Communication Systems
EE-351

Lectures 8 to 10

Is there any benefit of quadrature null effect?

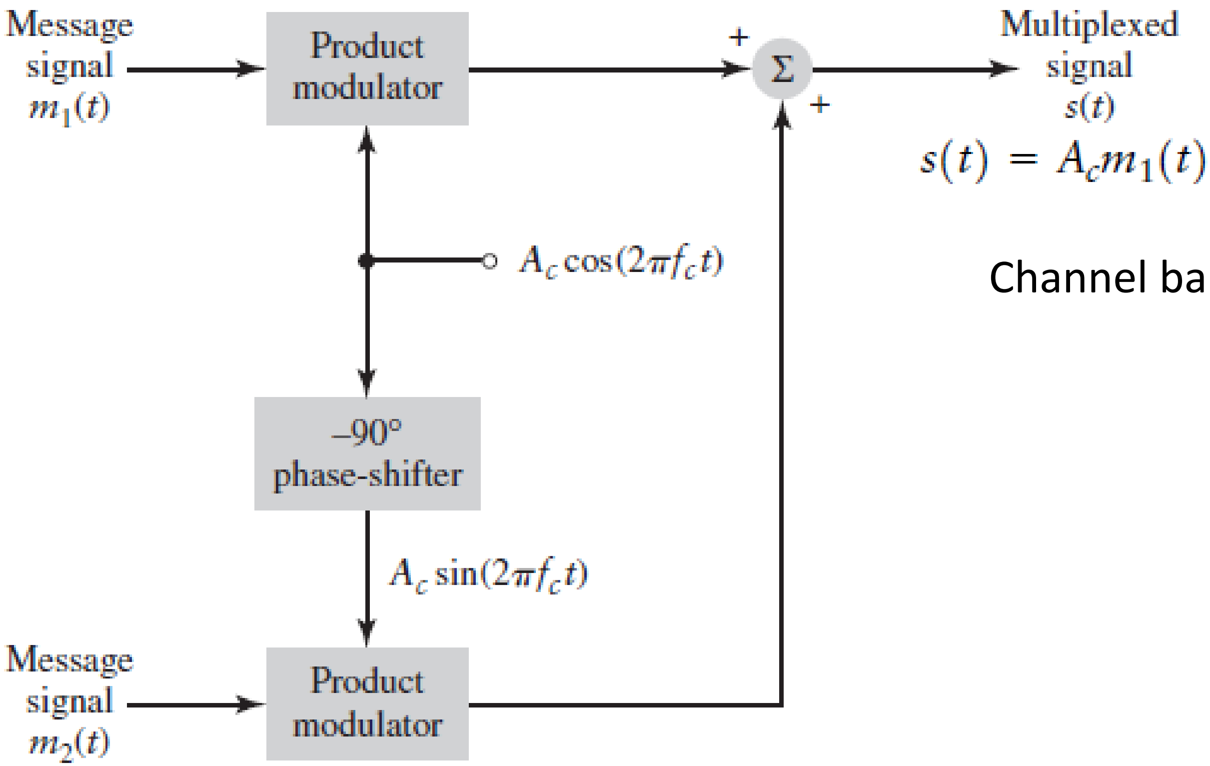
- Yes,
  - Quadrature Amplitude Modulation or
  - Quadrature Carrier Multiplexing

- Two signals with two carriers of same frequencies but different phases.

Motivation for QAM or QCM (A bandwidth-conservation system):

- The **quadrature null effect** of the coherent detector may also be put to good use in the construction of the so-called quadrature-carrier multiplexing or quadrature-amplitude modulation (QAM).
- This scheme enables two DSB-SC modulated waves (resulting from the application of two physically independent message signals) to occupy the same channel bandwidth.
- Yet it allows for the separation of the two message signals at the receiver output.

