               | 1                               | $-81 + 45\varphi$             | $276-171\varphi$                                | 0              | 4             | ++         | 1, 4                         | 1,4                         | 1, 4   | $I_1, I_4$   |           |
| d3   | $\varphi$                      | $1-\varphi$               | 1                               | -1                            | $-2+\varphi$                                    | 0              | 2             | + -        | 1, 1                         | 1, 1                        | 1, 1   | $\mathrm{I}_1,\mathrm{I}_1$                            |           |
| d4   | $1 + \varphi$                  | $-\varphi$                | 1                               | $-156-256\varphi$             | $-1447 - 2345\varphi$                           | 0              | 2             | -+         | 4, 1                         | 4, 1                        | 4,1    | $I_4, I_1$   |           |
| e1   | 0                              | $-1+\varphi$              | $\varphi$                       | $-85-138\varphi$              | $-594 - 961\varphi$                             | 0              | 1             | + -        | 1,1                          | 1,1                         | 1, 1   | $I_1, I_1$   |           |
| 641a                                       | a                              |                           |                                 | 64                            | $41a = (7 - 23\varphi) = 641a$ (2 iso           | ogeny classes) | )             |            |                              |                             |        |  | 641a      |
| a1   | $\varphi$                      | -1                        | 1                               | 0                             | 0   | 1              | 1             | + -        | 1                            | 1                           | 1      | $I_1$  |           |
| b1   |                                | $-1-\varphi$              | $\varphi$                       | -2                            | -1  | 0              | 1             | :<br>      | 1                            | 1                           | 1      | $I_1$  |           |
| 6411                                       | h                              |                           |                                 | 64                            | $41b = (16 - 23\varphi) = 641b$ (2 is           | ogeny classes  | )             |            |                              |                             |        |  | 641b      |
| al   |                                | $-1-\varphi$              | 1                               | $-\varphi$                    | 0   | 1              | 1             | -+         | 1                            | 1                           | 1      | $I_1$  |           |
| b1   |                                | $1+\varphi$               |                                 | $-3+2\varphi$                 | -4  | 0              | <u></u> -     | !' -<br>   | 1                            | 1                           | 1      | $\begin{array}{cccccccccccccccccccccccccccccccccccc$   | <u>-</u>  |
| 649  |                                | · ,                       |                                 | ·                             | $0a = (26 - \varphi) = 11a \cdot 59a$ (4 i      | sogeny classe  | a)            |            |                              |                             |        | 1  | 649a      |
| al   | $\frac{\mathbf{a}}{1+\varphi}$ | 1                         | 1                               | $-1+\varphi$                  | $\frac{9a - (20 - \varphi) - 11a \cdot 59a}{0}$ | 1              | 2             | ++         | 1,1                          | 1,1                         | 1,1    | $I_1, I_1$   | 0454      |
| $\begin{vmatrix} a_1 \\ a_2 \end{vmatrix}$ | $1+\varphi$<br>$1+\varphi$     | 1                         | 1                               | $-1 + \varphi$ $-6 + \varphi$ | $-6+4\varphi$                                   | 1              | $\frac{2}{4}$ | ++         | 2, 2                         | 2,2                         | 2,2    | $I_1, I_1$ $I_2, I_2$                                  |           |
| a3   | $1 + \varphi$                  | -1                        | $\varphi$                       | $-166-272\varphi$             | $1625 + 2633\varphi$                            | 1              | 4             | -+         | $\frac{2}{4}, \frac{2}{1}$   | 4, 1                        | 4, 1   | $I_2, I_2$ $I_4, I_1$                                  |           |
| a4   | $1+\varphi$                    | 1                         | 1                               | $-76+46\varphi$               | $-322 + 204\varphi$                             | 1              | 2             | + -        | 1, 4                         | 1,4                         | 1, 2   | $I_1, I_4$   |           |
| b1   | $\varphi$                      | 0                         | $\varphi$                       | 0                             | 0   |                | ' ·<br>  3    | ! - '<br>  | 1, 1                         | 1,1                         | 1,1    | $egin{array}{cccccccccccccccccccccccccccccccccccc$     |           |
| b2   | $\varphi$                      | 0                         | arphi                           | $-5-5\varphi$                 | $-9-10\varphi$                                  |                | 1             |            | 3, 3                         | 3,3                         | 1, 1   | $I_3, I_3$   |           |
| c1   | $\varphi$                      | · · · · · · · · · · · · · | <del>'</del> 1                  | $-7+4\varphi$                 | $-8+4\varphi$                                   | 0              | <br>- 2       | <u> </u>   | 1, 1                         | 1,1                         | 1, 1   | $I_1, I_1$   | <u>-</u>  |
| c2   | $1+\varphi$                    | $1-\varphi$               | 0                               | $-43-69\varphi$               | $-222-357\varphi$                               |                | 2             | - +        | 2, 2                         | 2, 2                        | 2,2    | $I_1, I_1$ $I_2, I_2$                                  |           |
| d1   | $\varphi$                      | $1-\varphi$               | $arphi$ $ec{arphi}$ $ec{arphi}$ | $-42 + 25\varphi$             | $-148 + 91\varphi$                              | 0              | 1             | i          | 1, 1                         | 1,1                         | 1, 1   | $I_1, I_1$   | <u>-</u>  |
| 6491                                       | b                              |                           |                                 | 649                           | $9b = (25 + \varphi) = 11b \cdot 59b$ (4 i      | sogeny classes | s)            |            |                              |                             |        |  | 649b      |
| a1   |                                | $-1-\varphi$              | $1+\varphi$                     | $-1-\varphi$                  | 0   | 1              | 2             | ++         | 1, 1                         | 1,1                         | 1,1    | $I_1, I_1$   |           |
| a2   |                                | $-1-\varphi$              |                                 | $-6-\varphi$                  | $3-4\varphi$                                    | 1              | $\frac{2}{4}$ | ++         | 2, 2                         | 2, 2                        | 2,2    | $I_1, I_1$ $I_2, I_2$                                  |           |
|  | 7<br>1                         |                           | $1+\varphi$                     | $-438 + 271\varphi$           | $4258 - 2634\varphi$                            | 1              | 4             | + -        | $\frac{-}{4}, \frac{-}{1}$   | 4, 1                        | 4, 1   | $I_4, I_1$   |           |
| a3   |                                |                           |                                 |                               |   |                |               |            |                              |                             |        |  |           |

|     | $a_1$              | $a_2$         | $a_3$         | $a_4$              | $a_6$                                   | r                  | T   | s        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$  | Kodaira                                    | Isogenies |
|-----|--------------------|---------------|---------------|--------------------|---|--------------------|-----|----------|------------------------------|-----------------------------|--------|--|-----------|
| 649 | b                  |               |               | 6                  | $49b = (25 + \varphi) = 11b \cdot 59b$  | (4 isogeny classes | s)  |          |                              |                             |        |  | 649b      |
| b1  | $1 + \varphi$      |               | $1 + \varphi$ | $-2\varphi$        | $-\varphi$                              | 0                  | 3   |          | 1, 1                         | 1, 1                        | 1, 1   | ${ m I}_1, { m I}_1$                       |           |
| b2  | $1+\varphi$        |               | $1+\varphi$   | $-10 + 3\varphi$   | $-19 + 9\varphi$                        | 0                  | 1   | <u> </u> | 3,3                          | 3,3                         | 1,1    | $I_3, I_3$                                 |           |
| c1  | •                  | $1+\varphi$   | 0             | $-2-\varphi$       | $-7-10\varphi$                          |                    | 2   | ++       | ,                            | 1,1                         | 1,1    | $I_1, I_1$                                 |           |
| c2  | $\dots \varphi_{}$ | 1             | 0             | $-112 + 69\varphi$ | $-579 + 357\varphi$                     |                    | 2   | + -<br>  | 2,2                          | 2,2                         | 2,2    | $I_2, I_2$                                 |           |
| d1  | $1+\varphi$        | 0             | $1+\varphi$   | $-17-27\varphi$    | $-57 - 92\varphi$                       | 0                  | 1   |          | 1,1                          | 1,1                         | 1,1    | $I_1, I_1$                                 |           |
| 649 | c                  |               |               | 64                 | $9c = (8 - 23\varphi) = 11b \cdot 59a$  | (3 isogeny classe  | es) |          |                              |                             |        |  | 649c      |
| a1  | 0                  | $-\varphi$    | $1+\varphi$   | $-5+4\varphi$      | $4-4\varphi$                            | 1                  | 1   |          | 1, 1                         | 1, 1                        | 1, 1   | ${ m I}_1, { m I}_1$                       |           |
| b1  | 0                  | -1            | $1+\varphi$   | $-\varphi$         | 0                                       | 1                  | 1   |          | 1, 1                         | 1,1                         | 1,1    | $I_1, I_1$                                 |           |
| c1  | 1 -                | $-1-\varphi$  | <u>i</u> -    | $-10+6\varphi$     | $-13 + 8\varphi$                        | 0                  | 4   | <u> </u> | 2, 2                         | 2,2                         | [2, 2] | $\overline{\mathrm{I}_{2},\mathrm{I}_{2}}$ |           |
| c2  |                    | -1-arphi      | 1             | arphi              | -1                                      | 0                  | 2   |          | 1, 1                         | 1, 1                        | 1,1    | $\mathrm{I}_1,\mathrm{I}_1$                |           |
| c3  | 1 -                | $-1-\varphi$  | 1             | $-155 + 91\varphi$ | $-841 + 530\varphi$                     | 0                  | 2   | ++       | 4, 1                         | 4, 1                        | 2, 1   | $I_4, I_1$                                 |           |
| c4  | $\varphi$          | 1             | 0             | $-49 - 74\varphi$  | $-261 - 422\varphi$                     | 0                  | 2   | ++       | 1,4                          | 1,4                         | 1,2    | $I_1, I_4$                                 |           |
| 649 | d                  |               |               | 64                 | $9d = (15 - 23\varphi) = 11a \cdot 59b$ | (3 isogeny class   | es) |          |                              |                             |        |  | 649d      |
| a1  | 0 -                | $-1+\varphi$  | $\varphi$     | $-1-4\varphi$      | $1+3\varphi$                            | 1                  | 1   |          | 1, 1                         | 1, 1                        | 1, 1   | $\mathrm{I}_1,\mathrm{I}_1$                |           |
| b1  | 0                  | -1            | $\varphi$     | $-1+\varphi$       | $1-\varphi$                             | 1                  | 1   |          | 1,1                          | 1,1                         | 1,1    | $oxed{I_1,I_1}$                            |           |
| c1  | 1                  | $1+\varphi$   |               | $-4-4\varphi$      | $-9-13\varphi$                          | 0                  | 4   | ++       | 2, 2                         | 2,2                         | [2, 2] | $\overline{\mathrm{I}_{2},\mathrm{I}_{2}}$ |           |
| c2  | 1                  | $1 + \varphi$ | 0             | $1+\varphi$        | 0                                       | 0                  | 2   |          | 1, 1                         | 1, 1                        | 1,1    | $\mathrm{I}_1,\mathrm{I}_1$                |           |
| c3  | 1                  | $1+\varphi$   | 0             | $-64 - 89\varphi$  | $-375-620\varphi$                       | 0                  | 2   | ++       | 4, 1                         | 4, 1                        | 2,1    | $I_4, I_1$                                 |           |
| c4  | $1+\varphi$        | $1-\varphi$   | 0             | $-123 + 74\varphi$ | $-683 + 422\varphi$                     | 0                  | 2   | ++       | 1,4                          | 1,4                         | 1,2    | $I_1, I_4$                                 |           |
| 655 | a                  |               |               | 659                | $5a = (14 - 23\varphi) = 5a \cdot 131a$ | (3 isogeny class   | es) |          |                              |                             |        |  | 655a      |
| a1  | $\varphi$          | $-\varphi$    | $\varphi$     | $-4-4\varphi$      | $4+6\varphi$                            | 1                  | 1   | <u> </u> | 1,3                          | 1,3                         | 1,3    | $I_1, I_3$                                 |           |
| b1  | $1 + \varphi$      | $1-\varphi$   | 1             | $1-\varphi$        | 0                                       | 1                  | 1   |          | 1, 1                         | 1, 1                        | 1,1    | $\mathrm{I}_1,\mathrm{I}_1$                |           |
| c1  | $\varphi$          | $1-\varphi$   | $\varphi$     | $1-3\varphi$       | $1-2\varphi$                            | 0                  | 2   | - +      | 1,3                          | 1,3                         | 1,1    | $I_1, I_3$                                 | [         |
| c2  | arphi              | $1-\varphi$   | $\varphi$     | $-9+2\varphi$      | $2-\varphi$                             | 0                  | 4   | ++       | 2,6                          | 2, 6                        | 2,2    | $I_2, I_6$                                 |           |
| c3  | arphi              | $1-\varphi$   | $\varphi$     | $-69 + 7\varphi$   | $-159 + 198\varphi$                     | 0                  | 2   | ++       | 1, 12                        | 1, 12                       | 1, 2   | $I_1, I_{12}$                              |           |
| c4  | $\varphi$          | $1-\varphi$   | $\varphi$     | $-109 + 77\varphi$ | $487 - 296\varphi$                      | 0                  | 2   | + -      | 4,3                          | 4,3                         | 2,1    | $I_4, I_3$                                 |           |
| 655 |                    |               |               | 65                 | $5b = (9 - 23\varphi) = 5a \cdot 131b$  | (3 isogeny classe  | es) |          |                              |                             |        |  | 655b      |
| a1  | $1+\varphi$        | -1            | $1 + \varphi$ | $-8+2\varphi$      | $10-7\varphi$                           | 1                  | 1   | <u> </u> | 1,3                          | 1,3                         | 1,3    | $I_1, I_3$                                 |           |
| b1  | $\varphi$          | 1             | 1             | 1                  | 0                                       | 1                  | 1   |          | 1, 1                         | 1,1                         | 1,1    | $I_1, I_1$                                 |           |
| c1  | $1+\varphi$        | 0             | $1+\varphi$   | $-2+\varphi$       | $-1+\varphi$                            | 0                  | 2   | + -      | 1,3                          | 1,3                         | 1,1    | $I_1, I_3$                                 |           |
| c2  | $1 + \varphi$      |               | $1+\varphi$   | $-7-4\varphi$      | 1                                       | 0                  | 4   | ++       | 2,6                          | 2,6                         | 2,2    | $I_2, I_6$                                 |           |
| c3  | $1+\varphi$        |               | $1+\varphi$   | $-62-9\varphi$     | $39-199\varphi$                         | 0                  | 2   | ++       | 1, 12                        | 1, 12                       | 1,2    | $I_1, I_{12}$                              |           |
| c4  | $1+\varphi$        | 0             | $1+\varphi$   | $-32 - 79\varphi$  | $191 + 295\varphi$                      | 0                  | 2   | -+       | 4,3                          | 4,3                         | 2, 1   | $I_4, I_3$                                 |           |

|          | $a_1$   | $a_2$                            | $a_3$            | $a_4$                                   | $a_6$  | r                                      | T   | $s \operatorname{ord}(\Delta)$                               | $\operatorname{ord}_{-}(j)$                        | $c_p$  | Kodaira  | Isogenies |
|----------|---|----------------------------------|------------------|---|--|--|---|--|--|--|--|-----------|
| 656      |   |                                  |                  |   | $656a = (28 - 4\varphi) = 2^2 \cdot 41a$       | (1 isogeny cla                         | nee)  | ,  |  | F  |  | 656a      |
| a1       |   | $-1-\varphi$                     | 0                | $-2+\varphi$                            | $\frac{-3 + 4\varphi}{}$                       | (1 isogeny ch                          | 3   | 8,1  | 1  | 3,1  | $IV^*, I_1$  | 0000      |
| a2       |   | $-1 - \varphi$<br>$-1 - \varphi$ | 0                | $-2+\varphi$ $-242+161\varphi$          | $-3 + 4\varphi$ $-1731 + 1060\varphi$          |  | 1   | $\begin{vmatrix} - & - & 0.1 \\ - & - & 8.3 \end{vmatrix}$   | 3  | 3, 1 $3, 1$  | $IV^*, I_1$<br>$IV^*, I_3$                                     |           |
|          |   | - P                              |                  | 212   1019                              | 1,01   10004                                   | "                                      | _   |  |  | 3,1  | 1, ,13   | _         |
| 656      | <u>b</u>  |                                  |                  |   | $656b = (24 + 4\varphi) = 2^2 \cdot 41b$       | (1 isogeny cla                         |   |  |  |  |  | -656b     |
| a1       | 0   | $1+\varphi$                      | 0                | $-2+\varphi$                            | $-1-4\varphi$                                  | 0                                      | 3   | 8,1  | 1  | 3, 1   | $IV^*, I_1$  |           |
| a2       | 0   | $1+\varphi$                      | 0                | $-82 - 159\varphi$                      | $-753 - 1220\varphi$                           | 0                                      | 1   | 8,3  | 3  | 3,1  | $IV^*, I_3$  |           |
| 671      | .a  |                                  |                  | 67                                      | $71a = (25 + 2\varphi) = 11a \cdot 61a$        | (2 isogeny cla                         | sses)   |  |  |  |  | 671a      |
| a1       | $\varphi$   | $-1-\varphi$                     | $\varphi$        | $1+2\varphi$                            | -2   | 1                                      | 1   | 5,1  | 5,1  | 5,1  | $I_5, I_1$   |           |
| b1       | $1+\varphi$   | $1+\varphi$                      | 0                | $1+3\varphi$                            | $1+\varphi$                                    | 1                                      | 1   | 1,1  | $\begin{bmatrix} 1,1 \end{bmatrix}$                | 1,1  | $I_1, I_1$   |           |
| 671      | b   |                                  |                  | 6                                       | $71b = (27 - 2\varphi) = 11b \cdot 61b$        | (2 isogeny cla                         | sses)   |  |  |  |  | 671b      |
| a1       | $1+\varphi$   | 1                                | 0                | $4-\varphi$                             | $2-2\varphi$                                   | 1                                      | 1   | - 5,1  | 5,1  | 5,1  | $I_5, I_1$   |           |
| b1       | $\varphi$   | $\varphi$                        | 1                | 0                                       | 0  | 1                                      | 1   | 1,1  | 1,1  | 1,1  | $I_1, I_1$   |           |
| 676      | la<br>la  |                                  |                  |   | $676a = (26) = 2 \cdot 13 \tag{3}$             | isogeny classes)                       | )   |  |  |  |  | 676a      |
| a1       | 1   | 1                                | $\varphi$        | $-3+\varphi$                            | $\frac{010a - (20) - 2 \cdot 13}{1 - \varphi}$ | 1                                      | 2   | ++ 1,2   | 1, 2   | 1,2  | $I_1, I_2$   |           |
| a2       | $\varphi$   | $1+\varphi$                      | $\varphi$        | $-32-52\varphi$                         | $102 + 164\varphi$                             | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | 2   | $\begin{vmatrix} + & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1$  | 2, 1   | 2, 1   | $I_1, I_2 \\ I_2, I_1$   |           |
| b1       | <u>'</u> <del>'</del> - 1                           | 0                                | 1                | <del>-</del>                            | 0  |  |   | $\begin{vmatrix} - & - & 1 \\ - & - & 1 \end{vmatrix}$       | - <del>'</del>                                     | $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 \end{vmatrix}$ | $I_1, I_1$   |           |
| b2       | 1   | 0                                | 1                | -5                                      | -8   | 0                                      | 3   | - 3,3  | 3,3  | 3,3  | $I_3, I_3$   |           |
| b3       | 1   | 0                                | 1                | -460                                    | -3830  | 0                                      | 1   | 1, 9   | 1,9  | 1,9  | $I_1, I_9$   |           |
| c1       | 1   | -1                               | 1                | -3                                      | 3  | 0                                      | 7   | 1, 7   | 1,7  | 1,7  | $ar{\mathrm{I}}_1,ar{\mathrm{I}}_7$                            |           |
| c2       | 1   | -1                               | 1                | -213                                    | -1257  | 0                                      | 1   | - 7, 1   | 7, 1   | 7,1  | $I_7, I_1$   |           |
| 684      | a   |                                  |                  | 68                                      | $44a = (6 - 24\varphi) = 2 \cdot 3 \cdot 19a$  | (3 isogeny cla                         | asses)  |  |  |  |  | 684a      |
| a1       | $1+\varphi$   | 0                                | $\varphi$        | $-117 + 64\varphi$                      | $547 - 333\varphi$                             | 1                                      | 2   | + + 1, 1, 6  | 1, 1, 6  | 1, 1, 6  | $I_{1}, I_{1}, I_{6}$  |           |
| a2       | $1+\varphi$   | 0                                | $\varphi$        | $-7+4\varphi$                           | $9-5\varphi$                                   | 1                                      | 2   | 2, 2, 3  | 2, 2, 3  | 2, 2, 3  | $I_2, I_2, I_3$  |           |
| b1       | 1   | $\varphi$                        | 0                | $-14 + 9\varphi$                        | $-23 + 13\varphi$                              | 0                                      | 2   | 2, 4, 1  | [2, 4, 1]  | [2, 2, 1]  | $I_2, I_4, I_1$  |           |
| b2       | $1+\varphi$   | -1                               | 0                | $-1557 + 960\varphi$                    | $-27279 + 16857\varphi$                        | 0                                      | 2   | + + 1, 2, 2  | 1,2,2  | 1, 2, 2  | $I_1, I_2, I_2$  |           |
| c1       | $1+\varphi$   | -1                               | $\varphi$        | $-20 + \varphi$                         | $-20+17\varphi$                                | 0                                      | 4   | + + 4, 2, 2  | 4, 2, 2  | 2, 2, 2  | $I_4, I_2, I_2$  |           |
| c2       | 1 '   | -1                               | $\varphi$        | $\varphi$                               | 200 + 551                                      | 0                                      | 4   | 2,4,1  | 2, 4, 1  | 2, 4, 1  | $I_2, I_4, I_1$  |           |
| c3<br>c4 | $\begin{vmatrix} 1+\varphi\\ \varphi \end{vmatrix}$ | $-1$ $1+\varphi$                 | $rac{arphi}{0}$ | $-80 - 89\varphi$ $-1674 + 1025\varphi$ | $328 + 551\varphi$ $-31153 + 19222\varphi$     | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{vmatrix} 2\\2 \end{vmatrix}$              | $\begin{vmatrix} ++ & 8, 1, 1 \\ ++ & 2, 1, 4 \end{vmatrix}$ | $\begin{bmatrix} 8, 1, 1 \\ 2, 1, 4 \end{bmatrix}$ | $\begin{bmatrix} 2, 1, 1 \\ 2, 1, 4 \end{bmatrix}$ | $\begin{bmatrix} I_8, I_1, I_1 \\ I_2, I_1, I_4 \end{bmatrix}$ |           |
|          |   | 1 + φ                            |                  | ·                                       | · ·  | <u> </u>                               |   |  | 2,1,1  | 2, 1, 1  | 12,11,14   | 00.41     |
| 684      | 1   |                                  |                  |   | $4b = (18 - 24\varphi) = 2 \cdot 3 \cdot 19b$  | (3 isogeny cl                          | ·   |  | 1  |  | T  | 684b      |
| a1       | $\varphi$   | $1-\varphi$                      |                  | $-52-66\varphi$                         | $214 + 332\varphi$                             | 1                                      | 2   | + + 1,1,6  | 1,1,6  | 1, 1, 6  | $I_1, I_1, I_6$  |           |
| a2       | $\frac{ }{ }$ $\frac{\varphi}{ }$                   | $1-\varphi$                      |                  | $-2-6\varphi$                           | $4+4\varphi$                                   | 1                                      | $\frac{ }{ } \frac{2}{ } \frac{ }{ } \frac{ }{ }$ | 2, 2, 3  | $\frac{ }{ }$ 2, 2, 3                              | 2,2,3  | $I_2, I_2, I_3$  |           |
| b1       | 1   | $1-\varphi$                      | 0                | $-5-9\varphi$                           | $-10 - 13\varphi$                              | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\frac{2}{2}$                                     | 2, 4, 1  | $\begin{bmatrix} 2, 4, 1 \\ 1, 2, 2 \end{bmatrix}$ | $\begin{bmatrix} 2, 2, 1 \\ 1, 2, 2 \end{bmatrix}$ | $I_2, I_4, I_1$  |           |
| b2       | $\varphi$   | $-\varphi$                       | 0                | $-597 - 960\varphi$                     | $-10422 - 16857\varphi$                        | 0                                      | 2   | + + 1, 2, 2  | 1, 2, 2  | 1, 2, 2  | $I_1, I_2, I_2$  |           |

|              | $a_1$         | $a_2$          | $a_3$                      | $a_4$                 | $a_6$   | r                | T      | s          | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$                      | Kodaira                                      | Isogenies    |
|--------------|---------------|----------------|----------------------------|-----------------------|---|------------------|--------|------------|------------------------------|-----------------------------|----------------------------|--|--------------|
| 00.41        |               |                |                            |                       |   |                  |        |            |                              |                             |                            |  | 00.41        |
| 684l         | <b>O</b>      |                |                            | 684                   | $4b = (18 - 24\varphi) = 2 \cdot 3 \cdot 19b$ | (3 isogeny cl    | asses) |            |                              |                             |                            |  | 684b         |
| c1           | $\varphi$     |                | $1 + \varphi$              | $-18-3\varphi$        | $-3-18\varphi$                                | 0                | 4      | ++         | 4, 2, 2                      | 4, 2, 2                     | 2, 2, 2                    | $I_4, I_2, I_2$                              |              |
| c2           | $\varphi$     |                | $1+\varphi$                | $2-3\varphi$          | $1-2\varphi$                                  | 0                | 4      |            | 2, 4, 1                      | 2, 4, 1                     | 2, 4, 1                    | $I_2, I_4, I_1$                              |              |
| c3           | $\varphi$     |                | $1+\varphi$                | $-168 + 87\varphi$    | $879 - 552\varphi$                            | 0                | 2      | ++         | 8, 1, 1                      | 8, 1, 1                     | 2, 1, 1                    | $I_8, I_1, I_1$                              |              |
| c4           | $1+\varphi$   | $-1+\varphi$   | φ                          | $-651 - 1024\varphi$  | $-12304 - 19873\varphi$                       | 0                | 2      | ++         | 2, 1, 4                      | 2, 1, 4                     | 2, 1, 4                    | $I_2, I_1, I_4$                              |              |
| 695a         | a             |                |                            | 69                    | $5a = (7 - 24\varphi) = 5a \cdot 139a$        | (3 isogeny cla   | sses)  |            |                              |                             |                            |  | 695a         |
| a1           | $1+\varphi$   | -1             | $1+\varphi$                | -3                    | $2-2\varphi$                                  | 1                | 1      |            | 1, 2                         | 1,2                         | 1,2                        | $I_1, I_2$                                   |              |
|              | $1+\varphi$   |                |                            | $-1 - 5\varphi$       | $1+3\varphi$                                  | 1                |        | '          | -1, 2                        | 1,2                         | -1,2                       | $I_1, I_2$                                   | <u>-</u>     |
|              | $1+\varphi$   |                | 0                          | $-11+10\varphi$       | $-2+32\varphi$                                | 1                | 1      |            | 3, 6                         | 3,6                         | 3, 2                       | $I_3, I_6$                                   |              |
| c1           | 1             | <del>-</del> - | $1+\varphi$                | $-27 + 15\varphi$     | $-63 + 38\varphi$                             | 0                | i      | ·<br>  + + | $-\frac{1}{2}, \frac{1}{2}$  | [-2,2]                      | $\frac{1}{2}, \frac{1}{2}$ | $I_2, I_2$                                   | <u>-</u>     |
| c2           | $\varphi$     |                | $1+\varphi$                | $-42-58\varphi$       | $154 + 254\varphi$                            | 0                | 4      | ++         | 4, 1                         | 4, 1                        | 4, 1                       | $I_4, I_1$                                   |              |
| c3           | $\varphi$     |                | $1+\varphi$                | $-2-3\varphi$         | $-2-3\varphi$                                 | 0                | 2      | -+         | 1, 1                         | 1,1                         | 1, 1                       | $I_1, I_1$                                   |              |
| c4           | $1+\varphi$   | $-\varphi$     | $1+\varphi$                | $-2865 + 1769\varphi$ | $-69988 + 43254\varphi$                       | 0                | 2      | +-         | 1, 4                         | 1,4                         | 1,4                        | $I_1, I_4$                                   |              |
|              |               |                |                            |                       |   |                  |        |            |                              |                             |                            |  |              |
| 695l         | o             |                |                            | 695                   | $5b = (17 - 24\varphi) = 5a \cdot 139b$       | (3 isogeny cla   | asses) |            |                              |                             |                            |  | 695b         |
| a1           | $\varphi$     | $-\varphi$     | $\varphi$                  | $-1-2\varphi$         | $1+\varphi$                                   | 1                | 1      |            | 1, 2                         | 1,2                         | 1, 2                       | $I_1, I_2$                                   |              |
| b1           | $\varphi$     | $1+\varphi$    | $1+\varphi$                | $-7+6\varphi$         | $9-5\varphi$                                  | 1                | 3      |            | 1, 2                         | [1, 2]                      | 1, 2                       | $I_1, I_2$                                   |              |
| b2           | $\varphi$     | $1+\varphi$    | $1+\varphi$                | $-2-9\varphi$         | $20-44\varphi$                                | 1                | 1      |            | 3, 6                         | 3, 6                        | 3, 2                       | $I_3, I_6$                                   |              |
| c1           | 1             | 0              | $\varphi$                  | $-11-16\varphi$       | $-24-39\varphi$                               | 0                | 4      | + +        | -2, 2                        | [2,2]                       | $\frac{1}{2}, \frac{1}{2}$ | $\bar{\mathrm{I}}_{2}, \bar{\mathrm{I}}_{2}$ |              |
| c2           | $1+\varphi$   | $-\varphi$     | $\varphi$                  | $-99 + 56\varphi$     | $409-255\varphi$                              | 0                | 4      | ++         | 4, 1                         | 4, 1                        | 4, 1                       | $I_4, I_1$                                   |              |
| c3           | $1 + \varphi$ | $-\varphi$     | $\varphi$                  | $-4+\varphi$          | $-4+2\varphi$                                 | 0                | 2      | +-         | 1, 1                         | 1, 1                        | 1, 1                       | $I_1, I_1$                                   |              |
| c4           | $\varphi$     | 0              | $\varphi$                  | $-1094 - 1771\varphi$ | $-26733 - 43255\varphi$                       | 0                | 2      | -+         | 1,4                          | 1,4                         | 1,4                        | $I_1, I_4$                                   |              |
| 701a         | a             |                |                            |                       | $701a = (27 - \varphi) = 701a$                | (1 isogeny class | a)     |            |                              |                             |                            |  | 701a         |
| al           | 1             | 0              | $1+\varphi$                | -1                    |   |                  | 3      |            | 1                            | 1                           | 1                          | $I_1$  | 1014         |
| a2           | 1             |                | $1+\varphi$<br>$1+\varphi$ | $4-5\varphi$          | $-\varphi$ $-13 + 5\varphi$                   |                  | 1      |            | 3                            | 3                           | 1                          | $I_3$  |              |
|              |               |                | · ·                        | ,                     |   | l                |        |            |                              |                             |                            |  |              |
| <b>701</b> k | b             |                |                            |                       | $701b = (26 + \varphi) = 701b$                | (1 isogeny class | s)     |            |                              |                             |                            |  | <b>701</b> b |
| a1           | 1             | 0              | $\varphi$                  | -arphi                | 0   | 0                | 3      |            | 1                            | 1                           | 1                          | $I_1$  |              |
| a2           | 1             | 0              | $\varphi$                  | 4arphi                | $-7-6\varphi$                                 | 0                | 1      |            | 3                            | 3                           | 1                          | $I_3$  |              |
| 704          | _             |                |                            |                       | (2) (2) (2)                                   | / <del>-</del> . |        |            |                              |                             |                            |  | 704          |
| 704a         |               |                |                            |                       | $04a = (8 - 24\varphi) = 2^3 \cdot 11a$       | (5 isogeny clas  |        |            |                              | 1                           |                            | T  | 704a         |
| a1<br>a2     | 0             | $1+\varphi$    | 0                          | $-1+2\varphi$         | 1   |                  | 2      | +-         | 1,4                          | 1                           | 1, 2                       | $I_1, III$                                   |              |
|              | 0             | $1+\varphi$    | 0                          | $-1-3\varphi$         | $4-\varphi$                                   | 1                | 2      | -+         | 2, 8                         | 2                           | 2,4                        | $I_2, I_1^*$                                 | 1            |

|   | $a_1$   | $a_2$  | $a_3$   | $a_4$  | $a_6$  | r   | T   | s  | $\operatorname{ord}(\Delta)$  | $\operatorname{ord}_{-}(j)$   | $c_p$   | Kodaira  | Isogenies    |
|---|---|--|---|--|--|---|---|--|---|---|---|--|--------------|
| 704   | a   |  |   |  | $704a = (8 - 24\varphi) = 2^3 \cdot 11a$   | (5 isogeny classes  | s)  |  |   |   |   |  | <b>7</b> 04a |
| b1  | 0   | 0  | 0   | $-5+3\varphi$  | $5-3\varphi$   | 1   | 4   | + -  | 1, 4  | 1   | 1,2   | $I_1, III$   |              |
| b2  | 0   | 0  | 0   | $-11-15\varphi$  | $22 + 36\varphi$   | 1   | 8   | ++   | 2, 8  | 2   | 2,4   | $\mathrm{I}_2,\mathrm{I}_1^*$  |              |
| b3  | 0   | 0  | 0   | $-151-255\varphi$  | $1422 + 2304\varphi$   | 1   | 4   | -+   | 1, 10   | 1   | 1,2   | $I_1, III^*$   |              |
| b4  | 0   | 0  | 0   | $-110 + 63\varphi$   | $-506 + 312\varphi$  | 1   | 4   | ++   | 4, 10   | 4   | 2,2   | $I_4, III^*$   |              |
| b5  | 0   | 0  | 0   | $-1750 + 1023\varphi$  | $-32482 + 19856\varphi$  | 1   | 2   | ++   | 2, 11   | 2   | 2, 1  | $I_2, II^*$  |              |
| b6  | 0   | 0  | 0   | $-70+63\varphi$  | $-786 + 480\varphi$  | 1   | 2   |  | 8, 11   | 8   | 2,1   | $I_8, II^*$  |              |
| c1  | 0   | $-1-\varphi$   | 0   | $-12+8\varphi$   | $-18+11\varphi$  | 0   | $\begin{bmatrix} 2 \end{bmatrix}$   | + -  | 1, 4  | 1   | [1, 2]  | $I_1, III$   | i            |
| c2  | 0   | 1  | 0   | $-44-68\varphi$  | $-220 - 352\varphi$  | 0   | 2   | -+   | 2, 8  | 2   | 2,2   | $I_2, I_1^*$   |              |
| d1  | 0   | $\varphi$  | 0   | $-2+2\varphi$  | $2-\varphi$  | 0   | 4   | + -  | 1, 4  | 1   | 1,2   | $I_1, III$   |              |
| d2  | 0   | arphi  | 0   | $-12+7\varphi$   | $-15+9\varphi$   | 0   | 4   | ++   | 2,8   | 2   | 2,2   | $I_2, I_1^*$   |              |
| d3  | 0   | $\varphi$  | 0   | $-192 + 107\varphi$  | $-1111 + 701\varphi$   | 0   | 2   | ++   | 1, 10   | 1   | 1,2   | $I_1, III^*$   |              |
| d4  | 0   | $1-\varphi$  | 0   | $-24-44\varphi$  | $-96 - 148\varphi$   | 0   | 2   | -+   | 4, 10   | 4   | 2,2   | $I_4, III^*$   |              |
| e1  | 0   | '<br>-φ  | 0   | $-3-\varphi$   | -2   | 0   | '<br>  4  | ++   | $\frac{1}{2}, \frac{1}{4}$  | 2   | [2, 2]  | $\overline{\mathrm{I}}_{2},\overline{\mathrm{III}}$  | <u>-</u>     |
| e2  | 0   | $-\varphi$   | 0   | $-28-36\varphi$  | $-72 - 120\varphi$   |   | 2   | ++   | 1, 8  | 1   | 1, 2  | $I_1, I_1^*$   |              |
| e3  | 0   | $-\varphi$   | 0   | $-3-6\varphi$  | $2+11\varphi$  |   | 4   | - +  | 4, 8  | 4   | 4, 2  | $I_4, I_1^*$   |              |
| e4  | 0   | $1-\varphi$  | 0   | $-197 + 122\varphi$  | $-1365 + 843\varphi$   |   | 2   | + -  | 1,8   | 1   | 1,4   | $I_1, I_1^*$   |              |
| 704   |   |  |   |  | $704b = (16 - 24\varphi) = 2^3 \cdot 11b$  | (5 isogeny classe   |   |  |   |   |   |  | 704b         |
| a1  | 0   | $-1-\varphi$   | 0   | 0  | $1+\varphi$  | 4   | 1 0   |  | 1 1   | 1 1   | 1 1   | T TIT  |              |
|   |   |  |   |  | The state of the s | 1   | 2   | -+   | 1, 4  | 1   | 1,2   | $I_1, III$   |              |
| a2  |   | $-1-\varphi$   | 0   | $-5+5\varphi$  | $8-3\varphi$   | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$  | $\begin{bmatrix} 2\\2\\ \end{bmatrix}$  | - +<br>  + -   | $\begin{array}{c} 1,4\\2,8\end{array}$  | $\begin{vmatrix} 1 \\ 2 \end{vmatrix}$  | $\begin{bmatrix} 1,2\\2,4 \end{bmatrix}$  | $I_1, I_1$<br>$I_2, I_1^*$   |              |
| b1  |   |  |   |  | The state of the s |   | ı   |  | $\frac{2,8}{1,4}$   |   | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$  |  | <u> </u>     |
| b1<br>b2  | 0   | $-1-\varphi$   | 0   | $ \begin{array}{r} -5 + 5\varphi \\ -2 - 3\varphi \\ -26 + 15\varphi \end{array} $   | $\begin{array}{c} 8 - 3\varphi \\ \hline 2 + 3\varphi \\ 58 - 36\varphi \end{array}$   | 1   | 2   | + -  | 2,8   | 2   | $\begin{bmatrix} 2,4\\ 1,2\\ 2,4 \end{bmatrix}$   | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   |              |
| b1  | $\frac{0}{0}$   | $ \begin{array}{c} -1 - \varphi \\ \hline 0 \\ 0 \\ 0 \end{array} $  | 0   | $-5 + 5\varphi \\ -2 - 3\varphi \\ -26 + 15\varphi \\ -406 + 255\varphi$   | $\begin{array}{c} 8-3\varphi\\ 2+3\varphi\\ 58-36\varphi\\ 3726-2304\varphi\end{array}$  | $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  | $\begin{bmatrix} 2 \\ -4 \end{bmatrix}$   | + -<br>- +   | $ \begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \end{array} $  | $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$  | $ \begin{array}{ c c c } \hline 2,4\\ 1,2\\ 2,4\\ 1,2\\ \end{array} $   | $\begin{array}{c c} & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ \end{array}$  |              |
| b1<br>b2<br>b3<br>b4                            | $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$   | $ \begin{array}{c} -1 - \varphi \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $  | 0 0   | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$  | $\begin{array}{c} 8 - 3\varphi \\ 2 + 3\varphi \\ 58 - 36\varphi \\ 3726 - 2304\varphi \\ -194 - 312\varphi \end{array}$   | $egin{bmatrix} 1 \ 1 \ 1 \end{bmatrix}$   | $\begin{bmatrix} 2 \\ 4 \\ 8 \\ 4 \\ 4 \end{bmatrix}$   | + -<br>- +<br>+ +  | $ \begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \end{array} $  | $\begin{array}{c c} 2 \\ \hline 1 \\ 2 \end{array}$   | $ \begin{array}{ c c c } \hline 2,4\\ 1,2\\ 2,4\\ 1,2\\ 2,2\\ \end{array} $   | $\begin{array}{ c c c c c }\hline & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ & I_4, III^* \\ \hline \end{array}$   |              |
| b1<br>b2<br>b3<br>b4<br>b5                      | $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$  | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | 0<br>0<br>0   | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$ $-727 - 1023\varphi$   | $\begin{array}{c} 8 - 3\varphi \\ 2 + 3\varphi \\ 58 - 36\varphi \\ 3726 - 2304\varphi \\ -194 - 312\varphi \\ -12626 - 19856\varphi \end{array}$  | 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1   | $\begin{bmatrix} 2 \\ 4 \\ 8 \\ 4 \\ 4 \\ 2 \end{bmatrix}$  | + -<br>- +<br>+ +<br>+ -                                 | $ \begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ 2,11 \end{array} $  | $\begin{array}{c c} 2 \\ \hline 1 \\ 2 \\ 1 \\ \end{array}$   | $\begin{array}{ c c c }\hline 2,4\\\hline 1,2\\\hline 2,4\\\hline 1,2\\\hline 2,2\\\hline 2,1\\\hline \end{array}$  | $\begin{array}{ c c c c c }\hline & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ & I_4, III^* \\ & I_2, II^* \\ \end{array}$   |              |
| b1<br>b2<br>b3<br>b4                            | 0<br>0<br>0<br>0  | $ \begin{array}{c} -1 - \varphi \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $  | 0<br>0<br>0<br>0  | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$  | $\begin{array}{c} 8 - 3\varphi \\ 2 + 3\varphi \\ 58 - 36\varphi \\ 3726 - 2304\varphi \\ -194 - 312\varphi \end{array}$   | 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1   | $\begin{bmatrix} 2 \\ 4 \\ 8 \\ 4 \\ 4 \end{bmatrix}$   | + -<br>- +<br>+ +<br>+ -<br>+ +                          | $ \begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \end{array} $  | $ \begin{array}{c cccc}  & 2 & & \\  & 1 & & \\  & 2 & & \\  & 1 & & \\  & 4 & & \\ \end{array} $   | $ \begin{array}{ c c c } \hline 2,4\\ 1,2\\ 2,4\\ 1,2\\ 2,2\\ \end{array} $   | $\begin{array}{ c c c c c }\hline & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ & I_4, III^* \\ \hline \end{array}$   |              |
| b1<br>b2<br>b3<br>b4<br>b5                      | 0<br>0<br>0<br>0<br>0   | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | 0<br>0<br>0<br>0<br>0   | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$ $-727 - 1023\varphi$   | $\begin{array}{c} 8 - 3\varphi \\ 2 + 3\varphi \\ 58 - 36\varphi \\ 3726 - 2304\varphi \\ -194 - 312\varphi \\ -12626 - 19856\varphi \end{array}$  | 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1   | $\begin{bmatrix} 2 \\ 4 \\ 8 \\ 4 \\ 4 \\ 2 \end{bmatrix}$  | + -<br>- +<br>+ +<br>+ -<br>+ +<br>+ +                   | $ \begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ 2,11 \end{array} $  | $ \begin{array}{c cccc}  & 2 & & \\  & 1 & & \\  & 2 & & \\  & 1 & & \\  & 4 & & 2 \end{array} $  | $\begin{array}{ c c c }\hline 2,4\\\hline 1,2\\\hline 2,4\\\hline 1,2\\\hline 2,2\\\hline 2,1\\\hline \end{array}$  | $\begin{array}{ c c c c c }\hline & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ & I_4, III^* \\ & I_2, II^* \\ \end{array}$   |              |
| b1   b2   b3   b4   b5   b6                     | 0<br>0<br>0<br>0<br>0<br>0  | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | 0<br>0<br>0<br>0<br>0<br>0  | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$ $-727 - 1023\varphi$ $-7 - 63\varphi$  | $\begin{array}{c} 8-3\varphi\\ 2+3\varphi\\ 58-36\varphi\\ 3726-2304\varphi\\ -194-312\varphi\\ -12626-19856\varphi\\ -306-480\varphi \end{array}$   | 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1   | $\begin{bmatrix} 2 \\ 4 \\ 8 \\ 4 \\ 4 \\ 2 \\ 2 \end{bmatrix}$   | + -<br>- +<br>+ +<br>+ -<br>+ +<br>+ +                   | 2,8<br>1,4<br>2,8<br>1,10<br>4,10<br>2,11<br>8,11   | 1 2 1 2 1 4 2 8   | $\begin{array}{ c c }\hline 2,4\\\hline 1,2\\2,4\\1,2\\2,2\\2,1\\2,1\\\end{array}$  | $\begin{array}{ c c c c }\hline & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ & I_1, III^*\\ & I_4, III^*\\ & I_2, II^*\\ & I_8, II^*\\ \hline \end{array}$  |              |
| b1 b2 b3 b4 b5 b6 c1                            |   | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | 0<br>0<br>0<br>0<br>0<br>0  | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$ $-727 - 1023\varphi$ $-7 - 63\varphi$ $-5 - 6\varphi$ $-112 + 68\varphi$   | $\begin{array}{c} 8-3\varphi\\ 2+3\varphi\\ 58-36\varphi\\ 3726-2304\varphi\\ -194-312\varphi\\ -12626-19856\varphi\\ -306-480\varphi\\ -12-18\varphi\\ -572+352\varphi \end{array}$   | 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1   | $ \begin{array}{ c c c } \hline 2 \\ \hline 4 \\ 8 \\ 4 \\ 4 \\ 2 \\ \hline 2 \\ \hline \end{array} $   | + -<br>- +<br>+ +<br>+ -<br>+ +<br>                      | $ \begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ 2,11 \\ 8,11 \\ \hline 1,4 \\ 2,8 \end{array} $   | 1 2 1 2 1 4 2 8 1 1   | $\begin{array}{ c c c }\hline 2,4\\\hline 1,2\\2,4\\1,2\\2,2\\2,1\\2,1\\\hline 1,2\\2,2\\\end{array}$   | $\begin{array}{ c c c c }\hline & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ & I_4, III^* \\ & I_2, II^* \\ & I_8, II^* \\ \hline & & I_1, III \\ & I_2, I_1^* \\ \hline & & I_2, I_1^* \\ \hline \end{array}$   |              |
| b1 b2 b3 b4 b5 b6 c1 c2 d1                      | 0<br>0<br>0<br>0<br>0<br>0<br>0   | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0   | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$ $-727 - 1023\varphi$ $-7 - 63\varphi$ $-5 - 6\varphi$ $-112 + 68\varphi$ $-2\varphi$   | $\begin{array}{c} 8 - 3\varphi \\ 2 + 3\varphi \\ 58 - 36\varphi \\ 3726 - 2304\varphi \\ -194 - 312\varphi \\ -12626 - 19856\varphi \\ -306 - 480\varphi \\ -12 - 18\varphi \\ -572 + 352\varphi \\ 1 + \varphi \end{array}$  | $\begin{array}{c c} & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 0 \\ & 0 \\ \end{array}$  | $ \begin{array}{ c c } \hline  & 2 \\  & 4 \\  & 8 \\  & 4 \\  & 4 \\  & 2 \\  & 3 \\  & 3 \\  & 4 \\  & 4 \\  & 4 \\  & 5 \\  & 5 \\  & 6 \\  &$ | + -<br>- +<br>+ +<br>+ -<br>+ +<br>+ -<br>- +<br>+ -     | $\begin{array}{c} 2,8 \\ \hline 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ 2,11 \\ 8,11 \\ \hline 1,4 \\ 2,8 \\ \hline 1,4 \end{array}$                                      | $ \begin{array}{c cccc}  & 2 & & \\  & 1 & & \\  & 2 & & \\  & 1 & & \\  & 4 & & \\  & 2 & & \\  & 8 & & \\  & & 1 & \\  & 2 & & \\  & & 1 & \\  & & 2 & \\  & & 1 & \\  & 1 & \\  &$   | $\begin{array}{ c c c }\hline 2,4\\\hline 1,2\\2,4\\1,2\\2,2\\2,1\\2,1\\\hline 1,2\\2,2\\\hline 1,2\\\end{array}$   | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   |              |
| b1 b2 b3 b4 b5 b6 c1 c2                         | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0  | $ \begin{array}{c} -1 - \varphi \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 \\ 1 - \varphi \\ 1 - \varphi \end{array} $  | 0<br>0<br>0<br>0<br>0<br>0<br>0   | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$ $-727 - 1023\varphi$ $-7 - 63\varphi$ $-5 - 6\varphi$ $-112 + 68\varphi$ $-2\varphi$ $-5 - 7\varphi$   | $\begin{array}{c} 8-3\varphi\\ 2+3\varphi\\ 58-36\varphi\\ 3726-2304\varphi\\ -194-312\varphi\\ -12626-19856\varphi\\ -306-480\varphi\\ -12-18\varphi\\ -572+352\varphi \end{array}$   | $ \begin{array}{c c} & 1 & \\ & 1 & \\ & 1 & \\ & 1 & \\ & 1 & \\ & 1 & \\ & 1 & \\ & 1 & \\ & 0 & \\ & 0 & \\ & 0 & \\ & 0 & \\ \end{array} $  | $ \begin{array}{ c c c } \hline  & 2 & \\  & 4 & \\  & 8 & \\  & 4 & \\  & 2 & \\  & 2 & \\  & 2 & \\  & 2 & \\  & 2 & \\  & 4 & \\ \end{array} $   | + -<br>+ +<br>+ -<br>+ +<br><br>- +<br>+ -               | $ \begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ 2,11 \\ 8,11 \\ \hline 1,4 \\ 2,8 \end{array} $   | 2<br>1<br>2<br>1<br>4<br>2<br>8<br>   | $\begin{array}{ c c c }\hline 2,4\\\hline 1,2\\2,4\\1,2\\2,2\\2,1\\2,1\\\hline 1,2\\2,2\\\end{array}$   | $\begin{array}{ c c c c }\hline & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ & I_1, III^*\\ & I_4, III^*\\ & I_2, II^*\\ & I_8, II^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ \end{array}$  |              |
| b1 b2 b3 b4 b5 b6 c1 c2 d1 d2                   | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0  | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0   | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$ $-727 - 1023\varphi$ $-7 - 63\varphi$ $-5 - 6\varphi$ $-112 + 68\varphi$ $-2\varphi$   | $\begin{array}{c} 8 - 3\varphi \\ 2 + 3\varphi \\ 58 - 36\varphi \\ 3726 - 2304\varphi \\ -194 - 312\varphi \\ -12626 - 19856\varphi \\ -306 - 480\varphi \\ -306 - 480\varphi \\ -12 - 18\varphi \\ -572 + 352\varphi \\ 1 + \varphi \\ -6 - 9\varphi \end{array}$  | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   | $ \begin{array}{ c c c } \hline 2 \\ 4 \\ 8 \\ 4 \\ 4 \\ 2 \\ 2 \\ \hline 2 \\ 4 \\ 4 \\ 4 \end{array} $  | + -<br>- +<br>+ +<br>+ -<br>+ +<br>+ -<br>- +<br>+ -     | $\begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ 2,11 \\ 8,11 \\ \hline 1,4 \\ 2,8 \\ \hline 1,4 \\ 2,8 \\ 1,10 \\ \end{array}$                           | $ \begin{array}{c cccc}  & 2 & & \\  & 1 & & \\  & 2 & & \\  & 1 & & \\  & 4 & & \\  & 2 & & \\  & 8 & & \\  & & 1 & \\  & 2 & & \\  & & 1 & \\  & 2 & & \\  & & 1 & \\  & 2 & & \\  & & 2 & \\  & 2 & \\  &$   | $\begin{array}{ c c c }\hline 2,4\\\hline 1,2\\2,4\\1,2\\2,2\\2,1\\\hline 2,1\\\hline 1,2\\2,2\\\hline 1,2\\2,2\\\end{array}$                                 | $\begin{array}{ c c c c }\hline & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ & I_1, III^*\\ & I_4, III^*\\ & I_2, II^*\\ \hline & I_8, II^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ \hline & I_1, III\\ \end{array}$  |              |
| b1 b2 b3 b4 b5 b6 c1 c2 d1 d2 d3 d4             |   | $ \begin{array}{c} -1 - \varphi \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 - \varphi \\ 1 - \varphi \\ 1 - \varphi \\ \varphi \end{array} $                    | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0                                    | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$ $-727 - 1023\varphi$ $-7 - 63\varphi$ $-5 - 6\varphi$ $-112 + 68\varphi$ $-2\varphi$ $-5 - 7\varphi$ $-85 - 107\varphi$ $-68 + 44\varphi$  | $\begin{array}{c} 8-3\varphi\\ 2+3\varphi\\ 58-36\varphi\\ 3726-2304\varphi\\ -194-312\varphi\\ -12626-19856\varphi\\ -306-480\varphi\\ -12-18\varphi\\ -572+352\varphi\\ 1+\varphi\\ -6-9\varphi\\ -410-701\varphi \end{array}$   | $\begin{array}{c c} & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ \end{array}$  | $ \begin{array}{ c c c } \hline 2 \\ -4 \\ 8 \\ 4 \\ 4 \\ 2 \\ 2 \\ -4 \\ 4 \\ 2 \\ \end{array} $   | + -<br>+ +<br>+ +<br>+ +<br><br>+ +<br>+ +<br>+ +<br>+ + | 2,8<br>1,4<br>2,8<br>1,10<br>4,10<br>2,11<br>8,11<br>1,4<br>2,8<br>1,4<br>2,8<br>1,4<br>2,8<br>1,4<br>2,8   | $\begin{array}{ c c c } & 2 & & \\ & 1 & \\ & 2 & \\ & 1 & \\ & 4 & \\ & 2 & \\ & 8 & \\ & & 1 & \\ & 2 & \\ & & 1 & \\ & 2 & \\ & & 1 & \\ \end{array}$  | $\begin{array}{ c c c c }\hline 2,4\\\hline 1,2\\2,4\\1,2\\2,2\\2,1\\\hline 2,1\\\hline 1,2\\2,2\\\hline 1,2\\2,2\\1,2\\2,2\\\end{array}$                     | $\begin{array}{ c c c c }\hline & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ & I_1, III^*\\ & I_4, III^*\\ & I_2, II^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ \hline & I_1, III^*\\ & I_2, I_1^*\\ \hline & I_1, III^*\\ & I_4, III^*\\ \hline & I_4, III^*\\ \hline \end{array}$  |              |
| b1 b2 b3 b4 b5 b6 c1 c2 d1 d2 d3 d4 e1          | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0 | $ \begin{array}{c} -1 - \varphi \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 - \varphi \end{array} $ |   | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$ $-727 - 1023\varphi$ $-7 - 63\varphi$ $-5 - 6\varphi$ $-112 + 68\varphi$ $-2\varphi$ $-5 - 7\varphi$ $-85 - 107\varphi$ $-68 + 44\varphi$ $-4 + \varphi$                                   | $\begin{array}{c} 8-3\varphi\\ 2+3\varphi\\ 58-36\varphi\\ 3726-2304\varphi\\ -194-312\varphi\\ -12626-19856\varphi\\ -306-480\varphi\\ -12-18\varphi\\ -572+352\varphi\\ \hline \\ 1+\varphi\\ -6-9\varphi\\ -410-701\varphi\\ -244+148\varphi\\ -2\end{array}$   | $\begin{array}{c c} & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ \end{array}$   | $ \begin{array}{ c c c c } \hline 2 \\ -4 \\ 8 \\ 4 \\ 4 \\ 2 \\ 2 \\ -4 \\ 4 \\ 2 \\ 2 \\ -4 \\ 4 \end{array} $  | + -<br>- +<br>+ +<br>+ +<br><br>- +<br>+ +<br>+ +        | $\begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ 2,11 \\ 8,11 \\ \hline 1,4 \\ 2,8 \\ \hline 1,4 \\ 2,8 \\ 1,10 \\ \hline 4,10 \\ \hline 2,4 \end{array}$ | $\begin{array}{ c c c } & 2 & & \\ & 1 & \\ & 2 & \\ & 1 & \\ & 4 & \\ & 2 & \\ & 8 & \\ & & 1 & \\ & 2 & \\ & & 1 & \\ & 2 & \\ & & 1 & \\ \end{array}$  | $\begin{array}{ c c c }\hline 2,4\\\hline 1,2\\2,4\\1,2\\2,2\\2,1\\\hline 2,1\\\hline 1,2\\2,2\\\hline 1,2\\2,2\\1,2\\2,2\\\hline 1,2\\2,2\\2,2\\\end{array}$ | $\begin{array}{ c c c c c }\hline & I_2, I_1^* \\ \hline & I_1, III \\ I_2, I_1^* \\ I_1, III^* \\ I_4, III^* \\ I_2, II^* \\ \hline & I_8, II^* \\ \hline & I_1, III \\ \hline & I_2, I_1^* \\ \hline & I_1, III \\ I_2, I_1^* \\ \hline & I_1, III^* \\ I_4, III^* \\ \hline & I_2, III \\ \hline \end{array}$   |              |
| b1 b2 b3 b4 b5 b6 c1 c2 d1 d2 d3 d4 e1 e2       | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0 | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  |   | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$ $-727 - 1023\varphi$ $-7 - 63\varphi$ $-5 - 6\varphi$ $-112 + 68\varphi$ $-2\varphi$ $-5 - 7\varphi$ $-85 - 107\varphi$ $-68 + 44\varphi$ $-4 + \varphi$ $-64 + 36\varphi$                 | $\begin{array}{c} 8-3\varphi\\ 2+3\varphi\\ 58-36\varphi\\ 3726-2304\varphi\\ -194-312\varphi\\ -12626-19856\varphi\\ -306-480\varphi\\ -12-18\varphi\\ -572+352\varphi\\ \hline \\ 1+\varphi\\ -6-9\varphi\\ -410-701\varphi\\ -244+148\varphi\\ \hline \\ -2\\ -192+120\varphi \end{array}$  | $ \begin{array}{c c} & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 0 \\ & $ | 2   | + -<br>- +<br>+ +<br>+ +<br><br>- +<br>+ +<br>+ +        | $\begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ 2,11 \\ 8,11 \\ \hline 1,4 \\ 2,8 \\ \hline 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ \hline 2,4 \\ 1,8 \end{array}$ | $\begin{array}{ c c c }\hline & 2 & & \\ \hline & 1 & \\ & 2 & \\ & 1 & \\ & 4 & \\ & 2 & \\ & 8 & \\ \hline & 1 & \\ & 2 & \\ & & 1 & \\ & 2 & \\ \hline & 1 & \\ & 2 & \\ & & 1 & \\ & & 2 & \\ \hline & 1 & \\ & 2 & \\ & & 1 & \\ & & 2 & \\ \hline & 1 & \\ & 2 & \\ & & 1 & \\ & & 2 & \\ & & 2 & \\ \hline & & 2 & \\ & 2 & \\ & $ | $\begin{array}{ c c c }\hline 2,4\\\hline 1,2\\2,4\\1,2\\2,2\\2,1\\\hline 2,1\\\hline 1,2\\2,2\\\hline 1,2\\2,2\\1,2\\1,2\\1,2\\1,2\\1,2\\1,2\\1,2\\1,2\\$    | $\begin{array}{ c c c c }\hline & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ & I_1, III^*\\ & I_4, III^*\\ & I_2, II^*\\ & I_8, II^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ \hline & I_1, III^*\\ \hline & I_2, III\\ & I_1, I_1^*\\ \hline \end{array}$ |              |
| b1 b2 b3 b4 b5 b6 c1 c2 d1 d2 d3 d4 e1          | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0 | $ \begin{array}{c} -1 - \varphi \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 + \varphi \\ 1 - \varphi \end{array} $ |   | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$ $-727 - 1023\varphi$ $-7 - 63\varphi$ $-5 - 6\varphi$ $-112 + 68\varphi$ $-2\varphi$ $-5 - 7\varphi$ $-85 - 107\varphi$ $-68 + 44\varphi$ $-4 + \varphi$                                   | $\begin{array}{c} 8-3\varphi\\ 2+3\varphi\\ 58-36\varphi\\ 3726-2304\varphi\\ -194-312\varphi\\ -12626-19856\varphi\\ -306-480\varphi\\ -12-18\varphi\\ -572+352\varphi\\ \hline \\ 1+\varphi\\ -6-9\varphi\\ -410-701\varphi\\ -244+148\varphi\\ -2\end{array}$   | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   | $ \begin{array}{ c c c } \hline 2 \\ -4 \\ 8 \\ 4 \\ 4 \\ 2 \\ \hline 2 \\ -4 \\ 4 \\ 2 \\ \hline 2 \\ -4 \\ 2 \\ 2 \\ -4 \\ 2 \\ 2 \\ -4 \\ 2 \\ 2 \\ -4 \\ 2 \\ 2 \\ -4 \\ -4 \\ 2 \\ -4 \\ -4 \\ -4 \\ -4 \\ -4 \\ -4 \\ -4 \\ -4$  | + -<br>- +<br>+ +<br>+ +<br><br>- +<br>+ +<br>+ +        | $\begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ 2,11 \\ 8,11 \\ \hline 1,4 \\ 2,8 \\ \hline 1,4 \\ 2,8 \\ 1,10 \\ \hline 4,10 \\ \hline 2,4 \end{array}$ | $\begin{array}{ c c c }\hline & 2 \\ & 1 \\ & 2 \\ & 1 \\ & 4 \\ & 2 \\ & 8 \\ \hline & 1 \\ & 2 \\ & 1 \\ & 4 \\ \hline & 2 \\ & 1 \\ & 4 \\ \hline & 2 \\ & 1 \\ & 1 \\ & 2 \\ & 1 \\ \end{array}$  | $\begin{array}{ c c c }\hline 2,4\\\hline 1,2\\2,4\\1,2\\2,2\\2,1\\\hline 2,1\\\hline 1,2\\2,2\\\hline 1,2\\2,2\\1,2\\2,2\\\hline 1,2\\2,2\\2,2\\\end{array}$ | $\begin{array}{ c c c c }\hline & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ & I_1, III^*\\ & I_4, III^*\\ & I_2, II^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ \hline & I_1, III^*\\ & I_2, I_1^*\\ \hline & I_1, III^*\\ & I_2, I_1^*\\ \hline & I_1, III^*\\ & I_4, III^*\\ \hline & I_4, III^*\\ \hline & I_4, III^*\\ \hline & I_4, I_1^*\\ & I_4, I_1^*\\ \end{array}$                                   |              |
| b1 b2 b3 b4 b5 b6 c1 c2 d1 d2 d3 d4 e1 e2 e3    | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0 | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0 | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$ $-727 - 1023\varphi$ $-7 - 63\varphi$ $-5 - 6\varphi$ $-112 + 68\varphi$ $-2\varphi$ $-5 - 7\varphi$ $-85 - 107\varphi$ $-68 + 44\varphi$ $-4 + \varphi$ $-64 + 36\varphi$ $-9 + 6\varphi$ | $\begin{array}{c} 8 - 3\varphi \\ 2 + 3\varphi \\ 58 - 36\varphi \\ 3726 - 2304\varphi \\ -194 - 312\varphi \\ -12626 - 19856\varphi \\ -306 - 480\varphi \\ -12 - 18\varphi \\ -572 + 352\varphi \\ \hline 1 + \varphi \\ -6 - 9\varphi \\ -410 - 701\varphi \\ -244 + 148\varphi \\ \hline -2 \\ -192 + 120\varphi \\ 13 - 11\varphi \end{array}$  | $ \begin{array}{c c} & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 0 \\ & $ | 2 4 8 4 4 2 2 2 - 4 4 2 2 4 4 2 2 4 2 2 - 4 4 2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 4   | + -<br>- +<br>+ -<br>+ +<br><br>- +<br>+ +<br>+ +        | 2,8<br>1,4<br>2,8<br>1,10<br>4,10<br>2,11<br>8,11<br>1,4<br>2,8<br>1,4<br>2,8<br>1,10<br>4,10<br>2,4<br>1,8<br>4,8  | $\begin{array}{ c c c } & 2 & & \\ & 1 & & \\ & 2 & \\ & 1 & \\ & 4 & \\ & 2 & \\ & 8 & \\ & & 1 & \\ & 2 & \\ & & 1 & \\ & 2 & \\ & & 1 & \\ & 4 & \\ & & 2 & \\ & & 1 & \\ & 4 & \\ & & 4 & \\ \end{array}$   | $\begin{array}{ c c c }\hline 2,4\\\hline 1,2\\2,4\\1,2\\2,2\\2,1\\\hline 2,1\\\hline 1,2\\2,2\\\hline 1,2\\2,2\\1,2\\4,2\\\end{array}$                       | $\begin{array}{ c c c c }\hline & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ & I_1, III^*\\ & I_4, III^*\\ & I_2, II^*\\ & I_8, II^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ \hline & I_1, III^*\\ \hline & I_2, III\\ & I_1, I_1^*\\ \hline \end{array}$ | 711a         |
| b1 b2 b3 b4 b5 b6 c1 c2 d1 d2 d3 d4 e1 e2 e3 e4 | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0 | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0<br>0 | $-5 + 5\varphi$ $-2 - 3\varphi$ $-26 + 15\varphi$ $-406 + 255\varphi$ $-47 - 63\varphi$ $-727 - 1023\varphi$ $-7 - 63\varphi$ $-5 - 6\varphi$ $-112 + 68\varphi$ $-2\varphi$ $-5 - 7\varphi$ $-85 - 107\varphi$ $-68 + 44\varphi$ $-4 + \varphi$ $-64 + 36\varphi$ $-9 + 6\varphi$ | $\begin{array}{c} 8-3\varphi\\ 2+3\varphi\\ 58-36\varphi\\ 3726-2304\varphi\\ -194-312\varphi\\ -12626-19856\varphi\\ -306-480\varphi\\ -12-18\varphi\\ -572+352\varphi\\ \hline \\ 1+\varphi\\ -6-9\varphi\\ -410-701\varphi\\ -244+148\varphi\\ -22\\ -192+120\varphi\\ 13-11\varphi\\ -522-843\varphi\\ \end{array}$  | $ \begin{array}{c c} & 1 \\ & 0 \\ & $ | 2 4 8 4 4 2 2 2 - 4 4 2 2 4 4 2 2 4 2 2 - 4 4 2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 2 4 4 4 2 4   | + -<br>- +<br>+ -<br>+ +<br><br>- +<br>+ +<br>+ +        | 2,8<br>1,4<br>2,8<br>1,10<br>4,10<br>2,11<br>8,11<br>1,4<br>2,8<br>1,4<br>2,8<br>1,10<br>4,10<br>2,4<br>1,8<br>4,8  | $\begin{array}{ c c c } & 2 & & \\ & 1 & & \\ & 2 & \\ & 1 & \\ & 4 & \\ & 2 & \\ & 8 & \\ & & 1 & \\ & 2 & \\ & & 1 & \\ & 2 & \\ & & 1 & \\ & 4 & \\ & & 2 & \\ & & 1 & \\ & 4 & \\ & & 4 & \\ \end{array}$   | $\begin{array}{ c c c }\hline 2,4\\\hline 1,2\\2,4\\1,2\\2,2\\2,1\\\hline 2,1\\\hline 1,2\\2,2\\\hline 1,2\\2,2\\1,2\\4,2\\\end{array}$                       | $\begin{array}{ c c c c }\hline & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ & I_1, III^*\\ & I_4, III^*\\ & I_2, II^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ \hline & I_1, III^*\\ & I_2, I_1^*\\ \hline & I_1, III^*\\ & I_2, I_1^*\\ \hline & I_1, III^*\\ & I_4, III^*\\ \hline & I_4, III^*\\ \hline & I_4, III^*\\ \hline & I_4, I_1^*\\ & I_4, I_1^*\\ \end{array}$                                   | 711a         |

|               |               |  |                     |                                       |  |  | Imi           | I        | 1/4)                         | 1 (1)  |   | T7 1 .   | т.            |
|---------------|---------------|--|---------------------|---------------------------------------|--|--|---------------|----------|------------------------------|--|---|--|---------------|
|               | $a_1$         | $a_2$  | $a_3$               | $a_4$                                 | $a_6$  | r                                      | T             | s        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                  | $c_p$   | Kodaira  | Isogenies     |
| 711           | a             |  |                     | 71                                    | $1a = (9 - 24\varphi) = 3 \cdot 79a $        | 2 isogeny classes                      | ()            |          |                              |  |   |  | 711a          |
| b1            | 0             | $-\varphi$   | $1 + \varphi$       | arphi                                 | $-1-\varphi$                                 | 1                                      | 1             | +-       | 1, 1                         | 1, 1   | 1,1   | ${ m I}_1, { m I}_1$                                 |               |
| 711           | b             |  |                     | 71:                                   | $1b = (15 - 24\varphi) = 3 \cdot 79b$        | (2 isogeny classes                     | s)            |          |                              |  |   |  | 711b          |
| a1            | 0             | 1  | $\varphi$           | $-4-5\varphi$                         | $4+5\varphi$                                 | 1                                      | 1             | -+       | 3, 1                         | 3, 1   | 3,1   | $I_3, I_1$   |               |
| b1            | 0             | $-1+\varphi$   | $\varphi$           | $1-\varphi$                           | -1   | 1                                      | 1             | -+       | 1,1                          | 1,1  | 1,1   | $I_1, I_1$   |               |
| 716           | a             |  |                     | 71                                    | $6a = (10 - 24\varphi) = 2 \cdot 179a$       | (1 isogeny class                       | s)            |          |                              |  |   |  | 716a          |
| a1            | $\varphi$     | -1   | $1+\varphi$         | $-2-2\varphi$                         | 1  | 1                                      | 1             |          | 1,4                          | 1,4  | 1,4   | $I_1, I_4$   |               |
| 716           | b             |  |                     | 71                                    | $6b = (14 - 24\varphi) = 2 \cdot 179b$       | (1 isogeny class                       | 1)            |          |                              |  |   |  | 716b          |
| a1            | $1 + \varphi$ | $-1-\varphi$   | $\varphi$           | -3                                    | $2-\varphi$                                  | 1                                      | 1             |          | 1,4                          | 1,4  | 1,4   | $I_1, I_4$   |               |
| 719           | a             |  |                     | 71                                    | $9a = (11 - 24\varphi) = 719a $              | 5 isogeny classes                      | )             | 1        |                              |  |   |  | 719a          |
| a1            | $\varphi$     | $\varphi$  | $\varphi$           | 0                                     | 0  | 1                                      | 1             |          | 1                            | 1  | 1   | $I_1$  |               |
| b1            | 0             | $-\varphi$   | $1+\varphi$         | $-1-3\varphi$                         | $2+4\varphi$                                 | 1                                      | 1             | - +      | 1                            | 1  | 1   | $I_1$  |               |
| c1            | 1             | $1+\varphi$  | $1+\varphi$         | $-10+8\varphi$                        | $14 - 8\varphi$                              | 1                                      | 1             | Ī — —    | 1                            | 1  | 1   | $I_1$  |               |
| d1            | 1             | $-1+\varphi$   | $\varphi$           | $-3-5\varphi$                         | $3+4\varphi$                                 | 0                                      | 4             | <u> </u> | 1                            | 1  | 1   | $I_1$  | <u> </u>      |
| d2            | $1 + \varphi$ | $\varphi$  | 1                   | $-69 + 43\varphi$                     | $273 - 169\varphi$                           |  | 4             | ++       | 2                            | 2  | 2   | $I_2$  |               |
| d3<br>d4      | •             | $-1 + \varphi$<br>$-1 + \varphi$                     | $\frac{1}{\varphi}$ | $-7578 + 4682\varphi$ $2 + 25\varphi$ | $302342 - 186859\varphi$<br>$38 + 21\varphi$ | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\frac{2}{2}$ | ++       | $\frac{1}{4}$                | $\begin{vmatrix} 1 \\ 4 \end{vmatrix}$       | $\begin{array}{ c c }\hline 1\\ 2 \end{array}$                    | $egin{array}{c} I_1 \ I_4 \end{array}$               |               |
| e1            |               | $1+\varphi$  | '                   | 1                                     | $-\varphi$                                   | 0                                      | 3             | ! -'<br> | · <del>-</del> ·<br>1        | <del>-</del><br>1                            | $\begin{bmatrix} -\frac{2}{1} \\ 1 \end{bmatrix}$                 | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ |               |
| e2            |               | $1+\varphi$ $1+\varphi$                              |                     | _9                                    | $-22-2\varphi$                               | 0                                      | 1             | -+       | _                            | 3  | 1   | $I_3$  |               |
| 719           | b             |  |                     | 71                                    | $.9b = (13 - 24\varphi) = 719b \tag{3}$      | 5 isogeny classes                      | )             |          |                              |  |   |  | 719b          |
| a1            | $1 + \varphi$ | $1 + \varphi$  | φ                   | $1+2\varphi$                          | $1+\varphi$                                  | 1                                      | 1             |          | 1                            | 1  | 1   | $I_1$  |               |
| b1            | 0             | $-1+\varphi$   | $\varphi$           | $-4+3\varphi$                         | $7-5\varphi$                                 | 1                                      | 1             | + -      | 1                            | 1  | 1   | $I_1$  |               |
| c1            | 1             | $-1-\varphi$   | $1+\varphi$         | $-3-7\varphi$                         | $9+14\varphi$                                | 1                                      | 1             | Ī — —    | 1                            | 1  | 1   | $I_1$  |               |
| d1            | 1             | $-\varphi$   | $1+\varphi$         | $-8+4\varphi$                         | $7-5\varphi$                                 | 0                                      | 4             | + -      | 1                            | 1  | 1   | $I_1$  |               |
| d2            |               | $-1+\varphi$   | 0                   | $-27-42\varphi$                       | $89 + 142\varphi$                            | 0                                      | 4             | ++       | 2                            | 2  | 2   | $I_2$  |               |
| d3            | $1 + \varphi$ | $\varphi$  | 0                   | $-2895 - 4681\varphi$                 | $110801 + 179281\varphi$                     | 0                                      | 2             | ++       | 1                            | 1  | 1   | $I_1$  |               |
| d4            | 1             |  | $1+\varphi$         | $27 - 26\varphi$                      | $59 - 22\varphi$                             | 0                                      | 2             | <u> </u> | 4                            | 4  | 2   | $I_4$  |               |
| e1            |               | $-1-\varphi$   | $\varphi$           | $2\varphi$                            | $-\varphi$                                   |  | 3             | +-       | 1                            | 1  | 1   | $I_1$  |               |
| e2 <b>725</b> |               | $-1-\varphi$   | φ                   | $-10+2\varphi$                        | -13  | (7:                                    | 1             | + -      | 3                            | 3  | 1   | $I_3$  | 725a          |
|               |               | 1  |                     |                                       | · · · · · · · · · · · · · · · · · · ·        | (5 isogeny classe                      | <u> </u>      | 1        | 1 9                          | 1  | 1.0   | T TTT  | 1 <b>2</b> 9a |
| a1            | 0             | -1<br>   | <sup>φ</sup>        | $-4+2\varphi$                         | $5-3\varphi$                                 | 1                                      | 1             |          | $\frac{1}{2}, \frac{3}{6}$   | 1<br>  | $\begin{vmatrix} 1,2\\ \overline{2} & \overline{4} \end{vmatrix}$ | $I_1, III$   |               |
| b1<br>b2      | 0             | $\begin{array}{c} -\varphi \\ 1-\varphi \end{array}$ | arphi               | $-98 - 148\varphi$ $4 - 2\varphi$     | $718 + 1150\varphi$                          | 1 1                                    | 1<br>1        |          | $3, 9 \\ 1, 7$               | $\begin{bmatrix} 3, 3 \\ 1, 1 \end{bmatrix}$ | $\begin{array}{ c c } 3,4 \\ 1,4 \end{array}$                     | $I_3, I_3^* $ $I_1, I_1^*$                           |               |

|                |             |              |               |                             |   |  | Imi | <u> </u>     | 1(A)                         | 1 (1)                       |       | T7 1 1                                | т .       |
|----------------|-------------|--------------|---------------|-----------------------------|---|--|-----|--------------|------------------------------|-----------------------------|-------|---------------------------------------|-----------|
|                | $a_1$       | $a_2$        | $a_3$         | $a_4$                       | $a_6$   |  | T   | s            | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$ | Kodaira                               | Isogenies |
| 725            | a           |              |               | 725a                        | $a = (30 - 5\varphi) = 5a^2 \cdot 29a$ (5 is  | ogeny class                            | es) |              |                              |                             |       |                                       | 725a      |
| c1             | 1           |              | $1 + \varphi$ | $-75-3\varphi$              | $148 - 177\varphi$                            | 0                                      | 4   | ++           | 4, 10                        | 4,4                         | 2,4   | $I_4, I_4^*$                          |           |
| c2             | 1           |              | $1+\varphi$   | $-25-28\varphi$             | $-57-92\varphi$                               | 0                                      | 4   | ++           | 2,8                          | 2, 2                        | 2,4   | $I_2, I_2^*$                          |           |
| c3             | 1           |              | $1+\varphi$   | $-3\varphi$                 | $-2-2\varphi$                                 | 0                                      | 2   |              | 1,7                          | 1,1                         | 1, 2  | $I_1, I_1^*$                          |           |
| c4             | $1+\varphi$ | 1            | 0             | $-5815 + 3545\varphi$       | $197580 - 121985\varphi$                      | 0                                      | 4   | ++           | 2, 14                        | 2,8                         | 2,4   | $I_2, I_8^*$                          |           |
| c5             | 1           |              | $1+\varphi$   | $-3300 - 3453\varphi$       | $101098 + 138148\varphi$                      | 0                                      | 2   | -+           | 1,22                         | 1, 16                       | 1,4   | $I_1, I_{16}^*$                       |           |
| c6             | 1           | -1           | $\varphi$     | $-4354667 + 2691332\varphi$ | $4112862887 - 2541889054\varphi$              | 0                                      | 2   | + -          | 1,10                         | 1,4                         | 1,4   | $I_1, I_4^*$                          |           |
| c7             | ,           | $-1-\varphi$ | $1+\varphi$   | $-2107 - 3387\varphi$       | $-68938 - 111514\varphi$                      | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 2   | ++           | 1,7                          | 1,1                         | 1,2   | $I_1, I_1^*$                          |           |
| c8             | 1           |              | $1+\varphi$   | $50 + 122\varphi$           | $73 - 777\varphi$                             | 0                                      | 2   | ļ — —        | 8,8                          | 8,2                         | 2,4   | $I_8, I_2^*$                          | <u> </u>  |
| d1             | 0           | 0            | $\varphi$     | $-35395 - 57415\varphi$     | $4889491 + 7909436\varphi$                    | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 1   |              | 7,7                          | 7, 1                        | 1,2   | $I_7, I_1^*$                          |           |
| $\frac{d2}{-}$ | 0           | 0            | φ _           | $-80 + 55\varphi$           | $-329 + 226\varphi$                           | 0                                      | 1   | ļ            | 1,13                         | 1, 7                        | 1,2   |                                       | <u> </u>  |
| e1             | 0           | $-1+\varphi$ | $\varphi$     | $-11-2\varphi$              | $-13-8\varphi$                                | 0                                      | 1   |              | 1,9                          | 1                           | 1, 2  | $I_1, III^*$                          |           |
| 725            | b           |              |               | 7250                        | $b = (25 + 5\varphi) = 5a^2 \cdot 29b$ (5 iso | ogeny class                            | es) |              |                              |                             |       |                                       | 725b      |
| a1             | 0           | -1           | $1+\varphi$   | $-2-2\varphi$               | $2+2\varphi$                                  | 1                                      | 1   |              | 1, 3                         | 1                           | 1, 2  | $I_1, III$                            |           |
| b1             | 0           | $-1+\varphi$ | $1+\varphi$   | $-246 + 148\varphi$         | $1868 - 1151\varphi$                          | 1                                      | 1   | i            | 3,9                          | 3,3                         | 3, 4  | $ar{\mathrm{I}}_3,ar{\mathrm{I}}_3^*$ | <u>-</u>  |
| b2             | 0           |              | $1+\varphi$   | $2+2\varphi$                | -arphi  | 1                                      | 1   |              | 1, 7                         | 1,1                         | 1, 4  | $I_1, I_1^*$                          |           |
| c1             | 1           | -1           | $\varphi$     | $-77 + 2\varphi$            | $-28+176\varphi$                              | 0                                      | 4   | i            | 4, 10                        | 4, 4                        | 2,4   | $I_4, I_4^*$                          | <u> </u>  |
| c2             | 1           | -1           | φ             | $-52+27\varphi$             | $-148 + 91\varphi$                            | 0                                      | 4   | ++           | 2,8                          | 2,2                         | 2,4   | $I_2, I_2^*$                          |           |
| c3             | 1           | -1           | $\varphi$     | $-2+2\varphi$               | $-3+\varphi$                                  | 0                                      | 2   |              | 1,7                          | 1, 1                        | 1, 2  | $I_1, I_1^*$                          |           |
| c4             | $\varphi$   | $-1-\varphi$ | $\varphi$     | $-2272 - 3544\varphi$       | $77866 + 125529\varphi$                       | 0                                      | 4   | ++           | 2, 14                        | 2,8                         | 2,4   | $I_2, I_8^*$                          |           |
| c5             | 1           | -1           | $\varphi$     | $-6752 + 3452\varphi$       | $239247 - 138149\varphi$                      | 0                                      | 2   | + -          | 1,22                         | 1, 16                       | 1,4   | $I_1,I_{16}^*$                        |           |
| c6             | 1           | -1           | $1 + \varphi$ | $-1663335 - 2691333\varphi$ | $1570973833 + 2541889053\varphi$              | 0                                      | 2   | -+           | 1, 10                        | 1, 4                        | 1,4   | $I_1, I_4^*$                          |           |
| c7             | $1+\varphi$ | 1            | 1             | $-5493 + 3387\varphi$       | $-185944 + 114900\varphi$                     | 0                                      | 2   | ++           | 1,7                          | 1,1                         | 1, 2  | $\mathrm{I}_1,\mathrm{I}_1^*$         |           |
| c8             | 1           | -1           | $\varphi$     | $173 - 123\varphi$          | $-703 + 776\varphi$                           | 0                                      | 2   | <u> </u>     | 8,8                          | 8,2                         | 2,4   | $I_8, I_2^*$                          | L         |
| d1             | 0           | 0            | $1 + \varphi$ | $-25-55\varphi$             | $-103 - 227\varphi$                           | 0                                      | 1   |              | 1, 13                        | 1,7                         | 1,2   | $I_1, I_7^*$                          |           |
| d2             | 0           | 0            | $1+\varphi$   | $-92810 + 57415\varphi$     | $12798927 - 7909437\varphi$                   | 0                                      | 1   |              | 7,7                          | 7,1                         | 1, 2  | $    I_7, I_1^* $                     |           |
| e1             | 0           | $-\varphi$   | $1 + \varphi$ | $-13 + 2\varphi$            | $-21+7\varphi$                                | 0                                      | 1   |              | 1,9                          | 1                           | 1,2   | $I_1, III^*$                          |           |
| 729            | a           |              |               |                             | $729a = (27) = 3^3$ (4 isogeny                | classes)                               |     |              |                              |                             |       |                                       | 729a      |
| a1             | 0           | 0            | 1             | -30                         | 63  | 1                                      | 3   |              | 5                            | 0                           | 3     | IV                                    |           |
| a2             | 0           | 0            | 1             | 0                           | 0   | 1                                      | 3   |              | 3                            | 0                           | 1     | II                                    |           |
| a3             | 0           | 0            | 1             | 0                           | -7  | 1                                      | 3   |              | 9                            | 0                           | 3     | $IV^*$                                |           |
| a4             | 0           | 0            | 1             | -270                        | -1708   | 1                                      | 1   | <u> </u> – – | 11                           | 0                           | 1     | II*                                   |           |
| b1             | 1           | -1           | $\varphi$     | $-3+\varphi$                | $3-2\varphi$                                  | 1                                      | 3   | -+           | 3                            | 0                           | 1     | II                                    |           |
| b2             | $\varphi$   | $-1-\varphi$ | $\varphi$     | $-17 - 32\varphi$           | $-48 - 78\varphi$                             | 1                                      | 1   | -+           | 9                            | 0                           | 3     | IV*                                   |           |
| c1             | $\varphi$   |              | $1 + \varphi$ | -2-4arphi                   | $3+5\varphi$                                  | 0                                      | 3   | -+           | 3                            | 0                           | 1     | II                                    |           |
| c2             | 1           |              | $1+\varphi$   | $-29 + 13\varphi$           | $-60 + 33\varphi$                             | 0                                      | 1   | -+           | 9                            | 0                           | 1     | IV*                                   |           |
| d1             | 0           |              | $1 + \varphi$ | 0                           | -arphi  | 0                                      | 3   |              | 3                            | 0                           | 1     | II                                    |           |
| d2             | 0           | 0            | $1 + \varphi$ | 0                           | $-14+6\varphi$                                | 0                                      | 1   |              | 9                            | 0                           | 1     | $IV^*$                                |           |

|  |               |              |               |                                       |  |  | Imi                                       | 1     | 1/ A )                       | 1 (:)                       |   | TZ 1 :   | т.           |
|--|---------------|--------------|---------------|---------------------------------------|--|--|---|-------|------------------------------|-----------------------------|---|--|--------------|
|  | $a_1$         | $a_2$        | $a_3$         | $a_4$                                 | $a_6$                                    | $\mid r \mid$                          | T   | s     | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$                                     | Kodaira  | Isogenies    |
| 739                                      | a             |              |               |                                       | $739a = (19 - 25\varphi) = 739a$         | (1 isogeny class)                      |   |       |                              |                             |   |  | 739a         |
| a1                                       | $1 + \varphi$ | 1            | 1             | $-3-4\varphi$                         | $1+2\varphi$                             | 1                                      | 1   | + -   | 1                            | 1                           | 1   | $I_1$  |              |
| 739                                      | b             |              |               |                                       | $739b = (6 - 25\varphi) = 739b$          | (1 isogeny class)                      |   |       |                              |                             |   |  | 739b         |
| a1                                       | $\varphi$     | $-1-\varphi$ | $1 + \varphi$ | $-8+4\varphi$                         | $10-7\varphi$                            | 1                                      | 1   | -+    | . 1                          | 1                           | 1   | $I_1$  |              |
| <b>751</b> 3                             | a             |              |               |                                       | $751a = (18 - 25\varphi) = 751a$         | (1 isogeny class)                      |   |       |                              |                             |   |  | <b>7</b> 51a |
| a1                                       | $\varphi$     | 0            | 0             | $-10+4\varphi$                        | $11-9\varphi$                            | 0                                      | 2   | +-    | 1                            | 1                           | 1   | $I_1$  |              |
| a2                                       | $1+\varphi$   | 0            | 0             | $-113 - 180\varphi$                   | $-931 - 1508\varphi$                     | 0                                      | 2   | -+    | 2                            | 2                           | 2   | $I_2$  |              |
| <b>751</b> ]                             | b             |              |               |                                       | $751b = (7 - 25\varphi) = 751b$          | (1 isogeny class)                      |   |       |                              |                             |   |  | <b>7</b> 51b |
| a1                                       | $1+\varphi$   | $-\varphi$   | 0             | $-6-4\varphi$                         | $2+9\varphi$                             | 0                                      | 2   | -+    | . 1                          | 1                           | 1   | $I_1$  |              |
| a2                                       | $\varphi$     | $1-\varphi$  | 0             | $-293 + 180\varphi$                   | $-2439 + 1508\varphi$                    | 0                                      | 2   | +-    | 2                            | 2                           | 2   | $I_2$  |              |
| 755                                      | a             |              |               | 7                                     | $55a = (28 - \varphi) = 5a \cdot 151a$   | (3 isogeny classe                      | s)  |       |                              |                             |   |  | 755a         |
| a1                                       | $\varphi$     | $-\varphi$   | $\varphi$     | $-5-7\varphi$                         | $8+12\varphi$                            | 1                                      | 1   |       | 1,5                          | 1,5                         | 1,5                                       | $I_1, I_5$                                       |              |
| b1                                       | 1             | 0            | $\varphi$     | 0                                     | 0  | 1                                      | 3   | Ī — — | 1,1                          | 1,1                         | [1, 1]                                    | $I_1, I_1$                                       |              |
| b2                                       | 1             | 0            | $\varphi$     | $-5\varphi$                           | $-7-8\varphi$                            | 1                                      | 1   |       | 3, 3                         | 3,3                         | 3,1                                       | $I_3, I_3$                                       |              |
| c1                                       | 1             | $1-\varphi$  | $\varphi$     | $-3\varphi$                           | $1+\varphi$                              | 0                                      | 4   | -+    | ,                            | 1, 2                        | 1,2                                       | $\mathrm{I}_1,\mathrm{I}_2$                      |              |
| c2                                       | 1             | $1-\varphi$  | $\varphi$     | $-5-8\varphi$                         | $-7-10\varphi$                           | 0                                      | 4   | ++    |                              | 2,4                         | 2, 2                                      | $I_2, I_4$                                       |              |
| c3                                       | 1             | $1-\varphi$  | $\varphi$     | $-85 - 113\varphi$                    | $-444 - 744\varphi$                      | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 2   | ++    |                              | 1,8                         | 1,2                                       | $I_1, I_8$                                       |              |
| c4                                       | $1+\varphi$   | 0            | 0             | $-83 + 54\varphi$                     | $-351 + 214\varphi$                      | 0                                      | 2   | + -   | 4,2                          | 4, 2                        | 2,2                                       | $I_4, I_2$                                       |              |
| 755                                      |               |              |               | 7                                     | $55b = (27 + \varphi) = 5a \cdot 151b$   | (3 isogeny classe                      | s)  |       |                              | I                           | 1   |  | 755b         |
| a1                                       | $1+\varphi$   | -1           | $1+\varphi$   | $-12 + 5\varphi$                      | $20-13\varphi$                           | 1                                      | 1   |       | 1,5                          | 1,5                         | 1,5                                       | $I_1, I_5$                                       |              |
| b1                                       | 1             |              | $1+\varphi$   | $-\varphi$                            | -arphi                                   | 1                                      | 3   | Ī — — | 1,1                          | 1,1                         | 1,1                                       | $I_1, I_1$                                       |              |
| b2                                       | 1             | 0            | $1+\varphi$   | $-5+4\varphi$                         | $-15 + 7\varphi$                         | 1                                      | 1   |       | 3,3                          | 3,3                         | 3,1                                       | $I_3, I_3$                                       |              |
| c1                                       | 1             |              | $1 + \varphi$ | $-3+2\varphi$                         | $2-2\varphi$                             | 0                                      | 4   | + -   | ,                            | 1, 2                        | 1,2                                       | $I_1, I_2$                                       |              |
| c2                                       | 1             |              | $1+\varphi$   | $-13+7\varphi$                        | $-17 + 9\varphi$                         | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 4   | ++    |                              | 2,4                         | 2,2                                       | $I_2, I_4$                                       |              |
| $\begin{bmatrix} c3 \\ c4 \end{bmatrix}$ | 1             |              | $1+\varphi$ 0 | $-198 + 112\varphi$ $-29 - 54\varphi$ | $-1188 + 743\varphi$ $-137 - 214\varphi$ | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\begin{array}{ c c } 2 \\ 2 \end{array}$ | ++    |                              | 1, 8 $4, 2$                 | $\begin{array}{c c} 1,2\\2,2 \end{array}$ | $egin{array}{c} I_1, I_8 \ I_4, I_2 \end{array}$ |              |
| C4                                       | φ             | $1-\varphi$  | 0             | $-29-34\varphi$                       | $-13t - 214\varphi$                      | 0                                      |   | +     | 4, 2                         | 4, 2                        | 2, 2                                      | 14,12  |              |
| 764                                      |               |              |               |                                       | $764a = (26 + 4\varphi) = 2 \cdot 191a$  | (1 isogeny class                       |   |       |                              |                             | Т   |  | 764a         |
| a1                                       | 1             | $\varphi$    | $\varphi$     | $-4+2\varphi$                         | $4-3\varphi$                             | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | $\frac{2}{2}$                             | - +   |                              | 1,4                         | $\begin{bmatrix} 1,4\\2&2 \end{bmatrix}$  | $I_1, I_4$                                       |              |
| a2                                       | 1             | φ            | φ             | $-64 + 42\varphi$                     | $248 - 151\varphi$                       | 1                                      | 2   | +-    | 2,2                          | 2, 2                        | 2,2                                       | $I_2, I_2$                                       |              |
| <b>764</b>                               |               |              |               |                                       | $764b = (30 - 4\varphi) = 2 \cdot 191b$  | (1 isogeny class                       |   | _     |                              |                             | ı   |  | <b>764</b> b |
| a1                                       | 1             |              | $1+\varphi$   | $-2-3\varphi$                         | $1+2\varphi$                             | 1                                      | 2   | + -   |                              | 1,4                         | 1,4                                       | $I_1, I_4$                                       |              |
| a2                                       | 1             | $1-\varphi$  | $1+\varphi$   | $-22-43\varphi$                       | $97 + 150\varphi$                        | 1                                      | 2   | -+    | 2,2                          | 2,2                         | 2,2                                       | $I_2, I_2$                                       |              |

|            |               |               |                    |                               |  |                   | Imi           |           | 1/ 4 )                       | 1 (:)                       |   | T/ 1 ·  | T .       |
|------------|---------------|---------------|--------------------|-------------------------------|--|-------------------|---------------|-----------|------------------------------|-----------------------------|---|---|-----------|
|            | $a_1$         | $a_2$         | $a_3$              | $a_4$                         | $a_6$                                  | r                 | T             | s         | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$                                   | Kodaira   | Isogenies |
| <b>769</b> | a             |               |                    | 76                            | $9a = (16 - 25\varphi) = 769a \tag{2}$ | isogeny classes   | )             |           |                              |                             |   |   | 769a      |
| a1<br>a2   | $\varphi$ 1   | $1-\varphi$   | $0 \\ 1 + \varphi$ | $-\varphi \\ -17 + 11\varphi$ | $0\\-30+18\varphi$                     | -                 | $\frac{2}{2}$ | ++        | $\frac{1}{2}$                | $\frac{1}{2}$               | $\begin{array}{c c} 1 \\ 2 \end{array}$ | $egin{array}{c} I_1 \ I_2 \end{array}$                    |           |
| b1         | :             | -1            | 0                  | $-4+2\varphi$                 | $5 - 3\varphi$                         | :                 | 1             | <u> </u>  | 1                            | 1 1                         | 1 1                                     | $\begin{array}{cccccccccccccccccccccccccccccccccccc$      |           |
| <b>769</b> | b             |               |                    | 76                            | $69b = (9 - 25\varphi) = 769b \tag{2}$ | isogeny classes)  |               |           |                              |                             |   | ı   | 769b      |
| a1         | $1+\varphi$   | 0             | 0                  | $-1+\varphi$                  | 0                                      |                   | 2             | ++        | 1                            | 1                           | 1                                       | $I_1$   |           |
| a2         | 1             | $1-\varphi$   | $\varphi$          | $-5-12\varphi$                | $-11-19\varphi$                        | 1                 | 2             | -+        | 2                            | 2                           | 2                                       | $I_2$   |           |
| b1         | φ             | $-\varphi$    | 0                  | $-2-2\varphi$                 | $2+3\varphi$                           | 1                 | 1             |           | 1                            | 1                           | 1                                       | $I_1$   |           |
| 775        | a             |               |                    | 775 <i>a</i>                  | $= (15 - 25\varphi) = 5a^2 \cdot 31a$  | (6 isogeny class  | es)           |           |                              |                             |   |   | 775a      |
| a1         | 0             | $1-\varphi$   | $1+\varphi$        | $-19-11\varphi$               | $34 + 36\varphi$                       | 1                 | 1             | -+        | 3, 2                         | 3                           | 3, 1                                    | $I_3, II$   |           |
| a2         | 0             | $1-\varphi$   | $1+\varphi$        | $1-\varphi$                   | $-\varphi$                             | 1                 | 1             | - +       | 1, 2                         | 1                           | 1,1                                     | $I_1, II$   |           |
| b1         | 0             | 1             | $1 + \varphi$      | -3                            | -2arphi                                | 1                 | 1             | -+        | 1, 4                         | 1                           | 1,3                                     | $I_1, IV$   |           |
| c1         | $1+\varphi$   | $1+\varphi$   | 0                  | $-65-104\varphi$              | $-488 - 792\varphi$                    | 0                 | 2             | - +       | 1,10                         | 1,4                         | 1,2                                     | $I_1, I_4^*$  |           |
| c2         | $\varphi$     | 1             | 0                  | $-138 + 62\varphi$            | $582 - 411\varphi$                     | 0                 | 4             | ++        | 2, 14                        | 2, 8                        | 2,4                                     | $I_2, I_8^*$  |           |
| c3         | 1             | $-1-\varphi$  | 1                  | $-14003 + 8665\varphi$        | $750731 - 464000\varphi$               | 0                 | 2             | + -       | 4, 10                        | 4,4                         | 2,2                                     | $\mathrm{I}_4,\mathrm{I}_4^*$                             |           |
| c4         | $  \varphi  $ | 1             | 0                  | $-263 + 12\varphi$            | $167 + 694\varphi$                     | 0                 | 2             | ++        | 1,22                         | 1,16                        | 1,4                                     | $I_1, I_{16}^*$   |           |
| d1         | 0             | $\varphi$     | $1+\varphi$        | $-33 + 17\varphi$             | $-75 + 44\varphi$                      | 0                 | 1             | -+        | 1,10                         | 1                           | 1,1                                     | $I_1, II^*$   |           |
| e1         | 0             | $-1-\varphi$  | $1+\varphi$        | $-1-4\varphi$                 | $2+4\varphi$                           | 0                 | 3             |           | 1,8                          | 1                           | 1,3                                     | $\overline{I_1,IV^*}$                                     |           |
| e2         |               | $-1-\varphi$  |                    | $-151-204\varphi$             | $-1048 - 1621\varphi$                  | 0                 | 1             | -+        | 3, 8                         | 3                           | 3, 1                                    | $I_3, IV^*$   |           |
| f1         | $\varphi$     | $1+\varphi$   | $\varphi$          | $-3+2\varphi$                 | $-4+\varphi$                           | 0                 | 2             | <br>  - + | 1, 6                         | 1                           | 1,2                                     | $\overline{\mathrm{I}}_{1},\overline{\mathrm{I}}_{0}^{*}$ |           |
| f2         | $1+\varphi$   | -1            | 0                  | $-354-556\varphi$             | $4635 + 7512\varphi$                   | 0                 | 4             | ++        | 4, 6                         | 4                           | 2,4                                     | $I_4, I_0^*$  |           |
| f3         | $\varphi$     | $1+\varphi$   | $\varphi$          | $-53 + 27\varphi$             | $-169 + 106\varphi$                    | 0                 | 4             | ++        | 2, 6                         | 2                           | 2, 4                                    | $I_2, I_0^*$  |           |
| f4         | 1             | $\varphi$     | 0                  | $-38001 - 61492\varphi$       | $5400356 + 8737966\varphi$             | 0                 | 2             | -+        | 2, 6                         | 2                           | 2,2                                     | $I_2, I_0^*$  |           |
| f5         | $\varphi$     | $1+\varphi$   | $\varphi$          | $-303 + 77\varphi$            | $1871 - 674\varphi$                    | 0                 | 2             | + -       | 8,6                          | 8                           | 2,2                                     | $I_8, I_0^*$  |           |
| f6         | 1             | 1             | $\varphi$          | $-5520 + 3412\varphi$         | $-187909 + 116136\varphi$              | 0                 | 2             | +-        | 1,6                          | 1                           | 1,2                                     | $I_1, I_0^*$  |           |
| 775        | b             |               |                    | 775 <i>b</i>                  | $= (10 - 25\varphi) = 5a^2 \cdot 31b$  | (6 isogeny classe | es)           |           |                              |                             |   |   | 775b      |
| a1         | 0             | $\varphi$     | $\varphi$          | $-30 + 11\varphi$             | $71 - 37\varphi$                       | 1                 | 1             | + -       | 3, 2                         | 3                           | 3, 1                                    | $I_3, II$   |           |
| a2         | 0             | $\varphi$     | $\varphi$          | arphi                         | 0                                      | 1                 | 1             | + -       | 1, 2                         | 1                           | 1,1                                     | $I_1, II$   |           |
| b1         | 0             | 1             | $\varphi$          | -3                            | $-1+\varphi$                           | 1                 | 1             | + -       | 1,4                          |                             | 1,3                                     | $oxed{I_1, IV}$   |           |
| c1         | $\varphi$     | $\varphi$     | 1                  | $-173 + 107\varphi$           | $-1002 + 620\varphi$                   | 0                 | 2             | + -       | 1, 10                        | 1,4                         | 1,2                                     | $\mathrm{I}_1,\mathrm{I}_4^*$                             |           |
| c2         | $1+\varphi$   | $1-\varphi$   | 0                  | $-76-62\varphi$               | $171 + 411\varphi$                     | 0                 | 4             | ++        | 2, 14                        | 2, 8                        | 2,4                                     | $I_2, I_8^*$  |           |
| c3         | 1             | $1+\varphi$   | 0                  | $-5338 - 8663\varphi$         | $281393 + 455336\varphi$               | 0                 | 2             | -+        | 4, 10                        | 4, 4                        | 2,2                                     | $\mathrm{I}_4,\mathrm{I}_4^*$                             |           |
| c4         | $1+\varphi$   | $1-\varphi$   | 0                  | $-251 - 12\varphi$            | $861 - 694\varphi$                     | 0                 | 2             | ++        | 1,22                         | 1,16                        | 1,4                                     | $I_1, I_{16}^*$   |           |
| d1         | 0             | $1-\varphi$   | $\varphi$          | $-16-17\varphi$               | $-30-45\varphi$                        | 0                 | 1             | + -       | 1, 10                        | 1                           | 1,1                                     | $I_1, II^*$   |           |
| e1         | 0             | $1+\varphi$   | $\varphi$          | $-6+6\varphi$                 | 1                                      | 0                 | 3             | + -       | 1,8                          | 1                           | 1,3                                     | $I_1, IV^*$   |           |
| e2         | 0             | $1 + \varphi$ | $\varphi$          | $-356 + 206\varphi$           | $-3024 + 1825\varphi$                  | 0                 | 1             | +-        | 3, 8                         | 3                           | 3, 1                                    | $I_3, IV^*$   |           |

| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   | $j) \mid c_p$                                | Kodaira                     | Isogenies |
|--|--|-----------------------------|-----------|
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$  |  |                             |           |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$  |  |                             |           |
|  |  |                             | 775b      |
|  | 2,4  |                             |           |
|  | 1, 2   | . 0                         |           |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | 2,4  |                             |           |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 2, 2   |                             |           |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | $\begin{array}{c c} 2,2\\ 1,2 \end{array}$   |                             |           |
| f6     1     1 $1 + \varphi$ $-2108 - 3413\varphi$ $-71773 - 116137\varphi$ $0$ $2$ $-+$ $1$   | 1, 2   | $I_1, I_0^*$                |           |
| <b>779a</b> = $(14 - 25\varphi) = 19b \cdot 41b$ (1 isogeny class)   |  |                             | 779a      |
|  | 0.1  | тт                          | 119a      |
| $ \begin{vmatrix} a1 & \varphi & 1 & 1 & -1 + 2\varphi & -2 + 2\varphi & 0 & 2 & -2, 1 & 2, 1 \\ a2 & 1 & -1 - \varphi & 0 & -234 + 145\varphi & -1611 + 996\varphi & 0 & 2 & + + & 1, 2 & 1, 2 \\ \end{vmatrix} $ | $\begin{bmatrix} 2, 1 \\ 1, 2 \end{bmatrix}$ |                             |           |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   | 1, 2   | $I_1, I_2$                  |           |
| <b>779b</b> $779b = (11 - 25\varphi) = 19a \cdot 41a$ (1 isogeny class)  |  |                             | 779b      |
| a1 $1 + \varphi$ $1 - \varphi$   | 2, 1   | $I_2, I_1$                  |           |
| a2   1 $1+\varphi$ 1 $-90-143\varphi$ $-705-1140\varphi$   0   2   ++ 1,2   1,2  | 1, 2   | $I_1, I_2$                  |           |
| <b>779c</b> $779c = (27 + 2\varphi) = 19b \cdot 41a$ (2 isogeny classes)   |  |                             | 779c      |
| a1 $1 	ext{ } 1 - \varphi 	ext{ } 1$   | 1,1  | $I_1, I_1$                  |           |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | $\frac{1}{2}, \frac{1}{3}$                   | $I_2, I_3$                  | ;         |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   | 6, 1   |                             |           |
|  | l  |                             |           |
| <b>779d</b> $779d = (29 - 2\varphi) = 19a \cdot 41b$ (2 isogeny classes)   |  |                             | 779d      |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   | 1, 1   | $\mathrm{I}_1,\mathrm{I}_1$ |           |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   | 2, 3   | $I_2, I_3$                  |           |
| b2   0 $1 + \varphi$ $\varphi$ $-707 + 436\varphi$ $-8596 + 5297\varphi$   0   1   $-$ 6, 1   6, 1   | 6, 1   | $I_6, I_1$                  |           |
|  |  |                             |           |
| $781a = (13 - 25\varphi) = 11b \cdot 71b \qquad (1 \text{ isogeny class})$   |  |                             | 781a      |
| a1 $\begin{vmatrix} 1+\varphi & -1-\varphi & 1+\varphi \\ \end{vmatrix}$ $-95-144\varphi$ $654+1048\varphi$ $\begin{vmatrix} 1 & 4 & -+ & 2,1 \\ \end{vmatrix}$ $2,1$  | 2, 1   |                             |           |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   |  | -/ -                        |           |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | 2,1  | $I_2, I_1$                  |           |
| a4 $\varphi$ -1 1+ $\varphi$ -483+281 $\varphi$ 4870 - 2862 $\varphi$ 1 2 + - 8,4 8,4  | 2,2  | $I_8, I_4$                  |           |
| <b>781b</b> $781b = (12 - 25\varphi) = 11a \cdot 71a$ (1 isogeny class)  |  |                             | 781b      |
| a1 $\varphi$ -1 $\varphi$ -237 + 142 $\varphi$ 1703 - 1049 $\varphi$ 1 4 + - 2,1 2,1   | 2, 1   | $I_2, I_1$                  |           |
| $\begin{vmatrix} 2 & 1+\varphi & -1-\varphi & \varphi & -216-273\varphi & 1936+2921\varphi & 1 & 4 & ++& 4,2 & 4,2 \end{vmatrix}$  | 2, 2   |                             |           |
| a3   $\varphi$ 1 1 + $\varphi$ -136262 - 220475 $\varphi$ 36749759 + 59462360 $\varphi$   1   2   + + 2,1   2,1  | 2, 1   | $\mathrm{I}_2,\mathrm{I}_1$ |           |
| a4 $  1 + \varphi - 1 - \varphi - \varphi  $ $-201 - 283\varphi$ $2009 + 2861\varphi$ $  1   2   -+ 8,4   8,4$   | 2,2  | $I_8, I_4$                  |           |

|          | $a_1$                                   | $a_2$                         | $a_3$             | $a_4$                         | $a_6$                                  | r                                      | T  | s   | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                   | $c_p$  | Kodaira  | Isogenies    |
|----------|---|-------------------------------|-------------------|-------------------------------|--|--|--|-----|------------------------------|---|--|--|--------------|
|          |   |                               |                   |                               |  |  |  |     |                              |   |  |  |              |
| 781      |   |                               |                   | 781                           | $c = (26 + 5\varphi) = 11b \cdot 71a$  | (2 isogeny classe                      | es)  |     |                              |   |  | <u> </u>   | 781c         |
| a1       |   | $-1-\varphi$                  |                   | -2                            | $-2\varphi$                            | 0                                      | 4  | + - |                              | 2,1   | 2, 1   | $I_2, I_1$   |              |
| a2       |   | $-1-\varphi$                  | ,                 | $-7-5\varphi$                 | $-4-5\varphi$                          | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\begin{vmatrix} 4 \\ 2 \end{vmatrix}$           | ++  |                              | 4, 2  | $\begin{bmatrix} 2,2\\ 2,1 \end{bmatrix}$    | $I_4, I_2$   |              |
| a3   a4  |   | $-1 - \varphi$ $-1 - \varphi$ | •                 | $-52 \\ -42 - 90\varphi$      | $-64 + 120\varphi$ $-260 - 402\varphi$ | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\begin{array}{ c c }\hline 2\\ 2\\ \end{array}$ | ++  | ,                            | 8, 1 $2, 4$                                   | $\begin{bmatrix} 2, 1 \\ 2, 2 \end{bmatrix}$ | $\begin{matrix} I_8, I_1 \\ I_2, I_4 \end{matrix}$           |              |
| b1       | 1 · · · · · · · · · · · · · · · · · · · | · <del>-</del> 0              | · <del>'-</del> - | $-4 + 2\varphi$               | $-4+2\varphi$                          | 1                                      | 2  |     | <del>-</del>                 | $\frac{1}{2}$ , $\frac{2}{2}$ , $\frac{4}{1}$ | $\begin{bmatrix} 2, 2 \\ 2, 1 \end{bmatrix}$ | $\begin{array}{c c} I_2, I_4 \\ \hline I_2, I_1 \end{array}$ | <u> </u>     |
| b2       | 1                                       | 0                             | $arphi \ arphi$   | $-4+2\varphi$ $-64+37\varphi$ | $-4 + 2\varphi$ $-233 + 142\varphi$    | 1                                      | $\frac{2}{2}$                                    | ++  | ,                            | 1, 2  | 1, 2   | $I_1, I_2$   |              |
|          |   |                               | ,                 |                               | ' '                                    |  |  |     | ,                            | ,   | ,  | 1) 2   |              |
| 781      | d                                       |                               |                   | 781                           | $d = (31 - 5\varphi) = 11a \cdot 71b$  | (2 isogeny classe                      | es)  |     |                              |   |  |  | 781d         |
| a1       | $\varphi$                               | -1                            | $\varphi$         | $-2\varphi$                   | $-1+\varphi$                           | 0                                      | 4  | - + |                              | 2,1   | 2,1  | $I_2, I_1$   |              |
| a2       | $\varphi$                               | -1                            | $\varphi$         | $-10 + 3\varphi$              | $-8+4\varphi$                          | 0                                      | 4  | ++  |                              | 4, 2  | 2,2  | $I_4, I_2$   |              |
| a3       | $\varphi$                               | -1                            | $\varphi$         | $-50-2\varphi$                | $57 - 121\varphi$                      | 0                                      | 2  | ++  |                              | 8, 1  | 2, 1   | $I_8, I_1$   |              |
| a4       | $  \frac{\varphi}{\varphi}$             | -1                            |                   | $-130 + 88\varphi$            | $-661 + 401\varphi$                    | 0                                      | 2  | + - |                              | $\frac{2}{1}$                                 | $\frac{1}{2}, \frac{2}{2}$                   | $I_2, I_4$   |              |
| b1       | 1                                       |                               | $1+\varphi$       | $-2-3\varphi$                 | $-2-3\varphi$                          | 1                                      | $\frac{2}{2}$                                    | -+  | ,                            | 2,1   | 2,1  | $I_2, I_1$   |              |
| b2       | 1                                       | 0                             | $1+\varphi$       | $-27 - 38\varphi$             | $-91 - 143\varphi$                     | 1                                      | 2  | ++  | 1,2                          | 1, 2  | 1,2  | $I_1, I_2$   |              |
| 784      | a                                       |                               |                   |                               | $784a = (28) = 2^2 \cdot 7 \tag{2 i}$  | sogeny classes)                        |  |     |                              |   |  |  | 784a         |
| a1       | 0                                       | 0                             | 0                 | $-8-8\varphi$                 | $-12-16\varphi$                        | 0                                      | 1  |     | 8,1                          | 1   | 1,1  | $IV^*, I_1$  |              |
| b1       | 0                                       | 0                             | 0                 | $-16-16\varphi$               | $33 + 44\varphi$                       | 0                                      | 4  | ++  | 4, 2                         | 2   | 1,2  | $\overline{\text{IV}}, \overline{\text{I}}_2$                |              |
| b2       | 0                                       | 0                             | 0                 | $-1270 - 2053\varphi$         | $33102 + 53560\varphi$                 | 0                                      | 2  | + + | 8,1                          | 1   | 1,1  | $IV^*, I_1$  |              |
| b3       | 0                                       | 0                             | 0                 | $-487 + 296\varphi$           | $4830 - 2984\varphi$                   | 0                                      | 2  | ++  |                              | 1   | 1,1  | $IV^*, I_1$  |              |
| b4       | 0                                       | 0                             | 0                 | $-11-11\varphi$               | $54 + 72\varphi$                       | 0                                      | 2  |     | 8,4                          | 4   | 1, 2   | $IV^*, I_4$  |              |
| 796      | a                                       |                               |                   | 79                            | $6a = (26 + 6\varphi) = 2 \cdot 199a$  | (1 isogeny class                       | )  |     |                              |   |  |  | <b>7</b> 96a |
| a1       | 1                                       | 1                             | 0                 | $-1-5\varphi$                 | $-6-9\varphi$                          | 0                                      | 3  |     | -, -                         | 1,9   | 1,9  | $I_1, I_9$   |              |
| a2       | 1                                       | 1                             | 0                 | $-281 - 365\varphi$           | $-2974 - 4513\varphi$                  | 0                                      | 1  |     | 3,3                          | 3,3   | 3, 3   | $I_3, I_3$   |              |
| 796      | b                                       |                               |                   | 79                            | $96b = (32 - 6\varphi) = 2 \cdot 199b$ | (1 isogeny class                       | )  |     |                              |   |  |  | 796b         |
| a1       | 1                                       | 1                             | 0                 | $-6+5\varphi$                 | $-15 + 9\varphi$                       | 0                                      | 3  |     | 1,9                          | 1,9   | 1,9  | $I_1, I_9$   |              |
| a2       | 1                                       | 1                             | 0                 | $-646 + 365\varphi$           | $-7487 + 4513\varphi$                  | 0                                      | 1  |     | 3,3                          | 3,3   | 3, 3   | $I_3, I_3$   |              |
| 801      | a                                       |                               |                   | 8                             | $01a = (30 - 3\varphi) = 3 \cdot 89a$  | (1 isogeny class)                      | )  |     |                              |   |  |  | 801a         |
| a1       | 1                                       | $-\varphi$                    | 0                 | $-1-\varphi$                  | 0                                      | 1                                      | 2  | + + | 2,1                          | 2, 1  | 2, 1   | $I_2, I_1$   |              |
| a2       | 1                                       | $-\varphi$                    | 0                 | $4+4\varphi$                  | $-3-7\varphi$                          |  | 2  |     |                              | 4, 2  | 2,2  | $I_4, I_2$   |              |
| 801      | h                                       |                               |                   | 0                             | 011 (27   2.4) 2 201                   | (1 iaa ma1- )                          |  |     |                              |   |  | 1  | 801b         |
|          |   | 1 .                           |                   |                               | $01b = (27 + 3\varphi) = 3 \cdot 89b$  | (1 isogeny class)                      |  | , . | 0.1                          | 0.1   | 0.1  | тт   | OOTD         |
| a1<br>a2 |   | $-1 + \varphi$ $-1 + \varphi$ | $0 \\ 0$          | $-2 + \varphi$ $8 - 4\varphi$ | $0\\-10+7\varphi$                      | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | $\begin{array}{ c c }\hline 2\\ 2\\ \end{array}$ | ++  |                              | 2, 1 $4, 2$                                   | $\begin{array}{ c c } 2,1\\ 2,2 \end{array}$ | $\begin{matrix} I_2,I_1\\I_4,I_2\end{matrix}$                |              |
| aΔ       | 1                                       | $-1+\varphi$                  | U                 | $6-4\varphi$                  | $-10 + i\varphi$                       | 1                                      |  |     | 4, 4                         | 4, 4  | ۷, ۷   | 14,12  |              |

