# Communication Systems
# EE-351

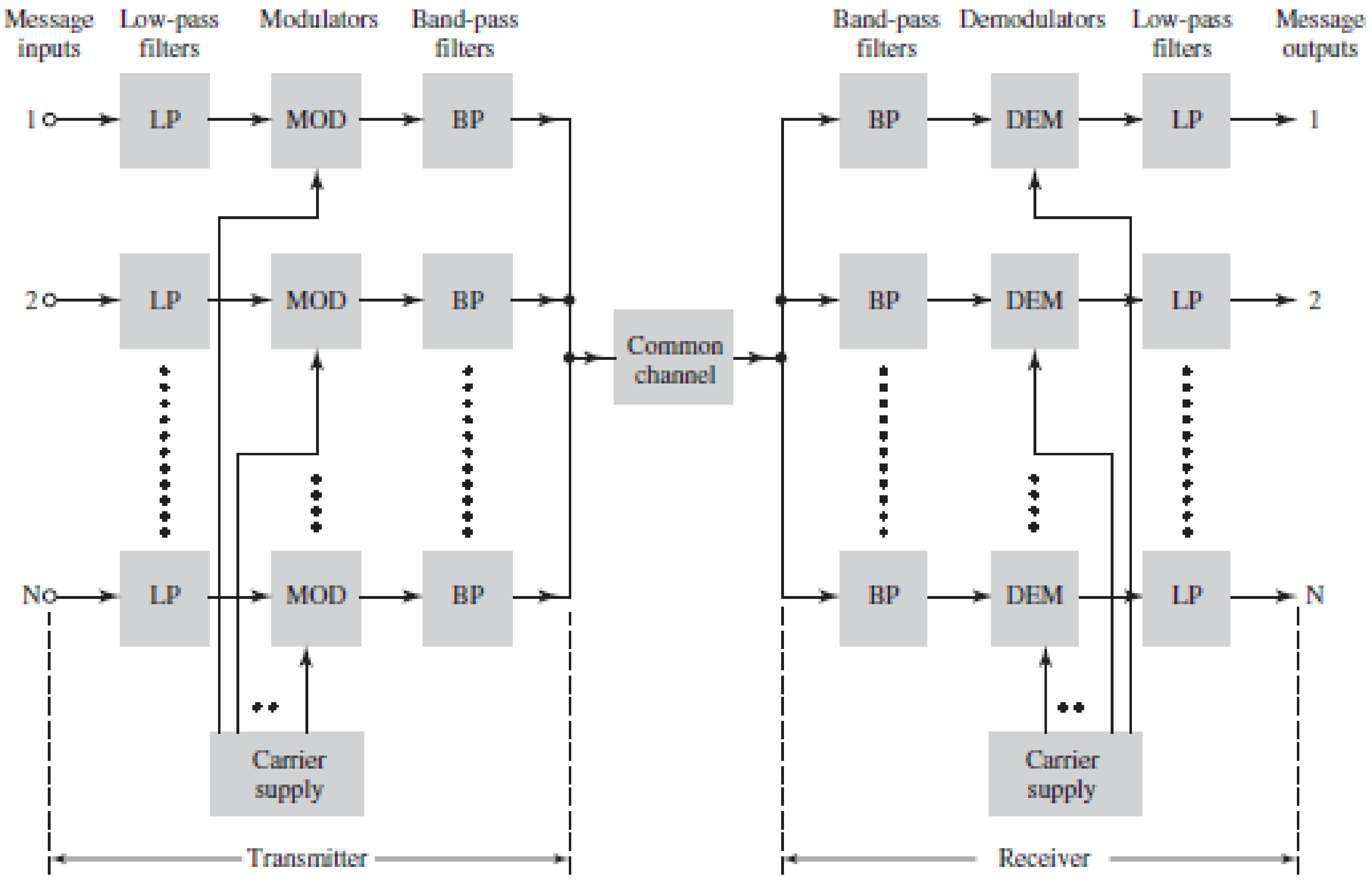
## Lectures 14 to 16

# Multiplexing:

- To transmit a number of independent signals by combining into a composite signal suitable for transmission over a common channel, an important signal processing operation used in analog communications is multiplexing.
- To transmit a number of these signals over the same channel (e.g. cable), the signals must be kept apart so that they do not interfere with each other, and thus they can be separated at the receiving end.
- This is done by separating the signals either in frequency or in time.

# Frequency Division Multiplexing (FDM):

- Voice frequencies transmitted over telephone systems, for example, range from 300 to 3100 Hz.



