Arbitrage Betting

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Abstract

This paper produces methodologies and ideas on how to do Arbitrage Betting. Arbitrage Betting is explained from first principles mathematically. Source Code for calculating arbitrage bets is also provided in Python. Pros and Cons of Arbitrage Betting are also discussed. Optimization of Arbitrage Betting is also discussed. Real World problems are solved in this paper.

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Chapter 1

Introduction

1.1 Introduction to Arbitrage Betting

Betting arbitrage ("sure bets", sports arbitrage) is an example of arbitrage arising on betting markets due to either bookmakers' differing opinions on event outcomes or errors. When conditions allow, by placing one bet per each outcome with different betting companies, the bettor can make a profit regardless of the outcome. Mathematically, arbitrage occurs when there are a set of odds, which represent all mutually exclusive outcomes that cover all state space possibilities (i.e. all outcomes) of an event, whose implied probabilities add up to less than 1. In the bettors' slang an arbitrage is often referred to as an arb; people who take advantage of these arbitrage opportunities are called arbers.

1.1.1 Theory of Arbitrage Betting

Consider the variables for a soccer betting market as follows: Let us suggest that there are there events in a soccer match.

P(Home Team Wins) = p_1
P(Draw) = p_2
P(Away Team Wins) = p_3

When betting a match a *bookmaker* will offer odds for each of the three possible outcomes.

Some examples of odds are as follows:

Odds(Home Team Wins) = o_1
Odds(Draw) = o_2
Odds(Away Team Wins) = o_3

The purpose of Arbitrage Betting is to find a set of odds that will allow us to make a profit regardless of the outcome of the match.

Before we can do this we need to understand the relationship between odds and probabilities. I will explain below how betting odds work.

1.1.1.1 Learning to Bet

Bet Amount/Punt Amount When betting on a match you will be asked to place a bet amount.
Odds are the ratio of the amount of money that you will win to the amount of money that you bet.
Probability is the likelihood of an event occurring.
Profit is the amount of money that you will win.
Return is the amount of money that you will win plus the amount of money that you bet.

Example: There is a soccer match between Manchester United vs Everton.

The odds for Manchester United to win are 1.5.
The odds for a draw are 3.5.
The odds for Everton to win are 5.

Now the ratio of the amount of money that you will win to the amount of money that you bet is as follows:

Manchester United to win = 1.5
Draw = 3.5
Everton to win = 5

You may be wondering how the odds are calculated.

The bookmaker will calculate the odds based on the probability of the event occurring. For interest sake Manchester United compared to Everton, Manchester United is the favourite to win. Everton is the underdog. The bookmaker will calculate the odds based on the probability of the event occurring. The bookmaker will calculate the probability of the event occurring by looking at the history of the teams. There are many factors that the bookmaker will take into account. Different bookmakers will have different odds for the same match. This is where we will take advantage later in our paper to do Arbitrage Betting.

Lets take a bet for Manchester United to win.

Bet Amount = R100
P(Manchester United to win) = 1.5

Our profit is calculated as follows:

$$\text{Profit} = (\text{Bet Amount} \times \text{P(Manchester United to win)}) - \text{Bet Amount}$$
$$\text{Profit} = (R100 \times 1.5) - R100$$
$$\text{Profit} = R50$$

$$\text{Return} = \text{Bet Amount} + \text{Profit}$$
$$\text{Return} = R100 + R50$$
$$\text{Return} = R150$$

As you can see we have made a profit of R50.
If we bet R100 on Manchester United to win we will win R150.

But what happens if Everton wins?
We will have a loss of R100.

Even a draw will result in a loss of R100.

This is why we need to find a set of odds that will allow us to make a profit regardless of the outcome of the match. This is when we will use Arbitrage Betting.

1.1.1.2 Betting on all the Outcomes

As we have seen above, we can make a profit by betting on Manchester United to win.

Let us see what happens if we bet on all the outcomes.
We will bet R100 on Manchester United to win, R100 on a draw and R100 on Everton to win in total

our investment is R300.