|   |                               |                               |                       |   |  | 1                                      | Imi                                  | T -         | $\operatorname{ord}(\Delta)$ | 1 (:)  |  | Kodaira  | T         |
|---|-------------------------------|-------------------------------|-----------------------|---|--|--|--------------------------------------|-------------|------------------------------|--|--|--|-----------|
|   | $a_1$                         | $a_2$                         | $a_3$                 | $a_4$   | $a_6$  | r                                      | T                                    | s           | $ord(\Delta)$                | $\operatorname{ord}_{-}(j)$                        | $c_p$  | Kodaira  | Isogenies |
| 809a                                    | a                             |                               |                       |   | $809a = (19 - 26\varphi) = 809a$                                     | (1 isogeny cla                         | ss)                                  |             |                              |  |  |  | 809a      |
| a1                                      | $1+\varphi$                   | 1                             | 0                     | $\varphi$   | 0  | 1                                      | 2                                    | ++          | 1                            | 1  | 1  | $I_1$  |           |
| a2                                      | $1+\varphi$                   | 1                             | 0                     | $-4\varphi$   | $-3-9\varphi$  | 1                                      | 2                                    | -+          | 2                            | 2  | 2  | $I_2$  |           |
| 8091                                    | b                             |                               |                       |   | $809b = (7 - 26\varphi) = 809b$                                      | (1 isogeny clas                        | s)                                   |             |                              |  |  |  | 809b      |
| a1                                      |                               | $-1-\varphi$                  | $\varphi$             | -1  | 0  | 1                                      | 2                                    | ++          | 1                            | 1  | 1  | $I_1$  |           |
| a2                                      | φ                             | $-1-\varphi$                  | φ                     | $-6+5\varphi$   | $-7 + 4\varphi$  | 1                                      | 2                                    | +-          | 2                            | 2  | 2  | $I_2$  |           |
| 811a                                    | a                             |                               |                       |   | $811a = (29 - \varphi) = 811a $                                      | 1 isogeny class                        | s)                                   |             |                              |  |  |  | 811a      |
| a1                                      | $1+\varphi$                   | •                             | $1+\varphi$           | $-2-\varphi$  | $-\varphi$   | 1                                      | 3                                    |             | 1                            | 1  | 1  | $I_1$  |           |
| a2                                      | $1+\varphi$                   | $-\varphi$                    | $1+\varphi$           | $8-6\varphi$  | $-9 + 3\varphi$  | 1                                      | 1                                    |             | 3                            | 3  | 3  | $I_3$  |           |
| 811                                     | b                             |                               |                       |   | $811b = (28 + \varphi) = 811b $                                      | 1 isogeny class                        | s)                                   |             |                              |  |  |  | 811b      |
| a1                                      | $\varphi$                     | 0                             | $\varphi$             | $-1-\varphi$  | 0  | 1                                      | 3                                    |             | 1                            | 1  | 1  | $I_1$  |           |
| a2                                      | φ                             | 0                             | φ                     | $4+4\varphi$  | $-5-4\varphi$  | 1                                      | 1                                    |             | 3                            | 3  | 3  | $I_3$  |           |
| 820                                     | a                             |                               |                       | 820   | $a = (18 - 26\varphi) = 2 \cdot 5a \cdot 41a$                        | (2 isogeny c                           | lasses                               | s)          |                              |  |  |  | 820a      |
| a1                                      |                               | $-1-\varphi$                  | 1                     | -1  | $1+\varphi$  | 1                                      | 2                                    |             | 2, 1, 2                      | 2, 1, 2  | 2, 1, 2  | $I_2, I_1, I_2$  |           |
| a2                                      |                               | $-1-\varphi$                  | 1                     | $-21 - 10\varphi$   | $49 + 45\varphi$   | 1                                      | 2                                    | ++          | 1,2,1                        | 1,2,1  | 1,2,1  | $I_1, I_2, I_1$  |           |
| b1<br>b2                                | $1+\varphi$                   | $1 - \varphi$                 | $1+\varphi$ 1         | $-36 - 61\varphi$ $-4010 - 6486\varphi$                             | $-182 - 297\varphi$ $-185337 - 299881\varphi$                        | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{vmatrix} 2\\2 \end{vmatrix}$ | <br>  + +   | 8, 1, 2 $4, 2, 1$            | $\begin{vmatrix} 8,1,2\\4,2,1 \end{vmatrix}$       | $\begin{vmatrix} 2,1,2\\2,2,1 \end{vmatrix}$               | $ \begin{vmatrix} I_8, I_1, I_2 \\ I_4, I_2, I_1 \end{vmatrix} $ |           |
|   |                               | Τ Ψ                           |                       | 1010 01004  | 100001 2000019   |  | _                                    | ' '         | -, -, -                      | 1,2,1  | 2,2,1  | 14,12,11   |           |
| 820                                     | <u>b</u>                      |                               |                       |   | $0b = (8 - 26\varphi) = 2 \cdot 5a \cdot 41b$                        | (2 isogeny cl                          | asses                                | )           |                              |  |  |  | 820b      |
| a1<br>a2                                | 1                             | $1+\varphi$                   | 0                     | $-1+2\varphi$   | 62 24.2  | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | 2                                    |             | 2, 1, 2                      | $\begin{bmatrix} 2, 1, 2 \\ 1, 2, 1 \end{bmatrix}$ | $\begin{bmatrix} 2, 1, 2 \\ 1, 2, 1 \end{bmatrix}$         | $I_2, I_1, I_2$  |           |
| b1                                      | $\frac{1}{\varphi}$           | $\frac{1+\varphi}{1-\varphi}$ | $\frac{0}{\varphi}$   | $ \begin{array}{r} -31 + 12\varphi \\ -95 + 59\varphi \end{array} $ | $ \begin{array}{r} 63 - 34\varphi \\ -478 + 296\varphi \end{array} $ |  | $\frac{ }{ } \frac{2}{2}$            |             | $-\frac{1,2,1}{8,1,2}$       | $\begin{bmatrix} 1, 2, 1 \\ 8, 1, 2 \end{bmatrix}$ | $\begin{array}{c c} 1, 2, 1 \\ \hline 2, 1, 2 \end{array}$ | $\begin{bmatrix} I_1, I_2, I_1 \\ I_8, I_1, I_2 \end{bmatrix}$   |           |
| b2                                      | $\frac{\varphi}{1}$           | $\varphi$                     | $\stackrel{arphi}{1}$ | $-10496 + 6486\varphi$  | $-485218 + 299881\varphi$  |  | 2                                    | ++          | 4, 2, 1                      | 4, 2, 1  | 2, 1, 2<br>2, 2, 1   | $I_{4}, I_{2}, I_{1}$  |           |
| 829                                     | a                             |                               |                       |   | $829a = (9 - 26\varphi) = 829a \tag{9}$                              | 3 isogeny class                        | ses)                                 |             |                              |  |  |  | 829a      |
| a1                                      | $\frac{1+\varphi}{1+\varphi}$ | $1-\varphi$                   | $\varphi$             | $-\varphi$  | 0  | 1                                      | 1                                    |             | 1                            | 1  | 1  | $I_1$  |           |
| b1                                      | $\varphi$                     | $-\varphi$                    |                       | $-2+2\varphi$   | $1-\varphi$  | 1                                      | 3                                    | .¦<br>  + - | 1                            | .'<br>  1  |  | $I_1$  |           |
| b2                                      | $\varphi$                     | $-\varphi$                    | 0                     | $-17-3\varphi$  | $-27-4\varphi$   | 1                                      | 1                                    | +-          | 3                            | 3  | 3  | $I_3$  |           |
| $\begin{bmatrix} c1 \\ c \end{bmatrix}$ | $1+\varphi$                   |                               | $1+\varphi$           | $-16 - 25\varphi$   | $33 + 54\varphi$   |  | 3                                    |             | 1                            |  | 1  | $I_1$  |           |
| c2                                      | φ                             | 0                             | $1+\varphi$           | $-270 + 162\varphi$   | $-1990 + 1236\varphi$  | 1                                      | 1                                    |             | 3                            | 3  | 3  | $I_3$  |           |
| 8291                                    | b                             |                               |                       |   | $829b = (17 - 26\varphi) = 829b $                                    | 3 isogeny class                        | ses)                                 |             |                              |  |  |  | 829b      |
| a1                                      | $\varphi$                     | 1                             | $1+\varphi$           | $-\varphi$  | $-\varphi$   | 1                                      | 1                                    |             | 1                            | 1  | 1  | $I_1$  |           |
| b1                                      | $1+\varphi$                   | -1                            | 0                     | $-2\varphi$   | $\varphi$  | 1                                      | 3                                    | - +         | 1                            | 1  | 1  | $I_1$  |           |
| b2                                      | $1+\varphi$                   | -1                            | 0                     | $-20 + 3\varphi$  | $-31+4\varphi$   | 1                                      | 1                                    | -+          | 3                            | 3  | 3  | $I_3$  |           |

|          | $a_1$  | $a_2$                          | $a_3$           | $a_4$  | $a_6$   | r                                      | T   | s         | $\operatorname{ord}(\Delta)$            | $\operatorname{ord}_{-}(j)$                                  | $c_p$  | Kodaira   | Isogenies    |
|----------|--|--------------------------------|-----------------|--|---|--|---|-----------|---|--|--|---|--------------|
|          |  |                                |                 |  |   | <u> </u>                               |   |           |   | 1  |  | 1   | 0001         |
| 829      | b  |                                |                 |  | $829b = (17 - 26\varphi) = 829b$                        | (3 isogeny c                           | lasses                                    | <u>s)</u> |   |  |  |   | 829b         |
| c1       | $\varphi$  | $1-\varphi$                    | $\varphi$       | $-39 + 23\varphi$                              | $88-55\varphi$  | 1                                      | 3   |           | _                                       | 1  | 1  | $I_1$   |              |
| c2       | $1+\varphi$  | $-\varphi$                     | φ               | $-107 - 164\varphi$                            | $-753 - 1237\varphi$                                    | 1                                      | 1   |           | 3                                       | 3  | 3  | $I_3$   |              |
| 836      | a  |                                |                 | 83   | $6a = (30 - 2\varphi) = 2 \cdot 11a \cdot 19b$          | (5 isogen                              | y clas                                    | sses)     |   |  |  |   | 836a         |
| a1       | $\varphi$  | 0                              | $1+\varphi$     | $-\varphi$                                     | 0   | 1                                      | 1   |           | 2, 1, 1                                 | 2, 1, 1  | 2, 1, 1  | $I_2, I_1, I_1$                                       |              |
| b1       | $\varphi$  | $-1+\varphi$                   | $1+\varphi$     | $37-49\varphi$                                 | $-108 + 137\varphi$                                     | 0                                      | 1   | Ī — —     | 7, 1, 9                                 | 7,1,9  | 1,1,1  | $  I_7, I_1, I_9  $                                   |              |
| c1       | 1  | $-\varphi$                     |                 | $-16 + 12\varphi$                              | $32-22\varphi$  | 0                                      | -¦<br>  5                                 | <u> </u>  | 2, 5, 1                                 | 2,5,1  | 2, 5, 1  | $  I_2, I_5, I_1  $                                   |              |
| c2       | $\varphi$  | $-1+\varphi$                   | $1 + \varphi$   | $-1257 - 2016\varphi$                          | $-33006 - 53012\varphi$                                 | 0                                      |   | 1         | 10, 1, 5                                | 10, 1, 5   | 10, 1, 1   | $I_{10}, I_1, I_5$                                    |              |
| d1       | $\varphi$  | $1+\varphi$                    | 1               | $-7+3\varphi$                                  | $8-2\varphi$  | 1                                      | 1   | Ī — —     | 2, 1, 7                                 | 2, 1, 7  | 2, 1, 7  | $  I_2, I_1, I_7  $                                   |              |
| e1       | $1+\varphi$  | $-\varphi$                     | <u>-</u> -      | -1   | 0   | 0                                      |   | <u> </u>  | 1, 1, 1                                 | 1,1,1  | 1, 1, 1  | $ $ $I_1, I_1, I_1$                                   | <u>-</u>     |
| e2       | $1+\varphi$  | -arphi                         | 1               | $4-5\varphi$                                   | $-4-2\varphi$   | 0                                      | 3   |           |   | 3, 3, 3  | 1, 3, 3  | $I_3, I_3, I_3$                                       |              |
| e3       | $1+\varphi$  | $-\varphi$                     | 1               | $-206 - 105\varphi$                            | $-1272 - 946\varphi$                                    | 0                                      | 1   |           | 9, 1, 1                                 | 9, 1, 1  | 1, 1, 1  | $I_9,I_1,I_1$   |              |
| 836      | b  |                                |                 | 83   | $6b = (28 + 2\varphi) = 2 \cdot 11b \cdot 19a$          | (5 isogen                              | y clas                                    | sses)     |   |  |  |   | <b>836</b> b |
| a1       | $1+\varphi$  | $-\varphi$                     | φ               | -arphi   | $1-\varphi$   | 1                                      | 1   | T         | 2, 1, 1                                 | 2, 1, 1  | 2, 1, 1  | $I_2, I_1, I_1$                                       |              |
| b1       | $1+\varphi$  | φ                              | $1+\varphi$     | $-11 + 48\varphi$                              | $78 - 101\varphi$                                       | 0                                      | <br>  1                                   | <u> </u>  | 7, 1, 9                                 | 7,1,9  | 1,1,1  | $  I_7, I_1, I_9  $                                   | <u>-</u>     |
| c1       |  | $-1+\varphi$                   |                 | $-4 - 12\varphi$                               | $10 + 22\varphi$  |  | -¦<br>  5                                 |           | $\frac{1}{2}, \frac{1}{5}, \frac{1}{1}$ | 2,5,1  | 2, 5, 1  | $  I_2, I_5, I_1  $                                   |              |
| c2       | $1+\varphi$  |                                | $1+\varphi$     | $-3272 + 2015\varphi$                          | $-84002 + 51754\varphi$                                 | 0                                      |   | 1         | 10, 1, 5                                | 10, 1, 5   | 10, 1, 1   | $I_{10}, I_1, I_5$                                    |              |
| d1       | $1+\varphi$  | $-1+\varphi$                   | $1+\varphi$     | $-6-3\varphi$                                  | $9-4\varphi$  | 1                                      | 1   | Ī — —     | 2, 1, 7                                 | [2, 1, 7]  | 2, 1, 7  | $  I_2, I_1, I_7  $                                   |              |
| e1       | $\varphi$  | 0                              | <u>-</u> -      | $-\varphi$                                     | 0   | 0                                      | 3   | <u> </u>  | 1, 1, 1                                 | 1,1,1  | 1, 1, 1  | $ $ $I_1, I_1, I_1$                                   |              |
| e2       | $\varphi$  | 0                              | 1               | 4arphi   | $-6+2\varphi$   | 0                                      | 3   |           | 3, 3, 3                                 | 3, 3, 3  | 1, 3, 3  | $I_3, I_3, I_3$                                       |              |
| e3       | $\varphi$  | 0                              | 1               | $-310 + 104\varphi$                            | $-2218 + 946\varphi$                                    | 0                                      | 1   |           | 9, 1, 1                                 | 9, 1, 1  | 1, 1, 1  | $I_9, I_1, I_1$                                       |              |
| 836      | $\mathbf{c}$   |                                |                 | 830  | $\delta c = (16 - 26\varphi) = 2 \cdot 11a \cdot 19a$   | (3 isoger                              | ny cla                                    | sses)     |   |  |  |   | 836c         |
| a1       | 1  | $\varphi$                      | 0               | $-4-3\varphi$                                  | $-3-7\varphi$   | 1                                      | 2   | ++        | 3, 1, 4                                 | 3, 1, 4  | 3, 1, 2  | $I_3, I_1, I_4$                                       |              |
| a2       | 1  | $\varphi$                      | 0               | $-24-23\varphi$                                | $33 + 69\varphi$  | 1                                      | 4   | 1         | 6, 2, 2                                 | 6, 2, 2  | 6, 2, 2  | $\mathrm{I}_6,\mathrm{I}_2,\mathrm{I}_2$              |              |
| a3       | 1  | $\varphi$                      | 0               | $-384 - 373\varphi$                            | $2621 + 5395\varphi$                                    | 1                                      | 2   | 1         | 3, 4, 1                                 | 3, 4, 1  | 3, 2, 1  | $I_3,I_4,I_1$   |              |
| a4       | 1  | $\varphi$                      | 0               | $16 + 7\varphi$                                |   | 1                                      | _ 2_                                      |           | 12,1,1                                  | 12, 1, 1   | 12,1,1   | $I_{12}, I_1, I_1$                                    |              |
| b1       | $1+\varphi$  | $-\varphi$                     | 0               | $-52 + 23\varphi$                              | $146 - 77\varphi$                                       | 0                                      | 6   |           | 1, 1, 4                                 | 1, 1, 4  | 1, 1, 4  | $I_1, I_1, I_4$                                       |              |
| b2       | 1  | 0                              | 0               | $-455 - 721\varphi$                            | $6918 + 11175\varphi$                                   | 0                                      | $\frac{6}{6}$                             |           | 2, 2, 2                                 | 2, 2, 2  | 2, 2, 2  | $I_2, I_2, I_2$                                       |              |
| b3       | $1+\varphi$  | $-\varphi$                     | 0               | $-87 + 28\varphi$                              | $-79 + \varphi$   | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 6   |           | 3, 3, 12                                | 3, 3, 12   | 1, 3, 12   | $I_3, I_3, I_{12}$                                    |              |
| b4<br>b5 | $1+\varphi$ 1  | $-\varphi$ 0                   | $0 \\ 0$        | $233 - 292\varphi$ $-6570 - 9161\varphi$       | $     113 + 769\varphi \\     -341043 - 534651\varphi $ | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{vmatrix} 6\\2 \end{vmatrix}$      |           | 6, 6, 6<br>9, 1, 4                      | 6, 6, 6<br>9, 1, 4   | 2, 6, 6  | $I_6, I_6, I_6$                                       |              |
| b6       | $1+\varphi$  | $-\varphi$                     | 0               | $-6570 - 9101\varphi$<br>$-5307 + 1348\varphi$ | $-341043 - 354051\varphi$ $-160183 + 57589\varphi$      |  | $\begin{vmatrix} 2 \\ 2 \end{vmatrix}$    | 1         | 9, 1, 4 $18, 2, 2$                      | $\begin{vmatrix} 9, 1, 4 \\ 18, 2, 2 \end{vmatrix}$          | $\begin{vmatrix} 1, 1, 4 \\ 2, 2, 2 \end{vmatrix}$ | $  I_9, I_1, I_4    I_{18}, I_2, I_2 $                |              |
| c1       | $\begin{vmatrix} 1 & 1 & \varphi \\ 1 & 1 \end{vmatrix}$ | $1+\varphi$                    |                 | $-71 + 37\varphi$                              | $-268 + 169\varphi$                                     |  | $\frac{1}{2} - \frac{2}{2} - \frac{2}{2}$ |           | $-\frac{10, 2, 2}{5, 1, 2}$             | $\begin{array}{c c} -10, 2, 2 \\ \hline 5, 1, 2 \end{array}$ | $\begin{bmatrix} 2, 2, 2 \\ 1, 1, 2 \end{bmatrix}$ | $  I_{18}, I_{2}, I_{2}  $<br>$  I_{5}, I_{1}, I_{2}$ | <u>_</u>     |
| c2       | 1  | $1 + \varphi$<br>$1 + \varphi$ | $arphi \ arphi$ | $-71 + 37\varphi$ $-71 + 47\varphi$            | $-208 + 109\varphi$ $-218 + 173\varphi$                 | 0                                      |   | 1         | 10, 1, 2 $10, 2, 1$                     | $\begin{bmatrix} 3, 1, 2 \\ 10, 2, 1 \end{bmatrix}$          | 1, 1, 2<br>2, 2, 1                                 | $I_{10}, I_{11}, I_{2}$<br>$I_{10}, I_{2}, I_{1}$     |              |
| 02       | 1  | 1   γ                          | Ψ               | π   π   ψ                                      | 210   119φ  | 0                                      |   |           | 10, 2, 1                                | 10,2,1   | 2,2,1  | 110, 12, 11   |              |

|  |  |  |   |   |   |  | Len   |  | 1/4)  | 1 (1)   |  |   |           |
|--|--|--|---|---|---|--|---|--|---|---|--|---|-----------|
|  | $a_1$  | $a_2$  | $a_3$   | $a_4$   | $a_6$   | r  | T   | s  | $\operatorname{ord}(\Delta)$  | $\operatorname{ord}_{-}(j)$   | $c_p$  | Kodaira   | Isogenies |
|  | _  |  |   |   |   |  |   |  |   |   |  |   |           |
| 836  | d  |  |   | 8366  | $d = (10 - 26\varphi) = 2 \cdot 11b \cdot 19b$  | (3 isogen                                      | y clas  | sses)  |   |   |  |   | 836d      |
| a1   | 1  | $1-\varphi$  | 0   | $-7+3\varphi$   | $-10 + 7\varphi$  | 1  | 2   | ++   |   | 3, 1, 4   | 3, 1, 2  | $I_3, I_1, I_4$   |           |
| a2   | 1  | $1-\varphi$  | 0   | $-47+23\varphi$   | $102-69\varphi$   | 1  | 4   |  | 6, 2, 2   | 6, 2, 2   | 6, 2, 2  | $I_6, I_2, I_2$   |           |
| a3   | 1  | $1-\varphi$  | 0   | $-757 + 373\varphi$   | $8016 - 5395\varphi$  | 1  | 2   | ++   | , ,   | 3, 4, 1   | 3, 2, 1  | $I_3, I_4, I_1$   |           |
| a4   | 1  | $1-\varphi$  | 0   | $23 - 7\varphi$   | $476 - 327\varphi$  | 1  | 2   |  | 12, 1, 1  | 12, 1, 1  | 12,1,1   | $I_{12}, I_1, I_1$  |           |
| b1   | $\varphi$  | 0  | 0   | $-29-23\varphi$   | $69 + 77\varphi$  | 0  | 6   | ++   | 1, 1, 4   | 1, 1, 4   | 1, 1, 4  | $I_1, I_1, I_4$   |           |
| b2   | 1  | 0  | 0   | $-1176 + 721\varphi$  | $18093 - 11175\varphi$  | 0  | 6   | + -  | 2, 2, 2   | 2, 2, 2   | 2, 2, 2  | $I_2, I_2, I_2$   |           |
| b3   | $\varphi$  | 0  | 0   | $-59-28\varphi$   | $-78 - \varphi$   | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$         | 6   | ++   | , ,   | 3, 3, 12  | 1, 3, 12   | $I_3, I_3, I_{12}$  |           |
| b4   | $\varphi$  | 0  | 0   | $-59 + 292\varphi$  | $882 - 769\varphi$  | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$         | 6   | +-   | 6, 6, 6   | 6, 6, 6   | 2, 6, 6  | $I_6, I_6, I_6$   |           |
| b5   | 1  | $0 \\ 0$   | $0 \\ 0$  | $-15731 + 9161\varphi$  | $-875694 + 534651\varphi \\ -102594 - 57589\varphi$   | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$         | $\begin{vmatrix} 2\\2 \end{vmatrix}$  | 1  | 9, 1, 4   | 9,1,4   | 1, 1, 4  | $I_9, I_1, I_4$   |           |
| b6   | $  \frac{\varphi}{1}$ $-$  |  |   | $-3959 - 1348\varphi$   |   | 0  | -'  |  | 18, 2, 2  | 18, 2, 2  | 2,2,2  | $  I_{18}, I_2, I_2  $  |           |
| $\begin{bmatrix} c1 \\ c \end{bmatrix}$              |  | $-1-\varphi$   | $\varphi$   | $-35-36\varphi$   | $-64 - 133\varphi$  | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$         | $\begin{vmatrix} 2 \\ 0 \end{vmatrix}$  | 1  | 5, 1, 2   | 5, 1, 2   | 1, 1, 2  | $I_5, I_1, I_2$   |           |
| c2   | 1  | $-1-\varphi$   | φ   | $-25 - 46\varphi$   | $-20 - 127\varphi$  | 0  | 2   | - +  | 10, 2, 1  | 10, 2, 1  | 2, 2, 1  | $I_{10}, I_2, I_1$  |           |
| 0.41   |  |  |   |   |   |  |   |  |   |   |  |   | 0.41      |
| 841  | a  |  |   |   | $841a = (29) = 29a \cdot 29b$   | (1 isogeny cl                                  | lass)   |  |   |   |  |   | 841a      |
| a1   | $\varphi$  |  | $1 + \varphi$   | $-2-\varphi$  | -arphi  | 1  | 2   | ++   | 1, 1  | 1,1   | 1, 1   | $I_1, I_1$  |           |
| a2   | $\varphi$  | 0  | $1+\varphi$   | $-7-11\varphi$  | $11 + 15\varphi$  | 1  | 2   | -+   | 2,2   | 2,2   | 2, 2   | $I_2, I_2$  |           |
|  |  |  |   |   |   |  |   |  |   |   |  |   |           |
| 845  | ล  |  |   | S   | $845a = (13 - 26\varphi) = 5a \cdot 13$   | (1 igogony                                     | alacc   | .)   |   |   |  |   | 845a      |
|  | a  |  |   |   | $545a - (15 20\varphi) - 5a \cdot 15$   | (1 isogeny                                     | Class   | )  |   |   |  |   | 049a      |
| a1   | 1  | 0  | 0   | -1  | $\frac{545u - (15 - 20\varphi) - 5u \cdot 15}{0}$   | (1 isogeny                                     | $\frac{\text{ciass}}{2}$  | ++   | 1,2   | 1,2   | 1, 2   | $I_1, I_2$  | 049a      |
| a1<br>a2   |  | 0  | 0   |   | . , ,   | , , , ,  | _   | <del></del>                                  | 1, 2<br>2, 4  | 1, 2<br>2, 4  | 1, 2<br>2, 2   | $\begin{matrix} I_1,I_2\\I_2,I_4 \end{matrix}$  | 049a      |
|  | 1  |  |   | -1  | 0   | 1  | 2   | ++   |   |   |  |   | 049a      |
|  | 1<br>1   |  |   | -1<br>4   | 0   | 1  | 2 2   | ++   |   |   |  |   | 855a      |
| a2   | 1<br>1   | 0  | 0   | $     \begin{array}{r}       -1 \\       4   \end{array} $ 855  | $0$ $1$ $5a = (27 + 6\varphi) = 5a \cdot 3 \cdot 19a$   | 1 1  | 2 2   | + +<br><br>ses)                              | 2,4   | 2,4   | 2,2  | $I_2, I_4$  |           |
| 855a   | 1<br>1   | $\varphi$  | $0$ $1+\varphi$   | -1<br>4   | 0 1   | 1 1 1 (5 isogeny                               | 2<br>2<br>v class   | ++   | 2, 2, 2   |   | 2,2,2  | $\begin{array}{ c c c c }\hline I_2,I_4\\ \hline &I_2,I_2,I_2\\ \hline \end{array}$   |           |
| 855a   | $\mathbf{a}$ $1$ $1$ $1$   | $\varphi$  | 0   | $ \begin{array}{r} -1\\ 4 \end{array} $ $ 855 $ $ -11 + 6\varphi $  | $6a = (27 + 6\varphi) = 5a \cdot 3 \cdot 19a$ $17 - 12\varphi$  | (5 isogeny                                     | $\begin{bmatrix} 2\\2 \end{bmatrix}$  | ++<br><br>ses)<br>++                         | 2,4   | 2,4   | 2,2  | $I_2, I_4$  |           |
| 855a a1 a2   | $\mathbf{a}$ $1 + \varphi$ $1 + \varphi$ $1 + \varphi$ $1 + \varphi$   | $\varphi$  | $\begin{array}{c} 0 \\ 1+\varphi \\ 1+\varphi \end{array}$  | $ \begin{array}{r} -1\\ 4 \end{array} $ $ 855 $ $ -11 + 6\varphi$ $ -1 + \varphi$   | $6a = (27 + 6\varphi) = 5a \cdot 3 \cdot 19a$ $17 - 12\varphi$ $0$  | (5 isogeny 1 1 1                               | $\begin{bmatrix} 2\\2 \end{bmatrix}$  | ++<br><br>ses)<br>++<br>+-                   | 2, 2, 2<br>1, 1, 1  | 2,4   | 2,2,2 1,1,1  | $\begin{array}{ c c c c c }\hline I_2,I_4\\ \hline I_2,I_2,I_2\\ I_1,I_1,I_1\\ \hline \end{array}$  |           |
| a2  855a  a1  a2  a3                                 | $\mathbf{a}$ $1 + \varphi$ $1 + \varphi$ $1 + \varphi$ $1 + \varphi$   | $\varphi$ $\varphi$ $\varphi$  | $\begin{array}{c} 1+\varphi\\ 1+\varphi\\ 1+\varphi\end{array}$   | $ \begin{array}{r} -1 \\ 4 \end{array} $ $ 855 $ $ -11 + 6\varphi \\ -1 + \varphi \\ -176 + 96\varphi $   |   | (5 isogeny 1 1 1 1 1                           | $\begin{array}{ c c }\hline 2\\2\\\hline \end{aligned}$   | ++<br><br>ses)<br>++<br>+-<br>++             | 2, 2, 2<br>1, 1, 1<br>1, 1, 4   | 2, 4<br>2, 2, 2<br>1, 1, 1<br>1, 1, 4   | 2, 2<br>2, 2, 2<br>1, 1, 1<br>1, 1, 2  | $\begin{array}{ c c c c c }\hline I_2,I_4\\ \hline &I_2,I_2,I_2\\ &I_1,I_1,I_1\\ &I_1,I_1,I_4\\ \hline \end{array}$   |           |
| 855;<br>a1<br>a2<br>a3<br>a4                         | $\mathbf{a}$ $1 + \varphi$   | $\begin{array}{c} \varphi \\ \varphi \\ \varphi \\ \varphi \\ -1 + \varphi \\ -\varphi \end{array}$  | $ \begin{array}{c} 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \\ 1 - \varphi \end{array} $   | $ \begin{array}{r} -1 \\ 4 \end{array} $ $ 855 $ $ -11 + 6\varphi \\ -1 + \varphi \\ -176 + 96\varphi \\ -24 - 39\varphi $  | $0$ $1$ $5a = (27 + 6\varphi) = 5a \cdot 3 \cdot 19a$ $17 - 12\varphi$ $0$ $1022 - 669\varphi$ $81 + 126\varphi$  | (5 isogeny   1   1   1   1   1   1   1   1   1 | 2   2   2     4   2   2   2   2   | ++<br><br>ses)<br>++<br>+-<br>++             | 2, 2, 2<br>1, 1, 1<br>1, 1, 4<br>4, 4, 1  | 2, 4<br>2, 2, 2<br>1, 1, 1<br>1, 1, 4<br>4, 4, 1  | 2, 2, 2<br>1, 1, 1<br>1, 1, 2<br>4, 2, 1   | $\begin{array}{ c c c c }\hline I_2,I_4\\ \hline &I_2,I_2,I_2\\ I_1,I_1,I_1\\ I_1,I_1,I_4\\ I_4,I_4,I_1\\ \hline \end{array}$   |           |
| 855a a1 a2 a3 a4 b1                                  | $\mathbf{a}$ $1 + \varphi$   | $\begin{matrix} \varphi \\ \varphi \\ \varphi \\ \varphi \\ -1 + \varphi \end{matrix}$   | $\begin{array}{c} 1+\varphi\\ 1+\varphi\\ 1+\varphi\\ 0 \end{array}$  | $ \begin{array}{r} -1\\ 4 \end{array} $ $ 855 $ $ -11 + 6\varphi$ $ -1 + \varphi$ $ -176 + 96\varphi$ $ -24 - 39\varphi$ $ -12611 - 20399\varphi$   | $\begin{array}{c} 0 \\ 1 \\ 6a = (27 + 6\varphi) = 5a \cdot 3 \cdot 19a \\ 17 - 12\varphi \\ 0 \\ 1022 - 669\varphi \\ 81 + 126\varphi \\ 1036052 + 1676373\varphi \end{array}$   | (5 isogeny   1   1   1   1   1   1   1   1   1 | $\begin{array}{ c c c }\hline 2\\2\\\hline \end{array}$   | ++<br><br>ses)<br>++<br>+-<br>++<br>-+<br>++ | 2,4<br>2,2,2<br>1,1,1<br>1,1,4<br>4,4,1<br>1,1,8  | $\begin{array}{ c c c c }\hline 2,4\\\hline &2,2,2\\1,1,1\\1,1,4\\4,4,1\\\hline &1,1,8\\\hline \end{array}$   | 2, 2, 2<br>1, 1, 1<br>1, 1, 2<br>4, 2, 1<br>1, 1, 2  | $\begin{array}{ c c c c }\hline I_2,I_4\\ \hline &I_2,I_2,I_2\\ I_1,I_1,I_1\\ I_1,I_1,I_4\\ I_4,I_4,I_1\\ \hline &I_1,I_1,I_8\\ \hline \end{array}$   |           |
| 855a a1 a2 a3 a4 b1 b2                               | $\mathbf{a}$ $1 + \varphi$   | $ \begin{array}{c} \varphi \\ \varphi \\ \varphi \\ -1 + \varphi \\ -\varphi \\ \varphi \end{array} $  | $ \begin{array}{c} 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \\                                  $   | $ \begin{array}{r} -1\\ 4 \end{array} $ $ 855 $ $ -11 + 6\varphi$ $ -1 + \varphi$ $ -176 + 96\varphi$ $ -24 - 39\varphi$ $ -12611 - 20399\varphi$ $ -138 + 48\varphi$   | $0$ $1$ $6a = (27 + 6\varphi) = 5a \cdot 3 \cdot 19a$ $17 - 12\varphi$ $0$ $1022 - 669\varphi$ $81 + 126\varphi$ $1036052 + 1676373\varphi$ $-552 + 430\varphi$   | (5 isogeny  1 1 1 1 1 1 0 0 0                  | $ \begin{vmatrix} 2\\2 \end{vmatrix} $  | ++<br><br>ses)<br>++<br>+-<br>++<br>-+<br>++ | 2, 4<br>2, 2, 2<br>1, 1, 1<br>1, 1, 4<br>4, 4, 1<br>1, 1, 8<br>2, 2, 4  | $ \begin{array}{ c c c c } \hline 2, 2, 2 \\ 1, 1, 1 \\ 1, 1, 4 \\ 4, 4, 1 \\ \hline -1, 1, 8 \\ 2, 2, 4 \end{array} $  | $ \begin{array}{ c c c } \hline 2,2,2\\ 1,1,1\\ 1,1,2\\ 4,2,1\\ \hline 1,1,2\\ 2,2,2\\ \end{array} $   | $\begin{array}{ c c c c c }\hline I_2,I_4\\ \hline &I_2,I_2,I_2\\ I_1,I_1,I_1\\ I_1,I_1,I_4\\ I_4,I_4,I_1\\ \hline &I_1,I_1,I_8\\ I_2,I_2,I_4\\ \hline \end{array}$   |           |
| 855a  a1 a2 a3 a4 b1 b2 b3                           | $ \begin{array}{c} 1\\ 1\\ \end{array} $ $ \begin{array}{c} 1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\1\\1\\1\end{array} $   | $ \begin{array}{c} \varphi \\ \varphi \\ \varphi \\ -1 + \varphi \\ -\varphi \\ \varphi \\ -1 \end{array} $  | $ \begin{array}{c} 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \\ \varphi \\ \varphi \end{array} $   | $ \begin{array}{r} -1\\ 4 \end{array} $ $ 855 $ $ -11 + 6\varphi$ $ -1 + \varphi$ $ -176 + 96\varphi$ $ -24 - 39\varphi$ $ -24 - 39\varphi$ $ -12611 - 20399\varphi$ $ -138 + 48\varphi$ $ -8 + 3\varphi$   | $0$ $1$ $6a = (27 + 6\varphi) = 5a \cdot 3 \cdot 19a$ $17 - 12\varphi$ $0$ $1022 - 669\varphi$ $81 + 126\varphi$ $1036052 + 1676373\varphi$ $-552 + 430\varphi$ $-9 + 6\varphi$   | (5 isogeny  1 1 1 1 1 1 0 0 0 0                | $ \begin{array}{ c c c } \hline 2 \\ 2 \end{array} $ $ \begin{array}{ c c c } \hline 4 \\ 2 \\ 2 \\ 2 \\ 4 \\ 4 \end{array} $   | ses) + + + + + + + + + + + + + + + + +       | 2,2,2<br>1,1,1<br>1,1,4<br>4,4,1<br>1,1,8<br>2,2,4<br>4,1,2   | $\begin{array}{ c c c }\hline 2,4\\\hline &2,2,2\\1,1,1\\1,1,4\\4,4,1\\\hline &1,1,8\\2,2,4\\4,1,2\\\hline \end{array}$   | $\begin{array}{ c c c }\hline 2,2\\\hline &2,2,2\\1,1,1\\1,1,2\\4,2,1\\\hline\hline &1,1,2\\2,2,2\\4,1,2\\\hline\end{array}$   | $\begin{array}{ c c c c c }\hline I_2,I_4\\ \hline &I_2,I_2,I_2\\ I_1,I_1,I_1\\ I_1,I_1,I_4\\ I_4,I_4,I_1\\ \hline &I_1,I_1,I_8\\ I_2,I_2,I_4\\ I_4,I_1,I_2\\ \hline \end{array}$   |           |
| 8553<br>a1<br>a2<br>a3<br>a4<br>b1<br>b2<br>b3<br>b4 | $\mathbf{a}$ $1 + \varphi$ $1 + \varphi$ $1 + \varphi$ $1 + \varphi$ $1$ $1 + \varphi$ $1$ $1 + \varphi$   | $ \begin{array}{c} \varphi \\ \varphi \\ \varphi \\ -1 + \varphi \\ -\varphi \\ \varphi \\ \varphi \\ -1 \\ 1 + \varphi \end{array} $  | $ \begin{array}{c} 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi \\ \varphi \\ \varphi \end{array} $   | $ \begin{array}{r} -1\\ 4 \end{array} $ $ 855 $ $ -11 + 6\varphi$ $ -1 + \varphi$ $ -176 + 96\varphi$ $ -24 - 39\varphi$ $ -12611 - 20399\varphi$ $ -138 + 48\varphi$ $ -8 + 3\varphi$ $ -13336 + 8235\varphi$  | $\begin{array}{c} 0\\ 1\\ 6a = (27 + 6\varphi) = 5a \cdot 3 \cdot 19a\\ 17 - 12\varphi\\ 0\\ 1022 - 669\varphi\\ 81 + 126\varphi\\ 1036052 + 1676373\varphi\\ -552 + 430\varphi\\ -9 + 6\varphi\\ -692335 + 427890\varphi \end{array}$  | (5 isogeny  1 1 1 1 1 1 0 0 0 0 0 0            | $\begin{array}{ c c c c }\hline 2\\2\\\hline \\ & 2\\\hline \\ & 4\\\hline & 4\\\hline & 2\\\hline & 2\\\hline & & 4\\\hline & & 4\\\hline & & 2\\\hline & & & 2\\\hline & & & 2\\\hline & & & & & 2\\\hline & & & & & & 2\\\hline & & & & & & & 2\\\hline & & & & & & & & \\\hline & & & & & & & & \\\hline & & & &$ | ses) + + + + + + + + + + + + + + + + +       | 2, 2, 2<br>1, 1, 1<br>1, 1, 4<br>4, 4, 1<br>1, 1, 8<br>2, 2, 4<br>4, 1, 2<br>1, 4, 2  | $\begin{array}{ c c c }\hline 2,4\\\hline &2,2,2\\1,1,1\\1,1,4\\4,4,1\\\hline &1,1,8\\2,2,4\\4,1,2\\1,4,2\\\hline\end{array}$   | 2, 2<br>1, 1, 1<br>1, 1, 2<br>4, 2, 1<br>1, 1, 2<br>2, 2, 2<br>4, 1, 2<br>1, 4, 2  | $\begin{array}{ c c c c }\hline I_2,I_4\\ \hline &I_2,I_2,I_2\\ I_1,I_1,I_1\\ I_1,I_1,I_4\\ I_4,I_4,I_1\\ \hline &I_1,I_1,I_8\\ I_2,I_2,I_4\\ I_4,I_1,I_2\\ I_1,I_4,I_2\\ \hline \end{array}$   |           |
| 8553 a1 a2 a3 a4 b1 b2 b3 b4 c1                      | $\mathbf{a}$ $1 + \varphi$ $1$ $1 + \varphi$ $1$ $1 + \varphi$ $1 + \varphi$   | $ \begin{array}{c} \varphi \\ \varphi \\ \varphi \\ -1 + \varphi \\ -\varphi \\ \varphi \\ -1 \\ 1 + \varphi \\ 1 \end{array} $  | $ \begin{array}{c} 1 + \varphi \\ \vdots \\ 1 + \varphi \end{array} $                           | $-1$ $4$ $855$ $-11 + 6\varphi$ $-1 + \varphi$ $-176 + 96\varphi$ $-24 - 39\varphi$ $-12611 - 20399\varphi$ $-138 + 48\varphi$ $-8 + 3\varphi$ $-8 + 3\varphi$ $-13336 + 8235\varphi$ $-3 + 2\varphi$   | $\begin{array}{c} 0\\ 1\\ 6a = (27 + 6\varphi) = 5a \cdot 3 \cdot 19a\\ 17 - 12\varphi\\ 0\\ 1022 - 669\varphi\\ 81 + 126\varphi\\ 1036052 + 1676373\varphi\\ -552 + 430\varphi\\ -9 + 6\varphi\\ -692335 + 427890\varphi\\ -4 \end{array}$   | (5 isogeny    1                                | $\begin{array}{ c c c c }\hline 2\\2\\\hline \\ & 2\\\hline \\ & 4\\\hline & 4\\\hline & 2\\\hline & 2\\\hline & & 4\\\hline & & 4\\\hline & & 2\\\hline & & & 2\\\hline & & & 2\\\hline & & & & & 2\\\hline & & & & & & 2\\\hline & & & & & & & 2\\\hline & & & & & & & & \\\hline & & & & & & & & \\\hline & & & &$ | ses) ++ ++ -+ ++ ++ ++ ++                    | 2, 4  2, 2, 2 1, 1, 1 1, 1, 4 4, 4, 1  1, 1, 8 2, 2, 4 4, 1, 2 1, 4, 2 1, 2, 1 2, 1, 2  | $\begin{array}{ c c c c }\hline 2,4\\ \hline & 2,2,2\\ 1,1,1\\ 1,1,4\\ 4,4,1\\ \hline & 1,1,8\\ 2,2,4\\ 4,1,2\\ \hline & 1,4,2\\ \hline & 1,2,1\\ \hline \end{array}$   | $\begin{array}{ c c c }\hline 2,2\\\hline &2,2,2\\\hline &1,1,1\\\hline &1,1,2\\\hline &4,2,1\\\hline &1,1,2\\\hline &2,2,2\\\hline &4,1,2\\\hline &1,4,2\\\hline &1,2,1\\\hline \end{array}$  | $\begin{array}{ c c c c c }\hline I_2,I_4\\ \hline &I_2,I_2,I_2\\ I_1,I_1,I_1\\ I_1,I_1,I_4\\ I_4,I_4,I_1\\ \hline &I_1,I_1,I_8\\ I_2,I_2,I_4\\ I_4,I_1,I_2\\ \hline &I_1,I_4,I_2\\ \hline &I_1,I_2,I_1\\ \hline \end{array}$   |           |
| 855a  a1 a2 a3 a4 b1 b2 b3 b4 c1 c2                  | $\mathbf{a}$ $1 + \varphi$ $1 + \varphi$ $1 + \varphi$ $1 + \varphi$ $1$ $1 + \varphi$ $1$ $1 + \varphi$ $1 + \varphi$ $\varphi$   | $ \begin{array}{c} \varphi \\ \varphi \\ \varphi \\ -1 + \varphi \\ -\varphi \\ \varphi \\ -1 \\ 1 + \varphi \\ 1 \\ -\varphi \end{array} $  | $ \begin{array}{c} 1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\0\\\vdots\\1+\varphi\\\varphi\\1+\varphi\\1+\varphi\end{array} $                             | $ \begin{array}{r} -1\\ 4 \end{array} $ $ 855 $ $ -11 + 6\varphi$ $ -1 + \varphi$ $ -176 + 96\varphi$ $ -24 - 39\varphi$ $ -12611 - 20399\varphi$ $ -138 + 48\varphi$ $ -8 + 3\varphi$ $ -8 + 3\varphi$ $ -13336 + 8235\varphi$ $ -3 + 2\varphi$ $ -36 + 207\varphi$            | $0$ $1$ $5a = (27 + 6\varphi) = 5a \cdot 3 \cdot 19a$ $17 - 12\varphi$ $0$ $1022 - 669\varphi$ $81 + 126\varphi$ $1036052 + 1676373\varphi$ $-552 + 430\varphi$ $-9 + 6\varphi$ $-692335 + 427890\varphi$ $-4$ $-2911 + 1798\varphi$  | (5 isogeny  1 1 1 1 1 0 0 0 0 0 0 0 0 0        | 2   2   2   4   4   2   2   2   2   2   | ses) ++ ++ -+ ++ ++ ++ ++                    | 2, 4  2, 2, 2  1, 1, 1  1, 1, 4  4, 4, 1  1, 1, 8  2, 2, 4  4, 1, 2  1, 4, 2  1, 2, 1  2, 1, 2  1, 2, 1                                     | $\begin{array}{ c c c }\hline 2,4\\\hline &2,2,2\\1,1,1\\1,1,4\\4,4,1\\\hline &1,1,8\\2,2,4\\4,1,2\\1,4,2\\\hline &1,2,1\\2,1,2,1\\\end{array}$   | $\begin{array}{ c c c }\hline 2,2,2\\\hline &1,1,1\\1,1,2\\4,2,1\\\hline\hline &1,1,2\\2,2,2\\4,1,2\\1,4,2\\\hline\hline &1,2,1\\2,1,2,1\\\end{array}$   | $\begin{array}{ c c c c }\hline I_2,I_4\\ \hline &I_2,I_2,I_2\\ I_1,I_1,I_1\\ I_1,I_1,I_4\\ I_4,I_4,I_1\\ \hline &I_1,I_1,I_8\\ I_2,I_2,I_4\\ I_4,I_1,I_2\\ I_1,I_4,I_2\\ \hline &I_1,I_2,I_1\\ I_2,I_1,I_2\\ \hline \end{array}$   |           |
| 855a  a1 a2 a3 a4 b1 b2 b3 b4 c1 c2 d1               | $\mathbf{a}$ $1 + \varphi$ $1 + \varphi$ $1 + \varphi$ $1 + \varphi$ $1$ $1 + \varphi$ $1$ $1 + \varphi$ $1 + \varphi$ $- \varphi$   | $ \begin{array}{c} \varphi \\ \varphi \\ \varphi \\ -1 + \varphi \\ -\varphi \\ \varphi \\ \varphi \\ -1 \\ 1 + \varphi \\ -\varphi \\ -\varphi \\ -\varphi \end{array} $  | $ \begin{array}{c} 1+\varphi\\1+\varphi\\1+\varphi\\0\\\hline\\1\\\varphi\\\varphi\\2\\1+\varphi\\1+\varphi\\1+\varphi\end{array} $                         | $-1$ $4$ $855$ $-11+6\varphi$ $-1+\varphi$ $-176+96\varphi$ $-24-39\varphi$ $-12611-20399\varphi$ $-138+48\varphi$ $-8+3\varphi$ $-8+3\varphi$ $-13336+8235\varphi$ $-3+2\varphi$ $-36+207\varphi$ $-1-3\varphi$  | $\begin{array}{c} 0\\ 1\\ 6a = (27+6\varphi) = 5a \cdot 3 \cdot 19a\\ 17-12\varphi\\ 0\\ 1022-669\varphi\\ 81+126\varphi\\ 1036052+1676373\varphi\\ -552+430\varphi\\ -9+6\varphi\\ -692335+427890\varphi\\ -4\\ -2911+1798\varphi\\ 1+\varphi \end{array}$   | (5 isogeny    1                                | 2 2 2 4 4 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2   | ses)  ++ ++ -+ ++ ++ ++ ++ ++ ++             | 2, 4  2, 2, 2  1, 1, 1  1, 1, 4  4, 4, 1  1, 1, 8  2, 2, 4  4, 1, 2  1, 4, 2  1, 2, 1  2, 1, 2, 1  1, 2, 1                                  | $\begin{array}{ c c c }\hline 2,4\\\hline &2,2,2\\1,1,1\\1,1,4\\4,4,1\\\hline &1,1,8\\2,2,4\\4,1,2\\1,4,2\\\hline &1,2,1\\2,1,2,1\\2,1,2,1\\\hline &1,2,1\\\end{array}$   | $\begin{array}{ c c c }\hline 2,2\\\hline &2,2,2\\1,1,1\\1,1,2\\4,2,1\\\hline &1,1,2\\2,2,2\\4,1,2\\1,4,2\\\hline &1,2,1\\2,1,2,1\\2,1,2\\\hline &1,2,1\\\end{array}$  | $\begin{array}{ c c c c }\hline I_2,I_4\\ \hline &I_2,I_2,I_2\\ I_1,I_1,I_1\\ I_1,I_1,I_4\\ I_4,I_4,I_1\\ \hline &I_1,I_1,I_8\\ I_2,I_2,I_4\\ I_4,I_1,I_2\\ I_1,I_4,I_2\\ \hline &I_1,I_2,I_1\\ I_2,I_1,I_2\\ \hline &I_1,I_2,I_1\\ \hline &I_1,I_2,I_1\\ \hline \end{array}$   |           |
| 855a  a1 a2 a3 a4 b1 b2 b3 b4 c1 c2 d1 d2 d3 d4      | $\mathbf{a}$ $1 + \varphi$ $1 + \varphi$ $1 + \varphi$ $1 + \varphi$ $1$ $1 + \varphi$ $1$ $1 + \varphi$ $1 + \varphi$ $- \varphi$ $\varphi$   | $ \begin{array}{c} \varphi \\ \varphi \\ \varphi \\ -1 + \varphi \\ -\varphi \\ \varphi \\ \varphi \\ -1 \\ 1 + \varphi \\ -\varphi \\ 1 - \varphi \\ -\varphi $ | $ \begin{array}{c} 1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\end{array} $                | $-1$ $4$ $855$ $-11 + 6\varphi$ $-1 + \varphi$ $-176 + 96\varphi$ $-24 - 39\varphi$ $-12611 - 20399\varphi$ $-138 + 48\varphi$ $-8 + 3\varphi$ $-13336 + 8235\varphi$ $-3 + 2\varphi$ $-336 + 207\varphi$ $-1 - 3\varphi$ $-6 - 3\varphi$ $-332 + 207\varphi$ $-51 - 48\varphi$ | $\begin{array}{c} 0\\ 1\\ 6a = (27+6\varphi) = 5a \cdot 3 \cdot 19a \\ 17-12\varphi\\ 0\\ 1022-669\varphi\\ 81+126\varphi\\ 1036052+1676373\varphi\\ -552+430\varphi\\ -9+6\varphi\\ -692335+427890\varphi\\ -4\\ -2911+1798\varphi\\ 1+\varphi\\ -3\varphi\\ 2547-1577\varphi\\ -171-210\varphi \end{array}$ | (5 isogeny    1                                | $\begin{array}{ c c c c }\hline 2\\2\\\hline \\ & 2\\\hline \\ & 4\\\hline & 4\\\hline & 2\\\hline & 4\\\hline & 4\\\hline & 2\\\hline & 2\\\hline & 2\\\hline & 4\\\hline \end{array}$                                     | ses) + + + + + + + + + + + + + + + + +       | 2,4  2,2,2 1,1,1 1,1,4 4,4,1 1,1,8 2,2,4 4,1,2 1,4,2 1,2,1 2,1,2 1,2,1 2,4,2 1,8,1 4,2,4  | $\begin{array}{ c c c }\hline 2,4\\\hline &2,2,2\\1,1,1\\1,1,4\\4,4,1\\\hline &1,1,8\\2,2,4\\4,1,2\\1,4,2\\\hline &1,2,1\\2,1,2,1\\2,1,2,1\\2,4,2\\1,8,1\\4,2,4\\\end{array}$   | $\begin{array}{ c c c }\hline 2,2\\\hline &2,2,2\\\hline &1,1,1\\\hline &1,1,2\\\hline &4,2,1\\\hline\hline &1,1,2\\\hline &2,2,2\\\hline &4,1,2\\\hline &1,4,2\\\hline\hline &1,2,1\\\hline &2,1,2\\\hline\hline &1,2,1\\\hline &2,4,2\\\hline &1,8,1\\\hline &2,2,4\\\hline \end{array}$ | $\begin{array}{ c c c c }\hline I_2,I_4\\ \hline &I_2,I_2,I_2\\ I_1,I_1,I_1\\ I_1,I_1,I_4\\ I_4,I_4,I_1\\ \hline &I_1,I_1,I_8\\ I_2,I_2,I_4\\ I_4,I_1,I_2\\ \hline &I_1,I_4,I_2\\ \hline &I_1,I_2,I_1\\ I_2,I_1,I_2\\ \hline &I_1,I_2,I_1\\ I_2,I_4,I_2\\ \hline \end{array}$   |           |
| 855a a1 a2 a3 a4 b1 b2 b3 b4 c1 c2 d1 d2 d3          | $\mathbf{a}$ $1 + \varphi$ $1 + \varphi$ $1 + \varphi$ $1 + \varphi$ $1$ $1$ $1 + \varphi$ $1$ $1 + \varphi$ $1 + \varphi$ $1$ $2$ $3$ $4$ $4$ $4$ $4$ $4$ $4$ $4$ $4$ $4$ $4$ | $ \begin{array}{c} \varphi \\ \varphi \\ \varphi \\ -1 + \varphi \\ -\varphi \\ \varphi \\ \varphi \\ -1 \\ 1 + \varphi \\ -\varphi \\ 1 - \varphi \\ -\varphi $ | $ \begin{array}{c} 1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\\vdots\\1+\varphi\\\varphi\\\varphi\\1+\varphi\\1+\varphi\\1+\varphi\\1+\varphi\end{array} $ | $-1$ $4$ $855$ $-11 + 6\varphi$ $-1 + \varphi$ $-176 + 96\varphi$ $-24 - 39\varphi$ $-12611 - 20399\varphi$ $-138 + 48\varphi$ $-8 + 3\varphi$ $-8 + 3\varphi$ $-13336 + 8235\varphi$ $-3 + 2\varphi$ $-36 + 207\varphi$ $-1 - 3\varphi$ $-6 - 3\varphi$ $-332 + 207\varphi$    | $\begin{array}{c} 0\\ 1\\ 6a = (27+6\varphi) = 5a \cdot 3 \cdot 19a\\ 17-12\varphi\\ 0\\ 1022-669\varphi\\ 81+126\varphi\\ 1036052+1676373\varphi\\ -552+430\varphi\\ -9+6\varphi\\ -692335+427890\varphi\\ -4\\ -2911+1798\varphi\\ 1+\varphi\\ -3\varphi\\ 2547-1577\varphi \end{array}$                    | (5 isogeny    1                                | 2 2 2 4 4 4 2 2 2 2 2 4 4 4 8 4 4 8 4   | ses) ++ +- ++ ++ ++ ++ ++ ++ ++ +- ++ ++     | 2, 2, 2<br>1, 1, 1<br>1, 1, 4<br>4, 4, 1<br>1, 1, 8<br>2, 2, 4<br>4, 1, 2<br>1, 4, 2<br>1, 2, 1<br>2, 1, 2<br>1, 2, 1<br>2, 4, 2<br>1, 8, 1 | $\begin{array}{ c c c }\hline 2,4\\\hline &2,2,2\\\hline 1,1,1\\\hline 1,1,4\\\hline 4,4,1\\\hline &1,1,8\\\hline 2,2,4\\\hline 4,1,2\\\hline &1,2,1\\\hline &2,1,2\\\hline\hline &1,2,1\\\hline &2,4,2\\\hline &1,2,1\\\hline &2,4,2\\\hline &1,8,1\\\hline \end{array}$ | $\begin{array}{ c c c }\hline 2,2\\\hline &2,2,2\\\hline &1,1,1\\\hline &1,1,2\\\hline &4,2,1\\\hline\hline &1,1,2\\\hline &2,2,2\\\hline &4,1,2\\\hline &1,4,2\\\hline\hline &1,2,1\\\hline &2,1,2\\\hline\hline &1,2,1\\\hline &2,4,2\\\hline &1,8,1\\\hline \end{array}$                | $\begin{array}{ c c c c }\hline I_2,I_4\\ \hline &I_2,I_2,I_2\\ I_1,I_1,I_1\\ I_1,I_1,I_4\\ I_4,I_4,I_1\\ \hline &I_1,I_1,I_8\\ I_2,I_2,I_4\\ I_4,I_1,I_2\\ I_1,I_4,I_2\\ \hline &I_1,I_2,I_1\\ I_2,I_1,I_2\\ \hline &I_1,I_2,I_1\\ I_2,I_4,I_2\\ \hline &I_1,I_2,I_1\\ I_2,I_4,I_2\\ \hline &I_1,I_2,I_1\\ I_2,I_4,I_2\\ I_1,I_8,I_1\\ \hline \end{array}$ |           |

|     | $a_1$         | $a_2$         | $a_3$         | $a_4$                   | $a_6$  | r          | T             | s   | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$  | Kodaira                                  | Isogenies    |
|-----|---------------|---------------|---------------|-------------------------|--|------------|---------------|-----|------------------------------|-----------------------------|--|--|--------------|
| 855 | a             |               |               | 855                     | $5a = (27 + 6\varphi) = 5a \cdot 3 \cdot 19a$  | (5 isogeny | classe        | es) |                              |                             |  |  | 855a         |
| e1  | φ             |               | $1+\varphi$   | $-3-6\varphi$           | $-5-7\varphi$                                  | 0          | 2             | -+  |                              | 3, 2, 1                     | 1, 2, 1  | $I_3, I_2, I_1$                          |              |
| e2  | φ             | 0             | $1+\varphi$   | $-3+9\varphi$           | $-23 - 22\varphi$                              | 0          | 2             | +-  | 6, 1, 2                      | 6, 1, 2                     | 2,1,2  | $I_6, I_1, I_2$                          |              |
| 855 | b             |               |               | 855                     | $5b = (33 - 6\varphi) = 5a \cdot 3 \cdot 19b$  | (5 isogeny | classe        | es) |                              |                             |  |  | <b>855</b> b |
| a1  | φ             | $-1+\varphi$  | $1+\varphi$   | $-5-7\varphi$           | $4+6\varphi$                                   | 1          | 4             | ++  | 2, 2, 2                      | 2, 2, 2                     | 2,2,2  | $I_2, I_2, I_2$                          |              |
| a2  | $\varphi$     | $-1+\varphi$  | $1+\varphi$   | -2arphi                 | $-1-\varphi$                                   | 1          | 2             | -+  | 1, 1, 1                      | 1, 1, 1                     | 1, 1, 1  | $\mathrm{I}_1,\mathrm{I}_1,\mathrm{I}_1$ |              |
| a3  | $\varphi$     | $-1+\varphi$  | $1+\varphi$   | $-80-97\varphi$         | $337 + 588\varphi$                             | 1          | 2             | ++  | 1, 1, 4                      | 1, 1, 4                     | 1, 1, 2  | $I_1, I_1, I_4$                          |              |
| a4  | 1             | $-\varphi$    | 0             | $-63 + 39\varphi$       | $207-126\varphi$                               | 1          | 2             | + - | 4, 4, 1                      | 4, 4, 1                     | 4, 2, 1  | $I_4, I_4, I_1$                          |              |
| b1  | $\varphi$     | 0             | 1             | $-33009 + 20398\varphi$ | $2712425 - 1676373\varphi$                     | 0          | 2             | + + | 1, 1, 8                      | 1,1,8                       | [1, 1, 2]  | $  I_1, I_1, I_8  $                      |              |
| b2  | 1             | $1-\varphi$   | $1+\varphi$   | $-90-49\varphi$         | $-122-431\varphi$                              | 0          | 4             | ++  | 2, 2, 4                      | 2, 2, 4                     | 2, 2, 2  | $I_2, I_2, I_4$                          |              |
| b3  | 1             | $1-\varphi$   | $1+\varphi$   | $-5-4\varphi$           | $-3-7\varphi$                                  | 0          | 4             |     | 4, 1, 2                      | 4, 1, 2                     | 4, 1, 2  | $I_4, I_1, I_2$                          |              |
| b4  | $\varphi$     | $-\varphi$    | $1 + \varphi$ | $-5100 - 8237\varphi$   | $-264445 - 427891\varphi$                      | 0          | 2             | ++  | 1, 4, 2                      | 1, 4, 2                     | 1, 4, 2  | $I_1, I_4, I_2$                          |              |
| c1  | $\varphi$     | $\varphi$     | $1+\varphi$   | $-3-\varphi$            | $-3-3\varphi$                                  | 0          | 2             | + - | 1, 2, 1                      | 1,2,1                       | [1, 2, 1]  | $  I_1, I_2, I_1  $                      |              |
| c2  | $1+\varphi$   | $1-\varphi$   | $\varphi$     | $-128 - 209\varphi$     | $-1112 - 1799\varphi$                          | 0          | 2             | -+  | 2, 1, 2                      | 2, 1, 2                     | 2, 1, 2  | $I_2, I_1, I_2$                          |              |
| d1  | $1+\varphi$   | -1            | $\varphi$     | $-3+\varphi$            | $3-2\varphi$                                   | 0          | 1 4           | ·   | 1, 2, 1                      | 1, 2, 1                     | 1, 2, 1  | $I_1, I_2, I_1$                          |              |
| d2  | $1+\varphi$   | -1            | $\varphi$     | $-8+\varphi$            | $-2+2\varphi$                                  | 0          | 8             | ++  | 2, 4, 2                      | 2, 4, 2                     | 2, 4, 2  | $I_2, I_4, I_2$                          |              |
| d3  | 1             | $\varphi$     | $\varphi$     | $-124 - 208\varphi$     | $971 + 1576\varphi$                            | 0          | 4             | - + | 1, 8, 1                      | 1,8,1                       | 1,8,1  | $I_1, I_8, I_1$                          |              |
| d4  | $1+\varphi$   | -1            | $\varphi$     | $-98+46\varphi$         | $-380 + 209\varphi$                            | 0          | 4             | ++  | 4, 2, 4                      | 4, 2, 4                     | 2, 2, 4  | $I_4, I_2, I_4$                          |              |
| d5  | 1             | $\varphi$     | $\varphi$     | $-654 - 1033\varphi$    | $-12360 - 19938\varphi$                        | 0          | 2             | -+  | 8, 1, 2                      | 8, 1, 2                     | 2, 1, 2  | $I_8, I_1, I_2$                          |              |
| d6  | $  \varphi  $ | $1+\varphi$   |               | $-10064 + 6224\varphi$  | $-458560 + 283403\varphi$                      | 0          | 2             | + - | 2, 1, 8                      | 2, 1, 8                     | 2, 1, 8  | $I_2, I_1, I_8$                          |              |
| e1  | $1+\varphi$   | $-\varphi$    | $\varphi$     | $-8+4\varphi$           | $-11+6\varphi$                                 | 0          | 2             | + - | 3, 2, 1                      | 3, 2, 1                     | [1, 2, 1]  | $  I_3, I_2, I_1  $                      |              |
| e2  | $1+\varphi$   | $-\varphi$    | -             | $7-11\varphi$           | $-44 + 21\varphi$                              | 0          | 2             | -+  | 6, 1, 2                      | 6, 1, 2                     | 2, 1, 2  | $I_6, I_1, I_2$                          |              |
| 869 | a             |               |               | 8                       | $869a = (30 - \varphi) = 11b \cdot 79a$        | (1 isogeny | class)        |     |                              |                             |  |  | 869a         |
| a1  | φ             | 1             | 1             | $-1-2\varphi$           | 1  | 1          | 2             | -+  | 2, 1                         | 2,1                         | 2, 1   | $I_2, I_1$                               |              |
| a2  | φ             | 1             | 1             | $-16 + 8\varphi$        | $23-12\varphi$                                 | 1          | 2             | + - | 4, 2                         | 4, 2                        | 4, 2   | $I_4, I_2$                               |              |
| 869 | b             |               |               | 8                       | $869b = (29 + \varphi) = 11a \cdot 79b$        | (1 isogeny | class)        |     |                              |                             |  |  | <b>869</b> b |
| a1  | $1+\varphi$   | $1-\varphi$   | 1             | $-3+\varphi$            | 1  | 1          | 2             | +-  | 2, 1                         | 2,1                         | 2,1  | $I_2, I_1$                               |              |
| a2  | $1+\varphi$   | $1 - \varphi$ | 1             | $-8-9\varphi$           | $11 + 12\varphi$                               | 1          | 2             | -+  | 4, 2                         | 4,2                         | 4, 2   | $I_4, I_2$                               |              |
| 880 | a             |               |               | 880                     | $a = (32 - 4\varphi) = 2^2 \cdot 5a \cdot 11b$ | (2 isogeny | class         | es) |                              |                             |  |  | 880a         |
| a1  | 0             | 1             | 0             | $-20-27\varphi$         | $48 + 79\varphi$                               |            | 6             | ++  | 1, 8, 2                      | 1,2                         | 1,3,2  | $I_1, IV^*, I_2$                         |              |
| a2  | 0             | 1             | 0             | $-2\varphi$             | $1+\varphi$                                    | 0          | 6             |     | 2, 4, 1                      | 2,1                         | 2, 3, 1  | $I_2, IV, I_1$                           |              |
| a3  | 0             | 1             | 0             | $18\varphi$             | $-27-11\varphi$                                | 0          | 2             |     | 6, 4, 3                      | 6,3                         | 2, 3, 1  | $I_6, IV, I_3$                           |              |
| a4  | 0             | $-1-\varphi$  | 0             | $-940 + 568\varphi$     | $-12804 + 7908\varphi$                         | 0          | 2             | ++  |                              | 3,6                         | 1,3,2  | $I_3, IV^*, I_6$                         |              |
| b1  | 0             | 0             |               | $-63 + 37\varphi$       | $-222 + 136\varphi$                            | 0          | $\frac{1}{1}$ | :'  |                              | 1,2                         | $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ | $\bar{I}_1, \bar{I}V^*, \bar{I}_2$       |              |
| b2  | 0             | 0             |               | $\varphi$               | $-1-\varphi$                                   | 0          | 2             |     |                              | 2,1                         | 2, 1, 1  | $I_2, IV, I_1$                           |              |
|     |               |               | -             | r                       | r  |            |               | 1   | , , ,                        |                             |  | 2/ -/ 1                                  | I            |

|     | $a_1$         | $a_2$        | $a_3$         | $a_4$                     | $a_6$  | r            | T      | s   | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$     | Kodaira                              | Isogenies |
|-----|---------------|--------------|---------------|---------------------------|--|--------------|--------|-----|------------------------------|-----------------------------|-----------|--------------------------------------|-----------|
| 880 | b             |              |               | 880                       | $b = (28 + 4\varphi) = 2^2 \cdot 5a \cdot 11a$ | (2 isogeny   | class  | es) |                              |                             |           |                                      | 880b      |
| a1  | 0             | 1            | 0             | $-47 + 27\varphi$         | $127-79\varphi$                                | 0            | 6      | ++  | 1, 8, 2                      | 1,2                         | 1, 3, 2   | $I_1, IV^*, I_2$                     |           |
| a2  | 0             | 1            | 0             | $-2+2\varphi$             | 2-arphi  | 0            | 6      |     | 2, 4, 1                      | 2,1                         | 2, 3, 1   | $I_2, IV, I_1$                       |           |
| a3  | 0             | 1            | 0             | $18-18\varphi$            | $-38 + 11\varphi$                              | 0            | 2      |     | 6, 4, 3                      | 6, 3                        | 2, 3, 1   | $I_6, IV, I_3$                       |           |
| a4  | 0             | $1+\varphi$  | 0             | $-373-566\varphi$         | $-5269 - 8475\varphi$                          | 0            | 2      | ++  | 3, 8, 6                      | 3,6                         | 1, 3, 2   | $I_3, IV^*, I_6$                     |           |
| b1  | 0             | 0            | 0             | $-26 - 37\varphi$         | $-86-136\varphi$                               | 0            | 2      | + + | 1, 8, 2                      | 1,2                         | [1, 1, 2] | $[\bar{I}_1, \bar{I}V^*, \bar{I}_2]$ |           |
| b2  | 0             | 0            | 0             | $1-\varphi$               | $-2+\varphi$                                   | 0            | 2      |     | 2, 4, 1                      | 2,1                         | 2, 1, 1   | $I_2, IV, I_1$                       |           |
| 881 | a             |              |               |                           | $881a = (19 - 27\varphi) = 881a $              | (1 isogeny o | class) |     |                              |                             |           |                                      | 881a      |
| a1  | 0             | 1            | $1+\varphi$   | $1-\varphi$               | $-\varphi$                                     | 0            | 3      |     | 1                            | 1                           | 1         | $I_1$                                |           |
| a2  | 0             |              | $1+\varphi$   | $-9+9\varphi$             | $-22 + 14\varphi$                              | 0            | 1      |     | 3                            | 3                           | 1         | $I_3$                                |           |
| 881 | b             |              |               |                           | $881b = (8 - 27\varphi) = 881b \tag{2}$        | l isogeny cl | lass)  |     |                              |                             |           |                                      | 881b      |
| a1  | 0             | 1            | $\varphi$     | $\varphi$                 | 0  | 0            | 3      |     | 1                            | 1                           | 1         | $I_1$                                |           |
| a2  | 0             | 1            |               | $-9\varphi$               | $-7-15\varphi$                                 | 0            | 1      |     | 3                            | 3                           | 1         | $I_3$                                |           |
| 891 | a             |              |               | 8                         | $891a = (9 - 27\varphi) = 3^2 \cdot 11a$       | (1 isogeny   | class) |     |                              |                             |           |                                      | 891a      |
| a1  | 1             | -1           | 1             | $-1394 - 2160\varphi$     | $36836 + 59299\varphi$                         | 0            | 4      | ++  | 2,10                         | 2,4                         | 2,4       | $I_2, I_4^*$                         |           |
| a2  | $1 + \varphi$ | 1            | $\varphi$     | $-38-4\varphi$            | $13 + 37\varphi$                               | 0            | 4      | ++  | 4, 8                         | 4, 2                        | 2, 4      | $I_4, I_2^*$                         |           |
| a3  | $1 + \varphi$ | 1            | $\varphi$     | 7-4arphi                  | $4+\varphi$                                    | 0            | 4      |     | 2,7                          | 2,1                         | 2,4       | $I_2, I_1^*$                         | 1         |
| a4  | $\varphi$     | $-1-\varphi$ | 0             | $-31950 + 19683\varphi$   | $2583819 - 1596591\varphi$                     | 0            | 2      | + - | 1, 14                        | 1,8                         | 1,4       | $I_1, I_8^*$                         |           |
| a5  |               | $-1-\varphi$ | 1             | $-148182 - 239756\varphi$ | $41857279 + 67726471\varphi$                   | 0            | 2      | -+  | 1, 8                         | 1,2                         | 1,4       | $I_1, I_2^*$                         |           |
| a6  | $1+\varphi$   | 1            | φ             | $-308 + 131\varphi$       | $-2309 + 1252\varphi$                          | 0            | 2      | ++  | 8,7                          | 8,1                         | 2,4       | $I_8, I_1^*$                         |           |
| 891 | b             |              |               | 8                         | $91b = (18 - 27\varphi) = 3^2 \cdot 11b$       | (1 isogeny   | class  | )   |                              |                             |           |                                      | 891b      |
| a1  | 1             | -1           | 1             | $-3554 + 2160\varphi$     | $96135 - 59299\varphi$                         | 0            | 4      | ++  | 2, 10                        | 2,4                         | 2,4       | $I_2, I_4^*$                         |           |
| a2  | $\varphi$     | $-1-\varphi$ | 1             | $-42+4\varphi$            | $92-41\varphi$                                 | 0            | 4      | ++  | 4, 8                         | 4,2                         | 2,4       | $I_4, I_2^*$                         |           |
| a3  | $\varphi$     | $-1-\varphi$ | 1             | $3+4\varphi$              | $2-5\varphi$                                   | 0            | 4      |     | 2,7                          | 2,1                         | 2,4       | $I_2, I_1^*$                         |           |
| a4  | $1 + \varphi$ | 1            | $1 + \varphi$ | $-12268 - 19683\varphi$   | $974960 + 1576908\varphi$                      | 0            | 2      | -+  | 1, 14                        | 1,8                         | 1,4       | $I_1, I_8^*$                         |           |
| a5  | $1+\varphi$   | 1            | $\varphi$     | $-387938 + 239756\varphi$ | $109195812 - 67486715\varphi$                  | 0            | 2      | + - | 1, 8                         | 1,2                         | 1,4       | $I_1, I_2^*$                         |           |
| a6  | φ             | $-1-\varphi$ | 1             | $-177 - 131\varphi$       | $-880 - 1121\varphi$                           | 0            | 2      | ++  | 8,7                          | 8,1                         | 2,4       | $I_8, I_1^*$                         |           |
| 895 | a             |              |               | 89                        | $5a = (29 + 2\varphi) = 5a \cdot 179a$         | (3 isogeny   | classe | s)  |                              |                             |           |                                      | 895a      |
| a1  | 0             | $-1-\varphi$ | $\varphi$     | $-4+\varphi$              | $5 + \varphi$                                  | 1            | 1      | -+  | 1, 1                         | 1,1                         | 1,1       | $I_1, I_1$                           |           |
| b1  | 0             | 0            | φ             | $1-\varphi$               | $1-\varphi$                                    | 1            | 1      | - + | 1,3                          | 1,3                         | 1,3       | $I_1, I_3$                           |           |
| c1  | 0             | $1+\varphi$  | $\varphi$     | $-12-13\varphi$           | $16 + 21\varphi$                               | 0            | 3      | - + | 1,3                          | 1,3                         | 1,1       | $I_1, I_3$                           |           |
| c2  | 0             | $1+\varphi$  | φ             | $8-33\varphi$             | $-19-2\varphi$                                 | 0            | 1      | -+  | 3,9                          | 3,9                         | 1,1       | $I_3, I_9$                           |           |
| 895 | b             |              |               | 89                        | $5b = (31 - 2\varphi) = 5a \cdot 179b$         | (3 isogeny   | classe | s)  |                              |                             |           |                                      | 895b      |
| a1  | 0             | $1+\varphi$  | $1+\varphi$   | $-4+\varphi$              | $2-2\varphi$                                   | 1            | 1      | +-  | 1,1                          | 1,1                         | 1,1       | $I_1, I_1$                           |           |
|     |               |              |               |                           |  | l            |        | 1   |                              | 1                           |           |                                      |           |

|          | $a_1$                                   | $a_2$                         | $a_3$       | $a_4$                                       | $a_6$  | r                                      | T                                    | s            | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                              | $c_p$  | Kodaira   | Isogenies |
|----------|---|-------------------------------|-------------|---|--|--|--------------------------------------|--------------|------------------------------|--|--|---|-----------|
| 895      | b                                       |                               |             | ۶   | $395b = (31 - 2\varphi) = 5a \cdot 179b$         | (3 isogeny                             | z clas                               | ses)         |                              |  |  |   | 895b      |
| b1       | 0                                       | 0                             | $1+\varphi$ | arphi                                       | 0  | 1                                      | 1                                    | + -          | 1,3                          | 1,3  | 1,3  | $I_1, I_3$  |           |
| c1       | 0                                       | $-1-\varphi$                  |             | $-26 + 15\varphi$                           | $63 - 36\varphi$                                 |  | 3                                    | <u> </u>     | 1,3                          | 1, 3   | $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 $ | $I_1, I_3$  |           |
| c2       | 0                                       | $-1-\varphi$                  | $1+\varphi$ | $-26 + 35\varphi$                           | $5-33\varphi$                                    | 0                                      | 1                                    | +-           | 3, 9                         | 3,9  | 1, 1   | $I_3, I_9$  |           |
| 899      | a                                       |                               |             | 8   | $99a = (17 - 27\varphi) = 29a \cdot 31b$         | (2 isogen                              | v cla                                | sses)        |                              |  |  |   | 899a      |
| a1       | 1                                       | 1                             | 0           | $-3-4\varphi$                               | $\frac{-5-7\varphi}{}$                           | 0                                      | 2                                    | + -          | 2, 1                         | 2,1  | 2,1  | $I_2, I_1$  |           |
| a2       | 1                                       | 1                             |             | $-48 - 69\varphi$                           | $-232 - 380\varphi$                              | 0                                      | 2                                    | ++           | 1,2                          | 1,2  | 1, 2   | $I_1, I_2$  |           |
| b1       | 1                                       |                               | $1+\varphi$ | $-3+2\varphi$                               | $-5+\varphi$                                     | 0                                      | 2                                    | + -          | 2, 1                         | 2,1  | 2,1  | $I_2, I_1$  |           |
| b2       | 1                                       | 1                             | $1+\varphi$ | $-8-8\varphi$                               | $-17 - 16\varphi$                                | 0                                      | 2                                    | - +          | 4, 2                         | 4,2  | 2,2  | $I_4, I_2$  |           |
| 899      | b                                       |                               |             | 8   | $99b = (10 - 27\varphi) = 29b \cdot 31a$         | (2 isogen                              | y clas                               | sses)        |                              |  |  |   | 899b      |
| a1       | 1                                       | 1                             | 0           | $-7+4\varphi$                               | $-12 + 7\varphi$                                 | 0                                      | 2                                    | <u> </u>     | 2, 1                         | 2,1  | 2,1  | $I_2, I_1$  |           |
| a2       | 1                                       | 1                             | 0           | $-117 + 69\varphi$                          | $-612 + 380\varphi$                              | 0                                      | 2                                    | ++           | 1,2                          | 1,2  | 1,2  | $I_1, I_2$  |           |
| b1       | 1                                       | 1                             | $\varphi$   | $-3\varphi$                                 | $-3-2\varphi$                                    | 0                                      | 2                                    | -+           | 2, 1                         | 2,1  | 2, 1   | $I_2, I_1$  |           |
| b2       | 1                                       | 1                             | φ           | $-15 + 7\varphi$                            | $-32 + 15\varphi$                                | 0                                      | 2                                    | +-           | 4, 2                         | 4,2  | 2,2  | $I_4, I_2$  |           |
| 899      | $\mathbf{c}$                            |                               |             |   | $899c = (28 + 5\varphi) = 29a \cdot 31a$         | (1 isoger                              | ıy cla                               | ss)          |                              |  |  |   | 899c      |
| a1       | $1+\varphi$                             | $1-\varphi$                   | 1           | $-3-7\varphi$                               | $4+6\varphi$                                     | 1                                      | 2                                    | T - +        | 2, 1                         | 2,1  | 2, 1   | $I_2, I_1$  |           |
| a2       | $\varphi$                               | $\varphi$                     | 0           | $-12 + 8\varphi$                            | $7-3\varphi$                                     | 1                                      | 2                                    | + -          | 4, 2                         | 4,2  | 4, 2   | $I_4, I_2$  |           |
| 899      | d                                       |                               |             |   | $899d = (33 - 5\varphi) = 29b \cdot 31b$         | (1 isogen                              | ıv cla                               | ss)          |                              |  |  |   | 899d      |
| a1       | $\varphi$                               | 1                             | 1           | $-9+6\varphi$                               | $\frac{10-6\varphi}{}$                           | 1                                      | 2                                    | + -          | 2,1                          | 2,1  | 2, 1   | $I_2, I_1$  |           |
| a2       | $1+\varphi$                             | $1 + \varphi$                 | 1           | $-4-5\varphi$                               | $-3-8\varphi$                                    | 1                                      | 2                                    | -+           | 4,2                          | 4,2  | 4, 2   | $I_4, I_2$  |           |
| 000      | _                                       |                               |             |   | 000 (00) 0 7 2 0                                 | (2.                                    | ,                                    | `            |                              |  |  |   | 0000      |
| 900      |   | 1                             | 1           | 09 09                                       | ` '  | (2 isogeny c                           | 1                                    | <del>´</del> | 0.6.10                       | 0.0.4  | 0.0.4  | т т т*  | 900a      |
| a1<br>a2 |   | $-1 - \varphi$ $-1 - \varphi$ | 1<br>1      | $-93 - 93\varphi$ $7 + 7\varphi$            | $-394 - 525\varphi$ $-34 - 45\varphi$            | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{vmatrix} 4\\2 \end{vmatrix}$ | ++           | 2, 6, 10 $4, 3, 8$           | $\begin{bmatrix} 2, 6, 4 \\ 4, 3, 2 \end{bmatrix}$       | 2, 2, 4<br>2, 1, 2                                       | $I_2, I_6, I_4^* $ $I_4, I_3, I_2^*$                                    |           |
| a3       |   | $-1-\varphi$                  | 1           | $-1668 - 1668\varphi$                       | $35516 + 47355\varphi$                           |  | 4                                    | + +          | 6, 2, 18                     | 6, 2, 12   | 2, 1, 2<br>2, 2, 4                                       | $I_6, I_2, I_{12}^*$  |           |
| a4       |   | $-1-\varphi$                  | 1           | $-68-68\varphi$                             | $956 + 1275\varphi$                              | 0                                      | 2                                    |              | 12, 1, 12                    | 12, 1, 6   | 2, 1, 2  | $I_{12}, I_1, I_6^*$  |           |
| a5       |   | $-1-\varphi$                  | 1           | $-26668 - 26668\varphi$                     | $2255516 + 3007355\varphi$                       | 0                                      | 2                                    |              | 3, 4, 12                     | 3, 4, 6  | 1, 2, 2  | $I_3, I_4, I_6^*$   |           |
| a6       |   | $-1-\varphi$                  | 1           | $-343 - 343\varphi$                         | $2906 + 3875\varphi$                             | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 2                                    |              | 1, 12, 8                     | 1, 12, 2   | 1, 2, 2  | $I_1, I_{12}, I_2^*$  |           |
| a7<br>a8 |   | $-1 - \varphi$ $-1 - \varphi$ | 1<br>1      | $-2268 - 2268\varphi$ $-1443 - 1443\varphi$ | $ 8156 + 10875\varphi \\ -27934 - 37245\varphi $ | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{vmatrix} 2\\2 \end{vmatrix}$ | ++           | 3, 1, 30 $1, 3, 14$          | $\begin{vmatrix} 3, 1, 24 \\ 1, 3, 8 \end{vmatrix}$      | $1, 1, 4 \\ 1, 1, 4$                                     | $ \begin{vmatrix} I_3, I_1, I_{24}^* \\ I_1, I_3, I_8^* \end{vmatrix} $ |           |
| b1       | 1 · · · · · · · · · · · · · · · · · · · | $-\frac{1}{0}$                | 0           | -28   | 272  |  | 10                                   | <u> </u>     | 10, 5, 6                     | $\begin{array}{c c} 1, 5, 5 \\ \hline 10, 5 \end{array}$ | 10,5,2   | $  I_{10}, I_{5}, I_{0}^{*}  $  |           |
| b2       | 1                                       | 0                             | 0           | -828  | 9072   |  | 10                                   | + +          | 5, 10, 6                     | 5,10   | 5, 10, 2   | $I_{5}, I_{10}, I_{0}^{*}$  |           |
| b3       | 1                                       | 0                             | 0           | -53   | -153   | 0                                      | 2                                    | ++           | 1, 2, 6                      | 1,2  | 1, 2, 2  | $I_1, I_2, I_0^*$   |           |
| b4       | 1                                       | 0                             | 0           | -3  | -3   | 0                                      | 2                                    |              | 2, 1, 6                      | 2,1  | 2, 1, 2  | $I_2, I_1, I_0^*$   |           |
|          |   |                               |             |   |  |  |                                      |              |                              |  |  |   |           |

|  | <i>a.</i>                   |               | 7.0               | a.         | <i>a.</i>           | 0.0                                     |  | T                                    | 1 6      | $\operatorname{ord}(\Delta$ | $ord_{-}(j)$ | $c_p$  | Kodaira  | Isogenies |
|--|-----------------------------|---------------|-------------------|------------|---------------------|---|--|--------------------------------------|----------|-----------------------------|--------------|--|--|-----------|
|  | $a_1$                       |               | $\frac{n_2}{n_2}$ | $a_3$      | $a_4$               | $a_6$                                   |  | 1                                    | 3        | Oru(\(\Delta\)              | )   Old_(J)  | $c_p$  | Rodaira  | Isogemes  |
| 905a                                       | ì                           |               |                   |            | 90                  | $5a = (11 - 27\varphi) = 5a \cdot 181b$ | (3 isogeny class                       | ses)                                 |          |                             |              |  |  | 905a      |
| a1   | 0                           | (             | φ                 | $\varphi$  | 0                   | 0                                       | 1                                      | 1                                    | -        | - 1,1                       | 1,1          | 1,1  | $I_1, I_1$   |           |
| b1   | 0                           | -1+6          | $\varphi$         | $\varphi$  | $-10+7\varphi$      | $23-13\varphi$                          | 1                                      | 7                                    | Ī —      | -1,7                        | 1,7          | [1,7]  | $I_1, I_7$   |           |
| b2   | 0                           | -1 + 9        | $\varphi_{-}$     | $\varphi$  | $-250 - 213\varphi$ | $-3440 - 1402\varphi$                   | _                                      | 1                                    | –        | - 7,1                       | 7,1          | 1,1  | $I_7, I_1$   |           |
| c1   | $\varphi$                   |               | 1                 | 0          | $-19 + 5\varphi$    | $23-12\varphi$                          | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 4                                    |          |                             | 4,1          | 4,1  | $I_4, I_1$   |           |
| c2<br>c3                                   | $\varphi$                   |               | 1<br>1            | $0 \\ 0$   | -4                  | $-5-\varphi$                            | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{vmatrix} 4\\2 \end{vmatrix}$ | 1        |                             | 2, 2 $1, 1$  | $\begin{bmatrix} 2, 2 \\ 1, 1 \end{bmatrix}$ | $\begin{matrix} I_2,I_2\\I_1,I_1\end{matrix}$      |           |
| $\begin{vmatrix} c_3 \\ c_4 \end{vmatrix}$ | $\varphi$                   |               | 1                 | 0          | $-69-5\varphi$      | $-253 - 34\varphi$                      |  | $\frac{2}{2}$                        |          | ,                           | 1,1 $1,4$    | $\begin{bmatrix} 1,1\\1,4 \end{bmatrix}$     | $I_1, I_1 \\ I_1, I_4$                             |           |
|  | <u> </u>                    |               |                   |            |                     |   |  |                                      |          | -, -                        | _, _, _      | _, _, _                                      | -1,-4  |           |
| 905h                                       | )                           |               |                   |            | 90                  | $5b = (16 - 27\varphi) = 5a \cdot 181a$ | (3 isogeny class                       | ses)                                 |          |                             |              |  |  | 905b      |
| a1   | 0                           | $1-\varsigma$ | $\varphi$ 1       | $+\varphi$ | 0                   | -arphi                                  | 1                                      | 1                                    | -        | -1, 1                       | 1, 1         | 1, 1   | $\mathrm{I}_1,\mathrm{I}_1$                        |           |
| b1   | 0                           |               |                   | $+\varphi$ | $-3-7\varphi$       | $10+12\varphi$                          | 1                                      | 7                                    | Ī —      | -, -                        | 1,7          | [1,7]  | $\bar{\mathrm{I}}_1, \bar{\mathrm{I}}_7$           | [         |
| b2   | 0                           |               | $\varphi_{-1}$    | $+\varphi$ | $-463 + 213\varphi$ | $-4842 + 1401\varphi$                   | _                                      | 1                                    | <u> </u> | - $7, 1$                    | 7,1          | 1,1  | $I_7, I_1$   |           |
|  | $1+\varphi$                 |               | •                 | 0          | $-14-5\varphi$      | $11 + 12\varphi$                        | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 4                                    |          |                             | 4, 1         | 4,1  | $I_4, I_1$   |           |
|  | $1+\varphi$                 |               | •                 | 0          | -4                  | $-6+\varphi$                            | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\frac{1}{2}$                        |          |                             | 2, 2         | $\begin{bmatrix} 2,2 \\ 1 \end{bmatrix}$     | $I_2, I_2$   |           |
|  | $1 + \varphi$ $1 + \varphi$ |               | ,                 | $0 \\ 0$   | $-74+5\varphi$      | $0 \\ -287 + 34\varphi$                 | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{vmatrix} 2\\2 \end{vmatrix}$ | 1        | ,                           | 1, 1 $1, 4$  | $\begin{array}{ c c } 1,1\\1,4 \end{array}$  | $\begin{matrix} I_1, I_1 \\ I_1, I_4 \end{matrix}$ |           |
| 01   | Ι   Ψ                       | 1             | Υ                 |            | ν τ τ ο φ           | 201   019                               |  |                                      | '        | 1, 1, 1                     | 1, 1         | 1, 1   | 11,14  |           |
| 909a                                       | ì                           |               |                   |            | 90                  | $09a = (15 - 27\varphi) = 3 \cdot 101a$ | (2 isogeny class                       | es)                                  |          |                             |              |  |  | 909a      |
| a1   | $1+\varphi$                 |               | 0                 | 1          | $-\varphi$          | 0                                       | 1                                      | 1                                    | -        | - 1,2                       | 1, 2         | 1, 2   | $I_1, I_2$   |           |
| b1   | $1+\varphi$                 | $1+\varsigma$ | $\varphi$         | 1          | $-31 + 13\varphi$   | $-83 + 26\varphi$                       | 0                                      | 1                                    |          | -1, 2                       | 1, 2         | 1,2  | $I_1, I_2$   |           |
| 909t                                       | )                           |               |                   |            | 9                   | $09b = (12 - 27\varphi) = 3 \cdot 101b$ | (2 isogeny classe                      | es)                                  |          |                             |              |  |  | 909b      |
| a1   | $\varphi$                   | 1 - 0         | $\varphi$         | 1          | 0                   | 0                                       |  | T -                                  | _        | - 1,2                       | 1, 2         | 1, 2   | $I_1, I_2$   |           |
| b1   | $\varphi$                   |               | <u>·</u><br>φ     | 0          | $-21 - 10\varphi$   | $-48-46\varphi$                         | 0                                      | <u>'</u>                             | ·        |                             | 1,2          | 1,2  | $I_1, I_2$   | <u>-</u>  |
| 916a                                       | ì                           |               |                   |            |                     | $916a = (34 - 6\varphi) = 2 \cdot 229a$ | (1 isogeny class                       | 3)                                   |          |                             |              |  |  | 916a      |
| a1   | $\varphi$                   |               | 0 1               | $+\varphi$ | $2-2\varphi$        | $-\varphi$                              | 1                                      | 1                                    | -        | - 4,1                       | 4,1          | 2,1  | $I_4, I_1$   |           |
| 916h                                       | )                           |               |                   |            |                     | $916b = (28 + 6\varphi) = 2 \cdot 229b$ | (1 isogeny class                       | 1)                                   |          |                             |              |  |  | 916b      |
| a1   | $1 + \varphi$               | <u> </u>      | $\varphi$         | $\varphi$  | 1                   | 0                                       | 1                                      | 1                                    | _        | - 4,1                       | 4,1          | 2, 1   | $I_4, I_1$   |           |
| 919a                                       | ì                           |               |                   |            |                     | $919a = (32 - 3\varphi) = 919a$         | (2 isogeny classes                     | )                                    |          |                             |              |  |  | 919a      |
|  | $1+\varphi$                 |               | $\varphi$         | $\varphi$  | $-1-\varphi$        | 0                                       | 1                                      | 3                                    | _        | _                           | 1            | 1  | $I_1$  |           |
| a2   | $1+\varphi$                 |               | $\varphi_{\perp}$ | $\varphi$  | $4-\varphi$         | $-5-8\varphi$                           | 1                                      | 1                                    | _        | - 3                         | 3            | 3  | $I_3$  |           |
|  | $1+\varphi$                 |               |                   | 1          | $1+2\varphi$        | $\varphi$                               |  | 2                                    | Ī —      |                             | 1            | 1  | $I_1$  |           |
| b2   | $1+\varphi$                 | $1+\varsigma$ | $\varphi$         | 1          | $-4+7\varphi$       | $-6+7\varphi$                           | 0                                      | 2                                    | +        | - 2                         | 2            | 2  | $I_2$  |           |

|  | $a_1$                 | $a_2$          | a             | 3                 | $a_4$                            | $a_6$  | r                                      | T              | s        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$                                       | Kodaira   | Isogenies |
|--|-----------------------|----------------|---------------|-------------------|----------------------------------|--|--|----------------|----------|------------------------------|-----------------------------|---|---|-----------|
|  |                       |                |               |                   |                                  |  |  |                |          |                              |                             |   |   |           |
| 9191                                     | )                     |                |               |                   |                                  | $919b = (29 + 3\varphi) = 919b$                            | (2 isogeny classes                     | s)             |          |                              |                             |   |   | 919b      |
| a1                                       | $\varphi$             |                | $1+\varsigma$ |                   | $-1-\varphi$                     | $-\varphi$   | 1                                      |                |          | 1                            | 1                           | 1   | $I_1$   |           |
| a2                                       | <del></del> <i>φ</i>  |                | 1 + 9         |                   | $4-\varphi$                      | $-13+7\varphi$   | 1                                      | -'             |          | 3                            | 3                           | 3   | I <sub>3</sub>  |           |
| b1 b2                                    | $\varphi$             | $\varphi$      |               | 0<br>0            | $arphi \ -4arphi$                | $0 \\ -5-6 \varphi$  | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ |                | +-       | $\frac{1}{2}$                | $\frac{1}{2}$               | $\begin{vmatrix} 1\\2 \end{vmatrix}$        | $egin{array}{c} I_1 \ I_2 \end{array}$                |           |
| 02                                       | φ                     | φ              |               | 0                 | $-4\varphi$                      | $-3-0\varphi$  | 0                                      |                | - +      |                              |                             |   | 12  |           |
| 931a                                     | <b>a</b>              |                |               |                   |                                  | $931a = (7 - 28\varphi) = 19a \cdot 7$                     | (1 isogeny class                       | :)             |          |                              |                             |   |   | 931a      |
| al                                       | 0                     | $1+\varphi$    |               | 1                 | $-7+6\varphi$                    | $\frac{301a - (1 - 20\varphi) - 19a^{-1}}{9 - 5\varphi}$   |  | <del>´</del>   |          | 2,1                          | 2,1                         | 2, 1  | $I_2, I_1$  |           |
| 41                                       |                       | - 1 γ          |               |                   | . 1 09                           | υ σγ   | 1                                      | 1 -            |          | -, -                         | 2, 1                        | 2, 1  | 12,11   |           |
| 9311                                     | )                     |                |               |                   |                                  | $931b = (21 - 28\varphi) = 19b \cdot 7$                    | (1 isogeny class                       | s)             |          |                              |                             |   |   | 931b      |
| al                                       |                       | $-1-\varphi$   |               | 1                 | $-2-4\varphi$                    | $\frac{6+10\varphi}{}$                                     | 1                                      | <del>í –</del> | T        | 2, 1                         | 2,1                         | 2, 1  | $I_2, I_1$  |           |
|  |                       | - r            |               |                   | r                                | · r  | -                                      |                |          | -, -                         | -, -                        | _,_   | -2;-1   |           |
| 944a                                     | a                     |                |               |                   | g                                | $044a = (20 - 28\varphi) = 2^2 \cdot 59a$                  | (1 isogeny clas                        | ss)            |          |                              |                             |   |   | 944a      |
| a1                                       | 0                     | $1+\varphi$    |               | 0                 | $-1+3\varphi$                    | $-1+2\varphi$  | 0                                      | <del></del>    | + -      | 4, 1                         | 1                           | 1, 1  | $IV, I_1$   |           |
| a2                                       | 0                     | $1+\varphi$    |               | 0                 | $-1-2\varphi$                    | $-5+\varphi$   | 0                                      |                | -+       |                              | 2                           | 1, 2  | $IV^*, I_2$   |           |
|  |                       |                |               |                   |                                  |  | ·                                      |                |          |                              |                             |   |   |           |
| 9441                                     |                       |                |               |                   |                                  | $944b = (8 - 28\varphi) = 2^2 \cdot 59b$                   | (1 isogeny class                       | s)             |          |                              |                             |   |   | 944b      |
| a1                                       |                       | $-1-\varphi$   |               | 0                 | $1-\varphi$                      | 0  | 0                                      |                | -+       |                              | 1                           | 1,1   | $IV, I_1$   |           |
| a2                                       | 0                     | $-1-\varphi$   |               | 0                 | $-4+4\varphi$                    | $-4\varphi$  | 0                                      | 2              | + -      | 8, 2                         | 2                           | 1,2   | $IV^*, I_2$   |           |
| 956a                                     | <b>a</b>              |                |               |                   | 0                                | $56a = (32 - 2\varphi) = 2 \cdot 239a$                     | (3 isogeny class                       | og)            |          |                              |                             |   |   | 956a      |
| a1                                       | $\frac{1}{1+\varphi}$ | $1-\varphi$    |               | <br>1             | $\frac{9}{-5+\varphi}$           | $\frac{30a = (32 - 2\varphi) = 2 \cdot 239a}{2 - \varphi}$ | (3 Isogeny class                       | т <sup>*</sup> |          | 2,1                          | 2,1                         | 2, 1  | Т. Т.   | 330a      |
|  |                       |                |               |                   |                                  |  |  | -              | - +      |                              | :                           |   | $\begin{bmatrix} I_2, I_1 \\ I_2, I_3 \end{bmatrix}$  |           |
| b1                                       | <del>-</del>          | 1<br>          |               | 0<br><del>-</del> | $-3-5\varphi$                    | $2+3\varphi$   | 1                                      | -'             | - +      |                              | 4,1                         | $\begin{vmatrix} 4,1 \\ -1,2 \end{vmatrix}$ | $\begin{bmatrix} I_4, I_1 \\ I_4 & I_1 \end{bmatrix}$ |           |
| $\begin{bmatrix} c1 \\ c2 \end{bmatrix}$ | $1 \\ 1 + \varphi$    | $-1 \\ 1$      |               | 1<br>o            | $-54 - 85\varphi$ $-3 + \varphi$ | $-268 - 434\varphi$ $-4 + \varphi$                         | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 1              | ++       | $1, 2 \\ 2, 1$               | $1, 2 \\ 2, 1$              | $\begin{array}{ c c } 1,2\\2,1\end{array}$  | $\begin{matrix} I_1,I_2\\I_2,I_1\end{matrix}$         |           |
| 02                                       | Ι Ι Ψ                 |                |               |                   | 0 1 9                            | Τ 1 Υ  |  |                | '        | 2, 1                         | 2,1                         | 2, 1  | 12,11   |           |
| 956l                                     | )                     |                |               |                   | 9                                | $56b = (30 + 2\varphi) = 2 \cdot 239b$                     | (3 isogeny classe                      | es)            |          |                              |                             |   |   | 956b      |
| a1                                       | $\varphi$             | 1              |               | 1                 | $-3-2\varphi$                    | $1+\varphi$  | 1                                      | - ´            | +-       | 2,1                          | 2,1                         | 2,1   | $I_2, I_1$  |           |
| b1                                       | $1+\varphi$           | $1-\varphi$    | 1+0           | <br>0             | $-8 + 3\varphi$                  | $5-4\varphi$   |  | -'<br>  1      | + -      |                              | $\frac{1}{4}, \frac{1}{4}$  | 4,1   | $oxed{I_4,I_1}$                                       |           |
| c1                                       | 1                     | <u>'</u><br>-1 | '             | -<br><br>1        | $-139 + 85\varphi$               | $-702 + 434\varphi$  |  | - '            | <u> </u> |                              | 1, 2                        | $\frac{1}{1,2}$                             | $I_1, I_2$  |           |
| c2                                       | _                     | $-1-\varphi$   |               | 1                 | $-2-\varphi$                     | -1   | 0                                      | 1              | -+       |                              | 2, 1                        | 2, 1  | $I_1, I_2$ $I_2, I_1$                                 |           |
|  |                       |                |               |                   |                                  |  | 1                                      |                |          |                              |                             |   |   |           |
| 961a                                     | <b>1</b>              |                |               |                   |                                  | $961a = (31) = 31a \cdot 31b$                              | (2 isogeny classes)                    | )              |          |                              |                             |   |   | 961a      |
| a1                                       | 0                     | $\varphi$      |               | 1                 | 2                                | $-2+\varphi$   | 1                                      | 1              |          | 3, 1                         | 3,1                         | 3,1   | $I_3, I_1$  |           |
| b1                                       | 0                     | 0              | 1 + 9         | 0                 | $5+8\varphi$                     | $-3-\varphi$   | 0                                      | 1              | j        | 1,5                          | 1,5                         | 1,1   | $I_1, I_5$  |           |
|  |                       |                |               |                   |                                  |  | I                                      |                | 1        |                              | 1                           | l .   | 1   |           |

|     | I  |                  |             |                            |  |              |            | 1                |                              |   |  | T  |           |
|-----|--|------------------|-------------|----------------------------|--|--------------|------------|------------------|------------------------------|---|--|--|-----------|
|     | $a_1$  | $a_2$            | $a_3$       | $a_4$                      | $a_6$  | r            | T          | s                | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                                 | $c_p$  | Kodaira  | Isogenies |
| 961 | b  |                  |             | 96                         | $1b = (34 - 5\varphi) = 31a^2 $ (3 isog        | eny classes) | )          |                  |                              |   |  |  | 961b      |
| a1  | φ  | 0                | 0           | $-16-7\varphi$             | $-4+24\varphi$                                 | 1            | 4          | -+               | 7                            | 1   | 4  | $I_1^*$  |           |
| a2  | 1  | 0                | 0           | $-984 + 594\varphi$        | $-13925 + 8580\varphi$                         | 1            | 4          | ++               | 8                            | 2   | 4  |  |           |
| a3  | $\varphi$  | 0                | 0           | $-836 - 1017\varphi$       | $-13992 - 21096\varphi$                        | 1            | 4          | ++               | 10                           | 4   | 4  | $ar{	ext{I}_4^*}$                                      |           |
| a4  | $1+\varphi$  | $-\varphi$       | 0           | $-107217 + 66265\varphi$   | $-15908726 + 9832128\varphi$                   | 1            | 2          | + -              | 7                            | 1   | 2  | $\mathrm{I}_1^*$                                       |           |
| a5  | $\varphi$  | 0                | 0           | $-1396 - 652\varphi$       | $-5575 - 27054\varphi$                         | 1            | 2          | + -              | 14                           | 8   | 4  | $I_8^*$  |           |
| a6  | 1  | $1-\varphi$      | $\varphi$   | $-515571 - 834228\varphi$  | $-270972315 - 438442397\varphi$                | 1            | 2          | - +              | 8                            | 2   | 2  | I <sub>2</sub>   |           |
| b1  | 0  | $1+\varphi$      | 1           | $-2+2\varphi$              | 1-arphi  | 1            | 1          | ++               | 2                            | 0   | 1  | II   |           |
| c1  | 0  | $-1+\varphi$     | 1           | $-95 + 42\varphi$          | $-332 + 192\varphi$                            | 0            | 1          | ++               | 8                            | 0   | 1  | $IV^*$   |           |
| 961 | $\mathbf{c}$   |                  |             | 96                         | $1c = (29 + 5\varphi) = 31b^2$ (3 isog         | eny classes) |            |                  |                              |   |  |  | 961c      |
| a1  | $1+\varphi$  | $-\varphi$       | 0           | $-23+7\varphi$             | $20 - 24\varphi$                               | 1            | 4          | + -              | 7                            | 1   | 4  | $\mathrm{I}_1^*$                                       |           |
| a2  | 1  | 0                | 0           | $-390-594\varphi$          | $-5345 - 8580\varphi$                          | 1            | 4          | ++               | 8                            | 2   | 4  |  |           |
| a3  | $1+\varphi$  | $-\varphi$       | 0           | $-1853 + 1017\varphi$      | $-35088 + 21096\varphi$                        | 1            | 4          | ++               | 10                           | 4   | 4  | ${\rm I}_4^{\overline{*}}$                             |           |
| a4  | $\varphi$  | 0                | 0           | $-40952 - 66265\varphi$    | $-6076598 - 9832128\varphi$                    | 1            | 2          | -+               |                              | 1   | 2  | $\mathrm{I}_1^*$                                       |           |
| a5  | $1+\varphi$  | $-\varphi$       | 0           | $-2048 + 652\varphi$       | $-32629 + 27054\varphi$                        | 1            | 2          | -+               | 14                           | 8   | 4  | I <sub>8</sub>   |           |
| a6  | 1  |                  | $1+\varphi$ | $-1349799 + 834227\varphi$ | $-709414712 + 438442396\varphi$                | 1            | 2          | + -              | 8                            | 2   | 2  | I <sub>2</sub>   |           |
| b1  | 0  | $-1-\varphi$     | 1           | -1                         | $1+2\varphi$                                   | 1            | 1          | ++               | 2                            | 0   | 1  | II   |           |
| c1  | 0  | $-\varphi$       | 1           | $-53-42\varphi$            | $-140 - 192\varphi$                            | 0            | 1          | ++               | 8                            | 0   | 1  | $IV^*$   |           |
| 964 | a  |                  |             | 964a                       | $a = (10 - 28\varphi) = 2 \cdot 241a$ (3 is    | ogeny class  | es)        |                  |                              |   |  |  | 964a      |
| a1  | φ  | $-\varphi$       | 1           | $-1-\varphi$               | φ  | 1            | 1          | + -              | 1, 1                         | 1,1   | 1,1  | $I_1, I_1$   |           |
| b1  | $\begin{vmatrix} 1 + \varphi \end{vmatrix}$                                  | $1+\varphi$      | $\varphi$   | $-1+3\varphi$              | $1+2\varphi$                                   | 1            | ' ·<br>  1 | <u></u><br>  + - | 7, 1                         | $\begin{array}{c c} -7 & -7 & -7 & -7 & -7 & -7 & -7 & -7 $ | [-7, 1]  | $I_7, I_1$   | <u>-</u>  |
| c1  | $\begin{vmatrix} \cdot & \cdot & \cdot & \cdot \\ & & \varphi \end{vmatrix}$ | $1+\varphi$      |             | $-4+5\varphi$              | 5-2arphi                                       |              | ' ·<br>  3 | <u></u><br>  + - | 1, 1                         | 1,1   | $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ | $\overline{\mathrm{I}}_{1},\overline{\mathrm{I}}_{1}$  | <u>-</u>  |
| c2  | $\varphi$  | $1+\varphi$      | 0           | $-24+10\varphi$            | $-52+16\varphi$                                | 0            | 3          | + -              | 3,3                          | 3, 3  | 3, 3   | $I_3, I_3$   |           |
| c3  | $1+\varphi$  | -1               | $\varphi$   | $-2005 - 2684\varphi$      | $-55913 - 86630\varphi$                        | 0            | 1          | + -              | 1, 1                         | 1,1   | 1,1  | $I_1, I_1$   |           |
| 964 | b  |                  |             | 964b                       | $\phi = (18 - 28\varphi) = 2 \cdot 241b$ (3 is | ogeny classe | es)        |                  |                              |   |  |  | 964b      |
| a1  | $1+\varphi$  | -1               | 1           | -2                         | $1-\varphi$                                    | 1            | 1          | -+               | 1, 1                         | 1,1   | 1,1  | $I_1, I_1$   |           |
| b1  | $\varphi$  | $\varphi$        | $\varphi$   | $-1-\varphi$               | $1-2\varphi$                                   | 1            | 1          |                  | 7, 1                         | 7,1   | 7,1  | $\overline{\mathrm{I}}_{7}, \overline{\mathrm{I}}_{1}$ |           |
| c1  | $1+\varphi$  | $-1+\varphi$     | $\varphi$   | $-1-4\varphi$              |  | 0            | 3          |                  | 1,1                          | 1,1   | 1,1  | $\overline{\mathrm{I}}_{1},\overline{\mathrm{I}}_{1}$  | <u> </u>  |
| c2  |  | $-1+\varphi$     | $\varphi$   | $-16-9\varphi$             | $-29-32\varphi$                                | 0            | 3          | -+               |                              | 3,3   | 3,3  | $I_3, I_3$   |           |
| c3  | $\varphi$  | $-\dot{\varphi}$ | $1+\varphi$ | $-4688 + 2682\varphi$      | $-142543 + 86629\varphi$                       | 0            | 1          | -+               | 1,1                          | 1,1   | 1,1  | $I_1, I_1$   |           |
| 971 | a  |                  |             | 97                         | $71a = (17 - 28\varphi) = 971a$ (1 iso         | ogeny class) |            |                  |                              |   |  |  | 971a      |
| a1  | φ  | $1-\varphi$      | $\varphi$   | $-6-11\varphi$             | $11 + 18\varphi$                               | 0            | 3          |                  | 1                            | 1   | 1  | $I_1$  |           |
| a2  | 1  | $\varphi$        | 1           | $-136 + 84\varphi$         | $-697 + 434\varphi$                            | 0            | 1          |                  | 3                            | 3   | 1  | $I_3$  |           |
|     |  |                  |             |                            |  |              |            |                  |                              |   |  |  |           |

