«به نام خدا»

تمرین سری چهارم سیگنال

امیرمحمد پیرحسین لو

9531014

ⓐ①

$$x(t) = \underbrace{e^{-3|t|}}_{p(t)} \underbrace{\sin(2t)}_{q(t)} \qquad X(j\omega) = \frac{1}{2\pi} P(j\omega) ** Q(j\omega)$$

$$Q(j\omega) = \frac{\pi}{j}\Big(\delta(\omega-2) - \delta(\omega+2)\Big)$$

$$P(j\omega) = F\left\{e^{-3t}u(t) + e^{+3t}u(-t)\right\} = \frac{1}{3+j\omega} + \frac{1}{3-j\omega} = \frac{6}{3+\omega^2}$$

$$\frac{Q(j\omega) ** P(j\omega)}{2\pi} = \frac{\pi}{2\pi j}\left(\frac{6}{3+(\omega-2)^2} - \frac{6}{3+(\omega+2)^2}\right) = \frac{3}{j}\left(\frac{1}{3+(\omega-2)^2} - \frac{1}{3+(\omega+2)^2}\right)$$

$$= X(j\omega)$$

جواب نهایی

ⓑ

$$e^{-2t}u(t) \xleftrightarrow{F} \frac{1}{2+j\omega} \Longrightarrow jte^{-2t}u(t) \xleftrightarrow{F} \left(\frac{1}{2+j\omega}\right)' = \frac{-j}{(2+j\omega)^2}$$

$$\frac{2}{j}\, jte^{-2t}u(t) \xleftrightarrow{F} \frac{-2}{(2+j\omega)^2} \Longrightarrow \frac{d}{dt}\left(2te^{-2t}u(t)\right) \xleftrightarrow{F} \boxed{\frac{-2j\omega}{(2+j\omega)^2}}$$

ⓒ

$$\cancel{p(t) \xleftrightarrow{F}}$$

$$\frac{1}{3+2t^2} = \frac{1}{2\sqrt{3}}\left[\underbrace{\frac{1}{\sqrt{3}+j\sqrt{2}t}}_{p(t)} + \underbrace{\frac{1}{\sqrt{3}-j\sqrt{2}t}}_{q(t)}\right]$$

duality

$$\frac{1}{\sqrt{3}+jt} \xleftrightarrow{F} 2\pi e^{+\sqrt{3}\omega} u(-\omega) \Longrightarrow p(j\omega) \xleftrightarrow{F} \frac{2\pi}{\sqrt{2}} e^{+\sqrt{\frac{3}{2}}\omega} u\left(\frac{-\omega}{\sqrt{2}}\right)$$

$$\Longrightarrow q(j\omega) \xleftrightarrow{F} \frac{2\pi}{\sqrt{2}} e^{-\sqrt{\frac{3}{2}}\frac{\omega}{\sqrt{2}}} u\left(\frac{\omega}{+\sqrt{2}}\right)$$

$$X(j\omega) = \frac{2\pi}{2\sqrt{3}}\left(P(j\omega) + Q(j\omega)\right) = \frac{2\pi}{2\sqrt{6}}\, e^{-\frac{\sqrt{3}}{\sqrt{2}}|\omega|} \boxed{= \frac{\pi}{\sqrt{6}}\, e^{-\sqrt{\frac{3}{2}}|\omega|}}$$

(d)

$$x(t) = \underbrace{\sin(\pi t)}_{p(t)}\Big[u(t) \underbrace{- u(t-1)}_{q(t)}\Big]$$

$$u(t) \xleftrightarrow{F} \frac{1}{j\omega} + \pi\,\delta(\omega) \qquad u(t-1) \xleftrightarrow{F} e^{-j\omega_0}\left(\frac{1}{j\omega} + \pi\delta(\omega)\right)$$

$$\sin(\pi t) \xleftrightarrow{F} \frac{\pi}{j}\Big(\delta(\omega-\pi) - \delta(\omega+\pi)\Big)$$

$$X(j\omega) = \frac{1}{2\pi}\left[P(j\omega) ** Q(j\omega)\right] = \frac{1}{2j}\left(1 - e^{-j\omega_0}\right)\left[\frac{1}{j(\omega-\pi)} + \pi\,\delta(\omega-\pi) - \frac{1}{j(\omega+\pi)} - \pi\,\delta(\omega+\pi)\right]$$

