Signals HW2 « in the name of GOD »

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(a) (1)

$$\pi(t) = \begin{cases} 1 & -0{,}5 < t < 0{,}5 \\ 0 & \text{o.w} \end{cases} = u(t+0{,}5) - u(t-0{,}5)$$

$$x(t) = \pi(t-0{,}5) - \pi(t-1{,}5) = u(t) - u(t-1) - u(t-1) + u(t-2)$$
$$= u(t) - 2u(t-1) + u(t-2)$$

پاسخ سیستم به $u(t) \leftarrow \int_{-\infty}^{t} h(\tau)\,d\tau$

$$y(t) = \int_{-\infty}^{t} e^{-t}\big(u(t) - u(t-1)\big)d\tau + \int_{-\infty}^{t-2} e^{-\tau}\big(u(\tau) - u(\tau-1)\big)d\tau$$
$$-2\int_{-\infty}^{t-1} e^{-\tau}\big(u(\tau) - u(\tau-1)\big)d\tau = \cancel{3 + 3e^{-1} - 2e^{-t} - 2e^{-(t-1)} - 2e^{-(t-2)}}$$
$$= \left(-e^{-t} - e^{-(t-2)} + 2e^{-(t-1)}\right)u(t) + \left(e^{-t} + e^{-(t-2)} - 2e^{-(t-1)}\right)u(t-1)$$

(b)

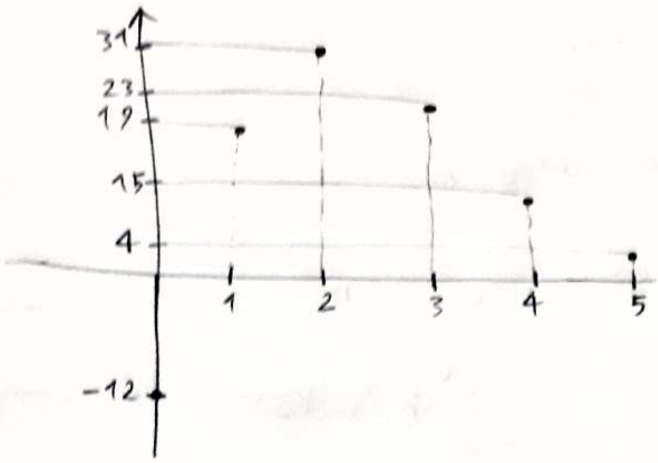
$$y(t) = \int x(\tau)\,h(t-\tau)\,d\tau \quad \cancel{\int x(t)\,\delta(t)\,d\tau}$$

از آن جا که $h(t) = \delta(t)$ عامل همانی در مضاحب conv است ⇐

$$y(t) = e^{-t}u(t) + \frac{1}{2}e^{-(t-1)}u(t-1) + \frac{3}{10}e^{-(t-2)}u(t-2) + \frac{2}{10}e^{-(t-3)}u(t-3)$$

② ⓐ

$$y[n] = 4x[n] + 3x[n-1] + 2x[n-2] + x[n-3]$$



ⓑ

$$y[n] = \sum x[k]h[n-k] = \sum_{-\infty}^{+\infty} h[k]x[n-k]$$

$$= \sum_{\varnothing}^{n} (0{,}8)^k u[k]u[n-k] - \sum_{\varnothing}^{n-6} (0{,}8)^k u[k]u[n-k-6]$$

$$= \underbrace{\frac{1-(0{,}8)^{n+1}}{0{,}2}}_{u[n]} - \frac{1-(0{,}8)^{n-5}}{0{,}2}\, u[n-6]$$

