Communication Systems
EE-351

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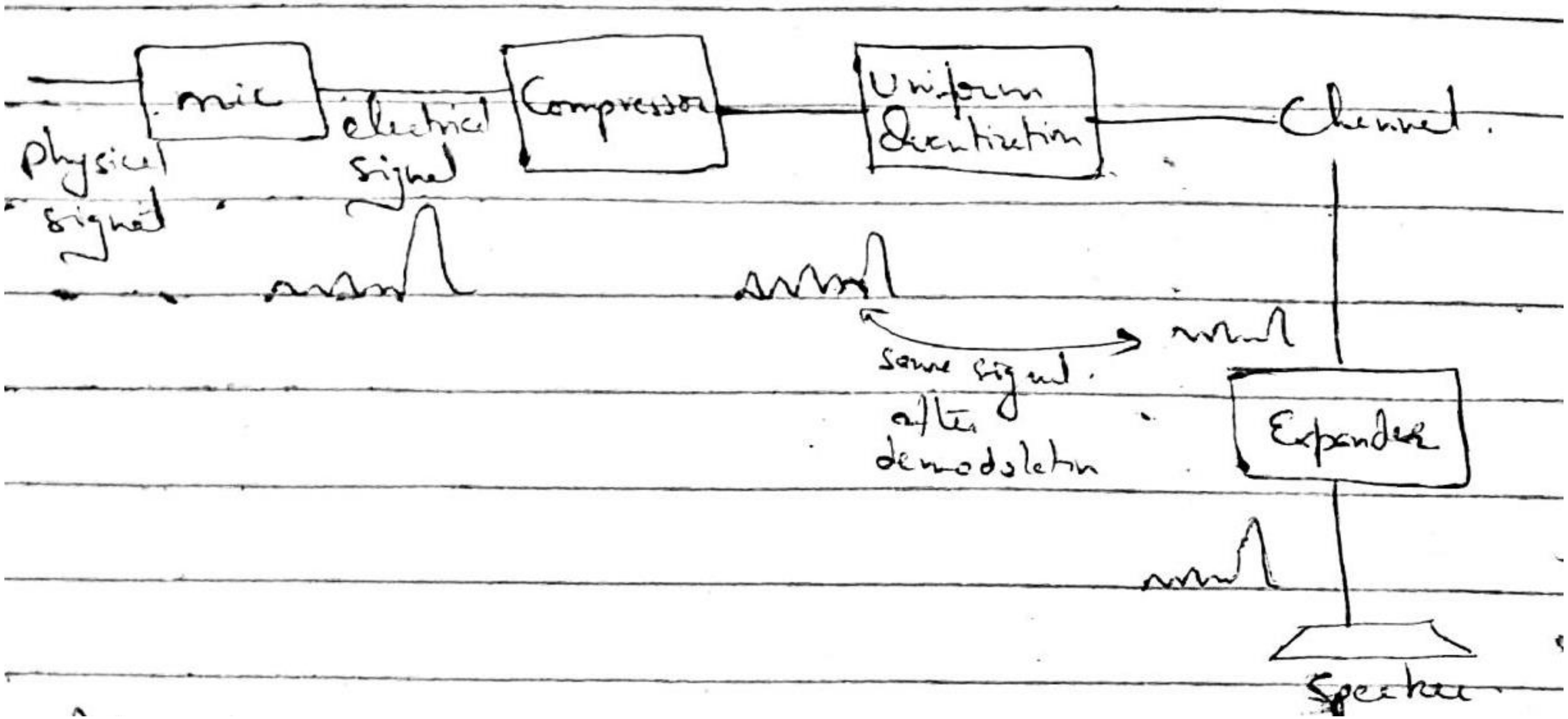
Lectures 34 to 36

Companding:

- Compressing + Expanding



Companding:

- Advantages:
  - Reduce dynamic range
    - Quantization error ↓
  - Step-size is reduced
  - SNR↑, efficiency↑

