Transfer reactions: the DWBA method

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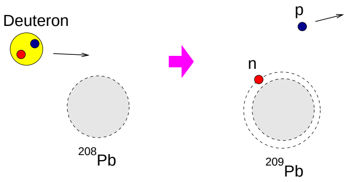
Material available at: `https://github.com/ammoro/RAON`

Outline

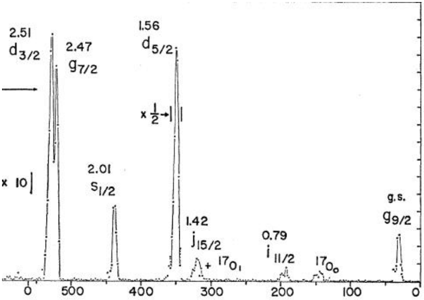
Example: d+^{208}Pb $\rightarrow$ p + ^{209}Pb

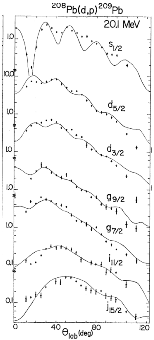


1. What do we measure in a transfer reaction?
   1. For a typical transfer reaction (e.g. d+^{208}Pb $\rightarrow$ p + ^{209}Pb), one measures the angular and energy distribution of outgoing fragments (e.g. protons).
   2. Additionally, one may collect information of decay products of ^{209}Pb (e.g. γ-rays, n, p, etc)
2. What information can we infer from a transfer reaction?
   1. Excitation energies of the residual nucleus (^{209}Pb).
   2. Angular momentum assignment.
   3. Single-particle content of populated states (i.e. spectroscopic factors).

What do we measure in a transfer reaction?

Example: $d + {}^{208}Pb \rightarrow p + {}^{209}Pb$ *Phys. Rev. 159 (1967) 1039*

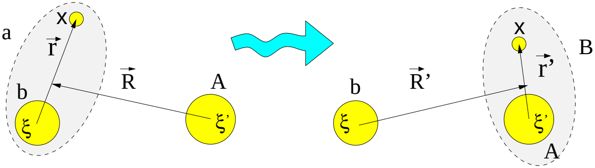




- The proton energy spectrum shows some peaks which reflect the energy spectrum of the residual nucleus (^{209}Pb).
- Each peak has a characteristic angular distribution, which depends on the structure of the associated state.
- The population probability will depend on the reaction dynamics and on the structure properties of these states.

DWBA method for transfer reactions

- Transfer process: $\underbrace{(b+x)}_{a} + A \longrightarrow b + \underbrace{(A+x)}_{B}$



- Complications arise with respect to inelastic scattering because now we have two different mass partitions involved

$$\underbrace{a+A}_{\alpha} \longrightarrow \underbrace{b+B}_{\beta}$$

Evaluation of scattering amplitude in Born approximation (post form)

- Projectile-target interaction in post representation:

$$V_\beta(\mathbf{R}',\mathbf{r}') = V_{xb} + U_{bA} = \underbrace{U_\beta(\mathbf{R}')}_{\text{Aux. pot.}} + \underbrace{[V_{xb} + U_{bA} - U_\beta(\mathbf{R}')]}_{\text{Resid. inter.}} \equiv U_\beta(\mathbf{R}') + \Delta V_\beta$$

- In DWBA, the scattering amplitude is $f_{\beta\alpha}(\theta) = -(\mu_\beta/2\pi\hbar^2)\mathcal{T}_{\beta,\alpha}$ with

$$\boxed{\mathcal{T}_{\beta,\alpha}(\theta) = \int \underbrace{\chi_\beta^{(-)*}(\mathbf{K}_\beta,\mathbf{R}')\,\Phi_\beta^*(\xi_\beta)}_{\text{final state}}\ \Delta V_\beta\ \underbrace{\chi_\alpha^{(+)}(\mathbf{K}_\alpha,\mathbf{R})\,\Phi_\alpha(\xi_\alpha)}_{\text{initial state}}\ \underbrace{d\xi_\beta\, d\mathbf{R}'}_{\text{(all coordinates)}}}$$

