



NUST School of Electrical Engineering and Computer Science

Faculty Member: _____

Date: _____

Semester: _____

Section: _____

Department of Electrical Engineering

EE-379: Control Systems

LAB 1: Laplace Transform, Transfer Functions and Control System Toolbox in MATLAB

Student name	Reg. No.	Lab Report Marks / 10	Viva Marks /5	Total/15

LAB 1: Laplace Transform, Transfer Function and Control System Toolbox in MATLAB

1. Objectives of Lab 1

The objectives of the first lab are:

- An introduction to MATLAB Control systems toolbox, which is one of the most popular software tools used by control engineers.
- An introduction to MATLAB Simulink, another very popular software tool
- Learn how to find Laplace and inverse Laplace transforms in MATLAB
- Find how to represent transfer functions (or models) in MATLAB and Simulink

2. Laplace and Inverse Laplace Transforms in MATLAB

Laplace transform is an important tool that is very useful when we work with differential equations. We will also use Laplace transforms in the control systems course. In this handout we will only show how to find the Laplace/ inverse Laplace transforms in MATLAB. The use of Laplace transforms in control systems theory will be shown in later weeks.

The steps to calculate the Laplace transform in MATLAB are:

- i. Define a symbolic variable for time using the keyword `syms`
- ii. Define the time domain function
- iii. Find the Laplace transform by using the function `laplace()`

In MATLAB, to display the result of any command you skip the semicolon at the end.

An example of the above steps is given below:

```
syms t; %Define variable in symbolic form
g1t = 5*t*exp(-5*t); %Function in time domain
G1s = laplace(g1t) %Find Laplace, notice no semicolon
```

The inverse Laplace transform can be found using the function `ilaplace()`, for example you can give the following command:

```
ilaplace(G1s)
```

You will agree that it is a lot easier to use a software tool to calculate the transforms in comparison to doing it manually.

Exercise 1

Find the Laplace transform of the following time domain functions:

- i. $g_1(t) = (t \sin 2t + e^{-2t})$
- ii. $g_2(t) = \sin 2t \cos 2t$
- iii. $g_3(t) = e^{-t} \cos(3t)$
- iv. $g_4(t) = -e^{-t} + 9te^{-t} + 5e^{-2t} + t - 2$
- v. $g_5(t) = 5t^2 \cos(3t + 45)$

Find the inverse Laplace transform of the following frequency domain functions.

Tip: You will have to start by defining a symbolic variable for frequency.

- i. $G_1(s) = \frac{1}{s(s+2)(s+3)}$
- ii. $G_2(s) = \frac{10}{(s+1)^2(s+3)}$
- iii. $G_3(s) = \frac{2(s+1)}{s(s^2+s+2)}$
- iv. $G_4(s) = \frac{s+1}{s(s+2)(s^2+2s+2)}$

NOTE: You can do the exercise in the command window or using a MATLAB script (m file). I suggest that you use the m file, because you can save it. This comment also applies to the rest of MATLAB exercises in all the labs.

3. MATLAB Control System Toolbox

MATLAB Control System Toolbox provides industry-standard algorithms and tools for systematically analyzing, designing, and tuning linear control systems. It is an important tool for control engineers and is widely used in industry and academia. We can access the Control System Toolbox by doing the following steps

- Open MATLAB
- Click Start (the start button inside MATLAB)
- Go to Toolboxes
- Click on the Control System Toolbox

