Communication Systems
EE-351

Lectures 28 to 30

Types of Modulation:

- Analog Modulation
  - Amplitude Modulation
  - Angle Modulation
    - Frequency Modulation
    - Phase Modulation
  - Pulse Modulation
    - Pulse Amplitude Modulation
    - Pulse Duration/width Modulation
    - Pulse Position Modulation

- Digital Modulation
  - High freq. modulation
    - Amplitude shift keying
    - Phase shift keying
    - Frequency shift keying
    - Binary phase-shift keying
  - Low freq. modulation
    - Pulse Code Modulation
    - Delta Modulation
    - Adaptive delta modulation

Pulse Modulation:

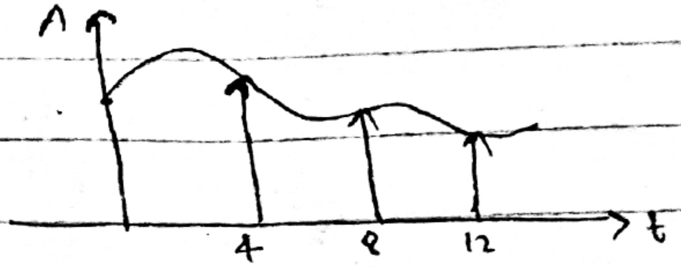
- Continuous wave modulation, some parameter of a sinusoidal carrier wave is varied continuously in accordance with the message signal.
- This is in direct contrast to pulse modulation.
- In *pulse modulation*, some parameter of a *pulse train* is varied in accordance with the message signal.
- Families of Pulse Modulation (depending on how the modulation is performed) are:
  - **Analog Pulse Modulation**
  - **Digital Pulse Modulation**
- In **analog pulse modulation**, a periodic pulse train is used as the carrier wave, and some characteristic feature of each pulse (e.g., amplitude, duration, or position) is varied in a continuous manner in accordance with the corresponding *sample* value of the message signal.
- Thus, in analog pulse modulation, information is transmitted basically in analog form, but the transmission takes place at discrete times.
- In **digital pulse modulation**, the message signal is represented in a form that is *discrete in both time and amplitude*, thereby permitting its transmission in digital form as a sequence of *coded pulses*.

Sampling Theorem:

- Converts analog signal into discrete form.
- For computer processing, real world analog signals are converted into digital form/signal.

Sampling Theorem:

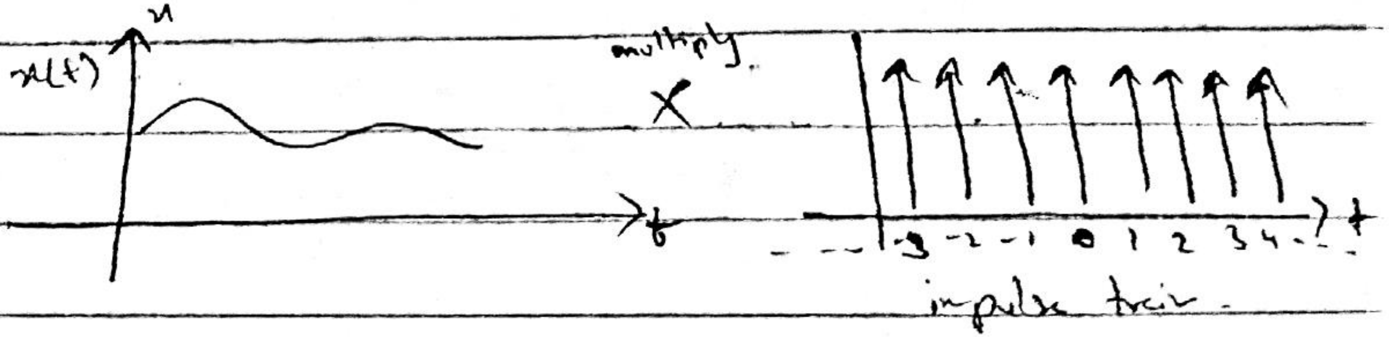
What should be this time interval?

