NYU Computer Science Bridge to Tandon Course	Winter 2021

Homework 2

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Question 5

a) Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.12.2

   (b)

$$\begin{array}{l} p \to (q \land r) \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Solution.

1.	$\neg q$	Hypothesis
2.	$\neg q \lor \neg r$	Domination law, 1
3.	$\neg(q \land r)$	De Morgan's law, 2
4.	$p \to (q \land r)$	Hypothesis
5.	$\neg p$	Modus tollens, 3, 4

■

   (e)

$$\begin{array}{l} p \lor q \\ \neg p \lor r \\ \neg q \\ \hline \therefore r \end{array}$$

Solution.

1.	$p \lor q$	Hypothesis
2.	$\neg p \lor r$	Hypothesis
3.	$q \lor r$	Resolution, 1, 2
4.	$\neg q$	Hypothesis
5.	r	Disjunctive syllogism, 3, 4

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2. Exercise 1.12.3

   (c)

$$\begin{array}{l} p \lor q \\ \neg p \\ \hline \therefore q \end{array}$$

   Solution.

1.	$p \lor q$	Hypothesis
2.	$\neg(\neg p) \lor q$	Double negation, 1
3.	$\neg p \to q$	Conditional identity, 2
4.	$\neg p$	Hypothesis
5.	q	Modus ponens, 3, 4

$\blacksquare$

3. Exercise 1.12.5

   (c)

$$\begin{array}{l} \text{I will buy a new car and a new house only if I get a job.} \\ \text{I am not going to get a job.} \\ \hline \therefore \text{I will not buy a new car.} \end{array}$$

   Solution.

   - j: I will get a job
   - c: I will buy a new car
   - h: I will buy a new house

   The form of the argument is

$$\begin{array}{l} (c \land h) \to j \\ \neg j \\ \hline \therefore \neg c \end{array}$$

   The argument is invalid. When $c = \text{T}$ and $h = j = \text{F}$, the hypotheses are both true and the conclusion is false. $\blacksquare$

   (d)

$$\begin{array}{l} \text{I will buy a new car and a new house only if I get a job.} \\ \text{I am not going to get a job.} \\ \text{I will buy a new house.} \\ \hline \therefore \text{I will not buy a new car.} \end{array}$$

   Solution.

   - j: I will get a job
   - c: I will buy a new car
   - h: I will buy a new house

   The form of the argument is

$$\begin{array}{l} (c \land h) \to j \\ \neg j \\ h \\ \hline \therefore \neg c \end{array}$$

The argument is valid.

1.	$(c \wedge h) \to j$	Hypothesis
2.	$\neg j$	Hypothesis
3.	$\neg(c \wedge h)$	Modus tollens, 1, 2
4.	$\neg c \vee \neg h$	De Morgan's law, 3
5.	h	Hypothesis
6.	$\neg c$	Disjunctive syllogism, 5, 6

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b) Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.13.3

   (b)

$$\begin{array}{l} \exists x\,(P(x) \vee Q(x)) \\ \exists x\,\neg Q(x) \\ \hline \therefore \exists x\,P(x) \end{array}$$

Solution.

	P	Q
a	F	T
b	F	F

The hypothesis $\exists x\,(P(x) \vee Q(x))$ is true when $x = a$; the hypothesis $\exists x\,\neg Q(x)$ is true when $x = b$. However, the conclusion is false. ■

2. Exercise 1.13.5

   (d)

$$\begin{array}{l} \text{Every student who missed class got a detention.} \\ \text{Penelope is a student in the class.} \\ \text{Penelope did not miss class.} \\ \hline \text{Penelope did not get a detention.} \end{array}$$

Solution.

- $M(x)$: x missed class
- $A(x)$: x received an A
- $D(x)$: x got a detention

The form of the argument is

$$\begin{array}{l} \forall x\,(M(x) \to D(x)) \\ \text{Penelope is a student in the class} \\ \neg M(\text{Penelope}) \\ \hline \therefore \neg D(\text{Penelope}) \end{array}$$

The argument is invalid. Suppose Penelope is the only student in the class, and let $M(\text{Penelope}) = \text{F}$ and $D(\text{Penelope}) = \text{T}$. Then the hypotheses are all true and the conclusion is false.

■

(e)

Every student who missed class or got a detention did not get an A.
Penelope is a student in the class.
Penelope got an A.

Penelope did not get a detention.

Solution.

- $M(x)$: x missed class
- $A(x)$: x received an A
- $D(x)$: x got a detention

The form of the argument is

$$\begin{array}{l} \forall x\,((M(x) \vee D(x)) \to \neg A(x)) \\ \text{Penelope is a student in the class} \\ A(\text{Penelope}) \\ \hline \therefore \neg D(\text{Penelope}) \end{array}$$

The argument is valid.

