

TOC.

Language (L) = collection of string

• DFA

$$M = (Q, \Sigma, \delta, q_0, f)$$

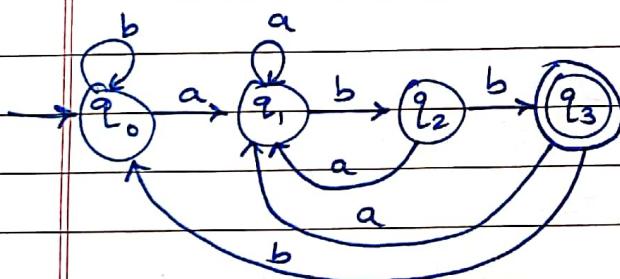
Transition fun"

- Type 1

 \rightarrow ends withover alphabet a, b .Design DFA $\Sigma = \{a, b\}$ which accept the string which ends with abb

$$\rightarrow q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{b} \underline{\underline{q_3}}$$

$$L = \{abb, aabb, babb, \dots\}$$



Transition diagram.

~~(0)~~(0) \rightarrow final state(1) \rightarrow non final state

$Q \setminus \Sigma$	a	b
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_1	q_0

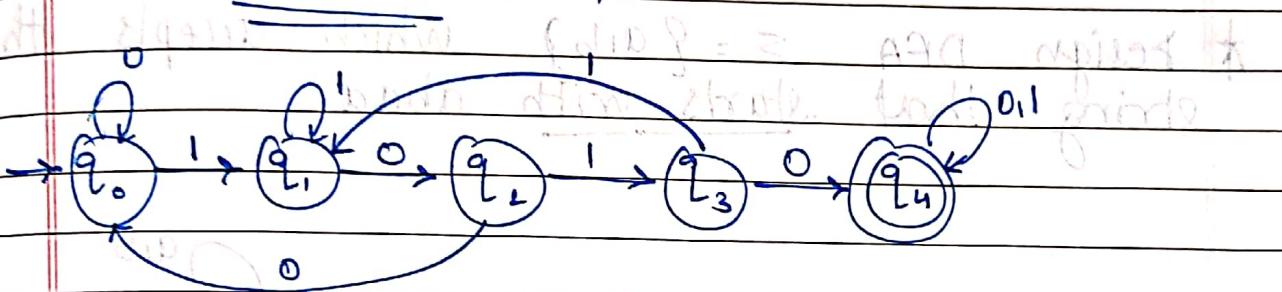
Transition table

- Type 2

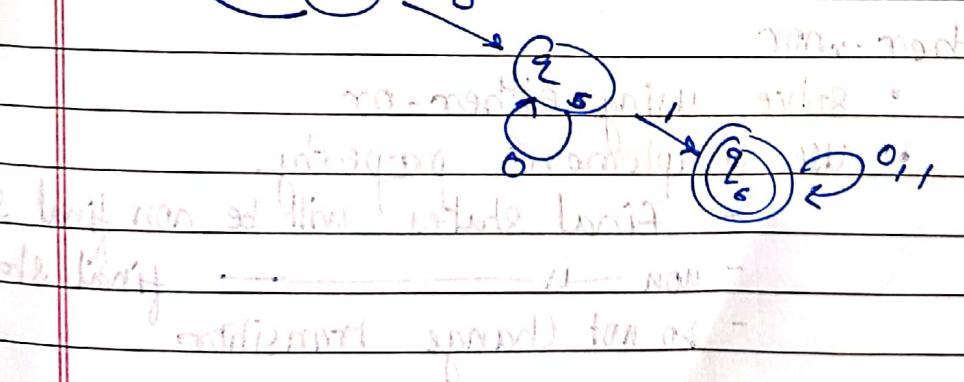
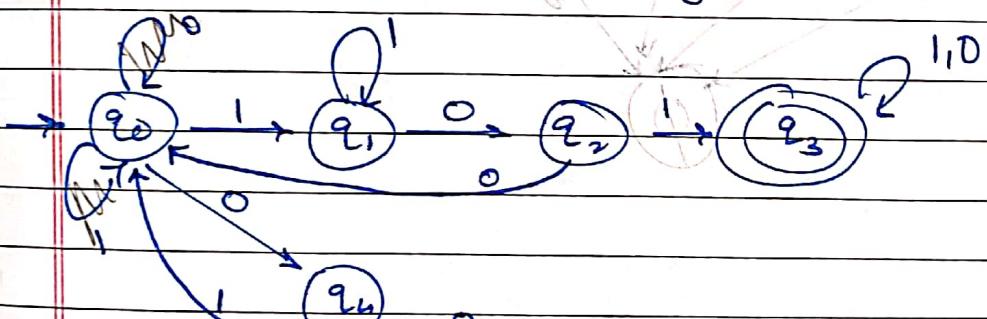
substring .

substring always gives self loop at the end.

* Design DFA $\Sigma = \{0, 1\}$ which accept those string which contains 1010



* Accept string that either contains 101 or 001 as substring .

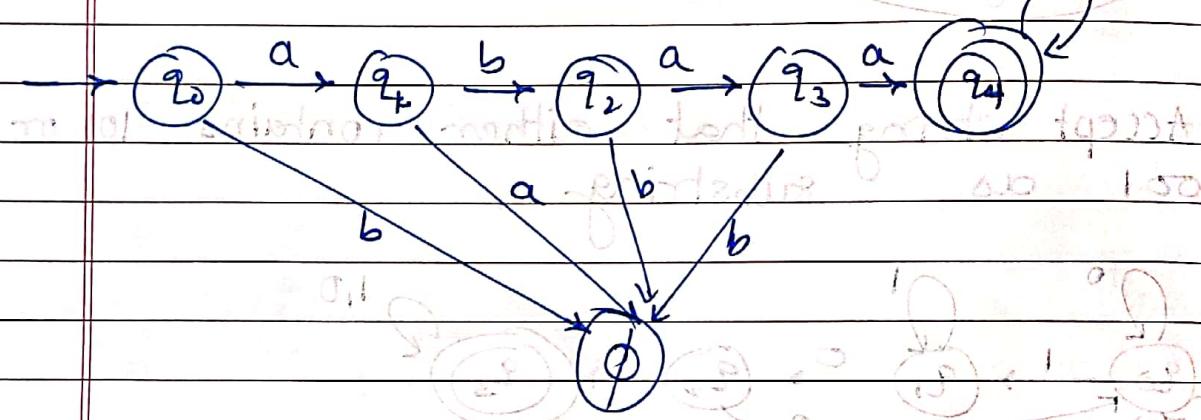


Type 3

→ Starts with the word $a^m b^n$

- add dead state (β)

* Design DFA $\Sigma = \{a, b\}$ which accepts the string that starts with abaa



Type 4

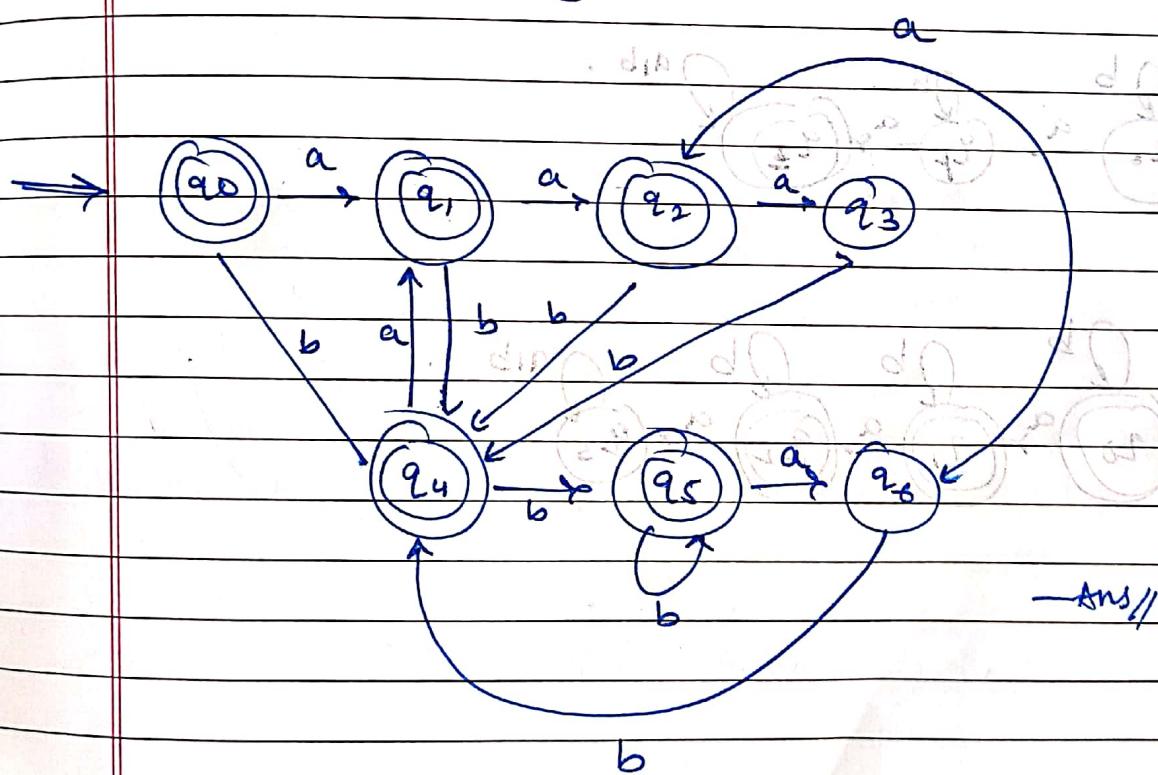
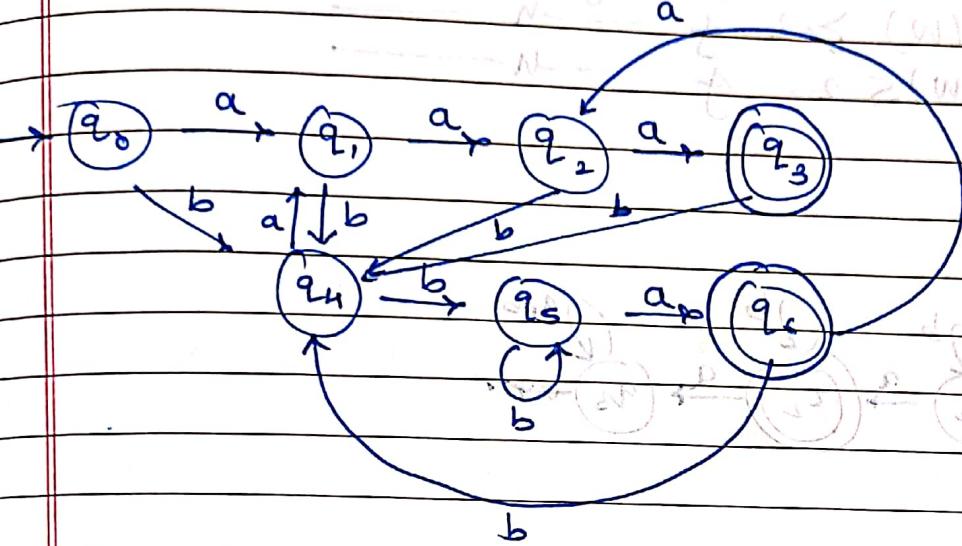
→ Neither - nor

