Discrete Event Simulator

Assignment 1

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October 2021

1 - Introduction:

The goal of this assignment is to study and analyze how M/M/1 and M/M/n queue models works based on Random selection and compare the results to theoretical ones.

2 - Background:

The M/M/1 Queue is the simplest stochastic process that can be used to model a queue. an M/M/1 queue represents the queue length in a system having a single server, where arrivals are determined by a Poisson process and job service times have an exponential distribution. The model name is written in Kendall's notation.
Job arrives λ and served at service rate μ. To be in equilibrium it should be λ<μ otherwise it would keep growing. An M/M/n is a multi-queue implementation of M/M/1. The distribution can be random or follow an algorithm.

3 - MM1

Considering Single-Server in 1 FIFO queue, using different arrival rate (the values that are used for these report are 0.5, 0.7, 0.9, 0.99) and assuming service rate as 1 and keep running the simulation time for 10000ms.

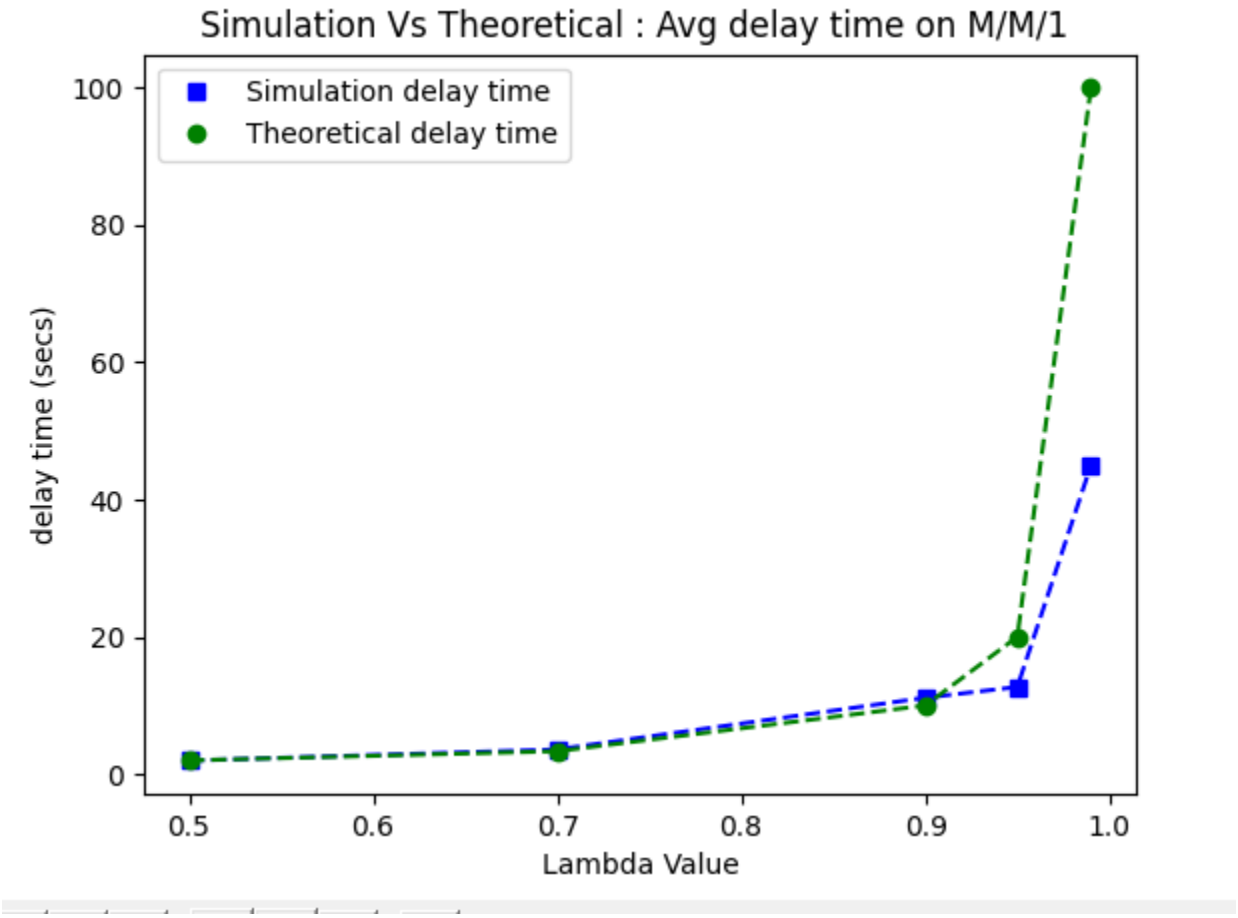
By considering the formula to define theoretically the mean time spend:

$$\frac{1}{(1-\lambda)}$$

The result has shown on the table below and by comparing the result between expected and simulated delay time, the relative error that comes it's not a lot at first since lambda was small but by increasing the value of lambda the waiting time of a job increases.

