Application of Matrix in Daily Life

$\boldsymbol{Q.1}$ $A\ manufacture\ produces\ 3-products\ A, B, C\ which\ he\ sells\ in\ the\ market\ annual\ sale\ volumes\ are\ indicates\ as\ follows$:

$Market$	$Product$		
	A	B	C
I	8,000	10,000	15,000
II	10,000	2,000	20,000

i) $If\ unit\ sale\ price\ of\ A, B\ \&\ C\ are\ TK\ 2.25, Tk\ 1.50, TK\ 1.25\ respectively, find\ the\ total\ revenue\ in\ each\ market\ with\ the\ help\ of\ matrices.$

ii) $If\ the\ unit\ costs\ of\ the\ above\ there\ products\ are\ TK, 1.60\ , TK\ 1.20\ \&\ TK\ 0.90\ respectively\ find\ the\ gross\ profit\ with\ the\ help\ of\ matrices.$

Solution(i):

$The\ total\ revenue\ in\ each\ market\ is\ given\ by\ the\ products\ matrix,$

$$= [2.25 \quad 1.50 \quad 1.25] \times \begin{bmatrix} 8,000 & 10,000 \\ 10,000 & 2,000 \\ 15,000 & 20,000 \end{bmatrix}$$

$$= [2.25 \times 8,000 + 1.50 \times 10,000 + 1.25 \times 15,000 \quad 2.25 \times 2,000 + 1.50 \times 2,000 + 1.25 \times 20,000]$$

$$= [18000 + 15000 + 18750 \quad 22,500 + 3000 + 25,000]$$

$$= [51,750 \quad 50,500]$$

$Total\ revenue\ from\ the\ Market\ I = Tk\ 51750$

$And \quad '' \quad `` \quad `` \quad `` \quad Market\ II = \ Tk\ 50,500$

Solution(ii):

Similarly, the total costs of products with the manufacturer sells in the market are:

$$= [1.60 \quad 1.20 \quad 0.90] \begin{bmatrix} 8,000 & 10,000 \\ 10,000 & 2,000 \\ 15,000 & 20,000 \end{bmatrix}$$

$$= [1.60 \times 8{,}000 + 1.20 \times 10{,}000 + 0.90 \times 15{,}000 \quad 1.60 \times 10{,}000 + 1.20 \times 2{,}000 + 0.90 \times 20{,}000]$$

$$= [1280 + 12000 + 13500 \quad 16000 + 2400 + 18000]$$

$$= [38{,}300 \quad 36{,}400]$$

The total cost of products each the manufacture sells in the market I & II Tk. 38,300

and Tk. 36,400 respectively,

$$\therefore \text{ } Required\ gross\ profit = (Tk\ 51{,}750 + Tk\ 50{,}500) - (Tk\ 38{,}300 + Tk\ 36{,}400)$$

$$= Tk\ 27{,}550$$ **Answer:**

Q. 2 A trust fund has Tk. 50,000 that is to be invested into two types of bonds. The first bond pays 5% interest per year and the second bond pays 6% interest per year. Using matrix multiplication, determine how to divide Tk. 50,000 among two types of bonds so as to obtain an annual total interest of Tk. 2780.

Solution:

Let, Tk 50,000 be divided into two parts $Tk. x$ & Tk $(50{,}000 - x)$ out of which first part is invested in first type of bonds and the second part is invested in second type of bonds.

The value of these bonds can be written in the form of a row matrix A.

