Deep Reinforcement Learning

Special Focus: Continuous Control with Deep Deterministic Policy Gradients

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Reinforcement Learning Recap

Policy Gradients

Deterministic Policy Gradient

Deep Deterministic Policy Gradient

Applications of Reinforcement Learning

Reinforcement Learning Recap

Problem Setting

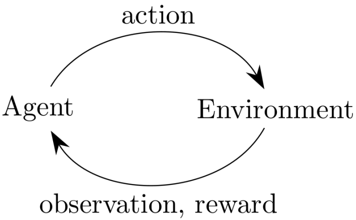


Figure 1: RL General Idea

- An agent is within an environment
- The agent is to complete some task and receive reward
- It solves this task over some amount of time steps

Silver (2015)

Markov Decision Processes

The environment in RL can be described as a Markov Decision Process

This relies on what's called a Markov State:

$$\mathbb{P}[S_{t+1}|S_t] = \mathbb{P}[S_{t+1}|S_1 ... S_t]$$

This means:

- The future is independent of the past, given the present
- The state is a sufficient statistic of the future
- All previous states can be thrown away and the same result will still be calculated

Note: For the Markov property to hold, the environment must be fully observable.

MDP: Observability

- In a *fully observable environment*, the agent's internal state is the same as the environment's internal state
  - i.e., the agent knows how the environment works exactly, and can therefore predict what each of its action will do with 100% accuracy
  - Put formally, the observation at time t is the same as both the agent's and environment's internal representations $S_t^e = O_t = S_t^a$
  - Can be represented with an MDP
- In a *partially observable environment*, the agent only indirectly observes the environment's state
  - The agent must construct it's own internal state based on its belief/construction of the environment state
  - Can be represented with a *Partially Observable Markov Decision Process*, POMDP

MDP: Now to the Markov Decision Process

"Decision" in *Markov* ***Decision*** *Process* means that actions need to be chosen–therefore, we add a policy to choose actions

An MDP can be represented as a tuple:

$$\langle S, A, P, R, \gamma \rangle$$

where:

S is the (finite) state space

A is a finite set of actions

P is the state transition matrix (matrix of state transition probabilities)

R is a reward function

γ is a discount factor, $\gamma \in [0, 1]$

MDP: Value Functions

The *State-Value Function* depends on the policy, and determines how good it is to be in a given state:

$$V^{\pi}(s) = \mathbb{E}_{\pi}(R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + ...|S_t)$$

"The expectation when we sample all actions according to this policy π"; the value of a state

The *Action-Value Function* is defined as how good it is to take a particular action when the agent is in a particular state:

$$Q^{\pi}(s,a) = \mathbb{E}_{\pi}\left(\sum_{k=0} \gamma^k R_{t+k+1} \middle| S_t = s, A_t = a\right)$$

"The expected return starting from state s, taking action a, and then following policy π"; the value of an action

MDP: Solving Reinforcement Learning

We want to maximize the value of our actions based on future reward:

$$V^*(s) = \max_a Q^*(s, a)$$

$$Q^*(s, a) = R_s^a + \gamma \sum_{s' \in S} P_{ss'}^a V^*(s')$$

We can nest these to get:

$$Q^*(s, a) = R_s^a + \gamma \sum_{s' \in S} P_{ss'}^a \max_a Q^*(s, a)$$

This is the *Bellman Optimality Equation* (note: it can be nested in the other direction too to solve for $V^*(s)$)

Solve this, and the reinforcement learning problem is solved.

Q-Learning

Method to solve Q^*

- Iteratively act through *episodes*
- *"Backpropagate"* reward in order to calculate Q values which tell the values of actions
- Take the maximum Q value at each time step
- Store Q values into a table

$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha_t \cdot \left(R_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t) \right)$$

Q-Learning – Expanded

DQN

- Uses a neural network to approximate Q^*
- Can be thought of as looking at a state row in the Q-table, then taking the argmax

Continuous Actions

- Discretizing the action space almost always leads to combinatorial explosion:
  - Consider discretizing the human arm (7-DoF) into Up/Straight/Down – $3^7 = 2187$ dimensional action space
- If we can't (or don't want to) discretize the action space, the Q-table becomes incalculable (would equate to an infinite length table)
- Therefore, Q-learning (and by extension DQN) will not work when dealing with continuous actions

