NUST School of Electrical Engineering and Computer Science

Faculty Member:______________ Date: ______________

Semester:_____________ Section: ______________

Department of Electrical Engineering

EE-379 : Control Systems

LAB 3: System Modeling

Student name	Reg. No.	Pre-lab Marks /5	Lab Report Marks /5	Viva Marks / 5	Total/15

Lab 3: System Modeling and Simulation in Simulink

1. Objectives

- Learn the modeling of a DC motor using first principles.
- Given a set of differential equations that describe a system, learn how to create a model in Simulink.
- Given a set of differential equations describing a system, learn how to find the transfer function of the system.

2. Introduction

Modeling is the process of finding a mathematical model of a system. A mathematical model is a description of a system and helps us to understand the behavior of that system. Obtaining a model is usually the first step in controller design.

In this handout we will see how we can derive a model using first principles. First principles are the established physical laws, e.g. Kirchhoff's voltage law, Newton's laws of motion, etc. Modeling based on first principles doesn't involve the value of system parameters like mass, spring constant etc. These parameter should either be known or have to be estimated using system identification techniques.

Dynamical systems have some quantity that changes with time, and therefore the rate of change of that quantity is often necessary to describe that dynamical system. For this reason, most of the dynamical systems can be modeled with a set of differential equations. In this handout we will derive the model of a DC motor using first principles and will see that it is also a set of differential equations.

In some cases, the model is provided by the system manufacturer/designer and we don't have to find out the differential equation ourselves.

Once we have a set of differential equations, we will see how to represent those differential equations (or the dynamical system) in Simulink. We will also learn how to find a transfer function of a system from a set of differential equations.

3. Modeling of DC Motor

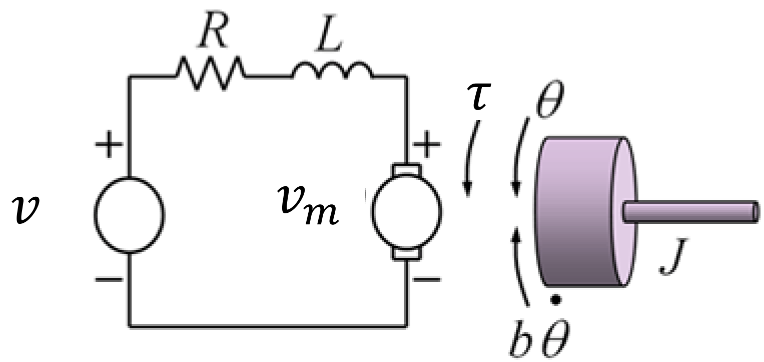


Figure 1: Model of DC Motor

Figure 1 shows a representation of a DC motor. The DC motor has an electrical part and mechanical part. When a voltage is applied to the motor, it affects the current in the armature of the motor, which in turn affects the speed of the motor.

Let us list the facts that we know about the motor

- The motor torque (τ) is proportionally related to the armature current (i)

  $$\tau = K_t i \tag{1}$$

  where K_t is a constant.

- The back emf (v_m) of the motor is proportionally related to the angular velocity ($\dot{\theta}$) of the motor

  $$v_m = K_e \dot{\theta} \tag{2}$$

  Where $v_m = K_e$ is a constant

- Using Kirchhoff's law we can find the equation for the electrical circuit of the DC motor

  $$L\frac{di}{dt} + Ri + v_m = v \tag{3}$$

- Using Newton's law we can find the relation between the torque and the angular velocity. The relation is

  $$J\ddot{\theta} + b\dot{\theta} = \tau \tag{4}$$

  where "J" is the moment of inertia of the armature and "b" is damping coefficient.

Now we have listed all the relations in terms of equations. We can easily manipulate the above equations to reduce the number of total equations.

Using equation (1) and (3) we get

$$L\frac{di}{dt} + Ri + K_e\dot{\theta} = v \qquad (5)$$

Using equation (2) and (4) we get

$$J\ddot{\theta} + b\dot{\theta} \ = K_t i \qquad (6)$$

The equations (5) and (6) describe the behavior of a DC motor and therefore represent the model of a DC motor.

Note that we haven't used any values of the parameters, e.g. resistance, inductance, constants, etc. Therefore, this model only represents a general motor. To work with a specific motor, we will have to use the values of these parameters.