(e)

$$x(t) = t\cos(\omega_0 t)$$

$$\cos(\omega_0 t) \xleftrightarrow{F} \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$

$$jt\cos(\omega_0 t) \xleftrightarrow{F} \pi\left(\quad '' \quad\right)'$$

$$\Longrightarrow X(j\omega) = -\pi j\left(\delta(\omega-\omega_0)' + \delta(\omega+\omega_0)'\right)$$

duality

$X(j\omega) = 3\,\delta(\omega-3)$ $\quad 2\pi\,\delta(t-t_0) \xrightarrow{F} 2\pi e^{-j\omega_0 t_0}$ ⓐ②

$e^{j\omega_0 t} \xleftrightarrow{F} 2\pi\,\delta(\omega-\omega_0) \Longrightarrow \frac{3}{2\pi} e^{3jt} \longleftrightarrow 3\,\delta(\omega-3)$

$\Longrightarrow x(t) =$

$X(j\omega) = \begin{cases} \cos(\omega) & |\omega| < \pi \\ 0 & o.w \end{cases}$ ⓑ

$X(j\omega) = 3\,\delta(\omega-3)$ ⓐ②

$e^{j\omega_0 t} \xleftrightarrow{F} 2\pi\,\delta(\omega-\omega_0) \Longrightarrow \frac{3}{2\pi} e^{j\omega_0 t} \xleftrightarrow{F} 3\,\delta(\omega-\omega_0) \quad \omega_0 = 3$

$\longrightarrow x(t) = \frac{3}{2\pi} e^{3jt}$

$X(j\omega) = \begin{cases} \cos(\omega) & |\omega| < \pi \\ 0 & o.w \end{cases}$ $\quad H(j\omega) = \begin{cases} 1 & |\omega| < \pi \\ 0 & o.w \end{cases}$ ⓑ

$X(j\omega) = H(j\omega)\,Y(j\omega)$ $\quad Y(j\omega) = \cos(\omega)$

$x = h ** y$ $\quad \cos(\omega_0 t) \xleftrightarrow{F} \pi\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right)$

$h = \frac{\sin(\pi t)}{\pi t}$ $\quad \hookrightarrow \pi\left(\delta(t-t_0) + \delta(t+t_0)\right) \xleftrightarrow{F} 2\pi \cos(-t_0\omega)$ duality

$t_0 = -1 \Longrightarrow \frac{1}{2}\left(\delta(t-1) + \delta(t+1)\right) \xleftrightarrow{F} \cos(\omega)$

$\Rightarrow y(t) = \frac{1}{2}\left(\delta(t-1) + \delta(t+1)\right)$

$$\boxed{x(t) = y ** h = \frac{\sin(\pi(t-1))}{2\pi(t-1)} + \frac{\sin(\pi(t+1))}{2\pi(t+1)}}$$

(c)

$$X(j\omega) = \pi e^{-|\omega|} = \pi\left(e^{-\omega}u(\omega) + e^{\omega}u(-\omega)\right)$$

$$e^{-at}u(t) \overset{F}{\longleftrightarrow} \frac{1}{a+j\omega} \Longrightarrow \frac{1}{a+jt} \overset{F}{\longleftrightarrow} \cancel{2\pi e^{at}u(-t)}$$

$$2\pi e^{+a\omega}u(-\omega)$$

$$\Rightarrow \pi e^{-\omega}u(\omega) \overset{F}{\longleftrightarrow} \frac{\frac{1}{2}}{+1+(-jt)}$$

$$\Rightarrow \pi e^{\omega}u(-\omega) \overset{F}{\longleftrightarrow} \frac{\frac{1}{2}}{1+jt}$$

$$\Longrightarrow x(t) = \frac{1}{1+t^2}$$

(d)

