

تہذیب اول درس آمار
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$$P(\text{سفید بودن مهر}) = \frac{3}{5}$$
$$\rho \text{ (لغفید بودن مهر)} = \frac{V}{15}$$

$$P(B|A) = ?$$
$$P = \frac{1}{R} \times \frac{1}{r} + \frac{1}{R} \times \frac{1}{r} = \frac{1}{R}$$

$g \rightarrow \text{gold}$

Diagram illustrating the formation of a virtual image by a convex lens. The object is placed between the lens and its focal point. The image formed is virtual, upright, and magnified.

$$b \begin{cases} 10 & \frac{3}{5} \\ 2 & \frac{2}{5} \end{cases}$$

$$10 \times \frac{3}{5} + 2 \times \frac{2}{5} = 4,8$$

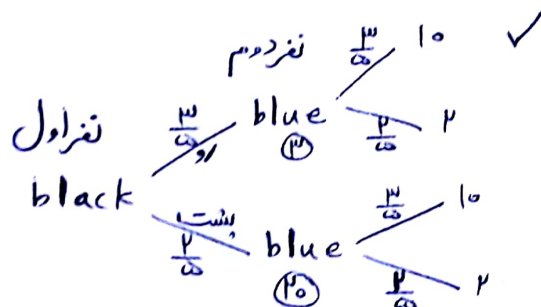
$$r = \begin{cases} 4 & \frac{3}{5} \\ 4 & \frac{2}{5} \end{cases}$$

$$4 \times \frac{3}{5} + 4 \times \frac{2}{5} = 4$$

$$bl \begin{cases} 3 & \frac{2}{5} \\ 20 & \frac{2}{5} \end{cases} \quad (4)$$

$$3 \times \frac{3}{5} + 20 \times \frac{2}{5} = 9,8$$

نفر اول سیاه را انتخاب می کند و نفر دوم ~~آبی~~ را زیرا به طور میانگین سکه سیاه امتیاز بیشتری نسبت به آبی و آبی امتیاز بیشتری نسبت به قرمز دارد. (مرد نفر بهترین انتخاب را دارند)



$$p(\text{برنده شدن نفر اول}) = \frac{3}{5} \times \frac{2}{5} + \frac{2}{5} \times (\frac{3}{5} + \frac{2}{5})$$

$$= \frac{12}{25}$$

$$p(\text{برنده شدن نفر دوم}) = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$$

ترجیح می دهیم نفر اول با سهم تا نفر دوم

$$P(\text{spam}) = 0,7 \quad P(\sim \text{spam}) = 0,3 \quad P(X) = 0,7 \quad P(X') = 0,3 \quad (5)$$

(الف)

$$P(X | \text{spam}) = \frac{P(X \cap \text{spam})}{P(\text{spam})} = \frac{0,4}{0,7} = \frac{4}{7}$$

$$P(X' | \text{spam}) = \frac{1}{7}$$

$$P(X | \text{spam}') = \frac{P(X \cap \text{spam}')}{P(\text{spam}')} = \frac{0,1}{0,3} = \frac{1}{3}$$

$$P(X' | \text{spam}') = \frac{2}{3}$$

data	spam probability
11	$\frac{6}{7}$
12	$\frac{6}{7}$
13	$\frac{1}{3}$
14	$\frac{1}{3}$
15	$\frac{6}{7}$
→ $\frac{6}{7}$	

$$P(\text{spam} | X) = \frac{P(\text{spam}) \cdot P(X | \text{spam})}{P(X)}$$

$$P(\text{spam} | X, Y') = \frac{P(X, Y' | \text{spam}) \times P(\text{spam})}{P(X, Y')}$$

(ب)

$$= \frac{P(X | \text{spam}) \times P(Y' | \text{spam}) \times P(\text{spam})}{P(X, Y')}$$

در صورت مستقل بودن X و Y' از هم.

$$= \frac{1}{P(X, Y')} \times P(X | \text{spam}) \times P(Y' | \text{spam}) \times P(\text{spam})$$

$$= \frac{1}{P(X, Y')} \times \frac{4}{V} \times \frac{5}{V} \times \frac{V}{10} = \frac{2}{V} \times \frac{1}{P(X, Y')} = A$$

