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# Event Detection Using Sensor Networks

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*Abstract*— **This paper investigates the use of a sensor network for detecting the presence of an event. The sensors monitor the signal emitted from the source and report the existence of the event when the received signal strength is above a certain threshold. In this paper we derive analytical expressions for the probability of false alarms and the probability of no detection as functions of the threshold. Subsequently, we determine the optimal threshold that trades off the probability of false alarms and the probability of no detection.**

## I. INTRODUCTION

This paper investigates the use of a wireless sensor network (WSN) for detecting the presence of an event source that releases a certain signal or substance in the environment which is then propagated over a large area. The concentration of the substance at the source location is assumed unknown. The sensor nodes are able to measure the substance concentration at their own locations but the measurements are noisy. Based on these concentration readings the sensor nodes have to decide whether they detected the event or not. The proposed sensor network can deal with a number of environmental monitoring and tracking applications including acoustic source localization, toxic source identification, early detection of fires and so on [1], [2], [3].

The motivation of this problem stems from the general problem of estimating the location of an event given the sensor measurements. In source location estimation, it is often beneficial to simply ignore measurements that are below a certain threshold. Applying a threshold significantly improves the accuracy of the location estimate [4]. Furthermore in the context of wireless sensor networks it conserves energy since only a fraction of the sensors will report to the sink, therefore, less power consumption. The natural question that arises then is what threshold to use. Choosing the "right" threshold is an optimization problem involving the probability of false alarms and the probability of miss: The sensor nodes are continuously monitoring the environment and their measurements are highly uncertain so there will be situations where they will be triggered by just noise. To avoid these highly undesirable situations of false alarms the threshold has to be large enough to minimize their probability of occurrence. On the other hand we do not want to compromise the possibility of an event going completely undetected so the threshold needs to be small enough to maximize the detection probability of our sensor nodes.

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The main contribution of this paper is the analytical evaluation of the probability of false alarms and the probability of no detection (event miss) in a wireless sensor network in terms of the local detection threshold used by each sensor node. The determination of this threshold is then solved as an optimization problem that minimizes a cost function involving the overall probability of error (either due to missed events or false alarms).

The paper is organized as follows. First, in Section II, we look at related work in sensor networks and distributed detection. In Section III, we present the model we have adopted and the underlying assumptions. Then in Section IV, we present the cost function we seek to minimize and the analytical evaluation of the probability of false alarms and the probability of no detection. In Section V, we present several simulation results. We conclude with Section VI where we also present plans for future work.

## II. RELATED WORK

Distributed detection using multiple sensors and optimal fusion rules has been extensively investigated for radar, sonar applications ([5], [6] and references therein). The objective in most studies is to develop computationally efficient algorithms at the sensors and at the fusion center. Optimality is usually studied under the Neyman-Pearson and Bayesian detection criteria [7], [8]. Both of these formulations however, require complete or partial knowledge of the joint densities (pdf) of the observations at the sensor nodes given the hypothesis. For independent identically distributed (iid) observations the joint pdf is easily derived as the product of the marginal pdf. In large-scale wireless sensor networks however, the signal generated by the event to be detected has unknown strength and varies spatially making sensor observations location-dependent and not identically distributed. Without the conditional independence assumption there is no guarantee that optimal decision rules can be derived in terms of thresholds for the likelihood ratio because the optimal solution is mathematically intractable (NP-hard) for sensor numbers exceeding 2 [9]. Consequently one needs to resort to suboptimal schemes and heuristics to achieve the desired objectives and the optimal decision rule for detection should be determined at the sensor node level sometimes even before deployment [10]. Following this line of reasoning our goal is to find a threshold for local detection at the sensor level that minimizes the total probability of error.

