RAON online School: Nuclear reactions with exotic nuclei

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Material available at: `https://github.com/ammoro/RAON`

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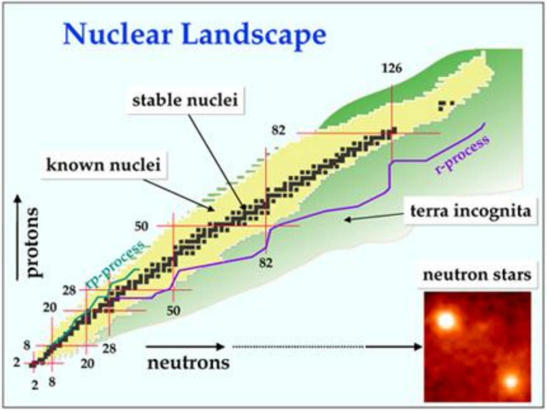
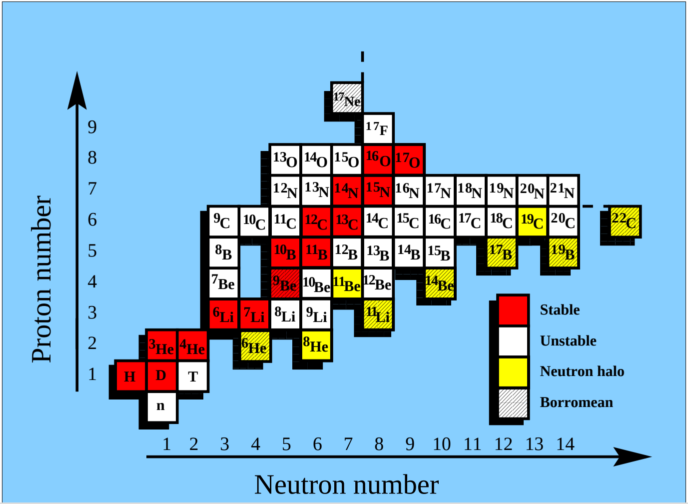
Unstable nuclei and the limits of stability



Note that:

- Not all unstable nuclei are weakly-bound.
- There are weakly-bound nuclei which are not unstable (eg. deuteron).

Light exotic nuclei: halo nuclei and Borromean systems



Light exotic nuclei: halo nuclei and Borromean systems

- Radioactive nuclei: they typically decay by β emission.

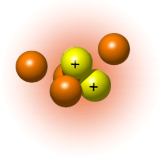
  E.g.: $^6\mathrm{He} \xrightarrow{\beta^-} {}^6\mathrm{Li}$ $\quad (\tau_{1/2} \simeq 807$ ms)

- Weakly bound: typical separation energies are around 1 MeV or less.

- Spatially extended

- Halo structure: one or two weakly bound nucleons (typically neutrons) with a large probability of presence beyond the range of the potential.

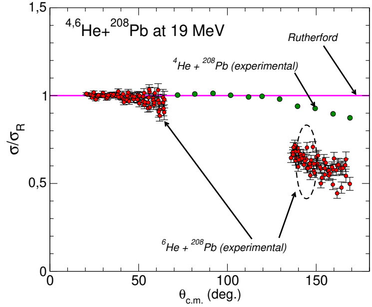
- Borromean nuclei: Three-body systems with no bound binary sub-systems.

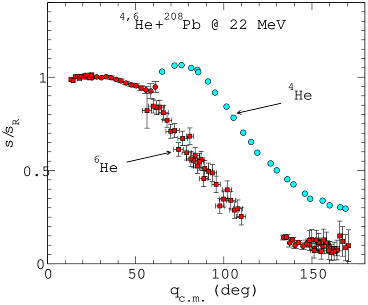


Signatures of weakly bound nuclei in reaction observables

Elastic scattering: Rutherford experiment...100 years later

For ^{4}He+^{208}Pb, the Coulomb barrier is $V_b \approx 21$ MeV

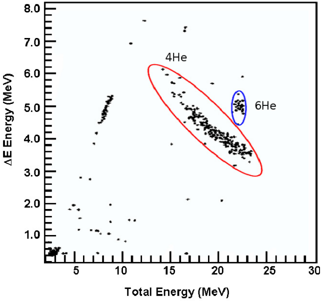


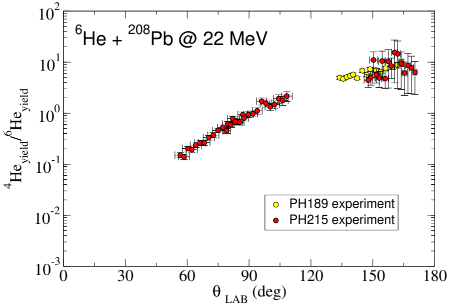


- ^{4}He follows Rutherford formula at 19 MeV but not at 22 MeV.
- ^{6}He drastically departs from Rutherford formula at both energies!

Large fragment production

$$\boxed{{}^{6}\mathrm{He} + {}^{208}\mathrm{Pb} \to \alpha + \mathrm{X}}$$





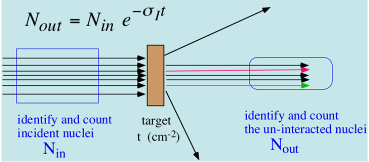
⇨At large angles, there are more α's than ^{6}He (elastic) !

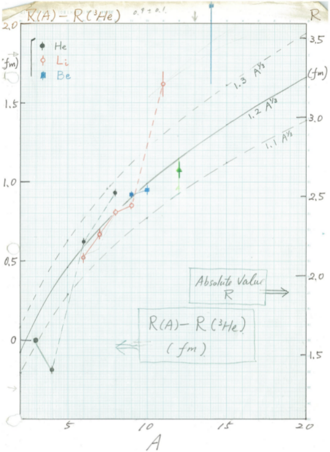
⇨What are the mechanisms behind the α producion and how can we compute it?

High-energy interaction cross sections with light targets

Interaction cross sections of nuclei on light targets and high energies are proportional to the size of the colliding nuclei.

$$\boxed{\sigma_I \simeq \pi(R_p + R_t)^2}$$



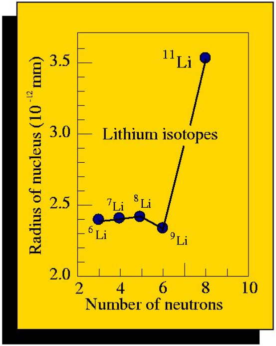


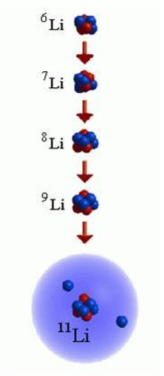
From I. Tanihata

High-energy interaction cross sections with light targets

Interaction cross sections of nuclei on light targets and high energies (hundreds MeV/nucleon) are proportional to the size of the colliding nuclei.

$$\boxed{\sigma_I \simeq \pi(R_p + R_t)^2}$$

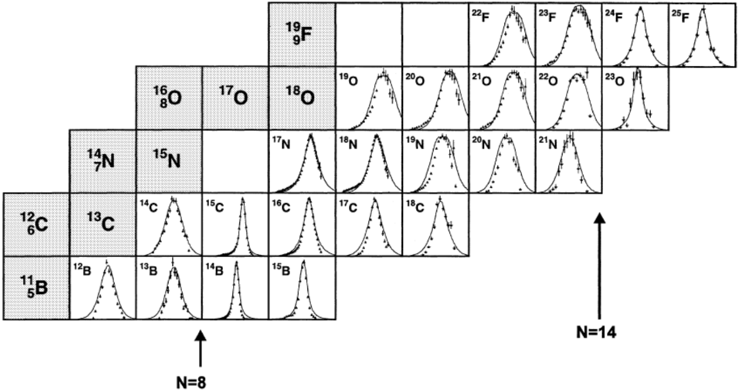




Tanihata *et al*, PRL55, 2676 (1985)

Momentum distributions in high-energy fragmentation reactions

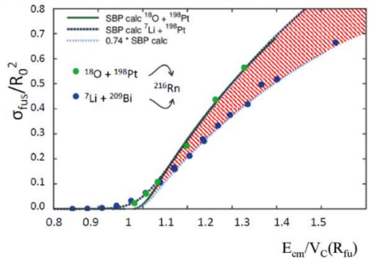
What do momentum distributions tell us about the size of the nucleus?