(e)

$$X(j\omega) = \frac{5j\omega+12}{j\omega+2}\cdot\frac{1}{j\omega+3} = \underbrace{\left(6 - \frac{j\omega}{2+j\omega}\right)}_{P(j\omega)}\underbrace{\left(\frac{1}{j\omega+3}\right)}_{q(j\omega)}$$

$$x(t) = p(t) ** q(t)$$

$$e^{-at}u(t) \overset{F}{\longleftrightarrow} \frac{1}{a+j\omega} \Longrightarrow q(t) = e^{-3t}u(t)$$

$$\left(e^{-at}(u(t))\right)' \overset{F}{\longleftrightarrow} \frac{j\omega}{a+j\omega} \Longrightarrow \frac{j\omega}{2+j\omega} \overset{F}{\longleftrightarrow} \left(e^{-2t}u(t)\right)' = -2e^{-2t}u(t) + e^{-2t}\delta(t)$$

$$\cancel{1 \overset{F}{\longleftrightarrow} 2\pi\delta(\omega)} \longleftrightarrow 6 \overset{F}{\longleftrightarrow} 12\pi\delta(\omega)$$

$$\delta(t) \overset{F}{\longleftrightarrow} 1 \Longrightarrow 6\delta(t) \overset{F}{\longleftrightarrow} 6$$

$$x(t) = p ** q = \left(\cancel{12\pi\delta(\omega)} + e^{-2t}\left(2u(t) - \delta(t)\right)\right) ** \left(e^{-3t}u(t)\right)$$

$$6\delta(t)$$

$$= \left(6\delta(t) + e^{-2t}\left(2u(t) - \delta(t)\right)\right) ** \left(e^{-3t}u(t)\right)$$

(a) (3)

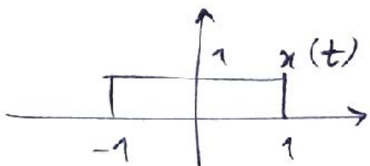
$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt = A\int_{-\frac{m}{2}}^{\frac{m}{2}} e^{-j\omega t}\, dt = \frac{A e^{-j\omega t}}{-j\omega}\Bigg|_{-\frac{m}{2}}^{\frac{m}{2}}$$

$$= \frac{A \times 2 \sin\left(\frac{j\omega m}{2}\right)}{+j\omega}$$

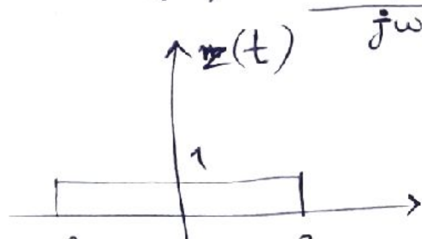
$$y(t) = -x(t-0{,}5) + \frac{3}{4} z(t-3{,}5)$$

$$Y(j\omega) = \frac{-2e^{-\frac{j\omega}{2}} \sin(j\omega) + \frac{3}{2} e^{-\frac{7j\omega}{2}} \sin(2j\omega)}{j\omega}$$

(b)



$$X(j\omega) = \frac{2 \sin(j\omega)}{j\omega}$$



$$Z(j\omega) = \frac{2 \sin(2j\omega)}{j\omega}$$

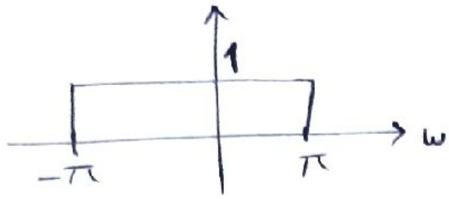
$$x(t) = \frac{\sin(\pi t)}{\pi t} \qquad p(t) = \cos(4\pi t) \qquad q(t) = \frac{\sin(5\pi t)}{\pi t}$$

$$H(\omega) = \begin{cases} 1 & \omega \geq 2\pi \\ \emptyset & \omega < 2\pi \end{cases}$$