4. Implementation of DC Motor Model in Simulink

In this section we will use the obtained model of the DC motor and implement it in Simulink. Although we are working with the model of a DC motor, our purpose is to show that how you can implement any model using differential equations in Simulink.

Follow the steps given below:

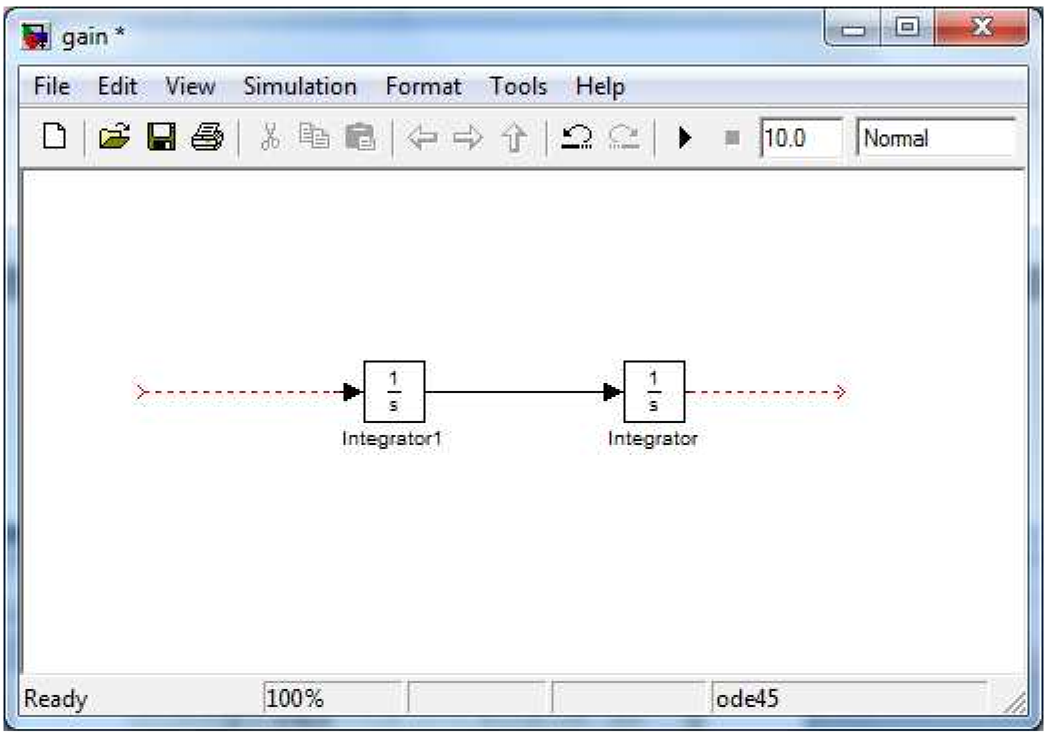
i. Open Simulink

ii. Go to File >> New >> Model to create open new model file.

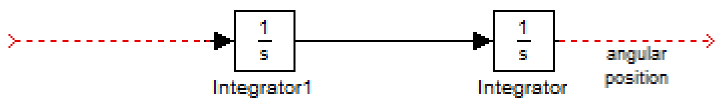
iii. First we will implement equation (6). The equation has a first order derivative and a second order derivative of the angular position θ. However, to represent this equation in Simulink we will not use any differentiators.

We never use differentiators to represent differential equations in Simulink, we only use integrators. The reason is that differentiation can increase the noise in signals.

