ПЗ 17

$$\frac{x'}{\cos t} - x = \cos^2 t \quad , x(\pi) = -1$$

Преобразование к интегральному уравнению Вольтерры:

$$x' - x\cos t = \cos^3 t \longrightarrow x' = x\cos t + \cos^3 t$$

$$x' - x\cos t = 0$$

$$x' = x\cos t$$

B. $\begin{cases} x(a) = x_a \\ x' = p(t)\cdot x + q(t) \end{cases} \Rightarrow p(t) = \cos t;\ q(t) = \cos^3 t$

$$x(t) = -1 + \int_\pi^t \cos(s)x(s)\,ds + \int_\pi^t \cos^3(s)\,ds =$$

$$= \int_\pi^t \cos(s)x(s)\,ds + \left(\sin s - \frac{\sin^3 s}{3}\right)\Bigg|_\pi^t - 1 =$$

$$= \int_\pi^t \cos(s)x(s)\,ds + \sin t - \frac{\sin^3 t}{3} - 1$$

Метод простых итераций = grigoryev_17.m

A. $\frac{dx}{dt} = x\cos t$

$$\int \frac{dx}{x} = \int \cos t\,dt$$

$$x_g = C_1 e^{\sin t} \qquad C_1 = C_1(x)$$

$$x' = e^{\sin t}C_1' + e^{\sin t}\cos t \cdot C_1$$

$$\frac{C_1' e^{\sin t} + C_1 \cos t\, e^{\sin t}}{\cos t} - \cancel{C_1 e^{\sin t}} = \cos^2 t$$

$$C_1' e^{\sin t} = \cos^3 t$$

$$C_1' = \frac{\cos^3 t}{e^{\sin t}}$$

$$\int dC_1 = \int \frac{\cos^3 t\,dt}{e^{\sin t}}$$

$$C_1 = \frac{\sin^2 t + 2\sin t + 1}{e^{\sin t}} + C$$

$$x_{o} = \sin^2 t + 2\sin t + 1 + Ce^{\sin t}$$

$$x(\pi) = 1 + Ce^0 = 1 + C = -1 \Rightarrow C = -2$$

$$x_r = \sin^2 t + 2\sin t + 1 - 2e^{\sin t}$$ ← A. Точное решение