**Practice Examples**
**DTFT and z-Transform**

## Summary and Rules for DTFT (Chapter-2):

1. Convolution sum is a property of LTI system and DTFT is a biproduct of passing exponential sequence (god sequences) through LTI system.

2. If sequence is absolute summable, it's DTFT exists for all $\omega$ i.e.,
$$\sum_{n=-\infty}^{\infty} |x[n]| < \infty$$
otherwise there will be some discontinuity in $X(e^{j\omega})$.

3. No DTFT for periodic sequences as they are not absolute summable as they can exist from $-\infty$ to $\infty$, $-\infty$ to 0 or 0 to $\infty$.
    a. Only way is to assume some $X(e^{j\omega})$ first and then find IDTFT to prove. It cannot be found directly using DTFT formula i.e.,
    $$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$
    b. For this, one thing to remember is that the assumed $X(e^{j\omega})$ is always in the form of impulses (derived from Fourier Series later in this course).

4. No DTFT for diverging sequences as they exist from $-\infty$ to $\infty$, $-\infty$ to 0 or 0 to $\infty$ also their magnitude tends to infinity.
    a. There is no way you can find DTFT, not by assuming $X(e^{j\omega})$ first nor directly by
    $$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$
    b. Only way is to take help from z-Transform.

5. DTFT for any finite sequence exists.

6. DTFT is the only tool used to plot frequency response to analyze the shape and frequency-dependent behavior of signal/system. Existence of DTFT is something else and plotting is something else.

7. Always remember to solve for DTFT formula with some geometric series.

**Summary and Rules for z-Transform (Chapter-3)**

1. z-Transform is the generalization of DTFT as it introduces a new parameter $r$ in its relationship i.e.,
$$z = re^{j\omega}$$
$$X\left(re^{j\omega}\right) = \sum_{n=-\infty}^{\infty} (x[n]r^{-n})e^{-j\omega n}$$

2. z-Transform gives the information about signals that may be not absolute summable and also divergent.

3. z-Transform does not exist for periodic sequences that extend from $-\infty$ to $\infty$ as there is no unique ROC. However, they may exist if the periodic sequence exists from either $-\infty$ to 0 or 0 to $\infty$. For example, $x[n] = \cos(\omega_0 n)$ does not have a z-transform but $x[n] = \cos(\omega_0 n)u[n]$ may have one.

4. Always remember to solve for z-Transform with some geometric series.

5. The condition of z-transform gives the bounds of ROC, while the entire solution
    a. Taking all the expressions to positive power of z,
    b. Solving and combining all the terms (if applicable) after LCM,
    c. Factorizing numerator and denominator with smallest powers of z,

   gives the actual location of poles and zeros.

6. In $X(z)$ numerator, if there is a standalone $z$ with power greater than any $z$ in denominator, there will be a pole at infinity. Similarly, if there is any standalone $z$ in denominator with power greater than any $z$ in numerator, there will be a zero at infinity.

**Practice**

1. Show that any discrete signal can have $\pi$ as maximum radians frequency.

2. There are 5 signals whose DTFT should be assumed first as given below. The general way will be learnt later in this course.
    a. $x[n] = 1, for\ all\ n$
    b. $x[n] = \cos(\omega_0 n)$
    c. $x[n] = \sin(\omega_0 n)$
    d. $x[n] = e^{j\omega_0 n}$
    e. $x[n] = \frac{\sin(\omega_c n)}{\pi n}$

