Differentiation of various Functions

Formula for Differentiation:

Polynomials	Trigonometric
1. $\frac{d}{dx}(c) = 0$ 2. $\frac{d}{dx}(x) = 1$ 3. $\frac{d}{dx}(cx) = c$ 4. $\frac{d}{dx}(x^n) = nx^{n-1}$ 5. $\frac{d}{dx}(cx^n) = ncx^{n-1}$ 6. $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$	1. $\frac{d}{dx}(sinx) = cosx$ 2. $\frac{d}{dx}(cosx) = -sinx$ 3. $\frac{d}{dx}(tanx) = sec^2x$ 4. $\frac{d}{dx}(cotx) = -cosec^2x$ 5. $\frac{d}{dx}(secx) = secx\ tanx$ 6. $\frac{d}{dx}(cosecx) = -cosecx\ cotx$ 7. $\frac{d}{dx}(sin^2x) = 2sinx\ cosx$ 8. $\frac{d}{dx}(cos^2x) = -2cosx\ sinx$ 9. $\frac{d}{dx}(tan^2x) = 2tanx\ sec^2x$ 10. $\frac{d}{dx}(cot^2x) = -2cotx\ cosec^2x$ 11. $\frac{d}{dx}(sec^2x) = 2\ sec^2x\ tanx$ 12. $\frac{d}{dx}(cosec^2x) = -2\ cosec^2x\ cotx$ 13. $\frac{d}{dx}(sinmx) = mcosmx$ **14.** $\frac{d}{dx}(cos3x) = -3sin3x$
Inverse Trigonometric	**Exponential and Logarithmic**
1. $\frac{d}{dx}(sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$ 2. $\frac{d}{dx}(cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$ 3. $\frac{d}{dx}(tan^{-1}x) = \frac{1}{1+x^2}$ 4. $\frac{d}{dx}(cot^{-1}x) = -\frac{1}{1+x^2}$ 5. $\frac{d}{dx}(sec^{-1}x) = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$ 6. $\frac{d}{dx}(cosec^{-1}x) = -\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	1. $\frac{d}{dx}(e^x) = e^x$ 2. $\frac{d}{dx}(a^x) = a^x\ln(a),$ 3. $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$, x>0 4. $\frac{d}{dx}(\ln\lvert x\rvert) = \frac{1}{x}$, x≠ 0 5. $\frac{d}{dx}(log_a x) = \frac{1}{xlna,}$, x>0 6. $\frac{d}{dx}(e^{mx}) = me^{mx}$

Differentiation is one of the most important processes in engineering mathematics. It is the study of the way in which functions change. The function may represent pressure, stress, volume or some other physical variable. For example, the pressure of a vessel may depend upon temperature. As the temperature of the vessel increases, then so does the pressure. Engineers often need to know the rate at which such a variable changes.

Differentiate following Functions

1) ***Find the differential coefficient of y where,***

(i) $y = 2x^{\frac{1}{2}} + 6x^{\frac{1}{3}} - 2x^{\frac{3}{2}}$; *(ii)* $y = \sqrt[3]{3x^2} - \frac{1}{\sqrt{5x}}$;

Solution: (i)

Given, $\boldsymbol{y = 2x^{\frac{1}{2}} + 6x^{\frac{1}{3}} - 2x^{\frac{3}{2}}}$ **…………………..(1)**

Differentiating (1) with respect to x, then

$$\frac{dy}{dx} = \frac{d}{dx}\left(\boldsymbol{2x^{\frac{1}{2}} + 6x^{\frac{1}{3}} - 2x^{\frac{3}{2}}}\right)$$

$$= \frac{d}{dx}\left(\boldsymbol{2x^{\frac{1}{2}}}\right) + \frac{d}{dx}\left(\boldsymbol{6x^{\frac{1}{3}}}\right) - \frac{d}{dx}\left(\boldsymbol{2x^{\frac{3}{2}}}\right)$$

$$= \boldsymbol{2.\frac{1}{2}.x^{\frac{1}{2}-1} + 6.\frac{1}{3}.x^{\frac{1}{3}-1} - 2.\frac{3}{2}.x^{\frac{3}{2}-1}} \qquad \left[\frac{d}{dx}(cx^n) = cnx^{n-1}\right]$$

$$= \boldsymbol{x^{-\frac{1}{2}} + 2x^{\frac{-2}{3}} - 3x^{\frac{1}{2}}}$$

$$= \frac{1}{x^{\frac{1}{2}}} + \frac{2}{x^{\frac{2}{3}}} - 3x^{\frac{1}{2}} \textbf{ Ans.}$$