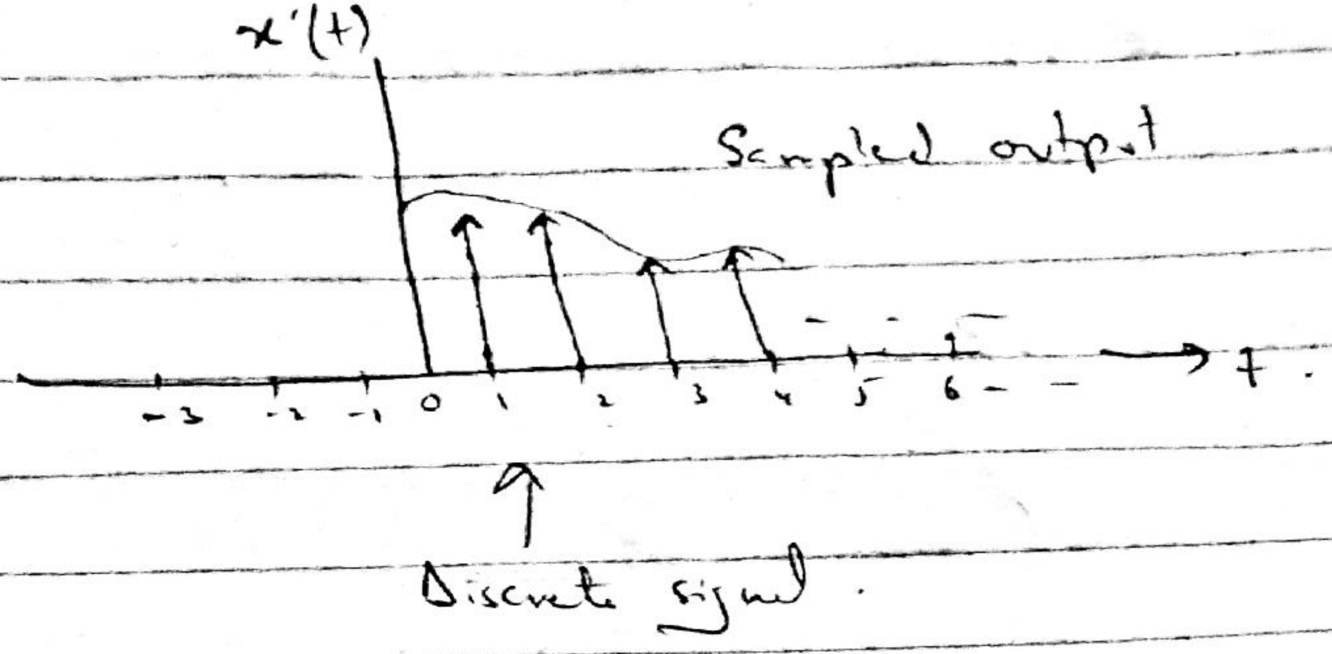


Can we reconstruct the signal?

Sample: Estimated value of signal at any instant of time.

How we can do sampling?





$x'(t) = x(t) \times$ impulse train

$\sum_{n=-\infty}^{\infty} \delta(t - nt_s)$ ← sampling instant

$t_s = 1$

δ shifted version.

$x'(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nt_s)$

$$X'(f) = F.T.\{x'(t)\}$$

$$X(f) \longleftrightarrow x(t)$$

$$\sum_{n=-\infty}^{\infty} \delta(t - nt_s) \xrightarrow{F.T} f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

$$X'(f) = X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

Constant as $t_s = 1$

$$= f_s \sum_{n=-\infty}^{\infty} X(f) * \delta(f - nf_s)$$

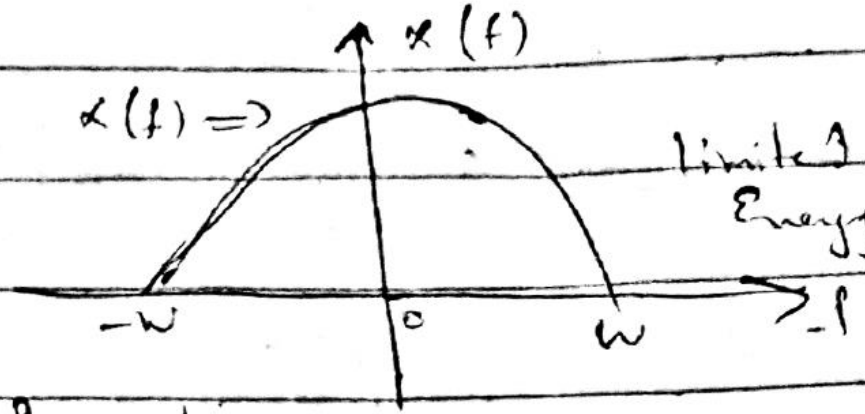
$$= f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

