9531014

«بنام خدا»

تمرین 5 سیگنال

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Subject :

Year : Month : Date :

a 1

$$X'(j\omega) = X(j\omega) + e^{-j\omega} X(j\omega)$$

$$= \left(1 + e^{-j\omega}\right) X(j\omega) = 2\cos\left(\frac{\omega}{2}\right) e^{-j\frac{\omega}{2}} X(j\omega)$$

$$\Longrightarrow \boxed{\omega_0' = \omega_0}$$

به دلیل ضرب شدن $X(j\omega)$ تنها در یک عدد

b 1

$$X'(j\omega) = j\omega X(j\omega) \Longrightarrow \boxed{\omega_0' = \omega_0}$$

c 1

$$X'(j\omega) = \frac{1}{2\pi} X(j\omega) ** X(j\omega)$$

convolution $\longrightarrow \omega_0' = 2\omega_0$

Kooshaa

$$X'(j\omega) = \frac{1}{2\pi} X(j\omega) ** \left(\delta(\omega-\omega_0) + \delta(\omega-\omega_0)\right)_{\times\pi}$$ d 1

$$= \frac{1}{2}\left(X(j(\omega-\omega_0)) + X(j(\omega+\omega_0))\right)$$

تابع در سمت چپ و راست ظاهر می شود. $\Leftarrow$ $\boxed{w_0' = 2w_0}$

$$X_a(j\omega) = \left[\frac{1}{j\omega} + \pi\delta(\omega)\right]\left(1 - e^{-3j\omega}\right)$$ a 2

دامنه محدود نیست $\Leftarrow$ sampling همراه با خطا است. در بازسازی خطا داریم.

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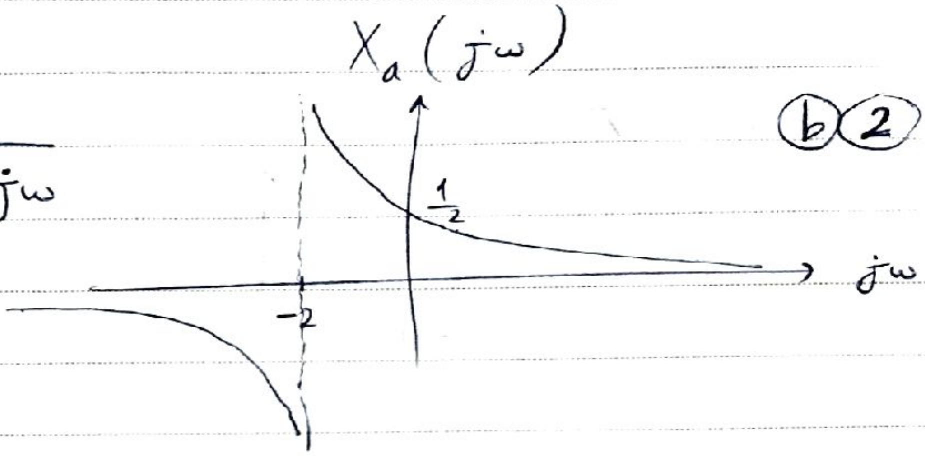
$$X_a(j\omega) = \frac{1}{2+j\omega}$$

b 2



دامنه نامحدود است $\Leftarrow$ sampling همراه با از دست دادن اطلاعات است.