λ	Expected Average	Simulated Average	Relative Error
0.5	2	1.96	0.02
0.7	3.33	3.59	0.07
0.9	10.0	11.13	0.1
0.95	20	12.7	0.57

0.99	100	44.98	1.22

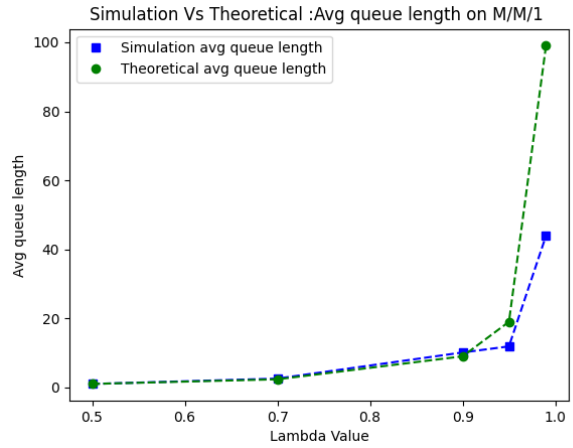


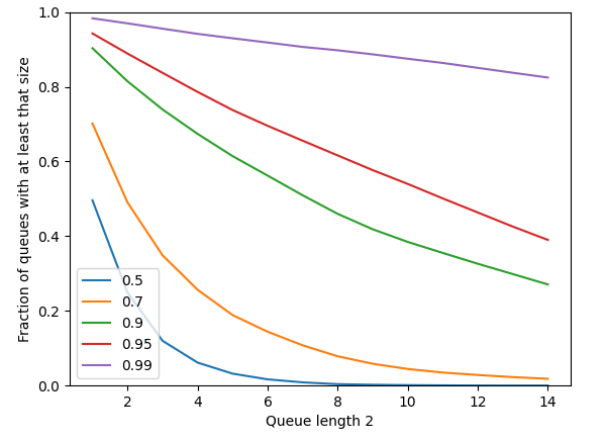
And by considering the formula for mean queue length

$$\frac{\lambda}{(1-\lambda)}$$

with increasing the lambda, the error grows.

λ	Expected Queue Length	Simulated Queue Length	Relative Error
0.5	1	0.99	0.01
0.7	2.33	2.55	0.09
0.9	9.0	10.12	0.11
0.95	19.0	11.87	0.6
0.99	99.0	43.93	1.25

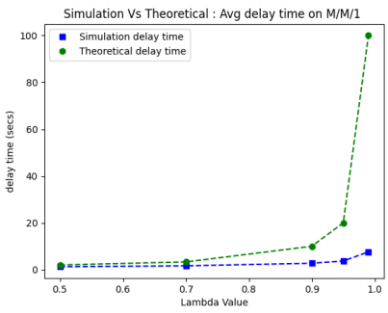


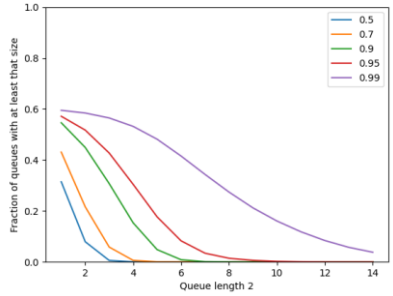


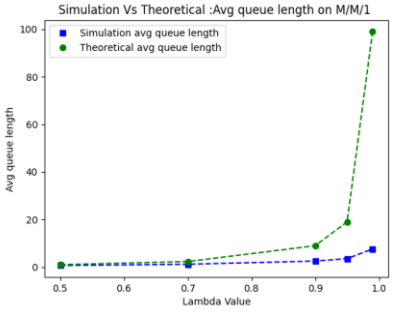
4- MMN – supermarket model (FIFO)

by assuming 30 servers and using supermarket model we have the possibility to select more than one queue and choose the shortest between them, it helps us to distribute better the jobs among the queues.

