NYU Computer Science Bridge to Tandon Course Winter 2021

Homework 1

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Question 1

A. Convert the following numbers to their decimal representation. Show your work.

1. 10011011_2

   Solution.

$$\begin{aligned} 10011011_2 &= 2^7 + 2^4 + 2^3 + 2^1 + 2^0 \\ &= 128 + 16 + 8 + 2 + 1 \\ &= \mathbf{155} \end{aligned}$$

■

2. 456_7

   Solution.

$$\begin{aligned} 456_7 &= 4 \times 7^2 + 5 \times 7^1 + 6 \times 7^0 \\ &= 196 + 35 + 6 \\ &= \mathbf{237} \end{aligned}$$

■

3. 38A_{16}

   Solution.

$$\begin{aligned} 38\text{A}_{16} &= 3 \times 16^2 + 8 \times 16^1 + 10 \times 16^0 \\ &= 768 + 128 + 10 \\ &= \mathbf{906} \end{aligned}$$

■

4. 2214_5

   Solution.

$$\begin{aligned} 2214_5 &= 2 \times 5^3 + 2 \times 5^2 + 1 \times 5^1 + 4 \times 5^0 \\ &= 250 + 50 + 5 + 4 \\ &= \mathbf{309} \end{aligned}$$

■

B. Convert the following numbers to their binary representation:

1. 69_{10}

   Solution. Recursively divide the quotients and collect the remainders

$$\left.\begin{array}{r|l} 2\overline{)69} & 1 \\ 2\overline{)34} & 0 \\ 2\overline{)17} & 1 \\ 2\overline{)8} & 0 \\ 2\overline{)4} & 0 \\ 2\overline{)2} & 0 \\ 2\overline{)1} & 1 \end{array}\right\} \mathbf{1000101_2}$$

   Thus the binary representation of 69_{10} is $\mathbf{1000101_2}$. ■

2. 485_{10}

   Solution. Recursively divide the quotients and collect the remainders

$$\left.\begin{array}{r|l} 2\overline{)485} & 1 \\ 2\overline{)242} & 0 \\ 2\overline{)121} & 1 \\ 2\overline{)60} & 0 \\ 2\overline{)30} & 0 \\ 2\overline{)15} & 1 \\ 2\overline{)7} & 1 \\ 2\overline{)3} & 1 \\ 2\overline{)1} & 1 \end{array}\right\} \mathbf{111100101_2}$$

   Thus the binary representation of 485_{10} is $\mathbf{111100101_2}$. ■

3. $6\mathrm{D}1\mathrm{A}_{16}$

   Solution. We first obtain the binary representation of each digit

$$\begin{aligned} 6_{16} &= 0110_2 \\ \mathrm{D}_{16} &= 1101_2 \\ 1_{16} &= 0001_2 \\ \mathrm{A}_{16} &= 1010_2 \end{aligned}$$

   Thus the binary representation of $6\mathrm{D}1\mathrm{A}_{16}$ is $\mathbf{110110100011010_2}$. ■

C. Convert the following numbers to their hexadecimal representation:

1. 1101011_2

   Solution. Since we have

$$
\begin{aligned}
0110_2 &= 6_{16} \\
1011_2 &= \mathrm{B}_{16}
\end{aligned}
$$

   thus the hexadecimal representation of 1101011_2 is $\mathbf{6B_{16}}$. ■

2. 895_{10}

   Solution. Recursively divide the quotients and collect the remainders

$$
\begin{aligned}
895 \div 16 &= 55 \text{ R15} \to \mathrm{F}_{16} \\
55 \div 16 &= 3 \text{ R7} \to 7_{16} \\
3 \div 16 &= 0 \text{ R3} \to 3_{16}
\end{aligned}
$$

   Thus the hexadecimal representation of 895_{10} is $\mathbf{37F_{16}}$. ■

Question 2

Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8 =$

   Solution.

$$\begin{array}{r} {\scriptstyle 1\,1\,1} \\ 7566_8 \\ +\ 4515_8 \\ \hline 14303_8 \end{array}$$

■

2. $10110011_2 + 1101_2 =$

   Solution.

$$\begin{array}{r} {\scriptstyle 1\,1\,1\,1\,1\,1} \\ 10110011_2 \\ +\ 00001101_2 \\ \hline 11000000_2 \end{array}$$

■

3. $7A66_{16} + 45C5_{16} =$

   Solution.