- Unusually narrow momentum distributions of the fragments occur for specific isotopes (e.g. ^{23}O $\rightarrow$ ^{22}O +n)
- A narrow momentum distribution is a signature of an extended spatial distribution

Complete fusion suppression at above-barrier energies

CF of weakly bound nuclei suppressed at energies above the Coulomb barrier



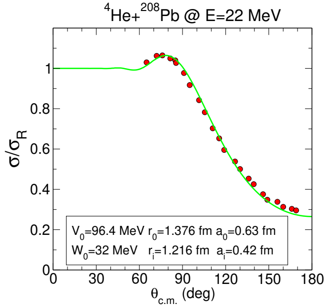
- Observed for weakly-bound projectiles (^{6,7,8}Li,^{9}Be)
- CF reduced by $\sim$30% with respect to BPM or CC calculations.

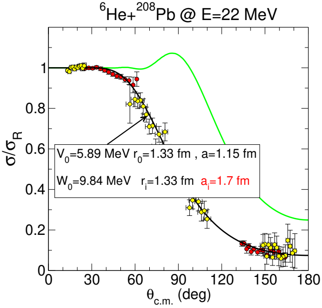
M. Dasgupta et al., PRC 70, 024606 (2004)

⇨ A widely accepted interpretation is that CF is mostly reduced by breakup and incomplete fusion.

Elastic scattering of weakly bound nuclei

How does the halo structure affect the elastic scattering?





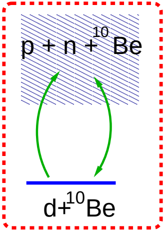
- ^{4}He+^{208}Pb shows typical Fresnel pattern and “standard” optical model parameters
- ^{6}He+^{208}Pb shows a prominent reduction in the elastic cross section, suggesting that part of the incident flux goes to non-elastic channels (eg. breakup)

  Understanding and disentangling these non-elastic channels requires going beyond the optical model (eg. coupled-channels method)

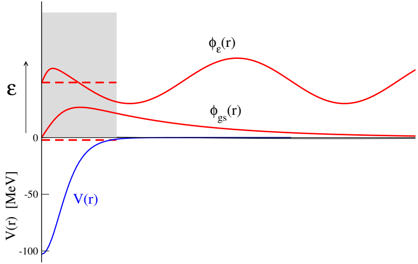
Inclusion of breakup channels: the CDCC method

Breakup modelspace

- In collisions involving weakly bound nuclei, excitation of unbound states (breakup channels) of the weakly-bound nucleus plays an important role.
- Reaction formalisms (DWBA, CC...) must be conveniently extended in order to incorporate the possibility of coupling to these breakup channels.



Bound versus scattering states

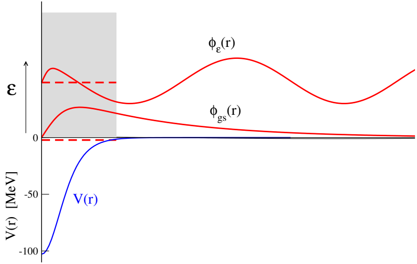


Continuum wavefunctions:

$$\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r}[Y_\ell(\hat{r}) \otimes \chi_s]_{jm}$$

$$\varepsilon = \frac{\hbar^2 k^2}{2\mu}$$

Bound versus scattering states



Continuum wavefunctions:

$$\boxed{\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r}[Y_\ell(\hat{r}) \otimes \chi_s]_{jm} \qquad \varepsilon = \frac{\hbar^2 k^2}{2\mu}}$$

Unbound states are not suitable for CC calculations:

- They have a continuous (infinite) distribution in energy.
- Non-normalizable: $\langle u_{k,\ell sj}(r)|u_{k',\ell sj}(r)\rangle \propto \delta(k-k')$

SOLUTION $\Rightarrow$ continuum discretization

The origins of CDCC

- Continuum discretization method proposed by G.H. Rawitscher [PRC9, 2210 (1974)] and Farrell, Vincent and Austern [Ann.Phys.(New York) 96, 333 (1976)] to describe deuteron scattering as an effective three-body problem $p + n + A$.

PHYSICAL REVIEW C VOLUME 9, NUMBER 6 JUNE 1974

Effect of deuteron breakup on elastic deuteron-nucleus scattering

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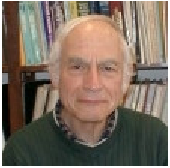
(Received 1 October 1973; revised manuscript received 4 March 1974)

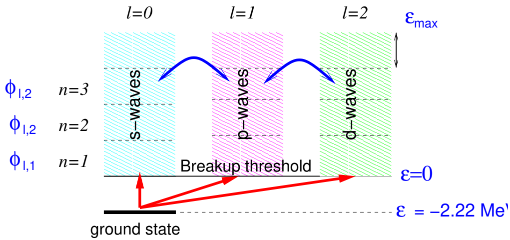
The properties of the transition matrix elements $V_{ab}(R)$ of the breakup potential V_N taken between states $\phi_a(\vec{r})$ and $\phi_b(r)$ are examined. Here $\phi_a(\vec{r})$ are eigenstates of the neutron-proton relative-motion Hamiltonian, and the eigenvalues of the energy ϵ_a are positive (continuum states) or negative (bound deuteron); $V_N(\vec{r}, \vec{R})$ is the sum of the phenomenological proton nucleus $V_{p-A}(|\vec{R} - \frac{1}{2}\vec{r}|)$ and neutron nucleus $V_{n-A}(|\vec{R} + \frac{1}{2}\vec{r}|)$ optical potentials evaluated for nucleon energies equal to half the incident deuteron energy. The bound-to-continuum transition matrix element for relative neutron-proton angular momenta $l=2$ are found to be comparable in magnitude to the ones for $l=0$ for values of ϵ_a larger than about 3 MeV, and both decrease only slowly with ϵ_a, suggesting that a large breakup spectrum is involved in deuteron-nucleus collisions. The effect of the various breakup transitions on the elastic phase shifts is estimated by numerically solving a set of coupled equations. These equations couple the functions $\chi_a(\vec{R})$ which are the coefficients of the expansion of the neutron-proton-nucleus wave function in a set of the $\phi_a(\vec{r})$'s. The equations are rendered manageable by performing a (rather crude) discretization in the neutron-proton relative-momentum variable k_a. Numer-



George Rawitscher (1928-2018)

- Full numerical implementation by Kyushu group (Sakuragi, Yahiro, Kamimura, and co.): Prog. Theor. Phys.(Kyoto) 68, 322 (1982)

Continuum discretization for deuteron scattering



- Select a number of angular momenta ($\ell = 0, \ldots, \ell_{\max}$).
- For each ℓ, set a maximum excitation energy $\varepsilon_{\max}$.
- Divide the interval $\varepsilon = 0 - \varepsilon_{\max}$ in a set of sub-intervals (*bins*).
- For each bin, calculate a representative wavefunction $\phi_{\ell m}(\mathbf{r})$.

