Structural Equation Modelling & Causal Inference

Day 1–Confirmatory Factor Analysis (CFA), Structural Regression (SR), and Multiple Group Analysis (MGA)

Ozan Aksoy

UCL Social Research Institute & NCRM
University College London

November 2, 2023

Topics

1. Confirmatory factor analysis (CFA)
2. Reflexive vs formative factors
3. Hybrid models (Sructural Regression)
4. Multiple Group Analysis
5. Structural invariance
6. Measurement invariance
7. Hybrid models (SR) with multiple groups

Latent variables

- Latent = unobserved variables
- Are they really "out there"? (reification)
- Latent variables can be
  - normal distributed (EFA, CFA, ...)
  - discrete distributed (latent class models–model based clustering)
- Errors/residuals are latent variables

1-Factor Confirmatory Factor Analysis (CFA)

Regression of 4 observed cont. items x_j on unobserved factor U

$$x_{ij} = \alpha_j + \lambda_j U_i + e_{ij}$$

$$\begin{aligned} U_i &= N(0, \sigma_U^2) & \qquad \mathrm{cov}(e_{ij}, e_{ih}) &= 0 \\ e_{ij} &= N(0, \sigma_j^2) & \qquad \mathrm{cov}(e_{ij}, U_i) &= 0 \end{aligned}$$

Terminology

- U is common factor, a continuous latent (unobserved) variable
- e_j are specific factors, also called errors or residuals
- $\lambda_j = \mathrm{cov}(x_j, U)/\mathrm{var}(U)$ are (unstandardized) factor loadings

R lavaan:

- specifies measurement as `U =~ x1+x2+x3+x4`
- the name `U` should not occur as a variable
- auto-fixes loading λ_1 of first item to 1 (identification)
- No SE and associated tests reported

CFA in R lavaan—input

ex_cfa1.inp

```
acfa <- 'A =~ A1 + A2 + A3 + A4
         A1 + A2 + A3 + A4 ~ 1'
fit.acfa <- sem(acfa, data = dcfa, estimator = 'MLR')
summary(fit.acfa, modindices = TRUE)
```

14 sample moments	12 free parameters
4 means	4 intercepts α_j
4 variances	3 loadings λ_j (λ_1 fixed)
6 covariances	4 error variances σ_j^2
	1 factor variance σ_U^2

14-12 $=$ 2 degrees of freedom: test goodness-of-fit

CFA in R—selected output

ex_cfa1

```
                   Estimate   Std.Err   z-value     P(>|z|)
A        =~
   A1                 1.000
   A2                 0.820     0.027    30.051       0.000
   A3                 0.348     0.027    12.981       0.000
   A4                 0.247     0.026     9.422       0.000
Intercepts:
   A1                 1.963     0.044    44.659       0.000
   A2                 2.771     0.040    69.718       0.000
   A3                 3.609     0.037    97.104       0.000
   A4                 4.412     0.034   130.938       0.000
Variances:
   A1                 0.071     0.056     1.281       0.200
   A2                 0.329     0.039     8.357       0.000
   A3                 1.156     0.051    22.538       0.000
   A4                 1.022     0.047    21.731       0.000
   A                  1.862     0.102    18.262       0.000
```

fitted values for (co)variances

```
Var(A2)     = LOAD(A2,A)^2 * Var(A) + RESVAR(A2) = 0.820^2*1.862+0.329 = 1.581
Cov(A1,A2) = LOAD(A1,A)*LOAD(A2,1) * Var(A)      = 0.820*1.000*1.862   = 1.526
```

Confirmatory factor analysis; model improvement

- Chi-Square Test of Model Fit: fitted vs saturated model Chi2(2)=17.1, p<.01.
- Conclusion: CFA(1)-model does not fit well (but be cautious to conclude only on χ^2)
- Restrictive assumption: all items have "1 thing in common", i.e., all error covariances fixed to 0
- Run `summary(fit.acfa, fit.measures = TRUE)`

modification indices (selection)

```
Modification Indices:
   lhs op rhs     mi mi.scaled   epc sepc.lv sepc.all sepc.nox
15  A1 ~~  A2 14.475    15.667 1.264   1.264    0.723    0.723
20  A3 ~~  A4 14.475    15.667 0.132   0.132    0.106    0.106
```

