Communication Systems
EE-351

Lectures 11 and 13

Problems:

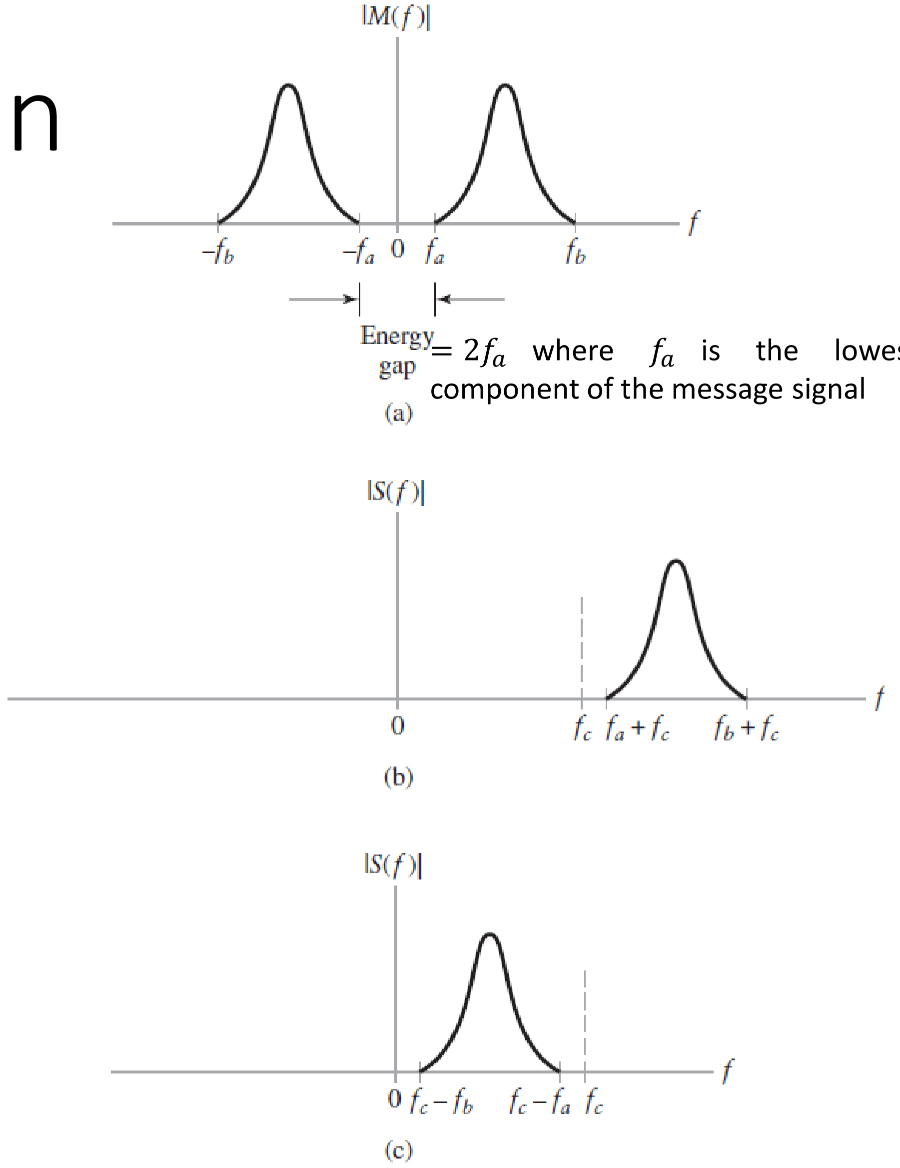
Consider an SSB signal when the modulating signal is $x(t)$. $\hat{x}(t)$ is the Hilbert transform of $x(t)$. What is the envelope of SSB?

Solution: $\sqrt{x^2(t) + \hat{x}^2(t)}$

A DSB-SC signal is generated using the carrier $\cos(\omega_c t + \theta)$ and modulating signal is $x(t)$. Find the envelope?

Solution: $|x(t)|$

Single Sideband Modulation (SSB):



$= 2f_a$ where f_a is the lowest frequency component of the message signal

FIGURE 3.18 (*a*) Spectrum of a message signal $m(t)$ with energy gap centered around zero frequency. Corresponding spectra of SSB-modulated waves using (*b*) upper sideband, and (*c*) lower sideband. In parts (*b*) and (*c*), the spectra are only shown for positive frequencies.

(a) For the upper SSB,

$$S(f) = \begin{cases} \frac{A_c}{2} M(f - f_c), & \text{for } \quad f \geq f_c \\ 0, & \text{for } 0 < f \leq f_c \end{cases}$$

(b) For the lower SSB,

$$S(f) = \begin{cases} 0, & \text{for } \quad f > f_c \\ \frac{A_c}{2} M(f - f_c), & \text{for } 0 < f \leq f_c \end{cases}$$

$$x(t) = m(t)\cos(2\pi F_c t) - \hat{m}(t)\sin(2\pi F_c t)$$

$$= \left[\frac{1}{2} M(F - F_c) + \frac{1}{2} M(F + F_c)\right] - [\frac{-j}{2}\widehat{M}(F - F_c) + \frac{j}{2}\widehat{M}(F + F_c)]$$

Modulators for SSB:

Frequency Discrimination Method:

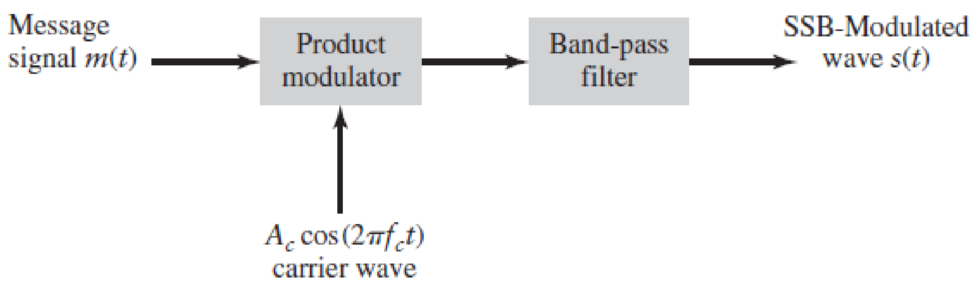


FIGURE 3.19 Frequency-discrimination scheme for the generation of a SSB modulated wave.

