RAON online School: Nuclear reactions with exotic nuclei

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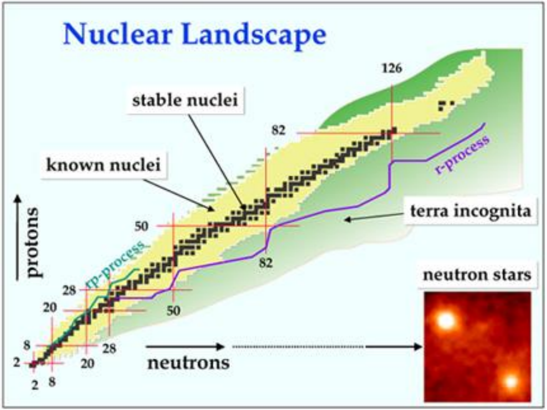
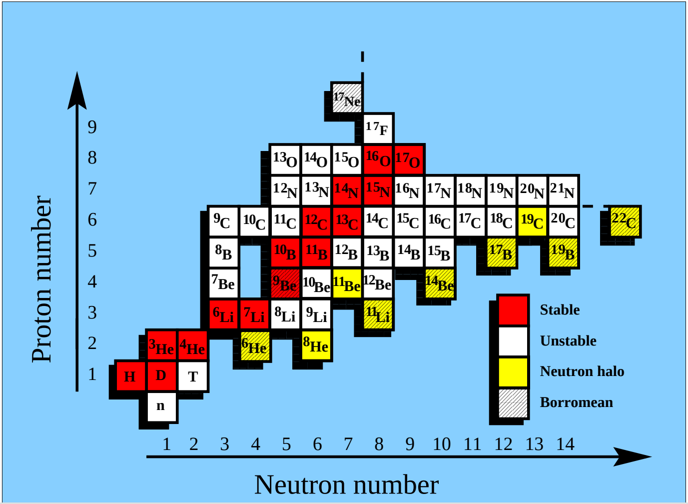
Unstable nuclei and the limits of stability



Note that:

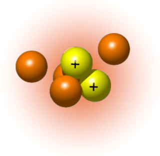
- Not all unstable nuclei are weakly-bound.
- There are weakly-bound nuclei which are not unstable (eg. deuteron).

Light exotic nuclei: halo nuclei and Borromean systems



Light exotic nuclei: halo nuclei and Borromean systems

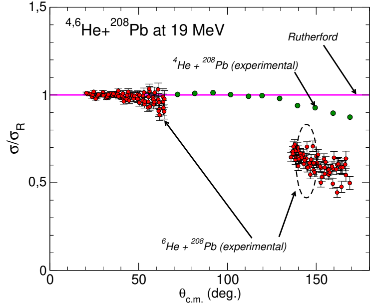
- Radioactive nuclei: they typically decay by β emission.
  E.g.: $^{6}\mathrm{He} \xrightarrow{\beta^-} {}^{6}\mathrm{Li}$ $\quad(\tau_{1/2} \simeq 807$ ms)
- Weakly bound: typical separation energies are around 1 MeV or less.
- Spatially extended
- Halo structure: one or two weakly bound nucleons (typically neutrons) with a large probability of presence beyond the range of the potential.
- Borromean nuclei: Three-body systems with no bound binary sub-systems.