# Angle Modulation:

# Angle Modulation:

- Another way of modulating a sinusoidal carrier wave—namely, **angle modulation**, in which the angle of the carrier wave is varied according to the information-bearing signal.
  - the amplitude of the carrier wave is maintained constant.

# Few basics:

- An important feature of angle modulation is that it can **provide better discrimination against noise and interference** than amplitude modulation.
  - this improvement in performance is achieved **at the expense of increased transmission bandwidth**.
- Angle modulation provides us with a <u>practical means of exchanging channel bandwidth for improved noise performance</u>.
- Such a tradeoff is not possible with amplitude modulation.
- Moreover, the improvement in noise performance in angle modulation is achieved at the **cost of increased system complexity** in both the transmitter and receiver.

- Definitions
    - Phase deviation, $\Delta\theta$
    - Frequency deviation, $\Delta f$
    - Modulation index, $\beta$
    - Relationship

    - Power
    - Bandwidth

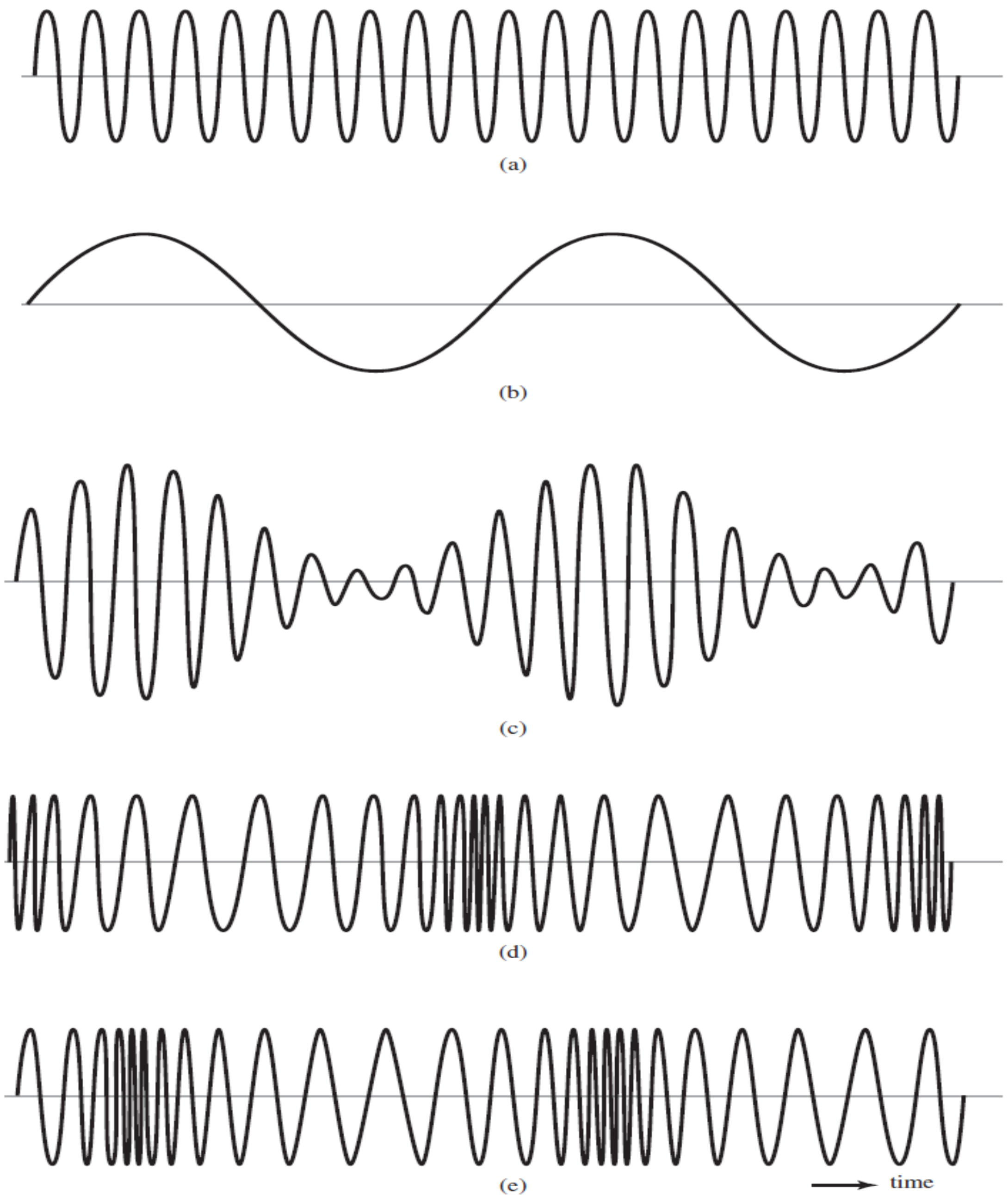



**FIGURE 4.1** Illustration of AM, PM, and FM waves produced by a single tone. (*a*) Carrier wave. (*b*) Sinusoidal modulating signal. (*c*) Amplitude-modulated signal. (*d*) Phase-modulated signal. (*e*) Frequency modulated signal.