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 7A66_{16} \\ +\ 45C5_{16} \\ \hline C02B_{16} \end{array}$$

■

4. $3022_5 - 2433_5 =$

   Solution.

$$\begin{array}{r} {\scriptstyle 2\,4\,1} \\ \cancel{3}\cancel{0}\cancel{2}2_5 \\ -\ 2433_5 \\ \hline 34_5 \end{array}$$

■

Question 3

A. Convert the following numbers to their 8-bit two's complement representation. Show your work.

1. 124_{10}

   Solution. We first obtain the binary representation of 124_{10}

$$\left.\begin{array}{r|l} 2\underline{)124} & 0 \\ 2\underline{)62} & 0 \\ 2\underline{)31} & 1 \\ 2\underline{)15} & 1 \\ 2\underline{)7} & 1 \\ 2\underline{)3} & 1 \\ 2\underline{)1} & 1 \end{array}\right\} \mathbf{1111100_2}$$

   Thus the 8-bit two's complement representation of 124_{10} is **01111100**. ■

2. -124_{10}

   Solution. To get the 8-bit two's complement representation of a negative number, we first "flip" the bits (swapping 0s and 1s) of its positive counterpart; the value of 1 is then added to the resulting value (ignoring the overflow). We already know that 124_{10} is represented by

$$01111100$$

   Flip the bits and we have

$$01111100 \rightarrow 10000011$$

   Finally, add 1 to the representation and we have

$$10000011_2 + 1_2 = 10000100_2$$

   Thus the 8-bit two's complement representation of -124_{10} is **10000100**. ■

3. 109_{10}

   Solution. We first obtain the binary representation of 109_{10}

$$\left.\begin{array}{r|l} 2\underline{)109} & 1 \\ 2\underline{)54} & 0 \\ 2\underline{)27} & 1 \\ 2\underline{)13} & 1 \\ 2\underline{)6} & 0 \\ 2\underline{)3} & 1 \\ 2\underline{)1} & 1 \end{array}\right\} \mathbf{1101101_2}$$

   Thus the 8-bit two's complement representation of 109_{10} is **01101101**. ■

4. -79_{10}

Solution. We first obtain the binary representation of 79_{10}

$$\left.\begin{array}{r|l} 2\underline{)79} & 1 \\ 2\underline{)39} & 1 \\ 2\underline{)19} & 1 \\ 2\underline{)9} & 1 \\ 2\underline{)4} & 0 \\ 2\underline{)2} & 0 \\ 2\underline{)1} & 1 \end{array}\right\} \mathbf{1001111_2}$$

Flip the bits and we have

$$01001111 \to 10110000$$

Finally, add 1 to the representation and we have

$$10110000_2 + 1_2 = 10110001_2$$

Thus the 8-bit two's complement representation of -79_{10} is **10110001**. ■

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{\text{8-bit 2's comp}}$

Solution. Since the leftmost bit is 0, we directly convert the binary representation to the decimal representation

$$\begin{aligned} 11110_2 &= 2^4 + 2^3 + 2^2 + 2^1 \\ &= 16 + 8 + 4 + 2 \\ &= 30 \end{aligned}$$

Thus the decimal representation of $00011110_{\text{8-bit 2's comp}}$ is **30**. ■

2. $11100110_{\text{8-bit 2's comp}}$

Solution. Since the leftmost bit is 1, we know that the number is negative. To convert a negative number into its decimal representation, we first subtract 1 from the 8-bit two's complement, flip the bits of the result, and then convert the flipped bits to its decimal representation.