- Conclusion: We could either free cov(e.a1,e.a2) or free cov(e.a4,e.a4) to get a much better fit.
- Substantively meaningful?
- Guesstimate of model chi2 after the modification: Chi2(2-1) = 17.1-14.75 = 2.4

Confirmatory factor analysis; free covariance

- In R-lavaan, cov(z1,z2) is specified by `z1 ~~ z2;`
- For depvars z, variance and covariances refer to the errors in z, not z itself!
- Recall: Variance and covariances for depvars z are NOT parameters, but *implied* statistics

ex_cfa3

```
acfa2 <- 'A =~ A1 + A2 + A3 + A4
          A1 + A2 + A3 + A4 ~ 1
          A1 ~~  A2'
         #A1 ~~ 0*A2 fixing covariance to 0, default
fit.acfa2 <- sem(acfa2, data = dcfa, estimator = 'MLR')
summary(fit.acfa2, fit.measures = TRUE)
```

Confirmatory factor analysis; free covariance

ex_cfa3 output (selected)

```
  Estimator                                         ML      Robust
  Minimum Function Test Statistic                1.235       1.389
  Degrees of freedom                                 1           1
  P-value (Chi-square)                           0.266       0.239
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  A =~
    A1                1.000
    A2                0.808    0.030   26.912    0.000
    A3                0.633    0.083    7.584    0.000
    A4                0.447    0.061    7.276    0.000
Covariances:
 .A1 ~~
   .A2                0.700    0.117    5.966    0.000
Intercepts:
   .A1                1.963    0.044   44.659    0.000
   .A2                2.771    0.040   69.718    0.000
   .A3                3.609    0.037   97.104    0.000
   .A4                4.412    0.034  130.938    0.000
    A                 0.000
Variances:
   .A1                0.910    0.137    6.628    0.000
   .A2                0.912    0.107    8.510    0.000
   .A3                0.972    0.064   15.250    0.000
   .A4                0.931    0.050   18.492    0.000
    A                 1.023    0.149    6.880    0.000
```

Methods of identification: ULI vs UFI vs . . .

Equivalent methods to identify factor model (Kline 2016)

- Unit Loading Identification (ULI): default in R
  - one loading fixed to 1, var(LV) free
  - intercepts of items free, E(LV) fixed to 0
  - Advise: use most reliable item as anchor
- Unit Factor Identification (UFI):
  - all loadings free, var(LV) fixed to 1
  - intercepts of items free, E(LV) fixed to 0
  - hard to implement with endogenous factors
  - not appropriate in multiple group or longitudinal analyses
- *Little, Siegers & Card (2006) (SLC):
  - average loading is 1
  - var(LV) free, e(LV) fixed to 0
  - meaningful if all items at same scale
- *Ozan's Favorite Identification (OFI):
  - for one item: loading fixed to 1, intercept fixed to 0
  - E(LV) and var(LV) free

Factor identification in R

ULI with implicit constraints

```
A =~ a1 + a2 + a3 + a4
A1 + A2 + A3 + A4 ~ 1
```

ULI with explicit constraints

```
A =~ 1*a1 + a2 + a3 + a4
A ~~ A
A1 + A2 + A3 + A4 ~ 1
```

ULI with explicit constraints---using u2 as anchor

```
A =~ NA*a1 + 1*a2 + a3 + a4
A ~~ A
A1 + A2 + A3 + A4 ~ 1
```

UFI loading of u1 free; var(U) fixed to 1

```
A =~ NA*a1 + a2 + a3 + a4
A ~~ 1*A
A1 + A2 + A3 + A4 ~ 1
```