# Angle Modulation:

- There are an infinite number of ways in which the angle, $\theta_i(t)$ may be varied in some manner with the message signal.
  - $\theta_i(t)$ denotes the angle of a modulated sinusoidal carrier at time t
- Only two commonly used methods are considered here:
  - Phase modulation
  - Frequency modulation

# Angle Modulation:

- Angle modulated wave: $s(t) = A_c cos\theta_i(t)$

Phase Modulation:

- Phase of carrier is modulated by message signal

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

- $\theta_i(t)$: instantaneous angle
- $k_p$: phase sensitivity factor of the modulator;
  - Unit: radians per volt on the assumption that message signal is in volts
- $2\pi f_c t$: angle of unmodulated carrier with the constant $\varphi_c$ set equal to zero for convenience of presentation;
- Phase modulated signal is $\therefore$ $s(t) = A_c\cos(2\pi f_c t + k_p m(t))$

# Angle Modulation:

Frequency Modulation:

- Frequency of carrier is modulated by message signal

$$f_i(t) = f_c + k_f m(t)$$

- $f_i(t)$: instantaneous frequency (frequency after modulation)
- $k_f$: frequency sensitivity factor of the modulator expressed in hertz per volt;
- $f_c$: frequency of unmodulated carrier

Also,

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

$$\therefore \theta_i(t) = 2\pi \int_0^t f_i(\tau)d\tau$$

Angle, $\theta_i(t) = 2\pi f_c t + \varphi\ _c$

$$\frac{d\theta_c(t)}{dt} = 2\pi f_c$$

$$\frac{d\theta_i(t)}{dt} = 2\pi f_i(t)$$

$$\theta_i(t) = 2\pi \int_0^t f_i(\tau) d\tau$$

# Angle Modulation:

- instantaneous phase of FM is

$$\theta_i(t) = 2\pi \int_0^t (f_c + k_f m(\tau)) d\tau = 2\pi f_c \mathrm{t} + 2\pi k_f \int_0^t m(\tau) d\tau$$

- Frequency modulated signal is $\therefore$ $s(t) = A_c \cos(2\pi f_c \mathrm{t} + 2\pi k_f \int_0^t m(\tau) d\tau)$

From the definition of $f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$, instantaneous freq. of PM is

$$f_i(t) = \frac{1}{2\pi} \frac{d(2\pi f_c t + k_p m(t))}{dt} = f_c + \frac{k_p}{2\pi} \frac{dm(t)}{dt}$$

# Summary of Basic Definitions in Angle Modulation:

**TABLE 4.1 *Summary of Basic Definitions in Angle Modulation***

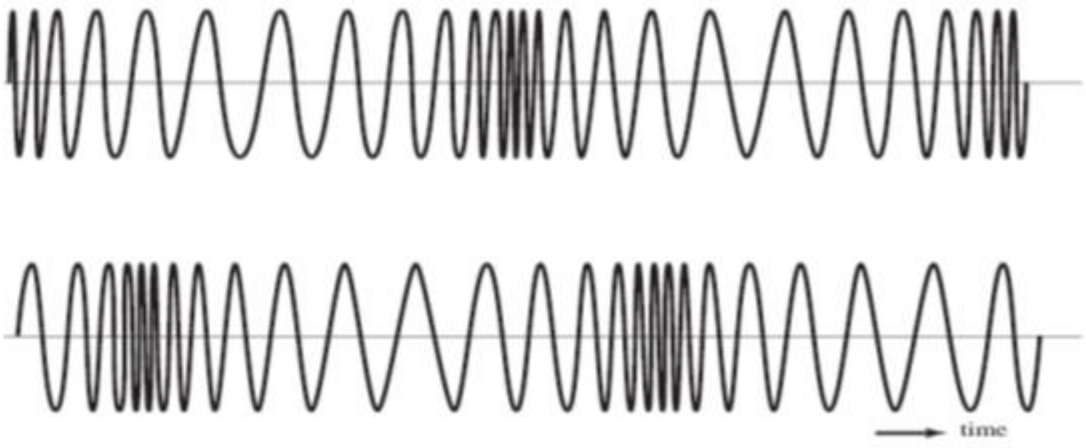
| | *Phase modulation* | *Frequency modulation* | *Comments* |
|---|---|---|---|
| Instantaneous phase $\theta_i(t)$ | $2\pi f_c t + k_p m(t)$ | $2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$ | $A_c$: carrier amplitude<br>$f_c$: carrier frequency<br>$m(t)$: message signal<br>$k_p$: phase-sensitivity factor<br>$k_f$: frequency-sensitivity factor |
| Instantaneous frequency $f_i(t)$ | $f_c + \frac{k_p}{2\pi}\frac{d}{dt}m(t)$ | $f_c + k_f m(t)$ | |
| Modulated wave $s(t)$ | $A_c \cos[2\pi f_c t + k_p m(t)]$ | $A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$ | |