1600

0

$L = 16$

$$\text{Step size} = \frac{V_{max} - V_{min}}{L}$$

$$= \frac{1600 - 0}{16}$$

$$\Delta = 100$$

After compression

160

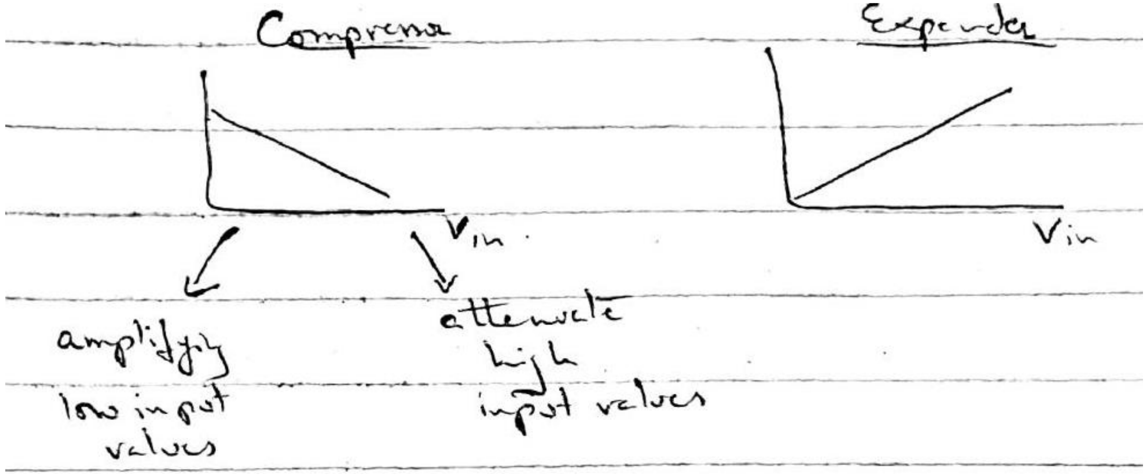
0

$$= \frac{160 - 0}{16} = 10$$

Characteristics of Companding:

$$Q_{e_{max}} = \left|\frac{\Delta}{2}\right|$$
$$\Delta\downarrow \Rightarrow Q_e \downarrow$$

Gain:

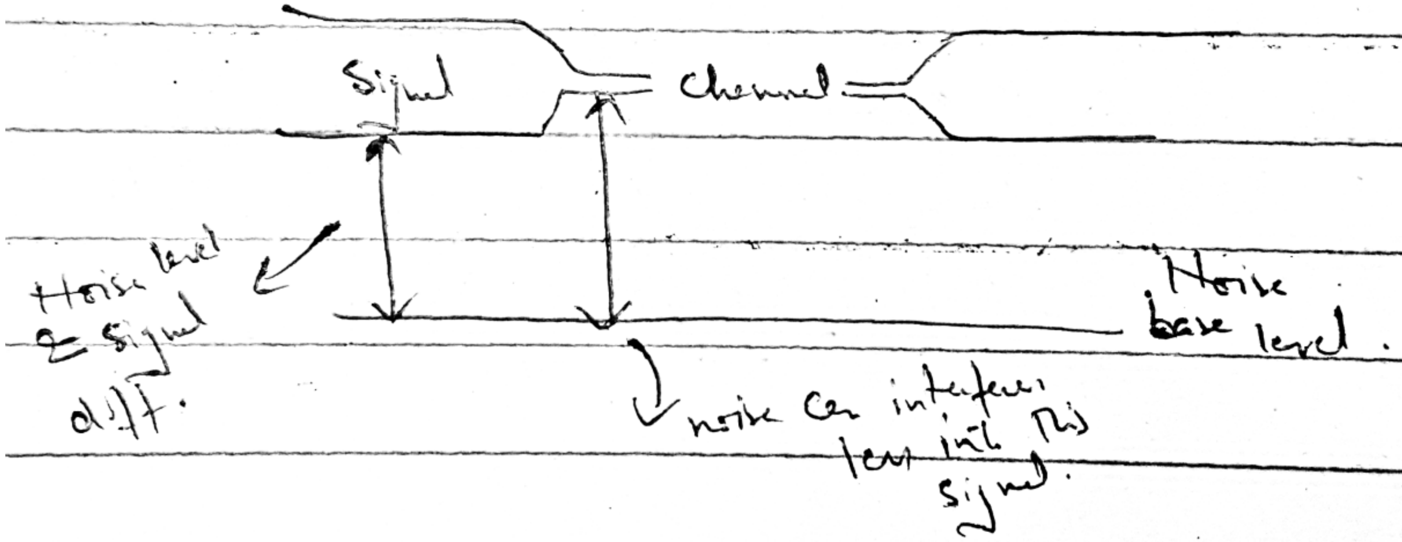


PCM → reduce the step size

$Q_e \downarrow$.