|          |               |  |     |                       |  |               |  |            | 1                                      | Imi                                  |              | 1/ 4 )                       | 1 ( 1)   | I  | TZ 1 :  | T .           |
|----------|---------------|--|-----|-----------------------|--|---------------|--|------------|--|--------------------------------------|--------------|------------------------------|--|--|---|---------------|
|          | $a_1$         | <i>a</i> :   | 2   | $a_3$                 | a.   | 4             | $a_6$  |            | r                                      | T                                    | s            | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                        | $c_p$  | Kodaira   | Isogenies     |
| 971      | b             |  |     |                       |  | 9             | $971b = (11 - 28\varphi) = 971b$                                 | (1 isoger  | ny cl                                  | ass)                                 |              |                              |  |  |   | 971b          |
| a1       | $1+\varphi$   | 0  | ) 1 | $1+\varphi$           | $-17+9\varphi$   |               | $\frac{29-19\varphi}{}$  |            | 0                                      | 3                                    |              | 1                            | 1  | 1  | $I_1$   |               |
| a2       | 1             | $1-\varphi$  | 2   | 1                     | $-52 - 84\varphi$  | ρ             | $-263 - 434\varphi$  |            | 0                                      | 1                                    |              | 3                            | 3  | 1  | $I_3$   |               |
| 979      | a             |  |     |                       |  | 979           | $0a = (13 - 28\varphi) = 11a \cdot 89a$                          | (1 isos    | geny                                   | class                                | )            |                              |  |  |   | 9 <b>7</b> 9a |
| a1       | $1+\varphi$   | $-\varphi$   | 2   | $\varphi$             | $-32-44\varphi$  |               | $-107 - 178\varphi$  |            | 0                                      | 2                                    | ++           | 4, 1                         | 4, 1   | 2, 1   | $\mathrm{I}_4,\mathrm{I}_1$   |               |
| a2       | $1+\varphi$   | $-\varphi$   | 2   | $\varphi$             | $-27 - 49\varphi$  | ρ             | $-90 - 173\varphi$   |            | 0                                      | 2                                    | -+           | 8,2                          | 8,2  | 2, 2   | $I_8, I_2$  |               |
| 979      | b             |  |     |                       |  | 979           | $9b = (15 - 28\varphi) = 11b \cdot 89b$                          | (1 isog    | geny                                   | class)                               | )            |                              |  |  |   | 979b          |
| a1       | $\varphi$     |  |     | $1+\varphi$           | $-75 + 42\varphi$  |               | $-285 + 177\varphi$  |            | 0                                      | 2                                    | ++           | 4, 1                         | 4, 1   | 2, 1   | $I_4, I_1$  |               |
| a2       | φ             |  | ) 1 | $1+\varphi$           | $-75 + 47\varphi$  | ρ             | $-263 + 172\varphi$  |            | 0                                      | 2                                    | +-           | 8,2                          | 8,2  | 2, 2   | $I_8, I_2$  |               |
| 979      | $\mathbf{c}$  |  |     |                       |  | 979           | $c = (35 - 6\varphi) = 11a \cdot 89b$                            | (2 isoge   | ny c                                   | lasses                               | )            |                              |  |  |   | 979c          |
| a1       | $\varphi$     |  |     | $1+\varphi$           |  |               | $-2\varphi$  |            | 1                                      | 2                                    | ++           | 2, 1                         | 2, 1   | 2, 1   | $I_2, I_1$  |               |
| a2       | <del></del>   |  |     | $1+\varphi$           | $-3-5\varphi$  |               | $6-\varphi$  |            | 1                                      | 2                                    | - +          | 4,2                          | 4,2  | 4,2  | $I_4, I_2$  |               |
| b1       | $\varphi$     | -1   | 1   | 0                     | $-53 + 33\varphi$  | ρ             | $-160 + 99\varphi$   |            | 0                                      | 1                                    |              | 1,1                          | 1,1  | 1,1  | $I_1, I_1$  |               |
| 979      | $\mathbf{d}$  |  |     |                       |  | 979           | $d = (29 + 6\varphi) = 11b \cdot 89a$                            | (2 isoge   | ny c                                   | lasses                               | 3)           |                              |  |  |   | 979d          |
| a1       | $1+\varphi$   | $1-\varphi$  |     | $\varphi$             | $-2-2\varphi$  |               | $-1+\varphi$   |            | 1                                      | 2                                    | ++           | 2, 1                         | 2, 1   | 2, 1   | $I_2, I_1$  |               |
|          | $1+\varphi$   | $1-\varphi$  |     |                       | $-7+3\varphi$  |               | 6  |            | 1                                      | 2                                    | + -          | 4,2                          | 4,2  | 4,2  | $I_4, I_2$  |               |
| b1       | $1+\varphi$   | $-1-\varphi$                                       | 2   | 0                     | $-20 - 33\varphi$  | ρ             | $-61 - 99\varphi$  |            | 0                                      | 1                                    |              | 1,1                          | 1,1  | 1,1  | $I_1, I_1$  |               |
| 980      | a             |  |     |                       |  | 980           | $0a = (14 - 28\varphi) = 2 \cdot 5a \cdot 7$                     | (1 isog    | geny                                   | class)                               | )            |                              |  |  |   | 980a          |
| a1       | 1             | -1   |     | 1                     | -88  |               | 317  |            | 0                                      | 2                                    |              | 1, 16, 1                     | 1, 16, 1   | 1, 2, 1  | $I_1, I_{16}, I_1$  |               |
| a2<br>a3 | 1             | -1 $-1$  |     | 1                     | -18  | $\frac{8}{2}$ | $-19 \\ -3$  |            | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 4                                    |              | 2, 8, 2                      | 2, 8, 2  | 2, 2, 2  | $I_2, I_8, I_2$   |               |
| a3<br>a4 | 1<br>1        | -1<br>-1   |     | 1<br>1                | -268   |               | -3 $-1619$   |            | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\frac{4}{2}$                        | 1            | $4, 4, 1 \\ 1, 4, 4$         | $\begin{bmatrix} 4, 4, 1 \\ 1, 4, 4 \end{bmatrix}$ | $\begin{vmatrix} 4, 2, 1 \\ 1, 2, 4 \end{vmatrix}$ | $\begin{bmatrix} I_4, I_4, I_1 \\ I_1, I_4, I_4 \end{bmatrix}$  |               |
|          |               |  |     |                       |  |               | (92 ) 001  | (0.        | 1                                      | \                                    |              |                              |  | , ,  | 1 1 1 1   | 0015          |
| 991      |               | 1  | -1  | 1 1                   | 1  |               | $991a = (32 - \varphi) = 991a$                                   | (2 isogeny |  | ,                                    |              | 1                            | 1  | 1  | т   | 991a          |
| a1       |               | $-1-\varphi$                                       |     |                       | -1   |               | -arphi   |            | ·!                                     | 1                                    |              | 1                            | 1 1  | 1  | $I_1$   |               |
| b1<br>b2 | $1+\varphi$ 1 | $1 - \varphi$ $1 + \varphi$                        |     | -                     | $ \begin{array}{c} -6 + \varphi \\ -43 - 68\varphi \end{array} $   |               | $\begin{array}{c} 2 - 3\varphi \\ -245 - 397\varphi \end{array}$ |            | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 2 2                                  | +-           | 1 2                          | $\begin{array}{ c c }\hline 1\\ 2\\ \end{array}$   | $\begin{vmatrix} 1\\2 \end{vmatrix}$               | $egin{array}{c} I_1 \\ I_2 \end{array}$   |               |
| 991      | b             |  |     |                       |  | 9             | $991b = (31 + \varphi) = 991b$                                   | (2 isogeny | clas                                   | ses)                                 |              |                              |  |  |   | 991b          |
| a1       | $1+\varphi$   | 1  | 1   | 1                     | (  | 0             | 0  |            | 1                                      | 1                                    |              | 1                            | 1  | 1  | $I_1$   |               |
| b1 b2    | $\varphi$ 1   | $ \begin{array}{c} 1 \\ -1 - \varphi \end{array} $ |     | $\varphi$ $1+\varphi$ | $ \begin{array}{c} -3 - 3\varphi \\ -112 + 69\varphi \end{array} $ |               | $ \begin{array}{c} 2\varphi \\ -530 + 327\varphi \end{array} $   |            | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{vmatrix} 2\\2 \end{vmatrix}$ | - +<br>  + - | 1<br>2                       | $\begin{vmatrix} 1 \\ 2 \end{vmatrix}$             | $\begin{vmatrix} 1 \\ 2 \end{vmatrix}$             | $egin{array}{cccc} I_1 & I_1 & I_2  |               |
|          |               |  |     |                       | 1  | ,             | · r  |            | 1                                      | <u> </u>                             |              |                              | 1  | 1  |   |               |

|          | $a_1$                 | $a_2$  | $a_3$                     | $a_4$                                       | $a_6$  | r                                      | T                                       | s        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$             | $c_p$                                   | Kodaira  | Isogenies |
|----------|-----------------------|--|---------------------------|---|--|--|---|----------|------------------------------|---|---|--|-----------|
| 995      | a                     |  |                           | 995   | $5a = (22 - 29\varphi) = 5a \cdot 199b$          | (1 isogeny clas                        | s)                                      |          |                              |   |   |  | 995a      |
| a1       |                       | $-1-\varphi$   | 1                         | $-1-\varphi$                                | 0  | 1                                      | 2                                       | -+       | 1, 2                         | 1, 2                                    | 1, 2                                    | $I_1, I_2$   |           |
| a2       | $1+\varphi$           | $-1-\varphi$   | 1                         | $-11-6\varphi$                              | $-2-16\varphi$                                   | 1                                      | 2                                       | ++       | 2, 1                         | 2,1                                     | 2,1                                     | $I_2, I_1$   |           |
| 995      | b                     |  |                           | 99  | $5b = (7 - 29\varphi) = 5a \cdot 199a$           | (1 isogeny class                       | s)                                      |          |                              |   |   |  | 995b      |
| a1       | $\varphi$             |  | 1                         | -1  | 0  | 1                                      | 2                                       | + -      | 1, 2                         | 1, 2                                    | 1,2                                     | $I_1, I_2$   |           |
| a2       | $\varphi$             | -1   | 1                         | $-16 + 5\varphi$                            | $-18 + 16\varphi$                                | 1                                      | 2                                       | ++       | 2,1                          | 2, 1                                    | 2,1                                     | $I_2, I_1$   |           |
| 100      | 4a                    |  |                           | 100   | $4a = (30 + 4\varphi) = 2 \cdot 251a \tag{4}$    | 4 isogeny classe                       | es)                                     |          |                              |   |   |  | 1004a     |
| a1       | $\varphi$             | 1  | 0                         | $-2-6\varphi$                               | $4+4\varphi$                                     | 1                                      | 1                                       |          | 4, 1                         | 4,1                                     | 2,1                                     | $I_4, I_1$   |           |
| b1       | $\varphi$             | $\varphi$  | 0                         | $-42 + 27\varphi$                           | $133 - 82\varphi$                                | 1                                      | 1                                       | + -      | 1, 1                         | 1,1                                     | 1,1                                     | $I_1, I_1$   |           |
| c1       | $\varphi$             | $-\varphi$   | $\varphi$                 | $-2-2\varphi$                               | $1+2\varphi$                                     | 1                                      | 3                                       | + -      | 3,1                          | 3,1                                     | [3,1]                                   | $I_3, I_1$   |           |
| c2       | 1                     | $1-\varphi$  | $\varphi$                 | $-193 + 121\varphi$                         | $-1352 + 835\varphi$                             | 1                                      | 1                                       | + -      | 1,3                          | 1,3                                     | 1,1                                     | $I_1, I_3$   |           |
| d1       | 1                     | ,  | $\varphi$                 | $-4+5\varphi$                               | $4-3\varphi$                                     | I                                      |   | + -      | 5, 1                         | 5, 1                                    | 5,1                                     | $I_5, I_1$   |           |
| d2       | 1                     | $-\varphi$   | φ                         | $-244 - 85\varphi$                          | $-1526 - 723\varphi$                             | 0                                      | 1                                       | + -      | 1, 5                         | 1,5                                     | 1,5                                     | $I_1, I_5$   |           |
| 100      | 4b                    |  |                           | 100   | $4b = (34 - 4\varphi) = 2 \cdot 251b \tag{4}$    | 4 isogeny classe                       | es)                                     |          |                              |   |   |  | 1004b     |
| a1       | $1+\varphi$           | $1-\varphi$  | 0                         | $-8+6\varphi$                               | $8-4\varphi$                                     | 1                                      | 1                                       |          | 4, 1                         | 4, 1                                    | 2,1                                     | $I_4, I_1$   |           |
| b1       | $1+\varphi$           | $1+\varphi$  | 1                         | $-15-24\varphi$                             | $25 + 41\varphi$                                 | 1                                      | 1                                       | - +      | 1, 1                         | 1,1                                     | 1,1                                     | $I_1, I_1$   |           |
| c1       | $1+\varphi$           | -1   | $1+\varphi$               | -4  | $3-3\varphi$                                     | 1                                      | 3                                       | -+       | 3,1                          | 3,1                                     | [3,1]                                   | $I_3, I_1$   |           |
| c2       | 1                     | $\varphi$  | $1 + \varphi$             | $-72 - 122\varphi$                          | $-517 - 836\varphi$                              | 1                                      | 1                                       | -+       | 1,3                          | 1,3                                     | 1,1                                     | $I_1, I_3$   |           |
| d1       |                       | $-1+\varphi$   |                           | $1-6\varphi$                                | $1+2\varphi$                                     | I                                      |   | -+       | 5, 1                         | 5, 1                                    | 5,1                                     | $I_5, I_1$   |           |
| d2       | 1                     | $-1+\varphi$   | $1+\varphi$               | $-329 + 84\varphi$                          | $-2249 + 722\varphi$                             | 0                                      | 1                                       | -+       | 1,5                          | 1,5                                     | 1,5                                     | $I_1, I_5$   |           |
| 1009     | 9a                    |  |                           | 10  | $09a = (8 - 29\varphi) = 1009a \tag{2}$          | isogeny classe                         | s)                                      |          |                              |   |   |  | 1009a     |
| a1       | $\varphi$             |  | $\varphi$                 | -2  | $1-\varphi$                                      | 1                                      | 2                                       | ++       | 1                            | 1                                       | 1                                       | $I_1$  |           |
| a2       | <del></del>           |  | $  \frac{\varphi}{z}$ $-$ | $-2-5\varphi$                               | $-6\varphi$                                      | 1                                      | 2                                       | _ +      | 2                            | 2                                       | 2                                       | I <sub>2</sub>                                       |           |
| b1<br>b2 | 1                     | _  | $0 \\ 1 + \varphi$        | $-4 - 3\varphi$ $-182 + 111\varphi$         | $3+6\varphi$                                     | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | $\begin{array}{c c} 4 \\ 4 \end{array}$ | +++      | $\frac{1}{2}$                | $\begin{array}{c c} 1 \\ 2 \end{array}$ | $\begin{array}{c c} 1 \\ 2 \end{array}$ | $egin{array}{c} I_1 \ I_2 \end{array}$               |           |
| b3       | $1+\varphi$ $\varphi$ |  | $1+\varphi$               | $-162 + 111\varphi$ $-19909 + 12304\varphi$ | $-1170 + 722\varphi$ $-1269178 + 784395\varphi$  | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | 2                                       | + +      | 1                            | 1                                       | $\begin{array}{c c} 2 \\ 1 \end{array}$ | $ \begin{array}{c}     I_2 \\     I_1 \end{array} $  |           |
| b4       | 1                     |  |                           | $-19 + 27\varphi$                           |  |  | 4                                       |          | 4                            | 4                                       | 4                                       | $I_4$  |           |
| 1009     | nh                    |  |                           | 100   | 001 (01 00 ) 10001 (6                            | •                                      |   |          |                              | 1                                       |   | ı  | 1009b     |
|          |                       | . 1  | 1                         |   | ,          | 2 isogeny classe                       |   |          | 1                            | 1                                       | 1                                       | т  | 10090     |
| a1<br>a2 |                       | $-1 - \varphi$ $-1 - \varphi$                                    |                           | $-2 - 2\varphi$ $-7 + 3\varphi$             | $ \begin{array}{c} 0\\ -6+5\varphi \end{array} $ | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | $\frac{2}{2}$                           | ++       | $\frac{1}{2}$                | $\begin{array}{c c} 1 \\ 2 \end{array}$ | $\frac{1}{2}$                           | $egin{array}{c} I_1 \ I_2 \end{array}$               |           |
| b1       | - · · · · · · 1       | $\begin{bmatrix} - & 1 & - & \gamma \\ - & 1 & -1 \end{bmatrix}$ | $\frac{1}{0}$             | $-7 + 3\varphi$                             | $9 - 6\varphi$                                   |  | - <del>-</del><br>  4                   | '<br>+ + | 1                            | !                                       |   | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ |           |
| b2       | $\varphi$             | $-1-\varphi$   | 0                         | $-70-111\varphi$                            | $-377 - 611\varphi$                              | 1                                      | 4                                       | ++       | 2                            | 2                                       | 2                                       | $I_2$  |           |
| b3       | $1+\varphi$           |  | $\varphi$                 | $-7605 - 12304\varphi$                      | $-492388 - 796699\varphi$                        | 1                                      | 2                                       | ++       | 1                            | 1                                       | 1                                       | $I_1$  |           |
| b4       | 1                     | -1   | 0                         | $8-27\varphi$                               | $14 - 58\varphi$                                 | 1                                      | 4                                       |          | 4                            | 4                                       | 4                                       | $I_4$  |           |

| $a_1$ | $a_2$ $a_3$ | $a_4$ | $a_6$ | $\mid r \mid$ | T | $s \operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_{p}$ | Kodaira Is | sogenies |
|-------|-------------|-------|-------|---------------|---|--------------------------------|-----------------------------|---------|------------|----------|

| 1024a  | $1024a = (32) = 2^5$ (10 isogeny cl | lasses) |   |       |    |   |               |                  | 1024a |
|--|-------------------------------------|---------|---|-------|----|---|---------------|------------------|-------|
| a1 0 0 0 —11   | 14                                  | 1       | 4 | ++    | 9  | 0 | 4             | $I_0^*$          |       |
| a2   0   | 0                                   | 1       | 4 | ++    | 6  | 0 | 2             | III              |       |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$                                       | -14                                 | 1       | 2 | ++    | 9  | 0 | 1             | $\mathrm{I}_0^*$ |       |
| a4   0   | 0                                   | 1 1     | 4 |       | 12 | 0 | 4             | $ m I_3^*$       |       |
| b1   0   | 2-arphi                             | 1       | 4 | + $+$ | 6  | 0 | 2             | III              |       |
| $b2 \mid 0 -1 - \varphi  0 \qquad -21 - 31\varphi$   | $84 + 137\varphi$                   | 1       | 2 | ++    | 9  | 0 | 2             | $\mathrm{I}_0^*$ |       |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$                                       | 96-59arphi                          | 1       | 4 | + -   | 9  | 0 | 2             | $\mathrm{I}_0^*$ |       |
| $  b4   0 $ $-1 0 $ $3-4\varphi$   | 5-4arphi                            | 1       | 2 | -+    | 12 | 0 | 4             | $I_3^*$          |       |
| $\begin{bmatrix} c1 & 0 & -1 & 0 \end{bmatrix}$ $\begin{bmatrix} -1 - \varphi \end{bmatrix}$ | $1+\varphi$                         | 1       | 4 | ++    | 6  | 0 | 2             | III              |       |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | $37 + 59\varphi$                    | 1       | 4 | -+    | 9  | 0 | 2             | $I_0^*$          |       |
| $\begin{bmatrix} c3 & 0 & 1+\varphi & 0 \\ \end{bmatrix}$                                    | $168-105\varphi$                    | 1       | 2 | ++    | 9  | 0 | 2             | $I_0^*$          |       |
| $\begin{vmatrix} c4 & 0 & -1 & 0 & -1 + 4\varphi \end{vmatrix}$                              | $1+4\varphi$                        | 1       | 2 | + -   | 12 | 0 | 4             | $I_3^*$          |       |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | $-1-\varphi$                        | 0       | 4 | ++    | 6  | 0 | 2             | III              |       |
| $d2 \mid 0 -1 - \varphi  0 \qquad -53 + 33\varphi$   | $-168 + 105\varphi$                 | 0       | 2 | ++    | 9  | 0 | 1             | $I_0^*$          |       |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | -1-4arphi                           | 0       | 4 | + -   | 12 | 0 | 4             | $I_3^*$          |       |
| d4   0   | $-37-59\varphi$                     | 0       | 2 | - +   | 9  | 0 | 2             | $I_0^*$          |       |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | -2+arphi                            | 0       | 4 | ++    | 6  | 0 | $\frac{1}{2}$ | III              | i     |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$                                       | $-5+4\varphi$                       | 0       | 4 | -+    | 12 | 0 | 4             | $I_3^*$          |       |
| e3   0   | $-84-137\varphi$                    | 0       | 2 | ++    | 9  | 0 | 1             | $I_0^*$          |       |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$                                       | $-96 + 59\varphi$                   | 0       | 2 | + -   | 9  | 0 | 2             | $\mathrm{I}_0^*$ |       |
| $  f1   0 -\varphi 0 $ $-11-10\varphi$   | $22 + 31\varphi$                    | 0       | 4 | ++    | 9  | 0 | 2             | $ m I_0^*$       |       |
| $  f2   0 -\varphi 0 $   | arphi                               | 0       | 4 | + +   | 6  | 0 | 2             | III              |       |
| f3   0 $-\varphi$ 0 $-1-5\varphi$  | $-3-2\varphi$                       | 0       | 2 | -+    | 12 | 0 | 2             | $I_3^*$          |       |
| f4   0   1 - $\varphi$ 0   | $-406 + 251\varphi$                 | 0       | 2 | +     | 9  | 0 | 2             | $\mathrm{I}_0^*$ |       |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | $53 - 31\varphi$                    | 0       | 4 | +++   | 9  | 0 | 2             | $\mathrm{I}_0^*$ |       |
| $ g2  0 -1 + \varphi 0$ $-1$   | 1-arphi                             | 0       | 4 | + +   | 6  | 0 | 2             | III              |       |
| $g3 \mid 0 -1 + \varphi  0 \qquad -6 + 5\varphi$   | $-5+2\varphi$                       | 0       | 2 | + -   | 12 | 0 | 2             | $I_3^*$          |       |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | $-155 - 251\varphi$                 | 0       | 2 | +     | 9  | 0 | 2             | $I_0^*$          |       |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | $-1+\varphi$                        | 0       | 4 | ++    | 6  | 0 | 2             | III              |       |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | $5-2\varphi$                        | 0       | 4 | + $-$ | 12 | 0 | 4             | $I_3^*$          |       |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | $155 + 251\varphi$                  | 0       | 2 | -+    | 9  | 0 | 1             | $I_0^*$          |       |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | $-53 + 31\varphi$                   | 0       | 2 | _++   | 9  | 0 | 2             | $\mathrm{I}_0^*$ |       |
| $  i1   0 \qquad \varphi  0 \qquad -1$   | -arphi                              | 0       | 4 | ++    | 6  | 0 |               | III              |       |
| i2   0 $\varphi$ 0 $-1-5\varphi$   | $3+2\varphi$                        | 0       | 4 | -+    | 12 | 0 | 4             | $I_3^*$          |       |
| i3   0 -1 + $\varphi$ 0 -85 + 53 $\varphi$   | $406-251\varphi$                    | 0       | 2 | + -   | 9  | 0 | 1             | $I_0^*$          |       |
| i4 0 $\varphi$ 0 $-11-10\varphi$   | $-22-31\varphi$                     | 0       | 2 | ++    | 9  | 0 | 2             | $I_0^*$          |       |

|                                   | I  |                         |   |  |   |  |                                      |                |                              |   |  | 1   |           |
|-----------------------------------|--|-------------------------|---|--|---|--|--------------------------------------|----------------|------------------------------|---|--|---|-----------|
|                                   | $a_1$  | $a_2$                   | $a_3$                                   | $a_4$  | $a_6$   |  | T                                    | s              | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                   | $c_p$  | Kodaira   | Isogenies |
|                                   |  |                         |   |  |   |  |                                      |                |                              |   |  |   |           |
| 102                               | 4a   |                         |   |  | $1024a = (32) = 2^5$ (10 iso  | geny classes)                          |                                      |                |                              |   |  |   | 1024a     |
| j1                                | 0  | 0                       | 0                                       | $-1-\varphi$                                 | 0   | 0                                      | 4                                    | ++             | 6                            | 0   | 2  | III   |           |
| j2                                | 0  | 0                       | 0                                       | $-11 - 11\varphi$                            | $-14-28\varphi$   | 0                                      | 2                                    | ++             | 9                            | 0   | 1  | $I_0^*$   |           |
| j3                                | 0  | 0                       | 0                                       | $-11-11\varphi$                              | $14 + 28\varphi$  | 0                                      | 2                                    | ++             | 9                            | 0   | 1  | $I_0^*$   |           |
| j4                                | 0  | 0                       | 0                                       | $4+4\varphi$                                 | 0   | 0                                      | 2                                    |                | 12                           | 0   | 2  | $I_3^*$   |           |
|                                   |  |                         |   |  |   |  |                                      |                |                              |   |  |   |           |
| 102                               | 5a   |                         |   | 1025   | $a = (35 - 5\varphi) = 5a^2 \cdot 41a$  | (8 isogeny class                       | ses)                                 |                |                              |   |  |   | 1025a     |
| a1                                | 1  | $-1-\varphi$            | $1+\varphi$                             | $-3+2\varphi$                                | $2-2\varphi$  | 1                                      | 1                                    | + -            | 1,4                          | 1   | 1,3  | $I_1, IV$   |           |
| b1                                | $1+\varphi$  | 0                       |   | $-1999 + 667\varphi$                         | $29838 - 25412\varphi$  |  | 2                                    | :<br>          | 2,11                         | $\frac{1}{2}, \frac{1}{5}$                    | [2, 2]   | $\overline{\mathrm{I}}_{2},\overline{\mathrm{I}}_{5}^{*}$ |           |
| b2                                | $1+\varphi$  | 0                       | 0                                       | $-124 + 42\varphi$                           | $463-412\varphi$  | 0                                      | 2                                    |                | 1, 16                        | 1,10  | 1,4  | $I_1, I_{10}^*$   |           |
| c1                                | $1+\varphi$  | -1                      |   | $-3-3\varphi$                                | $2+3\varphi$  | <u>-</u>   1                           | 1                                    | <br>  + -      | 1, 2                         | 1   | 1, 1   | $I_1, II$   |           |
| d1                                | ! <del>'</del> - <u>'</u> - !                                    |                         | $1+\varphi$                             | $-671 + 397\varphi$                          | $7679 - 4722\varphi$  | <u> </u><br>  1                        | '<br>  4                             | ! _ ·<br>  + + |                              | $\frac{1}{4}$ , $\frac{1}{2}$                 | $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 4 \end{bmatrix}$ | $\begin{array}{c c} I_4, I_2^* \end{array}$               |           |
| d2                                | $\varphi$  |                         | $1+\varphi$                             | $-24-25\varphi$                              | $-48 - 91\varphi$   |  | 4                                    | ++             | 2, 10                        | 2, 4  | 2, 4   | $I_2, I_4^*$  |           |
| d3                                | $\varphi$  |                         | $1+\varphi$                             | 1  | $-3-6\varphi$   | 1                                      | 2                                    |                | 1,8                          | 1,2   | 1,4  | $I_1, I_2^*$  |           |
| d4                                | $1+\varphi$  |                         | $1+\varphi$                             | $-72721 + 44905\varphi$                      | $8854328 - 5472168\varphi$  | 1                                      | 4                                    | ++             | 2, 7                         | 2, 1  | 2,4  | $I_2, I_1^*$  |           |
| d5                                | 1  | 0                       | $1+\varphi$                             | $-621 + 422\varphi$                          | $8354 - 4997\varphi$  | 1                                      | 2                                    |                | 8, 7                         | 8,1   | 8,4  | $I_8, I_1^*$  |           |
| d6                                | $  \varphi  $  | 0                       | $1+\varphi$                             | $-299-425\varphi$                            | $-3403 - 5381\varphi$   | 1                                      | 2                                    | ++             | 1, 14                        | 1,8   | 1,4  | $I_1, I_8^*$  |           |
| e1                                | 1  | $1+\varphi$             | $\varphi$                               | $-3+4\varphi$                                | $-6+2\varphi$   | 0                                      | 2                                    |                | 1,8                          | 1, 2  | 1,4  | $\overline{\mathrm{I}_{1},\mathrm{I}_{2}^{*}}$            | [         |
| e2                                | 1  | $1+\varphi$             | $\varphi$                               | $-228 - 146\varphi$                          | $909 + 1547\varphi$   | 0                                      | 2                                    | ++             | 6, 9                         | 6, 3  | 2,2  | $I_6, I_3^*$  |           |
| e3                                | 1  | $1+\varphi$             | $\varphi$                               | $22-21\varphi$                               | $34 + 47\varphi$  | 0                                      | 2                                    |                | 3, 12                        | 3, 6  | 1,4  | $I_3, I_6^*$  |           |
| e4                                | $1+\varphi$  | $1-\varphi$             | $\varphi$                               | $-479 + 292\varphi$                          | $-4993 + 3082\varphi$   | 0                                      | 2                                    | ++             | 2,7                          | 2, 1  | 2,2  | $I_2, I_1^*$  |           |
| f1                                | $\varphi$  | $\varphi$               | $1+\varphi$                             | $-31-46\varphi$                              | $-136 - 222\varphi$   | 0                                      | 1                                    | + -            | 1,8                          | 1   | 1,1  | $I_1, IV^*$   |           |
| g1                                | 0  | $1+\varphi$             | $1+\varphi$                             | $-201 - 249\varphi$                          | $1597 + 2272\varphi$  |  | 1                                    |                | 7,6                          | 7   | 1,1  | $I_7, I_0^*$  |           |
| g2                                | 0  | $1+\varphi$             | •                                       | $-1+\varphi$                                 | $-3-3\varphi$   | 0                                      | 1                                    |                | 1, 6                         | 1   | 1,1  | $I_1, I_0^*$  |           |
| h1                                | $\varphi$  | $1+\varphi$             |   | $-38 + 24\varphi$                            | $-113 + 68\varphi$  | 0                                      | 1                                    | + -            | 1,10                         | 1   | 1,1  | $I_1, II^*$   |           |
|                                   | ,  | · · ·                   | •                                       | ·  | · .   |  |                                      |                |                              |   | ,  |   |           |
| 102                               | 5h   |                         |   | 109  | $5b = (30 + 5\varphi) = 5a^2 \cdot 41b $                                      | 8 isogeny class                        | es)                                  |                |                              |   |  |   | 1025b     |
| a1                                | 1  | $1+\varphi$             | 1 + 10                                  | $-1-\varphi$                                 | $\frac{30 - (30 + 3\varphi) - 3a \cdot 410}{-\varphi}$                        | 1                                      | $\frac{(cs)}{1}$                     | -+             | 1,4                          | 1   | 1,3  | $I_1, IV$   | 10200     |
| b1                                |  | $1-\varphi$             | 1 · · · · · · · · · · · · · · · · · · · | $-1332 - 667\varphi$                         | $4426 + 25412\varphi$   | 1                                      | '                                    | !              | $\frac{1}{2}, \frac{1}{11}$  | $\frac{1}{2}, \frac{1}{5}$                    | $\begin{bmatrix} 1, 5 \\ 2, 2 \end{bmatrix}$           | $\begin{array}{c c} I_1, I_1 \\ I_2, I_5^* \end{array}$   |           |
| b2                                | $\varphi$  | $1-\varphi$ $1-\varphi$ | 0                                       | $-1332 - 607\varphi$ $-82 - 42\varphi$       | $4420 + 25412\varphi$ $51 + 412\varphi$                                       |  | $\begin{vmatrix} 2\\2 \end{vmatrix}$ | + +<br>        | $\frac{2,11}{1,16}$          | $\begin{bmatrix} 2, 5 \\ 1, 10 \end{bmatrix}$ | $\begin{bmatrix} 2, 2 \\ 1, 4 \end{bmatrix}$           | $I_{1}, I_{5}$ $I_{1}, I_{10}^{*}$                        |           |
| c1                                | $\frac{ \varphi }{ \varphi }$                                    |                         |   | $-6+3\varphi$                                | $5 - 3\varphi$  | º -<br>  1                             | <u> </u>                             | <del>!</del>   | 1, 10 $1, 2$                 | -;  | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$ |   |           |
|                                   | $\varphi$  | $-\varphi$              |   |  |   |  | <u>'</u> -                           | - +            |                              | 1   |  | $I_1, II$   |           |
| $\frac{d1}{d2}$                   | 1  | $0 \\ 0$                | $\varphi$                               | $-273 - 398\varphi \\ -27814 - 44907\varphi$ | $\begin{array}{c} 2958 + 4721\varphi \\ 3382161 + 5472167\varphi \end{array}$ | 1                                      | $\begin{vmatrix} 4\\4 \end{vmatrix}$ | +++            | $4, 8 \\ 2, 7$               | $4, 2 \\ 2, 1$                                | $\begin{array}{ c c } 4,4 \\ 2,4 \end{array}$          | $I_4, I_2^*$ $I_2$ $I_3$                                  |           |
| $\frac{\mathrm{d}z}{\mathrm{d}3}$ | $\begin{array}{ c c c c} & \varphi & \\ 1+\varphi & \end{array}$ |                         | $\varphi$                               | $-27814 - 44907\varphi$ $-48 + 23\varphi$    | $3382101 + 3472107\varphi$<br>$-138 + 90\varphi$                              | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | $\begin{vmatrix} 4\\4 \end{vmatrix}$ | +++            | 2, t $2, 10$                 | $2, 1 \\ 2, 4$                                | $\begin{bmatrix} 2,4\\2,4 \end{bmatrix}$               | $I_2, I_1^*$ $I_2, I_1^*$                                 |           |
| d4                                | $\begin{vmatrix} 1+\varphi\\1+\varphi \end{vmatrix}$             | $-\varphi$              | $\varphi$                               | $-46 + 23\varphi$ $2 - 2\varphi$             | $-138 + 90\varphi$ $-8 + 5\varphi$  | $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ | $\begin{vmatrix} 4\\2 \end{vmatrix}$ | <del>+</del> + | $\frac{2,10}{1,8}$           | 1, 2  | $\begin{bmatrix} 2, 4 \\ 1, 4 \end{bmatrix}$           | $I_2, I_4^* \ I_1, I_2^*$                                 |           |
| d5                                | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$           | $-\varphi$ $0$          | $arphi \ arphi$                         | $-198 - 423\varphi$                          | $3358 + 4996\varphi$  |  | $\frac{2}{2}$                        | l              | 8,7                          | 8,1   | 8,4  | $I_{1}, I_{2}$ $I_{8}, I_{1}^{*}$                         |           |
| d6                                | $1+\varphi$  | $-\varphi$              | arphi                                   | $-723 + 423\varphi$                          | $-8783 + 5380\varphi$   |  | $\frac{2}{2}$                        | ++             |                              | 1,8   | 1,4  | $I_{8}, I_{1} I_{1}, I_{8}^{*}$                           |           |
| 40                                | 1 1 4  | Ψ                       | Ψ                                       | 120   120φ                                   | 0100   0000φ  | 1                                      |                                      | 1 1            | -,                           | 1,0   | , -  | -1, -8  |           |

|   |  |  |   |   | T                                   |   | $\operatorname{ord}(\Delta)$                           | $\operatorname{ord}_{-}(j)$   |  | Kodaira  | Igomonica |
|---|--|--|---|---|-------------------------------------|---|--|---|--|--|-----------|
|   | $a_1$ $a_2$ $a_3$  | $a_4$  | $a_6$   | r   | 1                                   | 8   | $\operatorname{ord}(\Delta)$                           | ord_(j)   | $c_p$  | Rodana   | Isogenies |
| 102!  |  | 1025b  | $= (30 + 5\varphi) = 5a^2 \cdot 41b$ (8 iso   | geny cla                                      | sses)                               |   |  |   |  |  | 1025b     |
| e1  | $1  -1-\varphi \qquad \varphi$   | $-3\varphi$  | $-4+\varphi$  | 0   | 2                                   |   | 1,8  | 1, 2  | 1,4  | $I_1, I_2^*$   |           |
| e2<br>e3  | $ \begin{array}{ccc} 1 & -1 - \varphi & \varphi \\ 1 & -1 - \varphi & \varphi \end{array} $  | $-375 + 147\varphi$ $22\varphi$  | $2831 - 1694\varphi$ $81 - 69\varphi$   | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$        | $\frac{2}{2}$                       | ++  | 6, 9 $3, 12$   | 6, 3 $3, 6$   | $2, 2 \\ 1, 4$   | $I_6, I_3^*$   |           |
| e3<br>e4  | $egin{array}{cccc} 1 & -1-arphi & arphi \ arphi & 1 & 1+arphi \end{array}$   | $-186 - 294\varphi$  | $-1911 - 3083\varphi$   |   | $\frac{2}{2}$                       | ++  | $\frac{3,12}{2,7}$                                     | $\begin{bmatrix} 3, 6 \\ 2, 1 \end{bmatrix}$  | 2, 2   | $I_3, I_6^* $ $I_2, I_1^*$   |           |
| f1  | $1+\varphi  1+\varphi  1+\varphi$  | $-76 + 47\varphi$  | $-311 + 191\varphi$   |   | <del>-</del> -<br>1                 | '<br>  - +  | 1,8  |   | $\begin{vmatrix} 1 & -\frac{1}{2} - \frac{1}{2} \\ 1 & 1 \end{vmatrix}$  | $oxed{ egin{array}{c} I_1, IV^* \end{array} }$   |           |
| g1  | $\begin{array}{cccccccccccccccccccccccccccccccccccc$   | $-451 + 251\varphi$  | $4321 - 2523\varphi$  |   | ·<br>1                              | ' <del>'</del> - :<br>  | 7,6  | <del>-</del>  | $\begin{bmatrix} 1, 1 \\ 1, 1 \end{bmatrix}$   | $[ I_7, I_0^* ]$   |           |
| g2  | $0  -1 - \varphi \qquad \varphi$   | $-1+\varphi$   | $-4+2\varphi$   | 0   | 1                                   |   | 1, 6   | 1   | 1,1  | $I_1, I_0^*$   |           |
| h1  | $1+\varphi$ $-1+\varphi$ 0   | $-14 - 23\varphi$  | $-53-83\varphi$   | 0   | 1                                   | - +   | 1,10   | 1   | 1,1  | $I_1, II^*$  |           |
| 103   | 1a   | 1031   | $a = (10 - 29\varphi) = 1031a$ (2 isog  | geny cla                                      | sses)                               |   |  |   |  |  | 1031a     |
| a1  | $0  1+\varphi  \varphi$  | $-41835 + 25857\varphi$  | $-3877170 + 2396223\varphi$   | 0   | 1                                   | + -   | 1  | 1   | 1  | $I_1$  |           |
| b1  | $0  1 - \varphi  \varphi$  | $-3+2\varphi$  | $1-\varphi$   | 1   | 1                                   | + -   | 1  | 1   | 1  | $I_1$  |           |
| 103   | 1b   | 1031   | $b = (19 - 29\varphi) = 1031b$ (2 isog  | geny clas                                     | sses)                               |   |  |   |  |  | 1031b     |
| a1  | $0 -1 - \varphi  1 + \varphi$  | $-15979 - 25855\varphi$  | $-1464968 - 2370368\varphi$   | 0   | 1                                   | -+  | 1  | 1   | 1  | $I_1$  |           |
| b1  | $0 \qquad \varphi  1 + \varphi$  | $-1-2\varphi$  | 0   | 1   | 1                                   | -+  | 1  | 1   | 1  | $I_1$  |           |
| 1039  | 9a   | 1039   | $a = (11 - 29\varphi) = 1039a$ (3 isog  | geny cla                                      | sses)                               |   |  |   |  |  | 1039a     |
| a1  | $1+\varphi$ $-1-\varphi$ 1   | -1   | 0   | 1   | 1                                   | +-  | 1  | 1   | 1  | $I_1$  |           |
| b1  | $1 \qquad \varphi \qquad 1$  | $-21 + 8\varphi$   | $-38 + 14\varphi$   | 0   | 1                                   | + -   | 1  | 1   |  | $  I_1  $  |           |
| c1  |  |  |   |   |                                     | I — +   | 1  | 1   | 1 1  | $I_1$  |           |
|   | $1+\varphi$ $-1$ $\varphi$   | $-1-\varphi$   | -1  | 0   | 2                                   | '   |  |   |  | -1   |           |
|   | $ \begin{array}{cccc} 1 + \varphi & -1 & \varphi \\ 1 + \varphi & -1 & \varphi \end{array} $   | $ \begin{array}{c} -1 - \varphi \\ -16 + 9\varphi \end{array} $  | $-1$ $-24 + 14\varphi$  | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$        | 2                                   | + -   | 2  | 2   | 2  | $I_2$  |           |
|   | $1+\varphi$ $-1$ $\varphi$   | $-16 + 9\varphi$   | $-24 + 14\varphi$   | I   | 2                                   |   |  |   |  |  | 1039b     |
| c2  | $1+\varphi$ $-1$ $\varphi$   | $-16 + 9\varphi$   | $-24 + 14\varphi$   | 0   | 2                                   |   |  |   |  |  | 1039b     |
| 1039<br>a1<br>b1  | $1+\varphi$ $-1$ $\varphi$ $9\mathbf{b}$   | $-16 + 9\varphi$ $1039$  | $-24 + 14\varphi$ $b = (18 - 29\varphi) = 1039b \qquad (3 isogenetic statement)$  | geny clas                                     | sses)                               | +-  | 2  | 2   | 2  | $I_2$  | 1039b     |
| 1039<br>a1   b1   c1                                    | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $ \begin{array}{c} -16 + 9\varphi \\ \hline 1039 \\ -\varphi \\ -13 - 8\varphi \\ \hline -1 - \varphi \end{array} $  | $-24 + 14\varphi$ $b = (18 - 29\varphi) = 1039b \qquad (3 \text{ isog}$ $0$ $-24 - 14\varphi$ $-1 - \varphi$  | 0 geny clas                                   | 2<br>sses)<br>1<br>1<br>2           | + -<br>  - +<br>  - +   | 1 1 1  | 2<br>  1<br>  1<br>  1<br>  1   | 1 1 1 1 1 1 1 1 1  | $\begin{array}{c c} I_2 \\ \hline & I_1 \\ \hline & I_1 \\ \hline & I_1 \\ \hline & I_1 \\ \hline \end{array}$                                 | 1039b     |
| 1039<br>a1<br>b1  | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $ \begin{array}{r} -16 + 9\varphi \\ \hline 1039 \\ -\varphi \\ -13 - 8\varphi \end{array} $   | $-24 + 14\varphi$ $b = (18 - 29\varphi) = 1039b \qquad (3 \text{ isog}$ $0$ $-24 - 14\varphi$   | geny clas                                     | 2<br>sses)<br>1<br>1                | + -<br>  - +<br>  - +   | 1  | 1 1   | 1 1  | $egin{array}{cccccccccccccccccccccccccccccccccccc$   | 1039b     |
| 1039<br>a1   b1   c1                                    | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $ \begin{array}{r} -16 + 9\varphi \\ \hline 1039 \\ -\varphi \\ -13 - 8\varphi \\ \hline -1 - \varphi \\ -6 - 11\varphi \end{array} $ $ 1044a$   | $-24 + 14\varphi$ $b = (18 - 29\varphi) = 1039b \qquad (3 isog 0)$ $-24 - 14\varphi$ $-1 - \varphi$ $-10 - 15\varphi$   | 0 geny clas                                   | 2<br>sses)<br>1<br>                 | + -     - +     + -     + -     + -       + -       + -       + -       + -       + - | 1 1 1  | 2<br>  1<br>  1<br>  1<br>  1   | 1 1 1 1 1 1 1 1 1  | $\begin{array}{c c} I_2 \\ \hline & I_1 \\ \hline & I_1 \\ \hline & I_1 \\ \hline & I_1 \\ \hline \end{array}$                                 | 1039b     |
| 1039 a1 b1 c1 c2  | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $ \begin{array}{c} -16 + 9\varphi \\ \hline 1039 \\ -\varphi \\ -13 - 8\varphi \\ \hline -1 - \varphi \\ -6 - 11\varphi \end{array} $  | $-24 + 14\varphi$ $b = (18 - 29\varphi) = 1039b \qquad (3 isog 0)$ $-24 - 14\varphi$ $-1 - \varphi$ $-10 - 15\varphi$   | 0   1   0   0   0   0   0                     | 2<br>sses)<br>1<br>                 | + -<br>  - +<br>  - +<br>  + -<br>  - +   | 1 1 1  | 2<br>  1<br>  1<br>  1<br>  1   | 1 1 1 1 1 1 1 1 1  | $\begin{array}{c c} I_2 \\ \hline & I_1 \\ \hline & I_1 \\ \hline & I_1 \\ \hline & I_1 \\ \hline \end{array}$                                 |           |
| 1039  a1 b1 c1 c2  1044                                 | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $ \begin{array}{r} -16 + 9\varphi \\ \hline 1039 \\ -\varphi \\ -13 - 8\varphi \\ \hline -1 - \varphi \\ -6 - 11\varphi \end{array} $ $ 1044a$   | $-24 + 14\varphi$ $b = (18 - 29\varphi) = 1039b \qquad (3 \text{ isog})$ $-24 - 14\varphi$ $-1 - \varphi$ $-10 - 15\varphi$ $= (36 - 6\varphi) = 2 \cdot 3 \cdot 29a \qquad (5 \text{ isog})$   | 0   0   0   0   0   0   0   0   0   0         | 2<br>sses)<br>1<br>1<br>2<br>2<br>2 | + -<br>  - +<br>  - +<br>  + -<br>  - +   | 1<br>1<br>1<br>2                                       | 2<br>  1<br>  1<br>  1<br>  2   | 1 1 1 2  | $\begin{array}{c c} I_2 \\ \hline & I_1 \\ \hline & I_1 \\ \hline & I_1 \\ \hline & I_2 \\ \end{array}$  |           |
| c2   1039   a1   c1   c2   1044   a1   c2               | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $ \begin{array}{r} -16 + 9\varphi \\ \hline 1039 \\ -\varphi \\ -13 - 8\varphi \\ \hline -1 - \varphi \\ -6 - 11\varphi \end{array} $ $ \begin{array}{r} 1044a \\ -1 - 3\varphi \end{array} $  | $-24 + 14\varphi$ $b = (18 - 29\varphi) = 1039b \qquad (3 \text{ isog}$ $0$ $-24 - 14\varphi$ $-1 - \varphi$ $-10 - 15\varphi$ $= (36 - 6\varphi) = 2 \cdot 3 \cdot 29a \qquad (5 \text{ iso}$ $2 + 5\varphi$   | 0   1   0   0   0   0   0   0   1   1         | 2<br>sses)<br>1<br>1<br>2<br>2<br>2 | + -<br>  - +<br>  - +<br>  + -<br>  - +   | 2<br>1<br>1<br>2<br>1<br>2                             | 2   | 2  | $egin{array}{cccccccccccccccccccccccccccccccccccc$   |           |
| 1039  a1   b1   c1   c2    1044  a1   b1   c1   d1   d1 | $ \begin{array}{c ccccc} 1+\varphi & -1 & \varphi \\ \hline \mathbf{9b} \\ \hline \varphi & -1 & 1 \\ \hline 1 & 1-\varphi & 1 \\ \hline \varphi & -\varphi & 1+\varphi \\ \varphi & -\varphi & 1+\varphi \end{array} $ $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | $ \begin{array}{r} -16 + 9\varphi \\ \hline 1039 \\ -\varphi \\ -13 - 8\varphi \\ \hline -1 - \varphi \\ -6 - 11\varphi \end{array} $ $ \begin{array}{r} 1044a \\ -1 - 3\varphi \\ \hline 3 - 7\varphi \\ \hline -7 - 8\varphi \\ -5 - 12\varphi \end{array} $ | $-24 + 14\varphi$ $b = (18 - 29\varphi) = 1039b \qquad (3 \text{ isog})$ $-24 - 14\varphi$ $-1 - \varphi$ $-10 - 15\varphi$ $= (36 - 6\varphi) = 2 \cdot 3 \cdot 29a \qquad (5 \text{ iso})$ $2 + 5\varphi$ $-8 - 17\varphi$ $10 + 11\varphi$ $-12 - 22\varphi$ | 0   1   0   0   0   0   0   0   0   0         | 2<br>sses)<br>1<br>1<br>2<br>2<br>2 | + -<br>  - +<br>  - +<br>  + -<br>  - +   | $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $\begin{array}{ c c c }\hline & 2\\ \hline & 1\\ \hline & 1\\ \hline & 1\\ \hline & 2\\ \hline & 1,3,2\\ \hline & 5,3,1\\ \hline & 5,1,2\\ \hline & 1,1,6\\ \hline \end{array}$ | $\begin{array}{ c c c }\hline & 2 \\ \hline & 1 \\ \hline & 1 \\ \hline & 1 \\ \hline & 2 \\ \hline & 1, 3, 2 \\ \hline & 1, 1, 1 \\ \hline & 5, 1, 2 \\ \hline & 1, 1, 6 \\ \hline \end{array}$ | $egin{array}{c c} I_2 & I_1 & \\ \hline I_1 & I_1 & \\ \hline I_2 & \\ \hline I_1, I_3, I_2 & \\ \hline I_5, I_3, I_1 & \\ \hline \end{array}$ |           |
| 1039  a1 b1 c1 c2  1044  a1 b1 c1 c1 c1 c1              | $ \frac{1+\varphi}{9\mathbf{b}} $ $ \frac{\varphi}{1}  \frac{-1}{1}  \frac{1}{-\varphi}  \frac{1}{1} $ $ \frac{\varphi}{\varphi}  \frac{-\varphi}{1+\varphi}  \frac{1+\varphi}{\varphi} $ $ \frac{\mathbf{4a}}{1+\varphi}  \frac{1}{1+\varphi}  \frac{1}{1+\varphi}  \frac{1}{1+\varphi} $ $ \frac{\varphi}{1+\varphi}  \frac{1}{1+\varphi}  \frac{1}{1+\varphi} $ | $ \begin{array}{r} -16 + 9\varphi \\ \hline 1039 \\ -\varphi \\ -13 - 8\varphi \\ -1 - \varphi \\ -6 - 11\varphi \end{array} $ $ \begin{array}{r} 1044a \\ -1 - 3\varphi \\ 3 - 7\varphi \\ -7 - 8\varphi \end{array} $  | $-24 + 14\varphi$ $b = (18 - 29\varphi) = 1039b \qquad (3 \text{ isog}$ $0$ $-24 - 14\varphi$ $-1 - \varphi$ $-10 - 15\varphi$ $= (36 - 6\varphi) = 2 \cdot 3 \cdot 29a \qquad (5 \text{ iso}$ $2 + 5\varphi$ $-8 - 17\varphi$ $10 + 11\varphi$                 | 0   0   0   0   0   0   0   0   0   1   0   0 | 2 sses)  1                          | + -<br>  - +<br>  - +<br>  + -<br>  - +   | 1<br>1<br>2<br>1,3,2<br>5,3,1<br>5,1,2                 | $\begin{array}{ c c c }\hline & 2 \\ \hline & 1 \\ \hline & 1 \\ \hline & 1 \\ 2 \\ \hline & 1,3,2 \\ \hline & 5,3,1 \\ \hline & 5,1,2 \\ \hline \end{array}$                   | 1 1 1 2 1 2 1 1,3,2 1 1,1,1 1 5,1,2  | $I_{2}$ $I_{1}$ $I_{1}$ $I_{2}$ $I_{1}, I_{3}, I_{2}$ $I_{5}, I_{3}, I_{1}$ $I_{5}, I_{1}, I_{2}$  |           |

|  |               | $\overline{a_1}$ | $a_2$                    | $a_3$                       | $a_4$                                   | $a_6$   | r                                      |                  | T              | s          | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                                    | $c_p$   | Kodaira   | Isogenies     |
|--|---------------|------------------|--------------------------|-----------------------------|---|---|--|------------------|----------------|------------|------------------------------|--|---|---|---------------|
|  |               |                  |                          |                             | 4                                       |   |  |                  | 1 1            |            |                              | (3)  | P   |   | 8             |
| 104                                    | 4b            |                  |                          |                             | 10                                      | $044b = (30 + 6\varphi) = 2 \cdot 3 \cdot 29b$            | (5 isogeny                             | $_{\mathrm{cl}}$ | asses          | s)         |                              |  |   |   | <b>1044</b> b |
| a1                                     |               | $\varphi$        | -1                       | 1                           | $-3+2\varphi$                           | $7-5\varphi$  | 1                                      |                  | 1              |            | 1, 3, 2                      | 1, 3, 2  | 1, 3, 2   | $I_1, I_3, I_2$   |               |
| b1                                     |               | $\varphi$        | $\varphi$                | 0                           | $-7+10\varphi$                          | $-10 + 11\varphi$   | 0                                      | Ī                | 1              |            | 5, 3, 1                      | 5, 3, 1  | 1,1,1   | $  I_5, I_3, I_1  $   |               |
| c1                                     | $\frac{1}{1}$ | $\varphi$        | $\varphi$                | 1                           | $-15 + 9\varphi$                        | $29-18\varphi$  | 1                                      | i                | 1              |            | 5, 1, 2                      | [5, 1, 2]  | [5, 1, 2]   | $I_5, I_1, I_2$   | <u></u>       |
| d1                                     | 1+            |                  | 0                        | <u>-</u><br>1               |   | $-34+22\varphi$   | 0                                      |                  | 3              |            | 1, 1, 6                      | $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 $       | $\begin{bmatrix} 1, 1, 6 \end{bmatrix}$                         | $  I_1, I_1, I_6  $   |               |
| d2                                     |               | $\varphi$        | 0                        | 1                           | $-9657 + 5967\varphi$                   | $-429681 + 2655556\varphi$                                | 0                                      |                  | 1              |            | 3, 3, 2                      | 3, 3, 2  | 3, 1, 2   | $I_3, I_3, I_2$   |               |
| e1                                     |               | 1                | $-\varphi$               | $1+\varphi$                 | $-9 + 5\varphi$                         | $-15 + 8\varphi$  | 0                                      | Ī                | 1              |            | 1, 1, 3                      | 1,1,3  | 1,1,1   | $I_1, I_1, I_3$   |               |
| 104                                    | 5a            |                  |                          |                             | 1045                                    | $6a = (17 - 29\varphi) = 5a \cdot 11a \cdot 19b$          | (5 isoger                              | กระ              | clas           | ses)       |                              |  |   |   | 1045a         |
| al                                     | Ju            | 0 1              | $1-\varphi$              | /0                          |   | $\frac{a - (17 - 29\varphi) - 3a \cdot 11a \cdot 190}{0}$ | (3 Isoger 1                            | Ť                | 1              | — —        | 1, 1, 2                      | 1,1,2  | 1, 1, 2   | $I_1, I_1, I_2$   | 10100         |
| $\begin{bmatrix} a_1 \\ \end{bmatrix}$ |               | 1                |                          | $\frac{\varphi}{1+\varphi}$ |   | $41 + 59\varphi$  | 1                                      | -!               | - 6 -          |            |                              | $\begin{array}{c c} & 1, 1, 2 \\ \hline & 3, 2, 3 \end{array}$ | $\begin{bmatrix} 1, 1, 2 \\ -1, -1, 2 \\ 3, 2, 1 \end{bmatrix}$ | $\begin{array}{c c} I_1, I_1, I_2 \\ \hline I_3, I_2, I_3 \end{array}$                |               |
| b2                                     |               | 1                |                          | $1+\varphi$<br>$1+\varphi$  | •                                       | $208 - 18\varphi$   |  |                  | 6              | — +<br>+ = | 6, 1, 6                      | $\begin{bmatrix} 3, 2, 3 \\ 6, 1, 6 \end{bmatrix}$             | $\begin{bmatrix} 3, 2, 1 \\ 6, 1, 2 \end{bmatrix}$              | $I_6, I_1, I_6$   |               |
| b3                                     |               | 1                |                          | $1+\varphi$                 | •                                       | $-232 - 178\varphi$                                       | 0                                      |                  | 2              | - +        | , ,                          | 1, 6, 9  | 1, 6, 1   | $I_1, I_6, I_9$   |               |
| b4                                     |               | 1                |                          | $1+\varphi$                 |   | $-2029 - 556\varphi$                                      | 0                                      |                  | 2              | + -        | 2, 3, 18                     | 2, 3, 18   | 2, 3, 2   | $I_2, I_3, I_{18}$  |               |
| c1                                     |               | 0 -1             | $-\varphi$               | $\varphi$                   | $-87-136\varphi$                        | $646 + 1061\varphi$                                       | 0                                      | Ī                | 3              |            | 3, 1, 6                      | 3, 1, 6  | 1,1,6   | $  I_3, I_1, I_6  $   |               |
| c2                                     |               | 0                | -1                       | $\varphi$                   | $-2277 + 1389\varphi$                   | $-48094 + 29851\varphi$                                   | 0                                      |                  | 1              |            | 9, 3, 2                      | 9, 3, 2  | 1, 3, 2   | $I_9, I_3, I_2$   |               |
| d1                                     |               | $\varphi$ $-1$   |                          | $1+\varphi$                 | •                                       | $-3+\varphi$  | 0                                      |                  | $\overline{2}$ | -+         |                              | 1,2,1  | 1,2,1   | $I_1, I_2, I_1$   |               |
| d2                                     |               | 1                |                          | $1+\varphi$                 | $-379 + 234\varphi$                     | $-3243 + 2003\varphi$                                     | 0                                      | 1                | 2              | + -        | 2, 1, 2                      | 2,1,2  | 2,1,2   | $I_2,I_1,I_2$   |               |
| e1                                     | 1 +           |                  | $+\varphi$               | 0                           |   | $204 + 249\varphi$  | 0                                      |                  | 4              | ++         | , ,                          | 2, 2, 10   | 2, 2, 2   | $I_2, I_2, I_{10}$  |               |
| e2                                     | 1 +           |                  | $1+\varphi$              | 0                           | •                                       | $5-5\varphi$  | 0                                      |                  | 2              | -+         | , ,                          | 1, 4, 5  | 1, 2, 1   | $I_1, I_4, I_5$   |               |
| e3                                     | 1 .           | 1                | $-\varphi$               | 1                           | $-5201 - 8266\varphi$                   | $271905 + 439445\varphi$                                  | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ |                  | 2              | ++         | , ,                          | 4, 1, 5  | [2, 1, 1]   | $I_4, I_1, I_5$   |               |
| e4                                     | 1+            | $\varphi$ 1      | $1 + \varphi$            | 0                           | $-314 + 103\varphi$                     | $-1835 + 1625\varphi$                                     | 0                                      |                  | 2              | + -        | 1, 1, 20                     | 1, 1, 20   | 1,1,2   | $I_1, I_1, I_{20}$  |               |
| 104                                    | 5b            |                  |                          |                             | 1045                                    | $5b = (12 - 29\varphi) = 5a \cdot 11b \cdot 19a$          | (5 isoger                              | ny               | clas           | ses)       |                              |  |   |   | 1045b         |
| a1                                     |               | 0                | $\varphi$                | $1+\varphi$                 | 0                                       | $-\varphi$  | 1                                      |                  | 1              |            | 1, 1, 2                      | 1, 1, 2  | 1, 1, 2   | $I_1, I_1, I_2$   |               |
| b1                                     |               | 1                | 1                        | $\varphi$                   | $-41 + 23\varphi$                       | $101-60\varphi$   | 0                                      | Ī                | 6              | +-         | 3, 2, 3                      | 3, 2, 3  | 3, 2, 1   | $I_3, I_2, I_3$   |               |
| b2                                     |               | 1                | 1                        | $\varphi$                   | $-56-12\varphi$                         | $191 + 17\varphi$   | 0                                      |                  | 6              | -+         | 6, 1, 6                      | 6, 1, 6  | 6, 1, 2   | $I_6, I_1, I_6$   |               |
| b3                                     |               | 1                | 1                        | $\varphi$                   |   | $-409 + 177\varphi$                                       | 0                                      |                  | 2              |            | 1, 6, 9                      | 1, 6, 9  | 1, 6, 1   | $I_1, I_6, I_9$   |               |
| b4                                     |               | 1                | 1                        | $\varphi$                   | $214 - 307\varphi$                      | $-2584 + 555\varphi$                                      | 0                                      | 1                | 2              | +          | 2,3,18                       | 2, 3, 18   | 2,3,2   | $I_2, I_3, I_{18}$  |               |
| c1                                     |               |                  |                          | $1+\varphi$                 |   | $1483 - 925\varphi$                                       | 0                                      |                  | 3              |            | 3, 1, 6                      | 3, 1, 6  | 1,1,6   | $I_3, I_1, I_6$   |               |
| c2                                     | -,            | 0                | -1<br>                   | $1+\varphi$                 |   | $-18243 - 29852\varphi$                                   | 0                                      | -                | 1              |            | 9, 3, 2                      | 9, 3, 2  | 1,3,2   | $I_9, I_3, I_2$   | <u> </u>      |
| d1                                     | 1+            | · .              | 1                        | 1                           | $-2-\varphi$                            | $-3-3\varphi$   | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ |                  | 2              | +-         | 1, 2, 1                      | 1, 2, 1  | $\begin{bmatrix} 1, 2, 1 \\ 2, 1 \end{bmatrix}$                 | $I_1, I_2, I_1$   |               |
| $\frac{d2}{1}$                         |               | 1                | 1                        | · <del>9</del>              |   | $-1239 - 2004\varphi$                                     | 0                                      |                  | $-\frac{2}{4}$ |            |                              | 2,1,2  | $\frac{ 2,1,2 }{ 2,2,2 }$                                       | $ $ $I_2, I_1, I_2$   |               |
| e1                                     |               | $\varphi$        | $\varphi$                | 1                           |   | $641 - 383\varphi$  | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ |                  | 4              |            | 2, 2, 10                     | $\begin{bmatrix} 2, 2, 10 \\ 1, 4 \end{bmatrix}$               | $\begin{bmatrix} 2, 2, 2 \\ 1, 2, 1 \end{bmatrix}$              | $I_2, I_2, I_{10}$  |               |
| e2<br>e3                               |               | $\varphi$ $1 -1$ | $\varphi$                | 1<br>1                      | $-5 + 10\varphi$ $-13467 + 8266\varphi$ | $13 + \varphi$ $711350 - 439445\varphi$                   | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ |                  | $\frac{2}{2}$  |            | 1, 4, 5 $4, 1, 5$            | 1,4,5  | $\begin{bmatrix} 1, 2, 1 \\ 2, 1, 1 \end{bmatrix}$              | $I_1, I_4, I_5$   |               |
| e3<br>e4                               |               | $\varphi$        | $1+\varphi$<br>$\varphi$ | 1                           |   | $711500 - 459445\varphi$ $-97 - 1839\varphi$              |  |                  | $\frac{2}{2}$  |            | 4, 1, 3 $1, 1, 20$           | $\begin{array}{ c c c } 4, 1, 5 \\ 1, 1, 20 \end{array}$       | $\begin{bmatrix} 2, 1, 1 \\ 1, 1, 2 \end{bmatrix}$              | $\begin{array}{ c c c c c c }\hline I_4, I_1, I_5 \\ I_1, I_1, I_{20} \\ \end{array}$ |               |
| 0.4                                    |               | Ψ                | Ψ                        | 1                           | 210 100φ                                | σι 100σψ  | 0                                      |                  |                | Г          | 1, 1, 20                     | 1,1,20   | 1,1,2   | 11,11,120   |               |

|      |                                       |              |               |                         |  |                |        |       |                              | T                           | 1         | 1  |           |
|------|---------------------------------------|--------------|---------------|-------------------------|--|----------------|--------|-------|------------------------------|-----------------------------|-----------|--|-----------|
|      | $a_1$                                 | $a_2$        | $a_3$         | $a_4$                   | $a_6$  | r              | T      | s     | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$     | Kodaira                                  | Isogenies |
| 104  | 5c                                    |              |               | 1045                    | $5c = (31 + 3\varphi) = 5a \cdot 11b \cdot 19b$                | (2 isogeny     | classe | es)   |                              |                             |           |  | 1045c     |
| a1   | 1                                     | $1+\varphi$  | $1+\varphi$   | $-6+5\varphi$           | $6-4\varphi$   | 0              | 4      | + -   | 1, 2, 1                      | 1, 2, 1                     | 1, 2, 1   | $I_1, I_2, I_1$                          |           |
| a2   | 1                                     | $1+\varphi$  | $1 + \varphi$ | $-11+5\varphi$          | $-6-\varphi$   | 0              | 8      | ++    | 2, 4, 2                      | 2, 4, 2                     | 2, 4, 2   | $I_2, I_4, I_2$                          |           |
| a3   | 1                                     | $1+\varphi$  | $1 + \varphi$ | $34-35\varphi$          | $12\varphi$  | 0              | 4      | - +   | 1, 8, 1                      | 1, 8, 1                     | 1, 8, 1   | $I_1, I_8, I_1$                          |           |
| a4   | $\varphi$                             | -1           | $\varphi$     | $-140 - 188\varphi$     | $-1005 - 1565\varphi$  | 0              | 4      | ++    | 4, 2, 4                      | 4, 2, 4                     | 2, 2, 4   | $I_4, I_2, I_4$                          |           |
| a5   | . ,                                   | $-1-\varphi$ | $\varphi$     | $-13427 - 21722\varphi$ | $-1134777 - 1836101\varphi$                                    | 0              | 2      | -+    | 2, 1, 8                      | 2, 1, 8                     | 2, 1, 8   | $I_2, I_1, I_8$                          |           |
| a6   | $1+\varphi$                           | $1-\varphi$  | $1+\varphi$   | $-12990 + 8021\varphi$  | $-676864 + 418314\varphi$                                      | 0              | 2      | + -   | 8, 1, 2                      | 8, 1, 2                     | [2, 1, 2] | $I_8, I_1, I_2$                          | L         |
| b1   | 1                                     |              | $1 + \varphi$ | $-2-\varphi$            | -1   | 1              | 2      | ++    | 1, 1, 2                      | 1, 1, 2                     | 1, 1, 2   | $I_1, I_1, I_2$                          |           |
| b2   | 1                                     |              | $1 + \varphi$ | $-12+4\varphi$          | $10-9\varphi$  | 1              | 4      | ++    | 2, 2, 4                      | 2, 2, 4                     | 2, 2, 2   | $I_2, I_2, I_4$                          |           |
| b3   | 1                                     |              | $1 + \varphi$ | $-187 + 79\varphi$      | $885-669\varphi$   | 1              | 2      | ++    | 4, 1, 2                      | 4, 1, 2                     | 4, 1, 2   | $I_4, I_1, I_2$                          |           |
| b4   | 1                                     | 1            | $1+\varphi$   | $3+9\varphi$            | $39 - 25\varphi$   | 1              | 2      |       | 1,4,8                        | 1,4,8                       | 1, 2, 2   | $I_1, I_4, I_8$                          |           |
| 104  | 5d                                    |              |               | 1045                    | $d = (34 - 3\varphi) = 5a \cdot 11a \cdot 19a$                 | (2 isogeny     | classe | es)   |                              |                             |           |  | 1045d     |
| a1   | 1                                     | $-1-\varphi$ | $1+\varphi$   | $-2-4\varphi$           | $4+7\varphi$   | 0              | 4      | -+    | 1, 2, 1                      | 1, 2, 1                     | 1, 2, 1   | $I_1, I_2, I_1$                          |           |
| a2   | 1                                     | $-1-\varphi$ | $1 + \varphi$ | $-7-4\varphi$           | 4arphi   | 0              | 8      | ++    | 2, 4, 2                      | 2, 4, 2                     | 2, 4, 2   | $I_2, I_4, I_2$                          |           |
| a3   | 1                                     | $-1-\varphi$ |               | $-2+36\varphi$          | $14-49\varphi$   | 0              | 4      | + -   | 1, 8, 1                      | 1, 8, 1                     | 1, 8, 1   | $\mathrm{I}_1,\mathrm{I}_8,\mathrm{I}_1$ |           |
| a4   | $1+\varphi$                           | $-1-\varphi$ | · /           | $-328 + 186\varphi$     | $-2570 + 1564\varphi$  | 0              | 4      | ++    | 4, 2, 4                      | 4, 2, 4                     | 2, 2, 4   | $I_4, I_2, I_4$                          |           |
| a5   | $\varphi$                             | -1           | $1+\varphi$   | $-35148 + 21720\varphi$ | $-2970878 + 1836100\varphi$                                    | 0              | 2      | + -   | 2, 1, 8                      | 2, 1, 8                     | 2, 1, 8   | $I_2, I_1, I_8$                          |           |
| a6   | $\varphi$                             | 1            | $\varphi$     | $-4967 - 8023\varphi$   | $-258549 - 418315\varphi$                                      | 0              | 2      | - +   | 8, 1, 2                      | 8, 1, 2                     | 2, 1, 2   | $\mid I_8, I_1, I_2$                     |           |
| b1   | 1                                     | 1            | $\varphi$     | -2                      | -arphi   | 1              | 2      | ++    | 1, 1, 2                      | 1, 1, 2                     | 1, 1, 2   | $\mid I_1, I_1, I_2$                     |           |
| b2   | 1                                     | 1            | $\varphi$     | $-7-5\varphi$           | $2+8\varphi$   | 1              | 4      | ++    | 2, 2, 4                      | 2, 2, 4                     | 2, 2, 2   | $I_2, I_2, I_4$                          |           |
| b3   | 1                                     | 1            | $\varphi$     | $-107 - 80\varphi$      | $217 + 668\varphi$   | 1              | 2      | ++    | 4, 1, 2                      | 4, 1, 2                     | 4, 1, 2   | $I_4, I_1, I_2$                          |           |
| b4   | 1                                     | 1            | φ             | $13 - 10\varphi$        | $15 + 24\varphi$   | 1              | 2      |       | 1,4,8                        | 1,4,8                       | 1, 2, 2   | $I_1, I_4, I_8$                          |           |
| 1049 | 9a                                    |              |               | 10                      | $049a = (13 - 29\varphi) = 1049a $                             | (2 isogeny cla | sses)  |       |                              |                             |           |  | 1049a     |
| a1   |                                       | $-1+\varphi$ | $\varphi$     | $-1-\varphi$            | $-\varphi$   | 1              | 2      | ++    | 1                            | 1                           | 1         | $I_1$                                    |           |
| a2   | $\varphi$                             | $-1+\varphi$ | $\varphi$     | $-6-11\varphi$          | $-14-24\varphi$  | 1              | 2      | - +   | 2                            | 2                           | 2         | $I_2$                                    |           |
| b1   | 1                                     | $1+\varphi$  | $1+\varphi$   | $1+\varphi$             | 0  | 1              |        |       | 1                            | 1                           | 1         | $I_1$                                    |           |
| 1049 | 9b                                    |              |               | 10                      | $049b = (16 - 29\varphi) = 1049b $                             | 2 isogeny cla  | sses)  |       |                              |                             |           |  | 1049b     |
| a1   | $1+\varphi$                           | $\varphi$    | $\varphi$     | $-1+\varphi$            | 0  | 1              | 2      | ++    | 1                            | 1                           | 1         | $I_1$                                    |           |
| a2   | $1+\varphi$                           | $\varphi$    | $\varphi$     | $-16+11\varphi$         | $-27 + 18\varphi$  | 1              | 2      | + -   | 2                            | 2                           | 2         | $I_2$                                    |           |
| b1   |                                       | $-1-\varphi$ |               | 1                       | $-1-\varphi$   | 1              | 1      | ·<br> | 1                            | 1                           | 1         | $I_1$                                    | <u>-</u>  |
| 105  | <br>5a                                |              |               | 10                      | $55a = (33 - \varphi) = 5a \cdot 211a$                         | (2 isogeny cla | sees)  |       |                              |                             |           | 1  | 1055a     |
| al   | $\frac{\partial \mathbf{u}}{\varphi}$ | $-\varphi$   | $\varphi$     | $1-\varphi$             | $\frac{-34 - (33 - \varphi) - 34 \cdot 2114}{-34 - 34\varphi}$ |                | 1      | T     | 1,13                         | 1,13                        | 1,1       | $I_{1}, I_{13}$                          | 10000     |
| b1   | $1+\varphi$                           |              |               | $-7-11\varphi$          | $-18 - 29\varphi$  | 0              | 1 1    | '     | 1,1                          | 1,1                         | 1,1       | $I_1, I_1$                               |           |
| 105  | 5b                                    |              |               | 10                      | $055b = (32 + \varphi) = 5a \cdot 211b $                       | 2 isogeny cla  | sses)  | 1     |                              | 1                           | 1         | 1  | 1055b     |
| a1   | $\frac{1+\varphi}{1+\varphi}$         | 1            | $1+\varphi$   |                         | $\frac{-68+33\varphi}{-68+33\varphi}$                          |                | 1      |       | 1,13                         | 1,13                        | 1,1       | $I_{1}, I_{13}$                          |           |
| aı   | $1 + \varphi$                         | -1           | $1+\varphi$   | $-\varphi$              | $-00+39\varphi$  | U              | 1      |       | 1,13                         | 1,10                        | 1,1       | 11,113                                   |           |

|   | $a_1$                   | $a_2$        | $a_3$       | $a_4$                                  |   | $a_6$ $r$        | T               | s        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$  | Kodaira  | Isogenies |
|---|-------------------------|--------------|-------------|--|---|------------------|-----------------|----------|------------------------------|-----------------------------|--|--|-----------|
| 105   | $5\mathrm{b}$           |              |             | 10                                     | $55b = (32 + \varphi) = 5a \cdot 211b$  | (2 isogeny class | ses)            |          |                              |                             |  |  | 1055b     |
| b1  | $\varphi$               | $1-\varphi$  | 0           | $-18 + 11\varphi$                      | -47 + 29                                | $9\varphi$ 0     | 1               |          | 1,1                          | 1,1                         | 1,1  | $I_1, I_1$   |           |
| 106   | 9a                      |              |             | 1                                      | $1069a = (35 - 4\varphi) = 1069a$       | (1 isogeny clas  | s)              |          |                              |                             |  |  | 1069a     |
| a1  | 1                       | $-\varphi$   | 0           | $-5+4\varphi$                          | -8 + 6                                  | $4\varphi$ 0     | 1               |          | 1                            | 1                           | 1  | $I_1$  |           |
| 106   | 9b                      |              |             |  | $1069b = (31 + 4\varphi) = 1069b$       | (1 isogeny class | s)              |          |                              |                             |  |  | 1069b     |
| a1  | 1                       | $-1+\varphi$ | 0           | $-1-4\varphi$                          | -4-6                                    |                  | <del></del>     |          | 1                            | 1                           | 1  | $I_1$  |           |
| 107   | 6a                      |              |             | 10                                     | $76a = (8 - 30\varphi) = 2 \cdot 269a$  | (2 isogeny class | ses)            |          |                              | •                           | •  |  | 1076a     |
| a1  | $1+\varphi$             | $-1-\varphi$ | 0           | $-1+\varphi$                           |   | 1 1              | <del></del>     | +-       | 2,1                          | 2,1                         | 2,1  | $I_2, I_1$   |           |
| b1  | 1                       | -1           | 1           | $-6-\varphi$                           | 6+                                      | $2\varphi$   1   | 1               | + -      | 6, 1                         | 6, 1                        | 6,1  | $I_6, I_1$   |           |
| 107   | 6b                      |              |             | 10'                                    | $76b = (22 - 30\varphi) = 2 \cdot 269b$ | (2 isogeny clas  | sses)           |          |                              |                             |  |  | 1076b     |
| a1  | $\varphi$               | -1           | 0           | $-\varphi$                             |   | 1 1              | <del>- ´-</del> | -+       | 2, 1                         | 2, 1                        | 2,1  | $I_2, I_1$   |           |
| b1  | 1                       | -1           | 1           | $-7+\varphi$                           | 8-1                                     | $2\varphi$   1   | 1               | <u> </u> | 6, 1                         | 6,1                         | 6,1  | $I_6, I_1$   |           |
| 108   | 4a                      |              |             | 10                                     | $84a = (34 - 2\varphi) = 2 \cdot 271a$  | (6 isogeny class | ses)            |          |                              | 1                           |  |  | 1084a     |
| a1  | $1+\varphi$             | $1-\varphi$  | 0           | $-5+3\varphi$                          | 3 – 1                                   |                  | 1               | -+       | 1,1                          | 1,1                         | 1,1  | $I_1, I_1$   |           |
| b1  | $\varphi$               | $-\varphi$   | 0           | $-47 + 22\varphi$                      | 134 - 9                                 | $3\varphi$   0   | 1               |          | 15,1                         | 15,1                        | 1,1  | $I_{15}, I_1$                                      |           |
| c1  | $\varphi$               | $\varphi$    | $1+\varphi$ | $-\varphi$                             | 1-1                                     | $2\varphi$   1   | 1               | -+       | 5, 1                         | 5,1                         | 5,1  | $I_5, I_1$   |           |
| d1  | $1+\varphi$             | 0            |             | $-4-2\varphi$                          |   | $2\varphi$ 0     |                 | + -      | ,                            | 6, 1                        | 6,1  | $I_6, I_1$   |           |
| $\begin{array}{ c c } d2 \\ d3 \end{array}$ | $1+\varphi$             | 0            | 1 $1$       | $-44 - 42\varphi$ $-913 + 567\varphi$  | 128 + 16 $-12541 + 775$                 | •                |                 | ++       |                              | $3, 2 \\ 2, 3$              | $\begin{bmatrix} 3, 2 \\ 2, 3 \end{bmatrix}$ | $\begin{matrix} I_3, I_2 \\ I_2, I_3 \end{matrix}$ |           |
| d4  | $\varphi$ $1+\varphi$   | 0            |             | $-913 + 307\varphi$ $-174 + 18\varphi$ | -672 + 46                               | •                |                 | ++       |                              | 1, 6                        | $\begin{bmatrix} 2, 3 \\ 1, 6 \end{bmatrix}$ | $I_1, I_6$   |           |
| e1  | 1                       | $\varphi$    | $\varphi$   | φ                                      | -2 +                                    |                  | -'              | <u> </u> | <u>,</u> - <u>-</u>          | $\frac{1}{6}, \frac{1}{1}$  | 6,1  | $I_6, I_1$   |           |
| e2  | 1                       | $\varphi$    | $\varphi$   | $-140 + 61\varphi$                     | -682 + 34                               |                  | 1               | <u> </u> | 2,3                          | 2, 3                        | 2,3  | $I_2, I_3$   |           |
| f1  | 1                       |              | $1+\varphi$ | $-804 + 493\varphi$                    | 10375 - 641                             |                  |                 | - +      | ,                            | 3, 1                        | 3,1  | $I_3, I_1$   |           |
| f2  | $1+\varphi$             | $-\varphi$   | 0           | $-6487 - 10459\varphi$                 | -381223 - 61689                         | $8\varphi$ 0     | 1               | -+       | 1,3                          | 1,3                         | 1,3  | $I_1, I_3$   |           |
| 108   | 4b                      |              |             | 10                                     | $84b = (32 + 2\varphi) = 2 \cdot 271b$  | (6 isogeny class | ses)            | 1        |                              |                             |  | 1  | 1084b     |
| a1  | $\varphi$               | 1            | 0           | $-2-3\varphi$                          | 1+3                                     | $2\varphi$   1   | 1               | + -      | 1,1                          | 1,1                         | 1,1  | $I_1, I_1$   |           |
| b1  | $1+\varphi$             | -1           | 0           | $-25-22\varphi$                        | 41 + 96                                 | $3\varphi$   0   | 1               |          | 15, 1                        | 15,1                        | 1,1  | $I_{15}, I_1$                                      |           |
| c1  | $1+\varphi$             | $1+\varphi$  | $1+\varphi$ | $2\varphi$                             | 1 + 1                                   | $2\varphi$   1   | 1               | + -      | 5, 1                         | 5,1                         | 5,1  | $I_5, I_1$   |           |
| d1  | $\varphi$               | $1-\varphi$  | 1           | $-5+\varphi$                           | 2-1                                     | '                | 6               | - +      | ,                            | 6,1                         | 6,1  | $I_6, I_1$   |           |
| d2  | $\varphi$               | $1-\varphi$  | 1           | $-85 + 41\varphi$                      | 290 - 16                                | '                |                 | ++       |                              | 3, 2                        | 3,2  | $I_3, I_2$   |           |
| d3<br>d4                                    | $\varphi$ $1 + \varphi$ | $1-\varphi$  | 1<br>1      | $-155 - 19\varphi$ $-346 - 568\varphi$ | -206 - 46 $-4789 - 775$                 | '                |                 | ++       |                              | 1, 6 $2, 3$                 | $\begin{bmatrix} 1, 6 \\ 2, 3 \end{bmatrix}$ | $I_1, I_6$ $I_2, I_6$                              |           |
| u4  | $1 + \varphi$           | $-\varphi$   | 1           | $-540-500\varphi$                      | -4109 - 110                             | <u>-</u> Ψ       |                 | - +      | ۷, ن                         | ۷, ۵                        | ۷, ن   | $I_2, I_3$   |           |

|      |               |                  |                                |                           |   |            | T                                      | Τ.       | $\operatorname{ord}(\Delta)$          | $\operatorname{ord}_{-}(j)$                   |  | Kodaira                            | Tanmanina |
|------|---------------|------------------|--------------------------------|---------------------------|---|------------|--|----------|---------------------------------------|---|--|------------------------------------|-----------|
|      | $a_1$         | $a_2$            | $a_3$                          | $a_4$                     | $a_6$   | r          | 1                                      | s        | $\frac{\operatorname{ord}(\Delta)}{}$ |   | $c_p$  | Kodana                             | Isogenies |
| 1084 | <b>1</b> b    |                  |                                | 108                       | $84b = (32 + 2\varphi) = 2 \cdot 271b$          | (6 isogeny | class                                  | es)      |                                       |   |  |                                    | 1084b     |
| e1   | 1             |                  | $1 + \varphi$                  | $1-2\varphi$              | $-1-2\varphi$                                   | 0          | 3                                      |          | -, -                                  | 6,1   | 6, 1   | $I_6, I_1$                         |           |
| e2   | 1             | $1-\varphi$      | $1+\varphi$                    | $-79-62\varphi$           | $-341 - 342\varphi$                             | 0          | 1                                      |          | -2,3                                  | 2,3   | 2,3  | $I_2, I_3$                         |           |
| f1   | 1             | $1-\varphi$      | $\varphi$                      | $-310 - 494\varphi$       | $3957 + 6418\varphi$                            | 0          | 3                                      | + -      | - 3,1                                 | 3,1   | 3,1  | $ $ $I_3, I_1$                     |           |
| f2   | $\varphi$     | 0                | 0                              | $-16946 + 10459\varphi$   | $-998121 + 616898\varphi$                       | 0          | 1                                      | + -      | - 1,3                                 | 1,3   | 1,3  | $I_1, I_3$                         |           |
| 1089 | 9a            |                  |                                | 10                        | $89a = (33) = 3 \cdot 11a \cdot 11b $           | 3 isogeny  | classe                                 | es)      |                                       |   |  |                                    | 1089a     |
| a1   | 1             | 1                | 0                              | -146                      | 621   | 1          | 4                                      | +-       | - 4, 4, 3                             | 4, 4, 3                                       | 4, 4, 3  | $I_4, I_4, I_3$                    |           |
| a2   | 1             | 1                | 0                              | -11                       | 0   | 1          | 4                                      | +-       | -2, 2, 6                              | 2, 2, 6                                       | 2, 2, 6  | $I_2, I_2, I_6$                    |           |
| a3   | 1             | 1                | 0                              | -6                        | -9  | 1          | 2                                      | +-       | -1, 1, 3                              | 1, 1, 3                                       | 1, 1, 3  | $I_1, I_1, I_3$                    |           |
| a4   | 1             | 1                | 0                              | 44                        | 55  | 1          | 2                                      |          | -1, 1, 12                             | 1, 1, 12                                      | 1, 1, 12   | $I_1, I_1, I_{12}$                 |           |
| b1   | $1+\varphi$   | $1+\varphi$      | 0                              | $-30+19\varphi$           | $71-44\varphi$                                  | 1          | $\begin{vmatrix} 2 \end{vmatrix}$      | + -      | -2, 2, 1                              | 2, 2, 1                                       | 2, 2, 1  | $  I_2, I_2, I_1  $                | [         |
| b2   | $1+\varphi$   | $1+\varphi$      | 0                              | 4arphi                    | $2+\varphi$                                     | 1          | 2                                      |          | -1, 1, 2                              | 1, 1, 2                                       | 1, 1, 2  | $I_1, I_1, I_2$                    |           |
| c1   | $\varphi$     | $\varphi$        | 1                              | $-15-16\varphi$           | $24 + 30\varphi$                                | 1          | $\begin{vmatrix} 1 & 2 \end{vmatrix}$  | + -      | -2, 2, 1                              | 2, 2, 1                                       | [2, 2, 1]  | $I_2, I_2, I_1$                    |           |
| c2   | $\varphi$     | φ                | 1                              | -arphi                    | 0   | 1          | 2                                      |          |                                       | 1, 1, 2                                       | 1, 1, 2  | $I_1, I_1, I_2$                    |           |
| 1089 | 9b            |                  |                                | 108                       | $9b = (21 - 30\varphi) = 3 \cdot 11a^2$         | (3 isogeny | clas                                   | ses)     |                                       |   |  |                                    | 1089b     |
| a1   | 0             | -1               | $1+\varphi$                    | $-263 + 163\varphi$       | $-1847 + 1142\varphi$                           | 0          | 1                                      | Ī — -    | - 8,1                                 | 1   | 1, 1   | $IV^*, I_1$                        |           |
| b1   | 1             |                  | $1+\varphi$                    | $6-4\varphi$              | 5+3arphi  |            |  | †        |                                       | $\frac{1}{2}$ , $\frac{1}{2}$ , $\frac{1}{2}$ | $\frac{1}{1} - \frac{1}{2} - \frac{1}{1}$              | $  I_2^*, I_1  $                   |           |
| b2   | 1             |                  | $1 + \varphi$<br>$1 + \varphi$ | $-44-19\varphi$           | $46 + 100\varphi$                               |            | $\begin{vmatrix} 2 \\ 4 \end{vmatrix}$ | ++       |                                       | 4, 2  | $\frac{2}{4}, \frac{1}{2}$                             | $I_4^2, I_1$                       |           |
| b3   | 1             |                  | $1+\varphi$ $1+\varphi$        | $-589 - 364\varphi$       | $2119 + 7994\varphi$                            |            | 4                                      | + -      | ,                                     | 2, 4  | 4, 4   | $I_2^{*}, I_4$                     |           |
| b4   | 1             |                  | $1+\varphi$                    | $-299 + 86\varphi$        | $1597 - 1286\varphi$                            | 0          | 2                                      | + -      |                                       | 8,1   | 4, 1   | $I_8^*, I_1$                       |           |
|      | $1 + \varphi$ | -1               | 1                              | $-244566 - 395709\varphi$ | $88487886 + 143176430\varphi$                   | 0          | 4                                      | -        |                                       | 1,2   | 4, 2   | $I_1^*, I_2$                       |           |
|      |               | $-1+\varphi$     | 1                              | $-28939 + 17780\varphi$   | $-2206705 + 1364278\varphi$                     | 0          | 2                                      | + -      |                                       | 1,8   | 2,8  | $I_1^*, I_8$                       |           |
| c1   | 0             | $-1-\varphi$     | $1+\varphi$                    | $-4+4\varphi$             | $-5+\varphi$                                    | 0          | 1                                      | <u> </u> | 2,1                                   | 1   | 1,1  | $ $ II, I $_1$                     |           |
| 1089 | )c            |                  |                                | 109                       | $89c = (9 - 30\varphi) = 3 \cdot 11b^2 $        | (3 isogeny | class                                  | es)      |                                       |   | I.   | 1                                  | 1089c     |
| al   | 0             | -1               | $\varphi$                      | $-100 - 163\varphi$       | $\frac{-704 - 1143\varphi}{}$                   | 0          | 1                                      | T        | - 8,1                                 | 1   | 1, 1   | $IV^*, I_1$                        |           |
| b1   | <br>1         | <del>-</del> - 1 |                                | $3+3\varphi$              | $9-4\varphi$                                    |            | $\frac{1}{2}$                          | <u> </u> |                                       | $\frac{1}{2}$ , $\frac{1}{2}$ , $\frac{1}{1}$ | $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ | $ I_2^*, I_1 $                     |           |
| b2   | 1             | 1                | arphi                          | $-62+18\varphi$           | $147 - 101\varphi$                              |            | $\frac{2}{4}$                          | +-       |                                       | $\frac{2}{4}, \frac{1}{2}$                    | $\frac{2}{4}, \frac{1}{2}$                             | $I_{4}^{2}, I_{1}$                 |           |
| b3   | 1             | 1                | arphi                          | $-952 + 363\varphi$       | $10114 - 7995\varphi$                           |            | 4                                      |          |                                       | 2,4   | 4, 2 $4, 4$  | $I_{2}^{4}, I_{4}$                 |           |
| b4   | 1             | 1                | $\varphi$                      | $-212-87\varphi$          | $312 + 1285\varphi$                             |            | 2                                      | + -      |                                       | 8,1   | 4, 1   | $I_8^*, I_1$                       |           |
| b5   | $\varphi$     | $-\varphi$       | 1                              | $-640274 + 395708\varphi$ | $231664316 - 143176430\varphi$                  | 0          | 4                                      | + -      |                                       | 1, 2  | 4, 2   | $I_1^*, I_2$                       |           |
| b6   | $\varphi$     | $1+\varphi$      | $\varphi$                      | $-11159 - 17779\varphi$   | $-860207 - 1393217\varphi$                      | 0          | 2                                      | -        |                                       | 1,8   | 2,8  | $I_1^*, I_8$                       |           |
| c1   | 0             | $1+\varphi$      | $\varphi$                      | $-1-2\varphi$             | $-4-5\varphi$                                   | 0          | <br>  1                                | <u> </u> | - 2,1                                 | 1   | $\begin{bmatrix} 1, 1 \end{bmatrix}$                   | $ $ II, I $_1$                     | <u>-</u>  |
| 1100 | )a            |                  |                                | 1100a                     | $a = (10 - 30\varphi) = 2 \cdot 5a^2 \cdot 11a$ | (9 isoge   | nv cl                                  | asses)   |                                       | L   | I  | 1                                  | 1100a     |
| al   | $\varphi$     | $\varphi$        | 1                              | -2                        | $\frac{1-\varphi}{1-\varphi}$                   | 1          | 2                                      |          | - 1,2,3                               | 1,2   | 1, 2, 2  | $I_1, I_2, III$                    |           |
| a2   |               | $-1+\varphi$     | 1                              | $-192 + 117\varphi$       | $1 - \varphi$ $1284 - 795\varphi$               | 1          | $\begin{vmatrix} 2 \\ 2 \end{vmatrix}$ | +-       |                                       | 2, 1  | 2, 1, 2  | $I_1, I_2, III$<br>$I_2, I_1, III$ |           |
|      |               | - 1 7            |                                | 102   1119                | 1201 1000                                       | 1          |  | '        | -, -, -                               | _, -, -                                       | -, -, -  | -2,-1,1                            |           |

|     | $a_1$       | $a_2$        | $a_3$          | $a_4$                       | $a_6$   | r                                      | T             | s        | $\operatorname{ord}(\Delta)$             | $\operatorname{ord}_{-}(j)$         | $c_p$   | Kodaira  | Isogenies |
|-----|-------------|--------------|----------------|-----------------------------|---|--|---------------|----------|--|-------------------------------------|---|--|-----------|
| 110 | 0a          |              |                | 1100a                       | $= (10 - 30\varphi) = 2 \cdot 5a^2 \cdot 11a$   | (9 isog                                | geny o        | classes) |  |                                     |   |  | 1100a     |
| b1  | $1+\varphi$ | $1-\varphi$  | 0              | $5+\varphi$                 | $-31+60\varphi$                                 |  | 1             | <u> </u> | 9, 3, 2                                  | 9,3                                 | 1, 1, 1   | $I_9, I_3, II$                                 |           |
| b2  | $1+\varphi$ | $1-\varphi$  | 0              | $-10+6\varphi$              | $-22+12\varphi$                                 | 0                                      | 1             |          | 3, 1, 2                                  | 3, 1                                | 1, 1, 1   | $I_3, I_1, II$                                 |           |
| c1  | $1+\varphi$ | $1-\varphi$  | <u>.</u> 1     | $1-5\varphi$                | $12-9\varphi$                                   | 1                                      | $\frac{1}{1}$ | i – +    | 3, 2, 7                                  | 3, 2, 1                             | 3, 2, 4   | $I_3, I_2, I_1^*$                              |           |
| c2  | $1+\varphi$ | $1-\varphi$  | 1              | $-374 + 20\varphi$          | $2602 - 189\varphi$                             | 1                                      | 2             | -+       | 1, 6, 9                                  | 1, 6, 3                             | 1, 2, 4   | $I_1, I_6, I_3^*$                              |           |
| c3  | $1+\varphi$ | $1-\varphi$  | 1              | $-49 + 45\varphi$           | $152 - 89\varphi$                               | 1                                      | 2             | + -      | 6, 1, 8                                  | 6, 1, 2                             | 6, 1, 4   | $I_6, I_1, I_2^*$                              |           |
| c4  | $\varphi$   | $\varphi$    | 0              | $-30733 + 18965\varphi$     | $2441677 - 1508975\varphi$                      | 1                                      | 2             | +-       | 2, 3, 12                                 | 2, 3, 6                             | 2, 1, 4   | $I_2, I_3, I_6^{\overline{*}}$                 |           |
| d1  | $\varphi$   | $-1-\varphi$ | $1+\varphi$    | $-22 - 32\varphi$           | $82 + 126\varphi$                               | 0                                      | 5             |          | 5, 1, 4                                  | [5,1]                               | 5,1,1   | $ \tilde{I}_5, \tilde{I}_1, \tilde{IV} $       |           |
| d2  | $\varphi$   | $-1-\varphi$ | $\varphi$      | $3+11\varphi$               | $-14-16\varphi$                                 | 0                                      | 1             |          | 1, 5, 8                                  | 1,5                                 | 1, 1, 1   | $I_1, I_5, IV^*$                               |           |
| e1  | 1           | $-\varphi$   | 1              | $-534 + 316\varphi$         | $-5624 + 3501\varphi$                           | 0                                      | 7 - 4 -       | + +      | $\bar{2}, \bar{2}, \bar{12}$             | [2, 2, 6]                           | [2, 2, 4]   | $[I_2, I_2, I_6^*]$                            | [         |
| e2  | $\varphi$   | $-1+\varphi$ | $\varphi$      | $-3370 - 5148\varphi$       | $133542 + 215406\varphi$                        | 0                                      | 4             | ++       | 6, 6, 8                                  | 6, 6, 2                             | 2, 2, 4   | $I_6, I_6, I_2^*$                              |           |
| e3  | $\varphi$   | $-1+\varphi$ | $\varphi$      | $-20-23\varphi$             | $-53-79\varphi$                                 | 0                                      | 2             | -+       | 1, 4, 9                                  | 1, 4, 3                             | 1, 2, 2   | $I_1,I_4,I_3^{	ilde{*}}$                       |           |
| e4  | $\varphi$   | $-1+\varphi$ | $\varphi$      | $-170 - 348\varphi$         | $2022 + 3246\varphi$                            | 0                                      | 2             | - +      | 3, 12, 7                                 | 3, 12, 1                            | 1, 2, 2   | $I_3, I_{12}, I_1^*$                           |           |
| e5  | 1           | $-\varphi$   | 1              | $-534 + 366\varphi$         | $-5104 + 3261\varphi$                           | 0                                      | 2             | I .      | 4, 1, 18                                 | 4, 1, 12                            | 2, 1, 4   | $I_4, I_1, I_{12}^*$                           |           |
| e6  | 1           | $-\varphi$   | 1              | $-12409 + 6616\varphi$      | $593896 - 349739\varphi$                        | 0                                      | 2             |          | 12, 3, 10                                | 12, 3, 4                            | 2, 1, 4   | $I_{12}, I_3, I_4^*$                           |           |
| e7  | $1+\varphi$ | $1+\varphi$  | 0              | $-57865 + 35736\varphi$     | $-6304383 + 3896363\varphi$                     | 0                                      | 2             |          | 1, 1, 9                                  | 1, 1, 3                             | 1, 1, 2   | $I_1, I_1, I_3^*$                              |           |
| e8  | 1           | $-1+\varphi$ | 1              | $-2430508 - 3932586\varphi$ | $2773402247 + 4487459209\varphi$                | 0                                      | 2             | + +      | 3, 3, 7                                  | 3,3,1                               | 1,1,2   | $[I_3, I_3, I_1^*]$                            |           |
| f1  | $1+\varphi$ | 1            | $1 + \varphi$  | $-1+\varphi$                | 1   | 0                                      | 5             |          | 1, 5, 2                                  | 1,5                                 | 1, 5, 1   | $I_1, I_5, II$                                 |           |
| f2  | $1+\varphi$ | 1            | $1+\varphi$    | $-61-49\varphi$             | $-353 - 252\varphi$                             | 0                                      | 1             | l        | 5, 1, 10                                 | $\begin{bmatrix} 5,1 \end{bmatrix}$ | [5, 1, 1]   | $\begin{bmatrix} I_5, I_1, II^* \end{bmatrix}$ |           |
| g1  | $1+\varphi$ | 1            | $\varphi$      | $-23-41\varphi$             | $-114 - 179\varphi$                             | 0                                      | 2             | - +      | 1, 2, 11                                 | 1,2,5                               | 1, 2, 2   | $I_1, I_2, I_5^*$                              |           |
| g2  | $1+\varphi$ | 1            | $\varphi$      | $-1748 - 2466\varphi$       | $45711 + 71946\varphi$                          | 0                                      | 10            | -+       | 5, 10, 7                                 | 5, 10, 1                            | 5, 10, 2  | $I_5, I_{10}, I_1^*$                           |           |
| g3  |             | $-1-\varphi$ | 0              | $-18144 + 10714\varphi$     | $1090100 - 667232\varphi$                       | 0                                      | 10            | +-       | 10, 5, 8                                 | 10, 5, 2                            | 10, 5, 4  | $I_{10}, I_5, I_2^*$                           |           |
| g4  | $\varphi$   | $-1-\varphi$ | 0              | $-144 + 89\varphi$          | $-725 + 418\varphi$                             | 0                                      | 2             | + -      | 2, 1, 16                                 | 2, 1, 10                            | 2, 1, 4   | $I_2, I_1, I_{10}^*$                           |           |
| h1  | $1+\varphi$ | 0            | 0              | $-24 + 12\varphi$           | $-52 + 28\varphi$                               | 0                                      | 2             | -+       | 1, 2, 9                                  | 1, 2                                | 1, 2, 2   | $I_1, I_2, III^*$                              |           |
| h2  | $1+\varphi$ | 0            | 0              | $-374 + 212\varphi$         | $-3152 + 1978\varphi$                           | 0                                      | 2             | ++       | 2, 1, 9                                  | 2,1                                 | 2, 1, 2   | $I_2, I_1, III^*$                              |           |
| i1  | 1           | $1-\varphi$  | $\varphi$      | $-138 + 84\varphi$          | $653 - 400\varphi$                              | 0                                      | 3             |          | 3, 1, 8                                  | 3,1                                 | 1,1,3   | $[I_3, I_1, IV^*]$                             |           |
| i2  | $\varphi$   | $-\varphi$   | $\varphi$      | $27 + 33\varphi$            | $-707 - 1446\varphi$                            | 0                                      | 1             |          | 9, 3, 8                                  | 9, 3                                | 1, 3, 1   | $I_9, I_3, IV^*$                               |           |
| 110 | ՈՒ          |              |                | 11001                       | (20, 20, ) 2 5 2 111                            | (0.1                                   |               | 1        |  |                                     |   |  | 1100b     |
| _   |             |              |                |                             | $y = (20 - 30\varphi) = 2 \cdot 5a^2 \cdot 11b$ | (9 isog                                | 1             | · ·      | 0.0.=                                    | 0.6.1                               |   | T T T*   | TIOOD     |
| a1  | $\varphi$   | 1            | 1              | $-3+4\varphi$               | $3+9\varphi$                                    | 1                                      | 2             | +-       | 3, 2, 7                                  | 3, 2, 1                             | 3, 2, 4   | $I_3, I_2, I_1^*$                              |           |
| a2  | $\varphi$   | 1            | 1              | $-353-21\varphi$            | $2413 + 189\varphi$                             |  | $\frac{2}{2}$ | + -      | 1, 6, 9                                  | 1, 6, 3                             | 1, 2, 4   | $I_1, I_6, I_3^*$                              |           |
| a3  | $\varphi$   | 1            | 1              | $-3-46\varphi$              | $63 + 89\varphi$                                | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | $\frac{2}{2}$ | I .      | 6, 1, 8                                  | 6, 1, 2                             | 6, 1, 4   | $I_6, I_1, I_2^*$                              |           |
|     | $1+\varphi$ | $1+\varphi$  | <del>-</del> - | $-11768 - 18962\varphi$     | $913738 + 1478243\varphi$                       |  | 2             | - +      | $\frac{2}{1}, \frac{3}{2}, \frac{12}{2}$ | 2,3,6                               | $\frac{1}{1}$ $\frac{2}{1}$ $\frac{1}{2}$ $\frac{3}{2}$ | $I_2, I_3, I_6^*$                              |           |
| b1  | $1+\varphi$ | $1+\varphi$  | 0              | $-1+3\varphi$               | 1   |  | $\frac{2}{2}$ | + -      | 1, 2, 3                                  | 1,2                                 | 1, 2, 2   | $I_1, I_2, III$                                |           |
| b2  | 1           | $-\varphi$   | <u>l</u>       | $-75 - 117\varphi$          | $489 + 795\varphi$                              | _   1                                  | 2             | + +      | 2, 1, 3                                  | 2,1                                 | 2,1,2   | $I_2, I_1, III$                                |           |
| c1  | $\varphi$   | 1            | 0              | $6-\varphi$                 | $29-60\varphi$                                  | 0                                      | 1             |          | 9, 3, 2                                  | 9,3                                 | 1,1,1   | $I_9, I_3, II$                                 |           |
| c2  | $\varphi$   | 1            | 0              | $-4-6\varphi$               | $-10-12\varphi$                                 | _   0                                  | <u>  1</u>    |          | 3, 1, 2                                  | 3,1                                 | 1,1,1   | $I_3, I_1, II$                                 |           |
| d1  | $1+\varphi$ | 1            | 1              | $-53 + 32\varphi$           | $156 - 95\varphi$                               | 0                                      | 5             |          | 5, 1, 4                                  | 5, 1                                | 5, 1, 1   | $I_5, I_1, IV$                                 |           |
| d2  | $1+\varphi$ | 1            | 0              | $15-10\varphi$              | $-15+5\varphi$                                  | 0                                      | 1             |          | 1, 5, 8                                  | 1,5                                 | 1, 1, 1   | $I_1, I_5, IV^*$                               |           |

|  | $a_1$               | $a_2$         | $a_3$        | $a_4$                                 | $a_6$  | r                                      | T                                      | s                        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                  | $c_p$  | Kodaira   | Isogenies |
|--|---------------------|---------------|--------------|---------------------------------------|--|--|--|--------------------------|------------------------------|--|--|---|-----------|
|  |                     |               |              |                                       |  | l                                      |  |                          |                              |  |  |   |           |
| 110  | 0b                  |               |              | 1100b                                 | $\phi = (20 - 30\varphi) = 2 \cdot 5a^2 \cdot 11b$ | (9 isog                                | geny c                                 | lasses)                  |                              |  |  |   | 1100b     |
| e1   | 1                   | $-1+\varphi$  | 1            | $-218 - 316\varphi$                   | $-2123 - 3501\varphi$                              | 0                                      | 4                                      | ++                       | 2, 2, 12                     | 2, 2, 6                                      | 2, 2, 4  | $I_2, I_2, I_6^*$   |           |
| e2   | $1 + \varphi$       | $\varphi$     | $\varphi$    | $-8517 + 5148\varphi$                 | $354096 - 218776\varphi$                           | 0                                      | 4                                      | ++                       | 6, 6, 8                      | 6, 6, 2                                      | 2, 2, 4  | $I_6, I_6, I_2^*$   |           |
| e3   | $1 + \varphi$       | $\varphi$     | $\varphi$    | $-42 + 23\varphi$                     | $-109 + 59\varphi$                                 | 0                                      | 2                                      | + -                      | 1, 4, 9                      | 1,4,3  | 1, 2, 2  | $I_1,I_4,I_3^*$   |           |
| e4   | $1 + \varphi$       | $\varphi$     | $\varphi$    | $-517 + 348\varphi$                   | $5616 - 3416\varphi$                               | 0                                      | 2                                      | + -                      | 3, 12, 7                     | 3, 12, 1                                     | 1, 2, 2  | $I_3, I_{12}, I_1^*$  |           |
| e5   | 1                   | $-1+\varphi$  | 1            | $-168 - 366 \varphi$                  | $-1843 - 3261\varphi$                              | 0                                      | 2                                      | -+                       | 4, 1, 18                     | 4, 1, 12                                     | 2, 1, 4  | $I_4, I_1, I_{12}^*$  |           |
| e6   | 1                   | $-1+\varphi$  | 1            | $-5793 - 6616\varphi$                 | $244157 + 349739\varphi$                           | 0                                      | 2                                      | -+                       | 12, 3, 10                    | 12, 3, 4                                     | 2, 1, 4  | $I_{12}, I_3, I_4^*$  |           |
| e7   | $\varphi$           | $\varphi$     | 1            | $-22133 - 35733\varphi$               | $-2421622 - 3918495\varphi$                        | 0                                      | 2                                      | ++                       | 1, 1, 9                      | 1, 1, 3                                      | 1, 1, 2  | $I_1,I_1,I_3^*$   |           |
| e8   | 1                   | $-\varphi$    | 1            | $-6363094 + 3932586\varphi$           | $7260861456 - 4487459209\varphi$                   | 0                                      | 2                                      | ++                       | 3, 3, 7                      | 3, 3, 1                                      | 1, 1, 2  | $I_3, I_3, I_1^*$   |           |
| f1   | $\varphi$           | $-1-\varphi$  |              | $1-\varphi$                           | $1+\varphi$  | 0                                      | 5                                      |                          | 1, 5, 2                      | 1,5  | 1,5,1  | $\bar{\mathrm{I}}_{1}, \bar{\mathrm{I}}_{5}, \bar{\mathrm{II}}$ |           |
| f2   |                     | $-1-\varphi$  | 0            | $-109 + 49\varphi$                    | $-495 + 203\varphi$                                | 0                                      | 1                                      |                          | 5, 1, 10                     | 5, 1   | 5, 1, 1  | $I_5, I_1, II^*$  |           |
| g1   |                     | $-1-\varphi$  | 1            | $-64 + 41\varphi$                     | $-229 + 138\varphi$                                | 0                                      | 2                                      | + -                      | 1, 2, 11                     | [1, 2, 5]                                    | 1, 2, 2  | $[\bar{I}_1, \bar{I}_2, \bar{I}_5^*]$                           | <u>  </u> |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$ | $\varphi$           |               | 1            | $-4214 + 2466\varphi$                 | $121871 - 74412\varphi$                            | 0                                      | 10                                     | + -                      | 5, 10, 7                     | 5, 10, 1                                     | 5, 10, 2   | $I_5, I_{10}, I_1^*$  |           |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$ | $1+\varphi$         |               | $1+\varphi$  | $-7431 - 10714\varphi$                | $415437 + 656518\varphi$                           | 0                                      | 10                                     | - +                      | 10, 5, 8                     | 10, 5, 2                                     | 10, 5, 4   | $I_{10}, I_5, I_2^*$  |           |
| g4   | $1+\varphi$         |               | $1+\varphi$  | $-56-89\varphi$                       | $-363-507\varphi$                                  | 0                                      | 2                                      | -+                       | 2, 1, 16                     | 2, 1, 10                                     | $\begin{bmatrix} 2, 1, 4 \end{bmatrix}$            | $I_2, I_1, I_{10}^*$  |           |
| h1   |                     | $1-\varphi$   |              | $-12 - 12\varphi$                     | $-24 - 28\varphi$                                  | 0                                      | 1                                      | ' <del>'-</del><br>  + - | 1, 2, 9                      | 1,2  | $\begin{vmatrix} 1, 2, 2 \\ 1, 2, 2 \end{vmatrix}$ | $  I_1, I_2, III^*  $   |           |
| h2   | $\varphi$           |               | 0            | $-12 - 12\varphi$ $-162 - 212\varphi$ | $-24 - 26\varphi$ $-1174 - 1978\varphi$            | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{vmatrix} 2 \\ 2 \end{vmatrix}$ | ++                       | 2, 1, 9                      | $\begin{bmatrix} 1, 2 \\ 2, 1 \end{bmatrix}$ | $\begin{bmatrix} 1, 2, 2 \\ 2, 1, 2 \end{bmatrix}$ | $I_{1}, I_{2}, III$ $I_{2}, I_{1}, III^{*}$                     |           |
|  | $\frac{\varphi}{1}$ |               |              |                                       |  |  | <del>-</del>                           | <u> </u>                 |                              |  | . – – – – –  |   |           |
| i1   | 1                   | •             | $1+\varphi$  | $-54-85\varphi$                       | $253 + 399\varphi$                                 | 0                                      | 3                                      |                          | 3, 1, 8                      | $\begin{bmatrix} 3,1\\0,2\end{bmatrix}$      | 1,1,3  | $I_3, I_1, IV^*$  |           |
| i2   | $1+\varphi$         | -1            | $1+\varphi$  | $60 - 35\varphi$                      | $-2153 + 1445\varphi$                              | 0                                      | 1                                      |                          | 9, 3, 8                      | 9,3  | 1, 3, 1  | $I_9, I_3, IV^*$  |           |
|  |                     |               |              |                                       |  |  |  |                          |                              |  |  |   |           |
| 111  | 1a                  |               |              | 111                                   | $1a = (35 - 3\varphi) = 11b \cdot 101b$            | (3 isoge                               | ny cla                                 | asses)                   |                              |  |  |   | 1111a     |
| a1   | $\varphi$           | $\varphi$     | 0            | 1                                     | 0  | 1                                      | 2                                      | -+                       | 1, 1                         | 1, 1   | 1, 1   | ${ m I}_1, { m I}_1$  |           |
| a2   | $\varphi$           | $\varphi$     | 0            | -4                                    | $-1-5\varphi$                                      | 1                                      | 4                                      | ++                       | 2, 2                         | 2,2  | 2,2  | $I_2, I_2$  |           |
| a3   | $\varphi$           | $\varphi$     | 0            | $-39 + 25\varphi$                     | $115 - 80\varphi$                                  | 1                                      | 4                                      | + -                      | 4, 1                         | 4, 1   | 4, 1   | $\mathrm{I}_4,\mathrm{I}_1$                                     |           |
| a4   | $1+\varphi$         | $1-\varphi$   | 1            | $-176-278\varphi$                     | $-1730 - 2788\varphi$                              | 1                                      | 2                                      | ++                       | 1, 4                         | 1,4  | 1, 2   | $\mathrm{I}_1,\mathrm{I}_4$                                     |           |
| b1   | $\varphi$           | $-1+\varphi$  | $\varphi$    | $1-\varphi$                           | $-2+\varphi$                                       | 1                                      | 1                                      |                          | 1, 2                         | 1,2  | 1,2  | $I_1, I_2$  |           |
| c1   | $\varphi$           | $-\varphi$    | 1            | $-144 + 89\varphi$                    | $750 - 464\varphi$                                 | 0                                      | 3                                      |                          | 1, 2                         | 1,2  | 1,2  | $I_1, I_2$  | [         |
| c2   | ,                   | $-1+\varphi$  | 0            | $-64-65\varphi$                       | $-208-486\varphi$                                  | 0                                      | 1                                      |                          | 3, 6                         | 3,6  | 1,2  | $I_3, I_6$  |           |
|  |                     |               |              |                                       |  |  | 1                                      |                          |                              |  |  |   |           |
|  | 4 1                 |               |              |                                       |  |  |  |                          |                              |  |  |   | 1         |
| 111  |                     |               |              |                                       | $1b = (32 + 3\varphi) = 11a \cdot 101a$            | (3 isoge                               | ny cla                                 | asses)                   |                              |  |  |   | 1111b     |
| a1   | $1 + \varphi$       | $1+\varphi$   | 1            | $1+3\varphi$                          | $1+2\varphi$                                       | 1                                      | 2                                      | +-                       | 1, 1                         | 1, 1   | 1, 1   | ${ m I}_1, { m I}_1$  |           |
| a2   | $1 + \varphi$       | $1+\varphi$   | 1            | $-4+3\varphi$                         | $-5+2\varphi$                                      | 1                                      | 4                                      | ++                       | 2, 2                         | 2,2  | 2,2  | $\mathrm{I}_2,\mathrm{I}_2$                                     |           |
| a3   | $1 + \varphi$       | $1 + \varphi$ | 1            | $-14-22\varphi$                       | $11 + 42\varphi$                                   | 1                                      | 4                                      | -+                       | 4, 1                         | 4, 1   | 4,1  | $\mathrm{I}_4,\mathrm{I}_1$                                     |           |
| a4   | $\varphi$           | 1             | 1            | $-453 + 277\varphi$                   | $-4518 + 2788\varphi$                              | 1                                      | 2                                      | ++                       | 1,4                          | 1,4  | 1,2  | $I_1, I_4$  |           |
| b1   | $1+\varphi$         | $\varphi$     | $\varphi$    | $1+\varphi$                           | 0  | 1                                      | 1                                      |                          | 1, 2                         | 1,2  | 1,2  | $I_1, I_2$  | [         |
|  | $1+\varphi$         |               | · <u>-</u> - | $-55 - 90\varphi$                     | $286 + 464\varphi$                                 | 0                                      |  | '<br>                    | $\frac{1}{1}, \frac{1}{2}$   | 1,2  | -1,2   | $ I_1,I_2 $   |           |
| c2   | - Γ γ               |               |              | $-130 + 66\varphi$                    | $-629 + 421\varphi$                                |  | 1                                      |                          | 3, 6                         | 3,6  | 1, 2 $1, 2$  | $I_{1}, I_{2}$ $I_{3}, I_{6}$                                   |           |
| 02   | Ψ                   | ± 1 Ψ         | <u>-</u> ι γ | 100   00φ                             | 020   421ψ   |  |  |                          | 5,0                          | 0,0  | 1 -, -   | ±3, ±6  |           |