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③ ⓐ

$$y[n] = \sum_{-1}^{3} h[k]x[n-k] = \overset{1}{\cancel{h[-1]}}x[n+1] + \overset{3}{\cancel{h[0]}}x[n] + \overset{2}{\cancel{h[1]}}x[n-1]$$

$$+ \underset{-1}{\cancel{h[2]}}x[n-2] + \underset{1}{\cancel{h[3]}}x[n-3]$$

$$= 3\delta[n+1] - 2\delta[n] + 9\delta[n] \overset{①}{\cancel{-6\delta[n-1]}} + \overset{①}{\cancel{6\delta[n-1]}} - 4\delta[n-2]$$

$$-3\delta[n-2] + 2\delta[n-3] + 3\delta[n-3] - 2\delta[n-4]$$

$$= 3\delta[n+1] + 7\delta[n] - 7\delta[n-2] + 5\delta[n-3] - 2\delta[n-4]$$

ⓑ

$y[n] =$ @ماندبخش

$$= u\overset{*}{\cancel{[n+2]}} - \cancel{u[n-2]} + 3u[n+1] - 3u[n-3] + 2u[n] - 2u[n-4]$$

$$- u[n-1] + u[n-5] + \overset{①}{\cancel{u[n-2]}} - u[n-6]$$

$$= u[n+2] + 3u[n+1] + 2u[n] - u[n-1] - 3u[n-3] - 2u[n-4]$$
$$+ u[n-5] - u[n-6]$$

$$y(t) = x ** \underbrace{(h_1 + h_2 ** h_3 + h_2 ** h_4)}_{h_{eq}} \qquad \cancel{h_2 = h_3 = \delta(t)}$$

$$\cancel{h_{eq} = e^{-t}u(t) + \underbrace{\delta(t) ** \delta(t)}_{\delta(t)} + \underbrace{\delta(t) ** \delta(t-1)}_{\delta(t-1)} +}$$

$$\cancel{= e^{-t}u(t) + \delta(t) + \delta(t-1)}$$

$$h_{eq} = e^{-t}u(t) + tu(t) + 2(t-1)u(t-1) + (t-2)u(t-1)$$

theorem : $\delta[n] \longrightarrow h[n]$

$u[n] \longrightarrow \sum_{-\infty}^{n} h[k] \longrightarrow$ unit response

$$y_1(t) = \int_{-\infty}^{t} h_1(\tau)\,d\tau = \left[\int_{\cancel{-\infty}\to 0}^{t} e^{-\tau}\,\cancel{u(\tau)}_{\to 1}\,d\tau\right]u(t) = \left[-e^{-\tau}\Big|_{0}^{t}\right]u(t) = \boxed{-\left(e^{-t} - 1\right)u(t)}$$

$$w = x ** h_2 = ?$$

$$w = \int_{-\infty}^{t} u(\tau)\,d\tau - \int_{-\infty}^{t} u(\tau-1)\,d\tau = t\,u(t) - (t-1)\,u(t-1)$$

$$y_3 = w ** h_3 = \int_{-\infty}^{+\infty} \big(\tau u(\tau) - (\tau-1)u(\tau-1)\big)\big(u(t-\tau) - u(t-\tau-1)\big)\,d\tau$$

$$= \frac{t^2}{2}u(t) - \frac{(t-1)^2}{2}u(t-1) - \left(\frac{t^2}{2} - t + \frac{1}{2}\right)u(t-1) + \left(\frac{(t-1)^2}{2} - (t-1) + \frac{1}{2}\right)u(t-2)$$

$$y_4 = w ** h_4 = \cancel{\int_{-\infty}^{+\infty}} w ** \delta(t-1) = (t-1)\,u(t-1) - (t-2)\,u(t-2)$$

$$y(t) = x ** \underbrace{(h_1 + h_2 ** h_3 + h_2 ** h_4)}_{h_{eq}} \qquad \cancel{h_2 = h_3 = \delta(t)}$$ ④

$$\cancel{h_{eq} = e^{-t}u(t) + \underbrace{\delta(t) ** \delta(t)}_{\delta(t)} + \underbrace{\delta(t) ** \delta(t-1)}_{\delta(t-1)} +}$$

$$\cancel{= e^{-t}u(t) + \delta(t) + \delta(t-1)} \qquad h_{eq} = e^{-t}u(t) + tu(t) + 2(t-1)u(t-1) + (t-2)u(t-1)$$

theorem : $\delta[n] \longrightarrow h[n]$

$u[n] \longrightarrow \sum_{-\infty}^{n} h[k] \longrightarrow$ unit response

$$y_1(t) = \int_{-\infty}^{t} h_1(\tau)d\tau = \left[\int_{\underset{0}{-\infty}}^{t} e^{-\tau} \underset{1}{u(\tau)}\, d\tau\right] u(t) = \left[-e^{-\tau}\Big|_{0}^{t}\right] u(t) = -(e^{-t} - 1) \times u(t)$$