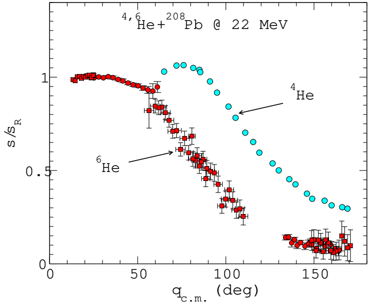


Signatures of weakly bound nuclei in reaction observables

Elastic scattering: Rutherford experiment...100 years later

For ^{4}He+^{208}Pb, the Coulomb barrier is $V_b \approx 21$ MeV

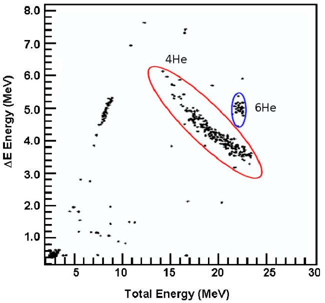


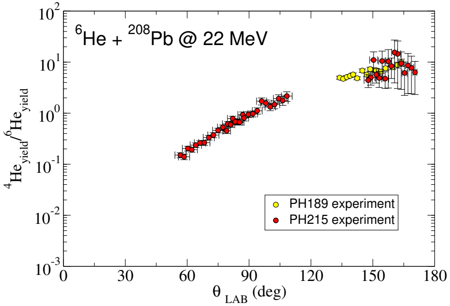


- ^{4}He follows Rutherford formula at 19 MeV but not at 22 MeV.
- ^{6}He drastically departs from Rutherford formula at both energies!

Large fragment production

$$\boxed{{}^{6}\mathrm{He} + {}^{208}\mathrm{Pb} \rightarrow \alpha + \mathrm{X}}$$





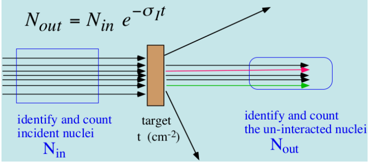
⇨At large angles, there are more α's than ^{6}He (elastic) !

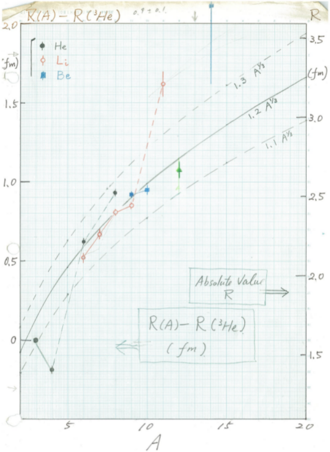
⇨What are the mechanisms behind the α producion and how can we compute it?

High-energy interaction cross sections with light targets

Interaction cross sections of nuclei on light targets and high energies are proportional to the size of the colliding nuclei.

$$\boxed{\sigma_I \simeq \pi(R_p + R_t)^2}$$



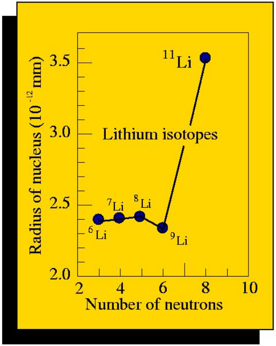


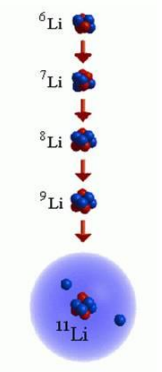
From I. Tanihata

High-energy interaction cross sections with light targets

Interaction cross sections of nuclei on light targets and high energies (hundreds MeV/nucleon) are proportional to the size of the colliding nuclei.

$$\boxed{\sigma_I \simeq \pi(R_p + R_t)^2}$$

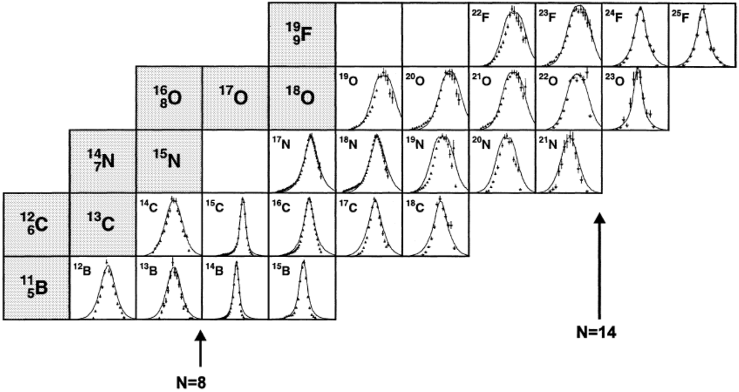




Tanihata *et al*, PRL55, 2676 (1985)

Momentum distributions in high-energy fragmentation reactions

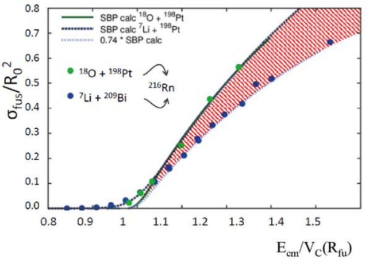
What do momentum distributions tell us about the size of the nucleus?