Q-Learning – Q-Table Example

State	Stay	Left	Right	Forward	Backward
0	0	-1	-1	0	0
1	0	0	-1	-1	10
2	0	0	-1	-1	10
3	0	-1	0	0	-1
4	0	0	0	0	10

Figure 2: Q-Table

Q-Learning – Deterministic Policies

- A deterministic policy can lead an agent into an infinite loop
- Imagine this rule from some policy:
  - "Whenever there is a wall to the north and south, go left"
- If we applied this policy to the problem pictured, the agent would get stuck
- This can be solved by using a stochastic policy and leaving ϵ active during test time

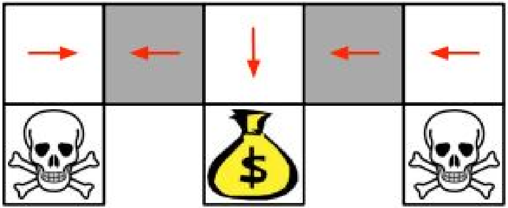


Figure 3: Deterministic Failure

Some Helpful Definitions

To follow everything to come, it is necessary to have a general grasp of the following concepts:

- **Agent**: what performs actions in the environment; wants to maximize future reward
- **Environment**: where the agent resides and what gives observations and reward; interacted with by agent
- **Reward**: R_t; reward at time step t, scalar
- **Observation**: O_t; what the environment shows the agent at step t, after an action
- **Action**: A_t; the action taken at step t, performed by agent
- **History**: sequence of observations, actions, and rewards up to current time step; i.e. $H_t = A_1, O_1, R_1 ... A_t, O_t, R_t$
- **State**: a function of history; $S_t = f(H_t)$; the information used to determine what happens next

Some Helpful Definitions

- **Fully observable**: the environment state equals the agent state; $S_t^a = S_t^e$; the agent knows the complete dynamics of the environment
- **Partially observable**: the agent must make an assumption about the environment because it doesn't know it's dynamics
- **Model**: the agent's internal representation of the environment
- **Policy**: π; what the agent uses to map states to actions; tells the agent what to do
- **Deterministic policy**: a state will always lead to a certain action; $\pi(s) = a$
- **Stochastic policy**: a state will yield a probability of actions to choose from; $\pi(a|s) = \mathbb{P}(A = a|S = s)$
- **State-Value Function**: tells the value of a state based on the expected future reward
- **Action-Value Function**: tells the value of an action based on expected future reward

Some Helpful Definitions: Types of RL Algorithms

- Value Based (e.g. Q-Learning)
  - No policy (implicit)
  - Learnt Value function
- Policy Based (e.g. Policy Gradient)
  - Learnt Policy
  - No value function
- ***Actor-Critic*** (e.g. DDPG)
  - Learnt Policy
  - Learnt Value function
- Model Based/Model Free
  - Learnt Policy and/or Value function
  - Based: has model; Free; no model

Policy Gradients

Policy Objective Function

- Plan: Given policy $\pi_\theta(s, a)$ with parameters θ, find the best θ
- How can we measure the goodness of a policy?
- We consider episodic environments with a start state
- The goodness of policy is the return gained when coming from the start state
- This is denoted by the performance objective:

$$J_1(\theta) = V^{\pi_\theta}(s_1) = \mathbb{E}_\pi(R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots | S_t)$$

We will find the gradient of the performance objective with respect to θ. This allows us to change θ in order to maximize the performance objective.

Adjusting with respect to a Score Function

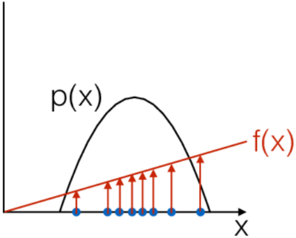


Figure 4: Evaluating f(x)

- Sampling x from $p(x)$
- Evaluating $f(x)$

Adjusting with respect to to a Score Function

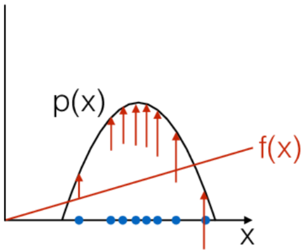


Figure 5: Pushing p(x) w.r.t f(x)

- Manipulate $p(x)$ according to magnitude of $f(x)$
- Re-normalize $p(x)$

Goal: Change p(x) such that it "produces" x which in turn result in high values in the score function. **But how to change** $p(x)$?