See Figure 1 for details

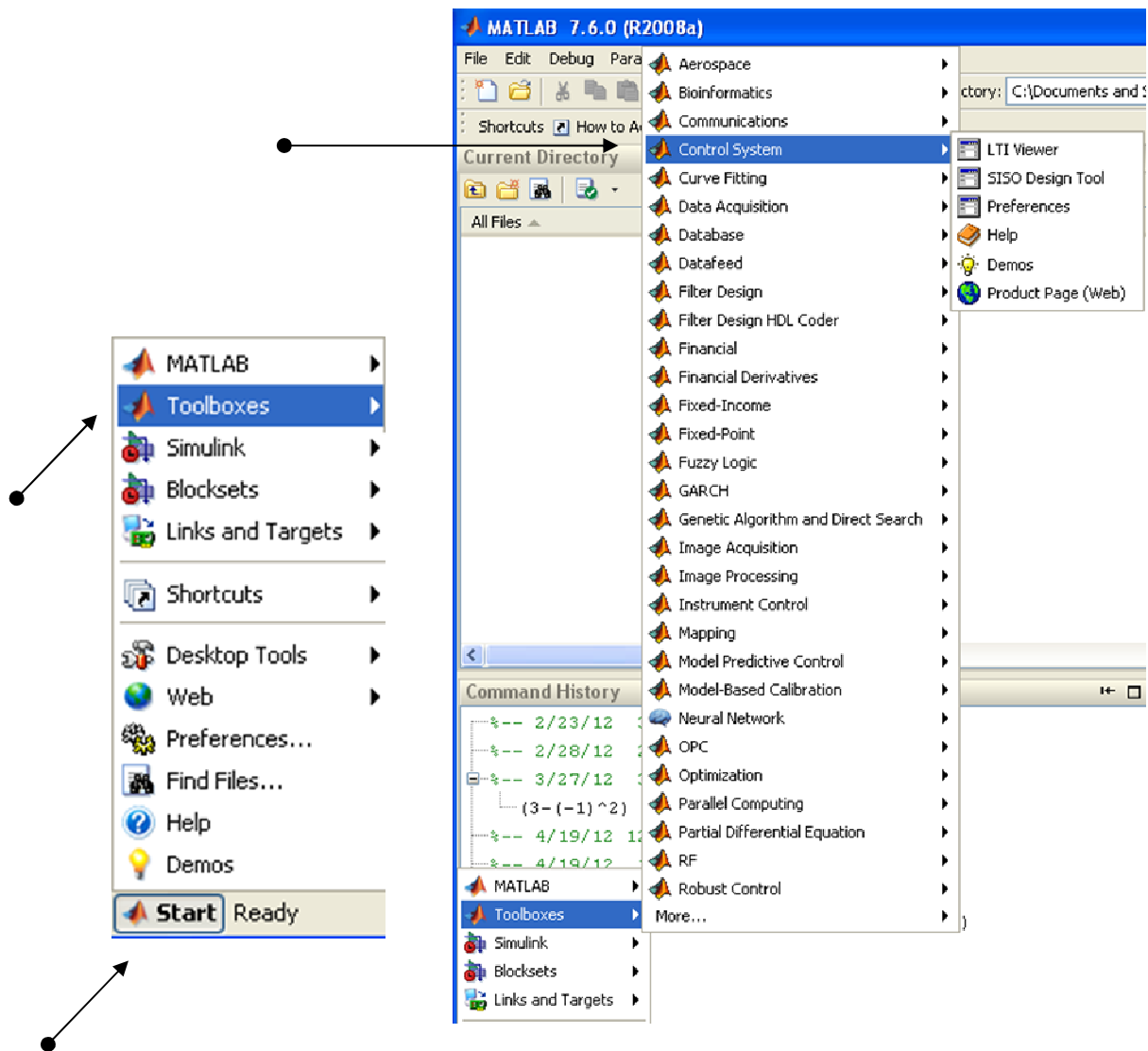


Figure 1: Accessing the MATLAB Control Systems Toolbox

In this handout we will introduce some of the basic functions in the Control Systems Toolbox. We will show you how to use the Control Systems Toolbox to enter the transfer function in MATLAB and how to plot the poles and zeros of a transfer functions. In future labs we will look at more advanced functionalities of the Control Systems Toolbox.

4. Transfer function in MATLAB

In this handout we will discuss two ways of entering transfer functions in MATLAB. The first way is by using the function `tf()`. For using this function you will need two arrays that represent the coefficients of the numerator polynomial and denominator polynomial of the transfer function. For example if you have the transfer function

$$\frac{5s + 10}{s^2 + 7s + 12}$$

then you can enter it into MATLAB with the following code:

```
num = [5 10];           %(coefficients of numerator)
den = [1 7 12];         %(coefficients of denominator)
G1 = tf(num,den)         %(make transfer function)
```

Exercise 2

Find the poles and zeros of the transfer function given above in your logbook. Remember the zeros are the roots of the numerator polynomial and poles are the roots of the denominator polynomial.

Another way to enter transfer functions in MATLAB is by using the function `zpk()`. This function lets you create a transfer function by just specifying the values of zeros, poles and the gain. For example, the transfer function given above can also be written in the form

$$\frac{5(s + 2)}{(s + 3)(s + 4)}$$

You can create this transfer function in MATLAB with the following code:

```
z = [-2];               %(zeros)
p = [-3 -4];            %(poles)
k = 5;                  %(gain)
G2 = zpk(z,p,k)          %(make transfer function)
```

Once you have entered a transfer function in any of the above forms, you can easily convert it to the other form. You can convert G2 to the usual form by using the command `tf(G2)`. You can also convert G1 to the zero-pole-gain form by using the command `zpk(G1)`.

