Department of Electrical Engineering

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Class/Section: BEE 12C **Semester:** 6

EE-330 Digital Signal Processing
Lab 4 # Frequency Response and Pole Zero Plots
(Taken from Signal processing first)

		PLO4-CLO4		PLO5-CLO5	PLO8-CLO6	PLO9-CLO7
Name	**Reg. No**	**Viva / Quiz / Lab Performance** **5 Marks**	**Analysis of data in Lab Report** **5 Marks**	**Modern Tool Usage** **5 Marks**	**Ethics and Safety** **5 Marks**	**Individual and Team Work** **5 Marks**
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Lab 4: Frequency Response and Pole Zero Plots of FIR/IIR Filters

Objectives

- PeZ will be used to create filters with complex conjugate poles and zeros.
- These types of filters are referred to as second-order filters.
- Second-order filters have a quadratic denominator polynomial with two roots.
- The lab's objective is to generate and analyze these types of filters.

Introduction

In order to build an intuitive understanding of the relationship between the location of poles and zeros in the z-domain, the impulse response $h[n]$ in the n-domain, and the frequency response $H(e^{j\omega})$ (the ω-domain), A graphical user interface (GUI) called **PeZ** was written in MATLAB for doing interactive explorations of the three domains[1]. **PeZ** is based on the system function, represented as a ratio of polynomials in z^{-1}, which can be expressed in either factored or expanded form as:

$$H(z) = \frac{B(z)}{A(z)} = G\frac{\prod_{k=1}^{M}(1 - z_k z^{-1})}{\prod_{\ell=1}^{N}(1 - p_\ell z^{-1})} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{\ell=1}^{N} a_\ell z^{-\ell}} \tag{1}$$

Software Used

- MATLAB R2022b

4 Introduction

In this part of the lab, you will use PeZ to create filters with complex conjugate poles and zeros. These are called *second-order filters* because the denominator polynomial is a quadratic with two roots.

4.1 PeZ

In order to build an intuitive understanding of the relationship between the location of poles and zeros in the z-domain, the impulse response $h[n]$ in the n-domain, and the frequency response $H(e^{j\omega})$ (the ω-domain), A graphical user interface (GUI) called **PeZ** was written in MATLAB for doing interactive explorations of the three domains[1]. **PeZ** is based on the system function, represented as a ratio of polynomials in z^{-1}, which can be expressed in either factored or expanded form as:

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Lab Task 1

Use the PeZ interface to implement the following second-order system:

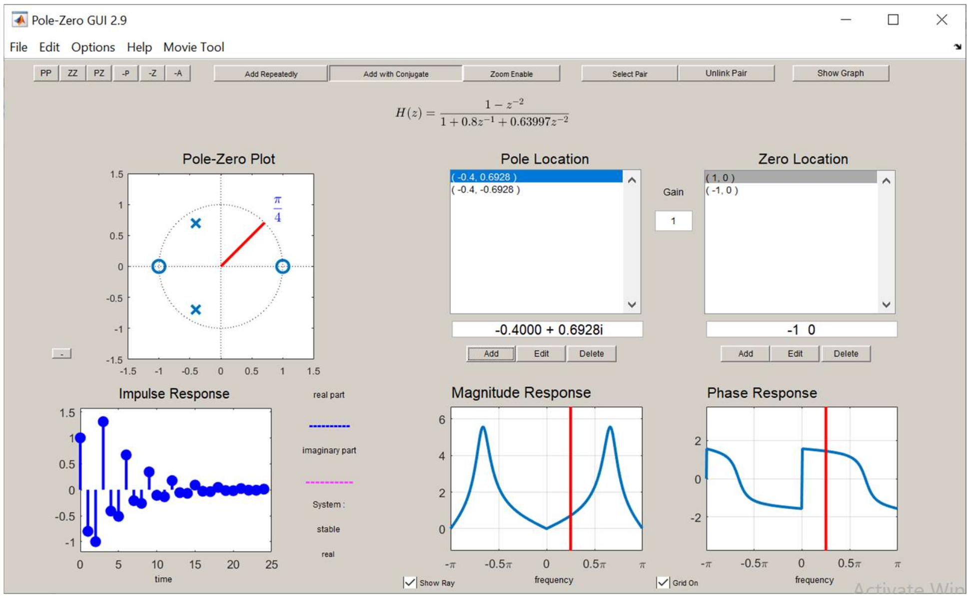
$$H(z) = \frac{1 - z^{-2}}{1 + 0.8z^{-1} + 0.64z^{-2}}$$

Look at the frequency response and determine what kind of filter you have?

