signal HW3-part1 in the name of God

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$T=3 \quad e^{j\left(\frac{2\pi t}{3}+\frac{\pi}{6}\right)} = \cos\left(\frac{2\pi t}{3}+\frac{\pi}{6}\right)+j\sin\left(\frac{2\pi t}{3}+\frac{\pi}{6}\right)$ (a)(1)

$$2\cos\left(\frac{2\pi t}{3}+\frac{\pi}{6}\right) = e^{j\left(\frac{2\pi t}{3}+\frac{\pi}{6}\right)} + e^{-j\left(\frac{2\pi t}{3}+\frac{\pi}{6}\right)} \qquad \omega_0=\frac{2\pi}{3}$$

$$'' \quad = e^{\frac{\pi j}{6}} e^{j\omega_0 t} + e^{-\frac{\pi j}{6}} e^{-\frac{2\pi t j}{6}}$$

$$e^{-j\omega_0 t}$$

$$\Longrightarrow a_1 = e^{\frac{\pi}{6}j} \qquad a_{-1} = \cancel{e^{-j\omega_0 t}}\, e^{-\frac{\pi}{6}j} \qquad a_k = 0 \quad k\neq 1,-1$$

$$T=4 \quad a_k = \frac{1}{4}\int_{-2}^{2} x(t) e^{-jk\omega_0 t}\,dt = \frac{1}{4}\int_{-2}^{0} x(t)e^{-jk\omega_0 t}\,dt + \cancel{\frac{1}{4}\int_{0}^{2} x(t)e^{-jk\omega_0 t}\,dt} \to 0 \qquad \text{(b)}$$

$$= \frac{1}{4}\int_{-2}^{0}\left(\frac{1}{2}t+1\right)e^{-jk\omega_0 t}\,dt = \left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\Bigg|_{-2}^{0} + \cancel{\frac{t\,e^{-jk\omega_0 t}}{-2jk\omega_0}\Bigg|_{-2}^{0} + \frac{1}{2}\int_{-2}^{0}\frac{e^{-jk\omega_0 t}}{jk\omega_0}\,dt}\right]$$

$$\cancel{=\frac{1}{4}\frac{e^{j2\omega_0}}{jk\omega_0}} \qquad \frac{1}{2}\left(t+\frac{1}{jk\omega_0}\right)e^{-jk\omega_0 t}\Bigg|_{-2}^{0} \times\frac{1}{4}$$

$$= \frac{1-e^{2\omega_0 kj}}{-4jk\omega_0} + \frac{1}{8jk\omega_0} - \frac{1}{8}\left(-2+\frac{1}{jk\omega_0}\right)e^{2k\omega_0 j} = \frac{e^{2kj\omega_0}-1}{8kj\omega_0} + \frac{1}{4}e^{2kj\omega_0}$$

(a) (2)

$$a_k = \frac{1}{T}\int_{-T_1}^{T_1} 1 \times e^{-jk\omega_0 t}\,dt = \left.\frac{e^{-jk\omega_0 t}}{-jk\omega_0 T}\right|_{-T_1}^{T_1} = \frac{e^{-jk\omega_0 T_1} - e^{jk\omega_0 T_1}}{-jk\omega_0 T}$$

$$= \frac{-2j\sin(k\omega_0 T_1)}{-jk\omega_0 T} = \begin{cases} \dfrac{\sin(k\omega_0 T_1)}{k\pi} & k \neq \varnothing \\[2ex] \dfrac{2T_1}{T} & k = \varnothing \end{cases}$$

(b)

$$g(t) = x(t - T_1) = x(t-1) \implies b_k = e^{-j\omega_0 t} a_k$$

$T_1 = 1$

$T = 4$

$$= e^{-j\omega_0 t}$$

$$b_k = \begin{cases} e^{-j\omega_0 k}\dfrac{\sin(k\omega_0 T_1)}{k\pi} & k \neq \varnothing \\[2ex] e^{-j\omega_0 t}\dfrac{2T_1}{T} & k = \varnothing \end{cases}$$

$$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$$

$$b_k = \begin{cases} e^{\frac{-j\pi k}{2}}\dfrac{\sin\left(\frac{k\pi}{2}\right)}{k\pi} & k \neq \varnothing \\[2ex] \dfrac{1}{2} & k = \varnothing \end{cases}$$

(c)

