Communication Systems
EE-351

Lecture 4

Fourier Transform (FT):

Consider an aperiodic continuous time signal x(t), its FT is given as:

$$X(F) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi Ft}dt$$

$$x(t) \xleftrightarrow[Fourier\ Transform\ pair]{} X(F)$$

$x(t)$ might be real but, $e^{-j2\pi Ft}$ is a complex quantity in $X(F) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi Ft}dt$, therefore, $X(F)$ in general is complex.

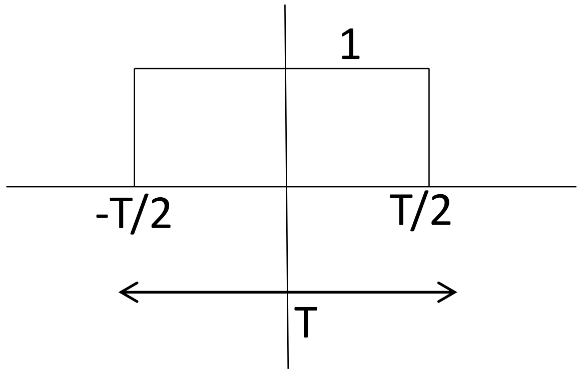
Two components of $X(F)$ are:

- Magnitude spectrum
- Phase spectrum

Given $X(F)$, the corresponding time domain signal x(t) is given by the inverse Fourier Transform as:

$$x(t) = \int_{-\infty}^{\infty} X(F)e^{j2\pi Ft}dF$$

Fourier Transform (example)



Consider the pulse x(t) defined as:

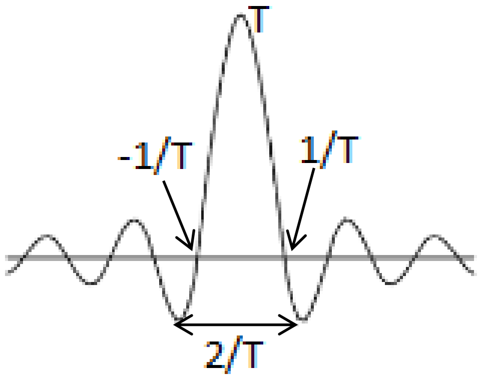
$$x(t) = P_T(t) = \begin{cases} 1, |t| \le {}^{T}\!/_{2} \\ 0, otherwise \end{cases}$$

$$X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi F t} dt = \int_{-\infty}^{\infty} P_T(t) e^{-j2\pi F t} dt$$

$$P_T(F) = \int_{-T/2}^{T/2} 1.\, e^{-j2\pi F t} dt = \frac{e^{-j2\pi F T/2} - e^{-j2\pi F(-\frac{T}{2})}}{-j2\pi F} = \frac{-2jsin(2\pi F T/2)}{-2j\pi F}$$

$$P_T(F) = \frac{\sin(\pi F T)}{\pi F}$$

Fourier Transform (example)



Null-null BW

$$P_T(F) = \frac{\mathrm{Tsin}(\pi F T)}{\pi F T} = Tsinc(FT)$$

At $F = 0, Tsinc(0) = T$

At $F = \frac{k}{T}, Tsinc\left(\frac{kT}{T}\right) = Tsinc(k) = 0$

$$P_T(t) \xleftrightarrow[Fourier\ Transform\ pair]{} P_T(F) = Tsinc(FT)$$

$$P_T(t) = \int_{-\infty}^{\infty} P_T(F)\, e^{j2\pi F t} dF = \int_{-\infty}^{\infty} Tsinc(FT)\, e^{j2\pi F t} dF$$

Fourier Transform (modulation property)

Modulation simply refers to:

Modulated signal $\rightarrow$ $x_m(t) = x(t)e^{j2\pi F_c t}$ $\leftarrow$ Carrier freq.

x(t) times a complex sinusoid with a very high freq.

If $x(t) \xleftrightarrow[Fourier\ Transform\ pair]{} X(F)$

$x_m(t) \longleftrightarrow ??$

$$X_m(F) = \int_{-\infty}^{\infty} x_m(t)e^{-j2\pi F t}dt = \int_{-\infty}^{\infty} x(t)e^{j2\pi F_c t}e^{-j2\pi F t}dt = \int_{-\infty}^{\infty} x(t)e^{-j2\pi(F-F_c)t}dt$$

Fourier Transform (modulation property)

$$X_m(F) = X(F - F_c)$$

Fourier Transform of modulated signal

Fourier Transform X(F) shifted by F_c

Modulation in time $\equiv$ shift in frequency by F_c

Linear time Invariant (LTI) system

Another principle of communication is the transmission of a signal through a linear system

Consider a signal x(t) given as an input to a LTI system

If system is linear

$$\underbrace{ax_1(t) + bx_2(t)}_{\text{Linear combination of inputs}} \rightarrow \underbrace{ay_1(t) + by_2(t)}_{\text{Linear combination of corresponding outputs}}$$

Linear time Invariant (LTI) system

If $\mathrm{x}(t) \rightarrow y(t)$

$$\underbrace{\mathrm{x}(t-t_o)}_{\text{Time shifted input}} \rightarrow \underbrace{y(t-t_o)}_{\text{Time shifted output}}$$

Therefore, linearity + time invariance = linear time invariant (LTI) system

An LTI system is characterized by impulse response h(t)

$$y(t) = x(t) * h(t)$$

Input signal

Convolution operator

This convolution operation is represented as:

$$\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$

$$Y(F) = X(F).H(F)$$ (multiplication in freq. domain)

Amplitude Modulation (AM)

Any communication system involves a carrier signal as:

$$c(t) = A_c \cos(2\pi F_c t)$$

A_c is Amplitude of the carrier and F_c is carrier frequency

Hence, AM signal is defined as:

$$x(t) = (1 + k_a m(t)) \times A_c \cos(2\pi F_c t)$$
$$x(t) = A_c(1 + k_a m(t)) \cos(2\pi F_c t)$$

$x(t)$ is amplitude modulated signal, k_a sensitivity of this AM signal, $m(t)$ is message signal or baseband signal

$\Longrightarrow$Amplitude of the carrier varies according to the message or Amplitude of the carrier is modulated according to the message