(a) For the upper SSB,

$$S(f) = \begin{cases} \frac{A_c}{2}M(f - f_c), & \text{for} \quad f \geq f_c \\ 0, & \text{for } 0 < f \leq f_c \end{cases}$$

(b) For the lower SSB,

$$S(f) = \begin{cases} 0, & \text{for} \quad f > f_c \\ \frac{A_c}{2}M(f - f_c), & \text{for } 0 < f \leq f_c \end{cases}$$

Phase Discrimination Method:

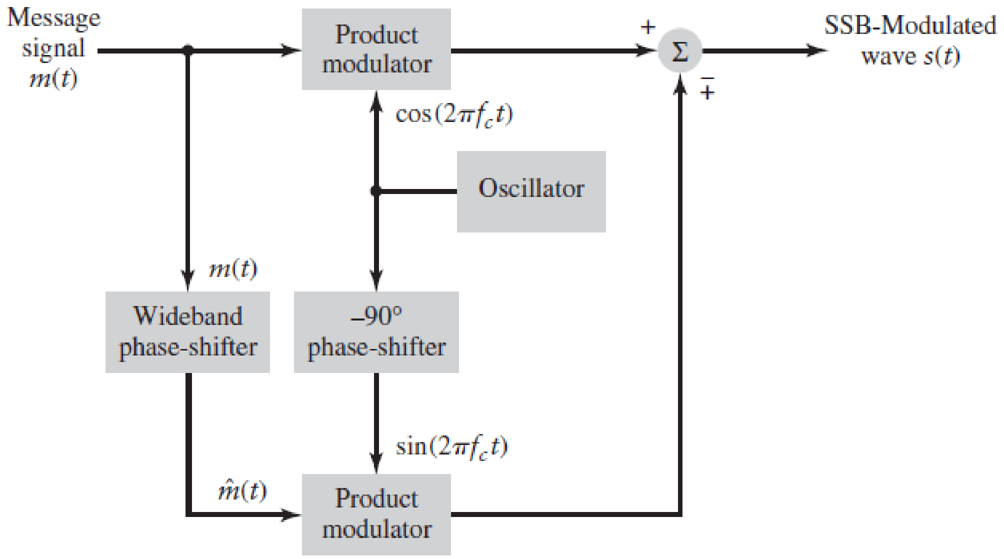


FIGURE 3.20 Phase discrimination method for generating a SSB-modulated wave. Note: The plus sign at the summing junction pertains to transmission of the lower sideband and the minus sign pertains to transmission of the upper sideband.

$$s(t) = \frac{A_c}{2}m(t)\cos(2\pi f_c t) \mp \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)$$

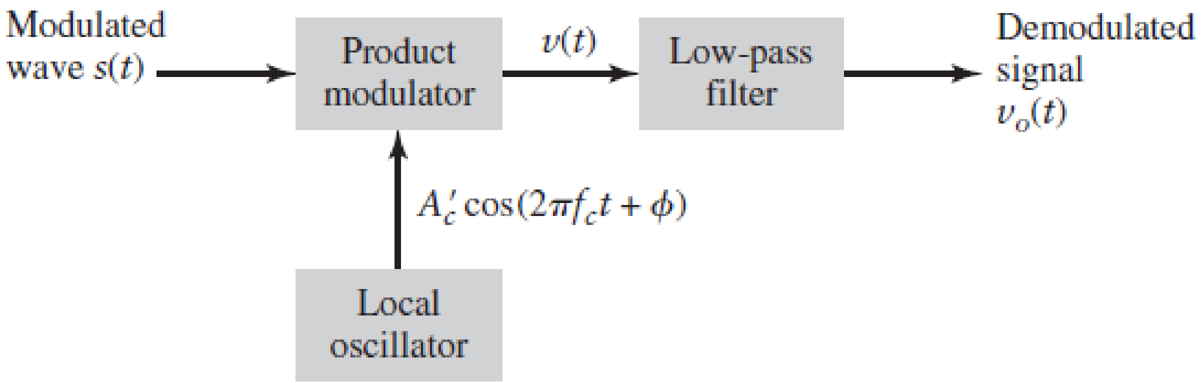
Modulators for SSB:

- **<u>Frequency Discrimination Method:</u>**
- Modulator consists of two components: product modulator followed by band-pass filter.
- Product modulator produces DSB-SC wave with two sidebands.
- BPF is design to transmit one of these bands.
- For the design of the band-pass filter to be practically feasible, there must be a certain separation between the two sidebands that is wide enough to accommodate the transition band of the band-pass filter.
- This requirement limits the applicability of SSB modulation to speech signals for which $f_a \approx 100\,Hz$ but rules it out for video signals and computer data whose spectral content extends down to almost zero frequency.
- BPF is the functional block.

- **<u>Phase Discrimination Method:</u>**
- This second SSB modulator consists of two parallel paths, one called the ***in-phase*** path and the other called the ***quadrature*** path.
- The role of the quadrature path embodying the wide-band phase shifter is merely to interfere with the in-phase path so as to eliminate power in one of the two sidebands, depending on whether upper SSB or lower SSB is the requirement.
- Wide-band phase-shifter is the functional block

Coherent Detection of SSB:

- In comparison to demodulation of DSB-SC (suppression of the carrier), the demodulation of SSB is further complicated by the additional suppression of the upper or lower sideband.
- The two sidebands share an important property: they are the ***images*** of each other with respect to the carrier.



- The coherent detector of Fig. 3.12 applies equally well to the demodulation of both DSB-SC and SSB;
- the only difference between these two applications is how the modulated wave is defined.

▶ **Drill Problem 3.13** Starting with Eq. (3.23) for a SSB modulated wave, show that the output produced by the coherent detector of Fig. 3.12 in response to this modulated wave is defined by

$$v_o(t) = \frac{A_c A_c'}{4} m(t)$$

Assume that the phase error $\phi = 0$ in Fig. 3.12. ◀

$$s(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) \mp \frac{A_c}{2} \widehat{m}(t) \sin 2\pi f_c t$$

$$r(t) = s(t).\, A_c \cos(2\pi F_c t)$$

$$= [\frac{A_c}{2} m(t) \cos(2\pi F_c t) \mp \frac{A_c}{2} \widehat{m}(t) \sin(2\pi F_c t)] \times A_c \cos(2\pi F_c t)$$

$$= \frac{{A_c}^2}{2} m(t) cos^2(2\pi F_c t) \mp \frac{{A_c}^2}{2} \widehat{m}(t) \sin(2\pi F_c t) \cos(2\pi F_c t)$$

$$= \frac{{A_c}^2}{4} m(t) + \frac{{A_c}^2 m(t) \cos(4\pi F_c t)}{4} \mp \frac{{A_c}^2 \widehat{m}(t) \sin(4\pi F_c t)}{4}$$

$$= \frac{{A_c}^2 m(t)}{4}$$

Repeat the problem by considering phase error:

$$s(t) \times A_c\cos(2\pi F_c t + \varphi)$$

$$= [\frac{A_c}{2} m(t)\cos(2\pi F_c t) \mp \frac{A_c}{2}\widehat{m}(t)\sin(2\pi F_c t)] \times A_c\cos(2\pi F_c t + \varphi)$$

$$cosAcosB = \frac{1}{2}(\cos(A+B) + \cos(A-B))$$

$$sinAcosB = \frac{1}{2}(\sin(A+B) + sin(A-B))$$

$$= \frac{{A_c}^2}{4} m(t)[\cos(4\pi F_c t + \varphi) + cos\varphi] \mp \frac{{A_c}^2\widehat{m}(t)[\sin(4\pi F_c t + \varphi) - sin\varphi]}{4}$$

$$= \frac{{A_c}^2}{4} m(t)cos\varphi \pm \frac{{A_c}^2}{4}\widehat{m}(t)sin\varphi$$

Frequency Translation:

- The basic operation performed in single sideband modulation is in fact a form of frequency translation, which is why single sideband modulation is sometimes referred to as *frequency changing*, *mixing*, or *heterodyning*.
- Suppose that we have a modulated wave $s_1(t)$ whose spectrum is centered on a carrier frequency f_1, and the requirement is to translate it upward or downward in frequency, such that the carrier frequency is changed from f_1 to a new value f_2.
- This requirement is accomplished by using a mixer (a functional block that consists of a product modulator followed by a band-pass filter).
- Conventional method for SSB modulator but with an important difference: the band-pass filter is now straightforward to design.

