Signals HW2 " in the name of GOD "
amirmohammad pirhosseinloo
9531094

(a) (1)

$$\pi(t) = \begin{cases} 1 & -0.5 < t < 0.5 \\ 0 & \text{o.w} \end{cases} = u(t+0.5) - u(t-0.5)$$

$$x(t) = \pi(t-0.5) - \pi(t-1.5) = u(t) - u(t-1) - u(t-1) + u(t-2)$$
$$= u(t) - 2u(t-1) + u(t-2)$$

پاسخ سیستم به $u(t) \leftarrow \int_{-\infty}^{t} h(\tau)d\tau$

$$y(t) = \int_{-\infty}^{t} e^{-\tau}(u(\tau)-u(\tau-1))d\tau + \int_{-\infty}^{t-2} e^{-\tau}(u(\tau)-u(\tau-1))d\tau$$
$$-2\int_{-\infty}^{t-1} e^{-\tau}(u(\tau)-u(\tau-1))d\tau = 3 + 3e^{-1} - 2e^{-t} - 2e^{-(t-1)} - 2e^{-(t-2)}$$

(b)

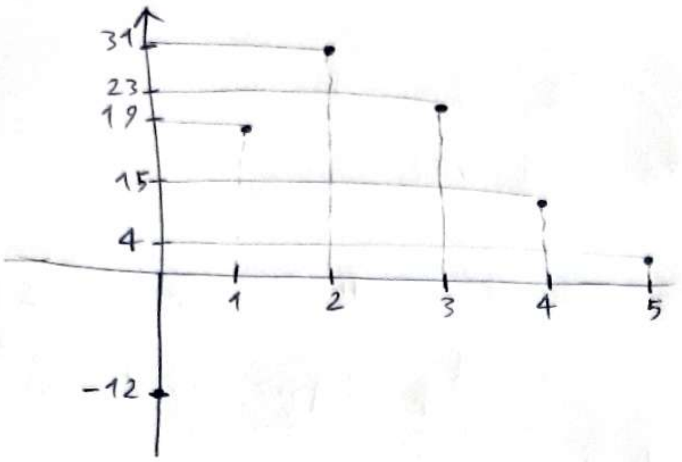
$$y(t) = \int x(\tau)h(t-\tau)d\tau \quad \cancel{\int x(t)\delta(t-\tau)d\tau}$$

از آن جا که $h(t) = \delta(t)$ عامل همانی در محاسبه conv است $\Leftarrow$

$$y(t) = e^{-t}u(t) + \frac{1}{2}e^{-(t-1)}u(t-1) + \frac{3}{10}e^{-(t-2)}u(t-2) + \frac{2}{10}e^{-(t-3)}u(t-3)$$

② ⓐ

$$y[n] = 4x[n] + 3x[n-1] + 2x[n-2] + x[n-3]$$



ⓑ

$$y[n] = \sum x[k]h[n-k] = \sum_{-\infty}^{+\infty} h[k]x[n-k]$$

$$= \sum_{0}^{n} (0.8)^k u[k]u[n-k] - \sum_{0}^{n-6} (0.8)^k u[k]u[n-k-6]$$

$$= \frac{1-(0.8)^{n+1}}{0.2} - \frac{1-(0.8)^{n-5}}{0.2}$$

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③ ⓐ

$$y[n] = \sum_{-1}^{3} h[k]x[n-k] = \overset{1}{\cancel{h[-1]}}x[n+1] + \overset{3}{\cancel{h[0]}}x[n] + \overset{2}{\cancel{h[1]}}x[n-1]$$
$$+ \underset{-1}{\cancel{h[2]}}x[n-2] + \underset{1}{\cancel{h[3]}}x[n-3]$$

$$= 3\delta[n+1] - 2\delta[n] + 9\delta[n] \overset{1}{\cancel{-6\delta[n-1]}} \overset{1}{\cancel{+6\delta[n-1]}} - 4\delta[n-2]$$
$$- 3\delta[n-2] + 2\delta[n-3] + 3\delta[n-3] - 2\delta[n-4]$$

$$= 3\delta[n+1] + 7\delta[n] - 7\delta[n-2] + 5\delta[n-3] - 2\delta[n-4]$$

ⓑ

$y[n] =$ مانندبخش ⓐ

$$= \overset{*}{\cancel{u[n+2]}} \overset{1}{\cancel{-u[n-2]}} + 3u[n+1] - 3u[n-3] + 2u[n] - 2u[n-4]$$
$$- u[n-1] + u[n-5] \overset{1}{\cancel{+u[n-2]}} - u[n-6]$$
$$= u[n+2] + 3u[n+1] + 2u[n] - u[n-1] - 3u[n-3] - 2u[n-4]$$
$$+ u[n-5] - u[n-6]$$