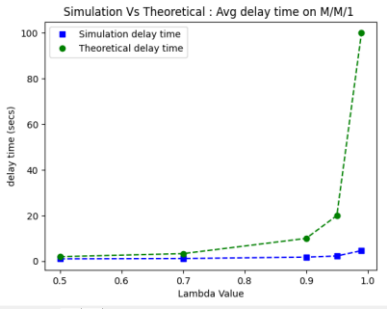
Choices	λ	Expected Average	Simulated Average
2	0.5	2	1.27
2	0.7	3.33	1.64
2	0.9	10.0	2.75
2	0.95	20.0	3.72
2	0.99	100.0	7.61

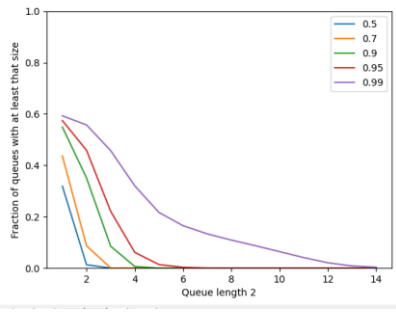


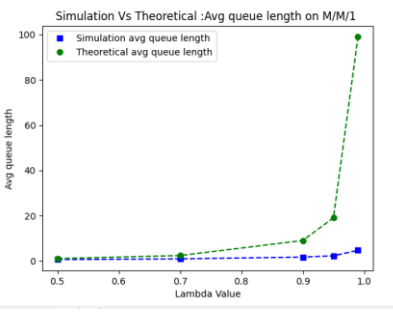




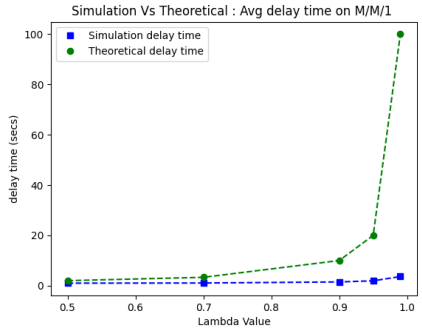
Choices	λ	Expected Average	Simulated Average
5	0.5	2.0	1.04
5	0.7	3.33	1.2
5	0.9	10.0	1.79
5	0.95	20.0	2.3
5	0.99	100.0	4.64

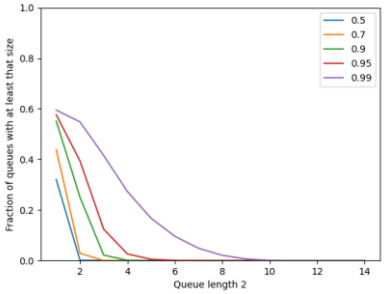


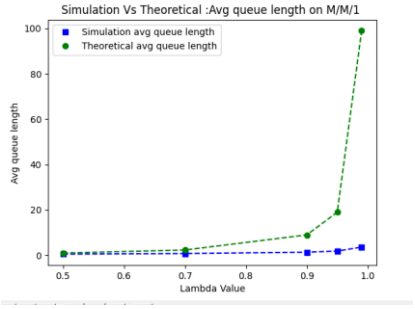




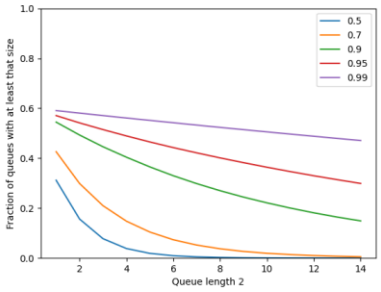
Choices	λ	Expected Average	Simulated Average
10	0.5	2.0	1.0
10	0.7	3.33	1.07
10	0.9	10.0	1.49
10	0.95	20.0	1.94
10	0.99	100.0	26.52

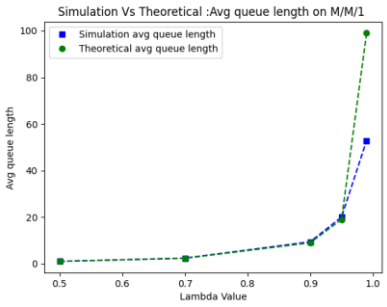


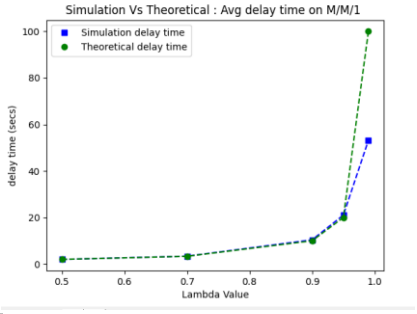




In random selection by consider the same amount of servers and same lambdas results are as following:







λ	Expected Average	Simulated Average
0.5	2.0	2.0
0.7	3.33	3.38
0.9	10.0	10.41
0.95	20.0	21.04
0.99	100.0	53.13

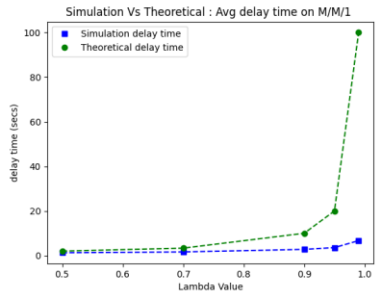
By comparing the results of random selection and supermarket model we can see that the results are decreasing significantly. In overall it's obvious that random selection performs not good and it proves that supermarket model is much better strategy than random selection.