Quadrature Carrier Multiplexing (QCM) Transmitter side:

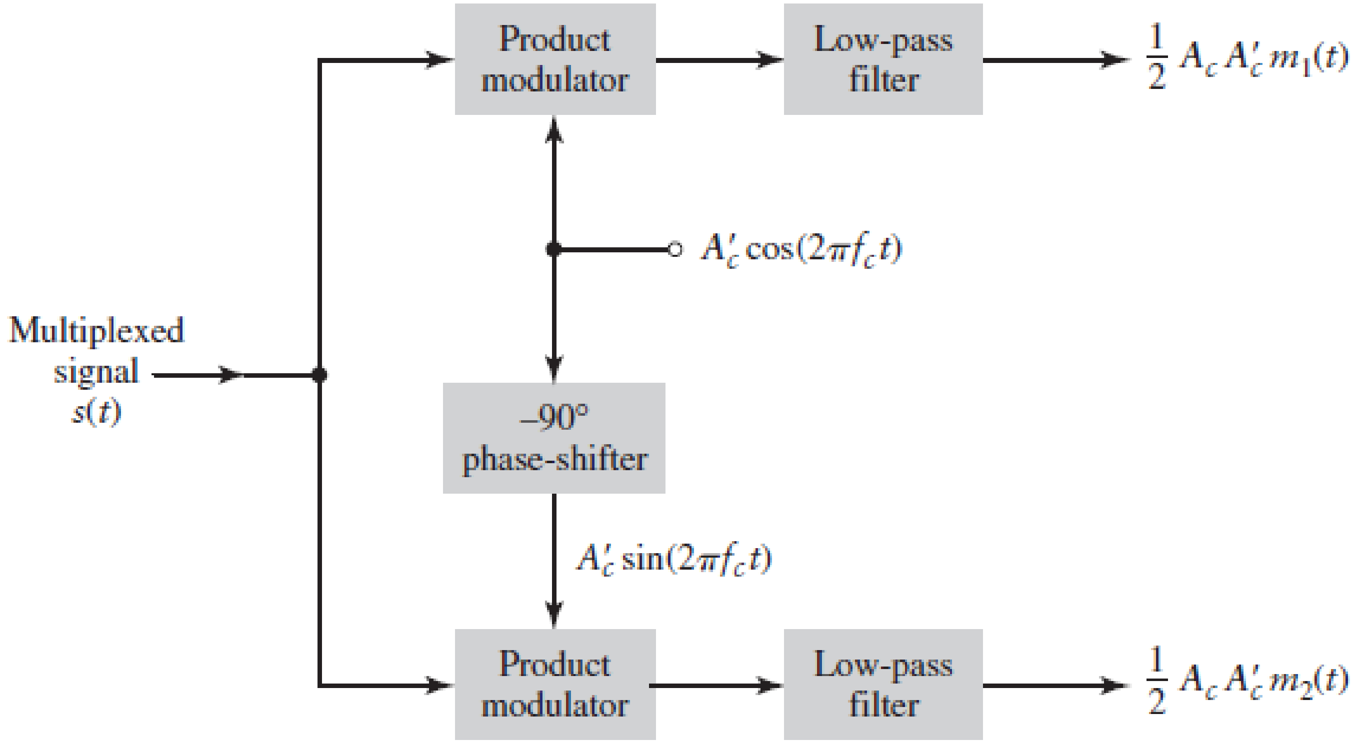


$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$

Channel bandwidth of $s(t)$ is 2W

Quadrature Carrier Multiplexing (QCM) Receiver side:

the multiplexed signal $s(t)$ is applied simultaneously to two separate coherent detectors that are supplied with two local carriers of the same frequency, but differing in phase by -90 degrees



Quadrature Carrier Multiplexing (QCM):

- For the system to operate satisfactorily, it is important to maintain the correct phase and frequency relationships between two oscillators (the oscillator used to generate the carriers in the transmitter and the corresponding local oscillator used in the receiver).
  - First Solution: Costas receiver is the best choice.
  - Second Solution: Another commonly used method is to send a pilot signal outside the passband of the modulated signal.
    - The pilot signal typically consists of a low-power sinusoidal tone whose frequency and phase are related to the carrier wave $c(t) = A_c 2\pi f_c t$
    - At the receiver, the pilot signal is extracted by means of a suitably tuned circuit and then translated to the correct frequency for use in the coherent detector.

