Signals and Systems - Assignment 6

Mohammad Khalaji - 9531025

June 2019

1 Question 1

1.a $x[n] = cos(\frac{\pi}{3}n + \frac{\pi}{6})$

$x[n]$ is periodic ($N = 6$) and therefore has a Fourier Series representation:

$$x[n] = \frac{1}{2}\Big(e^{j(\frac{\pi}{3}n+\frac{\pi}{6})} + e^{-j(\frac{\pi}{3}n+\frac{\pi}{6})}\Big)$$

Since $\frac{2\pi}{N} = \frac{\pi}{3}$:

$$a_1 = \frac{1}{2}e^{j\frac{\pi}{6}}$$

$$a_{-1} = \frac{1}{2}e^{-j\frac{\pi}{6}} = a_5$$

a_k is periodic with $N = 6$. $X(e^{j\omega})$ in one period ($0 < \omega < 2\pi$):

$$\hat{X}(e^{j\omega}) = 2\pi \sum_{k=0}^{5} a_k \delta(\omega - k\frac{\pi}{3})$$

$$\Rightarrow \hat{X}(e^{j\omega}) = \pi e^{j\frac{\pi}{6}}\delta(\omega - \frac{\pi}{3}) + \pi e^{j\frac{\pi}{6}}\delta(\omega + \frac{\pi}{3})$$

$X(e^{j\omega})$ is periodic with period 2π.

1.b $x[n] = 1,\ 0 \leq n \leq 10$

If we assume $y[n] = 1$ where $-5 \leq n \leq 5$, then $x[n] = y[n-5]$

$$Y(e^{j\omega}) = \frac{sin(\omega\frac{11}{2})}{sin(\frac{\omega}{2})}$$

Time shift:

$$X(e^{j\omega}) = e^{-5j\omega}\frac{sin(\omega\frac{11}{2})}{sin(\frac{\omega}{2})}$$

$X(e^{j\omega})$ is periodic with period 2π.

1.c $x[n] = \frac{sin(\frac{\pi}{6}n)}{\pi n}$

$X(e^{j\omega})$ is a low-pass filter with cutoff frequency $W = \frac{\pi}{6}$. It is periodic with period 2π.

1.d $x[n] = (0.5)^{|n|}u[-n-5]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty}(0.5)^{|n|}u[-n-5]e^{-jn\omega} = \sum_{n=-\infty}^{-5}(0.5)^{-n}e^{-jn\omega}$$

$$\Rightarrow X(e^{j\omega}) = \sum_{n=-\infty}^{-5}(\frac{1}{2}e^{j\omega})^{-n} = \sum_{n=5}^{\infty}(\frac{1}{2}e^{j\omega})^{n} = \sum_{m=0}^{\infty}(\frac{1}{2}e^{j\omega})^{m+5}$$

$$\Rightarrow X(e^{j\omega}) = \frac{1}{32}e^{j5\omega}\frac{1}{1-\frac{1}{2}e^{j\omega}}$$

1.e $x[n] = 2^n sin(\frac{\pi}{4}n)u[-n]$

First we calculate the Fourier Transform of $y[n] = x[-n] = -(\frac{1}{2})^n sin(\frac{\pi}{4}n)u[n]$, Then, from $Y(e^{j\omega}) = X(e^{-j\omega})$ we get the $X(e^{j\omega})$.

$$y[n] = -(\frac{1}{2})^n u[n] sin(\frac{\pi}{4}n) = r[n]s[n]$$

Periodic Convolution property:

$$\Rightarrow Y(e^{j\omega}) = \frac{1}{2\pi}(R(e^{j\omega}) * \hat{S}(e^{j\omega}))$$

$$s[n] = sin(\frac{\pi}{4}n) \Rightarrow \hat{S}(e^{j\omega}) = \frac{\pi}{j}\Big(\delta(\omega - \frac{\pi}{4}) - \delta(\omega + \frac{\pi}{4})\Big)$$

$$r[n] = -(\frac{1}{2})^n u[n] \Rightarrow R(e^{j\omega}) = \frac{-1}{1 - \frac{1}{2}e^{-j\omega}} = \frac{1}{\frac{1}{2}e^{-j\omega} - 1}$$

$$\Rightarrow Y(e^{j\omega}) = \frac{1}{2j}\Big(\frac{1}{\frac{1}{2}e^{-j(\omega - \frac{\pi}{4})} - 1} - \frac{1}{\frac{1}{2}e^{-j(\omega + \frac{\pi}{4})} - 1}\Big)$$

