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Bayesian Networks in Reliability: The Good, the Bad, and the Ugly

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Abstract. Bayesian network (BN) models gain more and more popularity as a tool in reliability analysis. In this paper we consider some of the properties of BNs that have made them popular, consider some of the recent developments, and also point to the most important remaining challenges when using BNs in reliability.

Keywords. Bayesian networks, reliability analysis, inference, hybrid models

1. The Good: The foundation of Bayesian Networks

A Bayesian Network (BN), [20,15], is a compact representation of a multivariate statistical distribution function. A BN encodes the probability density function governing a set of random variables $\{X_1, \ldots, X_n\}$ by specifying a set of conditional independence statements together with a set of conditional probability functions. More specifically, a BN consists of a qualitative part, a *directed acyclic graph* where the nodes mirror the random variables X_i, and a quantitative part, the set of conditional probability functions. An example of a BN over the variables $\{X_1, \ldots, X_5\}$ is shown in Figure 1, only the qualitative part is given. We call the nodes with outgoing edges pointing into a specific node the *parents* of that node, and say that X_j is a *descendant* of X_i if and only if there exists a directed path from X_i to X_j in the graph. In Figure 1, X_1 and X_2 are the parents of X_3, written $\mathrm{pa}(X_3) = \{X_1, X_2\}$ for short. Furthermore, $\mathrm{pa}(X_4) = \{X_3\}$ and since there are no directed path from X_4 to any of the other nodes, the descendants of X_4 are given by the empty set and, accordingly, its non-descendants are $\{X_1, X_2, X_3, X_5\}$.

The edges of the graph represents the assertion that a variable is conditionally independent of its non-descendants in the graph given its parents in the same graph; other conditional independence statements can be read off the graph by using the rules of *d-separation* [20]. The graph in Figure 1 does for instance assert that for all distributions compatible with it, we have that X_4 is conditionally independent of $\{X_1, X_2, X_5\}$ when conditioned on X_3.

When it comes to the quantitative part, each variable is described by the conditional probability function of that variable *given the parents* in the graph, i.e., the collection

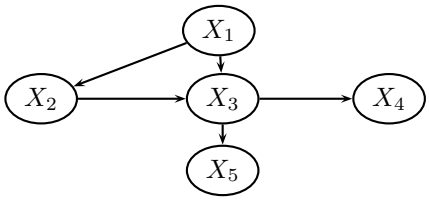


Figure 1. An example BN over the nodes $\{X_1, \ldots, X_5\}$. Only the qualitative part of the BN is shown.

of conditional probability functions $\{f(x_i|\mathrm{pa}\,(x_i))\}_{i=1}^{n}$ is required. The underlying assumptions of conditional independence encoded in the graph allow us to calculate the joint probability function as

$$f(x_1, , \ldots, x_n) = \prod_{i=1}^{n} f(x_i|\mathrm{pa}\,(x_i)). \tag{1}$$

BNs originated in the field of Artificial Intelligence, where it was used as a robust and efficient framework for reasoning with uncertain knowledge. The history of BNs in reliability can (at least) be traced back to [2] and [1]; the first real attempt to merge the efforts of the two communities is probably the work of [1], where he proposes the use of the GRAPHICAL-BELIEF tool for calculating reliability measures concerning a low pressure coolant injection system for a nuclear reactor.

Reliability analysts are more and more frequently choosing to use BNs as their modelling framework. Their choice is partly motivated by BNs being particularly easy to use in interaction with domain experts, a feature of high importance also in the reliability field [22]. This ease of use is obtained by seeing the BN as a model of causal influence, and although this interpretation is not necessarily correct in general, it can be defended if some additional assumptions are made [21]. Finally, it is worth noticing that BNs constitute a flexible class of models, as any joint statistical distribution can be represented by a BN. This can, for instance, be utilized to extend traditional fault-tree models to incorporate dependence between basic events (e.g., common-cause failures) [16]. The sound mathematical foundation, the ease of interpretation, and the usefulness in applications are "the good features" of Bayesian Networks.