1.	Penelope is a student in the class	Hypothesis
2.	$\forall x\,(M(x) \vee D(x) \to \neg A(x))$	Hypothesis
3.	$M(\text{Penelope}) \vee D(\text{Penelope}) \to \neg A(\text{Penelope})$	Universal instantiation
4.	$A(\text{Penelope})$	Hypothesis
5.	$\neg(M(\text{Penelope}) \vee D(\text{Penelope}))$	Modus tollens, 3, 4
6.	$\neg M(\text{Penelope}) \wedge \neg D(\text{Penelope})$	De Morgan's law, 5
7.	$\neg D(\text{Penelope})$	Simplification, 6

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Question 6

Solve Exercise 2.2.1, sections c, d, from the Discrete Math zyBook:

(c) If x is a real number and $x \leq 3$, then $12 - 7x + x^2 \geq 0$.

Proof. Since $x \leq 3$, we have $x - 3 \leq 0$ and $x - 4 \leq 0$. Hence

$$12 - 7x + x^2 = (x-3)(x-4) \geq 0.$$

■

(d) The product of two odd integers is an odd integer.

Proof. Let x and y be two odd integers. Then $x = 2m + 1$ for some integer m, and $y = 2n + 1$ for some integer n. Thus we have

$$\begin{aligned} xy &= (2m+1)(2n+1) \\ &= 4mn + 2m + 2n + 1 \\ &= 2(2mn + m + n) + 1. \end{aligned}$$

Since m and n are an integers, $2mn + m + n$ is also an integer. As $xy = 2k + 1$, where $k = 2mn + m + n$ is an integer, xy is odd. ■

Question 7

Solve Exercise 2.3.1, sections d, f, g, l, from the Discrete Math zyBook:

(d) For every integer n, if $n^2 - 2n + 7$ is even, then n is odd.

Proof. Proof by contrapositive. We assume that n is an even integer and show that $n^2 - 2n + 7$ is an odd integer. Let n be an even integer, then $n = 2k$ for some integer k. Then We have

$$\begin{aligned} n^2 - 2n + 7 &= (2k)^2 - 2(2k) + 7 \\ &= 4k^2 - 4k + 7 \\ &= 2(2k^2 - 2k + 3) + 1. \end{aligned}$$

Since k is an integer, $2k^2 - 2k + 3$ is also an integer. As $n^2 - 2n + 7 = 2t + 1$, where $t = 2k^2 - 2k + 3$ is an integer, $n^2 - 2n + 7$ is odd. ■

(f) For every non-zero real number x, if x is irrational, then $\frac{1}{x}$ is also irrational.

Proof. Proof by contrapositive. We assume that $\frac{1}{x}$ is rational and show that x is also rational. Suppose $\frac{1}{x}$ is rational, then $\frac{1}{x} = \frac{b}{a}$ for some two non-zero integers a and b (note that x is non-zero). Then $x = \frac{a}{b}$ is also rational by definition. ■

(g) For every pair of real numbers x and y, if $x^3 + xy^2 \leq x^2y + y^3$, then $x \leq y$.

Proof. Proof by contrapositive. Suppose $x > y$, then we have

$$\begin{aligned} x^3 + xy^2 &= x(x^2 + y^2) \\ &> y(x^2 + y^2) \\ &= x^2y + y^3. \end{aligned}$$

Note that the inequality holds since $x^2 + y^2$ must be positive. ■

(l) For every pair of real numbers x and y, if $x + y > 20$, then $x > 10$ or $y > 10$.

Proof. Proof by contrapositive. Suppose $x \leq 10$ and $y \leq 10$, then we have

$$x + y \leq 10 + 10 = 20.$$

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Question 8

Solve Exercise 2.4.1, sections c, e, from the Discrete Math zyBook:

(c) The average of three real numbers is greater than or equal to at least one of the numbers.

Proof. Proof by contradiction. Let a, b, and c be three real numbers and suppose that the average of these three real numbers $\frac{a+b+c}{3}$ is less than each one of the numbers. Then we have

$$\begin{aligned} a+b+c &> \frac{a+b+c}{3} + \frac{a+b+c}{3} + \frac{a+b+c}{3} \\ &= a+b+c, \end{aligned}$$

which is clearly a contradiction. ■

(e) There is no smallest integer.

Proof. Proof by contradiction. Suppose x is the smallest integer. However, $x-1$ is also an integer and clearly $x-1 < x$. This contradicts the premise that x is the smallest integer. ■

Question 9

Solve Exercise 2.5.1, section c, from the Discrete Math zyBook:

(c) If integers x and y have the same parity, then $x + y$ is even. The parity of a number tells whether the number is odd or even. If x and y have the same parity, they are either both even or both odd.

Proof. We consider two cases:

Case 1: x and y are both even. Then $x = 2m$ and $y = 2n$ for some two integers m and n. Thus we have

$$x + y = 2m + 2n = 2(m + n),$$

which is clearly also even.

Case 2: x and y are both odd. Then $x = 2m + 1$ and $y = 2n + 1$ for some two integers m and n. Thus we have

$$x + y = (2m + 1) + (2n + 1) = 2(m + n + 1),$$

which is clearly an even number. ■