

$$\text{big\_morph} \quad (\forall x y, f(x + y) = f(x) + f(y)) \rightarrow f(0) = \hat{0} \rightarrow f\left(\sum_{P(i)}^{i \leftarrow r} F(i)\right) = \sum_{P(i)}^{i \leftarrow r} f(F(i))$$

### Section Extensionality

$$\begin{aligned} \text{eq\_bigl} \quad & P_1 =_1 P_2 \rightarrow \sum_{P_1(i)}^{i \leftarrow r} F(i) = \sum_{P_2(i)}^{i \leftarrow r} F(i) \\ \text{eq\_bigr} \quad & (\forall i, P(i) \rightarrow F_1(i) = F_2(i)) \rightarrow \sum_{P(i)}^{i \leftarrow r} F_1(i) = \sum_{P(i)}^{i \leftarrow r} F_2(i) \\ \text{big\_nil} \quad & \sum_{P(i)}^{i \leftarrow \emptyset} F(i) = 0 \\ \text{big\_pred0} \quad & P =_1 \text{xpred0} \rightarrow \sum_{P(i)}^{i \leftarrow r} F(i) = 0 \\ \text{big\_pred1} \quad & P =_1 \text{pred1}(i) \rightarrow \prod_{P(j)}^i F(j) = G(i) \\ \text{big\_ord0} \quad & \sum_{P(i)}^{i < 0} F(i) = 0 \\ \text{big\_tnth} \quad & \sum_{P(i)}^{i \leftarrow r} F(i) = \sum_{P(r_i)}^{i < \text{size}(r)} F(r_i) \\ \text{big\_nat\_recl} \quad & m \leq n \rightarrow \sum_{m \leq i < n+1} F(i) = F(m) + \sum_{m \leq i < n} F(i+1) \\ \text{big\_ord\_recl} \quad & \sum_{i < n+1} F(i) = F(\text{ord0}) + \sum_{i < n} F(\text{lift}((n+1), \text{ord0}, i)) \\ \text{big\_const\_ord} \quad & \sum_{i < n} x = \text{iter}(n, (\lambda y. x + y), 0) \end{aligned}$$

### Section MonoidProperties

$$\text{big\_nat\_recr} \quad m \leq n \rightarrow \prod_{m \leq i < n+1} F(i) = \prod_{i < n} F(i) \times F(n)$$

### Section Abelian

$$\begin{aligned} \text{big\_split} \quad & \prod_{R(i)}^{i \leftarrow r} (F_1(i) \times F_2(i)) = \prod_{R(i)}^{i \leftarrow r} F_1(i) \times \prod_{R(i)}^{i \leftarrow r} F_2(i) \\ \text{bigU} \quad & A \cap B = \emptyset \rightarrow \prod_{i \in A \cup B} F(i) = (\prod_{i \in A} F(i)) \times (\prod_{i \in B} F(i)) \\ \text{partition\_big} \quad & (\forall i, P(i) \rightarrow Q(p(i))) \rightarrow \prod_{P(i)}^i F(i) = \prod_{Q(j)}^j \prod_{P(i) \text{ p}(i)=j}^i F(i) \\ \text{reindex\_onto} \quad & (\forall i, P(i) \rightarrow h(h'(i)) = i) \rightarrow \prod_{P(i)}^i F(i) = \prod_{P(h(j))}^j \prod_{h'(h(j))=j}^i F(h(j)) \\ \text{pair\_big} \quad & \prod_{P(i)}^i \prod_{Q(j)}^j F(i, j) = \prod_{P(p) Q(q)}^{(p, q)} F(p, q) \\ \text{exchange\_big} \quad & \prod_{P(i)}^{i \leftarrow r I} \prod_{Q(j)}^{j \leftarrow r J} F(i, j) = \prod_{Q(j)}^{j \leftarrow r J} \prod_{P(i)}^{i \leftarrow r I} F(i, j) \end{aligned}$$

### Section Distributivity

$$\begin{aligned} \text{big\_distr l} \quad & \sum_{P(i)}^{i \leftarrow r} F(i) \times a = \sum_{P(i)}^{i \leftarrow r} (F(i) \times a) \\ \text{big\_distr r} \quad & a \times \sum_{P(i)}^{i \leftarrow r} F(i) = \sum_{P(i)}^{i \leftarrow r} (a \times F(i)) \\ \text{big\_distr\_big\_dep} \quad & \prod_{P(i)}^i \sum_{Q(i, j)}^j F(i, j) = \sum_{f \in \text{pfamily}(j_0, P, Q)} \prod_{P(i)}^i F(i, f(i)) \\ \text{big\_distr\_big} \quad & \prod_{P(i)}^i \sum_{Q(j)}^j F(i, j) = \sum_{f \in \text{pfun\_on}(j_0, P, Q)} \prod_{P(i)}^i F(i, f(i)) \\ \text{bigA\_distr\_big} \quad & \prod_i \sum_{Q(j)}^i F(i, f(i)) = \sum_{f \in \text{ffun\_on}(Q)} \prod_i F(i, f(i)) \\ \text{bigA\_distr\_bigA} \quad & \prod_{i \in I} \sum_{j \in J} F(i, j) = \sum_{f \in J^I} \prod_{i \in I} F(i, f(i)) \end{aligned}$$

$\text{pfamily}(j_0, P, Q) \simeq$   
functions  $Q^P$

$$\begin{aligned} \text{partition\_big\_imset} \quad & \sum_{i \in A} F(i) = \sum_{i \in h \text{ e}: A} \sum_{h(i)=j}^{i \in A} F(i) \\ \text{big\_trivIset} \quad & \text{trivIset}(P) \rightarrow \sum_{x \in \text{cover}(P)} E(x) = \sum_{A \in P} \sum_{x \in A} E(x) \\ \text{partition\_disjoint\_bigcup} \quad & (\forall i, j, i \neq j \rightarrow F(i) \cap F(j) = \emptyset) \rightarrow \sum_{x \in \bigcup_i F(i)} E(x) = \sum_i \sum_{x \in F(i)} E(x) \end{aligned}$$