- ☞ Unusually narrow momentum distributions of the fragments occur for specific isotopes (e.g. $^{23}O \rightarrow$ ^{22}O +n)
- ☞ A narrow momentum distribution is a signature of an extended spatial distribution

Complete fusion suppression at above-barrier energies

CF of weakly bound nuclei suppressed at energies above the Coulomb barrier



- Observed for weakly-bound projectiles (^{6,7,8}Li,^{9}Be)
- CF reduced by $\sim 30\%$ with respect to BPM or CC calculations.

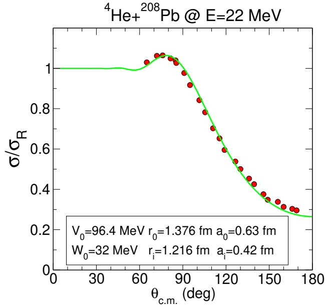
M. Dasgupta et al., PRC 70, 024606 (2004)

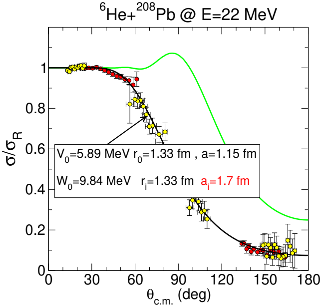
Common interpretation:

- ⇨ CF is mostly reduced by breakup and incomplete fusion (ICF)
- ⇨ ICF is modeled as two-step process: breakup followed by capture of one charged fragment (breakup-fusion, BF).

Normal versus halo nuclei

How does the halo structure affect the elastic scattering?





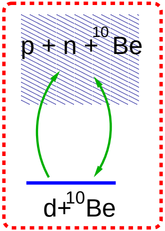
- ^{4}He+^{208}Pb shows typical Fresnel pattern and “standard” optical model parameters
- ^{6}He+^{208}Pb shows a prominent reduction in the elastic cross section, suggesting that part of the incident flux goes to non-elastic channels (eg. breakup)

  Understanding and disentangling these non-elastic channels requires going beyond the optical model (eg. coupled-channels method)

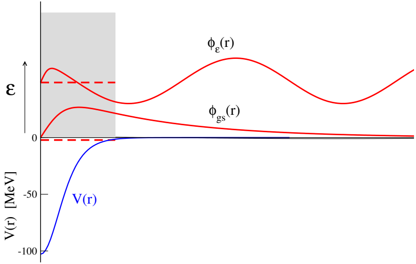
Inclusion of breakup channels: the CDCC method

Breakup modelspace

- In collisions involving weakly bound nuclei, excitation of unbound states (breakup channels) of the weakly-bound nucleus plays an important role.
- Reaction formalisms (DWBA, CC...) must be conveniently extended in order to incorporate the possibility of coupling to these breakup channels.



Bound versus scattering states

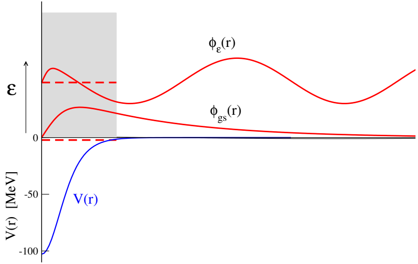


Continuum wavefunctions:

$$\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r}[Y_\ell(\hat{r}) \otimes \chi_s]_{jm}$$

$$\varepsilon = \frac{\hbar^2 k^2}{2\mu}$$

Bound versus scattering states



Continuum wavefunctions:

$$\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r}[Y_\ell(\hat{r}) \otimes \chi_s]_{jm}$$

$$\varepsilon = \frac{\hbar^2 k^2}{2\mu}$$

Unbound states are not suitable for CC calculations:

- They have a continuous (infinite) distribution in energy.
- Non-normalizable: $\langle u_{k,\ell sj}(r)|u_{k',\ell sj}(r)\rangle \propto \delta(k-k')$

SOLUTION $\Rightarrow$ continuum discretization

The origins of CDCC

- Continuum discretization method proposed by G.H. Rawitscher [PRC9, 2210 (1974)] and Farrell, Vincent and Austern [Ann.Phys.(New York) 96, 333 (1976)] to describe deuteron scattering as an effective three-body problem $p + n + A$.