$$\cancel{y_3(t) = \int_{-\infty}^{t} u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = u(t)} \Rightarrow$$ ~~برای y_3 و y_4 هم می‌توان از قضیه بالا استفاده کرد.~~

$$\cancel{y_3(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = u(t)}$$

$$\cancel{y_4(t) = \int_{-\infty}^{t} \delta(\tau-1)d\tau = u(t-1)}$$

ⓐ $h(t) = u(t+1) - u(t-1)$

$\cancel{h(\overset{+1}{2}) = u(3) -}$ $\to 0$

$h(-1) = \underset{1}{u(0)} - \cancel{u(-2)} = 1 \Longrightarrow$ memoryless X

$\Longrightarrow$ causal X

$\cancel{\sum_{-\infty}^{+\infty} |h(t)| = 2 < \infty} \Longrightarrow$ stable ✓

$\int_{-\infty}^{+\infty} |h(t)|\, dt = 2 < \infty$

memoryless ⑤

$$h(t) = \begin{cases} \text{هرچیزی} & t = 0 \\ 0 & t \neq 0 \end{cases}$$

causal

$$h(t) = \begin{cases} \text{هرچیزی} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

stability

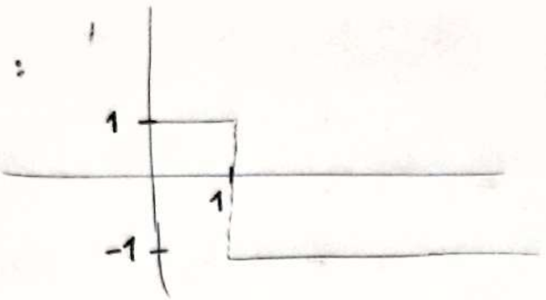
$$\int_{-\infty}^{+\infty} |h(t)|\, dt < \infty$$

(b)

$h(t) = u(t) - 2u(t-1)$

$h(5) = u(5) - 2u(4) = -1 \neq \varnothing \Longrightarrow$ memoryless X

$h(t):$



$h(t)\big|_{t<\varnothing} = \varnothing \Longrightarrow$ causal ✓

$\int_{-\infty}^{+\infty} |h(t)| = \infty \Longrightarrow$ stable X

(c)

$h(t) = e^{-2|t|}$ $\qquad h(-1) = e^{-2} \neq \varnothing \Longrightarrow$ memoryless X

causal X

$\int_{-\infty}^{+\infty} e^{-2|t|} dt = \int_{-\infty}^{\varnothing} e^{2t} dt + \int_{\varnothing}^{\infty} e^{-2t} dt = \frac{1}{2} + \frac{1}{2} = 1 < \infty \Longrightarrow$ stable ✓

(d)

$h(t) = \cos(\pi t) u(t)$ $\qquad h(t)\big|_{t<\varnothing} = \varnothing \Longrightarrow$ causal ✓

$h(1) = -1 \neq \varnothing \Longrightarrow$ memoryless X $\qquad \int_{-\infty}^{+\infty} |h(t)| dt = \int_{\cancel{-\infty}\ \varnothing}^{+\infty} |\cos(\pi t)| dt$

$= \infty \Longrightarrow$ stable X

(e)

$h[n] = 2^n u[-n]$

$h[-1] = \frac{1}{2} \neq \varnothing$ $\Longrightarrow$ memoryless X

$\Longrightarrow$ causal X

$\sum_{-\infty}^{+\infty} |2^n u[-n]| = \sum_{-\infty}^{\varnothing} 2^n$

$\cancel{\times\ 1(1-2}$

$= \sum_{\varnothing}^{\infty} \left(\frac{1}{2}\right)^n$

$= \frac{1}{1-\frac{1}{2}} = 2 < \infty$

stable ✓

(f)

$h[n] = e^{2n} u[n-1]$ $\qquad h[n]\big|_{n<\varnothing} = \varnothing \Longrightarrow$ causal ✓