- solve using either-or

• use complement property

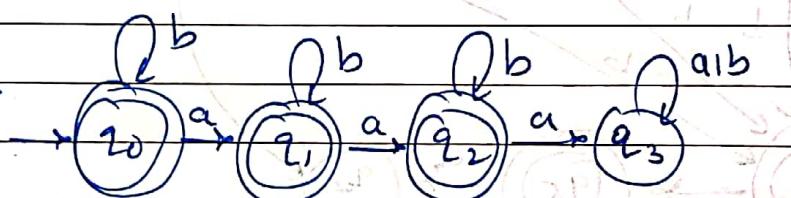
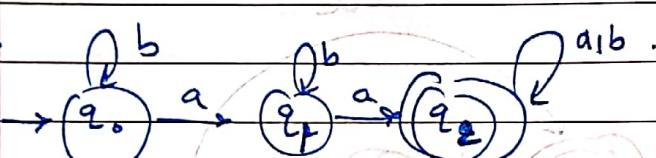
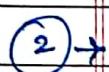
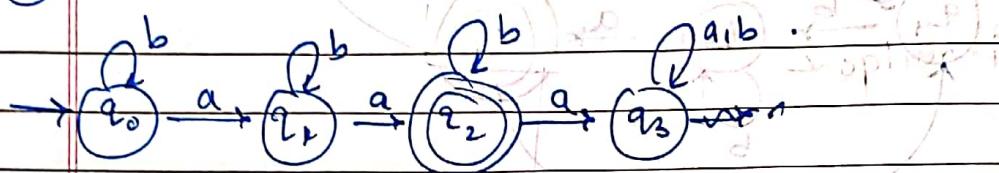
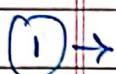
- final states will be non final states
- non - u - → final state
- do not change transitions

* Design DFA which accept string which neither ends with aaa nor bba.



* Design DFA $\Sigma = \{a, b\}$ which accepts

- ① $n_{\alpha}(W) = 32$ if any no. of 5
 - ② $n_{\alpha}(W) \geq 2$ if $\frac{1}{n}$
 - ③ $n_{\alpha}(W) \leq 2$ if $\frac{1}{n}$



* Design DFA that accepts binary no. $\Sigma = \{0, 1\}$ divisible by 3

(OR)

Divisibility by 3 test for binary no.

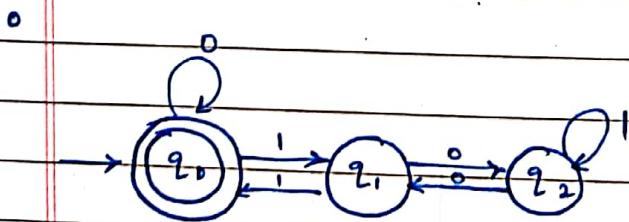
Ans:-

$q \setminus \Sigma$	0	1	
*00(2 ₀)	$\frac{000}{3} = 2_0$	$\frac{001}{3} = 2_1$	
01(2 ₁)	$\frac{010}{3} = 2_2$	$\frac{011}{3} = 2_0$	
10(2 ₂)	$\frac{100}{3} = 2_1$	$\frac{101}{3} = 2_2$	

Transition table

Binary no. Remainder ($\div 3$)

0	000	00 $\rightarrow 2_0$
1	001	01 $\rightarrow 2_1$
2	010	10 $\rightarrow 2_2$
3	011	00 $\rightarrow 2_0$
4	100	01 $\rightarrow 2_1$
5	101	10 $\rightarrow 2_2$
6	110	00 $\rightarrow 2_0$
7	111	01 $\rightarrow 2_1$



Transition diagram.

* DFA for decimal no. divisible by 4

Ans: $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

decimal no. remainder ($\div 4$) state

0	(0)	q_0
---	-------	-------

1	(1)	q_1
---	-------	-------

2	(2)	q_2
---	-------	-------

3	(3)	q_3
---	-------	-------

4	(0)	q_0
---	-------	-------

5	(1)	q_1
---	-------	-------

6	(2)	q_2
---	-------	-------

7	(3)	q_3
---	-------	-------

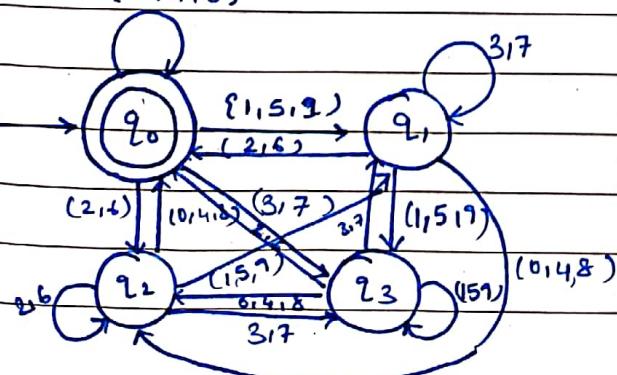
8	(0)	q_0
---	-------	-------

9	(1)	q_1
---	-------	-------

transition table

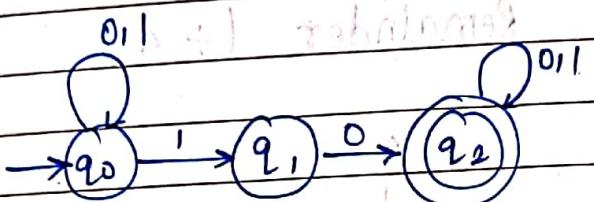
Σ	q_0	q_1	q_2	q_3
0	q_0	q_1	q_2	q_4
1	q_2	q_3	q_0	q_1
2	q_0	q_1	q_2	q_3
3	q_2	q_3	q_0	q_1

$\{0, 4, 8\}$



* NFA $M = (\mathcal{Q}, \Sigma, \delta, q_0, F)$ $\delta: \mathcal{Q} \times \Sigma \rightarrow \mathcal{Q}$
 It resides in multiple state at the same time

e.g.



\mathcal{Q} = Total no. of finite states

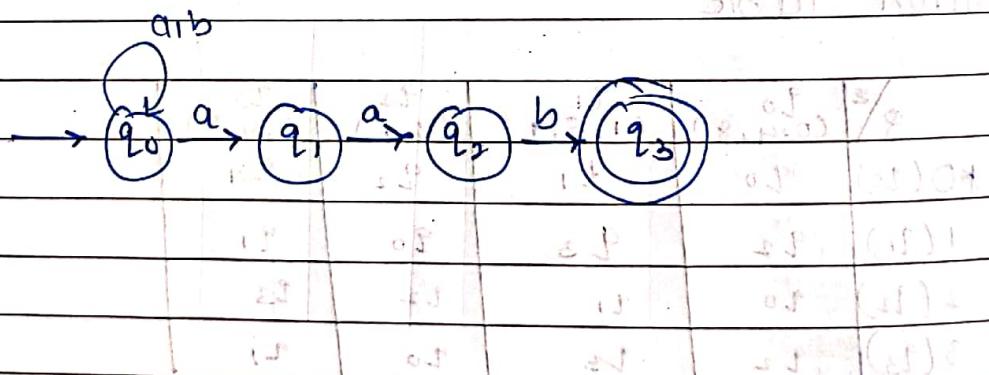
Σ = Input symbol

q_0 = Starting state

F = Set of final states

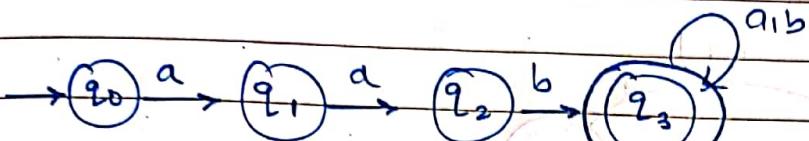
- Type 1 (ends with)
- Type 2 (Starts with)
- Type 3 (substring)

(1) ends with aab:



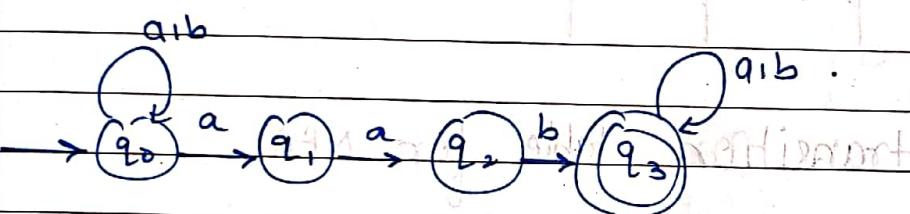
◦ Add self loop at start

(II) Starts with 'aab'



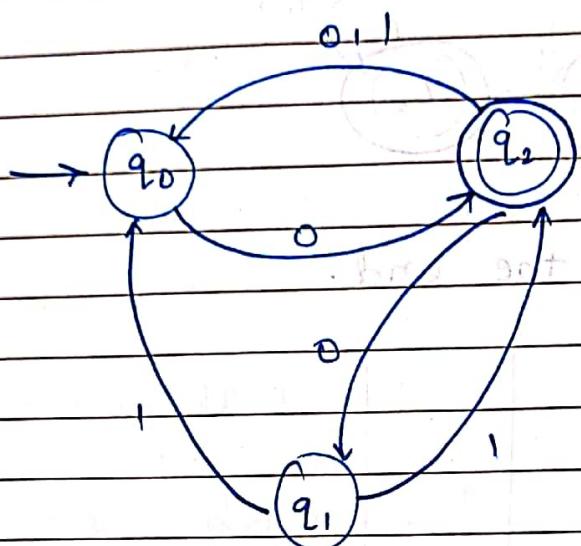
- add self loop at the end.

(III) substring 'aab'



- add self loop at start of at the end.