$$11100110_2 - 1_2 = 11100101_2$$

Flip the bits

$$11100101 \to 00011010$$

Finally we have

$$\begin{aligned} 11010_2 &= 2^4 + 2^3 + 2^1 \\ &= 16 + 8 + 2 \\ &= 26 \end{aligned}$$

Thus the decimal representation of $11100110_{\text{8-bit 2's comp}}$ is $\mathbf{-26}$. ■

3. $00101101_{\text{8-bit 2's comp}}$

   Solution. Since the leftmost bit is 0, we directly convert the binary representation to the decimal representation

$$
\begin{aligned}
101101_2 &= 2^5 + 2^3 + 2^2 + 2^0 \\
&= 32 + 8 + 4 + 1 \\
&= 45
\end{aligned}
$$

   Thus the decimal representation of $00101101_{\text{8-bit 2's comp}}$ is **45**. ■

4. $10011110_{\text{8-bit 2's comp}}$

   Solution. Since the leftmost bit is 1, we know that the number is negative. To convert a negative number into its decimal representation, we first subtract 1 from the 8-bit two's complement, flip the bits of the result, and then convert the flipped bits to its decimal representation.

$$10011110_2 - 1_2 = 10011101_2$$

   Flip the bits

$$10011101 \to 01100010$$

   Finally we have

$$
\begin{aligned}
1100010_2 &= 2^6 + 2^5 + 2^1 \\
&= 64 + 32 + 2 \\
&= 98
\end{aligned}
$$

   Thus the decimal representation of $11100110_{\text{8-bit 2's comp}}$ is $-\mathbf{98}$. ■

Question 4

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4

   (b) *Solution.*

p	q	$\neg(p \vee q)$
T	T	F
T	F	F
F	T	F
F	F	T

■

   (c) *Solution.*

p	q	r	$r \vee (p \wedge \neg q)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

■

2. Exercise 1.3.4

   (b) *Solution.*

p	q	$(p \to q) \to (q \to p)$
T	T	T
T	F	T
F	T	F
F	F	T

■

   (d) *Solution.*

p	q	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T
T	F	T
F	T	T
F	F	T

■

Question 5

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.7

   (b) *Solution.* $(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$ ■

   (c) *Solution.* $B \vee (D \wedge M)$ ■

2. Exercise 1.3.7

   (b) *Solution.* $(s \vee v) \to p$ ■

   (c) *Solution.* $p \to y$ ■

   (d) *Solution.* $p \leftrightarrow (s \wedge y)$ ■

   (e) *Solution.* $p \to (s \vee y)$ ■

3. Exercise 1.3.9

   (c) *Solution.* $c \to p$ ■

   (d) *Solution.* $c \to p$ ■

Question 6

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.3.6

   (b) *Solution.* If Joe is eligible for the honors program, then he maintains a B average. ■

   (c) *Solution.* If Rajiv can go on the roller coaster, then he is at least four feet tall. ■

   (d) *Solution.* If Rajiv is at least four feet tall, he can go on the roller coaster. ■

2. Exercise 1.3.10 (p: T, q: False, r: unknown)

   (c) $(p \vee r) \leftrightarrow (q \wedge r)$

   Solution. False. $(p \vee r)$ is true because p is true. $(q \wedge r)$ is false because q is false. Thus the biconditional proposition is false regardless of the truth value of r. ■

   (d) $(p \wedge r) \leftrightarrow (q \wedge r)$

   Solution. Unknown. Since q is false, $(q \wedge r)$ is false. However, the truth value of $(p \wedge r)$ depends on the truth value of r. ■

   (e) $p \to (r \vee q)$

   Solution. Unknown. The hypothesis is true, but the truth value of the conclusion depends on the truth value of r. ■

   (f) $(p \wedge q) \to r$

   Solution. True. The hypothesis is false, thus the conditional proposition is true regardless of the truth value of r. ■

Question 7

Solve Exercise 1.4.5, sections b – d, from the Discrete Math zyBook:

(b)
- If Sally did not get the job, then she was late for interview or did not update her resume.
- If Sally updated her resume and was not late for her interview, then she got the job.

