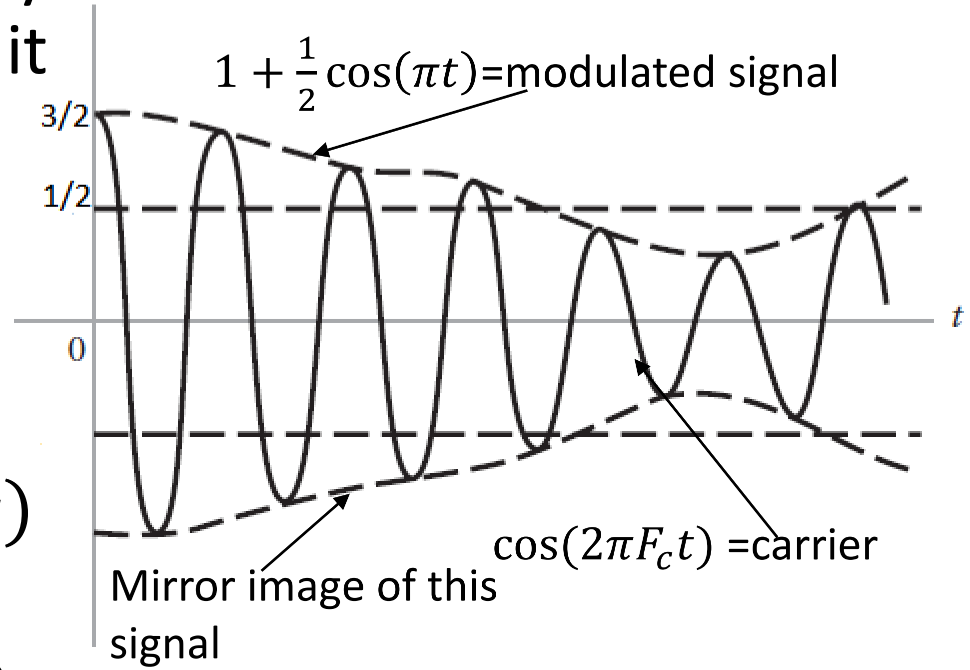
Communication Systems
EE-351

Lectures 5 and 6

Amplitude Modulation (AM)

- Modulation is nothing but, just variations. Amplitude of the carrier which is previously constant is now time varying in nature and it varies as per the message $m(t)$



<u>Example:</u>

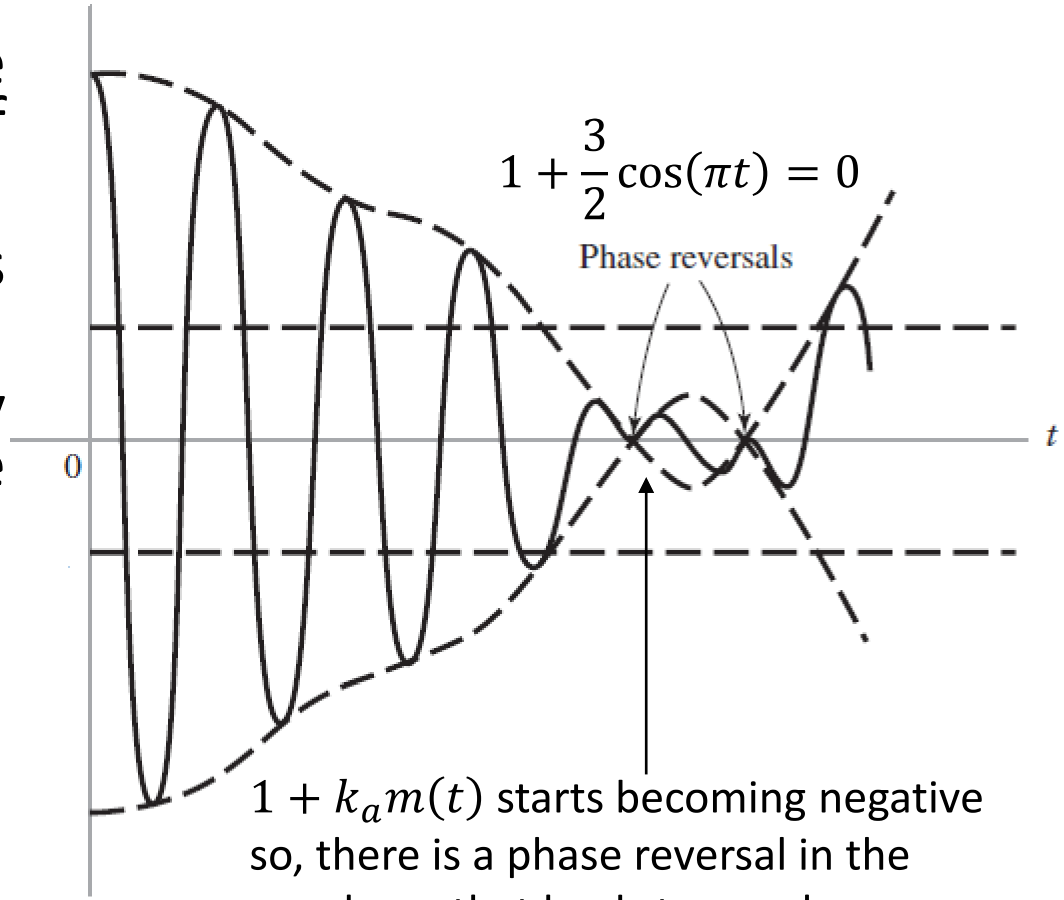
Let, $A_c = 1, k_a = 1/2, F_m = \frac{1}{2}$