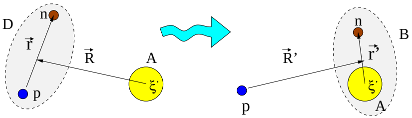
- Initial and final internal states:

$$\text{Initial state: } \Phi_\alpha(\xi_\alpha) = \varphi_a(\xi,\mathbf{r})\Phi_A(\xi') \qquad \xi_\alpha \equiv \{\xi,\xi',\mathbf{r}\}$$

$$\text{Final state: } \Phi_\beta(\xi_\beta) = \varphi_b(\xi)\Phi_B(\xi',\mathbf{r}') \qquad \xi_\beta \equiv \{\xi,\xi',\mathbf{r}'\}$$

- $\chi_{\alpha,\beta}^{(\pm)}$ are distorted waves for entrance and exit channels, obtained with appropriate optical potentials $U_\alpha(\mathbf{R})$, $U_\beta(\mathbf{R}')$

The important (d, p) case



- Introduce auxiliary potentials in entrance ($U_\alpha(\mathbf{R})$) and exit ($U_\beta(\mathbf{R}')$) channels.
- Projectile-target interaction: $V_\beta = V_{pn} + U_{pA} = U_{pB}(\mathbf{R}') + \underbrace{V_{pn} + U_{pA} - U_{pB}}_{\Delta V_\beta} \equiv U_\beta(\mathbf{R}') + \Delta V_\beta$
- Internal states:

$$\Phi_\alpha^{(0)}(\xi_\alpha) = \varphi_d(\mathbf{r})\phi_A(\xi') \qquad \xi_\alpha = \{\xi', \mathbf{r}\}$$
$$\Phi_\beta(\xi_\beta) = \Phi_B(\xi', \mathbf{r}') \qquad \xi_\beta = \{\xi', \mathbf{r}'\}$$

- Post-form DWBA transition amplitude:

$$\boxed{\mathcal{T}_{d,p}^{\text{DWBA}} = \int\int \chi_p^{(-)*}(\mathbf{K}_p, \mathbf{R}')\Phi_B(\xi', \mathbf{r}')\,(V_{pn} + U_{pA} - U_{pB})\chi_d^{(+)}(\mathbf{K}_d, \mathbf{R})\varphi_d(\mathbf{r})\phi_A(\xi')\,d\xi_\beta d\mathbf{R}'}$$

- For medium-mass/heavy targets: $U_{pA} \approx U_{pB} \Rightarrow V_{pn} + U_{pA} - U_{pB} \approx V_{pn}(\mathbf{r})$

(d, p) case: parentage decomposition of target nucleus

➩ We need to evaluate the overlap integral

$$\int d\xi' \; \phi_B^*(\xi', \mathbf{r}')\phi_A(\xi') \equiv \langle \phi_B | \phi_A \rangle$$

➩ Use the parentage decomposition of $B \to A + n$

$$\boxed{\Phi_B(\xi', \mathbf{r}') = \mathcal{A}_{BA}^{\ell j}\phi_A(\xi')\varphi_{nA}^{\ell j}(\mathbf{r}') + \sum_{A' \neq A} \mathcal{A}_{BA'}^{\ell' j'}\phi_{A'}(\xi')\varphi_{nA}^{\ell' j'}(\mathbf{r}')}$$

$$\Rightarrow \langle \phi_B | \phi_A \rangle = \mathcal{A}_{BA}^{\ell j}\varphi_{nA}^{\ell j}(\mathbf{r}')$$

- $\mathcal{A}_{BA}^{\ell j}$ = spectroscopic amplitude
- $|\mathcal{A}_{BA}^{\ell j}|^2 = S_{BA}^{\ell j}$ = spectroscopic factor
- $\varphi_{nA}^{\ell j}(\mathbf{r}')$ = single-particle wavefunction describing motion of n with respect to A.

☞ *The spectroscopic factor quantifies the single-particle content of a given physical state B, when described as A + n, with A in some specific state.*

Examples of parentage decomposition

1. Double-magic nucleus plus a single nucleon:

$$|^{209}\mathrm{Bi(g.s.)}\rangle_{9/2^-} \approx \left[|^{208}\mathrm{Pb}(0^+)\rangle \otimes |\pi 1h_{9/2}\rangle\right]_{9/2^-}$$

☞ *almost* single-particle configuration ($S^{\ell sj}_{IJ} \approx 1$).