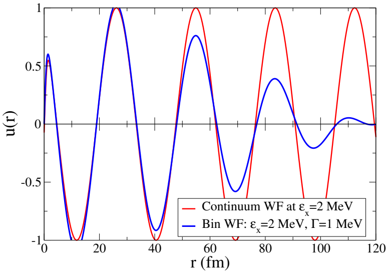
CDCC formalism: construction of the bin wavefunctions

Bin wavefunction:

$$\phi^{[k_1,k_2]}_{\ell jm}(\mathbf{r}) = \frac{u^{[k_1,k_2]}_{\ell j}(r)}{r}[Y_\ell(\hat{r})\otimes\chi_s]_{jm} \qquad [k_1,k_2] = \text{bin interval}$$

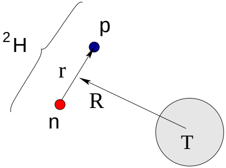
$$\boxed{u^{[k_1,k_2]}_{\ell sjm}(r) = \sqrt{\frac{2}{\pi N}} \int_{k_1}^{k_2} w(k) u_{k,\ell sj}(r) dk}$$

- k: linear momentum
- $u_{k,\ell sj}(r)$: scattering states (radial part)
- $w(k)$: weight function



CDCC formalism for deuteron scattering

- Hamiltonian: $H = T_{\mathbf{R}} + h_r(\mathbf{r}) + V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt})$
- Model wavefunction:



$$\Psi^{(+)}(\mathbf{R},\mathbf{r}) = \underbrace{\phi_{gs}(\mathbf{r})\chi_0(\mathbf{R})}_{\text{bound state}} + \underbrace{\sum_{n>0}^{N} \phi_n(\mathbf{r})\chi_n(\mathbf{R})}_{\text{discretized contimuum}}$$

  - Coupled equations: $[H - E]\Psi(\mathbf{R},\mathbf{r}) = 0$

$$\boxed{[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{R}) = \sum_{n'\neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})}$$

  - Coupling potentials:

$$V_{n;n'}(\mathbf{R}) = \int d\mathbf{r}\, \phi_n^*(\mathbf{r}) \left[V_{pt}(\mathbf{R} + \frac{\mathbf{r}}{2}) + V_{nt}(\mathbf{R} - \frac{\mathbf{r}}{2}) \right] \phi_{n'}(\mathbf{r})$$

Trivially equivalent local equivalent potential (TELP)

- From the elastic channel equation, a TELP can be defined as follows:

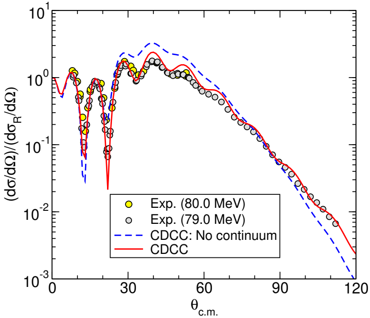
$$\left[E - \varepsilon_0 - \hat{T}_{\mathbf{R}} - V_{0,0}(\mathbf{R})\right]\chi_0(\mathbf{R}) = \sum_{i \neq 0} V_{i,0}(\mathbf{R})\chi_i(\mathbf{R}) \equiv U_{\mathrm{TELP}}(\mathbf{R})\chi_0(\mathbf{R}).$$

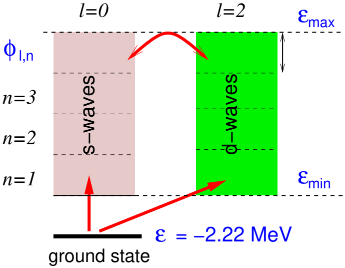
- In actual calculations, $U_{\mathrm{TELP}}(R)$ will depend on the total angular momentum, but a weighted average can be performed to obtain an approximate angular-momentum independent polarization potential
- A single channel calculation with the potential $U(\mathbf{R}) = V_{0,0}(\mathbf{R}) + U_{\mathrm{TELP}}(\mathbf{R})$ should reproduce approximately the elastic scattering cross section.

Applications of the CDCC formalism: d+ ^{58}Ni

Coupling to continuum states produce:

- Polarization of the projectile (modification of real part)
- Flux removal (absorption) from the elastic channel (imaginary part)

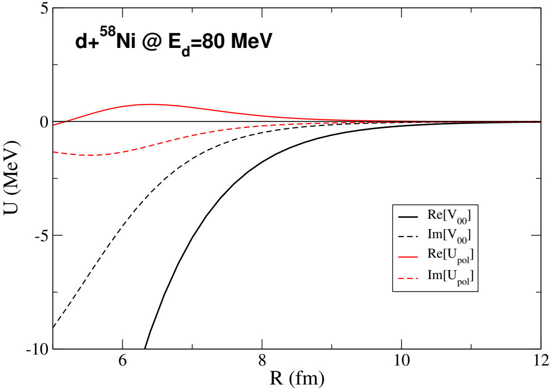




☞ No continuum ⇒ retain only the Watanabe potential:

$$V_{00}(\mathbf{R}) = \int d\mathbf{r}\, \phi^*_{gs}(\mathbf{r}) \left(V_{pt} + V_{nt}\right) \phi_{gs}(\mathbf{r})$$

Trivially equivalent local equivalent potential for d+^{58}Ni @ 80 MeV



For this reaction, the TELP is complex:

- The real part is repulsive (reduces projectile-target attraction)
- The imaginary part is absorptive (flux removal)

Two- and three-body breakup observables

- CDCC scattering amplitudes readily provide **two-body breakup** observables:

$$\boxed{\frac{d\sigma_n}{d\Omega_{\text{c.m.}}} = |f_{0,n}(\theta)|^2 \Rightarrow \frac{d^2\sigma}{d\Omega_{\text{c.m.}} d\epsilon_{pn}} \simeq \frac{1}{\Delta_n} \frac{d\sigma_n}{d\Omega_{\text{c.m.}}}}$$

  with:
  - Δ_n=width of the bin containing the relative energy ϵ_{pn}
  - $\Omega_{\text{c.m.}}$=C.M. scattering angle of the projectile c.m. (not easy to measure!)
- **Three-body observables** can be also calculated using a suitable combination of the scattering amplitudes and appropriate kinematical transformations (Tostevin, PRC63, 024617 (2001)):

$$\boxed{\frac{d^3\sigma}{d\Omega_n d\Omega_p dE_p}}$$

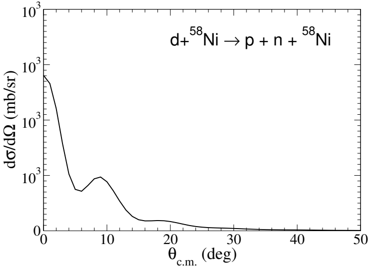
☞ **N.b.:** *These 3-body observables are not directly provided by FRESCO. They must be computed separately from the breakup scattering amplitudes.*

Two-body breakup observables: d+ ^{58}Ni → p+n+^{58}Ni

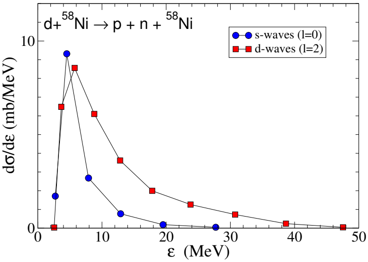
CDCC calculations for **d+ ^{58}Ni** at 80 MeV :