These definitions apply to all kinds of message signals, be they of the analog or digital kind.

**PROPERTY 1 Constancy of transmitted power** *From both Eqs. (4.4) and (4.7), we readily see that the amplitude of PM and FM waves is maintained at a constant value equal to the carrier amplitude $A_c$ for all time $t$, irrespective of the sensitivity factors $k_p$ and $k_f$. This property is well demonstrated by the PM wave of Fig. 4.1(d) and FM wave of Fig. 4.1(e). Consequently, the average transmitted power of angle-modulated waves is a constant, as shown by*

$$P_{\text{av}} = \frac{1}{2} A_c^2 \tag{4.8}$$

*where it is assumed that the load resistor is 1 ohm.*



**PROPERTY 2 Nonlinearity of the modulation process** *Another distinctive property of angle modulation is its nonlinear character. We say so because both PM and FM waves violate the principle of superposition. Suppose, for example, that the message signal $m(t)$ is made up of two different components $m_1(t)$ and $m_2(t)$, as shown by*

$$m(t) = m_1(t) + m_2(t)$$

*Let $s(t)$, $s_1(t)$, and $s_2(t)$ denote the PM waves produced by $m(t)$, $m_1(t)$, and $m_2(t)$ in accordance with Eq. (4.4), respectively. In light of this equation, we may express these PM waves as follows:*

$$s(t) = A_c \cos[2\pi f_c t + k_p(m_1(t) + m_2(t))]$$

$$s_1(t) = A_c \cos[2\pi f_c t + k_p m_1(t)]$$

*and*

$$s_2(t) = A_c \cos[2\pi f_c t + k_p m_2(t)]$$

*From these expressions, despite the fact that $m(t) = m_1(t) + m_2(t)$, we readily see that the principle of superposition is violated because*

$$s(t) \neq s_1(t) + s_2(t)$$

# Consider a specific example of sinusoidal modulation

Phase Modulation:

To prove max. phase deviation = modulation index

Let,

$$m(t) = A_m \sin(2\pi f_m t)$$

Sinusoidal message signal with amplitude $A_m$

$$\theta_i(t) = 2\pi f_c t + k_p m(t) = 2\pi f_c t + k_p A_m \sin(2\pi f_m t)$$

Hence, angle modulated signal is,

$$s(t) = A_c cos\theta_i(t) = A_c \cos(2\pi f_c t + k_p A_m \sin(2\pi f_m t))$$

Instantaneous Frequency is

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = \frac{1}{2\pi} \frac{d}{dt} [2\pi f_c t + k_p A_m \sin(2\pi f_m t)]$$

# Consider a specific example of sinusoidal modulation

$$= f_c + \frac{1}{2\pi} k_p A_m 2\pi f_m \cos(2\pi f_m t)$$

$$f_i(t) = f_c + k_p A_m f_m \cos(2\pi f_m t)$$

Frequency Deviation can be defined as:

$$\Delta f = max|f_i(t) - f_c| = max\left|k_p A_m f_m \cos(2\pi f_m t)\right|$$

*the maximum departure of the instantaneous frequency of the FM wave from the carrier frequency*

$$\Delta f = k_p A_m f_m$$

Modulation index, $\beta = \frac{\Delta f}{f_m} = \frac{k_p A_m f_m}{f_m} = k_p A_m$

*The ratio of the frequency deviation to the modulation frequency is commonly called the **modulation index of the FM wave**.*