2. Deformed core plus an extra nucleon:

$$|^{11}\mathrm{Be(gs)}\rangle_{1/2^+} = \alpha \left[|^{10}\mathrm{Be}(0^+)\rangle \otimes |\nu 2s_{1/2}\rangle\right]_{1/2^+} + \beta \left[|^{10}\mathrm{Be}(2^+)\rangle \otimes |\nu 1d_{5/2}\rangle\right]_{1/2^+} + \ldots$$

with $|\alpha|^2 + |\beta|^2 + \ldots \approx 1$

3. Due to indistinguishability of neutrons (or protons) the SF can be even larger than 1!

Scattering amplitude and cross sections

➯ In post form:

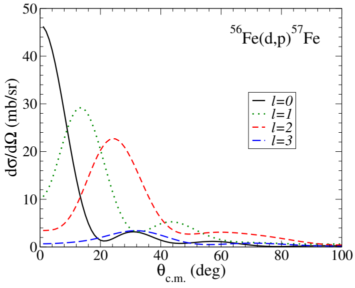
$$\boxed{\mathcal{T}_{d,p}^{\rm DWBA} = \mathcal{A}_{BA}^{\ell j} \int\int \chi_p^{(-)*}(\mathbf{K}_p, \mathbf{R}')\varphi_{nA}^{\ell j,*}(\mathbf{r}')\, V_{pn}(\mathbf{r})\, \chi_d^{(+)}(\mathbf{K}_d, \mathbf{R})\varphi_d(\mathbf{r})d\mathbf{r}'d\mathbf{R}'}$$

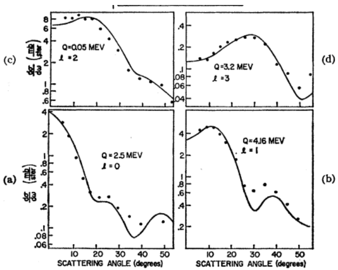
$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{(d,p)} = \frac{\mu_\alpha \mu_\beta}{(2\pi\hbar^2)^2} S_{BA}^{\ell j} \left|\int\int \chi_p^{(-)*}(\mathbf{K}_p, \mathbf{R}')\varphi_{nA}^{\ell j,*}(\mathbf{r}')\, V_{pn}(\mathbf{r})\, \chi_d^{(+)}(\mathbf{K}_d, \mathbf{R})\varphi_d(\mathbf{r})d\mathbf{r}'d\mathbf{R}'\right|^2}$$

$$|\mathcal{A}_{BA}^{\ell j}|^2 = S_{BA}^{\ell j} = \text{spectroscopic factor}$$

☞*In DWBA, the transfer cross section is proportional to the product of the projectile and target spectroscopic factors. Comparing the data with DWBA calculations, one can extract the values of* $S_{BA}^{\ell j}$

Orbital angular momentum sensitivity





☞ *Angular distributions of transfer cross sections are very sensitive to the single-particle configuration of the transferred nucleon/s.* $\Rightarrow \varphi_{nlj}(\mathbf{r})$

☞ From classical arguments, the angle of the first maximum appears at:

$$\theta_{\rm max} \approx \arcsin\left(\frac{\sqrt{\ell(\ell+1)}\hbar}{pR}\right)$$

Summary: What do we learn form a transfer experiment

1. Excitation energies of residual nucleus

➩ *The Q-value is related to the masses and excitation energies*

2. Spectroscopic factors (related to occupation numbers)

➩ In DWBA, $\sigma^{\ell jI} \propto S_{BA}^{\ell jI}$

3. Angular momentum of populated states.

➩ For heavy targets, the first maximum occurs at:

$$\theta_{\rm max} \approx \arcsin\left(\frac{\sqrt{\ell(\ell+1)}\hbar}{pR}\right)$$

^{1}H(^{11}Be,^{10}Be)^{2}H example

$$|^{11}\mathrm{Be}\rangle = \alpha\, |^{10}\ \mathrm{Be}(0^+) \otimes \nu 2s_{1/2}\rangle + \beta\, |^{10}\ \mathrm{Be}(2^+) \otimes \nu 1d_{5/2}\rangle + \ldots$$