Case 1: Manchester United wins.

$$\text{Winning} = \text{Bet Amount} \times \text{P(Manchester United to win)}$$
$$\text{Winning} = R100 \times 1.5$$
$$\text{Winning} = R150$$
$$\text{Return} = \text{Bet Amount} + \text{Winning}$$
$$\text{Return} = R100 + R150$$
$$\text{Return} = R250$$
$$\text{Profit/Loss} = \text{Return} - \text{TotalInvestment}$$
$$\text{Profit/Loss} = R250 - R300$$
$$\text{Loss} = -R50$$

Case 2: Draw.

$$\text{Winning} = \text{Bet Amount} \times \text{P(Draw)}$$
$$\text{Winning} = R100 \times 3.5$$
$$\text{Winning} = R350$$

$$\text{Return} = \text{Bet Amount} + \text{Winning}$$
$$\text{Return} = R100 + R350$$
$$\text{Return} = R450$$

$$\text{Profit/Loss} = \text{Return} - \text{TotalInvestment}$$
$$\text{Profit/Loss} = R450 - R300$$
$$\text{Profit} = R150$$

Case 3: Everton wins.

$$\text{Winning} = \text{Bet Amount} \times \text{P(Everton to win)}$$
$$\text{Winning} = R100 \times 5$$
$$\text{Winning} = R500$$

$$\text{Return} = \text{Bet Amount} + \text{Winning}$$
$$\text{Return} = R100 + R500$$
$$\text{Return} = R600$$

$$\text{Profit/Loss} = \text{Return} - \text{TotalInvestment}$$
$$\text{Profit/Loss} = R600 - R300$$
$$\text{Profit} = R300$$

As we can see by doing this there is times where we will make a profit and times where we will make a loss. In reality the odds wont be exactly the same as the ones that we have used in our example. It is more dangerous to bet on all the outcomes because the odds will be different.
There is still risk involved in betting on all the outcomes.
But now i will introduce you to Arbitrage Betting where we will make guaranteed profits.

1.1.1.3 Mathematics of Arbitrage Betting

Arbitrage Betting is a betting strategy that allows us to make a profit regardless of the outcome of the match.

In order to do Arbitrage Betting we need to find a set of odds that will allow us to make a profit regardless of the outcome of the match.

M_n	O_1	O_2	O_3
M_1	1.5	3.5	5
M_2	2	3.6	5
M_3	1.6	12	0.5

Table 1.1: Table of Odds

Assumption:

Let us generate 3 superficial game odds with 3 outcomes that can occur in a game. In the matrix we may say that all the odds computed are from the same bookmaker.

The odds are as follows:

Where M_n is the match number, O_1 is the odds of the first outcome, O_2 is the odds of the second outcome and O_3 is the odds of the third outcome.

How do we recognise Arbitrage Betting opportunities?

Let us denote A_n as the arbitrage probability of the M_n.

If the the sum of the A_n is < 1 then we have an Arbitrage Betting opportunity.

We can calculate the probability of the M_n as follows:

$$A_n = \sum_{O_n=1}^{N_O} \frac{1}{O_n} \tag{1.1}$$

Where N_O is the number of odds in the match and O_n is the current odds in the loop of the sum.

Example: Calculating the opportunity of a match

Let us calculate the arbitrage probability of the first match M_1.

$A_1 = \sum_{O_1=1}^{N_O} \frac{1}{O_1}$
$A_1 = \frac{1}{1.5} + \frac{1}{3.5} + \frac{1}{5}$
$A_1 = 1,15238095$
As we can see in A_1 we do not have an Arbitrage Betting opportunity.

Solving for the other matches we get the following:

$A_2 = 0.97777778$
$A_3 = 2,70833333$
As we can see in A_2 we have an Arbitrage Betting opportunity.

Since we now found an opportunity let us get to work on computing our profit for A_2.

Example: Calculating the Arbitrage Statistics for A_2

Let us calculate the arbitrage statistics for the second match M_2.

Given $A_2 = 0.97777778$

Since we have an Arbitrage Betting opportunity we can calculate the following:

$$Profit = (Investment/ArbitragePercentage) - Investment \tag{1.2}$$

Where $Investment$ is the amount of money that we are going to invest in the match.