Solution. Logically equivalent.

- $\neg j \to (l \lor \neg r)$
- $(r \land \neg l) \to j$

j	l	r	$\neg j \to (l \lor \neg r)$	$(r \land \neg l) \to j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

■

(c)
- If Sally got the job then she was not late for her interview.
- If Sally did not get the job, then she was late for her interview.

Solution. Not logically equivalent.

- $j \to \neg l$
- $\neg j \to l$

j	l	$j \to \neg l$	$\neg j \to l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

■

(d)
- If Sally updated her resume or she was not late for her interview, then she got the job.
- If Sally got the job, then she updated her resume and was not late for her interview.

Solution. Not logically equivalent.

- $(r \vee \neg l) \to j$
- $j \to (r \wedge \neg l)$

j	l	r	$(r \vee \neg l) \to j$	$j \to (r \wedge \neg l)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

■

Question 8

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.5.2

   (c) $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

   Solution.

$$\begin{aligned}(p \to q) \land (p \to r) &\equiv (\neg p \lor q) \land (\neg p \lor r) \\ &\equiv \neg p \lor (q \land r) \\ &\equiv p \to (q \land r)\end{aligned}$$

■

   (f) $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

   Solution.

$$\begin{aligned}\neg(p \lor (\neg p \land q)) &\equiv \neg((p \lor \neg p) \land (p \lor q)) \\ &\equiv \neg(T \land (p \lor q)) \\ &\equiv \neg(p \lor q) \\ &\equiv \neg p \land \neg q\end{aligned}$$

■

   (i) $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

   Solution.

$$\begin{aligned}(p \land q) \to r &\equiv \neg(p \land q) \lor r \\ &\equiv (\neg p \lor \neg q) \lor r \\ &\equiv \neg p \lor r \lor \neg q \\ &\equiv \neg(p \land \neg r) \lor \neg q \\ &\equiv (p \land \neg r) \to \neg q\end{aligned}$$

■

2. Exercise 1.5.3

   (c) $\neg r \land (\neg r \to p)$

   Solution.

$$\begin{aligned}\neg r \land (\neg r \to p) &\equiv \neg r \lor (\neg(\neg r) \lor p) \\ &\equiv \neg r \lor (r \lor p) \\ &\equiv (\neg r \lor r) \lor p \\ &\equiv \mathrm{T} \lor p \\ &\equiv \mathrm{T}\end{aligned}$$

■

(d) $\neg(p \to q) \to \neg q$

Solution.

$$\begin{aligned}
\neg(p \to q) \to \neg q &\equiv \neg(\neg p \lor q) \to \neg q \\
&\equiv \neg\neg(\neg p \lor q) \lor \neg q \\
&\equiv \neg p \lor q \lor \neg q \\
&\equiv \neg p \lor \mathrm{T} \\
&\equiv \mathrm{T}
\end{aligned}$$