➩ In DWBA:

$$\sigma(0^+) \propto |\alpha|^2; \quad \sigma(2^+) \propto |\beta|^2$$

^{1}H(^{11}Be,^{10}Be)^{2}H example

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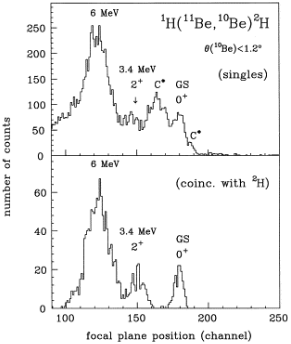
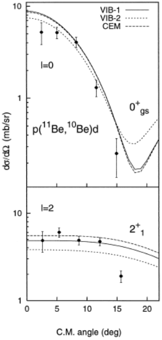
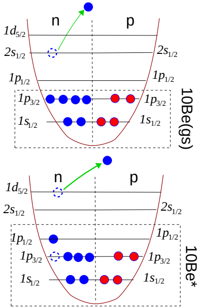
Fortier et al, PLB461, 22 (1999)

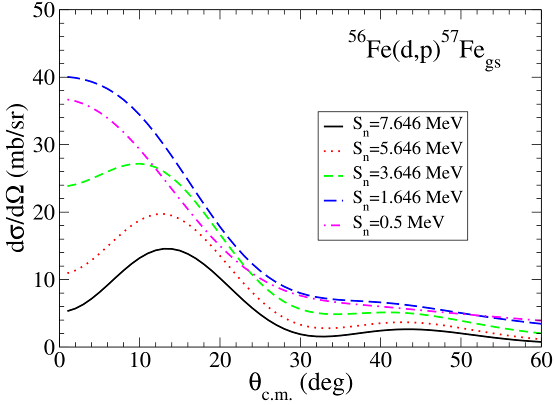






Transfer example: $^{56}Fe(d,p)^{57}Fe$

Dependence with binding energy:



Peripherality of transfer reactions: the ANC

- Recall the overlap function: $\langle \phi_B | \phi_A \rangle = \mathcal{A}^{\ell j}_{BA} \varphi^{\ell j}_{nA}(\mathbf{r}')$
- Outside the range of the nuclear potential:

$$\varphi^{\ell j}_{nA}(\mathbf{r}') \to b_{\ell,j} \frac{W_{-\eta,\ell+1/2}(2kr)}{r} \approx b_{\ell,j} e^{-kr} \qquad k = \sqrt{2\mu\epsilon_b}/\hbar$$

where $b_{\ell,j}$ is the single-particle asymptotic normalization coefficient.

- Then, outside the range of the nuclear potential:

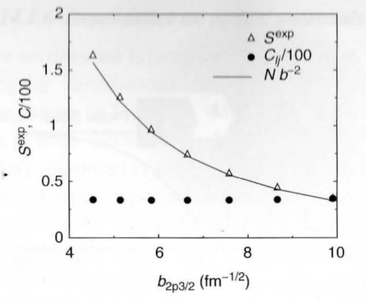
$$\langle \phi_B | \phi_A \rangle = \mathcal{A}^{\ell j}_{BA} b_{\ell,j} \frac{W_{-\eta,\ell+1/2}(2kr)}{r} \equiv C^{\ell j}_{BA} \frac{W_{-\eta,\ell+1/2}(2kr)}{r}$$

where $C^{\ell j}_{BA} = \mathcal{A}^{\ell j}_{BA} b_{\ell,j}$ is the asymptotic normalization coefficient of the $\langle \phi_B | \phi_A \rangle$ overlap.

☞ *Thus, outside the range of the nuclear potential, the overlap function is sensitive to the ANC $C^{\ell j}_{BA}$ rather than to the spectroscopic amplitude $\mathcal{A}^{\ell j}_{BA}$*

Peripherality of transfer reactions: the ANC

- For a peripheral transfer reaction, $d\sigma/d\Omega \propto |C^{\ell j}_{BA}|^2$.
- In DWBA, the ratio of the experimental and calculated cross sections will provide the quantity $|C^{\ell j}_{BA}|^2$.
- Since $|C^{\ell j}_{BA}|^2 = S^{\ell j}_{BA} b^2_{\ell,j}$, varying the parameters of the single-particle potential used to generate $\varphi^{\ell j}_{nA}(\mathbf{r}')$, will modify $b_{\ell,j}$ and also $S^{\ell j}_{BA}$ but their product ($|C^{\ell j}_{BA}|^2$) will remain roughly constant.



Quoted from Fig. 14.4 of Thompson and Nunes book