Communication Systems
EE-351

Lecture 19

BW of FM signals:

<u>Carson's Rule for BW of FM signal:</u>

- For large values of β, BW approaches ΔF
- For small values of β, BW $\approx 2F_m$

Combining these two principles, Carson's rule for BW of FM signals is given as:

$$BW \approx 2(1 + \frac{1}{\beta})\, \Delta F$$

- For large β, $\frac{1}{\beta} \approx 0 \Rightarrow BW \approx 2\, \Delta F$
- For small β, $\frac{1}{\beta} \gg 1 \Rightarrow BW \approx 2\, \frac{1}{\beta} \Delta F \approx 2 \frac{1}{\frac{\Delta F}{F_m}} \Delta F \approx 2\, F_m$

$$BW \approx 2(1 + \frac{1}{\beta})\, \Delta F \Rightarrow \text{BW} \approx 2\, \Delta F + \frac{2}{\beta} \Delta F \Rightarrow \text{BW} = 2\, \Delta F + 2\, F_m$$

TABLE A3.1 *Table of Bessel Functions*[a]

$J_n(x)$

$n \backslash x$	0.5	1	2	3	4	6	8	10	12
0	0.9385	0.7652	0.2239	−0.2601	−0.3971	0.1506	0.1717	−0.2459	0.0477
1	0.2423	0.4401	0.5767	0.3391	−0.0660	−0.2767	0.2346	0.0435	−0.2234
2	0.0306	0.1149	0.3528	0.4861	0.3641	−0.2429	−0.1130	0.2546	−0.0849
3	0.0026	0.0196	0.1289	0.3091	0.4302	0.1148	−0.2911	0.0584	0.1951
4	0.0002	0.0025	0.0340	0.1320	0.2811	0.3576	−0.1054	−0.2196	0.1825
5	—	0.0002	0.0070	0.0430	0.1321	0.3621	0.1858	−0.2341	−0.0735
6		—	0.0012	0.0114	0.0491	0.2458	0.3376	−0.0145	−0.2437
7			0.0002	0.0025	0.0152	0.1296	0.3206	0.2167	−0.1703
8			—	0.0005	0.0040	0.0565	0.2235	0.3179	0.0451
9				0.0001	0.0009	0.0212	0.1263	0.2919	0.2304
10				—	0.0002	0.0070	0.0608	0.2075	0.3005
11					—	0.0020	0.0256	0.1231	0.2704
12						0.0005	0.0096	0.0634	0.1953
13						0.0001	0.0033	0.0290	0.1201
14						—	0.0010	0.0120	0.0650

[a] For more extensive tables of Bessel functions, see Abramowitz and Stegun (1965, pp. 358–406).