* Conversion [NFA to DFA]



Ans Step 1

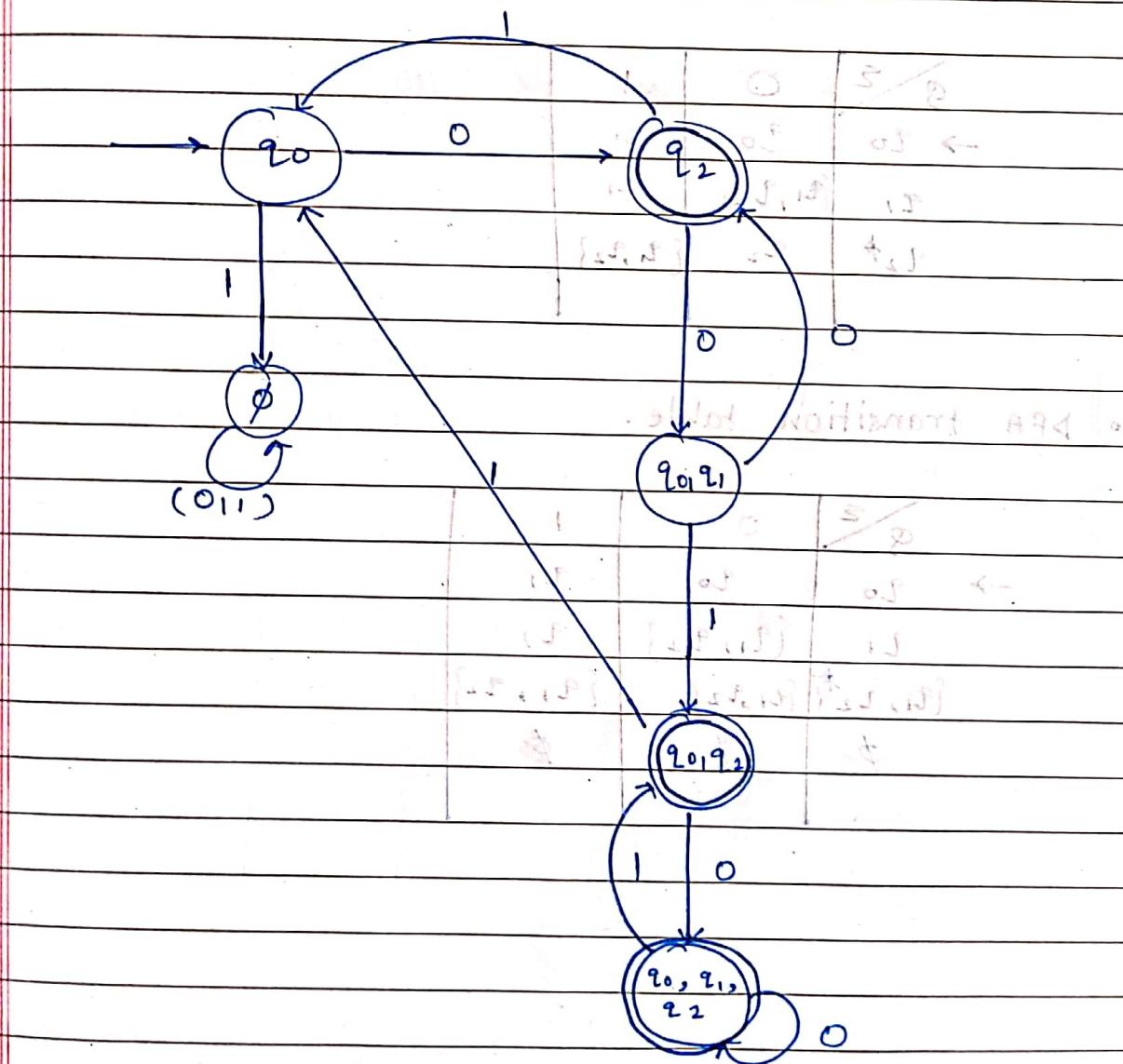
transition table for NFA.

$\delta \setminus \epsilon$	0	1
$\rightarrow q_0$	q_2	\emptyset
q_1	\emptyset	q_0, q_2
q_2^*	q_0, q_1	q_0

transition table for DFA :

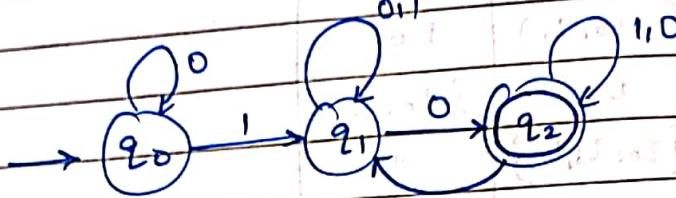
$\delta \setminus \epsilon$	0	1
q_0	q_2	\emptyset

\rightarrow	$\varnothing \setminus$	0	1	all elements
$\rightarrow 2_0$	2_2	\emptyset		
2_2^*	$\{2_0, 2_1\}$	2_0		
$\{2_0, 2_1\}^*$	2_2	$\{2_0, 2_2\}$		
$\{2_0, 2_2\}^*$	$\{2_0, 2_1, 2_2\}$	2_0		
$\{2_0, 2_1, 2_2\}^*$	$\{2_0, 2_1, 2_2\}$	$\{2_0, 2_2\}$		
\emptyset	\emptyset	\emptyset	\emptyset	all elements



*

convert NFA to DFA

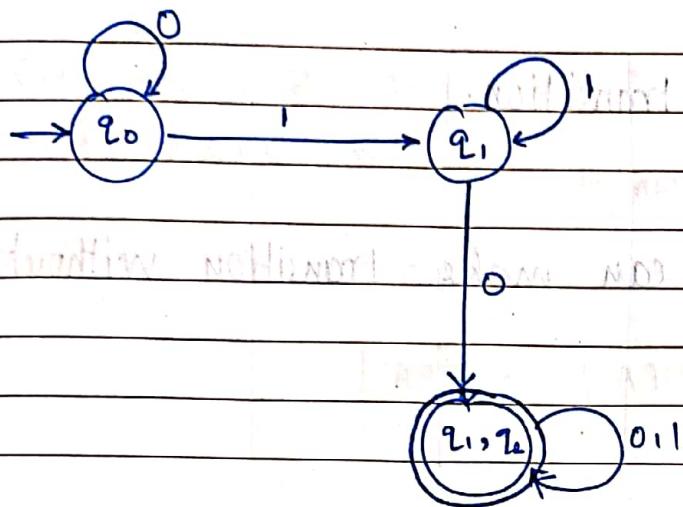


NFA transition table.

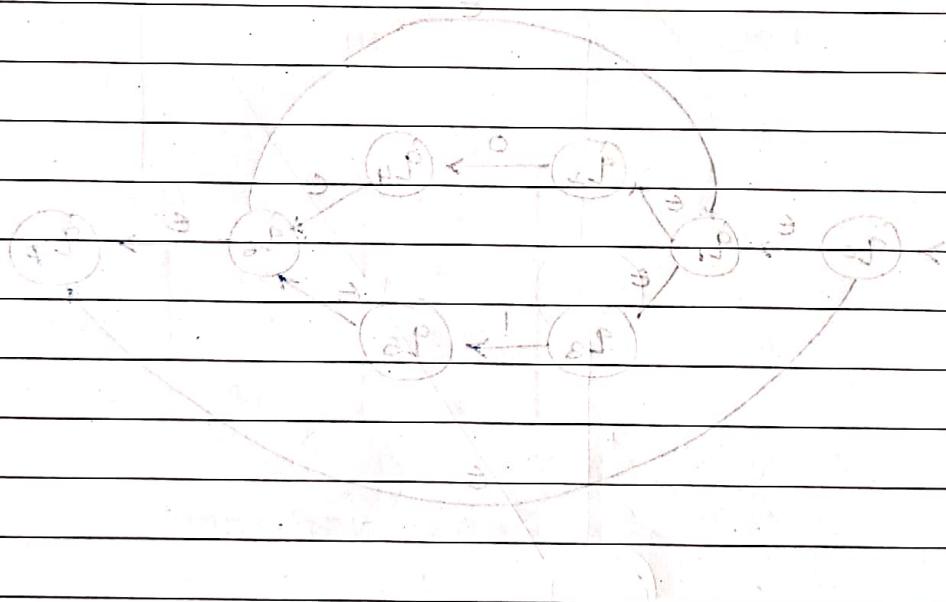
$q \setminus \epsilon$	0	1
$\rightarrow q_0$	q_0	q_1
q_1	$\{q_1, q_2\}$	q_1
q_2	q_2	$\{q_1, q_2\}$

DFA transition table.

$q \setminus \epsilon$	0	1
$\rightarrow q_0$	q_0	q_1
q_1	$\{q_1, q_2\}$	q_1
q_2	q_2	q_2



transition diagram of boundary 3



$$\left\{ \text{P} \in \text{P}(\text{P}(\text{P}(\text{P}(\text{P}))) \mid \text{P} \neq \text{P} \right\} \cup \{\text{P}\} \text{ or } \{\text{P}\} \cup \{\text{P}\}$$

$$\{ \text{exp}(\omega_1 P_1), \dots, \text{exp}(\omega_n P_n) \} = (\text{exp}(P))^k$$

$$\left(\frac{1}{2}P^{\frac{1}{2}}\right)^{-1} = \left(\frac{1}{2}P\right)^{-\frac{1}{2}}$$

State and District staff = - (25) to

Einzelne Pflanzteile (ab) * 3

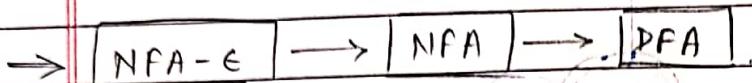
Is it true that $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f(k) = \int_0^\infty f(x) dx$?

$$\{F_i\} = \left(\frac{f_i}{f_0}\right)^{\frac{1}{n}}$$

* DFA with ϵ -transition. $M = (Q, \Sigma, \delta, q_0, F)$
 \downarrow
 null symbol