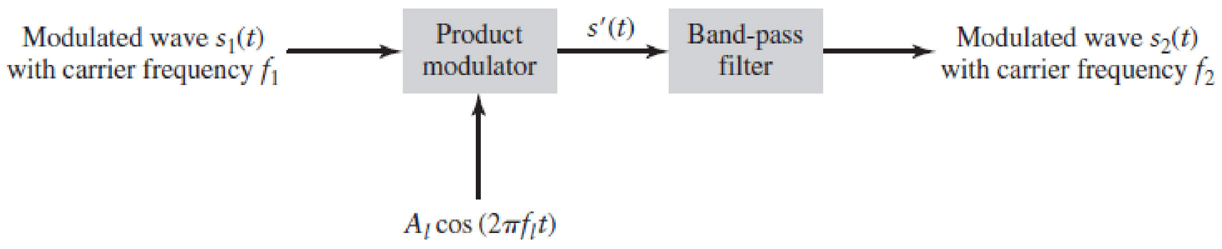


FIGURE 3.21 Block diagram of mixer.

Modulated signal with $f_c = f_1$

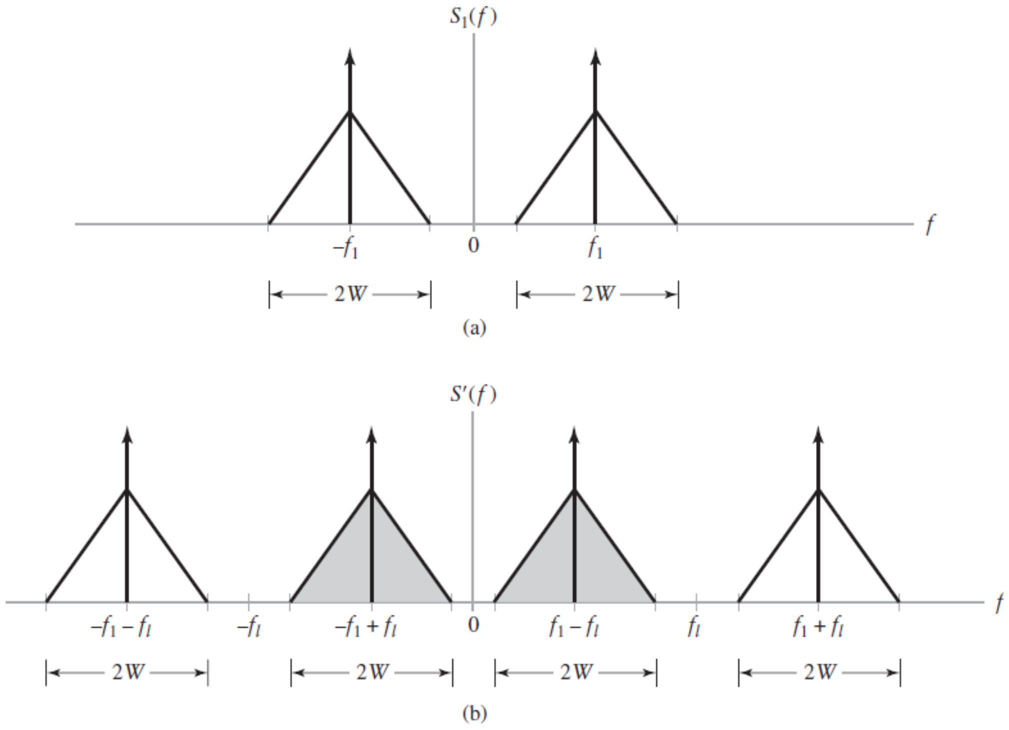


FIGURE 3.22 (*a*) Spectrum of modulated signal $s_1(t)$ at the mixer input. (*b*) Spectrum of the corresponding signal $s'(t)$ at the output of the product modulator in the mixer.

Frequency Translation:

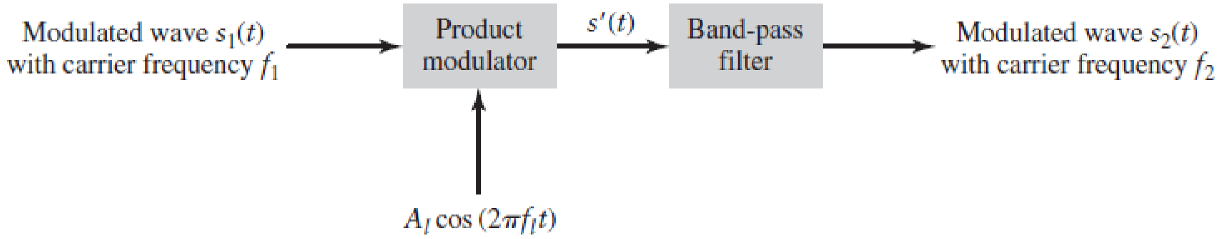


FIGURE 3.21 Block diagram of mixer.

- Two different situations can be identified, depending on the carrier frequency to be translated upward or downward.

Modulated signal with $f_c = f_1$

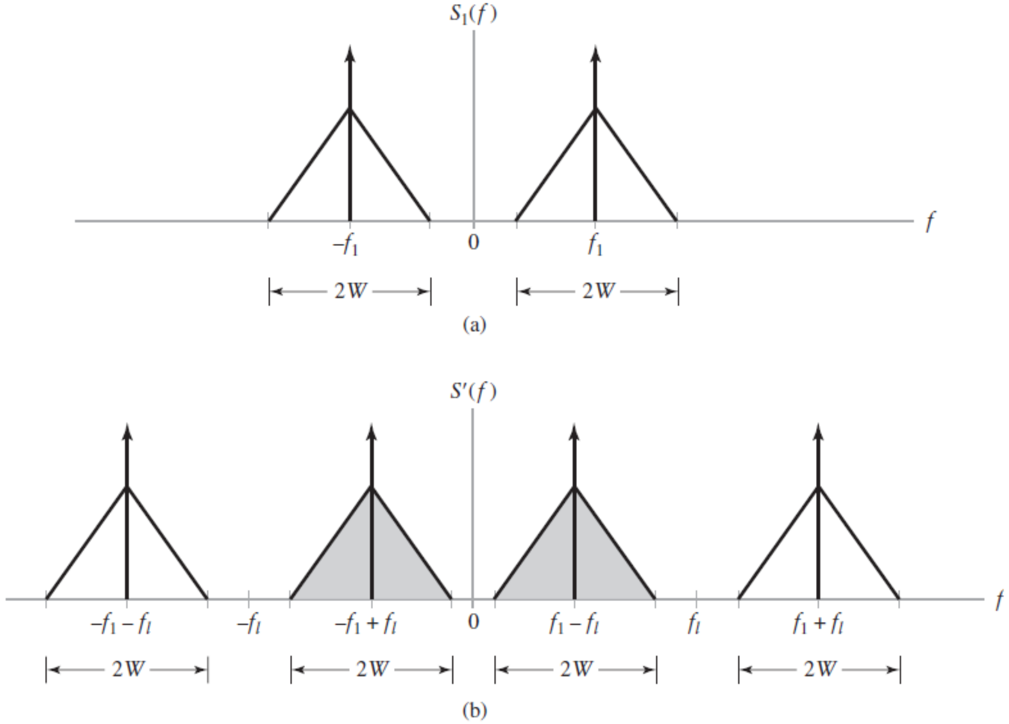


FIGURE 3.22 (*a*) Spectrum of modulated signal $s_1(t)$ at the mixer input. (*b*) Spectrum of the corresponding signal $s'(t)$ at the output of the product modulator in the mixer.