# Consider a specific example of sinusoidal modulation

Similarly,

$$\theta_i(t) = 2\pi f_c t + k_p m(t) = 2\pi f_c t + k_p A_m \sin(2\pi f_m t)$$

Phase Deviation can be defined as:

$$\Delta\theta = max|\theta_i(t) - 2\pi f_c t| = max\left|k_p A_m \sin(2\pi f_m t)\right| = k_p A_m = \beta$$

*the maximum departure of the angle $\theta_i(t)$ from the angle $2\pi f_c t$ of the unmodulated carrier.*

$$\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$$

The FM wave itself is given by

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

# Consider a specific example of sinusoidal modulation

Likewise, in case of Frequency modulation,

$$m(t) = A_m \cos(2\pi f_m t)$$
$$f_i(t) = f_c + k_f m(t)$$
$$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t)$$

$$\Delta f = k_F A_m$$

$\Delta f$ is frequency deviation

$$f_i(t) = f_c + \Delta f \cos(2\pi f_m t)$$
$$f_i(t)_{max} = f_c + \Delta f$$

And Modulation index, $\beta = \frac{\Delta F}{F_m} = \frac{k_F A_m}{F_m}$

Therefore,

$$\Delta\theta = \frac{k_F A_m}{F_m} = \beta$$

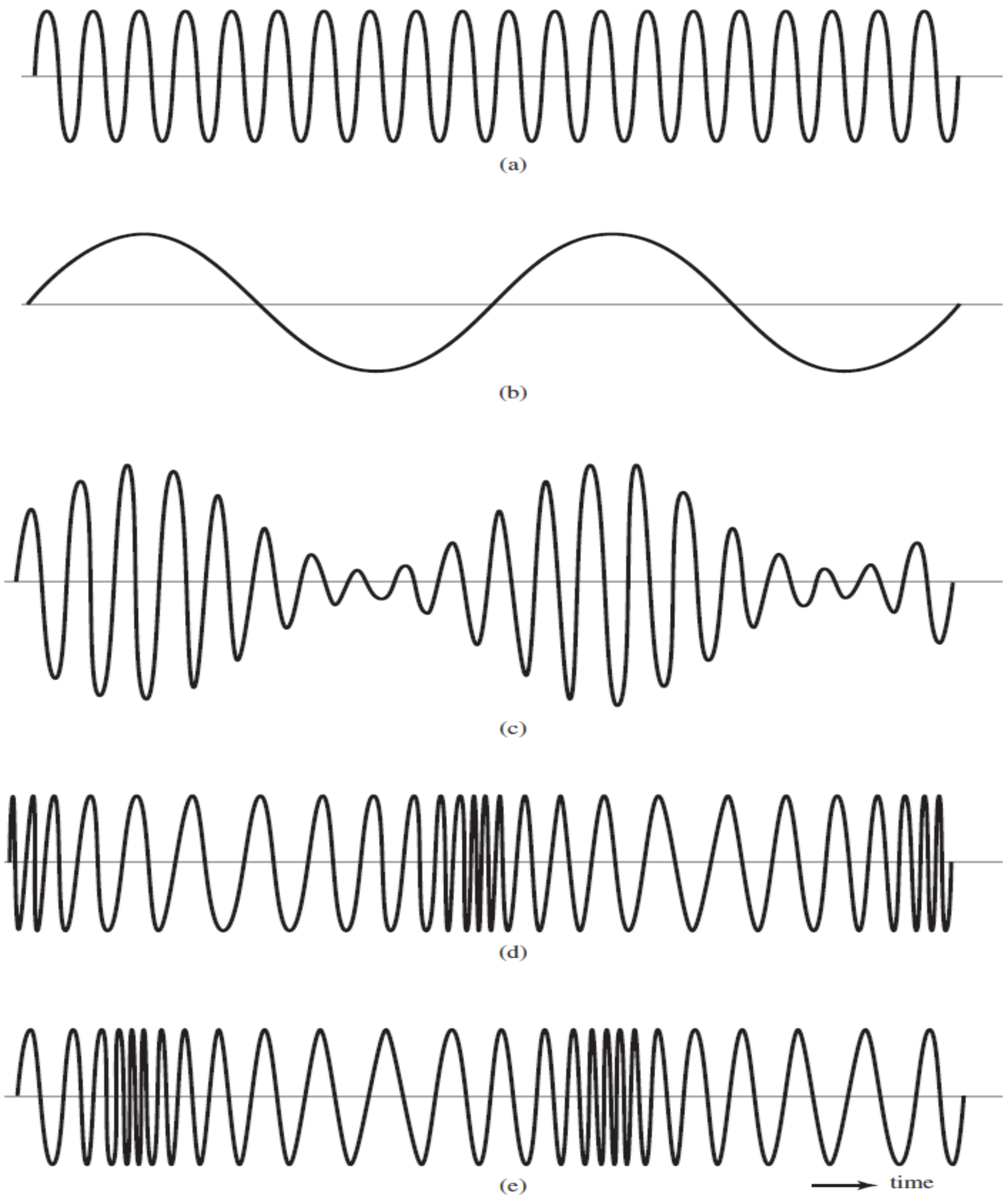
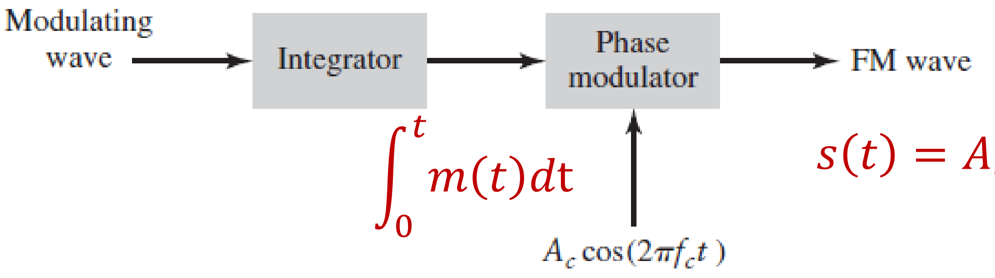