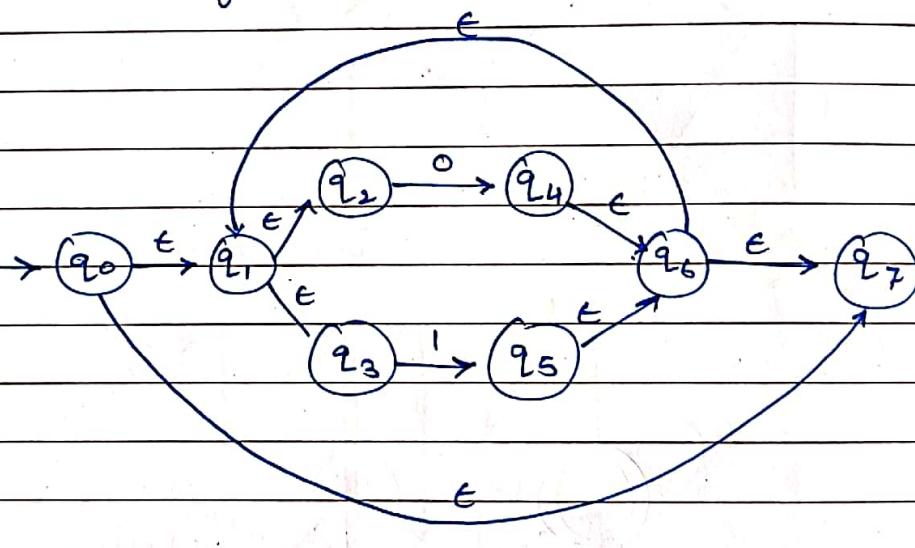
$$\delta : Q \times \Sigma \cup \{\epsilon\} \rightarrow 2^Q$$

A machine can make transition without input.



o ϵ -closure of q_i :

ϵ closure of q_i includes set of states
 reachable from q_i on ϵ moves



$$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$$

$$\epsilon^*(q_1) = \{q_1, q_2, q_3\}$$

$$\epsilon^*(q_2) = \{q_2\}$$

$$\epsilon^*(q_3) = \{q_3\}$$

$$\epsilon^*(q_4) = \{q_4, q_6, q_7, q_1, q_2, q_3\}$$

$$\epsilon^*(q_5) = \{q_5, q_6, q_7, q_1, q_2, q_3\}$$

$$\epsilon^*(q_6) = \{q_6, q_7, q_1, q_2, q_3\}$$

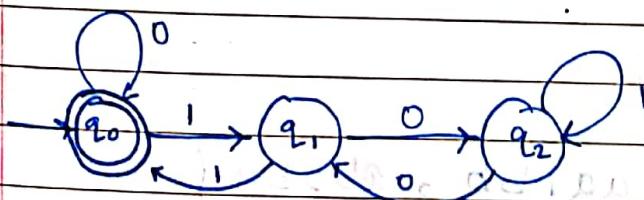
$$\epsilon^*(q_7) = \{q_7\}$$

* Construct minimal DFA in $\Sigma = \{0, 1\}$ which is completely divisible by 3.

Ans:-	decimal	Binary	Remainder ($\div 3$)	State
0	0	000	000	q_0
1	1	001	001	q_1
2	2	010	010	q_2
3	3	011	000	q_0
4	4	100	011	q_1
5	5	101	010	q_2
6	6	110	000	q_0
7	7	111	001	q_1

δ / Σ	0	1
(000) q_0	q_0	q_1
(001) q_1	q_2	q_0
(10) q_2	q_1	q_2

Transition Table



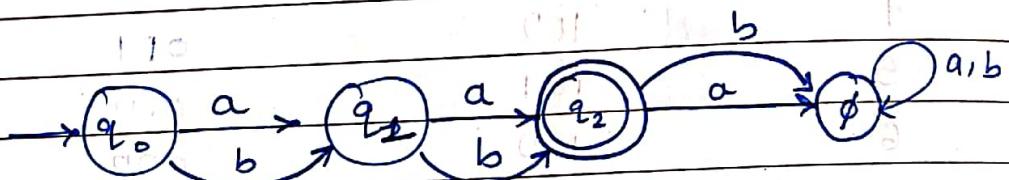
* Construct DFA for $\Sigma = \{a, b\}$ such that

- (i) length = 2
- (ii) length is atleast 2
- (iii) length is atmost 2

Ans:-

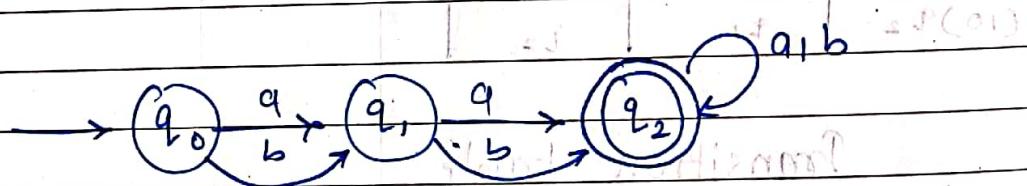
(i)

$$L = \{aa, ab, bb, ba\}$$



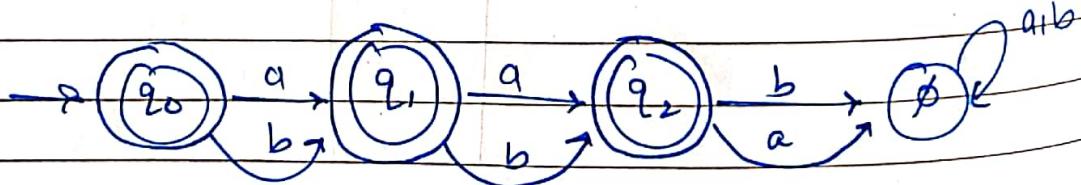
(ii) atleast 2

$$L = \{aa, aab, aba, \dots\}$$



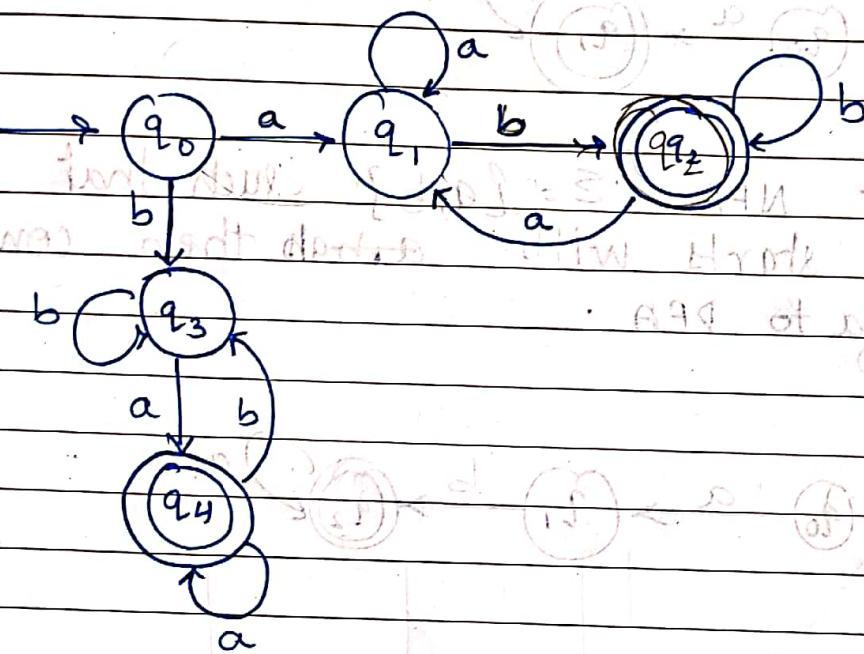
(iii) atmost 2

$$L = \{t, a, b, aa, bb, ab, ba\}$$



* Construct a minimal DFA $\{\Sigma = \{a, b\}\}$ which starts & ends with different symbol.

$$\rightarrow L = \{ab, ba, aab, baa, abab, \dots\}$$



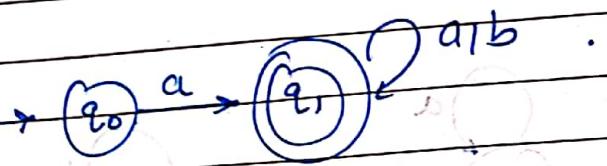
d	s	$\frac{s}{d}$
∅	1P	0P
1P	∅	1P
1P	1P	2P
2P	2P	1P

→ 100% DNA

d	s	$\frac{s}{d}$
∅	1P	0P
1P	∅	1P
1P	1P	2P
2P	2P	1P

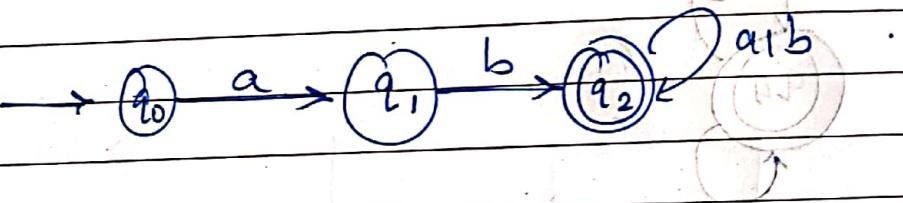
NFA

- * $L = \Sigma = \{a, b\}$ starts with a in $L = [a, aa, ab, aab, aba, \dots]$



- * Construct NFA $\Sigma = \{a, b\}$ such that all strings starts with ab then convert this nfa to DFA.

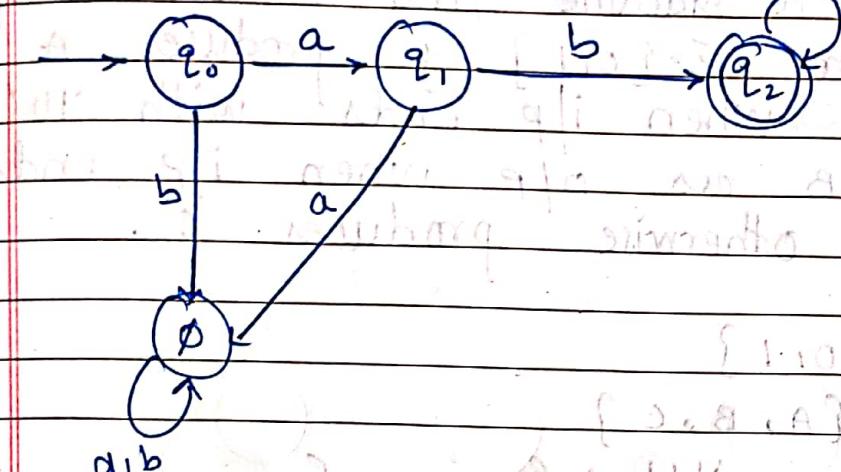
Ans:-



q / ϵ	a	b
q_0	q_1	\emptyset
q_1	\emptyset	q_2
q_2	q_2	q_2

NFA Table

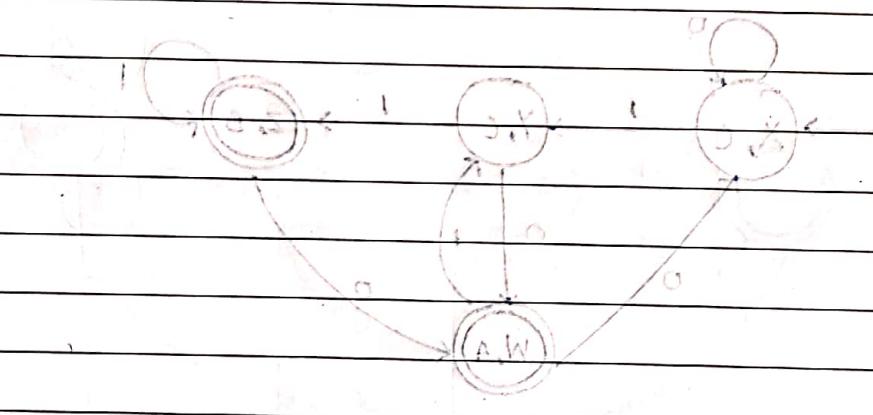
q / ϵ	a	b	
$\rightarrow q_0$	q_1	\emptyset	
q_1	\emptyset	q_2	
q_2	q_2	q_2	
\emptyset	\emptyset	\emptyset	