(i) *Up conversion*. In this form of mixing, the translated carrier frequency, denoted by f_2, is greater than the incoming carrier frequency f_1. The required local oscillator frequency f_l is therefore defined by

$$f_2 = f_1 + f_l$$

Solving for f_l, we therefore have

$$f_l = f_2 - f_1$$

In this situation, the unshaded part of the spectrum in Fig. 3.22(*b*) defines the up-converted signal $s_2(t)$, and the shaded part of this spectrum defines the *image signal* associated with $s_2(t)$, which is removed by the band-pass filter in Fig. 3.21. For obvious reasons, the mixer in this case is referred to as a *frequency-up converter*.

Frequency Translation:

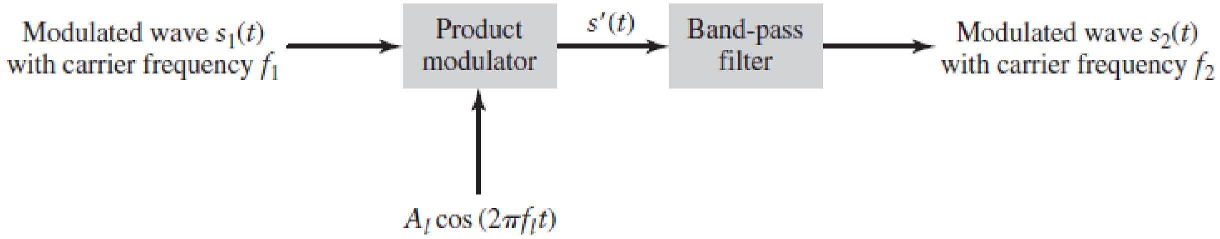


FIGURE 3.21 Block diagram of mixer.

Modulated signal with $f_c = f_1$

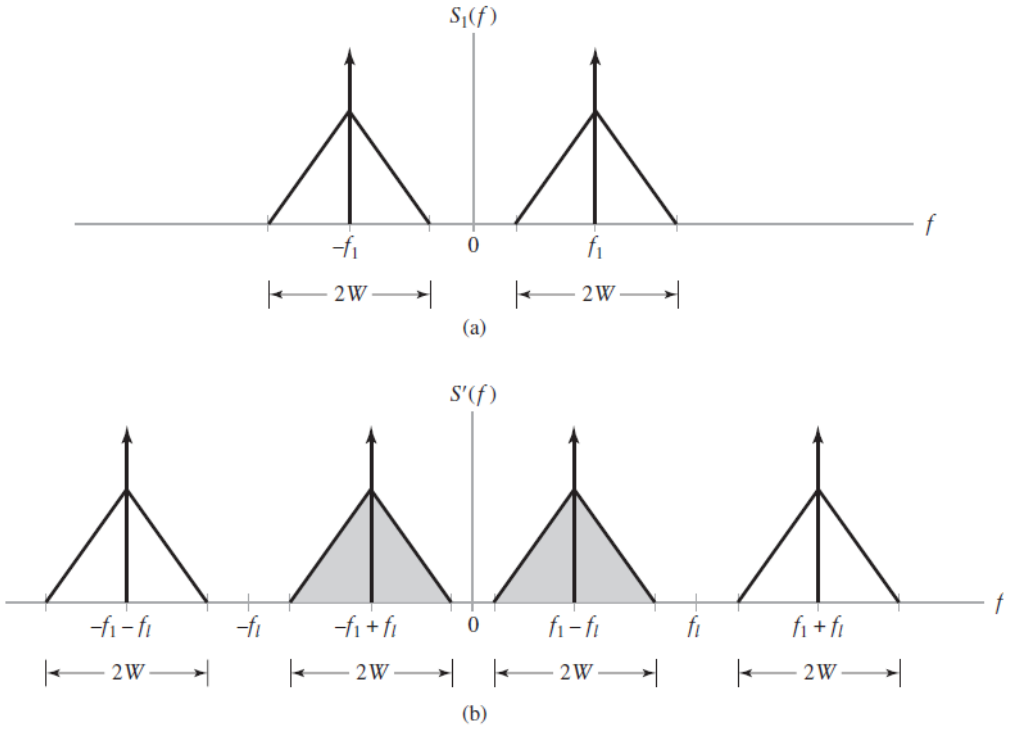


FIGURE 3.22 (*a*) Spectrum of modulated signal $s_1(t)$ at the mixer input. (*b*) Spectrum of the corresponding signal $s'(t)$ at the output of the product modulator in the mixer.

(ii) *Down conversion*. In this second form of mixing, the translated carrier frequency f_2 is smaller than the incoming carrier frequency f_1, as shown by

$$f_2 = f_1 - f_l$$

The required local oscillator frequency is therefore

$$f_l = f_1 - f_2$$

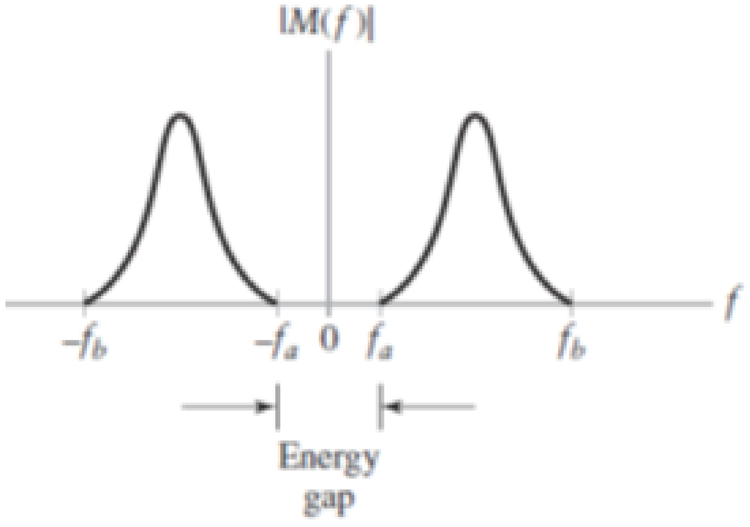
The shaded part of the spectrum defines the **down-converted signal**, and the unshaded part of this spectrum defines the associated image signal. This second mixer is referred to as a **frequency-down converter**.

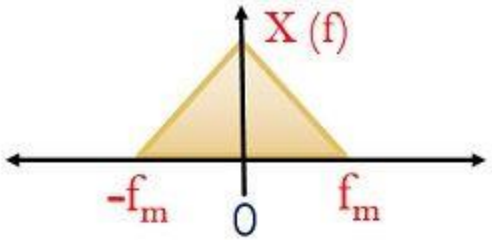
In order to avoid sideband overlap, what should be the carrier frequency, f_2 ?

