Matrix Algebra-02

Transpose of a matrix: If A is an $(m \times n)$ matrix $(n \times m)$ matrix is obtained from the matrix A by writing its rows as columns and its columns as rows is called the transpose of A and its denoted by the symbols $A^T or\ A^t\ or\ A'$

Ex: Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 4 & 6 \end{bmatrix}_{2\times 3}$ then $A^T = \begin{bmatrix} 1 & 3 \\ 0 & 4 \\ 2 & 6 \end{bmatrix}_{3\times 2}$

Complex Conjugate of a matrix: If A is an $(m \times n)$matrix over the complex field C. Then the conjugate of A is a matrix $\vec{A}$ whose elements are respectively the conjugate of the elements of A. that is,

If $A = [a_{ij}] then\ \vec{A} = [a_{ij}]$

Ex: If $A = \begin{bmatrix} 2 & i & 1+i \\ 0 & -i & 5 \end{bmatrix} then\ \vec{A} = \begin{bmatrix} 2 & -i & 1-i \\ 0 & i & 5 \end{bmatrix}$

Diagonal matrix: A square matrix whose elements $a_{ij} = 0$ for all all $i \neq j$ is called diagonal matrix. That is only the diagonal elements of the square matrix can be non-zero.

Ex: $A = \begin{bmatrix} 7 & 0 \\ 0 & 2 \end{bmatrix}\ and\ B = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

Scalar matrix: A diagonal matrix whose diagonal elements are all equal is called a $scalar$ matrix.

Example: $A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\ and\ B = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}\ are\ scalar$ matrix.

Idempotent matrix: A square matrix A is called an idempotent matrix if $A^2 = A$.

Example: A = $\begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ is an idempotent matrix

so $A^2 = \mathrm{A} \times \mathrm{A} = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 1+3-5 & -3-9+15 & -5-15+25 \\ -1-3+5 & 3+9-15 & 5+15-25 \\ 1+3-5 & -3-9+15 & -5-15+25 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} = A$$

Therefore, $A^2 = $ A, so given matrix is Idempotent matrix.

Periodic matrix: A square matrix A is called periodic if $A^{k+1} = A$, where k is a positive integer .

Theorem: If $A$and B are Idempotent matrix then AB is idempotent if $AB = BA$.

Proof:

Since A and B are Idempotent matrix then $A^2 = A$ and $B^2 = B$

Given $AB = BA$

Now$(AB)^2 = AB(AB)$

$= A(BA)B$

$= A(AB)B$

$= (AA)(BB)$

$= A^2.B^2$

$= AB$

So AB is Idempotent Matrix. (***Proved***).

Theorem: If A and B are Idempotent matrix then $A + B$ will be Idempotent if and only

if $AB = BA = 0$.

Proof:

Since A and B are idempotent matrix then$A^2 = A$ and $B^2 = B$

Now if $AB = BA = 0$, *then*

$(A + B)^2 = (A + B)(A + B)$

$= A^2 + AB + BA + B^2$

$= A^2 + B^2$

$= A + B$

So $A + B$ is an idempotent matrix.

Again if $A + B$ is an idempotent then

$$(A + B)^2 = A + B$$

$$=> A^2 + AB + BA + B^2 = A + B$$

$$=> A + AB + BA + B = A + B$$

$$=> A + B + AB + BA = A + B$$

$$=> AB + BA = 0$$

$$=> AB = -BA \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..(i)$$

Again, $AB = A^2$

$$= AA$$

$$= A(AB)$$

$$= A(-BA)$$

$$= -(AB)A$$

$$= -(-BA)A$$

$$= BA^2$$

$$= BA$$

$$AB = BA \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..(ii)$$

Now $(i + ii)$, we get,

$$=> 2BA = 0$$

$$=> AB = 0 = BA$$

So A+B is idempotent if and only if$AB = 0 = BA$

$(\boldsymbol{Proved})$

Home works:

1. Show that the matrix $A=\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$ is an $Idempotent$.

2. Show that the matrix $A=\begin{bmatrix}1 & -2 & -4\\ -1 & -3 & 4\\ 1 & -2 & -3\end{bmatrix}$ is not an $Idempotent$.

$\boldsymbol{Involutory\ Matrix}$**:** A square matrix A is called $involutory$ matrix if it satisfy the relation $A^2=I$ where I is the identity matrix.