Most of the recent research in distributed detection using sensor networks obtains asymptotic results as the number of measurements at each sensor goes to infinity [11] or as the number of sensors goes to infinity [10]. We believe that neither of the two assumptions is realistic in the sensor networks context as we experience them today. Having unlimited measurements at each sensor may never be possible given the severe energy constraints and so far the numbers of sensor nodes used in the testbeds and applications around the world do not justify the infinity assumption. Our work is most similar to [12] where the authors address the problem of finding the critical density of sensors for complete coverage. They also use a threshold but they assume this is given and they concentrate on finding the number of sensors for detecting a moving target. They do not provide any analytical evaluation of the probability of detection and the probability of false alarms in a sensor network. At this point it is worth pointing out that given a threshold our results could also be used to obtain the number of sensors required to achieve a certain probability of detection in a randomly deployed sensor network.

## III. Simulation Model

For the sensor network that detects an event we are going to make the following assumptions:

1) A set of $N$ sensor nodes is uniformly spread over a rectangular field $A$. The nodes are assumed stationary. The position of each node is denoted by $(x_i, y_i)$, $i = 1, \cdots, N$ and it is assumed that it is known through the use of a combination of GPS and localization algorithms.
2) The source of the event is located at a position $(x_s, y_s)$ which is also uniformly distributed inside the smaller area $B \subset A$ as shown in Figure 1.
3) The propagation of the signal from the source is uniform in all directions and there are no environmental changes throughout the propagation.

Assumption 1 is quite common and reasonable for sensor networks. Assumption 2 is rather technical. It is used to avoid the boundary effects and thus simplify the calculation of the area of the circles around the source. We point out that for large areas the probability of having a source on the boundary is very small and thus the effect of this assumption is negligible. Assumption 3 defines a propagation model that may be accurate for sources that emit sound or electromagnetic waves, but it is not very accurate for problems where an actual substance is released in the environment (for example in problems of environmental pollution).

For this paper, we assume that the measured intensity at the source is $c$ and as we move away from the source, the measured intensity is inversely proportional to the distance from the source raised to some power $\alpha \in \mathcal{A} \subset \mathbb{R}$ which depends on the environment. As a result, the measured

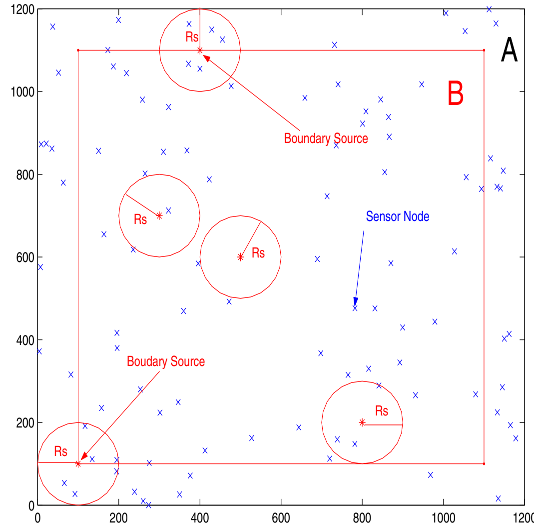


Fig. 1. A field with 100 randomly placed sensor nodes and 5 sources

intensity at any sensor $i$ is given by (1).

$$z_{i,t} = \frac{c}{r_i^{\alpha}} + w_{i,t}, \quad (1)$$

$i = 1, \cdots, N$, $t = 1, \cdots, M$. Measurement $z_{i,t}$ is the $t$-th sample at sensor $i$ while $r_i$ is the radial distance from the source, i.e.,

$$r_i = \sqrt{(x_i - x_s)^2 + (y_i - y_s)^2}$$

Finally, $w_{i,t}$ is additive Gaussian noise with zero mean and variance $\sigma_i^2$. At this point it is worth pointing out that (1) assumes that sensor measurements are uncorrelated with each other. Though this is a fairly simplistic assumption, it is possible to extend the results for scenarios where the sensor measurements are correlated and the correlation depends on the distance between the sensors.

