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$F[x] = \int_0^1 (s+x'')^2 (s^2+x)\,ds$, $k=4$ (множ-во четырежды непрерыв. дифф. функций)

$\eta = \eta(s) \in D_F$, $\eta(0)=\eta(1)=\eta'(0)=\eta'(1)=\eta''(0)=\eta''(1)=\eta'''(0)=\eta'''(1)=0$.

$x+t\eta \in D_F$; $t \in R$

$F[x+t\eta] = \int_0^1 (s+x''+t\eta'')^2 (s^2+x+t\eta)\,ds$

$$\left.\frac{dF[x+t\eta]}{dt}\right|_{t=0} = \left(\int_0^1 \left[2(s+x''+t\eta'')(s^2+x+t\eta)\eta'' + (s+x''+t\eta'')^2 \cdot \eta\right] ds \right)\Big|_{t=0} = 2\int_0^1 (s+x'')(s^2+x)\eta''\,ds + \int_0^1 (s+x'')^2 \eta\,ds =$$

$$= \underbrace{2(s+x'')(s^2+x)\eta'\Big|_0^1}_{=0} - 2\int_0^1 \left(x'''(s^2+x)+(s+x''')(2s+x')\right)\eta'\,ds +$$

$$+ \int_0^1 (s+x'')^2 \eta\,ds = \underbrace{-2\eta\left(x'''(s^2+x)+(s+x''')(2s+x')\right)\Big|_0^1}_{=0} + \int_0^1 (s+x'')^2 \eta\,ds +$$

$$+ 2\int_0^1 \left(x^{IV}(s^2+x)+x'''(2s+x')+x'''(2s+x')+(s+x'')(2+x'')\right)\eta\,ds =$$

$$= \int_0^1 2\left(x^{IV}(s^2+x)+2x'''(2s+x')+(x''+s)(2+2x''+s)\right)\eta\,ds = \left(F'[x], \eta\right)$$

градиент: $F'[x] = 2x^{IV}(x+s^2)+4x'''(x'+2s)+2(x''+s)(2x''+s+2)$

$F'[0] = 2s(s+2) = 2s^2+4s$; $F[0] = \int_0^1 s^4\,ds$

линейная часть приращ.: $F[x]-F[0] = (F'[0], x) + o(\|x\|)$ при $\|x\| \to 0$

$$\int_0^1 \left((s^2+2sx''+x''^2)(s^2+x)-s^4\right) ds = \int_0^1 (2s^2+4s)x\,ds + o(\|x\|)$$

$$\int_0^1 \left(s^2x+2s^3x''+2sxx''+s^2x''^2+xx''^2\right) ds = 2\int_0^1 s(s+2)x\,ds + o(\|x\|).$$