$Given\ by\ A = [x \quad 50{,}000 - x], which\ is\ a\ 1 \times 2\ matrix,$

$and\ the\ amount\ receive\ as\ interest\ per\ taka\ annually\ from\ these\ two\ types\ of\ bonds$

$can\ be\ written\ in\ the\ column\ matrix\ B\ given\ by,$

$$B = \begin{bmatrix} \frac{5}{100} \\ \frac{6}{100} \end{bmatrix} which\ is\ 2 \times 1\ matrix.$$

$here, the\ interest\ has\ been\ calculated\ per\ taka\ annually.$

$now\ the\ interest\ to\ be\ obtained\ annually\ is\ a\ single\ number, that\ is$

$$AB = [x \quad 50{,}000 - x] \begin{bmatrix} \frac{5}{100} \\ \frac{6}{100} \end{bmatrix}$$

$$= x \times \frac{5}{100} + (50{,}000 - x) \times \frac{6}{100}$$

$$= \left[\frac{5x}{100} + \frac{6(50{,}000 - x)}{100}\right]$$

$$= \left[\frac{5x + 300000 - 6x}{100}\right]$$

$$= \left[\frac{300000 - \mathrm{x}}{100}\right]$$

$$= \left[3000 - \frac{x}{100}\right]$$

$also, we\ are\ given\ that\ the\ annual\ interest = Tk. 2780$, $According\ \ to\ the\ question,$

$$\left[3000 - \frac{x}{100}\right] = [2780]$$

$$=> 3000 - \frac{x}{100} = 2780$$

$$=> x = 22{,}000$$

$Hence\ the\ required\ amount\ are\ Tk. 22{,}000\ and\ Tk\ (50000 - 22000).$

$That\ is\ Tk\ 22000\ and\ 28000$ ***Answer***:

Linear Mapping

Let, v and u be vector space over the same field k. A mapping $F: v \rightarrow u$ is called a linear mapping if it satisfies the following conditions:

1. For any $v, w \in V$; $F(v + w) = F(v) + F(w)$
2. For any $k \in K$; $F(Kv) = kF(v)$

Problem -01 : **Show that , the following mapping F are linear :**
$(\boldsymbol{i}) \boldsymbol{F}: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ $\boldsymbol{defined\ by\ F(x, y) = (x + y, x)}$
$(\boldsymbol{ii})$ $\boldsymbol{F}: \mathbb{R}^3 \rightarrow \mathbb{R}^2 \boldsymbol{defined\ by\ F(x, y, z) = 2x - 3y + 4z}$

Solution $(\boldsymbol{i})$:

$Let, v = (a, b)\ and\ w = (a', b')$

$Hence, v + w = (a + a', b + b')\ and\ kv = k(a, b) = (ka, kb)\ ;\ k \in \mathbb{R}$

$We\ have,\quad F(v) = (a + b, a)\ and\ F(w) = (a' + b', a')$

$$\begin{aligned} Then,\quad F(v + w) &= F(a + a', b + b') \\ &= (a + a' + b + b', a + a') \\ &= (a + b, a) + (a' + b', a') \\ &= F(v) + F(w) \end{aligned}$$

$$\begin{aligned} Again, F(kv) &= F(ka, kb) \\ &= (ka + kb, ka) \\ &= k(a + b, a) \\ &= kF(v) \end{aligned}$$

$Since\ \ v, w\ and\ k\ are\ arbitrary, So\ \ F\ is\ linear.$ $\quad (\boldsymbol{Showed})$

Solution $(\boldsymbol{ii})$: $Let, v = (a, b, c)\ and\ w = (a', b', c')$

$V + w = (a + a', b + b', c + c')\ and\ kv = k(a, b, c) = (ka, kb, kc);\ k \in \mathbb{R}$

$we\ have,\ F(v) = F(a, b, c) = 2a - 3b + 4c$

$F(w) = F(a', b', c') = 2a' - 3b' + 4c'$

$$now, \quad F(v+w) = F(a+a', b+b', c+c')$$
$$= 2(a+a') - 3(b+b') + 4(c+c')$$
$$= (2a - 3b + 4c) + (2a' - 3b' + 4c')$$
$$= F(v) + F(w)$$
$$Again, \; F(kv) = F(ka, kb, kc)$$
$$= 2ka - 3kb + 4kc$$
$$= 2a - 3b + 4c$$
$$= kF(v)$$
$since, v, w \; and \; k \; are \; arbitrary. \; So, F \; is \; linear.$ $\quad (\boldsymbol{Showed})$