FIGURE 4.1 Illustration of AM, PM, and FM waves produced by a single tone. (a) Carrier wave. (b) Sinusoidal modulating signal. (c) Amplitude-modulated signal. (d) Phase-modulated signal. (e) Frequency modulated signal.

# Relationship btw PM and FM waves:

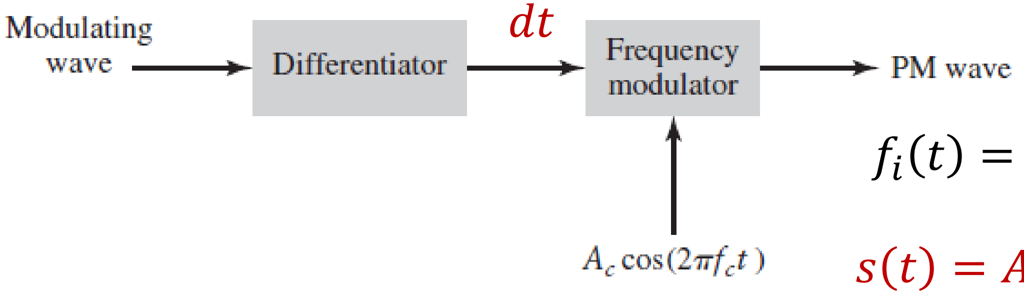



$$\int_0^t m(t)dt$$

$$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(t)dt)$$

(a)

$$\frac{dm(t)}{dt}$$




$$f_i(t) = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}$$

$$s(t) = A_c \cos(2\pi f_c t + k_p m(t))$$

(b)

**FIGURE 4.3** Illustration of the relationship between frequency modulation and phase modulation. (*a*) Scheme for generating an FM wave by using a phase modulator. (*b*) Scheme for generating a PM wave by using a frequency modulator.

# Relationship btw PM and FM waves:

- It follows therefore that phase modulation and frequency modulation are uniquely related to each other.
- This relationship, in turn, means that we may deduce the properties of phase modulation from those of frequency modulation and vice versa.
- For this reason, in this chapter we will be focusing much of the discussion on frequency modulation.

# Frequency Deviation and Modulation Index:

$$f_i(t) = f_c + k_f m(t)$$

$$\theta_i(t) = 2\pi f_c \mathrm{t} + 2\pi k_f \int_0^t m(t)d\mathrm{t}$$

$$\boldsymbol{s(t) = A_c cos\theta_i(t)}$$

$$s(t)_{FM} = A_c \cos(2\pi f_c \mathrm{t} + 2\pi k_f \int_0^t m(t)d\mathrm{t})$$