Quadrature Carrier Multiplexing (QCM)

- Consider two carriers:
  - $\cos(2\pi F_c t)$
  - $\sin(2\pi F_c t)$

Observe that the phase difference btw these two carriers is 90°, i.e.,

$$\sin\left(2\pi F_c t + {}^{\pi}/_{2}\right) = \cos(2\pi F_c t)$$

Hence, $\cos(2\pi F_c t)$ & $\sin(2\pi F_c t)$ are termed to be in quadrature.

- Consider a modulated signal as:
- $x(t) = A_c m_I(t)\cos(2\pi F_c t) + A_c m_Q(t)\sin(2\pi F_c t)$

$m_I(t)$ is the message modulated on $\cos(2\pi F_c t)$, in-phase carrier

$m_Q(t)$ is the message modulated on $\sin(2\pi F_c t)$, quadrature- carrier

Quadrature Carrier Multiplexing (QCM)

- Demodulation with $\cos(2\pi F_c t)$

$$x(t) \times A'_c \cos(2\pi F_c t)$$

$$= (A_c m_I(t)\cos(2\pi F_c t) + A_c m_Q(t)\ \sin(2\pi F_c t)) \times A'_c \cos(2\pi F_c t)$$

$$= \frac{1}{2} A_c A'_c m_I(t)(1 + \cos(4\pi F_c t)) + \frac{1}{2} A_c A'_c m_Q(t)(\sin(4\pi F_c t))$$

$$= \frac{A_c A'_c m_I(t)}{2} + \frac{A_c A'_c m_I(t)}{2}\cos(4\pi F_c t) + \frac{A_c A'_c m_Q(t)}{2}\sin(4\pi F_c t)$$

After passing through LPF, we have $\frac{A_c A'_c m_I(t)}{2}$ (recover the in-phase signal)

Quadrature Carrier Multiplexing (QCM)

- Demodulation with $\sin(2\pi F_c t)$

$$x(t) \times A'_c \sin(2\pi F_c t)$$
$$= (A_c m_I(t)\cos(2\pi F_c t) + A_c m_Q(t)\ \sin(2\pi F_c t)) \times A'_c \sin(2\pi F_c t)$$
$$= \frac{1}{2} A_c A'_c m_I(t)(\sin(4\pi F_c t)) + \frac{1}{2} A_c A'_c m_Q(t)(1 - \cos(4\pi F_c t))$$
$$= \frac{A_c A'_c m_I(t)}{2}\sin(4\pi F_c t) + \frac{A_c A'_c m_Q(t)}{2} - \frac{A_c A'_c m_Q(t)}{2}\cos(4\pi F_c t)$$

After passing through LPF, we have $\frac{A_c A'_c m_Q(t)}{2}$ (recover the quadrature-phase signal)

- Therefore, using orthogonal carriers $\cos(2\pi F_c t)\ \&\ \sin(2\pi F_c t)$, two parallel message signals $m_I(t)$ & $m_Q(t)$, can be transmitted on the same channel (sharing the same BW). This scheme is termed as QCM or QAM.
- Quadrature-carrier multiplexer is therefore a Bandwidth-conservation system