$h[1] = e^{2} \neq \varnothing \Longrightarrow$ memoryless X

$\sum_{-\infty}^{+\infty} |e^{2n} u[n-1]| = \sum_{1}^{\infty} e^{2n} = \infty \Longrightarrow$ stable X

$h[n] = \left(\frac{1}{2}\right)^n u[n]$ $\qquad h[n]\Big|_{n<0} = 0 \Longrightarrow$ Causal ✓ (g)

$h[1] = \frac{1}{2} \neq 0 \Longrightarrow$ memoryless ✗

$$\sum_{-\infty}^{+\infty} \left|\left(\frac{1}{2}\right)^n u[n]\right| = \sum_{0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{1-\frac{1}{2}} = 2 < \infty \Longrightarrow \text{stable} ✓$$

$h[n] = \cos\left(\frac{\pi}{2}n\right) u[n+3]$ (h)

$h[-2] = -1 \neq 0 \Longrightarrow$ memoryless ✗

Causal ✗

$$\sum_{-\infty}^{+\infty} \left|\cos\left(\frac{\pi}{2}n\right) u[n+3]\right| = \sum_{-3}^{\infty} \left|\cos\left(\frac{\pi}{2}n\right)\right| = \infty \Longrightarrow \text{stable ✗}$$

$y(t) = x(t) + \alpha_1 x(t-\tau_1) + \alpha_2 x(t-\tau_2)$ (6)

stability : درصورتی که x کران‌دار باشد، y پایدار خواهد بود.

causal است زیرا خروجی به ورودی در حال یا گذشته بستگی دارد.

linearity : ✓

$x_1 \longrightarrow y_1 \quad x_2 \longrightarrow y_2 \qquad x_3 = ax_1 + bx_2 \Longrightarrow y_3 = ay_1 + by_2$

time invariance : ✓

$y_1(t) = x_1(t) + \alpha_1 x_1(t-\tau_1) + \alpha_2 x_1(t-\tau_2)$

$x_2(t) = x_1(t-t_0)$

$y_2(t) = x_2(t) + \alpha_1 x_2(t-\tau_1) + \alpha_2 x_2(t-\tau_2)$

$= x_1(t-t_0) + \alpha_1 x_1(t-t_0-\tau_1) + \alpha_2 x_1(t-t_0-\tau_2)$

$= y_1(t-t_0)$ ✓

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step response $= w = \sum_{-\infty}^{n} h[k] = \int_{-\infty}^{t} h(\tau)\,d\tau$ (7)

(a) $w[n] = \sum_{-\infty}^{n} \left(\frac{1}{2}\right)^k u[k] = \sum_{0}^{n} \left(\frac{1}{2}\right)^k = \cancel{\frac{1}{1-\frac{1}{2}}} \; \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} = \boxed{2-\left(\frac{1}{2}\right)^n} u[n]$

(b) $\cancel{w[n]} = w(t) = \int_{-\infty}^{t} e^{-|\tau|}\,d\tau = \int_{-\infty}^{0} e^{\tau}\,d\tau + \int_{0}^{t} e^{-\tau}\,d\tau$

b

$$w(t) = \int_{-\infty}^{t} e^{-|\tau|}\, d\tau = \begin{cases} \int_{-\infty}^{0} e^{\tau} d\tau + \int_{0}^{t} e^{-\tau} d\tau & t > 0 \\ \int_{-\infty}^{t} e^{\tau} d\tau & t \leq 0 \end{cases}$$

$$= \begin{cases} e^{-t} & t > 0 \\ e^{t} & t \leq 0 \end{cases} = e^{-|t|}$$

$$w(t) = \begin{cases} \int_{-\infty}^{0} \delta(\tau) d\tau - \int_{-\infty}^{1} \delta(\tau-1) d\tau + \int_{0}^{t} \delta(\tau) d\tau - \int_{1}^{t} \delta(\tau-1) d\tau & t > 0 \\ \int_{-\infty}^{t} \delta(\tau) d\tau - \int_{-\infty}^{t} \delta(\tau-1) d\tau & t \leq 0 \end{cases}$$