Calculated the roots of poles and zeros with this code:

```
roots([1 0.8 .64])
roots([1 0 -1])
```

OUTPUT:



Band-stop filter obtained.

- Implement the following second-order system:

$$H(z) = \frac{64 + 80z^{-1} + 100z^{-2}}{1 + 0.8z^{-1} + 0.64z^{-2}}$$

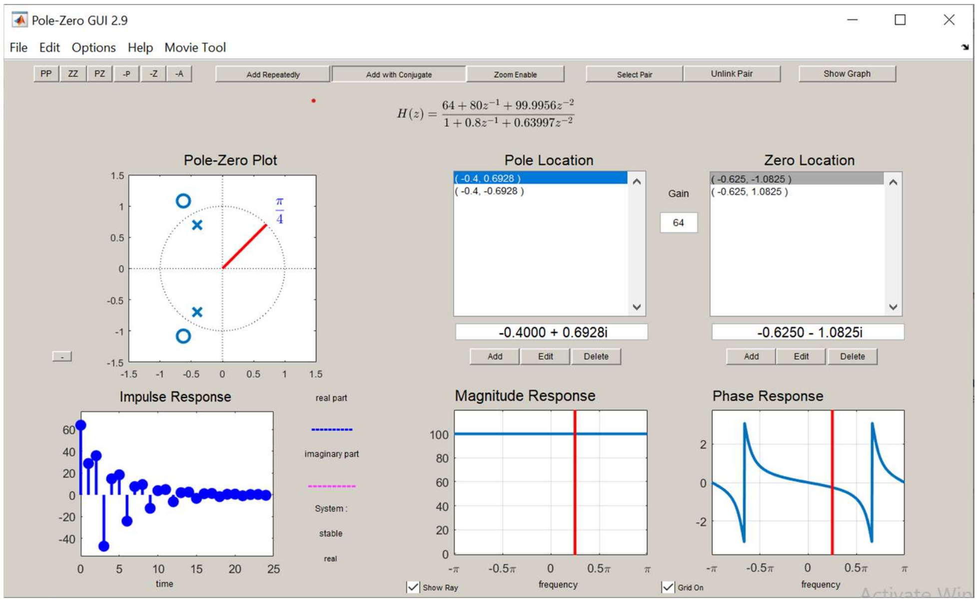
Look at the frequency response and determine what kind of filter you have.

Roots were calculated of poles and zeros by:

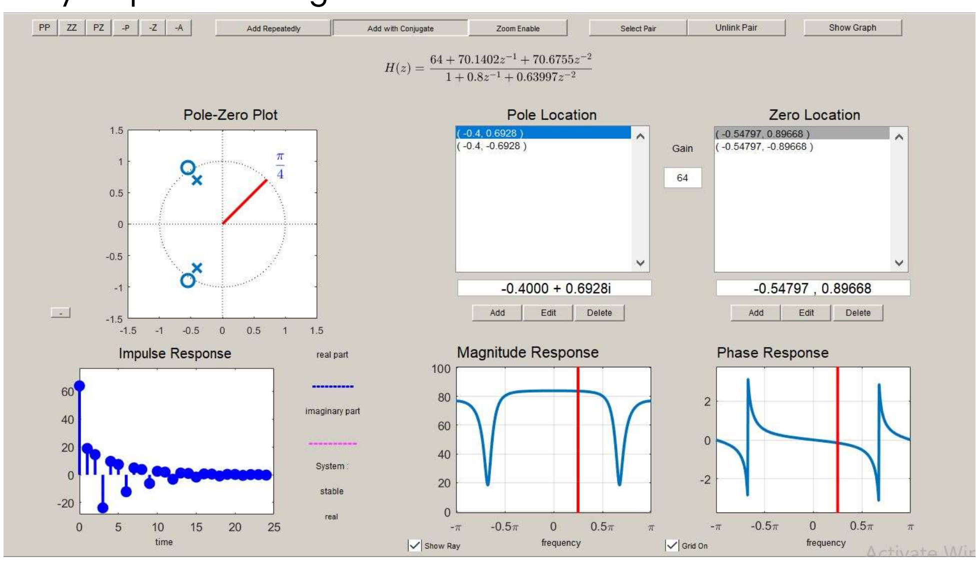
```
roots([64 80 100])
roots([1 0.8 0.64])
```