Exercise 3

Create the following transfer functions in MATLAB and convert them to the other form. Give each transfer function a different name, e.g. G1, G2 etc.

$$\frac{30s - 180}{(s^3 + 4s^2 + 13s + 7)}$$

$$\frac{s^3 + s + 1}{s^3 + s^2 + 6}$$

$$\frac{(s + 4)(s + 2 - 4i)(s + 2 + 4i)}{(s - 2)(s + 4)(s + 5i)(s - 5i)}$$

$$\frac{(s^3 + 11s^2 + 35s + 250)}{s^5 + 4s^4 + 39s^3 + 108s^2}$$

$$\frac{s^2 + 5s + 6}{(s^5 + 4s^3 + 15s + 35)}$$

$$\frac{s^2 - 1}{s^4 + 4s^2 + 6s + 4}$$

5. Pole zero plot in MATLAB

Once you have entered the transfer function in MATLAB, you can plot the poles and zeros of the transfer function by using the command `pzplot()`. Here is some example code

```
G1 = tf([5 10],[1 2 4 -3])    %(make transfer function)
pzplot(G1)                  %plot the poles and zeros
```

The output is shown in Figure 2.

Exercise 4

Plot the poles and zeros of any three transfer functions that you have already created in the MATLAB.

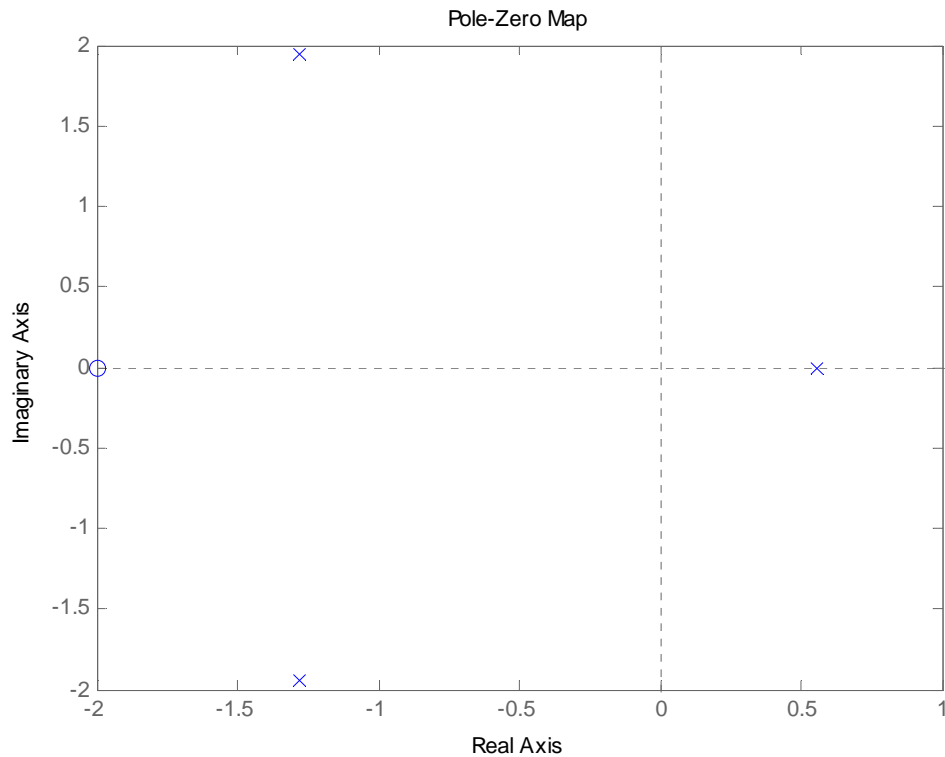


Figure 2: Plot of poles and zeros using the command pzplot()

6. Transfer function in Simulink

Simulink is a very useful component of MATLAB that allows you to create models in form of block diagrams and analyze/simulate these models. It is used by engineers all over the world. In this section, you will learn how to create a model in Simulink using a given transfer function.

To start Simulink click on the Simulink icon in the MATLAB window. See Figure 3 for details.