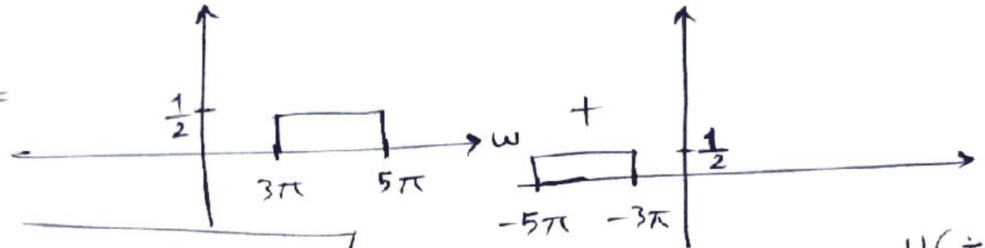
$$a(t) = x(t)\,p(t) \qquad A(j\omega) = \frac{1}{2\pi}\, X(j\omega) ** P(j\omega)$$

$$P(j\omega) = \pi\left(\delta(\omega - 4\pi) + \delta(\omega + 4\pi)\right)$$

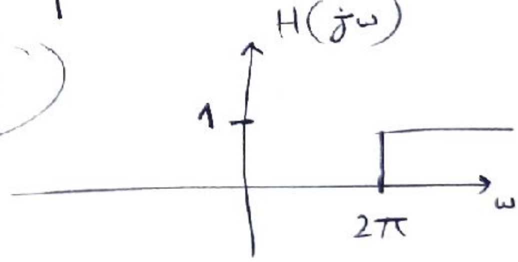
$X(j\omega) =$



$\Longrightarrow A(j\omega) =$

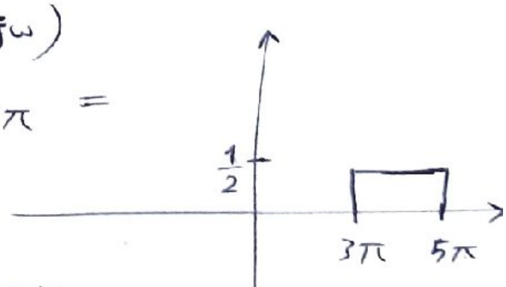


$$= \boxed{\begin{cases} \frac{1}{2} & 3\pi < |\omega| < 5\pi \\ \emptyset & \text{o.w} \end{cases}} \longleftarrow A(j\omega)$$



$$b(t) = h(t) ** a(t) \qquad B(j\omega) = A(j\omega) \cdot H(j\omega)$$

$$B(j\omega) = \begin{cases} \frac{1}{2} & 3\pi < \omega < 5\pi \\ \emptyset & \text{o.w} \end{cases} =$$



$$x_2(t) = \frac{\sin(Wt)}{\pi t} \xleftrightarrow{F} X_2(j\omega) = \begin{cases} 1 & |\omega| < W \\ \emptyset & \text{o.w} \end{cases}$$

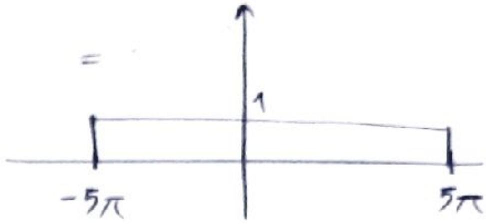
$$\frac{1}{2}\, \frac{\sin(\pi t)}{\pi t}\, e^{-j4\pi t} \xleftrightarrow{F} \begin{cases} \frac{1}{2} & 3\pi < \omega < 5\pi \\ \emptyset & \text{o.w} \end{cases}$$

$$\Longrightarrow \boxed{b(t) = \frac{1}{2}\, \frac{\sin(\pi t)}{\pi(t)}\, e^{-4\pi j t}}$$