Let us say that we are going to invest R100 in the match.

$Profit = (R100/0.97777778) - R100$
$Profit = R2.27$
As we can see we are going to make a profit of R2.27. With zero risk. We bet on all the outcomes and we are guaranteed to make a profit.

Now we need to find the stake for each outcome.
We can calculate the stake for each outcome as follows:

$$Individualbets = (Investment \times IndividualArbitragePercentage)/TotalArbitragePercentage$$

$$S_n = \frac{I \times A_{current}}{A_{total}} \tag{1.3}$$

Where S_n is the stake for the M_n at O_n odds, I is the investment, $A_{current}$ is the arbitrage percentage of the current O_n odd and A_{total} is the total arbitrage percentage of M_n.

Let us compute the stake for all the outcomes of M_2.

$S_1 = \frac{R100 \times \frac{1}{2}}{0.97777778}$
$S_1 = R51.14$
$S_2 = \frac{R100 \times \frac{1}{3.6}}{0.97777778}$
$S_2 = R38.41$
$S_3 = \frac{R100 \times \frac{1}{5}}{0.97777778}$
$S_3 = R20.45$
As we can see we are going to bet R51.14 on the first outcome, R38.41 on the second outcome and R20.45 on the third outcome.

To get a guaranteed profit of R2.27
Our return on investment is as follows:

$$ROI = \frac{Profit}{Investment} \times 100 \tag{1.4}$$

$ROI = \frac{R2.27}{R100} \times 100$
$ROI = 2.27\%$

Now that your are aware of the concept of Arbitrage Betting, In the real world you get O_n outcomes and M_n matches.
The odds are never given like that in the real world by the bookies they are always one step ahead of us.
But i am going to show you how to get the odds in the real world and introduce your to Arbitrage Betting in different bookmakers at once.
You can start looking for opportunities with this current knowledge but further in this paper we will look at how to automate the process of finding Arbitrage Betting opportunities. Talk about issues with the current solutions and how we can improve on them.
Hope your enjoyed my introduction to Arbitrage Betting.

Chapter 2

Betting in the Real World

2.1 Finding the most Optimized Arbitrage Bet

We have learnt how to find arbitrage bets in the previous section. Now we will learn how to find the most optimized arbitrage bet. This is done by finding the most optimized arbitrage bet in the following way:

We look at n amount of bookmakers B. We formulate a matrix of O_n odds for the same match M_n

I wrote the odds in a form $Ooddtype, matrixentrynumber$ in the matrix

$$M_n = \begin{bmatrix} B_1 & B_2 & B_3 & \dots & B_n \\ O_{1,1} & O_{2,3} & O_{3,5} & \dots & O_{4,7} \\ O_{1,2} & O_{2,4} & O_{3,6} & \dots & O_{4,8} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ O_{1,k} & O_{2,k+1} & O_{3,k+2} & \dots & O_{4,k+3} \end{bmatrix} \tag{2.1}$$

We then find the most optimized arbitrage bet by finding the maximum value for each row in the matrix M_n

Resulting in a matrix of O_n odds for the same match M_n

$$Opt_{M_n} = \begin{bmatrix} B_x & B_y & B_z & \dots & B_p \\ O_{1,maxentry} & O_{2,maxentry} & O_{3,maxentry} & \dots & O_{n,maxentry} \end{bmatrix} \tag{2.2}$$

Where B_{var} is the bookmaker with the highest odds for the match M_n

In theory the more bookmakers we have the more optimized our arbitrage bet will be. The more optimized bet results in yielding a higher profit for the arbitrage bet.

But always remember for it to be an arbitrage bet the A_n must be less than 1.

2.2 Python Class to Compute Arbitrage Bets

Using the Python Class we can compute arbitrage bets. The Python Class is called Arbitrage.
I wrote this class based on the mathmatical logic of arbitrage betting as discussed in the previous sections.