OUTPUT:

This one is an **all pass filter.**



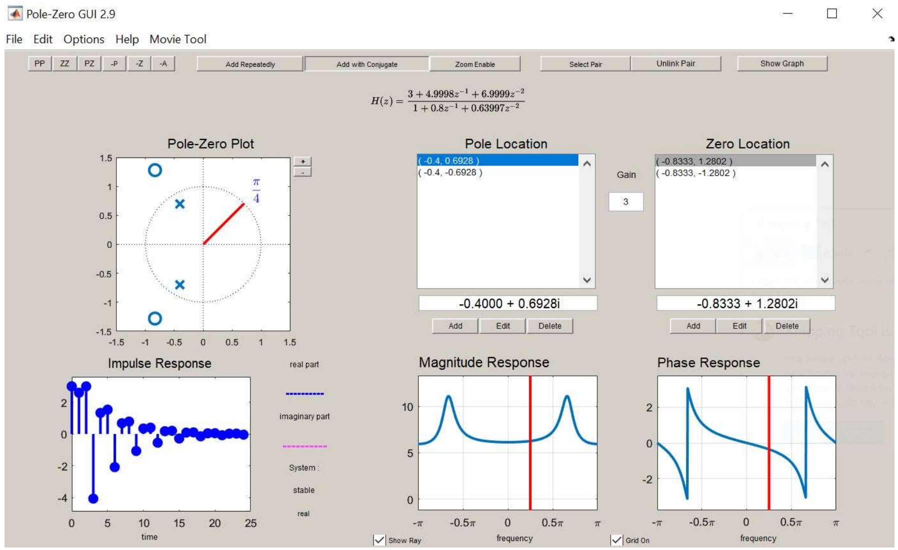
Now, use the mouse to "grab" the zero-pair and move the zeros to be exactly on the unit-circle at the same angle as the poles. Observe how the frequency response changes.



In addition, determine the H(z) for this filter. Describe the type of filter that you have now created.

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + 0.8z^{-1} + 0.64z^{-2}}$$

Taking random values for b0, b1 and b2 as 3, 5 and 7, and calculating their roots. **Band-stop filter response.**