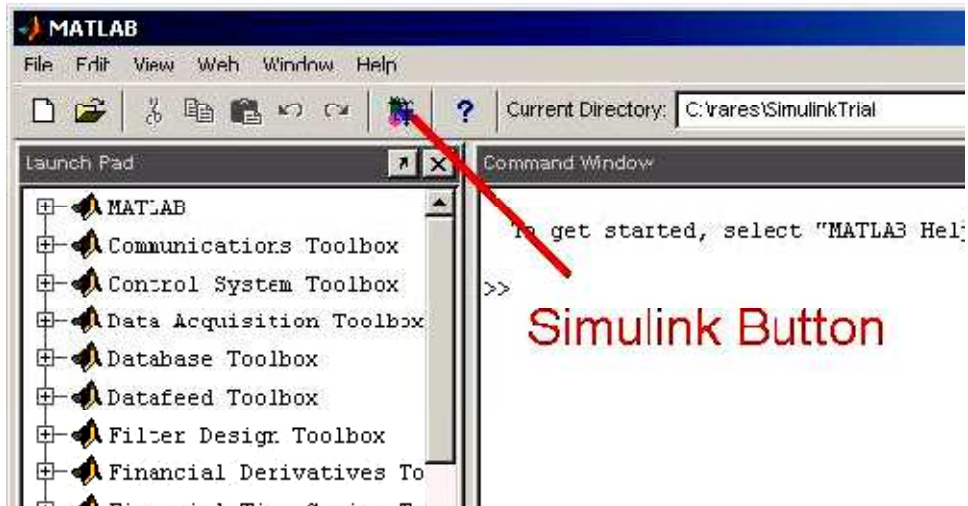


Figure 3: Starting Simulink

You will see the Simulink Library browser as shown in Figure 4.

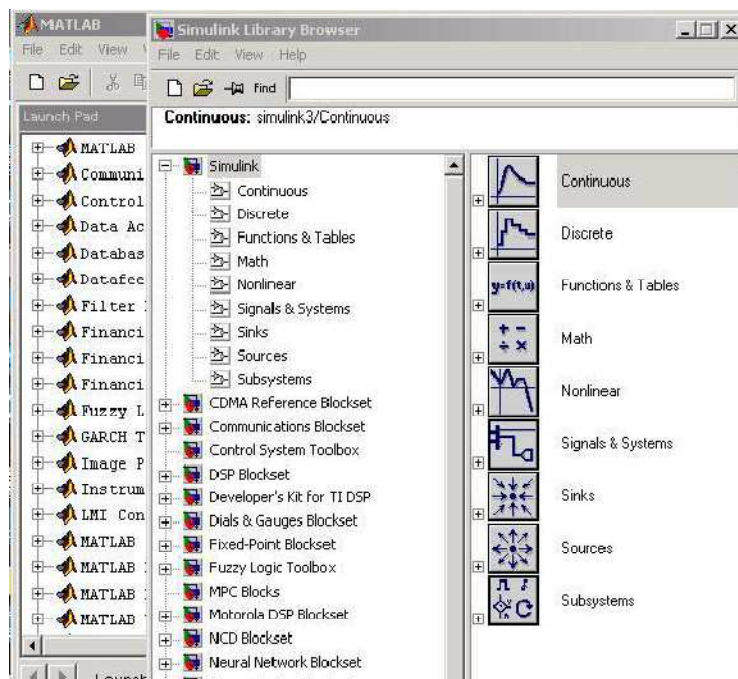


Figure 4: Simulink window

A Simulink file is called a Simulink model. Create a new model by going to File -> New -> Model.

To create a transfer function in Simulink, use the following steps:

- i. In the Simulink library browser click on Simulink
- ii. Click on Continuous

- iii. Drag the transfer function block to your new model window
- iv. Double click on transfer function block
- v. Enter the coefficients of Numerator and Denominator

See Figure 5 for details.