2 Question 2

2.a $x[1-n]+x[-1-n]$

$$x[n] \overset{\text{FT}}{\longleftrightarrow} X(e^{j\omega})$$

$$x[n+1] \overset{\text{FT}}{\longleftrightarrow} e^{j\omega}X(e^{j\omega})$$

$$x[-n+1] = x[1-n] \overset{\text{FT}}{\longleftrightarrow} e^{-j\omega}X(e^{-j\omega})$$

$$x[n-1] \overset{\text{FT}}{\longleftrightarrow} e^{-j\omega}X(e^{j\omega})$$

$$x[-n-1] = x[-1-n] \overset{\text{FT}}{\longleftrightarrow} e^{j\omega}X(e^{-j\omega})$$

$$\Rightarrow x[1-n]+x[-1-n] \overset{\text{FT}}{\longleftrightarrow} e^{-j\omega}X(e^{-j\omega}) + e^{j\omega}X(e^{-j\omega}) = 2cos(\omega)X(e^{-j\omega})$$

2.b $(n-1)^2x[n]$

$$x[n] \overset{\text{FT}}{\longleftrightarrow} X(e^{j\omega})$$

$$nx[n] \overset{\text{FT}}{\longleftrightarrow} j\frac{d}{d\omega}X(e^{j\omega})$$

$$n^2x[n] \overset{\text{FT}}{\longleftrightarrow} -\frac{d^2}{d\omega^2}X(e^{j\omega})$$

$$\Rightarrow (n-1)^2x[n] = (n^2-2n+1)x[n] \overset{\text{FT}}{\longleftrightarrow} -\frac{d^2}{d\omega^2}X(e^{j\omega}) - 2j\frac{d}{d\omega}X(e^{j\omega}) + X(e^{j\omega})$$

2.c $x^*[n]$

$$x[n] \overset{\text{FT}}{\longleftrightarrow} X(e^{j\omega})$$

$$x^*[n] \overset{\text{FT}}{\longleftrightarrow} X^*(e^{-j\omega})$$

$$x^*[-n] \overset{\text{FT}}{\longleftrightarrow} X^*(e^{j\omega})$$

$x[n]$ is real ($x[n] = x^*[n]$), then $X(e^{j\omega}) = X^*(e^{-j\omega})$, conjugating both sides, $X^*(e^{j\omega}) = X(e^{-j\omega})$.

3 Question 3

$$y[n] + y[n-1] + 0.89y[n-2] = x[n] + 2x[n-1]$$

Fourier Transform:

$$Y(e^{j\omega}) + e^{-j\omega}Y(e^{j\omega}) + 0.89e^{-2j\omega}Y(e^{j\omega}) = X(e^{j\omega}) + 2e^{-j\omega}X(e^{j\omega})$$

$$\Rightarrow H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 + 2e^{-j\omega}}{1 + e^{-j\omega} + 0.89e^{-2j\omega}}$$

3.a $x[n] = e^{j0.2\pi n}$

$$\hat{X}(e^{j\omega}) = 2\pi\delta(\omega - 0.2\pi)$$
$$\Rightarrow \hat{Y}(e^{j\omega}) = 2\pi\delta(\omega - 0.2\pi)H(e^{j0.2\pi})$$
$$\Rightarrow y[n] = e^{j0.2\pi n}H(e^{j0.2\pi})$$