Score Function Gradient Estimator

$$
\begin{aligned}
\nabla_\theta \mathbb{E}_x[f(x)] &= \nabla_\theta \sum_x p(x|\theta) f(x) && \text{definition of expectation} \quad (1)\\
&= \sum_x \nabla_\theta p(x|\theta) f(x) && \text{swap sum and gradient} \quad (2)\\
&= \sum_x p(x|\theta) \frac{\nabla_\theta p(x|\theta)}{p(x|\theta)} f(x) && \text{both multiply and divide by } p(x|\theta) \quad (3)\\
&= \sum_x p(x|\theta) \nabla_\theta \log p(x|\theta) f(x) && \text{use the fact that } \nabla_\theta \log(z) = \frac{1}{z}\nabla_\theta z \quad (4)\\
&= \mathbb{E}_x[f(x) \nabla_\theta \log p(x|\theta)] && \text{definition of expectation} \quad (5)
\end{aligned}
$$

Now we need to sample $x_i \sim \; p(x|\theta)$, and compute

$$\hat{g}_i = f(x_i) \nabla_\theta \log(p(x_i|\theta))$$

Score Function Gradient Estimator Intuition

$$\hat{g}_i = f(x_i)\nabla_\theta \log p(x_i|\theta)$$

- $f(x)$ measures how good the sample x is (score function)
- Stepping (ascending) in the direction $\hat{g}_i$ increments the log probability of the x, proportionally to the score
- x which yield good scores in f become more probable

Score Function Gradients in Context of Policies

In the context of policies the random variable x is a whole trajectory $\tau = (s_0, a_0, r_0, s_1, a_1, r_1, ..., s_{T-1}, a_{T-1}, r_{T-1}, s_T)$

Previous slide:

$$\hat{g}_i = f(x_i)\nabla_\theta \log p(x_i|\theta)$$

Now in the context of policies:

$$\nabla_\theta \mathbb{E}_\tau[R(\tau)] = \mathbb{E}_\tau[\nabla_\theta \log\ p(\tau|\theta)R(\tau)]$$

Now we detail $p(\tau|\theta)$:

$$p(\tau|\theta) = \mu(s_0) \prod_{t=0}^{T-1} [\pi(a_t|s_t, \theta)P(s_{t+1}, r_t|s_t, a_t)]$$

Score function Gradients in context of policies II

$$\log\, p(\tau|\theta) = \log\, \mu(s_0) + \sum_{t=0}^{T-1} [\log\, \pi(a_t|s_t,\theta) + \log\, P(s_{t+1}, r_t|s_t, a_t)]$$

Now differentiating with respect to θ

$$\nabla_\theta \log\, p(\tau|\theta) = \nabla_\theta \sum_{t=0}^{T-1} [\log\, \pi(a_t|s_t,\theta)$$

The gradient is not dependent on the state transition distribution $P(s_{t+1}, r_t|s_t, a_t)$.
Inserting back into the expectation yields:

$$\nabla_\theta \mathbb{E}_\tau[R] = \mathbb{E}_\tau[R \nabla_\theta \sum_{t=0}^{T-1} [\log\, \pi(a_t|s_t,\theta)]]$$

Stochastic Policy Gradient Theorem

- When using the state-action value function Q^π for R the policy gradient is:

$$\nabla_\theta \mathbb{E}_\tau[R] = \mathbb{E}_\tau[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t, \theta) Q^\pi(s_t, a_t)]$$

Silver et al. (2014)

Actor - Critic

Actor - Critic is an Architecture based on the Policy Gradient Theorem.