2. The Bad: Building quantitative models

Bayesian networks are quantitative stochastic models, and therefore requires quantitative parameters to be fully specified. This is obviously not a particularity for Bayesian networks, but since building the BN structure is such a simple and intuitive procedure, the burden of eliciting the quantitative part of the BN from experts often comes as a surprise to the reliability analyst. We therefore consider this to be "the bad part" of BNs' usage in reliability.

To elicit the quantitative part from experts, one must acquire all conditional distributions $\{f(x_i|\mathrm{pa}\,(x_i))\}_{i=1}^{n}$ in Equation (1). To get a feeling for the assessment burden, consider Figure 1, and assume all variables are discrete with k states. We now need to quantify

$$q_i = (k-1)\cdot k^{|\mathrm{pa}(x_i)|} \tag{2}$$

parameters to specify $f(x_i|\mathrm{pa}\,(x_i))$ for a fixed variable x_i, and therefore $q = \sum_i q_i$ to specify the full model. In total we need $q = 11$ parameters if $k = 2$ and $1\,179$ parameters if $k = 10$. Although the last number may be too large to handle the individual parameters in detail, the BN still attempts to keep the knowledge acquisition burden as low as possible (through the factorized representation of Equation (1)). If we had not utilized the BN structure, the full joint distribution would require $q = 31$ ($q = 99\,999$) parameters for $k = 2$ ($k = 10$). The parametrization is however not optimized; it is merely defined to be sufficient to encode *any* distribution compatible with the conditional independence statements encoded in the graph. Many researchers have therefore explored even more cost-efficient representations, including the *deterministic relations*, *noisy-OR relations* [12] and general *independence of causal influence* models [6], *logistic regression*, and the *IPF procedure* [25].

Finally, *vines* have been proposed as another natural modelling framework for the reliability analyst [4]. Using vines can dramatically simplify the elicitation of the quantitative parameters. Conditional rank correlations (realized by copulas) model the dependence structure among the variables, and is therefore the fundamental quantitative input when modelling with vines. Recent developments by Hanea and Kurowicka [11] extend these idea. The authors show how one can build non-parametric Bayesian networks (containing both discrete and continuous variables) while still using conditional rank correlations to define the quantitative part of the model.

3. The Ugly: Hybrid models

3.1. Background

BNs have found applications in domains like, e.g., software reliability [8], fault finding systems [13], and structural reliability [5], see [16] for an overview. A characteristic feature of these problem domains is that all variables are discrete (e.g., the variables' states are $\{\texttt{failed}, \texttt{operating}\}$). The preference for discrete variables in the BN community is mainly due to the technicalities of the calculation scheme. BNs are equipped with efficient algorithms for calculating arbitrary marginal distributions, say, $f(x_i, x_j, x_k)$ as well as conditional distributions, say, $f(x_i, x_j|x_k, x_\ell)$, but the base algorithm only works as long as all variables are discrete [14][1]. We note that the BNs' applicability in reliability analysis would be enormously limited if one would only consider discrete variables, and

[1] Some models containing discrete and *Gaussian* variables can also be handled. However, these models, called *conditional Gaussian* (CG) distributions [17], impose modelling restrictions we would like to avoid, and are therefore not considered here.

that the simplicity of making Bayesian network models does not go well together with the difficulties of inference in the models. Finding robust and computationally efficient inference techniques applicable for BNs containing both continuous and discrete variables (so-called *hybrid* BNs) is therefore a hot research area. Overviews of the current usage of BNs among practitioners show that the available techniques are relatively unknown however, and many consider the poor treatment of hybrid BNs the missing ingredient for BNs to become even more popular in the reliability community. We therefore dub hybrid models "the ugly part" of using BNs in reliability.