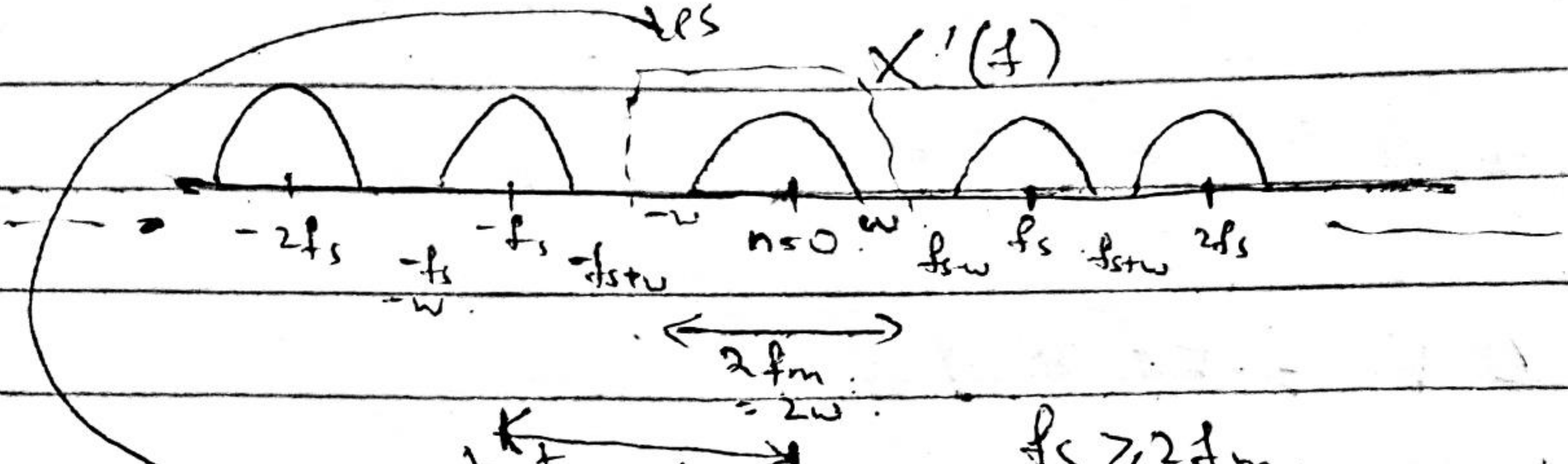
Let $x(t)$

↓

Band limited signal

↳ does not have high freq. values





to reconstruct the signal. f_s

$f_s \geq 2f_m$.

Hence, using unlimited BW to transmit a Bandlimited signal.

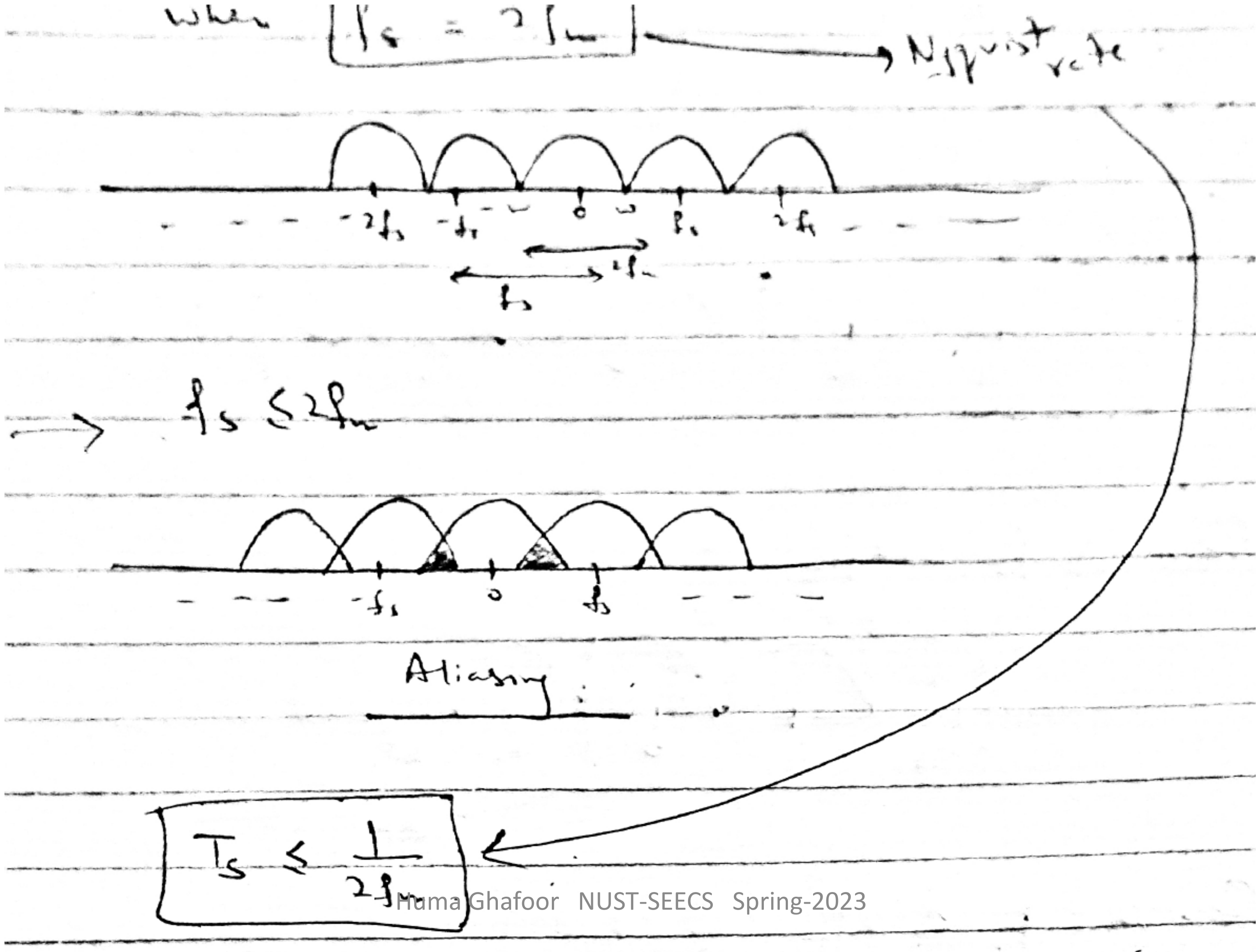
Sampling theorem.