$$\{q_0, q_1\} = \emptyset$$

$$\{q_2, q_3\} = \emptyset$$

$$\{q_0, q_1, q_2, q_3\} = \emptyset$$



A	I	O	Q
D	X	X	X
S	S	W	X
a	S	W	S
A	X	X	W

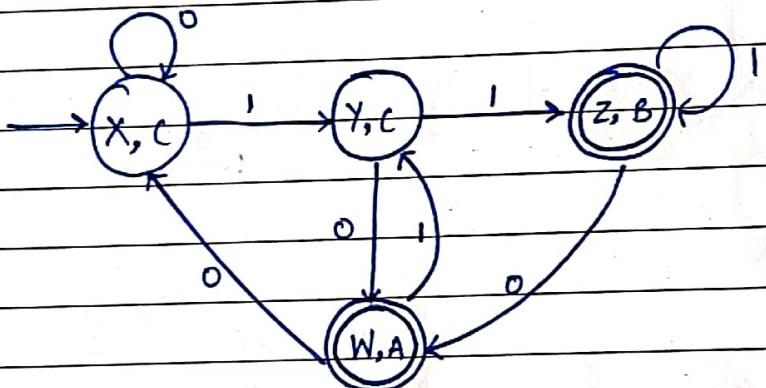
* Construct moore machine that takes set of all strings over $\Sigma = \{0, 1\}$ & produces A as o/p when i/p ends with '10' or produces B as o/p when i/p ends with '11' otherwise produces C.

Ans:- $\Sigma = \{0, 1\}$

$\Delta = \{A, B, C\}$

'10' $\rightarrow A$, '11' $\rightarrow B$, C

$L = \{10, 11, 010, 111, 011 \dots\}$



$\Sigma =$	0	1	Δ
$\rightarrow X$	X	Y	C
y	W	Z	C
*z	W	Z	B
*w	X	Y	A

* Construct a moore machine over $\Sigma = \{a, b\}$ such that all strings ending with 'baa' & printing 1 as output otherwise output is 0.

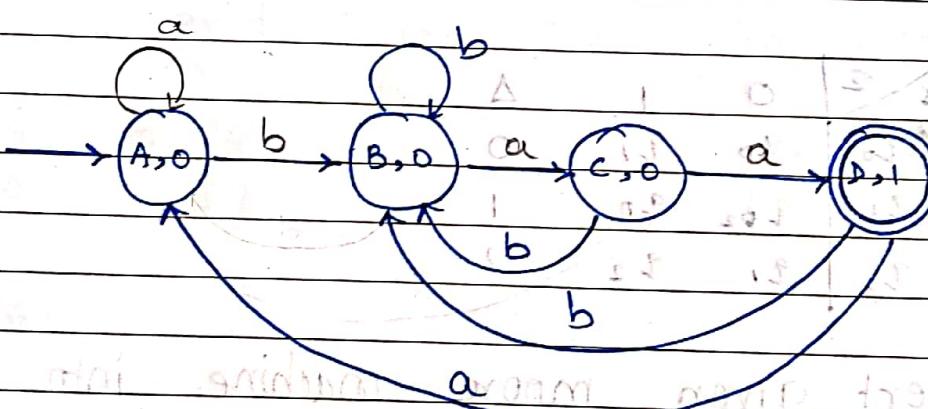
Ans:-

$$\Sigma = \{a, b\}$$

$$\Delta = \{0, 1\}$$

'baa' \rightarrow 1, 0 .

$$L = \{ \text{baa}, \text{abaa}, \text{bbaa} \dots \}$$

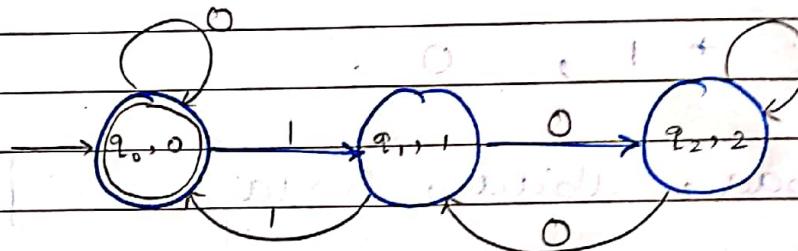


$\varnothing \Sigma$	a	b	Δ
$\rightarrow A$	A	B	0
B	C	B	0
C	D	B	0
D^*	A	B	1

* Construct a moore machine that takes binary no. as I/P & that binary is completely divisible by 3.

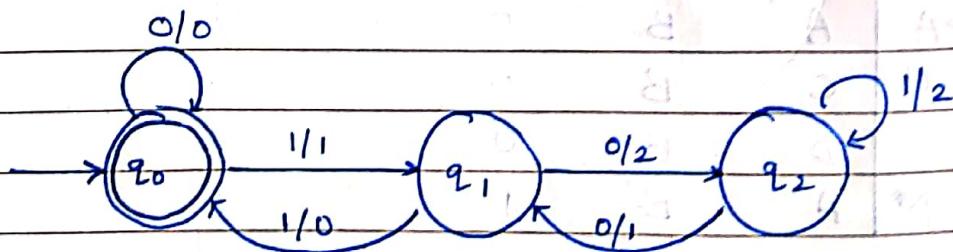
Ans:- $\Sigma = \{ 0, 1 \}$

$\Delta = \{ 0, 1, 2 \}$



δ / Σ	0	1	Δ
$\rightarrow * q_0$	q_0	q_1	0
q_1	q_{02}	q_0	1
q_2	q_1	q_2	0

→ Convert given moore machine into mealy machine



q₀ ²	0	0
q₀	(q ₀ , 0)	(q ₁ , 1)
q ₁	(q ₂ , 2)	(q ₀ , 0)
q ₂	(q ₁ , 1)	(q ₂ , 2)

* Construct delay machine that takes binary no. as i/p & produces 2's complement of that binary no. as o/p.

Assume the string is read from LSB to MSB & end carry will be discarded.