The Arbitrage class is defined as follows:

```
class Arbitrage:
  def __init__(self,odds,investment):self.odds=odds;self.investment=investment
  def is_arbitrage(self):
    arbitrageP=0
    for odd in self.odds:arbitrageP+=1/odd
    if arbitrageP<1:return True
    else:return False
  def calculate_arbitrage_percentage(self):
    arbitrageP=0
    for odd in self.odds:arbitrageP+=1/odd
    return arbitrageP
  def calculate_arbitrage_roi(self):return self.calculate_arbitrage_stats()['
                                          returnOnInvestment']
  def calculate_arbitrage_stats(self):
    arbitrageP=self.calculate_arbitrage_percentage();bArbitrage=self.is_arbitrage();
                                          actualProfit=self.investment/arbitrageP-
                                          self.investment;IndividualBets=[];
                                          returnOnInvestment=actualProfit/self.
                                          investment*100;returnOnInvestment=str(
                                          returnOnInvestment)+'%';Payouts=[]
    for odd in odds:IndividualBets.append(self.investment*(1/odd/arbitrageP));Payouts.
                                          append(self.investment*(1/odd/arbitrageP)*
                                          odd)
    return{'arbitrageP':arbitrageP,'bArbitrage':bArbitrage,'actualProfit':actualProfit
                                          ,'IndividualBets':IndividualBets,'
                                          returnOnInvestment':returnOnInvestment,'
                                          Payouts':Payouts}
```

Let us do an example of how to use the Arbitrage class.

Assumption: We have computed the best odds for a match from different bookmakers.

We have resulted in a matrix of odds for the same match.

$$M_1 = \begin{bmatrix} B_1 & B_2 & B_3 \\ O_1 = 2.6 & O_2 = 7.8 & O_3 = 5 \end{bmatrix} \tag{2.3}$$

Let our investment I be $R100$

```
odds=[2.6,7.8,5]
investment=100
objArbitrage = Arbitrage(odds, investments)
print(objArbitrage.calculate_arbitrage_stats())
```

The output of the above code is as follows:

```
{'arbitrageP': 0.7128205128205127, 'bArbitrage': True, 'actualProfit': 40.
                                          28776978417267, 'IndividualBets': [53.
                                          956834532374096, 17.985611510791372, 28.
                                          057553956834536], 'returnOnInvestment': '40
                                          .28776978417267%', 'Payouts': [140.
                                          28776978417267, 140.2877697841727, 140.
                                          28776978417267]}
```

2.2.0.1 Opportunities from this

The above code can be used to compute arbitrage bets.

We can use that code to compute arbitrage bets to compute the most optimized arbitrage bet.

If we have a matrix of different odds for the same match from different bookmakers. We can compute the most optimized arbitrage bet by finding the maximum value for each row in the matrix M_n

$$Opt_n = \begin{bmatrix} B_x & B_y & B_z & \dots & B_p \\ O_{1,maxentry} & O_{2,maxentry} & O_{3,maxentry} & \dots & O_{n,maxentry} \end{bmatrix}$$

We can introduce Data Science methodologies scrape data from different bookmakers and compute the most optimized arbitrage bet.
I wont be showing how to do that in this paper it is up to you to figure that out.

There is already websites that do this for you.

Example of a website that does this for you:
https://oddspedia.com/odds

There is also an web application i have developed that does this for you.

https://odds.adgstudios.co.za/

It is powered by the Oddspedia API.

Limitations of this method:
The websites are not always up to date.
Odds change all the time.
There are some bookmakers that are not on the websites.
Some bookmakers change the odds live during the match.
You will have to manually check the odds on the bookmakers website before placing the bet.
You will have to have many bookmakers accounts to place the arbitrage bet.
Money withdrawal from bookmakers can take a long time.