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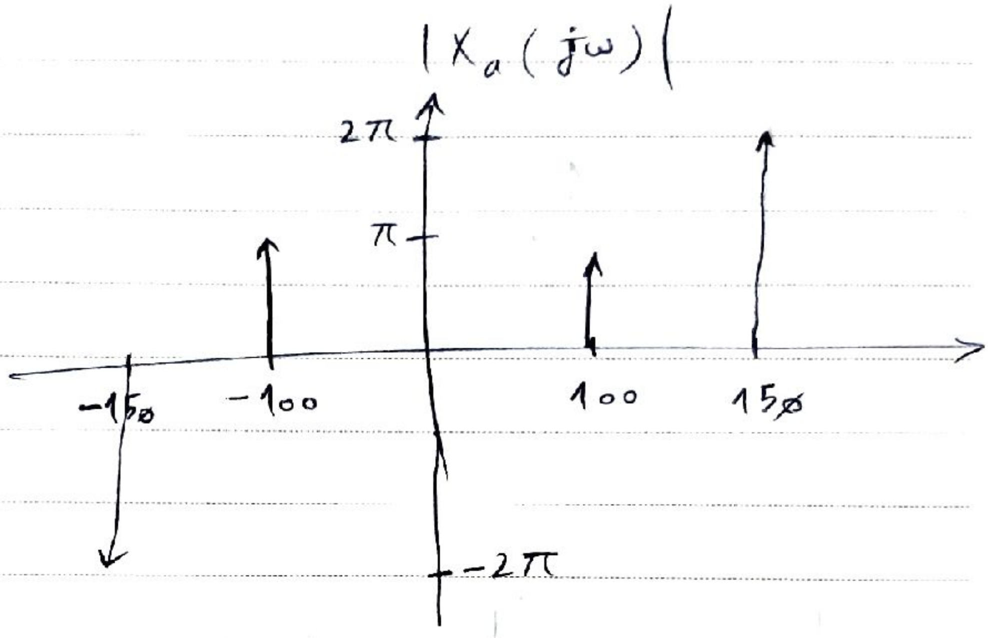
$$X_a(j\omega) = \pi\Big(\delta(\omega - 100\pi) + \delta(\omega + 100\pi)\Big) \qquad \text{c } 2$$

$$+ \frac{2\pi}{j}\Big(\delta(\omega - 150\pi) - \delta(\omega + 150\pi)\Big)$$

sampling without loss of information



$\omega_s > 300\pi$

دامنه محدود ✓

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d 2

$$X_a(j\omega) = \pi\Big(\delta(\omega-100\pi)+\delta(\omega+100\pi)\Big)$$

$$+\frac{\cancel{2\pi}}{j}\times\frac{1}{\cancel{2\pi}}\Big(\delta(\omega-150\pi)-\delta(\omega+150\pi)\Big) **$$

$$\frac{\pi}{j}\Big(\delta(\omega-200\pi)-\delta(\omega+200\pi)\Big)$$

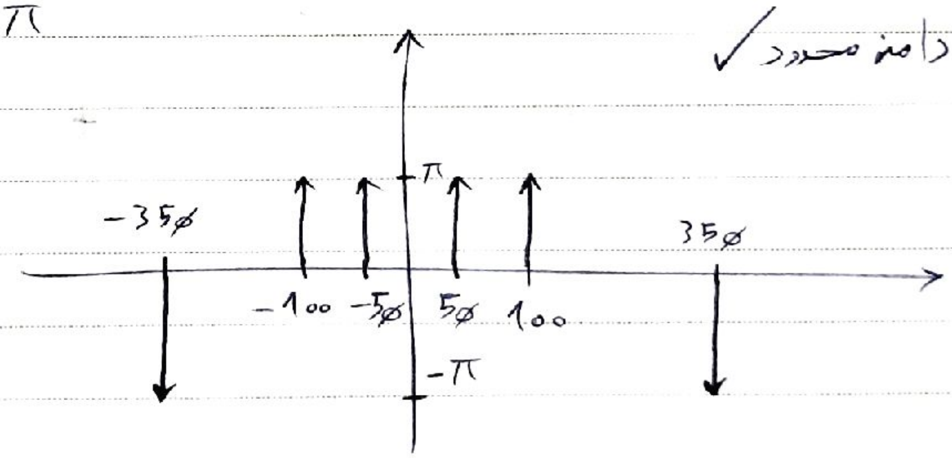
$$=\pi\Big(\delta(\omega-100\pi)+\delta(\omega+100\pi)\Big)$$

$$-\pi\Big(\delta(\omega-350\pi)-\delta(\omega+50\pi)-\delta(\omega-50\pi)+\delta(\omega+350\pi)\Big)$$

Sampling is done without loss of information

$\Longrightarrow \omega_s \geq 700\pi$

دامنه محدود ✓



$$x_a(t) = \cos(150\pi t) \qquad f_s = 200\,Hz \qquad T = \frac{1}{200} \quad (a)\ 4$$

