Assignment-1 CLO-1
BEE-12CD
Digital Signal Processing
Due Date: 15 March 2023

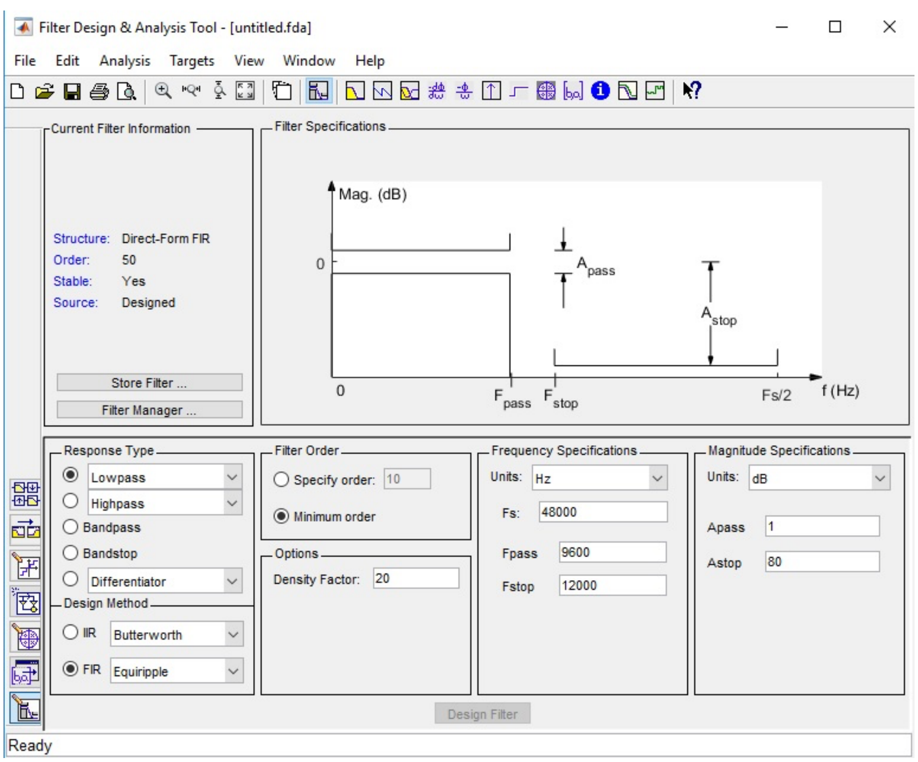
Getting familiarized with DTFT and z-Transform