|     |              | $a_1$      |      | $a_2$                            | $a_3$       |      | $a_4$                |              |                                   | $a_6$                    |          | r     | T             | s   | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$     | Kodaira              | Isogenies |
|-----|--------------|------------|------|----------------------------------|-------------|------|----------------------|--------------|-----------------------------------|--------------------------|----------|-------|---------------|-----|------------------------------|-----------------------------|-----------|----------------------|-----------|
| 111 | 10           | <b>;</b>   |      |                                  |             |      | 1:                   | 111c = (31   | $+6\varphi) = 11a \cdot 1$        | 101b                     | (1 isog  | geny  | class         | )   |                              |                             |           |                      | 1111c     |
| a1  |              | 1          |      | $-\varphi$                       | 0           |      | $-1+\varphi$         | ·            | ·                                 | 0                        | ·        | 1     | 2             | + - | 1,1                          | 1, 1                        | 1, 1      | $I_1, I_1$           |           |
| a2  |              | $\varphi$  | -1 - | $+\varphi$                       | $1+\varphi$ |      | $-5-11\varphi$       |              | 7 +                               | + 12 <i>φ</i>            |          | 1     | 2             | -+  | 2,2                          | 2,2                         | 2, 2      | $I_2, I_2$           |           |
| 111 | 10           |            |      |                                  |             |      | 11                   | 111d = (37)  | $-6\varphi) = 11b \cdot 1$        | 101a                     | (1 isog  | geny  | class         | )   |                              |                             |           |                      | 1111d     |
| a1  |              |            | -1 - |                                  | 0           |      | $-\varphi$           |              | 20                                | 0                        |          | 1     | 2             | -+  | 1, 1                         | 1,1                         | 1,1       | $I_1, I_1$           |           |
| a2  | 1            | $+\varphi$ |      | $\varphi$                        | $1+\varphi$ |      | $-15 + 10\varphi$    |              | 30 -                              | <u>- 18φ</u>             |          | 1     | 2             | + - | 2,2                          | 2,2                         | 2,2       | $I_2, I_2$           |           |
| 111 | <b>6</b> a   | ı          |      |                                  |             |      | 111                  | 6a = (18 -   | $-30\varphi) = 2 \cdot 3 \cdot 3$ | 31a                      | (5 isog  | eny   | classe        | es) |                              |                             |           |                      | 1116a     |
| a1  |              | $\varphi$  |      | 0                                | $\varphi$   |      | -1                   |              |                                   | 0                        |          | 1     | 2             | + - | 2, 1, 1                      | 2, 1, 1                     | 2, 1, 1   | $I_2,I_1,I_1$        |           |
| a2  |              | $\varphi$  |      | 0                                | $\varphi$   |      |                      |              |                                   | $-2\varphi$              | ]        | 1     | 2             | ++  | 1, 2, 2                      | 1,2,2                       | 1,2,2     | $I_1, I_2, I_2$      |           |
| b1  |              | 1          |      | $+\varphi$                       | 0           |      | $-8-8\varphi$        |              | -18 -                             |                          |          | 1     | $\frac{2}{2}$ | ++  | 1, 2, 4                      | 1, 2, 4                     | 1, 2, 4   | $I_1, I_2, I_4$      |           |
| b2  |              | 1          | 1 -  |                                  | 0           |      | $2+2\varphi$         |              |                                   | 0                        |          | 1     | 2             |     | 2, 1, 2                      | 2,1,2                       | 2,1,2     | $I_2, I_1, I_2$      |           |
| c1  |              | 1          | -1   | ,                                | 0           |      | $-186 + 109\varphi$  |              | 1129 -                            | ,                        |          | 0     | 2             |     | 2, 1, 3                      | 2, 1, 3                     | 2, 1, 1   | $I_2, I_1, I_3$      |           |
| c2  | <u>ا</u> ۔ ۔ | $\varphi$  |      | 1                                | 1           |      | $-388 - 569\varphi$  |              | -5184 - 8                         |                          |          | 0     | 2             | ++  | 1, 2, 6                      | 1,2,6                       | 1,2,2     | $I_1, I_2, I_6$      |           |
| d1  |              | 1          | -1   | $+\varphi$                       | 0           |      | $-340 - 534\varphi$  |              | 4340 + 7                          |                          |          | 0     | 2             |     | 12, 1, 2                     | 12, 1, 2                    | 2, 1, 2   | $I_{12}, I_1, I_2$   |           |
| d2  |              | $\varphi$  |      | $\varphi$                        | 0           | -365 | $522 - 59050\varphi$ |              | 5078244 + 8216                    |                          |          | 0     | 2             |     | 6, 2, 1                      | [6, 2, 1]                   | [2, 2, 1] | $I_6, I_2, I_1$      |           |
| e1  |              | $+\varphi$ |      | $-\varphi$                       | 0           |      | $-42-\varphi$        |              | 98                                | $+7\varphi$              |          | 1     | 2             |     | 3, 4, 2                      | 3, 4, 2                     | 3, 4, 2   | $I_3, I_4, I_2$      |           |
| e2  | 1            | $+\varphi$ |      | $-\varphi$                       | 0           |      | $-2-\varphi$         |              |                                   | $2-\varphi$              |          | 1     | 2             |     | 6, 2, 1                      | 6, 2, 1                     | 6, 2, 1   | $I_6, I_2, I_1$      |           |
| 111 | 6ł           | )          |      |                                  |             |      | 111                  | 16b = (12 -  | $-30\varphi) = 2 \cdot 3 \cdot 3$ | 31b                      | (5 isoge | eny ( | classe        | s)  |                              |                             |           |                      | 1116b     |
| a1  | 1            | $+\varphi$ |      | $-\varphi$                       | $1+\varphi$ |      | $-1-2\varphi$        | `            | - ,                               | $-\varphi$               |          | 1     | 2             | -+  | 2, 1, 1                      | 2, 1, 1                     | 2, 1, 1   | $I_2, I_1, I_1$      |           |
| a2  |              | $+\varphi$ |      |                                  | $1+\varphi$ |      | $-11-2\varphi$       |              | 10                                | $0+\varphi$              |          | 1     | 2             | ++  | 1, 2, 2                      | 1, 2, 2                     | 1, 2, 2   | $I_1, I_2, I_2$      |           |
| b1  |              | 1          | -1   | $-\varphi^-$                     | 1           |      | $-18+10\varphi$      |              | -27 +                             | $+17\varphi$             |          | 1     | $\frac{1}{2}$ | ++  | 1, 2, 4                      | [1, 2, 4]                   | [1, 2, 4] | $I_1, I_2, I_4$      |           |
| b2  |              |            | -1 - |                                  | 1           |      | 2                    |              |                                   | $3+\varphi$              |          | 1     | 2             |     | 2, 1, 2                      | 2, 1, 2                     | 2, 1, 2   | $I_2, I_1, I_2$      |           |
| c1  |              | 1          | 1 -  |                                  | 1           |      | $-78 - 107\varphi$   |              | 348 +                             | $595\varphi$             | ·        | 0     | $\frac{1}{2}$ | ·   | 2, 1, 3                      | 2,1,3                       | [2, 1, 1] | $I_2, I_1, I_3$      |           |
| c2  | 1            | $+\varphi$ |      |                                  | 1           |      | $-957 + 568\varphi$  |              | -13689 + 8                        |                          |          | 0     | 2             |     | 1, 2, 6                      | 1, 2, 6                     | 1, 2, 2   | $I_1, I_2, I_6$      |           |
| d1  |              | 1          |      | $-\varphi^-$                     | 0           |      | $-874 + 534\varphi$  |              | 11384 - 7                         | $70\overline{44}\varphi$ | ·        | 0     | $\bar{2}$     |     | 12, 1, 2                     | 12, 1, 2                    | [2, 1, 2] | $ I_{12}, I_1, I_2 $ |           |
| d2  | 1            | $+\varphi$ | 1 -  | $+\varphi$                       | 1           | -955 | $572 + 59053\varphi$ | 13           | 3354147 - 8253                    | $3373\varphi$            |          | 0     | 2             | ++  | 6, 2, 1                      | 6, 2, 1                     | 2, 2, 1   | $I_6, I_2, I_1$      |           |
| e1  |              | $\varphi$  |      | 0                                | 0           |      | $-43+\varphi$        |              |                                   | $-7\varphi$              | `        | 1     | 2             | ++  | 3, 4, 2                      | 3,4,2                       | [3, 4, 2] | $I_3, I_4, I_2$      |           |
| e2  |              | $\varphi$  |      | 0                                | 0           |      | •                    |              | 1                                 | $1+\varphi$              |          | 1     | 2             |     | 6, 2, 1                      | 6, 2, 1                     | 6, 2, 1   | $I_6, I_2, I_1$      |           |
| 112 | 1a           | ì          |      |                                  |             |      | -<br>-               | 1121a = (34) | $4 - \varphi) = 19b \cdot 5$      | 59 <i>b</i>              | (1 isoge | eny c | lass)         |     |                              |                             |           |                      | 1121a     |
| a1  |              | $\varphi$  |      | $-\varphi$                       | $1+\varphi$ |      | $-4-6\varphi$        | `            | -4                                | $-6\varphi$              |          | 0     | 2             | ++  | 1,1                          | 1,1                         | 1,1       | $I_1, I_1$           |           |
| a2  |              | 1          | 1 -  | $-\frac{\dot{\varphi}}{\varphi}$ | $1+\varphi$ |      | $-43 + 25\varphi$    |              | -155 +                            | $+94\varphi$             |          | 0     | 2             | + - | 2, 2                         | 2, 2                        | 2, 2      | $I_2, I_2$           |           |
| 112 | 1            | )          |      |                                  |             |      |                      | 1121b = (33) | $3 + \varphi) = 19a \cdot 5$      | 59a                      | (1 isoge | eny c | class)        |     |                              |                             |           |                      | 1121b     |
| a1  | 1            | $+\varphi$ |      | -1                               | $\varphi$   |      | $-9+4\varphi$        |              | -9                                | $+5\varphi$              |          | 0     | 2             | ++  | 1,1                          | 1,1                         | 1, 1      | $I_1, I_1$           |           |
| a2  |              | 1          |      | $\varphi$                        | $\varphi$   |      | $-17-26\varphi$      |              | -60 -                             | $-95\varphi$             |          | 0     | 2             | -+  | 2, 2                         | 2, 2                        | 2, 2      | $I_2, I_2$           |           |
|     |              |            |      |                                  |             |      |                      |              |                                   |                          | ı        | I     |               |     |                              | •                           |           | •                    |           |

| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$   |      | $a_1$         | $a_2$                 | $a_3$         | $\overline{a}$        | $a_6$                                      | r                                     | T   | s        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$ | Kodaira   | Isogenies |
|---|------|---------------|-----------------------|---------------|-----------------------|--|---------------------------------------|-----|----------|------------------------------|-----------------------------|-------|---|-----------|
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$   | 1124 | <br>1a        |                       |               |                       | $1124a - (16 - 30\alpha) - 2.281a$         | (1 isogeny clas                       | e)  |          |                              |                             |       | I   | 1124a     |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$   |      |               | $\varphi$             | 0             |                       | ` ' '                                      | · · · · · · · · · · · · · · · · · · · |     |          | 1, 1                         | 1,1                         | 1,1   | $I_1, I_1$  |           |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$   | 112  | 4h            |                       |               |                       | 1124b - (14 - 306) - 2.281b                | (1 isogony class                      | a)  |          |                              |                             |       |   | 1124h     |
| $ \begin{array}{ c c c c c c }\hline 1129a & 1129a & (38 - 7\varphi) = 1129a & (1 \text{ isogeny class}) & 1129a \\\hline a1 & 0 & -1 - \varphi & \varphi & -1 + \varphi & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 \\\hline 1129b & 1129b - (31 + 7\varphi) = 1129b & (1 \text{ isogeny class}) & 1129b \\\hline a1 & 0 & 1 + \varphi & 1 + \varphi & -1 + \varphi & -\varphi & 1 & 1 & - & 1 & 1 & 1 & 1 \\\hline 1151a & & & & & & & & & & & & & & & & & & \\\hline 1151a & & & & & & & & & & & & & & & & & & &$ |      |               | $-1 + \varphi$        | 1             |                       | . , ,                                      | · · · · · ·                           |     | l        | 1.1                          | 1.1                         | 1.1   | I <sub>1</sub> , I <sub>1</sub>                       | 11240     |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$   |      | · ·           | - ' <i>r</i>          |               |                       |  |                                       |     |          | -,-                          | _, _                        |       | -1, -1  | 1100-     |
| $ \begin{array}{ c c c c c c } \hline & & & & & & & & & & & & & & & & & & $   |      |               | 1                     |               | 1 .                   |  | ( )                                   |     |          | 1                            | 1                           | 1     | т   | 1129a     |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$   | al   |               | $\frac{-1-\varphi}{}$ | $\varphi$     | $\frac{-1+\varphi}{}$ | ρ Ι  | 1                                     | 1   |          | 1                            | 1                           | 1     | 11  |           |
| $ \begin{array}{c c c c c c c c c c c c c c c c c c c $   | 1129 | 9b            |                       |               |                       | $1129b = (31 + 7\varphi) = 1129b$          | (1 isogeny class)                     |     |          |                              |                             |       |   | 1129b     |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$  | a1   | 0             | $1 + \varphi$         | $1 + \varphi$ | $-1+\varphi$          | $\varphi$ $-\varphi$                       | 1                                     | 1   |          | 1                            | 1                           | 1     | $I_1$   |           |
| $ \begin{array}{c c c c c c c c c c c c c c c c c c c $   | 115  | 1a            |                       |               |                       | $1151a = (33 + 2\varphi) = 1151a$          | (2 isogeny classes                    | s)  |          |                              |                             |       |   | 1151a     |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$   |      | $1+\varphi$   | $-\varphi$            | 1             |                       | •  | -                                     |     |          | 1                            | 1                           |       |   |           |
| $ \begin{array}{c c c c c c c c c c c c c c c c c c c $   |      | $\varphi$     | $1-\varphi$           | 11            | $-42 + 25\varphi$     |  | 0                                     | 2   | + -      | 2                            | 2                           | 2     | $I_2$   |           |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$  | b1   | 0             | 1                     | 1             | <u>-4</u>             | $\rho$ 0                                   | 1                                     | 1   | -+       | 1                            | 1                           | 1     | $I_1$   |           |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$  | 115  | 1b            |                       |               |                       | $1151b = (35 - 2\varphi) = 1151b$          | (2 isogeny classes                    | ;)  |          |                              |                             |       |   | 1151b     |
| $ \begin{array}{c c c c c c c c c c c c c c c c c c c $   |      |               | 0                     | 1             | •                     | •  |                                       |     | +-       | 1                            | 1                           |       |   |           |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$  |      |               |                       |               |                       |  |                                       |     | <u> </u> |                              | 2                           | 2     | <del>-</del>  |           |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$  | b1   | 0             | 1                     | 1             | $-1+\varphi$          | $\rho = 0$                                 | 1                                     | 1   | +-       | 1                            | 1                           | 1     | $I_1$   |           |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   | 1150 | 6a            |                       |               |                       | $1156a = (34) = 2 \cdot 17 $               | (2 isogeny classes)                   |     |          |                              |                             |       |   | 1156a     |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$  | a1   | $1 + \varphi$ | $-1-\varphi$          | 1             | $-4-4\varphi$         | $\rho$ $-4-5\varphi$                       | 0                                     | 1   |          | 1,7                          | 1,7                         | 1,1   | $I_1, I_7$  |           |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$  |      | 1             |                       | -             |                       |  |                                       | _   |          |                              |                             |       |   |           |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   |      |               |                       |               |                       |  |                                       |     |          |                              |                             |       |   |           |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$  |      |               |                       |               |                       |  |                                       |     |          |                              |                             |       |   |           |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$   |      |               |                       |               |                       |  | , i                                   |     | ' '      | 0,2                          | 0,2                         | 0,2   | 13,12   | 11500     |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$  |      |               |                       |               |                       |  | ,                                     |     |          | 1 1                          | 1 1                         | 1 1   | т т   | 11390     |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$   | al   | $1+\varphi$   | 0                     | $\varphi$     | $-2+\varphi$          | $\rho = 1 - \varphi$                       | 1                                     | 1   |          | 1, 1                         | 1, 1                        | 1,1   | $I_1, I_1$  |           |
|   | 1159 | 9d            |                       |               |                       | $1159d = (22 - 31\varphi) = 19a \cdot 61b$ | (1 isogeny clas                       | ss) |          |                              |                             |       |   | 1159d     |
| $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$   | a1   | $\varphi$     | $1-\varphi$           | $1 + \varphi$ | $-3\varphi$           | $\rho$ 0                                   | 1                                     | 1   |          | 1, 1                         | 1,1                         | 1, 1  | $I_1, I_1$  |           |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$  | 1179 | 9a            |                       |               |                       | $1179a = (36 - 3\varphi) = 3 \cdot 131a$   | (3 isogeny classe                     | es) |          |                              |                             |       |   | 1179a     |
|   | a1   | $1 + \varphi$ | $-1+\varphi$          | $\varphi$     | $-1+\varphi$          | $\rho$ 0                                   | 1                                     | 1   | + -      | 1,3                          | 1,3                         | 1,3   | $I_1, I_3$  |           |
|   | b1   | 1             | $-\varphi$            | 1             | -6+4                  | ho 4 $-$ 4 $arphi$                         | 1                                     | 1   |          | 1, 2                         | 1, 2                        | 1, 2  | $\overline{\mathrm{I}_{1},\mathrm{I}_{2}}$            |           |
|   | c1   | 1             | $\varphi$             | 1             | $-4+2\varphi$         | $\rho$ $3-2\varphi$                        | 1                                     | 1   | + -      | 1,1                          | 1,1                         | 1,1   | $\overline{\mathrm{I}}_{1},\overline{\mathrm{I}}_{1}$ |           |

|                  |  |  |   |   |   |  | Imi  |                     | 1(A)                         | 1 (*)  |  | T/ 1 ·   | т .       |
|------------------|--|--|---|---|---|--|--|---------------------|------------------------------|--|--|--|-----------|
|                  | $a_1$  | $a_2$  | $a_3$                                       | $a_4$   | $a_6$   |  | T  | s                   | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                        | $c_p$  | Kodaira  | Isogenies |
| 117              | 9b   |  |   | 11  | $79b = (33 + 3\varphi) = 3 \cdot 131b$                                | (3 isogeny c                           | lasses   | )                   |                              |  |  |  | 1179b     |
| a1               | φ  | $1 + \varphi$  | 0   | 1   | $-1-\varphi$  | 1                                      | 1  | - +                 | 1,3                          | 1,3  | 1,3  | $I_1, I_3$   |           |
| b1               | 1  | $-1+\varphi$   | 1   | $-2-4\varphi$   | 4arphi  | 1                                      | 1  | <u> </u>            | 1, 2                         | 1,2  | 1, 2   | $I_1, I_2$   |           |
| c1               | 1  | $1-\varphi$  | 1   | $-2-2\varphi$   | $1+2\varphi$  | 1                                      | 1  | -+                  | 1,1                          | 1,1  | 1,1  | $I_1, I_1$   |           |
| 118              | 0a   |  |   | 1180  | $0a = (32 + 6\varphi) = 2 \cdot 5a \cdot 59b$                         | (4 isogeny                             | classe   | es)                 |                              |  |  |  | 1180a     |
| a1               | φ  | $1-\varphi$  | $1+\varphi$                                 | $1-\varphi$   | $-\varphi$  | 1                                      | 1  |                     | 2, 1, 1                      | 2, 1, 1  | 2, 1, 1  | $I_2,I_1,I_1$  |           |
| b1               | 1  | $\varphi$  | 1   | $-5-4\varphi$   | $4+3\varphi$  | 1                                      | 5  | 1                   | 2, 5, 1                      | [2, 5, 1]  | [2, 5, 1]  | $I_2, I_5, I_1$  |           |
| b2               | 1  |  | 1   | $-10 + 56\varphi$   | $-320 - 109\varphi$   | 1                                      | 1  |                     | 10, 1, 5                     | 10,1,5   | $\begin{bmatrix} 2, 1, 1 \\ \end{bmatrix}$                             | $I_{10}, I_1, I_5$   |           |
| c1               | 1  | r  | 1   | $-6873 + 4240\varphi$   | $255624 - 157991\varphi$  | 0                                      | 4  | ++                  | 2, 1, 4                      | 2, 1, 4  | 2, 1, 4  | $I_2, I_1, I_4$  |           |
| c2               | $\varphi$  | · <i>F</i>   | $\varphi$                                   | $-86 \\ -6$   | $162 - 310\varphi$  | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 4  | ++                  | 4, 2, 2                      | 4, 2, 2  | 2, 2, 2  | $I_4, I_2, I_2$  |           |
| c3<br>c4         | $\left  \begin{array}{c} \varphi \\ 1 \end{array} \right $   | $-1 + \varphi$ $-1 + \varphi$  | $rac{arphi}{1}$                            | $-6$ $-17708 - 28650\varphi$  | $ 2 - 6\varphi  -1730169 - 2799475\varphi $                           | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{array}{c c} 2 \\ 2 \end{array}$          | +-+                 | 8, 1, 1<br>2, 4, 1           | $\begin{bmatrix} 8, 1, 1 \\ 2, 4, 1 \end{bmatrix}$ | $\begin{bmatrix} 2, 1, 1 \\ 2, 4, 1 \end{bmatrix}$                     | $\begin{bmatrix} I_8, I_1, I_1 \\ I_2, I_4, I_1 \end{bmatrix}$         |           |
| d1               | 1  | <del>-</del>   |   | $-1+\varphi$  | 1100103 21004104  |  | $\frac{1}{1} - \frac{2}{6} - \frac{1}{6}$        | ' <u>'</u><br>  + - |                              | $\begin{bmatrix} 2, 4, 1 \\ 2, 3, 1 \end{bmatrix}$ | $\begin{bmatrix} 2, \frac{4}{2}, \frac{1}{2} \\ 2, 3, 1 \end{bmatrix}$ | $\begin{array}{c c} I_2, I_4, I_1 \\ \hline I_2, I_3, I_1 \end{array}$ |           |
| $\frac{d1}{d2}$  | 1  | $arphi \ arphi$  | $arphi \ arphi$                             | $-1 + \varphi$ $-1 - 9\varphi$  | $7+4\varphi$  |  | 6  | -+                  | 1, 6, 2                      | 1, 6, 2  | $\begin{bmatrix} 2, 3, 1 \\ 1, 6, 2 \end{bmatrix}$                     | $I_1, I_6, I_2$  |           |
| d3               | 1  |  | $\varphi$                                   | $-36+21\varphi$   | $-102 + 41\varphi$  |  | 2  | + -                 | 6, 1, 3                      | 6, 1, 3  | [6, 1, 1]  | $I_6, I_1, I_3$  |           |
| d4               | 1  |  | $\varphi$                                   | $-76-59\varphi$   | $-398 - 351\varphi$   | 0                                      | 2  | - +                 | 3, 2, 6                      | 3, 2, 6  | 3, 2, 2  | $I_3, I_2, I_6$  |           |
| 118              | 0b   |  |   | 1180  | $0b = (38 - 6\varphi) = 2 \cdot 5a \cdot 59a$                         | (4 isogeny                             | classe   | es)                 |                              |  |  |  | 1180b     |
| a1               | $1+\varphi$  | 0  | $\varphi$                                   | $1-\varphi$   | 0   |  | 1  |                     | 2, 1, 1                      | 2, 1, 1  | 2, 1, 1  | $I_2, I_1, I_1$  |           |
| b1               | ! <u>-</u> .<br>   |  | · <del>'</del> - ·<br>1                     | $-9+4\varphi$   | $7-3\varphi$  | 1                                      |  | '<br>               | 2, 5, 1                      | $\frac{1}{2}, \frac{1}{5}, \frac{1}{1}$            | [2, 5, 1]  | $  I_2, I_5, I_1  $  | <u>-</u>  |
| b2               | 1  | $1-\varphi$  | 1   | $46-56\varphi$  | $-429 + 109\varphi$   | 1                                      | 1  | 1                   | 10, 1, 5                     | 10, 1, 5   | 2, 1, 1  | $I_{10}, I_1, I_5$   |           |
| c1               | 1  | $-1+\varphi$   | 1   | $-2633 - 4240\varphi$   | $97633 + 157991\varphi$   | 0                                      | $\overline{4}$                                   | · ·<br>  + +        | 2, 1, 4                      | 2,1,4  | [2, 1, 4]  | $I_2, I_1, I_4$  | <u> </u>  |
| c2               | $1+\varphi$  | •  | $\varphi$                                   | -85   | $-148 + 224\varphi$   | 0                                      | 4  | ++                  | 4, 2, 2                      | 4, 2, 2  | 2, 2, 2  | $I_4, I_2, I_2$  |           |
| c3               | $1+\varphi$  | arphi  | $\varphi$                                   | -5  | -4  | 0                                      | 2  | -+                  | 8, 1, 1                      | 8, 1, 1  | 2, 1, 1  | $I_8, I_1, I_1$  |           |
| c4               | 1  | $-\varphi$   | 1   | $-46358 + 28650\varphi$   | $-4529644 + 2799475\varphi$   | 0                                      | 2  | + -                 | 2, 4, 1                      | [2, 4, 1]  | [2, 4, 1]  | $I_2, I_4, I_1$  |           |
| d1               | 1  | $1-\varphi$  | $1+\varphi$                                 | -2arphi   | 1-arphi   | 0                                      | 6  | -+                  | 2, 3, 1                      | 2, 3, 1  | [2, 3, 1]  | $I_2, I_3, I_1$  |           |
| 40               | 1  | $1-\varphi$  | $1+\varphi$                                 | $-10 + 8\varphi$  | $11-5\varphi$   | 0                                      | 6  | + -                 | 1, 6, 2                      | 1, 6, 2  | 1, 6, 2  | $I_1, I_6, I_2$  |           |
| d2               |  | - P  |   | -   |   |  | 0  | 1 .                 | 6, 1, 3                      | 6 1 9  | 611  | T T T  |           |
| d3               | 1  | $1-\varphi$  |   | $-15-22\varphi$   | $-61-42\varphi$   | 0                                      | 2  | -+                  | , ,                          | 6, 1, 3  | 6, 1, 1  | $I_6, I_1, I_3$  |           |
|                  | I  | $1-\varphi$  |   | $-15 - 22\varphi$ $-135 + 58\varphi$  | $ \begin{array}{r} -61 - 42\varphi \\ -749 + 350\varphi \end{array} $ | 0                                      | $\begin{array}{ c c }\hline 2\\ 2\\ \end{array}$ | +-                  | 3, 2, 6                      | 3, 2, 6  | 3, 2, 2  | $I_6, I_1, I_3$<br>$I_3, I_2, I_6$                                     |           |
| d3               | 1 1  | $1-\varphi$  |   | $-135 + 58\varphi$  | $-749 + 350\varphi$   |  | 2  | + -                 | , ,                          |  |  |  | 1181a     |
| d3<br>d4         | 1 1  | $1 - \varphi $ $1 - \varphi$   |   | $-135 + 58\varphi$  | $-749 + 350\varphi$   | 0                                      | 2  | + -                 | , ,                          |  |  |  | 1181a     |
| d3 d4 118        | 1<br>1<br><b>1a</b>  | $ \begin{array}{c} 1 - \varphi \\ 1 - \varphi \end{array} $ $ 1 - \varphi$   | $1+\varphi$                                 | $-135 + 58\varphi$ $11$   | $-749 + 350\varphi$ $81a = (20 - 31\varphi) = 1181a$                  | (2 isogeny c                           | 2 lasses   | )                   | 3, 2, 6                      | 3, 2, 6  | 3, 2, 2  | $I_3, I_2, I_6$  | 1181a     |
| d3 d4  118       | $\frac{1}{1}$ $\frac{1}{1}$ $\frac{\varphi}{1+\varphi}$  | $ \begin{array}{c} 1 - \varphi \\ 1 - \varphi \end{array} $ $ 1 - \varphi$   | $1 + \varphi$ $1$                           | $-135 + 58\varphi$ $11$ $-5 + 3\varphi$ $1 - \varphi$   |   | (2 isogeny c                           | 2 lasses 1 1 1 1                                 | )<br>               | 3, 2, 6                      | 3, 2, 6  | 3, 2, 2  | $I_3, I_2, I_6$ $I_1$  | 1181a     |
| d3 d4  118 a1 b1 | $\frac{1}{1}$ $\frac{1}{1}$ $\frac{\varphi}{1+\varphi}$  | $ \begin{array}{c} 1 - \varphi \\ 1 - \varphi \end{array} $ $ \begin{array}{c} 1 - \varphi \\ \hline \varphi \end{array} $ | $1 + \varphi$ $1$                           | $ \begin{array}{r} -135 + 58\varphi \\  & 11 \\  & -5 + 3\varphi \\ \hline  & 1 - \varphi \end{array} $ |   | (2 isogeny c                           | 2 lasses 1 1 1 1                                 | )<br>               | 3, 2, 6                      | 3, 2, 6  | 3, 2, 2  | $I_3, I_2, I_6$ $I_1$  |           |
| 118 a1 b1 118    | $ \begin{array}{c} 1\\ 1\\ \mathbf{1a} \end{array} $ $ \begin{array}{c} \varphi\\ 1+\varphi\\ \end{array} $ $ \begin{array}{c} \mathbf{1b}\\ \end{array} $ | $ \begin{array}{c} 1 - \varphi \\ 1 - \varphi \end{array} $ $ \begin{array}{c} 1 - \varphi \\ \hline \varphi \end{array} $ | $\frac{1}{1+\varphi}$ $\frac{1}{1+\varphi}$ | $-135 + 58\varphi$ $11$ $-5 + 3\varphi$ $1 - \varphi$ $11$  |   | (2 isogeny c                           | lasses 1 1 1 lasses                              | )<br>               | 3, 2, 6                      | 3,2,6  | 3, 2, 2  | $I_3, I_2, I_6$ $I_1$ $I_1$  |           |

| $a_1$   | $a_2$   | $a_3$   | $a_4$   | $a_6$  | r  | T  | s                                      | $\operatorname{ord}(\Delta)$             | $\operatorname{ord}_{-}(j)$   | $c_p$   | Kodaira  | Isogenies      |
|---|---|---|---|--|--|--|--|--|---|---|--|----------------|
|   |   |   | ~4  |  |  | 1-1  |  | 014(_)                                   | 014=(J)   |   | 110 00110  | _              |
| 1189a   |   |   | 118   | $89a = (35 - \varphi) = 29a \cdot 41b $  | 1 isogeny class  | s)   |  |  |   |   |  | 1189a          |
| a1 0  | 1   | $\varphi$   | $-74 + 45\varphi$   | $-313 + 192\varphi$  | 0  | 1  |  | 4, 1                                     | 4, 1  | 2, 1  | $I_4, I_1$   |                |
| 1189b   |   |   | 118   | $89b = (34 + \varphi) = 29b \cdot 41a $  | 1 isogeny class  | s)   |  |  |   |   |  | 1189b          |
| a1 0  | 1 1   | $+\varphi$  | $-29 - 45\varphi$   | $-121 - 193\varphi$  | 0  | 1  |  | 4, 1                                     | 4, 1  | 2,1   | $\mathrm{I}_4,\mathrm{I}_1$  |                |
| 1195a   |   |   | 1195  | • /  | (1 isogeny cla   | ss)  |  |  |   |   |  | 1195a          |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$  | 0   | 1   | $3-6\varphi$  | $-4-2\varphi$  | 1  |  | -+                                     | ,  | 1,6   | 1,2   | $I_1, I_6$   |                |
| a2 $1+\varphi$  | 0   | 1   | $-186 - 294\varphi$   | $-1915 - 3096\varphi$  | 1  | 2  | ++                                     | 2,3                                      | 2,3   | 2,1   | $I_2, I_3$   |                |
| 1195b   |   |   | 1195  | $5b = (18 - 31\varphi) = 5a \cdot 239a$  | (1 isogeny cla   | ss)  |  |  |   |   |  | 1195b          |
| $\begin{vmatrix} a1 & 1+\varphi \\ -2 & 1\end{vmatrix}$   | $-\varphi$  | 1   | $-3+5\varphi$   | $-6 + 2\varphi$  | 1  |  | + -                                    |  | 1,6   | 1,2   | $I_1, I_6$   |                |
| a2  | $1-\varphi$   | 1   | $-479 + 293\varphi$   | $-5011 + 3096\varphi$  | 1  | 2  | ++                                     | 2,3                                      | 2,3   | 2, 1  | $I_2, I_3$   |                |
| 1199a   |   |   | 1199  | $a = (17 - 31\varphi) = 11b \cdot 109b$  | (1 isogeny cla   | ss)  |  |  |   |   |  | 1199a          |
| a1 1  | $1-\varphi$   | 0   | $1-\varphi$   | $3-2\varphi$   | 0  | 3  |  | , -                                      | 1,3   | 1,3   | $I_1, I_3$   |                |
| a2   φ  | $-\varphi$  | 0   | $-116 - 187\varphi$   | $-879 - 1413\varphi$   | 0  | 1  |  | 3,1                                      | 3,1   | 3,1   | $I_3, I_1$   |                |
| 1199b   |   |   | 1100  | 1 (14 91) 11 100   | /1 • 1   | aga)   |  |  |   |   |  | 1199b          |
| 11000   |   |   | 1199  | $b = (14 - 31\varphi) = 11a \cdot 109a$  | (1 isogeny cla   | iss)   |  |  |   |   |  | 11990          |
| a1 1  | $\varphi$   | 0   | $\varphi$   | $1+2\varphi$   | 0  | 3  |  | , -                                      | 1,3   | 1,3   | $I_1, I_3$   | 11990          |
|   | $\varphi$ $-1$  | 0   |   |  | `  | 3  | <br>                                   | 1, 3<br>3, 1                             | 1, 3<br>3, 1  | 1, 3<br>3, 1  | $\begin{matrix} I_1,I_3\\I_3,I_1\end{matrix}$  | 11990          |
| a1 1  |   |   | $-303 + 187\varphi$   | $1 + 2\varphi$ $-2292 + 1413\varphi$   | 0  | 3 1  |  |  |   |   |  | 1199b<br>1201a |
| $\begin{bmatrix} a1 & 1 \\ a2 & 1+\varphi \end{bmatrix}$ <b>1201a</b>   |   | 0   | $-303 + 187\varphi$   | $1 + 2\varphi$ $-2292 + 1413\varphi$   | 0<br>0   | 3 1  | <br>                                   | 3,1                                      |   |   |  |                |
| $ \begin{array}{c cc} a1 & 1 \\ a2 & 1+\varphi \end{array} $ $ \begin{array}{c cc} 1201a & \varphi & \vdots $   | -1  | 0   | $-303 + 187\varphi$ $120$   |  | 0<br>0<br>2 isogeny classe   | 3<br>1   |  | 3, 1                                     | 3,1   | 3,1   | $I_3, I_1$   |                |
| $ \begin{array}{c cc} a1 & 1 \\ a2 & 1+\varphi \end{array} $ $ \begin{array}{c cc} 1201a & \varphi & \vdots $   | $-1$ $\varphi$ 1  | 0<br>+ φ  | $-303 + 187\varphi$ $120$ $-80 + 47\varphi$ $-2\varphi$   |  | 0<br>0<br>2 isogeny classe   | 3  | <u></u>                                | 3, 1                                     | 3, 1  | 3,1   | $I_3, I_1$ $I_1$   |                |
| $ \begin{array}{c cccc}                                 $   | $-1$ $\varphi$ 1  | $\frac{0}{+\varphi}$  | $-303 + 187\varphi$ $120$ $-80 + 47\varphi$ $-2\varphi$   |  | 0 0 2 isogeny classe 1 1 1 1   | 3  | <u></u>                                | 3,1                                      | 3, 1  | 3,1   | $I_3, I_1$ $I_1$   | 1201a          |
| $ \begin{array}{c cccc}                                 $   | $ \begin{array}{c c} -1 \\ \varphi & 1 \\ \hline -1 + \varphi \end{array} $   | $\frac{0}{+\varphi}$  | $-303 + 187\varphi$ $120$ $-80 + 47\varphi$ $-2\varphi$ $120$   | $1 + 2\varphi$ $-2292 + 1413\varphi$ $1a = (16 - 31\varphi) = 1201a \qquad (2\pi)$ $330 - 208\varphi$ $\varphi$ $1b = (15 - 31\varphi) = 1201b \qquad (2\pi)$  | 2 isogeny classe  1 1 isogeny classe 1   | 3 1 es) 1 - 1 - 1 s)                           |  | 3, 1<br>1<br>1                           | 3,1   | 3,1   | $I_3, I_1$ $I_1$ $I_1$   | 1201a          |
| $ \begin{array}{c cccc}                                 $   | $ \begin{array}{c cccc}  & -1 \\ \hline  & \varphi & 1 \\ \hline  & -1 + \varphi & 1 \\ \hline  & 1 + \varphi & 1 \end{array} $   | $ \begin{array}{c}  + \varphi \\ \hline  & 1 \end{array} $  | $ \begin{array}{r} \varphi \\ -303 + 187\varphi \end{array} $ $ \begin{array}{r} 120 \\ -80 + 47\varphi \\ -2\varphi \end{array} $ $ \begin{array}{r} 120 \\ -32 - 46\varphi \\ -2 + 2\varphi \end{array} $   |  | 2 isogeny classe  1 1 isogeny classe 1   | 3 1 1 es) 1 1 s) 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 | +<br><br>+ -                           | 3, 1<br>1<br>1                           | 3,1   | 3,1   | $I_3, I_1$ $I_1$ $I_1$ $I_1$   | 1201a          |
| $ \begin{array}{c cccc}                                 $   | $ \begin{array}{c cccc}  & -1 \\ \hline  & \varphi & 1 \\ \hline  & -1 + \varphi & 1 \\ \hline  & -\varphi & \\ \hline  & -1 & 1 \end{array} $  | $ \begin{array}{c} +\varphi \\ -1 \\ 1 \\ +\varphi \\ -1 \\ +\varphi \end{array} $  | $-303 + 187\varphi$ $120$ $-80 + 47\varphi$ $-2\varphi$ $120$ $-32 - 46\varphi$ $-2 + 2\varphi$ $1205$ $-11 - 7\varphi$   | $1 + 2\varphi$ $-2292 + 1413\varphi$ $1a = (16 - 31\varphi) = 1201a \qquad (2)$ $330 - 208\varphi$ $\varphi$ $1b = (15 - 31\varphi) = 1201b \qquad (2)$ $76 + 128\varphi$ $1 - \varphi$ $a = (33 + 4\varphi) = 5a \cdot 241a \qquad (4)$ $-14 - 15\varphi$   | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ 2 isogeny classe $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ 2 isogeny classe $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$   | 3 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2        | +<br><br>+ -                           | 3, 1                                     | 3,1   | 3,1   | $I_3, I_1$ $I_1$ $I_1$ $I_2, I_1$  | 1201a          |
| $ \begin{array}{c cccc}                                 $   | $ \begin{array}{c c} -1 \\ \varphi & 1 \\ -1 + \varphi \\ \hline 1 + \varphi & 1 \\ -\varphi \\ \end{array} $   | $ \begin{array}{c} +\varphi \\ -1 \\ \hline 1 \end{array} $ $ +\varphi \\ +\varphi \\ +\varphi \\ +\varphi \\ $                   | $   \begin{array}{r}                                     $  | $1 + 2\varphi$ $-2292 + 1413\varphi$ $1a = (16 - 31\varphi) = 1201a$ $230 - 208\varphi$ $\varphi$ $1b = (15 - 31\varphi) = 1201b$ $26 + 128\varphi$ $1 - \varphi$ $a = (33 + 4\varphi) = 5a \cdot 241a$ $-14 - 15\varphi$ $-1 - \varphi$   | 0   0   0     1     1     1     1     1     1     2     isogeny classe   0   0   0     0       1 | s) 1 1   ss)   1   1   ses)   2   2   2        | +                                      | 3, 1  1  1  2, 1  1, 2                   | 3, 1  1  1  1  1  1  1  1  1  1  1  1  1  | 3,1<br>1<br>1<br>1<br>1<br>2,1<br>1,2   | $\begin{array}{c c} I_{3}, I_{1} \\ \hline & I_{1} \\ \hline & I_{1} \\ \hline & I_{1} \\ \hline & I_{1}, I_{2} \\ \hline \end{array}$   | 1201a          |
| $ \begin{array}{c c c}                                  $   | $ \begin{array}{c cccc}  & -1 & & & \\  & \varphi & 1 & & \\ \hline  & -1 + \varphi & 1 & & \\ \hline  & 1 + \varphi & 1 & & \\  & -\varphi & & & \\ \hline  & -1 & 1 & & \\  & & -1 & 1 & \\  & & \varphi & 1 & & \\ \end{array} $ | $ \begin{array}{c} +\varphi \\ \hline 1 \\ +\varphi \\ \hline 1 \\ +\varphi \\ +\varphi \\ +\varphi \\ -\varphi \\ \end{array} $  | $ \begin{array}{r} \varphi \\ -303 + 187\varphi \\ \hline 120 \\ -80 + 47\varphi \\ -2\varphi \\ \hline 120 \\ -32 - 46\varphi \\ -2 + 2\varphi \\ \hline 1205 \\ -11 - 7\varphi \\ -1 - 2\varphi \\ -37936 + 23445\varphi \end{array} $                    | $1 + 2\varphi$ $-2292 + 1413\varphi$ $1a = (16 - 31\varphi) = 1201a \qquad (2\pi)$ $330 - 208\varphi$ $\varphi$ $1b = (15 - 31\varphi) = 1201b \qquad (2\pi)$ $76 + 128\varphi$ $1 - \varphi$ $1 - \varphi$ $a = (33 + 4\varphi) = 5a \cdot 241a \qquad (-14 - 15\varphi)$ $-1 - \varphi$ $3349874 - 2070337\varphi$                           | 0   0   0  | 3 1 1 es) 1 1 ses) 2 2 2 2 1 2                 | +                                      | 3, 1  1  1  1  2, 1  1, 2  1, 1          | $\begin{array}{ c c c c }\hline & 3,1\\ & 1\\ & 1\\ \hline & 1\\ \hline & 1\\ \hline & 1\\ \hline & 1,2\\ \hline & 1,1\\ \hline \end{array}$                                | 3,1  1  1  1  1  1  1  1,2  1,1   | $\begin{array}{c c} I_3, I_1 \\ \hline & I_1, I_2 \\ \hline & I_1, I_1 \\ \hline \end{array}$      | 1201a          |
| $ \begin{array}{ c c c c c c } \hline & & & & & 1 \\ & & & & 2 & & 1 + \varphi \\ \hline \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & 1 + \varphi \\ \hline & & & & 1 & & 1 + \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline & & & & 1 & & \varphi \\ \hline \end{array} $ | $ \begin{array}{c c} -1 \\ \hline \varphi & 1 \\ \hline -1 + \varphi \\ \hline 1 + \varphi & 1 \\ \hline -\varphi \\ \hline -1 & 1 \\ -1 & 1 \\ \hline \varphi & 1 \\ 1 + \varphi \end{array} $                                     | $ \begin{array}{c} + \varphi \\ \hline 1 \\ + \varphi \\ \hline 1 \\ + \varphi \\ + \varphi \\ + \varphi \\ \varphi \end{array} $ | $ \begin{array}{r} \varphi \\ -303 + 187\varphi \\ \hline 120 \\ -80 + 47\varphi \\ -2\varphi \\ \hline 120 \\ -32 - 46\varphi \\ -2 + 2\varphi \\ \hline 1205 \\ -11 - 7\varphi \\ -1 - 2\varphi \\ -37936 + 23445\varphi \\ -52 + 28\varphi \end{array} $ | $1 + 2\varphi$ $-2292 + 1413\varphi$ $1a = (16 - 31\varphi) = 1201a \qquad (2\pi)$ $330 - 208\varphi$ $\varphi$ $1b = (15 - 31\varphi) = 1201b \qquad (2\pi)$ $76 + 128\varphi$ $1 - \varphi$ $1 - \varphi$ $a = (33 + 4\varphi) = 5a \cdot 241a \qquad (3\pi)$ $-14 - 15\varphi$ $-1 - \varphi$ $3349874 - 2070337\varphi$ $147 - 101\varphi$ | 0   0   0     1     1     1     1     1     2  | 3 1 1 es) 1 1 ses) 2 2 4                       | +                                      | 3, 1  1  1  2, 1  1, 2  1, 1  2, 2       | $\begin{array}{ c c c }\hline & 3,1\\ \hline & 1\\ \hline & 1,2\\ \hline & 1,1\\ 2,2\\ \hline \end{array}$ | $\begin{array}{ c c c }\hline & 3,1\\ \hline & 1\\ \hline & 1\\ \hline & 1\\ \hline & 1\\ \hline & 1,2\\ \hline & 1,1\\ & 2,2\\ \hline \end{array}$ | $\begin{array}{c c} I_3, I_1 \\ \hline & I_1, I_2 \\ \hline & I_1, I_1 \\ \hline & I_2, I_2 \\ \hline \end{array}$ | 1201a          |
| $ \begin{array}{c c c}                                  $   | $ \begin{array}{c cccc}  & -1 & & & \\  & \varphi & 1 & & \\ \hline  & -1 + \varphi & 1 & & \\ \hline  & 1 + \varphi & 1 & & \\  & -\varphi & & & \\ \hline  & -1 & 1 & & \\  & & -1 & 1 & \\  & & \varphi & 1 & & \\ \end{array} $ | $ \begin{array}{c} +\varphi \\ \hline 1 \\ +\varphi \\ \hline 1 \\ +\varphi \\ +\varphi \\ +\varphi \\ -\varphi \\ \end{array} $  | $ \begin{array}{r} \varphi \\ -303 + 187\varphi \\ \hline 120 \\ -80 + 47\varphi \\ -2\varphi \\ \hline 120 \\ -32 - 46\varphi \\ -2 + 2\varphi \\ \hline 1205 \\ -11 - 7\varphi \\ -1 - 2\varphi \\ -37936 + 23445\varphi \end{array} $                    | $1 + 2\varphi$ $-2292 + 1413\varphi$ $1a = (16 - 31\varphi) = 1201a \qquad (2\pi)$ $330 - 208\varphi$ $\varphi$ $1b = (15 - 31\varphi) = 1201b \qquad (2\pi)$ $76 + 128\varphi$ $1 - \varphi$ $1 - \varphi$ $a = (33 + 4\varphi) = 5a \cdot 241a \qquad (-14 - 15\varphi)$ $-1 - \varphi$ $3349874 - 2070337\varphi$                           | 0   0   0  | 3 1 1 es) 1 1 ses) 2 2 2 2 1 2                 | <br>- + -<br><br>+ +<br><br>+ +<br>+ + | 3, 1  1  1  2, 1  1, 2  1, 1  2, 2  1, 4 | $\begin{array}{ c c c c }\hline & 3,1\\ & 1\\ & 1\\ \hline & 1\\ \hline & 1\\ \hline & 1\\ \hline & 1,2\\ \hline & 1,1\\ \hline \end{array}$                                | 3,1  1  1  1  1  1  1  1,2  1,1   | $\begin{array}{c c} I_3, I_1 \\ \hline & I_1, I_2 \\ \hline & I_1, I_1 \\ \hline \end{array}$      | 1201a          |

|   |  |   |  |   |   |  |  |   |   | 1   | ı   | 1  | 1                |
|---|--|---|--|---|---|--|--|---|---|---|---|--|------------------|
|   | $a_1$  | $a_2$   | $a_3$  | $a_4$   | $a_6$   | r                                      | T  | s   | $\operatorname{ord}(\Delta)$  | $\operatorname{ord}_{-}(j)$   | $c_p$   | Kodaira  | Isogenies        |
| 1208  | 5b   |   |  | 12  | $05b = (37 - 4\varphi) = 5a \cdot 241b$   | (2 isogeny                             | class  | es)   |   |   |   |  | $1205\mathrm{b}$ |
| a1  | $\varphi$  | $-\varphi$  | $\varphi$  | $-16+5\varphi$  | $-28 + 14\varphi$   | 0                                      | 2  | ++  | 2, 1  | 2,1   | 2, 1  | $I_2, I_1$   |                  |
| a2  | arphi  | $-\varphi$  | arphi  | _1<br>_1  | -1  | 0                                      | 2  |   |   | 1, 2  | 1, 2  | $I_1, I_2$   |                  |
| b1  | $1+\varphi$  | $1+\varphi$ .   | $1 + \varphi$  | $-14490 - 23444\varphi$   | $1256093 + 2032401\varphi$  | 0                                      | 2  | <u>-</u>  | 1,1   | 1,1   | 1,1   | $ $ $I_1, I_1$   | <u>-</u>         |
| b2  |  | $-1-\varphi$  | $\varphi$  | $-25-27\varphi$   | $71 + 128\varphi$   | 0                                      | 4  | ++  |   | 2,2   | 2,2   | $I_2, I_2$   |                  |
| b3  |  | $-1-\varphi$  | arphi  | -2arphi   | $1+3\varphi$  | 0                                      | 4  |   |   | 1,4   | 1,4   | $I_1, I_4$   |                  |
| b4  | 1 -  | $-1-\varphi$  | φ  | $-115 + 28\varphi$  | $-346 + 382\varphi$   | 0                                      | 2  | ++  | 4, 1  | 4,1   | 4, 1  | $I_4, I_1$   |                  |
| 1216  | ба   |   |  | 12  | $216a = (8 - 32\varphi) = 2^3 \cdot 19a$  | (4 isogeny                             | classe   | es)   |   |   |   |  | 1216a            |
| a1  | 0 -  | $-1+\varphi$  | 0  | $-1+\varphi$  | 0   | 1                                      | 2  | + -   | 1,4   | 1   | 1, 2  | $I_1, III$   |                  |
| a2  |  | $-1+\varphi$  | 0  | 4-4arphi  | $-8+4\varphi$   | 1                                      | 2  | - +   |   | 2   | 2, 4  | $I_2, I_1^*$   |                  |
| b1  |  | $1+\varphi$   |  | 2arphi  | $1+\varphi$   | 1                                      | ·<br>  4   | <u>-</u><br>  + -   |   | 1   | 1, 2  | $ I_1, III $   | <u>  </u>        |
| b2  | 0  | $1+\varphi$   | 0  | $-5-3\varphi$   | 3arphi  | 1                                      | 4  | ++  |   | 2   | 2,4   | $I_{2}, I_{1}^{*}$   |                  |
| b3  | 0  | $1+\varphi$   | 0  | $-85-63\varphi$   | $140 + 431\varphi$  | 1                                      | 2  | ++  |   | 1   | 1, 2  | $I_1, III*$  |                  |
| b4  | 0  | $1+\varphi$   | 0  | $-5-23\varphi$  | $-44-57\varphi$   | 1                                      | 2  | -+  | 4, 10   | 4   | 2, 2  | $I_4, III^*$   |                  |
| c1  | 0  | $1-\varphi$   | 0  | $-8-12\varphi$  | $-16 - 36\varphi$   | 0                                      | 1  | <u> </u>  | 3,11  | 3   | 1,1   | $I_3,II^*$   |                  |
| d1  | 0  | $1+\varphi$   | 0  | $-1+2\varphi$   | $-5+\varphi$  | 0                                      | 1  | Ī — —   | 1,11  | 1   | 1,1   | $I_1, II^*$  |                  |
|   |  |   |  |   |   |  |  |   |   |   |   |  |                  |
| 1216  | 6b   |   |  | 12  | $16b = (24 - 32\varphi) = 2^3 \cdot 19b$  | (4 isogeny                             | class  | es)   |   |   |   |  | 1216b            |
| 1216  |  | <b>—</b> (0   | 0  |   | $16b = (24 - 32\varphi) = 2^3 \cdot 19b$  | (4 isogeny                             |  | T .   | 1 4   | 1   | 1.2   | I, III   | 1216b            |
| a1  | 0  | $-\varphi$ $-\varphi$   | 0  | $-\varphi$  | 0   | 1                                      | 2  | -+  |   | 1 2   | 1, 2<br>2, 4  | $I_1, III$ $I_2, I_1^*$  | 1216b            |
| a1<br>a2  | 0  | $-\varphi$  | 0  | -arphi $4arphi$   | $\begin{matrix} 0 \\ -4 - 4\varphi \end{matrix}$  | 1 1                                    | 2 2  | - +<br>  + -  | 2,8   | 2   | 2,4   | $I_2, I_1^*$   | 1216b            |
| a1<br>a2<br>b1  | 0 0 0  | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$   | 0  | $ \begin{array}{c} -\varphi \\ 4\varphi \\ \hline 1 \end{array} $   | $ \begin{array}{c} 0 \\ -4 - 4\varphi \\ 1 \end{array} $  | 1 1                                    | $\begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$  | - +<br>  + -<br>  - +   | $\frac{2,8}{1,4}$   | $\begin{vmatrix} 2 \\ 1 \end{vmatrix}$  | $\frac{2,4}{1,2}$   | $\begin{bmatrix} & I_2, I_1^* \\ -& I_1, III \end{bmatrix}$  | 1216b            |
| a1<br>a2<br>b1<br>b2  | 0 - 0 - 0 - 0 - 0  | $ \begin{array}{ccc} -\varphi \\ -1 - \varphi \\ -1 - \varphi \end{array} $   | $\frac{0}{0}$  | $ \begin{array}{r} -\varphi \\ 4\varphi \\ 1\\ -9+5\varphi \end{array} $  | $\begin{matrix} 0 \\ -4 - 4\varphi \\ \hline 1 \\ 12 - 7\varphi \end{matrix}$   | 1 1 1 1                                | $\begin{array}{ c c c }\hline 2\\2\\4\\4\\\end{array}$   | - +<br>  + -<br>  - +<br>  + +  | $ \begin{array}{c} 2,8 \\ -1,4 \\ 2,8 \end{array} $   | $\begin{vmatrix} 2 \\ 1 \\ 2 \end{vmatrix}$   | $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$   | $\begin{array}{ c c c c c }\hline & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ \end{array}$  | 1216b            |
| a1<br>a2<br>b1<br>b2<br>b3                                  | 0<br>0<br>0<br>0<br>0<br>0<br>0  | $ \begin{array}{c} -\varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \end{array} $   | 0<br>0<br>0  | $ \begin{array}{r} -\varphi \\ 4\varphi \\ \hline 1 \\ -9 + 5\varphi \\ -149 + 65\varphi \end{array} $  | $   \begin{array}{r}     0 \\     -4 - 4\varphi \\     \hline     1 \\     12 - 7\varphi \\     720 - 495\varphi   \end{array} $                  | 1 1 1 1 1 1 1                          | $ \begin{array}{c c} 2 \\ 2 \\ 4 \\ 4 \\ 2 \end{array} $   | - +<br>  + -<br>  - +<br>  + +<br>  + +   | $ \begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \end{array} $  | $\begin{vmatrix} 2 \\ 1 \end{vmatrix}$  | $ \begin{array}{ c c c c c } \hline  & 2, 4 \\  & 1, 2 \\  & 2, 4 \\  & 1, 2 \end{array} $  | $\begin{array}{c c} & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ \end{array}$  | 1216b            |
| a1<br>a2<br>b1<br>b2  | 0<br>0<br>0<br>0<br>0<br>0<br>0  | $ \begin{array}{ccc} -\varphi \\ -1 - \varphi \\ -1 - \varphi \end{array} $   | $\frac{0}{0}$  | $ \begin{array}{r} -\varphi \\ 4\varphi \\ 1\\ -9+5\varphi \end{array} $  | $\begin{matrix} 0 \\ -4 - 4\varphi \\ \hline 1 \\ 12 - 7\varphi \end{matrix}$   | 1 1 1 1                                | $ \begin{array}{c c} 2 \\ 2 \\ 4 \\ 4 \\ 2 \\ 2 \end{array} $  | - +<br>  + -<br>  - +<br>  + +  | $ \begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \end{array} $  | $\begin{array}{c c} 2 \\ \hline 1 \\ 2 \\ 1 \end{array}$  | $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$   | $\begin{array}{ c c c c c }\hline & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ \end{array}$  | 1216b            |
| a1<br>a2<br>b1<br>b2<br>b3<br>b4                            | 0<br>0<br>0<br>0<br>0  | $ \begin{array}{ccc} -\varphi \\ -1-\varphi \\ -1-\varphi \\ -1-\varphi \\ -1-\varphi \\ -1-\varphi \end{array} $   | 0 0 0  |   | $   \begin{array}{r}     0 \\     -4 - 4\varphi \\ \hline     1 \\     12 - 7\varphi \\     720 - 495\varphi \\     -72 + 33\varphi \end{array} $ | 1<br>1<br>1<br>1<br>1<br>1             | $ \begin{array}{ c c c c } \hline 2 \\ 2 \\ \hline 4 \\ 4 \\ 2 \\ 2 \\ \hline 1 \\ \hline \end{array} $                          | - +<br>  + -<br>  - +<br>  + +<br>  + +   | 2,8<br>1,4<br>2,8<br>1,10<br>4,10<br>3,11   | 2<br>1 2<br>1 4   | $ \begin{array}{c c}  & 2,4 \\ \hline  & 1,2 \\  & 2,4 \\  & 1,2 \\  & 2,2 \\ \end{array} $   | $\begin{array}{c c} & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ & I_4, III^* \\ \hline \end{array}$   | 1216b            |
| a1<br>a2<br>b1<br>b2<br>b3<br>b4<br>c1<br>d1                | 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 - 0 -  | $ \begin{array}{cccc} -\varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \\ \varphi \end{array} $   | 0  |   | $ \begin{array}{r} 0 \\ -4 - 4\varphi \\ \hline 1 \\ 12 - 7\varphi \\ 720 - 495\varphi \\ -72 + 33\varphi \\ -52 + 36\varphi \\ -4 \end{array} $  | 1<br>1<br>1<br>1<br>1<br>1<br>1<br>0   | $ \begin{array}{ c c c c c } \hline 2 \\ 2 \\ 4 \\ 4 \\ 2 \\ 2 \\ \hline 1 \\ \hline 1 \end{array} $                             | - +<br>  + -<br>  - +<br>  + +<br>  + -<br>   | $ \begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ 3,11 \end{array} $  | $ \begin{array}{ c c c }  & 2 \\  & 1 \\  & 2 \\  & 1 \\  & 4 \\  & 3 \\ \end{array} $  | $ \begin{array}{c cccc}  & 2, 4 \\ \hline  & 1, 2 \\  & 2, 4 \\  & 1, 2 \\  & 2, 2 \\ \hline  & 1, 1 \\ \end{array} $   | $\begin{array}{ c c c }\hline & I_2, I_1^*\\\hline & I_1, III\\ & I_2, I_1^*\\ & I_1, III^*\\ & I_4, III^*\\\hline & I_3, II^*\\\hline \end{array}$  |                  |
| a1<br>a2<br>b1<br>b2<br>b3<br>b4<br>c1<br>d1                | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0   | $ \begin{array}{c} -\varphi \\ -1-\varphi \\ -1-$ | 0 0 0  |   | $0$ $-4 - 4\varphi$ $1$ $12 - 7\varphi$ $720 - 495\varphi$ $-72 + 33\varphi$ $-52 + 36\varphi$ $-4$ $0a = (36 - 2\varphi) = 2 \cdot 5a \cdot 61a$ | 1                                      | 2 2 4 4 4 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1  | - + + -   - + + + + + -   | $ \begin{array}{r} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ \hline 3,11 \\ \hline 1,11 \end{array} $  | 2   | 2,4<br>1,2<br>2,4<br>1,2<br>2,2<br>1,1<br>1,1   | $ \begin{array}{c c} & I_2, I_1^* \\ \hline -I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ & I_4, III^* \\ \hline -I_3, II^* \\ \hline & I_1, III^* \\ \hline \end{array} $  | 1216b            |
| a1 a2 b1 b2 b3 b4 c1 d1 a1 a2 a1                            | 0<br>0<br>0<br>0<br>0<br>0<br>0<br>0   | $ \begin{array}{c} -\varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \\ \hline -1 - \varphi \end{array} $  | 0  |   |   | 1 1 1 1 1 1 1 1 1 1 1 0 0 (6 isogen; 0 | 2 2 4 4 2 2 2 1 1 1 1 1 y class  | - +   + -   - +   + +   + +   + -   | $ \begin{array}{r} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ 3,11 \\ 1,11 \end{array} $ $ 5,3,1 $  | 2<br>1 2<br>1 4<br>1 3<br>1 5,3,1   | 2, 4<br>1, 2<br>2, 4<br>1, 2<br>2, 2<br>1, 1<br>1, 1  | $\begin{array}{ c c c }\hline & I_2, I_1^*\\ \hline & I_1, III\\ & I_2, I_1^*\\ & I_1, III^*\\ & I_4, III^*\\ \hline & I_3, II^*\\ \hline & I_1, III^*\\ \hline \end{array}$   |                  |
| a1 a2 b1 b2 b3 b4 c1 d1 | $ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $  | $ \begin{array}{c} -\varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \\ \hline -1 - \varphi \end{array} $ $ \begin{array}{c} \varphi \\ -1 - \varphi \\ \hline -1 - \varphi \end{array} $  | 0  |   |   | 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  | 2   2   4   4   2   2   1   1   1   1   2   2   2   1   1  | - +   + -   - +   + +   + +   + -                 + +     + +   +                   | $ \begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ 3,11 \\ 1,11 \\ \hline 5,3,1 \\ 2,3,1 \end{array} $   | 2<br>1 2<br>1 4<br>1 3<br>1 1<br>5,3,1<br>1 2,3,1   | $\begin{array}{ c c c c }\hline & 2,4\\ \hline & 1,2\\ & 2,4\\ & 1,2\\ & 2,2\\ \hline & 1,1\\ \hline & 1,1,1\\ \hline & 1,1,1\\ \hline & 2,3,1\\ \hline \end{array}$  | $\begin{array}{c c} & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ & I_4, III^* \\ \hline & I_3, II^* \\ \hline & I_1, III^* \\ \hline & I_2, I_3, I_1 \\ \hline & I_2, I_3, I_1 \\ \hline \end{array}$  |                  |
| a1 a2 b1 b2 b3 b4 c1 d1   1220  a1 b1 b2                    | $ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ \end{array} $ $ \begin{array}{c} 0 \\ \end{array} $ $ \begin{array}{c} 0 \\ \end{array} $ $ \begin{array}{c} 0 \\ \end{array} $                   | $ \begin{array}{c} -\varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \\ \hline -1 - \varphi \\ \hline -1 - \varphi \end{array} $   | 0  | $ \begin{array}{r} -\varphi \\ 4\varphi \\ \hline 1 \\ -9 + 5\varphi \\ -149 + 65\varphi \\ -29 + 25\varphi \\ -20 + 12\varphi \\ \hline 0 \\ 122 \\ -82 + 53\varphi \\ -2 + \varphi \\ -12 + 11\varphi \end{array} $               |   | 1                                      | 2   2   4   4   2   2   1   1   1   1     1     2   2  | - + + -         - + + + -                     - + + + -                             | $ \begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ 3,11 \\ 1,11 \end{array} $ $ \begin{array}{c} 5,3,1 \\ 2,3,1 \\ 1,6,2 \end{array} $   | $\begin{array}{ c c c }\hline & 2 \\ & 1 \\ & 2 \\ & 1 \\ & 4 \\ \hline & 3 \\ \hline & 1 \\ \hline & 5, 3, 1 \\ & 2, 3, 1 \\ & 1, 6, 2 \\ \hline \end{array}$  | $\begin{array}{ c c c }\hline & 2,4\\ \hline & 1,2\\ & 2,4\\ & 1,2\\ & 2,2\\ \hline & 1,1\\ \hline & 1,1\\ \hline & 1,1,1\\ \hline & 2,3,1\\ & 1,6,2\\ \hline \end{array}$                                  | $\begin{array}{c c} & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ & I_4, III^* \\ \hline & I_3, II^* \\ \hline & I_1, III^* \\ \hline & I_2, I_3, I_1 \\ & I_2, I_3, I_1 \\ & I_1, I_6, I_2 \\ \end{array}$   |                  |
| a1 a2 b1 b2 b3 b4 c1 d1   1220 a1 b2 c1 b2 c1               | $ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $  | $ \begin{array}{c} -\varphi \\ -1-\varphi \\ -1-\varphi \\ -1-\varphi \\ -1-\varphi \\ \hline -1-\varphi \\ \hline -1-\varphi \\ \hline -1-\varphi \\ \hline -1-\varphi \\ \varphi \\ -1+\varphi \\ \end{array} $   | $\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 $   | $ \begin{array}{r} -\varphi \\ 4\varphi \\ \hline 1 \\ -9 + 5\varphi \\ -149 + 65\varphi \\ -29 + 25\varphi \\ -20 + 12\varphi \\ \hline 0 \\ 122 \\ -82 + 53\varphi \\ -2 + \varphi \\ -12 + 11\varphi \\ -22\varphi \end{array} $ |   | 1                                      | $ \begin{array}{c c} 2 \\ 2 \\ 4 \\ 4 \\ 2 \\ 2 \\ 1 \\ 1 \end{array} $ $ \begin{array}{c c} 1 \\ 2 \\ 2 \\ 1 \\ 5 \end{array} $ | - +   + -   - +   + +   + +   + -                 + +   + -               + +   + - | $ \begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ \hline 3,11 \\ \hline 1,11 \end{array} $ $ \begin{array}{c} 5,3,1 \\ 2,3,1 \\ 1,6,2 \\ \hline 3,10,1 \end{array} $  | $\begin{array}{ c c c }\hline & 2 \\ & 1 \\ & 2 \\ & 1 \\ & 4 \\ \hline & 3 \\ \hline & 1 \\ \hline & 5,3,1 \\ & 2,3,1 \\ & 1,6,2 \\ \hline & 3,10,1 \\ \hline \end{array}$                               | $\begin{array}{ c c c }\hline & 2,4\\ \hline & 1,2\\ & 2,4\\ & 1,2\\ & 2,2\\ \hline & 1,1\\ \hline & 1,1\\ \hline & 1,1\\ \hline & 1,1,1\\ \hline & 2,3,1\\ & 1,6,2\\ \hline & 1,10,1\\ \hline \end{array}$ | $\begin{array}{c c} & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ & I_4, III^* \\ \hline & I_3, II^* \\ \hline & I_1, III^* \\ \hline & I_2, I_3, I_1 \\ & I_2, I_3, I_1 \\ & I_1, I_6, I_2 \\ \hline & I_3, I_{10}, I_1 \\ \end{array}$  |                  |
| a1 a2 b1 b2 b3 b4 c1 d1 b1 b2 c1 c1 c2 c1 c2                | $ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ | $ \begin{array}{c} -\varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \end{array} $ $ \begin{array}{c} \varphi \\ -1 - \varphi \\ \hline -1 - \varphi \\ \varphi \\ -1 + \varphi \\ -1 + \varphi \end{array} $  | $\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline \\ 0 \\ 0$ | $-\varphi \\ 4\varphi \\ 1 \\ -9 + 5\varphi \\ -149 + 65\varphi \\ -29 + 25\varphi \\ -20 + 12\varphi \\ 0$ $122 \\ -82 + 53\varphi \\ -2 + \varphi \\ -12 + 11\varphi \\ -22\varphi \\ -1050 + 803\varphi$                         |   | 1                                      | 2   2   4   4   2   2   1   1   1   1     1     2   2  | - + + -         - + + + -                     - + + + -                             | $\begin{array}{c} 2,8 \\ \hline 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ \hline 3,11 \\ \hline 1,11 \\ \hline \\ 5,3,1 \\ \hline 2,3,1 \\ 1,6,2 \\ \hline 3,10,1 \\ 15,2,5 \\ \end{array}$   | $\begin{array}{ c c c }\hline & 2 \\ & 1 \\ & 2 \\ & 1 \\ & 4 \\ \hline & 3 \\ \hline & 1 \\ \hline & 5, 3, 1 \\ & 1, 6, 2 \\ \hline & 3, 10, 1 \\ & 15, 2, 5 \\ \hline \end{array}$                      | $\begin{array}{ c c c }\hline & 2,4\\ \hline & 1,2\\ & 2,4\\ & 1,2\\ & 2,2\\ \hline & 1,1\\ \hline & 1,1\\ \hline & 1,1,1\\ \hline & 2,3,1\\ & 1,6,2\\ \hline & 1,10,1\\ & 1,2,5\\ \hline \end{array}$      | $\begin{array}{c c} & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ & I_4, III^* \\ \hline & I_3, II^* \\ \hline & I_1, III^* \\ \hline & I_2, I_3, I_1 \\ & I_2, I_3, I_1 \\ & I_1, I_6, I_2 \\ \hline & I_3, I_{10}, I_1 \\ & I_{15}, I_2, I_5 \\ \hline \end{array}$                                   |                  |
| a1 a2 b1 b2 b3 b4 c1 d1   1220 a1 b2 c1 c2 d1 c2            | $ \begin{array}{cccc} 0 & & & & \\ 0 & & & & \\ 0 & & & & \\ 0 & & & & \\ 0 & & & & \\ \end{array} $ $ \begin{array}{ccccc} 0 & & & & \\ 0 & & & & \\ \end{array} $ $ \begin{array}{ccccccccccccccccccccccccccccccccccc$   | $ \begin{array}{c} -\varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \end{array} $ $ \begin{array}{c} -1 - \varphi \\ \varphi \\ -1 - \varphi \\ -1 + \varphi \\ -1 - \varphi \end{array} $  | $\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 $   | $-\varphi \\ 4\varphi \\ 1 \\ -9 + 5\varphi \\ -149 + 65\varphi \\ -29 + 25\varphi \\ -20 + 12\varphi \\ 0 \\ 122 \\ -82 + 53\varphi \\ -2 + \varphi \\ -12 + 11\varphi \\ -22\varphi \\ -1050 + 803\varphi \\ -5 + 2\varphi$       |   | 1                                      | 2   2   4   4   2   2   1   1   1   1     1  | - +   + -   - +   + +   + +   + -                 + +   + -               + +   + - | $ \begin{array}{c} 2,8 \\ 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ 3,11 \\ 1,11 \end{array} $ $ \begin{array}{c} 5,3,1 \\ 2,3,1 \\ 1,6,2 \\ 3,10,1 \\ 15,2,5 \\ 5,2,1 \end{array} $  | $\begin{array}{ c c c }\hline & 2 \\ & 1 \\ & 2 \\ & 1 \\ & 4 \\ \hline & 3 \\ \hline & 1 \\ \hline & 5,3,1 \\ & 2,3,1 \\ & 1,6,2 \\ \hline & 3,10,1 \\ & 15,2,5 \\ \hline & 5,2,1 \\ \hline \end{array}$ | $\begin{array}{ c c c }\hline & 2,4\\ \hline & 1,2\\ & 2,4\\ & 1,2\\ & 2,2\\ \hline & 1,1\\ \hline & 1,1\\ \hline & 1,1\\ \hline & 1,1\\ \hline & 1,0,1\\ & 1,2,5\\ \hline & 5,2,1\\ \hline \end{array}$    | $\begin{array}{c c} & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ & I_4, III^* \\ \hline & I_3, II^* \\ \hline & I_1, III^* \\ \hline & I_2, I_3, I_1 \\ & I_2, I_3, I_1 \\ & I_1, I_6, I_2 \\ \hline & I_3, I_{10}, I_1 \\ & I_{15}, I_{2}, I_{5} \\ \hline & I_5, I_{2}, I_{1} \\ \hline \end{array}$ |                  |
| a1 a2 b1 b2 b3 b4 c1 d1 b1 b2 c1 c1 c2 c1 c2                | $ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} $ | $ \begin{array}{c} -\varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \\ -1 - \varphi \end{array} $ $ \begin{array}{c} \varphi \\ -1 - \varphi \\ \hline -1 - \varphi \\ \varphi \\ -1 + \varphi \\ -1 + \varphi \end{array} $  | $\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline \\ 0 \\ 0$ | $-\varphi \\ 4\varphi \\ 1 \\ -9 + 5\varphi \\ -149 + 65\varphi \\ -29 + 25\varphi \\ -20 + 12\varphi \\ 0$ $122 \\ -82 + 53\varphi \\ -2 + \varphi \\ -12 + 11\varphi \\ -22\varphi \\ -1050 + 803\varphi$                         |   | 1                                      | $ \begin{array}{c c} 2 \\ 2 \\ 4 \\ 4 \\ 2 \\ 2 \\ 1 \\ 1 \end{array} $ $ \begin{array}{c c} 1 \\ 2 \\ 2 \\ 1 \\ 5 \end{array} $ | - +   + -   - +   + +   + +   + -                 + +   + -               + +   + - | $\begin{array}{c} 2,8 \\ \hline 1,4 \\ 2,8 \\ 1,10 \\ 4,10 \\ \hline 3,11 \\ \hline 1,11 \\ \hline \\ 5,3,1 \\ \hline 2,3,1 \\ 1,6,2 \\ \hline 3,10,1 \\ \hline 15,2,5 \\ \hline 5,2,1 \\ \hline 3,1,1 \\ \hline \end{array}$ | $\begin{array}{ c c c }\hline & 2 \\ & 1 \\ & 2 \\ & 1 \\ & 4 \\ \hline & 3 \\ \hline & 1 \\ \hline & 5, 3, 1 \\ & 1, 6, 2 \\ \hline & 3, 10, 1 \\ & 15, 2, 5 \\ \hline \end{array}$                      | $\begin{array}{ c c c }\hline & 2,4\\ \hline & 1,2\\ & 2,4\\ & 1,2\\ & 2,2\\ \hline & 1,1\\ \hline & 1,1\\ \hline & 1,1,1\\ \hline & 2,3,1\\ & 1,6,2\\ \hline & 1,10,1\\ & 1,2,5\\ \hline \end{array}$      | $\begin{array}{c c} & I_2, I_1^* \\ \hline & I_1, III \\ & I_2, I_1^* \\ & I_1, III^* \\ & I_4, III^* \\ \hline & I_3, II^* \\ \hline & I_1, III^* \\ \hline & I_2, I_3, I_1 \\ & I_2, I_3, I_1 \\ & I_1, I_6, I_2 \\ \hline & I_3, I_{10}, I_1 \\ & I_{15}, I_2, I_5 \\ \hline \end{array}$                                   |                  |

|          | $a_1$             | $a_2$         | $a_5$             | $a_4$                                   | $a_6$   |              | r                                      | T             | s            | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$                      | Kodaira  | Isogenies |
|----------|-------------------|---------------|-------------------|---|---|--------------|--|---------------|--------------|------------------------------|-----------------------------|----------------------------|--|-----------|
| 122      | $\Omega_{\Omega}$ |               |                   | 1                                       |   | (c:          |  | 1             | \            |                              |                             |                            |  | 1220a     |
|          |                   |               |                   |   | $220a = (36 - 2\varphi) = 2 \cdot 5a \cdot 61a$       | (6 isog      |  |               |              | 0.1.1                        | 0.1.1                       | 0.1.1                      | T T T  | 1220a     |
| f1       | $\varphi$         | 0             | 1                 |   | $-1+2\varphi$   |              | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 6             | ++           | 6, 1, 1                      | 6, 1, 1                     | 6, 1, 1                    | $I_6, I_1, I_1$  |           |
| f2<br>f3 | $1+\varphi$       | $0 \\ 0$      | 1<br>1            | $-129 - 213\varphi$ $-265 + 140\varphi$ | $     1020 + 1652\varphi \\     -1873 + 1098\varphi $ |              | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\frac{6}{2}$ | - +          | 3, 2, 2<br>2, 3, 3           | 3, 2, 2<br>2, 3, 3          | 3, 2, 2                    | $I_3, I_2, I_2$  |           |
| f4       | $\varphi$         | 0             | 1                 | $-205 + 140\varphi$ $-275 + 120\varphi$ | $-1873 + 1098\varphi$ $-1837 + 1214\varphi$           |              | 0                                      | $\frac{2}{2}$ | + +<br>  - + | 1, 6, 6                      | 1, 6, 6                     | 2, 1, 3 $1, 2, 6$          | $\begin{bmatrix} I_2, I_3, I_3 \\ I_1, I_6, I_6 \end{bmatrix}$ |           |
| 14       | Ψ                 |               | 1                 | 210   120φ                              | 1037   1214φ  |              | U                                      |               | '            | 1,0,0                        | 1,0,0                       | 1, 2, 0                    | 11,16,16   |           |
| 122      | 0b                |               |                   | 1                                       | $220b = (34 + 2\varphi) = 2 \cdot 5a \cdot 61b$       | (6 isog      | eny                                    | class         | ses)         |                              |                             |                            |  | 1220b     |
| a1       | 1                 | $1+\varphi$   | 1                 | $-30-51\varphi$                         | $-160 - 261\varphi$                                   |              | 0                                      | 1             |              | 5, 3, 1                      | 5, 3, 1                     | 1, 1, 1                    | $I_5, I_3, I_1$  |           |
| b1       | $1+\varphi$       | $1+\varphi$   | 0                 | $2\varphi$                              | 0   |              | $\overline{1}$                         | 2             | ++           | 2, 3, 1                      | [2, 3, 1]                   | 2, 3, 1                    | $  I_2, I_3, I_1  $  | [         |
| b2       | $1+\varphi$       | $1+\varphi$   | 0                 | $-8\varphi$                             | $-14-26\varphi$                                       |              | 1                                      | 2             | - +          | 1, 6, 2                      | 1, 6, 2                     | 1, 6, 2                    | $I_1, I_6, I_2$  |           |
| c1       | $1+\varphi$       | $\varphi$     | $\varphi$         |   | 86  |              | $\bar{0}$                              | 5             | Ī — —        | 3, 10, 1                     | 3, 10, 1                    | 1, 10, 1                   | $  I_3, I_{10}, I_1  $   | [         |
| c2       | $1+\varphi$       | $\varphi$     | •                 |   | $-8299-10725\varphi$                                  |              | 0                                      | 1             |              | 15, 2, 5                     | 15, 2, 5                    | 1, 2, 5                    | $I_{15}, I_2, I_5$   |           |
| d1       | $1+\varphi$       | 1             |                   | $-3-2\varphi$                           | 1   |              | $1 \mid$                               | 1             |              | 5, 2, 1                      | [5, 2, 1]                   | 5, 2, 1                    | $I_5, I_2, I_1$  |           |
| e1       | $1+\varphi$       | -1            | $1+\varphi$       | $-\varphi$                              | -1-arphi  |              | 0                                      | 3             | Ī — —        | 3, 1, 1                      | [3, 1, 1]                   | 3, 1, 1                    | $  I_3, I_1, I_1  $  |           |
| e2       | $1+\varphi$       |               | $1+\varphi$       | •                                       | $-71-79\varphi$                                       |              | 0                                      | 1             |              | 4 0 0                        | 1, 3, 3                     | 1, 1, 3                    | $I_1, I_3, I_3$  |           |
| f1       | $1+\varphi$       | $-\varphi$    | 1                 | $-5-\varphi$                            | 1-2arphi  | ·            | 0                                      | 6             | ·<br>  + +   | 6, 1, 1                      | [6, 1, 1]                   | 6, 1, 1                    | $I_6, I_1, I_1$  | <u>  </u> |
| f2       | $\varphi$         | $1-\varphi$   | 1                 | ,                                       | $2672-1652\varphi$                                    |              | 0                                      | 6             | +-           |                              | 3, 2, 2                     | 3, 2, 2                    | $I_3, I_2, I_2$  |           |
| f3       | $1+\varphi$       | $-\varphi$    | 1                 | $-125-141\varphi$                       | $-775 - 1098\varphi$                                  |              | 0                                      | 2             | ++           | 2, 3, 3                      | 2, 3, 3                     | 2, 1, 3                    | $I_{2}, I_{3}, I_{3}$  |           |
| f4       | $1+\varphi$       | $-\varphi$    | 1                 | $-155 - 121\varphi$                     | $-623-1214\varphi$                                    |              | 0                                      | 2             | + -          | 1, 6, 6                      | 1, 6, 6                     | 1, 2, 6                    | $I_1, I_6, I_6$  |           |
| 122      | 5a                |               |                   |   | $1225a = (35) = 5a^2 \cdot 7$                         | (2 isogeny o | clas                                   | ses)          |              |                              |                             |                            |  | 1225a     |
| a1       | 0                 | -1            | 1                 | -148                                    | 748   | `            | 1                                      | 5             |              | 6,5                          | 5                           | 2, 5                       | $I_0^*, I_5$   |           |
| a2       | 0                 | -1            | 1                 | 2                                       | -2  |              | 1                                      | 1             |              | 6, 1                         | 1                           | 2, 3 $2, 1$                | $I_0^{*}, I_1$   |           |
| b1       | 0                 | $1-\varphi$   | <del>-</del><br>1 |   | $25 + 19\varphi$                                      |              | 0                                      | 1             | <u>-</u><br> | $\frac{12}{12}$              | 6,3                         | $\frac{1}{2}, \frac{1}{2}$ | $  I_6^*, I_3  $   |           |
| b2       | 0                 | $1 - \varphi$ | 1                 | $-656-657\varphi$                       | $9090 + 12339\varphi$                                 |              | 0                                      | 1             |              | 24, 1                        | 18,1                        | 2, 1 $2, 1$                | $I_{18}^*, I_1$  |           |
| b3       | 0                 | $1-\varphi$   | 1                 | ,                                       | $-10-11\varphi$                                       |              | 0                                      | 1             |              | 8, 1                         | 2,1                         | $\frac{2}{2}, \frac{1}{1}$ | $I_2^{*}, I_1$   |           |
|          |                   | •             |                   | · · · · · · · · · · · · · · · · · · ·   | ,   |              |  |               |              | ,                            |                             | ,                          | 2, 1   |           |
| 123      | 1a                |               |                   |   | $1231a = (23 - 32\varphi) = 1231a$                    | (1 isoge     | eny                                    | class         | )            |                              |                             |                            |  | 1231a     |
| a1       | $1+\varphi$       | 0             | 1                 | ,                                       | $-20-33\varphi$                                       |              | 0                                      | 2             | -+           | 1                            | 1                           | 1                          | $I_1$  |           |
| a2       | φ                 | 0             | 1                 | $-113 + 67\varphi$                      | $531 - 331\varphi$                                    |              | 0                                      | 2             | + -          | 2                            | 2                           | 2                          | $I_2$  |           |
| 123      | 1b                |               |                   |   | $1231b = (9 - 32\varphi) = 1231b$                     | (1 isoger    | ny o                                   | class)        |              |                              |                             |                            |  | 1231b     |
| a1       | $\varphi$         | $1-\varphi$   | 1                 | $-20 + 11\varphi$                       | $-53 + 33\varphi$                                     |              | 0                                      | 2             | + -          | 1                            | 1                           | 1                          | $I_1$  |           |
| a2       | $1+\varphi$       | $-\varphi$    | 1                 | $-46-68\varphi$                         | $200 + 331\varphi$                                    |              | 0                                      | 2             | -+           | 2                            | 2                           | 2                          | $I_2$  |           |
| 124      | 4a                |               |                   |   | $1244a = (22 - 32\varphi) = 2 \cdot 311a$             | (1 isog      | geny                                   | clas          | s)           |                              |                             |                            |  | 1244a     |
| a1       | $\varphi$         | $-1-\varphi$  | $1+\varphi$       | $-\varphi$                              | $-\varphi$  |              | 1                                      | 2             | -+           | 2, 1                         | 2,1                         | 2, 1                       | $I_2, I_1$   |           |
| a2       | (0                | $-1-\varphi$  | $1+\varphi$       | $-10-\varphi$                           | $6-9\varphi$  |              | 1                                      | 2             | ++           | 1, 2                         | 1,2                         | 1, 2                       | $I_1, I_2$   |           |
| az       | 7                 | r             | . ,               | ,                                       | '   | !            |  |               |              | ,                            | ,                           | ,                          | 1,2  |           |

|      | $a_1$                 | $a_2$        | $a_3$         | $a_4$                     | $a_6$  | r             | T     | s          | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$               | $c_p$ | Kodaira                                    | Isogenies |
|------|-----------------------|--------------|---------------|---------------------------|--|---------------|-------|------------|------------------------------|---|-------|--|-----------|
| 124  | $\overline{	ext{4b}}$ |              |               | 124                       | $44b = (10 - 32\varphi) = 2 \cdot 311b $       | 1 isogeny cla | ıss)  |            |                              |   |       |  | 1244b     |
| a1   | $1+\varphi$           | 1            | 1             | arphi                     | $\varphi$                                      | 1             | 2     | + -        | 2, 1                         | 2,1                                       | 2,1   | $I_2, I_1$                                 |           |
| a2   | $1+\varphi$           | 1            | 1             | $-10+\varphi$             | $-12 + 9\varphi$                               | 1             | 2     | ++         | 1, 2                         | 1,2                                       | 1, 2  | $I_1, I_2$                                 |           |
| 125  | 5a                    |              |               | 12556                     | $a = (11 - 32\varphi) = 5a \cdot 251a$ (       | 5 isogeny cla | sses) |            |                              |   |       |  | 1255a     |
| a1   | 0                     | $-\varphi$   | 1             | 0                         | 0  | 1             | 1     | -+         | 1, 1                         | 1,1                                       | 1, 1  | $I_1, I_1$                                 |           |
| b1   | 0                     | 0            | 1             | $-7-2\varphi$             | $-8-3\varphi$                                  | 1             | 1     | - +        | 1,3                          | 1,3                                       | 1, 1  | $I_1, I_3$                                 |           |
| c1   | 0                     | 0            | 1             | $-517 + 310\varphi$       | $5255 - 3255\varphi$                           | 0             | 5     | :<br>      | 1, 15                        | 1, 15                                     | 1, 15 | $I_1, I_{15}$                              | <u>-</u>  |
| c2   | 0                     | 0            | 1             | $-106361 - 172267\varphi$ | $-25423183 - 41135199\varphi$                  | 0             | 1     | - +        | 5, 3                         | 5,3                                       | 5, 3  | $I_5, I_3$                                 |           |
| d1   | 0                     | $-1-\varphi$ | 1             | -10                       | $-10+2\varphi$                                 | 0             | 1     | - +        | 1, 1                         | 1,1                                       | 1,1   | $I_1, I_1$                                 |           |
| e1   | $\varphi$             | $1+\varphi$  | $1+\varphi$   | $-9+5\varphi$             | $-13+7\varphi$                                 | 0             | 4     | :<br>  + + | $\frac{1}{2}, \frac{1}{2}$   | [2,2]                                     | 2, 2  | $I_2, I_2$                                 |           |
| e2   | arphi                 | $1+\varphi$  | $1+\varphi$   | 1                         | -1   | 0             | 2     | -+         | 1, 1                         | 1,1                                       | 1, 1  | $I_1, I_1$                                 |           |
| e3   | $\varphi$             | $1+\varphi$  |               | $-149 + 70\varphi$        | $-753 + 494\varphi$                            | 0             | 2     | ++         | 1, 4                         | 1,4                                       | 1, 2  | $I_1, I_4$                                 |           |
| e4   | φ                     | $1+\varphi$  | $1+\varphi$   | $-29 + 20\varphi$         | $59 - 32\varphi$                               | 0             | 2     | + -        | 4, 1                         | 4,1                                       | 2, 1  | $I_4, I_1$                                 |           |
| 1258 | 5b                    |              |               | 1255                      | $b = (21 - 32\varphi) = 5a \cdot 251b $        | 5 isogeny cla | sses) |            |                              |   |       |  | 1255b     |
| a1   | 0                     | $-1+\varphi$ | 1             | 0                         | 0  | 1             | 1     | + -        | 1, 1                         | 1,1                                       | 1, 1  | $I_1, I_1$                                 |           |
| b1   | 0                     | 0            | 1             | $-9+2\varphi$             | $-11 + 3\varphi$                               | 1             | 1     | + -        | 1,3                          | 1,3                                       | 1, 1  | $I_1, I_3$                                 |           |
| c1   | 0                     | 0            | 1             | $-207 - 310\varphi$       | $2000 + 3255\varphi$                           | 0             | 5     | :<br>  + - | 1, 15                        | 1, 15                                     | 1,15  | $I_1, I_{15}$                              | <u>  </u> |
| c2   | 0                     | 0            | 1             | $-278628 + 172267\varphi$ | $-66558382 + 41135199\varphi$                  | 0             | 1     | + -        | 5, 3                         | 5,3                                       | 5, 3  | $I_5, I_3$                                 |           |
| d1   | 0                     | $1+\varphi$  | 1             | $-11 + 2\varphi$          | $-19-\varphi$                                  | 0             | 1     | + -        | 1, 1                         | $\begin{vmatrix} 1 & 1 & 1 \end{vmatrix}$ | 1, 1  | $I_1, I_1$                                 |           |
| e1   | $1+\varphi$           | $-1+\varphi$ | 0             | $-4-4\varphi$             | $-5-12\varphi$                                 | 0             | 4     | + +        | 2, 2                         | $\begin{bmatrix} 2,2 \end{bmatrix}$       | 2, 2  | $I_2, I_2$                                 | [         |
| e2   | $1 + \varphi$         | •            | 0             | $1+\varphi$               | 0  | 0             | 2     | + -        | 1, 1                         | 1,1                                       | 1, 1  | $\mathrm{I}_1,\mathrm{I}_1$                |           |
| e3   | $1 + \varphi$         | ,            | 0             | $-79-69\varphi$           | $-248 - 574\varphi$                            | 0             | 2     | ++         | 1, 4                         | 1,4                                       | 1, 2  | $I_1, I_4$                                 |           |
| e4   | $1+\varphi$           | $-1+\varphi$ | 0             | $-9-19\varphi$            | $18 + 22\varphi$                               | 0             | 2     | -+         | 4,1                          | 4,1                                       | 2, 1  | $I_4, I_1$                                 |           |
| 127  | 1a                    |              |               | 1271                      | $a = (13 - 32\varphi) = 31a \cdot 41b $        | 3 isogeny cla | sses) |            |                              |   |       |  | 1271a     |
| a1   | 0                     | $-1+\varphi$ | $\varphi$     | $-49 + 30\varphi$         | $184 - 114\varphi$                             | 1             | 1     |            | 2, 1                         | 2,1                                       | 2, 1  | $I_2, I_1$                                 |           |
| b1   | $1+\varphi$           | $1-\varphi$  | $\varphi$     | $-2-4\varphi$             | $-3-4\varphi$                                  | 1             | 2     | - +        | 1, 2                         | $\begin{vmatrix} 1,2 \end{vmatrix}$       | 1, 2  | $I_1, I_2$                                 |           |
| b2   | $1+\varphi$           | $1-\varphi$  | $\varphi$     | $-37-44\varphi$           | $-142 - 198\varphi$                            | 1             | 2     | ++         | 2, 1                         | 2,1                                       | 2, 1  | $I_2, I_1$                                 |           |
| c1   | 1                     | $-\varphi$   | $\varphi$     | $1-4\varphi$              | $-2+\varphi$                                   | 0             | 2     | -+         | 3, 2                         | 3,2                                       | 1, 2  | $I_3, I_2$                                 |           |
| c2   | 1                     | $-\varphi$   | $\varphi$     | $-29 - 9\varphi$          | $3+77\varphi$                                  | 0             | 2     | ++         | 6, 1                         | 6, 1                                      | 2, 1  | $I_6, I_1$                                 |           |
| 127  | 1b                    |              |               | 1271                      | $b = (19 - 32\varphi) = 31b \cdot 41a \tag{4}$ | 3 isogeny cla | sses) |            |                              |   |       |  | 1271b     |
| a1   | 0                     | $-\varphi$   | $1 + \varphi$ | $-19-30\varphi$           | $70 + 113\varphi$                              | 1             | 1     |            | 2, 1                         | 2,1                                       | 2, 1  | $I_2, I_1$                                 |           |
| b1   | $\varphi$             | 1            | $1+\varphi$   | $-5+2\varphi$             | $-7+3\varphi$                                  | 1             | 2     | + -        | 1, 2                         | 1,2                                       | 1, 2  | $\overline{\mathrm{I}_{1},\mathrm{I}_{2}}$ |           |
| b2   | arphi                 |              | $1+\varphi$   | $-80 + 42\varphi$         | $-340 + 197\varphi$                            | 1             | 2     | + +        | 2, 1                         | 2,1                                       | 2, 1  | $I_2, I_1$                                 |           |
| c1   |                       | $-1+\varphi$ | •             | $-3+3\varphi$             | $-1-2\varphi$                                  | 0             | 2     | + -        | 3, 2                         | 3,2                                       | 1, 2  | $I_3, I_2$                                 |           |
| c2   | 1                     | $-1+\varphi$ | $1 + \varphi$ | $-38 + 8\varphi$          | $80-78\varphi$                                 | 0             | 2     | ++         | 6, 1                         | 6, 1                                      | 2, 1  | $I_6, I_1$                                 |           |

| $a_1$ $a_2$ $a_3$                                     | $a_4$              | $a_6$  | r          | T       | s          | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$   | Kodaira               | Isogenies |
|---|--------------------|--|------------|---------|------------|------------------------------|-----------------------------|---------|-----------------------|-----------|
|   |                    |  |            |         |            |                              |                             |         |                       |           |
| .271c   | 1271c =            | $(33 + 7\varphi) = 31a \cdot 41a \qquad (2)$   | isogeny cl | asses)  |            |                              |                             |         |                       | 1271c     |
| a1 0 0 $1 + \varphi$                                  | $1-\varphi$        | -2   | 1          | 1       |            | 2, 1                         | 2,1                         | 2,1     | $I_2, I_1$            |           |
| $b1 \mid \varphi \qquad 1 \qquad \varphi$             | $-5-7\varphi$      | $2+6\varphi$                                   | 1          |         | - +        | 3, 2                         | 3,2                         | 3,2     | $I_3, I_2$            |           |
| b2  | $-120 + 77\varphi$ | $-596 + 369\varphi$                            | 1          | 2       | +-         | 6, 1                         | 6, 1                        | 6,1     | $I_6, I_1$            |           |
| .271d   | 1271d =            | $= (40 - 7\varphi) = 31b \cdot 41b \qquad (2)$ | isogeny cl | asses)  |            |                              |                             |         |                       | 1271c     |
| a1 0 0 φ  | $\varphi$          | $-1-\varphi$                                   | 1          | 1       |            | 2, 1                         | 2, 1                        | 2, 1    | $I_2, I_1$            |           |
| $b1 \mid 1+\varphi  1-\varphi  1+\varphi$             | $-12+5\varphi$     | 8-7arphi                                       | 1          | 2       | ·<br>  + - | 3, 2                         | 3,2                         | 3,2     | $I_3, I_2$            | <u> </u>  |
| $b2 \qquad 1 \qquad 1 + \varphi  1 + \varphi$         | $-43-76\varphi$    | $-269 - 447\varphi$                            | 1          | 2       | -+         | 6, 1                         | 6, 1                        | 6, 1    | $I_6, I_1$            |           |
| 276a  | 1276a =            | $(14 - 32\varphi) = 2 \cdot 11b \cdot 29b$     | (1 isogeny | class   | ;)         |                              |                             |         |                       | 1276      |
| $a1 \qquad \varphi \qquad 1-\varphi \qquad 0$         | $-3+2\varphi$      | $\frac{1}{2-\varphi}$                          |            | 3       | Í – –      | 2, 3, 1                      | 2, 3, 1                     | 2, 3, 1 | $I_{2}, I_{3}, I_{1}$ |           |
| a2 $\varphi$ $1-\varphi$ 0                            | $27-18\varphi$     | $-12 + \varphi$                                | 1          | 1       |            | 6, 1, 3                      | 6, 1, 3                     | 2, 1, 1 | $I_6, I_1, I_3$       |           |
| .276b   | 1276b =            | $(18 - 32\varphi) = 2 \cdot 11a \cdot 29a$     | (1 isogeny | v class | s)         |                              |                             |         |                       | 1276l     |
| a1 $1+\varphi$ 0 0                                    | $-1-2\varphi$      | $1+\varphi$                                    | 1          | 3       |            | 2, 3, 1                      | 2, 3, 1                     | 2, 3, 1 | $I_2, I_3, I_1$       |           |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $9+18\varphi$      | $-11-\varphi$                                  | 1          | 1       |            | 6, 1, 3                      | 6, 1, 3                     | 2, 1, 1 | $I_6, I_1, I_3$       |           |
| .276c   | 1276c =            | $(34 + 4\varphi) = 2 \cdot 11b \cdot 29a$      | (1 isogeny | class   | )          |                              |                             |         |                       | 1276      |
| a1 $1 	 -1 	 1 + \varphi$                             | $-1-3\varphi$      | -4   | 1          | 2       | -+         | 3, 4, 1                      | 3, 4, 1                     | 3, 2, 1 | $I_3, I_4, I_1$       |           |
| a2 $1 	 -1 	 1 + \varphi$                             | $-21 + 17\varphi$  | $-48 + 16\varphi$                              | 1          | 2       | +-         | 6, 2, 2                      | 6, 2, 2                     | 6, 2, 2 | $I_6, I_2, I_2$       |           |
| .276d   | 1276d =            | $(38 - 4\varphi) = 2 \cdot 11a \cdot 29b$      | (1 isogeny | class   | )          |                              |                             |         |                       | 1276c     |
| a1 1 $-1$ $\varphi$                                   | $-3+2\varphi$      | $-3-\varphi$                                   | 1          | 2       | +-         | 3, 4, 1                      | 3, 4, 1                     | 3, 2, 1 | $I_{3}, I_{4}, I_{1}$ |           |
| a2  | $-3-18\varphi$     | $-31-17\varphi$                                | 1          | 2       | -+         | 6, 2, 2                      | 6, 2, 2                     | 6, 2, 2 | $I_6, I_2, I_2$       |           |
| .279a   | 1279a              | $= (15 - 32\varphi) = 1279a \tag{1}$           | isogeny c  | lass)   |            |                              |                             |         |                       | 1279i     |
| a1 $1+\varphi$ $-1+\varphi$ 1                         | $-4+3\varphi$      | $7-4\varphi$                                   | 1          | 3       |            | 1                            | 1                           | 1       | $I_1$                 |           |
| a2   1  | $-5-14\varphi$     | $-19 - 36\varphi$                              | 1          | 1       |            | 3                            | 3                           | 3       | $I_3$                 |           |
| .279b   | 1279b              | $= (17 - 32\varphi) = 1279b \tag{1}$           | isogeny cl | ass)    |            |                              |                             |         |                       | 12791     |
| a1 $\varphi$ 1+ $\varphi$ $\varphi$                   | $-1-2\varphi$      | 0  | 1          | 3       |            | 1                            | 1                           | 1       | $I_1$                 |           |
| T I I   |                    | $-54 + 35\varphi$                              | 1          | 1       | 1          | 3                            | 3                           | 3       | $I_3$                 | 1         |

| $a_1$ | $a_2 \ a_3$ | $a_4$ | $a_6$ | r | T | $s \operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$ | Kodaira | Isogenies |
|-------|-------------|-------|-------|---|---|--------------------------------|-----------------------------|-------|---------|-----------|
|-------|-------------|-------|-------|---|---|--------------------------------|-----------------------------|-------|---------|-----------|

| 128 | 0a   |                        | $1280a = (16 - 32\varphi) = 2^4 \cdot 5a$ | (12 isogeny classe | ses) |       |       |    |      |                               | 1280a |
|-----|--|------------------------|---|--------------------|------|-------|-------|----|------|-------------------------------|-------|
| a1  | $0 -1 - \varphi = 0$                                 | $1+\varphi$            | 0   | 1                  | 2    |       | 4, 1  | 1  | 1,1  | $II, I_1$                     |       |
| a2  | $0 -1 - \varphi = 0$                                 | $-4-4\varphi$          | $8+12\varphi$                             | 1                  | 4    | ++    | 8, 2  | 2  | 4, 2 | $\mathrm{I}_0^*,\mathrm{I}_2$ |       |
| a3  | $0  -1-\varphi  0$                                   | $-64-84\varphi$        | $344 + 580\varphi$                        | 1                  | 2    | ++    | 10, 1 | 1  | 2, 1 | $	ext{I}_2^*, 	ext{I}_1$      |       |
| a4  | 0 	 -1 	 0   | $-108 + 64\varphi$     | $536 - 332\varphi$                        | 1                  | 4    | ++    | 10, 4 | 4  | 4, 2 | $\mathrm{I}_2^*,\mathrm{I}_4$ |       |
| a5  | $0  1+\varphi  0$                                    | $-11733 + 7254\varphi$ | $573651 - 354535\varphi$                  | 1                  | 2    | + -   | 11, 2 | 2  | 4, 2 | $\mathrm{I}_3^*,\mathrm{I}_2$ |       |
| a6  | $\begin{vmatrix} 0 & -1 - \varphi & 0 \end{vmatrix}$ | $-144-204\varphi$      | $-952 - 1628\varphi$                      | 1                  | 2    | -+    | 11, 8 | 8  | 2,2  | $I_3^*, I_8$                  |       |
| b1  | $\begin{vmatrix} 0 & 1+\varphi & 0 \end{vmatrix}$    | $1+\varphi$            | 1   | 1                  | 2    | Ī — — | 4, 1  | 1  | 1,1  | $\mathrm{II},\mathrm{I}_1$    |       |
| b2  | $0  1+\varphi  0$                                    | $-9+6\varphi$          | $11-7\varphi$                             | 1                  | 4    | ++    | 8, 2  | 2  | 4, 2 | $\mathrm{I}_0^*,\mathrm{I}_2$ |       |
| b3  | $0  1+\varphi  0$                                    | $-149 + 86\varphi$     | $775-495\varphi$                          | 1                  | 2    | ++    | 10, 1 | 1  | 2,1  | $\mathrm{I}_2^*,\mathrm{I}_1$ |       |
| b4  | 0 	 -1 	 0   | $-44-64\varphi$        | $204 + 332\varphi$                        | 1                  | 4    | ++    | 10, 4 | 4  | 4, 2 | $\mathrm{I}_2^*,\mathrm{I}_4$ |       |
| b5  | $0  -1-\varphi  0$                                   | $-4480 - 7252\varphi$  | $223596 + 361788\varphi$                  | 1                  | 2    | -+    | 11, 2 | 2  | 4, 2 | $\mathrm{I}_3^*,\mathrm{I}_2$ |       |
| b6  | $\begin{vmatrix} 0 & 1+\varphi & 0 \end{vmatrix}$    | $-349 + 206\varphi$    | $-2929 + 1833\varphi$                     | 1                  | 2    | + -   | 11,8  | 8  | 2,2  | $I_3^*, I_8$                  |       |
| c1  | 0 -1 0   | -41                    | 116                                       | 0                  | 4    | + +   | 4,6   | 6  | 1,2  | $II, I_6$                     |       |
| c2  | 0 	 -1 	 0   | -1                     | 0   | 0                  | 4    | ++    | 4, 2  | 2  | 1,2  | $II, I_2$                     |       |
| c3  | $0  -1-\varphi  0$                                   | $-1257 - 2023\varphi$  | $33560 + 54293\varphi$                    | 0                  | 2    | ++    | 8,3   | 3  | 1,1  | $I_0^*, I_3$                  |       |
| c4  | $0  1+\varphi  0$                                    | $-3281 + 2025\varphi$  | $84572 - 52269\varphi$                    | 0                  | 2    | ++    | 8,3   | 3  | 1,1  | $I_0^*, I_3$                  |       |
| c5  | 0 	 -1 	 0   | $-11-5\varphi$         | $1-17\varphi$                             | 0                  | 2    | ++    | 8, 1  | 1  | 1, 1 | $\mathrm{I}_0^*,\mathrm{I}_1$ |       |
| c6  | 0 	 -1 	 0   | $-16+5\varphi$         | $-16 + 17\varphi$                         | 0                  | 2    | ++    | 8, 1  | 1  | 1,1  | $\mathrm{I}_0^*,\mathrm{I}_1$ |       |
| c7  | 0 	 -1 	 0   | -36                    | 140                                       | 0                  | 2    |       | 8, 12 | 12 | 1,2  | $I_0^*, I_{12}$               |       |
| c8  | $  \ 0 \ \ -1 \ \ 0$                                 | 4                      | -4  | 0                  | 2    |       | 8, 4  | 4  | 1,2  | $I_0^*, I_4$                  |       |
| d1  | $\begin{vmatrix} 0 & -1 - \varphi & 0 \end{vmatrix}$ | $1+\varphi$            | -1  | 0                  | 2    | Ī — — | 4, 1  | 1  | 1,1  | $\mathrm{II},\mathrm{I}_1$    |       |
| d2  | $0  -1-\varphi  0$                                   | $-9+6\varphi$          | $-11+7\varphi$                            | 0                  | 4    | ++    | 8, 2  | 2  | 2,2  | $\mathrm{I}_0^*,\mathrm{I}_2$ |       |
| d3  | 0 1 0  | $-44-64\varphi$        | $-204 - 332\varphi$                       | 0                  | 4    | ++    | 10, 4 | 4  | 4, 2 | $\mathrm{I}_2^*,\mathrm{I}_4$ |       |
| d4  | $0  -1-\varphi  0$                                   | $-149 + 86\varphi$     | $-775 + 495\varphi$                       | 0                  | 2    | ++    | 10, 1 | 1  | 2, 1 | $\mathrm{I}_2^*,\mathrm{I}_1$ |       |
| d5  | $0  -1-\varphi  0$                                   | $-349 + 206\varphi$    | $2929 - 1833\varphi$                      | 0                  | 2    | + -   | 11, 8 | 8  | 2,2  | $\mathrm{I}_3^*,\mathrm{I}_8$ |       |
| d6  | $\begin{vmatrix} 0 & 1+\varphi & 0 \end{vmatrix}$    | $-4480 - 7252\varphi$  | $-223596 - 361788\varphi$                 | 0                  | 2    | - +   | 11, 2 | 2  | 4,2  | $I_3^*, I_2$                  |       |
| e1  | $\begin{vmatrix} 0 & 1+\varphi & 0 \end{vmatrix}$    | $1+\varphi$            | 0   | 0                  | 2    | Ī — — | 4, 1  | 1  | 1,1  | $\mathrm{II},\mathrm{I}_1$    |       |
| e2  | $0  1+\varphi  0$                                    | $-4-4\varphi$          | $-8-12\varphi$                            | 0                  | 4    | ++    | 8, 2  | 2  | 2,2  | $\mathrm{I}_0^*,\mathrm{I}_2$ |       |
| e3  | 0 1 0  | $-108 + 64\varphi$     | $-536 + 332\varphi$                       | 0                  | 4    | ++    | 10, 4 | 4  | 4, 2 | $\mathrm{I}_2^*,\mathrm{I}_4$ |       |
| e4  | $0  1+\varphi  0$                                    | $-64-84\varphi$        | $-344 - 580\varphi$                       | 0                  | 2    | ++    | 10, 1 | 1  | 2,1  | $\mathrm{I}_2^*,\mathrm{I}_1$ |       |
| e5  | $0  1+\varphi  0$                                    | $-144-204\varphi$      | $952 + 1628\varphi$                       | 0                  | 2    | - +   | 11, 8 | 8  | 2,2  | $I_3^*, I_8$                  |       |
| e6  | $0  -1-\varphi  0$                                   | $-11733 + 7254\varphi$ | $-573651 + 354535\varphi$                 | 0                  | 2    | + -   | 11, 2 | 2  | 4, 2 | $\mathrm{I}_3^*,\mathrm{I}_2$ |       |

|  |  | $a_1$ | $a_2$ $a_3$ | $a_{4}$ | $a_6$ | r | T | s ( | $\operatorname{ord}(\Delta)$ | $ord_{-}(i)$ | $c_n$ | Kodaira | Isogenies |
|--|--|-------|-------------|---------|-------|---|---|-----|------------------------------|--------------|-------|---------|-----------|
|--|--|-------|-------------|---------|-------|---|---|-----|------------------------------|--------------|-------|---------|-----------|

| 128 | 0a |                |   |                         | $1280a = (16 - 32\varphi) = 2^4 \cdot 5a$ | (12 isogeny cla | sses | ()       |     |       |    |                        |  | 1280a |
|-----|----|----------------|---|-------------------------|---|-----------------|------|----------|-----|-------|----|------------------------|--|-------|
| f1  | 0  | 0              | 0 | $-2-2\varphi$           | $-1-2\varphi$                             | 0               | 4    |          | ++  | 4, 2  | 2  | 1, 2                   | $II, I_2$  |       |
| f2  | 0  | 0              | 0 | $-7-7\varphi$           | $6+12\varphi$                             | 0               | 4    |          | ++  | 8, 4  | 4  | 2,2                    | $I_0^*, I_4$                                     |       |
| f3  | 0  | 0              | 0 | $-59 + 37\varphi$       | $206-128\varphi$                          | 0               | 2    | 2        | + - | 8, 1  | 1  | 1,1                    | $I_0^*, I_1$                                     |       |
| f4  | 0  | 0              | 0 | $-107 - 107\varphi$     | $426 + 852\varphi$                        | 0               | 4    | Į.       | ++  | 10, 2 | 2  | 4, 2                   | $I_2^*, I_2$                                     |       |
| f5  | 0  | 0              | 0 | $-22-37\varphi$         | $-78 - 128\varphi$                        | 0               | 2    | ?        | - + | 8, 1  | 1  | 2,1                    | $I_0^*, I_1$                                     |       |
| f6  | 0  | 0              | 0 | $-8495 - 13736\varphi$  | $572810 + 926836\varphi$                  | 0               | 2    | ?        | -+  | 11, 1 | 1  | 4,1                    | $I_3^*, I_1$                                     |       |
| f7  | 0  | 0              | 0 | $13 + 13\varphi$        | $34 + 68\varphi$                          | 0               | 2    | ?        |     | 10, 8 | 8  | 2,2                    | $	ext{I}_2^*, 	ext{I}_8$                         |       |
| f8  | 0  | 0              | 0 | $-22231 + 13736\varphi$ | $-1499646 + 926836\varphi$                | 0               | 2    | :        | + - | 11, 1 | 1  | 2,1                    | $\overline{\mathrm{I}_{3}^{st}}, \mathrm{I}_{1}$ |       |
| g1  | 0  | 0              | 0 | -2-2arphi               | $1+2\varphi$                              | 0               | 4    |          | ++  | 4, 2  | 2  | 1,2                    | $II, I_2$  |       |
| g2  | 0  | 0              | 0 | $-7-7\varphi$           | $-6-12\varphi$                            | 0               | 4    | Ŀ        | ++  | 8, 4  | 4  | 2,2                    | $\mathrm{I}_0^*,\mathrm{I}_4$                    |       |
| g3  | 0  | 0              |   | $-22-37\varphi$         | $78 + 128\varphi$                         | 0               | 2    | ?        | - + | 8, 1  | 1  | 1,1                    | $\mathrm{I}_0^*,\mathrm{I}_1$                    |       |
| g4  | 0  | 0              | 0 | $-107 - 107\varphi$     | $-426 - 852\varphi$                       | 0               | 4    | Ŀ        | ++  | 10, 2 | 2  | 4, 2                   | $\mathrm{I}_2^*,\mathrm{I}_2$                    |       |
| g5  | 0  | 0              | - | $-59 + 37\varphi$       | $-206 + 128\varphi$                       | 0               | 2    |          | + - | 8, 1  | 1  | 2,1                    | $\mathrm{I}_0^*,\mathrm{I}_1$                    |       |
| g6  | 0  | 0              | - | $-22231 + 13736\varphi$ | $1499646 - 926836\varphi$                 | 0               | 2    | 2        | + - | 11, 1 | 1  | 4,1                    | $\mathrm{I}_3^*,\mathrm{I}_1$                    |       |
| g7  | 0  | 0              | 0 | $13 + 13\varphi$        | $-34-68\varphi$                           | 0               | 2    |          |     | 10, 8 | 8  | 2,2                    | $\mathrm{I}_2^*,\mathrm{I}_8$                    |       |
| g8  | 0  | 0              | 0 | $-8495 - 13736\varphi$  | $-572810 - 926836\varphi$                 | 0               | 2    | <u> </u> | _ + | 11,1  | 1  | $\lfloor 2, 1 \rfloor$ | $I_3^*, I_1$                                     |       |
| h1  | 0  | 0              | - | -107                    | 426                                       | 1               | 8    | 3        | ++  | 10, 2 | 2  | 4,2                    | $\mathrm{I}_2^*,\mathrm{I}_2$                    |       |
| h2  | 0  | 0              |   | -7                      | 6   | 1               | 8    | 3        | ++  | 8, 4  | 4  | 4, 4                   | $\mathrm{I}_0^*,\mathrm{I}_4$                    |       |
| h3  | 0  | 0              |   | -2                      | -1  | 1               | 4    |          | ++  | 4, 2  | 2  | 1, 2                   | $II, I_2$  |       |
| h4  | 0  | 0              | - | $-3254 - 5241\varphi$   | $135242 + 218784\varphi$                  | 1               | 4    | Ŀ        | -+  | 11, 1 | 1  | 2, 1                   | $\mathrm{I}_3^*,\mathrm{I}_1$                    |       |
| h5  | 0  | 0              |   | $-58198 + 35967\varphi$ | $6352610 - 3926128\varphi$                | 1               | 4    | Ŀ        | + - | 11, 1 | 1  | 2,1                    | $\mathrm{I}_3^*,\mathrm{I}_1$                    |       |
| h6  | 0  | 0              | - | $-7-15\varphi$          | $-22-28\varphi$                           | 1               | 2    |          | -+  | 8, 1  | 1  | 1, 1                   | $\mathrm{I}_0^*,\mathrm{I}_1$                    |       |
| h7  | 0  | 0              | - | $-155 + 96\varphi$      | $-874 + 540\varphi$                       | 1               | 2    |          | + - | 8, 1  | 1  | 1, 1                   | $\mathrm{I}_0^*,\mathrm{I}_1$                    |       |
| h8  | 0  | 0              | 0 | 13                      | 34  | 1               | 4    |          |     | 10,8  | 8  | 4,8                    | $I_{2}^{*}, I_{8}$                               |       |
| i1  | 0  | $1-\varphi$    | 0 | $-24-20\varphi$         | $52 + 76\varphi$                          | 0               | 8    | 3        | ++  | 10, 4 | 4  | 4,4                    | $\mathrm{I}_2^*,\mathrm{I}_4$                    |       |
| i2  | 0  | $1-\varphi$    | 0 | -4                      | $-4+4\varphi$                             | 0               | 4    |          | ++  | 8, 2  | 2  | 2, 2                   | $\mathrm{I}_0^*,\mathrm{I}_2$                    |       |
| i3  | 0  | $1-\varphi$    | 0 | 1                       | 0   | 0               | 2    | ?        |     | 4, 1  | 1  | 1, 1                   | $\mathrm{II},\mathrm{I}_1$                       |       |
| i4  | 0  | $-\varphi$     | 0 | $-1708 - 2772\varphi$   | $52788 + 85404\varphi$                    | 0               | 4    |          | -+  | 11, 2 | 2  | 4, 2                   | $\mathrm{I}_3^*,\mathrm{I}_2$                    |       |
| i5  | 0  | r              | 0 | $-144 + 60\varphi$      | $676-276\varphi$                          | 0               | 8    |          | + - | 11, 8 | 8  | 4,8                    | $\mathrm{I}_3^*,\mathrm{I}_8$                    |       |
| i6  | 0  | $\varphi$      | 0 | $-384 + 235\varphi$     | $-3315 + 2045\varphi$                     | 0               | 2    | ?        | ++  | 10, 1 | 1  | 4,1                    | $\mathrm{I}_2^*,\mathrm{I}_1$                    |       |
| j1  | 0  | $\varphi$      | 0 | $-16-21\varphi$         | $33 + 49\varphi$                          | 0               | 4    |          | ++  | 8,1   | 1  | [2,1]                  | $I_0^*, I_1$                                     |       |
| j2  | 0  | $\varphi$      | 0 | $-1-\varphi$            | 0   | 0               | 4    |          | ++  | 4, 2  | 2  | 1,2                    | $II, I_2$  |       |
| j3  | 0  | $\varphi$      | 0 | $-41-41\varphi$         | $-116 - 232\varphi$                       | 0               | 4    | Ŀ        | ++  | 4,6   | 6  | 1,6                    | $II, I_6$  |       |
| j4  | 0  | $-\varphi$     | 0 | $-8587 + 5306\varphi$   | $358254 - 221413\varphi$                  | 0               | 4    |          | ++  | 8,3   | 3  | 2,3                    | $I_0^*, I_3$                                     |       |
| j5  |    | $-1 + \varphi$ | 0 | $-37 + 21\varphi$       | $-82 + 49\varphi$                         | 0               | 2    |          | ++  | 8, 1  | 1  | 2, 1                   | $\mathrm{I}_0^*,\mathrm{I}_1$                    |       |
| j6  | 0  | $\varphi$      | 0 | $4+4\varphi$            | $4+8\varphi$                              | 0               | 4    | Ŀ        |     | 8, 4  | 4  | 2,4                    | $\mathrm{I}_0^*,\mathrm{I}_4$                    |       |
| j7  | 0  | r              | 0 | $-36-36\varphi$         | $-140 - 280\varphi$                       | 0               | 4    | - 1      |     | 8, 12 | 12 | 2, 12                  | $\mathrm{I}_0^*, \mathrm{I}_{12}$                |       |
| j8  | 0  | $1-\varphi$    | 0 | $-3281 - 5306\varphi$   | $-136841 - 221413\varphi$                 | 0               | 2    | ?        | ++  | 8,3   | 3  | 2,3                    | $I_0^*, I_3$                                     |       |

