HW6 «In the name of God»

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Year: Month: Date:

(a) (1)

$$X(e^{j\omega}) = e^{\frac{-j\omega\pi}{6}} \times \frac{2\pi}{2} \sum_{l=-\infty}^{+\infty} \left(\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l) \right)$$

$\omega_0 = \frac{\pi}{3}$

(b)

$$X(e^{j\omega}) = e^{-5j\omega} \frac{\sin\left(\frac{11\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$$

(c)

$$X(e^{j\omega}) = \begin{cases} 1 & 0 \leq |\omega| \leq \frac{\pi}{6} \\ 0 & \frac{\pi}{6} < |\omega| \leq \pi \end{cases}$$

(d)

$$y[n] = (0.5)^{|n-5|} u[n-5]$$

$$\Longrightarrow Y(e^{j\omega}) = \frac{e^{-5j\omega}}{1 - \frac{e^{-j\omega}}{2}}$$

$|n-5| \rightarrow |n+5|$

$$z[n] = y[-n] \Longrightarrow z[n] = (0.5)^{|n+5|} u[-n-5]$$

$$\Longrightarrow Z(e^{j\omega}) = \frac{e^{5j\omega}}{1 - \frac{e^{j\omega}}{2}}$$

$$x[n] = \begin{cases} 32\, z[n] & n \geq 0 \\ -5 \leq n < 0 \\ n \leq -5 \end{cases}$$

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(e)

(2)

(a)

$$Y(e^{j\omega}) = e^{-j\omega} X(e^{-j\omega}) + e^{j\omega} X(e^{-j\omega})$$

$$= X(e^{-j\omega}) \times 2\cos(\omega)$$

(b)

$$y[n] = (n-1)^2 x[n] = n^2 x[n] - 2n x[n] + x[n]$$

$$\Rightarrow Y(e^{j\omega}) = X(e^{j\omega}) - 2j \frac{d}{d\omega} X(e^{j\omega}) - \frac{d^2 X(e^{j\omega})}{d\omega^2}$$

Koosha

Subject :

Year : Month : Date :

$x = x^* \Longrightarrow x^*[-n] = x[-n]$ ©

$\Longrightarrow Y(e^{j\omega}) = X(e^{-j\omega})$

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③

$$H(e^{j\omega}) = \frac{1+2e^{-j\omega}}{1+e^{-j\omega}+0.89e^{-2j\omega}} = \frac{1+2e^{-j\omega}}{0.89(\quad s-\quad)(s+0.5-0.8j)}$$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) \quad \text{(a)}$$

$\rightarrow 2\pi \sum_{\ell=-\infty}^{+\infty} \delta(\omega - 0.2\pi - 2\pi\ell)$

$$= 2\pi \sum_{\ell=-\infty}^{+\infty} \delta(\omega - 0.2\pi - 2\pi\ell) \times \frac{1+2e^{-j(0.2\pi+2\pi\ell)}}{1+e^{-j(0.2\pi+2\pi\ell)}+0.89e^{-2j(0.2\pi+2\pi\ell)}}$$

$$= 2\pi \left[\sum_{\ell=-\infty}^{+\infty} \delta(\omega - 0.2\pi - 2\pi\ell)\right] \times \underbrace{\left[\frac{1+2e^{-j0.2\pi}}{1+e^{-j0.2\pi}+0.89e^{-2j0.2\pi}}\right]}_{A}$$

$\Longrightarrow y[n] = Ae^{j0.2\pi n}$

(b) از جواب قسمت قبل استفاده می‌کنیم:

جواب : $\text{Real}(Ae^{j0.2\pi n}) = A\cos(0.2\pi n)$

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Subject :
Year : Month : Date :

ⓒ اگر در قسمت الف ⓐ ورودی $e^{j0,3\pi n}$ بوده خروجی برابر $Ae^{j0,3\pi n}$ می شود. حال با استفاده از این قسمت و جواب ⓐ خواهیم داشت:

$$\text{پاسخ} = 2 \times \text{imaginary}\left(Ae^{j0,3\pi n}\right) = 2A\sin(0,3\pi n)$$