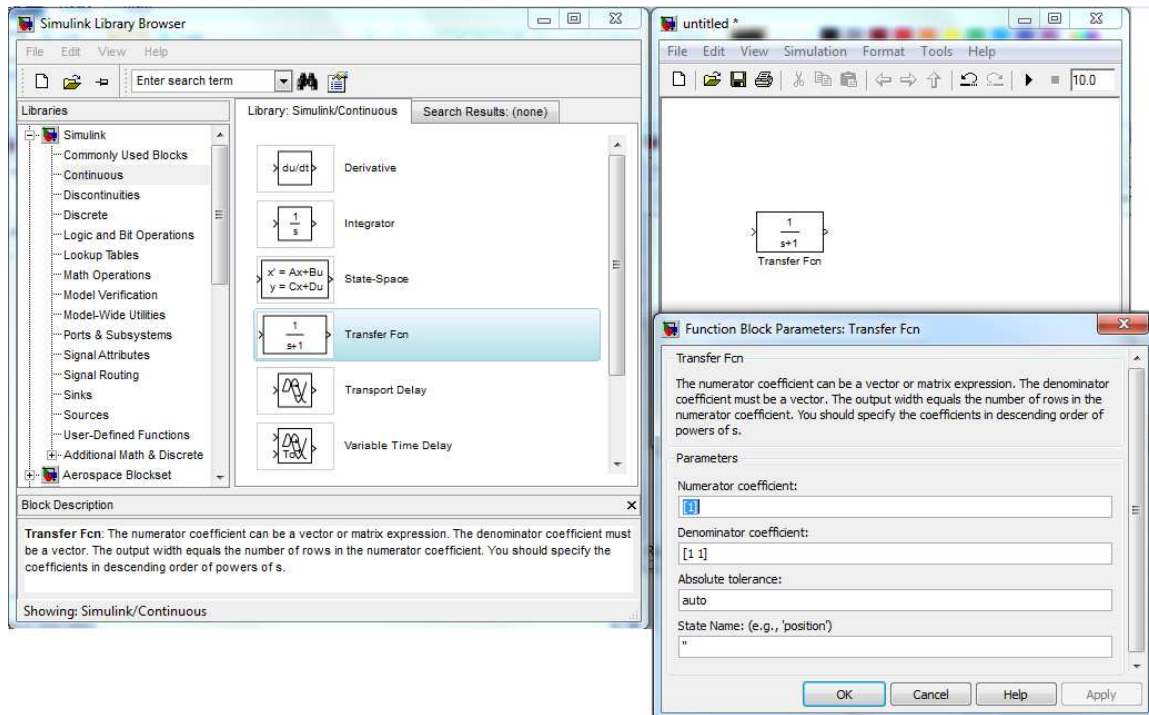


Figure 5: Transfer function block in Simulink

Exercise 5

Create a model for the following transfer functions in Simulink:

$$\frac{30s - 180}{(s^3 + 4s^2 + 13s + 7)}$$

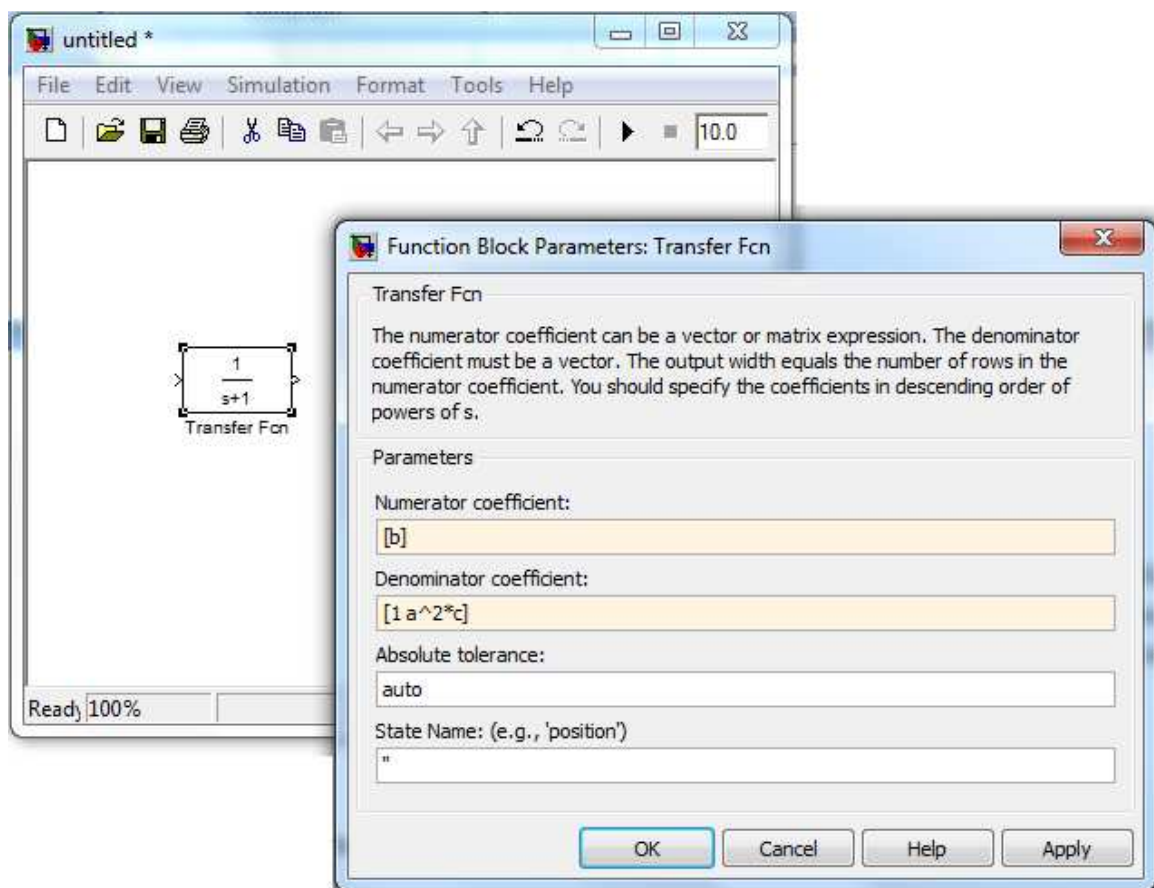
$$\frac{s^3 + s + 1}{s^3 + s^2 + 6}$$

Another very interesting feature in Simulink is that instead of using numerical values for coefficients, you can also use symbols/variables. This is useful if you want to keep your transfer functions in terms of physical parameters, e.g. mass, velocity, etc. Lets say that we want to create a model for the following transfer function

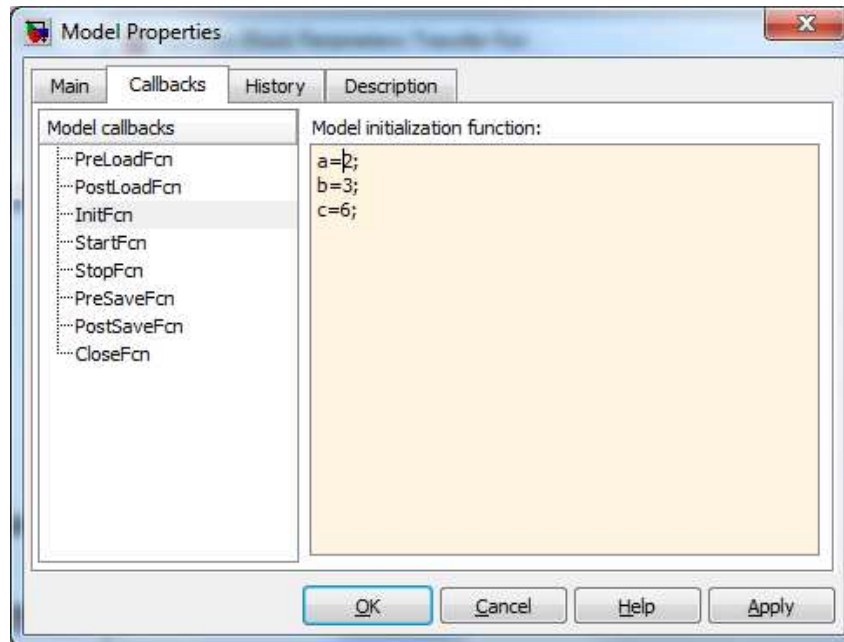
$$\frac{b}{s + a^2c}$$

where the parameter $a=2$, $b=3$ and $c=6$.

First we will create the transfer function block and enter the values shown in the figure below:



Now we have to specify the values of these parameters. For this go to File->Model properties->Callback->InitFcn. Here you can write a script to initialize the values of parameter. See the figure below for details.



Exercise 6

The transfer function of a pendulum on a cart is given by

$$\frac{\frac{mL}{q}s^2}{s^4 + \frac{b(1+mL^2)}{q}s^3 - \frac{(M+m)mgL}{q}s^2 - \frac{mgL}{q}s}$$

where

Mass of cart = $M = 0.5$ kg

Mass of pendulum = $m = 0.5$ kg

Friction of cart = $b = 0.1$ N/m/sec

Length of pendulum = $L = 0.3$ m

Inertia of pendulum = $I = 0.006$ kg m²

$q = [(M + m)(I + mL^2) - (mL)^2]$

Create a model for this transfer function in Simulink.

Also create an m file in to create this transfer function in MATLAB

In this handout we have only shown you how to create models in MATLAB and Simulink. In later weeks we will also analyze and simulate it.