3.b $x[n] = cos(0.2\pi n)$

$$\hat{X}(e^{j\omega}) = \pi\Big(\delta(\omega - 0.2\pi) + \delta(\omega + 0.2\pi)\Big)$$
$$\Rightarrow \hat{Y}(e^{j\omega}) = \pi\Big(\delta(\omega - 0.2\pi)H(e^{j0.2\pi}) + \delta(\omega + 0.2\pi)H(e^{-j0.2\pi})\Big)$$
$$\Rightarrow y[n] = \pi H(e^{j0.2\pi})\frac{1}{2\pi}e^{j0.2\pi n} + \pi H(e^{-j0.2\pi})\frac{1}{2\pi}e^{-j0.2\pi n}$$

3.c $x[n] = 2sin(0.3\pi n)$

$$\hat{X}(e^{j\omega}) = \frac{2\pi}{j}\Big(\delta(\omega - 0.3\pi) - \delta(\omega + 0.3\pi)\Big)$$
$$\Rightarrow \hat{Y}(e^{j\omega}) = \frac{2\pi}{j}\Big(\delta(\omega - 0.3\pi)H(e^{j0.3\pi}) - \delta(\omega + 0.3\pi)H(e^{-j0.3\pi})\Big)$$
$$\Rightarrow y[n] = \frac{2\pi}{j}H(e^{j0.3\pi})\frac{1}{2\pi}e^{j0.3\pi n} - \frac{2\pi}{j}H(e^{-j0.3\pi})\frac{1}{2\pi}e^{-j0.3\pi n}$$

3.d $x[n] = 3cos(0.1\pi n) - 5sin(0.2\pi n)$

$$\hat{X}(e^{j\omega}) = 3\pi\Big(\delta(\omega - 0.1\pi) + \delta(\omega + 0.1\pi)\Big) - 5\frac{\pi}{j}\Big(\delta(\omega - 0.2\pi) - \delta(\omega + 0.2\pi)\Big)$$

$$\Rightarrow \hat{Y}(e^{j\omega}) = 3\pi\Big(\delta(\omega - 0.1\pi)H(e^{j0.1\pi}) + \delta(\omega + 0.1\pi)H(e^{-j0.1\pi})\Big) - 5\frac{\pi}{j}\Big(\delta(\omega - 0.2\pi)H(e^{j0.2\pi}) - \delta(\omega + 0.2\pi)H(e^{-j0.2\pi})\Big)$$

$$\Rightarrow y[n] = \frac{3\pi}{2\pi}\Big(e^{j0.1\pi n}H(e^{j0.1\pi}) + e^{-j0.1\pi n}H(e^{-j0.1\pi})\Big) - 5\frac{\pi}{j}\frac{1}{2\pi}\Big(e^{j0.2\pi n}H(e^{j0.2\pi}) - e^{-j0.2\pi n}H(e^{-j0.2\pi})\Big)$$

4 Question 4

$$x[n] = \delta[n] \Rightarrow X(e^{j\omega}) = 1$$

Since $H(e^{j\omega})$ is a low-pass filter with cutoff frequency $\frac{\pi}{2}$:

$$h[n] = \frac{sin(\frac{\pi}{2}n)}{\pi n}$$

If we put the name $w[n]$ for output of the system:

$$y[n] = w[n] + (-1)^n w[n]$$

$$W(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = H(e^{j\omega})$$

$$w[n] \xleftrightarrow{\text{FT}} W(e^{j\omega}) = H(e^{j\omega})$$

$$(-1)^n w[n] \xleftrightarrow{\text{FT}} W(e^{j(\omega-\pi)}) = H(e^{j(\omega-\pi)})$$

$$\Rightarrow Y(e^{j\omega}) = H(e^{j\omega}) + H(e^{j(\omega-\pi)})$$

$$\Rightarrow y[n] = h[n] + (-1)^n h[n] = \begin{cases} 2h[n] & \text{n is even} \\ 0 & \text{n is odd} \end{cases} = \delta[n]$$

5 Question 5

Frequency Response of the first system:

$$w[n] = x[n] - x[n-1]$$
$$\Rightarrow W(e^{j\omega}) = X(e^{j\omega}) - e^{-j\omega}X(e^{j\omega})$$
$$\Rightarrow H_1(e^{j\omega}) = \frac{W(e^{j\omega})}{X(e^{j\omega})} = 1 - e^{-j\omega}$$