PHYSICAL REVIEW C VOLUME 9, NUMBER 6 JUNE 1974

Effect of deuteron breakup on elastic deuteron-nucleus scattering

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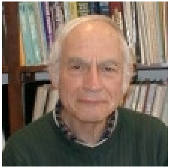
(Received 1 October 1973; revised manuscript received 4 March 1974)

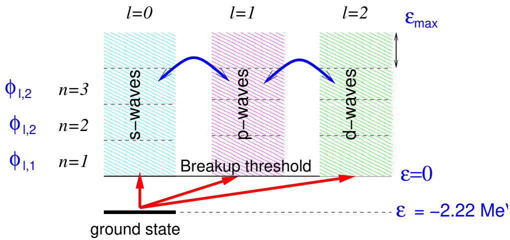
The properties of the transition matrix elements $V_{ab}(R)$ of the breakup potential V_N taken between states $\phi_a(\vec{r})$ and $\phi_b(r)$ are examined. Here $\phi_a(\vec{r})$ are eigenstates of the neutron-proton relative-motion Hamiltonian, and the eigenvalues of the energy ϵ_a are positive (continuum states) or negative (bound deuteron); $V_N(\vec{r}, \vec{R})$ is the sum of the phenomenological proton nucleus $V_{p-A}(|\vec{R} - \frac{1}{2}\vec{r}|)$ and neutron nucleus $V_{n-A}(|\vec{R} + \frac{1}{2}\vec{r}|)$ optical potentials evaluated for nucleon energies equal to half the incident deuteron energy. The bound-to-continuum transition matrix element for relative neutron-proton angular momenta $l = 2$ are found to be comparable in magnitude to the ones for $l = 0$ for values of ϵ_a larger than about 3 MeV, and both decrease only slowly with ϵ_a, suggesting that a large breakup spectrum is involved in deuteron-nucleus collisions. The effect of the various breakup transitions on the elastic phase shifts is estimated by numerically solving a set of coupled equations. These equations couple the functions $\chi_a(\vec{R})$ which are the coefficients of the expansion of the neutron-proton-nucleus wave function in a set of the $\phi_a(\vec{r})$'s. The equations are rendered manageable by performing a (rather crude) discretization in the neutron-proton relative-momentum variable k_a. Numer-



George Rawitscher (1928-2018)

- Full numerical implementation by Kyushu group (Sakuragi, Yahiro, Kamimura, and co.): Prog. Theor. Phys.(Kyoto) 68, 322 (1982)

Continuum discretization for deuteron scattering



- Select a number of angular momenta ($\ell = 0, \ldots, \ell_{\max}$).
- For each ℓ, set a maximum excitation energy $\varepsilon_{\max}$.
- Divide the interval $\varepsilon = 0 - \varepsilon_{\max}$ in a set of sub-intervals (*bins*).
- For each bin, calculate a representative wavefunction.

