NYU Computer Science Bridge to Tandon Course Winter 2021

Homework 6

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Question 5

Use the definition of Θ in order to show the following:

a. $5n^3 + 2n^2 + 3n = \Theta(n^3)$

Solution. First we will show that $5n^3 + 2n^2 + 3n = O(n^3)$. Let $c = 10$ and $n_0 = 1$, then for any $n \geq n_0$ we have

$$\begin{aligned} 5n^3 + 2n^2 + 3n &\leq 5n^3 + 2n^3 + 3n^3 \\ &= 10n^3 \\ &= cn^3. \end{aligned}$$

Then we will show that $5n^3 + 2n^2 + 3n = \Omega(n^3)$. Let $c = 5$ and $n_0 = 1$, then for any $n \geq n_0$ we have

$$\begin{aligned} 5n^3 + 2n^2 + 3n &\geq 5n^3 \\ &= cn^3. \end{aligned}$$

Since $5n^3 + 2n^2 + 3n = O(n^3)$ and $5n^3 + 2n^2 + 3n = \Omega(n^3)$, it follows that $5n^3 + 2n^2 + 3n = \Theta(n^3)$. ∎

b. $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

Solution. First we will show that $\sqrt{7n^2 + 2n - 8} = O(n)$. Let $c = 3$ and $n_0 = 1$, then for any $n \geq n_0$ we have

$$\begin{aligned} \sqrt{7n^2 + 2n - 8} &\leq \sqrt{7n^2 + 2n} \\ &\leq \sqrt{7n^2 + 2n^2} \\ &= \sqrt{9n^2} \\ &= 3n \\ &= cn. \end{aligned}$$

Then we will show that $\sqrt{7n^2+2n-8} = \Omega(n)$. Let $c = 1$ and $n_0 = 1$, then for any $n \geq n_0$ we have

$$\begin{aligned}\sqrt{7n^2+2n-8} &\geq \sqrt{7n^2+2n-8n}\\ &= \sqrt{7n^2-6n}\\ &= \sqrt{n(7n-6)}.\end{aligned}$$

Note that $7n - 6 \geq 7n - 6n = n$ for any $n \geq 1$, thus we have

$$\begin{aligned}\sqrt{n(7n-6)} &\geq \sqrt{n \cdot n}\\ &= \sqrt{n^2}\\ &= n\\ &= cn.\end{aligned}$$

Since $\sqrt{7n^2+2n-8} = O(n)$ and $\sqrt{7n^2+2n-8} = \Omega(n)$, it follows that $\sqrt{7n^2+2n-8} = \Theta(n)$. ■