$$= \begin{cases} \\ \end{cases}$$

c

$$w(t) = \int_{-\infty}^{t} [\delta(\tau) - \delta(\tau-1)]\, d\tau = \int_{-\infty}^{t} \delta(\tau) d\tau + \int_{-\infty}^{t} -\delta(\tau-1) d\tau$$

$$= \int_{-\infty}^{0} \delta(\tau) d\tau + \int_{0}^{t} \delta(\tau) d\tau + \int_{-\infty}^{1} -\delta(\tau-1) d\tau + \int_{1}^{t} -\delta(\tau-1) d\tau$$

$$= u(t) - u(t-1)$$

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$$x(t) = u(t-1{,}5) - u(t-2{,}5)$$

$$y(t) = u(t-2) + (t-1)\big(u(t-1) - u(t-2)\big) = (t-1)u(t-1) + (-t+2)u(t-2)$$

$$y(t) = (t-1)\,x(t+0{,}5) + \sum_{k=\emptyset}^{\infty} x(t-k-0{,}5)$$

$y(t) = h(t) \Leftarrow x(t) = \delta(t)$ اذا

$$h(t) = (t-1)\,\delta(t+0{,}5) + \sum_{k=\emptyset}^{\infty} \delta(t-k-0{,}5)$$

$$= -1{,}5\,\delta(t+0{,}5) + \sum_{k=\emptyset}^{\infty} \delta(t-k-0{,}5)$$

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9 a

$$x(t) = u(t) \qquad h(t) = \left(-\tfrac{1}{2}t + 1\right)\big(u(t) - u(t-2)\big)$$

$$y(t) = \int_{-\infty}^{+\infty} u(\tau)\left(-\tfrac{1}{2}t + \tfrac{1}{2}\tau + 1\right)\big(u(t-\tau) - u(t-\tau-2)\big)\,d\tau$$

$$= \int_{\emptyset}^{\infty} \left(-\tfrac{1}{2}(t-\tau) + 1\right)u(t-\tau)\,d\tau - \int_{\emptyset}^{\infty} \left(-\tfrac{1}{2}(t-\tau) + 1\right)u(t-\tau-2)\,d\tau$$

$$= \left[\int_{\emptyset}^{t} \left(-\tfrac{1}{2}(t-\tau) + 1\right)d\tau\right]u(t) - \left[\int_{\emptyset}^{t-2} \left(-\tfrac{1}{2}(t-\tau) + 1\right)d\tau\right]u(t-2)$$

$$= \left(t - \tfrac{1}{4}t^2\right)u(t) - \tfrac{1}{4}t^2 u(t-2)$$

$$x(t) = u(t) - u(t-1) + u(t-2) - 2u(t-4) + u(t-3)$$ (b)

$$y(t) = \int_{\varnothing}^{\infty} u(\tau)u(t-\tau)\,d\tau - \int_{\varnothing}^{\infty} u(\tau)u(t-\tau-1)\,d\tau - \int_{1}^{\infty} u(\tau-1)u(t-\tau)\,d\tau + \int_{1}^{\infty} u(\tau-1)u(t-\tau-1)\,d\tau$$

$$+ \int_{2}^{\infty} u(\tau-2)u(t-\tau)\,d\tau - \int_{2}^{\infty} u(\tau-2)u(t-\tau-1)\,d\tau - 2\int_{4}^{\infty} u(\tau-4)u(t-\tau)\,d\tau + 2\int_{4}^{\infty} u(\tau-4)u(t-\tau-1)\,d\tau$$

$$+ \int_{3}^{\infty} u(\tau-3)u(t-\tau)\,d\tau - \int_{3}^{\infty} u(\tau-3)u(t-\tau-1)\,d\tau$$

$$= t\,u(t) - (t-1)u(t-1) - (t-1)u(t) + (t-2)u(t-1) + (t-2)u(t)$$

$$- (t-3)u(t-1) - 2(t-4)u(t) + 2(t-5)u(t-1)$$

$$+ (t-3)u(t) - (t-4)u(t-1)$$

$$= \cancel{t u(t)}\; u(t)\left[4\right] + u(t-1)\left[-4\right]$$