1 The VBF kinematics for double Higgs

Let us consider two quarks that scatter and produce two quarks and two Higgs bosons in vector boson fusion:

$$q(p_1) + \bar{q}(p_2) \to q(p_3) + \bar{q}(p_4) + H(q_1) + H(q_2)\,, \tag{1}$$

where the Higgs bosons are on shell $q_1^2 = q_2^2 = m_H^2$.

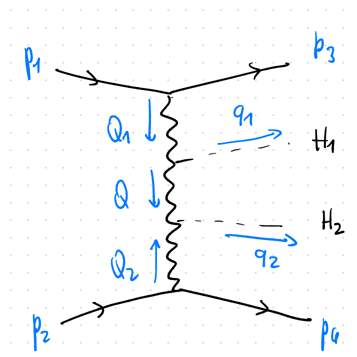

Figure 1: momenta assignment in the propagators

We are interested in studying the forward limit so it's useful to introduce light-cone coordinates

$$q^\mu = (q^+, q^-, \mathbf{q}_\perp)\,, \quad q^+ = \frac{q^0 + q^3}{\sqrt{2}}\,, \quad q^- = \frac{q^0 - q^3}{\sqrt{2}}\,, \quad \mathbf{q}_\perp = (q^1, q^2)\,. \tag{2}$$

Recall that in these coordinates the length of a vector q^μ is given by

$$q^\mu k_\mu = q^+ k^- + q^- k^+ - \mathbf{q}_\perp \cdot \mathbf{k}_\perp\,, \qquad q^\mu q_\mu = 2q^+ q^- - \mathbf{q}_\perp^2\,, \tag{3}$$

and the rapidity of a particle of momentum q^μ is given by

$$\eta_q = \frac{1}{2} \ln\left(\frac{q^+}{q^-}\right) \tag{4}$$

and therefore a central particle (ie with zero rapidity) has $q^+ = q^-$.

Let us pick for definiteness the incoming quarks such that they only have either plus or minus components

$$p_1^\mu = (0, p_1^-, \mathbf{0}_\perp)\,, \quad p_2^\mu = (p_2^+, 0, \mathbf{0}_\perp)\,. \tag{5}$$

In the VBF kinematics, the quarks (jets) are emitted forward and the vector bosons are emitted centrally.

Conditions from forward quarks

Forward jets means that the outgoing quarks must have little perpendicular components $\mathbf{p_i}_\perp \sim \lambda$ where $\lambda \ll 1$. Then for the outgoing quarks, since they are on-shell $p_i^2 = 0$, one finds that the suppressed light-cone component scales as λ^2 and is further suppressed by the large light-cone components:

$$\begin{aligned} p_3^\mu &= (p_3^+, p_3^-, \mathbf{p_3}_\perp) \quad \to \quad p_3^\mu p_{3\mu} = 0 = 2p_3^+ p_3^- - \mathbf{p_3}_\perp^2 \quad \to \quad p_3^+ = \frac{\mathbf{p_3}_\perp^2}{2p_3^-} \sim \mathcal{O}(\lambda^2) \\ p_4^\mu &= (p_4^+, p_4^-, \mathbf{p_4}_\perp) \quad \to \quad p_4^\mu p_{4\mu} = 0 = 2p_4^+ p_4^- - \mathbf{p_4}_\perp^2 \quad \to \quad p_4^- = \frac{\mathbf{p_4}_\perp^2}{2p_4^+} \sim \mathcal{O}(\lambda^2) \end{aligned} \tag{6}$$

Let us now study the momenta that flow in the vector bosons. We call the three momenta, from up to down, Q_1^μ, Q^μ, Q_2^μ. We assume that Q_1 and Q_2 go out from the fermion line and Q goes from the first Higgs to the second, such that

$$Q_1 + Q_2 = q_1 + q_2 \,.$$

For the two vector bosons attached to the quark lines, we have to $\mathcal{O}(\lambda)$

$$\begin{aligned} Q_1^\mu &= p_1^\mu - p_3^\mu = (-p_3^+, p_1^- - p_3^-, -\mathbf{p_3}_\perp) \sim (0,\ p_1^- - p_3^-, -\mathbf{p_3}_\perp) \\ Q_2^\mu &= p_2^\mu - p_4^\mu = (p_2^+ - p_4^+, -p_4^-, -\mathbf{p_4}_\perp) \sim (p_2^+ - p_4^+, 0,\ -\mathbf{p_4}_\perp) \end{aligned} \tag{7}$$

where we neglected everything that is of order λ^2. This implies that, to $\mathcal{O}(\lambda)$ we have

$$Q_1^2 \sim -\mathbf{p_3}_\perp^2 \,, \qquad Q_2^2 \sim -\mathbf{p_4}_\perp^2 \,. \tag{8}$$