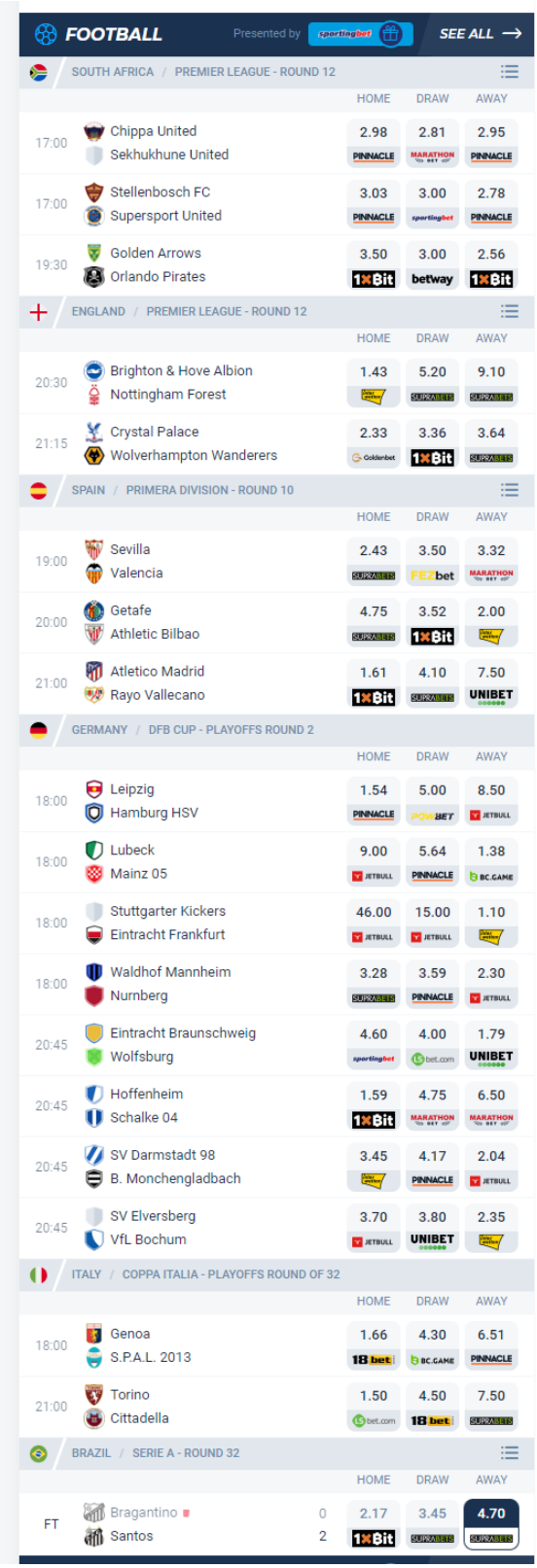

Figure 2.1: Oddspedia

Figure 2.2: https://odds.adgstudios.co.za/

Chapter 3

Simulation Analysis

This is just a chapter running a simulation to see how much money you can make if you quit your full time job and do this for a living.

3.1 Running an Arbitrage Betting Operation

Operation: Use the arbitrage betting method to make money. We start of with an Investment I of R100 per bet we do 10 Arbitrage A bets per day over time t. We try and make at least 1% ROI from each bet. We set our constaints as follows: $ROI \in [1\%, 3\%]$

Let us see how much money we can make if we do this for a year.

We will be putting back the profits in the bets each day so it can be reinvested and profits can grow exponentially.

We set up our experiment as follows:
Per bet we use a random function to generate a random number between 1% and 3% to determine the ROI of the bet.
We declare a D_p variable for profit per day.
We declare a D_t variable for total profit over time.
We declare a D_i variable for investment per day.
T is set for 365 days.

In reality there is chances of losing money. But for the sake of this experiment we will always make money.
We could also make more money by doing more bets per day. But for the sake of this experiment we will assume we do 10 bets per day.
The Return on Investment can be always be higher than 3%. But for the sake of this experiment we will assume $ROI \in [1\%, 3\%]$
From that we estimate a general formula to create a simulation of the arbitrage betting operation over T

$$D_p = \begin{cases} D_p + I \times ROI & \text{if } ROI \in [1\%, 3\%] \\ D_p & \text{if } ROI \notin [1\%, 3\%] \end{cases} \tag{3.1}$$

Once complete the simulation we can plot the results. Using the simulation we can see how much money we can make if we do this for a year.

Once done that we will fit the data into this growth formula here.

$$D_t = I * e^{(D_p \times T)} \tag{3.2}$$

We can now plot the results of the simulation and the growth formula.