LSC

```
A =~ NA*A1 + p2*A2 + p3*A3 + p4*A4
A1 ~ p1*A
(p1+p2+p3+p4) == 4
A1 + A2 + A3 + A4 ~ 1
```

OFI

```
A  =~ 1*A1 + A2 + A3 + A4
A  ~~ A
A  ~ NA*1
A1 ~  0*1
A2 + A3 + A4 ~ 1
```

Comparison of identification methods


```
                        ULI         UFI         OFI         LSC
H0 Value          -5595.655   -5595.655   -5595.655   -5595.655
A        BY
   A1                 1.000       1.011       1.000       1.385
   A2                 0.808       0.817       0.808       1.119
   A3                 0.633       0.640       0.632       0.876
   A4                 0.447       0.452       0.446       0.619
A1       WITH
   A2                 0.700       0.700       0.700       0.700
Intercepts
   A1                 1.963       1.963       0.000       1.963
   A2                 2.771       2.771       1.184       2.771
   A3                 3.609       3.609       2.367       3.609
   A4                 4.412       4.412       3.535       4.412
Mean
   A                  0.000       0.000       1.963       0.000
Variances
   A                  1.023       1.000       1.023       0.533
Residual Variances
   A1                 0.910       0.910       0.910       0.910
   A2                 0.912       0.912       0.912       0.912
   A3                 0.972       0.972       0.972       0.972
   A4                 0.931       0.931       0.931       0.931
```

- $\lambda_{UFI} = \frac{\lambda_{ULI}}{\sqrt{Var(F_{ULI})}}$

CFA: Multiple factors

Often $k > 1$ LVs measured with disjoint collections of items

- does each item contribute to measurement (high R^2)? (Convergent validity)
- does item measure the expected LV, not others?
- theoretical reluctance to cross load items on multiple LVs, unless a LV represents response effects (see MTMM below)
- consider joining highly correlated LVs (Discriminant validity)
- test discriminant validity by corr(LV1,LV2) $= 1$.
- R default: correlated factors
- to enforce uncorrelated factors: specify `LV1 ~~ 0*LV2`
- `estimator = 'MLR'` adjusts SEs and tests for non-normality

ex_cfa4

```
mcfa <-  'A =~ A1 + A2 + A3 + A4
          A1 ~~  A2
           B =~ B1 + B2 + B3
           C =~ C1 + C2 + C3'
fit.mcfa <- sem(mcfa, data = dcfa, estimator = 'MLR')
summary(fit.mcfa, fit.measures = TRUE, standardized = TRUE)
```

ex_cfa4.out (selected & edited fit indicators)

```
Chi-Square Test of Model Fit
          Value                                 33.359
          Degrees of Freedom                        31
          P-Value                                0.353
          CFI                                    0.999
          TLI                                    0.999
RMSEA (Root Mean Square Error Of Approximation)
          Estimate                               0.009
          90 Percent C.I.                        0.000  0.026
          Probability RMSEA <= .05               1.000
SRMR (Standardized Root Mean Square Residual)
          Value                                  0.018
---
inspect(fit.mcfa, 'r2')

   A1    A2    A3    A4    B1    B2    B3    C1    C2    C3
0.557 0.432 0.298 0.158 0.494 0.827 0.430 0.500 0.497 0.491

```

output (selected)

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  A =~
    A1                1.000                               1.037    0.746
    A2                0.796    0.025   31.742    0.000    0.826    0.657
    A3                0.618    0.060   10.275    0.000    0.642    0.546
    A4                0.408    0.041    9.866    0.000    0.423    0.397
  B =~
    B1                1.000                               0.997    0.703
    B2                2.001    0.101   19.897    0.000    1.996    0.909
    B3                0.901    0.045   20.057    0.000    0.899    0.656
  C =~
    C1                1.000                               1.001    0.707
    C2                0.984    0.062   15.929    0.000    0.985    0.705
    C3                0.966    0.062   15.665    0.000    0.967    0.701