Solution: (ii) Given,

$y = \sqrt[3]{3x^2} - \frac{1}{\sqrt{5x}}$ **…………………..(1)**

Differentiating (1) with respect to x, then

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt[3]{3x^2} - \frac{1}{\sqrt{5x}}\right)$$

$=\frac{d}{dx}(3x^2)^{\frac{1}{3}} - \frac{d}{dx}(5x)^{-\frac{1}{2}}$

$=\frac{1}{3}(3x^2)^{\frac{1}{3}-1} \times 3 \times 2x - \left(-\frac{1}{2}\right)(5x)^{-\frac{1}{2}-1}.5$

$=2x(3x^2)^{-\frac{2}{3}}+\frac{5}{2}(5x)^{-\frac{3}{2}}.$

$=\frac{2x}{(3x^2)^{2/3}} + \frac{5}{2}.\frac{1}{(5x)^{3/2.}}$

$= \frac{2x}{(9x^4)^{1/3}} + \frac{5}{2} \times \frac{1}{5x\sqrt{5x}}$

$= \frac{2x}{(9x.x^3)^{1/3}} + \frac{1}{2x\sqrt{5x}}$

$= \frac{2x}{(9x)^{1/3}(x^3)^{1/3}} + \frac{1}{2x\sqrt{5x}}$

$= \frac{2x}{\sqrt[3]{9x}.x} + \frac{1}{2x\sqrt{5x}}$

$= \frac{2}{\sqrt[3]{9x}} + \frac{1}{2x\sqrt{5x}}$ Ans.

Formula:

$$\frac{d}{dx}\{\boldsymbol{uv}\} = v\frac{d}{dx}(u) + u\frac{d}{dx}(v)$$

$$\frac{d}{dx}\left\{\frac{u}{v}\right\} = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$$

2) ***Find,*** $\boldsymbol{\frac{dy}{dx}}$ ***where*** *(i)* $y = (x^2+4)^2(2x^3-1)^3$ *and*

(ii) $y = (x^2+4)(2x^3-3)$

Solution: (i) Given,

$$\boldsymbol{y} = (x^2+4)^{\mathbf{2}}(2x^3-1)^{\mathbf{3}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Differentiating (1) with respect to x, then

$$\frac{dy}{dx} = \frac{d}{dx}\{(x^2+4)^{\mathbf{2}}(2x^3-1)^{\mathbf{3}}\}$$

$=(x^2+4)^2\frac{d}{dx}(2x^3-1)^3+(2x^3-1)^3\frac{d}{dx}(x^2+4)^2$

$=(x^2+4)^2\times 3(2x^3-1)^2\times(2\times 3x^2-0)+(2x^3-1)^3\times 2(x^2+4)\times(2x+0)$

$=3(x^2+4)^2(2x^3-1)^2\times 6x^2+2(2x^3-1)^3(x^2+4)\times 2x$

$=(x^2+4)^2(2x^3-1)^2\times 18x^2+(2x^3-1)^3(x^2+4)\times 4x$

$=2x(2x^3-1)^2(x^2+4)\{9x(x^2+4)+2(2x^3-1)\}$

$=2x(2x^3-1)^2(x^2+4)(9x^3+36x+4x^3-2)$

$\frac{dy}{dx}=2x(2x^3-1)^2(x^2+4)(13x^3+36x-2)$ Ans…..

Solution: (ii) Given, $y=(x^2+4)(2x^3-3)$……………(1)

Differentiating (1) with respect to x, then

$$\frac{dy}{dx}=\frac{d}{dx}\{(x^2+4)(2x^3-1)\}$$

$=(x^2+4)\frac{d}{dx}(2x^3-1)+(2x^3-1)\frac{d}{dx}(x^2+4)$

$=(x^2+4)\,6x^2+(2x^3-1)2x$

$=10x^4+10x^2-2x$

H.W. Find, $\frac{dy}{dx}$ where $\boldsymbol{y}=\{\boldsymbol{ln}(x^2)+4\}^{\mathbf{3}}(2x^3+1)^{\mathbf{4}}$.