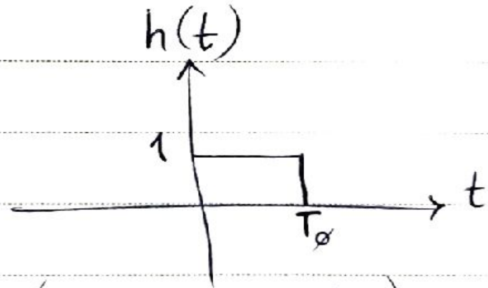
$$\omega_s = 2\pi f_s = 400\pi$$

$$X_s(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X_a\big(j(\omega - k\omega_0)\big) \qquad \omega_0 = \omega_s$$

$$X_a(j\omega) = \pi\big(\delta(\omega - 150\pi) + \delta(\omega + 150\pi)\big)$$

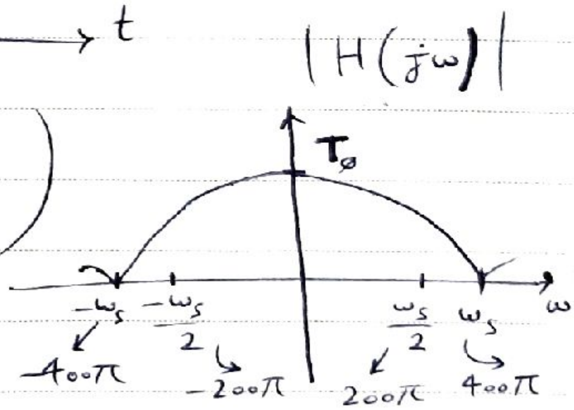
$$X_s(j\omega) = 200\sum_{-\infty}^{+\infty}\pi\big(\delta(\omega - 400\pi k - 150\pi) + \delta(\omega + 150\pi - 400\pi k)\big)$$

Zero order hold :

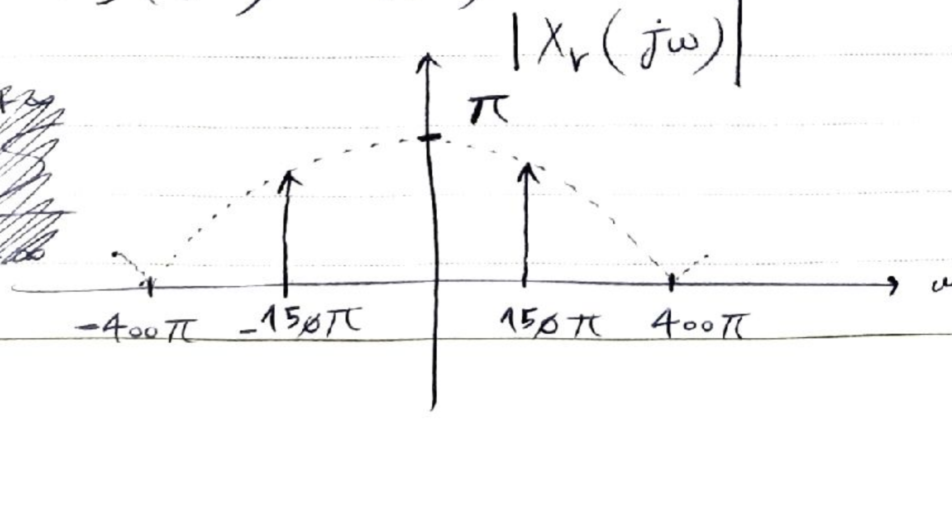


$$H(j\omega) = e^{\frac{-j\omega T_0}{2}}\left(\frac{2\sin\left(\frac{\omega T_0}{2}\right)}{\omega}\right)$$

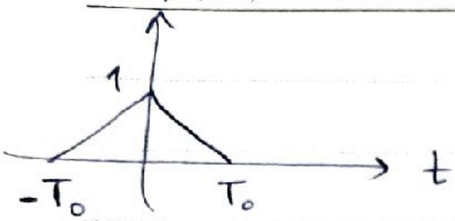
$$T_0 = \frac{2\pi}{\omega_s} = \frac{1}{200}$$