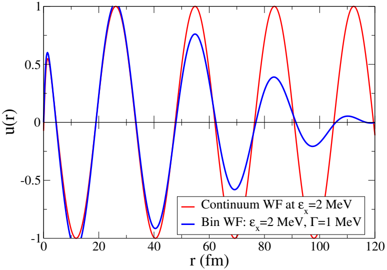
CDCC formalism: construction of the bin wavefunctions

Bin wavefunction:

$$\phi^{[k_1,k_2]}_{\ell j m}(\mathbf{r}) = \frac{u^{[k_1,k_2]}_{\ell j}(r)}{r}[Y_\ell(\hat{r}) \otimes \chi_s]_{jm} \qquad [k_1, k_2] = \text{bin interval}$$

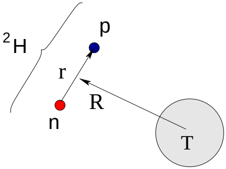
$$\boxed{u^{[k_1,k_2]}_{\ell sjm}(r) = \sqrt{\frac{2}{\pi N}} \int_{k_1}^{k_2} w(k) u_{k,\ell sj}(r) dk}$$

- k: linear momentum
- $u_{k,\ell sj}(r)$: scattering states (radial part)
- $w(k)$: weight function



CDCC formalism for deuteron scattering

- Hamiltonian: $H = T_{\mathbf{R}} + h_r(\mathbf{r}) + V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt})$
- Model wavefunction:



$$\Psi^{(+)}(\mathbf{R},\mathbf{r}) = \phi_{gs}(\mathbf{r})\chi_0(\mathbf{R}) + \sum_{n>0}^{N} \phi_n(\mathbf{r})\chi_n(\mathbf{R})$$

- Coupled equations: $[H - E]\Psi(\mathbf{R},\mathbf{r}) = 0$

$$\boxed{[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{R}) = \sum_{n'\neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})}$$

- Transition potentials:

$$V_{n;n'}(\mathbf{R}) = \int d\mathbf{r}\, \phi_n^*(\mathbf{r}) \left[V_{pt}(\mathbf{R} + \frac{\mathbf{r}}{2}) + V_{nt}(\mathbf{R} - \frac{\mathbf{r}}{2}) \right] \phi_{n'}(\mathbf{r})$$

Trivially equivalent local equivalent potential (TELP)

- From the elastic channel equation, a TELP can be defined as follows:

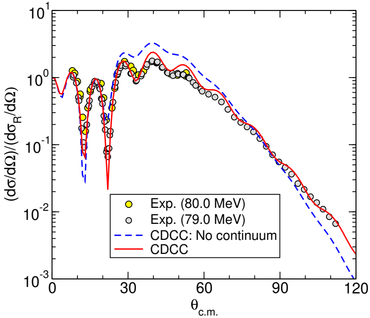
$$\left[E - \varepsilon_0 - \hat{T}_{\mathbf{R}} - V_{0,0}(\mathbf{R})\right]\chi_0(\mathbf{R}) = \sum_{i \neq 0} V_{i,0}(\mathbf{R})\chi_i(\mathbf{R}) \equiv U_{\mathrm{TELP}}(\mathbf{R})\chi_0(\mathbf{R}).$$

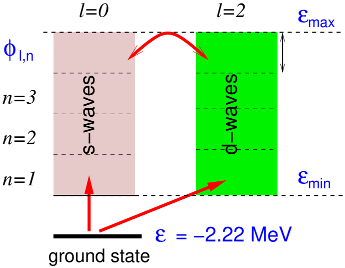
- In actual calculations, $U_{\mathrm{TELP}}(R)$ will depend on the total angular momentum, but a weighted average can be performed to obtain an approximate angular-momentum independent polarization potential
- A single channel calculation with the potential $U(\mathbf{R}) = V_{0,0}(\mathbf{R}) + U_{\mathrm{TELP}}(\mathbf{R})$ should reproduce approximately the elastic scattering cross section.

Applications of the CDCC formalism: d+ ^{58}Ni

Coupling to continuum states produce:

- Polarization of the projectile (modification of real part)
- Flux removal (absorption) from the elastic channel (imaginary part)