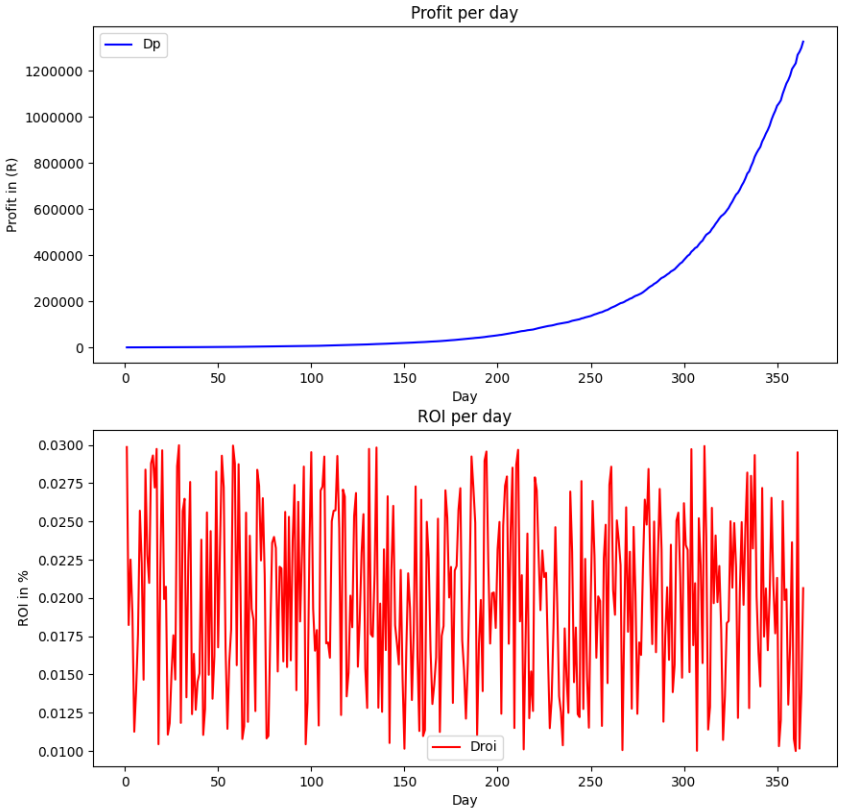

Figure 3.1: Plot of the simulation

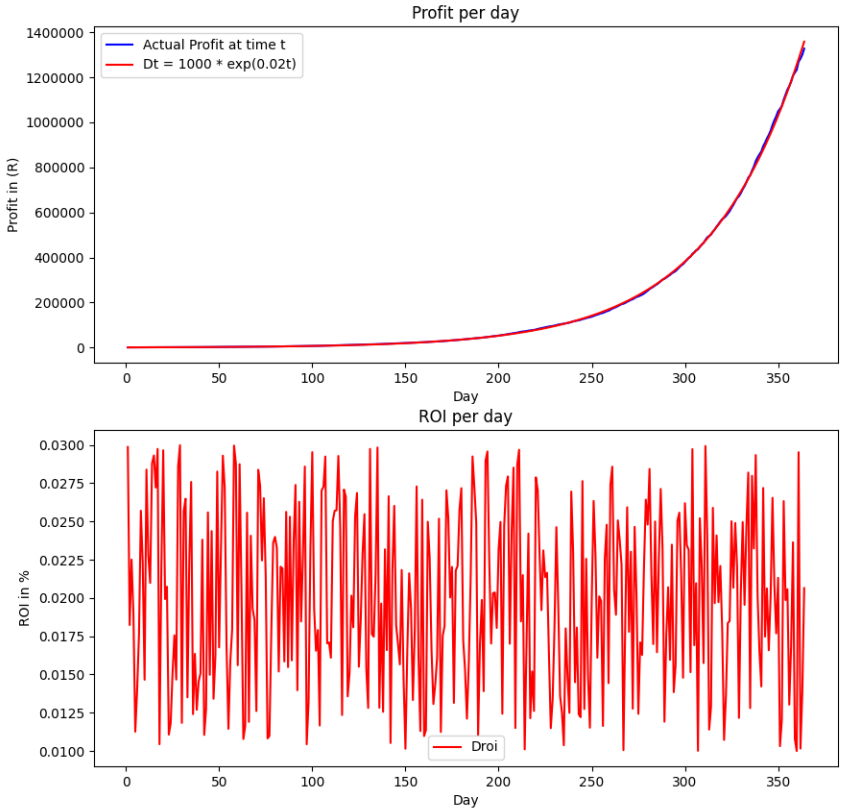

Figure 3.2: Plot of the simulation and the growth formula

Our best fit is $D_t = 1500 \times e^{0.02t}$

From this we can see that we can make a lot of money if we do this for a year.
Starting with an investment R1000 and R100 per bet doing 10 bets a day we can make R1,000,000 in a year.

It is also cool to see this model fit the data so well.

Running many iterations we can see that our model is very accurate.

Therefore we can see that if we increase the investment per bet we can make more money.
below is the code for the simulation.

```
import random

Dt = 0
I = 1000

Date = []
Dp = []
Droi = []

# get random values from 1%-3%
def get_random_roi():
    return random.uniform(0.01, 0.03)

for d in range(1, 365):
```

```
    Dt = d
    ROI = get_random_roi()
    I = I + (I * ROI)
    Droi.append(ROI)
    Date.append(Dt)
    Dp.append(I)

import seaborn as sns
import matplotlib.pyplot as plt

# plot Date,Dp and Droi sub graphs
fig, (ax1, ax2) = plt.subplots(2, 1, figsize=(10, 10))
ax1.plot(Date, Dp, 'b')
ax1.set_title('Profit per day')
ax1.set_xlabel('Day')
ax1.set_ylabel('Profit in (R)')
ax1.ticklabel_format(useOffset=False, style='plain')
#add legend
ax1.legend(['Dp'])
ax2.plot(Date, Droi, 'r')
ax2.legend(['Droi'])
ax2.set_title('ROI per day')
ax2.set_xlabel('Day')
ax2.set_ylabel('ROI in %')
ax2.ticklabel_format(useOffset=False, style='plain')
#save plot
fig.savefig('images/plot.png')



# curve fit Dp and Droi
import numpy as np
from scipy.optimize import curve_fit

def func(Dp, T):
    return 1000*np.exp(T*Dp)

xdata = np.array(Date)