Frequency Response of the second system:

$$H_2(e^{j\omega}) = \begin{cases} 1 & \frac{-\pi}{2} < \omega < \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

H_1 and H_2 are both periodic with period 2π

$$y[n] = w[n] * h_2[n] = x[n] * (h_1[n] * h_2[n])$$
$$\Rightarrow Y(e^{j\omega}) = X(e^{j\omega})\Big(H_1(e^{j\omega})H_2(e^{j\omega})\Big)$$

Frequency Response of the equivalent system:

$$\hat{H}(e^{j\omega}) = \hat{H}_1(e^{j\omega})\hat{H}_2(e^{j\omega}) = \begin{cases} 1 - e^{-j\omega} & \frac{-\pi}{2} < \omega < \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

The equivalent system filters $sin(0.6\pi n)$. So we can assume that input is $cos(0.4\pi n) + 2\delta[n-2]$.

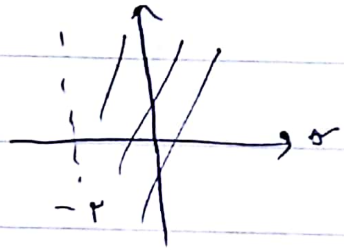
$$\hat{X}(e^{j\omega}) = \pi\Big(\delta(\omega - 0.4\pi) + \delta(\omega + 0.4\pi)\Big) + 2e^{-j2\omega} = \hat{X}_1(e^{j\omega}) + \hat{X}_2(e^{j\omega})$$

$$\hat{Y}_1(e^{j\omega}) = \hat{X}_1(e^{j\omega})\hat{H}(e^{j\omega}) = \pi(1 - e^{-j\omega})\Big(\delta(\omega - 0.4\pi) + \delta(\omega + 0.4\pi)\Big)$$
$$\Rightarrow y_1[n] = cos(0.4\pi(n)) - cos(0.4\pi(n-1))$$
$$\hat{Y}_2(e^{j\omega}) = \hat{X}_2(e^{j\omega})\hat{H}(e^{j\omega}) = 2e^{-j2\omega} - 2e^{-j3\omega} \qquad \frac{-\pi}{2} < \omega < \frac{\pi}{2}$$
$$\Rightarrow y_2[n] = \frac{1}{2\pi}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (2e^{-j2\omega} - 2e^{-j3\omega})e^{jn\omega}d\omega$$
$$\Rightarrow y_2[n] = \frac{1}{\pi}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} e^{j\omega(n-2)} - e^{j\omega(n-3)}d\omega$$
$$\Rightarrow y_2[n] = \frac{1}{\pi}\Big(\frac{2sin((n-2)\frac{\pi}{2})}{n-2} - \frac{2sin((n-3)\frac{\pi}{2})}{n-3}\Big)$$
$$y[n] = y_1[n] + y_2[n]$$

۴

$a)\ x(t)=e^{-۲t}u(t-1)$

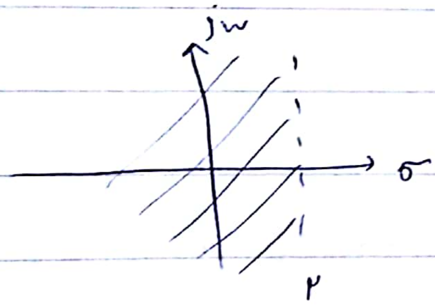
$$X(s)=\int_{-\infty}^{\infty}x(t)e^{-st}=\int_{1}^{\infty}e^{-۲t}e^{-st}dt=\frac{-1}{۲+s}e^{(۲+s)t}\Big|_{1}^{\infty}=0+\frac{e^{-(۲+s)}}{۲+s}$$