$f_2 > W$

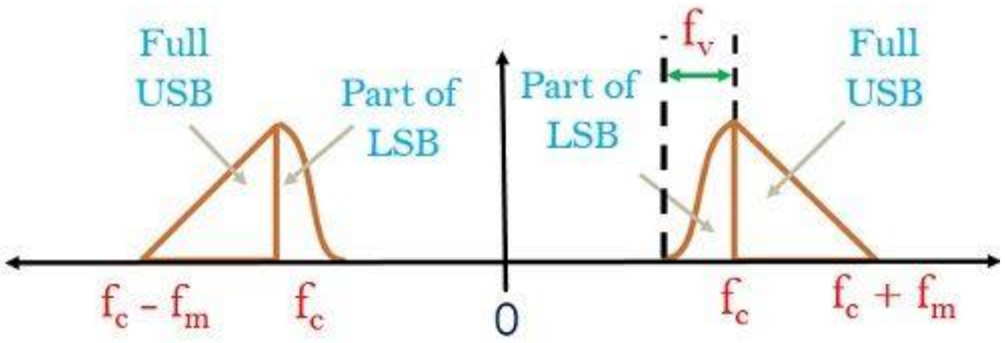
Vestigial Sideband (VSB) Modulation:

- Single-sideband modulation works satisfactorily for an information-bearing signal (e.g., speech signal) with an energy gap centered around zero frequency.
- For the spectrally efficient transmission of wideband signals, we have to look to a new method of modulation for two reasons:
  - Typically, the spectra of wideband signals (exemplified by television video signals and computer data) contain significant low frequencies, which make it impractical to use SSB modulation.
  - The spectral characteristics of wideband data befit the use of DSB-SC. However, DSB-SC requires a transmission bandwidth equal to twice the message bandwidth, which violates the bandwidth conservation requirement.

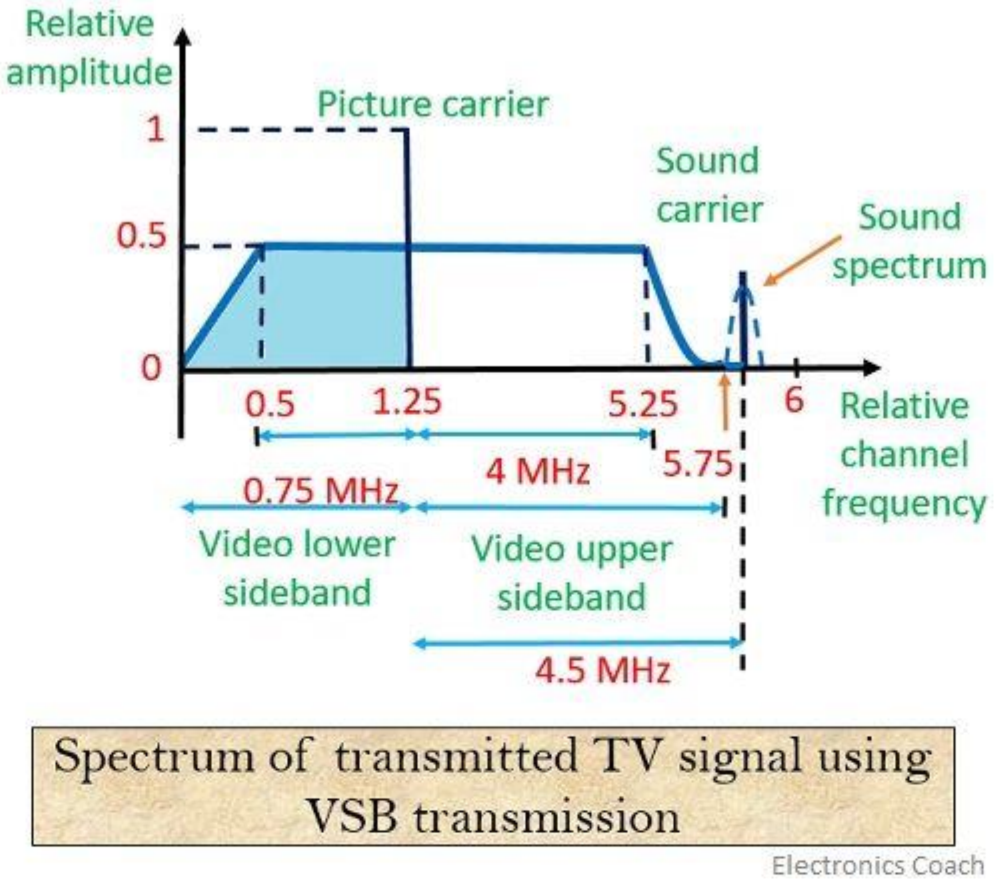




Spectrum of message signal



Spectrum of VSB signal



Spectrum of transmitted TV signal using VSB transmission

Source: Internet

Vestigial Sideband (VSB) Modulation:

- A scheme that lies somewhere between SSB and DSB-SC in its spectral characteristics.
- Vestigial sideband (VSB) modulation distinguishes itself from SSB modulation in two practical respects:
  - Instead of completely removing a sideband, a trace or *vestige* of that sideband is transmitted; hence, the name "vestigial sideband."
  - Instead of transmitting the other sideband in full, *almost* the whole of this second band is also transmitted.
- Accordingly, the transmission bandwidth of a VSB modulated signal is defined by

$$B_T = f_v + W$$

Where f_v is the vestige bandwidth and W is the message bandwidth.

Typically, f_v is 25 percent of W, which means that the VSB bandwidth lies between the SSB bandwidth, W, and DSB-SC bandwidth, 2W.

$$DSBSC > VSB > SSB$$

$$2f_m > f_m + f_v > f_m$$

Generation of VSB Modulation (Sideband Shaping Filter):

- For VSB modulation, the bandpass filter is referred to as a sideband shaping filter.

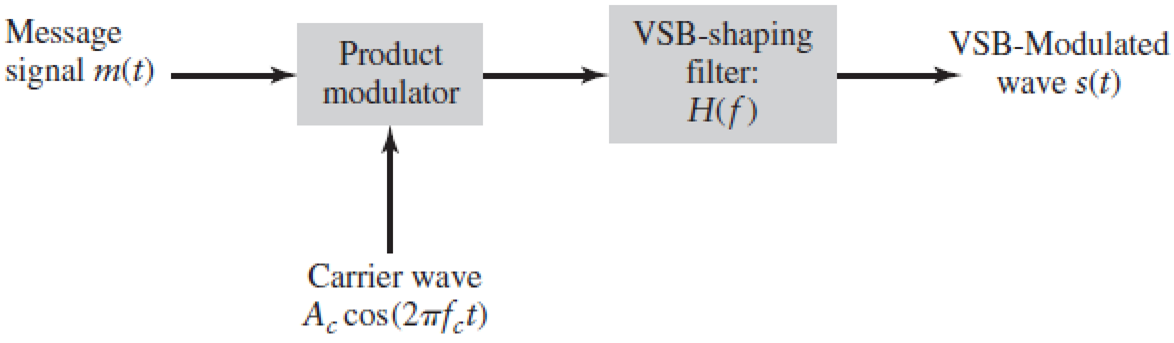


FIGURE 3.23 VSB modulator using frequency discrimination.