Problem: $how\ that\ A\ =\begin{bmatrix}-5 & -8 & 0\\ 3 & 5 & 0\\ 1 & 2 & -1\end{bmatrix}is\ a\ nilpotent\ matrix.$

Hints: $\boldsymbol{A^2=A.A=I}$

Nilpotent Matrix: A square matrix A is called a nilpotent matrix of order m if it satisfy the relation $A^m=0$ and $A^{m-1}\neq 0$ where m is a positive integer and 0 is a null matrix.

$Problem$: $Show\ that\ A\ =\begin{bmatrix}1 & 2 & 3\\ 1 & 2 & 3\\ -1 & -2 & -3\end{bmatrix}is\ a\ nilpotent\ matrix.$

Solution:

Given that, $A\ =\begin{bmatrix}1 & 2 & 3\\ 1 & 2 & 3\\ -1 & -2 & -3\end{bmatrix}$

$Now, A^2=A.A$

$$=\begin{bmatrix}1 & 2 & 3\\ 1 & 2 & 3\\ -1 & -2 & -3\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 1 & 2 & 3\\ -1 & -2 & -3\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+2\times1+3\times(-1) & 1\times2+2\times2+3\times(-2) & 1\times3+2\times3+3\times(-3)\\ 1\times1+2\times1+3\times(-1) & 1\times2+2\times2+3\times(-2) & 1\times3+2\times3+3\times(-3)\\ -1\times1+(-2)\times1+(-3)\times(-1) & -1\times2+(-2)\times2+(-3)\times(-2) & -1\times3+(-2)\times3+(-3)\times(-3)\end{bmatrix}$$

$$=\begin{bmatrix}1+2-3 & 2+4-6 & 3+6-9\\ 1+2-3 & 2+4-6 & 3+6-9\\ -1-2+3 & -2-4+6 & -3-6+9\end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 0 \quad where\ 0\ is\ the\ null\ matrix\ of\ order\ 3. that\ is\ A^2 = 0$

$but\ A \neq 0. Hence\ A\ is\ a\ nilpotent\ matrix\ of\ order\ 2.$

Symmetric matrix: A square matrix $A = [a_{ij}]$ is called symmetric matrix if $a_{ij} = a_{ji}$ for all i and j that is, $A = A'$.

Example: $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 5 \end{bmatrix}$ is an example of symmetric matrix.

Skew-symmetric matrix: A square matrix $A = [a_{ij}]$ is called symmetric matrix if $a_{ij} = -a_{ji}$ for all i and j that is, $A = -A'$.

Example: $A = \begin{bmatrix} 1 & 2 & -3 \\ -2 & 4 & 6 \\ 3 & -6 & 5 \end{bmatrix}$ is an example of skew-symmetric matrix.

Problem: Is given matrix

Theorem: Every square matrix can be written as a sum of symmetric and skew-symmetric matrices.

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

Where, $\frac{1}{2}(A + A')$ is symmetric part of A and $\frac{1}{2}(A - A')$ is skew-symmetric part of A.

Problem: Express the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 5 & 6 \\ -6 & 3 & 2 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrices.

Solution: Given, $A = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 5 & 6 \\ -6 & 3 & 2 \end{bmatrix}$ and $A' = \begin{bmatrix} 1 & 4 & -6 \\ 2 & 5 & 3 \\ -3 & 6 & 2 \end{bmatrix}$

We know,

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

Where, $\frac{1}{2}(A + A')$ is symmetric part of A and $\frac{1}{2}(A - A')$ is skew-symmetric part of A.