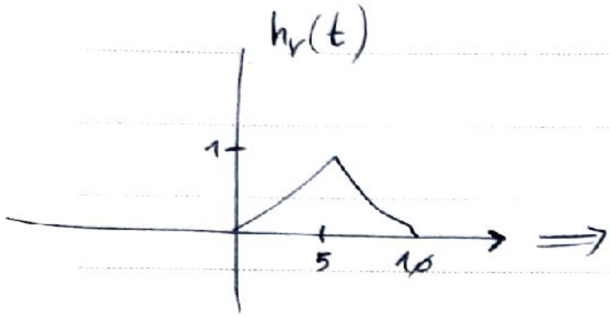
$$X_r(j\omega) = X_s(j\omega) \cdot H(j\omega)$$



(b) 4



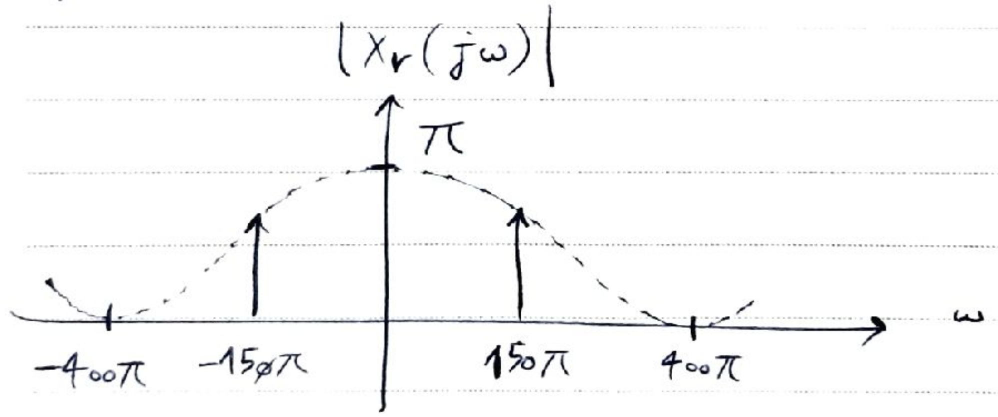
$$H_r(j\omega) = \frac{1}{T_0}\left(\frac{\sin\left(\frac{\omega T_0}{2}\right)}{\frac{\omega}{2}}\right)^2$$



$\Longrightarrow$

$$H_r(j\omega) = e^{-5j\omega} \times \frac{1}{5} \times \left(\frac{\sin\left(\frac{5\omega}{2}\right)}{\frac{\omega}{2}}\right)^2$$

$$X_r(j\omega) = X_s(j\omega) \cdot H_r(j\omega)$$



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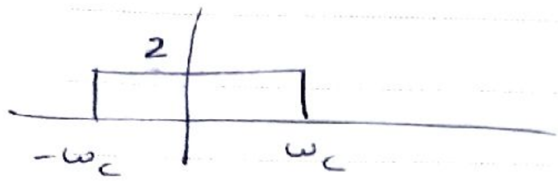
$H(j\omega)$



$\omega_c = 2\pi f_c = 300\pi$ (5)

$$Y(t) = g(t)\sin(300\pi t) = x(t)\sin^2(300\pi t)$$

$$= \underbrace{\left[\sin(200\pi t)\,\cancel{\sin^2} + 2\sin(300\pi t)\right]}_{Q(t)}\,\underbrace{\sin^2(300\pi t)}_{P(t)}$$

$$Q(j\omega) = \frac{\pi}{j}\Big(\delta(\omega-200\pi) - \delta(\omega+200\pi)\Big)$$

$$+\frac{2\pi}{j}\Big(\delta(\omega-300\pi) - \delta(\omega+300\pi)\Big)$$

$$P(j\omega) = \frac{1}{\cancel{2\pi}}\cdot\frac{\cancel{\pi}}{j}\cdot\frac{\pi}{j}\cdot\Big(\delta(\omega-300\pi) - \delta(\omega+300\pi)\Big) ** \Big(\delta(\omega-300\pi) - \delta(\omega+300\pi)\Big)$$

$$= \frac{\pi}{-2}\Big(\delta(\omega-600\pi) - 2\delta(\omega) + \delta(\omega+600\pi)\Big)$$

$$Y(j\omega) = \frac{Q(j\omega) ** P(j\omega)}{2\pi}$$

$$= \frac{1}{2\pi}\times\frac{\pi}{-2}\times\frac{\pi}{j}\Big(\delta(\omega-800\pi) - 2\delta(\omega-200\pi)$$