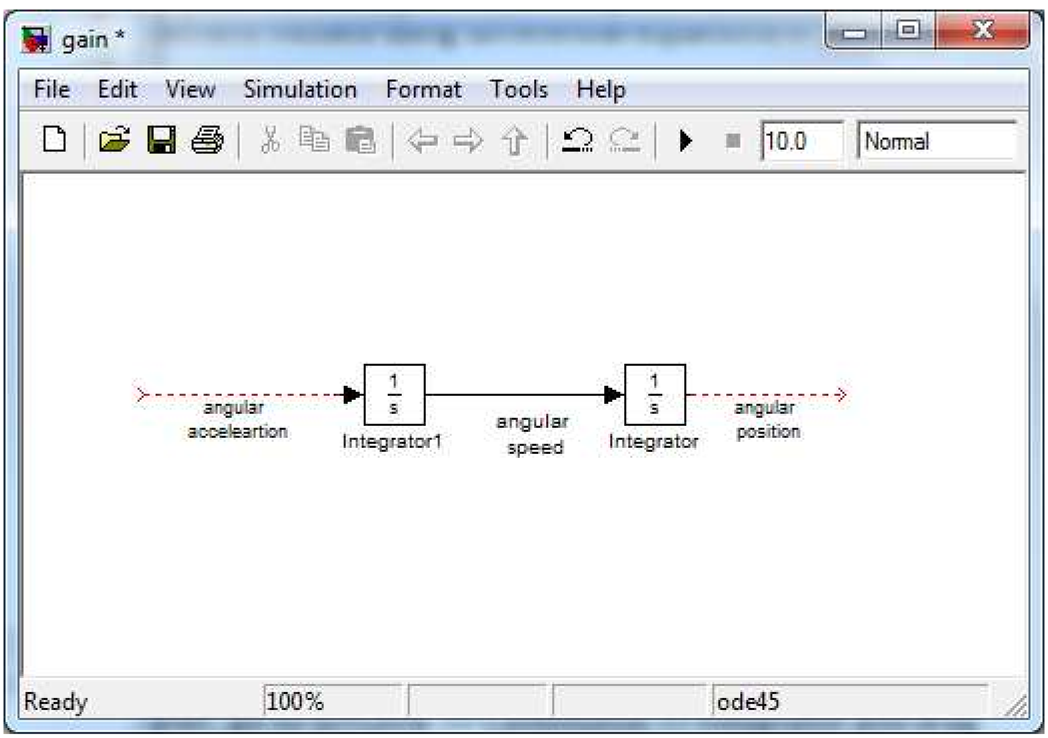
In the Simulink Library Browser, go to Simulink >> Continuous >> Integrator and drag two blocks into your file. Connect the two blocks as shown in the figure below. Also connect wires to the unconnected input and output of the integrator blocks.



iv. Lets label the output on the extreme right as the angular position as shown in the following figure. You can label a connecting wire by double clicking it.



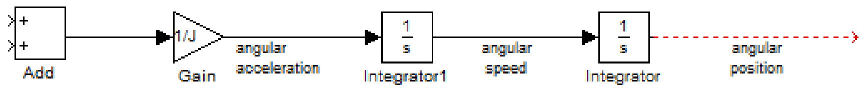
v. If the output on the right is the angular position, then because of the integrators the other two signals should be the angular speed and the angular acceleration. Label the connecting wires accordingly.



vi. I can rewrite equation (6) as

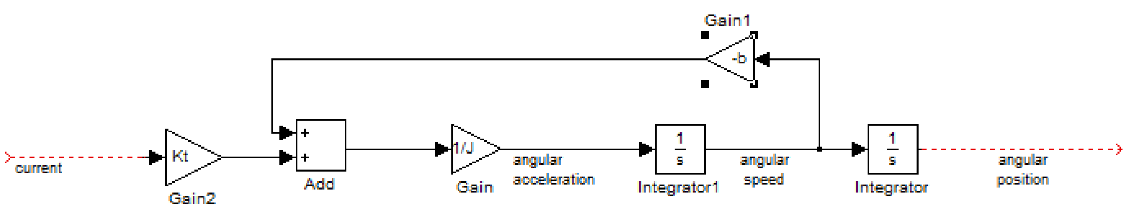
$$\ddot{\theta} = \frac{1}{J}(K_t i - b\dot{\theta}) \tag{7}$$

Considering $K_t i$ and $-b\dot{\theta}$ as two signals, it can be seen that the angular acceleration $\ddot{\theta}$ is the scaled sum of two signals which I can represent as shown in the figure below



In this block diagram we have used the Gain block and the Add block from the Simulink Library browser. To change the gain of the Gain block, just double click it and change the value.

To complete the equation, connect the inputs of the Add block. One input is a scaled value of the angular speed. The other input is scaled value of the current.