Covariances:
 .A1 ~~
   .A2                0.669    0.084    7.971    0.000    0.669    0.763
  A ~~
    B                 0.510    0.050   10.163    0.000    0.493    0.493
    C                 0.425    0.055    7.685    0.000    0.409    0.409
  B ~~
    C                 0.241    0.040    6.104    0.000    0.242    0.242
Variances:
    A                 1.076    0.114    9.401    0.000    1.000    1.000
    B                 0.994    0.089   11.228    0.000    1.000    1.000
    C                 1.002    0.092   10.844    0.000    1.000    1.000
```

How many items are needed . . .

- 3 items just-identify a 1-factor model without error covariances
  - perfect fit, cannot be falsified
  - too bad, cannot be proved wrong
- With multiple factors, 2 items per factor suffice for identification if
  - all items load on one factor
  - no error covariances
- more items theoretically compelling & more powerful tests, but requires large samples!

Reflexive vs formative factors

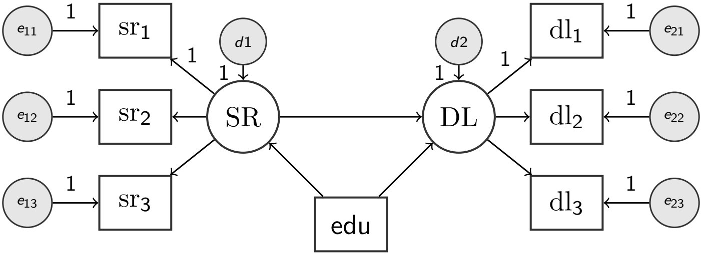
- Attitudes, preferences, expectations:
  - arrows from latent variables to *reflexive indicator*
  - F $=\sim$ items
- Resources & opportunities:
  - indicators (measures) constitute components of the latent variable, not reflections of the latent variable:
  - arrows from *formative indicators* to latent variables
  - no assumptions on correlations
  - F $\sim$ items or ad hoc additive scale
- SES reflects or is formed by income and status?
- Identification not always easy;

Structural regression (hybrid) models

measurement model(s) embedded in structural regression model

Example

- Sexrole attitudes (SR) measured by normative items sr_1, sr_2, sr_3 (coded 1=modern ... 5=traditional)
- Division of labor (DL) measured by dl for various activities dl_1, dl_2, dl_3 (coded 1=only wife ... 4=equal ...7=only husband)
- SR (fully? partly?) mediates effect of edu on DL



div labour input

```
M <- '
#measurement:
     SR =~ sr1 + sr2 + sr3 + sr4
    sr3 ~~ sr4 # included because of high MI
     DL =~ dl1 + dl2 + dl3 + dl4
    dl2 ~~ dl4 # included because of high MI
#structural
     DL ~ SR + edu
     SR ~ edu'
# fitting part of syntax omitted
```

divlabour output (selected & edited)

```
         Chi-sq                                   2469.938
         Degrees of Freedom                             36
         P-Value                                    0.0000

         CFI                                         0.982
         TLI                                         0.972
         RMSEA                                       0.034
         SRMR                                        0.023

                   Estimate     Std.Err      z-value    P(>|z|)
DL ~ SR              -0.400       0.050       -7.956      0.000
DL ~ EDU              0.089       0.015        5.751      0.000
SR ~ EDU             -0.132       0.012      -11.318      0.000
```

Edu $\rightarrow$ DL: Direct $= 0.089$; Indirect $= 0.053$

Model development

1. study groups-of-related-factors-at-time
   - if enough items: remove items with low R-sq (trouble makers)
   - if enough items: consider dropping cross-loading items
   - free error covariances that make substantive sense
2. simultaneous measurement analysis
3. impose causal relations among the latent and observed variables

Multiple Group Analysis (MGA)

- standard way to model interactions with categorical variables: independent groups
- usually: compare unstandardized parameters, not standardized
- structural parameters estimated for each group, constraints may be added
- measurement parameters constrained to be equal across groups ("strict invariance"), constraints may be removed
- assumes group membership is known/observed
- mixture/latent class modeling: group membership *unknown*
- most multiple group analyses not possible with correlation matrix (parameters already standardized)
- only feasible with few groups (excessive #parameters) consider collapsing categories, e.g., high vs low
- combine multiple grouping variables e.g., male Venusians, female Venusians, male Martians, ...

Multiple group data in R Lavaan

Individual data, all groups in one file

- existing variable `gender` specifying grouping in dataset
- obs with unlisted values on `gender` are omitted