- Continuum states with $\ell = 0, 2$.
- Proton and neutron intrinsic spins ignored.
- p/n+ ^{58}Ni from global optical potential.
- p+n simple Gaussian interaction describing deuteron g.s.

p + *n* c.m. angular distribution



Excitation energy distribution

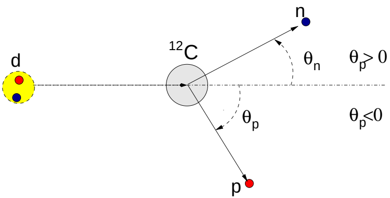


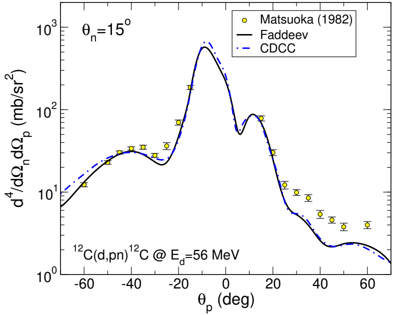
Breakup observables with CDCC: exclusive breakup of d+ ^{12}C → p+n+^{12}C

CDCC calculations for **d+ ^{12}C** at 56 MeV:

- Continuum states with $\ell \leq 8$ and ε_{max}=46 MeV.
- Proton and neutron intrinsic spins ignored
- p/n+ ^{58}Ni from Watson global optical potential
- p+n simple Gaussian interaction describing deuteron g.s.

Data: Matsuoka *et al.*, NPA391, 357 (1982).



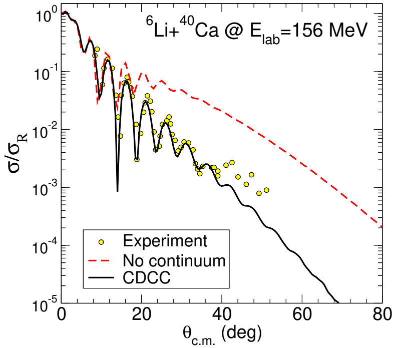


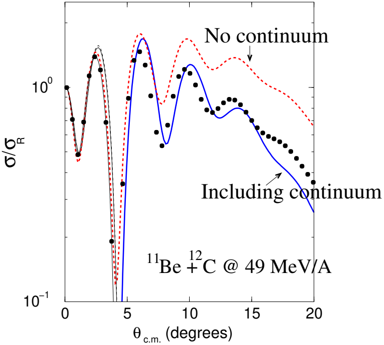
A. Deltuva et al, PRC 76, 064602 (2007)

Application of the CDCC method: ^{6}Li and ^{6}He scattering

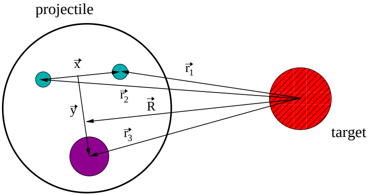
☞ The CDCC method has been also applied to nuclei with a cluster structure:

- ^{6}Li=α + d ($S_{\alpha,d}$=1.47 MeV)
- ^{11}Be=^{10}Be + n (S_n=0.504 MeV)





Extension to 3-body projectiles

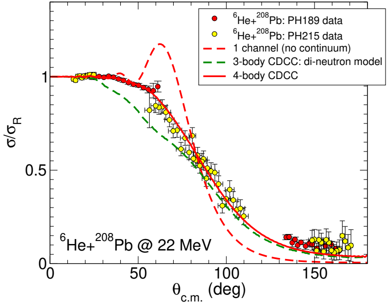


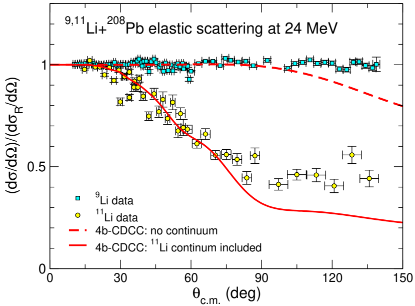
To extend the CDCC formalism, one needs to evaluate the new coupling potentials:

$$\boxed{V_{n;n'}(\mathbf{R}) = \int d\mathbf{r}\, \phi^*_n(\mathbf{x},\mathbf{y})\, \{V_{nt}(\mathbf{r}_1) + V_{nt}(\mathbf{r}_2) + V_{\alpha t}(\mathbf{r}_3)\}\, \phi_{n'}(\mathbf{x},\mathbf{y})}$$

- ☞ $\phi_n(\mathbf{x},\mathbf{y})$ three-body WFs for bound and continuum states: hyperspherical coordinates, Faddeev, etc (difficult to calculate!)
- ☞ 4b-CDCC calculations not included in FRESCO; require separate codes to compute the $\phi_n(\mathbf{x},\mathbf{y})$ wfs (e.g. FACE) and $V_{n;n'}(\mathbf{R})$ potentials

Four-body CDCC calculations for ^{6}He and ^{11}Li scattering





Data (LLN): NPA803, 30 (2008);PRC 84, 044604 (2011)
Calculations: PRC 80, 051601 (2009)

M Cubero et al, PRL109, 262701 (2012)

N.b.: 1-channel potential considers only g.s. → g.s. coupling potential:

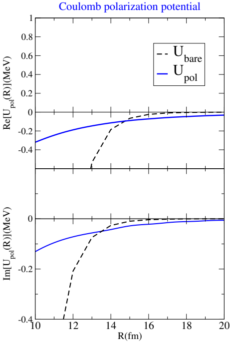
$$V_{00}(\mathbf{R}) = \int d\mathbf{r}\, \phi^*_{\mathrm{g.s.}}(\mathbf{x},\mathbf{y})\, \{V_{nt}(\mathbf{r}_1) + V_{nt}(\mathbf{r}_2) + V_{ct}(\mathbf{r}_3)\}\, \phi_{\mathrm{g.s.}}(\mathbf{x},\mathbf{y})$$

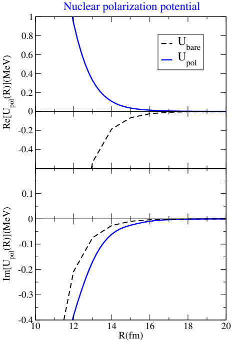
Polarization potential for ^{6}He+^{208}Pb: long-range effect

Trivially Equivalent Local Polarization potential (TELP):

$$\left[E - \varepsilon_0 - \hat{T}_{\mathbf{R}} - V_{0,0}(\mathbf{R})\right]\chi_0(\mathbf{R}) = \sum_{i \neq 0} V_{i,0}(\mathbf{R})\chi_i(\mathbf{R}) \equiv U_{\mathrm{TELP}}(\mathbf{R})\chi_0(\mathbf{R}).$$

Application to ^{6}He+^{208}Pb at 22 MeV



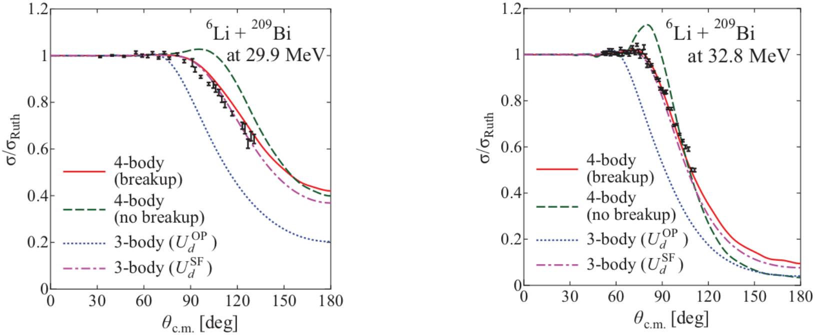


- Imaginary part of TELP is long-ranged and absorptive (explains the need for large imaginary diffuseness parameter in OM analysis)
- Real part is attractive for the Coulomb potential and repulsive for the nuclear couplings.

