Discussion on Domain and Range

Differential Calculus

In this section we will define and develop the concept of a "function," which is the basic mathematical object that scientists and mathematicians use to describe relationships between variable quantities. Functions play a central role in calculus and its applications.

SET: A set is a well defined collection of data or objects. There are two types of set

i) Finite Set   ii) Infinite Set

Finite Set: In mathematics, a finite set is a set that has a finite number of elements.

For example,

A= {a,b,c} , B = { p,q,s}

is a finite set with three elements. The number of elements of a finite set is a natural number (a non-negative integer) and is called the cardinality of the set.

Infinite Set: A set that is not finite is called infinite Set.

For Example: Integer $\mathbb{Z} = \{ \dots \dots .. -2, -1, 0, 1, 2, 3 \dots \dots .. \}$

Natural Number $\mathbb{N} = \{ 1, 2, 3, 4, \dots \dots \dots .. \}$

Real Number $\mathbb{R} = \{ \dots \dots ., -3, -2, -1.5, -1, 0, 1, 2, 3, 4, \dots \dots \}$

DEFINITION OF A FUNCTION

Many scientific laws and engineering principles describe how one quantity depends on another. This idea was formalized in 1673 by Gottfried Wilhelm Leibniz who coined the term *function* to indicate the dependence of one quantity on another, as described in the following definition.

Definition If a variable y depends on a variable x in such a way that each value of x determines exactly one value of y, then we say that ***y is a function of x***.

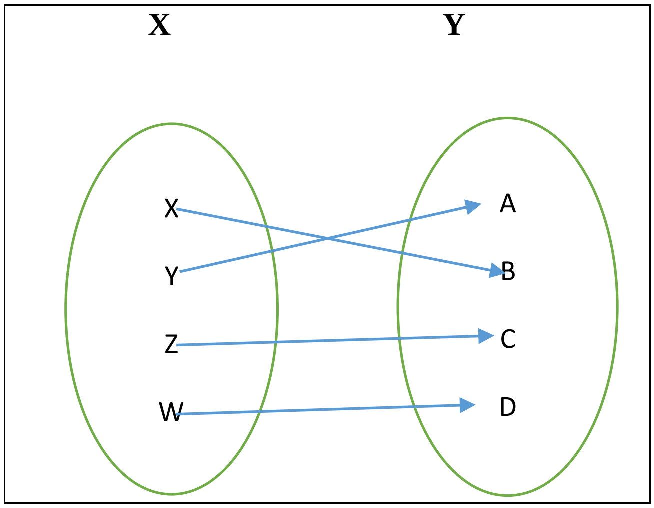
Definition A ***function*** f is a rule that associates a unique output with each input. If the input is denoted by x, then the output is denoted by $f(x)$ (read "f of x").

Function: If X and Y are two non-empty sets and f is a such rule that gives a unique $y \in Y$ for each $x \in X$ then f is called a function from the set X to the set Y. Here X is independent variable and Y is dependent variable. A function is a special relationship where each input has a single output.

Discussion on Domain and Range

We will see many ways to think about functions, but there are always three main parts:

- The input
- The relationship
- The output.



Properties of Function:

1) If X and Y are two sets then Must uses all elements of X.
2) If X and Y are two sets then one element of X and many elements of Y does not relation each others.
3) If X and Y are two sets then many elements of X and one element of Y relation.

Domain: If A is the set of all real values of x such that the formula or equation y = f(x) be defined or satisfied, then A is called domain of the formula.

$f: A \to B$, A is domain of f and B is codomain of f

Let, $x = \{1,2,3\}$ and $y = \{6,7,8,9,10\}$

$$y = f(x) = x + 5$$

Range={6,7,8}

$x = \{1,2,3\}$ and $z = \{6,7,8\}$

Range: If B is the set of all real values of y or f(x) corresponding each of the values or points x in domain A of the formula y= f(x) then B is called Range of the formula.

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Undefined form

i. $\frac{some\ thing}{0} = undefined$

ii. $\frac{1}{0} = undefined$

iii. $\sqrt{-ve} = undefined$

Example $* \boldsymbol{f(x)} = \frac{1}{x+2}, x + 2 = 0, x = -2$

$* \boldsymbol{f(x)} = \frac{1}{x-2}, x - 2 = 0, x = 2$

$* \boldsymbol{f(x)} = \frac{1}{3x-2}, 3x - 2 = 0, x = 2/3$

$* \boldsymbol{f(x)} = 2x + 1$

1. Find the domain & range of $\boldsymbol{f(x) = ax + b}$ ($\boldsymbol{a \neq 0}$, a not equal zero).

Solution: Here f(x) gives real values for all real values of x.

So, Domain = D_f = R(All real number) and

Again, $y = ax + b$

$ax = y - b$

$\Rightarrow x = \frac{y-b}{a}$

Here x gives real values for all real values of y.