$\boxed{f_s \geq 2 f_m}$

reconstruction is easy.

max freq of $x(f)$ i.e., W

Nyquist Rate: when $f_s = 2f_m$



Sampling Theorem:

$$X'(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

$$X'(f) \\ = \cdots + f_s X(f + 2f_s) + f_s X(f + f_s) + f_s X(f) + f_s X(f - f_s) \\ + f_s X(f - 2f_s) + \cdots$$

Frequency representation of sampled function of X.

Sampling Process:

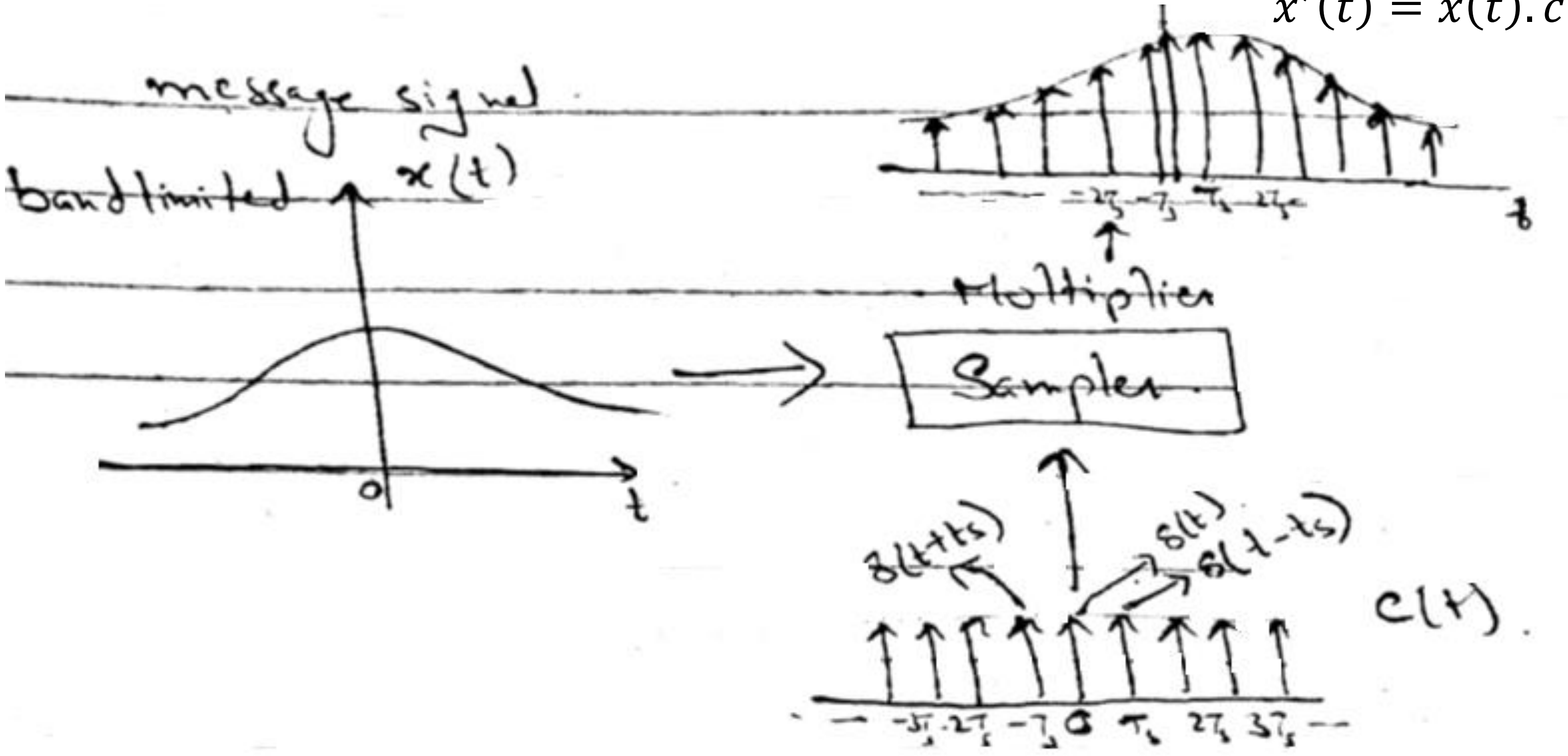
- Sampling process is basic to all pulse modulation systems. The *sampling process* is usually, but not exclusively, described in the time domain.
- *Analog pulse-modulation systems rely on the sampling process to maintain continuous amplitude representation of the message signal.*
- *In contrast, digital pulse-modulation systems use not only the sampling process but also the quantization process, which is non-reversible.*

- Making a signal periodic in the time domain has the effect of sampling the spectrum of the signal in the frequency domain.
- Sampling a signal in the time domain has the effect of making the spectrum of the signal periodic in the frequency domain. (duality property of Fourier transform)

Sampling Process:

- Continuous time signal to discrete time signal

$x'(t) = x(t).c(t)$

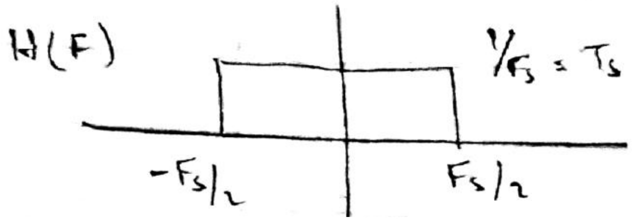


Impulse train $\sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

Periodic impulse train having fundamental time period, T_s (sampling interval)

Sampling:

$$H(f) = \begin{cases} T_s & if\ |F| \leq {}^{F_s}/_{2} \\ 0 & otherwise \end{cases}$$



$$h(t) = sinc(F_s t)$$

$$x'(t) \longrightarrow \boxed{h(t)} \longrightarrow x'(t) * h(t)$$
$$= x(t)$$

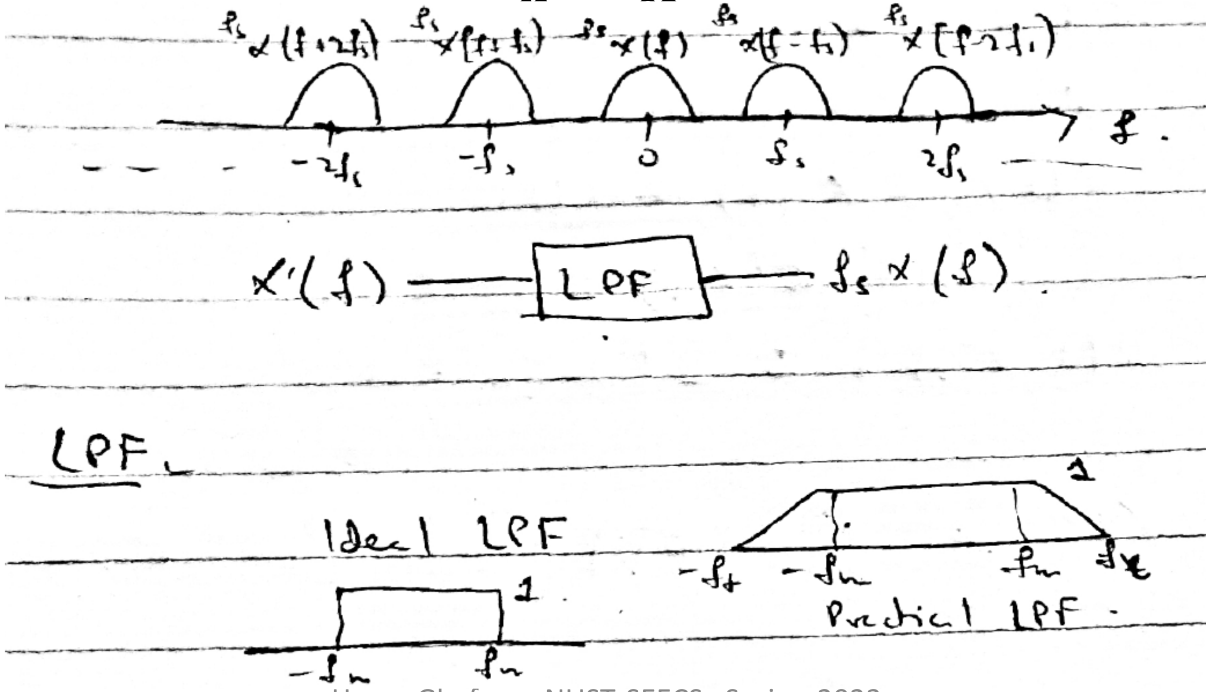
$$x'(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$$

Interpolation:

- Reconstruction of original signal from its sampled output.

$$X'(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$



► **Drill Problem 5.5** Specify the Nyquist rate and the Nyquist interval for each of the following signals:

(a) $g(t) = \text{sinc}(200t)$

(b) $g(t) = \text{sinc}^2(200t)$

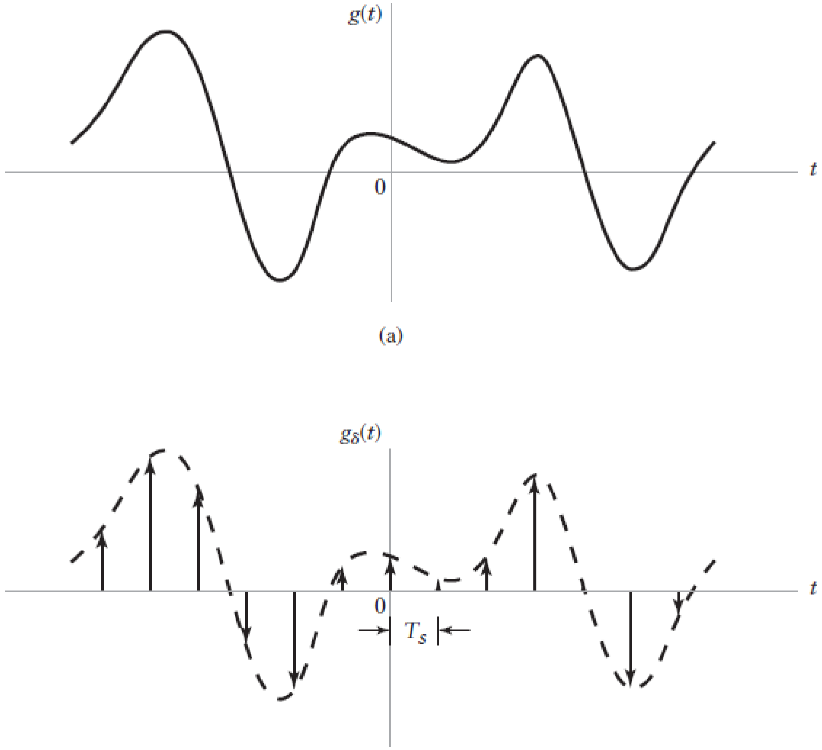
(c) $g(t) = \text{sinc}(200t) + \text{sinc}^2(200t)$ ◄