3) Find, $\frac{dy}{dx}$ ***where*** *(i)* $y=\frac{3-2x}{3+2x}$ *and (ii)* $\boldsymbol{y}=\frac{\boldsymbol{x^2}}{\sqrt{\boldsymbol{4-x^2}}}$.

Solution: (i) Given,

$y=\frac{3-2x}{3+2x}$…………………..(1)

Differentiating (1) with respect to x, then

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{3-2x}{3+2x}\right)$$

$$=\frac{(3+2x)\frac{d}{dx}(3-2x)-(3-2x)\frac{d}{dx}(3+2x)}{(\mathbf{3+2x})^2}$$

$$=\frac{(3+2x)(0-2)-(3-2x)(0+2)}{(\mathbf{3+2x})^2}$$

$$=\frac{-2(3+2x)-2(3-2x)}{(\mathbf{3+2x})^2}$$

$$=\frac{-6-4x-6+4x}{(\mathbf{3+2x})^2}$$

$$=\frac{-12}{(\mathbf{3+2x})^2} \qquad \text{Ans...}$$

<u>Solution</u>: (ii) Given,

$$\boldsymbol{y=\frac{x^2}{\sqrt{4-x^2}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

Differentiating (1) with respect to x, then

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{\boldsymbol{x^2}}{\sqrt{\boldsymbol{4-x^2}}}\right)$$

$$=\frac{\sqrt{\boldsymbol{4-x^2}}\,\frac{d}{dx}(\boldsymbol{x^2})-(\boldsymbol{x^2})\frac{d}{dx}\left(\sqrt{\boldsymbol{4-x^2}}\right)}{(\sqrt{\boldsymbol{4-x^2}})^2}$$

$$=\frac{2x.\sqrt{\boldsymbol{4-x^2}}-\frac{x^2}{2\sqrt{\boldsymbol{4-x^2}}}(0-2x)}{4-x^2}$$

$$=\frac{2x.\sqrt{\boldsymbol{4-x^2}}+\frac{x^3}{\sqrt{\boldsymbol{4-x^2}}}}{4-x^2}$$

$$=\frac{\frac{2x.(\boldsymbol{4-x^2})+x^3}{\sqrt{\boldsymbol{4-x^2}}}}{4-x^2}$$

$$=\frac{8x-\mathbf{2}x^3+x^3}{(4-x^2)\left(\sqrt{\boldsymbol{4-x^2}}\right)}$$

$$=\frac{8x-x^3}{(4-x^2)\left(\sqrt{\boldsymbol{4-x^2}}\right)}$$

$=\frac{8x-x^3}{(4-x^2)^{\frac{3}{2}}}$ **Ans…**

<u>H.W.</u> Find, $\frac{dy}{dx}$ where $\boldsymbol{y}=\frac{\mathbf{5x^4}}{\sqrt{\mathbf{4-x^4}}}$.

4) Find, (i) $\boldsymbol{\frac{dy}{dx}}$ ***where*** $\boldsymbol{y=\frac{u^2-1}{u^2+1}}$, $\boldsymbol{u=\sqrt[3]{x^2+2}}$.

(ii) $\boldsymbol{\frac{dy}{dt}}$***, when*** $\boldsymbol{t=\sqrt{2}}$ ***,Given*** $\boldsymbol{y=x^2-4x}$, $\boldsymbol{x=\sqrt{2t^2+1}}$.

<u>Solution</u>: (i) Given,

$$y=\frac{u^2-1}{u^2+1} \text{……………………… (1)}$$

$$u=\sqrt[3]{x^2+2} \text{ ……………..(2)}$$

Differentiating (1) with respect to u, then

$$\frac{dy}{du}=\frac{d}{du}\left(\frac{u^2-1}{u^2+1}\right)$$

$$=\frac{(u^2+1)\frac{d}{du}(u^2-1)-(u^2-1)\frac{d}{du}(u^2+1)}{(u^2+1)^2}$$

$$=\frac{(u^2+1)\,2u-(u^2-1)2u}{(u^2+1)^2}$$

$$\frac{dy}{du}=\frac{4u}{(u^2+1)^2}$$

Differentiating (2) with respect to x, then

$$\frac{du}{dx}=\frac{d}{dx}(\boldsymbol{x^2}+2)^{\frac{1}{3}}$$

$$=\frac{1}{3}(\boldsymbol{x^2}+2)^{\frac{1}{3}-1}\times 2x$$

$$\frac{du}{dx}=\frac{2x}{3}(\boldsymbol{x^2}+2)^{-\frac{2}{3}}$$

$$\therefore \frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

$= \frac{4u}{(u^2+1)^2} \times \frac{2x}{3} (\boldsymbol{x^2} + 2)^{-\frac{2}{3}}$