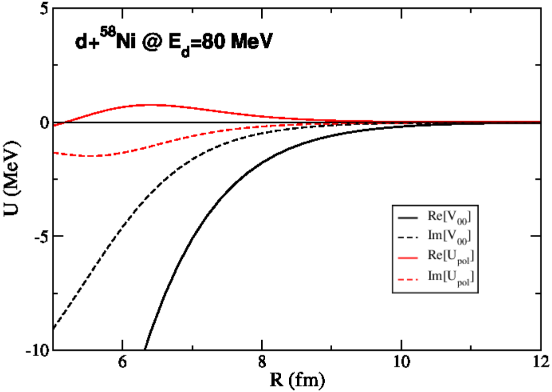




☞ No continuum ⇒ retain only the Watanabe potential:

$$V_{00}(\mathbf{R}) = \int d\mathbf{r}\phi^*_{\rm gs}(\mathbf{r})\left(V_{pt} + V_{nt}\right)\phi_{\rm gs}(\mathbf{r})$$

Trivially equivalent local equivalent potential for d+^{58}Ni @ 80 MeV



For this reaction, the TELP is complex:

- The real part is repulsive (reduces projectile-target attraction)
- The imaginary part is absorptive (flux removal)

Two- and three-body breakup observables

- CDCC scattering amplitudes readily provide **two-body breakup** observables:

$$\boxed{\frac{d\sigma_n}{d\Omega_{\rm c.m.}} = |f_{0,n}(\theta)|^2 \Rightarrow \frac{d^2\sigma}{d\Omega_{\rm c.m.} d\epsilon_{pn}} \simeq \frac{1}{\Delta_n}\frac{d\sigma_n}{d\Omega_{\rm c.m.}}}$$

  with:
  - Δ_n=width of the bin containing the relative energy ϵ_{pn}
  - $\Omega_{\rm c.m.}$=C.M. scattering angle of the projectile c.m. (not easy to measure!)
- **Three-body observables** can be also calculated using a suitable combination of the scattering amplitudes and appropriate kinematical transformations (Tostevin, PRC63, 024617 (2001)):

$$\boxed{\frac{d^3\sigma}{d\Omega_n d\Omega_p dE_p}}$$

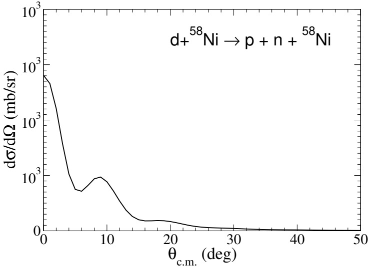
☞ **N.b.:** *These 3-body observables are not directly provided by FRESCO. They must be computed separately from the calculated amplitudes.*

Two-body breakup observables: d+ ^{58}Ni → p+n+^{58}Ni

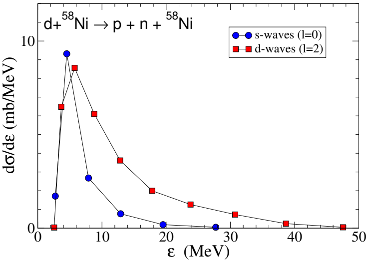
CDCC calculations for **d+ ^{58}Ni** at 80 MeV :

- Continuum states with $\ell = 0, 2$.
- Proton and neutron intrinsic spins ignored.
- p/n+ ^{58}Ni from global optical potential.
- p+n simple Gaussian interaction describing deuteron g.s.

$p + n$ c.m. angular distribution



Excitation energy distribution

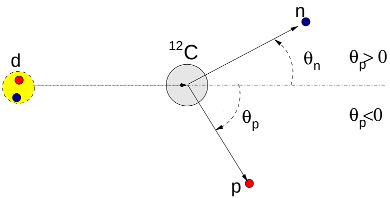


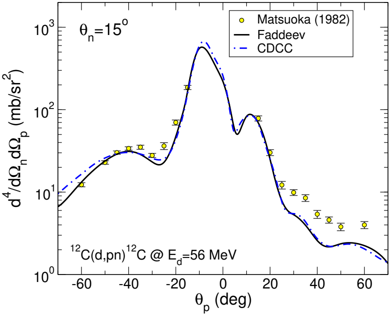
Breakup observables with CDCC: exclusive breakup of d+ ^{12}C → p+n+^{12}C

CDCC calculations for **d+ ^{12}C** at 56 MeV:

- Continuum states with $\ell \leq 8$ and ε_{max}=46 MeV.
- Proton and neutron intrinsic spins ignored
- p/n+ ^{58}Ni from Watson global optical potential
- p+n simple Gaussian interaction describing deuteron g.s.