```
fit <- sem(my.model, data = my.data, group = "gender")
```

Summary data obtained separately for each group

- Store covariance matrix and vector of means/sds per group as separate objects
- Declare those in the fitting stage with `list()` or `c()`

```
fit <- sem(model=my.model, sample.cov=list(covfem,covmale),
sample.mean=list(mfem,mmale), sample.nobs=list(200, 300))
```

Multiple group model specification in R

- Default in R is all parameters free in all groups
- Fixing or giving starting values:

```
my.model <- '
A =~ x1 + 0.5*x2 + c(0.6, 0.8)*x3
B =~ x4 + start(c(1.2, 0.6))*x5 + x6'
```

- Fixing parameters in all but one group:

```
mod <- 'F =~ i1 + c(1,NA,1,1)*i2 + i3'
```

- Constraining parameters across groups

```
A =~ x1 + c(a2,a2)*x2 + c(a3, a3)*x3
```

- Constraining groups of parameters to be equal across groups

```
my.model <- '
A =~ x1 + x2 + x3
B =~ x4 + x5 + x6'
fit <- cfa(my.model, data = d, group = "female",
group.equal = c("loadings"))
```

Constraining group of parameters continued

- `intercepts`: the intercepts of the observed variables
- `means`: the intercepts/means of the latent variables
- `residuals`: the residual variances of the observed variables
- `residual.covariances`: the residual covariances of the observed variables
- `lv.variances`: the (residual) variances of the latent variables
- `lv.covariances`: the (residual) covariances of the latent varibles
- `regressions`: all regression coefficients in the model

If you omit the `group.equal` argument, all parameters are free

I don't recommend using this shortcut, I prefer adding constraints directly in the model stage–unless you know what you are doing!!

Example MGA—with only observed variables

Example with only observed variables

By default, R, does not impose any constraints on regression coefficients, intercepts, (co)variances, thresholds. . . across groups

```
library(lavaan)
mod <- 'rincome ~ educ + age + female'
fit <- sem(mod, data=dgss[dgss$year==1988 | dgss$year==1998,],
           group = "year")
```

MGA—selected output


```
Group 1 [1988]:
Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  rincome ~
    educ              0.328    0.035    9.317    0.000
    age               0.034    0.008    4.555    0.000
    female           -1.710    0.201   -8.515    0.000
Intercepts:
   .rincome           4.188    0.591    7.080    0.000
Variances:
   .rincome           9.921    0.447   22.192    0.000

Group 2 [1998]:
Regressions:
  rincome ~
    educ              0.260    0.023   11.521    0.000
    age               0.043    0.005    8.884    0.000
    female           -1.106    0.120   -9.185    0.000
Intercepts:
   .rincome           5.493    0.375   14.643    0.000
Variances:
   .rincome           7.085    0.226   31.297    0.000
```

MGA—Imposing and testing structural invariance

to impose a cross-group equality constraint on a parameter

```
mod2 <- 'rincome ~ educ + age + c(bf,bf)*female'
```

to test invariance of a parameter across groups:

- Option 1: LR-test
  - Fit 2 models: with & without constraints
  - use `anova(fit, fit2)`
- Option 2: Wald test
  - attach labels to parameters (e.g., `bf88, bf98`)
  - fit the model without constraints
  - use `lavTestWald(m3, bf88==bf98)`