$$= \frac{8ux}{3(u^2+1)^2} \times \frac{1}{(x^2+2)^{\frac{2}{3}}}$$

$$= \frac{8ux}{3(u^2+1)^2} \times \frac{1}{u^2}$$

$= \frac{\mathbf{8x}}{\mathbf{3u(u^2+1)^2}}$ Ans…

<u>Solution</u>: (ii) Given,

$y = x^2 - 4x$ ……………………….(1)

$x = \sqrt{2t^2+1}$………………..(2)

Differentiating (1) with respect to x, then

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 - 4x)$$

$$=2x - 4$$

Differentiating (2) with respect to t, then

$$\frac{dx}{dt} = \frac{d}{dt}\left(\sqrt{2t^2+1}\right)$$

$$= \frac{1}{2\sqrt{2t^2+1}} \frac{d}{dt}(2t^2+1)$$

$$= \frac{1}{2\sqrt{2t^2+1}}(4t+0)$$

$$= \frac{2t}{\sqrt{2t^2+1}}$$

$\therefore \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$= (2x-4) \times \frac{2t}{\sqrt{2t^2+1}}$

$=\frac{4t(x-2)}{\sqrt{2t^2+1}}$

When, t=$\sqrt{2}$ then

$x=\sqrt{2(\sqrt{2})^2+1}$

$=\sqrt{2\times 2+1}$ $=\sqrt{5}$

Therefore,

$\frac{dy}{dt}=\frac{4\sqrt{\mathbf{2}}\,(\sqrt{\mathbf{5}}-\mathbf{2})}{\sqrt{\mathbf{5}}}$ Ans……

H.W. Find, (i) $\frac{dy}{dt}$ where $y=\frac{u^4-1}{u^4+5}, u=\sqrt[3]{t^2+2}$.

$$**\frac{\boldsymbol{d}}{\boldsymbol{dx}}(\boldsymbol{u})^{\boldsymbol{v}}=\boldsymbol{u}^{\boldsymbol{v}}\frac{\boldsymbol{d}}{\boldsymbol{dx}}(\boldsymbol{vlnu})**$$

5) ***Differentiate, (i)*** $\boldsymbol{y=(\sin x)^x}$ ***with to respect*** $\boldsymbol{z=x^{\sin x}}$

(ii) $\boldsymbol{y=ln\frac{1+\sqrt{x}}{1-\sqrt{x}}}$ ***with respect to*** $\boldsymbol{\sqrt{x^3}}$.

Solution: (i) Given,

y $=(\sin x)^x$…………………..(1)

$z=x^{\sin x}$……………………….(2)

Differentiating (1) with respect to x, Here

$\frac{dy}{dx}=\frac{d}{dx}sinx^x$

$=sinx^x\frac{d}{dx}\{x.\ln(sinx)\}$

$=sinx^x\{x\frac{d}{dx}\ln(sinx)+\ln(sinx)\frac{d}{dx}x\}$

$=sinx^x\{x\frac{1}{sinx}cosx+\ln(sinx)\}$

Differentiating (2) with respect to x, Here

$$\frac{dz}{dx} = \frac{d}{dx} x^{sinx}$$

$$=x^{\sin x} \frac{d}{dx} (\sin x \,.\, lnx)$$

$$=x^{\sin x}\{\sin x \frac{d}{dx} (lnx) + lnx \frac{d}{dx} (\sin x)\}$$

$$=x^{\sin x}\{\sin x \frac{1}{x} + lnx.\cos x\}$$

$$\frac{dy}{dz} = \frac{dy}{dx} . \frac{dx}{dz}$$

$$=sinx^{x}\{x \frac{1}{sinx} cosx + \ln(sinx)\} \frac{1}{x^{\sin x}\{\sin x \frac{1}{x} + lnx.\cos x\}}$$ Ans…..