a) Nyquist rate = 200 Hz, Nyquist interval = 5ms

b) Nyquist rate = 400 Hz, Nyquist interval = 2.5ms

c) Nyquist rate = 400 Hz, Nyquist interval = 2.5ms

Analog to Digital Conversion:



(b)

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

- Sampled signal =original signal × impulse train

$$m_\delta(t) = m(t) \times g_\delta(t)$$

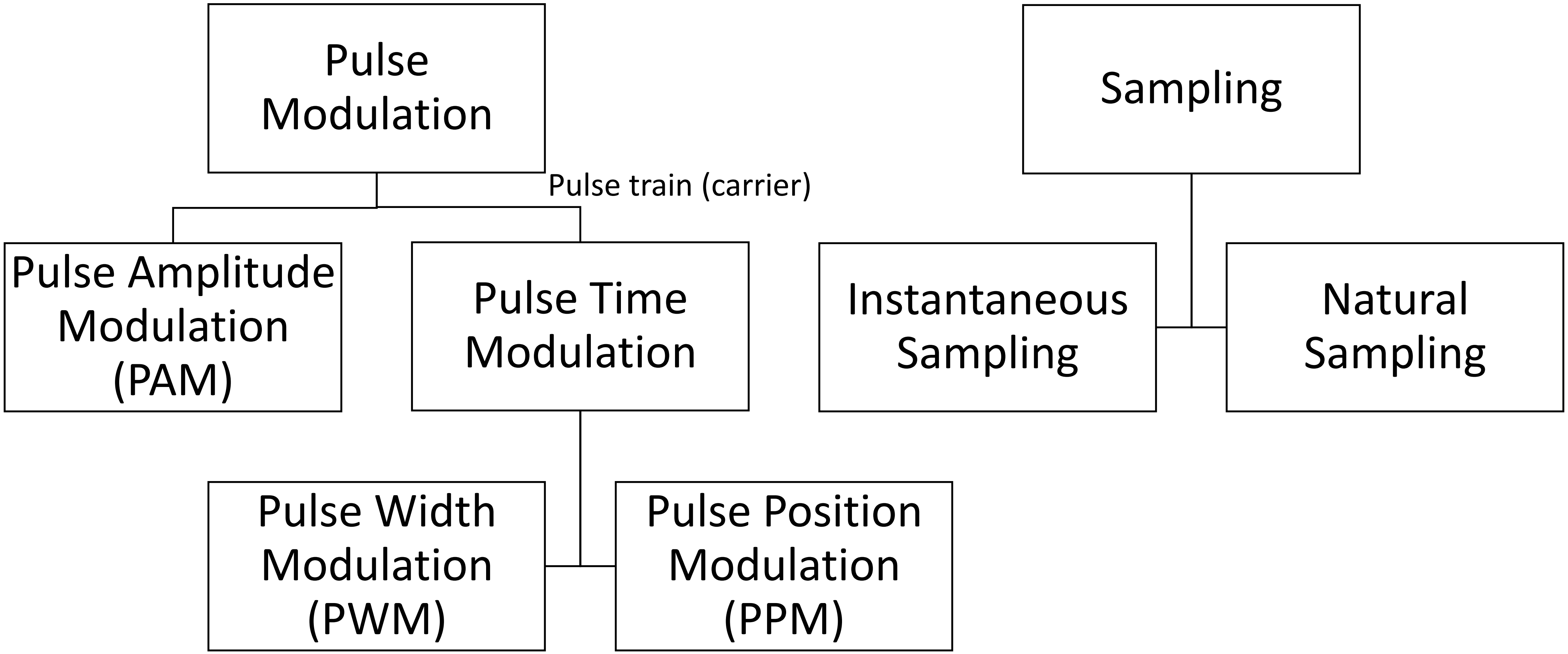
$$= m(t) \times \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s)$$