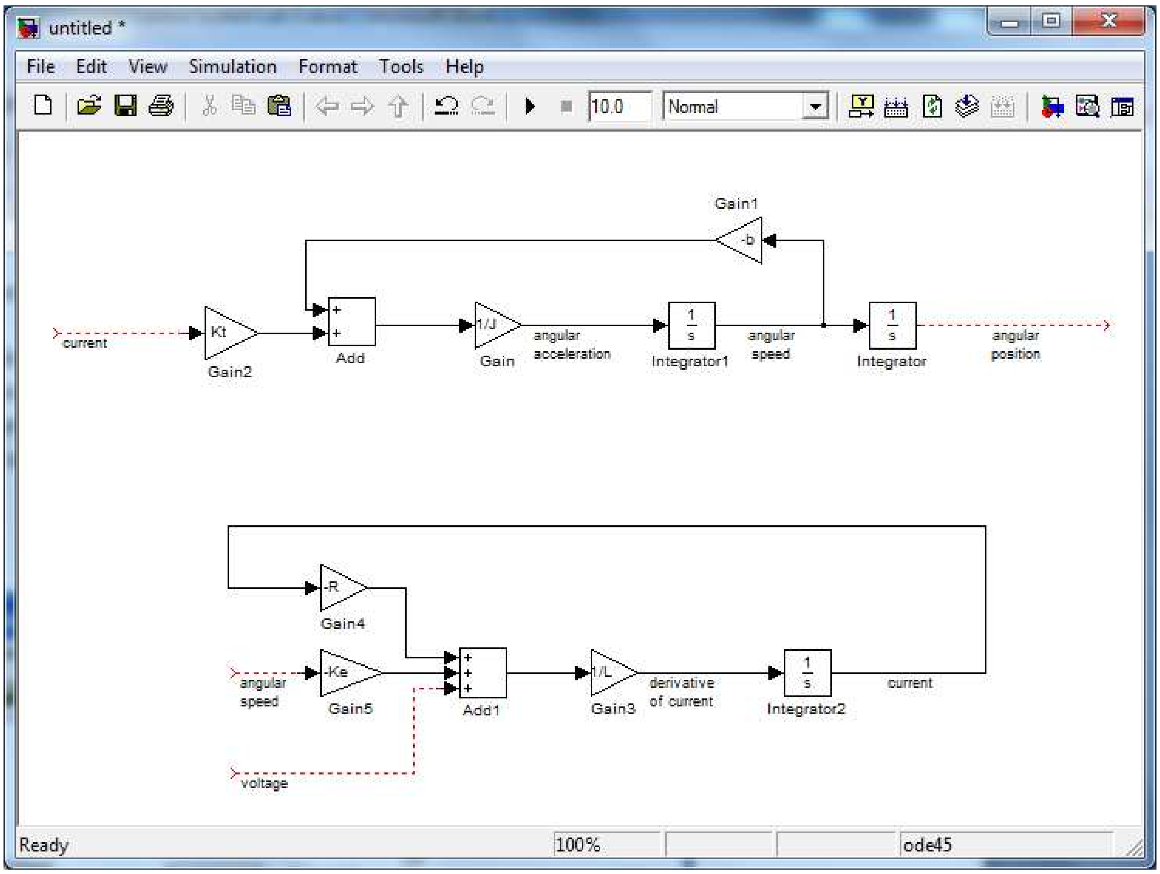
The reason why we needed equation (7) is a little bit complex and may need some experience and practice. However, in general it is useful to represent the highest order derivative in terms of the other variables.

Compare the block diagram with equation (6) or equation (7) and make sure that you understand that it represents the equation.

vii. Using a similar process we can also make a block diagram representation of equation (5). First we rewrite equation (5) as