|          |                         |                           |               |   |   | 1                                      | 1             | T  |  | 1             | Г   |           |
|----------|-------------------------|---------------------------|---------------|---|---|--|---------------|--|--|---------------|---|-----------|
|          | $a_1$                   | $a_2$                     | $a_3$         | $a_4$                                   | $a_6$   | r                                      | T             | $s \operatorname{ord}(\Delta)$                     | $\operatorname{ord}_{-}(j)$            | $c_p$         | Kodaira   | Isogenies |
| 1280     | 0a                      |                           |               | 12                                      | $280a = (16 - 32\varphi) = 2^4 \cdot 5a$          | (12 isogeny clas                       | sses)         |  |  |               |   | 1280a     |
| k1       | 0                       | $1-\varphi$               | 0             | $-37 + 21\varphi$                       | $82-49\varphi$                                    | 0                                      | 4             | ++ 8,1   | 1                                      | 2, 1          | $I_0^*, I_1$  |           |
| k2       | 0                       | $-\varphi$                | 0             | $-1-\varphi$                            | 0   | 0                                      | 4             | + + 4, 2   | 2                                      | 1,2           | $II, I_2$   |           |
| k3       | 0                       | $-\varphi$                | 0             | $-41-41\varphi$                         | $116 + 232\varphi$                                | 0                                      | 4             | ++4,6  | 6                                      | 1,6           | $II, I_6$   |           |
| k4       | 0                       | $-1+\varphi$              | 0             | $-3281 - 5306\varphi$                   | $136841 + 221413\varphi$                          | 0                                      | 4             | + + 8,3  | 3                                      | 2,3           | $I_0^*, I_3$  |           |
| k5       | 0                       | $-\varphi$                | 0             | $-16-21\varphi$                         | $-33-49\varphi$                                   | 0                                      | 2             | + + 8,1  | 1                                      | 2,1           | $I_0^*, I_1$  |           |
| k6       | 0                       | $-\varphi$                | 0             | $4+4\varphi$                            | $-4-8\varphi$                                     | 0                                      | 4             | 8,4  | 4                                      | 2,4           | $I_0^*, I_4$  |           |
| k7       | 0                       | $-\varphi$                | 0             | $-36-36\varphi$                         | $140 + 280\varphi$                                | 0                                      | 4             | 8, 12  | 12                                     | 2,12          | $I_0^*, I_{12}$   |           |
| k8       | 0                       | $\varphi$                 | 0             | $-8587 + 5306\varphi$                   | $-358254 + 221413\varphi$                         | 0                                      | 2             | + + 8,3  | 3                                      | 2,3           | $I_0^*, I_3$  |           |
| 11       | 0                       | $\varphi$                 | 0             | $-44 + 20\varphi$                       | $128-76\varphi$                                   | 0                                      | 8             | + + 10,4   | 4                                      | 4,4           | $I_{2}^{*}, I_{4}$                                      | ī         |
| 12       | 0                       | $\varphi$                 | 0             | -4                                      | -4arphi   | 0                                      | 4             | ++8,2  | 2                                      | 2,2           | $\overline{\mathrm{I}_0^*}, \mathrm{I}_2$               |           |
| 13       | 0                       | $\varphi$                 | 0             | 1                                       | 0   | 0                                      | 2             | 4,1  | 1                                      | 1, 1          | $II, I_1$   |           |
| 14       | 0                       | $-1+\varphi$              | 0             | $-4480 + 2772\varphi$                   | $138192 - 85404\varphi$                           | 0                                      | 4             | +-11,2   | 2                                      | 4, 2          | $I_3^*, I_2$  |           |
| 15       | 0                       | $\varphi$                 | 0             | $-84-60\varphi$                         | $400 + 276\varphi$                                | 0                                      | 8             | -+11,8   | 8                                      | 4,8           | $I_3^*, I_8$  |           |
| 16       | 0                       | $1-\varphi$               | 0             | $-149-235\varphi$                       | $-1270 - 2045\varphi$                             | 0                                      | 2             | ++10,1   | 1                                      | 4, 1          | $I_2^*, I_1$  |           |
| 1289     | 9a                      |                           |               | 1                                       | $289a = (25 - 33\varphi) = 1289a$                 | (2 isogeny class                       | ses)          |  |  |               |   | 1289a     |
| a1       | $1+\varphi$             | -1 1                      | $1 + \varphi$ | $-2\varphi$                             | -arphi  | 1                                      | 1             | 1  | 1                                      | 1             | $I_1$   |           |
| b1       |                         | $-1-\varphi$              |               | $-1+\varphi$                            | · · · · · · · · · · · · · · · · · · ·             | <del> </del><br>  1                    | 2             | + + 1  | ¦ 1                                    | <u> </u>      | $\stackrel{\downarrow}{ }$ $\stackrel{\downarrow}{I_1}$ |           |
| b2       |                         | $-1-\varphi$ $-1-\varphi$ | 1             | $-6+6\varphi$                           | $-8+4\varphi$                                     |  | $\frac{2}{2}$ | $\begin{vmatrix} + + & 1 \\ + - & 2 \end{vmatrix}$ | 2                                      | $\frac{1}{2}$ | $I_2$   |           |
| 100      |                         |                           |               | ·                                       | ·   |  | l             |  |  |               |   | 12221     |
| 1289     | 9b                      |                           |               | -                                       | $1289b = (8 - 33\varphi) = 1289b $                | 2 isogeny class                        | es)           |  |  |               |   | 1289b     |
| a1       | $\varphi$               | $-\varphi$                | $\varphi$     | 0                                       | 0   | 1                                      | 1             | 1  | 1                                      | 1             | $I_1$   |           |
| b1       | 1                       | $1+\varphi$               | 0             | arphi                                   | 0   | 1                                      | 2             | ++ 1   | 1                                      | 1             | $\bar{I}_1$   | [         |
| b2       | 1                       | $1+\varphi$               | 0             | -4arphi                                 | $-4-9\varphi$                                     | 1                                      | 2             | -+ 2   | 2                                      | 2             | $I_2$   |           |
| 129      | 1a                      |                           |               | 1                                       | $291a = (35 + 2\varphi) = 1291a $                 | (2 isogeny class                       | es)           |  |  |               |   | 1291a     |
| a1       | 1                       | 0                         | $\varphi$     | $-\varphi$                              | -1  | 1                                      | 1             | -+ 1   | 1                                      | 1             | $I_1$   |           |
| b1       | $1+\varphi$             | -1                        | 0             | $-45 - 66\varphi$                       | $193 + 304\varphi$                                | 1                                      | 1             | 1  | 1                                      | 1             | $I_1$   |           |
| 129      | 1b                      |                           |               | :                                       | $1291b = (37 - 2\varphi) = 1291b $                | 2 isogeny class                        | es)           |  | '                                      |               |   | 1291b     |
| a1       | 1                       | 0 1                       | $1+\varphi$   | -1                                      | $-1-\varphi$                                      |  | 1             | + - 1  | 1                                      | 1             | $I_1$   |           |
| b1       | $\varphi$               | -arphi                    | 0             | $-111 + 66\varphi$                      | $497 - 304\varphi$                                | 1                                      | <br>  1       | 1  | ¦  <br>  1                             | 1             | $I_1$   | <u>-</u>  |
| 129      |                         | · ·                       |               | ·                                       | <u> </u>  | isogeny classes)                       |               |  |  |               |   | 1296a     |
| a1       | $\frac{\mathbf{oa}}{0}$ | 0                         | 0             | $-24 - 24\varphi$                       | $\frac{1250u - (30) - 2 \cdot 3}{57 + 76\varphi}$ | 0                                      | 1             | ++ 4,8   | 2                                      | 1,4           | $IV, I_2^*$   |           |
| a1<br>a2 | 0                       | 0                         | 0             | $-24 - 24\varphi$ $-294 - 429\varphi$   | $37 + 70\varphi$ $3270 + 5368\varphi$             |  | $\frac{4}{2}$ |  |  | 1, 4 $1, 2$   |   |           |
|          |                         | 0                         | 0             | $-294 - 429\varphi$ $-723 + 429\varphi$ | •   | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ |               | ++ 8,7   | 1                                      |               | $IV^*, I_1^*$ $IV^*$ $I^*$                              |           |
| a3       | $0 \\ 0$                | 0                         | 0             | · · · · · · · · · · · · · · · · · · ·   | $8638 - 5368\varphi$ $246 + 328\varphi$           | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\frac{2}{2}$ | ++ 8,7   | $\begin{vmatrix} 1 \\ 4 \end{vmatrix}$ | 1, 2          | $IV^*, I_1^*$ $IV^*, I_1^*$                             |           |
| a4       | U                       | U                         | U             | $21 + 21\varphi$                        | $240 + 328\varphi$                                | 0                                      |               | 8,10   | 4                                      | 1,4           | $IV^*, I_4^*$   |           |

|     | $a_1$         | $a_2$         | $a_3$         | $a_4$                 | $a_6$  |                  | T              | 8        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$      | Kodaira                                  | Isogenies |
|-----|---------------|---------------|---------------|-----------------------|--|------------------|----------------|----------|------------------------------|-----------------------------|------------|--|-----------|
|     | 1             |               |               |                       | ***0   | '                | 1-1            | 1 -      | (—)                          | 3-3-(3)                     | - <i>p</i> |  | 1 8       |
| 129 | 6a            |               |               |                       | $1296a = (36) = 2^2 \cdot 3^2$                 | (2 isogeny class | ses)           |          |                              |                             |            |  | 1296a     |
| b1  | 0             | 0             | 0             | -15                   | 22   | 0                | 6              | ++       | 8,3                          | 0                           | 3, 2       | $IV^*, III$                              |           |
| b2  | 0             | 0             | 0             | 0                     | 1  | 0                | 6              |          | 4, 3                         | 0                           | 3, 2       | IV, III                                  |           |
| b3  | 0             | 0             | 0             | -135                  | -594   | 0                | 2              | ++       | 8,9                          | 0                           | 3, 2       | $IV^*, III^*$                            |           |
| b4  | 0             | 0             | 0             | 0                     | -27  | 0                | 2              |          | 4,9                          | 0                           | 3, 2       | IV, III*                                 |           |
|     |               |               |               |                       |  |                  |                |          |                              |                             |            |  |           |
| 130 | 5a            |               |               | 130                   | $35a = (9 - 33\varphi) = 5a \cdot 3 \cdot 29b$ | (4 isogeny       | class          | ses)     |                              |                             |            |  | 1305      |
| a1  | φ             | -1            | $\varphi$     | -4                    | $3-\varphi$                                    | 1                | 2              | ++       | 1,1,3                        | 1, 1, 3                     | 1, 1, 3    | $I_1, I_1, I_3$                          |           |
| a2  | $\varphi$     | -1            | $\varphi$     | $6-5\varphi$          | $10-4\varphi$                                  | 1                | 2              |          | 2, 2, 6                      | 2, 2, 6                     | 2, 2, 6    | $\mathrm{I}_2,\mathrm{I}_2,\mathrm{I}_6$ |           |
| b1  | 1             | $-1+\varphi$  | 1             | $-1-\varphi$          | 0  | 1                | $\overline{2}$ | + +      | 1, 1, 1                      | 1,1,1                       | 1,1,1      | $ $ $I_1, I_1, I_1$                      |           |
| b2  |               | $-1+\varphi$  | 1             | $-6-6\varphi$         | $-8-12\varphi$                                 | 1                | 4              | ++       | -2, 2, 2                     | 2, 2, 2                     | 2, 2, 2    | $I_2, I_2, I_2$                          |           |
| b3  | 1             | $-1+\varphi$  | 1             | $-66-111\varphi$      | $-434 - 690\varphi$                            | 1                | 2              | - +      | 4, 1, 1                      | 4, 1, 1                     | 4, 1, 1    | $I_4, I_1, I_1$                          |           |
| b4  | $1+\varphi$   | $\varphi$     | $\varphi$     | $-187 + 117\varphi$   | $-1129 + 696\varphi$                           | 1                | 2              | + -      | 1, 4, 4                      | 1,4,4                       | 1, 2, 2    | $\mid I_1, I_4, I_4$                     |           |
| c1  | $1+\varphi$   | $-1+\varphi$  | 0             | $-16+10\varphi$       | $39-25\varphi$                                 | 1                | 6              | + +      | -1, 3, 1                     | 1,3,1                       | [1, 3, 1]  | $  I_1, I_3, I_1  $                      |           |
| c2  | $1 + \varphi$ | $-1+\varphi$  | 0             | $-11 + 10\varphi$     | $51 - 32\varphi$                               | 1                | 6              |          | , - ,                        | 2, 6, 2                     | 2, 6, 2    | $I_2, I_6, I_2$                          |           |
| c3  | 1             | 1             | $\varphi$     | $-245 - 379\varphi$   | $-2785 - 4478\varphi$                          | 1                | 2              | ++       | , ,                          | 3, 1, 3                     | 1, 1, 1    | $I_3, I_1, I_3$                          |           |
| c4  | 1             | 1             | $\varphi$     | $-220 - 394\varphi$   | $-2823 - 4465\varphi$                          | 1                | 2              | _        | 6, 2, 6                      | 6, 2, 6                     | 2, 2, 2    | $\mid I_6, I_2, I_6$                     |           |
| d1  | $1 + \varphi$ | $1 + \varphi$ | 0             | $-3760 + 2320\varphi$ | $103727 - 64112\varphi$                        | 0                | 2              | ++       | 4, 1, 3                      | 4, 1, 3                     | 2, 1, 1    | $I_4, I_1, I_3$                          |           |
| d2  | 1             | $-\varphi$    | 1             | $-47 + \varphi$       | $54 - 93\varphi$                               | 0                | 4              | ++       | , ,                          | 2, 2, 6                     | 2, 2, 2    | $I_2, I_2, I_6$                          |           |
| d3  | 1             | $-\varphi$    | 1             | $-2+\varphi$          | $-3\varphi$                                    | 0                | 2              |          | , , -                        | 1, 4, 3                     | 1, 2, 1    | $I_1, I_4, I_3$                          |           |
| d4  | $\varphi$     | $-1+\varphi$  | φ             | $-1381 - 2222\varphi$ | $-37914 - 61359\varphi$                        | 0                | 2              | + +      | 1, 1, 12                     | 1, 1, 12                    | 1, 1, 2    | $I_1, I_1, I_{12}$                       |           |
|     |               |               |               |                       |  |                  |                |          |                              |                             |            |  |           |
| 130 | 5b            |               |               | 130                   | $5b = (24 - 33\varphi) = 5a \cdot 3 \cdot 29a$ | (4 isogeny       | clas           | ses)     |                              |                             |            |  | 1305      |
| a1  | $1 + \varphi$ | $-1-\varphi$  | $1 + \varphi$ | $-4-2\varphi$         | 2  | 1                | 2              | ++       | , ,                          | 1, 1, 3                     | 1, 1, 3    | $I_1, I_1, I_3$                          |           |
| a2  | $1+\varphi$   | $-1-\varphi$  | $1+\varphi$   | $1+3\varphi$          | $6+3\varphi$                                   | 1                | 2              | _        | 2, 2, 6                      | 2, 2, 6                     | 2, 2, 6    | $\mid I_2, I_2, I_6$                     |           |
| b1  | 1             | $-\varphi$    | 1             | $-2+\varphi$          | 0  | 1                | 2              | + +      | -1, 1, 1                     | 1, 1, 1                     | 1, 1, 1    | $\mid \ I_1,I_1,I_1$                     |           |
| b2  | 1             | $-\varphi$    | 1             | $-12+6\varphi$        | $-20+12\varphi$                                | 1                | 4              | ++       |                              | 2, 2, 2                     | 2, 2, 2    | $\mathrm{I}_2,\mathrm{I}_2,\mathrm{I}_2$ |           |
| b3  | 1             | $-\varphi$    | 1             | $-177 + 111\varphi$   | $-1124 + 690\varphi$                           | 1                | 2              | + -      |                              | 4, 1, 1                     | 4, 1, 1    | $I_4, I_1, I_1$                          |           |
| b4  | $\varphi$     | $-1+\varphi$  | $\varphi$     | $-71 - 117\varphi$    | $-479 - 767\varphi$                            | 1                | 2              | - +      | 1, 4, 4                      | 1,4,4                       | [1, 2, 2]  | $I_1, I_4, I_4$                          |           |
| c1  | $\varphi$     | $1+\varphi$   |               | $-7-9\varphi$         | $4+8\varphi$                                   | 1                | 6              | + +      | -1, 3, 1                     | 1, 3, 1                     | 1, 3, 1    | $I_1, I_3, I_1$                          |           |
| c2  | $\varphi$     | $1+\varphi$   |               | $-2-9\varphi$         | $9+20\varphi$                                  | 1                | 6              |          | , - ,                        | 2, 6, 2                     | 2, 6, 2    | $I_2, I_6, I_2$                          |           |
| c3  | 1             |               | $1+\varphi$   | $-624 + 378\varphi$   | $-7263 + 4477\varphi$                          | 1                | 2              | ++       | , ,                          | 3, 1, 3                     | 1, 1, 1    | $I_3, I_1, I_3$                          |           |
| c4  | 1             |               | $1+\varphi$   | $-614 + 393\varphi$   | $-7288 + 4464\varphi$                          | 1                | 2              | <u> </u> |                              | 6,2,6                       | 2,2,2      | $I_6, I_2, I_6$                          | <u> </u>  |
| d1  | $\varphi$     |               | 1             | $-1444 - 2317\varphi$ | $38740 + 62669\varphi$                         | 0                | 2              | 1        | 4, 1, 3                      | 4, 1, 3                     | 2,1,1      | $I_4, I_1, I_3$                          |           |
| d2  |               | $-1+\varphi$  | 1             | $-46-\varphi$         | $-39 + 93\varphi$                              | 0                | 4              |          | 2, 2, 6                      | 2, 2, 6                     | 2, 2, 2    | $I_2, I_2, I_6$                          |           |
| d3  |               | $-1+\varphi$  | 1             | $-1-\varphi$          | $-3+3\varphi$                                  | 0                | 2              |          | 1,4,3                        | 1,4,3                       | 1, 2, 1    | $I_1, I_4, I_3$                          |           |
| d4  | $1+\varphi$   | $\varphi$     | $\varphi$     | $-3602 + 2222\varphi$ | $-97051 + 59978\varphi$                        | 0                | 2              | + +      | -1, 1, 12                    | 1, 1, 12                    | 1, 1, 2    | $I_1, I_1, I_{12}$                       |           |

|          | $a_1$                                 | $a_2$                              | $a_3$              | $a_4$                                    | $a_6$                                       |  | T                                       | s            | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$               | $c_p$  | Kodaira   | Isogenies |
|----------|---------------------------------------|------------------------------------|--------------------|--|---|--|---|--------------|------------------------------|---|--|---|-----------|
| 1319     | 9a                                    |                                    |                    | 13                                       | $19a = (10 - 33\varphi) = 1319a$            | (1 isogeny class                       | ;)                                      |              |                              | 1   |  |   | 1319a     |
| a1       | $1+\varphi$                           |                                    | 0                  | $\varphi$                                | 0   | "                                      | 2                                       | +-           | 1                            | 1   | 1  | $I_1$   |           |
|          | $1+\varphi$                           | $-1+\varphi$                       | 0                  | $-4\varphi$                              | $-7-5\varphi$                               | 0                                      | 2                                       | -+           | 2                            | 2   | 2  | $I_2$   |           |
| 1319     | 9b                                    |                                    |                    |  |   | (1 isogeny class                       |   | 1            |                              | 1   | ı  |   | 1319b     |
| a1<br>a2 | $arphi \ arphi$                       | $1 + \varphi  1 \\ 1 + \varphi  1$ |                    | $0\\-5+5\varphi$                         | $-1 - \varphi$ $-8 + 4\varphi$              | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{array}{c c} 2 \\ 2 \end{array}$ | - +<br>  + - | $\frac{1}{2}$                | $\begin{array}{c c} 1 \\ 2 \end{array}$   | $\begin{array}{ c c }\hline 1\\ 2 \end{array}$ | $egin{array}{c} I_1 \ I_2 \end{array}$                |           |
| 132      |                                       | · r                                | . ,                | ·  | $321a = (38 - 3\varphi) = 1321a $           |  |   |              |                              |   |  |   | 1321a     |
| a1       | 0                                     | $\varphi$                          | $\varphi$          | $-9 + 7\varphi$                          | $-12 + 7\varphi$                            |  |   |              | 1                            | 1   | 1  | $I_1$   |           |
| 132      | 1b                                    |                                    |                    | 1:                                       | $321b = (35 + 3\varphi) = 1321b $           | 1 isogeny class)                       |   |              |                              | 1   |  | 1   | 1321b     |
| a1       | 0                                     | $1-\varphi$ 1                      | $1+\varphi$        | $-2-7\varphi$                            | $-5-8\varphi$                               |  | 1                                       |              | 1                            | 1   | 1  | $I_1$   |           |
| 132      | 4a                                    |                                    |                    | 132                                      | $4a = (40 - 6\varphi) = 2 \cdot 331a $      | 3 isogeny classe                       | es)                                     | •            |                              | •   |  | •   | 1324a     |
| a1       | 1                                     | -1 1                               | $1+\varphi$        | $-606 + 371\varphi$                      | $-6572 + 4063\varphi$                       | 0                                      | 1                                       | -+           | 7, 1                         | 7,1                                       | 1,1  | $I_7, I_1$  |           |
| b1       | 1                                     | 0                                  | 1                  | $-3+\varphi$                             | $3-2\varphi$                                |  | 1                                       | -+           | 5, 1                         | 5,1                                       | 5,1  | $\overline{I}_5,\overline{I}_1$                       |           |
| c1       | 1                                     | $1-\varphi$ 1                      |                    | $-1-3\varphi$                            | $1+\varphi$                                 |  |   | +            | 1,1                          | 1,1                                       | 1,1  | $\overline{\mathrm{I}}_{1},\overline{\mathrm{I}}_{1}$ |           |
| c2       | 1                                     | $1-\varphi$ 1                      |                    | $-6-13\varphi$                           | $-17 - 18\varphi$                           |  | 3<br>1                                  | -+           | 3, 3                         | 3, 3                                      | $\begin{bmatrix} 3, 3 \\ 0 \end{bmatrix}$      | $I_3, I_3$  |           |
| c3       | φ                                     | $-\varphi$ 1                       | $1+\varphi$        | $-3919 - 6285\varphi$                    | $-176675 - 285724\varphi$                   |  |   | -+           | 9,1                          | 9,1                                       | 9,1  | $I_9, I_1$  | 10041     |
| 132      |                                       |                                    |                    |  |   | 3 isogeny classe                       |   | ı            |                              | T   | I  |   | 1324b     |
| a1       | 1                                     | -1<br>                             | $\varphi$          | $-234 - 372\varphi$                      |   | 0                                      |   | + -          | 7, 1                         | 7,1                                       | 1,1  | $I_7, I_1$  |           |
| b1       | 1                                     | 0                                  | 1                  | $-2-\varphi$                             | $1+2\varphi$                                | 1                                      | ' :                                     | + -          | 5,1                          | 5,1                                       | 5,1  | $I_5, I_1$  |           |
| c1       | 1                                     | $\varphi$                          | $\varphi$          | $-3+2\varphi$                            | $3-2\varphi$                                |  | $\begin{vmatrix} 3 \\ 0 \end{vmatrix}$  | +-           | 1,1                          | 1,1                                       | $\begin{bmatrix} 1,1\\ 2,2 \end{bmatrix}$      | $I_1, I_1$  |           |
| c2<br>c3 | $1 \\ 1 + \varphi$                    | $arphi \ -1$                       | $\varphi$          | $-18 + 12\varphi$ $-10203 + 6283\varphi$ | $-34 + 17\varphi$ $-462398 + 285723\varphi$ | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 3                                       | +-           | 3, 3<br>9, 1                 | 3, 3<br>9, 1                              | $\begin{bmatrix} 3, 3 \\ 9, 1 \end{bmatrix}$   | $\begin{matrix} I_3,I_3\\I_9,I_1\end{matrix}$         |           |
| 133      | · · · · · · · · · · · · · · · · · · · |                                    | Ψ                  | · .                                      | $c = (11 - 33\varphi) = 11a^2 \cdot 11b$    |  |   | '            | J, 1                         | 0,1                                       | 0,1  | 19,11   | 1331c     |
| a1       |                                       | $-1+\varphi$ 1                     | 1 + 10             | $-9 - 15\varphi$                         | $\frac{12 + 19\varphi}{}$                   | ·                                      |   |              | 2,2                          | 2   | 2,1  | $I_2, II$   |           |
| b1       |                                       | $1+\varphi$ 1                      |                    | $2+\varphi$                              | $-1+2\varphi$                               |  | ' :                                     | !<br>        | $\frac{2}{2}, \frac{2}{4}$   | $\frac{1}{1} - \frac{2}{2} - \frac{2}{2}$ | $\begin{bmatrix} 2, 1 \\ 2, 3 \end{bmatrix}$   | $\overline{I_2,IV}$                                   |           |
| b2       |                                       | $1+\varphi$ 1                      |                    | $-83 + 36\varphi$                        | $-338+155\varphi$                           |  | 1                                       |              | 6, 4                         | 6   | 2, 1   | $I_6$ , IV  |           |
| c1       | 0                                     | $-\varphi$                         | 1                  | $2-\varphi$                              | $\varphi$                                   | 1                                      | 1                                       | <br>         | 3,3                          | 3   | 3,2  | $I_3, III$  |           |
| d1       | 0                                     | -1                                 | 1                  | $-3-\varphi$                             | $1-6\varphi$                                | <u>-</u><br>  1                        | 1                                       | <u> </u>     | 1,7                          | 1,1                                       | 1,4  | $ar{ar{I}}_1,ar{ar{I}}_1^*$                           |           |
| d2       | 0                                     | -1                                 | 1                  | $-103 - 31\varphi$                       | $11 + 844\varphi$                           | 1                                      | 1                                       |              | 5, 11                        | 5, 5                                      | 1,4  | $I_5, I_5^*$  |           |
| d3       | 0                                     | -1<br>                             | 1                  | $-78203 - 23461\varphi$                  | $-240119 + 9590534\varphi$                  | 1                                      | 1                                       | <u> </u>     | 1,7                          | 1,1                                       | 1,4  | $\frac{1}{1} \cdot \frac{I_1, I_1^*}{I_1 \cdot I_2}$  | <u> </u>  |
| e1       | 0                                     | $-\varphi$                         | 1                  | $13 - 12\varphi$                         | $-39 - 36\varphi$                           | 0                                      | 1                                       |              | 3,9                          | 3   | 1,2  | $I_3, III^*$  |           |
| f1       | $1+\varphi$                           | $-\varphi$                         | 0                  | $16+10\varphi$                           | $-43 + 17\varphi$                           |  | 1                                       |              | 2,10                         | 2   | $\begin{bmatrix} 2,1 \\ 6,1 \end{bmatrix}$     | $I_2, II^*$   |           |
| f2       | $1+\varphi$                           | $-\varphi$                         | · <del>-</del> - · | $-729 + 210\varphi$                      | $-6233 + 5904\varphi$                       | 0                                      | 1<br>                                   |              | $\frac{6,10}{2}$             | 6   | 6,1  | I <sub>6</sub> , II*                                  |           |
| g1       | $1+\varphi$                           | -1                                 | 1                  | $-39 - 19\varphi$                        | $-4 - 143\varphi$                           | 0                                      | 1                                       |              | 2, 8                         | 2   | 2,1  | $I_2, IV^*$   |           |

|          |                                      |  |                   |                                  |  |  |  | T          |                              |  | 1   |   |           |
|----------|--------------------------------------|--|-------------------|----------------------------------|--|--|--|------------|------------------------------|--|---|---|-----------|
|          | C                                    | $a_1$ $a_2$  | $a_3$             | $a_4$                            | $a_6$  | r                                      | T                                      | s          | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                  | $c_p$   | Kodaira   | Isogenies |
| 133      | 1d                                   |  |                   | 1331d                            | $= (22 - 33\varphi) = 11a \cdot 11b^2$           | (7 isogeny clas                        | sses)                                  |            |                              |  |   |   | 1331d     |
| a1       | 1 + 0                                | $\varphi$ $-1$   | $1+\varphi$       | $-5-\varphi$                     | $4-2\varphi$                                     | 1                                      | 1                                      |            | 2, 2                         | 2  | 1, 2  | $II, I_2$   |           |
| b1       | 1 + 6                                | $\varphi$ $-1+\varphi$                                   |                   | 3                                | -1   | 1                                      | 3                                      | <br>       | 4, 2                         | 2  | 3,2   | $\overline{\mathrm{IV}}, \overline{\mathrm{I}_2}$     | <u> </u>  |
| b2       | $1 + \epsilon$                       | $\varphi$ $-1+\varphi$                                   | 0                 | $-47 - 35\varphi$                | $-170 - 203\varphi$                              | 1                                      | 1                                      |            | 4,6                          | 6  | 1,2   | $IV, I_6$   |           |
| c1       |                                      | $0 -1 + \varphi$   | 1                 | $1+\varphi$                      | $1-\varphi$                                      | 1                                      | 1                                      |            | 3, 3                         | 3  | 2,3   | $ $ III, $I_3$  |           |
| d1       |                                      | 0 -1   | 1                 | $-4+\varphi$                     | $-5+6\varphi$                                    | 1                                      | 1                                      |            | 7,1                          | 1,1  | 4,1   | $oxed{I_1^*, I_1}$                                    | [         |
| d2       |                                      | 0 - 1  | 1                 | $-134 + 31\varphi$               | $855 - 844\varphi$                               | 1                                      | 1                                      |            | 11, 5                        | 5, 5   | 4, 1  | $I_5^*, I_5$  |           |
| d3       |                                      | 0 -1   | 1                 | $-101664 + 23461\varphi$         | $9350415 - 9590534\varphi$                       | 1                                      | 1                                      |            | 7,1                          | 1,1  | [4,1]   |   |           |
| e1       |                                      | $0 -1 + \varphi$   | 1                 | $1+12\varphi$                    | $-75 + 36\varphi$                                | 0                                      | 1                                      |            | 9,3                          | 3  | 2,1   | $III^*, I_3$  |           |
| f1       |                                      | $\varphi$ 0  | 0                 | $26-10\varphi$                   | $-26-17\varphi$                                  | 0                                      | 1                                      |            | ±0, <del>=</del>             | 2  | 1,2   | $\mathrm{II}^*,\mathrm{I}_2$                          |           |
| f2       |                                      | $\varphi = 0$  | 0                 | $-519 - 210\varphi$              | $-329 - 5904\varphi$                             | 0                                      | 1                                      |            | 10, 6                        | 6  | 1,6   | $II^*, I_6$   |           |
| g1       | (                                    | $\varphi$ $-\varphi$                                     | 1                 | $-57 + 18\varphi$                | $-147 + 143\varphi$                              | 0                                      | 1                                      |            | 8, 2                         | 2  | 1,2   | $IV^*, I_2$   |           |
| 101      | _                                    |  |                   |                                  |  |  |  |            |                              |  |   |   | 1015      |
| 134      |                                      |  |                   |                                  |  | 3 isogeny class                        | ses)                                   |            |                              |  |   | T   | 1345a     |
| a1       |                                      | $1 -1 - \varphi$   | $1+\varphi$       | $-3+\varphi$                     | 2-arphi  | 1                                      | 1                                      |            | 1, 1                         | 1,1  | 1,1   | $I_1, I_1$  |           |
| b1       | 1 + 6                                | •  | 0                 | $-14 + 7\varphi$                 | $13 - 10\varphi$                                 |  | 4                                      | ++         | ,                            | 2,6  | 2,2   | $I_2, I_6$  |           |
| b2       | 1+                                   | •  | -                 | $-4+2\varphi$                    | $-6+\varphi$                                     | 0                                      | 2                                      | ++         | ,                            | 1,3  | 1,1   | $I_1, I_3$  |           |
| b3       | 1+                                   | •  | 0                 | $11 - 43\varphi$                 | $153 - 15\varphi$                                | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 2                                      | -+         | , -                          | 4, 3   | 2,1   | $I_4, I_3$  |           |
| b4       | $\begin{vmatrix} 1+ i \end{vmatrix}$ | <del>-</del>   | 0                 | $-199 + 137\varphi$              | $1269 - 769\varphi$                              | 0                                      | 2                                      | + -        | '                            | 1,12   | $\frac{1}{1}, \frac{1}{2}$                    | $I_1, I_{12}$   | [<br>     |
| c1       | (                                    | $\varphi$ $-1-\varphi$                                   | $\varphi$         | $2-\varphi$                      |  | 1                                      | 1                                      |            | 1,3                          | 1,3  | 1,3   | $I_1, I_3$  |           |
| 134      | 5b                                   |  |                   | 1345b                            | $\phi = (34 + 7\varphi) = 5a \cdot 269a \tag{9}$ | 3 isogeny class                        | ses)                                   |            |                              |  |   |   | 1345b     |
| a1       |                                      | $1  1+\varphi$   | $1+\varphi$       | -2                               | -arphi   | 1                                      | 1                                      |            | 1, 1                         | 1, 1   | 1,1   | $I_1, I_1$  |           |
| b1       | :                                    | $\varphi$ $-1-\varphi$                                   | <del>'</del><br>φ | $-9-6\varphi$                    | $11 + 16\varphi$                                 |  | '<br>  4                               | :<br>      | $\frac{1}{2}, \frac{1}{6}$   | $\frac{1}{2}, \frac{1}{6}$                   | [2, 2]  | $\overline{\mathrm{I}}_{2},\overline{\mathrm{I}}_{6}$ | <u>-</u>  |
| b2       |                                      | $\varphi$ $-1-\varphi$                                   | $\varphi$         | -4-arphi                         | -2   | 0                                      | 2                                      | ++         | 1,3                          | 1,3  | 1,1   | $I_1, I_3$  |           |
| b3       |                                      | $\varphi$ $-1-\varphi$                                   | $\varphi$         | $-34+44\varphi$                  | $171-29\varphi$                                  | 0                                      | 2                                      | + -        | ,                            | 4,3  | 2,1   | $I_4, I_3$  |           |
| b4       |                                      | $\varphi$ $-1-\varphi$                                   | $\varphi$         | $-64 - 136\varphi$               | $563 + 905\varphi$                               | 0                                      | 2                                      | - +        | 1,12                         | 1,12   | 1,2   | $I_1, I_{12}$   |           |
| c1       | $1 + \epsilon$                       | $\varphi$ 1  | 0                 | $2+2\varphi$                     | $1+\varphi$                                      | 1                                      | 1                                      |            | 1,3                          | 1,3  | 1,3   | $I_1, I_3$  |           |
| 134      | 0 <sub>2</sub>                       |  |                   | 1340a                            | $s = (20 - 33\varphi) = 19b \cdot 71b $          | 2 isogeny clas                         | eoe)                                   |            |                              |  |   |   | 1349a     |
| a1       |                                      | (a 1   (a  | 1 + 40            |                                  | $\frac{1}{10000000000000000000000000000000000$   |  | <u> </u>                               |            | 1 1                          | 1 1  | 1 1   | тт  | 10404     |
|          |                                      | $\varphi$ $-1+\varphi$                                   |                   | $-1 - 3\varphi$                  |  | 1                                      | 1<br>                                  |            |                              | 1,1  | 1,1   | $I_1, I_1$  |           |
| b1<br>b2 |                                      | $ \varphi  -1 - \varphi \\ \varphi  -1 - \varphi $       | 1<br>1            | $-2 - 5\varphi$ $-7 - 5\varphi$  | $5 + 9\varphi$ $1 + 5\varphi$                    | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | $\begin{vmatrix} 4 \\ 8 \end{vmatrix}$ | - +<br>+ + | ,                            | $ \begin{array}{c c} 2,1\\ 4,2 \end{array} $ | $\begin{array}{ c c } 2,1 \\ 4,2 \end{array}$ | $I_2, I_1$  |           |
| b3       | ,                                    | $\varphi$ $-1 - \varphi$ $\varphi$ $-1 - \varphi$        | 1                 | $-7 - 9\varphi$ $-2 + 40\varphi$ | $\frac{1+3\varphi}{27-45\varphi}$                | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | 4                                      | + +        | 8, 1                         | 8, 1   | 8,1   | $\begin{matrix} I_4, I_2 \\ I_8, I_1 \end{matrix}$    |           |
| b4       | ,                                    | $ \begin{array}{cccc}                                  $ | 1                 | $-312+177\varphi$                | $-2342 + 1421\varphi$                            | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | 4                                      | ++         |                              | 2, 4   | 2,2   | $I_{2}, I_{4}$  |           |
| b5       |                                      | 1 -1   | 1                 | $-4987 + 2832\varphi$            | $-153628 + 93075\varphi$                         | 1                                      | 2                                      | ++         |                              | 1, 2   | 1,2   | $I_1, I_2$  |           |
| b6       | (                                    | $\varphi$ $-1-\varphi$                                   | 1                 | $-67-20\varphi$                  | $-591-599\varphi$                                | 1                                      | 2                                      |            | 1,8                          | 1,8  | 1,2   | $I_1, I_8$  |           |

|      |               |              |                |                       |   |            |        |          |                              | T                           | T                          | T  |           |
|------|---------------|--------------|----------------|-----------------------|---|------------|--------|----------|------------------------------|-----------------------------|----------------------------|--|-----------|
|      | $a_1$         | $a_2$        | $a_3$          | $a_4$                 | $a_6$   | r          | T      | s        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$                      | Kodaira                                  | Isogenies |
| 1349 | 9b            |              |                | 13                    | $349b = (13 - 33\varphi) = 19a \cdot 71a$       | (2 isogeny | clas   | ses)     |                              |                             |                            |  | 1349b     |
| a1   | $1 + \varphi$ | $\varphi$    | $1 + \varphi$  | $-3+2\varphi$         | $3-2\varphi$                                    | 1          | 1      |          | 1, 1                         | 1,1                         | 1,1                        | $I_1, I_1$                               |           |
| b1   | $1+\varphi$   | 1            | $\varphi$      | $-7+5\varphi$         | 7-4arphi  | 1          | 4      | + -      | 2,1                          | 2,1                         | $\frac{1}{2}, \frac{1}{1}$ | $I_2, I_1$                               |           |
| b2   | $1+\varphi$   | 1            | $\varphi$      | $-12+5\varphi$        | -6  | 1          | 8      | ++       | 4, 2                         | 4, 2                        | 4, 2                       | $I_4, I_2$                               |           |
| b3   | $1 + \varphi$ | 1            | $\varphi$      | $38-40\varphi$        | $20+5\varphi$                                   | 1          | 4      | -+       | 8, 1                         | 8,1                         | 8, 1                       | $I_8, I_1$                               |           |
| b4   | 1             | -1           | 1              | $-135 - 177\varphi$   | $-921-1421\varphi$                              | 1          | 4      | ++       | 2,4                          | 2,4                         | 2, 2                       | $I_2, I_4$                               |           |
| b5   | 1             | -1           | 1              | $-2155 - 2832\varphi$ | $-60553 - 93075\varphi$                         |            | 2      | ++       | 1,2                          | 1,2                         | 1, 2                       | $I_1, I_2$                               |           |
| b6   | $1+\varphi$   | 1            | φ              | $-87 + 20\varphi$     | $-1277 + 619\varphi$                            | 1          | 2      |          | 1,8                          | 1,8                         | 1,2                        | $I_1, I_8$                               |           |
| 1359 |               |              |                | 1                     | $359a = (18 - 33\varphi) = 3 \cdot 151a$        | (2 isogeny | class  | ses)     |                              |                             |                            |  | 1359a     |
| a1   | 0             | $-1+\varphi$ | 1              | -1                    | 0   | 1          | 1      | + -      | 1,1                          | 1,1                         | 1, 1                       | $  I_1, I_1$                             |           |
| b1   | 0             | $1+\varphi$  | 1              | $-8+6\varphi$         | $9-5\varphi$                                    | 0          | 5      | + -      | 1,5                          | 1,5                         | 1,5                        | $I_1, I_5$                               |           |
| b2   | 0             | $1+\varphi$  | 1              | $-1268 + 756\varphi$  | $-20349 + 12451\varphi$                         | 0          | 1      | +-       | 5, 1                         | 5, 1                        | 5, 1                       | $I_5, I_1$                               |           |
| 1359 | 9b            |              |                | 1                     | $359b = (15 - 33\varphi) = 3 \cdot 151b$        | (2 isogeny | class  | es)      |                              |                             |                            |  | 1359b     |
| a1   | 0             | $-\varphi$   | 1              | -1                    | 0   | 1          | 1      | -+       | 1,1                          | 1,1                         | 1,1                        | $I_1, I_1$                               |           |
| b1   | 0             | $-1-\varphi$ | <sub>1</sub> - | $-3-4\varphi$         | $7+10\varphi$                                   |            | 5      | <u> </u> | 1,5                          | 1,5                         | 1,5                        | $ I_1,I_5 $                              | <u></u>   |
| b2   |               | $-1-\varphi$ | 1              | $-513 - 754\varphi$   | $-7385-11696\varphi$                            | 0          | 1      | -+       | 5, 1                         | 5,1                         | 5, 1                       | $I_5, I_1$                               |           |
| 136  | 4a            |              |                | 13                    | $64a = (38 - 2\varphi) = 2 \cdot 11b \cdot 31b$ | (8 isogen  | y clas | sses)    |                              |                             |                            |  | 1364a     |
| a1   | $1 + \varphi$ | 1            | 0              | $1-\varphi$           | $-1+2\varphi$                                   | 0          | 5      |          | 5, 1, 1                      | 5, 1, 1                     | 5, 1, 1                    | $\mathrm{I}_5,\mathrm{I}_1,\mathrm{I}_1$ |           |
| a2   | $\varphi$     | $-1-\varphi$ | $\varphi$      | $-4414 + 2721\varphi$ | $-132131 + 81647\varphi$                        | 0          | 1      |          | 1, 5, 5                      | 1,5,5                       | 1,1,5                      | $I_1, I_5, I_5$                          |           |
| b1   | $1 + \varphi$ | $-\varphi$   | $\varphi$      | $-1-2\varphi$         | 0   | 1          | 1      | -+       | 1, 1, 1                      | 1, 1, 1                     | 1, 1, 1                    | $I_1,I_1,I_1$                            |           |
| c1   | $\varphi$     | $\varphi$    | 1              | $-3-2\varphi$         | 2-arphi   | 0          | 5      | Ī — +    | 5, 1, 1                      | [5, 1, 1]                   | 5, 1, 1                    | $  I_5, I_1, I_1$                        |           |
| c2   | $1 + \varphi$ | $1-\varphi$  | 0              | $-2848 - 4597\varphi$ | $-112042 - 181253\varphi$                       | 0          | 1      | -+       | 1, 5, 5                      | 1, 5, 5                     | 1, 1, 5                    | $I_1, I_5, I_5$                          |           |
| d1   | $1+\varphi$   | 1            | 1              | $-18-16\varphi$       | $-43-71\varphi$                                 | 0          | 2      | + -      | 2, 10, 1                     | [2, 10, 1]                  | [2, 2, 1]                  | $I_2, I_{10}, I_1$                       |           |
| d2   | $1+\varphi$   | 1            | 1              | $-178 - 336\varphi$   | $-2219 - 3655\varphi$                           | 0          | 2      | -+       | 4, 5, 2                      | 4, 5, 2                     | 4, 1, 2                    | $\mid I_4, I_5, I_2$                     |           |
| e1   | 1             | $\varphi$    | $\varphi$      | $-3-14\varphi$        | $15 + 12\varphi$                                | 1          | 3      | -+       | 3, 5, 1                      | 3, 5, 1                     | 3, 5, 1                    | $I_3, I_5, I_1$                          |           |
| e2   | 1             | $\varphi$    | $\varphi$      | $-58 - 14\varphi$     | $-276 + 3\varphi$                               | 1          | 1      | -+       | 1, 15, 3                     | 1,15,3                      | 1,15,3                     | $I_1, I_{15}, I_3$                       |           |
| f1   | 1             | $-\varphi$   | $1 + \varphi$  | $-11+4\varphi$        | $11-7\varphi$                                   | 1          | 1      | - +      | 1, 7, 1                      | 1, 7, 1                     | 1, 7, 1                    | $\mid \ I_1,I_7,I_1$                     |           |
| g1   | $\varphi$     | $1+\varphi$  |                | $2+\varphi$           | 1   | 0          | 3      | Ī — —    | 1, 3, 1                      | 1,3,1                       | 1, 3, 1                    | $I_1, I_3, I_1$                          |           |
| g2   | $1+\varphi$   | -1           | $\varphi$      | $-70-111\varphi$      | $-432-697\varphi$                               | 0          | 1      |          | 3, 1, 3                      | 3, 1, 3                     | 1, 1, 3                    | $I_3, I_1, I_3$                          |           |
| h1   | $1+\varphi$   | $-1+\varphi$ | $\varphi$      | $-5-5\varphi$         | $1+4\varphi$                                    | 0          | 3      | -+       | 1, 3, 1                      | 1, 3, 1                     | 1, 3, 1                    | $I_1, I_3, I_1$                          |           |
| h2   | $\varphi$     | $-\varphi$   | $1 + \varphi$  | $-1295 + 797\varphi$  | $-21363 + 13202\varphi$                         | 0          | 1      | -+       | 3, 1, 3                      | 3, 1, 3                     | 1, 1, 3                    | $I_3, I_1, I_3$                          |           |
| 136  | $4\mathrm{b}$ |              |                | 130                   | $64b = (36 + 2\varphi) = 2 \cdot 11a \cdot 31a$ | (8 isogen  | y cla  | sses)    |                              |                             |                            |  | 1364b     |
| a1   | $\varphi$     | $-1-\varphi$ | $\varphi$      | $-2+2\varphi$         | $2-4\varphi$                                    | 0          | 5      |          | 5, 1, 1                      | 5, 1, 1                     | 5, 1, 1                    | $I_5, I_1, I_1$                          |           |
| a2   | $1+\varphi$   | 1            | 0              | $-1692 - 2720\varphi$ | $-52176 - 84368\varphi$                         | 0          | 1      |          | 1, 5, 5                      | 1, 5, 5                     | 1, 1, 5                    | $I_1, I_5, I_5$                          |           |
| b1   | φ             | 0            | $1+\varphi$    | -2                    | $-\varphi$                                      | 1          | 1      | + -      | 1, 1, 1                      | 1,1,1                       | 1,1,1                      | $I_1,I_1,I_1$                            |           |

|                                     | $a_1$       | $a_2$              | $a_3$          | $a_4$                     | $a_6$  | r          | T      | s                 | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$  | Kodaira                                  | Isogenies     |
|-------------------------------------|-------------|--------------------|----------------|---------------------------|--|------------|--------|-------------------|------------------------------|-----------------------------|--|--|---------------|
| 1364                                | b           |                    |                | 1364                      | $b = (36 + 2\varphi) = 2 \cdot 11a \cdot 31a$  | (8 isogen  | y clas | sses)             |                              |                             |  |  | <b>1364</b> b |
| c1                                  | $1+\varphi$ | $1+\varphi$        | 0              | $-4+5\varphi$             | $4-\varphi$                                    | 0          | 5      | +-                | 5, 1, 1                      | 5, 1, 1                     | 5, 1, 1  | $I_5, I_1, I_1$                          |               |
| c2                                  | $\varphi$   | 1                  | 0              | $-7445 + 4597\varphi$     | $-293295 + 181253\varphi$                      | 0          | 1      | + -               | 1, 5, 5                      | 1, 5, 5                     | 1, 1, 5  | $I_1, I_5, I_5$                          |               |
| d1                                  | $\varphi$ - | $-1-\varphi$ 1     | $+\varphi$     | $-35+16\varphi$           | $-80 + 54\varphi$                              | 0          | 2      |                   | 2, 10, 1                     | 2, 10, 1                    | [2, 2, 1]  | $  I_2, I_{10}, I_1$                     |               |
| d2                                  | arphi -     | $-1-\varphi$ 1     | $+\varphi$     | $-515 + 336\varphi$       | $-5360 + 3318\varphi$                          | 0          | 2      | + -               | 4, 5, 2                      | 4, 5, 2                     | 4, 1, 2  | $I_4, I_5, I_2$                          |               |
| e1                                  | 1           | $1-\varphi$ 1      | $+\varphi$     | $-17+13\varphi$           | $27-13\varphi$                                 | 1          | 3      | + -               | 3, 5, 1                      | 3, 5, 1                     | [3, 5, 1]  | $  I_3, I_5, I_1$                        |               |
| e2                                  | 1           | $1-\varphi$ 1      | $+\varphi$     | $-72 + 13\varphi$         | $-273-4\varphi$                                | 1          | 1      | +-                | 1, 15, 3                     | 1, 15, 3                    | 1, 15, 3   | $I_1, I_{15}, I_3$                       |               |
| f1                                  | 1 -         | $-1+\varphi$       | $\varphi$      | $-6-5\varphi$             | 5+6arphi                                       | 1          | 1      | + -               | 1, 7, 1                      | 1,7,1                       | 1,7,1  | $ $ $I_1, I_7, I_1$                      |               |
| g1                                  | $1+\varphi$ | <u>·</u><br>_1 + φ | φ              | 1                         | arphi  |            |        | <u>-</u><br>      | 1, 3, 1                      | 1,3,1                       | 1, 3, 1  | $ I_1, I_3, I_1 $                        | <u>-</u>      |
| g2                                  | $\varphi$   | $-\varphi$ 1       |                | $-180 + 109\varphi$       | $-1129 + 696\varphi$                           | 0          | 1      |                   | 3, 1, 3                      | 3, 1, 3                     | 1, 1, 3  | $I_3, I_1, I_3$                          |               |
| h1                                  | . <i></i>   | $1+\varphi$        |                | $-9+6\varphi$             | $10-9\varphi$                                  | 0          |        |                   | 1, 3, 1                      | 1,3,1                       | $\begin{bmatrix} 1 & 1 & 3 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ | $ I_1, I_3, I_1 $                        | <u>-</u>      |
| 1                                   | $1+\varphi$ | -1                 | $\varphi$      | $-497 - 799\varphi$       | $-8160 - 13203\varphi$                         | 0          | 1      | + -               |                              | 3, 1, 3                     | 1, 1, 3  | $I_3, I_1, I_3$                          |               |
|                                     | · ·         |                    |                | ,                         | · .  |            |        | l                 |                              |                             |  |  |               |
| 1364                                | ·C          |                    |                | 1364                      | $4c = (34 + 8\varphi) = 2 \cdot 11a \cdot 31b$ | (4 isogeny | y clas | ses)              |                              |                             |  |  | 1364          |
| a1                                  | 1           | $-\varphi$         | 0              | $-29 + 17\varphi$         | $62-37\varphi$                                 | 1          | 2      | + -               | 2, 4, 1                      | 2, 4, 1                     | 2, 2, 1  | $I_2, I_4, I_1$                          |               |
| a2                                  | $\varphi$ - | $-1+\varphi$ 1     | $1+\varphi$    | $-68-105\varphi$          | $366 + 575\varphi$                             | 1          | 2      | -+                |                              | 4, 2, 2                     | 4, 2, 2  | $I_4, I_2, I_2$                          |               |
| b1                                  |             | $-1+\varphi$       | $\varphi$      | $-4-3\varphi$             | $3+2\varphi$                                   |            |        | <u>:</u>          | 2, 4, 1                      | [2, 4, 1]                   | [2, 4, 1]  | $  I_2, I_4, I_1  $                      | <u> </u>      |
| b2                                  | $\varphi$   | $\varphi$          | $\varphi$      | $-256-405\varphi$         | $2742 + 4427\varphi$                           | 1          | 2      | ++                | 1, 2, 2                      | 1, 2, 2                     | 1, 2, 2  | $I_1, I_2, I_2$                          |               |
| $\begin{bmatrix} c_1 \end{bmatrix}$ | $1+\varphi$ | $\varphi$          |                | $-12-2\varphi$            | $-24 - 20\varphi$                              | 0          | 4      | <u>-</u><br>  + - | 4, 4, 1                      | 4,4,1                       | [4, 4, 1]  | $  I_4, I_4, I_1  $                      | <u> </u>      |
| c2                                  |             | $-1+\varphi$       | 0              | $-874 + 512\varphi$       | $-11052 + 6744\varphi$                         | 0          | 4      | ++                | 2, 2, 2                      | 2, 2, 2                     | 2, 2, 2  | $I_2, I_2, I_2$                          |               |
| c3                                  |             | $1+\varphi$        | $\varphi$      | $-647903 + 400428\varphi$ | $-236063620 + 145895341\varphi$                | 0          | 2      | ++                | 1, 1, 1                      | 1, 1, 1                     | 1, 1, 1  | $I_1, I_1, I_1$                          |               |
| c4                                  | 1 -         | $-1+\varphi$ 1     | $+\varphi$     | $-9597 - 15488\varphi$    | $-689147 - 1115002\varphi$                     | 0          | 2      | -+                | 1, 1, 4                      | 1, 1, 4                     | 1, 1, 2  | $I_1, I_1, I_4$                          |               |
| d1                                  | 1           | $1-\varphi$ 1      | $+\varphi$     | $-3+3\varphi$             | $-1+\varphi$                                   | 0          | 6      | <br>  + _         | 2, 2, 3                      | [2, 2, 3]                   | 2, 2, 3  | $  I_2, I_2, I_3  $                      |               |
| d2                                  | 1           | $1-\varphi$ 1      |                | $-43 + 13\varphi$         | $-119 + 91\varphi$                             | 0          | 6      | ++                | 1, 1, 6                      | 1, 1, 6                     | 1, 1, 6  | $I_1, I_1, I_6$                          |               |
| d3                                  | 1           | $1-\varphi$ 1      | $+\varphi$     | $-198 + 103\varphi$       | $-1333 + 758\varphi$                           | 0          | 2      | + -               | 6, 6, 1                      | 6, 6, 1                     | 2, 6, 1  | $I_6, I_6, I_1$                          |               |
| d4                                  | $1+\varphi$ | 0                  | 1              | $-20864 + 12843\varphi$   | $-1361689 + 841360\varphi$                     | 0          | 2      | ++                | 3, 3, 2                      | 3, 3, 2                     | 1, 3, 2  | $I_3, I_3, I_2$                          |               |
| 1004                                |             |                    |                |                           |  |            |        |                   |                              |                             |  |  | 1001          |
| 1364                                | <b>d</b>    |                    |                | 1364                      | $d = (42 - 8\varphi) = 2 \cdot 11b \cdot 31a$  | (4 isogen  |        | ,                 |                              |                             |  |  | 13640         |
| a1                                  |             | $-1+\varphi$       | 0              | $-12-17\varphi$           | $25 + 37\varphi$                               |            | 2      | -+                |                              | 2, 4, 1                     |  | $I_2, I_4, I_1$                          |               |
| a2                                  | $1+\varphi$ | $\varphi$ 1        | $+\varphi$     | $-172 + 104\varphi$       | $1046 - 644\varphi$                            | 1          | 2      | + -               | 4, 2, 2                      | 4, 2, 2                     | 4, 2, 2  | $\mid I_4, I_2, I_2$                     |               |
| b1                                  | 1           | $-\varphi$ 1       | $+\varphi^{-}$ | $-7+2\varphi$             | 5-3arphi                                       | 1          | 2      | Ī — +             | 2, 4, 1                      | [2, 4, 1]                   | [2, 4, 1]  | $ $ $I_2, I_4, I_1$                      |               |
| b2                                  | $1+\varphi$ | $1+\varphi$        | $\varphi$      | $-660 + 407\varphi$       | $7575 - 4682\varphi$                           | 1          | 2      | ++                | 1, 2, 2                      | 1, 2, 2                     | 1, 2, 2  | $\mid I_1, I_2, I_2$                     |               |
| c1                                  | $\varphi$ - | $-1+\varphi$       | 1              | $-16+3\varphi$            | $-25+4\varphi$                                 | 0          | 4      | <u> </u>          | 4, 4, 1                      | 4,4,1                       | [4, 4, 1]  | $  I_4, I_4, I_1  $                      |               |
| c2                                  | $1+\varphi$ | $\varphi$          | 1              | $-362-511\varphi$         | $-4820 - 7618\varphi$                          | 0          | 4      | ++                | 2, 2, 2                      | 2, 2, 2                     | 2, 2, 2  | $\mathrm{I}_2,\mathrm{I}_2,\mathrm{I}_2$ |               |
| c3                                  | $\varphi$   | $\varphi$          | $\varphi$      | $-247478 - 400426\varphi$ | $-90321229 - 146142818\varphi$                 | 0          | 2      | ++                | 1, 1, 1                      | 1, 1, 1                     | 1, 1, 1  | $\mathrm{I}_1,\mathrm{I}_1,\mathrm{I}_1$ |               |
| c4                                  | 1           | $-\varphi$         | $\varphi$      | $-25084 + 15487\varphi$   | $-1804148 + 1115001\varphi$                    | 0          | 2      | + -               | 1, 1, 4                      | 1, 1, 4                     | 1, 1, 2  | $I_1,I_1,I_4$                            |               |

|     | $a_1$         | $a_2$        | $a_3$         | $a_4$                                 | $a_6$  |              | r    | T      | s   | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$   | Kodaira         | Isogenies |
|-----|---------------|--------------|---------------|---------------------------------------|--|--------------|------|--------|-----|------------------------------|-----------------------------|---------|-----------------|-----------|
| 100 | 4.1           |              |               |                                       |  |              |      |        |     |                              |                             |         |                 | 10041     |
| 136 | 4d            |              |               | 1364                                  | $4d = (42 - 8\varphi) = 2 \cdot 11b \cdot 31a$ | (4 isogen    | ус   | lasse  | s)  |                              |                             |         |                 | 1364d     |
| d1  | 1             | $\varphi$    | $\varphi$     | $1-4\varphi$                          | $1-2\varphi$                                   |              | 0    | 6      | -+  | 2, 2, 3                      | 2, 2, 3                     | 2, 2, 3 | $I_2, I_2, I_3$ |           |
| d2  | 1             | $\varphi$    | $\varphi$     | $-29-14\varphi$                       | $-27-92\varphi$                                | I            | 0    | 6      | ++  | 1, 1, 6                      | 1, 1, 6                     | 1, 1, 6 | $I_1, I_1, I_6$ |           |
| d3  | 1             | $\varphi$    | $\varphi$     | $-94-104\varphi$                      | $-574 - 759\varphi$                            | I            | 0    | 2      | -+  | 6, 6, 1                      | 6, 6, 1                     | 2, 6, 1 | $I_6, I_6, I_1$ |           |
| d4  | φ             | $1-\varphi$  | 1             | $-8020 - 12844\varphi$                | $-520329 - 841360\varphi$                      |              | 0    | 2      | ++  | 3, 3, 2                      | 3, 3, 2                     | 1, 3, 2 | $I_3, I_3, I_2$ |           |
| 136 | 9a            |              |               |                                       | 1369a = (37) = 37 (4 is                        | ogeny classe | es)  |        |     |                              |                             |         |                 | 1369a     |
| a1  | 0             | 1            | 1             | -3                                    | 1  |              | 1    | 3      | ++  | 1                            | 1                           | 1       | $I_1$           |           |
| a2  | 0             | 1            | 1             | -23                                   | -50  |              | 1    | 3      | ++  | 3                            | 3                           | 3       | $I_3$           |           |
| a3  | 0             | 1            | 1             | -1873                                 | -31833   |              | 1    | 1      | ++  | 1                            | 1                           | 1       | $I_1$           |           |
| b1  | 0             | 0            | 1             | -1                                    | 0  |              | 1    | 1      | + + | 1                            | 1                           | 1       | $I_1$           |           |
| c1  | 1             | $1-\varphi$  | 0             | $-4-6\varphi$                         | $-5-8\varphi$                                  |              | 0    | 2      | + + | 1                            | 1                           | 1       | $I_1$           |           |
| c2  | $1+\varphi$   | 0            | $\varphi$     | $-113 + 68\varphi$                    | $536 - 333\varphi$                             |              | 0    | 2      | + - | 2                            | 2                           | 2       | $I_2$           |           |
| d1  | 1             | $\varphi$    | 0             | $-10+6\varphi$                        | $-13 + 8\varphi$                               |              | 0    | 2      | + + | 1                            | 1                           | 1       | $I_1$           |           |
| d2  | $\varphi$     | $1-\varphi$  | $1 + \varphi$ | $-44-70\varphi$                       | $203 + 332\varphi$                             |              | 0    | 2      | -+  | 2                            | 2                           | 2       | $I_2$           |           |
|     |               |              |               |                                       |  |              |      |        |     |                              |                             |         |                 |           |
| 139 | 5a            |              |               | 139                                   | $5a = (39 - 3\varphi) = 5a \cdot 3 \cdot 31a$  | (4 isogeny   | y cl | lasses | s)  |                              |                             |         |                 | 1395a     |
| a1  | φ             | $-\varphi$   | 0             | $1+3\varphi$                          | 0  |              | 0    | 6      | + - | 3, 1, 2                      | 3, 1, 2                     | 3, 1, 2 | $I_3, I_1, I_2$ |           |
| a2  | $\varphi$     | $-\varphi$   | 0             | $-4-12\varphi$                        | $8+9\varphi$                                   |              | 0    | 6      | - + | 6, 2, 1                      | 6, 2, 1                     | 6, 2, 1 | $I_6, I_2, I_1$ |           |
| a3  | 1             | $1-\varphi$  | 0             | $-184 + 108\varphi$                   | $-1230 + 753\varphi$                           |              | 0    | 2      | + - | 1, 3, 6                      | 1, 3, 6                     | 1, 3, 2 | $I_1, I_3, I_6$ |           |
| a4  | $  \varphi  $ | $-\varphi$   | 0             | $-319 - 462\varphi$                   | $-3691 - 5796\varphi$                          |              | 0    | 2      | - + | 2, 6, 3                      | [2, 6, 3]                   | 2, 6, 1 | $I_2, I_6, I_3$ | <u> </u>  |
| b1  | $\varphi$     | $\varphi$    | 1             | $-23-12\varphi$                       | $35 + 26\varphi$                               |              | 1    | 2      | + + | 2, 1, 8                      | 2, 1, 8                     | 2, 1, 8 | $I_2, I_1, I_8$ |           |
| b2  | $  \varphi  $ | $\varphi$    | 1             | $2-2\varphi$                          | 1  |              | 1    | 2      |     | 1, 2, 4                      | 1, 2, 4                     | 1, 2, 4 | $I_1, I_2, I_4$ |           |
| c1  | $1+\varphi$   | $\varphi$    | 1             | $-27-40\varphi$                       | $-126 - 203\varphi$                            |              | 0    | 4      | + + | 2, 2, 4                      | [2, 2, 4]                   | 2, 2, 2 | $I_2, I_2, I_4$ |           |
| c2  | $\varphi$     | $-1+\varphi$ | 1             | $-9+5\varphi$                         | $-9+5\varphi$                                  |              | 0    | 2      | + - | 1, 1, 2                      | 1, 1, 2                     | 1, 1, 2 | $I_1,I_1,I_2$   |           |
| c3  | $\varphi$     | $-1+\varphi$ | 1             | $31-30\varphi$                        | $-89 + 45\varphi$                              |              | 0    | 2      | - + | 1, 4, 8                      | 1, 4, 8                     | 1, 2, 2 | $I_1, I_4, I_8$ |           |
| c4  | $1+\varphi$   | $\varphi$    | 1             | $-427-665\varphi$                     | $-6656 - 10813\varphi$                         |              | 0    | 2      | ++  | 4, 1, 2                      | 4, 1, 2                     | 2, 1, 2 | $I_4, I_1, I_2$ |           |
| d1  | $\varphi$     | $1-\varphi$  | 1             | $-9+5\varphi$                         | $8-5\varphi$                                   | ]            | 0    | 4      | + - | 1, 1, 2                      | [1, 1, 2]                   | 1, 1, 2 | $I_1, I_1, I_2$ |           |
| d2  | $1+\varphi$   | $-\varphi$   | 1             | $-12-15\varphi$                       | $21 + 35\varphi$                               |              | 0    | 8      | ++  | 2, 2, 4                      | 2, 2, 4                     | 2, 2, 4 | $I_2, I_2, I_4$ |           |
| d3  | $1+\varphi$   | $-\varphi$   | 1             | $-147 - 255\varphi$                   | $1401 + 2261\varphi$                           |              | 0    | 4      | - + | 1, 1, 8                      | 1, 1, 8                     | 1, 1, 8 | $I_1, I_1, I_8$ |           |
| d4  | $\varphi$     | $1-\varphi$  | 1             | $-139 + 80\varphi$                    | $-804 + 495\varphi$                            |              | 0    | 4      | ++  | 4, 4, 2                      | 4, 4, 2                     | 4, 2, 2 | $I_4, I_4, I_2$ |           |
| d5  | $1+\varphi$   | $-\varphi$   | 1             | $-217 - 330\varphi$                   | $-2273 - 3487\varphi$                          | 1            | 0    | 2      | - + | 2, 8, 1                      | 2, 8, 1                     | 2, 2, 1 | $I_2, I_8, I_1$ |           |
| d6  | $\varphi$     | $1-\varphi$  | 1             | $-2184 + 1370\varphi$                 | $-47758 + 29503\varphi$                        |              | 0    | 2      | +-  | 8, 2, 1                      | 8, 2, 1                     | 8, 2, 1 | $I_8, I_2, I_1$ |           |
| 139 | $5\mathrm{b}$ |              |               | 139                                   | $5b = (36 + 3\varphi) = 5a \cdot 3 \cdot 31b$  | (4 isogeny   | v cl | asses  | s)  |                              |                             |         |                 | 1395b     |
| a1  | $1+\varphi$   | -1           | 0             | $4-3\varphi$                          | 0  | , , ,        | 0    | 6      | -+  | 3, 1, 2                      | 3, 1, 2                     | 3, 1, 2 | $I_3, I_1, I_2$ |           |
| a2  | $1+\varphi$   | -1           | 0             | $-16+12\varphi$                       | $17-9\varphi$                                  |              | 0    | 6      | + - | 6, 2, 1                      | 6, 2, 1                     | 6, 2, 1 | $I_6, I_2, I_1$ |           |
| a3  | 1             | $\varphi$    | 0             | $-76-108\varphi$                      | $-477 - 753\varphi$                            |              | 0    | 2      | - + | 1, 3, 6                      | 1, 3, 6                     | 1, 3, 2 | $I_1, I_3, I_6$ |           |
| a4  | $1+\varphi$   | -1           | 0             | $-781 + 462\varphi$                   | $-9487 + 5796\varphi$                          |              | 0    | 2      | + - | 2, 6, 3                      | 2, 6, 3                     | 2, 6, 1 | $I_2, I_6, I_3$ |           |
|     | · · ·         |              |               | · · · · · · · · · · · · · · · · · · · | ,  |              |      | 1      | 1   |                              | <u> </u>                    |         |                 | I.        |

|                                   |               |               |                |                                       |  |  | Imi                                       |            | 1/ A \                       | 1 (:)   |   | IZ 1:                              | т .              |
|-----------------------------------|---------------|---------------|----------------|---------------------------------------|--|--|---|------------|------------------------------|---|---|------------------------------------|------------------|
|                                   | $a_1$         | $a_2$         | $a_3$          | $a_4$                                 | $a_6$  | r                                      | T   | s          | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                             | $c_p$   | Kodaira                            | Isogenies        |
| 139                               | $5\mathrm{b}$ |               |                | 1395                                  | $b = (36 + 3\varphi) = 5a \cdot 3 \cdot 31b \tag{4}$ | 4 isogeny                              | classe                                    | es)        |                              |   |   |                                    | $1395\mathrm{b}$ |
| b1                                | $1+\varphi$   | $1+\varphi$   | 0              | $-34+15\varphi$                       | $74 - 48\varphi$                                     | 1                                      | 2   | ++         | 2, 1, 8                      | 2, 1, 8   | 2, 1, 8   | $I_{2}, I_{1}, I_{8}$              |                  |
| b2                                | $1+\varphi$   | $1+\varphi$   | 0              | $1 + 5\varphi$                        | $4+3\varphi$   | 1                                      | 2   |            | 1, 2, 4                      | 1,2,4   | $\begin{bmatrix} 1, 2, 4 \end{bmatrix}$             | $I_1, I_2, I_4$                    |                  |
| c1                                |               | $-1+\varphi$  | 0              | $-68 + 41\varphi$                     | $-220 + 135\varphi$                                  | 0                                      | 4   | ++         | , ,                          | 2, 2, 4   | 2, 2, 2   | $I_2, I_2, I_4$                    |                  |
| c2                                | $1+\varphi$   | $\varphi$     | 0              | $-3-4\varphi$                         | $-9-14\varphi$                                       | 0                                      | 2   | - +        | 1, 1, 2                      | 1, 1, 2   | 1, 1, 2   | $I_1, I_1, I_2$                    |                  |
| c3                                | $1+\varphi$   | $\varphi$     | 0              | $2+31\varphi$                         | $-14 - 14\varphi$                                    | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 2   | + -        | 1, 4, 8                      | 1,4,8   | 1, 2, 2   | $I_1, I_4, I_8$                    |                  |
| c4                                | :             | $-1+\varphi$  | $-\frac{0}{1}$ | $-1093 + 666\varphi$                  | $-15710 + 9720\varphi$                               | 0                                      | $\frac{1}{1} - \frac{2}{4} - \frac{1}{4}$ | + +        | 4,1,2                        | $\frac{1}{1}$ $\frac{4}{1}$ $\frac{1}{2}$ $\frac{2}{1}$ | $\begin{bmatrix} 2, 1, 2 \\ -1, 1, 2 \end{bmatrix}$ | $ $ $I_4, I_1, I_2$                |                  |
| $\frac{d1}{d2}$                   | $1+\varphi$   | $0 \\ 0$      | 1<br>1         | $-4 - 6\varphi$ $-26 + 14\varphi$     | $3 + 5\varphi$ $56 - 35\varphi$                      | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 8   | - +        | 1, 1, 2<br>2, 2, 4           | $\begin{bmatrix} 1, 1, 2 \\ 2, 2, 4 \end{bmatrix}$      | $\begin{bmatrix} 1, 1, 2 \\ 2, 2, 4 \end{bmatrix}$  | $I_1, I_1, I_2$                    |                  |
| $\frac{\mathrm{d}z}{\mathrm{d}3}$ | $\varphi$     | 0             | 1              | $-20 + 14\varphi$ $-401 + 254\varphi$ | $3662 - 2261\varphi$                                 | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 4   | ++         | 1, 1, 8                      | $\begin{bmatrix} 2, 2, 4 \\ 1, 1, 8 \end{bmatrix}$      | $\begin{bmatrix} 2, 2, 4 \\ 1, 1, 8 \end{bmatrix}$  | $I_2, I_2, I_4  I_1, I_1, I_8$     |                  |
| d4                                | $1+\varphi$   | 0             | 1              | $-59 - 81\varphi$                     | $-309 - 495\varphi$                                  |  | 4   |            | 4, 4, 2                      | 4, 4, 2   | 4, 2, 2   | $I_1, I_1, I_8$<br>$I_4, I_4, I_2$ |                  |
| d5                                | $\varphi$     | 0             | 1              | $-546 + 329\varphi$                   | $-5760 + 3487\varphi$                                | 0                                      | 2   | + -        | 2, 8, 1                      | 2, 8, 1   | 2, 2, 1   | $I_2, I_8, I_1$                    |                  |
| d6                                | $1+\varphi$   | 0             | 1              | $-814-1371\varphi$                    | $-18255 - 29503 \varphi$                             | 0                                      | 2   | -+         | 8, 2, 1                      | 8, 2, 1   | 8, 2, 1   | $I_8, I_2, I_1$                    |                  |
| 139                               | 6a            |               |                | 1390                                  | $6a = (10 - 34\varphi) = 2 \cdot 349a \tag{4}$       | isogeny                                | classe                                    | s)         |                              |   |   |                                    | 1396a            |
| a1                                | 1             | 1 :           | $1+\varphi$    | $-24-3\varphi$                        | $-56-9\varphi$                                       | 1                                      | 1   | -+         | 5,1                          | 5, 1  | 1,1   | $I_5, I_1$                         |                  |
| b1                                | 1             |               | $1+\varphi$    | $-4+\varphi$                          | 2-2arphi   | <u>'</u><br>  1                        | 1 1                                       | <u></u>    | 2,1                          | 2,1   | 2,1   | $ I_2,I_1 $                        |                  |
| c1                                | 1             | 1             | 1              | $-1-\varphi$                          | 0  | 1                                      | $\begin{vmatrix} 1 & 1 \end{vmatrix}$     | <u> </u>   | 3,1                          | 3,1   | $\begin{bmatrix} 3, 1 \end{bmatrix}$                | $  I_3, I_1  $                     |                  |
| d1                                | $1+\varphi$   | $-\varphi$    | 1              | $-68 + 41\varphi$                     | $236-146\varphi$                                     | 0                                      | -3  | ·<br>  - + | 1,1                          | 1,1   | $\begin{bmatrix} -1, 1 \end{bmatrix}$               | $ I_1,I_1 $                        | i                |
| d2                                | 1             | 0             | 1              | $-36-45\varphi$                       | $-97 - 214\varphi$                                   | 0                                      | 3   | -+         | 3,3                          | 3,3   | 3,3   | $I_3, I_3$                         |                  |
| d3                                | 1             | $1-\varphi$ : | $1+\varphi$    | $-780941 - 1263591\varphi$            | $-505161029 - 817367715\varphi$                      | 0                                      | 1   | -+         | 1, 1                         | 1, 1  | 1, 1  | $I_1, I_1$                         |                  |
| 139                               | 6b            |               |                | 139                                   | $6b = (24 - 34\varphi) = 2 \cdot 349b \tag{4}$       | isogeny                                | classes                                   | s)         |                              |   |   |                                    | 1396b            |
| a1                                | 1             | 1             | $\varphi$      | $-26+2\varphi$                        | $-64 + 8\varphi$                                     | 1                                      | 1   | +-         | 5,1                          | 5, 1  | 1,1   | $I_5, I_1$                         |                  |
| b1                                | 1             | 0             | $\varphi$      | $-2-2\varphi$                         | $1+\varphi$  | 1                                      | 1 1                                       | Ī — —      | 2,1                          | 2,1   | [2, 1]  | $I_2, I_1$                         |                  |
| c1                                | 1             | 1             | 1              | $-2+\varphi$                          | 0  |  | 1 1                                       | +-         | 3,1                          | 3, 1  | 3,1   | $I_3, I_1$                         |                  |
| d1                                | $\varphi$     | 0             | 1              | $-26-42\varphi$                       | $90 + 146\varphi$                                    | 0                                      | 3   | + -        | 1,1                          | 1,1   | [1, 1]  | $  I_1, I_1$                       | [ ]              |
| d2                                | 1             | 0             | 1              | $-81+45\varphi$                       | $-311 + 214\varphi$                                  | 0                                      | 3   | + -        | 3, 3                         | 3,3   | 3,3   | $I_3, I_3$                         |                  |
| d3                                | 1             | $\varphi$     | $\varphi$      | $-2044531 + 1263590\varphi$           | $-1322528743 + 817367714\varphi$                     | 0                                      | 1   | + -        | 1, 1                         | 1, 1  | 1,1   | $I_1, I_1$                         |                  |
| 140                               | 5a            |               |                | 14                                    | $05a = (38 - \varphi) = 5a \cdot 281a \tag{1}$       | isogeny                                | class)                                    |            |                              |   |   |                                    | 1405a            |
| a1                                | 1             | 1             | $\varphi$      | $-15-22\varphi$                       | $-44 - 71\varphi$                                    | 0                                      |   | ++         |                              | 2, 1  | 2, 1  | $I_2, I_1$                         |                  |
| a2                                | 1             | 1             | $\varphi$      | $-2\varphi$                           | $-1-2\varphi$  | 0                                      | 2   |            | 1, 2                         | 1, 2  | 1,2   | $I_1, I_2$                         |                  |
| 140                               | $5\mathrm{b}$ |               |                | 14                                    | $05b = (37 + \varphi) = 5a \cdot 281b \tag{1}$       | isogeny                                | class)                                    |            |                              |   |   |                                    | 1405b            |
| a1                                | 1             | 1             | $1+\varphi$    | $-37 + 21\varphi$                     | $-115 + 70\varphi$                                   | 0                                      | 2   | ++         | 2, 1                         | 2,1   | 2, 1  | $I_2, I_1$                         |                  |
| a2                                | 1             | 1             | $1+\varphi$    | $-2+\varphi$                          | $-3+\varphi$   | 0                                      | 2   |            | 1, 2                         | 1, 2  | 1, 2  | $I_1, I_2$                         |                  |
| 142                               | 0a            |               |                | 14200                                 | $a = (12 - 34\varphi) = 2 \cdot 5a \cdot 71b$ (      | 7 isogeny                              | class                                     | ses)       |                              |   |   |                                    | 1420a            |
| a1                                | φ             | $\varphi$     | 0              | $5-2\varphi$                          | $-50 + 27\varphi$                                    | 0                                      | 1   |            | 11, 3, 1                     | 11, 3, 1  | 1,1,1   | $I_{11}, I_3, I_1$                 |                  |
|                                   |               |               |                | <u> </u>                              |  |  | 1   | 1          |                              | 1   | I   | 1                                  |                  |

|  |             |                                  |               |                             |  |  | _                                 | _                       |                              |  | Т                                       | Г  | 1                |
|--|-------------|----------------------------------|---------------|-----------------------------|--|--|-----------------------------------|-------------------------|------------------------------|--|---|--|------------------|
|  | $a_1$       | $a_2$                            | $a_3$         | $a_4$                       | $a_6$  | r                                      | T                                 |                         | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                  | $c_p$                                   | Kodaira  | Isogenies        |
| 1420                                       | 0a          |                                  |               | 142                         | $20a = (12 - 34\varphi) = 2 \cdot 5a \cdot 71b$          | (7 isoger                              | nv cla                            | asses)                  |                              |  |   |  | 1420a            |
| b1   | 1           | $\varphi$                        | 0             | 2arphi                      | 2  | 0                                      | Ť                                 | <del></del>             | 1, 5, 1                      | 1, 5, 1                                      | 1, 5, 1                                 | $I_1, I_5, I_1$                                      |                  |
| b2   | 1           | $\varphi$                        | 0             | $-95-58\varphi$             | $-499 - 388\varphi$                                      | 0                                      |                                   |                         | ت د ت                        | 5, 1, 5                                      | 1, 1, 5                                 | $I_5, I_1, I_5$                                      |                  |
| c1   | 1           | $1+\varphi$                      | 1             | $\varphi$                   | 2-arphi  | 1                                      | 1                                 | - <del> </del>          | 2, 3, 1                      | [2, 3, 1]                                    | [2, 3, 1]                               | $I_2, I_3, I_1$                                      |                  |
| d1   | $1+\varphi$ | <del>-</del> - 1                 | $\varphi$     | $-1-\varphi$                | $-\varphi$   |  | -¦ 1                              | - <del> </del><br>  — + | 4, 2, 1                      | [4, 2, 1]                                    | $\frac{1}{4}, \frac{1}{2}, \frac{1}{1}$ | $ I_4,I_2,I_1 $                                      | <u>-</u>  <br>   |
| d2   | $1+\varphi$ | 1                                | $\varphi$     | $-21-\varphi$               | 4-41arphi  | 1                                      | 4                                 | ++                      |                              | 2, 4, 2                                      | 2, 2, 2                                 | $I_2, I_4, I_2$                                      |                  |
| d3   | $\varphi$   | $-1-\varphi$                     | 0             | $-1654 + 1023\varphi$       | $30603 - 18918\varphi$                                   | 1                                      | 2                                 | + -                     | 1, 8, 1                      | 1, 8, 1                                      | 1, 2, 1                                 | $I_1, I_8, I_1$                                      |                  |
| d4   | 1           | -1                               | 1             | $-696 - 1119\varphi$        | $-13174-21318\varphi$                                    | 1                                      | 2                                 | ++                      | 1, 2, 4                      | 1, 2, 4                                      | 1, 2, 2                                 | $I_1, I_2, I_4$                                      |                  |
| e1   | 1           | 0                                | 0             | $-72+2\varphi$              | $108 - 206\varphi$                                       | 0                                      | $\begin{vmatrix} 1 \end{vmatrix}$ |                         | 1, 7, 1                      | 1,7,1  | [1, 1, 1]                               | $I_1, I_7, I_1$                                      |                  |
| $\bar{f}1$                                 | $1+\varphi$ | -1                               | $\varphi$     | $-4+\varphi$                | 5arphi   | 0                                      | 3                                 | - <u> </u>              | 2, 1, 3                      | [2, 1, 3]                                    | [2, 1, 3]                               | $I_2, I_1, I_3$                                      |                  |
| f2   | $\varphi$   | $1+\varphi$                      | 0             | $-2054 + 1272\varphi$       | $-42488 + 26252\varphi$                                  | 0                                      | 1                                 |                         | 6, 3, 1                      | 6, 3, 1                                      | 6, 1, 1                                 | $I_6, I_3, I_1$                                      |                  |
| g1   | $\varphi$   | $-\varphi$                       | $\varphi$     | $-41 - 85\varphi$           | $289 + 420\varphi$                                       | 0                                      | 9                                 |                         | 6, 9, 1                      | [6, 9, 1]                                    | [6, 9, 1]                               | $I_6, I_9, I_1$                                      |                  |
| g2   | $\varphi$   |                                  | arphi         | $9+190\varphi$              | $554 + 2415\varphi$                                      | 0                                      | 3                                 |                         | 18, 3, 3                     | 18, 3, 3                                     | 18, 3, 1                                | $I_{18}, I_3, I_3$                                   |                  |
| g3   | 1           | $1-\varphi$                      | $\varphi$     | $-55884 + 34272\varphi$     | $-5995669 + 3700506\varphi$                              | 0                                      | 1                                 |                         | 6, 1, 9                      | 6, 1, 9                                      | 6, 1, 1                                 | $I_6, I_1, I_9$                                      |                  |
| 1 404                                      | Ω1.         |                                  |               |                             |  | <i>'</i>                               |                                   |                         |                              |  |   |  | 1 4001           |
| 1420                                       |             |                                  |               |                             | $20b = (22 - 34\varphi) = 2 \cdot 5a \cdot 71a$          | (7 isoger                              | Ť                                 | <del></del>             |                              |  | I                                       | I  | 1420b            |
| a1   | $1+\varphi$ | $1+\varphi$                      | 1             | $3+5\varphi$                | $-20-21\varphi$  | 0                                      | 1                                 | _                       | 11, 3, 1                     | 11, 3, 1                                     | 1, 1, 1                                 | $I_{11}, I_3, I_1$                                   |                  |
| b1   | 1           | $1-\varphi$                      | 0             | $2-2\varphi$                | 2  | 0                                      | 5                                 |                         | -, -, -                      | 1, 5, 1                                      | 1, 5, 1                                 | $I_1, I_5, I_1$                                      |                  |
| b2   | 1           | $1-\varphi$                      | 0             | $-153 + 58\varphi$          | $-887 + 388\varphi$                                      | 0                                      | $\lfloor 1 \rfloor$               | _                       | 5, 1, 5                      | 5, 1, 5                                      | 1, 1, 5                                 | $I_5, I_1, I_5$                                      |                  |
| c1   | 1           | $-1-\varphi$                     | 0             | arphi                       | $1+\varphi$  | 1                                      | 1                                 |                         | 2, 3, 1                      | 2, 3, 1                                      | 2, 3, 1                                 | $I_2, I_3, I_1$                                      |                  |
| d1   | $\varphi$   | $-1-\varphi$                     | 1             | $-2+\varphi$                | 1  | 1                                      | 4                                 | + -                     | 4, 2, 1                      | [4, 2, 1]                                    | [4, 2, 1]                               | $I_4, I_2, I_1$                                      |                  |
| d2   | $\varphi$   | $-1-\varphi$                     | 1             | $-22+\varphi$               | $-15+40\varphi$  | 1                                      | 4                                 | ++                      | 2, 4, 2                      | 2, 4, 2                                      | 2, 2, 2                                 | $I_2, I_4, I_2$                                      |                  |
| d3   | $1+\varphi$ | 1                                | $1 + \varphi$ | $-632-1023\varphi$          | $11053 + 17895\varphi$                                   | 1                                      | 2                                 | - +                     | 1, 8, 1                      | 1, 8, 1                                      | 1, 2, 1                                 | $I_1, I_8, I_1$                                      |                  |
| d4   | 1           | -1                               | 1             | $-1815 + 1119\varphi$       | $-34492 + 21318\varphi$                                  | 1                                      | 2                                 | ++                      | 1, 2, 4                      | 1, 2, 4                                      | 1, 2, 2                                 | $I_1, I_2, I_4$                                      |                  |
| e1   | $\varphi$   | $-\varphi$                       | $1+\varphi$   | $-2-3\varphi$               | 5-6arphi   | 0                                      | 3                                 | _                       | 2, 1, 3                      | [2, 1, 3]                                    | [2, 1, 3]                               | $I_2, I_1, I_3$                                      |                  |
| e2   | $1+\varphi$ | $-1+\varphi$                     | $\varphi$     | $-784 - 1271\varphi$        | $-16723 - 27036\varphi$                                  | 0                                      | 1                                 | _                       | 6, 3, 1                      | 6, 3, 1                                      | 6, 1, 1                                 | $I_6, I_3, I_1$                                      |                  |
| f1   | 1           | 0                                | 0             | $-70-2\varphi$              | $-98 + 206\varphi$                                       | 0                                      | 1                                 |                         | 1, 7, 1                      | 1,7,1  | [ 1, 1, 1]                              | $I_1, I_7, I_1$                                      |                  |
| g1   | $1+\varphi$ | -1                               | $1+\varphi$   | $-126 + 83\varphi$          | $709 - 421\varphi$                                       | 0                                      | 9                                 | ī — —                   | 6, 9, 1                      | [6, 9, 1]                                    | [6, 9, 1]                               | $  I_6, I_9, I_1  $                                  |                  |
| g2   | $1+\varphi$ |                                  | $1+\varphi$   | $199-192\varphi$            | $2969 - 2416\varphi$                                     | 0                                      | 3                                 |                         | 18, 3, 3                     | 18, 3, 3                                     | 18, 3, 1                                | $I_{18}, I_3, I_3$                                   |                  |
| g3   | 1           | $\varphi$                        | $1 + \varphi$ | $-21612 - 34273\varphi$     | $-2295163 - 3700507\varphi$                              | 0                                      | 1                                 |                         | 6, 1, 9                      | 6, 1, 9                                      | 6, 1, 1                                 | $I_6, I_1, I_9$                                      |                  |
| 142  | 1 2         |                                  |               |                             | $1421a = (42 - 7\varphi) = 29a \cdot 7$                  | (1 isogeny                             | , alaa                            | (a)                     |                              |  |   |  | 1421a            |
| a1   |             | $1+\varphi$                      | 1 1           | -1                          | . , ,  | , , ,                                  | _                                 | <u> </u>                | 1,1                          | 1 1  | 1 1                                     | тт   | 1 <b>7</b> 210   |
| a1 a2                                      | arphi       |                                  |               | $4-10\varphi$               | -arphi -19 - 9arphi                                      | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ |                                   |                         |                              | $ \begin{array}{c c} 1,1\\ 3,3 \end{array} $ | $1, 1 \\ 1, 3$                          | $\begin{bmatrix} I_1, I_1 \\ I_3, I_3 \end{bmatrix}$ |                  |
| 142  | 1h          |                                  |               |                             | $1421b = (35 + 7\varphi) = 29b \cdot 7$                  | (1 isogeny                             | clas                              | s)                      |                              | •  | 1                                       |  | $1421\mathrm{b}$ |
| al   |             | $-1+\varphi$                     | 0             | $-1+\varphi$                | $\frac{14210 - (33 + 1\psi) - 230 \cdot 1}{2 - \varphi}$ | (1 isogeny                             | _                                 | <del></del>             | 1,1                          | 1,1  | 1,1                                     | $I_1, I_1$   | 11410            |
| $\begin{vmatrix} a_1 \\ a_2 \end{vmatrix}$ |             | $-1 + \varphi$<br>$-1 + \varphi$ | 0             | $-1+\varphi$ $-6+11\varphi$ | $2-\varphi$ $-10+2\varphi$                               | 1                                      |                                   |                         |                              | $\begin{bmatrix} 1,1\\3,3 \end{bmatrix}$     | 1, 1 $1, 3$                             | $I_1, I_1$ $I_3, I_3$                                |                  |
| W2   | - 1 Ψ       | - 1 4                            |               | σ γ 11φ                     | 10   24  | 1                                      | 1                                 |                         | 5,5                          | 5,5  | 1,0                                     | -3,-3  |                  |

|  | $a_1$  | $a_2$ $a_3$   | $a_4$   | $a_6$   |   | r   | T                               | s ord  | $l(\Delta)  \text{ord}_{-}(j)$   | $c_p$   | Kodaira  | Isogenies               |
|--|--|---|---|---|---|---|---------------------------------|--|--|---|--|-------------------------|
|  |  |   |   | ~~0   |   | .   | -1                              |  | ( <u>—</u> )   01 (1 )   |   | 110 daira  |                         |
| 142  |  |   |   | $424a = (40 - 4\varphi) = 2^2 \cdot 89a$  | (1 isogeny cl   |   |                                 |  |  |   |  | 1424a                   |
| a1<br>a2                                     | $ \begin{array}{c cccc} 0 & 1+ \\ 0 & 1+ \end{array} $   | •   | $-2 + \varphi$ $8 - 4\varphi$   | $0 \\ 4$  |   |   | I                               |  | $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$  | $\begin{vmatrix} 3, 1 \\ 3, 2 \end{vmatrix}$  | $IV, I_1$<br>$IV^*, I_2$   |                         |
| aΔ   | 0 1+   | $\frac{\varphi}{}$  | $6-4\varphi$  | 4   | -   | 1   | 2   -                           | 0  | , 2 2  | 3, 2  | 1 V , 12   |                         |
| 142  | 4b   |   | 1   | $424b = (36 + 4\varphi) = 2^2 \cdot 89b$  | (1 isogeny cla  | ass)                                      |                                 |  |  |   |  | 1424b                   |
| a1   | 0 -1-  | ,   | $-2+\varphi$  | 2   | I   |   |                                 |  | $\frac{1}{2}$  | 3, 1  | $IV, I_1$  |                         |
| a2   | 0 -1-  | $\varphi$ 0   | $3+6\varphi$  | $1-5\varphi$  |   | 1   | 2   -                           | 8  | 2  | 3, 2  | $IV^*, I_2$  |                         |
| 143  | 6a   |   | 14  | $436a = (14 - 34\varphi) = 2 \cdot 359a$  | (1 isogeny c  | lass)                                     |                                 |  |  |   |  | 1436a                   |
| a1   | $\varphi$ 1+   | $\varphi$ 1 + $\varphi$   | $-3+2\varphi$   | $2-2\varphi$  |   |   |                                 |  | $1, 1 \qquad 2, 1$   | 2,1   | $I_2, I_1$   |                         |
| a2   | $\varphi$ 1+   | $\varphi$ 1+ $\varphi$  | $-53 + 32\varphi$   | $164 - 104\varphi$  |   | 1   | 2 +                             | - + 1  | ,2 1,2   | 1, 2  | $I_1, I_2$   |                         |
| 143  | 6b   |   | 1   | $436b = (20 - 34\varphi) = 2 \cdot 359b$  | (1 isogeny cl   | lass)                                     |                                 |  |  |   |  | $1436 \mathrm{b}$       |
| a1   | $1+\varphi$ $-1+$  | $\varphi = 0$   | $-1-\varphi$  | 1   | · · · · · ·   | <del></del>                               | 2 +                             | 2  | $1, 1 \qquad 2, 1$   | 2,1   | $I_2, I_1$   |                         |
| a2   | $1+\varphi$ $-1+$  | $\varphi$ 0   | $-21 - 31\varphi$   | $51 + 82\varphi$  |   | 1   | 2 +                             | + 1  | , 2 1, 2   | 1, 2  | $I_1, I_2$   |                         |
| 143  | 9a   |   |   | $1439a = (37 + 2\varphi) = 1439a$   | (1 isogeny cla  | ass)                                      |                                 |  |  |   |  | 1439a                   |
| a1   | 1 -  | φ φ   | $-9-18\varphi$  | $-20-33\varphi$   | (   | 0   | 1 -                             |  | 1 1  | 1   | $I_1$  |                         |
| $\overline{143}$                             | 0.1  |   |   |   | •   |   |                                 |  | <u> </u>   |   |  |                         |
| TAO  | 9b   |   |   | $1439b = (39 - 2\varphi) = 1439b$   | (1 isogeny cla  | ss)                                       |                                 |  |  |   |  | 1439b                   |
| a1   | ı  | $\varphi$ 1+ $\varphi$  | $-27 + 17\varphi$   | $\frac{1439b = (39 - 2\varphi) = 1439b}{-53 + 32\varphi}$   | (1 isogeny cla  | -í  | 1 -                             |  | 1 1  | 1   | $I_1$  | 1439b                   |
|  | 1 -1+  | $\varphi$ 1+ $\varphi$  | $-27 + 17\varphi$   | $-53 + 32\varphi$   |   | 0   |                                 |  | 1 1  | 1   | $I_1$  | 1439b<br>1441a          |
| a1   | 1 -1+ <b>1a</b>  | $\varphi$ 1 + $\varphi$ 1   | $-27 + 17\varphi$   | *   | b (1 isogeny  | 0 class                                   | 5)                              |  | 1 1  | 1,1   | I <sub>1</sub>   |                         |
| a1<br><b>144</b>                             | 1 -1+ <b>1a</b>  | 1 1   | $-27 + 17\varphi$ $14$  | $-53 + 32\varphi$ $41a = (19 - 34\varphi) = 11b \cdot 131b$   | b (1 isogeny  | class                                     | 2 +                             | - + 1  |  |   |  |                         |
| a1<br>144<br>a1                              | $ \begin{array}{c c} 1 & -1 + \\ \hline \mathbf{1a} \\ 1 \\ 1 + \varphi & -1 +  \end{array} $  | 1 1   | $-27 + 17\varphi$ $14$ $-1 - \varphi$ $-17 + 10\varphi$   | $-53 + 32\varphi$ $41a = (19 - 34\varphi) = 11b \cdot 131b$ $-1 - \varphi$ $41 - 26\varphi$   | b (1 isogeny  | class                                     | 2 +                             | - + 1  | ,1 1,1   | 1,1   | $I_1, I_1$   |                         |
| a1 144 a1 a2                                 | $     \begin{array}{c cccc}                                 $  | 1 1   | $-27 + 17\varphi$ $14$ $-1 - \varphi$ $-17 + 10\varphi$   | $-53 + 32\varphi$ $41a = (19 - 34\varphi) = 11b \cdot 131b$ $-1 - \varphi$  | to (1 isogeny   | class 1 1 class                           | 3)<br>2 +<br>2 +<br>3)          | - + 1<br>2   | ,1 1,1   | 1,1   | $I_1, I_1$   | 1441a                   |
| a1  144  a1  a2  144                         | $     \begin{array}{c cccc}                                 $  | $ \begin{array}{ccc} 1 & 1 \\ \varphi & 1 + \varphi \end{array} $ $ \begin{array}{ccc} 1 & 1 \\ \end{array} $   | $-27 + 17\varphi$ $14$ $-1 - \varphi$ $-17 + 10\varphi$ $14$  | $-53 + 32\varphi$ $41a = (19 - 34\varphi) = 11b \cdot 131b$ $-1 - \varphi$ $41 - 26\varphi$ $41b = (15 - 34\varphi) = 11a \cdot 131a$   | to (1 isogeny   | class 1 1 class                           | 3)<br>2                         | - + 1<br>2<br>- + 1                                  | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  | 1,1 2,2                                       | $\begin{matrix} I_1,I_1\\I_2,I_2\end{matrix}$  | 1441a                   |
| a1  144  a1  a2  144  a1                     | $ \begin{array}{c c} 1 & -1 + \\ \hline \mathbf{1a} \\ 1 + \varphi & -1 + \\ \hline \mathbf{1b} \\  & \varphi & 1 + \\ \end{array} $ | $ \begin{array}{ccc} 1 & 1 \\ \varphi & 1 + \varphi \end{array} $ $ \begin{array}{ccc} 1 & 1 \\ \end{array} $   | $-27 + 17\varphi$ $14$ $-1 - \varphi$ $-17 + 10\varphi$ $14$ $-2 + \varphi$ $-5 - 10\varphi$  | $-53 + 32\varphi$ $41a = (19 - 34\varphi) = 11b \cdot 131b$ $-1 - \varphi$ $41 - 26\varphi$ $41b = (15 - 34\varphi) = 11a \cdot 131a$ $-2 + \varphi$  | to (1 isogeny   | class  class  class  1  1                 | (s) 2 + 2 + 4 (s) 2 + 2 - 4 (s) | - + 1<br>2<br>- + 1                                  | $ \begin{array}{c cccc} ,1 & 1,1 \\ ,2 & 2,2 \\ \end{array} $  | 1,1 2,2                                       | $I_1, I_1 \\ I_2, I_2$   | 1441a                   |
| a1  144  a1  a2  144  a1  a2                 | $1 -1 + 1$ 1a $1 + \varphi$ 1b $1 + \varphi$ 1c  | $ \begin{array}{ccc} 1 & 1 \\ \varphi & 1 + \varphi \end{array} $ $ \begin{array}{ccc} 1 & 1 \\ \end{array} $   | $-27 + 17\varphi$ $14$ $-1 - \varphi$ $-17 + 10\varphi$ $14$ $-2 + \varphi$ $-5 - 10\varphi$  | $-53 + 32\varphi$ $41a = (19 - 34\varphi) = 11b \cdot 131b$ $-1 - \varphi$ $41 - 26\varphi$ $41b = (15 - 34\varphi) = 11a \cdot 131a$ $-2 + \varphi$ $5 + 9\varphi$ $41c = (35 + 8\varphi) = 11b \cdot 131a$  | to (1 isogeny of (2 isogeny cl  | class class lasses                        | (s)<br>2                        | - + 1<br>2<br>- + 1<br>- + 2                         | $ \begin{array}{c cccc} ,1 & 1,1 \\ ,2 & 2,2 \\ \end{array} $  | 1,1 2,2                                       | $I_1, I_1 \\ I_2, I_2$   | 1441a<br>1441b          |
| 144 a1 a2 144 a1 a2 144 a1 a2 144            | $     \begin{array}{c cccc}                                 $  | $ \begin{array}{cccc} 1 & 1 \\ \varphi & 1 + \varphi \end{array} $ $ \begin{array}{cccc} 1 & 1 \\ \varphi & 1 \end{array} $ $ \begin{array}{cccc} 1 & \varphi \\ \varphi & \varphi \end{array} $  | $-27 + 17\varphi$ $14$ $-1 - \varphi$ $-17 + 10\varphi$ $14$ $-2 + \varphi$ $-5 - 10\varphi$ $14$ $1 - \varphi$ $-1 - \varphi$                | $-53 + 32\varphi$ $41a = (19 - 34\varphi) = 11b \cdot 131b$ $-1 - \varphi$ $41 - 26\varphi$ $41b = (15 - 34\varphi) = 11a \cdot 131a$ $-2 + \varphi$ $5 + 9\varphi$ $41c = (35 + 8\varphi) = 11b \cdot 131a$ $0$ $0$  | to (1 isogeny of the control of the | class  class  lasses  lasses              | (s)<br>2                        | - + 1<br>2<br>- + 1<br>- + 2                         | $\begin{array}{c ccccc} , 1 & & 1, 1 \\ , 2 & & 2, 2 \\ \end{array}$ $\begin{array}{c ccccccccccccccccccccccccccccccccccc$               | 1,1<br>2,2<br>1,1<br>2,2<br>2,1<br>1,1        | $\begin{array}{ c c c c c }\hline I_1, I_1 \\ I_2, I_2 \\ \hline & I_1, I_1 \\ I_2, I_2 \\ \hline & I_2, I_1 \\ \hline & I_1, I_1 \\ \hline \end{array}$ | 1441a<br>1441b          |
| a1  144  a1 a2  144  a1 a2  144  a1 a2       | $1 -1 +$ 1a $1 + \varphi$ 1b $\varphi$ 1 +  1c   | $ \begin{array}{cccc} 1 & 1 \\ \varphi & 1 + \varphi \end{array} $ $ \begin{array}{cccc} 1 & 1 \\ \varphi & 1 \end{array} $ $ \begin{array}{cccc} 1 & \varphi \\ \varphi & \varphi \end{array} $  | $-27 + 17\varphi$ $14$ $-1 - \varphi$ $-17 + 10\varphi$ $14$ $-2 + \varphi$ $-5 - 10\varphi$ $14$ $1 - \varphi$                               | $-53 + 32\varphi$ $41a = (19 - 34\varphi) = 11b \cdot 131b$ $-1 - \varphi$ $41 - 26\varphi$ $41b = (15 - 34\varphi) = 11a \cdot 131a$ $-2 + \varphi$ $5 + 9\varphi$ $41c = (35 + 8\varphi) = 11b \cdot 131a$ $0$  | to (1 isogeny of the control of the | class 1 1 class 1 1 0                     | s) 2 + 2 + + s) 8) 1            | - + 1<br>2<br>- + 1<br>- + 2                         | $\begin{array}{c cccc} , 1 & & 1, 1 \\ , 2 & & 2, 2 \\ \end{array}$ $\begin{array}{c ccccc} , 1 & & 1, 1 \\ , 2 & & 2, 2 \\ \end{array}$ | 1,1<br>2,2<br>1,1<br>2,2                      | $\begin{array}{ c c c c }\hline I_1,I_1\\I_2,I_2\\\hline &I_1,I_1\\I_2,I_2\\\hline &I_2,I_1\\\hline \end{array}$   | 1441a<br>1441b          |
| 144 a1 a2 144 a1 a2 144 a1 a2 144            | $ \begin{array}{c cccc} 1 & -1 + \\ \hline  1a \\ 1 + \varphi & -1 + \\ \hline  1b \\ \hline                                   $     | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$   | $-27 + 17\varphi$ $14$ $-1 - \varphi$ $-17 + 10\varphi$ $14$ $-2 + \varphi$ $-5 - 10\varphi$ $14$ $1 - \varphi$ $-1 - \varphi$ $9 + 9\varphi$ | $-53 + 32\varphi$ $41a = (19 - 34\varphi) = 11b \cdot 131b$ $-1 - \varphi$ $41 - 26\varphi$ $41b = (15 - 34\varphi) = 11a \cdot 131a$ $-2 + \varphi$ $5 + 9\varphi$ $41c = (35 + 8\varphi) = 11b \cdot 131a$ $0$ $0$  | to (1 isogeny of the control of the | class 1 1 1 1 class 1 1 1 0 0 0           | s)                              | - + 1<br>2<br>- + 1<br>- + 2                         | $\begin{array}{c ccccc} , 1 & & 1, 1 \\ , 2 & & 2, 2 \\ \end{array}$ $\begin{array}{c ccccccccccccccccccccccccccccccccccc$               | 1,1<br>2,2<br>1,1<br>2,2<br>2,1<br>1,1        | $\begin{array}{ c c c c c }\hline I_1, I_1 \\ I_2, I_2 \\ \hline & I_1, I_1 \\ I_2, I_2 \\ \hline & I_2, I_1 \\ \hline & I_1, I_1 \\ \hline \end{array}$ | 1441a<br>1441b          |
| a1  144  a1 a2  144  a1 a2  144  a1 b1 b1 b2 | $1 -1 +$ 1a $1 + \varphi$ 1b $\varphi$ 1 +  1c $0 + \varphi$ 1 +  1d   | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$   | $-27 + 17\varphi$ $14$ $-1 - \varphi$ $-17 + 10\varphi$ $14$ $-2 + \varphi$ $-5 - 10\varphi$ $14$ $1 - \varphi$ $-1 - \varphi$ $9 + 9\varphi$ | $-53 + 32\varphi$ $41a = (19 - 34\varphi) = 11b \cdot 131b$ $-1 - \varphi$ $41 - 26\varphi$ $41b = (15 - 34\varphi) = 11a \cdot 131a$ $-2 + \varphi$ $5 + 9\varphi$ $41c = (35 + 8\varphi) = 11b \cdot 131a$ $0$ $9 + 17\varphi$  | (2 isogeny cl   | class 1 1 1 1 class 1 1 1 0 0 0           | s) 2                            | - + 1<br>2<br>- + 1<br>- + 2<br>2<br>1<br>3          | $\begin{array}{c ccccc} , 1 & & 1, 1 \\ , 2 & & 2, 2 \\ \end{array}$ $\begin{array}{c ccccccccccccccccccccccccccccccccccc$               | 1,1<br>2,2<br>1,1<br>2,2<br>2,1<br>1,1        | $\begin{array}{ c c c c }\hline I_1,I_1\\I_2,I_2\\\hline &I_1,I_1\\I_2,I_2\\\hline &-I_2,I_1\\\hline &-I_1,I_1\\\hline \end{array}$                      | 1441a<br>1441b<br>1441c |
| 144 a1 a2 144 a1 a2 144 a1 b1 b2 144         | $ \begin{array}{c cccc} 1 & -1 + \\ \hline                                  $  | $ \begin{array}{cccc} 1 & 1 \\ \varphi & 1 + \varphi \end{array} $ $ \begin{array}{cccc} 1 & 1 \\ \varphi & 1 \end{array} $ $ \begin{array}{cccc} \frac{1}{\varphi} & \frac{\varphi}{\varphi} \end{array} $ $ \begin{array}{cccc} \varphi & \varphi \end{array} $ |   | $-53 + 32\varphi$ $41a = (19 - 34\varphi) = 11b \cdot 131b$ $-1 - \varphi$ $41 - 26\varphi$ $41b = (15 - 34\varphi) = 11a \cdot 131a$ $-2 + \varphi$ $5 + 9\varphi$ $41c = (35 + 8\varphi) = 11b \cdot 131a$ $0$ $9 + 17\varphi$ $41d = (43 - 8\varphi) = 11a \cdot 131b$ | (2 isogeny cl   | Class   1   1   1   1   1   1   1   1   1 | s) 2                            | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | $\begin{array}{c ccccc} , 1 & & 1, 1 \\ , 2 & & 2, 2 \\ \end{array}$ $\begin{array}{c ccccccccccccccccccccccccccccccccccc$               | 1,1<br>2,2<br>1,1<br>2,2<br>2,1<br>1,1<br>1,1 | $\begin{array}{ c c c c }\hline I_1,I_1\\I_2,I_2\\\hline I_1,I_1\\I_2,I_2\\\hline &I_1,I_1\\\hline &I_3,I_3\\\hline \end{array}$                         | 1441a<br>1441b<br>1441c |

|  | $a_1$                                  | $a_2$               | $a_3$        | $a_4$   | $a_6$   | r                                      | T  | s        | $\operatorname{ord}(\Delta)$            | $\operatorname{ord}_{-}(j)$                         | $c_p$   | Kodaira   | Isogenies          |
|--|--|---------------------|--------------|---|---|--|--|----------|---|---|---|---|--------------------|
| 44                                     | $A_{\mathbf{a}}$                       |                     |              | 144.  | $4a = (38) = 2 \cdot 19a \cdot 19b$ (4 is             | ogeny c                                | laccoc   | .)       |   |   |   |   | 1444               |
| a1                                     | φ                                      | $1+\varphi$         | 1 + 10       | $2-3\varphi$  | $\frac{140 - (90) - 2 \cdot 190 \cdot 190}{-\varphi}$ |  | 1  | ,<br>    | 3, 1, 1                                 | 3, 1, 1   | 3, 1, 1   | $I_3, I_1, I_1$   |                    |
|  | <u> </u>                               |                     | <del>-</del> |   |   | :                                      | ¦ ·  |          |   | !   |   | !   | <u> </u>           |
| b1                                     | <u> </u>                               | $-1+\varphi$        | 0            | $-1+4\varphi$   | $5-\varphi$   | 1                                      | 1<br> - <del>-</del>                             |          | $\frac{1}{3}, \frac{3}{4}, \frac{1}{5}$ | 1,3,1   | $\begin{bmatrix} 1, 3, 1 \\ -1, -1, -1 \end{bmatrix}$ | $\left \begin{array}{c} I_1, I_3, I_1 \\ \hline \end{array}\right $ | <u> </u>           |
| c1 $c2$                                | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | 1<br>1              | 1<br>1       | $0 \\ -70$  | -279  | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | 5  |          | 1, 1, 5<br>5, 5, 1                      | $\begin{bmatrix} 1, 1, 5 \\ 5, 5, 1 \end{bmatrix}$  | $1, 1, 5 \\ 1, 1, 1$                                  | $I_1, I_1, I_5$   |                    |
|  | <del>!</del>                           |                     |              |   |   | !                                      | ' ·  |          |   |   |   | $  I_5, I_5, I_1$   | <br>               |
| $\frac{d1}{d2}$                        | 1 1                                    | $0 \\ 0$            | 1<br>1       | -16   | 22<br>90  | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | $\begin{vmatrix} 3 \\ 3 \end{vmatrix}$           |          | 1, 1, 3<br>3, 3, 9                      | $\begin{vmatrix} 1,1,3\\3,3,9 \end{vmatrix}$        | 1, 1, 3<br>3, 3, 9                                    | $I_1, I_1, I_3$   |                    |
| d3                                     | 1                                      | 0                   | 1            | -86   | -2456   | 1                                      | ე<br>1   |          | 3, 3, 9 $1, 1, 27$                      | $\begin{bmatrix} 3, 3, 9 \\ 1, 1, 27 \end{bmatrix}$ | 3, 3, 9 $1, 1, 27$                                    | $\begin{bmatrix} I_3, I_3, I_9 \\ I_1, I_1, I_{27} \end{bmatrix}$   |                    |
| uo                                     | 1                                      |                     |              |   | 2400  | 1                                      | 1  |          | 1, 1, 21                                | 1,1,21  | 1,1,21  | 11, 11, 127   |                    |
| 44                                     | 4b                                     |                     |              | 1444l   | $b = (18 - 34\varphi) = 2 \cdot 19a^2 $ (6 i          | sogeny                                 | classe   | es)      |   |   |   |   | 1444               |
| a1                                     | φ                                      | 1                   | 0            | $-21+10\varphi$   | $22-41\varphi$  | 1                                      | 1  |          | 7, 3                                    | 1,3   | 4, 1  | $I_1^*, I_3$  |                    |
| a2                                     | $\varphi$                              | 1                   | 0            | $109 - 35\varphi$   | $493 + 393\varphi$                                    | 1                                      | 1  |          | 9, 9                                    | 3, 9  | 4, 1  | $I_3^*, I_9$  |                    |
| 3                                      | $\varphi$                              | 1                   | 0            | $-2931 - 2315\varphi$   | $37765 + 104233\varphi$                               | 1                                      | 1  |          | 15, 3                                   | 9, 3  | 4, 1  | $I_9^*, I_3$  |                    |
| a4                                     | $1+\varphi$                            | $1+\varphi$         | 0            | $-1038187 - 1649818\varphi$                                   | $757309343 + 1226929055\varphi$                       | 1                                      | 1  |          | 9, 1                                    | 3, 1  | 4, 1  | $I_3^*, I_1$  |                    |
| b1                                     | 1                                      | $-\varphi$          | 1            | $26-30\varphi$  | $120 - 177\varphi$                                    | 0                                      | 1  |          | 9,5                                     | 5   | 2,1   | $ $ III $^*$ , I $_5$   |                    |
| <br>c1                                 | 1                                      | $1-\varphi$         | 1            | $-3+5\varphi$   | $-10 + 11\varphi$                                     | 0                                      | 1  |          | 4,7                                     | 7   | 1,1   | $ $ IV, $I_7$   | [                  |
| <br>11                                 | 1                                      | 0                   | 1            | $2-2\varphi$  | 4-2arphi  | 1                                      | 1  |          | 3,5                                     | 5   | 2,5   | $ $ III, $I_5$  | <u>-</u>           |
| <br>e1                                 | !<br>  1                               | $-\varphi$          | $1+\varphi$  | $-21 - 42\varphi$   | $-77 - 127\varphi$                                    |  | '<br>  1   | <u>-</u> | 7,1                                     | 1,1   | $\frac{1}{2}, \frac{1}{1}$                            | $ \mathbf{I}_1^*, \mathbf{I}_1 $                                    | <u>-</u>           |
| e2                                     | $1+\varphi$                            | $1+\varphi$         | $\varphi$    | $-2553 + 1636\varphi$   | $-59466 + 37129\varphi$                               | 0                                      | 1  |          | 11, 5                                   | 5,5   | 2, 5  | $I_5^{*}, I_5$  |                    |
| ·<br>f1                                |  | $-1+\varphi$        | $1+\varphi$  | $-13 + 110\varphi$  | $-737 - 400\varphi$                                   | 0                                      | 1  |          | 10,7                                    | 7   | 1,7   | $  II^*, I_7$   | <u>.</u>           |
|  | I                                      |                     |              |   |   | I                                      |  |          |   | <u>I</u>  |   |   | 1                  |
| 44                                     | 4c                                     |                     |              | 1444  | $c = (16 - 34\varphi) = 2 \cdot 19b^2$ (6 i           | sogeny                                 | classe   | es)      |   |   |   |   | 14444              |
| a1                                     | $1+\varphi$                            | $1-\varphi$         | 0            | $-11-10\varphi$   | $-19 + 41\varphi$                                     | 1                                      | 1  |          | 7, 3                                    | 1,3   | 4, 1  | $I_1^*, I_3$  |                    |
| a2                                     | $1+\varphi$                            | $1-\varphi$         | 0            | $74 + 35\varphi$  | $886 - 393\varphi$                                    | 1                                      | 1  |          | 9,9                                     | 3, 9  | 4, 1  | $I_3^*, I_9$  |                    |
| a3                                     | $1+\varphi$                            | $1-\varphi$         | 0            | $-5246 + 2315\varphi$   | $141998 - 104233\varphi$                              | 1                                      | 1  |          | 15, 3                                   | 9, 3  | 4, 1  | $I_9^*, I_3$  |                    |
| 14<br>                                 | $\varphi$                              | $\_\_\_ \varphi_\_$ | 1            | $-2688009 + 1649821\varphi$                                   | $1988576226 - 1229617063\varphi$                      | _   1                                  | 1  |          | 9,1                                     | 3,1   | 4,1   | $I_3^*, I_1$  |                    |
| o1                                     | 1                                      | $-1+\varphi$        | 1            | $-4 + 30\varphi$  | $-57 + 177\varphi$                                    | 0                                      | 1  |          | 9,5                                     | 5   | 2,1   | $ $ III*, $I_5$   |                    |
| c1                                     | 1                                      | $\varphi$           | 1            | $2-5\varphi$  | 1-11arphi   | 0                                      | 1  |          | 4,7                                     | 7   | 1, 1  | $IV, I_7$   |                    |
| d1                                     | 1                                      | 0                   | 1            | $2\varphi$  | $2+2\varphi$  | 1                                      | 1  |          | 3, 5                                    | 5   | 2,5   | $ $ III, $I_5$  |                    |
| e1                                     | 1                                      | $-1+\varphi$        | $\varphi$    | $-62+41\varphi$   | $-203 + 126\varphi$                                   | 0                                      | 1  | Ī — —    | 7,1                                     | 1,1   | 2,1   | $ \mathrm{I}_1^*,\mathrm{I}_1 $                                     | [                  |
| e2                                     | $\varphi$                              | $\varphi$           | $\varphi$    | $-920 - 1634\varphi$  | $-23053 - 38048\varphi$                               | 0                                      | 1  |          | 11, 5                                   | 5,5   | 2,5   | $I_5^*, I_5$  |                    |
| f1                                     | 1                                      | $-\varphi$          | $\varphi$    | $98 - 111\varphi$   | $-1136 + 399\varphi$                                  | 0                                      | 1  |          | 10, 7                                   | 7   | 1,7   | $  II^*, I_7$   |                    |
|  | <b>.</b>                               |                     |              |   | · (1- 01) · (1-                                       |  |  | `        |   |   |   |   | 1445               |
| 11                                     | oa -                                   |                     |              |   | , ,   | isogeny                                |  | <u></u>  |   | 1 4   | 1.0   | T T   | $_{\perp}$ $1445a$ |
| .44                                    | 4                                      |                     |              |   |   | 1 7                                    | ٠,   | 1 1 1    | 1 /1                                    | 1 1 1   | 1 ()  |   | I .                |
| $\frac{44}{\mathbf{a1}}$ $\mathbf{a2}$ | 1<br>1                                 | 1<br>1              | $0 \\ 0$     | $     \begin{array}{r}       -8 \\       -3     \end{array} $ | $-13 \\ -22$  | 1                                      | $\begin{array}{ c c }\hline 2\\ 2\\ \end{array}$ | + +<br>  | $1, 4 \\ 2, 8$                          | $ \begin{array}{c c} 1,4\\2,8 \end{array} $         | $1,2 \\ 2,2$  | $\begin{matrix} I_1,I_4\\I_2,I_8\end{matrix}$                       |                    |