- Fourier transform of sampled signal is:

$$M_\delta(F) = M(F) * G_\delta(F)$$

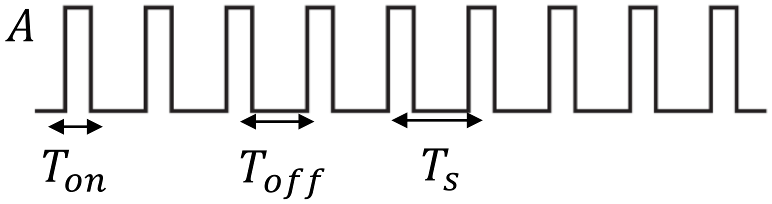
$G_\delta(F)$ is spectrum of $g_\delta(t)$ i.e. periodic signal with sampling interval T_s

Types of Sampling:

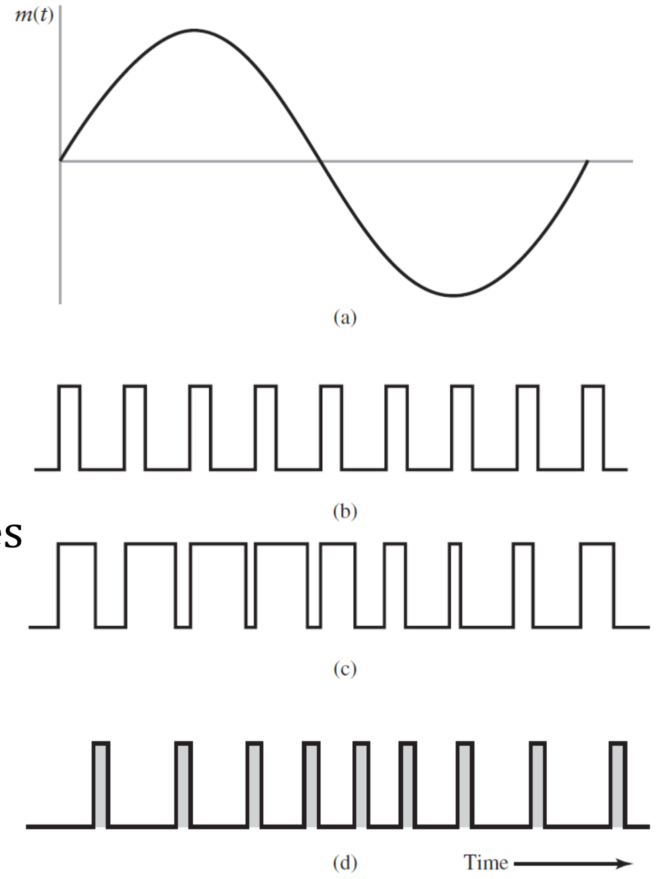


Types of Sampling:

A = Amplitude
T_{on}= pulse on time
T_{off}= pulse off time
T_s= difference between two samples or sampling time



Pulse Train



T_{on} changes
T_s= same
A = same

T_s changes
T_{on}= same
A = same

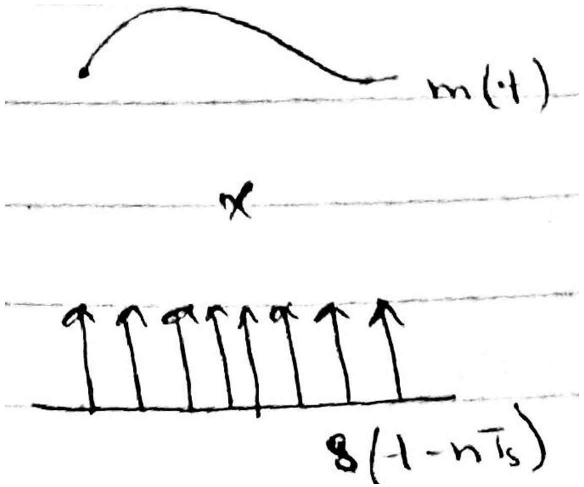
Instantaneous Sampling:

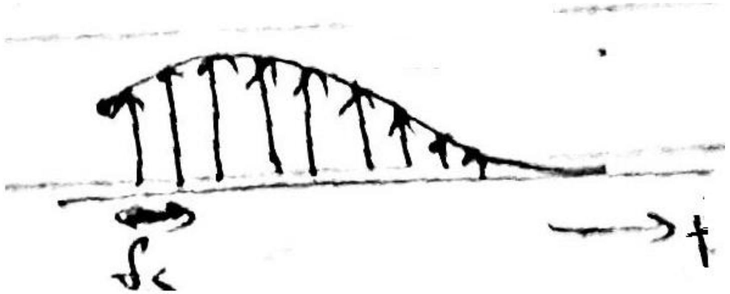
- Nyquist rate must be satisfied to generate these types of modulated signals.