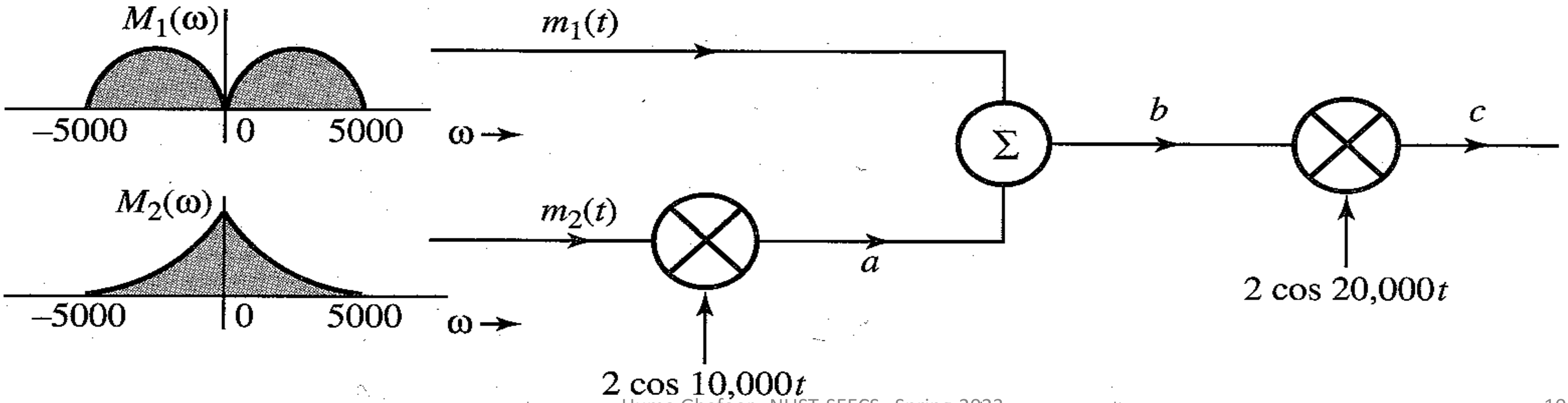
Problem form B.P. Lathi:

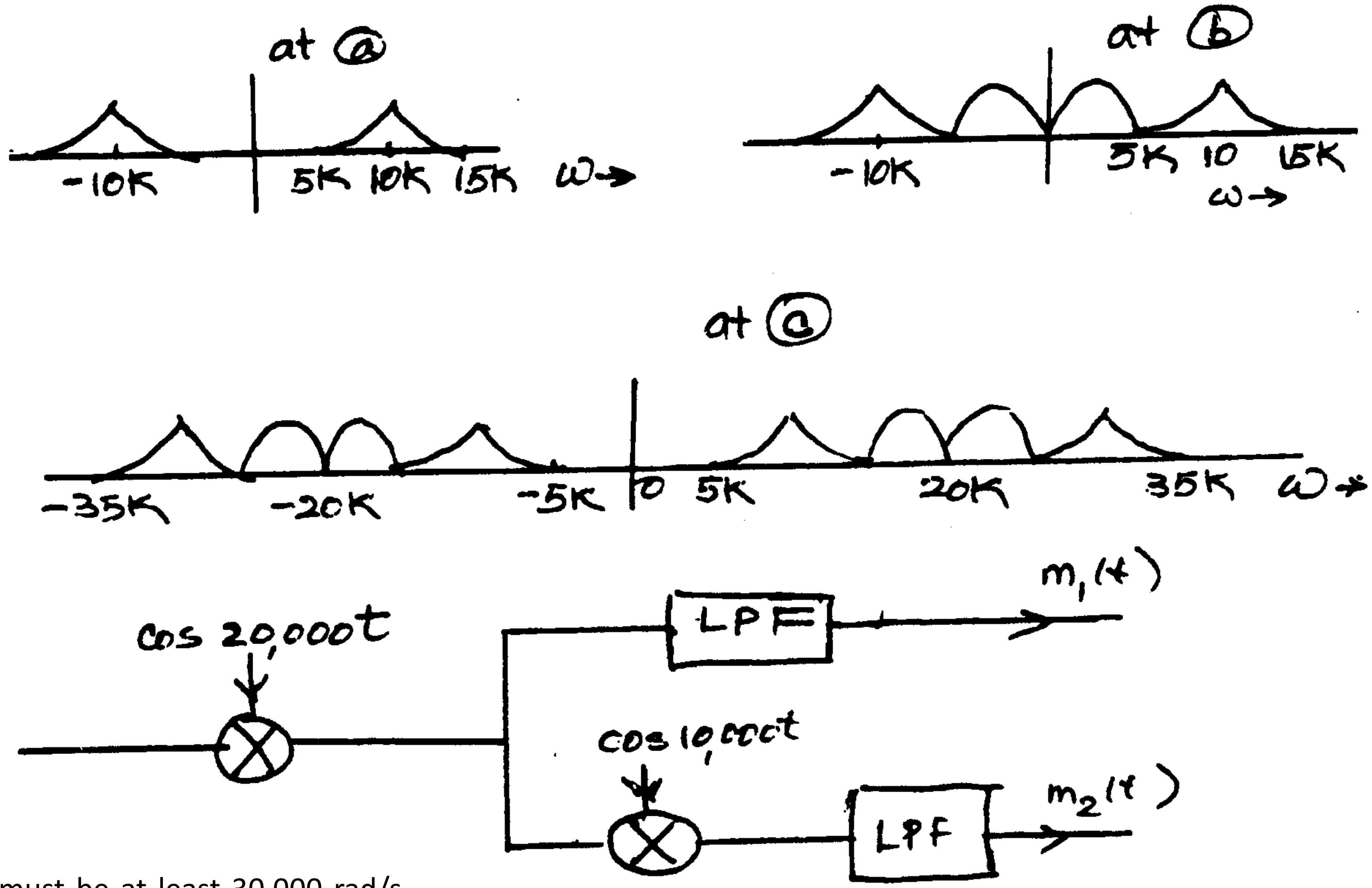
4.2-8 Two signals $m_1(t)$ and $m_2(t)$, both band-limited to 5000 rad/s, are to be transmitted simultaneously over a channel by the multiplexing scheme shown in Fig. P4.2-8. The signal at point b is the multiplexed signal, which now modulates a carrier of frequency 20,000 rad/s. The modulated signal at point c is transmitted over a channel.

