ПЗ 12

a) $X_n(t) = \dfrac{2nt}{1+n^2t^2}$, $X(t) = 0$, $[a;b] = [0;2]$

$$\rho_{C[0,2]}(X_n, 0) = \max_{0 \le t \le 2}\left(\dfrac{2nt}{1+n^2t^2}\right) \qquad \varphi(t) = \dfrac{2nt}{1+n^2t^2}$$

$$\varphi'(t) = \dfrac{2n-2n^3t^2}{(1+n^2t^2)^2} \qquad \varphi'=0 \Longleftrightarrow 2n-2n^3t^2 = 0$$

$$2n(1-nt^2) = 0$$

$\varphi(0) = 0$

$\varphi(2) = \dfrac{4n}{1+4n^2}$

$n \overset{?}{=} 0$ или $1-nt^2 = 0$

$n_0 = 0$ $\qquad nt^2 = 1$

$$t^2 = \dfrac{1}{n}$$

$\varphi(t_0) > \varphi(2)$?

$$t_0 = \dfrac{1}{\sqrt{n}}$$

$$\lim_{n \to \infty}\dfrac{4n}{1+4n^2} = 0 \qquad \lim_{n \to \infty}\dfrac{2n}{\sqrt{n}(1+n)} = 0 \qquad \varphi(t_0) = \dfrac{\dfrac{2n}{\sqrt{n}}}{1+\dfrac{n^2}{n}} =$$

порядок=2 $\qquad$ порядок = 1,5

$$= \dfrac{2n}{\sqrt{n}+n\sqrt{n}}$$

$\varphi(t_0)$ убывает медленнее $\Rightarrow \rho_{C[0;2]}(X_n, 0) =$

$$= \varphi(t_0) \xrightarrow[n \to \infty]{} 0 \Rightarrow \text{равномерн.}$$
$$\text{сходимость}$$

есть и
$\Rightarrow$ среднеквадратич. сходимость

б) $X_n(t) = \sqrt{t^2 + \dfrac{1}{n^2}}$, $X(t) = |t|$, $[a;b] = \left[-\dfrac{3}{2}; \dfrac{3}{2}\right]$.

$$\rho_{C\left[\frac{3}{2},\frac{3}{2}\right]}(X_n, |t|) = \max_{-\frac{3}{2} \le t \le \frac{3}{2}}\left|\sqrt{t^2 + \dfrac{1}{n^2}} - |t|\right| \ge 0 \Rightarrow \rho = \max_{-\frac{3}{2} \le t \le \frac{3}{2}}\left(\sqrt{t^2 + \dfrac{1}{n^2}} - |t|\right)$$

$$\varphi'(t) = \dfrac{nt}{\sqrt{n^2t^2+1}} - \dfrac{t}{|t|} \qquad \varphi'(t) = 0 \Longleftrightarrow \dfrac{nt}{\sqrt{n^2t^2+1}} - \dfrac{t}{|t|} = 0$$

$$\frac{|t|nt - t\sqrt{n^2t^2+1}}{|t|\sqrt{n^2t^2+1}} = 0 \qquad t \neq 0$$

$$|t|nt - t\sqrt{n^2t^2+1} = 0$$

$$t\left(n|t| - \sqrt{n^2t^2+1}\right) = 0 \qquad |:t \quad t\neq 0$$

$$n|t| - \sqrt{n^2t^2+1} = 0$$

$$n^2t^2 = n^2t^2+1$$

$$0 \neq 1 \implies \varphi'(t) \neq 0$$

$$\varphi\left(-\tfrac{3}{2}\right) = \sqrt{\tfrac{9}{4}+\tfrac{1}{n^2}} - \tfrac{3}{2}$$

$$\varphi\left(\tfrac{3}{2}\right) = \sqrt{\tfrac{9}{4}+\tfrac{1}{n^2}} - \tfrac{3}{2} \quad \Big\} \text{ равны} \implies \varphi(1) = \varphi(-1) \text{ и т.д.}$$

$$\varphi(0) = \frac{1}{n}$$

$$\varphi(1) = \sqrt{1+\tfrac{1}{n^2}} - 1$$

$$\lim_{n\to\infty}\left(\varphi(0)\right) = 0 \qquad \lim_{n\to\infty}\left(\varphi\left(-\tfrac{3}{2}\right)\right) = 0$$

~~$\varphi(0) < \varphi(1) < \varphi$~~ $\qquad \varphi\left(\tfrac{3}{2}\right) < \varphi(1) < \varphi(0) \quad (n=1)$

$\varphi(0)$ убывает медленнее $\implies \rho_{C[-\frac{3}{2};\frac{3}{2}]}(x_n, |t|) = \varphi(0)$

$\varphi(0) \xrightarrow[n\to\infty]{} 0 \implies \varphi(0) = \frac{1}{n}$ — расходящ. гармонич. ряд, нет равномерной сходимости

Сходимость в $L^2\left[-\tfrac{3}{2};\tfrac{3}{2}\right]$: $\quad \rho_{L^2[-\frac{3}{2};\frac{3}{2}]}(x_n, |t|) =$

$$= \sqrt{\int_{-\frac{3}{2}}^{\frac{3}{2}}\left(t^2+\frac{1}{n^2}-|t|\right)dt} = \sqrt{\left(\frac{nt\sqrt{1+n^2t^2}+\ln\left|\sqrt{1+n^2t^2}+nt\right|}{2n^2}\mp\frac{t^2}{2}\right)\Bigg|_{-\frac{3}{2}}^{\frac{3}{2}}} =$$

$$= \sqrt{\mp\frac{9}{8}+\frac{1}{2n^2}\left(\frac{3}{2}n\sqrt{1+\frac{9}{4}n^2}+\ln\left|\sqrt{1+\frac{9}{4}n^2}+\frac{3}{2}n\right|\right)\mp\frac{9}{8}-\frac{1}{2n^2}} \cdot$$

$$\cdot \left(-\frac{3}{2}n\sqrt{1+\frac{9}{4}n^2}+\ln\left|\sqrt{1+\frac{9}{4}n^2}-\frac{3}{2}n\right|\right)^{'} =$$

$$= \frac{1}{\sqrt{2}\,n}\cdot\sqrt{\frac{3}{2}n\left(\sqrt{1+\frac{9}{4}n^2}+\sqrt{1+\frac{9}{4}n^2}\right)+\ln\left|\frac{\sqrt{1+\frac{9}{4}n^2}+\frac{3}{2}n}{\sqrt{1+\frac{9}{4}n^2}-\frac{3}{2}n}\right|} =$$

$$= \sqrt{\frac{3}{2n}}\cdot\frac{1}{\sqrt{2}\,n}\cdot\sqrt{3\sqrt{1+\frac{9}{4}n^2}+\ln\left|\frac{\sqrt{1+\frac{9}{4}n^2}+\frac{3}{2}n}{\sqrt{1+\frac{9}{4}n^2}-\frac{3}{2}n}\right|}$$

$$\rho_{(n\to\infty)} \sim \frac{1}{n}\sqrt{3\sqrt{2}\,n+\ln\left|\frac{\sqrt{2}\,n+1,5n}{\sqrt{2}\,n-1,5n}\right|} =$$

$$= \frac{\sqrt{3\sqrt{2}\,n+\ln\left|\frac{\sqrt{2}+1,5}{\sqrt{2}-1,5}\right|}}{n} \sim \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}} - \text{расход.}$$
$$\text{ряд} \Rightarrow$$

$$\Rightarrow \text{ нет среднеквадратичной сходимости}$$