ПЗ 24

$$X = \frac{3e_5 + 3e_6 - e_7 - e_8}{2}, \quad y = \frac{e_4 + e_6}{3}$$

$$X - y = -\frac{e_4}{3} + \frac{3e_5}{2} + \left(\frac{3}{2} - \frac{1}{3}\right)e_6 - \frac{e_7}{2} - \frac{e_8}{2}$$

$$\rho(x,y) = \|X - y\| = \sqrt{(x-y, x-y)}, \quad \rho(x,y) = \sqrt{\frac{1}{9} + \frac{9}{4} + \left(\frac{7}{6}\right)^2 + \frac{1}{2}} =$$

$$= \sqrt{\frac{1}{9} + \frac{11}{4} + \frac{49}{36}} = \sqrt{\frac{4 + 99 + 49}{36}} = \sqrt{\frac{152}{36}} = \sqrt{\frac{38}{9}} = \frac{\sqrt{38}}{3}$$

Ответ: $\frac{\sqrt{38}}{3}$.

ПЗ 25

$x = -2e_{16} + 4\left(e_3 + \frac{e_5}{2}\right) - e_1, \quad y = \frac{e_1}{2} + e_2 - 3e_{16}, \quad \varphi = \angle(x, y)$

$\cos\varphi = \frac{(x,y)}{\|x\|\,\|y\|} \qquad (x,y) = (-2)\cdot(-3) + (-1)\cdot\frac{1}{2} = 6 - \frac{1}{2} = 5,5$

$\|x\| = \sqrt{4+16+4+1} = 5$

$\|y\| = \sqrt{\frac{1}{4}+1+9} = \sqrt{10\frac{1}{4}} = \sqrt{\frac{41}{4}} = \frac{\sqrt{41}}{2}$

$\cos\varphi = \frac{5\frac{1}{2}}{5\cdot\frac{\sqrt{41}}{2}} = \frac{2\cdot\frac{11}{2}}{5\cdot\sqrt{41}} = \frac{11}{5\sqrt{41}} = \frac{11\sqrt{41}}{205}$

Ответ: $\varphi = \arccos\left(\frac{11\sqrt{41}}{205}\right)$.

ПЗ 26

$x(t) = \cos^2 t,\quad y(t) = \sin 3t + \cos 3t,\quad$ 1) $(a;b) = (-\pi;\pi)$,

2) $(c;d) = (0;\pi)$.

$$(x,y) = \int_a^b x(t)\,y(t)\,dt$$

1) $$\int_{-\pi}^{\pi} \cos^2 t(\sin 3t + \cos 3t)\,dt = \int_{-\pi}^{\pi} \cos^2 t \sin 3t\,dt + \int_{-\pi}^{\pi} \cos^2 t \cos 3t\,dt =$$

$$= 0 + 2\int_0^{\pi} \cos^2 t \cos 3t\,dt = 2\left(\left(\cos^2 t \cdot \frac{\sin 3t}{3}\right)\Big|_0^{\pi} + \int_0^{\pi} \frac{\sin 3t}{3} \cdot 2\cos t \sin t\,dt\right) =$$

$$= \frac{2}{3}(\cos^2 t \sin 3t)\Big|_0^{\pi} + \frac{4}{3}\int_0^{\pi} \sin 3t \cos t \sin t\,dt = \frac{2}{3}(\cos^2 t \sin 3t)\Big|_0^{\pi} +$$

$$+ \frac{2}{3}\int_0^{\pi} \sin t(\sin 4t + \sin 2t)\,dt = \frac{2}{3}(\cos^2 t \sin 3t)\Big|_0^{\pi} + \frac{\cancel{2}}{3}\int_0^{\pi} \left(\frac{\cos 3t - \cos 5t + \cos t - \cos 3t}{\cancel{2}}\right)dt = \frac{2}{3}(\cos^2 t \sin 3t)\Big|_0^{\pi} + \frac{1}{3}\int_0^{\pi}(\cos t - \cos 5t)\,dt =$$

$$= \frac{2}{3}(\cos^2 t \sin 3t)\Big|_0^{\pi} + \frac{1}{3}\left(\sin t - \frac{\sin 5t}{5}\right)\Big|_0^{\pi} = \left(\frac{2}{3}\cos^2 t \sin 3t - \frac{\sin 5t}{15} + \frac{\sin t}{3}\right)\Big|_0^{\pi} = \frac{2\cos^2\pi \sin 3\pi}{3} - \frac{\sin 5\pi}{15} + \frac{\sin\pi}{3} - \frac{2\cos^2 0 \sin 0}{3} +$$

$$+ \frac{\sin 0}{15} - \frac{\sin 0}{3} = 0. \Rightarrow$$ X и Y ортогональны на $(a;b)$

2) $$\int_0^{\pi} \cos^2 t \sin 3t\,dt + \underbrace{\int_0^{\pi} \cos^2 t \cos 3t\,dt}_{=0} = \int_0^{\pi} \cos^2 t \sin 3t\,dt =$$

$$= \frac{1}{2}\int_0^{\pi}(\cos 2t + 1)\sin 3t\,dt = \frac{1}{2}\left(\int_0^{\pi} \cos 2t \sin 3t\,dt + \int_0^{\pi} \sin 3t\,dt\right) =$$

$$= \frac{1}{4}\int_0^{\pi}(\sin 5t + \sin t)\,dt - \frac{\cos 3t}{3}\Big|_0^{\pi} = -\left(\frac{\cos 5t}{20} + \frac{\cos 3t}{6} + \frac{\cos t}{4}\right)\Big|_0^{\pi} =$$

$$= \frac{\cos 0}{20} + \frac{\cos 0}{6} + \frac{\cos 0}{4} - \frac{\cos\pi}{20} - \frac{\cos\pi}{6} - \frac{\cos\pi}{4} = 2\left(\frac{1}{20} + \frac{1}{6} + \frac{1}{4}\right) = \frac{14}{15}.$$

X и Y не ортогон. на $(c;d)$