(a) Sketch signal spectra at points a, b, and c.

(b) What must be the bandwidth of the channel?

(c) Design a receiver to recover signals $m_1(t)$ and $m_2(t)$ from the modulated signal at point c.





Part b: must be at least 30,000 rad/s (from 5000 rad/s to 35,000 rad/s)

Fig. S4.2-8

Single Sideband Modulation (SSB):

- In suppressing the carrier, DSB-SC modulation takes care of a major limitation of AM that pertains to the wastage of transmitted power.
- To take care of the other major limitation of AM that pertains to channel bandwidth, we need to suppress one of the two sidebands in the DSB-SC modulated wave.
  - SSB modulation relies solely on the lower sideband or upper sideband to transmit the message signal across a communication channel.
    - Lower SSB
    - Upper SSB

Single Sideband Modulation (SSB):

Consider a modulated signal, $x(t) = m(t)\cos(2\pi F_c t)$

$X(F) = \frac{1}{2}M(F - F_c) + \frac{1}{2}M(F + F_c)$

LSB= Lower sideband $[-F_c, -F_c + F_m] \cup [F_c - F_m, F_c]$

USB= Upper sideband $[-F_c - F_m, -F_c] \cup [F_c, F_c + F_m\ ]$

Hence,

- Spectral efficiency of SSB=2 x spectral efficiency of DSB

- A rigorous derivation of SSB modulation theory that applies to an arbitrary message signal is rather demanding and therefore beyond the scope of this book.

Single Sideband Modulation (SSB):

To proceed then, consider a DSB-SC modulator using the sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t)$$

With the carrier $c(t) = A_c \cos(2\pi f_c t)$, the resulting DSB-SC modulated wave is defined by

$$\begin{aligned} S_{\text{DSB}}(t) &= c(t)m(t) \\ &= A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) \\ &= \frac{1}{2}A_c A_m \cos[2\pi(f_c + f_m)t] + \frac{1}{2}A_c A_m \cos[2\pi(f_c - f_m)t] \end{aligned} \tag{3.13}$$

which is characterized by two *side-frequencies*, one at $f_c + f_m$ and the other at $f_c - f_m$. Suppose that we would like to generate a sinusoidal SSB modulated wave that retains the upper side-frequency at $f_c + f_m$. Then, suppressing the second term in Eq. (3.13), we may express the upper SSB modulated wave as

$$S_{\text{USSB}}(t) = \frac{1}{2}A_c A_m \cos[2\pi(f_c + f_m)t] \tag{3.14}$$

Upper Single Sideband (USSB):