The sensor nodes are assumed to be in an energy-conserve state until triggered by the presence of the signal. When a sensor node detects something, i.e., it receives a measurement $z_{i,t} > T$, where $T$ is a threshold, it wakes up and takes a number of discrete measurements $M$ over a time interval and takes the mean of these measurements $\bar{z}_i$:

$$\overline{z_i} = \frac{1}{M} \sum_{t=1}^{M} z_{i,t} \quad (2)$$

Then, it compares this value to the threshold $T$ to decide whether to communicate this information to the sink and continue measuring or go back to sleep. This threshold will be the same for all sensor nodes in the field and will work in a distributed fashion in that each sensor node will use this threshold to decide whether it has detected the event or not. The mean is a sufficient statistic of the sensor data and is the uniformly most powerful test (UMP) in the Neyman-Pearson formulation for a single sensor in composite hypothesis one-sided detection [7].

Taking the mean before communicating the information to the sink is also justified as a way of reducing the amount of data flowing in the sensor network and saving both bandwidth and energy and therefore prolonging the lifetime of the network. Moreover it is shown in [4] that there is no loss in accuracy when it is used to estimate the source position compared to the case that every sensor measurement is used instead. Finally we assume a centralized approach where sensor readings from the alarmed sensors are gathered at the sink using a communication paradigm like directed diffusion [13]. The overall detection of the event will then be decided at the sink if at least one sensor node reports detection.

## IV. Analytical evaluation of Threshold

To solve for the optimal threshold we seek to minimize the following cost function $J$ representing the overall error in detection as a function of the threshold $T$:

$$J(T) = w \times P_{fa}(T) + (1-w) \times P_{nd}(T) \tag{3}$$

where $P_{fa}$ is the probability of false alarms, $P_{nd}$ is the probability of no detection and $0 \leq w \leq 1$ is a user specified weight that should be chosen with care according to the application. A small $w$ implies that the application can tolerate more false alarms but it cannot tolerate any missed events. For example, in networks that monitor for toxic terrorist attacks in crowded areas, $w$ could be set close to zero. On the other hand, larger values of $w$ imply that some missed events may be tolerated to reduce the cost of false alarms. For example, in applications such as environmental monitoring of large areas, false alarms may incur a significant cost because a response crew may have to travel to the suspected area. Also, in many cases, frequent false alarms may make the users simply ignore *all* alarms and as a result important events may go undetected. Finally we point out that alternatively one could also formulate the problem as a constrained optimization problem. For example, minimize the probability of false alarm subject to the probability of no detection being less than some value.

*Definition 1:* Let $H_0$ represent the noise-only hypothesis and $H_1$ represent the signal-present detection hypothesis using the statistical hypothesis testing formulation [7].

Below we go into the details of evaluating the probability of false alarms and the probability of no detection as a function of the threshold $T$.

### A. Probability of false alarms

In the absence of a source the sensors are measuring just noise which is Gaussian with distribution $N(0, \sigma_i^2)$. Therefore from (2) the distribution of the sample mean at each sensor node is also Gaussian with distribution $\overline{z_i} \sim N(0, \frac{\sigma_i^2}{M})$. The probability of false alarms is the probability that at least one of the sensors mistakenly reports the presence of a source and is given by the following equation:

$$P_{fa}(T) = 1 - \prod_{i=1}^{N} \Phi\left(\frac{T \times \sqrt{M}}{\sigma_i}\right) \tag{4}$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{y^2}{2}\right) dy$ is the probability the Standard Normal Gaussian random variable ($N(0,1)$) is less than $x$ and can be calculated from tables or using Matlab.

The derivation is outlined below:

$$\begin{aligned} P_{fa}(T) &= \Pr\{H_1 | H_0\} \\ &= \Pr\{(\overline{z_i} \geq T) \text{ for at least one sensor } i\} \\ &= 1 - \Pr\{(\overline{z_i} < T) \forall \text{ sensors } i\text{=1,...,N}\} \\ &= 1 - \prod_{i=1}^{N} \Pr\{\overline{z_i} < T\} \\ &= 1 - \prod_{i=1}^{N} \Phi\left(\frac{T \times \sqrt{M}}{\sigma_i}\right) \end{aligned}$$

where the last equality follows from the fact that noise samples are assumed to be uncorrelated between the sensor nodes and therefore are independent [14].