$$y(t) = x ** \underbrace{(h_1 + h_2 ** h_3 + h_2 ** h_4)}_{h_{eq}} \qquad h_2 = h_3 = \delta(t)$$ ④

$$h_{eq} = e^{-t}u(t) + \underbrace{\delta(t) ** \delta(t)}_{\delta(t)} + \underbrace{\delta(t) ** \delta(t-1)}_{\delta(t-1)}$$

$$= \boxed{e^{-t}u(t) + \delta(t) + \delta(t-1)}$$

theorem :
$$\delta[n] \longrightarrow h[n]$$
$$u[n] \longrightarrow \sum_{-\infty}^{n} h[k] \longrightarrow \text{unit response}$$

$$y_1(t) = \int_{-\infty}^{t} h_1(\tau)\,d\tau = \int_{\cancel{-\infty}\to 0}^{t} e^{-\tau}\cancel{u(\tau)}_{\to 1}\,d\tau = -e^{-\tau}\Big|_{0}^{t} = \boxed{-(e^{-t} \pm 1)}$$

$$\cancel{y_3(t) = \int_{-\infty}^{t}} u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t) \Longrightarrow$$ برای y_3 و y_4 هم می‌توان
از قضیه بالا استفاده کرد.

$$y_3(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t)$$

$$y_4(t) = \int_{-\infty}^{t} \delta(\tau-1)\,d\tau = u(t-1)$$

ⓐ $h(t) = u(t+1) - u(t-1)$

$\cancel{h(+1) = u(3) -}$

$h(-1) = \underbrace{u(0)}_{1} - \cancel{u(-2)} = 1 \Longrightarrow$ memory less X

$\Longrightarrow$ causal X

$\cancel{\sum_{-\infty}^{+\infty} |h(t)| = 2 < \infty} \Longrightarrow$ stable ✓

$$\int_{-\infty}^{+\infty} |h(t)|\,dt = 2 < \infty$$

memory less ⑤

$$h(t) = \begin{cases} \text{متغیر} & t = 0 \\ 0 & t \neq 0 \end{cases}$$

causal

$$h(t) = \begin{cases} \text{متغیر} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

stability

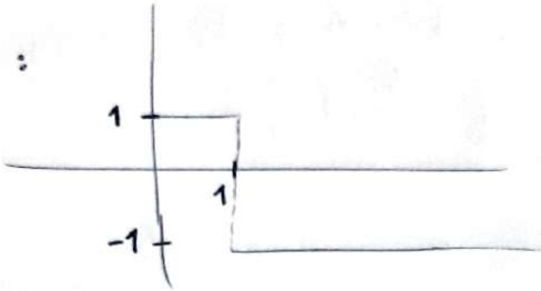
$$\int_{-\infty}^{+\infty} |h(t)|\,dt < \infty$$

(b)

$h(t) = u(t) - 2u(t-1)$

$h(5) = u(5) - 2u(4) = -1 \neq \emptyset \Longrightarrow$ memoryless X

$h(t):$



$h(t)\big|_{t<\emptyset} = \emptyset \Longrightarrow$ causal ✓

$\int_{-\infty}^{+\infty} |h(t)| = \infty \Longrightarrow$ stable X

(c)

$h(t) = e^{-2|t|}$ $\quad h(-1) = e^{-2} \neq \emptyset \Longrightarrow$ memoryless X

causal X

$$\int_{-\infty}^{+\infty} e^{-2|t|}\,dt = \int_{-\infty}^{\emptyset} e^{2t}\,dt + \int_{\emptyset}^{\infty} e^{-2t}\,dt = \frac{1}{2} + \frac{1}{2} = 1 < \infty \Longrightarrow \text{stable ✓}$$

(d)

$h(t) = \cos(\pi t)\,u(t)$ $\quad h(t)\big|_{t<\emptyset} = \emptyset \Longrightarrow$ causal ✓

$h(1) = -1 \neq \emptyset \Longrightarrow$ memoryless X

$$\int_{-\infty}^{+\infty} |h(t)|\,dt = \int_{\cancel{-\infty}\ \emptyset}^{+\infty} |\cos(\pi t)|\,dt$$

$= \infty \Longrightarrow$ stable X

(e)