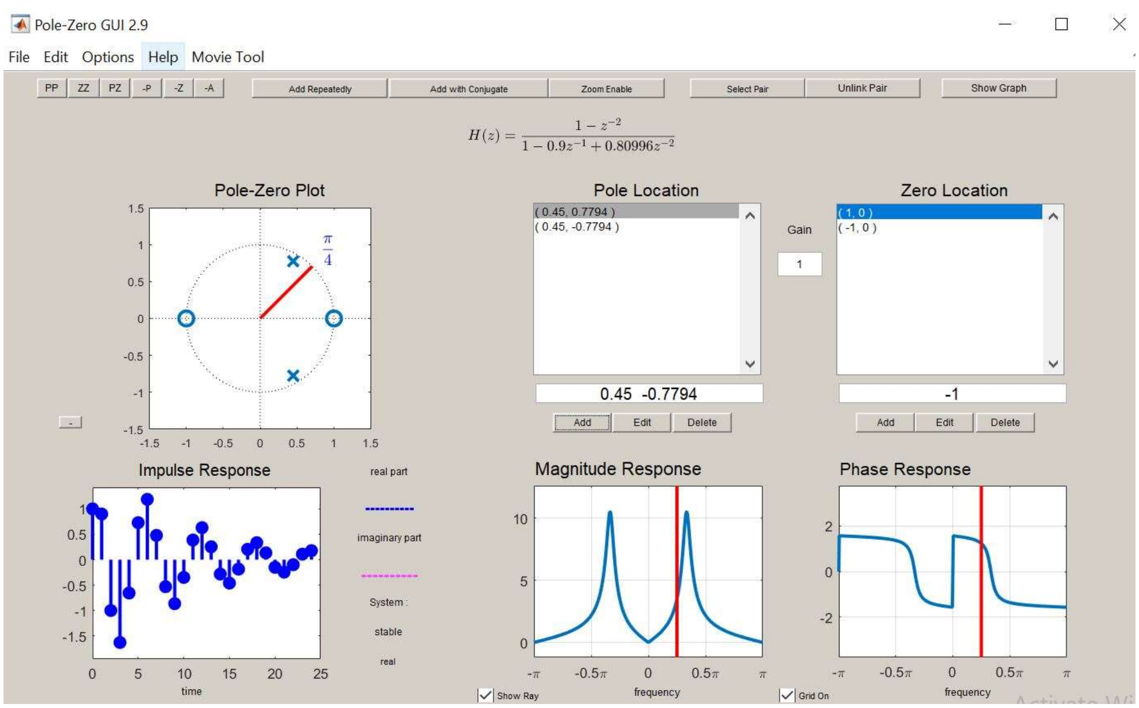


LAB TASK 2:

(a) Place following single pole-pairs

$$z = 0.9e^{\pm j\pi/3}, \text{ and zeros at } z = \pm 1.$$

Then determine the coefficients of the numerator and denominator of the resulting $H(z)$.



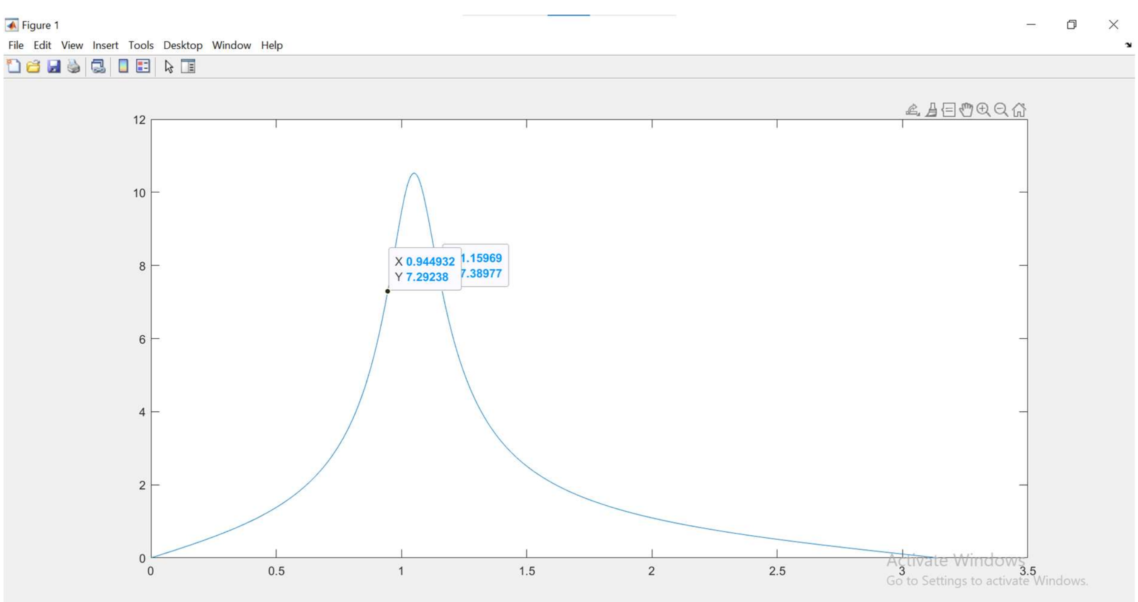
The numerator and denominator coefficients can be seen in the diagram.