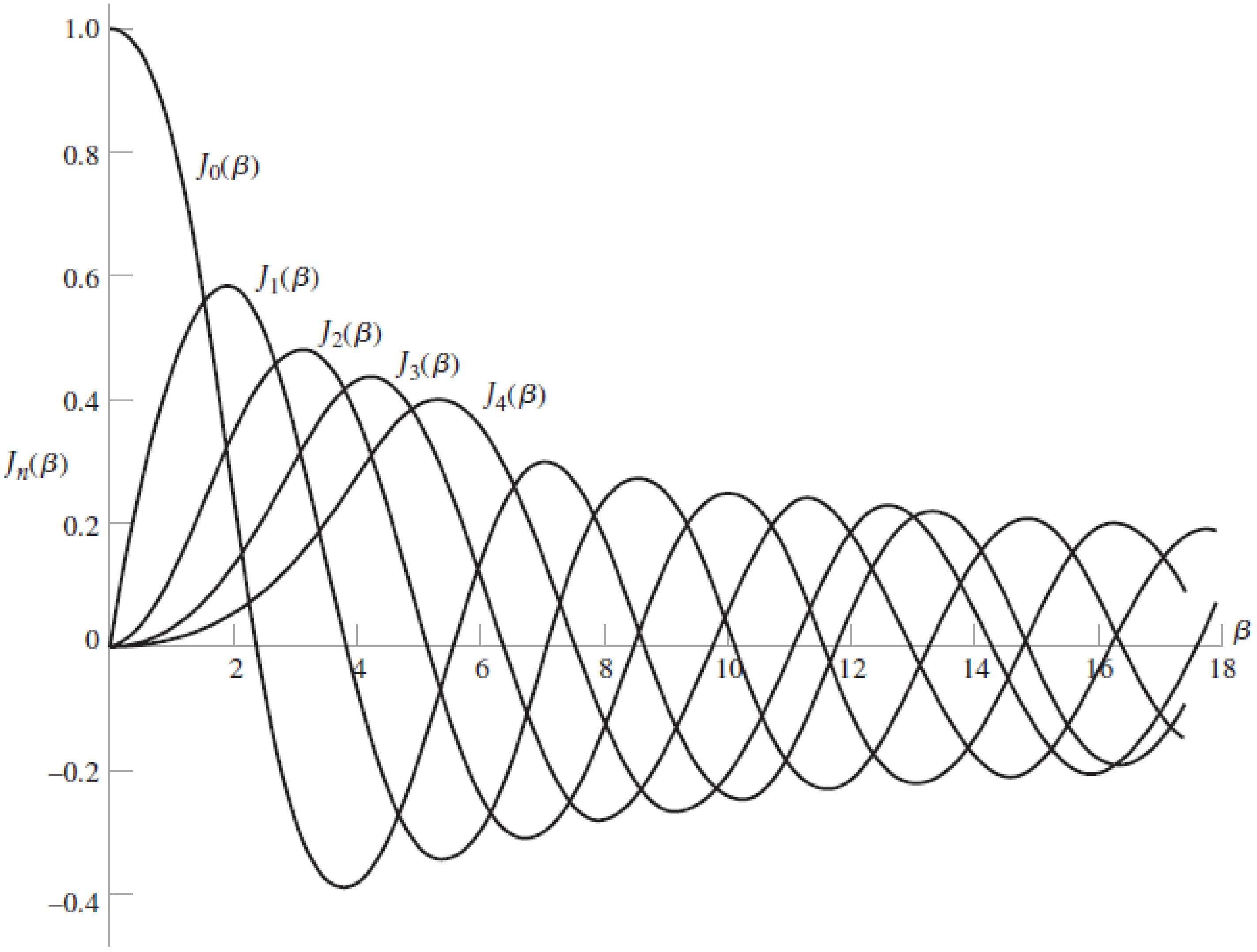

FIGURE 4.6 Plots of the Bessel function of the first kind, $J_n(\beta)$, for varying order n.

Power of FM signal:

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi(f_c + nf_m)t$$

$$s(t) = \cdots + A_c J_{-2}(\beta)\cos 2\pi(f_c - 2f_m)t - A_c J_{-1}(\beta)\cos 2\pi(f_c - f_m)t + A_c J_0(\beta)\cos 2\pi f_c t + A_c J_1(\beta)\cos 2\pi(f_c + f_m)t + A_c J_2(\beta)\cos 2\pi(f_c + 2f_m)t + \cdots$$

$$P_{total} = \cdots + \frac{A_c{}^2 J_{-2}{}^2(\beta)}{2} + \frac{A_c{}^2 J_{-1}{}^2(\beta)}{2} + \frac{A_c{}^2 J_0{}^2(\beta)}{2} + \frac{A_c{}^2 J_1{}^2(\beta)}{2} + \frac{A_c{}^2 J_2{}^2(\beta)}{2} + \cdots$$

$$= \frac{A_c{}^2}{2}\left[\ldots + J_{-2}{}^2(\beta) + J_{-1}{}^2(\beta) + J_0{}^2(\beta) + J_1{}^2(\beta) + J_2{}^2(\beta) + \cdots\right]$$

Power of FM signal:

$$= \frac{{A_c}^2}{2}$$

Property:

$$\sum_{n=-\infty}^{\infty} {J_n}^2(\beta) = 1$$

Power of FM signal after modulation = Power of carrier before modulation

$$s(t) = \frac{{A_c}^2}{2}$$

$$c(t) = A_c \cos(2\pi f_c t + \varphi)$$

$$P_T = \frac{{A_c}^2}{2}$$

Power of FM signal:

$$P_{c(after\ modulation)} = \frac{{A_c}^2 {J_0}^2(\beta)}{2} \text{ due to } A_c J_0(\beta)\cos 2\pi f_c t$$

If we reduce $\frac{{A_c}^2 {J_0}^2(\beta)}{2}$ to 0, we'll get 100% efficiency:

$$\eta = \frac{P_{SB}}{P_c + P_{SB}}$$

Eigen value of β when η = 100%,

$$J_0(\beta) = 0$$
$$\beta = 2.4, 5.5, 8.6, 11.8, \ldots$$