$h[n] = 2^n u[-n]$

$h[-1] = \frac{1}{2} \neq \emptyset \Longrightarrow$ memoryless X

$\Longrightarrow$ causal X

$$\sum_{-\infty}^{+\infty} |2^n u[-n]| = \sum_{-\infty}^{\emptyset} 2^n$$

$\cancel{\times\ 1(1-2}$

$$= \sum_{\emptyset}^{\infty} \left(\frac{1}{2}\right)^n$$

$$= \frac{1}{1-\frac{1}{2}} = 2 < \infty$$

stable ✓

(f)

$h[n] = e^{2n} u[n-1]$ $\quad h[n]\big|_{n<\emptyset} = \emptyset \Longrightarrow$ causal ✓

$h[1] = e^2 \neq \emptyset \Longrightarrow$ memoryless X

$$\sum_{-\infty}^{+\infty} |e^{2n} u[n-1]| = \sum_{1}^{\infty} e^{2n} = \infty \Longrightarrow \text{stable X}$$

$$h[n] = \left(\tfrac{1}{2}\right)^n u[n] \qquad h[n]\Big|_{n<\emptyset} = \emptyset \Longrightarrow \text{causal} \checkmark$$ (g)

$$h[1] = \tfrac{1}{2} \neq \emptyset \Longrightarrow \text{memoryless } \times \qquad \sum_{-\infty}^{+\infty} \left|\left(\tfrac{1}{2}\right)^n u[n]\right| = \sum_{\emptyset}^{\infty} \left(\tfrac{1}{2}\right)^n$$

$$= \frac{1}{1-\frac{1}{2}} = 2 < \infty \Longrightarrow \text{stable} \checkmark$$

$$h[n] = \cos\left(\tfrac{\pi}{2} n\right) u[n+3] \qquad \sum_{-\infty}^{+\infty} \left|\cos\left(\tfrac{\pi}{2} n\right) u[n+3]\right|$$ (h)

$$h[-2] = -1 \neq \emptyset \Longrightarrow \begin{array}{l}\text{memoryless } \times \\ \text{causal } \times\end{array} \qquad = \sum_{-3}^{\infty} \left|\cos\left(\tfrac{\pi}{2} n\right)\right| = \infty \Longrightarrow \text{stable } \times$$

$$y(t) = x(t) + \alpha_1 x(t-\tau_1) + \alpha_2 x(t-\tau_2)$$ (6)

stability : درصورتی که x کران‌دار باشد، y پایدار خواهد بود.

causal است زیرا خروجی به ورودی درحال یا گذشته بستگی دارد.

linearity : ✓

$$x_1 \longrightarrow y_1 \quad x_2 \longrightarrow y_2 \qquad x_3 = ax_1 + bx_2 \Longrightarrow y_3 = ay_1 + by_2$$

time invariance : ✓

$$y_1(t) = x_1(t) + \alpha_1 x_1(t-\tau_1) + \alpha_2 x_1(t-\tau_2)$$

$$x_2(t) = x_1(t-t_\emptyset)$$

$$y_2(t) = x_2(t) + \alpha_1 x_2(t-\tau_1) + \alpha_2 x_2(t-\tau_2)$$

$$= x_1(t-t_\emptyset) + \alpha_1 x_1(t-t_\emptyset-\tau_1) + \alpha_2 x_1(t-t_\emptyset-\tau_2)$$

$$= y_1(t-t_\emptyset) \checkmark$$

$$\text{step response} = w = \sum_{-\infty}^{n} h[k] = \int_{-\infty}^{t} h(\tau)\,d\tau$$ (7)

$$w[n] = \sum_{-\infty}^{n} \left(\tfrac{1}{2}\right)^k u[k] = \sum_{\emptyset}^{n} \left(\tfrac{1}{2}\right)^k = \cancel{\frac{1}{1-\frac{1}{2}}} \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} = \boxed{2 - \left(\tfrac{1}{2}\right)^n}$$ (a)

$$\cancel{w[n]} = w(t) = \int_{-\infty}^{t} e^{-|\tau|}\,d\tau = \int_{-\infty}^{\emptyset} e^{\tau}\,d\tau + \int_{\emptyset}^{t} e^{-\tau}\,d\tau$$ (b)

b

$$w(t) = \int_{-\infty}^{t} e^{-|\tau|}\, d\tau = \begin{cases} \int_{-\infty}^{0} e^{\tau}\, d\tau + \int_{0}^{t} e^{-\tau}\, d\tau & t > 0 \\ \int_{-\infty}^{t} e^{\tau}\, d\tau & t \leqslant 0 \end{cases}$$

$$= \begin{cases} e^{-t} & t > 0 \\ e^{t} & t \leqslant 0 \end{cases} = e^{-|t|}$$