|  | $a_1$              | $a_2$         | $a_3$             | $a_4$                                     | $a_6$  |  | T                                    | s               | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$            | Kodaira   | Isogenies |
|--|--------------------|---------------|-------------------|---|--|--|--------------------------------------|-----------------|------------------------------|-----------------------------|------------------|---|-----------|
|  |                    |               |                   |   | ~0   | '                                      | -                                    |                 | 314( <u>—</u> )              | 0142(3)                     |                  | 110 00110   | 1 9       |
| 147  |                    |               |                   |   |  | isogeny classes                        | s)                                   |                 |                              |                             |                  |   | 1471a     |
| a1   | $1+\varphi$        | $1-\varphi$   | $\varphi$         | $-41 + 22\varphi$                         | $-139 + 83\varphi$                               | 0                                      | 2                                    | + -             |                              | 1                           | 1                | $I_1$   |           |
| a2   | 1                  | $1+\varphi$   | <del>φ</del> -    | $-117 - 177\varphi$                       | $-983 - 1573\varphi$                             | 0                                      | 2                                    | - +<br>         |                              | 2                           | 2                | I <sub>2</sub>  |           |
| b1   | $1+\varphi$        |               | $\varphi$         | $-40 + 24\varphi$                         | $137 - 85\varphi$                                |  | 4                                    | + -             |                              | 1                           | 1                | $\mathbf{l}_1$  |           |
| b2<br>b3   | $1 \\ 1 + \varphi$ | 1<br>-1       | 0                 | $-32 - 39\varphi$ $-19867 - 32145\varphi$ | $92 + 141\varphi$ $2045295 + 3309357\varphi$     | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\begin{vmatrix} 4\\2 \end{vmatrix}$ | +++             |                              | 2                           | $\frac{2}{1}$    | $I_2$   |           |
| b4   | $1+\varphi$ 1      | -1<br>1       | $\frac{arphi}{0}$ | $-19007 - 32149\varphi$ $-17 - 49\varphi$ | $2045295 + 3509357\varphi$<br>$121 + 117\varphi$ |  | $\frac{2}{2}$                        | + +<br>- +      |                              | 4                           | $\frac{1}{2}$    | $egin{array}{c} I_1 \ I_4 \end{array}$                  |           |
| $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ |                    | $-1+\varphi$  |                   | $-3-5\varphi$                             | $-5 - 8\varphi$                                  |  | 1 1                                  | -<br> <br>  - + |                              |                             | - <del>-</del> - | ÷=  |           |
|  |                    |               |                   |   |  |  | <u>+</u>                             |                 |                              | <u>+</u><br>                |                  | $\begin{vmatrix} I_1 \\ \vdots \\ \vdots \end{vmatrix}$ |           |
| d1   | 0                  |               | 1                 | $\varphi$                                 | 0  | 1                                      | 1<br>                                | + -             | · 1                          | <u>1</u>                    | I                | I <sub>1</sub>  | ļ         |
| e1   | $1+\varphi$        | $-1+\varphi$  | 0                 | $-565 + 350\varphi$                       | $-5794 + 3581\varphi$                            | 0                                      | 1                                    | <u> </u>        | · 1                          | 1                           | 1                | I <sub>1</sub>  |           |
| f1   | 1                  | $\varphi$     | 1                 | $-2-4\varphi$                             | $-5-7\varphi$                                    | 0                                      | 1                                    |                 | 1                            | 1                           | 1                | $I_1$   |           |
| 147  | 1b                 |               |                   | 14  | $71b = (40 - 3\varphi) = 1471b \qquad (6$        | isogeny classes                        | s)                                   |                 |                              |                             |                  |   | 1471b     |
| a1   | φ                  |               | $1+\varphi$       | $-18-24\varphi$                           | $-56-84\varphi$                                  | 0                                      | 2                                    | -+              | . 1                          | 1                           | 1                | $I_1$   |           |
| a2   | 1                  | $-1-\varphi$  | $\varphi$         | $-295 + 178\varphi$                       | $-2261 + 1395\varphi$                            | 0                                      | 2                                    | + -             | 2                            | 2                           | 2                | $I_2$   |           |
| b1   | $\varphi$          | $1 + \varphi$ | 0                 | $-15-23\varphi$                           | $28 + 45\varphi$                                 | 0                                      | 4                                    | -+              |                              | 1                           | 1                | $I_1$   |           |
| b2   | 1                  | 1             | 0                 | $-71 + 39\varphi$                         | 233-141arphi                                     | 0                                      | 4                                    | ++              |                              | 2                           | 2                | $I_2$   |           |
| b3   | $\varphi$          | •             | $1+\varphi$       | $-52011 + 32143\varphi$                   | $5354652 - 3309358\varphi$                       | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 2                                    | ++              |                              | 1                           | 1                | $I_1$   |           |
| b4   | 1                  | 1             | 0                 | $-66 + 49\varphi$                         | $238 - 117\varphi$                               | 0                                      | 2                                    | + -<br>         |                              | 4                           | 2                | $\frac{1}{1}$ $\frac{I_4}{I_4}$                         |           |
| c1   | 1                  | $-\varphi$    | 0                 | $-8+5\varphi$                             | $-13 + 8\varphi$                                 | 1                                      | 1                                    | + -             | · I                          | <u> </u>                    | 1<br>            | $\frac{1}{1}$   |           |
| d1   | 0                  | $1-\varphi$   | _ 1               | $1-\varphi$                               | 0  | 1                                      | 1                                    | <u> </u>        | · 1                          | 1                           | 1                | $I_1$   |           |
| e1   |                    | $1+\varphi$   | $1+\varphi$       | $-216 - 349\varphi$                       | $-2563 - 4147\varphi$                            | 0                                      | 1                                    | + -             | 1                            | 1                           | 1                | $I_1$   |           |
| f1   | 1                  | $1-\varphi$   | 1                 | $-6+4\varphi$                             | $-12 + 7\varphi$                                 | 0                                      | 1                                    |                 | 1                            | 1                           | 1                | $I_1$   |           |
| 147  | 5a                 |               |                   | 1475                                      | $a = (25 - 35\varphi) = 5a^2 \cdot 59a$          | (8 isogeny class                       | ses)                                 |                 |                              |                             |                  |   | 1475a     |
| a1   | $1+\varphi$        | $\varphi$     | 1                 | $-1-\varphi$                              | 0  | 1                                      | 1                                    |                 | 3, 1                         | 1                           | 2,1              | $III, I_1$  |           |
| b1   | $\varphi$          | 1             | <u>-</u> -        | $-15 - 22\varphi$                         | $30 + 50\varphi$                                 | 1                                      | 2                                    | i<br>i + +      | 3, 2                         | 2                           | 2, 2             | $ \tilde{I}II, \tilde{I}_2 $                            |           |
| b2   | $\varphi$          | 1             | 1                 | $-2\varphi$                               | 0  | 1                                      | 2                                    | - +             | 3, 1                         | 1                           | 2, 1             | $III, I_1$  |           |
| c1   | 1                  | $-1+\varphi$  | 0                 | $-1770 + 1094\varphi$                     | $34516 - 21331\varphi$                           | 1                                      | 1                                    | Ī — —           | 9,1                          | 1 1                         | 2, 1             | $ $ III $^*$ , $I_1$                                    |           |
| d1   | $\varphi$          | $-\varphi$    | 0                 | arphi                                     | $-3+3\varphi$                                    | 1                                      | 3                                    | Ī — —           | 8,1                          | 1                           | 3,1              | $  IV^*, I_1  $   |           |
| d2   | $\varphi$          | $-\varphi$    | 0                 | $-175 + 101\varphi$                       | $-1078 + 628\varphi$                             | 1                                      | 1                                    |                 | 8,3                          | 3                           | 1,1              | $IV^*, I_3$   |           |
| e1   | 1                  | $1+\varphi$   | $\varphi$         | $1+\varphi$                               | 0  | 1                                      | 1                                    |                 | 2, 1                         | 1                           | 1,1              | $II, I_1$   |           |
| e2   | $1+\varphi$        | $1-\varphi$   | φ                 | $-90 + 54\varphi$                         | $337 - 208\varphi$                               | 1                                      | 1                                    | <u> </u>        | 2,3                          | 3                           | 1,3              | $II, I_3$   |           |
| f1   | $1+\varphi$        | -1            | $1+\varphi$       | $-136 + 82\varphi$                        | $-669 + 412\varphi$                              | 0                                      | 1                                    | <u> </u>        | 3,1                          | 1 1                         | 2,1              | $\mid$ III, I <sub>1</sub>                              |           |
| g1   | $1+\varphi$        | -1            | $\varphi$         | $3-7\varphi$                              | $-4-2\varphi$                                    | 0                                      | 2                                    | +               | ,                            | 1                           | 2,1              | $ $ $III^*, I_1$  |           |
| g2   | $1+\varphi$        | -1            | $\varphi$         | $-47 - 32\varphi$                         | $-104 - 177\varphi$                              | 0                                      | 2                                    | ++              | 9,2                          | 2                           | 2,2              | $III^*, I_2$  |           |
| h1   | 1                  | 0             | $1 + \varphi$     | $-6-3\varphi$                             | $-11-2\varphi$                                   | 0                                      | 1                                    |                 | 9, 1                         | 1                           | 2, 1             | $\mathrm{III}^*,\mathrm{I}_1$                           |           |

|                 | $a_1$         | $a_2$         | $a_3$         | $a_4$                     | $a_6$   | r                                      | T                                       | s        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$             | $c_p$     | Kodaira                         | Isogenies      |
|-----------------|---------------|---------------|---------------|---------------------------|---|--|---|----------|------------------------------|---|-----------|---------------------------------|----------------|
|                 | _             |               |               |                           |   |  |   |          |                              |   |           |                                 |                |
| 147             | 5b            |               |               | 147                       | $75b = (10 - 35\varphi) = 5a^2 \cdot 59b$     | (8 isogeny                             | clas                                    | ses)     |                              |   |           |                                 | 1475b          |
| a1              | $\varphi$     | $-1+\varphi$  | 0             | $-3+2\varphi$             | 5-3arphi                                      | 1                                      | 1                                       |          | 3,1                          | 1                                       | 2, 1      | $III, I_1$                      |                |
| b1              | $1+\varphi$   | $1-\varphi$   | 1             | $-37 + 21\varphi$         | $80-50\varphi$                                | 1                                      | $\begin{vmatrix} 1 & 2 \end{vmatrix}$   | + +      | 3,2                          | 2                                       | 2,2       | $ $ III, $I_2$                  |                |
| b2              | $1+\varphi$   | $1-\varphi$   | 1             | $-2+\varphi$              | 0   | 1                                      | 2                                       | + -      | 3,1                          | 1                                       | 2, 1      | $III, I_1$                      |                |
| c1              | 1             | $-\varphi$    | 0             | $-676 - 1094\varphi$      | $13185 + 21331\varphi$                        | 1                                      | 1                                       | Ī — —    | 9,1                          | 1                                       | 2,1       | $   III^*, I_1$                 |                |
| d1              | $1+\varphi$   | -1            | 0             | $1-\varphi$               | $-3\varphi$                                   | 1                                      | 3                                       | j        | 8,1                          | 1                                       | 3,1       | $ V^*, I_1 $                    |                |
| d2              | $1+\varphi$   | -1            | 0             | $-74-101\varphi$          | $-450 - 628\varphi$                           | 1                                      | 1                                       |          | 8,3                          | 3                                       | 1, 1      | $IV^*, I_3$                     |                |
| e1              | 1             | $-1-\varphi$  | $\varphi$     | 1                         | -1  | 1                                      | 1                                       | Ī — —    | 2,1                          | 1                                       | 1,1       | $ $ II, I $_1$                  |                |
| e2              | $\varphi$     | 1             | $1 + \varphi$ | $-35-56\varphi$           | $129 + 207\varphi$                            | 1                                      | 1                                       |          | 2,3                          | 3                                       | 1,3       | $II,I_3$                        |                |
| f1              | $\varphi$     | $-\varphi$    | $\varphi$     | $-52-84\varphi$           | $-256 - 413\varphi$                           | 0                                      | 1                                       | Ī — —    | 3,1                          | 1                                       | 2,1       | $ $ III, $I_1$                  |                |
| g1              | $\varphi$     | $-\varphi$    | $1+\varphi$   | $-3+5\varphi$             | $-6+\varphi$                                  | 0                                      | 2                                       | <u> </u> | 9,1                          | 1                                       | 2,1       | $   \text{III}^*, \text{I}_1  $ | <u>-</u>       |
| g2              | $\varphi$     |               | $1+\varphi$   | $-78 + 30\varphi$         | $-281 + 176\varphi$                           | 0                                      | 2                                       | ++       |                              | 2                                       | 2, 2      | $\mathrm{III}^*, \mathrm{I}_2$  |                |
| h1              | 1             | 0             | $\varphi$     | $-8+2\varphi$             | $-12 + \varphi$                               | 0                                      | 1                                       | Ī — —    | 9,1                          | 1                                       | 2,1       | $  III^*, I_1$                  |                |
|                 |               |               |               |                           |   | <u>'</u>                               |   |          |                              |   |           |                                 |                |
| 147             | 6a            |               |               | 147                       | $76a = (42 - 6\varphi) = 2 \cdot 3 \cdot 41a$ | (6 isogeny                             | class                                   | ses)     |                              |   |           |                                 | 1476a          |
| a1              | $\varphi$     | $1+\varphi$   | $\varphi$     | $1+2\varphi$              | $1+\varphi$                                   | 1                                      | 2                                       |          | 2, 2, 1                      | 2, 2, 1                                 | 2, 2, 1   | $I_2, I_2, I_1$                 |                |
| a2              | $\varphi$     | $1 + \varphi$ | $\varphi$     | $-9+2\varphi$             | $5-9\varphi$                                  | 1                                      | 2                                       | ++       | 1, 4, 2                      | 1, 4, 2                                 | 1, 4, 2   | $I_1, I_4, I_2$                 |                |
| b1              | 1             | $-\varphi$    | 0             | $-40-41\varphi$           | $-91 - 175\varphi$                            | 1                                      | 2                                       | + +      | 3, 2, 4                      | 3, 2, 4                                 | 1, 2, 4   | $  I_3, I_2, I_4  $             |                |
| b2              | 1             | $-\varphi$    | 0             | -arphi                    | $-3-7\varphi$                                 | 1                                      | 2                                       |          | 6, 1, 2                      | 6, 1, 2                                 | 2, 1, 2   | $I_6, I_1, I_2$                 |                |
| c1              | $1+\varphi$   | $\varphi$     | 0             | $-42 + 23\varphi$         | $126 - 77\varphi$                             | 1                                      | 4                                       | + +      | 2, 2, 2                      | [2, 2, 2]                               | [2, 2, 2] | $  I_2, I_2, I_2  $             |                |
| c2              | $1 + \varphi$ | $\varphi$     | 0             | $-2+3\varphi$             | $2-\varphi$                                   | 1                                      | 2                                       | ++       | , ,                          | 4, 1, 1                                 | 4, 1, 1   | $I_4, I_1, I_1$                 |                |
| c3              | $\varphi$     | $-1+\varphi$  | 0             | $-4469 + 2762\varphi$     | $137474 - 84963\varphi$                       | 1                                      | 2                                       | +-       |                              | 1, 4, 1                                 | 1, 2, 1   | $I_1, I_4, I_1$                 |                |
| c4              | 1             | $-1+\varphi$  | $1+\varphi$   | $-262 - 413\varphi$       | $2949 + 4748\varphi$                          | 1                                      | 2                                       | - +      |                              | 1,1,4                                   | 1,1,2     | $ $ $I_1, I_1, I_4$             |                |
| d1              | $\varphi$     | 0             | 0             | $-22+13\varphi$           | $45-28\varphi$                                | 0                                      | 6                                       | ++       | , ,                          | 2, 3, 1                                 | 2, 3, 1   | $I_2, I_3, I_1$                 |                |
| d2              | $1+\varphi$   | 0             | 0             | $-25-31\varphi$           | $59 + 94\varphi$                              | 0                                      | 6                                       | ++       |                              | 1, 6, 2                                 | 1, 6, 2   | $I_1, I_6, I_2$                 |                |
| d3              | $1 + \varphi$ | 0             | 0             | $-140 - 196\varphi$       | $-1136 - 1792\varphi$                         | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 2                                       | ++       |                              | 6, 1, 3                                 | 6, 1, 1   | $I_6, I_1, I_3$                 |                |
| d4              | <del>φ</del>  |               | 0             | $-1472 + 868\varphi$      | $-25176 + 15424\varphi$                       | 0                                      | 2                                       | + +      |                              | 3,2,6                                   | 3, 2, 2   | $I_3, I_2, I_6$                 |                |
| e1              | 1             | $1+\varphi$   | $\varphi$     | $1+2\varphi$              | $2+3\varphi$                                  | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 5                                       |          | ٠, ٠, ٠                      | 5, 1, 1                                 | 5, 1, 1   | $I_5, I_1, I_1$                 |                |
| e2              | 1             | $1+\varphi$   |               | $-79 - 108\varphi$        | $-544 - 805\varphi$                           | 0                                      | 1                                       |          |                              | 1,5,5                                   | 1,1,5     | $I_1, I_5, I_5$                 |                |
| $\int_{co}^{c}$ | $1+\varphi$   | $1+\varphi$   | $\varphi$     | $-323-271\varphi$         | $-3115 - 3796\varphi$                         | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\begin{vmatrix} 2 \\ 2 \end{vmatrix}$  | 1        | 14, 1, 1                     | 14, 1, 1                                | 14, 1, 1  | $I_{14}, I_1, I_1$              |                |
| f2              | $\varphi$     | $-1+\varphi$  | 0             | $-159063 - 257364\varphi$ | $-46511406 - 75257037\varphi$                 | 0                                      | 2                                       | + +      | 7, 2, 2                      | 7, 2, 2                                 | 7, 2, 2   | $I_7, I_2, I_2$                 |                |
| 147             | 6b            |               |               | 147                       | $76b = (36 + 6\varphi) = 2 \cdot 3 \cdot 41b$ | (6 isogeny                             | class                                   | ses)     |                              |   |           |                                 | 14 <b>7</b> 6b |
| a1              |               | $-1+\varphi$  | 1             | $2-\varphi$               | $-1+\varphi$                                  | 1                                      | 2                                       |          | 2, 2, 1                      | 2, 2, 1                                 | 2, 2, 1   | $I_{2}, I_{2}, I_{1}$           |                |
| a2              |               | $-1+\varphi$  | 1             | $-8-\varphi$              | $3+\varphi$                                   | 1                                      | 2                                       | ++       |                              | 1, 4, 2                                 | 1, 4, 2   | $I_1, I_4, I_2$                 |                |
| b1              |               | $-1+\varphi$  | 0             | $-81 + 41\varphi$         | $-266 + 175\varphi$                           | · '<br>  1                             | $\begin{vmatrix} -2 \\ 2 \end{vmatrix}$ | + +      |                              | $\frac{1}{3}, \frac{1}{2}, \frac{1}{4}$ | 1, 2, 4   | $  I_3, I_2, I_4  $             |                |
| b2              |               | $-1+\varphi$  | 0             | $-1+\varphi$              | $-10 + 7\varphi$                              | 1                                      | 2                                       |          |                              | 6, 1, 2                                 | 2, 1, 2   | $I_6, I_1, I_2$                 |                |

|     |               |              |               |                           |   |            | 1      |          |                              | 1                           |          | T                  | T         |
|-----|---------------|--------------|---------------|---------------------------|---|------------|--------|----------|------------------------------|-----------------------------|----------|--------------------|-----------|
|     | $a_1$         | $a_2$        | $a_3$         | $a_4$                     | $a_6$   | r          | T      | s        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$    | Kodaira            | Isogenies |
| 147 | $6\mathrm{b}$ |              |               | 14                        | $76b = (36 + 6\varphi) = 2 \cdot 3 \cdot 41b$ | (6 isogeny | class  | ses)     |                              |                             |          |                    | 1476b     |
| c1  | $\varphi$     | $-1+\varphi$ | 1             | $-21-22\varphi$           | $48 + 56\varphi$                              | 1          | 4      | ++       |                              | 2, 2, 2                     | 2, 2, 2  | $I_2,I_2,I_2$      |           |
| c2  | $\varphi$     |              | 1             | $-1-2\varphi$             | 0   | 1          | 2      | ++       | 4, 1, 1                      | 4, 1, 1                     | 4, 1, 1  | $I_4, I_1, I_1$    |           |
| c3  | $1+\varphi$   | $\varphi$    | 1             | $-1707 - 2761\varphi$     | $49749 + 80494\varphi$                        | 1          | 2      | -+       | 1, 4, 1                      | 1, 4, 1                     | 1, 2, 1  | $I_1, I_4, I_1$    |           |
| c4  | 1             | $-\varphi$   | $\varphi$     | $-674 + 412\varphi$       | $7698 - 4749\varphi$                          | _   1      | 2      | + -      |                              | 1, 1, 4                     | 1,1,2    | $I_1, I_1, I_4$    |           |
| d1  | $1+\varphi$   |              | 0             | $-9-13\varphi$            | $17 + 28\varphi$                              | 0          | 6      | + +      |                              | 2, 3, 1                     | 2, 3, 1  | $I_2, I_3, I_1$    |           |
| d2  | $\varphi$     | $1-\varphi$  | 0             | $-56 + 31\varphi$         | $153 - 94\varphi$                             | 0          | 6      | ++       | 1, 6, 2                      | 1, 6, 2                     | 1, 6, 2  | $I_1, I_6, I_2$    |           |
| d3  | $\varphi$     | $1-\varphi$  | 0             | $-336 + 196\varphi$       | $-2928 + 1792\varphi$                         | 0          | 2      | ++       | 6, 1, 3                      | 6, 1, 3                     | 6, 1, 1  | $I_6, I_1, I_3$    |           |
| d4  | $1+\varphi$   | $-\varphi$   | 0             | $-604 - 868\varphi$       | $-9752 - 15424\varphi$                        | 0          | 2      | ++       |                              | 3, 2, 6                     | 3, 2, 2  | $I_3, I_2, I_6$    |           |
| e1  |               | $-1-\varphi$ | $\varphi$     | $2-\varphi$               | $3-2\varphi$                                  | 0          |        |          | 0, -, -                      | 5, 1, 1                     | 5, 1, 1  | $I_5, I_1, I_1$    |           |
| e2  |               | $-1-\varphi$ | $\varphi$     | $-188 + 109\varphi$       | $-1161 + 696\varphi$                          | 0          | 1      |          | 1, 5, 5                      | 1, 5, 5                     | 1, 1, 5  | $I_1, I_5, I_5$    |           |
| f1  | $2-\varphi$   | $2-\varphi$  | $1-\varphi$   | $-594 + 271\varphi$       | $-6911 + 3796\varphi$                         | 0          |        | 1        | 14, 1, 1                     | 14, 1, 1                    | 14, 1, 1 | $I_{14}, I_1, I_1$ |           |
| f2  | $1-\varphi$   | $-\varphi$   | 0             | $-416427 + 257364\varphi$ | $-121768443 + 75257037\varphi$                | 0          | 2      | ++       | 7, 2, 2                      | 7, 2, 2                     | 7, 2, 2  | $I_7, I_2, I_2$    |           |
| 148 | 1a            |              |               | 1                         | $1481a = (39 - \varphi) = 1481a \qquad (2)$   | isogeny o  | lasses | s)       |                              |                             |          |                    | 1481a     |
| a1  | 0             | -1           | $\varphi$     | $1-\varphi$               | 0   | 1          | 1      |          | 1                            | 1                           | 1        | $I_1$              |           |
| b1  | 0             | $1+\varphi$  | $\varphi$     | $-30 + 20\varphi$         | $73 - 45\varphi$                              | 1          | 1      | <u> </u> | 1                            | 1                           | 1        | $I_1$              | <u> </u>  |
| 148 | 1b            |              |               |                           | $1481b = (38 + \varphi) = 1481b \qquad (2)$   | isogeny c  | lasses | )        |                              |                             |          |                    | 1481b     |
| a1  | 0             | -1           | $1 + \varphi$ | $\varphi$                 | $-\varphi$                                    | 1          | 1      |          | 1                            | 1                           | 1        | $I_1$              |           |
| b1  | 0             | $-1-\varphi$ | $1+\varphi$   | $-11-18\varphi$           | $39 + 63\varphi$                              | 1          | 1      |          | 1                            | 1                           | 1        | $I_1$              |           |
| 150 | 1a            |              |               | 1                         | $501a = (37 + 4\varphi) = 19a \cdot 79b$      | (1 isogen  | y clas | ss)      |                              |                             |          |                    | 1501a     |
| a1  | $\varphi$     | $-\varphi$   | 1             | $-4+3\varphi$             | $-6+3\varphi$                                 | 0          | 2      | + -      | 2,1                          | 2,1                         | 2, 1     | $I_2, I_1$         |           |
| a2  | $\varphi$     | $-\varphi$   | 1             | $-4-2\varphi$             | $-10 + 5\varphi$                              | 0          | 2      | -+       | 4, 2                         | 4,2                         | 2, 2     | $I_4, I_2$         |           |
| 150 | 1b            |              |               | 1                         | $501b = (41 - 4\varphi) = 19b \cdot 79a$      | (1 isogen  | y clas | s)       |                              |                             |          |                    | 1501b     |
| a1  | $1+\varphi$   | -1           | 1             | $-1-4\varphi$             | $-3-3\varphi$                                 | 0          | 2      | -+       | 2, 1                         | 2,1                         | 2, 1     | $I_2, I_1$         |           |
| a2  | $1+\varphi$   | -1           | 1             | $-6+\varphi$              | $-5-5\varphi$                                 | 0          | 2      | +-       | 4, 2                         | 4, 2                        | 2, 2     | $I_4, I_2$         |           |
| 151 | 1a            |              |               | 1                         | $511a = (22 - 35\varphi) = 1511a$             | (1 isogeny | class  | s)       |                              |                             |          |                    | 1511a     |
| a1  | $1+\varphi$   | 0            | $1+\varphi$   | $-1-2\varphi$             | $-\varphi$                                    | 0          | 3      |          | 1                            | 1                           | 1        | $I_1$              |           |
| a2  | $1+\varphi$   |              | $1+\varphi$   | $-1+3\varphi$             | $-11 + 11\varphi$                             | 0          | 1      |          | 3                            | 3                           | 1        | $I_3$              |           |
| 151 | 1b            |              |               | -                         | $1511b = (13 - 35\varphi) = 1511b$            | (1 isogeny | class  | 3)       |                              |                             |          |                    | 1511b     |
| a1  | $\varphi$     | $1-\varphi$  | $\varphi$     | -1                        | 0   | 0          | 3      |          | 1                            | 1                           | 1        | $I_1$              |           |
| a2  | φ             | $1-\varphi$  | φ             | $4-5\varphi$              | $1-12\varphi$                                 | 0          | 1      |          | 3                            | 3                           | 1        | $I_3$              |           |
| 151 | 6a            |              |               | 15                        | $516a = (40 - 2\varphi) = 2 \cdot 379a $      | 6 isogeny  | class  | es)      |                              |                             |          |                    | 1516a     |
| a1  | $\varphi$     | $-1+\varphi$ | $\varphi$     | $-5+2\varphi$             | $7-5\varphi$                                  | 1          | 1      | -+       | 1, 1                         | 1,1                         | 1, 1     | $I_1, I_1$         |           |
|     |               |              |               |                           |   |            |        |          |                              |                             |          |                    |           |

|          | I           |       |                   |                   |                 |                                |   |              |  |            | 1                                      |         |                              | 1                           | I         | T   | T         |
|----------|-------------|-------|-------------------|-------------------|-----------------|--------------------------------|---|--------------|--|------------|--|---------|------------------------------|-----------------------------|-----------|---|-----------|
|          | $a_1$       | L     | $a_2$             | a                 | 13              | $a_4$                          |   | $a_6$        | r                                      |            | T                                      | s       | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$     | Kodaira   | Isogenies |
| 151      | 6a          |       |                   |                   |                 | 15                             | $516a = (40 - 2\varphi) = 2 \cdot 379a$ | (6           | isogeny cla                            | sse        | es)                                    |         |                              |                             |           |   | 1516a     |
| b1       | $1+\varphi$ | -1    | $1-\varphi$       | (                 | 0               | $-5+5\varphi$                  | -11 + 4                                 | $4\varphi$   | 1                                      |            | 1                                      |         | 10, 1                        | 10, 1                       | 2, 1      | $I_{10},I_1$  |           |
| c1       | 1           |       | $1+\varphi$       | $\frac{1}{1+4}$   | <br>φ           | $-2\varphi$                    | -1 <i>-</i>                             | $\varphi$    | 1                                      | Ī          | 1                                      | - +     | 3,1                          | 3,1                         | 3,1       | $I_3, I_1$  |           |
| d1       | :           |       | $-\varphi$        |                   |                 | $-519 + 317\varphi$            | 5203 - 3218                             | $3\varphi$   | 0                                      | Ė          | 3                                      |         | -6, 1                        | [6, 1]                      | [6, 1]    | $\overline{\mathrm{I}}_{6},\overline{\mathrm{I}}_{1}$ | <u>.</u>  |
| d2       | 1           |       | 1                 |                   | 0               | $-4526 - 7303\varphi$          | -224131 - 362693                        | $1\varphi$   | 0                                      |            | 1                                      |         | 2, 3                         | 2,3                         | 2, 3      | $I_2, I_3$  |           |
| e1       | 1           |       | 1                 | (                 | 0               | $-85-80\varphi$                |   |              | 0                                      | Ī          | 3                                      | +       | 15,1                         | 15, 1                       | 15, 1     | $I_{15}, I_1$   |           |
| e2       | $  \varphi$ |       | $-\varphi$        | $1+\varphi$       | $\varphi$       | $-99628 + 61555\varphi$        | -14250801 + 8807450                     | $9\varphi$   | 0                                      |            | 1                                      | +       | 5,3                          | 5,3                         | 5,3       | $I_5, I_3$  |           |
| f1       | 1           |       | $1-\varphi$       | 4                 | $\varphi$       | $1 - \varphi$                  |   | $-\varphi$   | 0                                      |            | 3                                      |         | 3, 1                         | 3,1                         | 3, 1      | $I_3, I_1$  |           |
| f2       | 1           |       | $1-\varphi$       | 4                 | φ               | $-19 - 31\varphi$              | -66 - 89                                | $9\varphi$   | 0                                      |            | 1                                      |         | 1,3                          | 1,3                         | 1,3       | $I_1, I_3$  |           |
| 151      | <b>6</b> b  |       |                   |                   |                 | 1                              | $516b = (38 + 2\varphi) = 2 \cdot 379b$ | (6           | isogeny clas                           | sse        | es)                                    |         |                              |                             |           |   | 1516b     |
| a1       | $1+\varphi$ | ı     | $\varphi$         | 4                 | $\varphi$       | $-2-2\varphi$                  |   | 0            | 1                                      |            | 1                                      | + -     | 1,1                          | 1,1                         | 1,1       | $I_1, I_1$  |           |
| b1       | φ           |       | -1                | (                 | 0               | $-5\varphi$                    | -7 – 4                                  | $4\varphi$   | 1                                      | Ī          | 1                                      |         | 10, 1                        | 10, 1                       | 2, 1      | $I_{10}, I_1$   | [         |
| c1       | 1           |       | $-\varphi$        | <br>4             | <br>φ           | $-1+\varphi$                   |   | -1           | 1                                      | Ī          | 1                                      | +-      | 3,1                          | 3,1                         | 3,1       | $I_3, I_1$  |           |
| d1       | $1+\varphi$ |       | -1                | ·<br>4            | <br>φ           | $-201 - 319\varphi$            | 1986 + 321                              | <br>7φ       | 0                                      | Ť          | 3                                      |         | 6, 1                         | [6, 1]                      | [6, 1]    | $\overline{I}_6, \overline{I}_1$                      |           |
| d2       | 1           |       | 1                 | (                 | 0               | $-11829 + 7303\varphi$         | -586822 + 362693                        | $1\varphi$   | 0                                      |            | 1                                      |         | 2, 3                         | 2,3                         | 2, 3      | $I_2, I_3$  |           |
| e1       | 1           |       | 1                 | (                 | 0               | $-165 + 80\varphi$             | 781 - 544                               | •            | 0                                      | Ī          | 3                                      | + -     | 15,1                         | 15, 1                       | 15, 1     | $I_{15}, I_1$   |           |
| e2       | $1+\varphi$ |       | -1                | 4                 | $\varphi_{-}$ . | $-38072 - 61557\varphi$        | -5443350 - 8807453                      | $1\varphi$   | 0                                      |            | 1                                      | + -     | 5,3                          | 5,3                         | 5, 3      | $I_5, I_3$  |           |
| f1       | 1           |       | •                 | $1+\varphi$       | •               | 0                              |   | -1           | 0                                      |            | 3                                      |         | 3, 1                         | 3,1                         | 3, 1      | $I_3, I_1$  |           |
| f2       | 1           |       | φ                 | $1+\varphi$       | $\varphi$       | $-50 + 30\varphi$              | -155 + 88                               | $3\varphi$   | 0                                      |            | 1                                      |         | 1,3                          | 1,3                         | 1,3       | $I_1, I_3$  |           |
| 151      | 9a          |       |                   |                   |                 | 15                             | $519a = (21 - 35\varphi) = 31a \cdot 7$ | (3           | 3 isogeny cla                          | sse        | es)                                    |         |                              |                             |           |   | 1519a     |
| a1       | φ           | 1     | -1                | 4                 | $\varphi$       | $-5-7\varphi$                  | -3 - '                                  | $7\varphi$   | 1                                      |            | 2                                      | - +     | 3, 2                         | 3, 2                        | 3, 2      | $I_3, I_2$  |           |
| a2       | 1           | 1     | $1+\varphi$       | $1 + \varphi$     | $\varphi$       | $-109 + 72\varphi$             | 504 - 312                               | $2\varphi$   | 1                                      | Ĺ          | 2                                      | + -     | 6, 1                         | 6,1                         | 6, 1      | $I_6, I_1$  |           |
| b1       | 0           |       | $\varphi$         | -                 | 1               | $-1-4\varphi$                  | 4 + 2                                   | $2\varphi$   | 1                                      |            | 1                                      |         | 2,3                          | 2,3                         | 2,3       | $I_2, I_3$  |           |
| c1       | $1+\varphi$ |       | $1+\varphi$       | $1+\varphi$       | $\varphi$       | $-75 + 37\varphi$              | 260 - 160                               | ,            | 0                                      | Ī          | 4                                      | ++      | 4,1                          | 4,1                         | [-4, 1]   | $I_4, I_1$  |           |
| c2       | $1+\varphi$ |       | $1+\varphi$       | ,                 | •               | $-5+2\varphi$                  | 1-6                                     | $\Im arphi$  | 0                                      |            | 4                                      | ++      | 2, 2                         | 2, 2                        | 2, 2      | $I_2, I_2$  |           |
| c3       | $1+\varphi$ | 1     | $1+\varphi$       | $1+\varphi$       | $\varphi$       | $2\varphi$                     | 1005 175                                | $\varphi$    | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ |            | $\frac{2}{2}$                          | + -     | 1, 1                         | 1,1                         | 1,1       | $I_1, I_1$  |           |
| c4       | 1           |       | $-\varphi$        | 4                 | $\varphi$       | $-134 - 219\varphi$            | -1085 - 1758                            | $S\varphi$   | 0                                      |            | 2                                      | -+      | 1,4                          | 1,4                         | 1,4       | $I_1, I_4$  |           |
| 151      | <b>9</b> b  |       |                   |                   |                 | 1!                             | $519b = (14 - 35\varphi) = 31b \cdot 7$ | (3           | isogeny clas                           | sse        | es)                                    |         |                              |                             |           |   | 1519b     |
| a1       |             |       | $1-\varphi$       |                   |                 | $-12 + 5\varphi$               | -10 + 6                                 |              | 1                                      |            | 2                                      | + -     | 3, 2                         | 3, 2                        | 3, 2      | $I_3, I_2$  |           |
| a2       |             |       | $1 - \varphi$     | $\frac{1+4}{1+4}$ | φ               | $-38 - 71\varphi$              | 230 + 383                               |              | 1                                      | . <u>ļ</u> | 2                                      | - +     | 6, 1                         | [-6, 1]                     | 6,1       | $I_6, I_1$  |           |
| b1       | 0           | 1<br> | $1 - \varphi$     | :                 | 1               | $-5+4\varphi$                  | 6 – 5                                   | . <u>-</u> - | 1                                      |            | 1                                      |         | 2,3                          | 2,3                         | 2,3       | $I_2, I_3$  |           |
| c1       | $\varphi$   |       | -                 | $1+\varphi$       |                 | $-40-36\varphi$                | 103 + 120                               |              | 0                                      |            | 4                                      | ++      | 4, 1                         | 4,1                         | 4, 1      | $I_4, I_1$  |           |
| c2       | $\varphi$   |       | -                 | $1+\varphi$       |                 | $-5-\varphi$                   | -2 +                                    |              | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ |            | 4                                      | ++      | 2, 2                         | 2, 2                        | 2, 2      | $I_2, I_2$  |           |
| c3<br>c4 | $\varphi$   |       | $arphi \ 1+arphi$ | $1+\varphi$       |                 | $-\varphi$ $-353 + 218\varphi$ | -1 - $-2843 + 175$                      | •            | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ |            | $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ | _ +<br> | 1, 1 $1, 4$                  | 1,1<br>1,4                  | 1,1 $1,4$ | $I_1, I_1$  |           |
| C4       | 1           |       | $+\varphi$        | 1 + 4             | Ψ               | $-353 \pm 218\varphi$          | -2043 + 173                             | ıΨ           | 0                                      |            | ۷                                      | + -     | 1,4                          | 1,4                         | 1,4       | $I_1, I_4$  |           |

|              | a           | $a_2$                    | $a_3$      | $a_4$               | $a_6$   | r              | T      | s           | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$                                 | Kodaira                         | Isogenies |
|--------------|-------------|--------------------------|------------|---------------------|---|----------------|--------|-------------|------------------------------|-----------------------------|---------------------------------------|---------------------------------|-----------|
| 1504         |             |                          |            |                     |   |                |        | <u>'</u>    |                              |                             |                                       |                                 | 1500      |
| 1520         |             |                          |            |                     | $20a = (36 + 8\varphi) = 2^2 \cdot 5a \cdot 19a$  | (3 isoger      | ny cla | sses)       |                              |                             |                                       |                                 | 1520a     |
| a1           |             | -1                       | 0          | $-5+3\varphi$       | 7-4arphi  | 1              | 6      | +-          | 1, 4, 2                      | 1,2                         | 1, 3, 2                               | $I_1, IV, I_2$                  |           |
| a2           | -           | $0 -1 - \varphi$         | 0          | $-21-35\varphi$     | $96 + 153\varphi$                                 | 1              | 6      | -+          | , - ,                        | 2,1                         | 2, 3, 1                               | $I_2, IV^*, I_1$                |           |
| a3           | (           | -                        | 0          | $-65 + 43\varphi$   | $-225 + 132\varphi$                               | 1              | 2      | +-          | 3, 4, 6                      | 3,6                         | 3, 3, 2                               | $I_3, IV, I_6$                  |           |
| a4           |             | ) -1                     | 0          | $-40 - 7\varphi$    | $-560 + 277\varphi$                               |                | 2      | - +         |                              | 6,3                         | [6, 3, 1]                             | $I_6, IV^*, I_3$                |           |
| b1           |             | $-\varphi$               | 0          | $-9+3\varphi$       | $-10 + 10\varphi$                                 | 0              |        | ++          | , ,                          | 2,2                         | 2, 1, 2                               | $I_2, IV, I_2$                  |           |
| b2           | -           | $-\varphi$               | 0          | $-24-32\varphi$     | $60 + 148\varphi$                                 | 0              |        | -+          | 1, 8, 4                      | 1,4                         | 1,1,2                                 | $I_1, IV^*, I_4$                |           |
| b3           | -           | $-\varphi$               | 0          | $-19-2\varphi$      | $-6-13\varphi$                                    | 0              |        | ++          | 4, 8, 1                      | 4, 1                        | 2, 1, 1                               | $I_4, IV^*, I_1$                |           |
| b4           |             | $0  1-\varphi$           | 0          | $-869 + 538\varphi$ | $-12081 + 7467\varphi$                            | 0              |        | + -         | ′_ ′                         | 1,1                         | 1,1,1                                 | $I_1, IV^*, I_1$                |           |
| c1           |             | $0 -1 + \varphi$         | 0          | $-3-2\varphi$       | $4+2\varphi$                                      | 1              |        | + -         | 1, 4, 4                      | 1,4                         | 1, 3, 4                               | $I_1, IV, I_4$                  |           |
| c2           | (           | $0 -1 + \varphi$         | 0          | $-28 - 52\varphi$   | $124 + 192\varphi$                                | 1              | 2      | -+          | 2, 8, 2                      | 2,2                         | 2, 3, 2                               | $I_2, IV^*, I_2$                |           |
| 1520         | ՈՒ          |                          |            | 16                  | $520b = (44 - 8\varphi) = 2^2 \cdot 5a \cdot 19b$ | (2:            | 1      | )           |                              |                             |                                       |                                 | 1520b     |
|              |             | 2 1                      |            |                     | , , ,   | (3 isoger      |        | <del></del> | 1 1 0                        | 1.0                         | 1.0.0                                 | T 777 T                         | 19200     |
| a1           |             | ) —1                     | 0          | $-2-3\varphi$       | $3+4\varphi$                                      |                |        | - +         | , ,                          | 1,2                         | 1,3,2                                 | $I_1, IV, I_2$                  |           |
| a2           | (           | . ,                      | 0          | $-57 + 37\varphi$   | $192 - 117\varphi$                                | 1              |        | +-          | 2, 8, 1                      | 2,1                         | [2, 3, 1]                             | $I_2, IV^*, I_1$                |           |
| a3           |             | -1                       | 0          | $-22-43\varphi$     | $-93 - 132\varphi$                                | 1              |        | - +         | , ,                          | 3,6                         | $\frac{3, 3, 2}{6, 2, 1}$             | $I_3, IV, I_6$                  |           |
| a4           |             | ) -1                     |            | $-47 + 7\varphi$    | $-283 - 277\varphi$                               | 1              |        | + -         | ´_ ´                         | 6,3                         | [6, 3, 1]                             | $I_6, IV^*, I_3$                |           |
| b1           |             | $0 - 1 + \varphi$        | 0          | $-6-3\varphi$       | $-10\varphi$                                      | 0              |        | ++          | 2, 4, 2                      | 2,2                         | 2, 1, 2                               | $I_2, IV, I_2$                  |           |
| b2           | -           | $-1+\varphi$             | 0          | $-21+2\varphi$      | $-19+13\varphi$                                   | 0              |        | ++          | 4, 8, 1                      | 4,1                         | 2, 1, 1                               | $I_4, IV^*, I_1$                |           |
| b3           | -           | $-1+\varphi$             | 0          | $-56 + 32\varphi$   | $208 - 148\varphi$                                | 0              |        | + -         | 1, 8, 4                      | 1,4                         | 1,1,2                                 | $I_1, IV^*, I_4$                |           |
| b4           |             |                          | 0          | $-331 - 538\varphi$ | $-4614 - 7467\varphi$                             | 0              |        | - +         |                              | 1,1                         | $\begin{vmatrix} 1,1,1 \end{vmatrix}$ | $I_1, IV^*, I_1$                |           |
| c1           |             | $-\varphi$               | 0          | $-5+2\varphi$       | $6-2\varphi$                                      | 1              |        | -+          |                              | 1,4                         | 1,3,4                                 | $I_1, IV, I_4$                  |           |
| c2           | (           | $-\varphi$               | 0          | $-80 + 52\varphi$   | $316 - 192\varphi$                                | 1              | 2      | + -         | 2, 8, 2                      | 2,2                         | 2, 3, 2                               | $I_2, IV^*, I_2$                |           |
| <b>152</b> 3 | 1a          |                          |            |                     | $1521a = (39) = 3 \cdot 13 $                      | 4 isogeny clas | sses)  |             |                              |                             |                                       |                                 | 1521a     |
| a1           | 1           | 1 1                      | 0          | -19                 | 22  | 0              | 4      | ++          | 4, 1                         | 4,1                         | 4, 1                                  | $I_4, I_1$                      |           |
| a2           | 1           | 1 1                      | 0          | -4                  | -5  | 0              | 4      | ++          | 2, 2                         | 2,2                         | 2,2                                   | $I_2, I_2$                      |           |
| a3           | 1           | 1 1                      | 0          | 1                   | 0   | 0              | 2      |             | 1, 1                         | 1,1                         | 1,1                                   | $I_1, I_1$                      |           |
| a4           | 1           | 1 1                      | 0          | -69                 | -252  | 0              | 2      | ++          | 1,4                          | 1,4                         | 1,4                                   | I <sub>1</sub> , I <sub>4</sub> |           |
| b1           | $1+\varphi$ | $\rho = 1 - \varphi$     | 1          | $-1-\varphi$        |   | 1              | 2      | + +         | 1, 2                         | 1,2                         | 1, 2                                  | $ I_1,I_2 $                     |           |
| b2           | $1+\varphi$ | $\rho = 1 - \varphi$     | 1          | $-16-\varphi$       | $-19 + 24\varphi$                                 | 1              | 2      | ++          | 2, 1                         | 2,1                         | 2,1                                   | $I_2, I_1$                      |           |
| c1           | <br>4       | $\rho$ 1                 | 1          | -1                  | -1  | 1              | 1 2    | + +         | 1, 2                         | 1,2                         | 1, 2                                  | $I_1, I_2$                      |           |
| c2           | 4           |                          | 1          | -16                 | $5-24\varphi$                                     | 1              | 2      | ++          | 2, 1                         | 2,1                         | 2,1                                   | $I_2, I_1$                      |           |
| d1           |             | $) -1 + \varphi$         | 1          | $-\varphi$          | $-4-5\varphi$                                     | 0              | 1      | <u> </u>    | 1,5                          | 1,5                         | 1,1                                   | $I_1, I_5$                      |           |
| 150          |             |                          |            |                     |   |                | -      |             |                              | I                           | l                                     | 1                               | 1505      |
| 152          | oa          |                          |            |                     | $1525a = (20 - 35\varphi) = 5a^2 \cdot 61a$       | (1 isoger      | ny cla | <del></del> |                              | T -                         | Ι .                                   |                                 | 1525a     |
| a1           | 4           |                          |            | $-28-26\varphi$     | $52 + 63\varphi$                                  | 1              | 4      | ++          |                              | 2,2                         | 4, 2                                  | $I_2^*, I_2$                    |           |
| a2           | 4           | •                        |            | $-453 - 426\varphi$ | $4477 + 5663\varphi$                              | 1              |        | ++          | 7, 1                         | 1,1                         | 4, 1                                  | $I_1^*, I_1$                    |           |
| a3           | 4           | •                        |            | $-3-\varphi$        | $-3-2\varphi$                                     | 1              |        | ++          | 7,1                          | 1,1                         | 4, 1                                  | $I_1^*, I_1$                    |           |
| a4           | 4           | $\rho = 1 + \varphi = 1$ | $+\varphi$ | $-3-26\varphi$      | $147 + 123\varphi$                                | 1              | 2      |             | 10, 4                        | 4,4                         | 4,4                                   | $I_4^*, I_4$                    |           |

|     |                 |                      |               |                       |   |                 | Τ.    | . 1            | - ( - )                      |                             | I      |   |           |
|-----|-----------------|----------------------|---------------|-----------------------|---|-----------------|-------|----------------|------------------------------|-----------------------------|--------|---|-----------|
|     |                 | $a_1$ $a_2$          | $a_3$         | $a_4$                 | $a_6$                                     | r               | T     |                | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$  | Kodaira   | Isogenies |
| 152 | 5b              |                      |               | 152                   | $35b = (15 - 35\varphi) = 5a^2 \cdot 61b$ | (1 isogeny cla  | ss)   |                |                              |                             |        |   | 1525b     |
| a1  | $1+\varsigma$   | $o$ $-1+\varphi$     | 0             | $-879 + 427\varphi$   | $11447 - 6543\varphi$                     | 1               | 2     | ++             | 7, 1                         | 1,1                         | 4,1    | $I_1^*, I_1$  |           |
| a2  | $1+\varsigma$   | $o -1 + \varphi$     | 0             | $-54 + 27\varphi$     | $197-118\varphi$                          | 1               | 4     |                | 8, 2                         | 2, 2                        | 4, 2   | $I_2^*, I_2$  |           |
| a3  |                 | $o -1 + \varphi$     |               | $-4+2\varphi$         | $2-3\varphi$                              | 1               | 2     |                |                              | 1, 1                        | 4,1    | $I_1^*, I_1$  |           |
| a4  | $1+\varsigma$   | $\rho$ $-1+\varphi$  | 0             | $-29 + 27\varphi$     | $327 - 153\varphi$                        | 1               | 2     |                | 10,4                         | 4,4                         | 4,4    | $I_4^*, I_4$  |           |
| 152 | 9a              |                      |               | 15296                 | $a = (16 - 35\varphi) = 11b \cdot 139a$   | (5 isogeny cla  | sses) | )              |                              |                             |        |   | 1529a     |
| a1  | 1               | $0 -1 - \varphi$     |               | $-2+3\varphi$         | $2-2\varphi$                              | 1               | 3     | 1              | 1, 1                         | 1,1                         | 1,1    | $I_1, I_1$  |           |
| a2  |                 | $0 -1 - \varphi$     | 1             | $8-7\varphi$          | $9-8\varphi$                              | 1               | 3     |                | 3, 3                         | 3,3                         | 1,3    | $I_3, I_3$  |           |
| a3  | (               | 0 1                  | 1             | $-999 - 1588\varphi$  | $-23045 - 37208\varphi$                   | 1               | 1     | _              | 9,1                          | 9,1                         | 1,1    | $I_9, I_1$  |           |
| b1  |                 | 1 0                  | 0             | $-28 + 11\varphi$     | $54 - 37\varphi$                          | 1               | 1     | -+             | 7, 1                         | 7, 1                        | 7,1    | $\mathrm{I}_7,\mathrm{I}_1$                           |           |
| c1  |                 | $1 - 1 - \varphi$    | 0             | $-1+\varphi$          | 1   | 1               | 1     | <del>- +</del> | 1,1                          | 1,1                         | 1,1    | $I_1, I_1$  |           |
| d1  | ]               | 0 -1                 | 1             | $-2-4\varphi$         | $3+6\varphi$                              | 1               | 1     |                | 1, 1                         | 1,1                         | [1, 1] | $I_1, I_1$  |           |
| e1  | $\frac{1}{1+9}$ | $\rho$ $-1+\varphi$  | 1             | 1                     | arphi                                     | 0               | 2     | Ī — —          | 1, 1                         | 1, 1                        | 1,1    | $\overline{\mathrm{I}}_{1},\overline{\mathrm{I}}_{1}$ |           |
| e2  |                 | $\rho$ $-1+\varphi$  |               | $-4-5\varphi$         | $2+3\varphi$                              | 0               | 4     | ++             | 2, 2                         | 2, 2                        | 2,2    | $I_2, I_2$  |           |
| e3  | ,               | $\rho$ $-\varphi$    | 0             | $-68 + 39\varphi$     | $228-142\varphi$                          | 0               | 2     | ++             | 1, 4                         | 1,4                         | 1,2    | $I_1, I_4$  |           |
| e4  | $ 1+\zeta$      | $\rho$ $-1+\varphi$  | 1             | $-69-80\varphi$       | $238 + 425\varphi$                        | 0               | 2     | ++             | 4, 1                         | 4, 1                        | 2,1    | $I_4, I_1$  |           |
| 152 | <b>9</b> b      |                      |               | 1529                  | $b = (19 - 35\varphi) = 11a \cdot 139b$   | (5 isogeny clas | sses) | )              |                              |                             |        |   | 1529b     |
| a1  |                 | $0  1+\varphi$       | 1             | $-\varphi$            | 0   | 1               | 3     |                | 1, 1                         | 1,1                         | 1,1    | $I_1, I_1$  |           |
| a2  |                 | $0  1 + \varphi$     |               | 9arphi                | $1+16\varphi$                             | 1               | 3     |                | 3, 3                         | 3,3                         | 1,3    | $I_3, I_3$  |           |
| a3  | (               | 0 1                  |               | $-2587 + 1588\varphi$ | $-60253 + 37208\varphi$                   | 1               | 1     |                | 9, 1                         | 9, 1                        | 1,1    | $I_9, I_1$  |           |
| b1  |                 | 1 0                  | 0             | $-17-11\varphi$       | $17 + 37\varphi$                          | 1               | 1     | + -            | 7, 1                         | 7, 1                        | 7,1    | $I_7, I_1$  |           |
| c1  |                 | $1 + \varphi$        | 1             | $-1+\varphi$          | 0   | 1               | 1     | + -            | 1, 1                         | 1,1                         | 1,1    | $I_1, I_1$  |           |
| d1  |                 | 0 -1                 | 1             | $-6+4\varphi$         | $9-6\varphi$                              | 1               | 1     |                | 1, 1                         | 1,1                         | 1,1    | $I_1, I_1$  |           |
| e1  | <u>-</u>        | $\rho$ $1+\varphi$   | $\varphi$     | 1+arphi               | 1   | 0               | 2     | i              | 1, 1                         | 1,1                         | 1,1    | $\overline{\mathrm{I}}_{1},\overline{\mathrm{I}}_{1}$ |           |
| e2  | 1               | $o  1 + \varphi$     | -             | $-9+6\varphi$         | $10-7\varphi$                             | 0               | 4     | 1              |                              | 2, 2                        | 2,2    | $I_2, I_2$  |           |
| e3  | $1+\varsigma$   | o $-1$               | 0             | $-29-39\varphi$       | $86 + 142\varphi$                         | 0               | 2     | ++             | 1, 4                         | 1,4                         | 1, 2   | $I_1, I_4$  |           |
| e4  | ς γ             | $\rho$ 1 + $\varphi$ | $\varphi$     | $-149 + 81\varphi$    | $743 - 494\varphi$                        | 0               | 2     | ++             | 4, 1                         | 4, 1                        | 2,1    | $I_4, I_1$  |           |
| 152 | 9c              |                      |               | 1529                  | $c = (42 - 5\varphi) = 11a \cdot 139a$    | (2 isogeny clas | sses) |                |                              |                             |        |   | 1529c     |
| a1  |                 | 1 1                  | ,             | $-1-\varphi$          | 0   | 1               | 3     | T              | 1,1                          | 1,1                         | 1,1    | $I_1, I_1$  |           |
| a2  | <u> </u>        | 1 1                  | '             | $9+4\varphi$          | $3-2\varphi$                              | 1               | 1     | _              | 3,3                          | 3,3                         | 1,3    | $I_3, I_3$  |           |
| b1  |                 |                      | $1 + \varphi$ | $-5+2\varphi$         | $5-4\varphi$                              | 1               | 2     |                |                              | 1,1                         | 1,1    | $I_1, I_1$  |           |
| b2  | ١ ٧             | $\rho$ $-1-\varphi$  | $1+\varphi$   | $-17-27\varphi$       | $60 + 96\varphi$                          | 1               | 2     | -+             | 2, 2                         | 2,2                         | 2,2    | $I_2, I_2$  |           |
| 152 | 9d              |                      |               | 1529                  | $0d = (37 + 5\varphi) = 11b \cdot 139b$   | (2 isogeny clas | sses) |                |                              |                             |        |   | 1529d     |
| a1  |                 |                      | $1+\varphi$   | -2                    | $-\varphi$                                | 1               | 3     |                | 1, 1                         | 1,1                         | 1,1    | $I_1, I_1$  |           |
| a2  |                 | 1 1                  | $1 + \varphi$ | $13-5\varphi$         | $1 + \varphi$                             | 1               | 1     |                | 3, 3                         | 3,3                         | 1,3    | $I_3, I_3$  |           |
| •   |                 |                      |               |                       |   |                 |       | •              |                              |                             |        |   |           |

|      | $a_1$       | $a_2$         | $a_3$         | $a_4$                     | $a_6$  | r               | T    | s   | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$ | Kodaira   | Isogenies |
|------|-------------|---------------|---------------|---------------------------|--|-----------------|------|-----|------------------------------|-----------------------------|-------|---|-----------|
| 1529 | 9d          |               |               | 1529d                     | $= (37 + 5\varphi) = 11b \cdot 139b \tag{2}$   | 2 isogeny class | ses) |     |                              |                             |       |   | 1529d     |
| b1   | 1           | -1            | $\varphi$     | $-2-3\varphi$             | $2+3\varphi$                                   | 1               | 2    | ++  | 1, 1                         | 1,1                         | 1,1   | $I_1, I_1$  |           |
| b2   | $1+\varphi$ | 1             | 1             | $-43 + 27\varphi$         | $114 - 70\varphi$                              | 1               | 2    | + - | 2, 2                         | 2, 2                        | 2,2   | $I_2, I_2$  |           |
| 1539 | 9a          |               |               | 1539                      | $9a = (9 - 36\varphi) = 3^2 \cdot 19a \tag{1}$ | l isogeny class | s)   |     |                              |                             |       |   | 1539a     |
| a1   | 1           | -1            | $1 + \varphi$ | $-63161 - 102101\varphi$  | $11625398 + 18810017\varphi$                   | 1               | 6    | + + | 3,8                          | 3, 2                        | 3, 4  | $I_3, I_2^*$                                      |           |
| a2   | $1+\varphi$ | 1             | 0             | $-546 - 948\varphi$       | $9763 + 15407\varphi$                          | 1               | 6    |     | 6, 7                         | 6, 1                        | 6,4   | $I_6, I_1^*$                                      |           |
|      | $1+\varphi$ | 1             | 0             | $-186-138\varphi$         | $475 + 1457\varphi$                            | 1               | 2    | ++  | 1, 12                        | 1,6                         | 1,4   | $I_1, I_6^*$                                      |           |
| a4   | $1+\varphi$ | 1             | 0             | $-6-3\varphi$             | $16 + 44\varphi$                               | 1               | 2    |     | 2,9                          | 2,3                         | 2,4   | $I_2, I_3^*$                                      |           |
| 1539 | 9b          |               |               | 1539                      | $b = (27 - 36\varphi) = 3^2 \cdot 19b $        | 1 isogeny clas  | s)   |     |                              |                             |       |   | 1539b     |
| a1   | 1           | -1            | $\varphi$     | $-165261 + 102100\varphi$ | $30435416 - 18810018\varphi$                   | 1               | 6    | ++  | 3, 8                         | 3, 2                        | 3,4   | $I_3, I_2^*$                                      |           |
| a2   | •           | $-1-\varphi$  | $\varphi$     | $-326 + 139\varphi$       | $2257 - 1596\varphi$                           | 1               | 2    | ++  | 1,12                         | 1, 6                        | 1,4   | $I_1, I_6^*$                                      |           |
| a3   | ,           | $-1-\varphi$  | $\varphi$     | $-11+4\varphi$            | $70-48\varphi$                                 | 1               | 2    |     | 2,9                          | 2, 3                        | 2,4   | $I_2, I_3^*$                                      |           |
| a4   | φ           | $-1-\varphi$  | φ             | $-1496 + 949\varphi$      | $26665 - 16356\varphi$                         | 1               | 6    |     | 6, 7                         | 6, 1                        | 6,4   | $I_6, I_1^*$                                      |           |
| 1549 | 9a          |               |               | 1549                      |  | sogeny classes  | s)   |     |                              |                             |       | 1   | 1549a     |
| a1   | $\varphi$   | $1+\varphi$   | $1+\varphi$   | $-1+\varphi$              | -1   | 1               | 1    | + - | 1                            | 1                           | 1     | $I_1$   |           |
| b1   | 1           | $\varphi$     | $1+\varphi$   | $-82 + 50\varphi$         | $343 - 213\varphi$                             | 1               | 1    | + - | 1                            | 1                           | 1     | $I_1$   |           |
| 1549 | 9b          |               |               | 154                       | $9b = (38 + 3\varphi) = 1549b$ (2 i            | sogeny classes  | s)   |     |                              |                             |       |   | 1549b     |
| a1   | $1+\varphi$ | $-1+\varphi$  | 0             | 0                         | $-\varphi$                                     | 1               | 1    | -+  | 1                            | 1                           | 1     | $I_1$   |           |
| b1   | 1           | $1-\varphi$   | φ             | $-31 - 51\varphi$         | $131 + 212\varphi$                             | 1               | 1    | -+  | 1                            | 1                           | 1     | $I_1$   |           |
| 1555 | 5a          |               |               | 1555a                     | $a = (37 + 6\varphi) = 5a \cdot 311b \tag{3}$  | isogeny class   | es)  |     |                              |                             |       |   | 1555a     |
| a1   | 0           | 1             | $\varphi$     | $-7+4\varphi$             | $6-4\varphi$                                   | 1               | 1    | + - | 1, 2                         | 1, 2                        | 1,2   | $I_1, I_2$  |           |
| b1   | 0           | $-1-\varphi$  | $\varphi$     | $-7-6\varphi$             | $13 + 23\varphi$                               | 1               | 1    | + - | 1,8                          | 1,8                         | 1,8   | $\overline{\mathbf{I}}_1,\overline{\mathbf{I}}_8$ |           |
| c1   | 1           | $1+\varphi$   | $\varphi$     | 2arphi                    | $1+\varphi$                                    | 0               | 4    | + - | 1, 2                         | 1, 2                        | 1,2   | $I_1, I_2$  |           |
| c2   | 1           | $1 + \varphi$ | $\varphi$     | $-5-3\varphi$             | $-1+2\varphi$                                  | 0               | 4    | + + | 2, 4                         | 2,4                         | 2,2   | $I_2, I_4$  |           |
| c3   | 1           | $1+\varphi$   | $\varphi$     | $-85-58\varphi$           | $106 + 406\varphi$                             | 0               | 2    | ++  | 1,8                          | 1,8                         | 1,2   | $I_1, I_8$  |           |
| c4   | 1           | $1+\varphi$   | φ             | $-5-28\varphi$            | $-56 - 78\varphi$                              | 0               | 2    | -+  | 4,2                          | 4,2                         | 2,2   | $I_4, I_2$  |           |
| 1555 | 5b          |               |               | 1555b                     | $= (43 - 6\varphi) = 5a \cdot 311a \qquad (3$  | isogeny class   | es)  |     |                              |                             |       |   | 1555b     |
| a1   | 0           | 1             | $1 + \varphi$ | $-3-4\varphi$             | $2+3\varphi$                                   | 1               | 1    | - + | 1, 2                         | 1, 2                        | 1, 2  | $I_1,I_2$   |           |
| b1   | 0           | $1+\varphi$   | $1+\varphi$   | $-14 + 8\varphi$          | $22-17\varphi$                                 | 1               | 1    | - + | 1,8                          | 1,8                         | 1,8   | $\overline{I}_1, \overline{I}_8$                  |           |
| c1   | 1           | $-1-\varphi$  | $\varphi$     | $1-\varphi$               | 1  | 0               | 4    | _ + | 1, 2                         | 1, 2                        | 1,2   | $I_1, I_2$  | [         |
| c2   | 1           | $-1-\varphi$  | $\varphi$     | $-9+4\varphi$             | $10-6\varphi$                                  | 0               | 4    | ++  | 2, 4                         | 2, 4                        | 2,2   | $I_2, I_4$  |           |
| c3   |             | $-1-\varphi$  | arphi         | $-144 + 59\varphi$        | $656-465\varphi$                               | 0               | 2    | ++  | 1, 8                         | 1,8                         | 1,2   | $I_1, I_8$  |           |
| c4   | 1           | $-1-\varphi$  | $\varphi$     | $-34 + 29\varphi$         | $-100 + 49\varphi$                             | 0               | 2    | + - | 4, 2                         | 4, 2                        | 2,2   | $I_4, I_2$  |           |

|          | $a_1$                         | $a_2$          | $a_3$         | $a_4$   | $a_6$   | r                                      |                                      | $s \operatorname{ord}(\Delta)$                               | $\operatorname{ord}_{-}(j)$                        | $c_p$  | Kodaira  | Isogenies |
|----------|-------------------------------|----------------|---------------|---|---|--|--------------------------------------|--|--|--|--|-----------|
|          |                               |                |               | · · · · · · · · · · · · · · · · · · ·                 |   |  | ' '                                  |  | (3)  | P  |  |           |
| 155      | 6a                            |                |               | 155   | $66a = (26 - 36\varphi) = 2 \cdot 389a$                       | (2 isogeny                             | class                                | es)  |  |  |  | 1556a     |
| a1       | $\varphi$                     | $1+\varphi$    | $\varphi$     | $-20 + 14\varphi$                                     | $-46 + 28\varphi$   |  | 1                                    | +-5,1  | 5,1  | 1,1  | $I_5, I_1$   |           |
| b1       | <u>:</u>                      | <u>-</u><br>-1 |               | $-1-\varphi$  |   |  | -¦<br>  1                            | $\begin{array}{cccccccccccccccccccccccccccccccccccc$         | 1,1  | 1,1  | $  I_1, I_1  $   |           |
|          |                               |                | - ' 7         | <u> </u>  |   |  |                                      |  |  |  | -17-1  |           |
| 155      | 6b                            |                |               | 155   | $56b = (10 - 36\varphi) = 2 \cdot 389b$                       | (2 isogeny                             | class                                | es)  |  |  |  | 1556b     |
| a1       | $\frac{1+\varphi}{1+\varphi}$ | $-1 + \omega$  | 1             | $-7-13\varphi$  | $\frac{-24 - 35\varphi}{}$                                    | ·                                      | 1                                    | -+5,1  | 5,1  | 1, 1   | $I_5, I_1$   |           |
| b1       | 1                             | -1             |               | -1  | $1-\varphi$   | :                                      | -!                                   | $\begin{vmatrix} - & - & - & - & - & - & - & - & - & - $     | 1,1  | 1,1  | $  I_1, I_1  $   |           |
| 01       |                               |                | φ             | 1   | Ι Ψ   | 1                                      | 1                                    | 1, 1, 1  | 1,1  | 1,1  | 11,11  |           |
| 157      | 1a                            |                |               | 15  | $71a = (25 - 36\varphi) = 1571a $                             | 2 isogeny                              | classe                               | s)   |  |  |  | 1571a     |
| a1       | $\varphi$                     | $-\varphi$     | 1             | $-\varphi$  | 0   | 1                                      | 1                                    | -+ 1   | 1  | 1  | $I_1$  |           |
| b1       | 1                             | -arphi         | $\varphi$     | $-23 + 12\varphi$                                     | $-55 + 33\varphi$   | 0                                      | <br>  1                              | <u>-</u> 1   | 1  | 1  | $  I_1 $   |           |
|          |                               | ,              | ,             | · ,   |   |  |                                      |  |  | 1  | 1  |           |
| 157      | 1h                            |                |               | 15  | $571b = (11 - 36\varphi) = 1571b $                            | 2 isogeny                              | classo                               | a)   |  |  |  | 1571b     |
| a1       | $\frac{1}{1+\varphi}$         | -1             | 1             |   | $\frac{0.000 - (11 - 30\varphi) - 13710}{0}$                  |  | _                                    | ı´ .   | 1  | 1  | $I_1$  | 10110     |
|          | <u>-</u>                      |                |               |   |   |  | -'                                   |  |  | <del>-</del>                                       |  |           |
| b1       | 1                             | $-1+\varphi$   | $1+\varphi$   | $-11-13\varphi$                                       | $-22 - 34\varphi$   | 0                                      | 1                                    | +- 1   | 1  | 1  | $I_1$  |           |
| 158      | Λa                            |                |               | 1500  | 0- (20   4-)  | (C:                                    | 1                                    | )  |  |  |  | 1580a     |
|          |                               | 1 :            | 1 .           |   | $0a = (38 + 4\varphi) = 2 \cdot 5a \cdot 79a$                 | (6 isogen                              |                                      |  | 0.0.1  | 0.0.1  |  | 1960a     |
| a1<br>a2 | 1<br>1                        | -1 1 $-1$ 1    |               | $-2 \\ -32 + 20\varphi$                               | $\begin{array}{c} 2 - 2\varphi \\ 90 - 56\varphi \end{array}$ |  | $\begin{vmatrix} 2\\2 \end{vmatrix}$ | $\begin{vmatrix} -+ & 2, 2, 1 \\ +- & 1, 4, 2 \end{vmatrix}$ | $\begin{bmatrix} 2, 2, 1 \\ 1, 4, 2 \end{bmatrix}$ | 2, 2, 1 $1, 2, 2$                                  | $I_2, I_2, I_1 \\ I_1, I_4, I_2$                                       |           |
| b1       |                               | $-1+\varphi$   | $\frac{1}{0}$ | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | $5-3\varphi$  |  | -!                                   | $\begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 $     | $\frac{1}{4}, \frac{1}{2}, \frac{1}{1}$            | $\begin{bmatrix} 1, 2, 2 \\ 4, 2, 1 \end{bmatrix}$ | $\begin{array}{c c} I_1, I_4, I_2 \\ \hline I_4, I_2, I_1 \end{array}$ | <u> </u>  |
| b2       |                               | $-1+\varphi$   | 0             | $-63 + 21\varphi$                                     | $229 - 111\varphi$  |  |                                      | ++2,1,2  | 2, 1, 2  | 2, 1, 2  | $I_2, I_1, I_2$  |           |
| c1       | $1+\varphi$                   |                | $\varphi$     | $-26 - 17\varphi$                                     | $1+66\varphi$   |  |                                      | $\begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 $     | $\frac{1}{4}, \frac{1}{2}, \frac{1}{3}$            | $\frac{1}{4}, \frac{1}{2}, \frac{1}{3}$            | $  I_4, I_2, I_3  $  |           |
| c2       | $1+\varphi$                   | ,              | φ             | $-246 + 123\varphi$                                   | $-1451 + 990\varphi$  |  | 6                                    | +-2,1,6  | 2, 1, 6  | 2, 1, 6  | $I_2, I_1, I_6$  |           |
| c3       | arphi                         | -arphi :       |               | $-8114 + 5001\varphi$                                 | $-332283 + 205350\varphi$                                     | 0                                      | 2                                    | -+12,6,1   | 12, 6, 1   | 12, 2, 1   | $I_{12}, I_6, I_1$   |           |
| c4       | 1                             | $1-\varphi$ :  | $1+\varphi$   | $-889174 + 549544\varphi$                             | $-379937519 + 234814293\varphi$                               | 0                                      | 2                                    | +-6,3,2  | 6, 3, 2  | 6, 1, 2  | $I_6, I_3, I_2$  |           |
| d1       | $1+\varphi$                   | $-\varphi$     | 1             | $1-23\varphi$   | $23 + 34\varphi$  |  | 6                                    | -+6,4,1  | [6, 4, 1]  | [6, 2, 1]  | $  I_6, I_4, I_1  $  |           |
| d2       | 1                             | 0              | 1             | $-1065 - 1708\varphi$                                 | $25108 + 40628\varphi$  | 0                                      | 6                                    | + + 3, 2, 2  | 3, 2, 2  | 3, 2, 2  | $I_3, I_2, I_2$  |           |
| d3       | $1 + \varphi$                 | $-\varphi$     | 1             | $-2679 + 1307\varphi$                                 | $-56517 + 38006\varphi$                                       | 0                                      | 2                                    | + + 1, 6, 6  | 1, 6, 6  | 1, 2, 6  | $I_1, I_6, I_6$  |           |
| d4       | $1+\varphi$                   | $-\varphi$     | 1             | $-179 + 57\varphi$                                    | $-1017 + 506\varphi$  | 0                                      | 2                                    | -+ 2, 12, 3  | 2, 12, 3   | [2, 2, 3]  | $I_2, I_{12}, I_3$   |           |
| e1       | $\varphi$                     | $1 + \varphi$  | 1             | $-18-3\varphi$  | 23  | 0                                      | 6                                    | -+2,2,1  | 2, 2, 1  | 2, 2, 1  | $I_2,I_2,I_1$  |           |
| e2       | 1                             | 1              | 1             | $-1313 + 809\varphi$                                  | $20954 - 12950\varphi$  | 0                                      | 6                                    | + + 1, 1, 2  | 1, 1, 2  | 1, 1, 2  | $I_1, I_1, I_2$  |           |
| e3       | $\varphi$                     | $1+\varphi$    | 1             | $-8-28\varphi$  | $29 - 110\varphi$   | 0                                      | 6                                    | -+6,6,3  | 6, 6, 3  | 6, 2, 3  | $I_6, I_6, I_3$  |           |
| e4       | $\varphi$                     | $1+\varphi$    | 1             | $-408 - 228\varphi$                                   | $-1411 - 4830\varphi$   | 0                                      | 6                                    | ++3,3,6  | 3, 3, 6  | 3, 1, 6  | $I_3, I_3, I_6$  |           |
| e5       | $1+\varphi$                   | -1             | -             | $-7486 - 12094\varphi$                                | $-475203 - 768665\varphi$                                     | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 2                                    | -+2,18,1   | 2, 18, 1   | 2, 2, 1  | $I_2, I_{18}, I_1$   |           |
| e6       | 1                             | arphi .        | $1+\varphi$   | $-819998 - 1326668\varphi$                            | $-544089077 - 880354256\varphi$                               | 0                                      | 2                                    | ++1,9,2  | 1, 9, 2  | 1, 1, 2  | $I_1, I_9, I_2$  |           |

|                 | $a_1$   | $a_2$  | $a_3$              | $a_4$  | $a_6$   | r                                      | T  | s ord   | $(\Delta) \mid \operatorname{ord}_{-}(j)$ | $c_p$   | Kodaira  | Isogenies |
|-----------------|---|--|--------------------|--|---|--|--|---|---|---|--|-----------|
| 1 20            | 0   |  |                    |  |   |  |  |   |   |   |  | 1500      |
| 158             | ua  |  |                    |  | ( ) (   | isoger                                 | ıy cla   |   |   |   |  | 1580a     |
| f1              | $\varphi$   | 1  | $\varphi$          | $2-6\varphi$   | $12-11\varphi$  | 0                                      | 8  | -+8,4   |   | 8, 4, 1   | $I_8, I_4, I_1$  |           |
| f2              | $\varphi$   | 1  | $\varphi$          | $-78-6\varphi$                                       | $44-251\varphi$   | 0                                      | 8  | + + 4.8   | 1 1 1                                     | 4, 8, 2   | $I_4, I_8, I_2$  |           |
| f3              | $\varphi$   | 1  | $\varphi$          | $-858 + 494\varphi$                                  | $10592 - 6891\varphi$                                       | 0                                      | 4  | +-2,1   |   | 2, 16, 1  | $I_2, I_{16}, I_1$   |           |
| f4              | $1+\varphi$                                       | $1+\varphi$  | $\varphi$          | $-2672 - 4261\varphi$                                | $-103485 - 167306\varphi$                                   | 0                                      | 4  | ++2,4   |   | 2, 4, 2   | $I_2, I_4, I_4$  |           |
| f5              | $\varphi$   | $-1+\varphi$   | 0                  | $-1988320 - 3217173\varphi$                          | $-2053491225 - 3322618594\varphi$                           | 0                                      | 2  | -+1,2   | 1 1 1                                     | 1, 2, 2   | $I_1, I_2, I_2$  |           |
| f6              | φ   | 1  | φ                  | $-3328 + 1194\varphi$                                | $-79056 + 35959\varphi$                                     | 0                                      | 2  | +-1,2   | 8 1,2,8                                   | 1,2,2   | $I_1, I_2, I_8$  |           |
| 158             | 0b  |  |                    | 1580   | $b = (42 - 4\varphi) = 2 \cdot 5a \cdot 79b \tag{6}$        | isogen                                 | y cla  | sses)   |   |   |  | 1580b     |
| a1              | 1   | -1   | $\varphi$          | $-1-\varphi$   | $1+\varphi$   | 1                                      | 2  | +-2,2   | 1, 1  2, 2, 1                             | 2,2,1   | $I_2, I_2, I_1$  |           |
| a2              | 1   | -1   | $\varphi$          | $-11-21\varphi$                                      | $35 + 55\varphi$  | 1                                      | 2  | -+ 1, 4   |   | 1, 2, 2   | $I_1, I_4, I_2$  |           |
| b1              | $1+\varphi$                                       | $\varphi$  | · <del>'</del> - · |  | 1   | 1                                      | 2  | + - 4, 2  |   | 4,2,1   | $I_4, I_2, I_1$  | <u>-</u>  |
| b2              | $1+\varphi$                                       | $\varphi$  | 1                  | $-42-20\varphi$                                      | $97 + 48\varphi$  | 1                                      | 2  | + + 2, 1  |   | 2, 1, 2   | $I_2, I_1, I_2$  |           |
| c1              |   | $\frac{r}{0}$  | <del>-</del><br>1  | $-21 + 22\varphi$                                    | $57 - 34\varphi$  | 0                                      | 6  | + - 6, 4  |   | 6,2,1   | $I_6, I_4, I_1$  |           |
| c2              | $\begin{array}{c c} & \varphi \\ & 1 \end{array}$ | 0  | 1                  | $-21 + 22\varphi$ $-2773 + 1708\varphi$              | $65736 - 40628\varphi$                                      |  | 6  | $\begin{vmatrix} + & 0, 4 \\ + & 3, 2 \end{vmatrix}$      |   | $\begin{bmatrix} 0, 2, 1 \\ 3, 2, 2 \end{bmatrix}$      | $I_{3}, I_{2}, I_{2}$  |           |
| c3              |   | 0  | 1                  | $-1371 - 1308\varphi$                                | $-18511 - 38006\varphi$                                     | 0                                      | 2  | + + 1,6   |   | 1, 2, 6   | $I_1, I_6, I_6$  |           |
| c4              | $\varphi$   | 0  | 1                  | $-121 - 58\varphi$                                   | $-511 - 506\varphi$   | 0                                      | 2  | $\begin{vmatrix} + & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 &$ |   | $\begin{array}{ c c c } 1, 2, 0 \\ 2, 2, 3 \end{array}$ | $I_1, I_6, I_6$<br>$I_2, I_{12}, I_3$                                |           |
|                 | $\varphi$   |  |                    |  |   |  | <del>-</del>                                   | .' ' - '  | -''                                       |   |  |           |
| $\frac{d1}{d2}$ | $\varphi$   | $1 + \varphi$ $1 + \varphi$                            | $0 \\ 0$           | $-42 + 18\varphi$                                    | $84 - 92\varphi$ $-584 - 1236\varphi$                       | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 6  | $\begin{vmatrix} + - & 4, 2 \\ - + & 2, 1 \end{vmatrix}$  |   | $\begin{vmatrix} 4, 2, 3 \\ 2, 1, 6 \end{vmatrix}$      | $I_4, I_2, I_3$  |           |
| d3              | $\varphi$   | $1+\varphi$ $-1$                                       |                    | $-122 - 122\varphi$ $-3112 - 5003\varphi$            | $-384 - 1230\varphi$ $-126932 - 205351\varphi$              | 0                                      | 2  | $\begin{vmatrix} -+ & 2, 1 \\ +- & 12, \end{vmatrix}$     |   | $\begin{vmatrix} 2,1,6\\12,2,1 \end{vmatrix}$           | $I_2, I_1, I_6$  |           |
| d4              | $1+\varphi$                                       |  | $\varphi$          | $-3112 - 5003\varphi$ $-339629 - 549545\varphi$      | $-120932 - 203331\varphi$ $-145123225 - 234814294\varphi$   | 0                                      | $\frac{2}{2}$                                  | $\begin{vmatrix} + & 12, \\ - & 6, 3 \end{vmatrix}$       |   | $\begin{bmatrix} 12, 2, 1 \\ 6, 1, 2 \end{bmatrix}$     | $ \begin{vmatrix} I_{12}, I_6, I_1 \\ I_6, I_3, I_2 \end{vmatrix} $  |           |
|                 | <u> </u>  | <del>-</del> -   | $\varphi$          |  |   |  | <u>-</u>                                       | .'  |   |   |  |           |
| e1              | $1+\varphi$                                       | $-1+\varphi$   | -                  | $-23+3\varphi$                                       | $49-23\varphi$  | 0                                      | 6  | +-2,2   |   | 2, 2, 1   | $I_2, I_2, I_1$  |           |
| e2              | 1 1 1   | 1  | 1                  | $-504 - 809\varphi$                                  | $8004 + 12950\varphi$                                       | 0                                      | 6  | + + 1,1   |   | 1, 1, 2   | $I_1, I_1, I_2$  |           |
| e3              |   | $-1+\varphi$   | •                  | $-38+28\varphi$                                      | $-15 + 72\varphi$   | 0                                      | 6  | +-6,6   |   | 6, 2, 3   | $I_6, I_6, I_3$  |           |
| e4 $ e5$        |   | $-1+\varphi$   | •                  | $-638 + 228\varphi \\ -19578 + 12092\varphi$         | $-5375 + 4192\varphi$ $-1243867 + 768664\varphi$            | 0                                      | $\frac{0}{2}$                                  | + + 3,3   |   | $\frac{3,1,6}{2,2,1}$                                   | $I_3, I_3, I_6$  |           |
| e6              | $\begin{array}{c c} & \varphi \\ & 1 \end{array}$ | $ \begin{array}{c} -\varphi \\ 1-\varphi \end{array} $ | $\varphi$          | $-19578 + 12092\varphi$ $-2146665 + 1326667\varphi$  | $-1243807 + 708004\varphi$ $-1424443332 + 880354255\varphi$ | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\frac{2}{2}$                                  | $\begin{vmatrix} + - & 2, 1 \\ + + & 1, 9 \end{vmatrix}$  |   | $\begin{vmatrix} 2, 2, 1 \\ 1, 1, 2 \end{vmatrix}$      | $ \begin{array}{c c} I_2, I_{18}, I_1 \\ I_1, I_9, I_2 \end{array} $ |           |
|                 |   |  | $\varphi$          |  |   |  | <u>'</u>                                       | .' ' - '  |   |   |  |           |
| f1              | $1+\varphi$                                       | $1-\varphi$  | •                  | $-4+4\varphi$  | $1+10\varphi$   | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 8  | +-8,4   |   | 8, 4, 1   | $I_8, I_4, I_1$  |           |
| f2              | $1+\varphi$                                       | $1-\varphi$  |                    | $-84+4\varphi$                                       | $-207 + 250\varphi$   | 0                                      | 8  | ++ 4,8  |   | 4, 8, 2   | $I_4, I_8, I_2$  |           |
| f3              | $1+\varphi$                                       | •  | $1+\varphi$        | $-364 - 496\varphi$                                  | $3701 + 6890\varphi$  | 0                                      | 4  | -+2,1   |   | 2, 16, 1  | $I_2, I_{16}, I_1$   |           |
| f4<br>f5        | $\varphi$   | $\varphi$  | $\varphi$          | $-6936 + 4263\varphi$ $-5205493 + 3217174\varphi$    | $-259594 + 160371\varphi$                                   | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{array}{ c c }\hline 4\\ 2 \end{array}$ | + + 2,4   |   | $\begin{bmatrix} 2, 4, 2 \\ 1, 2, 2 \end{bmatrix}$      | $I_2, I_4, I_4$  |           |
| f6              |   | $\varphi$  |                    | $-3203493 + 3217174\varphi$<br>$-2134 - 1196\varphi$ | $-5372892646 + 3320630274\varphi \\ -43097 - 35960\varphi$  |  | $\frac{2}{2}$                                  |   |   | 1, 2, 2   | $I_1, I_2, I_2$  |           |
| 10              | $1+\varphi$                                       | $1-\varphi$  | $1+\varphi$        | $-2134 - 1190\varphi$                                | $-45097 - 35900\varphi$                                     | 0                                      |  | -+1,2   | 8 1,2,8                                   | 1,2,2   | $I_1, I_2, I_8$  |           |
| 158             | 4a  |  |                    | 15846  | $a = (12 - 36\varphi) = 2^2 \cdot 3 \cdot 11a \tag{3}$      | 3 isoge                                | nv cla   | asses)  |   |   |  | 1584a     |
| a1              |   | $-1+\varphi$   | 0                  | $-6+\varphi$   | $\frac{1-(12-60\varphi)-2-6-11\alpha}{8-6\varphi}$          |  | Ť.   | · · ·   | , 2 2, 2                                  | 2, 3, 2   | $I_2, IV, I_2$   |           |
| a1<br>a2        |   | $-1 + \varphi$<br>$-\varphi$                           | 0                  | $-6 + \varphi$ $-592 + 368\varphi$                   | $6-6\varphi$ $6412 - 3964\varphi$                           | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | $\begin{vmatrix} 4\\2 \end{vmatrix}$           | $\begin{vmatrix} + + & 2,4 \\ + - & 1,8 \end{vmatrix}$    |   | $\begin{bmatrix} 2, 3, 2 \\ 1, 3, 1 \end{bmatrix}$      | $\begin{bmatrix} I_2, IV, I_2 \\ I_1, IV^*, I_1 \end{bmatrix}$       |           |
| a2<br>a3        |   | $-\varphi$ $-1+\varphi$                                | 0                  | $-392 + 308\varphi$ $-21 - 14\varphi$                | $0412 - 3904\varphi$ $29 + 45\varphi$                       | 1                                      | $\frac{2}{2}$                                  | $\begin{vmatrix} + - & 1, 6 \\ + + & 4, 8 \end{vmatrix}$  |   | $\begin{bmatrix} 1, 3, 1 \\ 4, 3, 1 \end{bmatrix}$      | $\begin{bmatrix} I_1, IV, I_1 \\ I_4, IV^*, I_1 \end{bmatrix}$       |           |
| a4              |   | $-1 + \varphi$<br>$-1 + \varphi$                       | 0                  | $-21 - 14\varphi$ $-16 - 24\varphi$                  | $-32 - 84\varphi$   | 1                                      | $\frac{2}{2}$                                  | $\begin{vmatrix} + + & 4, 6 \\ - + & 1, 8 \end{vmatrix}$  |   | 1, 3, 1   | $\begin{bmatrix} 1_4, 1_V, 1_1 \\ I_1, IV^*, I_4 \end{bmatrix}$      |           |
| a4              | l U   | $-1 + \varphi$   | U                  | $-10-24\varphi$                                      | $-32-34\varphi$   | 1                                      |  | 1, c  | , - 1, 4                                  | 1,0,4   | 11,1V,14   |           |

|   | $a_1$   | $a_2$  | $a_3$   | $a_4$  | $a_6$   | r  | T   | s  | $\operatorname{ord}(\Delta)$   | $\operatorname{ord}_{-}(j)$  | $c_p$   | Kodaira   | Isogenies |
|---|---|--|---|--|---|--|---|--|--|--|---|---|-----------|
|   |   |  |   |  |   |  | •   |  |  |  |   |   |           |
| 158   | 4a  |  |   | 158  | $34a = (12 - 36\varphi) = 2^2 \cdot 3 \cdot 11a$  | (3 isoge   | ny cla                                      | asses)   |  |  |   |   | 1584a     |
| b1  | 0   | $-1-\varphi$   | 0   | $-3+3\varphi$  | 0   | 1  | 2   | +-   | 1, 4, 2  | 1,2  | 1, 3, 2   | $I_1, IV, I_2$  |           |
| b2  | 0   | 1  | 0   | $-16-31\varphi$  | $52 + 83\varphi$  | 1  | 2   | -+   | 2, 8, 1  | 2,1  | 2, 3, 1   | $I_2, IV^*, I_1$  |           |
| c1  | 0   | $\varphi$  | 0   | $-12+5\varphi$   | $16-14\varphi$  | 0  | 2   | + -  | 1,4,4  | 1,4  | [1, 1, 2]   | $ \tilde{I}_1, \tilde{IV}, \tilde{I}_4 $  |           |
| c2  | 0   | $1-\varphi$  | 0   | $-145 - 239\varphi$  | $-1270 - 2065\varphi$   | 0  | 2   | -+   | 2, 8, 2  | 2,2  | 2, 1, 2   | $I_2, IV^*, I_2$  |           |
| <b>1 F</b> O  | 41  |  |   |  |   |  |   |  |  |  |   |   | 12041     |
| 158   | 5 <b>4</b> b  |  |   |  | $84b = (24 - 36\varphi) = 2^2 \cdot 3 \cdot 11b$  | (3 isoger  | ny cla                                      | sses)  |  |  |   |   | 1584b     |
| a1  | 0   | $-\varphi$   | 0   | $-5-\varphi$   | $2+6\varphi$  | 1  | 4   | ++   | 2, 4, 2  | 2,2  | 2, 3, 2   | $I_2, IV, I_2$  |           |
| a2  | 0   | $-1+\varphi$   | 0   | $-224 - 368\varphi$  | $2448 + 3964\varphi$  | 1  | 2   | -+   | 1, 8, 1  | 1,1  | 1, 3, 1   | $I_1, IV^*, I_1$  |           |
| a3  | 0   | $-\varphi$   | 0   | $-35 + 14\varphi$  | $74-45\varphi$  | 1  | 2   | ++   | 4, 8, 1  | 4,1  | 4, 3, 1   | $I_4, IV^*, I_1$  |           |
| a4  | 0   | $-\varphi$   | 0   | $-40 + 24\varphi$  | $-116 + 84\varphi$  | 1  | 2   | + -  | 1,8,4  | 1,4  | 1,3,2   | $I_1, IV^*, I_4$  |           |
| b1  | 0   | $1+\varphi$  | 0   | $-1-\varphi$   | $-1-2\varphi$   | 1  | 2   | -+   | 1, 4, 2  | 1,2  | 1, 3, 2   | $I_1, IV, I_2$  |           |
| b2  | 0   | 1  | 0   | $-47 + 31\varphi$  | $135 - 83\varphi$   | 1  | 2   | + -  | 2, 8, 1  | 2,1  | 2, 3, 1   | $I_2, IV^*, I_1$  |           |
| c1  | 0   | $1-\varphi$  | 0   | $-7-5\varphi$  | $2+14\varphi$   | 0  | 2   | - +  | 1,4,4  | 1,4  | [1, 1, 2]   | $ $ $I_1, IV, I_4$  |           |
| 0.1   |   |  |   |  |   |  | 1   | 1  |  |  |   | T TTT# T  | 1         |
| c2  | 0   | $\varphi$  | 0   | $-384 + 239\varphi$  | $-3335 + 2065\varphi$   | 0  | 2   | +-   | 2,8,2  | 2,2  | 2,1,2   | $\boxed{I_2, IV^*, I_2}$  |           |
|   | 5a  | ·  | 0   | 159  | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$   | (7 isoge   |   | 1  |  | ,  |   |   | 1595a     |
| c2 159 a1   | 95a   | $-1+\varphi$   | φ   | $ \begin{array}{c} 159 \\ -6 - 10\varphi \end{array} $   | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$   | (7 isoge   | eny cl                                      | asses)   | 1,1,8  | 1,1,8  | 1,1,8   | $I_{1}, I_{1}, I_{8}$   | 1595a     |
| c2  159  a1  a2   | $\begin{array}{ c c c } \mathbf{5a} \\ \hline 1+\varphi \\ 1+\varphi \end{array}$   | $-1 + \varphi$ $-1 + \varphi$  | $\varphi$   | $   \begin{array}{r}     159 \\     -6 - 10\varphi \\     -131 - 135\varphi   \end{array} $  | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$  | (7 isoge 0 0                                     | eny cl                                      | asses)   | 1, 1, 8<br>2, 2, 4   | 1, 1, 8<br>2, 2, 4   | 1, 1, 8<br>2, 2, 4  | $\begin{matrix} I_1, I_1, I_8 \\ I_2, I_2, I_4 \end{matrix}$  | 1595a     |
| c2  159  a1 a2 a3                                       | $\begin{array}{c c} 7 & $ | $-1 + \varphi$ $-1 + \varphi$ $1 + \varphi$  | $\varphi$ $\varphi$ 0   | $   \begin{array}{r}     159 \\     -6 - 10\varphi \\     -131 - 135\varphi \\     -72533 - 117358\varphi   \end{array} $  | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$  | (7 isoge   | eny cl 4 4 2                                | asses)   -+ ++ ++                              | 1, 1, 8 $2, 2, 4$ $1, 1, 2$  | 1, 1, 8<br>2, 2, 4<br>1, 1, 2  | 1, 1, 8 $2, 2, 4$ $1, 1, 2$   | $\begin{array}{c c} I_1,I_1,I_8 \\ I_2,I_2,I_4 \\ I_1,I_1,I_2 \end{array}$  | 1595a     |
| a1<br>a2<br>a3<br>a4                                    | $\begin{array}{ c c c } \mathbf{5a} \\ \hline 1+\varphi \\ 1+\varphi \end{array}$   | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ 1 + \varphi \\ -\varphi \end{array} $   | $ \begin{array}{c} \varphi \\ \varphi \\ 0 \\ 1+\varphi \end{array} $   | $   \begin{array}{r}                                     $   | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$ $-108646 + 67069\varphi$   | (7 isoge 0 0 0 0 0 0 0                           | eny cl                                      | asses)   | 1, 1, 8<br>2, 2, 4<br>1, 1, 2<br>4, 4, 2   | 1,1,8<br>2,2,4<br>1,1,2<br>4,4,2   | 1,1,8<br>2,2,4<br>1,1,2<br>2,4,2  | $\begin{array}{c} I_1,I_1,I_8\\ I_2,I_2,I_4\\ I_1,I_1,I_2\\ I_4,I_4,I_2 \end{array}$  | 1595a     |
| c2  159  a1 a2 a3                                       | $\begin{array}{c c} 7 & $ | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ 1 + \varphi \\ -\varphi \end{array} $   | $\varphi$ $\varphi$ 0   | $   \begin{array}{r}     159 \\     -6 - 10\varphi \\     -131 - 135\varphi \\     -72533 - 117358\varphi   \end{array} $  | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$  | (7 isoge   | eny cl 4 4 2                                | asses)   -+ ++ ++                              | 1, 1, 8 $2, 2, 4$ $1, 1, 2$  | 1, 1, 8<br>2, 2, 4<br>1, 1, 2  | 1, 1, 8 $2, 2, 4$ $1, 1, 2$   | $\begin{array}{c c} I_1,I_1,I_8 \\ I_2,I_2,I_4 \\ I_1,I_1,I_2 \end{array}$  | 1595a     |
| a1<br>a2<br>a3<br>a4                                    | $\begin{array}{c c} \mathbf{5a} \\ 1+\varphi \\ 1+\varphi \\ \varphi \\ \varphi \end{array}$  | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ 1 + \varphi \\ -\varphi \end{array} $   | $ \begin{array}{c} \varphi \\ \varphi \\ 0 \\ 1+\varphi \end{array} $   | $   \begin{array}{r}                                     $   | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$ $-108646 + 67069\varphi$   | (7 isoge 0 0 0 0 0 0 0                           | eny cl 4 4 2 2                              | asses)   -+ ++ ++                              | 1, 1, 8<br>2, 2, 4<br>1, 1, 2<br>4, 4, 2<br>2, 1, 3  | 1,1,8<br>2,2,4<br>1,1,2<br>4,4,2   | 1,1,8<br>2,2,4<br>1,1,2<br>2,4,2  | $\begin{array}{c} I_1,I_1,I_8\\ I_2,I_2,I_4\\ I_1,I_1,I_2\\ I_4,I_4,I_2 \end{array}$  | 1595a     |
| 159 a1 a2 a3 a4 b1                                      | $\begin{array}{c c} \mathbf{5a} \\ \hline 1+\varphi \\ 1+\varphi \\ \varphi \\ \hline \varphi \\ \hline \end{array}$  | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ 1 + \varphi \\ -\varphi \\ 1 - \varphi \end{array} $  | $ \begin{array}{c} \varphi \\ \varphi \\ 0 \\ 1+\varphi \\ 1+\varphi \end{array} $  | $   \begin{array}{r}                                     $   | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$ $-108646 + 67069\varphi$ $2 - 2\varphi$  | (7 isoge   0   0   0   0   0   1                 | eny cl 4 4 2 2 1                            | asses)   - +   + +   + +                       | $   \begin{array}{c}     1, 1, 8 \\     2, 2, 4 \\     1, 1, 2 \\     4, 4, 2 \\     \hline     2, 1, 3   \end{array} $  | 1,1,8<br>2,2,4<br>1,1,2<br>4,4,2<br>2,1,3  | $ \begin{array}{c} 1, 1, 8 \\ 2, 2, 4 \\ 1, 1, 2 \\ 2, 4, 2 \\ 2, 1, 3 \end{array} $  | $ \begin{vmatrix} I_1, I_1, I_8 \\ I_2, I_2, I_4 \\ I_1, I_1, I_2 \\ I_4, I_4, I_2 \\ \hline I_2, I_1, I_3 \end{vmatrix} $  | 1595a     |
| 159 a1 a2 a3 a4 b1 c1                                   | $\begin{array}{c c} \mathbf{75a} \\ \hline 1+\varphi \\ 1+\varphi \\ \varphi \\ \hline -\varphi \\ \hline -0 \\ \hline \end{array}$   | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ 1 + \varphi \\ -\varphi \\ 1 - \varphi \end{array} $  | $\varphi \\ \varphi \\ 0 \\ 1 + \varphi \\ 1 + \varphi \\ 1 + \varphi$  | $   \begin{array}{r}                                     $   | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$ $-108646 + 67069\varphi$ $2 - 2\varphi$ $11 + 10\varphi$ $161 - 110\varphi$ $185264 - 114496\varphi$   | (7 isoge 0 0 0 0 0 1 1 1 1                       | eny cl 4 4 2 2 1 1 6                        | asses)   - +   + +   + +   + -     - +         | $ \begin{array}{c} 1,1,8\\2,2,4\\1,1,2\\4,4,2\\2,1,3\\3,1,4 \end{array} $  | $ \begin{array}{ c c c } \hline 1,1,8\\2,2,4\\1,1,2\\4,4,2\\\hline &2,1,3\\\hline &3,1,4\\\hline \end{array} $   | $ \begin{array}{c} 1,1,8\\2,2,4\\1,1,2\\2,4,2\\2,1,3\\3,1,2 \end{array} $   | $ \begin{vmatrix} I_1, I_1, I_8 \\ I_2, I_2, I_4 \\ I_1, I_1, I_2 \\ I_4, I_4, I_2 \\ & I_2, I_1, I_3 \\ & I_3, I_1, I_4 \end{vmatrix} $  | 1595a     |
| 159 a1 a2 a3 a4 b1 c1 c2 c3 c4                          | $\begin{vmatrix} 1+\varphi \\ 1+\varphi \\ \varphi \\  \end{vmatrix} = 0$   | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ 1 + \varphi \\ -\varphi \\ 1 - \varphi \end{array} $  | $\varphi \\ \varphi \\ 0 \\ 1 + \varphi \\ 1 + \varphi \\ 1 \\ 1$   | $   \begin{array}{r}                                     $   | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$ $-108646 + 67069\varphi$ $2 - 2\varphi$ $11 + 10\varphi$ $161 - 110\varphi$ $185264 - 114496\varphi$ $-631 - 1266\varphi$  | (7 isoge 0 0 0 0 0 1 1 1 1 1 1 1                 | eny cl 4 4 2 2 1 1 6 12                     | asses)   - +                                   | $ \begin{array}{c} 1,1,8\\2,2,4\\1,1,2\\4,4,2\\\hline{2,1,3}\\\hline{3,1,4}\\6,2,2 \end{array} $   | $ \begin{array}{c cccc} 1,1,8 \\ 2,2,4 \\ 1,1,2 \\ 4,4,2 \\ \hline 2,1,3 \\ \hline 3,1,4 \\ 6,2,2 \end{array} $  | $ \begin{array}{c} 1,1,8\\2,2,4\\1,1,2\\2,4,2\\\hline2,1,3\\3,1,2\\6,2,2\end{array} $   | $\begin{array}{ c c c c c }\hline I_1,I_1,I_8\\ I_2,I_2,I_4\\ I_1,I_1,I_2\\ I_4,I_4,I_2\\ \hline I_2,I_1,I_3\\ \hline I_3,I_1,I_4\\ I_6,I_2,I_2\\ \end{array}$  | 1595a     |
| 159 a1 a2 a3 a4 b1 c1 c2 c3 c4 c5                       | $\begin{array}{c c} \mathbf{5a} \\ \hline 1+\varphi \\ 1+\varphi \\ \varphi \\ \hline                              $  | $ \begin{array}{c} -1+\varphi \\ -1+\varphi \\ 1+\varphi \\ -\varphi \\ 1-\varphi \\ -\varphi \\ -1 \end{array} $  | $\begin{array}{c} \varphi \\ \varphi \\ 0 \\ 1+\varphi \\ \hline 1+\varphi \\ \hline 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array}$                         | $ \begin{array}{r}                                     $   | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$ $-108646 + 67069\varphi$ $2 - 2\varphi$ $11 + 10\varphi$ $161 - 110\varphi$ $185264 - 114496\varphi$ $-631 - 1266\varphi$ $-201 - 178\varphi$  | (7 isoge 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1   | eny cl 4 4 2 2 1 1 6 12 6                   | asses) -+ ++ ++                                | $ \begin{array}{c} 1,1,8\\2,2,4\\1,1,2\\4,4,2\\\hline{2,1,3}\\\hline{3,1,4}\\6,2,2\\3,4,1\\12,1,1\\1,3,12\end{array} $   | $\begin{array}{ c c c }\hline 1,1,8\\2,2,4\\1,1,2\\4,4,2\\\hline\hline 2,1,3\\\hline 3,1,4\\6,2,2\\3,4,1\\12,1,1\\1,3,12\\\hline\end{array}$   | $1,1,8 \\ 2,2,4 \\ 1,1,2 \\ 2,4,2 \\ \hline 2,1,3 \\ \hline 3,1,2 \\ 6,2,2 \\ 3,2,1 \\ 12,1,1 \\ 1,1,2$   | $ \begin{bmatrix} I_1, I_1, I_8 \\ I_2, I_2, I_4 \\ I_1, I_1, I_2 \\ I_4, I_4, I_2 \\ \hline I_2, I_1, I_3 \\ \hline I_3, I_1, I_4 \\ I_6, I_2, I_2 \\ I_3, I_4, I_1 \\ I_{12}, I_1, I_1 \\ I_1, I_3, I_{12} \end{bmatrix} $  | 1595a     |
| 159 a1 a2 a3 a4 b1 c1 c2 c3 c4 c5 c6                    | $\begin{array}{c c} \mathbf{5a} \\ 1+\varphi \\ 1+\varphi \\ \varphi \\ \hline \\ 0 \\ \hline \\ 1 \\ 1+\varphi \\ 1 \end{array}$   | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ 1 + \varphi \\ \hline                                  $  | $ \begin{array}{c} \varphi \\ \varphi \\ 0 \\ 1+\varphi \\ 1+\varphi \\ 1 \\ 1 \\ 1 \\ 1 \end{array} $  | $   \begin{array}{r}                                     $   | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$ $-108646 + 67069\varphi$ $2 - 2\varphi$ $11 + 10\varphi$ $161 - 110\varphi$ $185264 - 114496\varphi$ $-631 - 1266\varphi$ $-201 - 178\varphi$ $-8701 - 12678\varphi$   | (7 isoge 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1   | eny cl 4 4 2 2 1 1 6 12 6 6 2 4             | asses)   - + + + + + + - + + + + + + + + + + + | $\begin{array}{c} 1,1,8\\ 2,2,4\\ 1,1,2\\ 4,4,2\\ \hline 2,1,3\\ \hline 3,1,4\\ 6,2,2\\ 3,4,1\\ 12,1,1\\ 1,3,12\\ 2,6,6 \end{array}$   | $\begin{array}{ c c c }\hline 1,1,8\\2,2,4\\1,1,2\\4,4,2\\\hline\hline 2,1,3\\\hline 3,1,4\\6,2,2\\3,4,1\\12,1,1\\1,3,12\\2,6,6\\\hline\end{array}$  | $ \begin{array}{c} 1,1,8\\2,2,4\\1,1,2\\2,4,2\\\hline{2,1,3}\\3,1,2\\6,2,2\\3,2,1\\12,1,1\\1,1,2\\2,2,2\end{array} $  | $\begin{array}{ c c c c }\hline I_1,I_1,I_8\\ I_2,I_2,I_4\\ I_1,I_1,I_2\\ I_4,I_4,I_2\\ \hline I_2,I_1,I_3\\ \hline I_3,I_1,I_4\\ I_6,I_2,I_2\\ I_3,I_4,I_1\\ I_{12},I_1,I_1\\ I_1,I_3,I_{12}\\ I_2,I_6,I_6\\ \end{array}$  | 1595a     |
| c2  159 a1 a2 a3 a4 b1 c1 c2 c3 c4 c5 c6 c7             | $\begin{array}{c c} \mathbf{5a} \\ \hline 1+\varphi \\ 1+\varphi \\ \varphi \\ \hline                              $  | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ 1 + \varphi \\                                  $   | $\begin{array}{c} \varphi \\ \varphi \\ 0 \\ 1+\varphi \\ 1+\varphi \\ \end{array}$   | $ \begin{array}{r} -6 - 10\varphi \\ -131 - 135\varphi \\ -72533 - 117358\varphi \\ -3855 + 2350\varphi \\ -2 + \varphi \\ -9 - 8\varphi \\ -59 + 17\varphi \\ -5474 + 3394\varphi \\ -154 - 118\varphi \\ -9 - 63\varphi \\ -634 - 688\varphi \\ -1284 - 263\varphi \end{array} $   | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$ $-108646 + 67069\varphi$ $2 - 2\varphi$ $11 + 10\varphi$ $161 - 110\varphi$ $185264 - 114496\varphi$ $-631 - 1266\varphi$ $-201 - 178\varphi$ $-8701 - 12678\varphi$ $1589 - 19808\varphi$   | (7 isoge 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1   | eny cl 4 4 2 2 1 1 6 12 6 6 2 4 2           | asses) -+++ ++++ ++- ++- ++- ++-               | $ \begin{array}{r} 1,1,8\\2,2,4\\1,1,2\\4,4,2\\\hline{2,1,3}\\\hline{3,1,4}\\6,2,2\\3,4,1\\12,1,1\\1,3,12\\2,6,6\\1,12,3\end{array} $  | $ \begin{array}{ c c c }\hline 1,1,8\\2,2,4\\1,1,2\\4,4,2\\\hline\hline 2,1,3\\\hline 3,1,4\\6,2,2\\3,4,1\\12,1,1\\1,3,12\\2,6,6\\1,12,3\\\hline \end{array}$  | $1,1,8 \\ 2,2,4 \\ 1,1,2 \\ 2,4,2 \\ \hline 2,1,3 \\ \hline 3,1,2 \\ 6,2,2 \\ 3,2,1 \\ 12,1,1 \\ 1,1,2 \\ 2,2,2 \\ 1,2,1$   | $\begin{array}{ c c c c c }\hline I_1,I_1,I_8\\ I_2,I_2,I_4\\ I_1,I_1,I_2\\ I_4,I_4,I_2\\ \hline I_2,I_1,I_3\\ \hline I_3,I_1,I_4\\ I_6,I_2,I_2\\ I_3,I_4,I_1\\ I_{12},I_1,I_1\\ I_{11},I_3,I_{12}\\ I_{2},I_6,I_6\\ I_{11},I_{12},I_3\\ \hline \end{array}$  | 1595a     |
| c2  159 a1 a2 a3 a4 b1 c1 c2 c3 c4 c5 c6 c7 c8          | $\begin{array}{c c} \mathbf{5a} \\ \hline & 1+\varphi \\ & 1+\varphi \\ & \varphi \\ \hline & & \varphi \\ \hline & & & 1\\ & & 1\\ & & 1\\ & & 1\\ & & 1\\ & & 1\\ \end{array}$  | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ 1 + \varphi \\ -\varphi \\ 1 - \varphi \end{array} $ $ \begin{array}{c} \varphi \\ \varphi \\ -1 \\ \varphi \\ \varphi \\ \varphi \end{array} $ | $\varphi \\ \varphi \\ 0 \\ 1 + \varphi \\ 1 + \varphi$ $1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1$   | $ \begin{array}{r}                                     $   | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$ $-108646 + 67069\varphi$ $2 - 2\varphi$ $11 + 10\varphi$ $161 - 110\varphi$ $185264 - 114496\varphi$ $-631 - 1266\varphi$ $-201 - 178\varphi$ $-8701 - 12678\varphi$ $1589 - 19808\varphi$ $-8931320 - 14444725\varphi$  | (7 isoge 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1   | eny cl 4 4 2 2 1 1 6 12 6 6 2 4             | asses)  - + + + + + + + + + + + + + + + +      | $ \begin{array}{c} 1,1,8\\2,2,4\\1,1,2\\4,4,2\\\hline{2,1,3}\\\hline{3,1,4}\\6,2,2\\3,4,1\\12,1,1\\1,3,12\\2,6,6\\1,12,3\\4,3,3\\\end{array} $   | $\begin{array}{ c c c }\hline 1,1,8\\2,2,4\\1,1,2\\4,4,2\\\hline 2,1,3\\\hline 3,1,4\\6,2,2\\3,4,1\\12,1,1\\1,3,12\\2,6,6\\1,12,3\\4,3,3\\\hline\end{array}$   | $\begin{array}{c} 1,1,8\\2,2,4\\1,1,2\\2,4,2\\2,1,3\\3,1,2\\6,2,2\\3,2,1\\12,1,1\\1,1,2\\2,2,2\\1,2,1\\4,1,1\\\end{array}$  | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$  | 1595a     |
| c2  159 a1 a2 a3 a4 b1 c1 c2 c3 c4 c5 c6 c7 c8 d1       | $\begin{vmatrix} 1+\varphi \\ 1+\varphi \\ \varphi \\  \end{vmatrix} \begin{vmatrix} 0 \\ 1+\varphi \\ \varphi \\  \end{vmatrix}$   | $ \begin{array}{c} -1 + \varphi \\ -1 + \varphi \\ 1 + \varphi \\                                  $   | $\varphi \\ \varphi \\ 0 \\ 1 + \varphi \\ 1 + \varphi \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\$   | $ \begin{array}{r}                                     $   | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$ $-108646 + 67069\varphi$ $2 - 2\varphi$ $11 + 10\varphi$ $161 - 110\varphi$ $185264 - 114496\varphi$ $-631 - 1266\varphi$ $-201 - 178\varphi$ $-8701 - 12678\varphi$ $1589 - 19808\varphi$ $-8931320 - 14444725\varphi$ $-2 + 6\varphi$                                      | (7 isoge 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1   | eny cl 4 4 2 2 1 1 6 12 6 6 2 4 2           | asses)  - + + + + + + + + + + + + + + + +      | $\begin{array}{c} 1,1,8\\ 2,2,4\\ 1,1,2\\ -\frac{4}{2},\frac{4}{2},\frac{2}{2},\frac{2}{2},\frac{2}{3},\frac{3}{4},\frac{1}{4}\\ 6,2,2\\ 3,4,1\\ 12,1,1\\ 1,3,12\\ 2,6,6\\ 1,12,3\\ \frac{4}{3},\frac{3}{3},\frac{3}{1},\frac{3}{3},\frac{4}{4}\\ \end{array}$ | $ \begin{array}{ c c c }\hline 1,1,8\\2,2,4\\1,1,2\\4,4,2\\\hline\hline 2,1,3\\\hline 3,1,4\\6,2,2\\3,4,1\\12,1,1\\1,3,12\\2,6,6\\1,12,3\\4,3,3\\\hline\hline 1,3,4\\ \end{array} $                      | $\begin{array}{c} 1,1,8\\ 2,2,4\\ 1,1,2\\ \underline{2,4,2}\\ \underline{2,1,3}\\ \overline{3,1,2}\\ 6,2,2\\ 3,2,1\\ 12,1,1\\ 1,1,2\\ 2,2,2\\ 1,2,1\\ 4,1,1\\ \overline{1,3,2}\\ \end{array}$ | $ \begin{bmatrix} I_1, I_1, I_8 \\ I_2, I_2, I_4 \\ I_1, I_1, I_2 \\ I_4, I_4, I_2 \\ \hline I_2, I_1, I_3 \\ \hline I_3, I_1, I_4 \\ I_6, I_2, I_2 \\ I_3, I_4, I_1 \\ I_{12}, I_1, I_1 \\ I_1, I_3, I_{12} \\ I_2, I_6, I_6 \\ I_1, I_{12}, I_3 \\ \hline I_4, I_3, I_3 \\ \hline I_1, I_3, I_4 \end{bmatrix} $ | 1595a     |
| c2  159 a1 a2 a3 a4 b1 c1 c2 c3 c4 c5 c6 c7 c8 d1 d2    | $\begin{array}{c c} \mathbf{5a} \\ 1+\varphi \\ 1+\varphi \\ \varphi \\ \hline \\ 0 \\ \hline \\ 1 \\ 1 \\ 1 \\ 1 \\ \varphi \\ \end{array}$  | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $\begin{array}{c} \varphi \\ \varphi \\ 0 \\ 1+\varphi \\ 1+\varphi \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ \varphi \\ \end{array}$                    | $ \begin{array}{r}                                     $   | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$ $-108646 + 67069\varphi$ $2 - 2\varphi$ $11 + 10\varphi$ $161 - 110\varphi$ $185264 - 114496\varphi$ $-631 - 1266\varphi$ $-201 - 178\varphi$ $-8701 - 12678\varphi$ $1589 - 19808\varphi$ $-8931320 - 14444725\varphi$ $-2 + 6\varphi$ $-37 + 66\varphi$                    | (7 isoge   0   0   0   0     1   1   1   1   1   | eny cl 4 4 2 2 1 1 6 12 6 6 2 4 2 2 1 6 6 6 | asses)  -+ ++ ++ + ++ ++ ++ -+ ++ ++ ++ ++     | $\begin{array}{c} 1,1,8\\ 2,2,4\\ 1,1,2\\ 4,4,2\\ \hline 2,1,3\\ \hline 3,1,4\\ 6,2,2\\ 3,4,1\\ 12,1,1\\ 1,3,12\\ 2,6,6\\ 1,12,3\\ 4,3,3\\ \hline 1,3,4\\ 2,6,2\\ \end{array}$   | $ \begin{array}{ c c c }\hline 1,1,8\\2,2,4\\1,1,2\\4,4,2\\\hline\hline 2,1,3\\\hline 3,1,4\\6,2,2\\3,4,1\\12,1,1\\1,3,12\\2,6,6\\1,12,3\\4,3,3\\\hline\hline 1,3,4\\2,6,2\\ \end{array} $               | $ \begin{array}{c} 1,1,8\\2,2,4\\1,1,2\\2,4,2\\\hline{2,1,3}\\\hline{3,1,2}\\6,2,2\\3,2,1\\12,1,1\\1,1,2\\2,2,2\\1,2,1\\4,1,1\\\hline{1,3,2}\\2,6,2\end{array} $                              | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$  | 1595a     |
| c2  159 a1 a2 a3 a4 b1 c1 c2 c3 c4 c5 c6 c7 c8 d1 d2 d3 | $\begin{array}{ c c c } \hline \mathbf{5a} \\ \hline & 1+\varphi \\ & \varphi \\ \hline & \varphi \\ \hline & 0 \\ \hline & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & 1 \\ & \varphi \\ \hline & \varphi \\ \hline \end{array}$  | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $\varphi \\ \varphi \\ 0 \\ 1+\varphi \\ 1+\varphi \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ \varphi$  | $ \begin{array}{c}                                     $   | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$ $-108646 + 67069\varphi$ $2 - 2\varphi$ $11 + 10\varphi$ $161 - 110\varphi$ $185264 - 114496\varphi$ $-631 - 1266\varphi$ $-201 - 178\varphi$ $-8701 - 12678\varphi$ $1589 - 19808\varphi$ $-8931320 - 14444725\varphi$ $-2 + 6\varphi$ $-37 + 66\varphi$ $-891 + 66\varphi$ | (7 isoge 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1   | eny cl 4 4 2 2 1 1 6 12 6 6 2 4 2 2 6 6 2   | asses)  -++ ++ ++ ++ ++                        | $\begin{array}{c} 1,1,8\\ 2,2,4\\ 1,1,2\\ 4,4,2\\ \hline 2,1,3\\ \hline 3,1,4\\ 6,2,2\\ 3,4,1\\ 12,1,1\\ 1,3,12\\ 2,6,6\\ 1,12,3\\ 4,3,3\\ \hline 1,3,4\\ 2,6,2\\ 3,1,12\\ \end{array}$  | $ \begin{array}{ c c c }\hline 1,1,8\\2,2,4\\1,1,2\\4,4,2\\\hline\hline 2,1,3\\\hline 3,1,4\\6,2,2\\3,4,1\\12,1,1\\1,3,12\\2,6,6\\1,12,3\\4,3,3\\\hline\hline 1,3,4\\2,6,2\\3,1,12\\\hline \end{array} $ | $\begin{array}{c} 1,1,8\\ 2,2,4\\ 1,1,2\\ 2,4,2\\ \hline 2,1,3\\ \hline 3,1,2\\ 6,2,2\\ 3,2,1\\ 12,1,1\\ 1,1,2\\ 2,2,2\\ 1,2,1\\ 4,1,1\\ \hline 1,3,2\\ 2,6,2\\ 1,1,2\\ \end{array}$          | $ \begin{array}{ c c c c c }\hline I_1,I_1,I_8\\ I_2,I_2,I_4\\ I_1,I_1,I_2\\ I_4,I_4,I_2\\ \hline I_2,I_1,I_3\\ \hline I_3,I_1,I_4\\ I_6,I_2,I_2\\ I_3,I_4,I_1\\ I_{12},I_1,I_1\\ I_1,I_3,I_{12}\\ I_2,I_6,I_6\\ I_1,I_{12},I_3\\ I_4,I_3,I_3\\ \hline I_1,I_3,I_4\\ I_2,I_6,I_2\\ I_3,I_1,I_{12}\\ \end{array} $ | 1595a     |
| c2  159 a1 a2 a3 a4 b1 c1 c2 c3 c4 c5 c6 c7 c8 d1 d2    | $\begin{array}{ c c c } \hline \mathbf{5a} \\ \hline & 1+\varphi \\ & 1+\varphi \\ & \varphi \\ \hline & \varphi \\ \hline & & 0 \\ \hline & & 1 \\ & & 1 \\ & & 1 \\ & & 1 \\ & & 1 \\ & & 1 \\ & & 1 \\ & & 1 \\ & & \varphi \\ & & \varphi \\ \end{array}$   | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $\begin{array}{c} \varphi \\ \varphi \\ 0 \\ 1+\varphi \\ 1+\varphi \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ \varphi \\ \varphi \end{array}$            | $\begin{array}{c} 159 \\ -6-10\varphi \\ -131-135\varphi \\ -72533-117358\varphi \\ -3855+2350\varphi \\ -2+\varphi \\ -9-8\varphi \\ -59+17\varphi \\ -5474+3394\varphi \\ -154-118\varphi \\ -9-63\varphi \\ -634-688\varphi \\ -1284-263\varphi \\ -53308-85520\varphi \\ -1-8\varphi \\ -26+17\varphi \\ -166+17\varphi \\ -14005+8656\varphi \end{array}$ | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$ $-108646 + 67069\varphi$ $2 - 2\varphi$ $11 + 10\varphi$ $161 - 110\varphi$ $185264 - 114496\varphi$ $-631 - 1266\varphi$ $-201 - 178\varphi$ $-8701 - 12678\varphi$ $1589 - 19808\varphi$ $-8931320 - 14444725\varphi$ $-2 + 6\varphi$ $-37 + 66\varphi$                    | (7 isoge 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1   | eny cl 4 4 2 2 1 1 6 12 6 6 2 4 2 2 1 6 6 6 | asses)  -++ ++ ++ ++ ++                        | $\begin{array}{c} 1,1,8\\ 2,2,4\\ 1,1,2\\ 4,4,2\\ \hline 2,1,3\\ \hline 3,1,4\\ 6,2,2\\ 3,4,1\\ 12,1,1\\ 1,3,12\\ 2,6,6\\ 1,12,3\\ 4,3,3\\ \hline 1,3,4\\ 2,6,2\\ \end{array}$   | $ \begin{array}{ c c c }\hline 1,1,8\\2,2,4\\1,1,2\\4,4,2\\\hline\hline 2,1,3\\\hline 3,1,4\\6,2,2\\3,4,1\\12,1,1\\1,3,12\\2,6,6\\1,12,3\\4,3,3\\\hline\hline 1,3,4\\2,6,2\\ \end{array} $               | $ \begin{array}{c} 1,1,8\\2,2,4\\1,1,2\\2,4,2\\\hline{2,1,3}\\\hline{3,1,2}\\6,2,2\\3,2,1\\12,1,1\\1,1,2\\2,2,2\\1,2,1\\4,1,1\\\hline{1,3,2}\\2,6,2\end{array} $                              | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$  | 1595a     |
| c2  159 a1 a2 a3 a4 b1 c1 c2 c3 c4 c5 c6 c7 c8 d1 d2 d3 | $\begin{vmatrix} 1+\varphi \\ 1+\varphi \\ \varphi \\ \varphi \\ \end{vmatrix} \begin{vmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ \varphi \\ \varphi \\ \varphi \\ \varphi \end{vmatrix}$  | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $\begin{array}{c} \varphi \\ \varphi \\ 0 \\ 1+\varphi \\ 1+\varphi \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ \varphi \\ \varphi \\ \varphi \end{array}$ | $ \begin{array}{c}                                     $   | $5a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ $-19 - 23\varphi$ $-844 - 1198\varphi$ $-14380743 - 23268531\varphi$ $-108646 + 67069\varphi$ $2 - 2\varphi$ $11 + 10\varphi$ $161 - 110\varphi$ $185264 - 114496\varphi$ $-631 - 1266\varphi$ $-201 - 178\varphi$ $-8701 - 12678\varphi$ $1589 - 19808\varphi$ $-8931320 - 14444725\varphi$ $-2 + 6\varphi$ $-37 + 66\varphi$ $-891 + 66\varphi$ | (7 isoge   0   0   0   0   1   1   1   1   1   1 | eny cl 4 4 2 2 1 1 6 12 6 6 2 4 2 2 6 6 2   | asses)  -++ ++ ++ ++ ++                        | $\begin{array}{c} 1,1,8\\ 2,2,4\\ 1,1,2\\ 4,4,2\\ \hline 2,1,3\\ \hline 3,1,4\\ 6,2,2\\ 3,4,1\\ 12,1,1\\ 1,3,12\\ 2,6,6\\ 1,12,3\\ 4,3,3\\ \hline 1,3,4\\ 2,6,2\\ 3,1,12\\ \end{array}$  | $ \begin{array}{ c c c }\hline 1,1,8\\2,2,4\\1,1,2\\4,4,2\\\hline\hline 2,1,3\\\hline 3,1,4\\6,2,2\\3,4,1\\12,1,1\\1,3,12\\2,6,6\\1,12,3\\4,3,3\\\hline\hline 1,3,4\\2,6,2\\3,1,12\\\hline \end{array} $ | $\begin{array}{c} 1,1,8\\ 2,2,4\\ 1,1,2\\ 2,4,2\\ \hline 2,1,3\\ \hline 3,1,2\\ 6,2,2\\ 3,2,1\\ 12,1,1\\ 1,1,2\\ 2,2,2\\ 1,2,1\\ 4,1,1\\ \hline 1,3,2\\ 2,6,2\\ 1,1,2\\ \end{array}$          | $ \begin{array}{ c c c c c }\hline I_1,I_1,I_8\\ I_2,I_2,I_4\\ I_1,I_1,I_2\\ I_4,I_4,I_2\\ \hline I_2,I_1,I_3\\ \hline I_3,I_1,I_4\\ I_6,I_2,I_2\\ I_3,I_4,I_1\\ I_{12},I_1,I_1\\ I_1,I_3,I_{12}\\ I_2,I_6,I_6\\ I_1,I_{12},I_3\\ I_4,I_3,I_3\\ \hline I_1,I_3,I_4\\ I_2,I_6,I_2\\ I_3,I_1,I_{12}\\ \end{array} $ | 1595a     |

