Solution to Assignment 1, Problem 1(a)

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Softmax Prove that $softmax$ is invariant to constant offsets in the input, that is, for any input vector x and any constant c,

$$\text{softmax}(\mathbf{x}) = \text{softmax}(\mathbf{x} + c), \tag{1}$$

where $(\mathbf{x} + c)$ means adding the constant c to every dimension of $\mathbf{x}$.

$$\text{softmax}(\mathbf{x})_j = \frac{e^{x_j}}{\sum_{k=1} e^{x_k}} \tag{2}$$

Solution:

$$\text{softmax}(\mathbf{x} + c)_j = \frac{e^{(x_j+c)}}{\sum_{k=1} e^{(x_k+c)}} \tag{3}$$

$$= \frac{e^c}{e^c} \frac{e^{(x_j)}}{sum_{k=1} e^{(x_j)}} \tag{4}$$

$$= \text{softmax}(\mathbf{x})_j \tag{5}$$