So Range R_f = R(All real number)

2. Find the domain and range of $f(x) = \frac{1}{ax+b}, a \neq 0$

Solution:

Here, $f(x)$ is not defined for $ax + b$=0 or $x = -\frac{b}{a}$ and f(x) gives real values that is defined for all real values for x except, $x = -\frac{b}{a}$ of domain of f or $D_f = \mathbb{R} - \left\{-\frac{b}{a}\right\}$

Again,

$y = \frac{1}{ax+b}, a \neq 0$

$\Rightarrow ax + b = \frac{1}{y}$

$\Rightarrow ax = \frac{1}{y} - b$

$\Rightarrow x = \frac{1}{a}(\frac{1}{y} - b)$

Here, x gives real values for all real values of y, except $y = 0$

So range of f,

$$R_f = \mathbb{R} - \{0\}$$

3. Find the domain and range of $f(x) = \frac{3}{2x+7}$.

Solution: Here $f(x)$ is not defined for $2x + 7 = 0$, or $x = -\frac{7}{2}$ and $f(x)$ gives real values that is defined for all real values of x, except $x = -\frac{7}{2}$, for Domain of f

$$D_f = \mathbb{R} - \left\{-\frac{7}{2}\right\}$$

Again,

$y = \frac{3}{2x+7}$

$\Rightarrow 2x + 7 = \frac{3}{y}$

$\Rightarrow 2x = (\frac{3}{y} - 7)$

$\Rightarrow x = \frac{1}{2}\left(\frac{3}{y} - 7\right)$

Here, x gives real values for all real values of y, except $y = 0$

So range of f,

$$R_f = \mathbb{R} - \{0\}.$$

4. Find the domain and range of $f(x) = \frac{ax+b}{cx+d}, [c \neq 0, a \neq 0]$

Solution: Here $f(x)$ is not defined for $cx + d = 0$, or $x = -\frac{d}{c}$ and $f(x)$ gives real values that is defined for all real values of x, except $x = -\frac{d}{c}$, for Domain of f

$$D_f = \mathbb{R} - \left\{-\frac{d}{c}\right\}$$

Again,

$y = \dfrac{ax+b}{cx+d}$

$\Rightarrow y(cx+d) = ax+b$

$\Rightarrow cxy + dy = ax + b$

$\Rightarrow cxy - ax = b - dy$

$\Rightarrow (cy-a)x = b - dy$

$\Rightarrow x = \dfrac{b-dy}{cy-a}$

Here, x gives real values for all real values of y, except $cy - a = 0$ or $y = \frac{a}{c}$

So range of f,

$$R_f = \mathbb{R} - \left\{\frac{a}{c}\right\}.$$

5. Find the domain and range of $f(x) = \frac{3x+2}{5x+3}$.

Solution: Here $f(x)$ is not defined for $5x + 3 = 0$, or $x = -\frac{3}{5}$ and $f(x)$ gives real values that is defiened for all real values of x, except, $x = -\frac{3}{5}$, for Domain of f

$$D_f = \mathbb{R} - \left\{-\frac{3}{5}\right\}$$

Again, $y = \frac{3x+2}{5x+3}$

$\Rightarrow y(5x+3)$=$3x+2$

$\Rightarrow 5xy + 3y$=$3x+2$

$\Rightarrow 5xy - 3x$=$2 - 3y$

$\Rightarrow x(5y-3)$=$2 - 3y$

$\Rightarrow x = \dfrac{2-3y}{5y-3}$

Here, x gives real values for all real values of y, except $5y - 3 = 0$ or $y = \frac{3}{5}$

So range of f,

$$R_f = \mathbb{R} - \left\{\frac{3}{5}\right\}.$$

6. Find the domain and range of $f(x) = \frac{x^2-a^2}{x-a}$

Solution: Given, $f(x) = \frac{x^2-a^2}{x-a}$

Here, $f(x)$ gives real values for all real values of x except, $x = a$.

So, the domain of this function, $D_f = R - \{a\}$

Again,

$$y = f(x) = \frac{x^2 - a^2}{x - a}$$

$$\Rightarrow y = x + a, where\ x \neq a$$

$$\Rightarrow x = y - a, where\ y \neq 2a\ and\ x \neq a$$

Here, x gives all real values for all real values of y except $y = 2a$

So, the range of this function, $R_f = R - \{2a\}$.

7. Find the domain and range of $f(x) = \frac{x^2-9}{x-3}$

Solution: Given, $f(x) = \frac{x^2-9}{x-3}$

Here, $f(x)$ gives real values for all real values of x except, $x = 3$.

So, the domain of this function, $D_f = R - \{3\}$

Again,

$$y = f(x) = \frac{x^2 - 9}{x - 3}$$

$$\Rightarrow y = x + 3, where\ x \neq 3$$

$$\Rightarrow x = y - 3, where\ y \neq 6\ and\ x \neq 3$$

Here, x gives all real values for all real values of y except $y = 6$

So, the range of this function, $R_f = R - \{6\}$.

Discussion on Domain and Range

H. W. Find the domain and range for the following functions

$(i)\ f(x) = \frac{x+2}{5x+1}$, $(ii)\ f(x) = \frac{3x}{2x+3}$; $(iii)\ f(x) = \frac{x}{9x+3}$

$(iv)\ \ f(x) = \frac{5}{\frac{3}{2}x+1}$ $(v) f(x) = \frac{x^2-4}{x-2}$; $(vi)\ f(x) = \frac{x-3}{2x+1}$

Answer:

$(i)\ D_f = \mathbb{R} - \left\{-\frac{1}{5}\right\}, R_f = \mathbb{R} - \left\{\frac{1}{5}\right\}$; $(ii)\ D_f = \mathbb{R} - \left\{-\frac{3}{2}\right\}, R_f = \mathbb{R} - \left\{\frac{3}{2}\right\}$;

$(iii) D_f = \mathbb{R} - \left\{-\frac{1}{3}\right\}, R_f = \mathbb{R} - \left\{\frac{1}{9}\right\}$