|  | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_6$ | r | T | s | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$ | Kodaira | Isogenies |
|--|-------|-------|-------|-------|-------|---|---|---|------------------------------|-----------------------------|-------|---------|-----------|

| 159 | 5a          |              |               | 1595                    | $a = (39 + 2\varphi) = 5a \cdot 11b \cdot 29b$ | (7 isoger | ıy cla | sses)       |           |           |                     | 1595a |
|-----|-------------|--------------|---------------|-------------------------|--|-----------|--------|-------------|-----------|-----------|---------------------|-------|
| f1  | φ           | 1            | $\varphi$     | $-18-31\varphi$         | $49 + 79\varphi$                               | 0         | 4      | -+1,1,4     | 1, 1, 4   | 1, 1, 4   | $I_1,I_1,I_4$       |       |
| f2  | 1           | $-1-\varphi$ | $1 + \varphi$ | $-23 + 8\varphi$        | $-8+15\varphi$                                 | 0         | 8      | + + 2, 2, 8 | 2, 2, 8   | 2, 2, 8   | $I_2, I_2, I_8$     |       |
| f3  | 1           | $-1-\varphi$ | $1 + \varphi$ | $-173 + 123\varphi$     | $1103 - 656\varphi$                            | 0         | 4      | +-1,1,16    | 1, 1, 16  | 1, 1, 16  | $I_1, I_1, I_{16}$  |       |
| f4  | 1           | $-1-\varphi$ | $1 + \varphi$ | $-273 + 133\varphi$     | $-1883 + 1090\varphi$                          | 0         | 4      | + + 4, 4, 4 | 4, 4, 4   | 2, 4, 4   | $I_4, I_4, I_4$     |       |
| f5  | $\varphi$   | 1            | $\varphi$     | $-1773 - 2806\varphi$   | $-54488 - 87791\varphi$                        | 0         | 2      | -+2,8,2     | 2, 8, 2   | 2, 8, 2   | $I_2, I_8, I_2$     |       |
| f6  | $1+\varphi$ | $-1-\varphi$ | $1 + \varphi$ | $-28164 + 17408\varphi$ | $-2135494 + 1319783\varphi$                    | 0         | 2      | +-8,2,2     | 8, 2, 2   | 2, 2, 2   | $I_8, I_2, I_2$     |       |
| g1  | $\varphi$   | 0            | $\varphi$     | $-44 + 29\varphi$       | $133 - 82\varphi$                              | 0         | 2      | + - 5, 2, 1 | [5, 2, 1] | [1, 2, 1] | $  I_5, I_2, I_1  $ | [     |
| g2  | $\varphi$   | 0            | $\varphi$     | $-24+4\varphi$          | $286 - 179\varphi$                             | 0         | 2      | -+10,1,2    | 10, 1, 2  | 2, 1, 2   | $I_{10}, I_1, I_2$  |       |

| 159 | $5\mathrm{b}$ |              |               | 1595                      | $b = (41 - 2\varphi) = 5a \cdot 11a \cdot 29a$ | (7 isoger | ny cla | sses) |          |           |          |                      | 1595b |
|-----|---------------|--------------|---------------|---------------------------|--|-----------|--------|-------|----------|-----------|----------|----------------------|-------|
| a1  | $\varphi$     | $1+\varphi$  | 0             | $-15 + 11\varphi$         | $-32+17\varphi$                                | 0         | 4      | + -   | 1, 1, 8  | 1, 1, 8   | 1, 1, 8  | $I_1, I_1, I_8$      |       |
| a2  | $\varphi$     | $1+\varphi$  | 0             | $-265 + 136\varphi$       | $-1907 + 1067\varphi$                          | 0         | 4      | ++    | 2, 2, 4  | 2, 2, 4   | 2, 2, 4  | $I_2, I_2, I_4$      |       |
| a3  | $1+\varphi$   | $-1+\varphi$ | $\varphi$     | $-189893 + 117359\varphi$ | $-37342022 + 23078638\varphi$                  | 0         | 2      | ++    | 1, 1, 2  | 1, 1, 2   | 1, 1, 2  | $I_1, I_1, I_2$      |       |
| a4  | $1+\varphi$   | -1           | $\varphi$     | $-1504 - 2352\varphi$     | $-41576 - 67070\varphi$                        | 0         | 2      | -+    | 4, 4, 2  | 4, 4, 2   | 2, 4, 2  | $I_4, I_4, I_2$      |       |
| b1  | 0             | $\varphi$    | $\varphi$     | $-1-\varphi$              | $1+\varphi$                                    |           | 1      | [     | 2, 1, 3  | 2, 1, 3   | 2, 1, 3  | $\mid \ I_2,I_1,I_3$ |       |
| c1  | 1             | $1-\varphi$  | 1             | $-17 + 8\varphi$          | $21 - 10\varphi$                               | 1         | 6      | + -   | 3, 1, 4  | [3, 1, 4] | 3, 1, 2  | $  I_3, I_1, I_4  $  |       |
| c2  | 1             | $1-\varphi$  | 1             | $-42-17\varphi$           | $51 + 110\varphi$                              | 1         | 12     | ++    | 6, 2, 2  | 6, 2, 2   | 6, 2, 2  | $I_6, I_2, I_2$      |       |
| c3  | $\varphi$     | $-\varphi$   | 1             | $-2079 - 3395\varphi$     | $70768 + 114496\varphi$                        | 1         | 6      | -+    | 3, 4, 1  | 3, 4, 1   | 3, 2, 1  | $I_3, I_4, I_1$      |       |
| c4  | 1             | $1-\varphi$  | 1             | $-272 + 118\varphi$       | $-1897 + 1266\varphi$                          | 1         | 6      | ++    | 12, 1, 1 | 12, 1, 1  | 12, 1, 1 | $I_{12}, I_1, I_1$   |       |
| c5  | 1             | $1-\varphi$  | 1             | $-72+63\varphi$           | $-379 + 178\varphi$                            | 1         | 2      | + -   | 1, 3, 12 | 1, 3, 12  | 1, 1, 2  | $I_1, I_3, I_{12}$   |       |
| c6  | 1             | $1-\varphi$  | 1             | $-1322 + 688\varphi$      | $-21379 + 12678\varphi$                        | 1         | 4      | ++    | 2, 6, 6  | 2, 6, 6   | 2, 2, 2  | $I_2, I_6, I_6$      |       |
| c7  | 1             | $1-\varphi$  | 1             | $-1547 + 263\varphi$      | $-18219 + 19808\varphi$                        | 1         | 2      | 1     | 1, 12, 3 | 1, 12, 3  | 1, 2, 1  | $I_1, I_{12}, I_3$   |       |
| c8  | $1+\varphi$   | 0            | $1+\varphi$   | $-138828 + 85518\varphi$  | $-23376045 + 14444724\varphi$                  | 1         | 2      | ++    | 4, 3, 3  | 4, 3, 3   | 4, 1, 1  | $I_4, I_3, I_3$      |       |
| d1  | $1+\varphi$   | $-1+\varphi$ | 1             | $-10 + 9\varphi$          | $23-16\varphi$                                 | 1         | 6      | + -   | 1, 3, 4  | 1, 3, 4   | 1, 3, 2  | $  I_1, I_3, I_4  $  |       |
| d2  | $1+\varphi$   | $-1+\varphi$ | 1             | $-10-16\varphi$           | $23-76\varphi$                                 | 1         | 6      | -+    | 2, 6, 2  | 2, 6, 2   | 2, 6, 2  | $I_2, I_6, I_2$      |       |
| d3  | $1+\varphi$   | $-1+\varphi$ | 1             | $-150-16\varphi$          | $-691-216\varphi$                              | 1         | 2      | + -   | 3, 1, 12 | 3, 1, 12  | 1, 1, 2  | $I_3, I_1, I_{12}$   |       |
| d4  | 1             | 1            | $1 + \varphi$ | $-5349 - 8657\varphi$     | $-288778 - 467083\varphi$                      | 1         | 2      | - +   | 6, 2, 6  | 6, 2, 6   | 2, 2, 2  | $I_6, I_2, I_6$      |       |
| e1  | $1+\varphi$   | $1-\varphi$  | $\varphi$     | -3-arphi                  | $-4+\varphi$                                   | 0         | 2      | Ī — + | 1, 2, 3  | [1, 2, 3] | 1, 2, 1  | $I_1, I_2, I_3$      | Ī     |
| e2  | $1+\varphi$   | $1-\varphi$  | $\varphi$     | $-38 + 29\varphi$         | $-143 + 87\varphi$                             | 0         | 2      | + -   | 2, 1, 6  | 2, 1, 6   | 2, 1, 2  | $I_2, I_1, I_6$      |       |
| f1  | $1 + \varphi$ | $1-\varphi$  | $1 + \varphi$ | $-49 + 29\varphi$         | $128 - 80\varphi$                              | 0         | 4      | + -   | 1, 1, 4  | 1, 1, 4   | 1, 1, 4  | $I_1,I_1,I_4$        |       |
| f2  | 1             | $1+\varphi$  | $1+\varphi$   | $-15-7\varphi$            | $-7-24\varphi$                                 | 0         | 8      | ++    | 2, 2, 8  | 2, 2, 8   | 2, 2, 8  | $I_2, I_2, I_8$      |       |
| f3  | 1             | $1+\varphi$  | $1+\varphi$   | $-50-122\varphi$          | $398 + 532\varphi$                             | 0         | 4      | -+    | 1, 1, 16 | 1, 1, 16  | 1, 1, 16 | $I_1, I_1, I_{16}$   |       |
| f4  | 1             | $1+\varphi$  | $1+\varphi$   | $-140 - 132\varphi$       | $-932 - 1224\varphi$                           | 0         | 4      | ++    | 4, 4, 4  | 4, 4, 4   | 2, 4, 4  | $I_4, I_4, I_4$      |       |
| f5  | $1+\varphi$   | $1-\varphi$  | $1+\varphi$   | $-4579 + 2804\varphi$     | $-142279 + 87790\varphi$                       | 0         | 2      | + -   | 2, 8, 2  | 2, 8, 2   | 2, 8, 2  | $I_2, I_8, I_2$      |       |
| f6  | $\varphi$     | -1           | $\varphi$     | $-10754 - 17410\varphi$   | $-815710 - 1319784\varphi$                     | 0         | 2      | - +   | 8, 2, 2  | 8, 2, 2   | 2, 2, 2  | $I_8, I_2, I_2$      |       |
| g1  | $1+\varphi$   | $-\varphi$   | $1+\varphi$   | $-15-31\varphi$           | $51 + 81\varphi$                               | 0         | 2      | Ī — + | 5, 2, 1  | 5, 2, 1   | 1, 2, 1  | $  I_5, I_2, I_1  $  | Ī     |
| g2  | $1+\varphi$   | $-\varphi$   | $1+\varphi$   | $-20-6\varphi$            | $107 + 178\varphi$                             | 0         | 2      | +-    | 10, 1, 2 | 10, 1, 2  | 2, 1, 2  | $I_{10}, I_1, I_2$   |       |

|     | $a_1$       | $a_2$ $a_3$                           | $a_4$               | $a_6$   | r              | T                                  | s          | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$     | Kodaira                | Isogenies |
|-----|-------------|---------------------------------------|---------------------|---|----------------|------------------------------------|------------|------------------------------|-----------------------------|-----------|------------------------|-----------|
| 159 | 5 <i>c</i>  |                                       | 1505 a —            | $= (23 - 36\varphi) = 5a \cdot 11a \cdot 29b$ | (Figogra       | ov ele                             | agog)      |                              |                             |           |                        | 1595c     |
|     |             |                                       |                     | . , ,   | (5 isoge       | <u> </u>                           |            | T 1 4                        | F 1 4                       | F 1 0     |                        | 10300     |
| a1  | 1           | $-\varphi$ $\varphi$                  | $-224 + 138\varphi$ | $1449 - 907\varphi$                           |                | $\frac{2}{2}$                      | l          | 5, 1, 4                      | 5, 1, 4                     | 5, 1, 2   | $I_5, I_1, I_4$        |           |
| a2  | 1           | $-\varphi$ $\varphi$                  | $-249 + 88\varphi$  | $1414 - 1102\varphi$                          | 1              | 2                                  |            | 10, 2, 2                     | 10, 2, 2                    | 10, 2, 2  | $  I_{10}, I_2, I_2  $ |           |
| b1  | $\varphi$   | $-1$ $\varphi$                        | $-64-102\varphi$    | $373 + 603\varphi$                            | 1              | 2                                  | ++         | , ,                          | 1, 2, 2                     | 1, 2, 2   | $I_1, I_2, I_2$        |           |
| b2  | 1           | $1+\varphi$ $1+\varphi$               | $1-\varphi$         | -arphi  | 1              | 2                                  | <u> </u>   | 2, 1, 1                      | [2, 1, 1]                   | [2, 1, 1] | $I_2, I_1, I_1$        |           |
| c1  | $\varphi$   | $1+\varphi$ $1+\varphi$               | 2arphi              | $1 + \varphi$                                 | 1              | 2                                  | + -        | , ,                          | 1, 1, 2                     | 1, 1, 2   | $I_1, I_1, I_2$        |           |
| c2  | $\varphi$   | $1+\varphi$ $1+\varphi$               | $-5-3\varphi$       | -2  | 1              | 4                                  | ++         | 2, 2, 4                      | 2, 2, 4                     | 2, 2, 2   | $I_2, I_2, I_4$        |           |
| c3  | $\varphi$   | $1+\varphi$ $1+\varphi$               | $-80-53\varphi$     | $83 + 350\varphi$                             | 1              | 2                                  | ++         | 1, 4, 2                      | 1, 4, 2                     | 1, 4, 2   | $I_1, I_4, I_2$        |           |
| c4  | $\varphi$   | $1+\varphi$ $1+\varphi$               | $-10-33\varphi$     | $-79-114\varphi$                              | 1              | 2                                  | -+         | 4, 1, 8                      | 4, 1, 8                     | 2, 1, 2   | $I_4, I_1, I_8$        |           |
| d1  | $1+\varphi$ | $-\varphi$ 1+ $\varphi$               | $-19 + 2\varphi$    | $21-4\varphi$                                 | 1              | 6                                  | + +        | 1, 2, 6                      | 1, 2, 6                     | 1, 2, 6   | $  I_1, I_2, I_6  $    |           |
| d2  | $1+\varphi$ | $-\varphi$ 1+ $\varphi$               | $1-3\varphi$        | 2-2arphi                                      | 1              | 6                                  |            | 2, 1, 3                      | 2, 1, 3                     | 2, 1, 3   | $I_2, I_1, I_3$        |           |
| d3  | $1+\varphi$ | $-\varphi$ 1+ $\varphi$               | $-394 + 202\varphi$ | $-3424 + 2001\varphi$                         | 1              | 2                                  | ++         | 3, 6, 2                      | 3, 6, 2                     | 1, 2, 2   | $I_3, I_6, I_2$        |           |
| d4  | $1+\varphi$ | $-\varphi$ 1+ $\varphi$               | $-19+12\varphi$     | $-79 + 40\varphi$                             | 1              | 2                                  |            | 6, 3, 1                      | 6, 3, 1                     | 2, 1, 1   | $I_6, I_3, I_1$        |           |
| e1  | $\varphi$   | $1  1 + \varphi$                      | $-5-2\varphi$       | $-3+3\varphi$                                 | 1              | $\begin{vmatrix} -2 \end{vmatrix}$ | :<br>  + - | 1, 1, 4                      | 1,1,4                       | 1, 1, 2   | $ I_1, I_1, I_4 $      | <u> </u>  |
| e2  | $1+\varphi$ | $1+\varphi$ $1+\varphi$               | $-213 - 345\varphi$ | $1982 + 3208\varphi$                          | 1              | 2                                  | - +        | 2, 2, 2                      | 2, 2, 2                     | 2, 2, 2   | $I_2, I_2, I_2$        |           |
| 159 |             | 1 . 1 .                               |                     | $= (13 - 36\varphi) = 5a \cdot 11b \cdot 29a$ | t (5 isoge     |                                    |            | F 1 4                        | F 1 4                       | F 1 0     | T T T                  | 1595d     |
| a1  |             | $-1+\varphi$ $1+\varphi$              | $-86 - 139\varphi$  | $542 + 906\varphi$                            |                | $\frac{2}{2}$                      | -+         |                              | 5, 1, 4                     | 5,1,2     | $I_5, I_1, I_4$        |           |
| a2  |             | $-1+\varphi  1+\varphi$               | $-161 - 89\varphi$  | $312 + 1101\varphi$                           |                | 2                                  |            | 10, 2, 2                     | 10, 2, 2                    | 10, 2, 2  | $  I_{10}, I_2, I_2  $ | <u> </u>  |
| b1  |             | $-1-\varphi$ $1+\varphi$              | $-166 + 100\varphi$ | $976 - 604\varphi$                            | 1              | 2                                  | ++         | , ,                          | 1, 2, 2                     | 1, 2, 2   | $I_1, I_2, I_2$        |           |
| b2  | 1           | $-1-\varphi$ $1+\varphi$              | $-1+2\varphi$       | -2arphi                                       |                | 2                                  |            | 2, 1, 1                      | 2, 1, 1                     | [2, 1, 1] | $I_2, I_1, I_1$        |           |
| c1  | $1+\varphi$ | $-1+\varphi$ 0                        | $2-\varphi$         | 0   | 1              | 2                                  | -+         | 1, 1, 2                      | 1, 1, 2                     | 1, 1, 2   | $I_1, I_1, I_2$        |           |
| c2  | $1+\varphi$ | $-1+\varphi$ 0                        | $-8+4\varphi$       | $11-9\varphi$                                 | 1              | 4                                  | ++         | 2, 2, 4                      | 2, 2, 4                     | 2, 2, 2   | $I_2, I_2, I_4$        |           |
| c3  | $1+\varphi$ | $-1+\varphi$ 0                        | $-133 + 54\varphi$  | $621-484\varphi$                              | 1              | 2                                  | ++         | 1, 4, 2                      | 1, 4, 2                     | 1, 4, 2   | $I_1, I_4, I_2$        |           |
| c4  | $1+\varphi$ | $-1+\varphi$ 0                        | $-43 + 34\varphi$   | $-115 + 70\varphi$                            | 1              | 2                                  | + -        | 4, 1, 8                      | 4, 1, 8                     | 2, 1, 2   | $I_4, I_1, I_8$        |           |
| d1  | $\varphi$   | $0 \qquad \varphi$                    | $-15-4\varphi$      | $18 + 3\varphi$                               | 1              | 6                                  | + +        | 1, 2, 6                      | [1, 2, 6]                   | 1, 2, 6   | $  I_1, I_2, I_6  $    |           |
| d2  | $\varphi$   | $0 \qquad \varphi$                    | arphi               | $1+\varphi$                                   | 1              | 6                                  |            | 2, 1, 3                      | 2, 1, 3                     | 2, 1, 3   | $I_2, I_1, I_3$        |           |
| d3  | $\varphi$   | $0 \qquad \varphi$                    | $-190 - 204\varphi$ | $-1422 - 2002\varphi$                         | 1              | 2                                  | ++         | 3, 6, 2                      | 3, 6, 2                     | 1, 2, 2   | $I_3, I_6, I_2$        |           |
| d4  | $\varphi$   | $0 \qquad \varphi$                    | $-5-14\varphi$      | $-38-41\varphi$                               | 1              | 2                                  |            | 6, 3, 1                      | 6, 3, 1                     | 2, 1, 1   | $I_6, I_3, I_1$        |           |
| e1  | $1+\varphi$ | $1-\varphi$ $\varphi$                 | -6                  | $1-4\varphi$                                  | 1              | $\begin{bmatrix} -2 \end{bmatrix}$ |            | 1, 1, 4                      | 1,1,4                       | 1, 1, 2   | $  I_1, I_1, I_4  $    |           |
| e2  | $\varphi$   | $\varphi$ 1+ $\varphi$                | $-560 + 346\varphi$ | $6095 - 3768 \varphi$                         | 1              | 2                                  | + -        | 2, 2, 2                      | 2, 2, 2                     | 2, 2, 2   | $I_2, I_2, I_2$        |           |
|     |             | · · · · · · · · · · · · · · · · · · · | •                   | •   | l .            |                                    | l          |                              |                             |           |                        |           |
| 160 |             |                                       |                     |   | 14 isogeny cla |                                    | I          |                              | T                           |           | T                      | 1600a     |
| a1  | 0           | $1-\varphi$ 0                         | $-9-6\varphi$       | $11 + 15\varphi$                              | 1              | 2                                  | ++         | 8,3                          | 0                           | 4, 2      | $I_1^*, III$           |           |
| a2  | 0           | $1-\varphi$ 0                         | $1-\varphi$         | 1   | 1              | 2                                  |            | 4,3                          | 0                           | 2,2       | III, III               |           |
| b1  | 0           | arphi 0                               | $-15+6\varphi$      | $26-15\varphi$                                | 1              | 2                                  | ++         | 8,3                          | 0                           | 4, 2      | $I_1^*, III$           |           |
| b2  | 0           | arphi 0                               | arphi               | 1   | 1              | 2                                  |            | 4,3                          | 0                           | 2, 2      | III, III               |           |
|     |             |                                       |                     |   |                |                                    |            |                              |                             |           |                        |           |

|          | $a_1$                                  | $a_2$ $a_3$  | $a_4$   | $a_6$   | r                                      | T                                       | s        | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$            | $c_p$          | Kodaira   | Isogenies |
|----------|--|--|---|---|--|---|----------|------------------------------|--|----------------|---|-----------|
| 160      | )a                                     |  | 16  | $500a = (40) = 2^3 \cdot 5a^2$ (14 isogeny of   | classes)                               |   |          |                              |  |                |   | 1600a     |
| d1       | 0                                      | 1 0  | -28   | 48  | 1                                      | 4                                       | ++       | 8,6                          | 0                                      | 4, 4           | $I_1^*, I_0^*$  |           |
| d2       | 0                                      | 1 0  | -3  | -2  | 1                                      | 4                                       | ++       | 4,6                          | 0                                      | 2,4            | $III, I_0^*$  |           |
| d3       | 0                                      | $1+\varphi = 0$  | $-856-1384\varphi$                            | $17600 + 28464\varphi$                          | 1                                      | 2                                       | - +      | 10, 6                        | 0                                      | 2, 2           | $III^*, I_0^*$  |           |
| d4       | 0                                      | $-1-\varphi$ 0   | $-2241 + 1386\varphi$                         | $48305 - 29849\varphi$                          | 1                                      | 2                                       | + -      | 10, 6                        | 0                                      | 2, 2           | $III^*, I_0^*$  |           |
| d5       | 0                                      | 1 0  | -3-25arphi                                    | -37-55arphi                                     |  | 2                                       | -+       | 8,6                          | 0                                      | 2,2            | $I_1^*, I_0^*$  |           |
| d6       |  | 1 0  | $-28 + 25\varphi$                             | $-92 + 55\varphi$                               | 1                                      | 2                                       | + -      | 8,6                          | 0                                      | 2,2            | $I_1^*, I_0^*$  |           |
| e1       | 0                                      | $-1-\varphi$ 0   | $-541 + 321\varphi$                           | $5680 - 3479\varphi$                            | 0                                      | 4                                       | ++       | 10, 10                       | 4                                      | 2,4            | $\prod_{-*}^*, I_4^*$   |           |
| e2       | 0                                      | 1 0  | $-23-25\varphi$                               | $-57-95\varphi$                                 | 0                                      | 4                                       | ++       | 8,8                          | 2                                      | 2, 4           | $I_1^*, I_2^*$  |           |
| e3       | 0                                      | 1 0  | 2   | $-2-5\varphi$                                   | 0                                      | 4                                       |          | 4,7                          | 1                                      | 2,4            | $III, I_1^*$  |           |
| e4       | 0                                      | -1  0  | $-58668 + 36265\varphi$                       | $6449872 - 3986235\varphi$                      | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 2                                       | + -      | 11,8                         | 2                                      | 1,4            | $II^*, I_2^*$   |           |
| e5       | 0                                      | $\begin{array}{cc} 1 & 0 \\ 1 & 0 \end{array}$                                   | $-723 - 1025\varphi$                          | $12543 + 19405\varphi$                          | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 2                                       |          | 11, 14                       | 8                                      | $1, 4 \\ 2, 4$ | $II^*, I_8^*$   |           |
| e6       | 0                                      |  | $-323 - 425\varphi$                           | $-3657 - 5595\varphi$                           | 0                                      | 2<br> - <del>-</del>                    |          | 10,7                         | 1 1                                    |                | $\frac{1}{1} \cdot \frac{\text{III}^*, \text{I}_1^*}{1}$                |           |
| f1       | 0                                      | 1 0  | $-48 + 25\varphi$                             | $-152 + 95\varphi$                              | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 4                                       | ++       | 8,8                          | 2                                      | 2, 4           | $\begin{bmatrix} I_1^*, I_2^* \\ III I^* \end{bmatrix}$                 |           |
| f2<br>f3 | 0                                      | $\begin{array}{ccc} 1 & 0 \\ 1 & 1 & 0 \end{array}$                              | $-221 - 319\varphi$                           | $-7 + 5\varphi$ $1980 + 3159\varphi$            | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 4                                       |          | 4,7                          | $\frac{1}{4}$                          | $2, 4 \\ 2, 4$ | $III, I_1^*$  |           |
| f4       | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $     \begin{array}{ccc}       1 + \varphi & 0 \\       -1 & 0     \end{array} $ | $-221 - 319\varphi$ $-22403 - 36265\varphi$   | $1980 + 3139\varphi$ $2463637 + 3986235\varphi$ | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | $\begin{array}{c c} 4 \\ 2 \end{array}$ | ++       | 10, 10 $11, 8$               | $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ | 1, 4           | $\begin{array}{c c} III^*, I_4^* \\ II^*, I_2^* \end{array}$            |           |
| f5       | 0                                      | $\begin{array}{ccc} -1 & 0 \\ 1 & 0 \end{array}$                                 | $-22403 - 30203\varphi$ $-1748 + 1025\varphi$ | $31948 - 19405\varphi$                          |  | $\frac{2}{2}$                           |          | 11, 0 $11, 14$               | 8                                      | 1,4 $1,4$      | $II^*, I_2^*$<br>$II^*, I_8^*$  |           |
| f6       | 0                                      | 1 0  | $-748 + 425\varphi$                           | $-9252 + 5595\varphi$                           |  | $\frac{2}{2}$                           |          | 10,7                         | 1                                      | 2, 4           | $\begin{array}{ c c }\hline & 11 & 18 \\ & 111^*, 1_1^* \\ \end{array}$ |           |
| g1       |  | $-\varphi$ 0   | $-43 - 38\varphi$                             | $\frac{22+279\varphi}{}$                        |  | 1                                       | -''-<br> | 11,8                         | 0                                      | 1, 1           | II*, IV*  | <u>-</u>  |
| h1       | 0                                      | $-1+\varphi$ 0   | $-81 + 38\varphi$                             | $301-279\varphi$                                | 0                                      | 1                                       |          | 11,8                         | 0                                      | 1, 1           | II*,IV*   | <u>  </u> |
| i1       | 0                                      | -1 	 0   | $-16-24\varphi$                               | $-36-64\varphi$                                 | 0                                      | 1                                       |          | 11, 2                        | 0                                      | 1,1            | $ $ $II^*, II$  |           |
| j1       | 0                                      | -1  0  | $-40 + 24\varphi$                             | $-100+64\varphi$                                | 0                                      | 1                                       |          | 11,2                         | 0                                      | 1,1            | $ $ $II^*, II$  |           |
| k1       | 0                                      | 0 0  | 5   | _10   | 0                                      | 1                                       |          | 11,4                         | 0                                      | 1,1            | II*, IV   |           |
| 11       | 0                                      | $-1-\varphi$ 0   | $-76-104\varphi$                              | $-360-564\varphi$                               | 0                                      | 2                                       | ++       | 8,9                          | 0                                      | 2, 2           | $I_1^*, III^*$  |           |
| 12       | 0                                      | -1 	 0   | $7-5\varphi$                                  | $-18 + 10\varphi$                               | 0                                      | 2                                       |          | 4,9                          | 0                                      | 2,2            | III, III*   |           |
| m1       | 0                                      | $1+\varphi = 0$  | $-181 + 106\varphi$                           | $-1105 + 669\varphi$                            | 0                                      | 2                                       | ++       | 8,9                          | 0                                      | 2, 2           | $I_1^*, III^*$  |           |
| m2       | 0                                      | -1 0   | $2+5\varphi$                                  | $-8-10\varphi$                                  | 0                                      | $\begin{bmatrix} 2 \end{bmatrix}$       |          | 4,9                          | 0                                      | 2,2            | III, III*   |           |
| n1       | 0                                      | 0 0  | $-35-35\varphi$                               | $90 + 120\varphi$                               | 0                                      | 4                                       | ++       | 8, 10                        | 4                                      | 2,4            | $\mid I_1^*, I_4^*$   |           |
| n2       | 0                                      | 0 0  | $-10-10\varphi$                               | $-15-20\varphi$                                 | 0                                      | 4                                       | ++       | 4, 8                         | 2                                      | 2, 4           | $III, I_2^*$  |           |
| n3       | 0                                      | 0 0  | $-535-535\varphi$                             | $6390 + 8520\varphi$                            | 0                                      | 4                                       |          | 10, 8                        | 2                                      | 2,4            | $III^*, I_2^*$  |           |
| n4       | 0                                      | 0 0  | $-42475 - 68680\varphi$                       | $6404310 + 10362280\varphi$                     | 0                                      | 2                                       | -+       | 11, 7                        | 1                                      | 1,4            | $II^*, I_1^*$   |           |
| n5       | 0                                      | 0 0  | $-111155 + 68680\varphi$                      | $16766590 - 10362280\varphi$                    | 0                                      | 2                                       | + -      | 11,7                         | 1                                      | 1, 4           | $II^*, I_1^*$   |           |
| n6       | 0                                      | 0 0  | $-110 - 185\varphi$                           | $-890 - 1420\varphi$                            | $\begin{vmatrix} 0 \\ 0 \end{vmatrix}$ | 2                                       | -+       | 8,7                          | 1                                      | 4,2            | $I_1^*, I_1^*$  |           |
| n7       | 0                                      | 0 0  | $-295+185\varphi$                             | $-2310 + 1420\varphi$                           | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 2                                       | + -      | 8,7                          | 1                                      | $\frac{4}{2}$  | $I_1^*, I_1^*$  |           |
| n8       | 0                                      | 0 0  | $65 + 65\varphi$                              | $510 + 680\varphi$                              | 0                                      | 2                                       |          | 10, 14                       | 8                                      | 2,4            | $III^*, I_8^*$  |           |
| 160      | 4a                                     |  | 1604  | $a = (22 - 36\varphi) = 2 \cdot 401a$ (5 isoger | ny class                               | es)                                     |          |                              |  |                |   | 1604a     |
|          |  |  |   |   |  |   |          |                              | 1                                      |                | I   |           |

 $9-4\varphi$ 

- - 4, 1

4, 1

2, 1

 $\mathrm{I}_4,\mathrm{I}_1$ 

 $-7+6\varphi$ 

 $a1 \quad 1 \quad 1 + \varphi \quad 0$ 

|     | _   |              |       |                |                       |   |                  |           | ITI           | _    | 1( A )                       | 1 (:)  | _  | V - 1-:                     | T         |
|-----|---|--------------|-------|----------------|-----------------------|---|------------------|-----------|---------------|------|------------------------------|--|--|-----------------------------|-----------|
|     | $a_1$   | $a_2$        | 2     | $a_3$          | $a_4$                 | $a_6$   |                  |           | T             | s    | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                  | $c_p$                                    | Kodaira                     | Isogenies |
| 160 | 4a  |              |       |                | 160                   | $4a = (22 - 36\varphi) = 2 \cdot 401a$                | (5 isogeny clas  | ss∈       | es)           |      |                              |  |  |                             | 1604a     |
| b1  | 1   | _            |       | 0              | $-104 + 65\varphi$    | $509 - 315\varphi$                                    |                  | )         | 5             |      | 5, 1                         | 5, 1   | 5,1                                      | $I_5, I_1$                  |           |
| b2  | $\varphi$   | $-1-\varphi$ |       | 0              | $-353 - 697\varphi$   | $-5673 - 9467\varphi$                                 |                  | )         | 1             |      | 1,5                          | 1,5  | 1,5                                      | $I_1, I_5$                  |           |
| c1  | $\varphi$   |              |       | $1+\varphi$    | $-19 + 5\varphi$      | $25-13\varphi$  |                  | -         | 2             | ++   | 8, 1                         | 8,1  | 8,1                                      | $I_8, I_1$                  |           |
| c2  | 1   | $-1-\varphi$ |       | $\varphi$      | $-1792 + 1109\varphi$ | $34429 - 21278\varphi$                                | 1                | -         | 2             | + -  | $\frac{4}{2}$                | 4,2  | 4,2                                      | $I_4, I_2$                  | <u> </u>  |
| d1  | $1+\varphi$                                       |              |       | $\varphi$      | $-1-\varphi$          | -1<br>1 10  | 1                |           | 2             | ++   | 2, 1                         | 2,1  | $\begin{bmatrix} 2,1\\ 1 \end{bmatrix}$  | $I_2, I_1$                  |           |
| d2  | $1+\varphi$                                       | $1-\varphi$  |       | <del>-</del> - | $-11-\varphi$         | $1 - 12\varphi$                                       |                  | - '-      | 2             | ++   | 1,2                          | $\frac{1}{1}$ , $\frac{1}{2}$                | $\frac{1}{1}, \frac{2}{2}$               | $I_1, I_2$                  |           |
| e1  | $\begin{array}{c c} & \varphi \\ & 1 \end{array}$ | $-1-\varphi$ |       | 0              | $-8 - \varphi$        | -7  |                  |           | $\frac{2}{2}$ | ++   | 2, 1                         | $\begin{bmatrix} 2, 1 \\ 1, 2 \end{bmatrix}$ | $\begin{bmatrix} 2,1\\1,2 \end{bmatrix}$ | $I_2, I_1$                  |           |
| e2  | 1   | -1           | -     | 0              | $-587 + 362\varphi$   | $-6285 + 3884\varphi$                                 | 0                | ,         |               | +-   | 1, 2                         | 1, 2   | 1, 2                                     | $I_1, I_2$                  |           |
| 160 | 4b  |              |       |                | 160                   | $4b = (14 - 36\varphi) = 2 \cdot 401b$                | (5 isogeny clas  | sse       | es)           |      |                              |  |  |                             | 1604b     |
| a1  | 1   | $-1-\varphi$ | ,     | 1              | $-3-4\varphi$         | $7+9\varphi$  | 1                |           | 1             |      | 4, 1                         | 4, 1   | 2, 1                                     | $I_4, I_1$                  |           |
| b1  | 1   | -1           |       | 0              | $-39-65\varphi$       | $194 + 315\varphi$                                    | 0                | - ;-<br>) | 5             | <br> | 5, 1                         | 5,1  | 5,1                                      | $ar{I}_5,ar{I}_1$           |           |
| b2  | $1+\varphi$                                       | 1            |       | $1+\varphi$    | $-1051 + 697\varphi$  | $-16191 + 10164\varphi$                               | 0                | )         | 1             |      | 1, 5                         | 1,5  | 1, 5                                     | $\mathrm{I}_1,\mathrm{I}_5$ |           |
| c1  | $1+\varphi$                                       | $1-\varphi$  | ,     | $\varphi$      | $-13-7\varphi$        | $13+12\varphi$  | 1                | -         | 2             | ++   | 8, 1                         | 8,1  | 8,1                                      | $I_8, I_1$                  |           |
| c2  | 1   | $1+\varphi$  | )<br> | $\varphi$      | $-683 - 1108\varphi$  | $12468 + 20169\varphi$                                | 1                | .         | 2             | - +  | 4, 2                         | 4, 2   | 4,2                                      | $I_4, I_2$                  |           |
| d1  | $\varphi$   |              |       | $1+\varphi$    | $-1-\varphi$          | $-1-\varphi$  |                  | -         | 2             | ++   | 2, 1                         | 2,1  | 2,1                                      | $I_2, I_1$                  |           |
| d2  | $\varphi$   | 1            |       | $1+\varphi$    | $-11-\varphi$         | $-11 + 11\varphi$                                     | 1                | -         | 2             | ++   | 1,2                          | 1,2  | 1,2                                      | $I_1, I_2$                  |           |
| e1  | $1+\varphi$                                       |              |       | $1+\varphi$    | $-10 + \varphi$       | $-17 + \varphi$                                       |                  |           | 2             | ++   | 2, 1                         | 2,1  | 2,1                                      | $I_2, I_1$                  |           |
| e2  | 1   | -1           | -     | 0              | $-225 - 362\varphi$   | $-2401 - 3884\varphi$                                 | 0                | )         | 2             | -+   | 1, 2                         | 1,2  | 1,2                                      | $I_1, I_2$                  |           |
| 161 | 6a  |              |       |                | 1616                  | $3a = (20 - 36\varphi) = 2^2 \cdot 101a$              | (2 isogeny cla   | ass       | es)           |      |                              |  |  |                             | 1616a     |
| a1  | 0   | 0            | )     | 0              | $-3-\varphi$          | $2+\varphi$   | 1                |           | 2             | ++   | 1,4                          | 1  | 1,3                                      | $I_1, IV$                   |           |
| a2  | 0   | 0            | )     | 0              | $-3+4\varphi$         | $2+8\varphi$  | 1                | .         | 2             | + -  | 2,8                          | 2  | 2,3                                      | $I_2, IV^*$                 |           |
| b1  | 0   | 1            |       | 0              | $-4-6\varphi$         | $-7-11\varphi$  | 0                | )         | 2             | ++   | 1, 4                         | 1  | 1,1                                      | $I_1, IV$                   |           |
| b2  | 0   | $-1-\varphi$ | )     | 0              | $12 - 8\varphi$       | $-4+4\varphi$   | 0                | )         | 2             | - +  | 2,8                          | 2  | 2,1                                      | $I_2, IV^*$                 |           |
| 161 | <b>6</b> b  |              |       |                | 1610                  | $6b = (16 - 36\varphi) = 2^2 \cdot 101b$              | (2 isogeny cla   | asse      | es)           |      |                              |  |  |                             | 1616b     |
| a1  | 0   | 0            | )     | 0              | $-4+\varphi$          | $3-\varphi$   |                  | _         | 2             | ++   | 1,4                          | 1  | 1,3                                      | $I_1, IV$                   |           |
| a2  | 0   | 0            |       | 0              | $1-4\varphi$          | $10-8\varphi$   |                  | - 1       | 2             | -+   | 2, 8                         | 2  | 2, 3                                     | $I_2$ , $IV^*$              |           |
| b1  | 0   | 1            |       | 0              | $-10+6\varphi$        | $-18 + 11\varphi$                                     | 0                | )         | 2             | ++   | 1, 4                         | 1  | 1,1                                      | $I_1, IV$                   | [         |
| b2  | 0   | $1+\varphi$  | ,     | 0              | $3+10\varphi$         | $3+5\varphi$  | 0                | )         | 2             | + -  | 2,8                          | 2  | 2,1                                      | $I_2, IV^*$                 |           |
| 161 | 9a  |              |       |                | 16.                   | $19a = (17 - 36\varphi) = 1619a$                      | (2 isogeny class | sses      | s)            |      |                              |  |  |                             | 1619a     |
| a1  |   | $-1-\varphi$ | )     | 1              | $-2-2\varphi$         | $\frac{2+4\varphi}{}$                                 | ·                |           | 1             | + -  | 1                            | 1  | 1  | $I_1$                       |           |
| b1  | $1+\varphi$                                       |              |       | 1              | $-1+\varphi$          | 0   |                  | -  -      | 1             | + -  | 1                            | 1  | 1  | $I_1$                       |           |
| 161 | 9b  |              |       |                | 16                    | $19b = (19 - 36\varphi) = 1619b$                      | (2 isogeny class | ses       | 3)            |      |                              | 1  |  |                             | 1619b     |
| al  | $1+\varphi$                                       | 1            |       | (0             | $-4+2\varphi$         | $\frac{100 - (10 - 60\varphi) - 10100}{2 - 2\varphi}$ | ·                |           | 1             | -+   | 1                            | 1  | 1  | $I_1$                       |           |
| aı  | Ι - Ι - Ψ   |              | -     | φ              | π ι 2γ                | 2 2Ψ  | 1                |           |               | 1    | 1                            | 1  | 1  | -1                          |           |

|              | $a_1$           | $a_2$       | $a_3$                          | $a_4$                       | $a_6$   | r         | T             | s      | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$    | Kodaira  | Isogenies |
|--------------|-----------------|-------------|--------------------------------|-----------------------------|---|-----------|---------------|--------|------------------------------|-----------------------------|----------|--|-----------|
| 1619         | 9b              |             |                                | 1                           | $1619b = (19 - 36\varphi) = 1619b $             | (2 isogen | y clas        | ses)   |                              |                             |          |  | 1619b     |
| b1           | $\varphi$       | 1           | 1                              | $1-2\varphi$                | 0   | 1         | 1             | -+     | 1                            | 1                           | 1        | $I_1$  |           |
| 1620         | )a              |             |                                | 16                          | $20a = (18 - 36\varphi) = 2 \cdot 5a \cdot 3^2$ | (3 isoge  | ny cl         | asses) |                              |                             |          |  | 1620a     |
| a1           | 1               | -1          | 1                              | -128                        | 587   | 1         | 6             | ++     | 3, 3, 4                      | 3, 4                        | 3, 2, 2  | $I_3$ , III, $I_4$                                 |           |
| a2           | 1               | -1          | 1                              | -8                          | 11  | 1         | 6             |        | 6, 3, 2                      | 6, 2                        | 6, 2, 2  | $I_6, III, I_2$                                    |           |
| a3           | 1               | -1          | 1                              | -218                        | -269  | 1         | 2             | ++     | 1, 9, 12                     | 1,12                        | 1, 2, 2  | $I_1, III^*, I_{12}$                               |           |
| a4           | 1               | -1          | 1                              | 52                          | -53   | 1         | 2             |        | 2, 9, 6                      | 2,6                         | 2, 2, 2  | $I_2, III^*, I_6$                                  |           |
| b1           | 1               | -1          | 1                              | -3002                       | 63929   | 0         | 12            | + +    | 6, 8, 12                     | [6, 2, 12]                  | 6, 4, 12 | $I_6, I_2^*, I_{12}$                               | Ī         |
| b2           | 1               | -1          | 1                              | -617                        | 5231  | 0         | 2             | ++     | 1, 18, 2                     | 1, 12, 2                    | 1, 4, 2  | $I_1, I_{12}^*, I_2$                               |           |
| b3           | 1               | -1          | 1                              | -167                        | -709  | 0         | 4             | ++     | 2, 12, 4                     | 2, 6, 4                     | 2, 4, 4  | $I_2, I_6^*, I_4$                                  |           |
| b4           | 1               | -1          | 1                              | -122                        | 1721  | 0         | 12            |        | 12, 7, 6                     | 12, 1, 6                    | 12, 4, 6 | $I_{12}, I_1^*, I_6$                               |           |
| b5           | 1               | -1          | 1                              | 13                          | -61   | 0         | 4             |        | 4, 9, 2                      | 4, 3, 2                     | 4, 4, 2  | $I_4, I_3^*, I_2$                                  |           |
| b6           | 1               | -1          | 1                              | -48002                      | 4059929   | 0         | 6             | ++     | 3, 10, 6                     | 3, 4, 6                     | 3, 4, 6  | $I_3, I_4^*, I_6$                                  |           |
| b7           | 1               | -1          | 1                              | -4082                       | 14681   | 0         | 6             | ++     | 3, 7, 24                     | 3, 1, 24                    | 3, 4, 24 | $I_3, I_1^*, I_{24}$                               |           |
| b8           | 1               | -1          | 1                              | -2597                       | -50281  | 0         | 2             | ++     | 1, 9, 8                      | 1, 3, 8                     | 1, 4, 8  | $I_1, I_3^*, I_8$                                  |           |
| c1           | 1               | -1          | 0                              | -24                         | 18  | 0         | 6             | + +    | 1, 3, 12                     | 1,12                        | 1, 2, 12 | $ $ $I_1$ , $III$ , $I_{12}$                       | [         |
| c2           | 1               | -1          | 0                              | 6                           | 0   | 0         | 6             |        | 2, 3, 6                      | 2,6                         | 2, 2, 6  | $I_2, III, I_6$                                    |           |
| c3           | 1               | -1          | 0                              | -1149                       | -14707  | 0         | 2             | ++     | 3, 9, 4                      | 3, 4                        | 3, 2, 4  | $I_3, III^*, I_4$                                  |           |
| c4           | 1               | -1          | 0                              | -69                         | -235  | 0         | 2             |        | 6, 9, 2                      | 6, 2                        | 6, 2, 2  | $I_6, III^*, I_2$                                  |           |
| 162          | la              |             |                                |                             | $1621a = (28 - 37\varphi) = 1621a$              | (1 isoger | nv cla        | ıss)   |                              |                             |          |  | 1621a     |
| a1           |                 | $1-\varphi$ | $\varphi$                      | $-4 + \varphi$              | $-6+3\varphi$                                   | 0         | 1             |        | 1                            | 1                           | 1        | $I_1$  |           |
| 162          | 1b              |             |                                |                             | $1621b = (9 - 37\varphi) = 1621b$               | (1 isogen | v clas        | ss)    |                              |                             |          |  | 1621b     |
| a1           | $1+\varphi$     | 0           | $1+\varphi$                    | $-3-3\varphi$               | $-3-4\varphi$                                   | 0         | 1             |        | 1                            | 1                           | 1        | $I_1$  |           |
| 1630         | <br>6a          |             |                                | 1                           | $636a = (38 + 6\varphi) = 2 \cdot 409a$         | (4 isogen | v clas        | sses)  |                              |                             |          |  | 1636a     |
| a1           | $\varphi$       | 0           | $1+\varphi$                    | $-340 - 514\varphi$         | $\frac{4156 + 6789\varphi}{}$                   | 0         | 2             | ++     | 5, 2                         | 5,2                         | 1, 2     | $I_5, I_2$   |           |
| a2           | arphi           |             | $1 + \varphi$<br>$1 + \varphi$ | $-20-34\varphi$             | $60 + 101\varphi$                               |           | $\frac{2}{2}$ |        | 10, 1                        | 10, 1                       | 2, 1     | $I_{10}, I_{1}$                                    |           |
| <del>-</del> |                 |             |                                |                             |   | :         |               |        |                              | <del>:</del> '              | <u> </u> |  |           |
| b1           | $\dots \varphi$ |             | $1+\varphi$                    | $-4-\varphi$                | $1-\varphi$                                     |           |               | -+     | $\frac{4,1}{1}$              | 4,1                         | 4,1      | $I_4, I_1$   |           |
| c1           | 1               | 0           | 0                              | $-33-53\varphi$             | $134 + 219\varphi$                              | 0         | 3             | - +    | 2, 1                         | 2, 1                        | 2, 1     | $I_2, I_1$   |           |
| c2           | $1+\varphi$     | $-\varphi$  | 0                              | $-501 + 298\varphi$         | $-5099 + 3165\varphi$                           | 0         | 3             | -+     | 6, 3                         | 6, 3                        | 6, 3     | $I_6, I_3$   |           |
| c3           | 1               | $\varphi$   | $1+\varphi$                    | $-1887057 + 1166258\varphi$ | $-1172701355 + 724769281\varphi$                | 0         | 1             | - +    | $\frac{2,1}{2}$              | $\frac{1}{2}, \frac{1}{2}$  | 2,1      | $I_2, I_1$   |           |
| d1           | $1 + \varphi$   | 0           | 0                              | $-2-8\varphi$               | $-12-12\varphi$                                 | 1         | 1             | - +    | 10, 1                        | 10, 1                       | 10, 1    | $I_{10}, I_1$                                      |           |
| 1630         | 6b              |             |                                | 1                           | $636b = (44 - 6\varphi) = 2 \cdot 409b$         | (4 isogen | y clas        | sses)  |                              |                             |          |  | 1636b     |
| a1           | $1+\varphi$     | $-\varphi$  | $\varphi$                      | $-853 + 512\varphi$         | $10946 - 6790\varphi$                           | 0         | 2             | ++     | 5, 2                         | 5, 2                        | 1, 2     | $I_5, I_2$   |           |
| a2           | $1+\varphi$     | $-\varphi$  | $\varphi$                      | $-53+32\varphi$             | $162-102\varphi$                                | 0         | 2             |        | 10, 1                        | 10, 1                       | 2, 1     | $I_{10}, I_1$                                      |           |
| b1           | $1+\varphi$     |             | '<br>φ                         | $-4-\varphi$                |   | 1         |               | + -    | $\frac{1}{4,1}$              | 4,1                         | 4,1      | $egin{array}{cccccccccccccccccccccccccccccccccccc$ |           |
| 71           | - · Y           | - Y         | Υ                              | Ι Ψ                         | 1   |           |               |        | -, -                         | -, -                        | -, -     | *4,*1  |           |

|   |   |  |  |  |   |  | Leni   |  | 7/4)   | 1 (1)   |   |  |           |
|---|---|--|--|--|---|--|--|--|--|---|---|--|-----------|
|   | $a_1$   | $a_2$  | $a_3$  | $a_4$  | $a_6$   | r  | T  | s  | $\operatorname{ord}(\Delta)$   | $\operatorname{ord}_{-}(j)$   | $c_p$   | Kodaira  | Isogenies |
| 163   | 6b  |  |  | 163  | $6b = (44 - 6\varphi) = 2 \cdot 409b$ (4 is   | ogeny class  | ses)   |  |  |   |   |  | 1636b     |
| c1  | 1   | 0  | 0  | $-86 + 53\varphi$  | $353-219\varphi$  | 0  | 3  | +-   | 2, 1   | 2,1   | 2, 1  | $I_2, I_1$   |           |
| c2  | $\varphi$   | 0  | 0  | $-203 - 298\varphi$  | $-1934 - 3165\varphi$   | 0  | 3  | +-   | 6, 3   | 6, 3  | 6, 3  | $I_6, I_3$   |           |
| c3  | 1   | $1-\varphi$  | $\varphi$  | $-720798 - 1166259\varphi$   | $-447932073 - 724769282\varphi$   | 0  | 1  | + -  | 2,1  | $\begin{bmatrix} 2,1 \end{bmatrix}$   | 2, 1  | $I_2, I_1$   |           |
| d1  | $\varphi$   | $1-\varphi$  | 0  | $-10 + 8\varphi$   | $-24 + 12\varphi$   | 1  | 1  | +-   | 10, 1  | 10, 1   | 10, 1   | $I_{10}, I_1$  |           |
| 1639  | 9a  |  |  | 163  | $9a = (41 - \varphi) = 11b \cdot 149a \tag{1}$  | isogeny cla  | ass)   |  |  |   |   |  | 1639a     |
| a1  | 1   | $1+\varphi$  | 1  | $-5+2\varphi$  | $2-3\varphi$  | 0  | 4  | ++   | 2, 2   | 2,2   | 2, 2  | $I_2, I_2$   |           |
| a2  | 1   | $1+\varphi$  | 1  | 2arphi   | arphi   | 0  | 2  | + -  | 1, 1   | 1,1   | 1, 1  | $\mathrm{I}_1,\mathrm{I}_1$  |           |
| a3  | $1+\varphi$   | $1-\varphi$  | 1  | $-464 + 282\varphi$  | $4245 - 2621\varphi$  | 0  | 4  | ++   | 4, 1   | 4,1   | 4, 1  | $I_4, I_1$   |           |
| a4  | 1   | $1+\varphi$  | 1  | $-10-18\varphi$  | $-26-71\varphi$   | 0  | 2  | -+   | 1,4  | 1,4   | 1,4   | $I_1, I_4$   |           |
| 1639  | 9b  |  |  | 163  | $39b = (40 + \varphi) = 11a \cdot 149b \tag{1}$   | isogeny cla  | ıss)   |  |  |   |   |  | 1639b     |
| a1  | 1   | $-1-\varphi$   | 0  | -4   | $3+4\varphi$  | 0  | 4  | ++   | 2, 2   | 2,2   | 2, 2  | $I_2, I_2$   |           |
| a2  | 1   | $-1-\varphi$   | 0  | 1  | 0   | 0  | 2  | -+   | 1, 1   | 1,1   | 1, 1  | $I_1, I_1$   |           |
| a3  | $\varphi$   | 1  | 1  | $-181 - 283\varphi$  | $1624 + 2621\varphi$  | 0  | 4  | ++   | 4, 1   | 4,1   | 4, 1  | $I_4, I_1$   |           |
| a4  | 1   | $-1-\varphi$   | 0  | $-29 + 20\varphi$  | $-68 + 52\varphi$   | 0  | 2  | + -  | 1,4  | 1,4   | 1,4   | $I_1, I_4$   |           |
| 1639  | 9c  |  |  | 1639c  | $e = (27 - 37\varphi) = 11a \cdot 149a \tag{2}$   | isogeny cla  | asses)   |  |  |   |   |  | 1639c     |
| a1  | 0   | 0  | $\varphi$  | $-\varphi$   | 0   | 1  | 1  | -+   | 1, 1   | 1,1   | 1, 1  | $I_1,I_1$  |           |
| b1  |   |  |  |  |   |  |  |  |  | ,   |   |  |           |
| ~ I   | 0   | 1  | $\varphi$  | $-5+2\varphi$  | $3-2\varphi$  | 0  | 3  | - +  | 1, 1   | $\mid 1, 1 \mid$  | 1, 1  | $oxed{I}_1,oxed{I}_1$  |           |
| b2  | 0   | 1 1  | $arphi \ arphi$  | $-5 + 2\varphi$ $-5 - 18\varphi$   | $\begin{array}{c} 3 - 2\varphi \\ -4 - 53\varphi \end{array}$   | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$   | 3  | -+   | $   \begin{array}{c}     1, 1 \\     3, 3   \end{array} $  | $ \begin{array}{c c} 1,1\\ 3,3 \end{array} $  | $1, 1 \\ 1, 1$  | $\begin{bmatrix} I_1, I_1 \\ I_3, I_3 \end{bmatrix}$   |           |
|   | 0   |  | •  | $-5-18\varphi$   | $-4-53\varphi$  |  | 1  | 1  |  |   |   |  | 1639d     |
| b2  | 0   | 1  | φ  | $-5 - 18\varphi$ $1639a$   | $-4-53\varphi$  | 0  | 1  | 1  |  |   | 1,1   | $I_3, I_3$   | 1639d     |
| 1639  | 9d  | 0 1  | $\frac{\varphi}{\varphi}$ $1 + \varphi$                  | $ \begin{array}{r} -5 - 18\varphi \\ \hline 1639a \\ -1 + \varphi \end{array} $  | $-4 - 53\varphi$ $l = (10 - 37\varphi) = 11b \cdot 149b \qquad (2$ $-\varphi$   | isogeny cla  | asses)   | +-   | 3,3  | 3,3   | 1,1   | $I_3, I_3$ $I_1, I_1$  | 1639d     |
| 1639  | 9 <b>d</b>  | 1<br>0<br>1  | φ  | $-5 - 18\varphi$ $1639a$   | $-4 - 53\varphi$ $d = (10 - 37\varphi) = 11b \cdot 149b \qquad (2)$   | isogeny cla  | 1 asses)   | -+   | 3,3  | 3,3   | 1,1   | $I_3, I_3$   | 1639d     |
| 1639<br>a1<br>b1  | 9d  | 1<br>0<br>1  | $\frac{1+\varphi}{1+\varphi}$                            | $-5 - 18\varphi$ $1639a$ $-1 + \varphi$ $-3 - 2\varphi$ $-23 + 18\varphi$  | $-4 - 53\varphi$ $d = (10 - 37\varphi) = 11b \cdot 149b \qquad (2)$ $-\varphi$ $1 + \varphi$ $-57 + 52\varphi$  | isogeny cla  | 1 asses) 1 3 1   | +-   | 3, 3<br>   | 3,3   | 1,1   | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   | 1639d     |
| b2  1639  a1  b1  b2  b2  | 9d  | 1<br>0<br>1  | $\frac{1+\varphi}{1+\varphi}$                            | $-5 - 18\varphi$ $1639a$ $-1 + \varphi$ $-3 - 2\varphi$ $-23 + 18\varphi$  | $-4 - 53\varphi$ $d = (10 - 37\varphi) = 11b \cdot 149b \qquad (2)$ $-\varphi$ $1 + \varphi$ $-57 + 52\varphi$  | isogeny cla  | 1 asses) 1 3 1   | +-   | 3, 3<br>   | 3,3   | 1,1   | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   |           |
| 163:  a1 b1 b2  165:  | 9d<br>0<br>0<br>0   | 0 1 1 1 1  | $ \begin{array}{c}                                     $ | $ \begin{array}{r} -5 - 18\varphi \\ \hline 1639a \\ -1 + \varphi \\ \hline -3 - 2\varphi \\ -23 + 18\varphi \end{array} $   | $-4 - 53\varphi$ $l = (10 - 37\varphi) = 11b \cdot 149b \qquad (2$ $-\varphi$ $1 + \varphi$ $-57 + 52\varphi$ $a = (26 - 37\varphi) = 5a \cdot 331a \qquad (6$  | isogeny cla  0 0 0 classification of the control of | 1 asses) 1 3 1 sses)                                       | - +<br>  + -<br>  + -<br>  + -                                 | 3, 3<br>   | 3,3<br>1,1<br>1,1<br>3,3  | 1, 1  | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   |           |
| 1639  a1   b1   b2    1650  a1   a1   contact   contact | $ \begin{array}{c} 0 \\ \hline \mathbf{9d} \\ \hline 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} \mathbf{5a} \\ \hline 1+\varphi \end{array} $   | $ \begin{array}{c} 1 \\ 0 \\ 1 \\ 1 \end{array} $ $ 1 - \varphi$   | $ \begin{array}{c}                                     $ | $-5 - 18\varphi$ $1639a$ $-1 + \varphi$ $-3 - 2\varphi$ $-23 + 18\varphi$ $1655a$ $1 - 5\varphi$   | $-4 - 53\varphi$ $l = (10 - 37\varphi) = 11b \cdot 149b \qquad (2$ $-\varphi$ $1 + \varphi$ $-57 + 52\varphi$ $a = (26 - 37\varphi) = 5a \cdot 331a \qquad (6$  | isogeny cla  0 0 0 classification of the control of | 1 asses) 1 3 1 sses) 1 1 1                                 | - +<br>  + -<br>  + -<br>  + -                                 | 3, 3  -1, 1 -1, 1 3, 3  -1, 7 -1, 1 -1, 1  | 3,3       1,1       1,1       3,3   | 1,1<br>1,1<br>1,1<br>1,1                                    | $\begin{array}{ c c c c c }\hline I_3, I_3\\ \hline & I_1, I_1\\ \hline & I_1, I_1\\ \hline & I_3, I_3\\ \hline & I_1, I_7\\ \hline & & \\ \hline \end{array}$   |           |
| 163:  a1 b1 b2  165: a1 b1 b1 b1  | $ \begin{array}{c} 0 \\ \hline \mathbf{9d} \\ \hline                                   $  | $ \begin{array}{c c} 1 \\ \hline  & 0 \\ \hline  & 1 \\ 1 \\ \hline  & 1 \\ \hline  & -\varphi \end{array} $   | $ \begin{array}{c}                                     $ | $-5 - 18\varphi$ $1639a$ $-1 + \varphi$ $-3 - 2\varphi$ $-23 + 18\varphi$ $1655a$ $1 - 5\varphi$ $-22 + 14\varphi$   | $-4 - 53\varphi$ $d = (10 - 37\varphi) = 11b \cdot 149b \qquad (2)$ $-\varphi$ $1 + \varphi$ $-57 + 52\varphi$ $a = (26 - 37\varphi) = 5a \cdot 331a \qquad (6)$ $3$  | isogeny cla  0 0 0 classification of the control of | 1 asses) 1 3 1 sses) 1 1 1                                 | +-   | 3, 3<br>1, 1<br>1, 1<br>3, 3<br>1, 7<br>1, 1   | 3,3       1,1       1,1       3,3   | 1,1<br>1,1<br>1,1<br>1,1<br>1,7<br>1,7                      | $\begin{array}{c c} I_3, I_3 \\ \hline & I_1, I_1 \\ \hline & I_1, I_1 \\ \hline & I_3, I_3 \\ \hline & I_1, I_7 \\ \hline & I_1, I_1 \\ \hline \end{array}$   |           |
| b2  1639  a1 b1 b2  1650  a1 b1 c1 c1   | $ \begin{array}{c} 0 \\ \hline  9d \\ \hline  0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 7 \\ \hline  1 + \varphi \\ \hline  1 + \varphi \\ \hline  1 + \varphi \\ \end{array} $                           | $ \begin{array}{c} 1 \\ 0 \\ 1 \\ 1 \end{array} $ $ \begin{array}{c} 1 - \varphi \\ -\varphi \\ -1 \end{array} $                                     | $ \begin{array}{c}                                     $ | $-5 - 18\varphi$ $1639a$ $-1 + \varphi$ $-3 - 2\varphi$ $-23 + 18\varphi$ $1655a$ $1 - 5\varphi$ $-22 + 14\varphi$ $-267 - 390\varphi$   | $-4 - 53\varphi$ $d = (10 - 37\varphi) = 11b \cdot 149b \qquad (2)$ $-\varphi$ $1 + \varphi$ $-57 + 52\varphi$ $a = (26 - 37\varphi) = 5a \cdot 331a \qquad (6)$ $3$ $43 - 26\varphi$ $2885 + 4744\varphi$  | 0   isogeny cla   0   0     0         1     1     0     0  | 1 asses) 1 3 1 sses) 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 | - +<br>  + -<br>  + -<br>  + -<br>  - +                        | 1,1<br>1,1<br>3,3<br>1,7<br>1,1<br>1,13  | 1,1       1,1       1,1       3,3   | 1,1<br>1,1<br>1,1<br>1,1<br>1,7<br>1,1<br>1,1               | $\begin{array}{ c c c c c }\hline I_3, I_3\\ \hline & I_1, I_1\\ \hline & I_1, I_1\\ \hline & I_3, I_3\\ \hline & I_1, I_7\\ \hline & I_1, I_1\\ \hline & I_1, I_{13}\\ \hline \end{array}$  |           |
| 163:  a1 b1 b2  165:  a1 c1 d1 d1   | $ \begin{array}{c} 0 \\ \hline \mathbf{9d} \\ \hline 0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} \mathbf{5a} \\ \hline 1 + \varphi \\ 1 + \varphi \end{array} $  | $ \begin{array}{c} 1 \\ 0 \\ 1 \\ 1 \end{array} $ $ \begin{array}{c} 1 - \varphi \\ \varphi \\ 1 \\ - 1 + \varphi \end{array} $                      | $ \begin{array}{c}                                     $ | $-5 - 18\varphi$ $1639a$ $-1 + \varphi$ $-3 - 2\varphi$ $-23 + 18\varphi$ $1655a$ $1 - 5\varphi$ $-22 + 14\varphi$ $-267 - 390\varphi$ $-2 - 2\varphi$   | $-4 - 53\varphi$ $d = (10 - 37\varphi) = 11b \cdot 149b \qquad (2)$ $-\varphi$ $1 + \varphi$ $-57 + 52\varphi$ $a = (26 - 37\varphi) = 5a \cdot 331a \qquad (6)$ $3$ $43 - 26\varphi$ $2885 + 4744\varphi$ $1$  | 0   isogeny cla   0   0     1     1     0     1        | 1 asses) 1 3 1 sses) 1                                     | - +<br>  + -<br>  + -<br>  - +<br>  - +                        | 3, 3  1, 1  1, 1  3, 3  1, 7  1, 1  1, 13  1, 13   | 1,1   1,1   3,3     1,7     1,13     1,11   | 1,1<br>1,1<br>1,1<br>1,1<br>1,1<br>1,1<br>1,1               | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   |           |
| 1639  a1   b1   b2    1650  a1   c1   d1   d2   | $ \begin{array}{c} 0 \\ \hline 9d \\ \hline 0 \\ \hline 0 \\ 0 \end{array} $ $ \begin{array}{c} 5a \\ \hline 1 + \varphi \\ \hline 1 + \varphi \\ \hline 1 + \varphi \\ \hline                                  $ | $ \begin{array}{c} 1 \\ 0 \\ 1 \\ 1 \end{array} $ $ \begin{array}{c} 1 - \varphi \\ - \varphi \\ - 1 \\ - 1 + \varphi \\ - 1 + \varphi \end{array} $ | $ \begin{array}{c}                                     $ | $-5 - 18\varphi$ $1639a$ $-1 + \varphi$ $-3 - 2\varphi$ $-23 + 18\varphi$ $1655a$ $1 - 5\varphi$ $-22 + 14\varphi$ $-267 - 390\varphi$ $-2 - 2\varphi$ $-7 - 22\varphi$ $-6 - 5\varphi$ $-1$                   | $-4 - 53\varphi$ $d = (10 - 37\varphi) = 11b \cdot 149b \qquad (2)$ $-\varphi$ $1 + \varphi$ $-57 + 52\varphi$ $a = (26 - 37\varphi) = 5a \cdot 331a \qquad (6)$ $3$ $43 - 26\varphi$ $2885 + 4744\varphi$ $1$ $-39 - 62\varphi$ $-2 - 6\varphi$ $-1 - \varphi$                     | 0   isogeny cla   0   0   0   1   1   1   1   1   1   1  | 1 asses) 1 3 1 sses) 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 | - +<br>  + -<br>  + -<br>  - +<br>  - +<br>  - +               | $ \begin{array}{c} 3,3 \\ \hline 1,1 \\ 1,1 \\ 3,3 \\ \hline 1,7 \\ 1,1 \\ 1,13 \\ \hline 1,1 \\ 3,3 \\ \hline 2,2 \\ 1,1 \\ \end{array} $ | $\begin{array}{ c c c c c }\hline & 3,3 \\ \hline & 1,1 \\ \hline & 1,1 \\ \hline & 3,3 \\ \hline & & 1,7 \\ \hline & & 1,1 \\ \hline & & 1,13 \\ \hline & & & 1,1 \\ \hline & & & 3,3 \\ \hline & & & & 2,2 \\ \hline & & & & 1,1 \\ \hline \end{array}$ | 1,1<br>1,1<br>1,1<br>1,1<br>1,1<br>1,1<br>1,1<br>1,1<br>3,1 | $\begin{array}{ c c c c c }\hline I_3, I_3\\ \hline & I_1, I_1\\ \hline & I_3, I_3\\ \hline & I_3, I_3\\ \hline \end{array}$   |           |
| 1639  a1   b1   b2    1650  a1   c1   d1   d2   e1  | $ \begin{array}{c} 0 \\ \hline  9d \\ \hline  0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ \hline  1 \\                               $   | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $ \begin{array}{c}                                     $ | $-5 - 18\varphi$ $1639a$ $-1 + \varphi$ $-3 - 2\varphi$ $-23 + 18\varphi$ $1655a$ $1 - 5\varphi$ $-22 + 14\varphi$ $-267 - 390\varphi$ $-2 - 2\varphi$ $-7 - 22\varphi$ $-6 - 5\varphi$ $-1$ $-91 - 75\varphi$ | $-4 - 53\varphi$ $d = (10 - 37\varphi) = 11b \cdot 149b \qquad (2)$ $-\varphi$ $1 + \varphi$ $-57 + 52\varphi$ $a = (26 - 37\varphi) = 5a \cdot 331a \qquad (6)$ $3$ $43 - 26\varphi$ $2885 + 4744\varphi$ $1$ $-39 - 62\varphi$ $-2 - 6\varphi$ $-1 - \varphi$ $-203 - 522\varphi$ | isogeny cla  | 1 3 1 sses) 1 1 1 1 1 3 1 1 4                              | - +<br>  + -<br>  + -<br>  - +<br> <br>  - +<br>  - +          | 1,1<br>1,1<br>3,3<br>1,7<br>1,1<br>1,13<br>1,1<br>3,3<br>2,2<br>1,1<br>1,4   | 1,1       1,1       1,1       3,3         1,7       1,1       1,13       1,1       3,3  | 1,1<br>1,1<br>1,1<br>1,1<br>1,1<br>1,1<br>1,1<br>1,1        | $\begin{array}{ c c c c c }\hline I_3, I_3\\ \hline & I_1, I_1\\ \hline & I_2, I_2\\ \hline & I_1, I_1\\ \hline & I_1, I_1\\ \hline & I_2, I_2\\ \hline & I_1, I_1\\ \hline & I_1, I_4\\ \hline \end{array}$ |           |
| 1639  a1   b1   b2    1650  a1   c1   d1   d2   e1   e2   | $ \begin{array}{c} 0 \\ \hline  9d \\ \hline  0 \\ 0 \\ 0 \end{array} $ $ \begin{array}{c} 0 \\ 0 \\ \hline  1 \\                               $   | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $ \begin{array}{c}                                     $ | $-5 - 18\varphi$ $1639a$ $-1 + \varphi$ $-3 - 2\varphi$ $-23 + 18\varphi$ $1655a$ $1 - 5\varphi$ $-22 + 14\varphi$ $-267 - 390\varphi$ $-2 - 2\varphi$ $-7 - 22\varphi$ $-6 - 5\varphi$ $-1$                   | $-4 - 53\varphi$ $d = (10 - 37\varphi) = 11b \cdot 149b \qquad (2)$ $-\varphi$ $1 + \varphi$ $-57 + 52\varphi$ $a = (26 - 37\varphi) = 5a \cdot 331a \qquad (6)$ $3$ $43 - 26\varphi$ $2885 + 4744\varphi$ $1$ $-39 - 62\varphi$ $-2 - 6\varphi$ $-1 - \varphi$                     | 0   isogeny cla   1   0   0     1   1   1   1   0   0  | 1 asses) 1 sses) 1 1 1 1 1 1 4 2                           | - +<br>  + -<br>  + -<br>  - +<br> <br>  - +<br>  - +<br>  + + | $ \begin{array}{c} 3,3 \\ \hline 1,1 \\ 1,1 \\ 3,3 \\ \hline 1,7 \\ 1,1 \\ 1,13 \\ \hline 1,1 \\ 3,3 \\ \hline 2,2 \\ 1,1 \\ \end{array} $ | $\begin{array}{ c c c c c }\hline & 3,3 \\ \hline & 1,1 \\ \hline & 1,1 \\ \hline & 3,3 \\ \hline & & 1,7 \\ \hline & & 1,1 \\ \hline & & 1,13 \\ \hline & & & 1,1 \\ \hline & & & 3,3 \\ \hline & & & & 2,2 \\ \hline & & & & 1,1 \\ \hline \end{array}$ | 1,1<br>1,1<br>1,1<br>1,1<br>1,1<br>1,1<br>1,1<br>1,1        | $\begin{array}{ c c c c c }\hline I_3, I_3\\ \hline & I_1, I_1\\ \hline & I_1, I_1\\ \hline & I_3, I_3\\ \hline & I_1, I_1\\ \hline & I_1, I_1\\ \hline & I_1, I_1\\ \hline & I_1, I_1\\ \hline & I_3, I_3\\ \hline & I_2, I_2\\ & I_1, I_1\\ \hline \end{array}$  |           |

|          |                       |              |                  |                                  |   |                   | Impl          |                   | 1( <b>A</b> )                | 1 (:)                       | l _                                      | I/ - 1-:  | T:               |
|----------|-----------------------|--------------|------------------|----------------------------------|---|-------------------|---------------|-------------------|------------------------------|-----------------------------|--|---|------------------|
|          | $a_1$                 | $a_2$        | $a_3$            | $a_4$                            | $a_6$   |                   | T             | s                 | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$                                    | Kodaira   | Isogenies        |
| 165      | 5b                    |              |                  | 165                              | $5b = (11 - 37\varphi) = 5a \cdot 331b$                       | (6 isogeny class  | ses)          |                   |                              |                             |  |   | $1655\mathrm{b}$ |
| a1       | $1+\varphi$           | 0            | 1                | $-4+4\varphi$                    | 3   | 1                 | 1             | + -               | 1,7                          | 1,7                         | 1,7                                      | $I_1, I_7$  |                  |
| b1       | $\varphi$             | 0            | 0                | $-8-14\varphi$                   | $17 + 26\varphi$  | 1                 | 1             | <u> </u>          | 1,1                          | 1,1                         | 1,1                                      | $\overline{\mathrm{I}}_{1},\overline{\mathrm{I}}_{1}$ |                  |
| c1       | 1                     | <br>-1       | $\varphi$        | $-656 + 389\varphi$              | $7630 - 4745\varphi$  | 0                 | 1             | <u> </u>          | 1,13                         | 1,13                        | 1,1                                      | $I_1, I_{13}$   | <u>-</u>         |
| d1       | $\varphi$             | $1+\varphi$  | <br>φ            | $-4+3\varphi$                    | $3-2\varphi$  |                   |               | <u> </u>          | 1, 1                         | 1,1                         | 1,1                                      | $oxed{I_1, I_1}$                                      |                  |
| d2       | $\varphi$             | $1+\varphi$  | $\varphi$        | $-29+23\varphi$                  | $-79 + 55\varphi$   | 1                 | 1             | + -               | 3,3                          | 3, 3                        | 3, 1                                     | $I_3, I_3$  |                  |
| e1       | $1+\varphi$           | 1            | 1                | $-10+5\varphi$                   | $-17+10\varphi$   | 0                 | 4             | + +               | 2, 2                         | 2,2                         | [2, 2]                                   | $I_2, I_2$  |                  |
| e2       | $1+\varphi$           | 1            | 1                | 0                                | -1  | 0                 | 2             | -+                | 1, 1                         | 1, 1                        | 1,1                                      | $I_1, I_1$  |                  |
| e3       | $1+\varphi$           | 1            | 1                | $-165 + 75\varphi$               | $-889 + 596\varphi$   | 0                 | 2             | ++                | 1, 4                         | 1, 4                        | 1, 2                                     | $I_1, I_4$  |                  |
| e4       | $1 + \varphi$         | 1            | 1                | $-15+15\varphi$                  | $11 + 4\varphi$   | 0                 | 2             | + -               | 4, 1                         | 4, 1                        | 2, 1                                     | $I_4, I_1$  |                  |
| f1       | 1                     | $-1-\varphi$ | $\varphi$        | $\varphi$                        | -arphi  | 1                 | 1             | Ī — —             | 1,1                          | 1,1                         | 1,1                                      | $I_1, I_1$  |                  |
| 166      | 1a                    |              |                  | 166                              | $1a = (43 - 4\varphi) = 11b \cdot 151b$                       | (2 isogeny class  | ses)          |                   |                              |                             |  |   | 1661a            |
| a1       | 1                     | 1            | φ                | 0                                | $\varphi$   | 1                 | 2             | + -               | 2, 1                         | 2,1                         | 2, 1                                     | $I_2, I_1$  |                  |
| a2       | $1 + \varphi$         | $-1+\varphi$ | 0                | $-63 + 38\varphi$                | $-194 + 120\varphi$   | 1                 | 2             | ++                |                              | 1, 2                        | 1,2                                      | $I_1, I_2$  |                  |
| b1       | 1                     | 0            | $\varphi$        | $-13-3\varphi$                   | $17 + 5\varphi$   | 1                 | 6             | <u>-</u><br>  + - | $\frac{1}{2}, \frac{1}{1}$   | $\frac{1}{2}, \frac{1}{1}$  | $\frac{1}{2}, \frac{1}{1}$               | $I_2, I_1$  | <u>  </u>        |
| b2       | $\varphi$             | 0            | $\varphi$        | $-519 - 837\varphi$              | $8505 + 13760\varphi$   | 1                 | 6             | -+                | $\frac{1}{1}, \frac{1}{2}$   | 1, 2                        | 1, 2                                     | $I_1, I_2$  |                  |
| b3       | 1                     | 0            | $\varphi$        | $-13 + 22\varphi$                | $10+73\varphi$  | 1                 | 2             | + -               | 6, 3                         | 6, 3                        | 2, 3                                     | $I_6, I_3$  |                  |
| b4       | 1                     | 0            | $\varphi$        | $-18-163\varphi$                 | $282 + 820\varphi$  | 1                 | 2             | -+                | 3, 6                         | 3, 6                        | 1,6                                      | $I_3, I_6$  |                  |
| 166      | 1b                    |              |                  | 166                              | $1b = (39 + 4\varphi) = 11a \cdot 151a$                       | (2 isogeny clas   | ses)          | 1                 |                              |                             |  |   | 1661b            |
| a1       | 1                     | 1            | $1+\varphi$      | -arphi                           | $1-2\varphi$  |                   | 2             | l – +             | 2,1                          | 2,1                         | 2, 1                                     | $I_2, I_1$  |                  |
| a2       | $\varphi$             |              | $1+\varphi$      | $-26-37 \overset{\prime}{arphi}$ | $-112-184\varphi$   | 1                 | 2             | ++                | 1, 2                         | 1,2                         | 1,2                                      | $I_1, I_2$  |                  |
| b1       | 1                     |              | $1+\varphi$      | $-16 + 2\varphi$                 | 22-6arphi   | 1                 | 6             | <u> </u>          | $\frac{1}{2,1}$              | $\frac{1}{2}, \frac{1}{1}$  | $\frac{1}{2}, \frac{1}{1}$               | $I_2, I_1$  | <u>  </u>        |
| b2       | $1+\varphi$           |              | $1+\varphi$      | $-1356 + 835\varphi$             | $22265 - 13761\varphi$  | 1                 | 6             | + -               | 1, 2                         | 1, 2                        | 1, 2                                     | $I_1, I_2$  |                  |
| b3       | 1                     |              | $1+\varphi$      | $9-23\varphi$                    | $83-74\varphi$  | 1                 | 2             | - +               | 6, 3                         | 6, 3                        | 2,3                                      | $I_6, I_3$  |                  |
| b4       | 1                     |              | $1+\varphi$      | $-181 + 162\varphi$              | $1102 - 821\varphi$   | 1                 | 2             | + -               | 3, 6                         | 3,6                         | 1,6                                      | $I_{3}, I_{6}$  |                  |
| 166      | 1c                    |              |                  | 166                              | $1c = (45 - 7\varphi) = 11b \cdot 151a$                       | (2 isogeny class  | ses)          |                   |                              |                             |  |   | 1661c            |
| a1       | φ                     | $-1-\varphi$ | 1                | $-3-6\varphi$                    | $7+12\varphi$   | 1                 | 2             | -+                | 2, 1                         | 2, 1                        | 2, 1                                     | $I_2, I_1$  |                  |
| a2       | 1                     | -1           | 1                | $-14 + 8\varphi$                 | $-14+10\varphi$   | 1                 | 2             | + -               | 4, 2                         | 4, 2                        | 4, 2                                     | $I_4, I_2$  |                  |
| b1       | $\varphi$             | $-\varphi$   | $\varphi$        | $-\varphi$                       | 0   | 1                 | 1             | <u> </u>          | 1,1                          | 1,1                         | 1,1                                      | $I_1, I_1$  |                  |
| 166      | 1d                    |              |                  | 166                              | $1d = (38 + 7\varphi) = 11a \cdot 151b$                       | (2 isogeny clas   | ses)          |                   |                              |                             |  |   | 1661d            |
| al       | $\frac{1}{1+\varphi}$ | 1            | /0               | $\frac{-9+6\varphi}{}$           | $\frac{1a - (66 + 1\varphi) - 11a \cdot 1616}{10 - 6\varphi}$ | 1                 | 2             | T + -             | 2,1                          | 2, 1                        | 2,1                                      | $I_2, I_1$  |                  |
| a1<br>a2 | $1+\varphi$ 1         | -1           | $rac{arphi}{1}$ | $-9 + 6\varphi$ $-6 - 8\varphi$  | $-4 - 10\varphi$  | 1                 | $\frac{2}{2}$ | <del>+</del> -    |                              | 4, 2                        | $\begin{bmatrix} 2,1\\4,2 \end{bmatrix}$ | $I_{2}, I_{1}$ $I_{4}, I_{2}$                         |                  |
| b1       | $1+\varphi$           | -1           | $1+\varphi$      | $-1-\varphi$                     | -arphi  | 1                 | 1             | <del></del>       | 1, 1                         | 1,1                         | 1,1                                      | $I_1, I_1$  |                  |
| 167      | 6a                    |              |                  | 167                              | $76a = (42 - 2\varphi) = 2 \cdot 419a$                        | (5 isogeny classe | es)           | •                 |                              |                             |  |   | 1676a            |
| a1       | $1+\varphi$           | <b>-</b> ω   | $1+\varphi$      | $-12-17\varphi$                  | $-70-20\varphi$   |                   | <del></del>   | l                 | 16, 1                        | 16, 1                       | 2,1                                      | $I_{16}, I_{1}$                                       |                  |
|          | - ' 7                 | 7            | - ' 7            | 12 114                           |   |                   |               |                   |                              |                             | , -, -                                   | -10, -1   |                  |

|     | $a_1$            |                | $l_2$          | $a_3$       |                    | $\overline{a_4}$ | $a_6$                                  |         |                     | T      | s                 | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$   | $c_p$ | Kodaira   | Isogenies |
|-----|------------------|----------------|----------------|-------------|--------------------|------------------|--|---------|---------------------|--------|-------------------|------------------------------|-------------------------------|-------|---|-----------|
| 167 |                  |                | ~              |             |                    |                  |  |         |                     |        |                   | 014(_)                       | 9142(3)                       |       | TTO GGITG   |           |
| 167 |                  |                |                |             |                    |                  | $6a = (42 - 2\varphi) = 2 \cdot 419a$  |         | sogeny classe       |        | 1                 |                              |                               |       | 1   | 1676a     |
| b1  | $\varphi$        |                |                |             |                    | 0                | 0                                      |         | 1                   | 1      | ļ — -             |                              | 1,1                           | 1,1   | $I_1, I_1$  |           |
| c1  | $1+\varphi$      | $-1-\zeta$     | $\rho$ :       | $1+\varphi$ | -1-3               | $\varphi$        | $-1-\varphi$                           |         | 1                   | 1      | <u> </u>          | - 1,1                        | 1,1                           | 1,1   | $I_1, I_1$  |           |
| d1  | $1+\varphi$      | $1-\varsigma$  | $\rho_{-}$     | $\varphi$   | -2 - 4             | arphi            | $2+\varphi$                            |         | 1                   | 1      | - +               | - 5,1                        | 5,1                           | 5,1   | $I_5, I_1$  |           |
| e1  | 1                | - ' 7          |                |             | -36 + 18           | •                | $83 - 51\varphi$                       |         |                     | 7      | Ī — -             | 7, 1                         | 7,1                           | 7,1   | $I_7, I_1$  |           |
| e2  | 1                | $1+\varsigma$  | ρ .            | $1+\varphi$ | 64 - 52            | $2\varphi$       | $-6271 + 3347\varphi$                  |         | 1                   | 1      |                   | - 1,7                        | 1,7                           | 1,1   | $I_1, I_7$  |           |
| 167 | $6\mathrm{b}$    |                |                |             |                    | 167              | $76b = (40 + 2\varphi) = 2 \cdot 419b$ | (5 is   | ogeny classe        | s)     |                   |                              |                               |       |   | 1676b     |
| a1  | $\varphi$        |                | 0              | $\varphi$   | -27 + 15           | ίφ               | $-89 + 19\varphi$                      |         | 1                   | 1      |                   | - 16,1                       | 16, 1                         | 2,1   | $I_{16}, I_{1}$   |           |
| b1  | $1+\varphi$      | $-1+\zeta$     | <br>ρ          | 1           | $-1 + \frac{1}{2}$ | $\varphi$        | $2-\varphi$                            |         | 1                   | 1      | Ī — —             | - 1,1                        | 1,1                           | 1,1   | $\overline{I_1}, \overline{I_1}$  |           |
| c1  | $\varphi$        |                | 1              | $\varphi$   | -2 + 6             | $\varphi$        | -1                                     |         | 1                   | 1      | <u> </u>          | - 1,1                        | 1,1                           | 1,1   | $\overline{\mathbf{I}}_1,\overline{\mathbf{I}}_1$   | <u>-</u>  |
| d1  | $\varphi$        |                | 1              | $1+\varphi$ | -5 + 2             | $^{-}$ $^{-}$    | 3-2arphi                               |         | 1                   | 1      | <u> </u>          | 5,1                          | [5,1]                         | 5, 1  | $\overline{ }$ | <u>-</u>  |
| e1  | 1                | $-1-\zeta$     | ρ.             | $1+\varphi$ | -19 - 17           | <br>'φ           | $51 + 67\varphi$                       |         | <del>-</del><br>  1 | 7      | <u> </u>          | 7,1                          | $\frac{1}{7}$ , $\frac{1}{1}$ | 7,1   | $\bar{I}_7, \bar{I}_1$  |           |
| e2  | 1                | $-1-\zeta$     | ρ :            | $1+\varphi$ | 11 + 53            | $\varphi$        | $-2935 - 3401\varphi$                  |         | 1                   | 1      |                   | - 1,7                        | 1,7                           | 1,1   | $I_1, I_7$  |           |
| 168 | 1a               |                |                |             |                    | 1                | $681a = (41) = 41a \cdot 41b$          | (4 isog | geny classes)       |        |                   |                              |                               |       |   | 1681a     |
| a1  | 0                | $1-\varsigma$  | ρ              | $\varphi$   | -1 - 8             | $\varphi$        | $-1-11\varphi$                         |         | 1                   | 1      |                   | - 2,3                        | 2,3                           | 2,3   | $I_2, I_3$  |           |
| b1  | 0                |                |                | $1+\varphi$ | -9 + 8             | <br>Βφ           | $-12+10\varphi$                        |         |                     | 1      | i<br>i            | 3, 2                         | 3,2                           | 3,2   | $\overline{I}_3, \overline{I}_2$  |           |
| c1  | 1                |                |                | $1+\varphi$ | -96 - 151          | <br>.φ           | $708 + 1146\varphi$                    |         | 0                   | 2      | <u>-</u><br>  + + | - 1,3                        | 1,3                           | 1,1   | $\overline{ }$  |           |
| c2  | $1+\varphi$      | $1+\zeta$      | $\rho_{\perp}$ | arphi       | 39 - 52            | $^{.}arphi_{-}$  | $80 - 8\varphi$                        |         | 0                   | 2      | <u> </u>          | 2,6                          | 2,6                           | 2,2   | $I_2, I_6$  |           |
| d1  | 1                | $-1 + \zeta$   | ρ              | $\varphi$   | -246 + 150         |                  | $1855 - 1147\varphi$                   |         | 0                   | 2      | + +               | ,                            | 3,1                           | [1,1] | $I_3, I_1$  |           |
| d2  | $\varphi$        | ζ              | ρ              | $\varphi$   | -16 + 54           | $\varphi$        | $140 - 7\varphi$                       |         | 0                   | 2      |                   | 6,2                          | 6, 2                          | 2,2   | $I_6, I_2$  |           |
| 168 | 1b               |                |                |             |                    | 16               | $81b = (24 - 37\varphi) = 41b^2$       | (3 iso  | geny classes        | 3)     |                   |                              |                               |       |   | 1681b     |
| a1  | 0                |                |                | $\varphi$   | -16 - 20           | $\varphi$        | $-67-79\varphi$                        |         | 0                   | 1      |                   | •                            | 1                             | 2     | $I_1^*$   |           |
| a2  | 0                | $1+\varsigma$  | ρ              | $\varphi$   | -1366 - 40         | )φ               | $22560 + 5872\varphi$                  |         | 0                   | 1      | <u> </u>          | - 13                         | 7                             | 2     | I <sub>7</sub>  |           |
| b1  | 1                |                | 1              | 0           | -89 + 49           | )φ<br>           | $298 + 184\varphi$                     |         | 0                   | 1      | + -               | - 10                         | 0                             | 1     | II*   |           |
| c1  | $\varphi$        |                | 1              | $1+\varphi$ | $-3 + \frac{1}{2}$ | $\varphi$        | -3                                     |         | 1                   | 1      | +-                | - 4                          | 0                             | 3     | IV  |           |
| 168 | 1c               |                |                |             |                    | 16               | $81c = (13 - 37\varphi) = 41a^2$       | (3 iso  | geny classes        | ;)     |                   |                              |                               |       |   | 1681c     |
| a1  | 0                | $-1-\varsigma$ | ρ              | $1+\varphi$ | -37 + 22           |                  | $-109 + 57\varphi$                     |         | 0                   | 1      |                   | - 7                          | 1                             | 2     | $\mathrm{I}_1^*$  |           |
| a2  |                  | $-1-\zeta$     |                |             | -1407 + 42         | $2\varphi$       | $29839 - 5914\varphi$                  |         | 0                   | 1      |                   | - 13                         | 7                             | 2     | I <sub>7</sub>  |           |
| b1  | 1                |                | 1              | 0           | -40 - 49           | arphi            | $482 - 184\varphi$                     |         | 0                   | 1      | <u> </u>          | - 10                         | 0                             | 1     |   |           |
| c1  | $1+\varphi$      | $1-\varsigma$  | ρ              | $\varphi$   | -1 - 3             | arphi            | $-2-\varphi$                           |         | 1                   | 1      | +                 | - 4                          | 0                             | 3     | IV  |           |
| 168 | $\overline{4}$ a |                |                |             |                    | 168              | $84a = (46 - 8\varphi) = 2 \cdot 421a$ | (1 i    | isogeny class       | <br>s) |                   |                              |                               |       |   | 1684a     |
| a1  | $1+\varphi$      |                | 0              | $\varphi$   |                    | -2               | $1-\varphi$                            |         |                     | 1      |                   | - 2,1                        | 2,1                           | 2,1   | $I_2, I_1$  |           |
|     | · <i>r</i>       |                |                | г           |                    |                  | r                                      |         |                     |        | 1                 | ,                            | ,                             | ,     | 2, 1  |           |

|          | $a_1$              | $a_2$        | $a_3$              | $a_4$                                 | $a_6$   |           | r                                      | T              | s          | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$               | $c_p$             | Kodaira  | Isogenies |
|----------|--------------------|--------------|--------------------|---------------------------------------|---|-----------|--|----------------|------------|------------------------------|---|-------------------|--|-----------|
| 168      | 4b                 |              |                    | 1                                     | $684b = (38 + 8\varphi) = 2 \cdot 421b$                     | (1 isogen | ıy cl                                  | ass)           |            |                              |   |                   |  | 1684b     |
| a1       | φ                  | $1-\varphi$  | $1 + \varphi$      | $-1-2\varphi$                         | 0   |           | 1                                      | 1              |            | 2, 1                         | 2,1                                       | 2, 1              | $I_2, I_1$   |           |
| 169      | 1a                 |              |                    | 169                                   | $1a = (14 - 37\varphi) = 19a \cdot 89a$                     | (4 isoger | ny cl                                  | lasse          | s)         |                              |   |                   |  | 1691a     |
| a1       | φ -                | $-1-\varphi$ | $\varphi$          | $-44 + 27\varphi$                     | $135 - 84\varphi$   |           | 1                                      | 1              |            | 1, 1                         | 1,1                                       | 1, 1              | $I_1, I_1$   |           |
| b1       | $\varphi$ -        | $-1-\varphi$ | $1+\varphi$        | $-14-18\varphi$                       | $36 + 59\varphi$  |           | $\overline{1}$                         | $\bar{2}$      | ++         | 4,1                          | 4,1                                       | [-4, 1]           | $  I_4, I_1$   |           |
| b2       | $1+\varphi$        | 1            | 1                  | $-1280 - 2074\varphi$                 | $32432 + 52476\varphi$                                      |           | 1                                      | $2 \mid$       | +          | 2, 2                         | 2,2                                       | 2, 2              | $I_2, I_2$   |           |
| c1       | $\varphi$          | 1            | 1                  | $-6+2\varphi$                         | $3-4\varphi$  |           | 1                                      | 2              | ++         | 2, 1                         | 2,1                                       | 2, 1              | $I_2, I_1$   |           |
| c2       | 1 -                | $-1-\varphi$ | 0                  | $-629 + 390\varphi$                   | $7175 - 4435\varphi$  |           | 1                                      | 2              | +-         | 1,2                          | 1,2                                       | 1, 2              | $I_1, I_2$   |           |
| d1       | $1+\varphi$ -      | $-1-\varphi$ | $1 + \varphi$      | $-2\varphi$                           | $-\varphi$  |           | 1                                      | 1              |            | 1, 1                         | 1,1                                       | 1, 1              | $I_1, I_1$   |           |
| 169      | 1b                 |              |                    | 169                                   | $01b = (23 - 37\varphi) = 19b \cdot 89b$                    | (4 isogen | ny cl                                  | asse:          | s)         |                              |   |                   |  | 1691b     |
| a1       | $1+\varphi$        | 1            | 0                  | $-16-26\varphi$                       | $35 + 57\varphi$  |           | 1                                      | 1              |            | 1,1                          | 1,1                                       | 1, 1              | $I_1, I_1$   |           |
| b1       | $1+\varphi$        | 1            | 1                  | $-31+18\varphi$                       | $65-42\varphi$  |           | $\overline{1}$                         | $\bar{2}^{-1}$ | ++         | 4,1                          | 4,1                                       | 4,1               | $I_4, I_1$   |           |
| b2       | $\varphi$ -        | $-1-\varphi$ | $1 + \varphi$      | $-3355 + 2074\varphi$                 | $88262 - 54551\varphi$                                      |           | 1                                      | 2              | + -        | 2, 2                         | 2,2                                       | 2, 2              | $I_2, I_2$   |           |
| c1       | $1+\varphi$        | $1-\varphi$  | 1                  | $-4-3\varphi$                         | $-1+4\varphi$   |           | $\overline{1}$                         | $\bar{2}$      | ++         | 2, 1                         | [2,1]                                     | 2, 1              | $ $ $I_2,I_1$  |           |
| c2       | 1                  | $1+\varphi$  | 1                  | $-240 - 388\varphi$                   | $2500 + 4046\varphi$  |           | 1                                      | 2              | -+         | 1, 2                         | 1, 2                                      | 1, 2              | $I_1, I_2$   |           |
| d1       | $\varphi$          | -1           | $\varphi$          | 0                                     | 0   |           | 1                                      | 1              |            | 1, 1                         | 1,1                                       | 1, 1              | $I_1, I_1$   |           |
| 169      | 1c                 |              |                    | 16                                    | $91c = (44 - 5\varphi) = 19a \cdot 89b$                     | (2 isogen | y cla                                  | asses          | )          |                              |   |                   |  | 1691c     |
| a1       | 1                  | -1           | 1                  | $-5+\varphi$                          | $-3+\varphi$  |           | 1                                      | 2              | ++         | 2,1                          | 2,1                                       | 2, 1              | $I_2, I_1$   |           |
| a2       | 1                  | -1           | 1                  | $-5-4\varphi$                         | $-3+11\varphi$  |           | 1                                      | 2              | -+         | 4, 2                         | 4,2                                       | 4, 2              | $I_4, I_2$   |           |
| b1       | 1                  | 1            | 1                  | $-5+\varphi$                          | 2-arphi   |           | 0                                      | $\overline{4}$ | ++         | 2, 1                         | 2,1                                       | 2, 1              | $I_2, I_1$   |           |
| b2       | 1                  | 1            | 1                  | $-10-4\varphi$                        | $-10-13\varphi$   |           | 0                                      | 4              | ++         | 4, 2                         | 4, 2                                      | 4, 2              | $I_4, I_2$   |           |
| b3       | 1                  | 1            | 1                  | $-50 + 41\varphi$                     | $-232 + 81\varphi$  |           | 0                                      | 2              | + -        | 8, 1                         | 8,1                                       | 8, 1              | $I_8, I_1$   |           |
| b4       | 1                  | 1            | 1                  | $-50 - 129\varphi$                    | $-456 - 775\varphi$   |           | 0                                      | 2              |            | 2,4                          | 2,4                                       | 2,4               | $I_2, I_4$   |           |
| 169      |                    |              |                    |                                       | $91d = (39 + 5\varphi) = 19b \cdot 89a$                     | (2 isogen | y cla                                  |                | /          |                              |   |                   |  | 1691d     |
| a1       | 1                  | -1           | 1                  | $-4-\varphi$                          | $-2-\varphi$  |           | 1                                      | 2              | ++         | 2, 1                         | 2,1                                       | 2, 1              | $I_2, I_1$   |           |
| a2       | 1                  | -1           | <del>.</del> .     | $-9+4\varphi$                         | $8-11\varphi$   |           | 1                                      | 2              | +-         | 4,2                          | 4,2                                       | 4,2               | $I_4, I_2$   | <u> </u>  |
| b1       | 1                  | 1            | 1                  | $-4-\varphi$                          | $1+\varphi$   |           | 0                                      | 4              | ++         | 2, 1                         | $\begin{bmatrix} 2,1\\ 4,2 \end{bmatrix}$ | 2, 1              | $I_2, I_1$   |           |
| b2<br>b3 | 1                  | 1            | 1                  | $-14 + 4\varphi$ $-9 - 41\varphi$     | $-23 + 13\varphi$   |           | 0                                      | 4 2            | ++         | $4, 2 \\ 8, 1$               | 4, 2                                      | 4,2               | $I_4, I_2$   |           |
| b4       | 1<br>1             | 1<br>1       | 1<br>1             | $-9 - 41\varphi$ $-179 + 129\varphi$  | $-151 - 81\varphi$ $-1231 + 775\varphi$                     |           | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\frac{2}{2}$  | - +<br>+ - | 2, 4                         | $8, 1 \\ 2, 4$                            | $8, 1 \\ 2, 4$    | $\begin{bmatrix} I_8, I_1 \\ I_2, I_4 \end{bmatrix}$                           |           |
|          |                    |              |                    |                                       | ·   |           |  |                |            | _, _                         | _, _                                      | _, _,             | -2)-4  | 1705      |
| 170      |                    |              | 4 .                |                                       | $\underline{a = (16 - 37\varphi) = 5a \cdot 11a \cdot 31a}$ | ` ;       | - 1                                    |                |            | 1.1.0                        |   |                   |  | 1705a     |
| a1       | $1+\varphi$        |              | $1+\varphi$        | $-33 + 19\varphi$                     | $-96 + 58\varphi$   |           | $\frac{0}{0}$                          | $\frac{2}{4}$  |            | 1, 1, 3                      | 1, 1, 3                                   | 1, 1, 1           | $I_1, I_1, I_3$  |           |
| a2       | 1<br>1             | $-1 \\ -1$   | 0                  | $-32 - 41\varphi$ $-457 - 691\varphi$ | $-100 - 155\varphi$ $-6495 - 10615\varphi$                  |           | 0                                      | 4              | ++         |                              | 2, 2, 6                                   | 2, 2, 2           | $I_2, I_2, I_6$  |           |
| a3<br>a4 | $1 \\ 1 + \varphi$ |              | $0 \\ 1 + \varphi$ | $-457 - 691\varphi$ $57 - 41\varphi$  | $-6495 - 10615\varphi$ $-416 + 228\varphi$                  |           | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | 2 2            | ++         |                              | 1, 4, 3 $4, 1, 12$                        | 1, 4, 1 $4, 1, 2$ | $\begin{array}{ c c c c c } & I_1, I_4, I_3 \\ & I_4, I_1, I_{12} \end{array}$ |           |
| a4       | $1+\varphi$        | 1            | $\tau + \varphi$   | $57 - 41\varphi$                      | $-410 + 226\varphi$   |           | U                                      | 4              |            | 4, 1, 12                     | 4, 1, 12                                  | 4, 1, 4           | 14,11,112  |           |