ROC: $۲+Re\{s\}>0 \rightarrow Re\{s\}>-۲$

$b)\ x(t)=e^{۲t}u(-t+1)$

$$X(s)=\int_{-\infty}^{1}e^{۲t}e^{-st}dt=\frac{1}{۲-s}e^{(۲-s)t}\Big|_{-\infty}^{1}=\frac{e^{(۲-s)}}{۲-s}$$

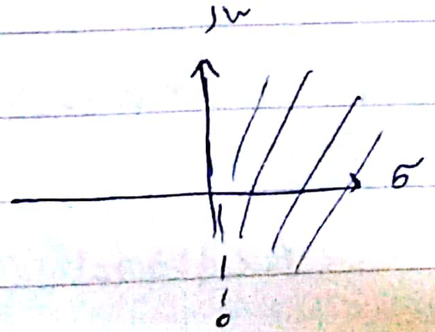


ROC: $۲-Re\{s\}>0 \quad R\{s\}<۲$

$c)\ x(t)=u(t)-۲u(t-1)$

$$X(s)=\int_{-\infty}^{\infty}(u(t)-۲u(t-1))e^{-st}=\int_{0}^{\infty}e^{-st}-۲\int_{1}^{\infty}e^{-st}dt$$

$$=\frac{-1}{s}e^{-st}\Big|_{0}^{\infty}+\frac{۲}{s}e^{-st}\Big|_{1}^{\infty}=\frac{1}{s}-\frac{۲}{s}e^{-s}$$



$Re\{s\}>0$

d) $x(t) = \cos(2t)u(t) + 2\sin(3t)u(t)$ ۹

$$X(s) = \frac{1}{2}\int_0^{\infty} (e^{2t} + e^{-2t}) e^{-st} dt + \frac{2}{2j}\int_0^{\infty} (e^{3t} - e^{-3t}) e^{-st} dt =$$

$$\frac{1}{2}\left(\frac{1}{2j-s} e^{(2j-s)t} - \frac{1}{2j+s} e^{-(2j+s)t}\right)\Big|_0^{\infty} + \frac{1}{j}\left(\frac{1}{3j-s} e^{(3j-s)t} + \frac{R}{3j+s} e^{-(3j+s)t}\right)\Big|_0^{\infty}$$

$$= \frac{1}{2}\left(0 - \frac{1}{2j-s} + \frac{1}{2j+s}\right) + \frac{1}{j}\left(0 - \frac{1}{3j-s} - \frac{1}{3j+s}\right)$$

$Re\{s\} > 0$:Roc

۲)

a) $X(s) = \dfrac{s-۲}{s^{۲}+۳s+۲}$, $x(t)$ is casual

$$X(s) = \frac{A}{s+۲} + \frac{B}{s+۱} = \frac{۴}{s+۲} - \frac{۳}{s+۱}$$

$A = \lim_{s \to -۲} (s+۲) X(s) = ۴$

$B = \lim_{s \to -۱} (s+۱) X(s) = -۳$

علّی پس سمت راستی

$s+۲ = ۰ \rightarrow s = -۲$

$s+۱ = ۰ \rightarrow s = -۱$

$Re\{s\} > -۱$

$j\omega$ را شامل شده پس پایدار است و تبدیل فوریه دارد.

$$\xrightarrow{FT} X(e^{j\omega}) = \frac{-۳}{۱+j\omega} + \frac{۴}{۲+j\omega}$$

b) $X(s) = \dfrac{s}{s^{۲}-۱}$, $x(t)$ is casual

$$X(s) = \frac{A}{s-۱} + \frac{B}{s+۱} = \frac{۱/۲}{s-۱} + \frac{۱/۲}{s+۱}$$

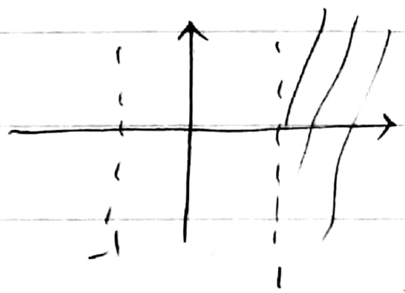
$A = \lim_{s \to ۱} (s-۱) X(s) = \frac{۱}{۲}$

$B = \lim_{s \to -۱} (s+۱) X(s) = \frac{۱}{۲}$

علّی است پس سمت راستی

$s-۱ = ۰ \rightarrow s = ۱$

$s+۱ = ۰ \rightarrow s = -۱$



$Re\{s\} > ۱$

شامل $j\omega$ نیست پس پایدار نیست و تبدیل فوریه ندارد.