- *Actor* adjusts the parameters θ of the policy π
- This is done by ascent of the policy gradient
- The real $Q^{\pi}(s, a)$ is unknown
- Therefore function $Q^{w}(s, a)$ with parameters w is approximated (e.g with deep neural network)
- A *critic* estimates parameters the action-value function Q^{w}

Deterministic Policy Gradient

Policies in Continuous Action Space

Problem:

- The action value is in $\mathbb{R}$
- At every step this requires to evaluate the action-value function Q globally over (at least a subset of) $\mathbb{R}$
- This is infeasible

Solution:

- Do not maximize over Q
- But move policy in direction of Q
- The policy is now deterministic, giving a real valued number

Policies in Continuous Action Space

Specifically:

$$\theta^{k+1} = \theta + \alpha \mathbb{E}_{s \sim \rho^{\mu^k}} \left[\nabla_\theta Q^{\mu^k}(s, \mu_\theta(s)) \right]$$

where $\mu_\theta(s)$ is the deterministic policy

Now applying the chain rule:

$$\theta^{k+1} = \theta + \alpha \mathbb{E}_{s \sim \rho^{\mu^k}} \left[\nabla_\theta \mu_\theta(s) \nabla_a Q^{\mu^k}(s, a)|_{a=\mu_\theta(s)} \right]$$

Determinisitc Policy Gradient Theorem

The deterministic policy gradient now is:

$$\nabla_\theta J(\mu_\theta) = \mathbb{E}_{s\sim\rho^\mu}\left[\nabla_\theta \mu_\theta(s)\nabla_a Q^\mu(s,a)|_{a=\mu_\theta(s)}\right]$$

Silver et al. (2014)

Deep Deterministic Policy Gradient

From DQN to Deep Deterministic Policy Gradient – Code!

DDPG is very similar to DQN implementation-wise – just with some added bells and whistles. **If you plan to implement DDPG, you might want to start with DQN.**

- Define an environment with observations, rewards and actions.
- Repeatedly act in the environment using the current policy & store experiences.
- Q network as value function approximator, optimized using the Bellman equation.
- **New:** Policy network for continuous actions, optimized using policy gradient.
- Online & target network split. **New:** Soft updates.

```
import tensorflow as tf
```

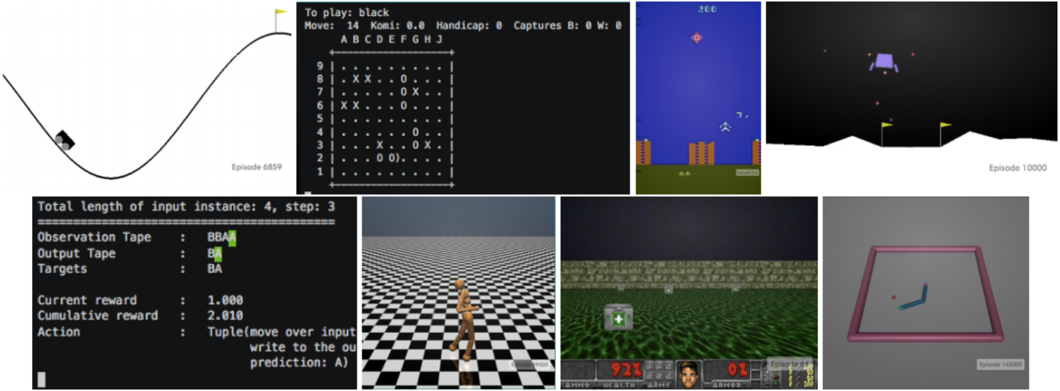


Figure 6: OpenAI Gym Environments (Brockman et al. 2016)[1]

[1]github.com/openai/gym

OpenAI Gym: API

```python
import gym
```

Sensible standardized interface for RL environments. When creating custom environments, building on top of its specifications might make sense.

```python
env = gym.make('LunarLanderContinuous-v2')
env.observation_space  # e.g. float vector, 3D array...
env.action_space  # e.g. integer, float vector...
env.reset()
action = env.action_space.sample()
state, reward, done, info = env.step(action)
env.render()
```

Off-Policy Reinforcement Learning: Generating Samples

Act in the environment following the current policy to generate experiences, store them.

```python
from collections import deque
memory = deque([], maxlen=1e6)  # Note: Random access is O(n)!
policy = lambda state: env.action_space.sample()
done = True
while True:
  if done:
    state = env.reset()
  action = policy(state)
  state_, reward, done, _ = env.step(action)
  memory.append((state, action, reward, state_))
  state = state_
```

The Critic (aka Value Network)

DDPG: $Q(s, a) \to q$

State & action to single Q value.

Lillicrap et al. (2015)

DQN: $Q(s) \to \vec{q}$

State to Q vector, one value per action.