|            |               |              |               |                       |   | T                                      |         |                 |       |                              | T                           | T         | T  |           |
|------------|---------------|--------------|---------------|-----------------------|---|--|---------|-----------------|-------|------------------------------|-----------------------------|-----------|--|-----------|
|            | $a_1$         | $a_2$        | $a_3$         | $a_4$                 | $a_6$   | r                                      | ·       | T               | s     | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$     | Kodaira                                  | Isogenies |
| 170        | 5a            |              |               | 1705                  | $a = (16 - 37\varphi) = 5a \cdot 11a \cdot 31e$ | a (6 isog                              | geny    | cla             | sses) |                              |                             |           |  | 1705a     |
| b1         | $1+\varphi$   | 1            | $1 + \varphi$ | $-2-\varphi$          | $-\varphi$                                      | 1                                      |         | 1               |       | 1, 1, 3                      | 1, 1, 3                     | 1, 1, 3   | $I_1, I_1, I_3$                          |           |
| c1         | 1             | $\varphi$    | 1             | $4\varphi$            | 2arphi  | 1                                      | .   - ; | 3 [             |       | 1, 3, 1                      | 1,3,1                       | 1, 3, 1   | $  I_1, I_3, I_1  $                      |           |
| c2         | $1+\varphi$   | -1           | 1             | $-840 + 511\varphi$   | $-10683 + 6590\varphi$                          | 1                                      | .       | 1               |       | 3, 1, 3                      | 3, 1, 3                     | 1, 1, 1   | $I_3, I_1, I_3$                          |           |
| d1         | 1             | $\varphi$    | 1             | $1-6\varphi$          | $-9 + 34\varphi$                                | 1                                      | .       | 1               |       | 1, 1, 13                     | 1, 1, 13                    | 1,1,13    | $  I_1, I_1, I_{13}$                     |           |
| e1         | $1+\varphi$   | $-\varphi$   | $1+\varphi$   | $-2-2\varphi$         | -1-arphi  | 1                                      | .   -   | $\overline{2}$  | ++    | 1, 1, 1                      | 1,1,1                       | 1,1,1     | $ $ $I_1, I_1, I_1$                      |           |
| e2         | $1+\varphi$   | $-\varphi$   | $1 + \varphi$ | $-97-7\varphi$        | $347 + 38\varphi$                               | 1                                      | .   .   | 4               | ++    | 1, 1, 4                      | 1, 1, 4                     | 1, 1, 4   | $I_1,I_1,I_4$                            |           |
| e3         | $1+\varphi$   |              |               | $-7-2\varphi$         | 4   | 1                                      | - 1     | 4               | + +   | 2, 2, 2                      | 2, 2, 2                     | 2, 2, 2   | $\mathrm{I}_2,\mathrm{I}_2,\mathrm{I}_2$ |           |
| e4         | $1+\varphi$   | $-\varphi$   | $1+\varphi$   | $3+3\varphi$          | $21 + 6\varphi$                                 | 1                                      | .   :   | $2 \mid$        |       | 4, 4, 1                      | 4, 4, 1                     | [2, 2, 1] | $  I_4, I_4, I_1$                        |           |
| f1         | 1             | 1            | 1             | $2-\varphi$           | $1-\varphi$                                     | 1                                      |         | 1               |       | 3, 1, 1                      | 3, 1, 1                     | 3, 1, 1   | $I_3, I_1, I_1$                          |           |
| 170        | 5b            |              |               | 1705                  | $b = (21 - 37\varphi) = 5a \cdot 11b \cdot 31b$ | o (6 isog                              | geny    | clas            | sses) |                              |                             |           |  | 1705b     |
| a1         | φ             | $-1-\varphi$ | 0             | $-13-19\varphi$       | $-24-39\varphi$                                 | 0                                      | )       | 2               | ++    | 1, 1, 3                      | 1,1,3                       | 1, 1, 1   | $I_1, I_1, I_3$                          |           |
| a2         | 1             | -1           | 0             | $-73 + 41\varphi$     | $-255 + 155\varphi$                             | 0                                      | )   .   | 4               | + +   | 2, 2, 6                      | 2, 2, 6                     | 2, 2, 2   | $I_2, I_2, I_6$                          |           |
| a3         | 1             | -1           | 0             | $-1148 + 691\varphi$  | $-17110 + 10615\varphi$                         | 0                                      | - 1     | 2               |       | 1, 4, 3                      | 1, 4, 3                     | 1, 4, 1   | $I_1, I_4, I_3$                          |           |
| a4         | $  \varphi  $ | $-1-\varphi$ | 0             | $17 + 41\varphi$      | $-204 - 269\varphi$                             | 0                                      |         | 2               |       | 4, 1, 12                     | 4, 1, 12                    | 4, 1, 2   | $I_4, I_1, I_{12}$                       |           |
| b1         | $  \varphi  $ | $-1-\varphi$ | 0             | $-2+\varphi$          | 2   | 1                                      | .       | 1               |       | 1, 1, 3                      | 1,1,3                       | 1,1,3     | $\mid I_1, I_1, I_3 \mid$                |           |
| c1         | 1             | $1-\varphi$  | 1             | $4-4\varphi$          | $2-2\varphi$                                    | 1                                      | .   - : | 3               |       | 1, 3, 1                      | 1, 3, 1                     | 1, 3, 1   | $  I_1, I_3, I_1  $                      |           |
| c2         | $  \varphi  $ | -arphi       | 1             | $-328-512\varphi$     | $-4093 - 6590\varphi$                           | 1                                      | .       | 1               |       | 3, 1, 3                      | 3, 1, 3                     | 1, 1, 1   | $I_3, I_1, I_3$                          |           |
| d1         | 1             | $1-\varphi$  | 1             | $-5+6\varphi$         | $25-34\varphi$                                  | 1                                      | .       | 1               |       | 1, 1, 13                     | 1,1,13                      | 1,1,13    | $  I_1, I_1, I_{13}  $                   |           |
| e1         | $\varphi$     | 0            | $\varphi$     | -2                    | —1  | 1                                      | .       | $2\overline{ }$ | ++    | 1, 1, 1                      | 1,1,1                       | 1,1,1     | $ $ $I_1,I_1,I_1$                        |           |
| e2         | $\varphi$     | 0            | $\varphi$     | $-102 + 5\varphi$     | $386 - 39\varphi$                               | 1                                      | .   .   | 4               | + +   | 1, 1, 4                      | 1, 1, 4                     | 1, 1, 4   | $I_1, I_1, I_4$                          |           |
| e3         | $\varphi$     | 0            | arphi         | -7                    | $5-\varphi$                                     | 1                                      | - 1     | 4               | + +   | 2, 2, 2                      | 2, 2, 2                     | 2, 2, 2   | $\mathrm{I}_2,\mathrm{I}_2,\mathrm{I}_2$ |           |
| e4         | $\varphi$     | 0            | $\varphi$     | $8-5\varphi$          | $28-7\varphi$                                   | 1                                      | .   _   | 2               |       | 4, 4, 1                      | 4, 4, 1                     | 2, 2, 1   | $\mid I_4, I_4, I_1$                     |           |
| f1         | 1             | 1            | 1             | $1+\varphi$           | arphi   | 1                                      |         | 1               |       | 3, 1, 1                      | 3, 1, 1                     | 3, 1, 1   | $I_3,I_1,I_1$                            |           |
| <b>170</b> | 5c            |              |               | 170                   | $5c = (9 - 38\varphi) = 5a \cdot 11a \cdot 31b$ | (5 isoge                               | eny     | clas            | ses)  |                              |                             |           |  | 1705c     |
| a1         |               | $-1-\varphi$ | 1             | $-22 + 8\varphi$      | $42-22\varphi$                                  | 1                                      | - 1     | 2               | ++    | 1, 2, 8                      | 1, 2, 8                     | 1, 2, 8   | $\mathrm{I}_1,\mathrm{I}_2,\mathrm{I}_8$ |           |
| a2         | $  \varphi  $ | $-1-\varphi$ | 1             | $-2+3\varphi$         | -2  | 1                                      | .   _ : | $2 \mid$        | + -   | 2, 1, 4                      | 2, 1, 4                     | [2, 1, 4] | $I_2, I_1, I_4$                          |           |
| b1         | $\varphi$     | $\varphi$    | 0             | $1+\varphi$           | 0   | 1                                      | .       | $2\overline{ }$ |       | 1, 1, 1                      | 1, 1, 1                     | 1,1,1     | $ $ $I_1,I_1,I_1$                        |           |
| b2         | $\varphi$     | $\varphi$    | 0             | $-4-4\varphi$         | $-6-11\varphi$                                  | 1                                      | .   .   | $4 \mid$        | + +   | 2, 2, 2                      | 2, 2, 2                     | 2, 2, 2   | $\mathrm{I}_2,\mathrm{I}_2,\mathrm{I}_2$ |           |
| b3         | 1             | $-1+\varphi$ | 0             | $-53 + 29\varphi$     | $-139 + 88\varphi$                              | 1                                      | - 1     | 2               |       | 1, 4, 4                      | 1, 4, 4                     | 1, 4, 2   | $I_1, I_4, I_4$                          |           |
| b4         | $\varphi$     | $\varphi$    | 0             | $-69 - 74\varphi$     | $-285 - 538\varphi$                             | 1                                      | .       | $\frac{2}{2}$   |       | 4, 1, 1                      | 4,1,1                       | [2, 1, 1] | $ $ $I_4, I_1, I_1$                      |           |
| c1         | $\varphi$     | $1+\varphi$  | $\varphi$     | $-30-30\varphi$       | $42 + 93\varphi$                                | 0                                      |         | 6               |       | 6, 1, 2                      | 6, 1, 2                     | 6, 1, 2   | $I_6, I_1, I_2$                          |           |
| c2         | $1+\varphi$   | -1           | 0             | $-2454 - 3962\varphi$ | $88215 + 142725\varphi$                         | 0                                      |         | $\frac{6}{2}$   |       | 3, 2, 4                      | 3, 2, 4                     | 3, 2, 2   | $I_3, I_2, I_4$                          |           |
| c3         | 1             | 1            | $\varphi$     | $-4514 + 2669\varphi$ | $-136322 + 84773\varphi$                        | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | - 1     | $\frac{2}{2}$   |       | 1, 6, 12                     | 1, 6, 12                    | 1, 6, 2   | $I_1, I_6, I_{12}$                       |           |
| c4         | 1             |              | $\varphi$     | $-4454 + 2764\varphi$ | $-136591 + 84469\varphi$                        |  | -!      | $\frac{2}{2}$   |       | 2, 3, 6                      | 2, 3, 6                     | 2,3,2     | $  I_2, I_3, I_6  $                      |           |
| d1         |               | $-1+\varphi$ | 1             | $-21+6\varphi$        | $45-22\varphi$                                  | 1                                      | - 1     | $\frac{2}{2}$   |       | 1, 2, 4                      | 1,2,4                       | 1,2,4     | $I_1, I_2, I_4$                          |           |
| d2         | 1             | $-1+\varphi$ | 1             | $-1+\varphi$          | 1   | 1                                      | .   '   | 2               | + -   | 2, 1, 2                      | 2, 1, 2                     | 2, 1, 2   | $I_2, I_1, I_2$                          |           |

|                                  | $a_1$  | $a_2$  | $a_3$                   | $a_4$  | $a_6$  |                  | T   | s   | $\operatorname{ord}(\Delta)$                          | $\operatorname{ord}_{-}(j)$                          | $c_p$                                | Kodaira  | Isogenies |
|----------------------------------|--|--|-------------------------|--|--|------------------|---|---|---|--|--------------------------------------|--|-----------|
|                                  |  |  |                         |  |  |                  | ' '   |   |   | (3)  | P                                    |  |           |
| 170                              | )5c  |  |                         | 1705a  | $c = (9 - 38\varphi) = 5a \cdot 11a \cdot 31b$   | (5 isogen        | y clas  | ses)  |   |  |                                      |  | 1705c     |
| e1                               | 0  | -1   | $1+\varphi$             | 0  | $-1-2\varphi$  | 0                | 3   | <u> </u>  | 1, 1, 3   | 1,1,3  | 1,1,3                                | $I_1, I_1, I_3$  |           |
| e2                               | 0  |  | $1+\varphi$             | $-40 - 70\varphi$  | $-193 - 303\varphi$  | 0                | 1   |   | 3, 3, 1   | 3, 3, 1  | 1, 3, 1                              | $I_3, I_3, I_1$  |           |
|                                  |  |  |                         |  |  |                  |   |   |   |  |                                      |  |           |
| <b>170</b>                       | 5d   |  |                         | 1705d  | $I = (29 - 38\varphi) = 5a \cdot 11b \cdot 31a$  | (5 isoger        | y clas  | sses)   |   |  |                                      |  | 1705d     |
| a1                               | $1+\varphi$  | 1  | $\varphi$               | $-14-8\varphi$   | $6+14\varphi$  | 1                | 2   | ++  | 1, 2, 8   | 1, 2, 8  | 1, 2, 8                              | $I_1,I_2,I_8$  |           |
| a2                               | $1+\varphi$  | 1  | $\varphi$               | $1-3\varphi$   | $-1-3\varphi$  | 1                | 2   | -+  | 2, 1, 4   | 2, 1, 4  | 2, 1, 4                              | $I_2, I_1, I_4$  |           |
| b1                               | $1+\varphi$  | $1+\varphi$  | 1                       | $2+2\varphi$   | 2arphi   | 1                | 2   |   | 1, 1, 1   | 1,1,1  | [1, 1, 1]                            | $ $ $I_1,I_1,I_1$  |           |
| b2                               | $1+\varphi$  | $1+\varphi$  | 1                       | $-8+7\varphi$  | $-12 + 8\varphi$   | 1                | 4   | ++  | 2, 2, 2   | 2, 2, 2  | 2, 2, 2                              | $I_2, I_2, I_2$  |           |
| b3                               | 1  | $-\varphi$   | 0                       | $-24-29\varphi$  | $-51-88\varphi$  | 1                | 2   | ++  | 1, 4, 4   | 1, 4, 4  | 1, 4, 2                              | $\mathrm{I}_1,\mathrm{I}_4,\mathrm{I}_4$   |           |
| b4                               | $1+\varphi$  | $1+\varphi$  | 1                       | $-143 + 77\varphi$   | $-748 + 470\varphi$  | 1                | 2   | ++  | 4, 1, 1   | 4,1,1  | [2, 1, 1]                            | $\mid I_4, I_1, I_1$   |           |
| c1                               | $\varphi$  | $-\varphi$   | 0                       | $-6416 + 3962\varphi$  | $230940 - 142725\varphi$   | 0                | 6   | ++  | 3, 2, 4   | 3, 2, 4  | 3, 2, 2                              | $I_3, I_2, I_4$  |           |
| c2                               | $1+\varphi$  | $-1+\varphi$   | 1                       | $-61 + 31\varphi$  | $227-154\varphi$   | 0                | 6   | -+  | 6, 1, 2   | 6, 1, 2  | 6, 1, 2                              | $I_6, I_1, I_2$  |           |
| c3                               | 1  |  | $1+\varphi$             | $-1845 - 2670\varphi$  | $-51549 - 84774\varphi$  | 0                | 2   | 1   | 1, 6, 12  | 1, 6, 12   | 1, 6, 2                              | $I_1, I_6, I_{12}$   |           |
| c4                               | 1  | 1  | $1+\varphi$             | $-1690 - 2765\varphi$  | $-52122 - 84470\varphi$  | 0                | 2   | -+  | 2, 3, 6   | [2, 3, 6]  | 2, 3, 2                              | $I_2, I_3, I_6$  |           |
| d1                               | 1  | $-\varphi$   | 1                       | $-15-6\varphi$   | $23 + 22\varphi$   | 1                | 2   | + +   | 1, 2, 4   | [1, 2, 4]  | [1, 2, 4]                            | $  I_1, I_2, I_4  $  |           |
| d2                               | 1  | $-\varphi$   | 1                       | -arphi   | 1  | 1                | 2   | -+  | 2, 1, 2   | 2, 1, 2  | 2, 1, 2                              | $I_2, I_1, I_2$  |           |
| e1                               | 0  | -1   | $\varphi$               | 0  | $-2+\varphi$   | 0                | 3   | <u> </u>  | 1, 1, 3   | 1,1,3  | [1, 1, 3]                            | $  I_1, I_1, I_3  $  |           |
| e2                               | 0  | -1   | φ                       | $-110 + 70\varphi$   | $-495 + 302\varphi$  | 0                | 1   |   | 3, 3, 1   | 3, 3, 1  | 1, 3, 1                              | $I_3, I_3, I_1$  |           |
|                                  |  |  |                         |  |  |                  |   |   |   |  |                                      |  |           |
| 171                              |  |  |                         | 171  | $1a = (19 - 37\varphi) = 29a \cdot 59b$  | (3 isogeny       |   | es)   |   |  |                                      |  | 1711a     |
| a1                               | $1+\varphi$  |  | $1+\varphi$             | $-\varphi$   | $-\varphi$   | 1                | 2   | -+  | 1, 1  | 1, 1   | 1,1                                  | $I_1, I_1$   |           |
| a2                               | $1+\varphi$  | 1  | $1+\varphi$             | $-5+4\varphi$  | 4-2arphi   | 1                | 2   | + -   | 2, 2  | 2,2  | 2,2                                  | $I_2, I_2$   |           |
|                                  | $\varphi$  | 1  | $1 + \varphi$           | $-3-2\varphi$  | $-4-3\varphi$  | 1                | 1   | + -   | 1, 1  | 1,1  | 1, 1                                 | $I_1, I_1$   |           |
| b1                               |  |  |                         |  |  |                  |   |   |   | 1 1 1  | 1,1                                  | $I_1, I_1$   | <u> </u>  |
|                                  |  | $-1+\varphi$   |                         | $-12-17\varphi$  | $24+35\varphi$   | 1                | 4   | - +   | 1, 1  | 1,1  | 1 1,1                                |  |           |
| b1<br>c1<br>c2                   | $\varphi$ 1  | $-1 + \varphi$ $-\varphi$  | $1+\varphi$             | $-12 - 17\varphi$ $-183 + 109\varphi$  | $24 + 35\varphi$ $1053 - 647\varphi$   |                  | 8   | - +<br>  + +  | $1, 1 \\ 2, 2$  | $\begin{array}{ c c } & 1,1 \\ & 2,2 \end{array}$    |                                      |  |           |
| c1                               |  | •  | $1 + \varphi$ $\varphi$ | $-183 + 109\varphi$  | $1053 - 647\varphi$  |                  |   |   | $ \begin{array}{c} 1, 1 \\ 2, 2 \\ 1, 1 \end{array} $ | $\begin{bmatrix} 1, 1 \\ 2, 2 \\ 1, 1 \end{bmatrix}$ | 2, 2                                 | $I_2, I_2$   |           |
| c1<br>c2                         | $\begin{vmatrix} 1 \\ 1+\varphi \end{vmatrix}$   | $-\varphi$   | •                       | •  | •  | 1                | 8   | ++  | 2, 2  | 2, 2   |                                      | $\begin{matrix} \mathrm{I}_2,\mathrm{I}_2\\ \mathrm{I}_1,\mathrm{I}_1 \end{matrix}$            |           |
| c1<br>c2<br>c3                   | $ \begin{vmatrix} 1 \\ 1 + \varphi \\ \varphi \end{vmatrix} $                            | $-\varphi \\ 1+\varphi$  | $\varphi$               | $-183 + 109\varphi \\ -19871 + 12284\varphi$   | $1053 - 647\varphi$ $1264809 - 781692\varphi$  | 1 1              | 8<br>4  | +++   | $2, 2 \\ 1, 1$  | 2, 2<br>1, 1   | 2, 2<br>1, 1                         | $I_2, I_2$   |           |
| c1<br>c2<br>c3<br>c4             | $ \begin{vmatrix} 1 \\ 1 + \varphi \\ \varphi \\ \varphi \end{vmatrix} $                 | $-\varphi$ $1 + \varphi$ $-1 + \varphi$  | $\varphi \ 0$           | $-183 + 109\varphi$ $-19871 + 12284\varphi$ $-62 - 42\varphi$  | $\begin{array}{c} 1053 - 647\varphi \\ 1264809 - 781692\varphi \\ -69 - 277\varphi \end{array}$  | 1 1              | 8<br>4<br>4   | ++++++  | 2, 2 $1, 1$ $4, 4$                                    | 2, 2<br>1, 1<br>4, 4                                 | 2, 2<br>1, 1<br>4, 2                 | $\begin{bmatrix} I_2, I_2 \\ I_1, I_1 \\ I_4, I_4 \end{bmatrix}$                               |           |
| c1<br>c2<br>c3<br>c4<br>c5<br>c6 | $ \begin{array}{c c} 1\\ 1+\varphi\\ \varphi\\ \varphi\\ \varphi\\ \varphi \end{array} $ | $ \begin{array}{c} -\varphi \\ 1+\varphi \\ -1+\varphi \\ -1+\varphi \end{array} $                             | $\varphi$ 0 0           | $-183 + 109\varphi$ $-19871 + 12284\varphi$ $-62 - 42\varphi$ $-537 - 887\varphi$ $13 + 163\varphi$  | $1053 - 647\varphi$ $1264809 - 781692\varphi$ $-69 - 277\varphi$ $-9351 - 15580\varphi$ $-847 - 1114\varphi$   | 1<br>1<br>1<br>1 | 8<br>4<br>4<br>2<br>2   | + +<br>+ -<br>+ +<br>- +<br>+ -   | 2, 2<br>1, 1<br>4, 4<br>8, 2                          | 2, 2<br>1, 1<br>4, 4<br>8, 2                         | 2, 2<br>1, 1<br>4, 2<br>8, 2         | $\begin{matrix} I_2, I_2 \\ I_1, I_1 \\ I_4, I_4 \\ I_8, I_2 \end{matrix}$                     | 4844      |
| c1<br>c2<br>c3<br>c4<br>c5<br>c6 | $ \begin{array}{c c} 1 \\ 1 + \varphi \\ \varphi \\ \varphi \\ \varphi \end{array} $     | $ \begin{array}{c} -\varphi \\ 1+\varphi \\ -1+\varphi \\ -1+\varphi \\ -1+\varphi \end{array} $               | $\varphi$ 0 0 0         | $-183 + 109\varphi$ $-19871 + 12284\varphi$ $-62 - 42\varphi$ $-537 - 887\varphi$ $13 + 163\varphi$  | $1053 - 647\varphi$ $1264809 - 781692\varphi$ $-69 - 277\varphi$ $-9351 - 15580\varphi$ $-847 - 1114\varphi$ $1b = (18 - 37\varphi) = 29b \cdot 59a$ | 1 1              | 8 4 4 2 2 2 classe  | + +<br>+ -<br>+ +<br>- +<br>+ -   | 2, 2<br>1, 1<br>4, 4<br>8, 2<br>2, 8                  | 2, 2<br>1, 1<br>4, 4<br>8, 2<br>2, 8                 | 2, 2<br>1, 1<br>4, 2<br>8, 2<br>2, 2 | $I_2, I_2 \\ I_1, I_1 \\ I_4, I_4 \\ I_8, I_2 \\ I_2, I_8$                                     | 1711b     |
| c1<br>c2<br>c3<br>c4<br>c5<br>c6 | $ \begin{array}{c c} 1 \\ 1 + \varphi \\ \varphi \\ \varphi \\ \varphi \end{array} $     | $ \begin{array}{c} -\varphi \\ 1+\varphi \\ -1+\varphi \\ -1+\varphi \\ -1+\varphi \\ -1-\varphi \end{array} $ | $\varphi$ 0 0 0 0       | $   \begin{array}{r}     -183 + 109\varphi \\     -19871 + 12284\varphi \\     -62 - 42\varphi \\     -537 - 887\varphi \\     13 + 163\varphi   \end{array} $ |  | (3 isogeny 1     | $\begin{array}{ c c }\hline 8\\ 4\\ 4\\ 2\\ 2\\ \hline \end{array}$ | + + + + - + - + + - + - + + - + + - + + - + + - + + + + + + + + + + + + + + + + + | 2, 2<br>1, 1<br>4, 4<br>8, 2<br>2, 8                  | 2, 2<br>1, 1<br>4, 4<br>8, 2<br>2, 8                 | 2, 2<br>1, 1<br>4, 2<br>8, 2<br>2, 2 | $ \begin{array}{c c} I_2, I_2 \\ I_1, I_1 \\ I_4, I_4 \\ I_8, I_2 \\ I_2, I_8 \\ \end{array} $ | 1711b     |
| c1<br>c2<br>c3<br>c4<br>c5<br>c6 | $ \begin{array}{c c} 1 \\ 1 + \varphi \\ \varphi \\ \varphi \\ \varphi \end{array} $     | $ \begin{array}{c} -\varphi \\ 1+\varphi \\ -1+\varphi \\ -1+\varphi \\ -1+\varphi \end{array} $               | $\varphi$ 0 0 0         | $   \begin{array}{r}     -183 + 109\varphi \\     -19871 + 12284\varphi \\     -62 - 42\varphi \\     -537 - 887\varphi \\     13 + 163\varphi   \end{array} $ | $1053 - 647\varphi$ $1264809 - 781692\varphi$ $-69 - 277\varphi$ $-9351 - 15580\varphi$ $-847 - 1114\varphi$ $1b = (18 - 37\varphi) = 29b \cdot 59a$ | 1<br>1<br>1<br>1 | 8 4 4 2 2 2 classe  | + + + + - + + - + + - + + - es)   | 2, 2<br>1, 1<br>4, 4<br>8, 2<br>2, 8                  | 2, 2<br>1, 1<br>4, 4<br>8, 2<br>2, 8                 | 2, 2<br>1, 1<br>4, 2<br>8, 2<br>2, 2 | $I_2, I_2 \\ I_1, I_1 \\ I_4, I_4 \\ I_8, I_2 \\ I_2, I_8$                                     | 1711b     |

|          | $a_1$   | $a_2$                 | $a_3$         | $a_4$                                    | $a_6$   |  | T                                    | 8              | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                                | $c_p$          | Kodaira                                       | Isogenies |
|----------|---|-----------------------|---------------|--|---|--|--------------------------------------|----------------|------------------------------|--|----------------|---|-----------|
|          |   |                       |               |  |   |  |                                      | "              | Ord(\(\Delta\)               |  | $c_p$          | Rodana  | , ,       |
| 171      |   |                       |               |  | $= (18 - 37\varphi) = 29b \cdot 59a$          | (3 isogeny cla                         | sses)                                |                |                              |  |                |   | 1711b     |
| c1       | $1+\varphi$   | $\varphi$             | 1             | $-29 + 18\varphi$                        | $76-47\varphi$                                | 1                                      | 4                                    | + -            | 1, 1                         | 1,1  | 1, 1           | $I_1, I_1$                                    |           |
| c2       |   | $-1+\varphi$          | $\varphi$     | $-73 - 110\varphi$                       | $407 + 646\varphi$                            | $\frac{1}{1}$                          | 8                                    | ++             | 2, 2                         | 2, 2   | 2, 2           | $I_2, I_2$                                    |           |
| c3       | $\varphi$   | $\varphi$             | $\varphi$     | $-7590 - 12282\varphi$                   | $478423 + 774103\varphi$                      |  | 4                                    | - +            | 1, 1                         | 1,1  | 1, 1           | $I_1, I_1$                                    |           |
| c4<br>c5 | $1+\varphi$   | $\varphi$             | 1<br>1        | $-104 + 43\varphi$ $176 - 162\varphi$    | $-304 + 215\varphi$ $-2124 + 1127\varphi$     | 1                                      | $\begin{vmatrix} 4\\2 \end{vmatrix}$ | ++             | $4, 4 \\ 2, 8$               | 4, 4 $2, 8$  | $4, 2 \\ 2, 2$ | $I_4, I_4$                                    |           |
| c6       | $\begin{array}{ c c c } 1+\varphi \\ 1+\varphi \end{array}$ | $arphi \ arphi$       | 1             | $-170 - 102\varphi$ $-1424 + 888\varphi$ | $-2124 + 1127\varphi$ $-24044 + 15043\varphi$ | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | $\frac{2}{2}$                        | - +<br>  + -   | 8, 2                         | 8,2  | 8, 2           | $\begin{matrix} I_2,I_8\\I_8,I_2\end{matrix}$ |           |
| 171      |   | ,                     |               | ,  | $= (40 + 3\varphi) = 29a \cdot 59a$           | (3 isogeny clas                        | sses)                                | <u> </u>       |                              | ,  |                | 0, 2  | 1711c     |
| a1       | 0   | (0                    | $1+\varphi$   | $\frac{-7+5\varphi}{}$                   | $\frac{(16+9\varphi)}{-8+4\varphi}$           | 1                                      | 1                                    | + -            | 1,1                          | 1,1  | 1,1            | $I_1, I_1$                                    |           |
| b1       | $\begin{vmatrix} 1 & 1 & 0 \\ 1 & \varphi \end{vmatrix}$    | $   \frac{r}{1}$ $ 1$ | $\varphi$     | $-2+\varphi$                             | $-2+\varphi$                                  | 1                                      | 2                                    | ! <u> </u>     | 1,1                          | $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ | 1,1            | $I_1, I_1$                                    |           |
| b2       | $1+\varphi$   | $1+\varphi$           | $\varphi$     | $-4-8\varphi$                            | $2+\varphi$ $2+4\varphi$                      | 1                                      | $\frac{2}{2}$                        | _ +            | 2, 2                         | $\begin{bmatrix} 1,1\\2,2 \end{bmatrix}$                   | 2, 2           | $I_1, I_1$ $I_2, I_2$                         |           |
| c1       | 0   |                       | $1+\varphi$   | $-8+5\varphi$                            | $13-9\varphi$                                 |  | 3                                    | .'             | 1,1                          | $  \begin{array}{cccccccccccccccccccccccccccccccccccc$     | 1,1            | $I_1, I_1$                                    |           |
| c2       |   | $-1-\varphi$          | •             | $-1+11\varphi$                           | $-21-23\varphi$                               | 0                                      | 1                                    | + -            | 3, 3                         | 3,3  | 1, 1           | $I_3, I_3$                                    |           |
| 171      | 1d  | <u> </u>              | -             | 1711 <i>d</i>                            | $= (43 - 3\varphi) = 29b \cdot 59b$           | (3 isogeny clas                        | sses)                                | 1              |                              |  |                |   | 1711d     |
| a1       | 0   | $1-\varphi$           | φ             | $-2-5\varphi$                            | $-3-5\varphi$                                 | 1                                      | 1                                    | -+             | 1,1                          | 1,1  | 1, 1           | $I_1, I_1$                                    |           |
| b1       | $1+\varphi$   | $1-\varphi$           |               | $-1-3\varphi$                            | $-1-2\varphi$                                 |  | 2                                    | .'<br>    –  + | 1,1                          | 1, 1   | 1,1            | $oxed{I_1, I_1}$                              |           |
| b2       | $\varphi$   | $\varphi$             | $\varphi$     | $-15+10\varphi$                          | $29-18\varphi$                                | 1                                      | 2                                    | + -            | 2, 2                         | 2,2  | 2,2            | $I_2, I_2$                                    |           |
| c1       | 0   | -1                    | $\varphi$     | $-3-5\varphi$                            | $5+8\varphi$                                  | 0                                      | 3                                    | - +            | 1, 1                         | 1,1  | 1,1            | $\bar{\mathbf{I}}_1, \bar{\mathbf{I}}_1$      |           |
| c2       | 0   | $1 + \varphi$         | $\varphi$     | $9-9\varphi$                             | $-34 + 12\varphi$                             | 0                                      | 1                                    | -+             | 3, 3                         | 3,3  | 1, 1           | $I_3, I_3$                                    |           |
| 171      | 9a  |                       |               | 1719                                     | $a = (39 + 6\varphi) = 3 \cdot 191a$          | (1 isogeny cla                         | ss)                                  |                |                              |  |                |   | 1719a     |
| a1       | φ   | 1                     | $\varphi$     | $1-\varphi$                              | $-5+2\varphi$                                 | 0                                      | 1                                    |                | 1,5                          | 1,5  | 1, 1           | $I_1, I_5$                                    |           |
| 171      | 9b  |                       |               | 1719                                     | $b = (45 - 6\varphi) = 3 \cdot 191b$          | (1 isogeny cla                         | ss)                                  |                |                              |  |                |   | 1719b     |
| a1       | $1+\varphi$   | $1-\varphi$           | $1+\varphi$   | $-\varphi$                               | $-3-3\varphi$                                 | 0                                      | 1                                    |                | 1,5                          | 1,5  | 1, 1           | $I_1, I_5$                                    |           |
| 172      | 1a  |                       |               | 172                                      | $21a = (42 - \varphi) = 1721a$                | (1 isogeny class                       | s)                                   |                |                              |  |                |   | 1721a     |
| a1       | 1   | $1-\varphi$           | 0             | $-\varphi$                               | 0   | 1                                      | 2                                    | ++             | 1                            | 1  | 1              | $I_1$   |           |
| a2       | 1   | $1-\varphi$           | 0             | $4\varphi$                               | $-4 + \varphi$                                | 1                                      | 2                                    | + -            | 2                            | 2  | 2              | $I_2$   |           |
| 172      | 1b  |                       |               | 172                                      | $21b = (41 + \varphi) = 1721b$                | (1 isogeny class                       | s)                                   |                |                              |  |                |   | 1721b     |
| a1       | 1   | $\varphi$             | 0             | $-1+\varphi$                             | 0   | 1                                      | 2                                    | ++             | 1                            | 1  | 1              | $I_1$   |           |
| a2       | 1   | φ                     | 0             | $4-4\varphi$                             | $-3-\varphi$                                  | 1                                      | 2                                    | - +            | 2                            | 2  | 2              | $I_2$   |           |
| 172      | 4a  |                       |               |  | $= (10 - 38\varphi) = 2 \cdot 431a$           | (4 isogeny cla                         | sses)                                |                |                              | , ,  |                |   | 1724a     |
| a1       | $1+\varphi$   | $1-\varphi$           | 0             | $-232 + 144\varphi$                      | $-1728 + 1088\varphi$                         | 0                                      | 1                                    |                | 12,1                         | 12,1   | 2, 1           | $I_{12}, I_1$                                 |           |
| b1       | $1+\varphi$   | $1-\varphi$           | $\varphi$     | $-12+19\varphi$                          | $29-8\varphi$                                 | 1                                      | 1                                    |                | 11,1                         | 11,1   | 11, 1          | $I_{11}, I_1$                                 |           |
| c1       | $\varphi$   | $\varphi$             | $1 + \varphi$ | $-3-4\varphi$                            | $1+\varphi$                                   | 1                                      | 1                                    |                | 3, 1                         | 3,1  | 3, 1           | $I_3, I_1$                                    |           |
| L        | l   |                       |               |  |   |  | 1                                    |                |                              |  |                | 1   | 1         |

|   |   |                    |                                    |  |  | Imi                                    | l -          | 1( A )                       | 1 (:)  | _   | I/ - 1-:   | T         |
|---|---|--------------------|------------------------------------|--|--|--|--------------|------------------------------|--|---|--|-----------|
|   | $a_1$ $a_2$   | $a_3$              | $a_4$                              | $a_6$                                    | r                                      | T                                      | s            | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                  | $c_p$                                     | Kodaira  | Isogenies |
| 172   | 4a  |                    | 172                                | $24a = (10 - 38\varphi) = 2 \cdot 431a$  | (4 isogeny clas                        | sses)                                  |              |                              |  |   |  | 1724a     |
| d1  | $1 -1 - \varphi$  | $\varphi$          | $-5-9\varphi$                      | $14 + 23\varphi$                         | 1                                      | 1                                      |              | 2, 1                         | 2, 1   | 2, 1                                      | $I_2, I_1$   |           |
| 172   | $4\mathrm{b}$   |                    | 172                                | $24b = (28 - 38\varphi) = 2 \cdot 431b $ | 4 isogeny clas                         | sses)                                  |              |                              |  |   |  | 1724b     |
| a1  | $\varphi$ 1   | 0                  | $-88 - 144\varphi$                 | $-640 - 1088\varphi$                     | 0                                      | 1                                      |              | 12, 1                        | 12, 1  | 2, 1                                      | $I_{12},I_1$                                       |           |
| b1  | $\varphi$ 1   | $1+\varphi$        | $8-21\varphi$                      | $21 + 7\varphi$                          | 1                                      | 1                                      |              | 11, 1                        | 11, 1  | 11, 1                                     | $I_{11},I_1$                                       |           |
| c1  | $1+\varphi$ $1+\varphi$   | $1+\varphi$        | $-6+5\varphi$                      | $7-4\varphi$                             | 1                                      | 1                                      |              | 3, 1                         | 3,1  | 3,1                                       | $I_3, I_1$   |           |
| d1  | $1  1+\varphi$  | $\varphi$          | $-14 + 10\varphi$                  | $23-14\varphi$                           | 1                                      | 1                                      |              | 2, 1                         | 2,1  | 2, 1                                      | $I_2, I_1$   |           |
| 174   | 5a  |                    | 17                                 | $45a = (39 + 7\varphi) = 5a \cdot 349a$  | (1 isogeny cla                         | ass)                                   |              |                              |  |   |  | 1745a     |
| a1  | $1 - \varphi$   | 0                  | $-3-6\varphi$                      | $7+11\varphi$                            | 1                                      | 1                                      |              | 1, 2                         | 1, 2   | 1, 2                                      | $I_1, I_2$   |           |
| 174   | <br>5b  |                    | 17                                 | $745b = (46 - 7\varphi) = 5a \cdot 349b$ | (1 isogeny cla                         | ass)                                   |              |                              |  |   |  | 1745b     |
| a1  | $1 -1 + \varphi$  | 0                  | $-9+6\varphi$                      | $18 - 11\varphi$                         | 1                                      | 1                                      |              | 1, 2                         | 1, 2   | 1, 2                                      | $I_1, I_2$   |           |
| 175   | 6a  |                    | 175                                | $66a = (26 - 38\varphi) = 2 \cdot 439a$  | (6 isogeny clas                        | sses)                                  |              |                              |  |   |  | 1756a     |
| a1  | $\varphi$ $\varphi$   | 1                  | -1                                 | $-\varphi$                               | 1                                      | 1                                      | ++           | 1, 1                         | 1,1  | 1,1                                       | $\mathrm{I}_1,\mathrm{I}_1$                        |           |
| b1  | $1 -1 - \varphi$  | 1                  | $2\varphi$                         | $-1-\varphi$                             | 1                                      | 2                                      | + -          | 2, 1                         | [2, 1]                                       | 2,1                                       | $I_2, I_1$   |           |
| b2  | $1+\varphi$ $-1-\varphi$  | <u>-</u> 1         | $-54 + 32\varphi$                  | $-170 + 105\varphi$                      | 1                                      | 2                                      | + +          | 1,2                          | 1,2  | $\frac{1}{1}$ $\frac{1}{1}$ $\frac{2}{1}$ | $I_1, I_2$   | <br>      |
| c1  | $\varphi$ $-1+\varphi$  | φ                  | $-628 - 1001\varphi$               | $11079 + 17899\varphi$                   | 1<br>                                  | 1<br>                                  | + +          | 5,1                          | 5,1<br>                                      | 1,1                                       | $I_5, I_1$   | <br>      |
| d1  |   | $1+\varphi$        | $-5 - 5\varphi$                    | $5+5\varphi$                             | 1<br>                                  | <u> </u>                               | + +          | 5,1                          | 5,1  | 5,1                                       | $I_5, I_1$   |           |
| e1  | $1+\varphi$ 0   | <del>-</del>       | $-5 - 7\varphi$                    | $5+8\varphi$                             | 1                                      | 1                                      | + +<br>      | $\frac{3}{1}$                | 3,1  | 3,1                                       | $I_3, I_1$   |           |
| $\begin{array}{ c c }\hline f1 \\ f2 \end{array}$ | $ \varphi \qquad 1 - \varphi \\ 1 \qquad \qquad \varphi $             | $0 \\ 1 + \varphi$ | $-5 - 8\varphi$ $-91 + 52\varphi$  | $8 + 13\varphi$ $-371 + 223\varphi$      | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\begin{vmatrix} 3 \\ 3 \end{vmatrix}$ | ++++         | 1, 1 $3, 3$                  | $ \begin{array}{c c} 1,1\\ 3,3 \end{array} $ | $\begin{bmatrix} 1,1\\3,3 \end{bmatrix}$  | $\begin{matrix} I_1, I_1 \\ I_3, I_3 \end{matrix}$ |           |
| f3  | $1+\varphi$ $-1$  |                    | $-49401 + 30485\varphi$            | $-4949315 + 3058688\varphi$              | 0                                      | 1                                      | ++           | 1,1                          | 1,1  | 1,1                                       | $I_1, I_1$   |           |
| 175   | 6b  |                    | 175                                | $56b = (12 - 38\varphi) = 2 \cdot 439b $ | 6 isogeny clas                         | sses)                                  |              |                              |  |   |  | 1756b     |
| a1  | $1+\varphi$ $1+\varphi$   | 0                  | $3\varphi$                         | φ  | 1                                      | 1                                      | ++           | 1, 1                         | 1,1  | 1,1                                       | $\mathrm{I}_1,\mathrm{I}_1$                        |           |
|   | $1  1+\varphi$  | 0                  | 2                                  | 0  |  |  |              |                              |  |   | $I_2, I_1$   |           |
| b2  | $\varphi$ $-1$  | <del>1</del>       | $-21 - 33\varphi$                  | $-65 - 105\varphi$                       | 1                                      | 2                                      | + +          |                              | 1,2  | $\frac{1}{1}$                             | $I_1, I_2$   | <br>      |
| c1  | $1+\varphi \qquad \varphi$  |                    | $-1628 + 1001\varphi$              | $29979 - 18527\varphi$                   | 1                                      | 1<br>                                  | + +          | 5,1                          | 5,1<br>                                      | 1,1                                       | $I_5, I_1$   |           |
| d1  | $1 + \varphi  -1 - \varphi$   | <del>φ</del>       | $-9 + 3\varphi$                    | $11 - 6\varphi$                          | 1                                      |  | + +          | 5,1                          | 5,1  | 5,1                                       | $I_5, I_1$   | <br>      |
| e1  | $\varphi$ 1 – $\varphi$   | 0                  | $-12 + 7\varphi$                   | $13-8\varphi$                            | 1                                      | 1                                      | + +          | 3,1                          | 3,1  | 3,1                                       | $I_3, I_1$   | <br>      |
| f1<br>f2  | $ \begin{array}{ccc} 1 + \varphi & 0 \\ 1 & 1 - \varphi \end{array} $ | $0 \\ \varphi$     | $-13 + 8\varphi$ $-38 - 53\varphi$ | $21 - 13\varphi$ $-147 - 224\varphi$     | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ | $\frac{3}{3}$                          | + +<br>  + + | 1, 1 $3, 3$                  | $ \begin{array}{c c} 1,1\\ 3,3 \end{array} $ | $\begin{bmatrix} 1,1\\3,3 \end{bmatrix}$  | $\begin{matrix} I_1, I_1 \\ I_3, I_3 \end{matrix}$ |           |
| f3  | $\varphi$ $-\varphi$  | $\varphi$          | $-18914 - 30487\varphi$            | $-1890626 - 3058689\varphi$              | 0                                      | 1                                      | ++           | 1, 1                         | 1,1  | 1, 1                                      | $I_1, I_1$   |           |

|          | $a_1$              | $a_2$        | $a_3$       | $a_4$                | $a_6$                                      | r                                      |           | Γ     | s     | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$                                | Kodaira  | Isogenies |
|----------|--------------------|--------------|-------------|----------------------|--|--|-----------|-------|-------|------------------------------|-----------------------------|--------------------------------------|--|-----------|
| 176      | 4a                 |              |             |                      | $1764a = (42) = 2 \cdot 3 \cdot 7$         | (5 isogeny cla                         | asses     | s)    |       |                              |                             |                                      |  | 1764a     |
| a1       | $1+\varphi$        | $\varphi$    | $1+\varphi$ | $-34-34\varphi$      | $-128 - 182\varphi$                        | 0                                      | 2         | 2     | ++    | 4, 5, 1                      | 4, 5, 1                     | 2, 1, 1                              | $I_4, I_5, I_1$  |           |
| a2       | $1+\varphi$        | $\varphi$    | $1+\varphi$ | $-14-14\varphi$      | $-232 - 314\varphi$                        | 0                                      | 2         | 2     |       | 2, 10, 2                     | 2, 10, 2                    | 2, 2, 2                              | $I_2, I_{10}, I_2$   |           |
| b1       | $1+\varphi$ -      | $-1-\varphi$ | 0           | $-16-16\varphi$      | 0  | 0                                      |           |       | ++    | 8, 1, 3                      | 8,1,3                       | [2, 1, 1]                            | $  I_8, I_1, I_3  $  |           |
| b2       | $1+\varphi$ -      | $-1-\varphi$ | 0           | $64 + 64\varphi$     | $-48 - 64\varphi$                          | 0                                      | 2         | 2     |       | 4, 2, 6                      | 4, 2, 6                     | 2, 2, 2                              | $\mid I_4, I_2, I_6$   |           |
| c1       | 1                  | 0            | $\varphi$   | $-11-5\varphi$       | $-16-10\varphi$                            | 0                                      | - 1       | - 1   | + $+$ | 6, 1, 1                      | 6, 1, 1                     | 2, 1, 1                              | $I_6,I_1,I_1$  |           |
| c2       | $\dots \varphi_{}$ | 0            | φ           | $-556 - 896\varphi$  | $-9669 - 15645\varphi$                     |  | 2         | 2     | ++    | 3, 2, 2                      | 3,2,2                       | 1,2,2                                | $I_3, I_2, I_2$  |           |
| d1       | 1                  |              | $1+\varphi$ | $-16 + 4\varphi$     | $-26 + 9\varphi$                           | 0                                      |           | 2     | ++    | 6, 1, 1                      | 6, 1, 1                     | 2, 1, 1                              | $I_6, I_1, I_1$  |           |
| d2       | $1+\varphi$        |              | $1+\varphi$ | $-1452 + 894\varphi$ | $-25314 + 15644\varphi$                    | 0                                      | -'        | 2     | ++    | 3, 2, 2                      | 3,2,2                       | 1,2,2                                | $I_3, I_2, I_2$  |           |
| e1       | 1                  | 1            | 1           | -84                  | 261  |  | 8         |       | ++    | 4, 4, 2                      | 4, 4, 2                     | 4, 4, 2                              | $I_4, I_4, I_2$  |           |
| e2       | 1                  | 1            | 1           | -4                   | 5  | 1                                      | 8         |       |       | 8, 2, 1                      | 8, 2, 1                     | 8, 2, 1                              | $I_8, I_2, I_1$  |           |
| e3       | 1<br>1             | 1<br>1       | 1<br>1      | -1344 $-104$         | 18405<br>101                               | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | 4         |       | ++    | 2, 2, 1                      | 2, 2, 1                     | 2, 2, 1                              | $I_2, I_2, I_1$  |           |
| e4<br>e5 | 1                  | 1            | 1           | -104 $-914$          | -10915                                     | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$ | 2         |       | ++++  | 2, 8, 4 $1, 4, 8$            | 2, 8, 4 $1, 4, 8$           | 2, 8, 4 $1, 4, 8$                    | $\begin{bmatrix} I_2, I_8, I_4 \\ I_1, I_4, I_8 \end{bmatrix}$ |           |
| e6       | 1                  | 1            | 1           | -914 386             | -10913 $1277$                              | 1                                      | - 1       | 2     |       | 1, 4, 6 $1, 16, 2$           | 1, 4, 8<br>1, 16, 2         | 1, 4, 6 $1, 16, 2$                   | $I_1, I_4, I_8$<br>$I_1, I_{16}, I_2$                          |           |
|          |                    |              |             | 900                  | 1211                                       | 1                                      |           |       |       | 1,10,2                       | 1,10,2                      | 1,10,2                               | 11,116,12  |           |
| 1769     | 9 <b>c</b>         |              |             |                      | $1769c = (47 - 8\varphi) = 29b \cdot 61b$  | (1 isogen                              | y cl      | ass)  | )     |                              |                             |                                      |  | 1769c     |
| a1       | 1                  | $-\varphi$   | 0           | $-39-20\varphi$      | $106 + 100\varphi$                         | 0                                      | - 1       | 1     | ++    | 4, 1                         | 4,1                         | 4, 1                                 | $I_4, I_1$   |           |
| a2       | 1                  | $-\varphi$   | 0           | 1                    | 0  | 0                                      | - 1       | 2     |       | 1, 1                         | 1,1                         | 1, 1                                 | $I_1, I_1$   |           |
| a3       | 1                  | $-\varphi$   | 0           | -4                   | $-1+4\varphi$                              | 0                                      | 4         |       | ++    | 2, 2                         | 2,2                         | 2,2                                  | $I_2, I_2$   |           |
| a4       | 1                  | $-\varphi$   | 0           | $-49 + 20\varphi$    | $-152 + 84\varphi$                         | 0                                      |           | 2     | ++    | 1,4                          | 1,4                         | 1,4                                  | $I_1, I_4$   |           |
| 1769     | 9d                 |              |             |                      | $1769d = (39 + 8\varphi) = 29a \cdot 61a$  | ı (1 isogen                            | ıy cl     | lass  | )     |                              |                             |                                      |  | 1769d     |
| a1       | 1 -                | $-1+\varphi$ | 0           | $-59 + 20\varphi$    | $206 - 100\varphi$                         | 0                                      | 4         | 1     | ++    | 4,1                          | 4,1                         | 4, 1                                 | $I_4, I_1$   |           |
| a2       |                    | $-1+\varphi$ | 0           | 1                    | 0  | 0                                      | 2         | 2     |       | 1, 1                         | 1,1                         | 1, 1                                 | $I_1, I_1$   |           |
| a3       |                    | $-1+\varphi$ | 0           | -4                   | $3-4\varphi$                               | 0                                      | 4         |       | ++    | 2, 2                         | 2,2                         | 2, 2                                 | $I_2, I_2$   |           |
| a4       | 1 -                | $-1+\varphi$ | 0           | $-29 - 20\varphi$    | $-68 - 84\varphi$                          | 0                                      | 2         | 2     | ++    | 1,4                          | 1,4                         | 1,4                                  | $I_1, I_4$   |           |
| 177      | 5a                 |              |             | 1                    | $.775a = (45 - 5\varphi) = 5a^2 \cdot 71a$ | (3 isogeny                             | y cla     | asse  | s)    |                              |                             |                                      |  | 1775a     |
| a1       | φ -                | $-1-\varphi$ | $1+\varphi$ | $-\varphi$           | 0  | 1                                      | 1         | 1     |       | 3, 1                         | 1                           | 2, 1                                 | $III, I_1$   |           |
| b1       | $1+\varphi$        | 1            | $1+\varphi$ | $4-4\varphi$         | $-3-2\varphi$                              |  | 1         | i - i |       | 9,1                          | 1                           | 2,1                                  | $  III^*, I_1$   |           |
| c1       | $\varphi$ -        | $-1+\varphi$ | 1           | $-2-3\varphi$        | -3-5arphi                                  | 0                                      | - 2       | 2 [   | + -   | 6, 1                         | 1                           | [2, 1]                               | $I_0^*, I_1$   |           |
| c2       | 1                  | $-\varphi$   | $\varphi$   | $-281 + 178\varphi$  | $2076 - 1283\varphi$                       | 0                                      | 2         |       | + -   | 6, 3                         | 3                           | 2, 1                                 | $I_0^*, I_3$   |           |
| c3       |                    | $-1+\varphi$ | 1           | $-102-78\varphi$     | $297 + 145\varphi$                         | 0                                      | 2         | 2     | -+    | 6, 6                         | 6                           | 2, 2                                 | $I_0^*, I_6$   |           |
| c4       | φ -                | $-1+\varphi$ | 1           | $-27 - 53\varphi$    | $-143 - 225\varphi$                        | 0                                      | 2         | 2     | -+    | 6, 2                         | 2                           | 2,2                                  | $I_0^*, I_2$   |           |
| 177!     | 5b                 |              |             | -                    | $1775b = (40 + 5\varphi) = 5a^2 \cdot 71b$ | (3 isogeny                             | cla       | sses  | s)    |                              |                             |                                      |  | 1775b     |
| a1       | $1+\varphi$        | 1            | 1           | $\varphi$            | 1  | 1                                      | 1         | . 1   |       | 3, 1                         | 1                           | 2, 1                                 | $III, I_1$   |           |
| b1       |                    | $-1-\varphi$ |             | $1+4\varphi$         | $-5-2\varphi$                              | 1                                      | -¦<br>  1 | i - ¦ |       | 9, 1                         | 1                           | $\begin{bmatrix} 2, 1 \end{bmatrix}$ | $  III^*, I_1$   | <u>-</u>  |
|          | r                  | т            |             | - 1 17               | ·  |  |           |       |       | - / -                        | _                           | , -                                  | , -1   |           |

|  | $a_1$ $a_2$ $a_3$  | $a_4$  | $a_6$  | r                            | T  | s                                    | $\operatorname{ord}(\Delta)$                           | $\operatorname{ord}_{-}(j)$                             | $c_p$  | Kodaira  | Isogenies |
|--|--|--|--|------------------------------|--|--------------------------------------|--|---|--|--|-----------|
| 1775   | <br>5b   | 1'   | $775b = (40 + 5\varphi) = 5a^2 \cdot 71b$  | (3 isogeny cla               | asses)   |                                      |  |   |  |  | 1775b     |
| c1   | $1+\varphi$ $\varphi$ 0  | $-4+4\varphi$  | $-5+3\varphi$  | 0                            | 2  | -+                                   | 6, 1   | 1   | 2,1  | $I_0^*, I_1$   |           |
| c2   | $1 -1 + \varphi  1 + \varphi$  | $-103 - 179\varphi$  | $793 + 1282\varphi$  | 0                            | 2  | -+                                   | 6, 3   | 3   | 2, 1   | $I_0^*, I_3$   |           |
| c3   | $1+\varphi$ $\varphi$ 0  | $-179 + 79\varphi$   | $520 - 247\varphi$   | 0                            | 2  | + -                                  | 6, 6   | 6   | 2,2  | $I_0^*, I_6$   |           |
| c4   | $1+\varphi$ $\varphi$ 0  | $-79 + 54\varphi$  | $-315 + 198\varphi$  | 0                            | 2  | +-                                   | 6, 2   | 2   | 2,2  | $I_0^*, I_2$   |           |
| 1780   | )a   | 178  | $80a = (24 - 38\varphi) = 2 \cdot 5a \cdot 89a$  | (2 isogeny o                 | classe   | s)                                   |  |   |  |  | 1780a     |
|  | $1+\varphi$ $-1-\varphi$ 1   | $-12+6\varphi$   | $18-11\varphi$   | 1                            | 2  | ++                                   | 2, 4, 1  | 2, 4, 1   | 2, 4, 1  | $I_2, I_4, I_1$  |           |
| a2   | $1+\varphi$ $-1-\varphi$ 1   | $-22+16\varphi$  | $-22+13\varphi$  | 1                            | 2  | + -                                  | 1, 8, 2  | 1,8,2   | 1, 8, 2  | $I_1, I_8, I_2$  |           |
| b1   | $1 - 1 - \varphi$ $\varphi$  | $-5+\varphi$   | $4+\varphi$  | 1                            | 2  | + +                                  | 6, 2, 1  | [6, 2, 1]   | 6, 2, 1  | $I_6, I_2, I_1$  |           |
| b2   | $\varphi$ 1 1+ $\varphi$   | $-129 - 214\varphi$  | $1027 + 1658\varphi$   | 1                            | 2  | - +                                  | 3, 4, 2  | 3, 4, 2   | 3, 2, 2  | $I_3,I_4,I_2$  |           |
| 1780   | 0b   | 178  | $80b = (14 - 38\varphi) = 2 \cdot 5a \cdot 89b$  | (2 isogeny o                 | lasses   | s)                                   |  |   |  |  | 1780b     |
| a1   | $\varphi$ -1 1   | $-5-7\varphi$  | $7+11\varphi$  | 1                            | 2  | ++                                   | 2, 4, 1  | 2, 4, 1   | 2, 4, 1  | $I_{2}, I_{4}, I_{1}$  |           |
| a2   | $\varphi$ -1 1   | $-5-17\varphi$   | $-9-13\varphi$   | 1                            | 2  | -+                                   | 1, 8, 2  | 1,8,2   | 1, 8, 2  | $I_{1}, I_{8}, I_{2}$  |           |
| b1   | $1  1 + \varphi  \varphi$  | -4   | $1-2\varphi$   | 1                            | 2  | + +                                  | 6, 2, 1  | [6, 2, 1]   | [6, 2, 1]  | $[I_6, I_2, I_1]$  | Ī         |
| b2   | $1+\varphi$ $1-\varphi$ $\varphi$  | $-342 + 212\varphi$  | $2686 - 1659\varphi$   | 1                            | 2  | + -                                  | 3, 4, 2  | 3, 4, 2   | 3, 2, 2  | $I_3, I_4, I_2$  |           |
| 1789   | 9a   |  | $1789a = (23 - 38\varphi) = 1789a$   | (1 isogeny cl                | ass)   |                                      |  |   |  |  | 1789a     |
| a1   | $1+\varphi$ $1+\varphi$ $1+\varphi$  | $1+\varphi$  | 0  | 1                            | 1  |                                      | 1  | 1   | 1  | $I_1$  |           |
| 1789   | 9b   |  | $1789b = (15 - 38\varphi) = 1789b$   | (1 isogeny cla               | ass)   |                                      |  |   |  |  | 1789b     |
| a1   | $\varphi$ $\varphi$ $1+\varphi$  | 0  | -1   | 1                            | 1  |                                      | 1  | 1   | 1  | $I_1$  |           |
| 1791   | la   | 1  | $791a = (39 + 9\varphi) = 3 \cdot 199a$  | (4 isogeny cla               | sses)  |                                      |  | 1   | 1  |  | 1791a     |
| a1   | $1+\varphi$ $-1-\varphi$ 1   | $-5291 + 3269\varphi$  | $-173505 + 107232\varphi$  | 0                            | 2  | ++                                   | 1, 2   | 1, 2  | 1, 2   | $I_1, I_2$   |           |
| a2   | $1 -1 - \varphi$ 1   | $-48 + 31\varphi$  | $-150 + 92\varphi$   | 0                            | 2  |                                      | 2, 1   | 2,1   | 2, 1   | $I_2, I_1$   |           |
|  |  |  |  |                              |  |                                      |  |   |  |  |           |
| b1   | $0 \qquad -\varphi \qquad 1$   | $-1+2\varphi$  | -arphi   | 1                            | 1  | + -                                  | 1,2  | 1,2   | 1,2  | $I_1, I_2$   |           |
|  |  | $-1 + 2\varphi$ $-5 - 7\varphi$  | $-\varphi$ $5+8\omega$   |                              | <del>!</del>   | + -<br>  + +                         | $\frac{1,2}{2,2}$                                      | $\frac{1}{2}$   | $\frac{1}{1}$ $\frac{1,2}{2,2}$ -  | $I_1, I_2$ $I_2, I_3$  | <br>      |
| c1   | $\begin{array}{cccccccccccccccccccccccccccccccccccc$   | $-5-7\varphi$  | $5+8\varphi$   | 0                            | 4  | '<br>  + +                           | -2, 2  | 2,2   | 2, 2   | $ar{\mathrm{I}}_2,ar{\mathrm{I}}_2$  | <u>-</u>  |
| $\begin{bmatrix} c1 \\ c2 \end{bmatrix}$       | $ \begin{array}{ccc} 1 & -1 + \varphi & \varphi \\ 1 & -1 + \varphi & \varphi \end{array} $  | $ \begin{array}{c} -5 - 7\varphi \\ -2\varphi \end{array} $  | $5 + 8\varphi$ $-1 - \varphi$  |                              | <del>!</del>   | '<br>  + +<br>  - +                  | 2, 2<br>1, 1   | $\begin{bmatrix} 2, 2 \\ 1, 1 \end{bmatrix}$            | 2, 2 1, 1  | $\begin{bmatrix} I_2, I_2 \\ I_1, I_1 \end{bmatrix}$   | <u> </u>  |
| $\begin{bmatrix} c1 \\ c2 \\ c3 \end{bmatrix}$ | $\begin{array}{cccc} & 1 & -1 + \varphi & \varphi \\ & 1 & -1 + \varphi & \varphi \\ & 1 + \varphi & \varphi & 1 \end{array}$                            | $-5 - 7\varphi$ $-2\varphi$ $-53 + 35\varphi$  | $5 + 8\varphi$ $-1 - \varphi$ $187 - 111\varphi$   |                              | $\begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}$                | '<br>  + +<br>  - +<br>  + -         | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$ | $\begin{bmatrix} 2,2\\ 1,1\\ 2,1 \end{bmatrix}$                                    | $\begin{bmatrix} I_2, I_2 \\ I_1, I_1 \\ I_4, I_1 \end{bmatrix}$                                     |           |
| c1<br>c2<br>c3<br>c4                           | $ \begin{array}{ccccc} 1 & -1 + \varphi & \varphi \\ 1 & -1 + \varphi & \varphi \\ 1 + \varphi & \varphi & 1 \\ 1 & -1 + \varphi & \varphi \end{array} $ | $-5-7\varphi\\-2\varphi\\-53+35\varphi\\-85-102\varphi$  | $5 + 8\varphi$ $-1 - \varphi$ $187 - 111\varphi$ $376 + 660\varphi$  | 0 0 0 0                      | $\begin{array}{ c c c }\hline 4\\ 2\\ 2\\ 2\\ \end{array}$ | + +<br>- +<br>+ -<br>+ +             | $ \begin{array}{c} 2,2\\ 1,1\\ 4,1\\ 1,4 \end{array} $ | 2, 2<br>1, 1<br>4, 1<br>1, 4                            | $ \begin{array}{ c c c } \hline 2,2\\ 1,1\\ 2,1\\ 1,2 \end{array} $                | $\begin{bmatrix} I_2, I_2 \\ I_1, I_1 \\ I_4, I_1 \\ I_1, I_4 \end{bmatrix}$                         |           |
| c1 c2 c3 c4 d1                                 | $\begin{array}{cccc} & 1 & -1 + \varphi & \varphi \\ & 1 & -1 + \varphi & \varphi \\ & 1 + \varphi & \varphi & 1 \end{array}$                            | $-5-7\varphi\\-2\varphi\\-53+35\varphi\\-85-102\varphi$  | $5 + 8\varphi$ $-1 - \varphi$ $187 - 111\varphi$   |                              | $\begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}$                | '<br>  + +<br>  - +<br>  + -         | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$ | $\begin{bmatrix} 2,2\\ 1,1\\ 2,1 \end{bmatrix}$                                    | $\begin{bmatrix} I_2, I_2 \\ I_1, I_1 \\ I_4, I_1 \end{bmatrix}$                                     |           |
| c1 c2 c3 c4 d1                                 | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $ \begin{array}{r} -5 - 7\varphi \\ -2\varphi \\ -53 + 35\varphi \\ -85 - 102\varphi \\ -70 - 17\varphi \\ -5 - 2\varphi \end{array} $ | $\begin{array}{c} 5 + 8\varphi \\ -1 - \varphi \\ 187 - 111\varphi \\ 376 + 660\varphi \\ 13 - 263\varphi \end{array}$                         |                              | 4 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2                    | + +  <br>  - +  <br>  + -  <br>  + + | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | 2,2<br>1,1<br>4,1<br>1,4<br>1,2                         | $ \begin{array}{ c c c } \hline 2,2\\ 1,1\\ 2,1\\ 1,2\\ \hline 1,2\\ \end{array} $ | $\begin{bmatrix} I_2, I_2 \\ I_1, I_1 \\ I_4, I_1 \\ I_1, I_4 \\ \vdots \\ I_1, I_2 \end{bmatrix}$   | 1791b     |
| c1   c2   c3   c4   d1   d2                    | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  |  | $5 + 8\varphi$ $-1 - \varphi$ $187 - 111\varphi$ $376 + 660\varphi$ $13 - 263\varphi$ $-1 - 6\varphi$ $791b = (48 - 9\varphi) = 3 \cdot 199b$  | 0 0 0 0 0 0 0 (4 isogeny cla | 4  | ++ +- ++ ++                          | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | 2,2<br>1,1<br>4,1<br>1,4<br>1,2<br>2,1                  | 2, 2<br>1, 1<br>2, 1<br>1, 2<br>1, 2<br>2, 1                                       | $\begin{bmatrix} I_2, I_2 \\ I_1, I_1 \\ I_4, I_1 \\ I_1, I_4 \\ I_1, I_2 \\ I_2, I_1 \end{bmatrix}$ | 1791b     |
| c1   c2   c3   c4   d1   d2                    | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | $ \begin{array}{r} -5 - 7\varphi \\ -2\varphi \\ -53 + 35\varphi \\ -85 - 102\varphi \\ -70 - 17\varphi \\ -5 - 2\varphi \end{array} $ | $\begin{array}{c} 5 + 8\varphi \\ -1 - \varphi \\ 187 - 111\varphi \\ 376 + 660\varphi \\ \hline 13 - 263\varphi \\ -1 - 6\varphi \end{array}$ |                              | 4 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2                    | + +  <br>  - +  <br>  + -  <br>  + + | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$  | 2,2<br>1,1<br>4,1<br>1,4<br>1,2                         | $ \begin{array}{ c c c } \hline 2,2\\ 1,1\\ 2,1\\ 1,2\\ \hline 1,2\\ \end{array} $ | $\begin{bmatrix} I_2, I_2 \\ I_1, I_1 \\ I_4, I_1 \\ I_1, I_4 \\ \vdots \\ I_1, I_2 \end{bmatrix}$   | 1791b     |

|     |     |             |      |              |             |                                       |          |                       |        |                     |                           |       |          | T     | ITI           | _    | J(A)                         | 1 (:)                       | _       | W - 1-:                     | T:        |
|-----|-----|-------------|------|--------------|-------------|---------------------------------------|----------|-----------------------|--------|---------------------|---------------------------|-------|----------|-------|---------------|------|------------------------------|-----------------------------|---------|-----------------------------|-----------|
|     |     | $a_1$       |      | $a_2$        | $a_{i}$     | 3                                     |          | $a_4$                 | Į.     |                     | a                         | 6     |          | r     | T             | s    | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$ | $c_p$   | Kodaira                     | Isogenies |
| 179 | 1b  | •           |      |              |             |                                       |          | 1                     | 791b   | $= (48 - 9\varphi$  | $) = 3 \cdot 199b$        |       | (4 isoge | ny cl | asses]        | )    |                              |                             |         |                             | 1791b     |
| c1  |     | 1           | _    | -φ           | $1+\varphi$ | )                                     | _        | $12+6\varphi$         |        |                     | 13 - 90                   | ρ     |          | 0     | 4             | ++   | 2, 2                         | 2, 2                        | 2, 2    | $I_2, I_2$                  |           |
| c2  |     | 1           |      |              | $1+\varphi$ | )                                     |          | $-2+\varphi$          |        |                     | -                         |       |          | 0     | 2             | + -  | 1, 1                         | 1, 1                        | 1,1     | $I_1, I_1$                  |           |
| c3  |     | $\varphi$   | -1 + |              | C           |                                       |          | $9-34\varphi$         |        |                     | 61 + 920                  |       |          | 0     | 2             | -+   | 4, 1                         | 4, 1                        | 2, 1    | $\mathrm{I}_4,\mathrm{I}_1$ |           |
| c4  |     | 1           |      | $-\varphi$   | $1+\varphi$ | )<br>                                 | <br>-187 | $' + 101\varphi$      |        |                     | 1036 - 6619               | ρ     |          | 0     | 2             | ++   | 1,4                          | 1,4                         | 1,2     | $I_1, I_4$                  |           |
| d1  |     | $\varphi$   | 1 –  |              | $\varphi$   | )                                     | -8       | $35 + 15\varphi$      |        |                     | -249 + 2629               |       |          | 0     | 2             | ++   | 1, 2                         | 1, 2                        | 1,2     | $I_1, I_2$                  |           |
| d2  |     | $\varphi$   | 1 -  | - φ          | φ           | 2                                     |          | -5                    |        |                     | -6 + 59                   | ρ     |          | 0     | 2             |      | 2,1                          | 2, 1                        | 2,1     | $I_2, I_1$                  |           |
| 179 | 5a  | _           |      |              |             |                                       |          | 17                    | 795a : | $=(44-3\varphi)$    | $=5a\cdot 359b$           |       | (2 isoge | eny c | lasses        | s)   |                              |                             |         |                             | 1795a     |
| a1  |     | $\varphi$   | 1 -  | - φ          | $1+\varphi$ | )                                     |          | $-7-3\varphi$         |        |                     | -3 + 80                   | ρ     |          | 1     | 6             | + -  | 1,6                          | 1,6                         | 1,6     | $I_1, I_6$                  |           |
| a2  | 1 - | $+ \varphi$ |      |              | $1+\varphi$ |                                       | -295     | $-481\varphi$         |        |                     | 3712 + 6008               |       |          | 1     | 6             | -+   | 2,3                          | 2,3                         | 2,3     | $I_2, I_3$                  |           |
| a3  |     | 1           |      | $\varphi$    | C           | )                                     |          | $+ 1444\varphi$       |        |                     | 685 + 31329               |       |          | 1     | 2             | + -  | 3, 2                         | 3, 2                        | 1, 2    | $I_3, I_2$                  |           |
| a4  |     | $\varphi$   | 1 –  | $\varphi$    | $1+\varphi$ | )                                     | -387     | $\gamma + 132\varphi$ |        | -                   | 2879 + 21459              | ρ     |          | 1     | 2             | -+   | 6, 1                         | 6, 1                        | 2,1     | $I_6, I_1$                  |           |
| b1  |     | 1           |      | $-\varphi$   | 1           | i                                     | <br>     | $-1+\varphi$          |        |                     |                           | 1     |          | 1     | $\frac{1}{2}$ | + -  | 1,2                          | 1, 2                        | [1, 2]  | $ $ $I_1, I_2$              |           |
| b2  |     | 1           |      | $-\varphi$   | 1           | L                                     | _        | $16+6\varphi$         | 1      |                     | -29 + 169                 | ρ     |          | 1     | 2             | ++   | 2, 1                         | 2, 1                        | 2, 1    | $I_2, I_1$                  |           |
| 179 | 5b  | •           |      |              |             |                                       |          | 17                    | 795b = | $= (41 + 3\varphi)$ | $=5a\cdot 359a$           |       | (2 isoge | eny c | lasses        | s)   |                              |                             |         |                             | 1795b     |
| a1  | 1 - | $+\varphi$  |      | 0            | φ           | )                                     |          | $-9+\varphi$          |        |                     | 6 - 90                    | ρ     |          | 1     | 6             | -+   | 1,6                          | 1,6                         | 1,6     | $I_1, I_6$                  |           |
| a2  |     | $\varphi$   |      | 0            | φ           |                                       | -774     | $+479\varphi$         |        |                     | 9721 - 6009               |       |          | 1     | 6             | +-   | 2,3                          | 2,3                         | 2,3     | $I_2, I_3$                  |           |
| a3  |     | 1           | 1 –  | - φ          | C           |                                       | -887     | $-1444\varphi$        |        | -19                 | 356 - 31329               | ρ     |          | 1     | 2             | -+   | 3, 2                         | 3, 2                        | 1, 2    | $I_3, I_2$                  |           |
| a4  | 1 - | $+\varphi$  |      | 0            | $\varphi$   | 2                                     | <br>-254 | $-134\varphi$         |        | -                   | -733 - 2146               | ρ     |          | 1     | 2             | +-   | 6, 1                         | 6,1                         | 2,1     | $I_6, I_1$                  |           |
| b1  |     |             | -1 + |              | 1           | Ĺ                                     |          | $-\varphi$            |        |                     |                           | _     |          | 1     | 2             | -+   | 1, 2                         | 1, 2                        | [1, 2]  | $I_1, I_2$                  |           |
| b2  |     | 1           | -1 + | - φ          | 1           | L                                     | _        | $10-6\varphi$         | 1      |                     | -13 - 16                  | ρ     |          | 1     | 2             | ++   | 2,1                          | 2, 1                        | 2, 1    | $I_2, I_1$                  |           |
| 179 | 6a  | _           |      |              |             |                                       |          | 1                     | 1796a  | a = (22 - 38)       | $\varphi) = 2 \cdot 449e$ | ı     | (1 isog  | geny  | class)        | )    |                              |                             |         |                             | 1796a     |
| a1  |     | $\varphi$   |      | $\varphi$    | φ           | )                                     | -4       | $2-64\varphi$         |        |                     | 157 + 254                 | ρ     |          | 0     | 4             | ++   | 2,2                          | 2, 2                        | 2, 2    | $I_2, I_2$                  |           |
| a2  | 1 - | $+\varphi$  |      |              | $1+\varphi$ | )                                     | -4427    | $-7159\varphi$        |        | 2138                | 41 + 3460014              |       |          | 0     | 2             | ++   | 1, 1                         | 1, 1                        | 1, 1    | $\mathrm{I}_1,\mathrm{I}_1$ |           |
| a3  |     |             | -1 + |              | $\varphi$   | )                                     |          | $3-3\varphi$          |        |                     | -2 + 9                    |       |          | 0     | 4             |      | 4, 1                         | 4, 1                        | 4, 1    | $\mathrm{I}_4,\mathrm{I}_1$ |           |
| a4  |     | 1           | -1 + | - φ          | φ           | · · · · · · · · · · · · · · · · · · · | -14      | $7+77\varphi$         |        |                     | -688 + 443                | ρ     |          | 0     | 2             | ++   | 1,4                          | 1,4                         | 1,4     | $I_1, I_4$                  |           |
| 179 | 6b  | •           |      |              |             |                                       |          | 1                     | 1796b  | o = (16 - 38)       | $\varphi) = 2 \cdot 449l$ | )     | (1 isog  | geny  | class)        |      |                              |                             |         |                             | 1796b     |
| a1  |     | $\varphi$   |      | 1            | φ           | )                                     | -11584   | $+7157\varphi$        |        | 5598                | 43 - 346002               | ρ     |          | 0     | 2             | ++   | 1,1                          | 1, 1                        | 1,1     | $I_1, I_1$                  |           |
| a2  | 1 - | $+ \varphi$ | 1 +  |              | $\varphi$   | )                                     |          | $05 + 66\varphi$      |        |                     | 476 - 2956                |       |          | 0     | 4             | ++   | 2, 2                         | 2, 2                        | 2, 2    | $I_2, I_2$                  |           |
| a3  |     | 1           |      |              | $1+\varphi$ | )                                     |          | $2\varphi$            |        |                     | -1 - 20                   |       |          | 0     | 4             |      | 4, 1                         | 4, 1                        | 4, 1    | $\mathrm{I}_4,\mathrm{I}_1$ |           |
| a4  |     | 1           |      | $-\varphi$   | $1+\varphi$ | )                                     | -7       | $70-78\varphi$        |        |                     | -245 - 4444               | ρ     |          | 0     | 2             | ++   | 1,4                          | 1,4                         | 1,4     | $I_1, I_4$                  |           |
| 180 | 4a  | _           |      |              |             |                                       |          | 1804                  | 4a =   | $(20 - 38\varphi)$  | $= 2 \cdot 11a \cdot 41$  | a     | (3 iso   | ogeny | class         | ses) |                              |                             |         |                             | 1804a     |
| a1  | 1 - | $+\varphi$  | 1 -  | - φ          | 1           | L                                     |          | $6-4\varphi$          |        |                     | 10 - 50                   | ρ     |          | 0     | 5             |      | 5, 2, 1                      | 5, 2, 1                     | 5, 2, 1 | $I_{5}, I_{2}, I_{1}$       |           |
| a2  |     |             | 1 –  |              | 1           | L                                     | -519     | $+251\varphi$         |        | _                   | 5312 + 2959               | ρ     |          | 0     | 1             |      | 1, 10, 5                     | 1, 10, 5                    | 1, 2, 5 | $I_1, I_{10}, I_5$          |           |
| b1  |     | 1           | -1-  | - <b>-</b> - | $\varphi$   | )                                     | <br>     | $1-3\varphi$          |        |                     | 4 + 36                    | <br>ρ |          | 1     | 1             |      | 1, 8, 1                      | 1,8,1                       | 1,8,1   | $  I_1, I_8, I_1  $         |           |
|     |     |             |      | •            |             |                                       |          |                       |        |                     |                           |       |          | 1     | L             |      |                              | 1                           |         | 1                           |           |

|                               | 1   |   |  |  |   |  |   |   | T   | 1  | 1   | 1         |
|-------------------------------|---|---|--|--|---|--|---|---|---|--|---|-----------|
|                               | $a_1$ $a_2$   | $a_3$   | $a_4$  | $a_6$  | r   | T  | s   | $\operatorname{ord}(\Delta)$  | $\operatorname{ord}_{-}(j)$   | $c_p$  | Kodaira   | Isogenies |
| 180                           | 4a  |   | 1804a = (2)  | $20 - 38\varphi) = 2 \cdot 11a \cdot 41a$  | (3 isoger   | nv cla   | sses)   |   |   |  |   | 1804      |
| c1                            | T   | $\varphi$   | -1   | 1  | 1   |  | <del></del>   | 1, 2, 1   | 1, 2, 1   | 1, 2, 1  | $I_1, I_2, I_1$   |           |
|                               | · ·   | ·   |  |  |   |  |   |   |   |  |   |           |
| 180                           | 4b  |   | 1804b = (  | $18 - 38\varphi) = 2 \cdot 11b \cdot 41b$  | (3 isoger   | ny clas  | sses)   |   |   |  |   | 1804      |
| a1                            | $\varphi$ 1   | 1   | $3+3\varphi$   | $5+5\varphi$   | 0   |  | 1   | 5, 2, 1   | 5, 2, 1   | 5, 2, 1  | $I_5, I_2, I_1$   |           |
| a2                            | $\begin{vmatrix} \varphi & 1 \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \end{vmatrix}$  | 1   | $-267 - 252\varphi$  | $-2353 - 2959\varphi$  | 0   |  | -   | 1,10,5  | $\frac{1}{1}$ 1, 10, 5  | 1,2,5  | $  I_1, I_{10}, I_5  $  | <u> </u>  |
| b1                            | $\begin{vmatrix} 1 & 1+\varphi \\ \vdots & \vdots & \vdots \end{vmatrix}$   | $\varphi$   | $-2+4\varphi$  |  | _   1   | 1  |   |   | 1,8,1   | 1,8,1  | $I_1, I_8, I_1$   | ļ         |
| c1                            | $\varphi \qquad -\varphi  1$  | $+\varphi$  | $-2\varphi$  | $1-\varphi$  | 1   | 1  |   | 1, 2, 1   | 1, 2, 1   | 1, 2, 1  | $I_1, I_2, I_1$   |           |
| 180                           | Λc  |   | 1804a — (  | $46 - 6\varphi) = 2 \cdot 11a \cdot 41b$   | (4 isogen   | v alog   | gog)  |   |   |  |   | 1804      |
|                               | 1   | 1.7   | $-4-7\varphi$  | . ,  | Ť   |  | T .   | 1 1 1   | 1 1 1   | 1 1 1  | ттт   | 1004      |
| a1                            | <u> </u>  |   | <u>.</u>   | $4+6\varphi$   | 1   |  | - '   | 1, 1, 1   | 1,1,1   | 1,1,1  | $  I_1, I_1, I_1$   | <u> </u>  |
| b1<br>b2                      | $\begin{array}{ccc} 1+\varphi & 0 \\ 1+\varphi & 0 \end{array}$   | 1<br>1  | $\begin{array}{c} 1-5\varphi \\ -199-165\varphi \end{array}$   | $-4 - 2\varphi$ $-1412 - 1682\varphi$  | $\begin{vmatrix} 1 \\ 1 \end{vmatrix}$                                  |  | - +   | 1, 9, 1<br>3, 3, 3  | $\begin{bmatrix} 1, 9, 1 \\ 3, 3, 3 \end{bmatrix}$  | $\begin{vmatrix} 1, 9, 1 \\ 1, 3, 1 \end{vmatrix}$   | $\begin{bmatrix} I_1, I_9, I_1 \\ I_3, I_3, I_3 \end{bmatrix}$  |           |
|                               |   |   |  | $-1412 - 1002\varphi$ $-3$   | '   |  | - '   |   |   | ''   |   | <br>      |
| c1<br>c2                      | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$   |   | $-2 - \varphi$ $-227 + 64\varphi$  | $-3 \\ -1285 + 415\varphi$   | $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$                                  |  | -+  | 1, 1, 3<br>3, 3, 1  | $\begin{bmatrix} 1, 1, 3 \\ 3, 3, 1 \end{bmatrix}$  | $\begin{vmatrix} 1,1,3\\1,3,1 \end{vmatrix}$   | $\begin{array}{ c c c c } & I_1, I_1, I_3 \\ & I_3, I_3, I_1 \end{array}$   |           |
| $\frac{cz}{d1}$               | $\begin{bmatrix} 1 & 1 & \varphi & 1 & 1 & \varphi & 1 \\ 1 & -1 & 1 & -1 & \varphi & 1 \end{bmatrix}$  |   | $-2 - \varphi$   | <del>'</del> :   | <sup> </sup> - <u>-</u> 1   | · -; ·   | - :   | 1, 3, 1   | $\begin{array}{c c} 1 & 3, 3, 1 \\ \hline 1, 3, 1 \end{array}$  | $\begin{bmatrix} 1, 3, 1 \\ 1, 3, 1 \end{bmatrix}$   | :   | <u> </u>  |
| u1                            | $1 - 1 + \varphi$   | φ   | $-z-\varphi$   | $1-2\varphi$   | 1   | 1  | +   | 1, 3, 1   | 1, 3, 1   | 1, 3, 1  | $I_1, I_3, I_1$   |           |
|                               |   |   |  |  |   |  |   |   |   |  |   |           |
| 180                           | 4d  |   | 1804d = 0  | $40 + 600 = 2 \cdot 11b \cdot 41a$   | (4 isogen   | v clas   | ses)  |   |   |  |   | 1804      |
| 180                           | 1   | (2)   | <u> </u>   | $40 + 6\varphi) = 2 \cdot 11b \cdot 41a$ $11 - 7\varphi$   | (4 isogen   |  | <del></del>   | 111   | 1 1 1   | 1 1 1  | I, I, I,  | 1804      |
| a1                            | $\varphi$ 0   |   | $-9+5\varphi$  | $11 - 7\varphi$  | 1   | 1  | +-  |   | 1,1,1   | 1,1,1  | $I_1, I_1, I_1$   | 1804      |
| a1<br>b1                      | $\frac{\varphi}{\varphi} = \frac{0}{1-\varphi}$   | 1   | $-9 + 5\varphi$ $-3 + 4\varphi$  | $11 - 7\varphi$ $-6 + 2\varphi$  | 1   | 1 3  | +-  | 1, 9, 1   | 1,9,1   | 1,9,1  | $ $ $I_1, I_9, I_1$   | 18040     |
| a1<br>b1<br>b2                | $ \begin{array}{c cccc} \varphi & 0 \\ \hline & \varphi & 1 - \varphi \\ & \varphi & 1 - \varphi \end{array} $  | 1   | $-9 + 5\varphi$ $-3 + 4\varphi$ $-363 + 164\varphi$  | $ \begin{array}{r} 11 - 7\varphi \\ -6 + 2\varphi \\ -3094 + 1682\varphi \end{array} $   | 1 1 1   | 1 3 1  | +-  | 1, 9, 1<br>3, 3, 3  | 1,9,1   | $\begin{bmatrix} 1, 9, 1 \\ 1, 3, 1 \end{bmatrix}$   | $\begin{array}{ c c c c } & I_1, I_9, I_1 \\ & I_3, I_3, I_3 \end{array}$   | 1804      |
| a1<br>b1<br>b2<br>c1          | $ \begin{array}{c cccc} \varphi & 0 \\ \hline & \varphi & 1 - \varphi \\ \hline & \varphi & 1 - \varphi \\ \hline & \varphi & 1 + \varphi \\ \hline \end{array} $   | 1 1 1   | $-9 + 5\varphi$ $-3 + 4\varphi$ $-363 + 164\varphi$ $-1 + \varphi$   | $   \begin{array}{r}     11 - 7\varphi \\     -6 + 2\varphi \\     -3094 + 1682\varphi \\     -2 - 2\varphi   \end{array} $  |   | $\begin{array}{c c} & 1 \\ \hline & 3 \\ \hline & 1 \\ \hline & 3 \\ \hline & 3 \\ \end{array}$  | +-  | $ \begin{array}{c} 1, 9, 1 \\ 3, 3, 3 \\ -1, 1, 3 \end{array} $   | $ \begin{array}{c c} 1 & 1, 9, 1 \\ 3, 3, 3 \\ \hline 1, 1, 3 \end{array} $   | $\begin{array}{ c c c c c c }\hline 1,9,1\\ 1,3,1\\ \hline 1,1,3\\ \hline \end{array}$   | $ \begin{vmatrix} I_1, I_9, I_1 \\ I_3, I_3, I_3 \\ I_1, I_1, I_3 \end{vmatrix} $   | 1804      |
| a1<br>b1<br>b2<br>c1<br>c2    | $ \begin{vmatrix} \varphi & 0 \\ \varphi & 1 - \varphi \\ \varphi & 1 - \varphi \\                                $   | 1   | $-9 + 5\varphi$ $-3 + 4\varphi$ $-363 + 164\varphi$ $-1 + \varphi$ $-161 - 64\varphi$  | $   \begin{array}{r}     11 - 7\varphi \\     -6 + 2\varphi \\     -3094 + 1682\varphi \\     -2 - 2\varphi \\     -934 - 642\varphi   \end{array} $                 | $ \begin{array}{c c}  & 1 \\  & 1 \\  & 1 \\  & 0 \\  & 1 \end{array} $ | $\begin{array}{c c} \hline 1 \\ \hline 3 \\ \hline 1 \\ \hline 3 \\ \hline 1 \\ \hline \end{array}$  | +-  | $ \begin{array}{c} 1, 9, 1 \\ 3, 3, 3 \\ \hline 1, 1, 3 \\ 3, 3, 1 \end{array} $  | $ \begin{array}{c c}  & 1,9,1 \\  & 3,3,3 \\  & 1,1,3 \\  & 3,3,1 \end{array} $   | $\begin{array}{ c c c c }\hline 1,9,1\\ 1,3,1\\ \hline 1,1,3\\ 1,3,1\\ \hline \end{array}$   | $ \begin{vmatrix} I_1, I_9, I_1 \\ I_3, I_3, I_3 \end{vmatrix} $ $ \begin{vmatrix} I_1, I_1, I_3 \\ I_3, I_3, I_1 \end{vmatrix} $   | 1804      |
| a1<br>b1<br>b2<br>c1          | $ \begin{array}{c cccc} \varphi & 0 \\ \hline & \varphi & 1 - \varphi \\ \hline & \varphi & 1 - \varphi \\ \hline & \varphi & 1 + \varphi \\ \hline \end{array} $   | 1   | $-9 + 5\varphi$ $-3 + 4\varphi$ $-363 + 164\varphi$ $-1 + \varphi$   | $   \begin{array}{r}     11 - 7\varphi \\     -6 + 2\varphi \\     -3094 + 1682\varphi \\     -2 - 2\varphi   \end{array} $  |   | $\begin{array}{c c} \hline 1 \\ \hline 3 \\ \hline 1 \\ \hline 3 \\ \hline 1 \\ \hline \end{array}$  | +-  | $ \begin{array}{c} 1, 9, 1 \\ 3, 3, 3 \\ -1, 1, 3 \end{array} $   | $ \begin{array}{c c} 1 & 1, 9, 1 \\ 3, 3, 3 \\ \hline 1, 1, 3 \end{array} $   | $\begin{array}{ c c c c c c }\hline 1,9,1\\ 1,3,1\\ \hline 1,1,3\\ \hline \end{array}$   | $ \begin{vmatrix} I_1, I_9, I_1 \\ I_3, I_3, I_3 \\ I_1, I_1, I_3 \end{vmatrix} $   | 1804      |
| a1<br>b1<br>b2<br>c1<br>c2    | $ \begin{vmatrix} \varphi & 0 \\ \hline \varphi & 1 - \varphi \\ \hline \varphi & 1 - \varphi \\ \end{vmatrix} $ $ \begin{vmatrix} \varphi & 1 + \varphi \\ \hline \varphi & 1 + \varphi \\ \hline 1 & -\varphi & 1 \end{vmatrix} $   | 1   |  | $   \begin{array}{r}     11 - 7\varphi \\     -6 + 2\varphi \\     -3094 + 1682\varphi \\     -2 - 2\varphi \\     -934 - 642\varphi   \end{array} $                 | $ \begin{array}{c c}  & 1 \\  & 1 \\  & 1 \\  & 0 \\  & 1 \end{array} $ | 3<br>1<br>3<br>1<br>1<br>1   | + -<br>  + -<br>  + -<br>  + -<br>  + -   | $ \begin{array}{c} 1, 9, 1 \\ 3, 3, 3 \\ \hline 1, 1, 3 \\ 3, 3, 1 \end{array} $  | $ \begin{array}{c c}  & 1,9,1 \\  & 3,3,3 \\  & 1,1,3 \\  & 3,3,1 \end{array} $   | $\begin{array}{ c c c c }\hline 1,9,1\\ 1,3,1\\ \hline 1,1,3\\ 1,3,1\\ \hline \end{array}$   | $ \begin{vmatrix} I_1, I_9, I_1 \\ I_3, I_3, I_3 \end{vmatrix} $ $ \begin{vmatrix} I_1, I_1, I_3 \\ I_3, I_3, I_1 \end{vmatrix} $   |           |
| a1 b1 b2 c1 c2 d1 a1          | $ \begin{vmatrix} \varphi & 0 \\ \hline \varphi & 1 - \varphi \\ \hline \varphi & 1 - \varphi \\ \end{vmatrix} $ $ \begin{vmatrix} \varphi & 1 + \varphi \\ \hline \varphi & 1 + \varphi \\ \hline 1 & -\varphi & 1 \end{vmatrix} $   | $ \begin{array}{c} 1\\ 1\\ 1\\ 1\\ +\varphi \end{array} $   | $-9 + 5\varphi$ $-3 + 4\varphi$ $-363 + 164\varphi$ $-1 + \varphi$ $-161 - 64\varphi$ $-3$ $1805a = (1 - 35 - \varphi)$  | $11 - 7\varphi$ $-6 + 2\varphi$ $-3094 + 1682\varphi$ $-2 - 2\varphi$ $-934 - 642\varphi$ $-1 + \varphi$ $9 - 38\varphi) = 5a \cdot 19a \cdot 19b$ $-33 + 79\varphi$ |   | 1   3   1   3   1   1   1   1   1   1  | + -<br>  + -<br>  + -<br>  + -<br>  + -   | 1, 9, 1<br>3, 3, 3<br>1, 1, 3<br>3, 3, 1<br>1, 3, 1   | 1,9,1<br>3,3,3<br>1,1,3<br>3,3,1<br>1,3,1<br>1,3,1  | 1,9,1<br>1,3,1<br>1,1,3<br>1,3,1<br>1,3,1  | $ \begin{array}{c c} I_1, I_9, I_1 \\ I_3, I_3, I_3 \\ \hline I_1, I_1, I_3 \\ I_3, I_3, I_1 \\ \hline I_1, I_3, I_1 \\ \end{array} $   |           |
| a1 b1 b2 c1 c2 d1 a1 a2       | $ \begin{array}{c cccc}  & \varphi & 0 \\ \hline  & \varphi & 1 - \varphi \\  & \varphi & 1 + \varphi \\ \hline  & \varphi & 1 + \varphi \\ \hline  & 1 & -\varphi & 1 \end{array} $ $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | $ \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ + \varphi \end{array} $  | $-9 + 5\varphi$ $-3 + 4\varphi$ $-363 + 164\varphi$ $-1 + \varphi$ $-161 - 64\varphi$ $-3$ $1805a = (14)$ $-35 - \varphi$ $-25 + 14\varphi$  |  | 1   | $ \begin{array}{c c}  & 1 \\ \hline  & 3 \\  & 1 \\ \hline  & 3 \\  & 1 \\ \hline  & 1 \end{array} $ ny cla  | + -   + -   + -   + -   + -   + -   + -     + + -     + + -     + + -     + + -     + + -     + + -     + + -     + | 1,9,1<br>3,3,3<br>1,1,3<br>3,3,1<br>1,3,1<br>4,2,2<br>2,1,1   | $ \begin{array}{c c} \hline 1,9,1\\ 3,3,3\\ \hline -1,1,3\\ 3,3,1\\ \hline -1,3,1 \end{array} $ $ \begin{array}{c c} 4,2,2\\ 2,1,1 \end{array} $                                  | 1,9,1<br>1,3,1<br>1,1,3<br>1,3,1<br>1,3,1<br>2,2,2<br>2,1,1  | $\begin{array}{c c} I_1, I_9, I_1 \\ I_3, I_3, I_3 \\ \hline I_1, I_1, I_3 \\ I_3, I_3, I_1 \\ \hline I_1, I_3, I_1 \\ \hline I_4, I_2, I_2 \\ I_2, I_1, I_1 \\ \end{array}$  |           |
| a1 b1 b2 c1 c2 d1 s2 a3       | $ \begin{array}{c cccc} \varphi & 0 \\ \varphi & 1 - \varphi \\ \varphi & 1 + \varphi \\ \hline \varphi & 1 + \varphi \\ \hline 1 & -\varphi & 1 \end{array} $ $ \begin{array}{c ccccc} \mathbf{5a} \\ \varphi & -\varphi \\ 1 + \varphi & -1 + \varphi \end{array} $   | $ \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ + \varphi \end{array} $   | $-9 + 5\varphi$ $-3 + 4\varphi$ $-363 + 164\varphi$ $-1 + \varphi$ $-161 - 64\varphi$ $-3$ $1805a = (1$ $-35 - \varphi$ $-25 + 14\varphi$ $-1106 - 1763\varphi$  |  | (2 isoge 0 0 0 0  | $ \begin{array}{c c}  & 1 \\ \hline  & 3 \\  & 1 \\ \hline  & 3 \\  & 1 \\ \hline  & 1 \end{array} $ ny cla $ \begin{array}{c c}  & 4 \\  & 2 \\  & 4 \end{array} $  | + -   + -   + -   + -   + -   + +   + -   + +   + +   + +   + +   + +   + +   + +   + +   + +   | 1,9,1<br>3,3,3<br>1,1,3<br>3,3,1<br>1,3,1<br>4,2,2<br>2,1,1<br>2,4,1  | 1,9,1<br>3,3,3<br>1,1,3<br>3,3,1<br>1,3,1<br>4,2,2<br>2,1,1<br>2,4,1  | 1,9,1<br>1,3,1<br>1,1,3<br>1,3,1<br>1,3,1<br>1,3,1<br>2,2,2<br>2,1,1<br>2,4,1  | $\begin{array}{c c} I_1,I_9,I_1\\ I_3,I_3,I_3\\ \hline I_1,I_1,I_3\\ I_3,I_3,I_1\\ \hline I_1,I_3,I_1\\ \hline I_2,I_1,I_1\\ I_2,I_4,I_1\\ \end{array}$   |           |
| a1 b1 b2 c1 c2 d1 a2 a3 a4    | $ \begin{array}{c cccc} \varphi & 0 \\ \varphi & 1 - \varphi \\ \varphi & 1 + \varphi \\ \varphi & 1 + \varphi \end{array} $ $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$   | $-9 + 5\varphi$ $-3 + 4\varphi$ $-363 + 164\varphi$ $-1 + \varphi$ $-161 - 64\varphi$ $-3$ $1805a = (1$ $-35 - \varphi$ $-25 + 14\varphi$ $-1106 - 1763\varphi$ $15 - 36\varphi$                         | $     \begin{array}{r}                                     $   | (2 isoge 0 0 0 0 0 0  | $ \begin{array}{c c}  & 1 \\ \hline  & 3 \\  & 1 \\ \hline  & 3 \\  & 1 \\ \hline  & 1 \end{array} $ any cla $ \begin{array}{c c}  & 4 \\  & 2 \\  & 4 \\  & 2 \end{array} $   | + -   + -   + -   + -   + -   + -   + -   + -   + -   + -   + -   + -   + + -   + + +   - +   + -   + +   - +   + -   | 1,9,1<br>3,3,3<br>1,1,3<br>3,3,1<br>1,3,1<br>4,2,2<br>2,1,1<br>2,4,1<br>8,1,4   | 1,9,1<br>3,3,3<br>1,1,3<br>3,3,1<br>1,3,1<br>4,2,2<br>2,1,1<br>2,4,1<br>8,1,4   | $ \begin{vmatrix} 1,9,1\\1,3,1\\1,1,3\\1,3,1\\1,3,1\\1,3,1\\2,1,4\\2$ | $ \begin{vmatrix} I_1, I_9, I_1 \\ I_3, I_3, I_3 \end{vmatrix} $ $ \begin{vmatrix} I_1, I_1, I_3 \\ I_3, I_3, I_1 \end{vmatrix} $ $ \begin{vmatrix} I_1, I_2, I_2 \\ I_2, I_1, I_1 \\ I_2, I_4, I_1 \\ I_8, I_1, I_4 \end{vmatrix} $        |           |
| a1 b1 b2 c1 c2 d1 a2 a3 a4 b1 | $ \begin{array}{c cccc} \varphi & 0 \\ \hline \varphi & 1 - \varphi \\ \varphi & 1 + \varphi \\ \hline \varphi & 1 + \varphi \\ \hline 1 & -\varphi & 1 \end{array} $ $ \begin{array}{c ccccc} \bullet & -\varphi \\ \varphi & -\varphi \\ 1 + \varphi & -1 + \varphi \\ \hline \varphi & -\varphi \\ \hline 1 + \varphi & -1 + \varphi \end{array} $ | $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ | $-9 + 5\varphi$ $-3 + 4\varphi$ $-363 + 164\varphi$ $-1 + \varphi$ $-161 - 64\varphi$ $-3$ $1805a = (1$ $-35 - \varphi$ $-25 + 14\varphi$ $-1106 - 1763\varphi$ $15 - 36\varphi$ $-36$                   | $     \begin{array}{r}                                     $   | (2 isoge   0   0   0   0   0   0   0   0   0                            | $ \begin{array}{c c}  & 1 \\ \hline  & 3 \\  & 1 \\ \hline  & 3 \\  & 1 \\ \hline  & 1 \\ \hline  & 1 \\  \end{array} $ ny cla $ \begin{array}{c c}  & 4 \\  & 2 \\  & 4 \\  & 2 \\ \hline  & 4 \\ \end{array} $   | + -   + -   + -   + -   + -   + +   +   + +   +   + +   + | 1,9,1<br>3,3,3<br>1,1,3<br>3,3,1<br>1,3,1<br>4,2,2<br>2,1,1<br>2,4,1<br>8,1,4<br>2,4,2  | $\begin{array}{ c c c c }\hline 1,9,1\\ 3,3,3\\ \hline 1,1,3\\ 3,3,1\\ \hline 1,3,1\\ \hline \end{array}$ $\begin{array}{ c c c c c c c c c c c c c c c c c c c$                  | $\begin{array}{ c c c }\hline 1,9,1\\ 1,3,1\\\hline 1,1,3\\\hline 1,1,3\\\hline 1,3,1\\\hline 1,3,1\\\hline 2,1,4\\\hline 2,2,2\\\hline \end{array}$   | $\begin{array}{c c} & I_1, I_9, I_1 \\ & I_3, I_3, I_3 \\ & I_1, I_1, I_3 \\ & I_3, I_3, I_1 \\ & I_1, I_3, I_1 \\ \\ & I_2, I_1, I_1 \\ & I_2, I_4, I_1 \\ & I_8, I_1, I_4 \\ & I_2, I_4, I_2 \\ \end{array}$                              |           |
| 180 a1 c2 d1 a2 a3 a4 b1 b2   | $ \begin{array}{c cccc}  & \varphi & 0 \\ \hline  & \varphi & 1 - \varphi \\  & \varphi & 1 + \varphi \\ \hline  & \varphi & 1 + \varphi \\ \hline  & 1 & -\varphi & 1 \end{array} $ $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$   | $-9 + 5\varphi$ $-3 + 4\varphi$ $-363 + 164\varphi$ $-1 + \varphi$ $-161 - 64\varphi$ $-3$ $1805a = (1$ $-35 - \varphi$ $-25 + 14\varphi$ $-1106 - 1763\varphi$ $15 - 36\varphi$ $-36$ $-11 - 15\varphi$ |  | (2 isoge   0   0   0   0   0   0   0   0   0                            | $ \begin{array}{c c}  & 1 \\ \hline  & 3 \\  & 1 \\ \hline  & 3 \\  & 1 \\ \hline  & 1 \end{array} $ $ \begin{array}{c c}  & 3 \\  & 1 \\ \hline  & 1 \end{array} $ $ \begin{array}{c c}  & 4 \\  & 2 \\  & 4 \\  & 2 \end{array} $  | sses) ++ +- +- +- +- +- +- +- +- +- ++ +- ++ +- ++ +- ++ +- ++ +- ++  | $\begin{array}{c} 1,9,1\\ 3,3,3\\ -1,1,3\\ 3,3,1\\ 1,3,1 \end{array}$ $\begin{array}{c} 4,2,2\\ 2,1,1\\ 2,4,1\\ 8,1,4\\ -\frac{2}{2},\frac{4}{2},2\\ 1,2,1 \end{array}$ | $\begin{array}{c c} \hline 1,9,1\\ 3,3,3\\ \hline 1,1,3\\ 3,3,1\\ \hline 1,3,1\\ \hline 1,3,1\\ \hline 4,2,2\\ 2,1,1\\ 2,4,1\\ 8,1,4\\ \hline 2,4,2\\ 1,2,1\\ \hline \end{array}$ | $\begin{array}{ c c c }\hline 1,9,1\\ 1,3,1\\\hline 1,1,3\\\hline 1,1,3,1\\\hline 1,3,1\\\hline 1,3,1\\\hline 2,2,2\\2,1,1\\2,4,1\\\hline 2,1,4\\\hline 2,2,2\\1,2,1\\\hline \end{array}$  | $\begin{array}{c c} I_1,I_9,I_1\\ I_3,I_3,I_3\\ \hline I_1,I_1,I_3\\ I_3,I_3,I_1\\ \hline I_1,I_3,I_1\\ \hline I_2,I_4,I_1\\ I_2,I_4,I_1\\ I_3,I_4,I_2\\ \hline I_2,I_4,I_2\\ I_1,I_2,I_1\\ \hline I_2,I_4,I_2\\ I_1,I_2,I_1\\ \end{array}$ |           |
| 180 a1 a2 a3 a4 b1            | $ \begin{array}{c cccc}  & \varphi & 0 \\ \hline  & \varphi & 1 - \varphi \\  & \varphi & 1 + \varphi \\ \hline  & \varphi & 1 + \varphi \\ \hline  & 1 & -\varphi & 1 \end{array} $ $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$   | $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ $\frac{1}{1}$ | $-9 + 5\varphi$ $-3 + 4\varphi$ $-363 + 164\varphi$ $-1 + \varphi$ $-161 - 64\varphi$ $-3$ $1805a = (1$ $-35 - \varphi$ $-25 + 14\varphi$ $-1106 - 1763\varphi$ $15 - 36\varphi$ $-36$                   | $     \begin{array}{r}                                     $   | (2 isoge   0   0   0   0   0   0   0   0   0                            | $ \begin{array}{ c c c c } \hline & 1 & & & \\ \hline & 3 & & \\ \hline & 1 & & \\ \hline & 3 & & \\ \hline & 1 & & \\ \hline & 3 & & \\ \hline & 1 & & \\ \hline & 4 & & \\ \hline & 2 & & \\ \hline & 3 & & \\ \hline & 5 & & \\ $ | + -   + -   + -   + -   + -   + +   +   + +   +   + +   + | 1,9,1<br>3,3,3<br>1,1,3<br>3,3,1<br>1,3,1<br>4,2,2<br>2,1,1<br>2,4,1<br>8,1,4<br>2,4,2<br>1,2,1<br>4,2,1  | $\begin{array}{ c c c c }\hline 1,9,1\\ 3,3,3\\ \hline 1,1,3\\ 3,3,1\\ \hline 1,3,1\\ \hline \end{array}$ $\begin{array}{ c c c c c c c c c c c c c c c c c c c$                  | $\begin{array}{ c c c }\hline 1,9,1\\ 1,3,1\\\hline 1,1,3\\\hline 1,1,3\\\hline 1,3,1\\\hline 1,3,1\\\hline 2,1,4\\\hline 2,2,2\\\hline \end{array}$   | $\begin{array}{c c} & I_1, I_9, I_1 \\ & I_3, I_3, I_3 \\ & I_1, I_1, I_3 \\ & I_3, I_3, I_1 \\ & I_1, I_3, I_1 \\ \\ & I_2, I_1, I_1 \\ & I_2, I_4, I_1 \\ & I_8, I_1, I_4 \\ & I_2, I_4, I_2 \\ \end{array}$                              | 18046     |

|                              |  |  |  |   |  |  |                     |              | -/.                          | 1  |         |                                    |                |
|------------------------------|--|--|--|---|--|--|---------------------|--------------|------------------------------|--|---------|------------------------------------|----------------|
|                              | $a_1$  | $a_2$  | $a_3$  | $a_4$   | $a_6$  | r  | T                   | s            | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(j)$                  | $c_p$   | Kodaira                            | Isogenies      |
| 1805b                        |  |  | $1805b = (43 - \varphi) = 5a \cdot 19a^2 \qquad (1 \text{ isogeny class})$ |   |  |  |                     |              |                              |  | 1805b   |                                    |                |
| a1                           | $1+\varphi$  | $1-\varphi$  | $1+\varphi$  | $-1-22\varphi$  | $14 + 18\varphi$   | 0  | 4                   | -+           | 7, 3                         | 1,3  | 4,3     | $I_{1}^{*}, I_{3}$                 |                |
| a2                           | $1+\varphi$  | $1-\varphi$  | $1+\varphi$  | $-86-62\varphi$   | $-311 - 411\varphi$  | 0  | 4                   | ++           | 8,6                          | 2,6  | 4, 6    | $I_{2}^{*}, I_{6}$                 |                |
| a3                           | 1  | $1+\varphi$  | $1+\varphi$  | $-4822 - 7758\varphi$   | $239035 + 386868\varphi$   | 0  | 4                   | - +          | 9, 1                         | 3, 1   | 4, 1    | $\mathrm{I}_3^*,\mathrm{I}_1$      |                |
| a4                           | $1+\varphi$  | $1-\varphi$  | $1+\varphi$  | $-1136 - 1087\varphi$   | $-14246 - 28841\varphi$  |  | 2                   | ++           | 10, 3                        | 4,3  | 4, 3    | $I_4^*, I_3$                       |                |
| a5                           | $\varphi$  | $\varphi$  | $\varphi$  | $-15073 + 9087\varphi$  | $-825532 + 511721\varphi$  | 0  | 4                   | ++           | 12, 2                        | 6, 2   | 4, 2    | $I_6^*, I_2$                       |                |
| a6                           | $1+\varphi$  |  | $1+\varphi$  | $-396 + 323\varphi$   | $-5296 + 1023\varphi$  | 0  | 2                   | + -          | 7, 12                        | 1, 12  | 2,12    | $I_1^*, I_{12}$                    |                |
| a7                           | $1+\varphi$  |  | $1+\varphi$  | $-3301 - 407\varphi$  | $-50069 + 22700\varphi$  |  | 2                   | ++           | 18, 1                        | 12, 1  | 4, 1    | $I_{12}^*, I_1$                    |                |
| a8                           | 1  | $-1+\varphi$   | φ  | $-1641775 + 1014657\varphi$   | $-951092944 + 587807956\varphi$  | 0  | 2                   | + -          | 9,4                          | 3,4  | 2,4     | $I_3^*, I_4$                       |                |
| 180                          | 5c   |  |  | 180   | $05c = (42 + \varphi) = 5a \cdot 19b^2 \tag{1 i}$  | isogeny clas   | ss)                 |              |                              |  |         |                                    | 1805c          |
| a1                           | φ  | 1  | $\varphi$  | $-21+20\varphi$   | $33-19\varphi$   | 0  | 4                   | + -          | 7, 3                         | 1,3  | 4,3     | $I_{1}^{*}, I_{3}$                 |                |
| a2                           | $\varphi$  | 1  | $\varphi$  | $-146 + 60\varphi$  | $-721 + 410\varphi$  | 0  | 4                   | ++           | 8, 6                         | 2,6  | 4,6     | $I_{2}^{*}, I_{6}$                 |                |
| a3                           | 1  | $-1-\varphi$   | $1+\varphi$  | $-12581 + 7759\varphi$  | $638484 - 394628\varphi$   | 0  | 4                   | + -          | 9, 1                         | 3, 1   | 4, 1    | $ar{\mathrm{I}_3^*}, \mathrm{I}_1$ |                |
| a4                           | $\varphi$  | 1  | $\varphi$  | $-2221 + 1085\varphi$   | $-43086 + 28840\varphi$  | 0  | 2                   | ++           | 10, 3                        | 4,3  | 4,3     | $I_4^*, I_3$                       |                |
| a5                           | $1+\varphi$  | $1+\varphi$  | $\varphi$  | $-5985-9085\varphi$   | $-322897 - 526793\varphi$  | 0  | 4                   | ++           | 12, 2                        | 6, 2   | 4, 2    | $I_6^*, I_2$                       |                |
| a6                           | $\varphi$  | 1  | $\varphi$  | $-71-325\varphi$  | $-4272 - 1024\varphi$  | 0  | 2                   | - +          | 7,12                         | 1,12   | 2, 12   | $I_{1}^{*}, I_{12}$                |                |
| a7                           | $\varphi$  | 1  | $\varphi$  | $-3706 + 405\varphi$  | $-27368 - 22701\varphi$  | 0  | 2                   | ++           | 18, 1                        | 12, 1  | 4, 1    | $I_{12}^*, I_1$                    |                |
| a8                           | 1  | $-\varphi$   | $1+\varphi$  | $-627118 - 1014658\varphi$  | $-363284988 - 587807957\varphi$  | 0  | 2                   | -+           | 9, 4                         | 3,4  | 2,4     | $I_3^*, I_4$                       |                |
| 182                          | 9a   |  |  | 1829  | $9a = (41 + 4\varphi) = 31a \cdot 59b \tag{1}$   | isogeny cla  | ıss)                |              |                              |  |         |                                    | 1829a          |
| a1                           | $1+\varphi$  | $-\varphi$   | $\varphi$  | $-20 - 28\varphi$   | $-52-89\varphi$  | 0  | 2                   | -+           | 1, 2                         | 1, 2   | 1, 2    | $I_1, I_2$                         |                |
| a2                           | $1+\varphi$  | $-\varphi$   | $\varphi$  | $-60-3\varphi$  | $60-160\varphi$  | 0  | 2                   | +-           | 2, 4                         | 2,4  | 2, 2    | $I_2, I_4$                         |                |
| 182                          | 9b   |  |  | 1829  | $9b = (45 - 4\varphi) = 31b \cdot 59a \tag{1}$   | isogeny cla  | ss)                 |              |                              |  |         |                                    | 1829b          |
| a1                           | φ  | 0  | $1+\varphi$  | $-47 + 26\varphi$   | $-141 + 88\varphi$   | 0  | 2                   | + -          | 1, 2                         | 1,2  | 1, 2    | $I_1, I_2$                         |                |
| a2                           | $\varphi$  |  | $1+\varphi$  | $-62 + \varphi$   | $-100 + 159\varphi$  |  | 2                   | -+           | 2, 4                         | 2,4  | 2,2     | $I_2, I_4$                         |                |
|                              | 9 <sub>C</sub>   |  |  | 1829  | $0c = (28 - 39\varphi) = 31b \cdot 59b \tag{1}$  | isogeny cla  | ass)                |              |                              |  |         |                                    | 1829c          |
| 182                          | $\sigma$   |  |  |   |  |  |                     |              | 2.4                          | 9.1  | 2, 1    | $I_2, I_1$                         |                |
|                              |  | $-1-\varphi$   | 0  | $-37 - 26 \varphi$  | $-34 - 118 \omega$   | 0  | 2                   | + +          | 2.1                          | 1 Z. I                                       |         |                                    |                |
| 182<br>a1<br>a2              | φ  | $-1 - \varphi$ $-1 - \varphi$                                  | 0<br>0   | $-37 - 26\varphi$ $-2 - \varphi$  | $-34 - 118\varphi$ $-\varphi$  | 0 0  | $\frac{2}{2}$       | + +<br>  + - | $2, 1 \\ 1, 2$               | $ \begin{array}{c c} 2,1\\ 1,2 \end{array} $ | 1, 2    | $I_1, I_2$                         |                |
| a1                           | φ<br>φ   |  |  | $-2-\varphi$  | $-\varphi$   | 0  | 2                   |              |                              |  |         |                                    | 1829d          |
| a1<br>a2<br><b>182</b>       | $egin{array}{c} arphi \ arphi \end{array}$ 9d  | $-1-\varphi$   | 0  | $-2 - \varphi$ $1829$   | $\frac{-\varphi}{d = (11 - 39\varphi) = 31a \cdot 59a} \tag{1}$  | 0<br>1 isogeny cl  | ass)                | +-           | 1,2                          | 1,2  | 1,2     | $I_1, I_2$                         | 1829d          |
| a1<br>a2                     | φ<br>φ   | $-1-\varphi$   |  | $-2-\varphi$  | $-\varphi$   | 1 isogeny cla  | 2                   |              |                              |  |         |                                    | 1829d          |
| a1<br>a2<br><b>182</b><br>a1 | $ \begin{array}{c} \varphi \\ \varphi \end{array} $ $ \begin{array}{c} \mathbf{9d} \\ 1+\varphi \\ 1+\varphi \end{array} $ | $-1-\varphi$   | $0$ $1 + \varphi$  | $ \begin{array}{r} -2 - \varphi \\ \hline 1829 \\ -64 + 26\varphi \\ -4 + \varphi \end{array} $ | $\frac{-\varphi}{d = (11 - 39\varphi) = 31a \cdot 59a \qquad (1)}$ $-216 + 144\varphi$ $-5 + 2\varphi$   | 1 isogeny cla  | 2<br>ass)<br>2<br>2 | + -          | 2,1                          | 2,1  | 2,1     | $I_1, I_2$ $I_2, I_1$              | 1829d<br>1831a |
| a1<br>a2<br>182<br>a1<br>a2  | $ \begin{array}{c} \varphi \\ \varphi \end{array} $ $ \begin{array}{c} \mathbf{9d} \\ 1+\varphi \\ 1+\varphi \end{array} $ | $-1-\varphi$   | $0$ $1 + \varphi$  | $ \begin{array}{r} -2 - \varphi \\ \hline 1829 \\ -64 + 26\varphi \\ -4 + \varphi \end{array} $ | $\frac{-\varphi}{d = (11 - 39\varphi) = 31a \cdot 59a \qquad (1)}$ $-216 + 144\varphi$ $-5 + 2\varphi$   | 1 isogeny classony cl | 2<br>ass)<br>2<br>2 | + -          | 2,1                          | 2,1  | 2,1     | $I_1, I_2$ $I_2, I_1$              |                |
| a1 a2 182 a1 a2 183          | $ \begin{array}{c} \varphi \\ \varphi \end{array} $ 9d $ \begin{array}{c} 1+\varphi \\ 1+\varphi \end{array} $ 1a          | $ \begin{array}{c} -1 - \varphi \\ \hline 1 \\ 1 \end{array} $ | $ \begin{array}{c} 1+\varphi\\1+\varphi\end{array} $                       | $ \begin{array}{r} -2 - \varphi \\ 1829 \\ -64 + 26\varphi \\ -4 + \varphi \end{array} $ 183    | $\frac{d = (11 - 39\varphi) = 31a \cdot 59a}{-216 + 144\varphi} $ (1 $ \frac{-216 + 144\varphi}{-5 + 2\varphi} $ (2) $ 1a = (40 + 7\varphi) = 1831a $ (4 iso | 1 isogeny classon ogeny classon  | ass) 2 2 2 es)      | ++-          | 2,1                          | 2,1  | 2,1 1,2 | $I_1, I_2$ $I_2, I_1$ $I_1, I_2$   |                |

| <i>a</i> .1 | a2 a3       | Q <sub>A</sub> | 0.6   | r     | T | s | $\operatorname{ord}(\Delta)$ | $\operatorname{ord}_{-}(i)$   | C ==    | Kodaira | Isogenies |
|-------------|-------------|----------------|-------|-------|---|---|------------------------------|-------------------------------|---------|---------|-----------|
| $  u_1  $   | $a_2$ $a_3$ | $a_4$          | $a_6$ | , , I | 1 | 3 | $\operatorname{ord}(\Delta)$ | $ \operatorname{ord}_{-}(J) $ | $ c_p $ | Rodana  | isogemes  |

1831a

d2

1

 $-\varphi$  1

 $1831a = (40 + 7\varphi) = 1831a$  (4 isogeny classes)

1831a

|    |                      |                | ( ) (       |   |   |      |   |   |       |  |
|----|----------------------|----------------|-------------|---|---|------|---|---|-------|--|
| d1 | $1  -1 + \varphi  1$ | -1             | -arphi      | 0 | 2 | +- 1 | 1 | 1 | $I_1$ |  |
| d2 | 1 $-1 + \varphi$ 1   | $-6-10\varphi$ | -12-21arphi | 0 | 2 | -+ 2 | 2 | 2 | $I_2$ |  |

1831b 1831b  $1831b = (47 - 7\varphi) = 1831b$ (4 isogeny classes)  $3-2\varphi$  $0 -1 + \varphi 1$ 2  $-2+\varphi$ 1 1 1  $I_1$ -+ $-169 - 273\varphi$  $-1581 - 2559\varphi$  $I_1$ b1 $-1 \varphi$ c1 $1-3\varphi$  $I_1$  $1-\varphi$  1  $-1+\varphi$ d11  $-\varphi$  1 -10 1 1 1  $I_1$ 

 $-33+21\varphi$ 

 $-16+10\varphi$ 

0

2

+ -

2

2

 $I_2$ 

2