(b) Make a plot of the frequency response (magnitude only) with **freqz** and measure the width of the peak versus frequency. This presents a problem because we must define how to measure width. The usual definition is to measure the width at the "3-dB level." In order to do this, the measurement must be made with respect to the peak value of the frequency response. If the peak value is H_{max} , then the "3-dB level" is at 0.707 H_{max} [4].

```
z = 0.9*exp(j*pi/3);
num = [1 0 -1];
den = [1 -0.9 0.81];
[h, w] = freqz(num, den);
plot(w, abs(h));
db3 = max(abs(h) * 0.707)
```

OUTPUT:

```
db3_level =

    7.4417
```



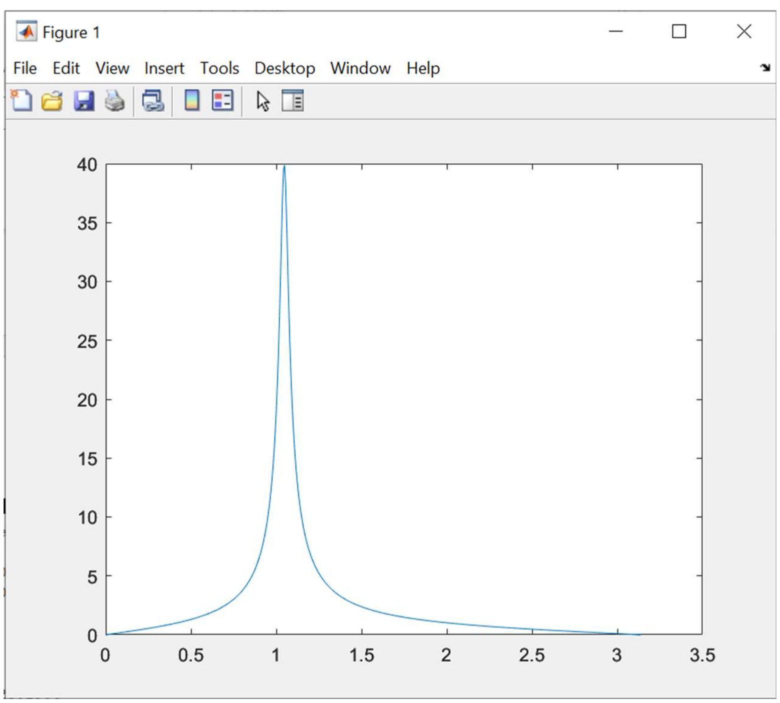
Width can be calculated as:
Width = 1.15 - 0.94 = **0.21**

(c) Move the pole-pair so that the angles remain fixed at ±π/3, but the radius is r = 0.95 and r = 0.975. In each case, measure the 3-db width of the peak. Using these measured values, create a formula for the width that is proportional to (1 - r), e.g., the following works quite well PeakWidth ≈ K (1 - r)/√r where K is a constant of proportionality.

```
pp = 0.975 * exp(1i * pi / 3);
pp;conj(pp);
b = [1 0 -1];
a = [1 -0.975 0.95];
[h, w] = freqz(b, a);
plot(w, abs(h))
db3 = max(abs(h) * 0.707);
display(db3)
```

OUTPUT:

```
poles =

   0.4875 + 0.8444i
   0.4875 - 0.8444i


db3 =

   28.1906
```