Application to ^{6}Li scattering

Example: ^{6}Li+^{209}Bi around Coulomb barrier

- 4-body CDCC: ^{6}Li=α+p+n
- 3-body CDCC: ^{6}Li=α+d

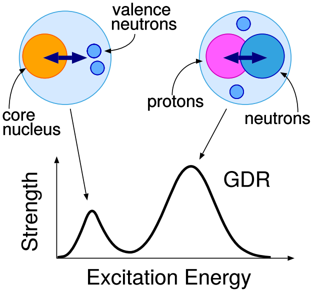


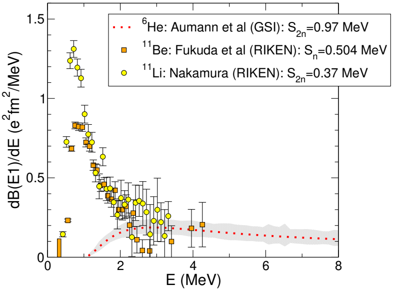
Watanabe *et al*, PRC 86, 031601(R) (2012)

Exploring the continuum with breakup reactions

1. Coulomb dissociation experiments
   - Semiclassical description: Alder and Winther
   - Quantum-mechanical description
2. Exploring continuum structures: resonances and virtual states

Electric response of weakly-bound nuclei





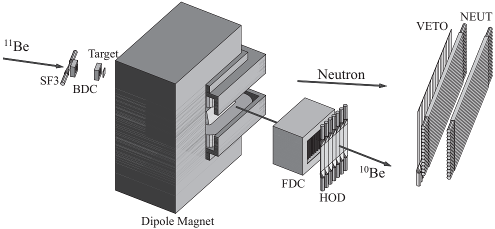
- The $E\lambda$ response can be quantified through the $B(E\lambda)$ probability:

$$\boxed{B(E\lambda; i \to f) = \frac{1}{2I_i + 1} |\langle \Psi_f \| \mathcal{M}(E\lambda) \| \Psi_i \rangle|^2}$$

- Neutron-halo nuclei have large $B(E1)$ strengths near threshold

How to probe/extract the $B(E1)$ of halo nuclei?

Example: ^{11}Be+^{208}Pb $\rightarrow$ ^{10}Be+ n+ ^{208}Pb measured at RIKEN (69 MeV/u).
Fukuda et al, PRC70, 054606 (2004))



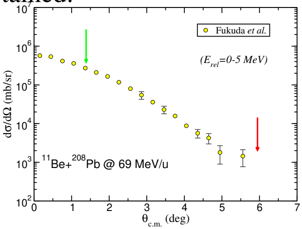
- ^{11}Be excitation energy can be reconstructed from core-neutron coincidences (*invariant mass method*)

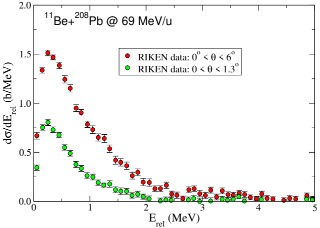
What observables are measured in Coulomb dissociation experiments?

- Experimentally, one measures angular and relative energy distribution of the ^{11}Be* system:

$$\boxed{\frac{d^2\sigma}{d\Omega\, dE}}$$

- Integrating over the angle or energy, single differential cross sections are obtained:





- In the Coulomb dominated region (i.e. small angles), the breakup cross section is expected to be dominated by the $dB(E\lambda)/dE$ distribution, but we need a theory that relates both observables.

Semiclassical 1st order Eλ excitation (Alder & Winther) (akin EPM method)

- For Eλ excitation to bound states ($0 \to n$):

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{0\to n} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{B(E\lambda, 0 \to n)}{e^2 a_0^{2\lambda-2}} f_\lambda(\theta,\xi)} \qquad \xi_{0\to n} = \frac{(E_n - E_0)}{\hbar}\frac{a_0}{v}$$

- For continuum states (breakup):

$$\boxed{\frac{d\sigma(E\lambda)}{d\Omega dE} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{1}{e^2 a_0^{2\lambda-2}} \frac{dB(E\lambda)}{dE} \frac{df_\lambda(\theta,\xi)}{d\Omega}}$$

☞ $dB(E\lambda)/dE$ can be extracted from small-angle Coulomb dissociation data.

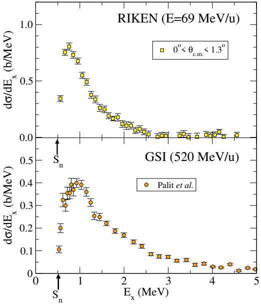
$$\boxed{\frac{d\sigma}{dE}(\theta < \theta_{\max}) = \int_0^{\theta_{\max}} \frac{d\sigma(E\lambda)}{d\Omega dE} d\Omega \propto \frac{dB(E\lambda)}{dE}}$$

Extracting $B(E1)$ of ^{11}Be from ^{11}Be+^{208}Pb Coulomb dissociation

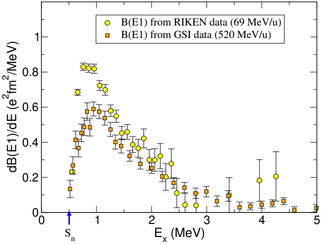
Common assumptions:

- Breakup dominated by Coulomb excitation (mostly E1).
- Nuclear excitation, if present, can be estimated and added incoherently

☞ If the assumptions above are fulfilled, the extracted $dB(E\lambda)dE$ should be independent of the incident energy and target employed, since it reflects a structure property of the projectile.



$$\frac{dB(E\lambda)}{dE} \propto \frac{d\sigma}{dE}$$



RIKEN: Fukuda et al, PRC70 (2004) 054606
GSI: Palit et al, PRC68 (2003) 034318

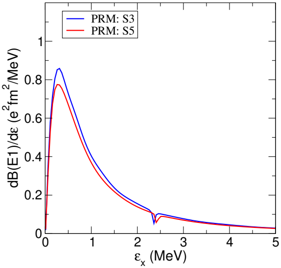
☞ *The extracted $dB(E\lambda)/dE$ distributions are reasonably compatible, but with apparent differences at the peak*

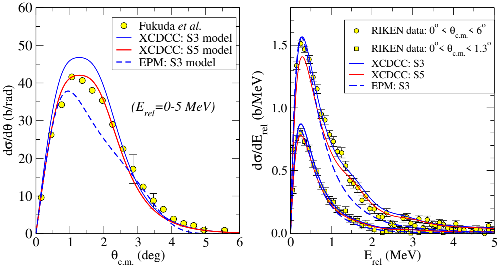
CDCC analysis of Coulomb dissociation data

- Nuclear excitation not negligible, even for small θ
- Nuclear contribution interferes with Coulomb
- Higher-order couplings can affect the cross sections (E2, E3...)

☞ These ingredients can be naturally incorporated within the CDCC method (at the expense of more complexity!)