$\Rightarrow A > B$

$$P(\text{spam}' | X, Y') = \frac{1}{10} \times \frac{1}{P(X, Y')} = B$$

$\checkmark \approx 2, 12$

$$P(\text{spam} | X, Y) = \frac{4}{100} \times \frac{1}{P(X, Y)} = A$$

$$P(\text{spam}' | X, Y) = \frac{1}{10} \times \frac{1}{P(X, Y)} = B$$

$\Rightarrow A > B$

$\checkmark \approx 4, 12$

$$P(\text{spam} | X', Y) = \frac{1}{100} \times \frac{1}{P(X', Y)} = A$$

$$P(\text{spam}' | X', Y) = \frac{1}{10} \times \frac{1}{P(X', Y)} = B$$

$\Rightarrow B > A$

$\checkmark \approx 4, 14$

$$P(\text{spam} | X', Y') = \frac{1}{12} \times \frac{1}{P(X', Y')} = A$$

$$P(\text{spam}' | X', Y') = \frac{1}{10} \times \frac{1}{P(X', Y')} = B$$

$\Rightarrow A > B$

$\checkmark \approx 7, 12$

data	F	D	spam probability	
11	1	0	spam	0,9P
12	1	0	"	0,9P 0,9P
13	0	1	not spam	0,18P
14	0	1	"	0,18P 0,18P
15	1	1	spam	0,17P

$$P(\text{spam} | F, M, C, I', D', Tr', Th) \quad (2)$$

$$= \frac{1}{P(F, M, C, I', D', Tr', Th)} \times \underbrace{0,1V \times \frac{4}{V} \times \frac{F}{V} \times \frac{F}{V} \times \frac{4}{V} \times \frac{5}{V} \times \frac{5}{V} \times \frac{V}{V}}_{0,18P}$$

$$= A$$

$$P(\text{spam}' | F, M, C, I', D', Tr', Th)$$

$$= \frac{1}{P(F, M, C, I', D', Tr', Th)} \times \underbrace{0,1^3 \times \frac{1}{P} \times \frac{1}{P} \times \frac{P}{P} \times \frac{1}{P} \times \frac{1}{P} \times \frac{1}{P} \times \frac{P}{P}}_{\frac{0,001}{PV}}$$

$$= B$$

$$\Rightarrow A > B \quad \textcircled{I} \quad C = 11,741$$

مقدار ثابت
constant

$$P(\text{spam} | F, M, C', I', D', Tr, Th) = \overset{\uparrow}{C} \times 0,1V \times \frac{4}{V} \times \frac{F}{V} \times \frac{F}{V} \times \frac{4}{V} \times \frac{5}{V} \times \frac{5}{V} \times \frac{V}{V} = 0,020VC = A$$

$$P(\text{spam}' | \quad \quad \quad) = C \times 0,1^3 \times \frac{1}{P} \times \frac{1}{P} \times \frac{1}{P} \times \frac{1}{P} \times \frac{1}{P} \times \frac{P}{P} \times \frac{P}{P}$$

$$= \frac{0,001}{PV} \times C = B$$

$$\Rightarrow B < A \quad \textcircled{II} \quad C = 27,1V$$

$$P(\text{spam} | F', M', C', I, D, Tr, Th) = C \times 0,1V \times \frac{1}{V} \times \frac{P}{V} \times \frac{P}{V} \times \frac{1}{V} \times \frac{P}{V} \times \frac{P}{V} \times \frac{V}{V} = \frac{0,001P}{V} \times C = A$$

$$P(\text{spam}' | \quad \quad \quad) = C \times 0,1^3 \times \frac{P}{P} \times \frac{P}{P} \times \frac{1}{P} \times \frac{P}{P} \times \frac{P}{P} \times \frac{P}{P} \times \frac{P}{P}$$

$$= 0,001^3 \times C = B \Rightarrow B > A \quad \textcircled{III} \quad C = 100,1V$$

data	F	D	spam probability
11	1	0	spam
12	1	0	"
13	0	1	not spam
14	0	1	"
15	1	1	spam

0,92

~~0,92~~ 0,92

0,12

~~0,12~~ 0,12

0,12

$$P(\text{spam} | F, M, C, I', D', Tr', Th)$$

(2)

$$= \frac{1}{P(F, M, C, I', D', Tr', Th)} \times 0,12 \times \frac{4}{V} \times \frac{F}{V} \times \frac{F}{V} \times \frac{4}{V} \times \frac{D}{V} \times \frac{D}{V} \times \frac{V}{V}$$

0,12

= A

$$P(\text{spam}' | F, M, C, I', D', Tr', Th)$$

$$= \frac{1}{P(F, M, C, I', D', Tr', Th)} \times 0,07 \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F}$$