### B. Probability of no detection

In Fig. 2 we see a graphical interpretation of the probability of no detection in a randomly created field of 100 sensor nodes. Each circle denotes the detection area of each sensor node as defined by the threshold in the absence of noise and the shaded area represents the locations where an event would go completely undetected. The sensor measurements have a Gaussian distribution $\overline{z_i} \sim N(\frac{c}{r_i^{\alpha}}, \frac{\sigma_i^2}{M})$ and they are spatially correlated based on the sensor positions and the distance from the source.

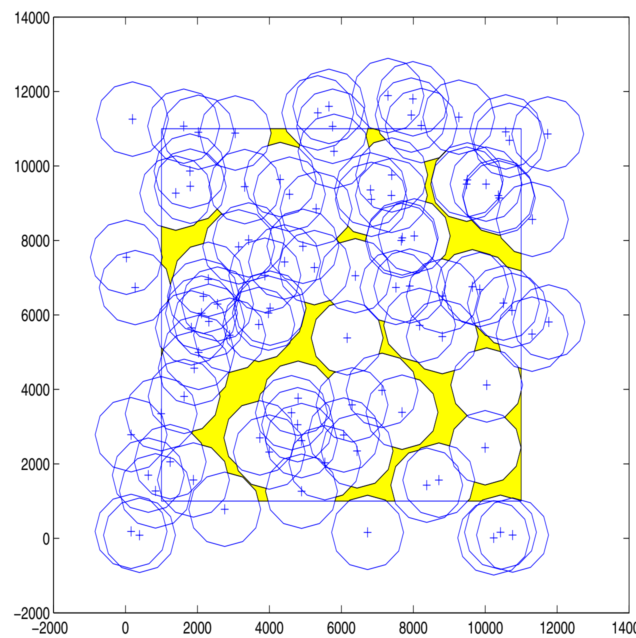


Fig. 2. Graphical interpretation of Probability of no detection

*Definition 2:* Let $R_s(T)$ denote the radius of a disk centered at the source defined by the prescribed threshold $T$ and the source attenuation model (see Fig. 1).

For the model given in (1) $R_s$ is given by

$$R_s(T) = \sqrt[\alpha]{\frac{c}{T}}. \quad (5)$$

*Definition 3:* Let $S_i$ denote the event that sensor node $i$ falls inside the disk of radius $R_s(T)$ centered at the source (see Fig. 1).

Using independence, the probability of no detection is given by

$$P_{nd}(T) = \prod_{i=1}^{N}(1 - P_{D_i}) \quad (6)$$

where $P_{D_i}$ is the probability of detection by sensor $i$ and is computed by conditioning on the event $S_i$:

$$P_{D_i} = \Pr\{H_1|S_i\} \times \Pr\{S_i\} + \Pr\{H_1|\overline{S_i}\} \times \Pr\{\overline{S_i}\} \quad (7)$$

where $\Pr\{\overline{S_i}\} = 1 - \Pr\{S_i\}$. Now using assumption 2,

$$\Pr\{S_i\} = \frac{\pi R_s^2}{A} \quad (8)$$

since the sensor nodes are uniformly distributed so all points in area $A$ are equally probable [14]. $\Pr\{H_1|\overline{S_i}\}$ is the probability that sensor $i$ detects the source given that the source is outside its coverage area. For values of $\alpha$ that are large enough, the signal measurement ($\frac{c}{r^\alpha}$) away from the source becomes zero and thus, this probability can be approximated with the probability of false alarm for sensor node $i$ which is given by (4). Finally we can derive $\Pr\{H_1|S_i\}$ by conditioning on the radial distance from the source. In the derivation below $R \in [0, R_s(T)]$ is a random variable representing the radial distance from the source with pdf $f_{R|S_i}(r|S_i) = \frac{2r}{R_s^2}$. This choice of pdf makes all points in the disk around the source equally probable.