E.g.: CDCC analysis based on two-body ^{10}Be+n model: PLB 811 (2020) 135959



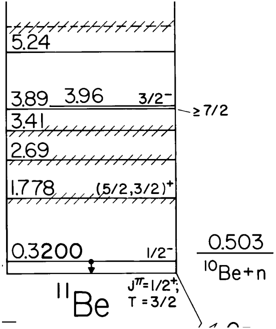


☞ Different structure models yield different $B(E1)$ strengths and hence different breakup cross sections

☞ Comparison with the angular distribution evidences the deficiencies of the semiclassical EPM model

Continuum resonances

The continuum spectrum is not "homogeneous"; it contains in general energy regions with special structures, such as resonances and virtual states

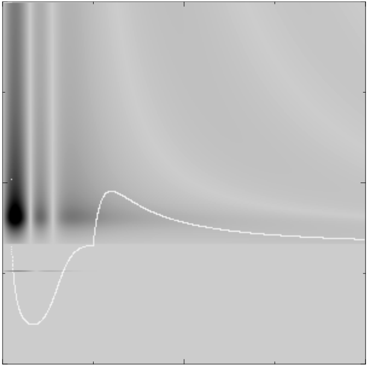


What is a resonance?

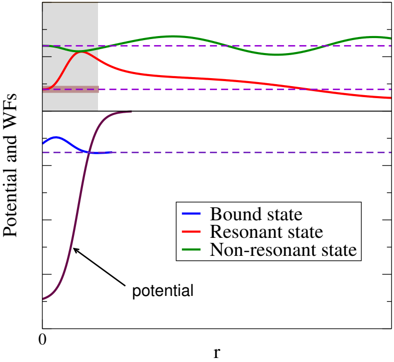
- It is a pole of the S-matrix in the complex energy plane.
- It is a structure on the continuum which may, or may not, produce a maximum in the cross section, depending on the reaction mechanism and the phase space available.
- The resonance occurs in the range of energies for which the phase shift is close to $\pi/2$.
- In this range of energies, continuum wavefunctions have a large probability of being in the radial range of the potential.
- The continuum wavefunctions are not square normalizable. For practical applications, a normalized wavepacket (or "bin") can be constructed to represent the resonance.

Distinctive features of a resonance

In the energy range of the resonance, the continuum wavefunctions have a large probability of being within the range of the potential.

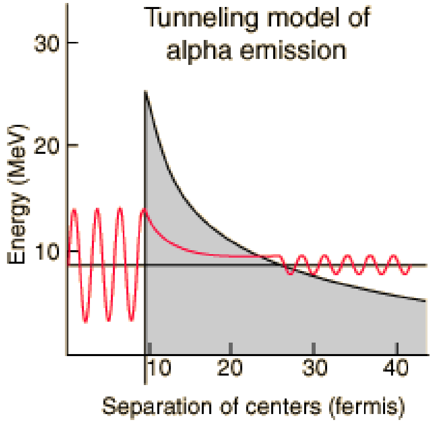


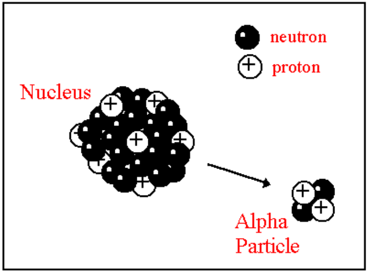
(Courtesy of C. Dasso)



Distinctive features of a resonance

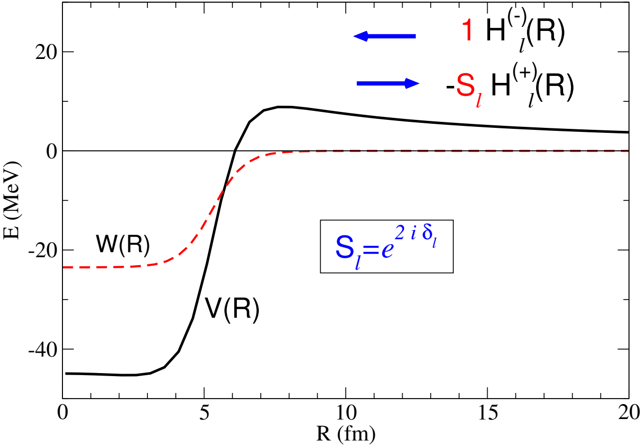
The decay of the resonance is also behind the α-decay phenomenon:



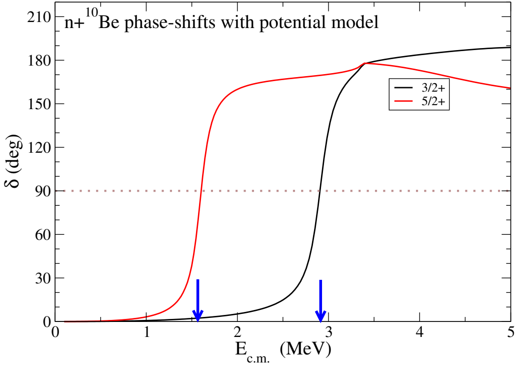


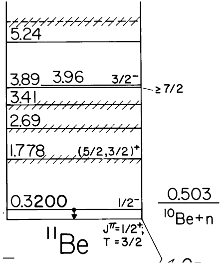
Resonances and phase-shifts

$$\chi_l(R) \to 1\, H^{(-)}_l(R) - S_l\, H^{(+)}_l(R)$$



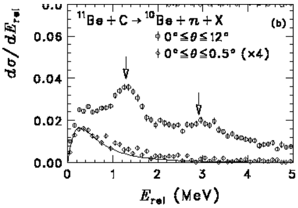
Resonances and phase-shifts

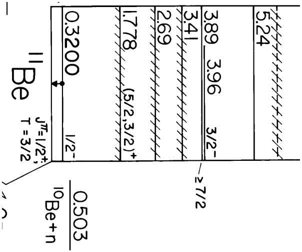




Studying resonances in nuclear breakup experiments

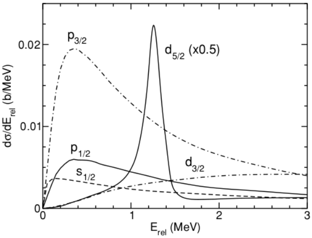
RIKEN data

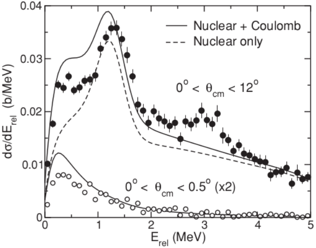




Fukuda *et al*, PRC70 (2004) 054606)

CDCC analysis



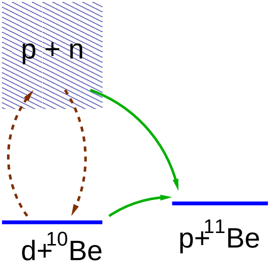


Howell *et al.*, JPG31 (2005) S1881

Transfer reactions with weakly bound nuclei

Transfer reactions with weakly bound nuclei

- DWBA approximates the total WF by the elastic channel and assumes that the transfer occurs in one step (Born approximation).
- For weakly bound projectiles (eg. deuterons), breakup is an important channel and can influence the transfer process.