3.2. An example model

We will consider a very simple hybrid BN to exemplify why inference in hybrid BNs is difficult, and to show how approximate techniques can be used. The model is shown in Figure 2, where we have 4 binary variables ($T_1, \ldots, T_4$) and two continuous variables (Z_1 and Z_2).

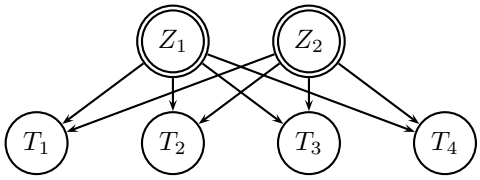


Figure 2. A model for the analysis of human reliability. A subject's ability to perform four different tasks $T_1, \ldots, T_4$ are influenced by the two explanatory variables Z_1 and Z_2. The explanatory variables are drawn with double-line to signify that these variables are continuous.

This model, which can be interpreted as a factor analyzer for binary data, was called a *latent trait* model in [3]. In reliability, similar models can be used to predict humans' ability to perform some tasks in a given environment (we are extending ideas from the THERP methodology[2] here). With this interpretation, T_i is a person's ability to correctly perform task i ($i = 1, \ldots, 4$) and T_i takes on the values 1 ("success") or 0 ("failure"). Each T_i is influenced by a set of explanatory variables, Z_j, $j = 1, 2$. The goal of the model is to quantify the effect the explanatory variables have on the observable ones, and to predict a subject's ability to perform the tasks $T_1, \ldots, T_4$. We have a mixture of both discrete and continuous variables in this model, and this will eventually lead to problems when trying to use this model for inference.

Assume first that the explanatory variables are used to model the environment, that the environment can be considered constant between subjects, and that it can be disclosed in advance (that is, the variables are observed before inference is performed). An example of such a factor can for instance be *"Lack of lighting"*, with the assumption that the luminous flux can be measured in advance, and that it affects different people in the same way. Each T_i is given by logistic regression, meaning that we have $P(T_i = 1|\boldsymbol{z}) = (1 + \exp(-\boldsymbol{w}_i'\boldsymbol{z}))^{-1}$ for a given set of weights $\boldsymbol{w}_i$. Here element j of $\boldsymbol{w}_i$ quantifies how covariate j influences a person's ability to perform task i. As long as $\boldsymbol{Z}$ is observed, this

[2] THERP: Technique for Human Error Rate Prediction [24]

is a simple generalized linear model, where T_k is conditionally independent of T_l given $\boldsymbol{Z}$. Therefore, inference in this model can be handled; note that $\boldsymbol{Z}$ simply can be regarded as a tool to fill in the probability tables for each T_i in this case.

Next, assume that some of the explanatory variables are used to model subject-specific properties, like a subject's likelihood for omitting a step in a procedure (this is one of the explanatory variables often used when employing the THERP methodology, see, e.g., [23]). It seems natural to assume that these explanatory variables are unobserved, and for the case of simplicity, to give them Gaussian distributions a priori, $Z_j \sim \mathcal{N}(\mu_j, \sigma_j^2)$.

Assume that we are interested in calculating the likelihood of an observation $\boldsymbol{e} = \{T_1 = 1, T_2 = 1, T_3 = 1, T_4 = 1\}$ (i.e., $\boldsymbol{Z}$ is unobserved) as well as the joint posterior distribution $f(z_1, z_2|\boldsymbol{e})$. It is straight forward to see that the likelihood is given by

$$P(\boldsymbol{e}) = \frac{1}{2\pi\sigma_1\sigma_2} \int_{\mathbb{R}^2} \frac{\exp\left(-\sum_{j=1}^{2} \frac{(z_j-\mu_j)^2}{2\sigma_j^2}\right)}{\prod_{i=1}^{4}\left\{1+\exp(-\boldsymbol{w}_i^{\mathrm{T}}\boldsymbol{z})\right\}}\, d\boldsymbol{z}, \tag{3}$$

but unfortunately this integral has no known analytic representation in general. Hence, we cannot calculate the likelihood of the observation in this model. Note that this a consequence not of the modelling language (the use of a BN), but of the model itself.