ydata = np.array(Dp)

popt, pcov = curve_fit(func, xdata, ydata, maxfev=5000)

#print equation
print('Dt = 1000 * exp(T * Dp)')
# print Dt at 1
equation = 'Dt = 1000 * exp('+str(round(popt[0],2))+'t)'

# plot curve fit Dp and Droi sub graphs
fig, (ax1, ax2) = plt.subplots(2, 1, figsize=(10, 10))
ax1.plot(xdata, ydata, 'b', label='data')
ax1.plot(xdata, func(xdata, *popt), 'r-', label='fit')
ax1.set_title('Profit per day')
ax1.set_xlabel('Day')
ax1.set_ylabel('Profit in (R)')
ax1.ticklabel_format(useOffset=False, style='plain')
#add legend
ax1.legend(['Actual Profit at time t', equation])
ax2.plot(Date, Droi, 'r')
ax2.legend(['Droi'])
ax2.set_title('ROI per day')
ax2.set_xlabel('Day')
ax2.set_ylabel('ROI in %')
ax2.ticklabel_format(useOffset=False, style='plain')
plt.show()

#save plot
fig.savefig('images/fit.png')
```

3.2 Conclusion

In conclusion we can see that we can make a lot of money if we do this for a year. There is also so much risk involved as the odds can change at any time. Multiple bets can also be made at the same time. This can also be done with other sports. If the odds change some losses can be made. But if the odds change in our favour we can make a lot of money.
You will have to manage and records where your money is going. This can be done with a spreadsheet or database
Finally I will like to tell your that you can get banned from the bookies if you do this. So be careful but there is always ways you can trick the algorithm/system. I think this could be a good way to make money. But you will have to do a lot of research and testing.
This can become a full time job.
Once scaled up you can get other people to do the betting for you.