Scanned by CamScanner

C) $X(s) = \frac{s+1}{s^2-4s+3}$, $x(t)$ is anti_casual

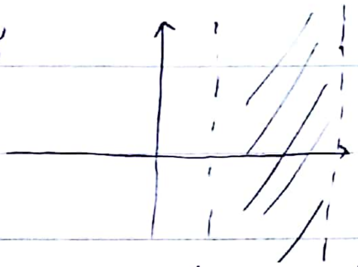
$$X(s) = \frac{A}{(s-1)} + \frac{B}{(s-3)} = \frac{-1}{s-1} + \frac{2}{s-3}$$

$$A = \lim_{s\to 1} (s-1)X(s) = -1 \qquad B = \lim_{s\to 3} (s-3)X(s) = 2$$

الف) ۲ حالت داریم ← غیرعلی پس سمت راستی نیست

$s-1=0 \to s=1$

$s-3=0 \to s=3$



$1 < Re\{s\} < 3$ که شامل $j\omega$ نمی‌شود پس پایدار نیست و تبدیل فوریه ندارد.

ب) شامل محور است پس پایدار است و تبدیل فوریه دارد

$$X(j\omega) = \frac{-1}{j\omega-1} + \frac{2}{j\omega-3}$$

D) $X(s) = \frac{s^2-s}{s^2-s-6}$ $x(t)$ is anti_casual

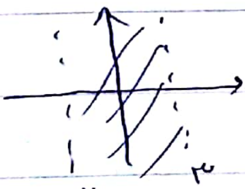
$$X(s) = \frac{A}{s-3} + \frac{B}{s+2} = \frac{6/5}{s-3} - \frac{6/5}{s+2}$$

$$A = \lim_{s\to 3} (s-3)X(s) = \frac{6}{5} \qquad B = \lim_{s\to -2} (s+2)X(s) = -\frac{6}{5}$$

الف) غیرعلی پس سمت راستی نیست

$s-3=0 \to s=3$

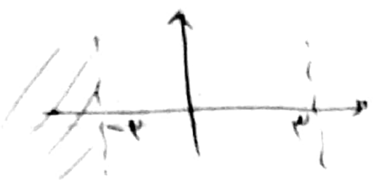
$s+2=0 \to s=-2$



$-2 < Re\{s\} < 3$ شامل محور پس پایدار و دارای تبدیل فوریه

$$X(j\omega) = \frac{6/5}{j\omega-3} - \frac{6/5}{j\omega+2}$$

ب) $Re\{s\}<-2$ شامل محور نمی‌شود ناپایدار است و تبدیل فوریه ندارد



۱۸)

$$a)\ X(s)=\frac{1}{s^2+3s+2}=\frac{A}{s+2}+\frac{B}{s+1}=\frac{1}{s+1}-\frac{1}{s+2}$$

$$A=\lim_{s\to -1}(s+1)X(s)=1 \qquad B=\lim_{s\to -2}(s+2)X(s)=-1$$

$$L^{-1}\left\{\frac{1}{s+1}\Big|_{Roc:Re\{s\}>-1}\right\}=e^{-t}u(t)$$

$$L^{-1}\left\{\frac{1}{s+2}\Big|_{Roc:Re\{s\}>-1>-2}\right\}=e^{-2t}u(t)$$

$$\rightarrow x(t)=e^{-t}u(t)-e^{-2t}u(t)$$

$$b)\ X(s)=\frac{1}{s+1}-\frac{1}{s+2}$$

$$L^{-1}\left\{\frac{1}{s+1}\Big|_{Re\{s\}<-1}\right\}=-e^{-t}u(-t)$$

$$\rightarrow x(t)=-e^{-t}u(-t)-e^{-2t}u(t)$$

8)