$T = 6$

$$a_k = \frac{1}{6}\int_{-2}^{-1} e^{jk\omega_0 t}\,dt - \frac{1}{6}\int_{1}^{2} e^{jk\omega_0 t}\,dt$$

$$= \frac{1}{6}\left(\frac{e^{-jk\omega_0} - e^{-2jk\omega_\varnothing} - e^{2jk\omega_\varnothing} + e^{jk\omega_0}}{jk\omega_\varnothing}\right) = \frac{2\cos(k\omega_\varnothing) - 2\cos(2k\omega_\varnothing)}{6jk\omega_\varnothing}$$

$$= \begin{cases} \dfrac{2\cos\left(\frac{k\pi}{3}\right) - 2\cos\left(\frac{2k\pi}{3}\right)}{2k\pi j} & k = \varnothing \\ \varnothing & k = \varnothing \end{cases}$$

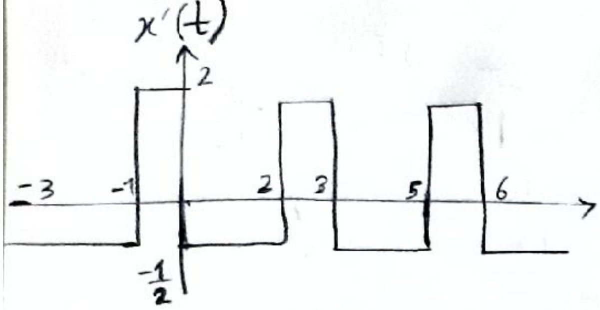
/ راه دوم:

$$y(t) = x\left(t+\tfrac{3}{2}\right) - x\left(t-\tfrac{3}{2}\right)$$ || معادل

قسمت الف

$$T_1 = \tfrac{1}{2} \quad T = 6 \quad \Longrightarrow a_k = \frac{\sin\left(\frac{k\pi}{6}\right)}{k\pi} \times 2j \sin\left(\frac{k\pi}{2}\right)$$

$$a_\varnothing = \varnothing$$



(d)

$$x'(t) \Longrightarrow b_k = \frac{1 - e^{-\frac{jk2\pi}{3}}}{jk\pi}$$

$$\Longrightarrow a_k = \cancel{e^{jk\omega_0}}\, \frac{1}{jk\omega_\varnothing} \left(\frac{1 - e^{-jk\omega_\varnothing}}{jk\pi}\right)$$

$$= \begin{cases} \dfrac{1 - e^{-jk\omega_\varnothing}}{k^2\pi^2} \times \dfrac{3}{2} & k \neq \varnothing \\ \dfrac{2}{3jk\omega_\varnothing} & k = \varnothing \end{cases}$$

solution

$$\frac{2}{3}\int_{-1}^{\varnothing} e^{jk\omega_\varnothing t}\, dt = \frac{1 - e^{-jk\omega_\varnothing}}{jk\pi} = b_k$$

$$b_\varnothing = \frac{2}{3}$$

$$y(t) = \sum_{-2}^{2} a_k H(jk\omega_0)\, e^{jk\omega_0 t} \qquad \omega_0 = 4\pi$$ (3)

$$H(-2\omega_0 j) = 2e^{\frac{2\pi}{5}j} \qquad H(2\omega_0 j) = 2e^{\frac{-2\pi}{5}j}$$

$$H(-\omega_0 j) = 6e^{\frac{\pi}{5}j} \qquad H(\omega_0 j) = 6e^{\frac{-\pi}{5}j}$$

$$H(0) = 10$$

$$y = \underbrace{\frac{e^{\frac{2\pi}{5}j}}{2}}_{b_{-2}} e^{-2j\omega_0 t} + \underbrace{3e^{\frac{\pi}{5}j}}_{b_{-1}} e^{-j\omega_0 t} + \underbrace{10}_{b_0} + \underbrace{3e^{\frac{-\pi}{5}j}}_{b_1} e^{j\omega_0 t}$$

$$+ \underbrace{\frac{e^{\frac{-2\pi}{5}j}}{2}}_{b_2} e^{2j\omega_0 t}$$

$$\frac{1}{T}\int_T |x(t)|^2\, dt = \sum_{-\infty}^{+\infty} |a_k|^2 \qquad \omega_0 = 2\pi = \frac{2\pi}{T} \implies T = 1$$ (4)

$$y = \sum_{-\infty}^{+\infty} a_k H(jk\omega_0)\, e^{jk\omega_0 t} = \sum_{-5}^{-4} a_k \cancelto{10}{H(jk\omega_0)}\, e^{jk\omega_0 t}$$

↳ periodic with $T=1$

$$+ \sum_{4}^{5} a_k \cancelto{10}{H(jk\omega_0)}\, e^{jk\omega_0 t}$$

$$\frac{1}{1}\int_{T=1} |y(t)|^2\, dt = \sum_{-5}^{-4} (10a_k)^2 + \sum_{4}^{5} (10a_k)^2 = 100\left(\frac{2}{(32)^2} + \frac{2}{(16)^2} \cancel{+ \frac{2}{8}}\right)$$

$$= \frac{1000}{1024}$$