ⓓ مانند قسمت های قبل:

$$\text{پاسخ} = 3A\cos(0,1\pi n) - 5A\sin(0,2\pi n)$$

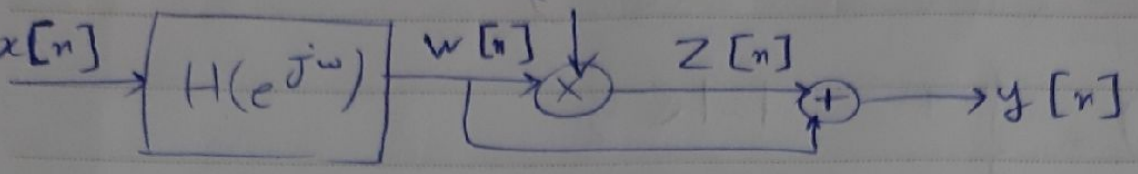
Koosha

Subject :

Year : Month : Date :

④

$(-1)^n = e^{j\pi n}$



$$X(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{+\infty} 1$$

$$W(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega}) = H(e^{j\omega})$$

$X(e^{j\omega}) \rightarrow 1$

$$e^{j\omega_0 n} x[n] \xrightarrow{FT} X(e^{j(\omega-\omega_0)})$$

$$\Rightarrow Z(e^{j\omega}) = W(e^{j(\omega-\pi)}) = H(e^{j(\omega-\pi)})$$

$$y[n] = w[n] + z[n]$$

$$\longrightarrow Y(e^{j\omega}) = W(e^{j\omega}) + Z(e^{j\omega})$$

$$= H(e^{j\omega}) + H(e^{j(\omega-\pi)})$$

$$H(e^{j\omega}) \xrightarrow{IFT} h[n] = \frac{\sin(\frac{\pi}{2}n)}{\pi n}$$

$$\longrightarrow y[n] = \frac{\sin(\frac{\pi}{2}n)}{\pi n} + e^{j\pi n} \frac{\sin(\frac{\pi}{2}n)}{\pi n}$$

$$= \frac{\sin(\frac{\pi}{2}n)}{\pi n} (1 + e^{j\pi n})$$

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Subject :
Year : Month : Date :

(5)

$$W(e^{j\omega}) = (1-e^{-j\omega})\, X(e^{j\omega})$$

$$H_2(e^{j\omega}) = \begin{cases} 1 & |\omega| \le \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < |\omega| \le \pi \end{cases}$$

$$Y(e^{j\omega}) = W(e^{j\omega}) H_2(e^{j\omega}) = (1-e^{-j\omega}) X(e^{j\omega}) H_2(e^{j\omega})$$

$$X(e^{j\omega}) = \pi \sum_{l=-\infty}^{+\infty} \left(\delta(\omega - \underset{0.4\pi}{\omega_0} - 2\pi l) + \delta(\omega + \underset{0.4\pi}{\omega_0} - 2\pi l)\right)$$
$$+ \frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \left(\delta(\omega - \underset{0.6\pi}{\omega_0} - 2\pi l) - \delta(\omega + \underset{0.6\pi}{\omega_0} - 2\pi l)\right)$$
$$+ 2e^{-2j\omega}$$

به خاطر $H_2(e^{j\omega})$، سهای خارج از $-\frac{\pi}{2}, \frac{\pi}{2}$ حذف می‌شوند.

$$\Rightarrow Y(e^{j\omega}) = \Big[\pi\,\delta(\omega - 0.4\pi)(1-e^{-j\omega}) + \pi\,\delta(\omega + 0.4\pi)(1-e^{-j\omega}) + 2e^{-2j\omega}(1-e^{-j\omega})\Big] H_2(e^{j\omega})$$

$$\Rightarrow Y(e^{j\omega}) = \pi\,\delta(\omega - 0.4\pi)\left(1 - e^{-j0.4\pi}\right) + \pi\,\delta(\omega + 0.4\pi)\left(1 - e^{j0.4\pi}\right) + \left(2e^{-2j\omega} - 2e^{-3j\omega}\right) H_2(e^{j\omega})$$

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$$\Rightarrow y[n] = \frac{(1 - e^{-j0.4\pi})}{2} e^{j0.4\pi} + \frac{(1 - e^{j0.4\pi})}{2} e^{-j0.4\pi} + \frac{2 \sin(0.5\pi(n-2))}{\pi(n-2)} - 2 \frac{\sin(0.5\pi(n-3))}{\pi(n-3)}$$