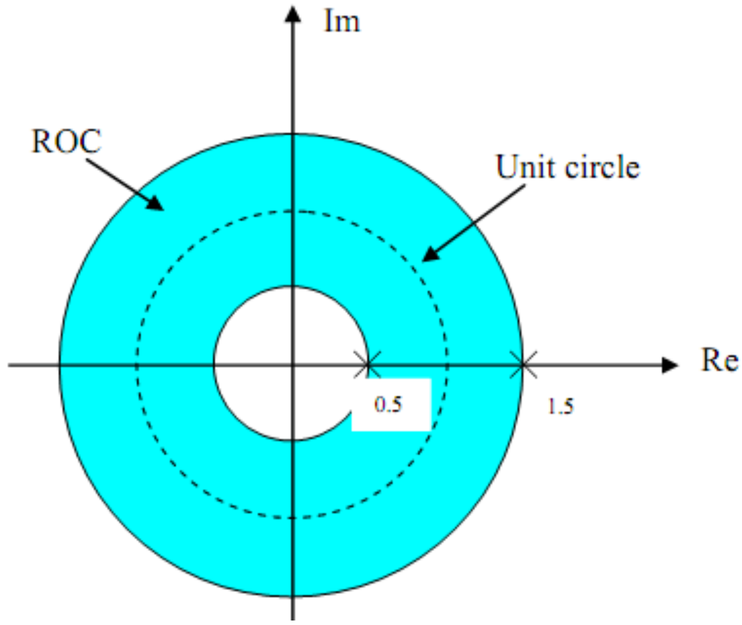
    To further summarize, periodic signals, complex periodic, constant and sinc.

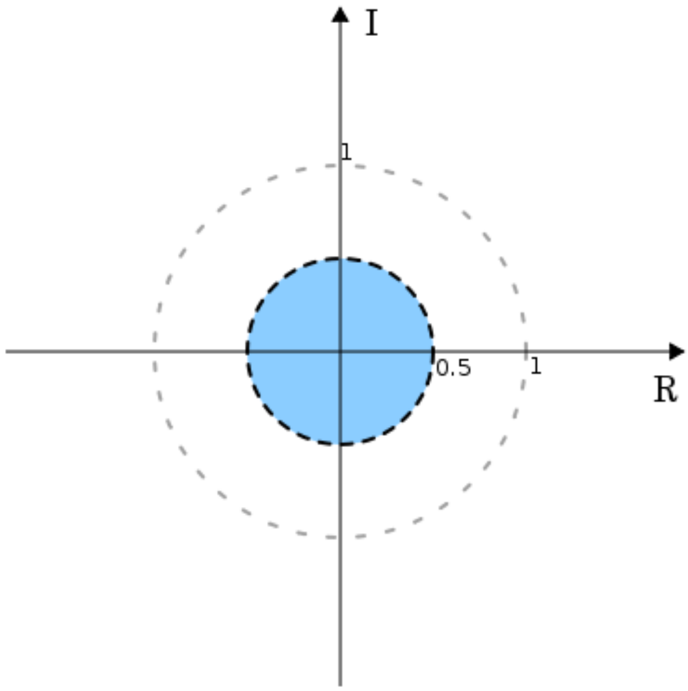
3. Find DTFT and then z-Transform of
    a. $x[n] = \cos(\omega_0 n)u[n]$
    b. $x[n] = e^{j\omega_0 n}u[-n]$

4. Find z-Transform of
    a. $x[n]$ =[2 0 0 5 6 7] with $x[-2] = 2$, the starting point.
    b. $x[n] = \left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n \ for \begin{bmatrix} n = -5\ to\ 5 \\ 0\ otherwise \end{bmatrix}$