$$C)\ X(s)=\frac{(s-1)(s-2)}{(s+1)(s+2)(s+3)}=\frac{\overset{3}{\cancel{A}}}{s+1}+\frac{\overset{-12}{\cancel{B}}}{s+2}+\frac{\overset{10}{\cancel{C}}}{s+3}$$

$$A=\lim_{s\to -1}(s+1)X(s)=3 \qquad B=\lim_{s\to -2}(s+2)X(s)=-12$$

$$C=\lim_{s\to -3}(s+3)X(s)=10 \qquad \text{casual}\ \ Re\{s\}>-1$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s+1}\Big|_{Re\{s\}>-1}\right\}=e^{-t}u(t)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s+2}\Big|_{Re\{s\}>-1>-2}\right\}=e^{-2t}u(t) \qquad \mathcal{L}^{-1}\left\{\frac{1}{s+3}\Big|_{Re\{s\}>-1>-3}\right\}=e^{-3t}u(t)$$

$$x(t)=3e^{-t}u(t)-12e^{-2t}u(t)+10e^{-3t}u(t)$$

$$D)\ X(s)=\frac{s+4}{(s+1)^2+4}=\frac{\overset{\frac{5+2j}{4j}}{\cancel{A}}}{s-(-1+2j)}+\frac{\overset{\frac{5-2j}{-4j}}{\cancel{B}}}{s-(-1-2j)}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s-(-1+2j)}\Big|_{Re\{s\}>0>-1}\right\}=e^{(-1+2j)t}u(t)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s-(-1-2j)}\Big|_{Re\{s\}>0>-1}\right\}=e^{(-1-2j)t}u(t)$$

$$x(t)=\frac{5+2j}{4j}e^{(-1+2j)t}u(t)+\frac{5+2j}{-4j}e^{(-1-2j)t}u(t)$$

15)

a) $H(s) = \dfrac{s+1}{s+3} = 1 - \dfrac{2}{s+3}$

$* \ F(s) = 1 \xrightarrow{L^{-1}} f(t) = \delta(t) \text{ at } t=0$

$* \ F(s) = \dfrac{1}{s+\alpha} \xrightarrow{\mathcal{L}^{-1}} f(t) = e^{-\alpha t}u(t)$

$\xrightarrow{L^{-1}} H(t) = \delta(t) - 2e^{-3t}u(t) \qquad Re\{s\} > -3$

b) $H(s) = \dfrac{s^2+1}{(s+1)(s+2)} = \dfrac{A}{s+1} + \dfrac{B}{s+2} = \dfrac{2}{s+1} + \dfrac{5}{s+2}$

$A = \lim_{s \to -1} (s+1)H(s) = 2 \qquad B = \lim_{s \to -2} (s+2)H(s) = 5$

$* \ F(s) = \dfrac{1}{s+\alpha} \xrightarrow{\mathcal{L}^{-1}} f(t) = e^{-\alpha t}u(t)$

$H(t) = 2e^{-t}u(t) + 5e^{-2t}u(t) \qquad Re\{s\} > -1$

c) $H(s) = \dfrac{s^2-1}{(s+3)(s^2+2s+2)} = \dfrac{A}{s+3} + \dfrac{Bs+C}{s^2+2s+2} = \dfrac{(A+B)s^2 + (2A+3B+C)s + (A+2C)}{(s+3)(s^2+2s+2)}$

$A + B = 1$

$2A + 3B + C = 0 \quad \longrightarrow \begin{cases} A = 3 \\ B = C = -2 \end{cases}$

$A + 2C = -1$

$* \ F(s) = \dfrac{1}{s+\alpha} \xrightarrow{\mathcal{L}^{-1}} e^{-\alpha t}u(t) \qquad * \ \dfrac{s+a}{(s+a)^2 + b^2} \xrightarrow{\mathcal{L}^{-1}} e^{-\alpha t}\cos bt$

$H(t) = 3e^{-3t}u(t) - 2e^{-t}\cos t \qquad Re\{s\} > -1$

Stbo

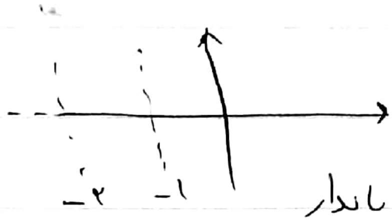
DATE ....