0,07
FV

= B

$$\Rightarrow A > B \quad \textcircled{I} \quad C = 11,42$$

constant مقدار ثابت

$$P(\text{spam} | F, M, C', I', D', Tr, Th) = C \times 0,12 \times \frac{4}{V} \times \frac{F}{V} \times \frac{F}{V} \times \frac{4}{V} \times \frac{D}{V} \times \frac{D}{V} \times \frac{V}{V} = 0,12 \times C = A$$

$$P(\text{spam}' | \text{ " }) = C \times 0,07 \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F} = \frac{0,07}{FV} \times C = B$$

$$\Rightarrow B < A \quad \textcircled{II} \quad C = 13,12$$

$$P(\text{spam} | F', M', C', I, D, Tr, Th) = C \times 0,12 \times \frac{1}{V} \times \frac{1}{V} \times \frac{1}{V} \times \frac{1}{V} \times \frac{1}{V} \times \frac{1}{V} \times \frac{1}{V} = \frac{0,12}{V} \times C = A$$

$$P(\text{spam}' | \text{ " }) = C \times 0,07 \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F} \times \frac{1}{F} = 0,07 \times C = B \Rightarrow B > A \quad \textcircled{III} \quad C = 13,12$$

$$P(\text{spam} | F, M', C, I, D, Tr, Th) = C \times 0.1 \times \frac{1}{V} \times \frac{P}{V} \times \frac{F}{V} \times \frac{C}{V} \times \frac{P}{V} \times \frac{P}{V} \times \frac{V}{V}$$

$$= 0.001V \times C = A$$

$$P(\text{spam}' | \dots) = C \times 0.1 \times \frac{P}{V} \times \frac{P}{V} \times \frac{P}{V} \times \frac{1}{V} \times \frac{P}{V} \times \frac{P}{V} \times \frac{P}{V}$$

$$= 0.01P \times C = B \Rightarrow B > A \quad \textcircled{IV} \quad C = 4V, 24$$

$$P(\text{spam} | F, M', C', I, D, Tr, Th) = C \times 0.1 \times \frac{C}{V} \times \frac{P}{V} \times \frac{P}{V} \times \frac{1}{V} \times \frac{P \times P}{V \times V} \times \frac{V}{V}$$

$$= 0.001P \times C = A$$

$$P(\text{spam}' | \dots) = C \times 0.1 \times \frac{1}{V} \times \frac{P}{V} \times \frac{1}{V} \times \frac{P}{V} \times \frac{P}{V} \times \frac{P}{V} \times \frac{P}{V}$$

$$= 0.004C \times C = B \Rightarrow B > A \quad \textcircled{V} \quad C \approx 129, 17$$

data	spam probability		
11	0.99		Spam
12	0.94		Spam
13	0.98		not spam
14	0.88		not spam
15	0.84		not spam

I
 II
 III
 IV
 V

data	F	F, D	complete set	real label
11	$\frac{6}{7} = 0.85$	0.94	0.99	spam
12	$\frac{6}{7} = 0.85$	0.94	0.94	spam
13	$\frac{1}{3} = 0.33$	0.88	0.04	not spam
14	$\frac{1}{3} = 0.33$	0.88	0.14	"
15	$\frac{6}{7} = 0.85$	0.84	0.14	"

صاحتمال spam بودن مستند

به وضع $complete set$ بهتر پیش بینی کرده و عملکرد بهتری دارد.
هر چه قدر که تناقض های سنجش بیشتر شود و فضای نمونه ای بزرگ تر شود، جواب های
دقیق تری به دست می آید.

$$P(C|A \cap B) = \frac{P(C \cap A \cap B)}{P(A \cap B)} \quad (1) \quad P(B|A) = \frac{P(A \cap B)}{P(A)} \quad (2) \quad (6 \text{ الف})$$

$$P(A) \times P(B|A) \times P(C|A \cap B) \stackrel{(1)}{=} \cancel{P(A)} \times \frac{\cancel{P(A \cap B)}}{\cancel{P(A)}} \times \frac{P(C \cap A \cap B)}{\cancel{P(A \cap B)}} \stackrel{(2)}{=} P(A \cap B \cap C)$$

$$P(R \cap I \cap G \cap D) = P(D) \times P(I) \times P(G|I, D) \times \underbrace{P(R|G)}_{P(R|I \cap D \cap G)} \quad (ب)$$

$$P(I \cap D \cap G) = P(I) \times P(D) \times P(G|I, D)$$

$$P(I_0) = 0,7 \quad P(I_1) = 0,3$$

طبق جدول 0,01

$$P(R_1 | I_1 \cap D_0 \cap G_1) = \frac{\cancel{P(D_0)} \times \cancel{P(I_1)} \times \cancel{P(G_1 | I_1, D_0)} \times P(R_1 | G_1)}{\cancel{P(I_1)} \times \cancel{P(D_0)} \times \cancel{P(G_1 | I_1, D_0)}}$$

$$= 0,01$$

طبق مثال بالا

$$P(R_1 | I_1, D_0, G_1) = P(R_1 | G_1) = 0,19$$

$$P(R_2 | \quad \quad \quad) = P(R_2 | G_2) = 0,8$$