$$\frac{di}{dt} = \frac{1}{L}(v - Ri - K_e\dot{\theta}) \quad (8)$$

We represent this equation it in Simulink as shown in the figure below

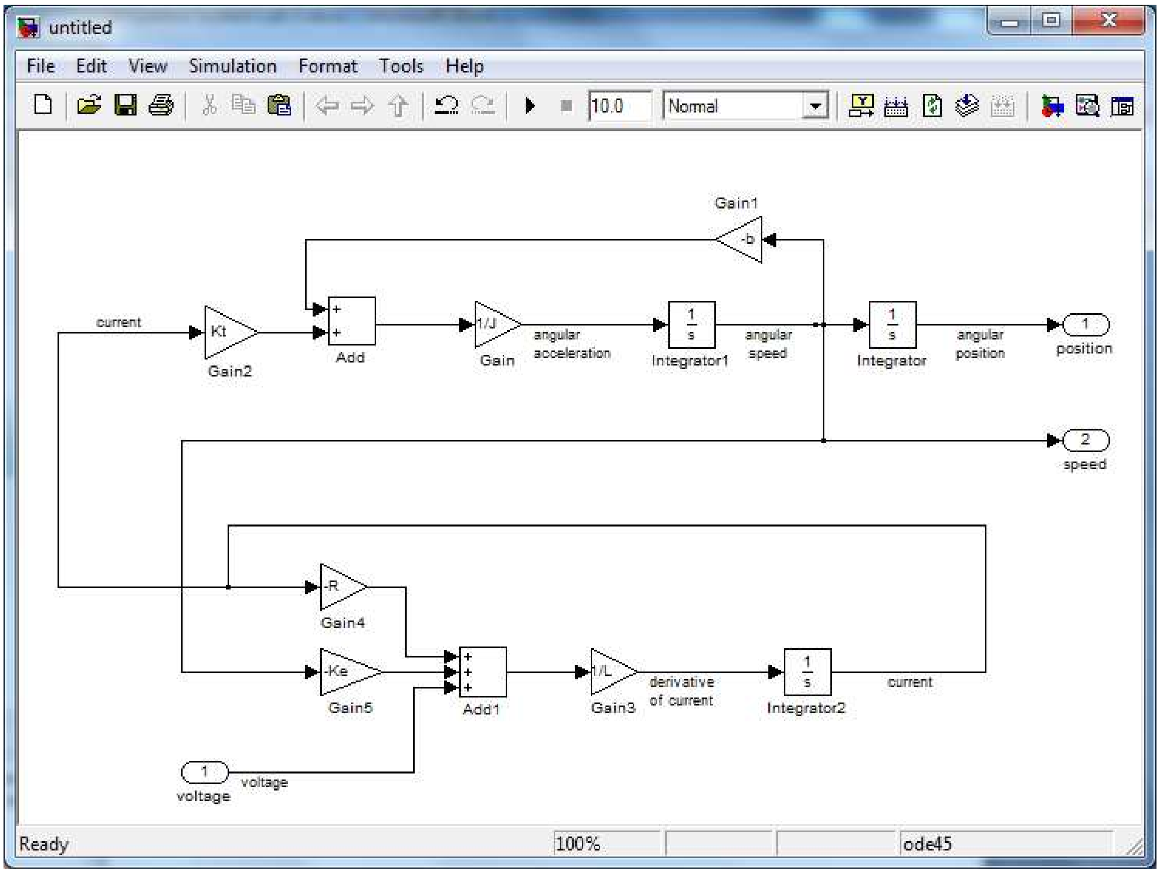


For this we need an Add block with three inputs. This can be done by inserting an Add block with 2 inputs, double clicking it to open its properties, and putting three plus signs instead of two.

Compare the block diagram with equation (5) or equation (8) and make sure that you understand that it represents the equation.

viii. As both equations have current and angular speed as variables, we connect them together. We also add input and output ports. We know that voltage is an input and the output of the motor is the position and the speed.