For the rest of this section we will consider some of the simpler schemes for approximating the calculations in Equation (3).

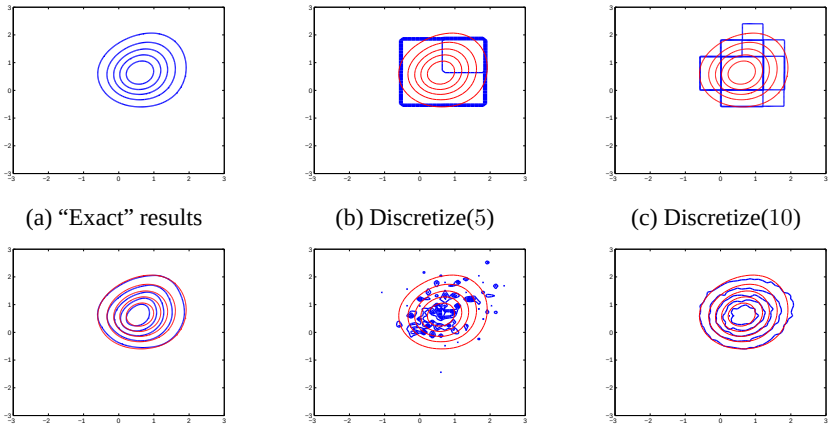
3.3. Approximative inference

We will use the model in Figure 2 as our running example, and for each approximative method we will calculate both the likelihood of the observation, $P(\boldsymbol{e})$, as well as the posterior distribution over the explanatory variables, $f(\boldsymbol{z}|\boldsymbol{e})$. In Figure 3 part (a) we have calculated this posterior using a numerical integration scheme (with 200 $\times$ 200 integration grid) for obtaining what we will consider the gold-standard solution to what the posterior should look like. The parameters in the example are $\begin{bmatrix}\boldsymbol{w}_1 & \boldsymbol{w}_2 & \boldsymbol{w}_3 & \boldsymbol{w}_4\end{bmatrix} = \begin{bmatrix}+2 & +1 & -1 & +1\\ -1 & +1 & +1 & +2\end{bmatrix}$, $\mu_1 = \mu_2 = 0$, and $\sigma_1^2 = \sigma_2^2 = 1$, which gave a likelihood of 0.0695.

There are several different approaches to efficient inference in hybrid BNs. At a high level, we may divide them into two categories:

1. Change the distribution functions, so that the mathematical operations (e.g., the integral in Equation (3)) can be handled analytically.
2. Approximate the difficult operations by sampling or some other numerical method.

The simplest approach to continuous variables is to simply *discretize* them. The idea is to divide the support of any distribution function into r intervals I_k, $k = 1, \ldots, r$, and thereby translate the hybrid BN into a discrete one. A random variable $T \in [a, b]$ would then be recoded into a new variable, say T'; $T' \in \{\texttt{Low} : T \in I_1,\ \texttt{Medium} : T \in I_2,\ \texttt{High} : T \in I_3\}$ if we chose to use $r = 3$ intervals. Mathematically, this corresponds



(a) "Exact" results (b) Discretize(5) (c) Discretize(10)

(d) MTEs (e) MCMC (10^3) (f) MCMC (10^6)

Figure 3. Results of some approaches to approximate $f(\boldsymbol{z}|\boldsymbol{e})$; see text for details.

to replacing the density functions in Equation (3) with piecewise constant approximations. The granularity of the model is controlled by choosing a "good" number of states for the discrete variable; larger number of states will improve model expressibility, but at the expense of computational efficiency (very simply put, the computational burden grows at least as fast as the number of cells in each conditional probability table, refer to Equation (2), and in practice much faster). Attempts to automatically choose the number of states have been developed, see, e.g., [19]. Figure 3 (b) gives the joint posterior when each continuous variable is discretized into 5 states, and Figure 3 (c) gives the same for 10-state discretization. The calculated likelihoods were 0.0686 and 0.0694, respectively.