Solution: (ii) Given,

$$y = \ln \frac{1+\sqrt{x}}{1-\sqrt{x}} \text{………………(1)}$$

$$z = \sqrt{x^3} \text{………………..(2)}$$

Differentiating (1) with respect to x, Here

$$\frac{dy}{dx} = \frac{d}{dx} (\ln \frac{1+\sqrt{x}}{1-\sqrt{x}})$$

$$=\frac{d}{dx}\{\ln\left(1+\sqrt{x}\right) - \ln\left(1-\sqrt{x}\right)\}$$

$$= \frac{1}{1+\sqrt{x}} . \frac{1}{2\sqrt{x}} - \frac{1}{1-\sqrt{x}}(-\frac{1}{2\sqrt{x}})$$

$$= \frac{1}{\sqrt{x}\,(1-x)}.$$

Again,

Differentiating (2) with respect to x, Here

$$\frac{dz}{dx} = \frac{d}{dx} (\sqrt{x^3})$$

$$=\frac{d}{dx}(x^{3/2})$$

$$= {}^3/_2 . x^{3/2-1}$$

$$= \frac{3\sqrt{x}}{2}$$

$$\frac{dy}{dz} = \frac{dy}{dx} . \frac{dx}{dz}$$

$$= \frac{1}{\sqrt{x}\,(1-x)} . \frac{3\sqrt{x}}{2}$$

$$\frac{dy}{dz} = \frac{2}{3x\,(1-x)}$$ Ans…..

6) ***Find the differential coefficient of y with respect to x where,***

(i) $y = sinx^{sinx}$; *(ii)* $y = x^{cos^{-1}x} + x^{lnx}$; *(iii)* $y = \frac{7\sqrt{2x^3+1}\,sin\,5x}{(2x+3)(x-1)}$; *(iv)* $ln(x+y) = xy$

Solution: (i) Given, $y = sinx^{sinx}$

Differentiating with respect to x, Here

$$\frac{dy}{dx} = sinx^{sinx} \frac{d}{dx}\{sinx \ln(sinx)\}$$
$$= sinx^{sinx} \left\{\ln(sinx)\frac{d}{dx}(sinx) + sinx\frac{d}{dx}\ln(sinx)\right\}$$

$$= sinx^{sinx}\{\ln(sinx)\, cosx + cosx\}$$

$$= sinx^{sinx} cosx\{\ln(sinx) + 1\}$$ Ans.

Solution: (ii) Given, $y = x^{\cos^{-1}x} + x^{lnx}$

Differentiating with respect to x, then

$$\frac{dy}{dx} = x^{\cos^{-1}x}\frac{d}{dx}(\cos^{-1}x\, lnx) + x^{lnx}\frac{d}{dx}(lnx\, lnx)$$

$$= x^{\cos^{-1}x}\left\{lnx\frac{d}{dx}\cos^{-1}x + \cos^{-1}x\frac{d}{dx}lnx\right\} + x^{lnx}\{lnx\frac{d}{dx}lnx + lnx\frac{d}{dx}lnx\}$$

$$= x^{\cos^{-1}x}\left\{\frac{\cos^{-1}x}{x} - \frac{lnx}{\sqrt{1-x^2}}\right\} + x^{lnx}\left\{\frac{lnx}{x} + \frac{lnx}{x}\right\}$$

$$= x^{\cos^{-1}x}\left\{\frac{\cos^{-1}x}{x} - \frac{lnx}{\sqrt{1-x^2}}\right\} + x^{lnx} \times \frac{2lnx}{x} \text{ Ans.}$$

Solution: (iii) Given, $y = \frac{7\sqrt{2x^3+1}\sin 5x}{(2x+3)(x-1)}$ …………………(1)