Noise Interference decreases

↓ how?



Quantization Error:

$$Q_e = x_q(kT_s) - x(kT_s)$$

Q_e = quantized value – sampled value

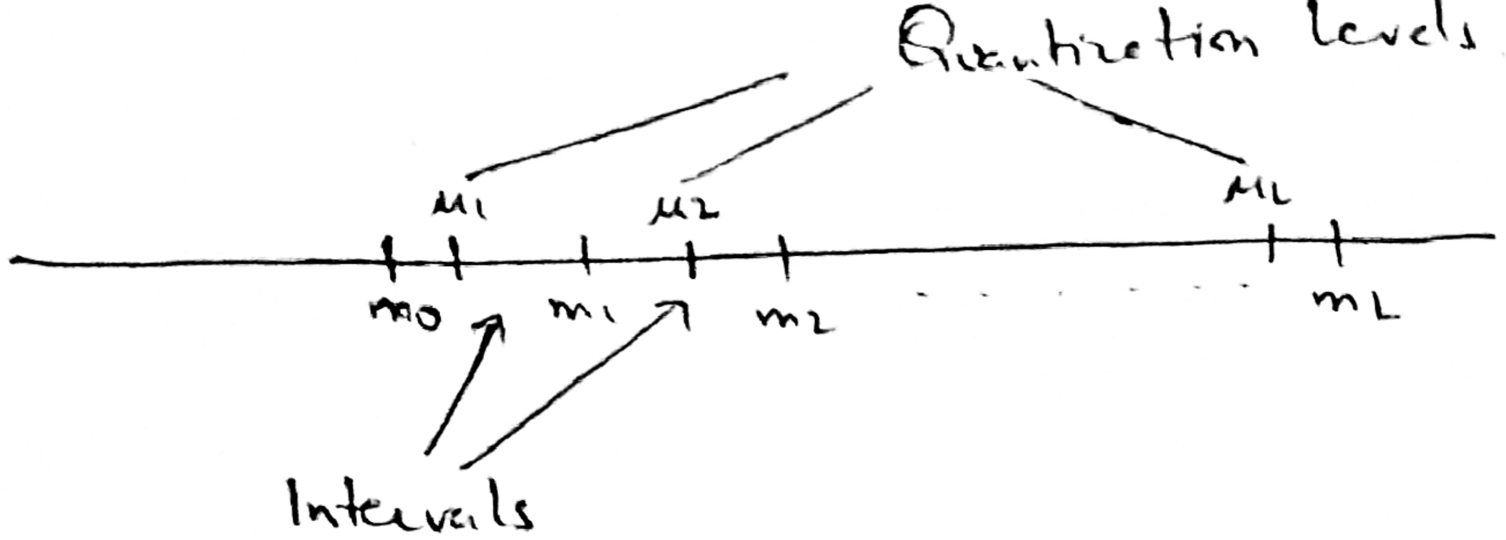
$Q_{e_{max}} = \left|\frac{\Delta}{2}\right| \Rightarrow -\frac{\Delta}{2}$ to $\frac{\Delta}{2}$

Quantization error → uniformly distributed function in $(-\frac{\Delta}{2}, \frac{\Delta}{2})$

Step size $= \frac{V_{max}-V_{min}}{L}$

Optimal Quantizer:
Lloyd-Max Quantizer:

⇒ minimizes Quantization Error



$$v_k = g(m) \quad \text{if} \quad m_{k-1} \leq m < m_k .$$

(o/p) quantized value

Optimal Quantizer:
Lloyd-Max Quantizer:

To design an optimal Quantizer

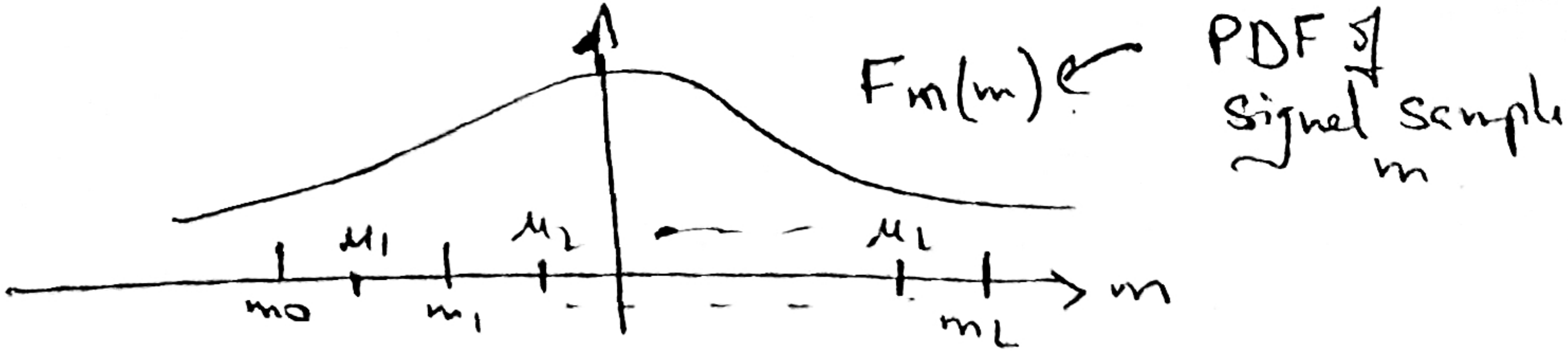
Choose Quantization levels (QL) & Quantization Intervals (QI) that minimize the QE.

$m_0, m_1, \ldots, m_L$ ⟵ Interval Boundaries.

$\mu_1, \mu_2, \ldots, \mu_L$ ⟵ QL.

How Lloyd-max algorithm optimally choose these levels/intervals?

Probability Density function (PDF) of signal sample m.



How Lloyd-max algorithm optimally choose these levels/intervals?

$[m_{k-1}, m_k]$ kth interval

μ_k Quantization level in kth interval

→ find both QI & QL in an iterative manner.

Given the intervals → find QL (optimal)

Given the levels → find QI (optimal)

Summary:

Step A. Quantization Intervals $\Rightarrow$ Levels

Step B. Levels $\Rightarrow$ Intervals

iteratively repeat until get final quantizer

or

convergence

↳ both QI & QL are approx. constant.

Optimal Quantizer:
Lloyd-Max Quantizer:

Condition:

Choose such that Quantization Error (average square value) is minimized

$$\int_{m_{k-1}}^{m_k} (\mu_k - m)^2 F_m(m)\,dm$$

Average quantization error for the interval $[m_{k-1}, m_k]$

$$= \frac{\partial}{\partial \mu_k} \int_{m_{k-1}}^{m_k} (\mu_k - m)^2 \, \underline{F_m(m)} \, dm$$

Using it to compute average.

$$= \int_{m_{k-1}}^{m_k} \frac{\partial}{\partial \mu_k} (\mu_k - m)^2 F_m(m) \, dm$$

$$\Rightarrow \int_{m_{k-1}}^{m_k} 2(\mu_k - m) F_m(m) \, dm \underbrace{= 0}$$

to find μ_k for which mean squared error is minimized.

$$\int_{m_{k-1}}^{m_k} 2(\mu_k - m) F_m(m) \, dm = 0$$

$$\Longrightarrow \int_{m_{k-1}}^{m_k} (\mu_k - m)\, F_m(m)\, dm = 0$$

$$\int_{m_{k-1}}^{m_k} \mu_k\, F_m(m)\, dm = \int_{m_{k-1}}^{m_k} m\, F_m(m)\, dm$$

$\Longrightarrow$ μ_k does not depend on m

$$\mu_k \int_{m_{k-1}}^{m_k} F_m(m)\, dm = \int_{m_{k-1}}^{m_k} m\, F_m(m)\, dm$$

$$\mu_k = \frac{\int_{m_{k-1}}^{m_k} m\, F_m(m)\, dm}{\int_{m_{k-1}}^{m_k} F_m(m)\, dm}$$

optimal value.