Problem

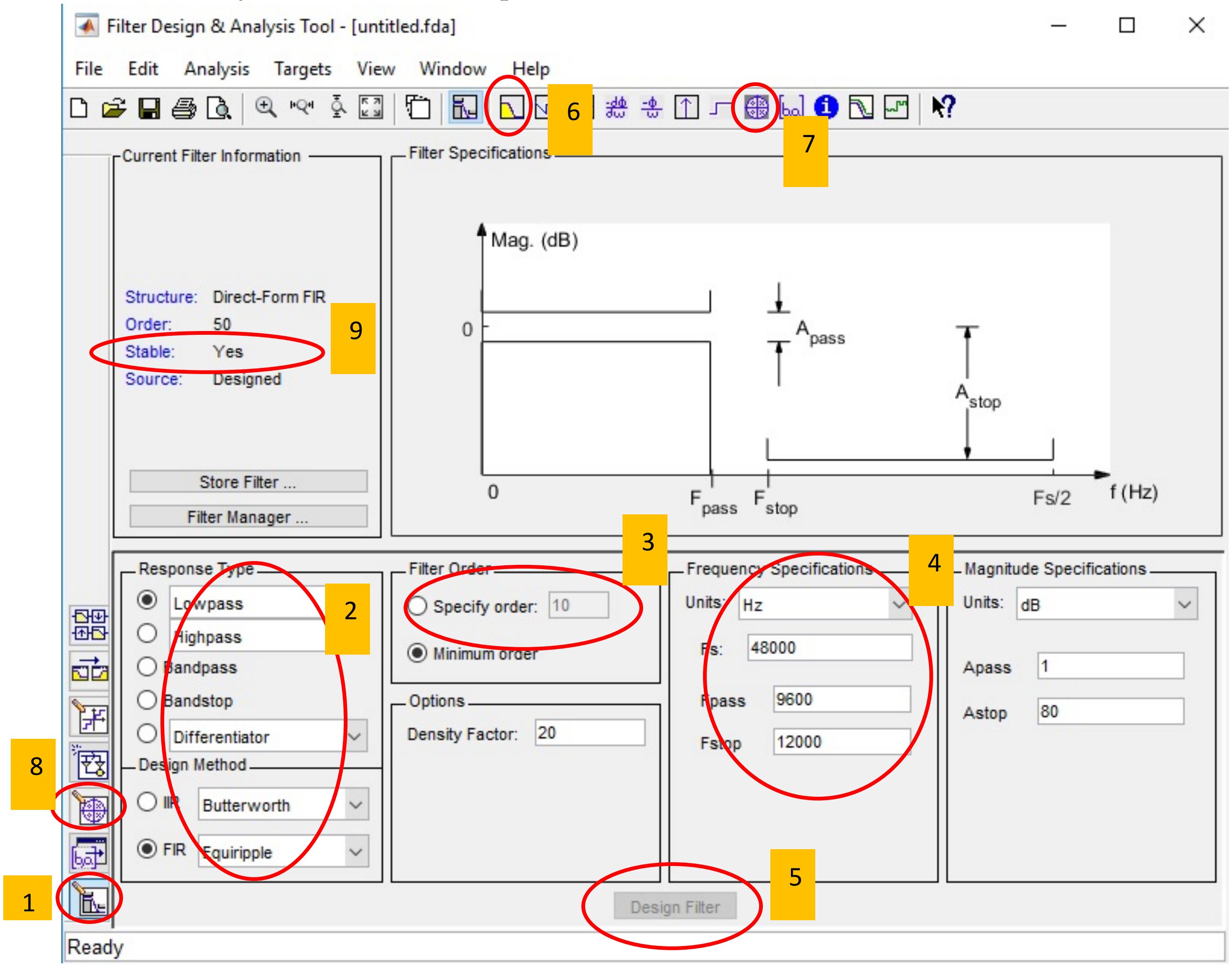
You have studied DTFT and z-Transform and their significance in signal/system visualization. In class lectures we have generally analyzed the sequences with notation $x[n]$ (not assuming it to an input to LTI system). However, same structure in time-domain can be assumed to be a system with response $h[n]$. You aim of this assignment should be understand the concept of system designing with special attributes in time and frequency domain. We will focus right now on frequency domain. And learn to construct any system (or may be sequence) and modify their properties by changing entities in their frequency domain representation.

Warmup

We will start with Matlab's established design tool called "fdatool". So, open Matlab and type "fdatool" in the command prompt and you will see figure like given below:



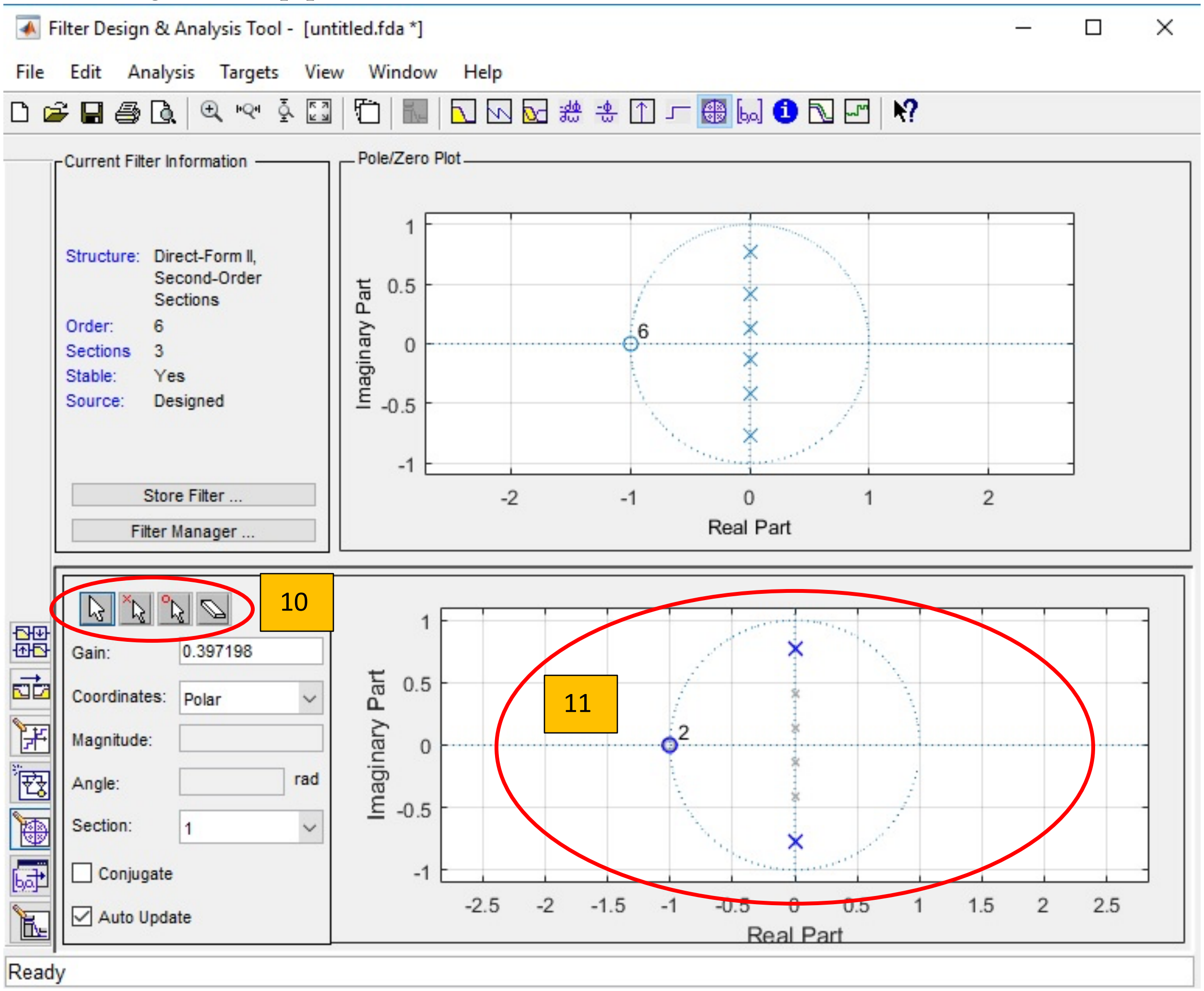
This is advanced designing tool especially for frequency selective filters. However, our aim is not to understand these filters but to understand the change in their behavior using pole-zero placements. There are lots of options but you should focus on a few as shown below; turn by turn select these options



1. Select designing
2. Select filter type e.g., select “lowpass”, IIR (Buttorworth)
3. Select order e.g., 6
4. Select units, you should select “Normalized” which is frequency in rad. And then select cutoff $\omega_c = 0.5$ which is equal to ${}^{\pi}/_{2}$ rad.
5. Press “Design Filter”.
6. Press to visualize frequency response $H(e^{j\omega})$ for this system.
7. See pole-zero plot.
8. Press to edit pole-zero location.

9. Forget about all description and focus on “stable” flag.

Now in (8) you will be able to add/remove pole or zero or on the screen, you should be able to drag and drop poles and zeros as shown below as (10) and (11).