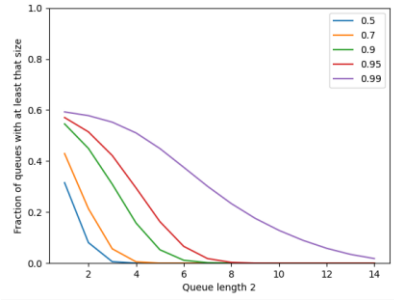
MMN – supermarket model (SJF)

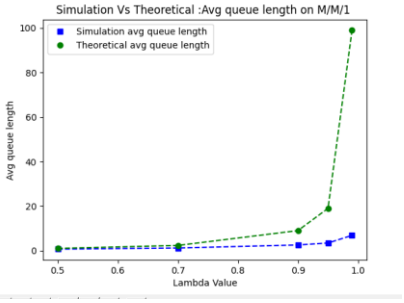
In this approach, d is chosen from the queues, the job is assigned to the shortest queue, and the jobs are further sorted in the queue based on the job size which we get from the duration of each job (difference between arrival time and service time).

Assumption of servers and d values are the same as the one we had in FIFO approach, 30 servers and d value of 2,5,10.

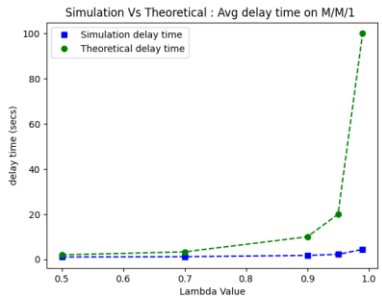
Choices	λ	Expected Average	Simulated Average
2	0.5	2	1.27
2	0.7	3.33	1.63
2	0.9	10.0	2.78
2	0.95	20.0	3.58
2	0.99	100.0	6.7

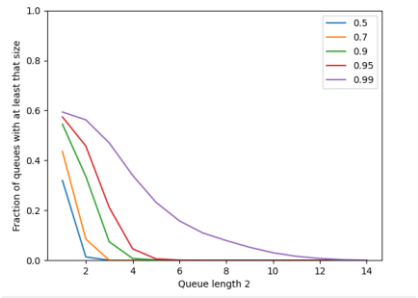


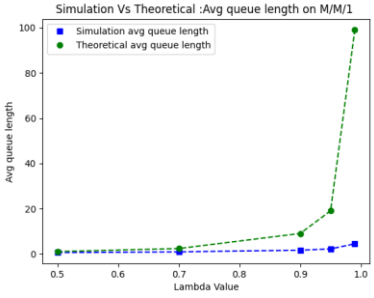




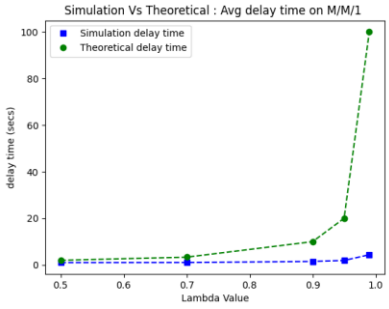
Choices	λ	Expected Average	Simulated Average
5	0.5	2.0	1.04
5	0.7	3.33	1.19
5	0.9	10.0	1.75
5	0.95	20.0	2.24
5	0.99	100.0	4.43

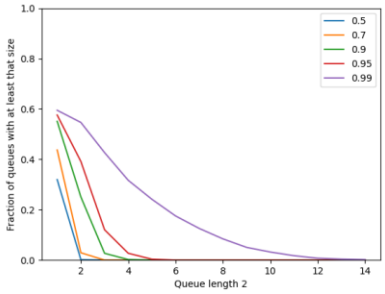


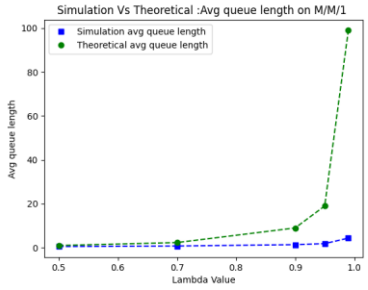




Choices	λ	Expected Average	Simulated Average
10	0.5	2.0	1.0
10	0.7	3.33	1.06
10	0.9	10.0	1.5
10	0.95	20.0	1.92
10	0.99	100.0	4.38







By observing the result we can see that the result of this approach is quite similar to the results we obtained of FIFO approach. using SJF It's useful when the job time known already.