Mnih et al. (2015)

```python
def make_critic(states, actions, name):
  with tf.variable_scope(name) as scope:
    net = tf.layers.dense(states, 400, tf.nn.relu)  # Feature extract
    net = tf.concat([net, actions], axis=1)
    net = tf.layers.dense(net, 300, tf.nn.relu)  # Value estimate
    q = tf.layers.dense(net, 1)  # shape (BATCHSIZE, 1)
    return tf.squeeze(q), get_variables(scope)
```

Training Q-Networks: Bellman Approximation

DQN vs. DDQN vs. DDPG – fine differences in estimating future reward.

$$y^{DQN} = r_t + \gamma max_a Q'(s_{t+1}, a) \qquad \text{Greedy estimate.} \tag{6}$$

$$y^{DDQN} = r_t + \gamma Q'(s_{t+1}, argmax_a Q(s_{t+1}, a)) \qquad \text{Estimate by online policy.} \tag{7}$$

$$y^{DDPG} = r_t + \gamma Q'(s_{t+1}, \mu'(s_{t+1})) \qquad \text{Estimate by detached policy.} \tag{8}$$

Mnih et al. (2015), Van Hasselt, Guez, and Silver (2016), Lillicrap et al. (2015)

Training the DDPG Critic: Bellman Approximation & Mean Squared Error

The critic is optimized to minimize the mean squared error loss between its output and the Bellman approximation.

$$y = r_t + \gamma Q'(s_{t+1}, \mu'(s_{t+1})) \qquad \text{Critic Target} \tag{9}$$

$$\mathbb{L} = \frac{1}{N}\sum^{N}(Q(s_t, a_t) - y)^2 \qquad \text{Critic Loss} \tag{10}$$

```
# critic, _ = make_critic(states, actions, 'online')
# critic_, _ = make_critic(states_, actor_, 'target')
def train_critic(critic, critic_, terminals, rewards):
  targets = tf.where(terminals, rewards, rewards + .99 * critic_)
  mse = tf.reduce_mean(tf.squared(targets - critic))
  return tf.train.AdamOptimizer(1e-3).minimize(mse)
```

The Actor (aka Policy Network)

DDPG: $\mu(s) \rightarrow a$

Vector of continues action values.

Lillicrap et al. (2015)

DQN: $argmax_a Q(s, a) \rightarrow a$

Greedy discrete action selection.

Mnih et al. (2015)

```
def make_actor(states, n_actions, name):
  with tf.variable_scope(name) as scope:
    net = dense(states, 400, tf.nn.relu)
    net = dense(net, 300, tf.nn.relu)
    y = dense(net, n_actions, tf.nn.tanh)  # Action scaling.
    return y, get_variables(scope)
```

Training the Actor (Policy Gradient Ascent)

Ascend the gradients of the critic network with respect to the online actor's actions.

$$\begin{aligned} \Delta_{\theta^\mu} J &\approx \Delta_{\theta^\mu} Q(s_t, a) & a = \mu(s_t|\theta^\mu) \quad (11) \\ &= \Delta_a \; Q(s_t, a) \Delta_{\theta^\mu} a & F'(x) = f'(g(x))g'(x) \quad (12) \end{aligned}$$

```python
# actor, thetaMu = make_actor(states, 4, 'online')
# critic, _ = make_critic(states, actor, 'online')
def train_actor(actor, thetaMu, critic):
  value_gradient, = tf.gradients(critic, actor)
  policy_gradients = tf.gradients(actor, thetaMu, -value_gradient)
  mapping = zip(policy_gradients, thetaMu)
  return tf.train.AdamOptimizer(1e-4).apply_gradients(mapping)
```

Target Network Updates

```
# _, theta = make_critic(states, actions, 'online')
# _, theta_ = make_critic(states_, actor_, 'target')
```

Hard Updates: Common in DQN implementations and on initial initialization.

```
def make_hard_update(theta, theta_):
  return [dst.assign(src) for src, dst in zip(theta, theta_)]
```

Soft updates: Slowly follow online parameters, prevents oscillation.

```
def make_soft_update(theta, theta_, tau=1e-3):
  return [dst.assign(tau * src + (1 - tau) * dst)
          for src, dst in zip(theta, theta_)]
```

What kind of monster did we just create?