$$m(t) = \cos(2\pi F_m t) = \cos\left(\frac{2\pi t}{2}\right) = \cos(\pi t)$$

AM signal $x(t) = (1 + \frac{1}{2}\cos(\pi t))\cos(2\pi F_c t)$

Amplitude Modulation (AM)

- Peaks of the carrier follow the message signal termed as the envelope of transmitted signal
- Therefore, the envelope contains information about the message signal.
- $F_c \gg F_m$, where F_m is message frequency or max. frequency component of the message
- Consider another example where $k_a = 3/2$ i.e., high sensitivity factor
- $1 + k_a m(t) = \underbrace{1 + \frac{3}{2}\cos(\pi t)}_{\text{max } 5/2, \text{ min } -1/2}$



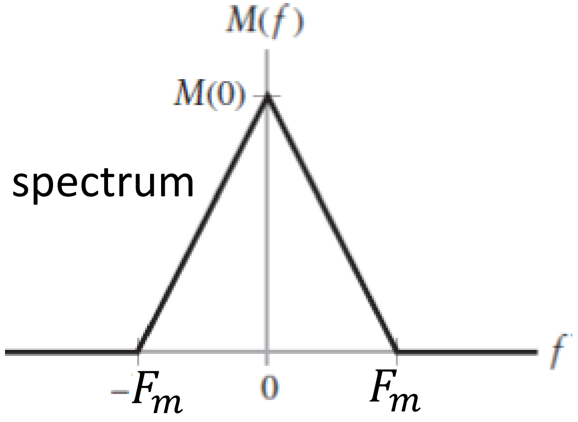
$1 + k_a m(t)$ starts becoming negative so, there is a phase reversal in the envelope, that leads to envelope distortion

Amplitude Modulation (AM)

- k_a should be chosen properly to avoid envelope distortion, in particular $\left(1 + k_a m(t)\right) \geq 0$ i.e., condition for no distortion
- For no distortion, modulation index i.e., $\mu \leq 1$, if $\mu \geq 1$, carrier is said to be over-modulated, i.e., envelope distortion which leads to phase reversal
- In first example, when $k_a = 1/2$, $\mu = \frac{1}{2} < 1$ $\Longrightarrow$ No envelope distortion
- In second example, when $k_a = 3/2$, $\mu = \frac{3}{2} > 1$ $\Longrightarrow$overmodulated signal
- Hence, modulation index = sensitivity factor $\times$ Amplitude

Spectrum of an AM signal

Message spectrum



- Consider a message signal $m(t)$ with spectrum M(F)
- Now, amplitude modulated signal,

Carrier freq.

$$x(t) = A_c(1 + k_a m(t))\cos(2\pi F_c t)$$
$$= \underbrace{A_c\cos(2\pi F_c t)}_{\text{Carrier component}} + A_c k_a m(t)\cos(2\pi F_c t)$$