$$s(t)_{PM} = A_c \cos(2\pi f_c t + k_p m(t))$$

$$f_{i_{max}} - f_{i_{min}} = 2\Delta f$$ Carrier swing

$$(f_c + k_f m(t))_{max} - (f_c + k_f m(t))_{min}$$

$$k_f(m(t)_{max} - m(t)_{min}) = 2\Delta f$$

# Frequency Deviation and Modulation Index:

$$m(t) = A_m \cos(2\pi f_m t)$$

$$k_f(A_m \cos(2\pi f_m t)_{max} - A_m \cos(2\pi f_m t)_{min}) = 2\Delta f$$

$$k_f(2A_m) = 2\Delta f$$

$\Delta f = k_f A_m$ Frequency Deviation

$$s(t) = A_c\cos(2\pi f_c \mathrm{t} + 2\pi k_f \int_0^t m(t)d\mathrm{t})$$

$$s(t) = A_c\cos(2\pi f_c \mathrm{t} + 2\pi(k_f \int_0^t m(t)d\mathrm{t})_{max})$$

# Frequency Deviation and Modulation Index:

$$s(t) = A_c \cos(2\pi f_c \mathrm{t} + \beta \sin(2\pi f_m t))$$

$$\beta = \frac{k_f A_m}{f_m}$$ modulation index

$$\beta = 2\pi(k_f \int_0^t m(t) d\mathrm{t})_{max}$$

$$m(t) = A_m \cos(2\pi f_m t)$$

$$\beta = 2\pi(k_f \int_0^t A_m \cos(2\pi f_m t)\, d\mathrm{t})_{max}$$

$$\beta = \frac{2\pi k_f A_m}{2\pi f_m} \sin(2\pi f_m t)_{max}$$

$$\beta = \frac{k_f A_m}{f_m} = \frac{\Delta f}{f_m} = \Delta\theta$$

# Frequency/Phase Deviation and Modulation Index:

Frequency Deviation:

$$\beta = \frac{k_f A_m}{f_m} = \frac{\Delta f}{f_m}$$

$$k_f = \frac{Hz}{volts}$$

$$f_i(t) = f_c + k_f m(t)$$

$$\Delta f = k_f A_m = \frac{Hz}{volts} \times volts$$

Phase Deviation:

$$s(t)_{PM} = A_c \cos\left(2\pi f_c t + k_p m(t)\right)$$

$$s(t) = A_c \cos\left(2\pi f_c \mathrm{t} + k_p A_m \cos(2\pi f_m t)\right)$$

$\beta = k_p A_m$ modulation index

$$s(t) = A_c \cos(2\pi f_c \mathrm{t} + \beta\cos(2\pi f_m t))$$

# Difference btw FM and PM:

- $\Delta f = k_f A_m$
- $\Delta f = k_p A_m F_m$

- $\Delta\theta = \frac{k_F A_m}{F_m}$
- $\Delta\theta = k_p A_m$

- $\beta = \frac{k_f A_m}{f_m}$
- $\beta = k_p\, A_m$

- Freq. deviation is used to calculate the BW

# Example 1:

$$s(t) = 6\cos[2\pi 10^6 t + 4\sin(2\pi 10^3 t)]$$

Find ratio of max. freq. deviation to max. phase deviation?

# Examples 2 and 3 (from B. P. Lathi):

$$s(t) = 10\cos(\omega_c t + 5sin2000\pi t)$$

Consider it as PM signal with sensitivity factor = 10. Find the modulating signal m(t)?

Consider it as PM signal with sensitivity factor = 10. Find the modulating signal m(t)?

$$\varphi_{\mathrm{EM}}(t) = 10\cos\left(\omega_c t + 5\sin 3000t + 10\sin 2000\pi t\right)$$

# Example 4 (from B. P. Lathi):

An angle-modulated signal with carrier frequency $\omega_c = 2\pi \times 10^5$ is described by the equation

$$\varphi_{\mathrm{EM}}(t) = 10 \cos\left(\omega_c t + 5 \sin 3000t + 10 \sin 2000\pi t\right)$$

**(a)** Find the power of the modulated signal.
**(b)** Find the frequency deviation $\Delta f$.
**(c)** Find the deviation ratio $\beta$.
**(d)** Find the phase deviation $\Delta\phi$.
**(e)** Estimate the bandwidth of $\varphi_{\mathrm{EM}}(t)$.

# Generation of Frequency Modulated Signal:

<u>Narrowband FM Generation (indirect method):</u>

Consider the message signal, $m(t) = A_m \cos(2\pi f_m t)$

Let the modulated FM signal be,

$$s(t) = Acos[(2\pi F_c t) + \beta \sin(2\pi F_m t)]$$

where, $\beta = \frac{k_F A_m}{F_m}$

Consider $\beta \ll 1 \Rightarrow \frac{\Delta F}{F_m} \ll 1 \Rightarrow \Delta F \ll F_m$

This is termed as narrowband FM signal (a signal for which $\beta$ is very small).