$$\begin{aligned}\Pr\{H_1|S_i\} &= \int_0^{R_s(T)} \Pr\{H_1|S_i, R = r\} f_{R|S_i}(r|S_i)dr \\ &= \int_0^{R_s(T)} \left[1 - \Phi\left(\left(T - \frac{c}{r^\alpha}\right)\frac{\sqrt{M}}{\sigma_i}\right)\right]\frac{2r}{R_s^2}dr \quad (9)\end{aligned}$$

Substituting (9),(8),(4) and (5) in (7) yields $P_{D_i}$. Then by substituting this in (6) we get the probability of no detection as a function of the threshold $T$. It is worth pointing out that in high SNR situations $\Pr\{H_1|S_i\} = 1$ so the probability of detection $P_{D_i}$ is given by the following equation:

$$P_{D_i} = \Pr\{S_i\} + (1 - \Pr\{S_i\})\, P_{fa_i} \quad (10)$$

The $P_{nd}(T)$ requires knowledge of $c$ the amplitude at the source which is usually unknown. Since this parameter is needed in advance to determine the optimal threshold for detection we decided to use the smallest value that would cause an alarm for the specific substance we aim to detect. This ensures that our optimal threshold $T_{opt}$ will be small enough to detect concentrations equal or greater than $c$. This is also justified in composite hypothesis testing where it is shown that the probability of detection increases with increasing the unknown parameter we aim to detect [7].

## V. RESULTS

For all subsequent experiments we used a square sensor field of $1km \times 1km$ and assume that the sensor measurements were given by:

$$z_{i,t} = \frac{10^6}{r_i^2} + w_{i,t} \quad (11)$$

where $i = 1, \cdots, N$, $t = 1, \cdots, M$, $r_i^2 = (x_s - x_i)^2 + (y_s - y_i)^2$ and $w_{i,t} = N(0, \sigma^2)$, $\forall i$, $\forall t$. The experimental results reported were obtained by taking the average over 100 randomly created sensor fields. For obtaining the experimental probability of false alarms ($P_{fa}$) the sensor nodes were simply exposed to noise and we counted the times that at least one sensor node reported the presence of an event source. For obtaining the probability of no detection ($P_{nd}$) we randomly placed a source in each sensor field 100 times and counted the number of times that no sensor node reported the presence of the source. For all the experiments we used the Matlab package.

### A. Probability of false alarms

In the first set of experiments we investigated the validity of (4) derived in section IV. In Fig. 3 we plot the analytical and experimental $P_{fa}$ vs. the threshold $T$ for different numbers of sensor fields- $N$ = 1, 10 and 100. In Fig. 4 we do the same but this time we use a fixed sensor field with 100 sensor nodes and investigate the effect of changing the noise variance- $\sigma^2$ =1,10,100. It is evident from the 2 plots that our experimental results are very well in agreement with the analytical ones for all situations tested proving the validity of (4).