VSB modulated signal is $x(t) = m(t)cos(2\pi f_c t) * h(t)$

Hence, frequency response of $x(t) = m(t)cos(2\pi f_c t) * h(t)$ is

$$X(F) = \left[\frac{1}{2}M(F - F_c) + \frac{1}{2}M(F + F_c)\right].H\ (F)$$

Generation of VSB Modulation (Sideband Shaping Filter):

- The spectrum shaping is defined by the transfer function of the filter, which is denoted by $H(f)$.
- The only requirement that the sideband shaping performed by $H(f)$ must satisfy is that the transmitted vestige compensates for the spectral portion missing from the other sideband.
- This requirement ensures that coherent detection of the VSB modulated wave recovers a replica of the message signal, except for amplitude scaling.

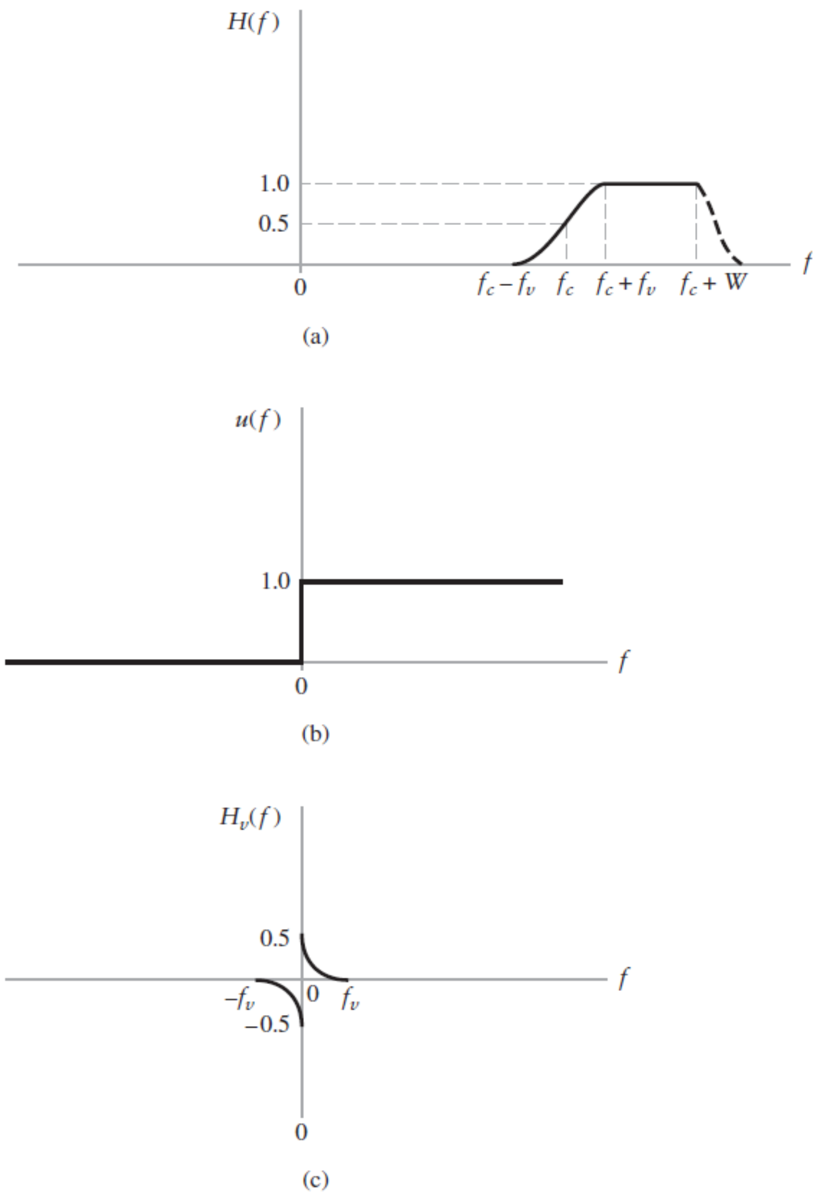
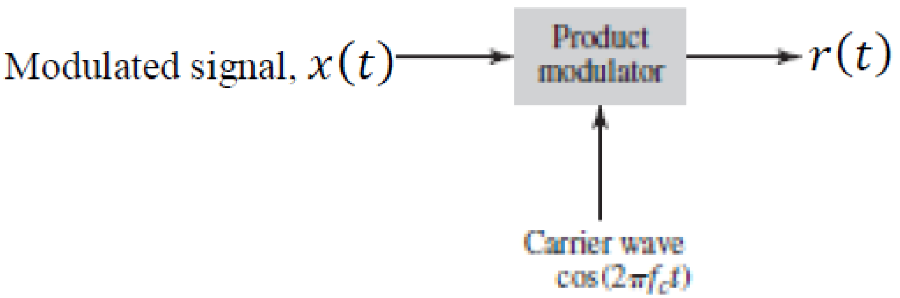


FIGURE 3.24 (*a*) Amplitude response of sideband-shaping filter; only the positive-frequency portion is shown, the dashed part of the amplitude response is arbitrary. (*b*) Unit-step function defined in the frequency domain. (*c*) Low-pass transfer function $H_v(f)$.

VSB Demodulator:

At the receiver (demodulator),



$$r(t) = x(t)cos(2\pi f_c t)$$

$$R(F) = \left[\frac{1}{2}X(F - F_c) + \frac{1}{2}X(F + F_c)\right]$$

VSB Demodulator:

$$R(F) = \left[\frac{1}{2}X(F - F_c) + \frac{1}{2}X(F + F_c)\right]$$

$$= \frac{1}{2}\left\{\left[\frac{1}{2}M(F - 2F_c) + \frac{1}{2}M(F)\right].H\,(F - F_c)\right\} + \frac{1}{2}\left\{\left[\frac{1}{2}M(F) + \frac{1}{2}M(F + 2F_c)\right].H\,(F + F_c)\right\}$$

$$= \underbrace{\frac{1}{4}M(F - 2F_c)H\,(F - F_c)} + \frac{1}{4}M(F)H\,(F - F_c) + \frac{1}{4}M(F)H\,(F + F_c) + \underbrace{\frac{1}{4}M(F + 2F_c)H\,(F + F_c)}$$

These two components correspond to $2F_c$

VSB Demodulator:

Therefore, after filter out,

$$R(F) = \frac{1}{4} M(F)\{H\,(F - F_c) + H\,(F + F_c)\}$$

$$R(F) = \frac{1}{4} M(F)$$

Set $H\,(F - F_c) + H\,(F + F_c) = 1$ (non-ideal VSB filter must satisfied this condition)

This property is satisfied if VSB filter is symmetric.

Complex Envelope:

- The term “baseband” is used to designate the band of frequencies representing the original signal as delivered by a source of information.

Why Complex Envelope?

- The carrier frequency is large compared to the message bandwidth, which makes the processing of a modulated wave on a digital computer a difficult proposition.

Objective of Complex Envelope:

- The objective is to process a modulated wave on a computer, the efficient procedure is to do the processing on the baseband version of the modulated wave rather than directly on the modulated wave itself.