- Now, let's look at the spectra of these two components

$$\cos(2\pi F_c t) = \frac{e^{j2\pi F_c t} + e^{-j2\pi F_c t}}{2}$$

  - $e^{j2\pi F_c t}$ has Fourier transform
  - $e^{j2\pi F_c t} \leftrightarrow \delta(F - F_c)$
  - $e^{-j2\pi F_c t} \leftrightarrow \delta(F + F_c)$

$$\frac{1}{2}\{\delta(F - F_c) + \delta(F + F_c)\}$$

Spectrum of an AM signal

- $A_c \cos(2\pi F_c t) \leftrightarrow \frac{A_c}{2}\delta(F - F_c) + \frac{A_c}{2}\delta(F + F_c)$
- Now, look at the other component i.e.,

$$A_c k_a m(t).\cos(2\pi F_c t)$$

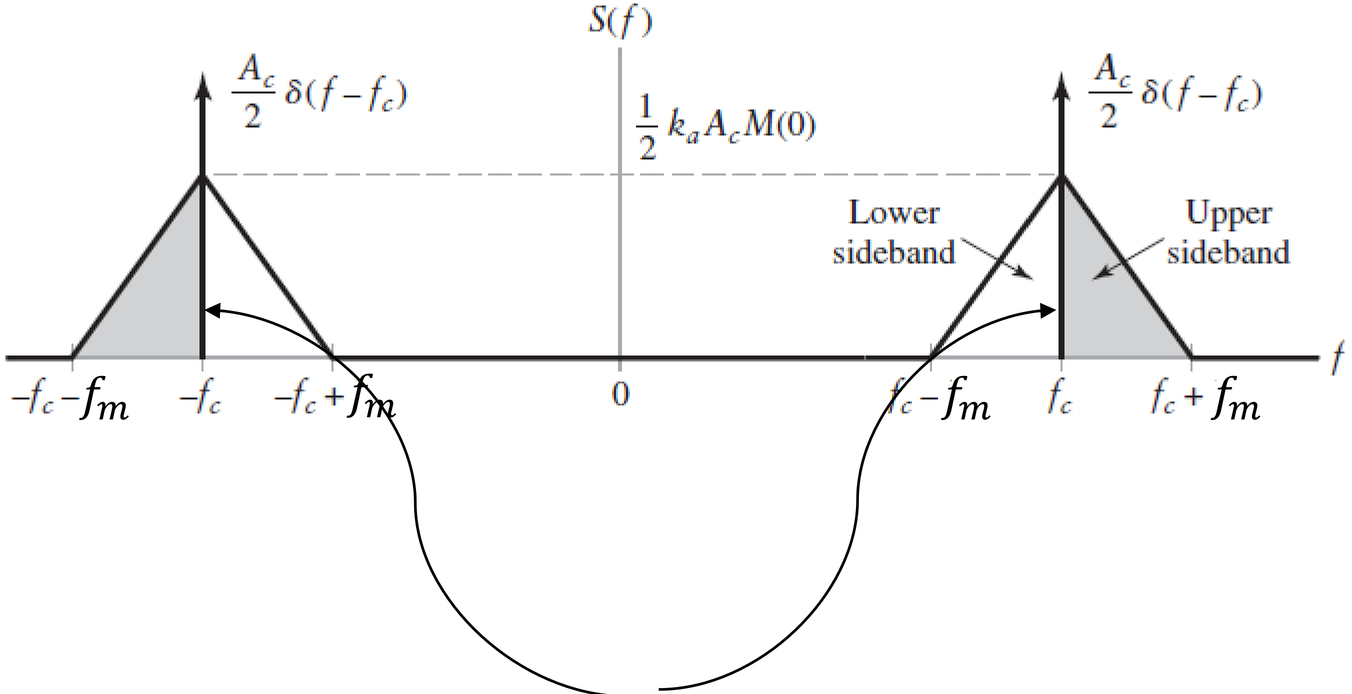
$$A_c k_a M(F) \quad * \quad \frac{1}{2}\{\delta(F - F_c) + \delta(F + F_c)\}$$

- $X(F) = \frac{A_c k_a}{2} \underbrace{M(F) * \delta(F - F_c)}_{\text{Shift M(F) to } F_c} + \frac{A_c k_a}{2} \underbrace{M(F) * \delta(F + F_c)}_{\text{Shift M(F) to } -F_c}$