So, $\frac{1}{2}(A+A') = \frac{1}{2}\left\{\begin{bmatrix} 1 & 2 & -3 \\ 4 & 5 & 6 \\ -6 & 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 4 & -6 \\ 2 & 5 & 3 \\ -3 & 6 & 2 \end{bmatrix}\right\}$

$$= \frac{1}{2}\left\{\begin{bmatrix} 1+1 & 2+4 & -3-6 \\ 4+2 & 5+5 & 6+3 \\ -6-3 & 3+6 & 2+2 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 6 & -9 \\ 6 & 10 & 9 \\ -9 & 9 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & -\frac{9}{2} \\ 3 & 5 & \frac{9}{2} \\ -\frac{9}{2} & \frac{9}{2} & 2 \end{bmatrix}$$

Therefore, symmetric part of A is

$= \begin{bmatrix} 1 & 3 & -\frac{9}{2} \\ 3 & 5 & \frac{9}{2} \\ -\frac{9}{2} & \frac{9}{2} & 2 \end{bmatrix}$.

Again, $\frac{1}{2}(A-A') = \frac{1}{2}\left\{\begin{bmatrix} 1 & 2 & -3 \\ 4 & 5 & 6 \\ -6 & 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 4 & -6 \\ 2 & 5 & 3 \\ -3 & 6 & 2 \end{bmatrix}\right\}$

$$= \frac{1}{2}\left\{\begin{bmatrix} 1-1 & 2-4 & -3+6 \\ 4-2 & 5-5 & 6-3 \\ -6+3 & 3-6 & 2-2 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 3 \\ -3 & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & \frac{3}{2} \\ 1 & 0 & \frac{3}{2} \\ -\frac{3}{2} & -\frac{3}{2} & 0 \end{bmatrix}$$

Therefore, skew-symmetric part of A is

$$=\begin{bmatrix} 0 & -1 & \frac{3}{2} \\ 1 & 0 & \frac{3}{2} \\ -\frac{3}{2} & -\frac{3}{2} & 0 \end{bmatrix}.$$

Therefore, $A=\begin{bmatrix} 1 & 2 & -3 \\ 4 & 5 & 6 \\ -6 & 3 & 2 \end{bmatrix}=\begin{bmatrix} 1 & 3 & -\frac{9}{2} \\ 3 & 5 & \frac{9}{2} \\ -\frac{9}{2} & \frac{9}{2} & 2 \end{bmatrix}+\begin{bmatrix} 0 & -1 & \frac{3}{2} \\ 1 & 0 & \frac{3}{2} \\ -\frac{3}{2} & -\frac{3}{2} & 0 \end{bmatrix}$

$$= \text{symmetric part of } A + \text{skewsymmetric part of } A\,.$$

Home works: Find the symmetric and skew-symmetric matrices for the following matrices

1. $\begin{bmatrix} 0 & 2 & 4 \\ 5 & 3 & 5 \\ 9 & 8 & 0 \end{bmatrix}$;
2. $\begin{bmatrix} 1 & 2 & 8 \\ -1 & 3 & -6 \\ 5 & 8 & 0 \end{bmatrix}$;
3. $\begin{bmatrix} -1 & 2 & -3 \\ 2 & -3 & -6 \\ -\frac{1}{2} & \frac{2}{3} & 9 \end{bmatrix}$.

Orthogonal Matrix: A square matrix A is called orthogonal matrix if $AA'=I$ where I is an identity matrix and A' is the transposed of A.

A real square matrix A is said to be orthogonal if AA^T=$A^TA=I\ (identity)$.

Theorem: If A and B are orthogonal matrix each of order n then the matrix AB and BA are also orthogonal.

Proof:

Since A and B are n rowed orthogonal matrix then AA^T=A^TA=I_n and BB^T=B^TB=I_n

The matrix product AB is also a square matrix of order n and $(AB)^T$(AB) =$(B^TA^T)AB$

=$B^T(A^TA)$B

=$B^T.I_n$ B

=B^TB =I_n

Thus AB is orthogonal matrix of order n.

Similarly (BA)$(BA)^T$=(BA)(A^TB^T)

$=B(AA^T)B^T$

$= B\, I_n B^T$

$= BB^T \quad = I_n$

$\therefore$BA is an orthogonal matrix of order n.