Baseband representation of a modulated wave:

- A generic, linear modulated wave is

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

$$c(t) = \cos(2\pi f_c t)$$

be the carrier wave with frequency f_c, and

$$\hat{c}(t) = \sin(2\pi f_c t)$$

TABLE 3.1 *Different Forms of Linear Modulation as Special Cases of Eq. (3.39), assuming unit carrier amplitude*

Type of modulation	*In-phase component* $s_I(t)$	*Quadrature component* $s_Q(t)$	*Comments*
AM	$1 + k_a m(t)$	0	k_a = amplitude sensitivity $m(t)$ = message signal
DSB-SC	$m(t)$	0	
SSB:			
(a) Upper sideband transmitted	$\frac{1}{2}m(t)$	$\frac{1}{2}\hat{m}(t)$	$\hat{m}(t)$ = Hilbert transform of $m(t)$ (see part (i) of footnote 4)[4]
(b) Lower sideband transmitted	$\frac{1}{2}m(t)$	$-\frac{1}{2}\hat{m}(t)$	
VSB:			
(a) Vestige of lower sideband transmitted	$\frac{1}{2}m(t)$	$\frac{1}{2}m'(t)$	$m'(t)$ = response of filter with transfer function $H_Q(f)$ due to message signal $m(t)$. The $H_Q(f)$ is defined by the formula (see part (ii) of footnote 4)
(b) Vestige of upper sideband transmitted	$\frac{1}{2}m(t)$	$-\frac{1}{2}m'(t)$	$H_Q(f) = -j[H(f + f_c) - H(f - f_c)]$ where $H(f)$ is the transfer function of the VSB sideband shaping filter.

Baseband representation of a modulated wave:

- The information content of the message signal and the way in which the modulation strategy is implemented are fully described by the **in-phase component** in both **AM and DSB-SC** or in the combination of the **in-phase component and the quadrature component** in both **SSB** and **VSB**.

- Moreover, the orthogonality of $s_I(t)$ and $s_Q(t)$ with respect to each other prompts us to introduce a new signal called the **complex envelope** of the modulated wave which is formally defined by:

$$\tilde{s}(t) = s_I(t) + js_Q(t)$$

Purpose of Complex Envelope:

- The use of complex envelope is intended merely to simplify signal processing operations on band-pass signals, which are exemplified by modulated waves based on a sinusoidal carrier.

$$c(t) = \cos(2\pi f_c t)$$

be the carrier wave with frequency f_c, and

$$\hat{c}(t) = \sin(2\pi f_c t)$$

$$\begin{aligned} \widetilde{c}(t) &= c(t) + j\hat{c}(t) \\ &= \cos(2\pi f_c t) + j\sin(2\pi f_c t) \\ &= \exp(j2\pi f_c t) \end{aligned}$$

- Accordingly, the modulated wave $s(t)$ is itself defined by

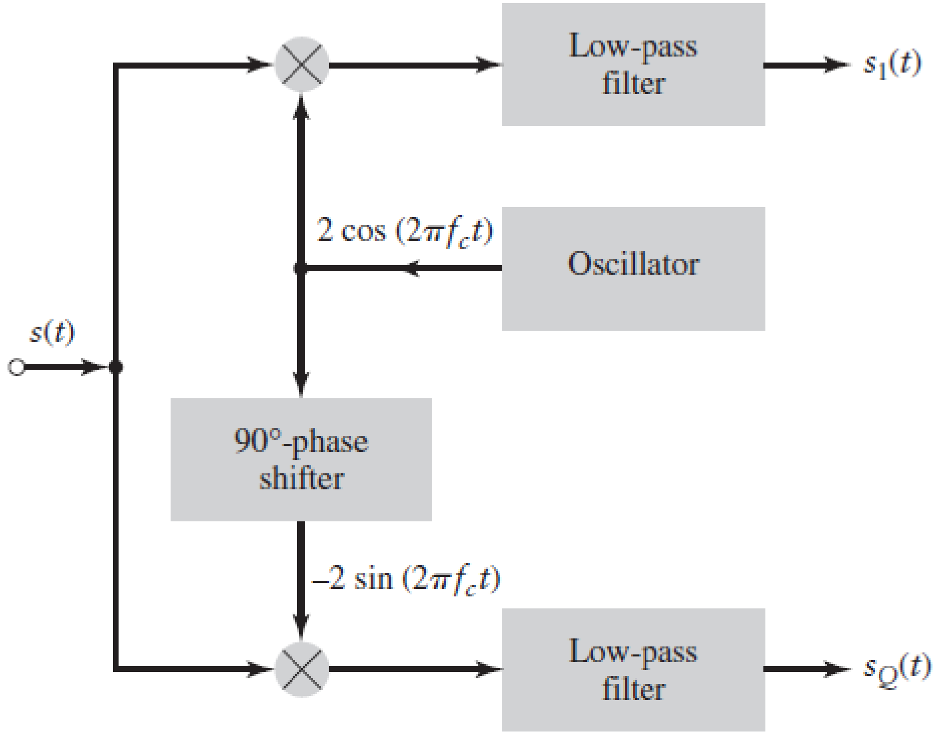
$$
\begin{aligned}
s(t) &= \mathrm{Re}[\tilde{s}(t)\tilde{c}(t)] \\
&= \mathrm{Re}[\tilde{s}(t)\exp(j2\pi f_c t)]
\end{aligned}
$$

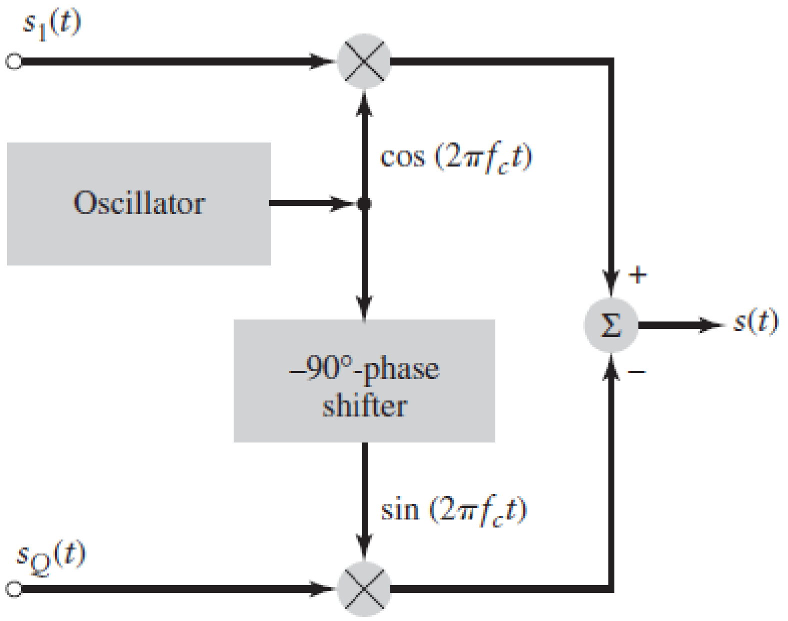
Advantage of Complex Envelope:

Now we can see the practical advantage of the complex envelope $\tilde{s}(t)$ over the real-valued modulated wave $s(t)$:

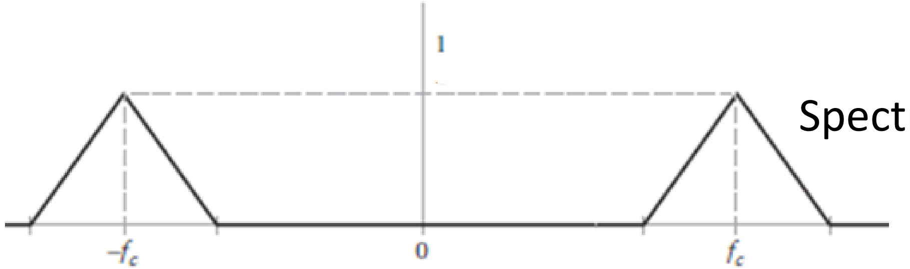
1. The highest frequency component of $s(t)$ may be as large as $f_c + W$, where f_c is the carrier frequency and W is the message bandwidth.
2. On the other hand, the highest frequency component of $\tilde{s}(t)$ is considerably smaller, being limited by the message bandwidth W.