$$\Sigma = \{0, 1\}$$

$$\Delta = \{0, 1\}$$

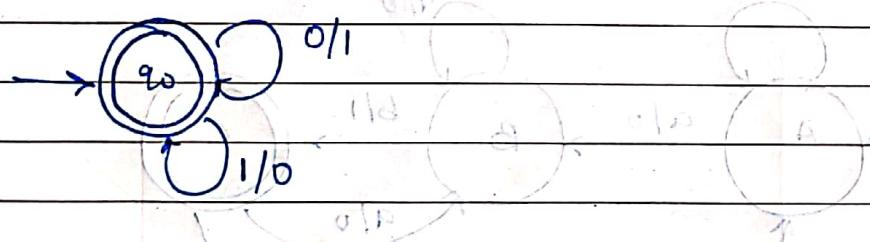
11101100

00010011

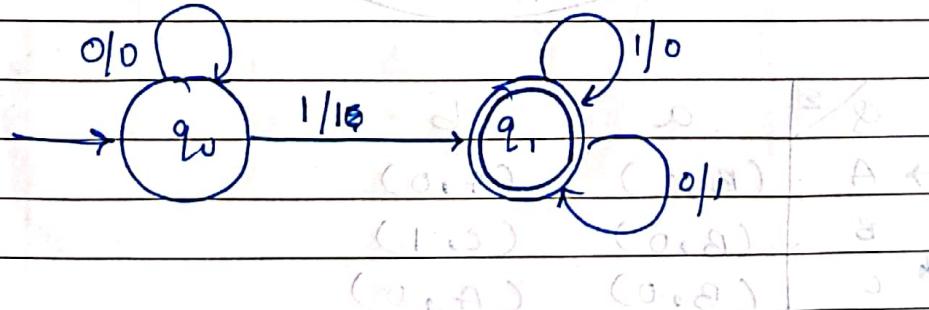
0001

00010100

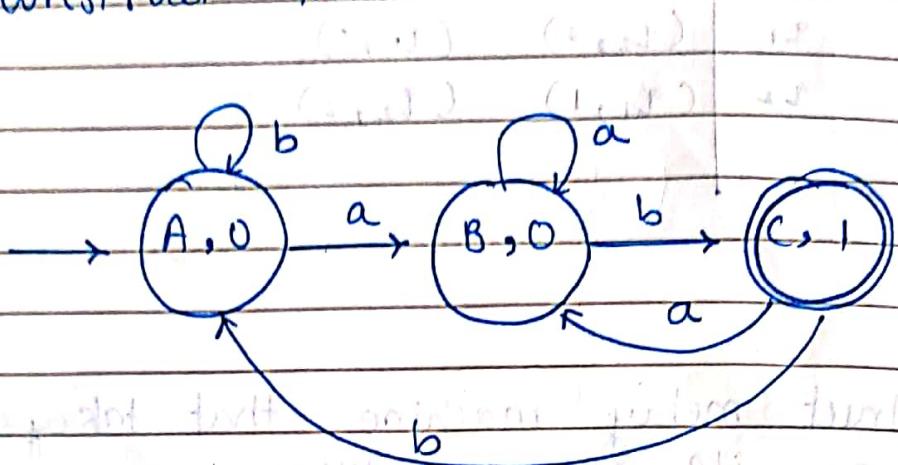
1's complement



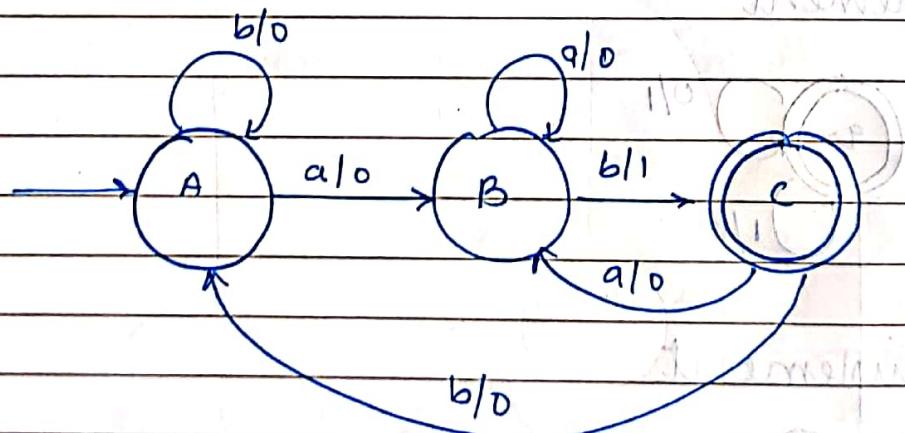
2's complement



* Convert following moore machine into mealy
& construct the table as well

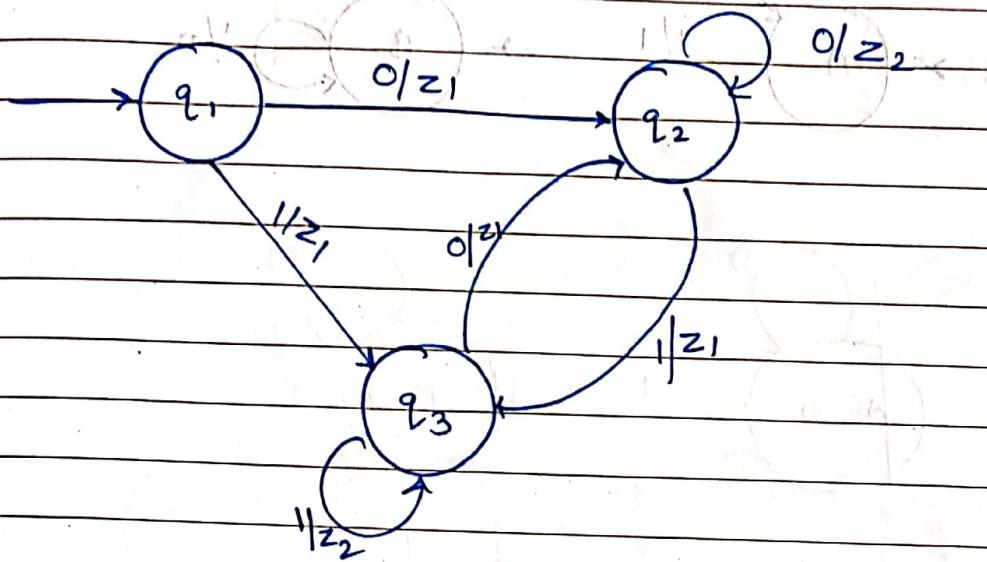


\varnothing/ϵ	a	b	c
$\rightarrow A$	B	C	A
B	B	C	B
* C	B	A	C

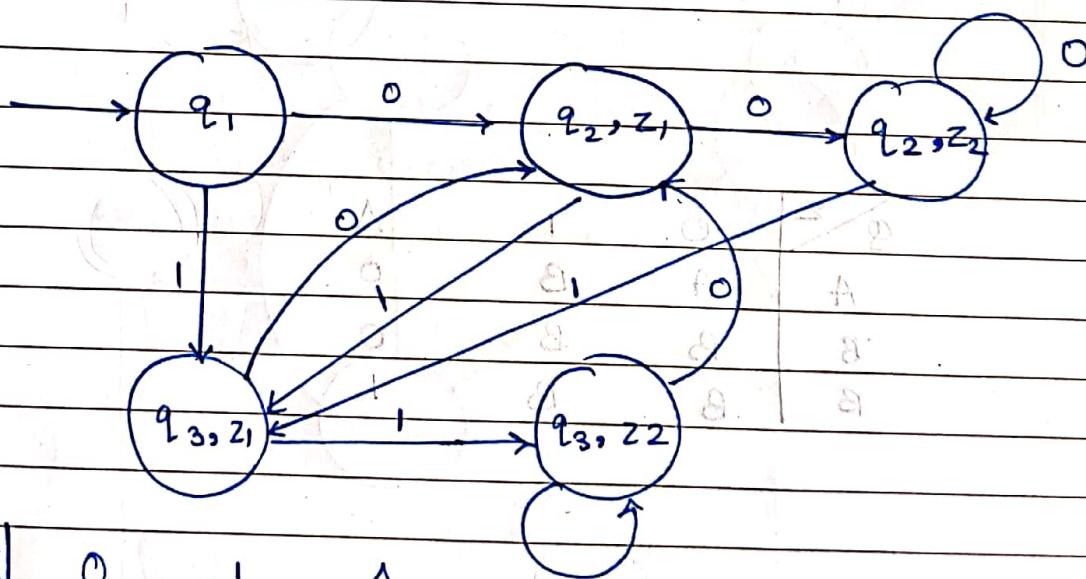


\varnothing/ϵ	a	b	c
$\rightarrow A$	(B, 0)	(A, 0)	
B	(B, 0)	(C, 1)	
* C	(B, 0)	(A, 0)	

* Convert given mealy machine to moore machine & also construct graph or transition table for both machines.

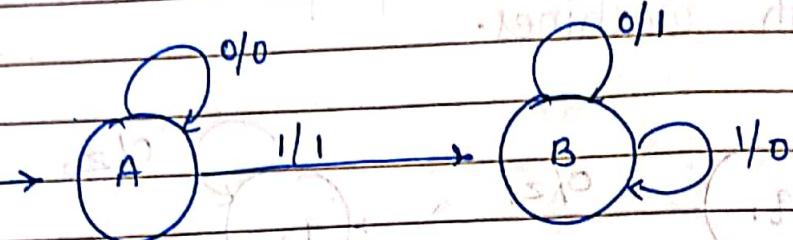


Ans:-

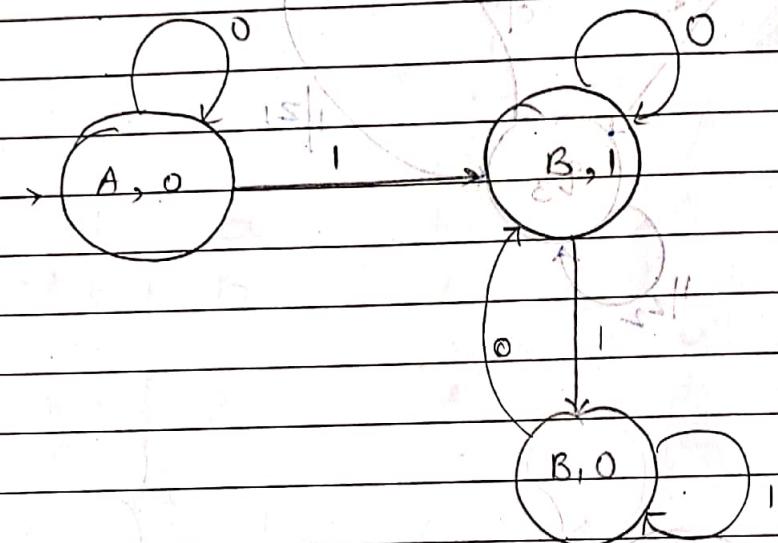


$q \in \Sigma$	0	1	Δ
q_1	q_2	q_3	-
q_2	q_2	q_3	z_1
q_2	q_2	q_3	z_2
q_3	q_2	q_3	z_1
q_3	q_2	q_3	z_2

* Mealy to moore machine.



Ans!-

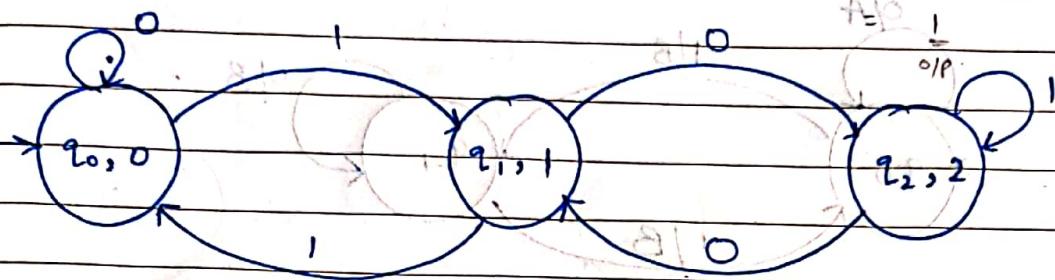


δ / Σ	0	1	Δ
A	A	B	0
B	B	B	0
B	B	B	1

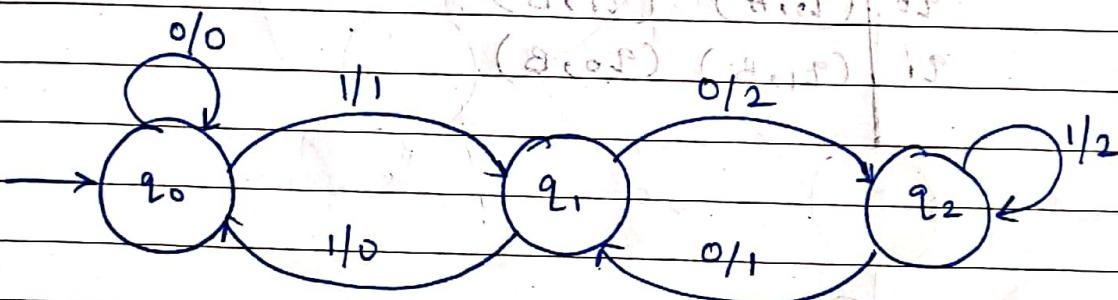
IMP note formation
in exam.

$$\lambda'(q, a) = \lambda(\delta(q, a))$$

* Moore to mealy \rightarrow ^{eg} $\lambda(q_0, 1) = \lambda(\delta(q_0, 1)) = \lambda(q_1)$