$$+\delta(\omega+400\pi) - \delta(\omega-400\pi) + 2\delta(\omega+200\pi)$$

$$-\delta(\omega+800\pi)\Big)$$

$$+\frac{1}{2\pi}\times\frac{\pi}{-2}\times\frac{2\pi}{j}\Big(\delta(\omega-900\pi) - 2\delta(\omega-300\pi)\cancel{-2\delta(\omega-300\pi)} + \delta(\omega+300\pi)$$

$$-\delta(\omega+300\pi) + 2\delta(\omega+300\pi) - \delta(\omega+900\pi)\Big)$$

خارج از $300\pi , -300\pi$ را حذف می‌کند

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$$\tilde{Y}(j\omega) \times H(j\omega) = \frac{\pi}{-2j}\left(-2\,\delta(\omega-200\pi)+2\,\delta(\omega+200\pi)\right)$$

$$-\frac{\pi}{j}\left(-3\,\delta(\omega-300\pi)+3\delta(\omega+300\pi)\right)$$

$$= +\frac{\pi}{j}\left(\delta(\omega-200\pi)-\delta(\omega+200\pi)\right)$$

$$+\frac{3\pi}{j}\left(\delta(\omega-300\pi)-\delta(\omega+300\pi)\right)$$

$$\text{output} = \sin(200\pi t) + 3\sin(300\pi t)$$

Koosha

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$N=5$ $\omega_0 = \frac{2\pi}{N} = \frac{2\pi}{5}$ (a) (7)

$$a_r = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\, e^{-j\omega_0 r n} = \frac{1}{5}\sum_{n=\emptyset}^{4} n e^{-j\frac{2\pi}{5} r n}$$

$$a_0 = \frac{1}{5}\sum_{n=\langle N\rangle} x[n] = \frac{1}{5}\times 10 = 2$$

$$a_1 = \frac{1}{5}\left(\emptyset + e^{-j\omega_0} + 2e^{-2j\omega_0} + 3e^{-3j\omega_0} + 4e^{-4j\omega_0}\right)$$

$$a_2 = \frac{1}{5}\left(\emptyset + e^{-2j\omega_0} + 2e^{-4j\omega_0} + 3e^{-6j\omega_0} + 4e^{-8j\omega_0}\right)$$

$$a_3 = \frac{1}{5}\left(\emptyset + e^{-3j\omega_0} + 2e^{-6j\omega_0} + 3e^{-9j\omega_0} + 4e^{-12j\omega_0}\right)$$

$$a_4 = \frac{1}{5}\left(\emptyset + e^{-4j\omega_0} + 2e^{-8j\omega_0} + 3e^{-12j\omega_0} + 4e^{-16j\omega_0}\right)$$

$\cos\left(\frac{n\pi}{2}\right) \longrightarrow \omega_\emptyset = \frac{\pi}{2} = \frac{2\pi}{4} \Rightarrow N=4$ (b) (7)

$$a_r = \frac{1}{4}\sum_{n=\emptyset}^{3} \cos\left(\frac{n\pi}{2}\right) e^{jr\omega_0 n}$$

$$= \frac{1}{4}\left[1 + \emptyset - e^{2j\frac{\pi}{2}r} + \emptyset\right] = \frac{1}{4} - \frac{1}{4} e^{j\pi r}$$

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$$\sin(n\pi) \longrightarrow \omega_0 = \pi = \frac{2\pi}{2} \Longrightarrow N=2$$

$$b_r = \frac{1}{2} \sum_{n=\varnothing}^{1} \sin(n\pi) e^{jr\omega_0 n} = \frac{1}{2}\left(\varnothing + \varnothing\right) = \varnothing$$

$$1 = \cos(2\pi n) \longrightarrow \omega_0 = 2\pi \Longrightarrow N=1$$

$$c_r = \frac{1}{1} \sum_{n=\varnothing}^{\varnothing} \cos(2\pi n) e^{jr\omega_\varnothing n} = \cancel{e^{jr2\pi}}\ 1$$

$$\Longrightarrow \text{cofficients} = 1 + \frac{1}{4} - \frac{1}{4} e^{j\pi r}$$

$$r=\varnothing \longrightarrow 1 \qquad r=1 \longrightarrow \frac{3}{2} \qquad r=3 \longrightarrow \frac{3}{2}$$