```
m3 <- 'rincome ~ educ + age + c(bf88, bf98)*female'
f3 <- sem(m3, data=dgss[dgss$year==1988 | dgss$year==1998,],
          group = "year")
lavTestWald(f3, constraints = "bf88==bf98")
> $stat: 6.65253; $df: 1; $p.value: 0.009901513
```

MGA—Structural invariance (cont)

Wald and LR tests for hypothesis that ...
$b(\text{edu} \mid 88) = b(\text{edu} \mid 98), b(\text{age} \mid 88) = b(\text{age} \mid 98), b(\text{fem} \mid 88) = b(\text{fem} \mid 98)$

```
m5 <- 'rincome ~ c(be88, be98)*educ + c(ba88,ba98)*age +
                 c(bf88, bf98)*female'
f5 <- sem(m5, data=dgss[dgss$year==1988 | dgss$year==1998,],
          group = "year")
c  <- 'be88 == be98
       ba88 == ba98
       bf88 == bf98'
# Wald test:
lavTestWald(f5, constraints = c)
> $stat: 10.43722, $df: 3, $p.value = 0.01519285

# LR test:
m6 <- 'rincome ~ c(be, be)*educ + c(ba,ba)*age +
                 c(bf, bf)*female'
f6 <- sem(m6, data=dgss[dgss$year==1988 | dgss$year==1998,],
          group = "year")
anova(f5, f6)
> Chi Square Difference Test
>     Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
> f5   0 56370 56430  0.000
> f6   3 56375 56417 10.407     10.407       3     0.0154 *
```

MGA—another example on structural invariance

Tests of other parameters are possible

Example: same residual variance for men and women in regression of income on human capital

```
m7 <- 'rincome ~ educ + age
       rincome ~~ c(vem, vef)*rincome' # residual variance
f7 <- sem(m7, data=dgss[dgss$year==1998,], group = "female")
lavTestWald(f7, constraints = "vem==vef")
$stat
[1] 31.09278
$df
[1] 1
$p.value
[1] 2.459852e-08
$se
[1] "standard"
```

MGA—measurement invariance

measurement invariance

for all (groups of) people, the same relation between measurement (item) and measured (latent)

important concern in cross-cultural research

Equality constraints	Configural invar.	Weak invar.	Strong Invar.	Strict Invar.
Model structure	Yes	Yes	Yes	Yes
Unst'dized Item loadings	No	Yes	Yes	Yes
Unst'dized Item intercepts	No	No	Yes	Yes
(Co)var's of Item residuals	No	No	No	Yes

Measurement invariance can (and should) be tested!

Weak invariance is required to compare variances and covariances of latent variables across groups

Strong invariance is required to compare means of latent variables across groups

If invariance does not hold, impose partial invariance at least

MGA—measurement invariance

Additional assumptions/constraints on latent variables:

- mean of exogenous LV: 0 in group 1, free in other groups
- intercept of endogenous LV: 0 in group 1, free in other groups
- variance of exogenous LV: free in all groups
- variance of residuals of endogenous LV: free in all groups

Crossvalidation

Many models modified from initial specification based on misfit

Statistical theory (e.g. p-values) assume ex-ante hypotheses formulated independent of data.

Risk: overfitting. p-values/modelfit too optimistic.

Cross-validation

- Random split data in calibration (2/3) and validation sample (1/3)
- Develop model in calibration sample
- Check the model in validation sample

Possibility: test whether the same model fits in the two samples: all corresponding parameters the same in the two samples

CFA with one factor, two groups

One factor with three items, two groups, without measurement invariance: df=0

One factor with three items, two groups, with full measurement invariance w.r.t. intercepts and loadings, not w.r.t. error variances: df = 18-14 = 4 (details see below)

18 sample moments	14 free parameters (assuming invariance)
6 means	3 intercepts: one for each item
6 variances	2 loadings
6 (= 2.3) covariances	6 error variances:
	(one for each item in each group)
	2 factor variances (one for each group)
	1 factor mean (fixed to 0 in ref group)

Verify yourself: With three groups, assuming measurement invariance: df = 27-19 = 8

MGA—Models with(out) measurement invariance

no invariance