Theorem: if A is an orthogonal matrix then A^{-1} is also orthogonal.

Proof:

If A is orthogonal then $AA^T = A^T A = I$ (Identity Matrix)

$\Rightarrow (AA^T)^{-1} = (A^T A)^{-1} = (I)^{-1}$

$\Rightarrow (A^T)^{-1} A^{-1} = A^{-1} (A^T)^{-1} = I$

$\Rightarrow (A^{-1})^T A^{-1} = A^{-1} (A^{-1})^T = I$

$[\because I^{-1} = I, (A^T)^{-1} = (A^{-1})^T]$

So, A-1 is orthogonal by definition. That is inverse of an orthogonal matrix is also orthogonal.

(Proved)

Example: $A = \begin{bmatrix} 1/3 & 2/3 & -2/3 \\ 2/3 & 1/3 & 2/3 \\ 2/3 & -2/3 & -1/3 \end{bmatrix}$ is an $Orthogonal\ Matrix$

Since $A.A^T = \begin{bmatrix} 1/3 & 2/3 & -2/3 \\ 2/3 & 1/3 & 2/3 \\ 2/3 & -2/3 & -1/3 \end{bmatrix} \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ -2/3 & 2/3 & -2/3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$

Problem.3 verify that $(\boldsymbol{AB})^t = \boldsymbol{B^t A^t}\ \boldsymbol{When\ A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{3} & \mathbf{-2} & \mathbf{1} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{2} & \mathbf{0} \\ \mathbf{-1} & \mathbf{1} \end{bmatrix}$

Solution:

$$Given\ that, AB = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+2\times2+3\times(-1) & 1\times2+2\times0+3\times1 \\ 3\times1+(-2)\times2+1\times(-1) & 3\times2+(-2)\times0+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4-3 & 2+0+3 \\ 3-4-1 & 6-0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 \\ -2 & 7 \end{bmatrix}$$

$$\therefore (AB)^t = \begin{bmatrix} 2 & 5 \\ -2 & 7 \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & -2 \\ 5 & 7 \end{bmatrix}$$

$$Again, A^t = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \end{bmatrix}$$

$$and\ B^t = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\therefore B^t A^t = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+2\times2+(-1)\times3 & 1\times3+2\times(-2)+(-1)\times1 \\ 2\times1+0\times2+1\times3 & 2\times3+0\times(-2)+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4-3 & 3-4-1 \\ 2+0+3 & 6+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ 5 & 7 \end{bmatrix}$$

$$= (AB)^t \quad (\boldsymbol{Verified})$$

$Problem$: $Show\ that\ the\ Matrix\ A = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}\ is\ Orthogonal\ Matrix.$

Solution:

$$let\ , \qquad A' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

$$Now, AA' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} -1\times(-1)+2\times2+2\times2 & -1\times2+2\times(-1)+2\times2 & 2\times(-1)+2\times2+2\times(-1) \\ 2\times(-1)+(-1)\times2+2\times2 & 2\times2+(-1)\times(-1)+2\times2 & 2\times2+(-1)\times2+2\times(-1) \\ 2\times(-1)+2\times2+(-1)\times2 & 2\times2+2\times(-1)+(-1)\times2 & 2\times2+2\times2+(-1)\times(-1) \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 1+4+4 & -2-2+4 & -2+4-2 \\ -2-2+4 & 4+1+4 & 4-2-2 \\ -2+4-2 & 4-2-2 & 4+4+1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} \frac{9}{9} & 0 & 0 \\ 0 & \frac{9}{9} & 0 \\ 0 & 0 & \frac{9}{9} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$\therefore\ AA' = I \qquad$ $(\boldsymbol{Verified})$

$\boldsymbol{Problem}$: $Verify\ that,$

$\boldsymbol{i})\ (AB)^t = B^tA^t\ when, \qquad A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & 1 & 8 \end{bmatrix}\ B = \begin{bmatrix} 4 & 1 & 0 \\ 2 & -3 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

$\boldsymbol{ii})\ If\ A = \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}\ then, \qquad AA' = I = A'A$