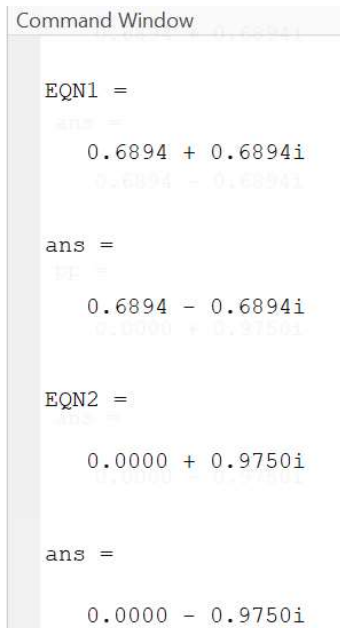


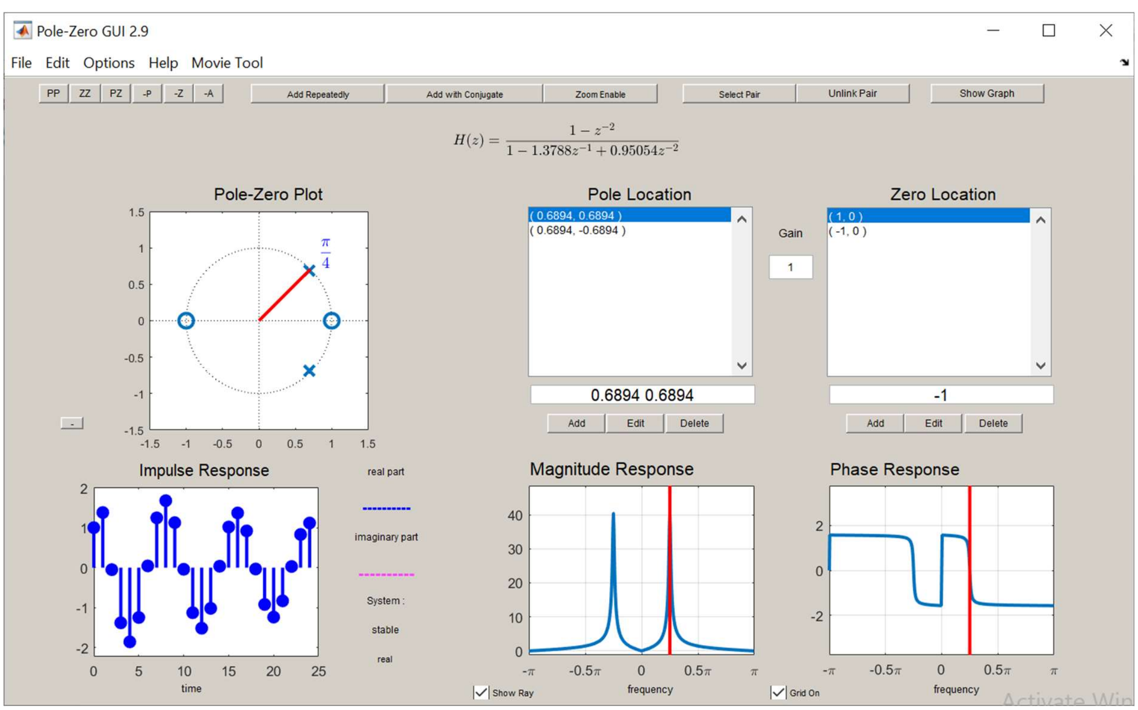
Peak width = K (1 - r)/√r. (r = 0.95 and 0.975) **K = 1.95**

(d) Move the pole-pair so that its radius remains fixed and the angles change from ±π/3 to ±π/4 and then to ±π/2. State a formula for the peak location as a function of the pole location.

```
EQN1 = 0.975 * exp(1i * pi / 4)
EQN1; conj(EQN1)
EQN2 = 0.975 * exp(1i * pi / 2)
EQN2; conj(EQN2)
```

OUTPUT:





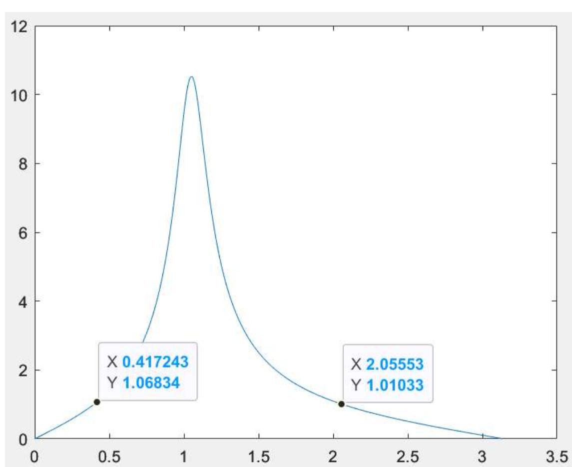
Peak Location =
It is at $\tan^{-1}(y/x)$. Both y and x are imaginary numbers.