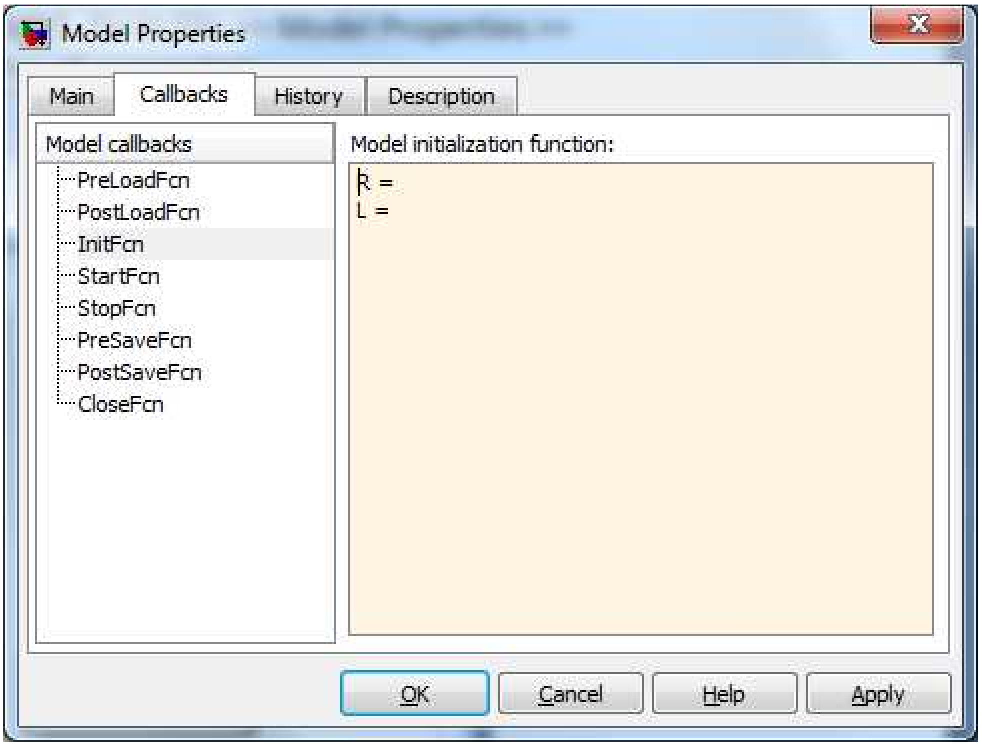
See the figure below for details.



You can insert the input and output blocks by using the Simulink Library browser. Go to Simulink >> Ports and subsystems.

This completes the representation of both differential equations.

ix. Now we will insert the values of the system parameters. We can initiatilize the parameters in the same way as we did in lab 1. Go to File >> Model Properties >> Callbacks >> InitFcn and enter the values for all parameters.