# Generation of Frequency Modulated Signal:

For such a narrowband FM signal,

$$s(t) = A_c cos[(2\pi F_c t) + \beta \sin(2\pi F_m t)]$$
$$= A_c \cos(2\pi F_c t) \cos(\beta \sin(2\pi F_m t))$$
$$- A_c \sin(2\pi F_c t) \sin(\beta \sin(2\pi F_m t))$$

For small $\beta$ , we have

$$\cos(\beta \sin(2\pi F_m t)) \approx 1$$

$$\sin(\beta \sin(2\pi F_m t)) \approx \beta \sin(2\pi F_m t)$$

Hence,

$$s(t) = A_c \cos(2\pi F_c t) - A_c \sin(2\pi F_c t) \beta \sin(2\pi F_m t)$$

Approximation for narrowband modulated FM signal.

# Indirect Method for Generating Narrowband FM:

This approximation ($s(t) = A_c \cos(2\pi F_c t) - A_c \sin(2\pi F_c t)\, \beta \sin(2\pi F_m t)$) can be employed for narrowband FM generation.

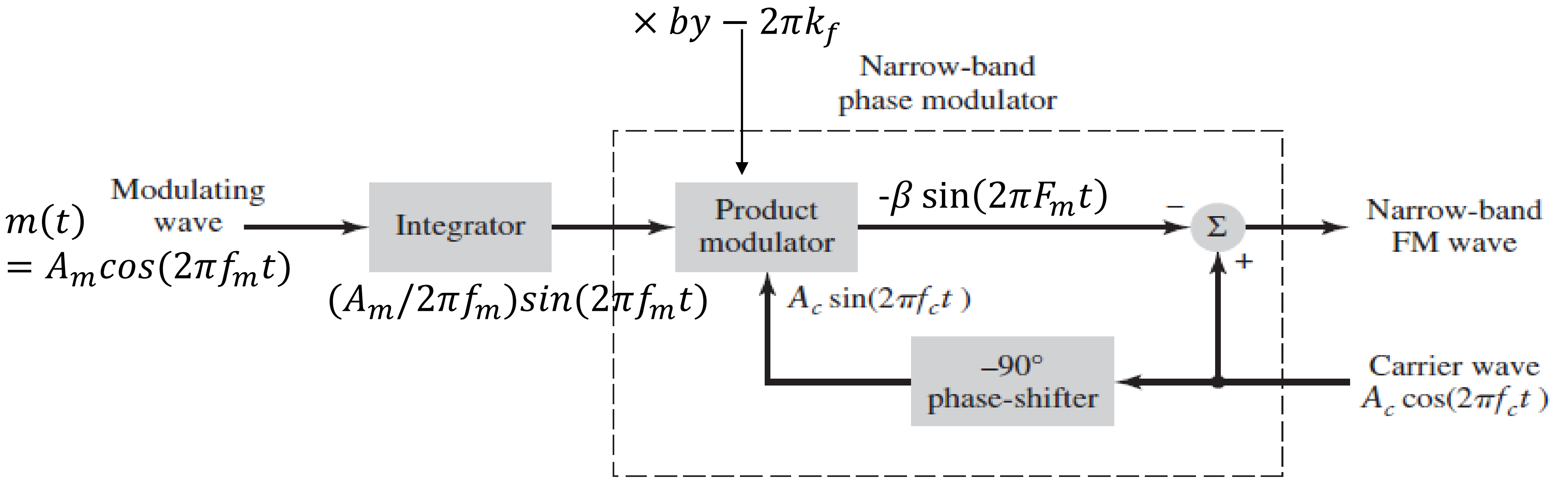



FIGURE 4.4 Block diagram of an indirect method for generating a narrow-band FM wave.

# Conclusion:

1. The envelope contains a *residual* amplitude modulation that varies with time.
2. The angle $\theta_i(t)$ contains *harmonic distortion* in the form of third- and higher order harmonics of the modulation frequency $f_m$.

# Examples:

$$m(t) = sinc(4\pi 10^4 t)$$

Is freq. modulated with sensitivity factor $10^3$Hz/volts. Calculate max. instantaneous freq. when carrier freq. is 1MHz.

- Find modulation index of an FM signal with carrier swing of 10kHz and freq. of m(t) = 8kHz?
- Carrier swing: $f_{i_{max}} - f_{i_{min}} = 2\Delta f$