Passband & Stopband

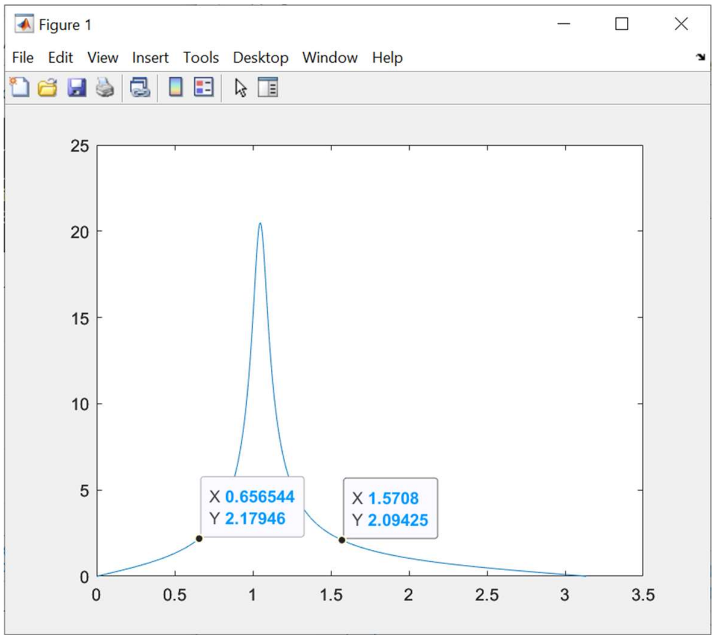
(a) Determine the stopband regions for three of the filters designed in the previous section. Use the cases where the pole angles are $\pm\pi/3$ and the radii are r = 0.9, 0.95 and 0.975. In each case, measure the frequency regions of the two stopbands. There is one lower stopband for $0 \le \omega \le \omega s1$ and one upper stopband for $\omega s2 \le \omega \le \pi$.

```
%First filter
b = [1 0 -1];
a = [1 -0.9 0.81];
[h, w] = freqz(b, a);
plot(w, abs(h))
db3 = max(abs(h) * 0.1);
display(db3)

%Second filter
b = [1 0 -1];
a = [1 -0.95 0.90246];
[h, w] = freqz(b, a);
plot(w, abs(h))
db3 = max(abs(h) * 0.1);
display(db3)

%Third filter
b = [1 0 -1];
a = [1 -0.9748 0.95006];
[h, w] = freqz(b, a);
plot(w, abs(h));
```

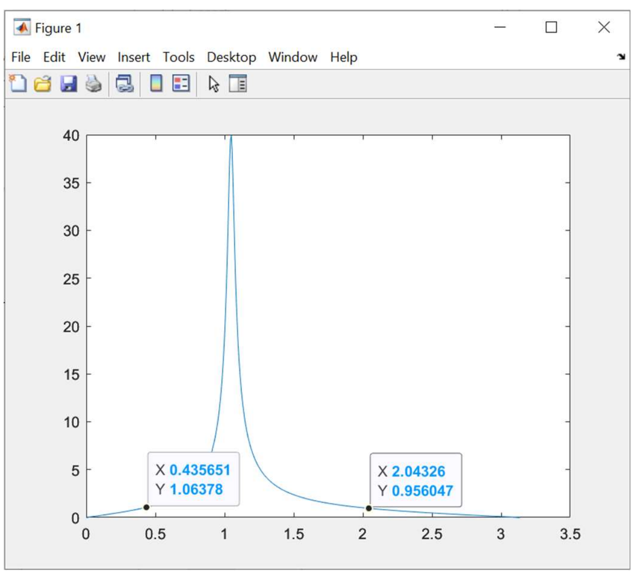
Part 1:



Part 2:



Part 3:



(c) Usually, filter design becomes difficult when we want the passband and stopband edges to be very close to one another. The difference between neighboring passband and stopband edges is called the Transition Width. Therefore, summarize the measurements of the previous two parts in a table that lists the two transition widths for each filter versus r. Does it depend on r?

From the measurements in the previous two parts, we can see that the transition widths (both for the passband and stopband) decrease as r increases. Therefore, the transition widths depend on r, and they become narrower as the value of r increases.

Conclusion

This lab explored the use of PeZ to generate second-order filters with complex conjugate poles and zeros. These filters are characterized by a quadratic denominator polynomial with two roots. The objectives of the lab were to create and analyze these filters using MATLAB, with a focus on understanding their frequency response and pole-zero plots. Overall, the lab was a valuable learning experience for exploring the properties of second-order filters.