Moral et al. [18] developed a framework for approximating any hybrid distribution arbitrarily well by employing *mixtures of truncated exponential* (MTE) distributions, and they also showed how the BNs' efficient calculation scheme can be extended to handle the MTE distributions. The main idea is again to divide the support of any distribution function into intervals I_k, and approximate each part of the distribution by a sum of truncated exponential functions; each exponential is linear in its argument:

$$\tilde{f}(x|\boldsymbol{\theta}) = a_0^{(k)}(\boldsymbol{\theta}) + \sum_{i=1}^{m} a_i^{(k)}(\boldsymbol{\theta}) \exp\left(b_i^{(k)}(\boldsymbol{\theta})x\right) \text{ for } x \in I_k.$$

We typically see $1 \leq r \leq 4$ and $0 \leq m \leq 2$ in applications; notice that setting $m = 0$ gives us the standard discretization. Clever choices for values of the parameters $\{a_0^{(k)}(\boldsymbol{\theta}), a_i^{(k)}(\boldsymbol{\theta}), b_i^{(k)}(\boldsymbol{\theta})\}$ to make $\tilde{f}$ as close as possible to the original distribution (in the KL-sense) are tabulated for many standard distributions in [7]. Parameter values from [7] were also used to generate the results in Figure 3 (d); each distribution was divided into 2 intervals, and within each interval a sum of $m = 2$ exponential terms were used[3].

[3]Note that this means that the density of each Gaussian, which is *quadratic* in the exponential function, is approximated by sums of terms that are *linear* in the exponential function.

The likelihood was calculated as 0.0695, and the approximation to the joint $f(\boldsymbol{z}|\boldsymbol{e})$ is also extraordinary good, even in the tails of the distribution.

Markov Chain Monte Carlo [9] is a *sampling scheme* for approximating any distribution by simulation[4]. Of particular interest for the BN community is BUGS [10], which is a modelling language that takes as its input a BN model, and estimates any posterior probability from this model using sampling. The approach is very general, and proceeds by generating random samples from the target distribution (this can be done even when the algebraic form of the function is unknown), and then approximate the target distribution by the empirical distribution of the samples. For the results given in Figure 3 (e) and (f), 10^3 and 10^6 samples were generated from $f(\boldsymbol{z}|\boldsymbol{e})$ respectively, giving likelihoods of 0.0690 (10^3 samples) and 0.0692 (10^6 samples).

If we compare the results, we see that the likelihoods calculated by the different methods are roughly the same, and comparable to the "exact" result. However, Figure 3 shows fairly large differences between the approximation of $f(\boldsymbol{z}|\boldsymbol{e})$ given by the three mentioned methods. The quality of the approximation is far better using MTEs than if we use standard discretization. This is particularly true in the tails of the distribution, and since reliability analysts often are considering infrequent events, this is an important finding for practitioners in reliability. MCMC simulation is a very popular method for approximate inference, but care must be taken so that enough samples are used to achieve high quality approximations of the tails of the distributions.

4. Conclusions

In this paper we have briefly described why BNs have become a popular modelling framework for reliability analysts. The main reasons are (in our experience) the intuitive representation and the modelling flexibility of BNs; these properties make a BN a well-suited tool for cooperating with domain experts. Furthermore, the efficient calculation scheme is (if applicable) an advantage when building complex models; real life models containing thousands of variables are not uncommon, and speed of inference is therefore of utter importance.

We then turned to challenges when using BNs, and pinpointed the quantification process as the bottleneck of the modelling phase. We also touched briefly upon inference in hybrid models, which by many is seen as the Achilles' heal of the BN framework.

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