<u>$\boldsymbol{Solution\ of\ (i)}$</u>

<u>Given that,</u>

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & 1 & 8 \end{bmatrix} \qquad \text{and}\ \ B = \begin{bmatrix} 4 & 1 & 0 \\ 2 & -3 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$Now, AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 4 & 1 & 8 \end{bmatrix} \begin{bmatrix} 4 & 1 & 0 \\ 2 & -3 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 4 + (-1)\times 2 + 0\times 1 & 1\times 1 + (-1)\times(-3) + 0\times 1 & 1\times 0 + (-1)\times 1 + 0\times(-1) \\ 2\times 4 + 1\times 2 + 3\times 1 & 2\times 1 + 1\times(-3) + 3\times 1 & 2\times 0 + 1\times 1 + 3\times(-1) \\ 4\times 4 + 1\times 2 + 8\times 1 & 4\times 1 + 1\times(-3) + 8\times 1 & 4\times 0 + 1\times 1 + 8\times(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 4-2+0 & 1+3+0 & 0-1+0 \\ 8+2+3 & 2-3+3 & 0+1-3 \\ 16+2+8 & 4-3+8 & 0+1-8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & -1 \\ 13 & 2 & -2 \\ 26 & 9 & -7 \end{bmatrix}$$

$$\therefore\ (AB)^t = \begin{bmatrix} 2 & 4 & -1 \\ 13 & 2 & -2 \\ 26 & 9 & -7 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 1 & 1 \\ 0 & 3 & 8 \end{bmatrix},\ B^t = \begin{bmatrix} 4 & 2 & 1 \\ 1 & -3 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\therefore B^tA^t = \begin{bmatrix} 4 & 2 & 1 \\ 1 & -3 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ -1 & 1 & 1 \\ 0 & 3 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 4\times 1 + 2\times -1 + 1\times 0 & 4\times 2 + 2\times 1 + 1\times 3 & 4\times 4 + 2\times 1 + 1\times 8 \\ 1\times 1 + (-3)\times(-1) + 1\times 0 & 1\times 2 + (-3)\times 1 + 1\times 3 & 1\times 4 + (-3)\times 1 + 1\times 8 \\ 0\times 1 + 1\times(-1) + (-1)\times 0 & 0\times 2 + 1\times 1 + (-1)\times 3 & 0\times 4 + 1\times 1 + (-1)\times 8 \end{bmatrix}$$

$$= \begin{bmatrix} 4-2+0 & 8+2+3 & 16+2+8 \\ 1+3+0 & 2-3+3 & 4-3+8 \\ 0-1+0 & 0+1-3 & 0+1-8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 13 & 26 \\ 4 & 2 & 9 \\ -1 & -2 & -7 \end{bmatrix}$$

$$\therefore (AB)^t = B^tA^t$$

<u>Solution of (ii):</u>

$$Given\ that\ , A' = \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$$

Here, $AA' = \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix} \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$

$$= \begin{bmatrix} cos\alpha cos\alpha + sin\alpha sin\alpha & cos\alpha(-sin\alpha) + sin\alpha cos\alpha \\ -sin\alpha cos\alpha + cos\alpha sin\alpha & -sin\alpha(-sin\alpha) + cos\alpha cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} cos^2\alpha + sin^2\alpha & -sin\alpha cos\alpha + sin\alpha cos\alpha \\ -sin\alpha cos\alpha + sin\alpha cos\alpha & sin^2\alpha + cos^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Again,

$$A'A = \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix} \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} cos\alpha cos\alpha + (-sin\alpha)(-sin\alpha) & cos\alpha sin\alpha + (-sin\alpha)cos\alpha \\ sin\alpha cos\alpha + cos\alpha(-sin\alpha) & sin\alpha sin\alpha + cos\alpha cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} cos^2\alpha + sin^2\alpha & sin\alpha cos\alpha - sin\alpha cos\alpha \\ sin\alpha cos\alpha - sin\alpha cos\alpha & sin^2\alpha + cos^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

$\therefore AA' = I = A'A$ $\qquad\qquad [\boldsymbol{proved}]$