The cosine term in Eq. (3.14) includes the sum of two angles—namely, $2\pi f_c t$ and $2\pi f_m t$. Therefore, expanding the cosine term in Eq. (3.14) using a well-known trigonometric identity, we have

$$S_{\mathrm{USSB}}(t) = \frac{1}{2}A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) - \frac{1}{2}A_c A_m \sin(2\pi f_c t)\sin(2\pi f_m t) \qquad (3.15)$$

Lower Single Sideband (LSSB):

If, on the other hand, we were to retain the lower side-frequency at $f_c - f_m$ in the DSB-SC modulated wave of Eq. (3.13), then we would have a lower SSB modulated wave defined by

$$S_{\mathrm{LSSB}}(t) = \frac{1}{2}A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) + \frac{1}{2}A_c A_m \sin(2\pi f_c t)\sin(2\pi f_m t) \qquad (3.16)$$

Single Sideband Modulation (SSB):

Examining Eqs. (3.15) and (3.16), we see that they differ from each other in only one respect: the minus sign in Eq. (3.15) is replaced with the plus sign in Eq. (3.16). Accordingly, we may combine these two equations and thereby define a sinusoidal SSB modulated wave as follows:

$$S_{\mathrm{SSB}}(t) = \frac{1}{2}A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t) \mp \frac{1}{2}A_c A_m \sin(2\pi f_c t)\sin(2\pi f_m t) \qquad (3.17)$$

where the plus sign applies to lower SSB and the minus sign applies to upper SSB.

Single Sideband Modulation (SSB):

$$S_{\mathrm{SSB}}(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) \mp \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t)$$

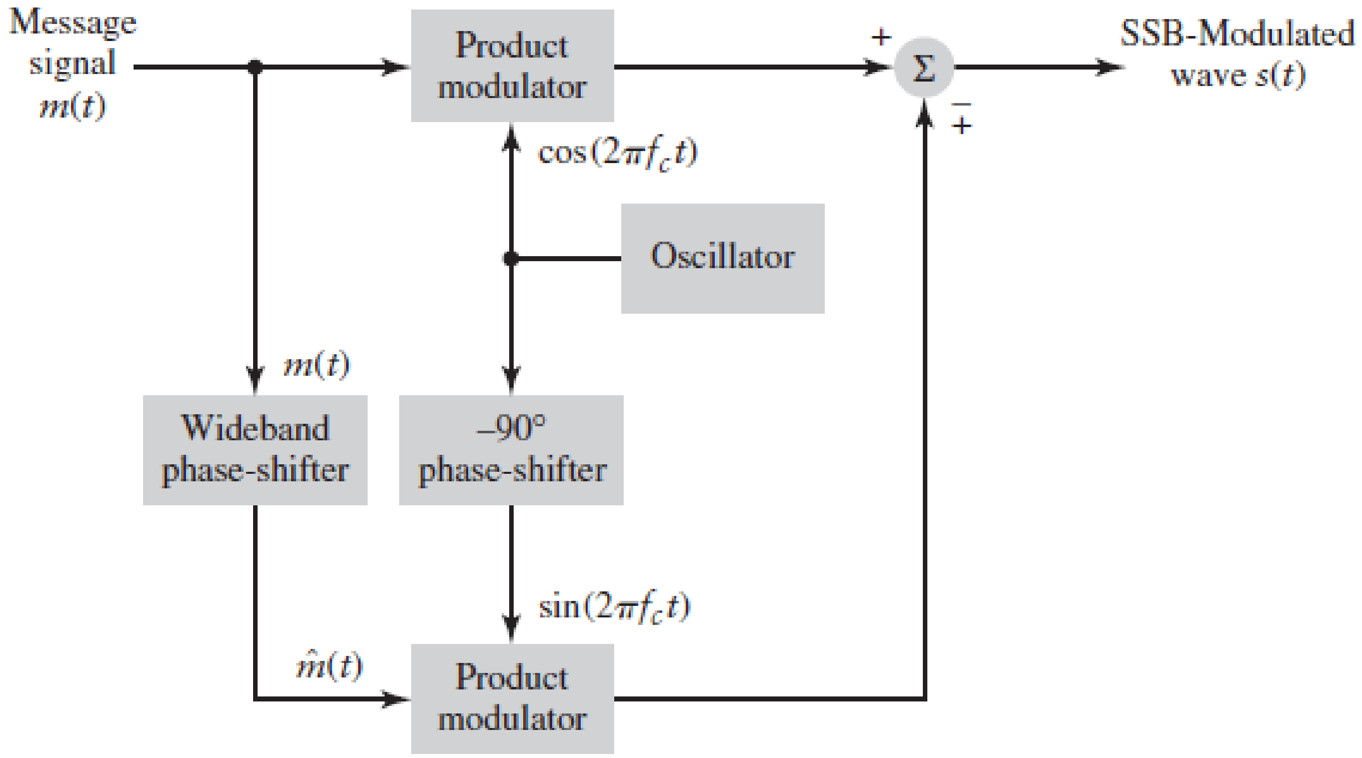
In both technical and practical terms, the observation we have just made is very important for two reasons:

1. We know from Fourier analysis that *under appropriate conditions, the Fourier series representation of a periodic signal converges to the Fourier transform of a nonperiodic signal*; see Appendix 2 for details.
2. *The signal $\hat{m}(t)$ is the Hilbert transform of the signal $m(t)$*. Basically, a *Hilbert transformer* is a system whose transfer function is defined by

$$H(f) = -j\,\mathrm{sgn}(f) \tag{3.22}$$

where $\mathrm{sgn}(f)$ is the signum function; for the definition of the signum function see Section 2.4. In words, the Hilbert transformer is a *wide-band phase-shifter* whose frequency response is characterized in two parts as follows (see Problem 2.52):

Single Sideband Modulation (SSB):

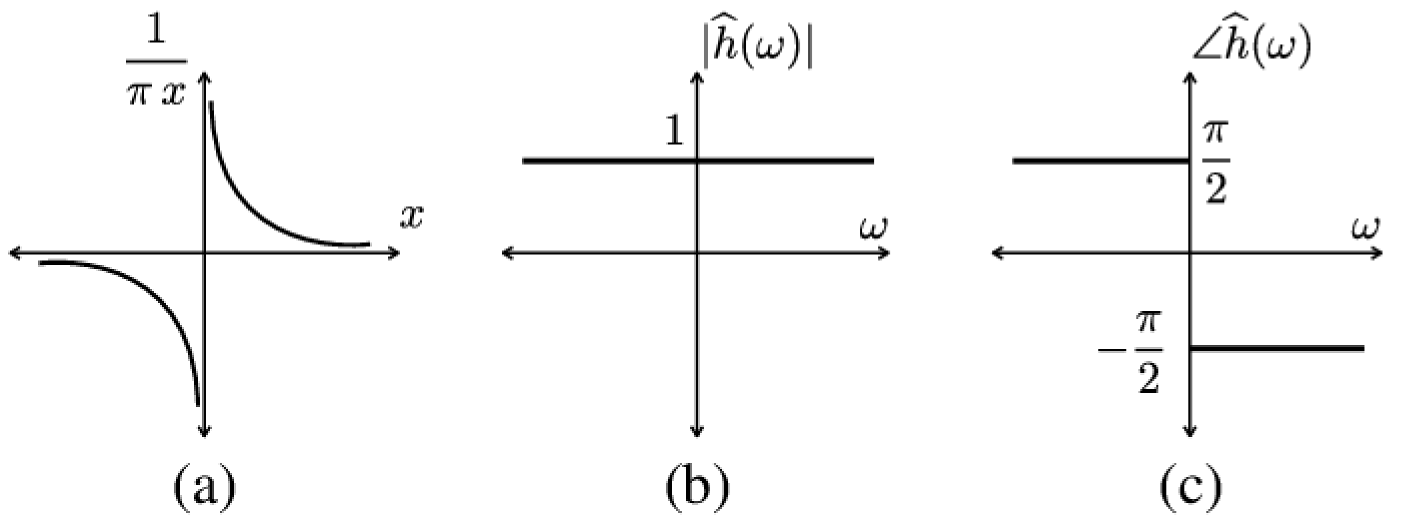


Hilbert Transform:

LTI system

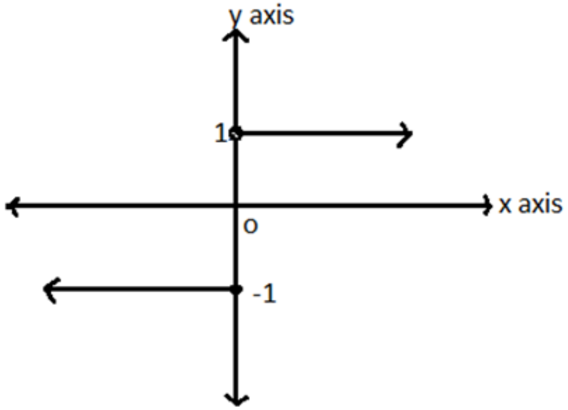
- Hilbert transformer is characterized by an impulse response as every LTI system is characterized by.

Hilbert Transform:



Characterization of the Hilbert transformer: (a) impulse response, (b) magnitude spectrum, and (c) phase spectrum.

Hilbert Transform (HT):



x-axis represents frequency, signum function expressed into unit step function

$$\hat{m}(t) = m(t) * h_{HT}(t)$$
$$h_{HT}(t) \leftrightarrow H_{HT}(F)$$

- $\hat{m}(t)$ is Hilbert transform of $m(t)$ or phase shifted version of m(t)
- $h_{HT}(t)$ is impulse response of HT
- $H_{HT}(F)$ Fourier transform of the impulse response i.e., $h_{HT}(t)$ of HT.

$$H_{HT}(F) = -jsgn(F)$$

$$sgn(F) = \begin{cases} 1, & F > 0 \\ 0, & F = 0 \\ -1, & F < 0 \end{cases}$$

Hilbert Transform (HT):

$$H_{HT}(F) = \begin{cases} -j, & F > 0 \\ 0, & F = 0 \\ j, & F < 0 \end{cases}$$

From derivative property of the Fourier Transform,

$$x(t) \leftrightarrow X(F)$$

$$\frac{dx(t)}{dt} \leftrightarrow j2\pi F X(F)$$

Derivative of signum function is:

$$\frac{d}{dt}\big(sgn(t)\big) = 2\delta(t)$$

FT of $\left\{\frac{d}{dt}\big(sgn(t)\big)\right\}$ =FT(2$\delta(t)$) $=$ 2FT($\delta(t)$ =2

Hilbert Transform (HT):

$$\text{FT}\left(\frac{dx(t)}{dt}\right) \leftrightarrow j2\pi F\ .\ \text{FT}\big(sgn(t)\big)$$

$$\Rightarrow j2\pi F\ .\ \text{FT}\big(sgn(t)\big) = 2$$

$$\Rightarrow \text{FT}\big(sgn(t)\big) = \frac{2}{j2\pi F} = \frac{1}{j\pi F}$$

$$sgn(t) \leftrightarrow \frac{1}{j\pi F}$$

From Duality property of Fourier Transform,

$$x(t) \leftrightarrow X(F)$$

$$X(t) \leftrightarrow x(-F)$$

$$\therefore \frac{1}{j\pi t} \leftrightarrow sgn(-F)$$

Hilbert Transform (HT):

$$\frac{1}{j\pi t} \leftrightarrow -sgn(F) \text{ (due to odd function)}$$

$$\frac{1}{\pi t} \leftrightarrow -jsgn(F)$$

Hence, $h_{HT}(t) = \frac{1}{\pi t}$

$$\widehat{m}(t) = m(t) * \frac{1}{\pi t}$$

$$\widehat{M}(F) = M(F). -jsgn(F)$$