$$x(t) = m(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} m(nT_s)\delta(t - nT_s)$$

$$X(f) = f_s \sum_{n=-\infty}^{\infty} M(f - nf_s)$$

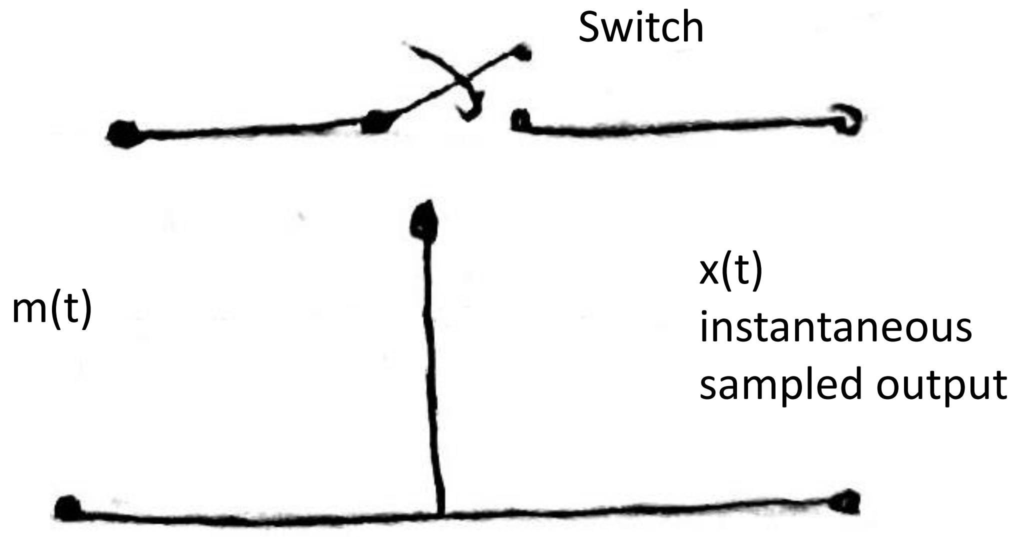




Duration (width) of this pulse is tending to zero, ∴ pulse becomes the impulse

Instantaneous Sampling:

- Limitations:
  - Practically not possible
  - Noise interference is high (due to zero width)
  - Power is low
    - Can't be transmitted over long distance

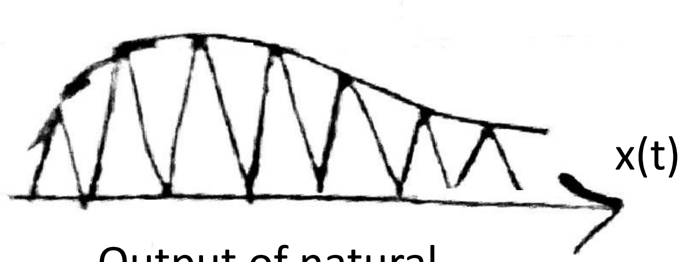


Natural Sampling:

$$x(t) = m(t) \times p(t)$$

$$= \sum_{n=-\infty}^{\infty} \frac{AT}{T_s} sinc(nf_s t)\, e^{j2\pi n f_s t} m(t)$$

- $e^{j2\pi n f_s t} m(t) = M(f - nf_s)$ (freq. shift property)
- $x(t) = \frac{AT}{T_s} \sum_{n=-\infty}^{\infty} sinc(nf_s t) M(f - nf_s)$



Output of natural sampling

Example:

- A signal $x(t) = 10cos10\pi t$ is sampled at 14 Hz rate. To recover the original signal the cutoff frequency of ideal filter should be ?

$$x(t) = 10\cos(2\pi f_m t)$$

$$f_m = 5 \text{ Hz}$$

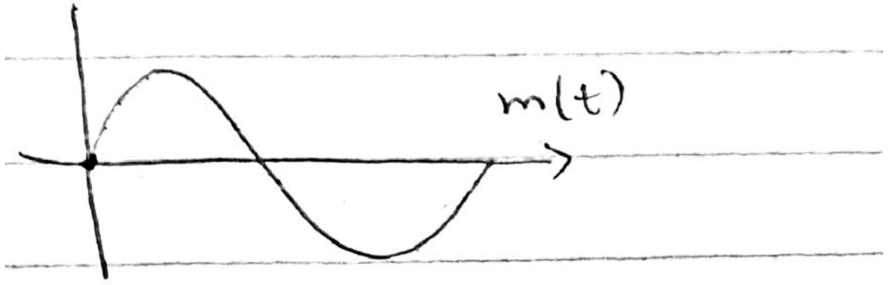
$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$$

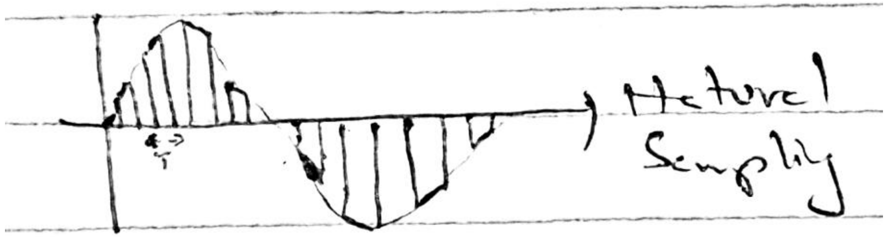
$$X(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

$$= 14 \sum_{n=-\infty}^{\infty} X(f - n14)$$

Flat-Top Sampling:

- Limitations:





- Pulse of duration T same
- Amplitude varies
- Changes amplitude with every pulse
    - Power calculation is difficult

Flat-top sampling:
Constant amplitude of each pulse