۱) a) $H(s) = \frac{s-1}{s^2+3s+2} = \frac{A}{(s+1)} + \frac{B}{s+2} = \frac{-2}{s+1} + \frac{3}{s+2}$

$A = \lim_{s \to -1} (s+1)H(s) = -2 \qquad B = \lim_{s \to -2} (s+2)H(s) = 3$

$s+2=0 \to s=-2$

$s+1=0 \to s=-1$



برای اینکه هم علّی باشد و هم پایدار باید هم سمت راستی باشد و هم شامل محور $j\omega$ که در اینجا اگر $Re\{s\}>-1$ باشد هر ۲ این شرط‌ها برقرار اند

b) $H(s) = \frac{s(s+1)}{s^2+2s+5}$

$s^2+2s+5=0 \to s=-1+2j$

$\searrow s=-1-2j$

قطب $Re\{s\}=-1$

اگر $Re\{s\}>-1$ باشد هم سمت راستی است هم شامل $j\omega$ پس هم علّی است هم پایدار

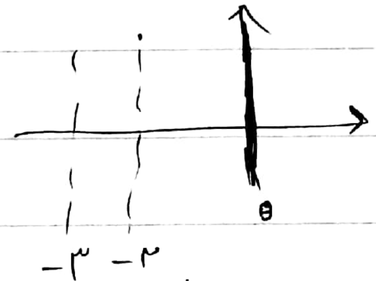
c) $H(s) = \frac{s^2+1}{s(s^2+5s+6)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$

$A = \lim_{s \to 0} sH(s) = \frac{1}{6} \qquad B = \lim_{s \to -2} (s+2)H(s) = -\frac{5}{2} \qquad C = \lim_{s \to -3}(s+3)H(s) = \frac{10}{3}$

$s=0$

$s+2=0 \to s=-2$

$s+3=0 \to s=-3$



در هر کدام از حالت‌ها $j\omega$ نمی‌تواند شامل ROC شود چون $s=0$ یک قطب است پس حالت پایداری رخ نمی‌دهد.

d) $H(s) = \frac{s+3}{s^2-4s+10} \Rightarrow s^2-4s+10=0 \rightarrow 3+2j$

$\searrow 3-2j$

قطب $Re\{s\}=3$

اگر سمت راستی باشد شامل محور نمی‌شود ۳

و اگر شامل شود سمت راستی نخواهد بود پس علی بودن و پایداری باهم رخ نمی‌دهد.

$$\frac{d^2y(t)}{dt^2} = -\frac{2\,dy(t)}{dt} - a\,y(t) + x(t)$$ ۱۲

a) $(s^2+2s+a)\,Y(s) = X(s)$

$$\rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2+2s+a}$$

b) برای اینکه سیستم پایدار باشد باید شامل سز باشد پس $Re\{s\}=0$ باید
بخشی از ROC آن باشد

$x^2+2s+a=0$

$\Delta = 4-4a$ ریشه۱ $\frac{-2+\sqrt{4-4a}}{2} = -1+\sqrt{1-a}$

ریشه۲ $\frac{-2-\sqrt{4-4a}}{2} = -1-\sqrt{1-a}$

if $\Delta<0$ $a>1$ $Re\{s\}>-1 \rightarrow$ پایدار

if $\Delta=0$ $a=1$ $Re\{s\}>-1 \rightarrow$ پایدار

if $\Delta>0$ $0<a<1$ $Re\{s\}>-1+\sqrt{1-a} \rightarrow$ پایدار

if $\Delta>0$ $a<0$ $-1-\sqrt{1-a}<Re\{s\}<-1+\sqrt{1-a} \rightarrow$ پایدار