```
ms <- 'SR =~ sr1 + sr2 + sr3 + sr4'
fs <- sem(ms, data=dh95, group = "female", missing = "ML")
```

strong invariance

```
msi <- 'SR =~ sr1 + c(a,a)*sr2 + c(b,b)*sr3 + c(c,c)*sr4
        sr1 ~ c(i1,i1)*1
        sr2 ~ c(i2,i2)*1
        sr3 ~ c(i3,i3)*1
        sr4 ~ c(i4,i4)*1
        SR  ~ c(0, NA)*1'
fsi <- sem(msi, data=dh95, group = "female", missing = "ML")
```

MGA—Coefficients from strong measurement (in)variance


```
                    with invariance     without invariance
                          M        F          M        F
SR      =~
   SR1                1.000    1.000      1.000    1.000
   SR2                1.154    1.154      1.141    1.378
   SR3                0.493    0.493      0.497    0.520
   SR4                0.847    0.847      0.821    0.943
Means
   SR                 0.000    0.289      0.000    0.000
Intercepts
   SR1                2.690    2.690      2.837    2.267
   SR2                2.568    2.568      2.485    2.318
   SR3                1.494    1.494      1.499    1.348
   SR4                2.131    2.131      2.110    1.909
Variances
   SR                 0.777    0.709      0.801    0.586

   SR1                1.293    1.191      1.254    1.214
   SR2                0.975    0.992      0.958    0.879
   SR3                0.793    0.590      0.792    0.599
   SR4                0.979    1.017      0.989    1.023
```

MGA—Testing strong measurement invariance

- Likelihood ratio test:
  - C: Model with invariance
    `output: Model chi2=168.896, df=10`
  - U: Model without invariance
    `output: Model chi2=92.367, df=4`
  - Compute the test
    `manual: LR=168.896-92.367=76.529, df=10-4=6, p<.001`
    `by R: anova(fs, fsi)`
- Wald test
  - Fit the model that does not assume invariance,
  - Label parameters
  - Use `lavTestWald()` function with
    - loadings: LM1==LF1, LM2==LF2, ...
    - intercepts: IM1==IM2, IM2==IM3....
- Score testing
  - Fit the model that assumes invariance
  - Ask for modification indices `mi <- modindices(fsi, free.remove=FALSE)`
  - Check for big modindices for intercepts and loadings

MGA—A refined analysis

- Tests of invariance makes sense if unconstrained model fits the data reasonably well
- Here, the unconstrained model does not fit well:
  `output: Model Chi2=92.367, df=4, p=.0000`
- Check modification indices: free cov(sr1,sr2) or cov(sr3,sr4)
- Results with this modification :
  `output: Model Chi2=1.007, df=2, p=.6044`
  We lost 2 df: cov(sr1,sr2) is estimated in both groups!
- Fit the model with MI in which cov(sr1,sr2) is freed
  `output: Model Chi2=96.02, df=6, p<.001`
- Recompute the LR test for strong MI
  `manual: LR=96.02-1.007=17.38, df=6-2=4, p<.001`

MGA—Partial measurement invariance

if strong invariance is rejected?

- check in model with MI for big modindices per group for "intercepts" or "loadings" — interpretation: the parameter should be allowed to be free in that group, ie, not invariant
- think whether such modifications make substantive sense
- check fit after modification is applied (may be repeated)
- if fit ok: conclude partial weak/strong invariance
  still allows comparison of LV across groups
- otherwise: too bad

MGA—Example of Partial strong measurement invariance

Procedure:

- Modification indices: relax constraint on intercept sr1
- Rejoice: Now model fits well.