Play around and get familiarized. Delete the poles/zeros and see what happens, add the poles/zeros and see what happens to the magnitude response by pressing (6). Keep an eye on Gain, Magnitude and Angle in the left sub-window. Try to make system unstable by placing poles outside unit circle etc.

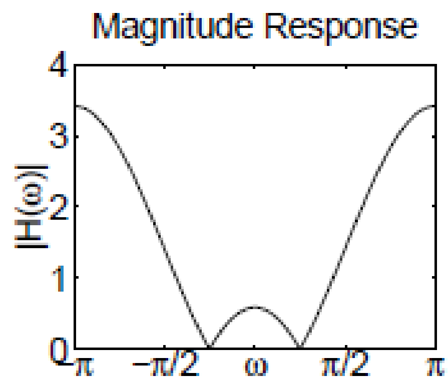
Design

1. Any transfer function with M-zeros and N-poles can be represented in z-transform as

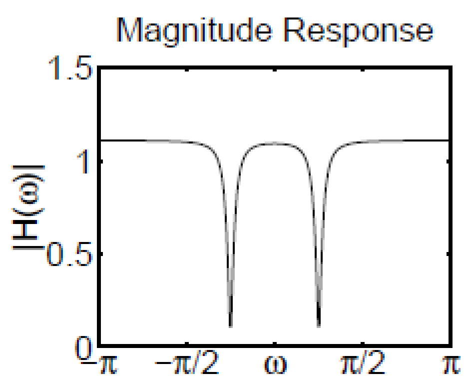
$$H(z) = \frac{(z - z_1)(z - z_2) \ldots (z - z_M)}{(z - p_1)(z - p_2) \ldots (z - p_N)} \qquad eq(1)$$

Remember, DTFT and z-transform are linked with $z = re^{j\omega}$.

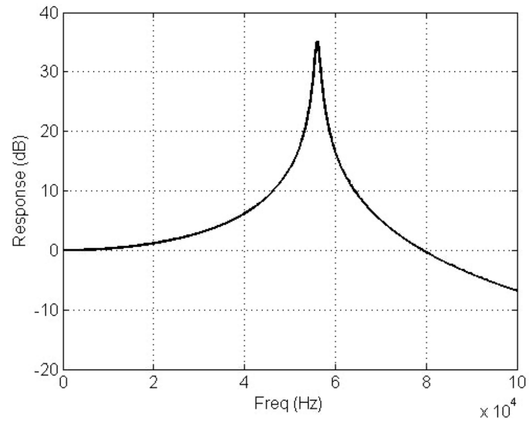
Your task is to design as system with the magnitude response as given below. Note that the dip to zero comes at around $\omega = \pm {}^{\pi}/_{4}$ rad. Generate as certain pole-zero configuration to get exactly this shape. Keep in mind that Matlab shows the gain of magnitude response (after pressing (6)) in dbs i.e., $20 \log\left|H\left(e^{j\omega}\right)\right|$ and the response in the figure below is different, so you should be careful in getting correct values on y-axis. [Hint: design maximally flat IIR filter using (2) with 2-zeros and 2-poles and then delete/add poles and zeros using (8) till you get desired shape]



2. Write complete transfer function $H(z)$ by hand for part-1.
3. What should be done with poles and zeros to make the response look like below at $\omega = \pm {}^{\pi}/_{4}$ rad and why?



4. Repeat part-3 but get the following response approximately at 50 kHz (convert that to rad)



5. Find poles and zeros of the following transfer function by hand and write in the form of eq. (1), apply to "fdatool" and get the magnitude response.

$$H(z) = \frac{z^2 - 2\cos(\pi/4)z + 1}{z^2 - 0.5\cos(\pi/4)z + 0.25} \qquad eq(2)$$

Deliverables

1. Complete report in hard copy. Paste all relevant screen shots of the "fdatool" as you work on your tasks.
2. There won't be any time extension.
3. You can work in a group of two. In report clearly mention the names of all group members.