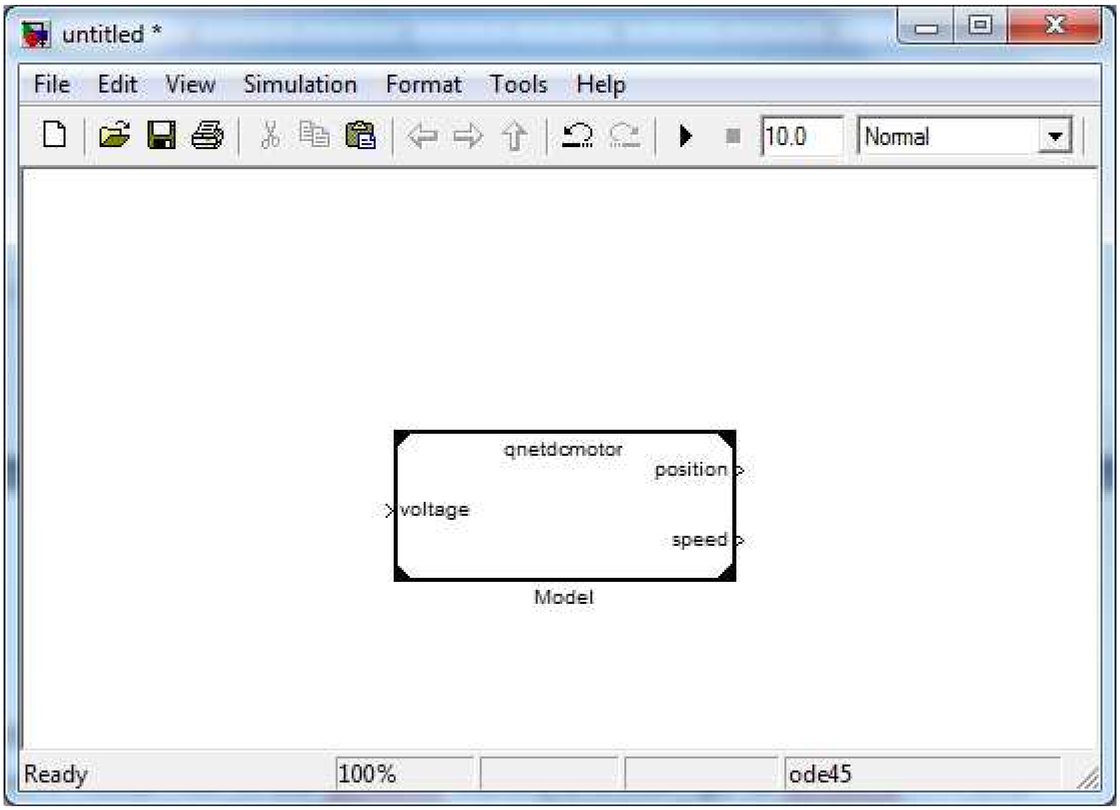


The parameters we will use here are given in the table below. They are the parameters for the QNET DC motor that we have in our control laboratory.

Parameter	**Symbol**	**Value**	**Units**
Motor Armature Resistance	R	8.70	Ohms
Motor Torque constant	Kt	0.0334	N-m
Motor back emf constant	Ke	0.0334	V/rad/s
Moment of the inertia of the motor rotor	J	1.80e-6	Kg-m^2
Armature Inductance	L	0.000	H
Damping ratio of mechanical System	b	0.000	Nms

x. Save the file as “qnetdcmotor_yourname.mdl”.

xi. Create a new model file. In the Simulink Library Browser, go to Simulink >> Ports and Subsystems >> Model and drag a Model block to your file. Double click the model block and enter the name of the dc motor model you have just created.



Now we can analyze and simulate this model. However, we will do this in latter labs.

If you want to look at the details of the model at any time, just double click the model block.

5. Modeling of the Rotary Inverted Pendulum

Figure: Rotary Inverted Pendulum

The modeling of a rotary inverted pendulum is too advanced for this course. Therefore, we will provide you with the set of differential equations that represent the rotary inverted pendulum. The equations are:

$$a\ddot{\theta} - b\ddot{\alpha} + e\dot{\theta} = fv \quad (9)$$
$$-b\ddot{\theta} + c\ddot{\alpha} - d\alpha = 0 \quad (10)$$

where θ is the angle of the driving is motor, α is the angle of pendulum arm, v is the voltage of the motor in the pendulum system.

$$a = J_{eq} + mr^2 + \eta_g K_g^2 J_m$$
$$b = mLr$$
$$c = \frac{4}{3} mL^2$$
$$d = mgL$$
$$f = \frac{\eta_m \eta_g K_t K_g}{R}$$

The description of the parameter in the above equations is given in the table below

Parameter	Symbol	Value	Unit
Moment of inertia of arm and pendulum	J_{eq}	1.84e-6	kg-m^2
Mass of pendulum	m	0.0270	kg
Rotating arm length	r	0.0826	m
Gear ratio	K_g	1	-
Moment of inertia of rotor	J_m	1.80e-4	kg-m^2
Gear box efficiency	η_g	1	-
Motor efficiency	η_m	0.69	-
Half-length of pendulum	L	0.0955	m
Gravity acceleration	g	9.8	m/s^2
Motor torque constant	K_t	0.0334	N-m
Armature resistance	R	8.7	ohms

Exercise 1

Using equations (9) and (10), make a model of the rotary inverted pendulum in Simulink. For the parameters, use the values given in the table above, which are for the QNET inverted pendulum available in our control laboratory.

Exercise 2 (Pre-lab)

Use the set differential equations and the Laplace transforms to find the following transfer functions (Perform this on paper and attach scans/photos in lab report).

i. Transfer function between the speed and voltage of the motor
ii. Transfer function between the position and voltage of the motor
iii. Transfer function between the position of the pendulum and voltage of the of the attached motor

Exercise 3

Using the things you have learnt in lab 1, create all the transfer functions of Exercise 2 in MATLAB.

Note: Always use m files when working in MATLAB so that you can save them.

Exercise 4

Using the things you have learnt in lab 1, create all the transfer functions of Exercise 2 in Simulink using the Transfer function block.

Exercise 5

Using the things you have learnt in lab 2, create all the transfer functions of Exercise 2 in LabVIEW.

Expected files at the end of the lab:

- Two Simulink files, in which the model is made with integrator blocks.
- Three Matlab scripts (m files) for the transfer functions.
- Three Simulink files, in which the model is made with transfer function blocks.
- Three LabVIEW files for the transfer functions.