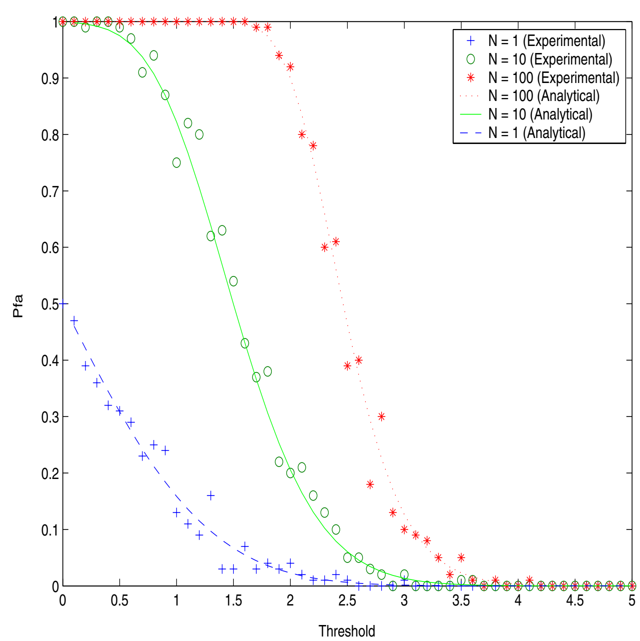


Fig. 3. Probability of false alarms vs. threshold for different numbers of sensor nodes

As it can be observed from Fig. 3, $P_{fa}$ increases with the number of sensors involved. This is expected because

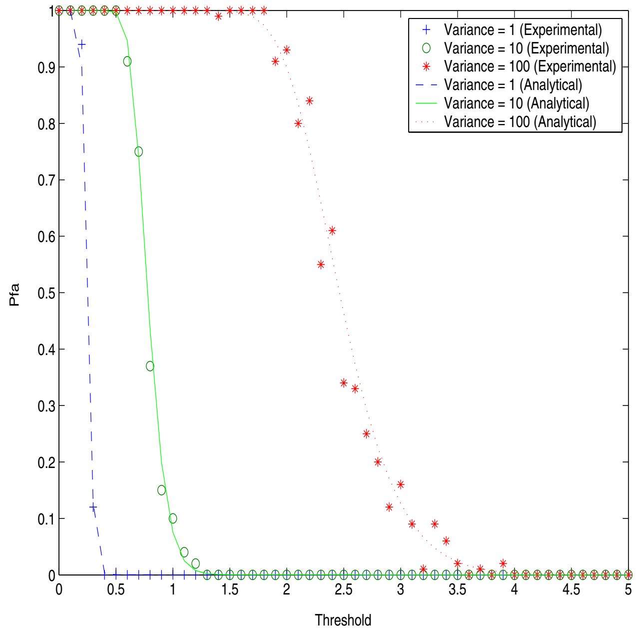


Fig. 4. Probability of false alarms vs. threshold for different noise variance conditions

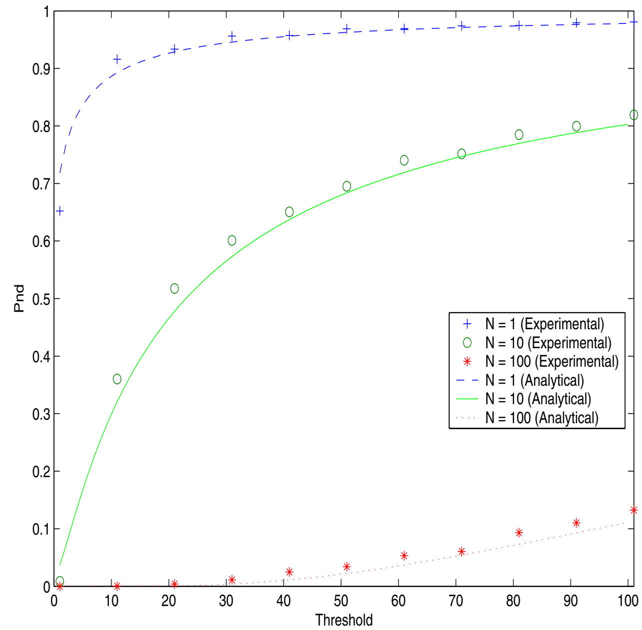


Fig. 5. Probability of no detection vs. threshold for different numbers of sensor nodes

the more sensors we have the more likely it becomes for one of the sensors to mistakenly report the presence of an event. Also from Fig. 4, it is evident that $P_{fa}$ increases with noise variance. This is also expected because noise is what causes the false alarms in the first place. In both of the above situations we would need to increase the threshold to minimize the probability of false alarms. Another way to compensate for the effects of noise would be to increase the number of measurements $M$ before averaging because as it can be seen from (4) this acts as a counterweight to noise variance $\sigma_i^2$.