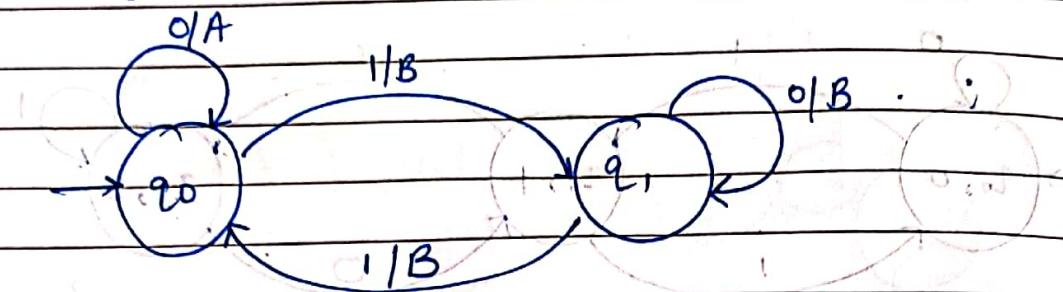
Ans-



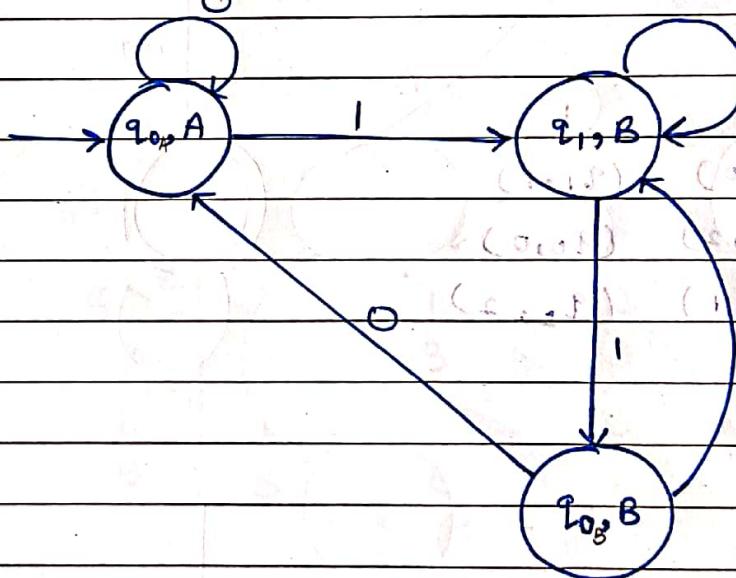
q	Σ	0	1
q_0	$(q_0, 0)$	$\rightarrow (q_1, 1)$	
q_1	$(q_2, 2)$	$\rightarrow (q_0, 0)$	
q_2	$(q_1, 1)$	$\rightarrow (q_2, 2)$	

A	B	C	D
A	1/0	0/0	0/0
B	1/0	0/0	0/0
C	0/0	1/0	1/0

* Mealy to Moore.

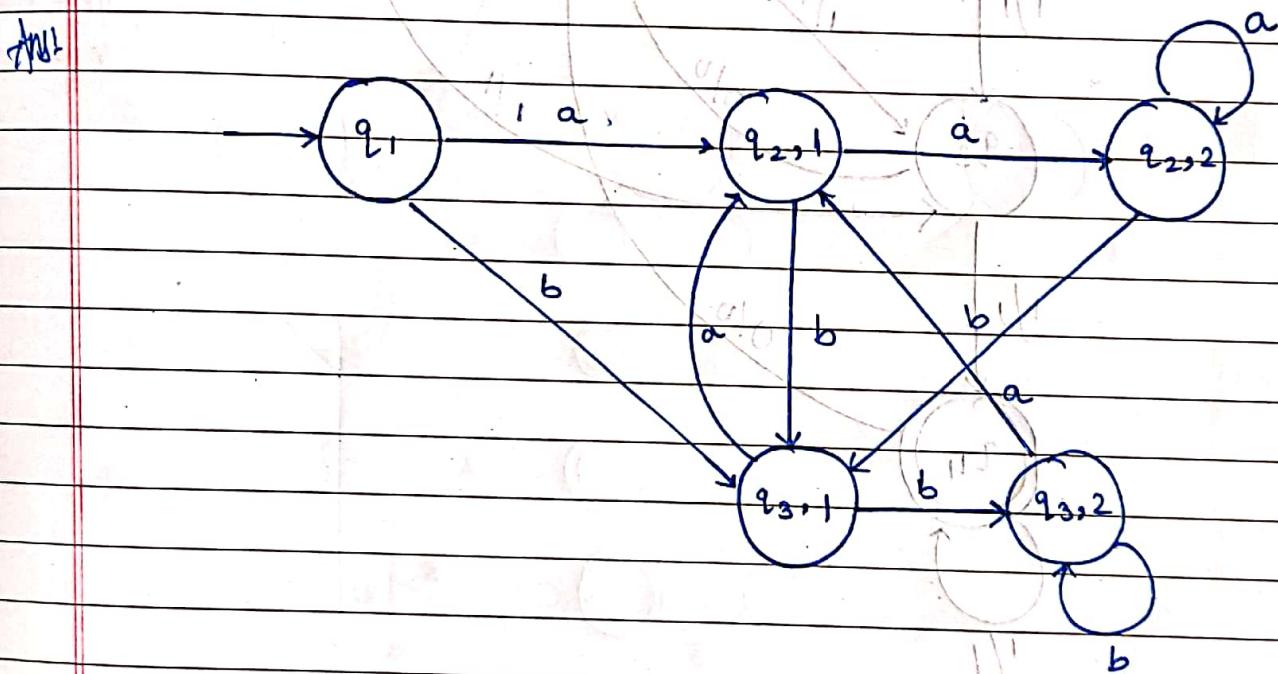
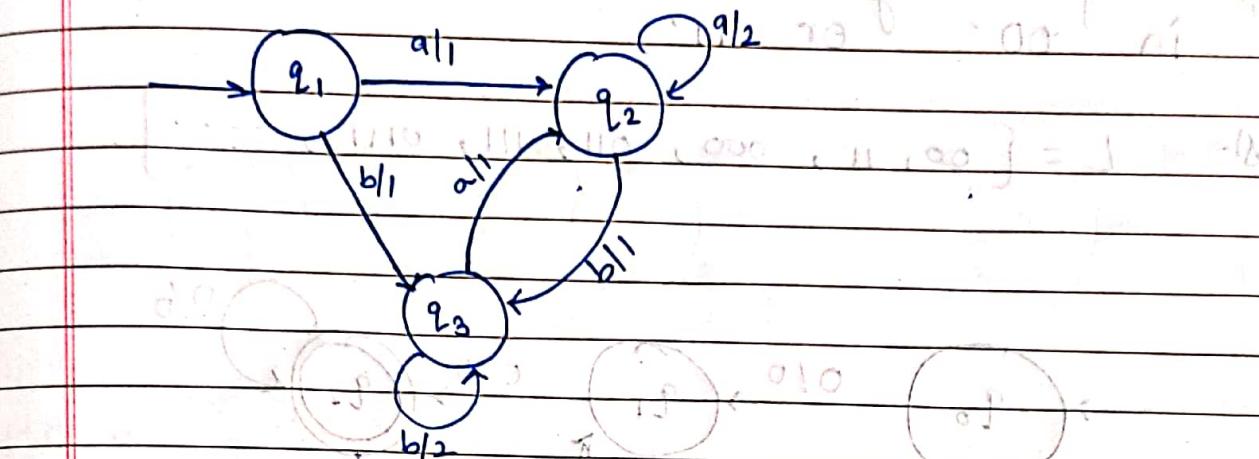


δ / ϵ	0	1
q_0	(q_0, A)	(q_1, B)
q_1	(q_1, B)	(q_0, B)



δ / ϵ	0	1	Δ
q_{0A}	q_{0A}	q_1	A
q_{0B}	q_{0A}	q_1	B
q_1	q_1	q_{0B}	B

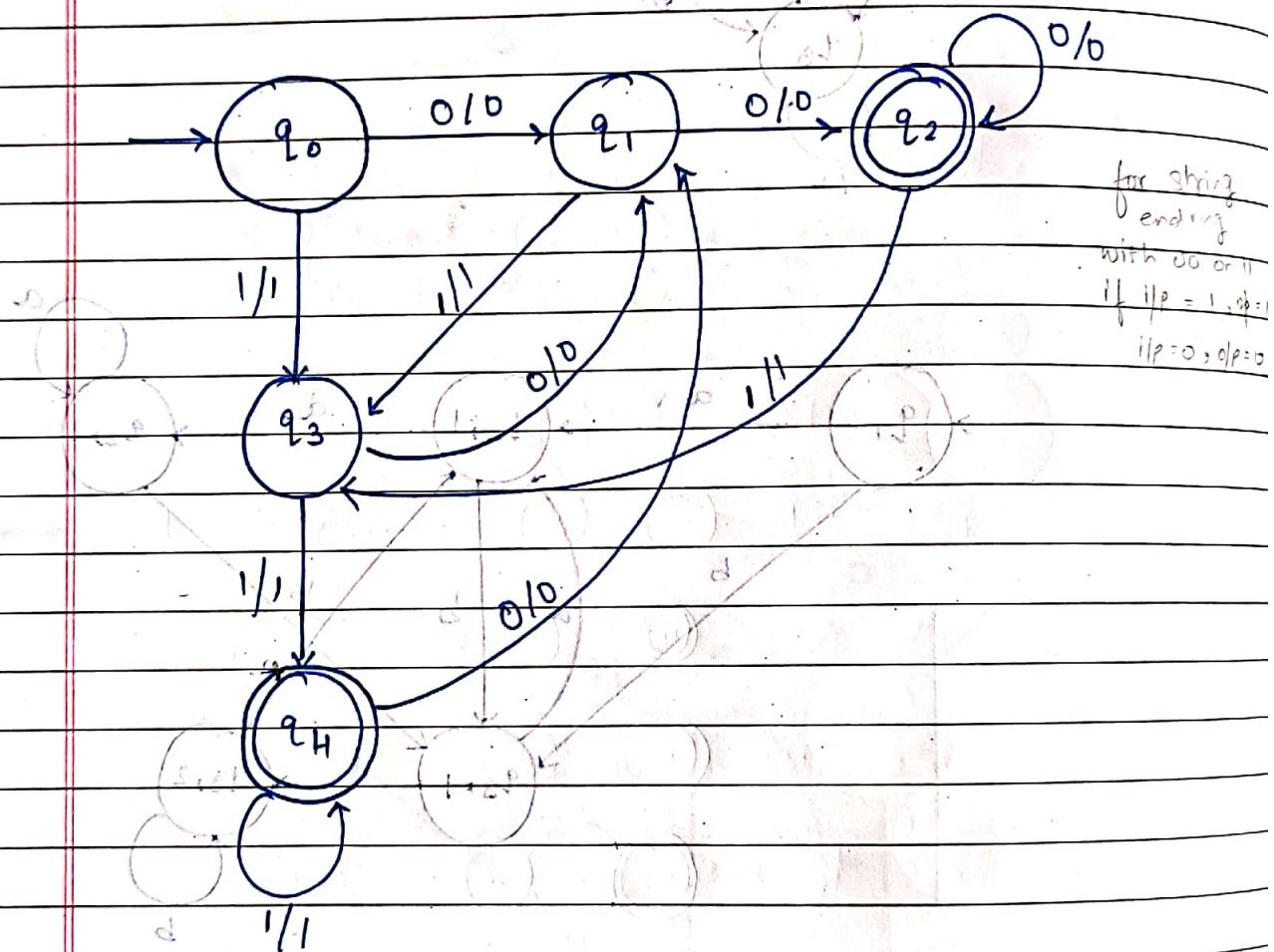
* Convert



State	Input a	Input b
(q1)	(q2,1)	(q3,1)
(q2,1)	(q2,2)	(q1)
(q2,2)	(q2,1)	(q1)
(q3,1)	(q2,1)	(q1)
(q3,2)	(q2,1)	(q1)