Taking logarithm both sides of (1)

$$lny = \ln\{\frac{7\sqrt{2x^3+1}\sin 5x}{(2x+3)(x-1)}\}$$

$$lny = \ln\left(7\sqrt{2x^3+1}\sin 5x\right) - \ln\{(2x+3)(x-1)\}$$

$$lny = ln7 + \ln(2x^3+1)^{\frac{1}{2}} + ln\sin 5x - ln(2x+3) - ln(x-1)$$

$$lny = ln7 + \frac{1}{2}\ln(2x^3+1) + ln\sin 5x - ln(2x+3) - ln(x-1)$$

Differentiating with respect to x, then

$$\frac{1}{y} \times \frac{dy}{dx} = 0 + \frac{6x^2}{2(2x^3+1)} + 5\cot 5x - \frac{2}{2x+3} - \frac{1}{x-1}$$

$$\therefore \frac{dy}{dx} = \frac{7\sqrt{2x^3+1}\sin 5x}{(2x+3)(x-1)}\left[\frac{6x^2}{2(2x^3+1)} + 5\cot 5x - \frac{2}{2x+3} - \frac{1}{x-1}\right] \text{ Ans.}$$

<u>**Solution:**</u> (iv) Given, $\ln(x+y) = xy$ ………………………(1)

Differentiating with respect to x both sides of (1), then

$$\frac{d}{dx}\ln(x+y) = \frac{d}{dx}\,xy$$

$$\Rightarrow \frac{1}{x+y}\left(1 + \frac{dy}{dx}\right) = y + x \times \frac{dy}{dx}$$

$$\Rightarrow \frac{1}{x+y} + \left(\frac{1}{x+y}\frac{dy}{dx}\right) = y + x \times \frac{dy}{dx}$$

$$\Rightarrow \frac{1}{x+y} \times \frac{dy}{dx} - x \times \frac{dy}{dx} = y - \frac{1}{x+y}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{x+y} - x\right) = \frac{xy + y^2 - 1}{x+y}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1-x^2-xy}{x+y}\right)=\frac{xy+y^2-1}{x+y}$$

$$\therefore \frac{dy}{dx}=\frac{xy+y^2-1}{1-x^2-xy} \text{ Ans.}$$

7) ***Differentiate,***

(i) $\tan^{-1} x$ with respect to $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

(ii) $e^{\sin^{-1}x}$ with respect to $\cos 3x$.

(iii)$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ with respect to $tanx$

(iv) $\tan^{-1}\frac{2x}{1-x^2}$ with respect to $\sin^{-1}\frac{2x}{1+x^2}$.

<u>Solution</u>: (i) let, $y=\tan^{-1} x$ and $z=\tan^{-1}\frac{\sqrt{1+x^2}-1}{\boldsymbol{x}}$

Here, $\mathrm{y}=\tan^{-1} x$

$$\frac{dy}{dx}=\frac{1}{1+x^2}$$

Again, $\mathrm{z}=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{\boldsymbol{x}}\right)$

$=\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$ **let** $\boldsymbol{x}=\boldsymbol{tan}\theta$

$=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$

$=\tan^{-1}\left(\frac{\frac{1}{\cos\theta}-1}{\tan\theta}\right)$

$=\tan^{-1}\left(\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right)$

$=\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$=\tan^{-1}\left(\frac{2sin^2\frac{\theta}{2}}{\mathbf{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$

$=\tan^{-1}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)$

$=\tan^{-1}\tan\frac{\theta}{2} = \frac{\theta}{2}$

$$z = \frac{1}{2}\tan^{-1}x$$

$$\frac{dz}{dx} = \frac{1}{2(1+x^2)}$$

Therefore,

$$\frac{dy}{dz} = \frac{dy}{dx}.\frac{dx}{dz}$$

$=\frac{1}{(1+x^2)}\times 2(1+x^2) = 2$ Ans.

<u>Solution</u>: (ii) Let,

$y = e^{\sin^{-1}x}$ …………….(1)

and $z = cos\,3x$…………….(2)

Differentiating (1) with respect to x, then

$$\frac{dy}{dx} = e^{\sin^{-1}x}.\frac{1}{\sqrt{1-x^2}}$$

Again, Differentiating (2) with respect to x, then

$$\frac{dz}{dx} = -3\,\mathbf{\cos 3}\boldsymbol{x}$$

Therefore,

$$\frac{dy}{dz} = \frac{dy}{dx}.\frac{dx}{dz}$$

$= \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}} \cdot \frac{1}{-3\cos 3x}$

$\therefore \frac{dy}{dz} = \frac{e^{\sin^{-1}x}}{-3\cos 3x . \sqrt{1-x^2}}$. Ans.