### B. Probability of no detection

In the second set of experiments we investigated the validity of (6) derived in section IV. In Fig. 5 we plot the analytical and experimental $P_{nd}$ vs. the threshold $T$ for different numbers of sensor fields- $N$ = 1, 10 and 100. In Fig. 6 we do the same but this time we use a fixed sensor field with 100 sensor nodes and investigate the effect of changing the initial concentration at the source $c$. It is evident from the 2 plots that our experimental results are very well in agreement with the analytical ones for all situations tested proving the validity of (6). The small deviation in the order of $2\%$ observed in the plots can be attributed to not having enough samples in the experiments to achieve truly uniform distribution of the sensor nodes.

As it can be observed from Fig. 5, the probability of no detection depends largely on the number of sensor nodes in the sensor field. This is expected because the more sensor nodes we have the better coverage we achieve. Noise variance on the other hand does not affect the $P_{nd}$. Even under very noisy conditions our sensor nodes have a big probability of detecting a source if they are located in the source neighborhood. Increasing the threshold $T$ increases the $P_{nd}$ in all cases.

In section IV we made the claim that even if we do not know the initial concentration $c$ we can use the minimum concentration we wish to detect in our optimization problem. In Fig. 6 it is evident that by doing so we will pick a threshold that will allow us to detect all larger concentrations as well. For example let's say that we wanted to detect a minimum $c = 500,000$ with a $P_{nd} \simeq 0.1$. From the plot we would choose $T = 40$ for this problem. This threshold value allows us to detect $c = 1,000,000$ with $P_{nd} \simeq 0.01$ and anything above $c = 2,000,000$ with $P_{nd} \simeq 0$.

## VI. CONCLUSIONS AND FUTURE WORK

We investigate a sensor network that monitors for the presence of an event. The network uses a threshold at the sensor level to decide for the existence of the event. We obtain the optimal threshold that minimizes the error involving the probability of false alarms and the probability of no detection. The correctness of the derived equations is shown through simulations of different test case scenario.

For our future work we plan to investigate the possibility of each sensor node adjusting its threshold dynamically to accommodate "holes" in detection caused by the initial random deployment as well as node failures. Another topic of future research will be fusion rules to determine the presence of a source based on the number of alarmed sensors. We would like this to work in a distributed fashion where a cluster of sensor nodes makes the decision and reports detection of the source with a single message. In addition, we also plan to investigate the possibility of having multiple simultaneous events in the sensor field.

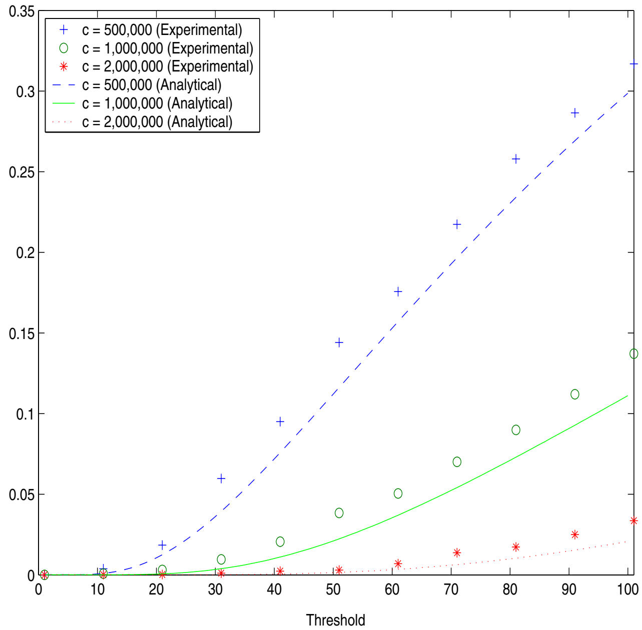


Fig. 6. Probability of no detection vs. threshold for different $c$ values

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