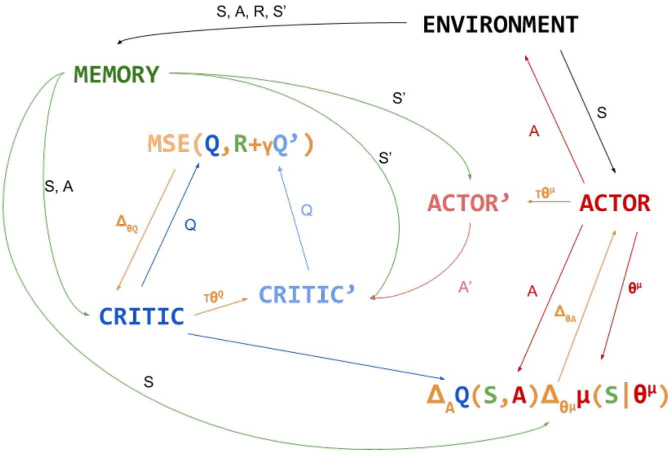


Figure 7: DDPG Dataflow Graph – TensorBoard failed us

Exploration in Continuous Environments

DQN: ϵ-greedy – only for discrete actions.
DDPG: Gaussian continuous through time with friction θ and diffusion σ (Uhlenbeck and Ornstein 1930).

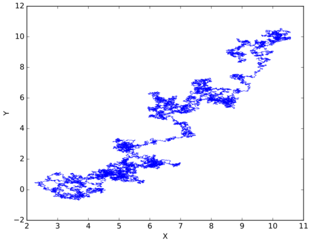


Figure 8: Prototypical Process

```python
def noise(n, theta=.15, sigma=.2):
  state = tf.Variable(tf.zeros((n,)))
  noise = -theta * state + sigma * tf.random_normal((n,))
  return state.assign_add(noise)
```

Do it yourself DDPG

All of the above and more at **github.com/ahoereth/ddpg** $\rightarrow$ **Lander.ipynb**

- Exhaustively documented. Would recommend if you are interested in Deep RL.
- Critic & actor, online & target networks with soft & hard updates.
- Batch normalization – disabled because it didn't improve performance.
- Threaded feeding and training:
    - Main thread can focus on generating new experiences.
    - Some threads feed samples from the memory to the TensorFlow graph.
    - Some threads train the network as scheduled by the agent.
- TensorBoard logs with (not so pretty) graph of whats going on.

Applications of Reinforcement Learning

Cutting Edge Applications of Reinforcement Learning

We will discuss:

- Safety in Robotics and Reinforcement Learning
- Poker
- Multiple Agents

Fisac et al. (2017); Li (2017); Heinrich and Silver (2016); Lowe et al. (2017)

The General Problem of Robotics

- The world is full of noise
  - Great for neural networks!
- Simulations can't simulate the full range and accuracy of the real world, so training actual robots is best
- Danger to break or destroy robot or property
  - Robots are expensive

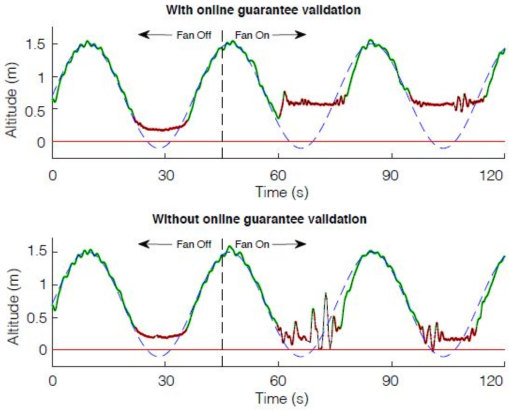
Keeping Robots Safe

- Since neural networks are “black boxes,” it is hard to pinpoint areas where training might lead to dangerous situations based on the weights of the model
- Safety has typically been guaranteed by a manual fallback mechanism or making the environment safe

Safety Framework

Solution:
Combines both a safety net and a bayesian mechanism online to deal with sudden changes in the environment

Youtube link:
https://youtu.be/WAAxyeSk2bw



The Cutting Edge

Let's now look at two applications recently published (last year and last month)

- Poker playing
- Reinforcement learning with multiple agents

Poker as an RL Problem

- Imperfect information game – the hands of other players are unknown, as well as the values of upcoming cards
- Multi-agent zero-sum game – Nash Equilibrium exists, but is incalculable

NFSP (Neural Fictitious Self Play)