Analyzer and Synthesizer of modulated wave:



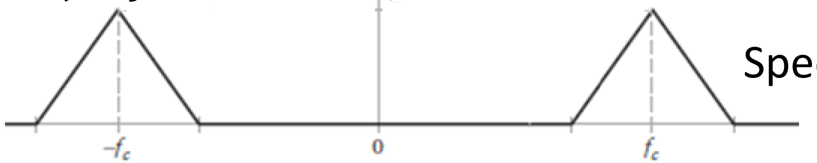


Complex Envelope:



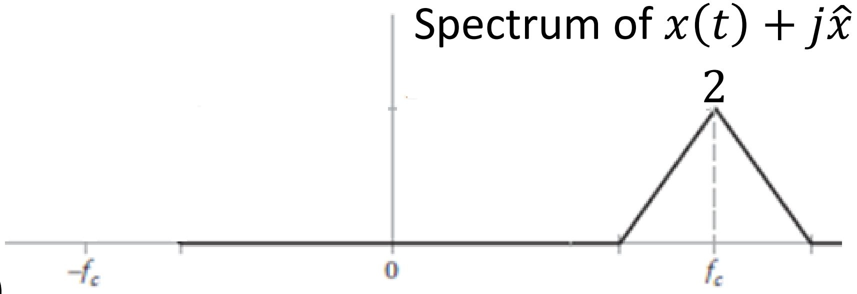
Spectrum of $\mathrm{x}(t)$

$\mathrm{j} \times j = -1$ $\quad -\mathrm{j} \times j = 1$



Spectrum of $j\hat{x}(t)$

Spectrum of $x(t) + j\hat{x}(t)$



Consider a passband signal $x(t)$,

$$x(t) \leftrightarrow X(F)$$

Consider another signal $x(t) + j\hat{x}(t)$
is a pre-envelope (complex signal)

$\hat{x}(t)$ is Hilbert transform of $x(t)$

$$X(F) = j\hat{X}(F)$$
$$X(F) = j(-jsgn(F)X(F))$$

Spectrum of $x(t) + j\hat{x}(t)$ = spectrum of $x(t)$ + spectrum of $j\hat{x}(t)$

Hence, complex baseband equivalent of passband signal $x(t)$ or complex envelope of $x(t)$ is $\tilde{x}(t)$

$\tilde{x}(t) = (x(t) + j\hat{x}(t))e^{-j2\pi F_c t}$

$e^{-j2\pi F_c t}$ shifts the spectrum to the baseband i.e., around F=0.

$\mathrm{Re}\{\tilde{x}(t)e^{j2\pi F_c t}\} = \mathrm{Re}\{(x(t) + j\hat{x}(t))e^{-j2\pi F_c t}e^{j2\pi F_c t}\} = \mathrm{Re}\{x(t) + j\hat{x}(t)\} = x(t)$

Complex envelope is just the frequency shifted version of pre-envelope.

Pre-Envelope (+ve frequencies):

$$X(F) = j(-jsgn(F)X(F))$$
$$X(F) = -(-1)\,\mathrm{sgn}(F)\,X(F)$$

Pre-envelope is

$$x(t) + j\hat{x}(t)$$
$$X(F) + \mathrm{sgn}(F)\,X(F)$$
$$((1 + \mathrm{sgn}(F))X(F)$$
$$sgn(F) = \begin{cases} 1, & F > 0 \\ 0, & F = 0 \\ -1, & F < 0 \end{cases}$$
$$x(preenvelope)(F) = \begin{cases} 2X(F), & F > 0 \\ X(F), & F = 0 \\ 0, & F < 0 \end{cases}$$

Pre-Envelope (-ve frequencies):

$$x(t) - j\hat{x}(t)$$
$$X(F) - \text{sgn}(F)\,X(F)$$
$$((1 - \text{sgn}(F))X(F)$$

$$sgn(F) = \begin{cases} 1, & F > 0 \\ 0, & F = 0 \\ -1, & F < 0 \end{cases}$$

$$x(preenvelope)(F) = \begin{cases} 0, & F > 0 \\ X(F), & F = 0 \\ 2X(F), & F < 0 \end{cases}$$

Complex Envelope of QCM:

$$s(t) = s_I(t)c(t) - s_Q(t)\hat{c}(t)$$

$$x(t) = x_I(t)\cos(2\pi F_c t) - x_Q(t)\sin(2\pi F_c t)$$

$$x(t) = x_I(t)[\frac{e^{j2\pi F_c t} + e^{-j2\pi F_c t}}{2}] - x_Q(t)[\frac{e^{j2\pi F_c t} - e^{-j2\pi F_c t}}{2j}]$$

Positive freq. band

Negative freq. band

$$= \underbrace{\frac{1}{2}[x_I(t) + jx_Q(t)]e^{j2\pi F_c t}}_{\text{Positive freq. band}} + \underbrace{\frac{1}{2}[x_I(t) - jx_Q(t)]e^{-j2\pi F_c t}}_{\text{Negative freq. band}}$$

Spectrum of complex pre-envelope, $x_p(t)$ = twice the spectrum corresponding to the positive frequency band

$$x_p(t) = x_I(t) + jx_Q(t)e^{j2\pi F_c t}$$

Complex envelope, $\tilde{x}(t) = x_p(t)e^{-j2\pi F_c t}$

$\tilde{x}(t) = x_I(t) + jx_Q$ i.e., complex envelope or complex baseband equivalent of QCM signal

Examples:

1. $A_c cos\omega_c t$

$= A_c\left[\frac{e^{j\omega_c t}+e^{-j\omega_c t}}{2}\right] = A_c e^{j\omega_c t}$ (pre-envelope)

Complex envelope?

$$= A_c e^{j\omega_c t}\ e^{-j\omega_c t}$$

$$= A_c$$

2.

$A_c[1+\mathrm{m(t)}]cos\omega_c t$

$A_c[1+\mathrm{m(t)}]\ e^{j\omega_c t}$ (pre-envelope)

$A_c[1+\mathrm{m(t)}]$ (complex envelope)

Example:

$$x(t) = \cos(2\pi 103t).\, u(t) \ \ centered\ at\ 100Hz$$

$$x_p(t) = u(t)e^{j2\pi 103t}$$

$$x_c(t) = u(t)e^{j6\pi t}$$