```
msi2 <- 'SR =~ sr1 + c(a,a)*sr2 + c(b,b)*sr3 + c(c,c)*sr4
        sr1 ~ c(i1,i1)*1
        sr2 ~ c(i2,i2)*1
        sr3 ~ c(i3,i3)*1
        sr4 ~ c(i4,i4)*1
        SR  ~ c(0, NA)*1
        sr1 ~~ sr2'
fsi2 <- sem(msi2, data=dh95, group = "female", missing = "ML")
print(mi <- modindices(fsi2, free.remove=FALSE))
# selected indices:
lhs op rhs    block     mi    epc sepc.lv sepc.all sepc.nox
...
5  sr1 ~1         1 41.560  0.213   0.213    0.148    0.148
...
20 sr1 ~1         2 38.090 -0.195  -0.195   -0.141   -0.141
```

MGA—Structural tests

Once (partial) measurement invariance ("same meaning"') established, it becomes meaningful to test for gender difference in the mean and variance of the latent variables SR:

```
msi3 <- 'SR =~ sr1 + c(a,a)*sr2 + c(b,b)*sr3 + c(c,c)*sr4
sr1 ~ c(i1,i12)*1
sr2 ~ c(i2,i2)*1
sr3 ~ c(i3,i3)*1
sr4 ~ c(i4,i4)*1
SR  ~ c(0, NA)*1
sr1 ~~ sr2'
msi4 <- 'SR =~ sr1 + c(a,a)*sr2 + c(b,b)*sr3 + c(c,c)*sr4
sr1 ~ c(i1,i12)*1
sr2 ~ c(i2,i2)*1
sr3 ~ c(i3,i3)*1
sr4 ~ c(i4,i4)*1
SR  ~ c(ma, ma)*1
SR ~~ c(va, va)*SR
sr1 ~~ sr2'
fsi3 <- sem(msi3, data=dh95, group = "female", missing = "ML")
fsi4 <- sem(msi4, data=dh95, group = "female", missing = "ML")
anova(fsi3, fsi4)
```

Conclusion:

- gender diffs in means and variances of SR: LR Chi2(3)=28.277, p2s<.001

Some notes on measurement invariance

- See also: Hancock and Mueller Ch5; Kline Ch16
- In this example, we ignored that men and women were actually husbands and wives. Next lecture!
- It is also possible to test for measurement invariance by
  - Satorra-Bentler variant of the likelihood ratio test
  - This is preferred for non-normal data...

Hybrid models with multiple groups

- Example: Sexrole attitudes as mediator for the effect of edu on division of household labor
- After testing for (strict, partial) measurement invariance for attitudes (and similarly for division of labor), we may
  - compare the structural coefficients,
  - test whether the indirect effect of edu on divlabor via attitudes are the same.
  - ...

```
mh <- '
SR =~ sr1 + c(a,a)*sr2 + c(b,b)*sr3 + c(c,c)*sr4
sr1 ~ c(i1,i12)*1
sr2 ~ c(i2,i2)*1
sr3 ~ c(i3,i3)*1
sr4 ~ c(i4,i4)*1
sr1 ~~ sr2
SR  ~ c(0, NA)*1

DL =~ dl1 + c(d,d)*dl2 + c(e,e)*dl3 + c(f,f)*dl4
dl2 ~~ dl4
dl1 ~ c(id1,id1)*1
dl2 ~ c(id2,id2)*1
dl3 ~ c(id3,id3)*1
dl4 ~ c(id4,id4)*1
DL  ~ c(0, NA)*1

DL ~ c(m1,f1)*SR + c(m2,f2)*edu
SR ~              c(m3,f3)*edu'
f <- sem(mh, data=dh95, group = "female", missing = "ML")
lavTestWald(f, constraints = "m1*m3 == f1*f3")
> $stat: 0.6709185, $df: 1, $p.value: 0.4127316
```