<u>Solution</u>: (iii) Let,

$y = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$...............(1)

$z = tanx$ ....(2)

Differentiating (1) with respect to x, then

$\frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}x)$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

Again, $y = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

$=\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$ **let $x = tan\theta$**

$= \tan^{-1}\frac{\sec\theta - 1}{\tan\theta}$

$= \tan^{-1}\left(\frac{\frac{1}{\cos\theta}-1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right)$

$= \tan^{-1}\frac{1-\cos\theta}{\sin\theta}$

$= \tan^{-1}\frac{2sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$= \tan^{-1}\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

$= \tan^{-1} \tan\frac{\theta}{2} = \frac{\theta}{2}$

$\therefore z = \frac{1}{2}\tan^{-1}x$

$\frac{dz}{dx} = \frac{1}{2(1+x^2)}$

Therefore,

$\frac{dz}{dy} = \frac{dz}{dx}.\frac{dx}{dy}$

$=\frac{1}{2(1+x^2)}\times(1+x^2) = \frac{1}{2}$ Ans.

Solution: (iv) Let, $y = \sin^{-1}\frac{\boldsymbol{2x}}{\boldsymbol{1+x^2}}$ ……………(1)

$z = \tan^{-1}\frac{\boldsymbol{2x}}{\boldsymbol{1-x^2}}$ ……………(2)

Here,

$y = \sin^{-1}\frac{2x}{1+x^2}$ let $x = tan\theta$

$=\sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta}$

$=\sin^{-1}\frac{2\sin\theta.\cos\theta}{\cos^2\theta+\sin^2\theta}$

$= \sin^{-1}\sin 2\theta = 2\theta$

$y = 2\tan^{-1}x$

$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$

Again, $z = \tan^{-1}\frac{2x}{1-x^2}$

$= \tan^{-1}\frac{\boldsymbol{2}tan\theta}{\boldsymbol{1}-\tan^2\theta}$

$= \tan^{-1}\left(\frac{2\sin\theta.\cos\theta}{\cos^2\theta-\sin^2\theta}\right)$

$= \tan^{-1} \tan 2\theta$

$= 2\theta$

$z = 2\tan^{-1} x$

$\therefore \frac{dz}{dx} = \frac{2}{1+x^2}$

Therefore,

$$\frac{dy}{dz} = \frac{dy}{dx} \cdot \frac{dx}{dz} \qquad = \frac{2}{1+x^2} \cdot \frac{1+x^2}{2} = 1 \quad \text{Ans.......}$$

H. W. Differentiate, $\boldsymbol{x^{\sin x}}$ **with respect to** $\boldsymbol{\sin^{-1} x}$.

8) Find, $\frac{dy}{dx}$ where $\boldsymbol{y = sec\left\{\frac{1}{2} ln(x^2 + a^2)\right\}}$.

Solution: Given,

$$y = sec\left\{\frac{1}{2} ln(x^2 + a^2)\right\} = secz \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Where, $z = \frac{1}{2} ln(x^2 + a^2) = \frac{1}{2} lns$…………………(2)

Where, $s = (x^2 + a^2)$ …………………(3)

Differentiating (1) with respect to z, then

$$\frac{dy}{dz} = \frac{d}{dz}(secz) = secztanz$$

Differentiating (2) with respect to s, then

$\frac{dz}{ds} = \frac{d}{ds}(\frac{1}{2} lns)$

$= \frac{1}{2} \cdot \frac{1}{s}$

$= \frac{1}{2s}$

Differentiating (3) with respect to x, then

$$\frac{ds}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{ds} \cdot \frac{ds}{dx}$$

$=\sec z \, . \tan z \, . \frac{1}{2s} 2x$

$=\sec \left\{\frac{1}{2} \boldsymbol{ln}(\boldsymbol{x^2} + \boldsymbol{a^2})\right\} . \tan \left\{\frac{1}{2} \boldsymbol{ln}(\boldsymbol{x^2} + \boldsymbol{a^2})\right\} . \frac{1}{2(\boldsymbol{x^2}+\boldsymbol{a^2})} 2x$

$=\frac{x}{(\boldsymbol{x^2}+\boldsymbol{a^2})} \sec \left\{\frac{1}{2} \boldsymbol{ln}(\boldsymbol{x^2} + \boldsymbol{a^2})\right\} . \tan \left\{\frac{1}{2} \boldsymbol{ln}(\boldsymbol{x^2} + \boldsymbol{a^2})\right\}$ **Ans……….**