Signals and systems
Homework #5

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Spring 97-98
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Deadline : 3 Khordad, 1398 [23:55]

- Homeworks will not be accepted after the deadline.
- For theoretical problems, gather them in a single ***.pdf** file.
- For the matlab problems, provide both these materials:
    - **codes [*.m files]**
    - a simple **report** that includes all plots and screenshots.
- Notice that the homeworks will be **checked by plagiarism detectors**, avoid any similarities.
- Matlab problems and theoretical problems will be graded separately (both will be graded out of 100), but their weights may be different and is determined by the course professor.

Question 1 (12 points)

Imagine that x(t) is a signal with nyquest rate of W_0, provide the nyquest rate for the following signals:

(a) $x(t) + x(t-1)$

(b) $\frac{dx(t)}{dt}$

(c) $x^2(t)$

(d) $x(t)\cos(\omega_0 t)$

Question 2 (15 points)

Indicate which of the following signals can be sampled without any loss of information? For signals that can be sampled properly, determine the minimum sampling rate that can be used.

(a) $x_a(t) = u(t) - u(t-3)$

(b) $x_a(t) = e^{-2t}u(t)$

(c) $x_a(t) = \cos(100\pi t) + 2\sin(150\pi t)$

(d) $x_a(t) = \cos(100\pi t) + 2\sin(150\pi t)\sin(200\pi t)$

(e) $x_a(t) = e^{-t}\cos(100\pi t)$

Question 3 (15 points)

A sinusoidal signal $x_a(t) = \sin(2\pi f_a t)$ with a frequency of $f_a = 1$ kHz is sampled using a sampling rate of $f_s = 2.4$ kHz to obtain a discrete-time signal $x[n]$.

(a) Manually sketch the signal $x_a(t)$ for the time interval $0 < t < 5$ ms.

(b) Show the sample amplitudes of the discrete-time signal $x[n]$ on the sketch of $x_a(t)$.

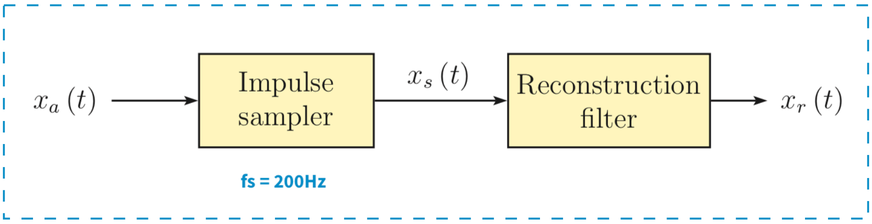
(c) Find three alternative frequencies for the analog signal that result in the same discrete- time signal $x[n]$ when sampled with the sampling rate $f_s = 2.4$ kHz.

(d) Sketch the frequency spectrum of the analog signal for the original sinusoid and each of the three alternative frequencies.

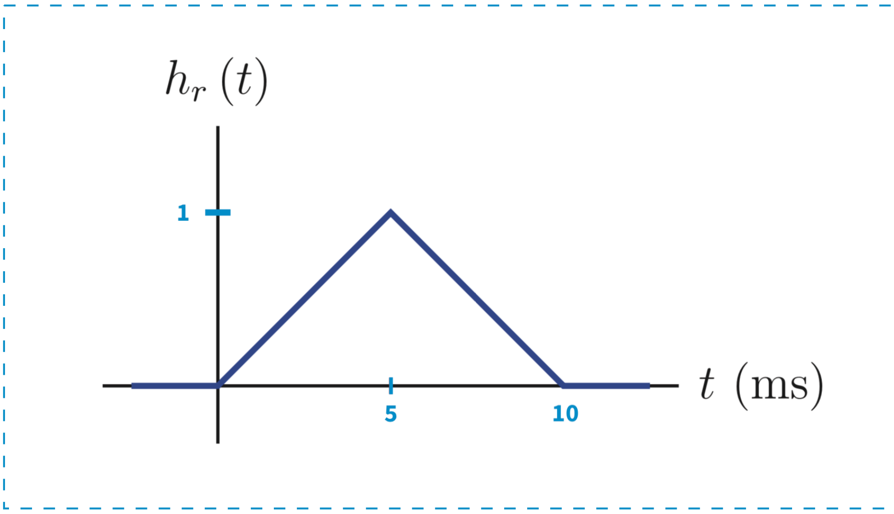
(e) For each of the signals and corresponding spectra in part (a), determine the DTFT spectrum of the discrete-time signal that results from sampling the analog signal with a sampling rate of $f_s = 2.4$ kHz.

Question 4 (15 points)

The signal $x_a(t) = \cos(150\pi t)$ is impulse-sampled with a sampling rate of $f_s = 200$ Hz and applied to a zero-order hold reconstruction filter as shown below



(a) Sketch the signal at the output of the reconstruction filter.
(b) Repeat the problem the reconstruction filter is a delayed first-order hold filter with the impulse response shown below



Question 5 (10 points)

Consider the following signals:

$$x(t) = \sin(200\pi t) + 2\sin(300\pi t)$$

$$g(t) = x(t)\sin(300\pi t)$$

if we pass the $g(t)\sin(300\pi t)$ from and ideal low-pass filter with stop frequency of 300π and gain of 2, determine the output signal.

Question 6 (15 points)

A message $m(t)$ with a bandwidth of $B = 2$ kHz modulates a cosine carrier of frequency 10 kHz to obtain a modulated signal $s(t) = m(t) \cos(20 \times 10^3 \pi t)$.

(a) What is the maximum frequency of $s(t)$? What would be the values of the sampling frequency f_s in Hz, according to the Nyquist sampling condition, that could be used to sample $s(t)$?

(b) Assume the spectrum of $m(t)$ is a triangle, with maximum amplitude 1, carefully plot the spectrum of $s(t)$. Would it be possible to sample $s(t)$ with a sampling frequency $f_s = 10$ kHz and recover the original message? Obtain the spectrum of the sampled signal and show how you would recover $m(t)$, if possible.

Question 7 (18 points)

Determine the discrete Fourier series for the following periodic signals

(a) $x[n] = \{\ldots, 3, 4, \underset{\substack{\uparrow \\ n=0}}{0}, 1, 2, 3, 4, 0, 1, 2, 3, 4, \ldots\}$

(b) $x[n] = 1 + \cos\left(\frac{n\pi}{2}\right) + \sin\left(n\pi\right)$

(c) $x[n] = \cos\left(\frac{\mathrm{n}\pi}{3}\right)\sin\left(\frac{2\pi}{3}n\right)$

Question 8 (15 points)

Fourier series coefficients of x[n] are represented by a_k, determine the Fourier series coefficients for the following signals:

(a) $y\left[n\right] = \begin{cases} x\left[\frac{n}{2}\right] & n\, is\, even \\ 0 & otherwise \end{cases}$

(b) $y\left[n\right] = x^*\left[-n+1\right]$

(c) $y\left[n\right] = \left(-1\right)^n x\left[n\right]$

Question 9 (10 points)

Consider applying $x\left[n\right] = \sum_{k=-\infty}^{+\infty} \delta\left(n-8k\right)$ to a system with frequency response of $H\left(j\omega\right)$. Determine the output.