$$r=2 \longrightarrow 1$$

$$N = 6 = \gcd(6,3) \qquad \omega_\varnothing = \frac{2\pi}{6} = \frac{\pi}{3}$$ C 7

$$a_r = \frac{1}{6} \sum_{n=\varnothing}^{5} \cos\left(\frac{n\pi}{3}\right) \sin\left(\frac{2\pi n}{3}\right) e^{jr\omega_\varnothing n}$$

$$a_r = \frac{1}{6}\left(\varnothing + \frac{\sqrt{3}}{4} e^{\frac{jr\pi}{3}} + \frac{\sqrt{3}}{4} e^{\frac{jr2\pi}{3}} + \varnothing + -\frac{\sqrt{3}}{4} e^{\frac{jr4\pi}{3}} + -\frac{\sqrt{3}}{4} e^{\frac{jr5\pi}{3}}\right)$$

Koosha

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$m = 2 \quad b_k = \frac{1}{2} a_k$ (a) 8

$P[n] = x^*[n] \Longrightarrow c_k = a^*_{-k}$ (b) 8

$Q[n] = P[-n] = x^*[-n] \Longrightarrow d_k = a^*_k = c_{-k}$

$Y[n] = Q[n-1] = x^*[-n+1] \Longrightarrow b_k = d_k e^{-jk\omega_0}$

$\Longrightarrow b_k = a^*_k e^{-jk\omega_0}$

$y[n] = (-1)^n x[n]$ (c) 8

$$e^{j\pi n} = \cos(\pi n) + \cancel{j\sin(\pi n)}^{\,0} = \begin{cases} 1 & n = 2k \\ -1 & n = 2k+1 \end{cases}$$

$$\left.\begin{array}{l} \Longrightarrow y[n] = e^{j\pi n} x[n] \\ \text{فرمول}: e^{jM\omega_0 n} x[n] \xleftrightarrow{FS} a_{k-M} \\ M\omega_0 = \pi \Longrightarrow M = \frac{\pi}{\omega_0} \end{array}\right\} \Longrightarrow y[n] \xleftrightarrow{FS} a_{k-\frac{\pi}{\omega_0}} = a_{k-\frac{N}{2}}$$

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$$y[n] = \sum_{k=\langle N\rangle} a_k H\left(e^{j\omega_0 k}\right) e^{jk\omega_0 n}$$ 2

$$H\left(e^{j\omega}\right) = \sum_{n=-\infty}^{+\infty} h[n] e^{-j\omega n}$$

$\longrightarrow \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$

fourier series cofficient of $x[n]$:

$$a_r = \frac{1}{8} \sum_{n=\langle N\rangle} x[n] e^{jr\omega_0 n} = \frac{1}{8}$$

$$H\left(e^{j\omega}\right) = \cancel{\frac{6}{\pi}\omega e^{-\frac{3}{2}j\omega} - \frac{6}{\pi}\omega e^{-\frac{3}{2}j\omega}}$$

$$\cancel{= \frac{6}{\pi}}$$

$$= \begin{cases} \frac{6}{\pi}|\omega| e^{-\frac{3}{2}j\omega} & \frac{\pi}{6} < |\omega| < \frac{\pi}{3} \\ 0 & \text{o.w} \end{cases}$$

$$\longrightarrow y[n] = \sum_{k=0}^{7} \frac{1}{8} H\left(e^{jk\omega_0}\right) e^{jk\omega_0 n}$$

$\left(\omega_0 = \frac{\pi}{4}\right)$

طبق این فرض :

$$0 < k\omega_0 < \frac{7\pi}{4}$$

از طرفی باید $\frac{\pi}{6} < |k\omega_0| < \frac{\pi}{3}$ باشد. $\Leftarrow$

$$y[n] = \sum_{k=1}^{1} \frac{1}{8} \times \frac{6}{\pi} k \times \frac{\pi}{4} \times e^{-\frac{3}{2}jk\frac{\pi}{4}} \cdot e^{jk\frac{\pi}{4}n}$$

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$$\Longrightarrow y[n] = \frac{3}{16} e^{-\frac{3\pi}{8} j} e^{\frac{\pi}{4} n j}$$

$$= \frac{3}{16} e^{\pi \left(-\frac{3}{8} + \frac{n}{4}\right) j}$$