- Applying neural networks to the concept of "Fictitious Self Play"
  - FSP = Choose the best response to the opponent's average behavior
- Approaches Nash Equilibrium as it learns

NFSP Poker: Architecture

- Remembers state transitions and the agent's best responses in two separate memories M_{RL} and M_{SL}
  - State transitions used for RL; Best responses used for supervised learning
- M_{RL} uses an off-policy deep RL algorithm to learn the best policy from the state transitions
- M_{SL} uses a feedforward net to learn the average play (in order to do fictitious self play)
- Target network for stability and has an explore parameter

NFSP Poker: Performance

- Comparable to other AIs based on expert knowledge representation (classic AI)
  - e.g. Smooth Upper Confidence Bounds and Counterfactual Regret Minimization+

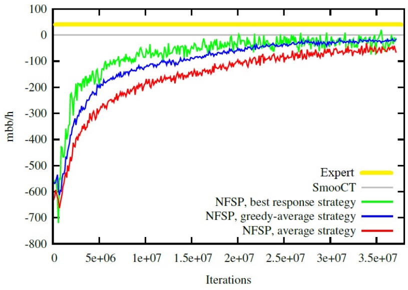


Figure 9: Poker Performance

The Problem

- Multiple agents affect the environment
  - Agent can't accurately predict environment because it is no longer based on its policy alone
  - Significantly increases the variability in policy gradient algorithms – this is because the reward in normal policy gradients is only conditioned on the agent's own actions

The Solution

- Actor-Critic with “centralized” training and “decentralized” execution.
  - The actor can not contain information about the other actors at both training and test time (would require additional assumptions)
  - Solve this by supplying the critic with the policies of all agents (centralized), while the actor remains isolated
  - At test time, only actors are used (decentralized)
  - “Since the centralized critic function explicitly uses the decision-making policies of other agents, we additionally show that agents can learn approximate models of other agents online and effectively use them in their own policy learning procedure”
- Ensemble of policies to make each individual agent robust to changes in other agents’ policies
- Named: MADDPG

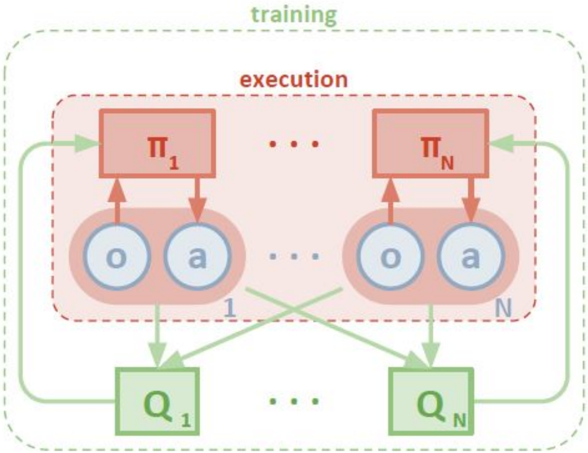


Figure 10: MultiNetwork

Multiagent Reinforcement Learning: Performance

- Trained on a battery of cooperative and competitive multi-agent tasks
- Outperformed DDPG significantly
- Youtube link: youtu.be/QCmBo91Wy64 (1:55)

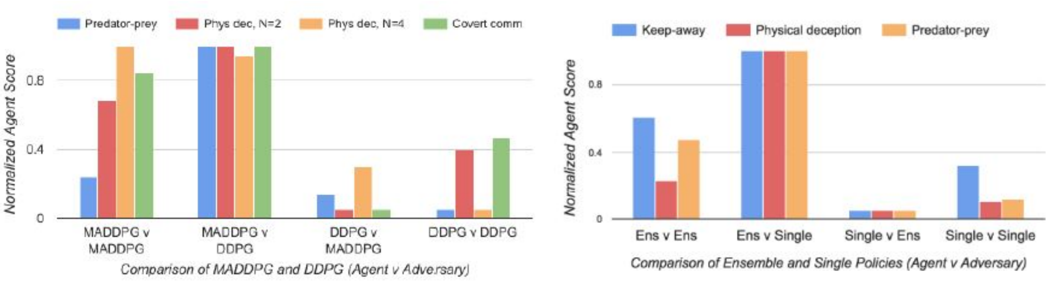


Figure 11: MADDPG Performance

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