- $\Psi^{(+)}_{\mathbf{K}_d}(\mathbf{R},\mathbf{r})$ includes breakup components, but these are lost when we make the DWBA approximation ($\Psi^{(+)} \approx \chi^{(+)}_d(\mathbf{K}_d,\mathbf{R})\varphi_d(\mathbf{r})$) $\Rightarrow$ need to go beyond DWBA

Adiabatic distorted wave approximation (ADWA)

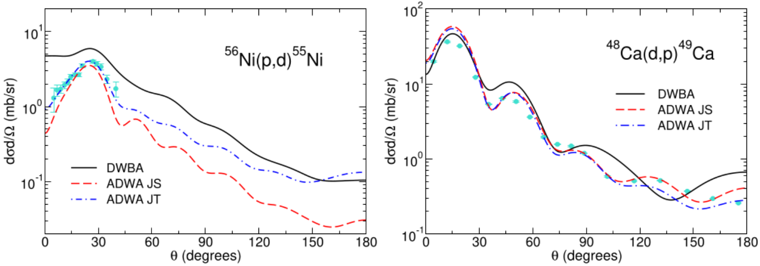
- $\chi^{(+)}_d(\mathbf{K}_d, \mathbf{R})$ describes deuteron elastic scattering but, for the (d, p) transfer matrix element, we need only $\Psi^{(+)}_{\mathbf{K}_d}(\mathbf{R}, \mathbf{r})$ for small $|\mathbf{r}|$
- R.C. Johnson and col. have derived an approximation of $\Psi^{(+)}_{\mathbf{K}_d}(\mathbf{R}, \mathbf{r})$ valid for $r \approx 0$, which includes the effect of deuteron breakup effectively (adiabatic approx.):
  1. Zero-range approximation (Johnson-Soper):[(Johnson,Soper,PRC1, 976 (1970)]

$$\boxed{U^{JS}(R) = U_{pA}(R) + U_{nA}(R)} \quad \Rightarrow \chi^{JS}_d(\mathbf{R})$$

  2. Finite-range version (Johnson–Tandy): [Johnson & Tandy, NPA235 (1974) 56]

$$\boxed{U^{JT}(R) = \frac{\langle \varphi_{pn}(\mathbf{r})|V_{pn}(U_{nA} + U_{pA})|\varphi_{pn}(\mathbf{r})\rangle}{\langle \phi_{pn}(\mathbf{r})|V_{pn}|\phi_{pn}(\mathbf{r})\rangle}}$$

DWBA vs ADWA



From Timofeyuk and, Progress in Particle and Nuclear Physics 111 (2020) 103738

CDCC-BA approximation

- Exact transition amplitude for a general $A(d,p)B$ process:

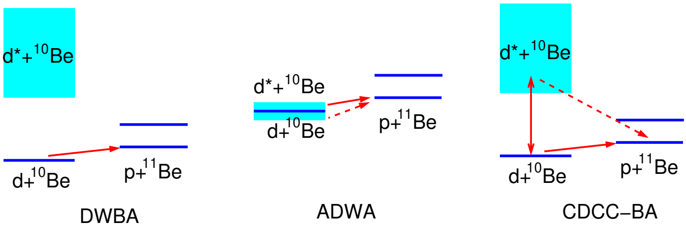
$$\boxed{\mathcal{T}^{\mathrm{CDCC}}_{d,p} = C^{\ell j}_{BA} \int\int \chi^{(-)*}_{p}(\mathbf{K}_p, \mathbf{R}')\varphi^{\ell j,*}_{nA}(\mathbf{r}')\; V_{pn}(\mathbf{r})\; \underbrace{\Psi^{(+)}_{\mathbf{K}_\alpha}(\mathbf{R}_\alpha, \xi_\alpha)}\, d\mathbf{r}'\, d\mathbf{R}'}$$

- Use CDCC approximation for $\Psi^{(+)}_{\mathbf{K}_\alpha}$:

$$\Psi^{(+)}_{\mathbf{K}_\alpha} \approx \Psi^{\mathrm{CDCC}} = \underbrace{\chi_0(\mathbf{R})\phi_0(\mathbf{r})}_{\text{elastic}} + \sum_{n',j,\pi} \underbrace{\phi^{j\pi}_{n'}(k_{n'}, \mathbf{r})\chi_{n',j,\pi}(\mathbf{R})}_{\text{breakup}}$$

- Unlike the DWBA and ADWA methods, coupling to deuteron breakup states is included explicitly.

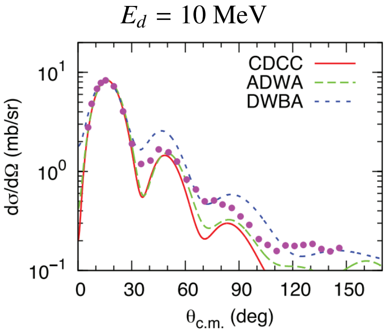
DWBA, ADWA and CDCC-BA compared

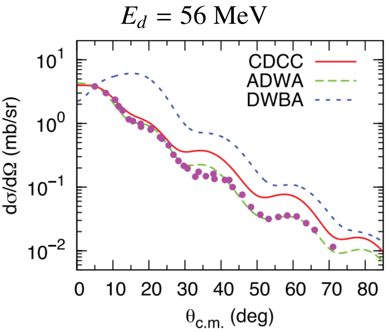


Gómez-Camacho and A.M.M., *A Pedestrian Approach to the Theory of Transfer Reactions: Application to Weakly-Bound and Unbound Exotic Nuclei*, Lecture Notes in Physics, vol 879.

DWBA vs ADWA vs CDCC

Example: ^{58}Ni(d,p)^{59}Ni





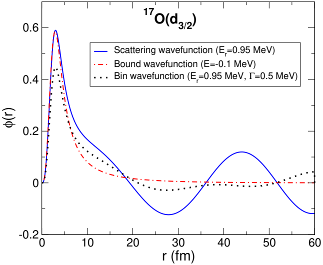
☞*CDCC and ADWA provide better description of the data and lead also to more realistic spectroscopic information (e.g. spectroscopic factors)*

Pang *et al*, PRC 90, 044611 (2014)

Exploring resonances from transfer reactions

- Calculation of transfer to unbound states in DWBA and ADWA poses numerical problems due to the oscillatory behaviour of unbound wavefunctions.
- A regularization method must be applied:
  - Representing the resonance by a weakly bound state with the same quantum numbers
  - Vincent & Fortune contour integration in the complex radius plane (PRC2 (1970) 782)
  - Representing the resonance by a continuum *bin*

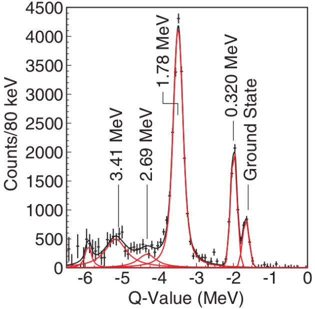
E.g.: $d_{3/2}$ resonance in ^{17}O resonance at $E_r = 0.95$ MeV



Exploring resonances from transfer reactions

- Inclusion of continuum states in DWBA poses numerical problems due to the oscillatory behaviour of unbound wavefunctions
- Regularization method must be applied, such as representing the resonance by a wavepacket (continuum *bin*, as in the CDCC method)

E.g.: ^{11}Be resonance at E_x = 1.78 MeV from ^{10}Be(d,p)^{11}Be   Schmitt et al, PRC88, 064612 (2013)



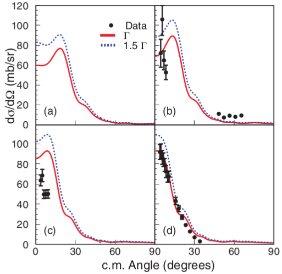
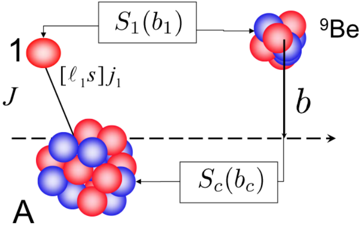


FIG. 8. (Color online) Differential cross sections are presented for transfer to the first resonance in ^{11}Be at 1.78 MeV via the ^{10}Be(d,p) reaction in inverse kinematics at deuteron energies of (a) 12 MeV, (b) 15 MeV, (c) 18 MeV, and (d) 21.4 MeV. The curves are from FR-ADWA calculations using (solid line) an energy bin that is the same width as for the resonance used in the calculation and (dotted line) with a width 1.5 times that value. At 12 MeV the protons were too low in energy to extract an angular distribution.