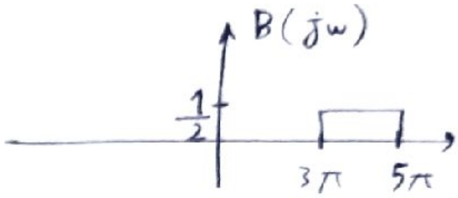
$c(t) = b(t) \cdot q(t) \qquad C(j\omega) = \frac{1}{2\pi} B(j\omega) ** q(j\omega)$

$$\frac{\sin(Wt)}{\pi t} \xleftrightarrow{F} \begin{cases} 1 & |\omega| < W \\ 0 & \text{o.w} \end{cases} \longrightarrow q(j\omega) = \begin{cases} 1 & |\omega| < 5\pi \\ 0 & \text{o.w} \end{cases}$$

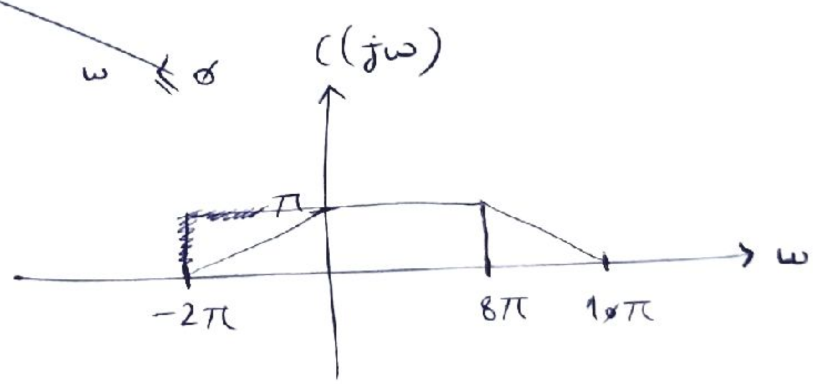
$$q(j\omega) ** B(j\omega) = \int_{-\infty}^{+\infty} B(j\tau)\, q(j(\omega-\tau))\, d\tau \quad =$$



$$= \frac{1}{2} \int_{3\pi}^{5\pi} q(j(\omega-\tau))\, d\tau$$



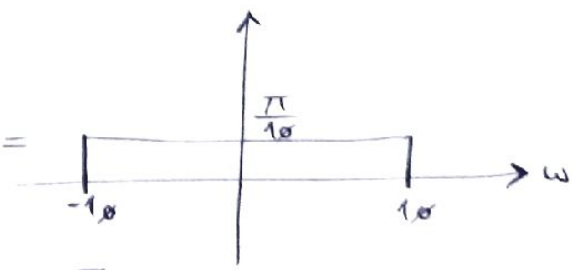
$$= \cancel{\begin{cases} 0 & \omega > 10\pi \\ \pi & 0 < \omega \leq 10\pi \\ & \omega \leq 0 \end{cases}}$$



5

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$

$$x(t) = \frac{\sin(10t) \times \pi}{\pi \times 10 t} \Longrightarrow X(j\omega) =$$



$$|X(j\omega)| = \frac{\pi}{10}$$

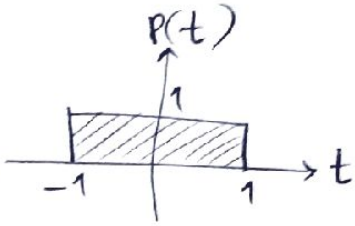
$$\frac{1}{2\pi} \int_{\cancel{-\infty} \to -10}^{\cancel{+\infty} \to 10} \frac{\pi^2}{100}\, d\omega = \boxed{\frac{\pi}{10}}$$

6

$$p(t) = \begin{cases} 1 & |t| < 1 \\ \emptyset & \text{o.w} \end{cases} \Longrightarrow P(j\omega) = \frac{2\sin(j\omega)}{j\omega}$$

$$P(\omega) = \frac{2\sin(\omega)}{\omega}$$

$$\int_{-\infty}^{+\infty} |p(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |P(\omega)|^2 \, d\omega$$



$$\Longrightarrow \int_{-\infty}^{+\infty} \left| \frac{\sin(\omega)}{\omega} \right|^2 d\omega = \frac{2\pi}{4} \underbrace{\int_{\cancel{-\infty} \to -1}^{\cancel{+\infty} \to 1} |p(t)|^2 \, dt}_{2} = \boxed{\pi}$$