Problem-02(HW) : **Show that, the following mapping F are linear :**

$(\boldsymbol{i}) \; \boldsymbol{F} = \mathbb{R}^2 \rightarrow \mathbb{R}^2 \; \boldsymbol{defined \; by \; F(x, y) = (x+y, x+y)}$

$(\boldsymbol{ii}) \; \boldsymbol{F} = \mathbb{R}^3 \rightarrow \mathbb{R}^2 \boldsymbol{defined \; by \; F(x, y, z) = (x+y, y+z)}$

$\boldsymbol{Solution} \; (\boldsymbol{i})$:

$$Let, v = (a, b) \; and \; w = (a', b')$$
$$v + w = a + a', b + b'$$
$$k(v) = k(a, b) = ka, kb \; ; \; k \in \mathbb{R}$$
$$we \; have, \quad F(v) = (a+b, a+b) \; and \; F(w) = (a'+b', a'+b')$$
$$Then, \quad F(v+w) = F(a+a', b+b')$$
$$= (a+a'+b+b', a+a'+b+b')$$
$$= (a+b, a+b) + (a'+b', a'+b')$$
$$= F(v) + F(w)$$
$$Again, \; F(kv) = F(ka + kb)$$
$$= (ka + kb, ka + kb)$$
$$= k(a+b, a+b)$$
$$= kF(v)$$
$Since, \; v, w \; and \; k \; are \; arbitrary. \; So, F \; is \; linear.$ $\quad (\boldsymbol{Showed})$

Solution (ii) : $let,\ v = (a, b, c)\ and\ w = (a', b', c')$

$$v + w = (a + a', b + b', c + c')$$

$$kv = k(a, b, c) = (ka, kb, kc)\ ;\ k \in \mathbb{R}$$

$we\ have, F(v) = F(a, b, c) = (a + b, b + c)$

$and\ F(w) = F(a', b', c') = (a' + b', b' + c')$

$$\begin{aligned} Then,\ F(v + w) &= F(a + a', b + b', c + c') \\ &= (a + a' + b + b', b + b' + c + c') \\ &= (a + b, b + c) + (a' + b', b' + c') \\ &= F(v) + F(w) \end{aligned}$$

$$\begin{aligned} Again, F(kv) &= F(ka, kb, kc) \\ &= (ka + kb, kb + kc) \\ &= k(a + b, b + c) \\ &= kF(v) \end{aligned}$$

$Since,\ v, w\ and\ k\ are\ arbitrary.\ So, F\ is\ linear.$ $(\boldsymbol{Showed})$

Problem- 03: Use a formula to define each of the following functions from $\mathbb{R}$ *in to* $\mathbb{R}$.

i) To each number let f assign its cube.

ii) To each number let g assign the number 5.

iii)To each positive number let h assign its space and to each non positive number let h assign the number 6 , also find the value of each function *at* $4, -2, 0$.

Solution 1: $Since\ f\ assigns\ to\ any\ number\ x\ its\ cube\ x^3, we\ can\ define\ f\ by\ f(x) = x^3$

$Also\ f(4) = 64, f(-2) = -8, f(0) = 0.$

Solution 2: $Since\ g\ assign\ 5\ to\ any\ number\ x, we\ can\ define\ g(x) = 5.$

$Thus, g(4) = 5, \quad g(-2) = 5, \quad g(0) = 0.$

Solution 3: $Two\ differential\ rules\ are\ used\ to\ has\ follows:$

$$h(x) = \begin{cases} x^2, & if \quad x > 0 \\ 6, & if \quad x \leq 0 \end{cases}$$

$Since\ 4 > 0, h\ (4)\ =\ 4^2\ = 16. On\ the\ other\ hand\ -2, 0\ \leq\ 0\ and\ h\ (-2) = 6, h(0) = 6.$