For outgoing Higgs momentum q_1, the momentum of the central vector boson is instead

$$Q = Q_1 - q_1 = (-q_1^+, p_1^- - p_3^- - q_1^-, -(\mathbf{p_3}_\perp + \mathbf{q_1}_\perp)) = (Q^+, Q^-, \mathbf{Q}_\perp) \,. \tag{9}$$

But we can also parametrise the W boson momentum with the momentum of the second Higgs boson

$$Q = q_2 - Q_2 = (q_2^+ - (p_2^+ - p_4^+), q_2^-, (\mathbf{p_4}_\perp + \mathbf{q_2}_\perp)) = (Q^+, Q^-, \mathbf{Q}_\perp) \tag{10}$$

such that equating eqs. (9) and (10) we get

$$p_2^+ - p_4^+ - q_2^+ = q_1^+ \,, \quad q_2^- = p_1^- - p_3^- - q_1^- \,, \quad (\mathbf{p_4}_\perp + \mathbf{q_2}_\perp) = -(\mathbf{p_3}_\perp + \mathbf{q_1}_\perp) \equiv \mathbf{Q}_\perp \tag{11}$$

and therefore for the $+/-$ components we have

$$q_2^+ + q_1^+ = p_2^+ - p_4^+ \,, \quad q_1^- + q_2^- = p_1^- - p_3^- \,. \tag{12}$$

In addition, the on-shell condition for the two Higgs bosons gives

$$q_i^2 = 2 q_i^+ q_i^- - \mathbf{q_i}_\perp^2 = m_H^2 \quad \to \quad 2 q_i^+ q_i^- = \mathbf{q_i}_\perp^2 + m_H^2 \,, \qquad \text{for } \ i = 1, 2 \tag{13}$$

and the square of the momentum flowing in the intermediate W propagator is

$$\begin{aligned} Q^2 &= -2 q_1^+ (p_1^- - p_3^- - q_1^-) - (\mathbf{p_3}_\perp + \mathbf{q_1}_\perp)^2 \\ &= 2 q_1^+ q_1^- - 2 q_1^+ (p_1^- - p_3^-) - (\mathbf{p_3}_\perp + \mathbf{q_1}_\perp)^2 \\ &= \mathbf{q_1}_\perp^2 + m_H^2 - 2 q_1^+ (p_1^- - p_3^-) - \mathbf{Q}_\perp^2 \end{aligned} \tag{14}$$

or equivalently

$$\begin{aligned} Q^2 &= -2 q_2^- (p_2^+ - p_4^+ - q_2^+) - (\mathbf{p_4}_\perp + \mathbf{q_2}_\perp)^2 \\ &= \mathbf{q_2}_\perp^2 + m_H^2 - 2 q_2^- (p_2^+ - p_4^+) - \mathbf{Q}_\perp^2 \end{aligned} \tag{15}$$

Central Higgs boson

In the VBF kinematics, the Higgs are produced centrally, so $q_i^+ \sim q_i^-$ for $i = 1, 2$ such that eq (12) implies *(pay attention to the signs!)*

$$q_2^+ + q_1^+ = q_1^- + q_2^- = p_2^+ - p_4^+ = p_1^- - p_3^- \,. \tag{16}$$

and the on-shell condition becomes

$$2 (q_i^+)^2 = \mathbf{q_i}_\perp^2 + m_H^2 \,, \qquad \text{for } \ i = 1, 2 \,. \tag{17}$$

If the two Higges are central and on-shell, in the CoM frame the situation is perfectly symmetric and we have $q_1^+ = q_2^+ \equiv q^+$ and $\mathbf{q_1}_\perp = \mathbf{q_2}_\perp = \mathbf{q}_\perp$, which gives in turn `[LT: please check this in particular]`

$$2q^+ = 2q_2^+ = 2q_1^+ = 2q_1^- = 2q_2^- = p_2^+ - p_4^+ = p_1^- - p_3^- \,, \tag{18}$$

and the momentum in the central W propagator becomes

$$Q^2 = \mathbf{q_2}_\perp^2 + m_H^2 - 4(q_2^+)^2 - \mathbf{Q}_\perp^2 = \mathbf{q_1}_\perp^2 + m_H^2 - 4(q_1^+)^2 - \mathbf{Q}_\perp^2 \tag{19}$$

and using the on-shell condition eq. (17) one gets

$$Q^2 = -\left[\mathbf{q}_\perp^2 + m_H^2 + \mathbf{Q}_\perp^2\right] \tag{20}$$

How do the integrals change

....to be continued....