$$\cancel{w(t) = \begin{cases} \int_{-\infty}^{0} \delta(\tau)\, d\tau - \int_{-\infty}^{1} \delta(\tau-1)\, d\tau + \int_{0}^{t} \delta(\tau)\, d\tau - \int_{1}^{t} \delta(\tau-1)\, d\tau & t > 0 \\ \int_{-\infty}^{t} \delta(\tau)\, d\tau - \int_{-\infty}^{t} \delta(\tau-1)\, d\tau & t \leqslant 0 \end{cases}}$$

$$\cancel{= \begin{cases} \\ \end{cases}}$$

c

$$w(t) = \int_{-\infty}^{t} [\delta(\tau) - \delta(\tau-1)]\, d\tau = \int_{-\infty}^{t} \delta(\tau)\, d\tau + \int_{-\infty}^{t} -\delta(\tau-1)\, d\tau$$

$$= \int_{-\infty}^{0} \delta(\tau)\, d\tau + \int_{0}^{t} \delta(\tau)\, d\tau + \int_{-\infty}^{1} -\delta(\tau-1)\, d\tau + \int_{1}^{t} -\delta(\tau-1)\, d\tau$$

$$= u(t) - u(t-1)$$

(8)

$$x(t)=u(t-1{,}5)-u(t-2{,}5)$$

$$y(t)=u(t-2)+(t-1)\big(u(t-1)-u(t-2)\big)=(t-1)u(t-1)+(-t+2)u(t-2)$$

$$y(t)=(t-1)\,x(t+0{,}5)+\sum_{k=\varnothing}^{\infty}x(t-k-0{,}5)$$

$$y(t)=h(t)\Leftarrow x(t)=\delta(t)$$

$$h(t)=(t-1)\,\delta(t+0{,}5)+\sum_{k=\varnothing}^{\infty}\delta(t-k-0{,}5)$$

$$=-1{,}5\,\delta(t+0{,}5)+\sum_{k=\varnothing}^{\infty}\delta(t-k-0{,}5)$$

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(9)
(a)

$$x(t)=u(t)\qquad h(t)=\left(-\tfrac{1}{2}t+1\right)\big(u(t)-u(t-2)\big)$$

$$y(t)=\int_{-\infty}^{+\infty}u(\tau)\left(-\tfrac{1}{2}t+\tfrac{1}{2}\tau+1\right)\big(u(t-\tau)-u(t-\tau-2)\big)\,d\tau$$

$$=\int_{\varnothing}^{\infty}\left(-\tfrac{1}{2}(t-\tau)+1\right)u(t-\tau)\,d\tau-\int_{\varnothing}^{\infty}\left(-\tfrac{1}{2}(t-\tau)+1\right)u(t-\tau-2)\,d\tau$$

$$=\left[\int_{\varnothing}^{t}\left(-\tfrac{1}{2}(t-\tau)+1\right)d\tau\right]u(t)-\left[\int_{\varnothing}^{t-2}\left(-\tfrac{1}{2}(t-\tau)+1\right)d\tau\right]u(t-2)$$

$$=\left(t-\tfrac{1}{4}t^{2}\right)u(t)-\tfrac{1}{4}t^{2}u(t-2)$$

ⓑ

$$x(t) = u(t) - u(t-1) + u(t-2) - 2u(t-4) + u(t-3)$$

$$y(t) = \int_{\varnothing}^{\infty} u(\tau)u(t-\tau)\,d\tau - \int_{\varnothing}^{\infty} u(\tau)u(t-\tau-1)\,d\tau - \int_{1}^{\infty} u(\tau-1)u(t-\tau)\,d\tau + \int_{1}^{\infty} u(\tau-1)u(t-\tau-1)\,d\tau$$

$$+ \int_{2}^{\infty} u(\tau-2)u(t-\tau)\,d\tau - \int_{2}^{\infty} u(\tau-2)u(t-\tau-1)\,d\tau - 2\int_{4}^{\infty} u(\tau-4)u(t-\tau)\,d\tau + 2\int_{4}^{\infty} u(\tau-4)u(t-\tau-1)\,d\tau$$

$$+ \int_{3}^{\infty} u(\tau-3)u(t-\tau)\,d\tau - \int_{3}^{\infty} u(\tau-3)u(t-\tau-1)\,d\tau$$

$$= t\,u(t) - (t-1)u(t-1) - (t-1)u(t) + (t-2)u(t-1) + (t-2)u(t)$$

$$- (t-3)u(t-1) - 2(t-4)u(t) + 2(t-5)u(t-1)$$

$$+ (t-3)u(t) - (t-4)u(t-1)$$

$$= \cancel{t\,u(t)}\; u(t)\left[4\right] + u(t-1)\left[-4\right]$$