■

Question 9

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.6.3

   (c) *Solution.* $\exists x\,(x = x^2)$ ■

   (d) *Solution.* $\forall x\,(x \leq x^2)$ ■

2. Exercise 1.7.4

   (b) *Solution.* $\forall x\,(\neg S(x) \land W(x))$ ■

   (c) *Solution.* $\forall x\,(S(x) \rightarrow \neg W(x))$ ■

   (d) *Solution.* $\exists x\,(S(x) \land W(x))$ ■

Question 10

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.7.9

	$P(x)$	$Q(x)$	$R(x)$
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

(c) $\exists x\,((x = c) \to P(x))$

Solution. True. Example: a. ■

(d) $\exists x\,(Q(x) \land R(x))$

Solution. True. Example: e. ■

(e) $Q(a) \land P(d)$

Solution. True. ■

(f) $\forall x\,((x \neq b) \to Q(x))$

Solution. True. ■

(g) $\forall x\,(P(x) \lor R(x))$

Solution. False. Counterexample: c. ■

(h) $\forall x\,(R(x) \to P(x))$

Solution. True. ■

(i) $\exists x\,(Q(x) \lor R(x))$

Solution. True. Example: a. ■

2. Exercise 1.9.2

P	1	2	3
1	T	F	T
2	T	F	T
3	T	T	F

Q	1	2	3
1	F	F	F
2	T	T	T
3	T	F	F

S	1	2	3
1	F	F	F
2	F	F	F
3	F	F	F

(b) $\exists x\, \forall y\, Q(x, y)$

Solution. True. Let $x = 2$, then $Q(x, 1)$, $Q(x, 2)$, and $Q(x, 3)$ are all true. ■

(c) $\exists x\, \forall y\, P(y, x)$

Solution. True. Let $x = 1$, then $P(1, x)$, $P(2, x)$, and $P(3, x)$ are all true. ■

(d) $\exists x\, \exists y\, S(x, y)$

Solution. False. There is no pair (x, y) such that $S(x, y)$ is true. ■

(e) $\forall x\, \exists y\, Q(x, y)$

Solution. False. Let $x = 1$, then there is no y such that $Q(1, y)$ is true. ■

(f) $\forall x\, \exists y\, P(x, y)$

Solution. True. When $x = 1$, let $y = 1$. When $x = 2$, let $y = 1$. When $3 = 1$, let $y = 1$ ■

(g) $\forall x\, \forall y\, P(x, y)$

Solution. False. Counterexample: $(x, y) = (1, 2)$. ■

(h) $\exists x\, \exists y\, Q(x, y)$

Solution. True. Example: $(x, y) = (2, 1)$. ■

(i) $\forall x\, \forall y\, \neg S(x, y)$

Solution. True. ■

Question 11

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.10.4

   (c) There are two numbers whose sum is equal to their product.

   Solution. $\exists x\, \exists y\, (x + y = xy)$ ■

   (d) The ratio of every two positive numbers is also positive.

   Solution. $\forall x\, \forall y\, ((x > 0 \land y > 0) \to x/y > 0)$ ■

   (e) The reciprocal of every positive number less than one is greater than one.

   Solution. $\forall x\, ((x > 0) \land (x < 1) \to 1/x > 1)$ ■

   (f) There is no smallest number.

   Solution. $\forall x\, \exists y\, (x > y)$ ■

   (g) Every number besides 0 has a multiplicative inverse.

   Solution. $\forall x\, \exists y\, ((x \neq 0) \to xy = 1)$ ■

2. Exercise 1.10.7

   - $P(x, y)$: x knows y's phone number. (A person may or may not know their own phone number.)
   - $D(x)$: x missed the deadline.
   - $N(x)$: x is a new employee.

   (c) There is at least one new employee who missed the deadline.

   Solution. $\exists x\, (N(x) \land D(x))$ ■

   (d) Sam knows the phone number of everyone who missed the deadline.

   Solution. $\forall x\, (D(x) \to P(\text{Sam}, x))$ ■

   (e) There is a new employee who knows everyone's phone number.

   Solution. $\exists x\, \forall y\, (N(x) \land P(x, y))$ ■

   (f) Exactly one new employee missed the deadline.

   Solution. $\exists x\, \forall y\, ((N(x) \land D(x)) \land (((y \neq x) \land N(y)) \to \neg D(y)))$ ■

3. Exercise 1.10.10

   (c) Every student has taken at least one class besides Math 101.

   Solution. $\forall x\, \exists y\, ((y \neq \text{Math 101}) \wedge T(x,y))$ ■

   (d) There is a student who has taken every math class besides Math 101.

   Solution. $\exists x\, \forall y\, ((y \neq \text{Math 101}) \rightarrow T(x,y))$ ■

   (e) Everyone besides Sam has taken at least two different math classes.