Knock-out reactions

Spectroscopic from momentum distributions

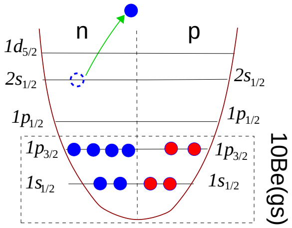
- Fast-moving projectile on a (typically) light target.
- One nucleon suddenly removed (absorbed) due to its interaction with the target.
- The remaining residue remains unchanged and is detected.
- The momentum of the core is related to that of the removed nucleon because, in the rest frame of the projectile, $\vec{P} = 0$

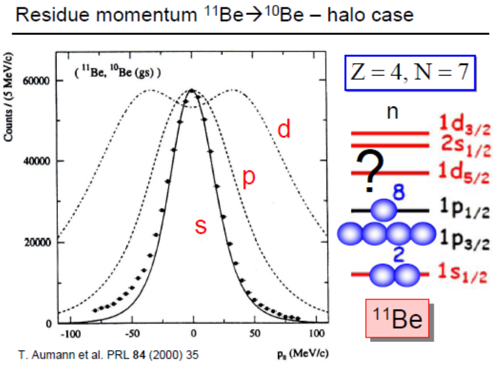


$$\boxed{\vec{P} = \vec{p}_c + \vec{p}_1 = 0}$$

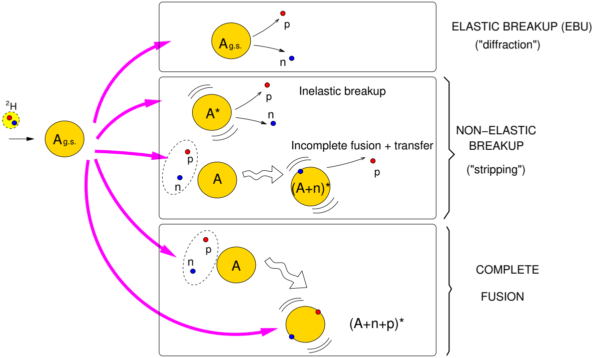
Angular momentum sensitivity of momentum distributions

- The shape is determined by the orbital angular momentum ℓ.
- The magnitude is determined by the amount of $s_{1/2}$ (spectroscopic factor)





Stripping and diffraction contributions to inclusive breakup cross sections



☞*The singles (inclusive) cross section of a given fragment will contain in general diffraction and stripping components*

Stripping cross section within a semiclassical (eikonal) theory

At high energies, one can use the sudden, eikonal approximations to obtain simple formulas for the stripping and diffraction parts of the inclusive breakup cross section for a inclusive reaction of the form $a + B \to b + X$, with $a = b + x$:

Stripping:

$$\boxed{\sigma_{\rm sp}^{\rm str} = 2\pi \int bdb \int d\mathbf{r}\, |\varphi_{bx}(\mathbf{r})|^2 (1 - |S_x(b_x)|)^2 |S_{bA}(b_b)|^2}$$

Diffraction:

$$\boxed{\sigma_{\rm sp}^{\rm diff} = 2\pi \int bdb \left[\langle\varphi_{bx}|\, |S_b S_x|^2 |\varphi_{bx}\rangle - |\langle\varphi_{bx}|S_b S_x|\varphi_{bx}\rangle|^2\right].}$$

- $|S_b(b_b)|^2$=probability of survival of the core.
- $1 - |S_x(b_x)|^2$=probability of absorption of the valence particle.

Extraction of SFs from knockout reactions

- Agreement theory vs experiment quantified with the reduction factor:

$$\boxed{R_s = \frac{\sigma_{\rm theor}}{\sigma_{\rm exp}}}$$

with

$$\boxed{\sigma_{\rm theor} = \sum_{n\ell j} S^{a}_{bx}(I;n\ell j)\sigma_{\rm sp}(I;n\ell j)} \qquad \boxed{\sigma_{\rm sp}(I;n\ell j) = \sigma^{\rm EBU}_{\rm sp} + \sigma^{\rm NEB}_{\rm sp}}$$

☞ $R_s < 1 \Rightarrow$ possible correlations (long-range, short-range, tensor,...) not included in $\sigma_{\rm theor}$?

☞ R_s strongly dependent on $\Delta S = S_p - S_n$.

Extraction of SFs from knockout reactions

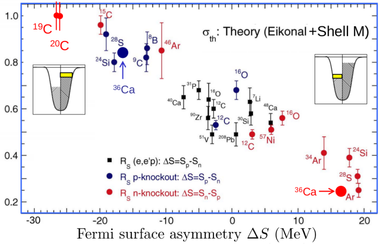
- Agreement theory vs experiment quantified with the reduction factor:

$$\boxed{R_s = \frac{\sigma_{\text{theor}}}{\sigma_{\text{exp}}}}$$

with

$$\boxed{\sigma_{\text{theor}} = \sum_{n\ell j} S^{a}_{bx}(I;n\ell j)\sigma_{\text{sp}}(I;n\ell j)} \qquad \boxed{\sigma_{\text{sp}}(I;n\ell j) = \sigma^{\text{EBU}}_{\text{sp}} + \sigma^{\text{NEB}}_{\text{sp}}}$$

Measurements at the two Fermi surfaces



-Gade et al, PRC 77, 044306 (2008)

Tostevin, PRC90,057602(2014)

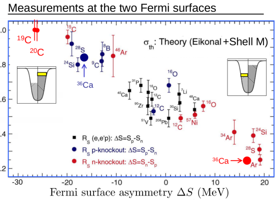
☞ $R_s < 1 \Rightarrow$ possible correlations (long-range, short-range, tensor,...) not included in σ_{theor}?

☞ R_s strongly dependent on $\Delta S = S_p - S_n$.

Extraction of SFs from knockout reactions

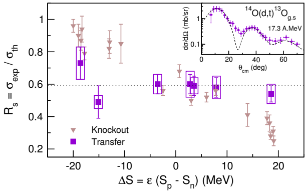
...however, this behaviour has not been corroborated by other probes, such as transfer or proton-induced knockout reactions (p, pN)

HI knockout (∼100 MeV/u)
Tostevin, PRC90,057602(2014)



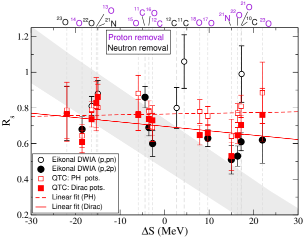
- Reaction model: eikonal + adiabatic
- R_s strongly dependent on $S_p - S_n$.

Low-energy transfer
Flavigny, PRL110, 122503(2013)



- Reaction model: ADWA, DWBA, CRC
- R_s ∼ constant.

(p, pN) @ 200-400 MeV/u
Aumann, PPNP118,103847(2021)



- Reaction models: DWIA, TC
- R_s ∼ constant.

☞ Similar results from RIKEN
Wakase, PTEP 021D01 (2018)

R_s from knockout disagree with those from transfer and (p, pN) ⇒ description of reaction mechanism?