5. What is the maximum number of possibilities of ROCs for signals/systems? Do zeros affect ROC?

6. Estimate the type of signal using following ROCs


Im
ROC
Unit circle
Re
0.5
1.5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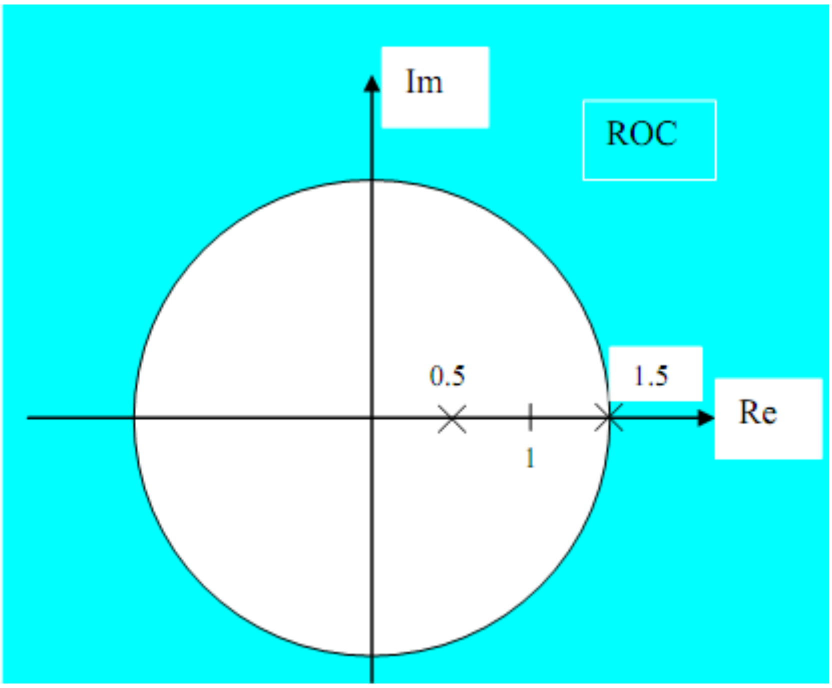
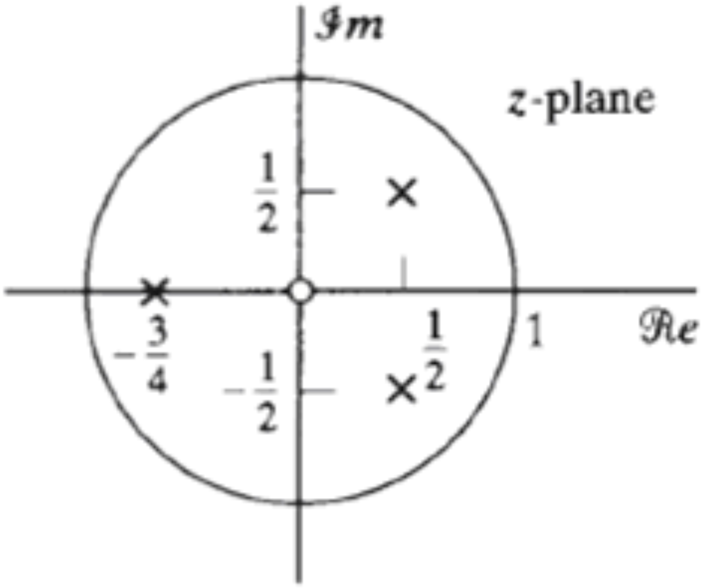



7. What are the possibilities for the ROCs of the following pole-zero plot. Also estimate the signal for each possibility

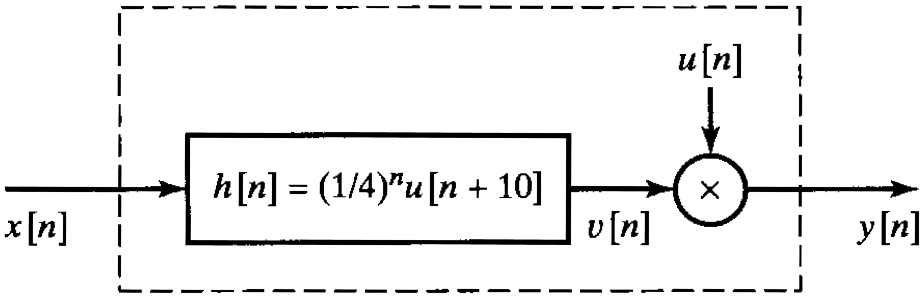



8. We know that LTI system can be expressed by two ways i.e., convolution sum and Linear Constant Coefficient Difference Equation;
   a. What is the way of finding frequency response of system $H(e^{j\omega})$ through convolution sum?

b. What is the way of finding frequency response of the system $H(e^{j\omega})$ through difference equation? Find the frequency response of the following equation

$$y[n] = \frac{1}{2}y[n-1] = x[n] + 2x[n-1] + x[n-2]$$

9. For the following system answer the following




a. Is the overall system LTI?
b. Is the overall system causal?
c. Is the overall system stable in BIBO sense?

10. **Additional Practice**: Do problems 2.4, 2.7, 2.42, 2.45, 3.3, 3.32, 3.40