   Solution. $\forall x\, \exists y\, \exists z\, ((x \neq \text{Sam}) \rightarrow ((y \neq z) \wedge T(x,y) \wedge T(x,z)))$ ■

   (f) Sam has taken exactly two math classes.

   Solution. $\exists x \exists y \forall z ((x \neq y) \wedge T(\text{Sam}, x) \wedge T(\text{Sam}, y) \wedge (((z \neq x) \wedge (z \neq y)) \rightarrow \neg T(\text{Sam}, z)))$ ■

Question 12

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.8.2

   - $P(x)$: x was given the placebo
   - $D(x)$: x was given the medication
   - $M(x)$: x had migraines

   (b) Every patient was given the medication or the placebo or both.

   Solution.

   - $\forall x\, (D(x) \lor P(x))$
   - Negation: $\neg\forall x\, (D(x) \lor P(x))$
   - Applying De Morgan's law: $\exists x\, (\neg D(x) \land \neg P(x))$
   - English: There is a patient who was not given the medication and the placebo.

   ■

   (c) There is a patient who took the medication and had migraines.

   Solution.

   - $\exists x\, (D(x) \land M(x))$
   - Negation: $\neg\exists x\, (D(x) \land M(x))$
   - Applying De Morgan's law: $\forall x\, (\neg D(x) \lor \neg M(x))$
   - English: Every patient either did not took the medication or did not have migraines (or both).

   ■

   (d) Every patient who took the placebo had migraines.

   Solution.

   - $\forall x\, (P(x) \to M(x))$
   - Negation: $\neg\forall x\, (P(x) \to M(x))$
   - Applying De Morgan's law: $\exists x\, (P(x) \land \neg M(x))$
   - English: There is a patient who took the placebo and did not have migraines.

   ■

(e) There is a patient who had migraines and was given the placebo.

Solution.

- $\exists x\,(M(x) \wedge P(x))$
- Negation: $\neg\exists x\,(M(x) \wedge P(x))$
- Applying De Morgan's law: $\forall x\,(\neg M(x) \vee \neg P(x))$
- English: Every patient either did not had migraines or was not given the placebo (or both).

■

2. Exercise 1.9.4

(c) $\exists x\,\forall y\,(P(x,y) \to Q(x,y))$

Solution.

$$\begin{aligned}\neg\exists x\,\forall y\,(P(x,y) \to Q(x,y)) &\equiv \forall x\,\exists y\,\neg(\neg P(x,y) \vee Q(x,y))\\ &\equiv \forall x\,\exists y\,(P(x,y) \wedge \neg Q(x,y))\end{aligned}$$

■

(d) $\exists x\,\forall y\,(P(x,y) \leftrightarrow P(y,x))$

Solution.

$$\begin{aligned}\neg\exists x\,\forall y\,(P(x,y) \leftrightarrow P(y,x)) &\equiv \forall x\,\exists y\,\neg((P(x,y) \to P(y,x)) \wedge ((P(y,x) \to P(x,y)))\\ &\equiv \forall x\,\exists y\,\neg((\neg P(x,y) \vee P(y,x)) \wedge (\neg P(y,x) \vee P(x,y)))\\ &\equiv \forall x\,\exists y\,(\neg(\neg P(x,y) \vee P(y,x)) \vee \neg(\neg P(y,x) \vee P(x,y)))\\ &\equiv \forall x\,\exists y\,((P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y)))\end{aligned}$$

■

(e) $\exists x\,\exists y\,P(x,y) \wedge \forall x \forall y\,Q(x,y)$

Solution.

$$\begin{aligned}\neg(\exists x\,\exists y\,P(x,y) \wedge \forall x \forall y\,Q(x,y)) &\equiv \neg\exists x\,\exists y\,P(x,y) \vee \neg\forall x \forall y\,Q(x,y)\\ &\equiv \forall x\,\forall y\,\neg P(x,y) \vee \exists x\,\exists y\,\neg Q(x,y)\end{aligned}$$

■