How to derive these equations?

① $V_f = V_0 + at$

② $S = \left(\frac{V_0 + V_f}{2}\right)t$

③ $S = V_0 t + \frac{1}{2}at^2$

④ $V_f^2 = V_0^2 + 2as$

Variables and basic concepts

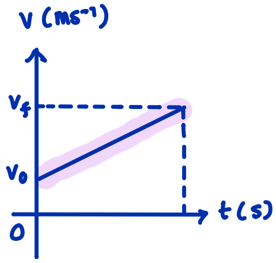
$S \rightarrow$ displacement
$v \rightarrow$ velocity
$a \rightarrow$ acceleration
$V_f \rightarrow$ final velocity
$V_0 \rightarrow$ initial velocity
$t \rightarrow$ time

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\text{acceleration} = \frac{\Delta\text{velocity}}{\text{time}}$$

* In kinematic, acceleration is always CONSTANT

① $V_f = V_0 + at$



$$\text{gradient} = \frac{\Delta V}{\Delta t}$$

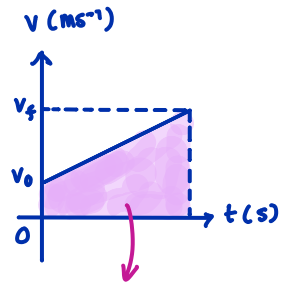
$$a = \frac{\Delta V}{\Delta t}$$

$$a = \frac{V_f - V_0}{t}$$

$$at = V_f - V_0$$

$$V_f = V_0 + at$$

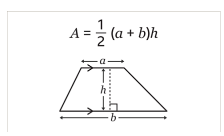
② $S = \left(\frac{V_0 + V_f}{2}\right)t$



$$\text{area under graph} = \frac{1}{2}(V_0 + V_f)t$$

$$S = \left(\frac{V_0 + V_f}{2}\right)t$$

area under graph = area of trapezium



ⓐ

$\bar{V}$ = average velocity, $\frac{V_0 + V_f}{2}$

$$\bar{V} = \frac{S}{t}$$

$$\frac{V_0 + V_f}{2} = \frac{S}{t}$$

$$S = \left(\frac{V_0 + V_f}{2}\right)t$$

③ $S = V_0 t + \frac{1}{2}at^2$

from previous equations: $V_f = V_0 + at$ — ①
$S = \left(\frac{V_0 + V_f}{2}\right)t$ — ②

from ②,

$$S = \left(\frac{V_0 + V_f}{2}\right)t$$

$$\frac{2S}{t} = V_0 + V_f$$

$$V_f = \frac{2S}{t} - V_0$$

equate to ①

$$V_f = V_f$$

$$\frac{2S}{t} - V_0 = V_0 + at$$

$$\frac{2S}{t} = 2V_0 + at$$

$$2S = 2V_0 t + at^2$$

$$S = V_0 t + \frac{1}{2}at^2$$

④ $V_f^2 = V_0^2 + 2as$

from previous equations: $V_f = V_0 + at$ — ①
$S = \left(\frac{V_0 + V_f}{2}\right)t$ — ②

from ①,

$$V_f = V_0 + at$$

$$V_f - V_0 = at$$

$$t = \frac{V_f - V_0}{a}$$

from ②,

$$S = \left(\frac{V_0 + V_f}{2}\right)t$$

$$t = \frac{2S}{V_0 + V_f}$$

$$t = t$$

$$\frac{V_f - V_0}{a} = \frac{2S}{V_0 + V_f}$$

$$V_f^2 - V_0^2 = 2as$$

$$V_f^2 = V_0^2 + 2as$$