Image Processing Lab

Batch 9:

Ajay Viswanathan [09EC3509]

Sourin Sutradhar [09EC3514]

Experiment 3

Discrete Wavelet Transformation (DWT and IDWT)

Problem Objective:

Write C/C++ modular functions to perform the following operations on the given test images. All functions must support at least 24-bit RGB, and 8-bit grayscale image formats.

a) Three level 2D discrete wavelet transform (DWT) using the Haar, Db2, 9/7 CDF wavelet filter banks
b) Reconstruction (wavelet synthesis) using the subbands to produce the Reconstructed Image.
c) Calculate Peak Signal to Noise Ratio (PSNR) between original input image and the reconstructed image.

Brief Theory:

A function $\psi \in L^2(\mathbb{R})$ is called an orthonormal wavelet if it can be used to define a Hilbert basis, that is a complete orthonormal system, for the Hilbert space $L^2(\mathbb{R})$ of square integrable functions. The Hilbert basis is constructed as the family of functions $\{\psi_{jk} : j, k \in \mathbb{Z}\}$ by means of dyadic translations and dilations of ψ,

$$\psi_{jk}(x) = 2^{j/2}\psi(2^j x - k)$$

for integers $j, k \in \mathbb{Z}$. This family is an orthonormal system if it is orthonormal under the inner product

$$\langle \psi_{jk}, \psi_{lm} \rangle = \delta_{jl}\delta_{km}$$

where δ_{jl} is the Kronecker delta and $\langle f, g \rangle$ is the standard inner product $\langle f, g \rangle = \int_{-\infty}^{\infty} f(x)\overline{g(x)}dx$ on $L^2(\mathbb{R})$. The requirement of completeness is that every function $h \in L^2(\mathbb{R})$ may be expanded in the basis as

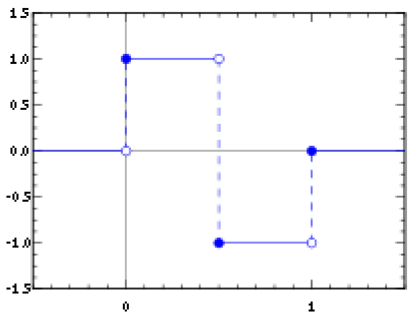
$$h(x) = \sum_{j,k=-\infty}^{\infty} c_{jk}\psi_{jk}(x)$$

with convergence of the series understood to be convergence in norm. Such a representation of a function f is known as a wavelet series. This implies that an orthonormal wavelet is self-dual.

Haar Wavelet

The Haar sequence was proposed in 1909 by Alfréd Haar. Haar used these functions to give an example of a countable orthonormal system for the space of square-integrable functions on the real line. The study of wavelets, and even the term "wavelet", did not come until much later. As a special case of the Daubechies wavelet, the Haar wavelet is also known as Db1.

The Haar wavelet is also the simplest possible wavelet. The technical disadvantage of the Haar wavelet is that it is not continuous, and therefore not differentiable. This property can, however, be an advantage for the analysis of signals with sudden transitions, such as monitoring of tool failure in machines.



The Haar wavelet

The Haar wavelet's mother wavelet function $\psi(t)$ can be described as

$$\psi(t) = \begin{cases} 1 & 0 \le t < 1/2, \\ -1 & 1/2 \le t < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Its scaling function $\phi(t)$ can be described as

$$\phi(t) = \begin{cases} 1 & 0 \leq t < 1, \\ 0 & \text{otherwise.} \end{cases}$$

In functional analysis, the Haar system denotes the set of Haar wavelets

$$\{t \mapsto \psi_{n,k}(t) = 2^{\frac{n}{2}} \psi(2^n t - k); n \in \mathbb{N}, 0 \leq k < 2^n\}.$$

In Hilbert space terms, this constitutes a complete orthogonal system for the functions on the unit interval. There is a related Rademacher system of sums of Haar functions, which is an orthogonal system but not complete.[3][4]

The Haar system (with the natural ordering) is further a Schauder basis for the space $L^p[0,1]$ for $1 \leq p < +\infty$. This basis is unconditional for p > 1.

Results:

Original image

Transformed image

Recovered image

The PSNR observed for this image was : 75.1614 dB

Observation and Inference:

1. The HH quadrant always contains the least amount of image data. The LH and HL quadrants are actually x and y gradient operators.
2. There is significant data loss when working with float values and type-casting them to 8bit chars.
3. Wavelet compression is a form of data compression well suited for image compression. This is used by JPEG2000, DjVu for images and CineForm, Ogg Tarkin for video. Wavelet compression can be either lossless of lossy.
4. Other types are Mexican Hat wavelet, triangular wavelet etc.