Multiplication in time domain

Convolution in freq. domain

Spectrum of an AM signal

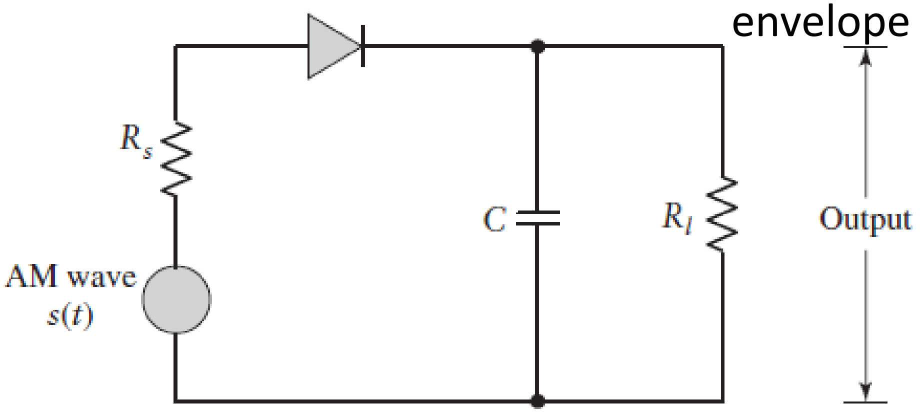


These two impulses correspond to the carrier wave & the two triangular bands correspond to the bands of the spectra of the original message shifted to F_c and $-F_c$

Envelope Detection

- Detects envelope of the transmitted signal to recover the message
- It works only when there is no distortion of envelope i.e., $\mu \leq 1$
- It employs a simple circuit known as envelope detector

V_o tracks the peaks of envelope



$V_{in} > V_o \implies$ Diode is forward biased

$V_{in} < V_o \implies$ Diode is reverse biased

Time constant, $\tau = R_l C$

$$\frac{1}{F_c} \ll \tau \ll \frac{1}{F_m}$$

Power of an AM signal

$$x(t) = (1 + k_a m(t)) \times A_c \cos(2\pi F_c t)$$
$$= \underbrace{A_c \cos(2\pi F_c t)}_{\text{Pure carrier}} + \underbrace{A_c k_a m(t) \cos(2\pi F_c t)}_{\text{carrier modulated by signal (message)}}$$

Power of $A_c \cos(2\pi F_c t) = \frac{{A_c}^2}{2}$

Consider $A_c k_a m(t) \cos(2\pi F_c t) \approx$ sinusoid with amplitude $A_c k_a m(t)$

The assumption is not true, when $m(t)$ is multiplied with a sinusoid signal, the resultant is no longer sinusoid.

Since, $m(t)$ is varying at a much slower rate than $\cos(2\pi F_c t)$, i.e., $F_c \gg F_m$

Therefore, instantaneous power can be approximated as $\frac{1}{2} {k_a}^2 {A_c}^2 m^2(t)$

Power of an AM signal

Average power can be obtained as:

Average power = $E\left\{\frac{1}{2}{k_a}^2{A_c}^2m^2(t)\right\}$, E stands for expectation

= $\frac{1}{2}{k_a}^2{A_c}^2\ E\{m^2(t)\} = \frac{1}{2}{k_a}^2{A_c}^2P_m$ (where P_m is power of baseband $m(t)$)

Total power of AM signal = $\frac{{A_c}^2}{2} + \frac{1}{2}{k_a}^2{A_c}^2P_m$

Efficiency of AM signal

Efficiency = $\dfrac{\text{power in carrier modulated by message}}{\text{total power}}$

$$\eta = \frac{\frac{1}{2}{k_a}^2{A_c}^2P_m}{\frac{1}{2}{A_c}^2 + \frac{1}{2}{k_a}^2{A_c}^2P_m} = \frac{{k_a}^2P_m}{1 + {k_a}^2P_m}$$

Consider now a specific case of sinusoidal message signal

$$m(t) = A_m\cos(2\pi F_m t)$$

$$P_m = \frac{1}{2}{A_m}^2$$

Efficiency of AM signal

$$\mu = k_a A_m$$

$$\frac{1}{2}\mu^2 = \frac{1}{2}{k_a}^2{A_m}^2$$

$$\frac{1}{2}\mu^2 = \frac{1}{2}{A_m}^2{k_a}^2$$

$$\frac{1}{2}\mu^2 = P_m{k_a}^2$$

$$\eta = \frac{\frac{1}{2}\mu^2}{1 + \frac{1}{2}\mu^2} = \frac{\mu^2}{2 + \mu^2}$$

Another imp. relationship for efficiency of AM signal or efficiency of sinusoidal modulation

Efficiency of AM signal

$$\eta = \frac{\mu^2}{2+\mu^2} = 1 - \underbrace{\frac{2}{2+\mu^2}}_{\text{Decreasing with } \mu}$$

$\eta = 1 - \frac{2}{2+\mu^2}$ whole thing is increasing

As μ increases, the η improves

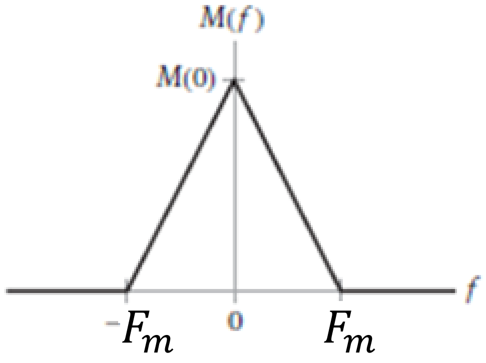
Interestingly, for envelope detection, max. value of $\mu = 1$

$$\eta_{max} = \frac{\mu^2}{2+\mu^2} |\mu = 1$$

η_{max} =1/3 i.e., 33%

This tells us efficiency of AM is very poor, approx. 67% of the energy is basically wasted.