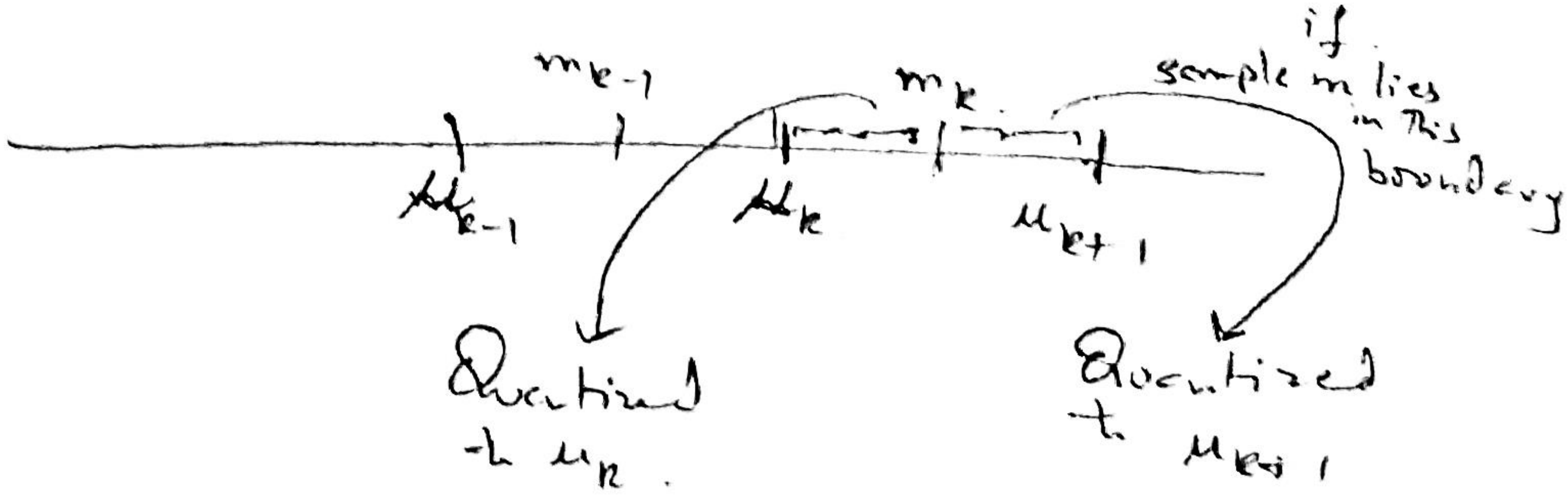
Step B

Given the level $\longrightarrow$ find interval ? (optimal)

$m_0, m_1, m_2, \ldots, m_L$.

can be ignored as determined from dynamic range.

$m_1, m_2, \ldots m_{L-1}$??


if
sample m lies
in this
boundary
Quantized
to μk.
Quantized
to μk+1

if $g(m) = \mu_{k+1}$,

sq. error $= (m - \mu_{k+1})^2$

if $g(m) = \mu_k$

sq. error $= (m - \mu_k)^2$

choose μ_k only if

$$(m - \mu_k)^2 \leq (m - \mu_{k+1})^2$$

$$\cancel{m^2} + \mu_k^2 - 2m\mu_k \leq \cancel{m^2} + \mu_{k+1}^2 - 2m\mu_{k+1}$$

$$m \leq \frac{1}{2}(\mu_{k+1} + \mu_k)$$

$$\therefore \boxed{m_k = \frac{1}{2}(\mu_k + \mu_{k+1})}$$

Example:

- Design a 3-level mid-tread quantizer

$$F_m(m) = (1 - m^2) \qquad [-2,2]$$

Midpoint,

$$\mu_k = \frac{\int mF_m(m)dm}{\int F_m(m)dm}$$

$$\mu_R = \frac{\int_{-2}^{-0.67} m(1-m^2)\,dm}{\int_{-2}^{-0.67} (1-m^2)\,dm} = \frac{\int_{-2}^{-0.67} (m-m^3)\,dm}{\int_{-2}^{-0.67} (1-m^2)\,dm}$$

$$= \frac{\left.\frac{m^2}{2} - \frac{m^4}{4}\right|_{-2}^{-0.67}}{\left.m - \frac{m^3}{3}\right|_{-2}^{-0.67}}$$