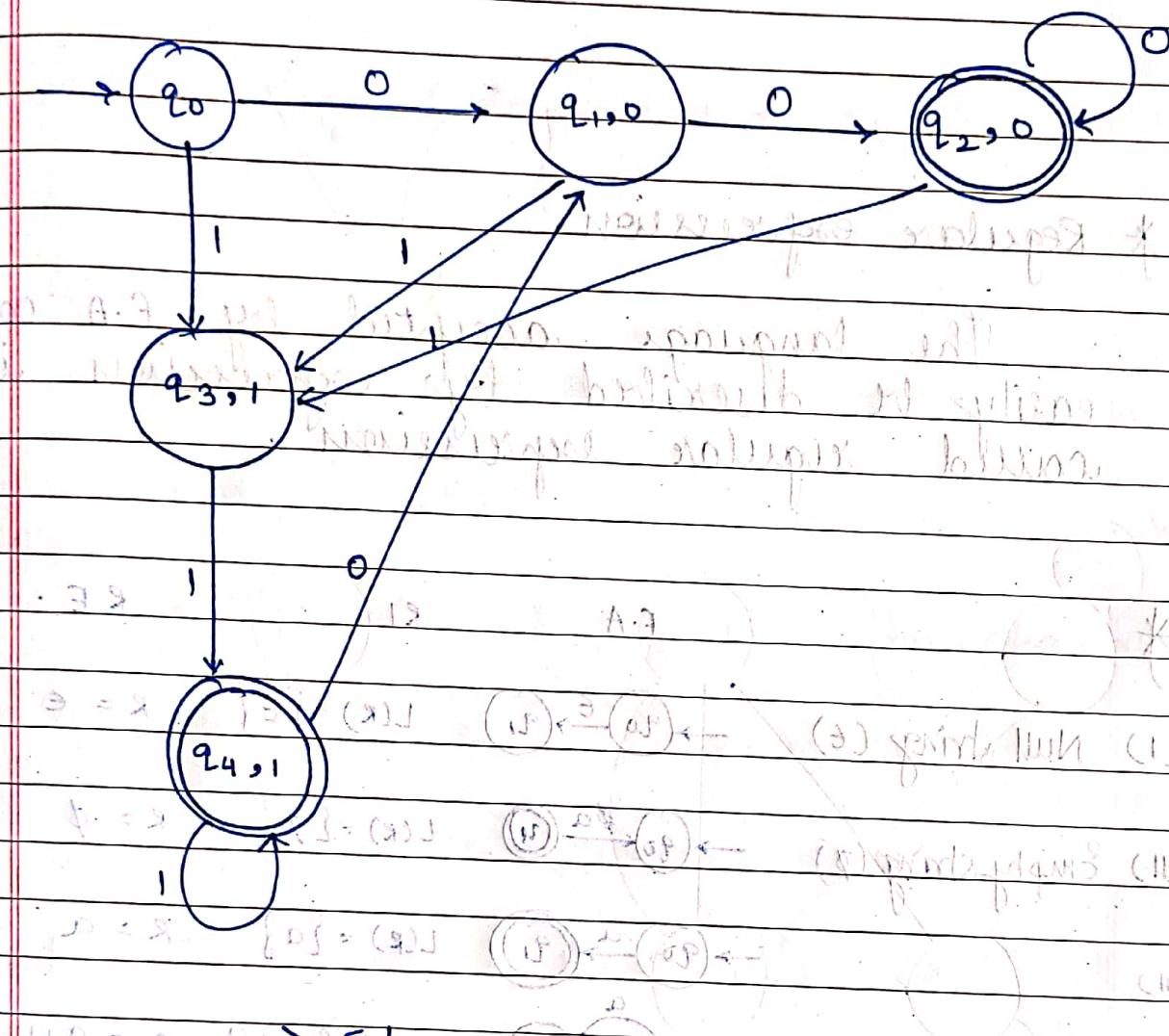
* Design mealy machine that accepts string ending in '00' or '11'

Ans- $L = \{00, 11, 000, 011, 111, 0111, \dots\}$



δ	Σ	0	1
$\rightarrow q_0$		$(q_1, 0)$	$(q_3, 1)$
q_1		$(q_2, 0)$	$(q_3, 1)$
$* q_2$		$(q_2, 0)$	$(q_3, 1)$
q_3		$(q_1, 0)$	$(q_4, 1)$
$* q_4$		$(q_1, 0)$	$(q_4, 1)$

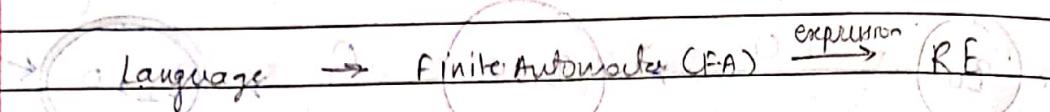
→ moore machine



$d = 2$	$\{q_0\} \Sigma$	q_0	q_1	q_2	q_3	Δ	$(+)$ initial	(V)
$\rightarrow q_0$		q_0	q_1	q_2	q_3	-		
q_1		q_2	q_3	0				
$d = 2$	$\{q_2\}$	q_2	q_3	0				
q_3		q_1	q_2	q_3	1			
$d = 2$	$\{q_3\}$	q_1	q_2	q_3	-1			

UNIT 11

Regular Expression (RE)



* Regular expression

The language accepted by F.A. can easily be described by expression is called regular expression.

* F.A RL RE

(I) Null string (ϵ) $\xrightarrow{q_0 \xrightarrow{\epsilon} q_1}$ $L(R) = \{\epsilon\}$ $R = \epsilon$

(II) Empty string (\emptyset) $\xrightarrow{q_0 \xleftarrow{\#a} q_1}$ $L(R) = \{\emptyset\}$ $R = \emptyset$

(III) $\xrightarrow{q_0 \xrightarrow{a} q_1}$ $L(R) = \{a\}$ $R = a$

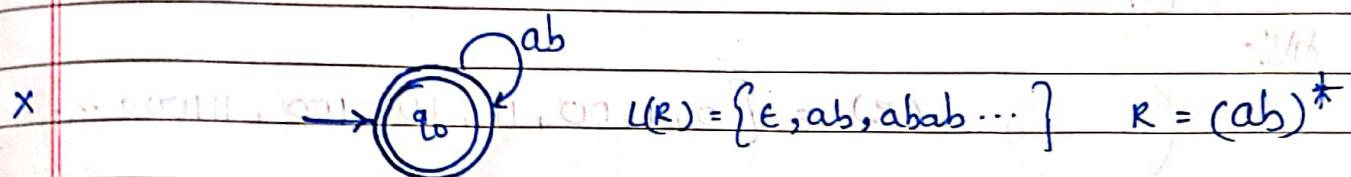
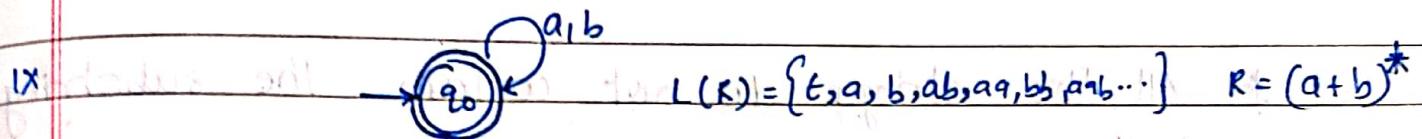
(IV) Union/OR (+) $\xrightarrow{q_0 \xrightarrow{a} q_1 \quad | \quad q_0 \xrightarrow{b} q_1}$ $L(R) = \{a, b\}$ $R = a + b$

(V) Concatenation (\cdot) $\xrightarrow{q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2}$ $L(R) = \{ab\}$ $R = ab$

(VI) closure (*) $\xrightarrow{q_0 \xrightarrow{a} q_0}$ $L(R) = \{\epsilon, a, aa, \dots\}$ $R = a^*$

(VII) +ve closure $\xrightarrow{q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_0}$ $L(R) = \{a, aa, aaa, \dots\}$ $R = aa^* \text{ or } a^*$

(VIII) $\xrightarrow{q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_3 \xrightarrow{a} q_2 \xrightarrow{b} q_0}$ $L(R) = \{ab, ba\}$ $R = ab + ba$



Q* Write R.E. for the language accepting all strings that can take any no. of 'a' + 'b'

Ans: $R.E = (a+b)^*$

$$L(R) = \{\epsilon, a, b, ab, ba, aa, bb, aba, aabb, \dots\}$$

* Starting with 'a' & ending with 'b'

Ans: $L(R) = \{10, 110, 100, 10100010, \dots\}$

$$R.E = ((1+0)^*)^0, 01, 10$$

* Starting with 'a' but not having consecutive 'b'

Ans: $L(R) = \{a, ab, aba, aab, aaba, aaa, \dots\}$

$$R.E = (a+ab)^*$$

$$\text{to } (a+1)^* \text{ to } = 32$$

* All the strings do not contain the substring "01"

Ans:-

$$L(R) = \{0, 1, 00, 11, 10, 100, 111000 \dots\}$$

$$R.E = 1^* 0^*$$

* At least 2 consecutive '0'

Ans:- $L(R) = \{aa, aab, baa, aab, babaaba \dots\}$

$$R.E = (a+b)^* aa(a+b)^*$$

inverzor
Q3a - * The set of strings over alphabet $\{0, 1\}$ that have at least one '01'.

$$L(R) = \{1, 10, 01, 110, 11, 011 \dots\}$$

$$R.E = (1+0)^* 1 (1+0)^*$$

* Atmost one '01'

$$L = \{\epsilon, 0, 1, 00, 01, 10, 0001, 1000\}$$

$$RE = 0^* (1+\epsilon) 0^*$$

* ending with '00' & begining with '1'

$$R.E = 1(0+1)^* 00$$