Double Sideband (DSB) Modulation:



$$x(t) = A_c m(t) \cos(2\pi F_c t)$$

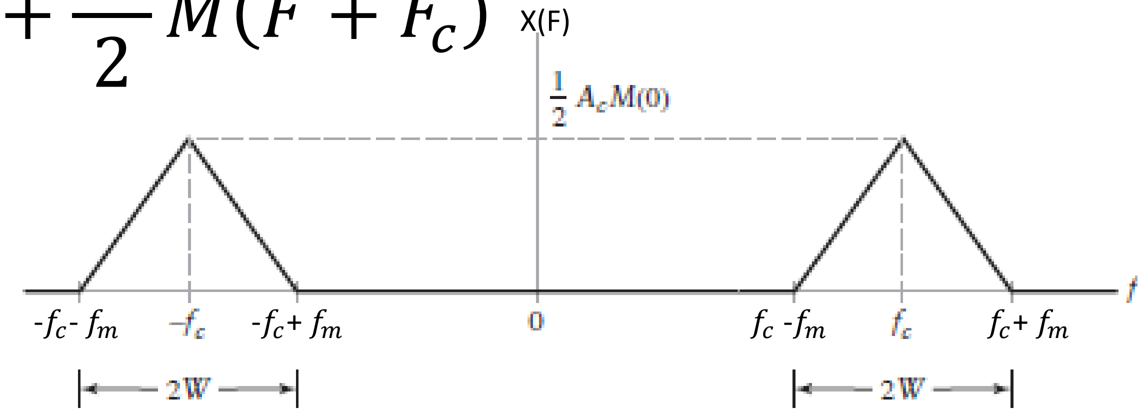
- There is no pure carrier component therefore, it is termed as double sideband-suppressed carrier, DSB-SC.

$$x(t) = m(t) \times A_c \cos(2\pi F_c t)$$

$$X(F) = M(F) * \left\{ \frac{A_c}{2} \delta(F - F_c) + \frac{A_c}{2} \delta(F + F_c) \right\}$$

$$X(F) = \frac{A_c}{2} M(F - F_c) + \frac{A_c}{2} M(F + F_c)$$

➢ Spectrum of the DSB-SC signal



Demodulation of DSB-SC signal:

- known as coherent detection or synchronous demodulation.
- Demodulation is carried out by multiplying modulated signal (i.e., incoming signal) with a coherently generated carrier $\cos(2\pi F_c t)$ at the receiver.

$$r(t) = x(t).\cos(2\pi F_c t)$$

$$= A_c m(t)\cos(2\pi F_c t) \times \cos(2\pi F_c t)$$

$$= A_c m(t) cos^2(2\pi F_c t)$$

$$= A_c m(t)\left[\frac{1+\cos(4\pi F_c t)}{2}\right]$$

$$= \underbrace{\frac{A_c m(t)}{2}}_{\text{Baseband centered at F=0}} + \underbrace{\frac{1}{2} A_c m(t)\cos(4\pi F_c t)}_{\text{centered at freq. } 2F_c}$$

Demodulation of DSB-SC signal:

$$r(t) = \frac{A_c m(t)}{2} + \frac{1}{2} A_c m(t) \cos(4\pi F_c t)$$

$$R(F) = \frac{1}{2} A_c M(F) + M(F) * \left\{ \frac{A_c}{4} \delta(F - 2F_c) + \frac{A_c}{4} \delta(F + 2F_c) \right\}$$

$$= \frac{1}{2} A_c M(F) + \frac{A_c}{4} M(F - 2F_c) + \frac{A_c}{4} M(F + 2F_c)$$

So, choosing LPF of suitable bandwidth $F_m \leq W \leq 2F_c - F_m$, $\frac{A_c m(t)}{2}$ can be separated from $\frac{1}{2} A_c \mathrm{m(t)} \cos(4\pi F_c t)$