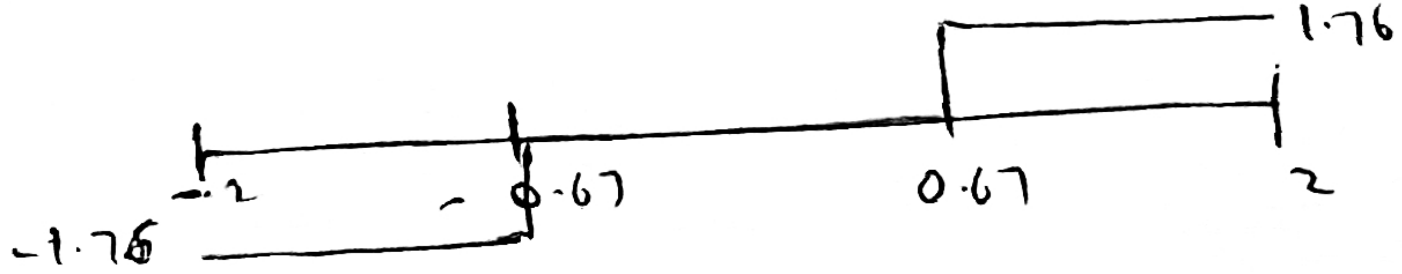
$$= \frac{\frac{(-0.67)^2}{2} - \frac{(-0.67)^4}{4} - \left[\frac{(-2)^2}{2} - \frac{(-2)^4}{4}\right]}{-0.67 - \frac{(-0.67)^3}{3} - \left[-2 - \frac{(-2)^3}{3}\right]}$$

$$= \frac{0.22 - 0.05 - [2-4]}{-0.67 + 0.1 + 2 - 2.67} = \frac{2.17}{-1.24} = -1.75$$

$$\mu_R = \int_{-0.67}^{0.67} \frac{m(1-m^2)\,dm}{(1-m^2)\,dm} = \frac{0.22-0.05-0.22+0.05}{0.67-0.10+0.67-0.10} = 0.$$

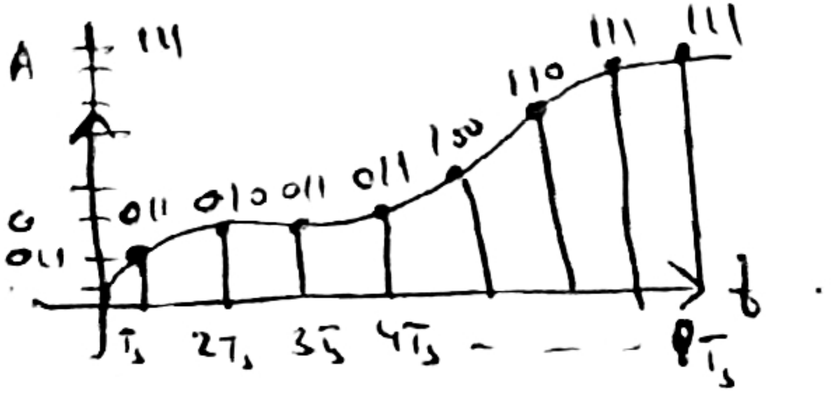
$$\mu_R = \int_{0.67}^{2} \frac{m(1-m^2)\,dm}{(1-m^2)\,dm} = \frac{-2.17}{-1.23} = 1.76.$$

mid-tread



Conclusion:

- Drawbacks:
  - Large dynamic range
    - Large number of levels to quantize
      - Number of bits large
        - BW large
  - Same no. of levels
    - Step size large
      - Quantization error large
  - Large redundant bits are encoded
    - Large number of redundant bits are transmitting
    - each and every bit requires BW for propagation



7Ts & 8Ts using 111.

3Ts & 4Ts using 011.

110 → 111.

2 bits are redundant.