Data: Matsuoka *et al.*, NPA391, 357 (1982).

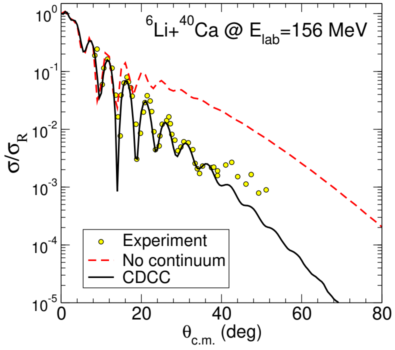


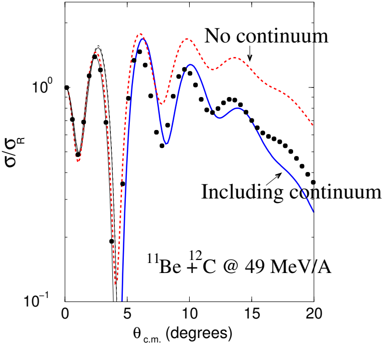


A. Deltuva et al, PRC 76, 064602 (2007)

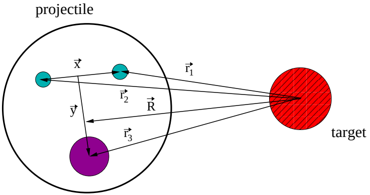
Application of the CDCC method: ^{6}Li and ^{6}He scattering

- The CDCC has been also applied to nuclei with a cluster structure:
  - $^6\text{Li}=\alpha + \text{d}$ $\quad (S_{\alpha,d}=1.47$ MeV)
  - $^{11}\text{Be}={}^{10}\text{Be} + \text{n}$ ($S_n=0.504$ MeV)





Extension to 3-body projectiles

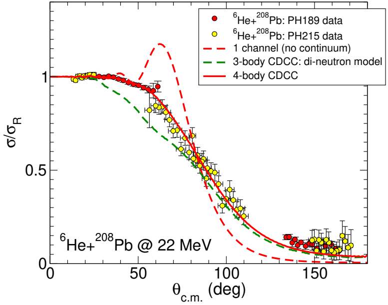


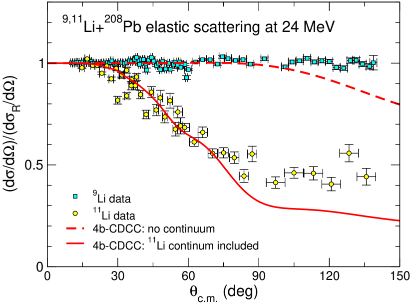
To extend the CDCC formalism, one needs to evaluate the new coupling potentials:

$$\boxed{V_{n;n'}(\mathbf{R}) = \int d\mathbf{r}\, \phi_n^*(\mathbf{x},\mathbf{y}) \left\{V_{nt}(\mathbf{r}_1) + V_{nt}(\mathbf{r}_2) + V_{\alpha t}(\mathbf{r}_3)\right\} \phi_{n'}(\mathbf{x},\mathbf{y})}$$

- $\phi_n(\mathbf{x},\mathbf{y})$ three-body WFs for bound and continuum states: hyperspherical coordinates, Faddeev, etc (difficult to calculate!)
- 4b-CDCC calculations not included in FRESCO; require separate codes to compute the $\phi_n(\mathbf{x},\mathbf{y})$ wfs (e.g. FACE) and $V_{n;n'}(\mathbf{R})$ potentials

Four-body CDCC calculations for ^{6}He and ^{11}Li scattering





Data (LLN): NPA803, 30 (2008);PRC 84, 044604 (2011)
Calculations: PRC 80, 051601 (2009)

M Cubero et al, PRL109, 262701 (2012)

N.b.: 1-channel potential considers only g.s. → g.s. coupling potential:

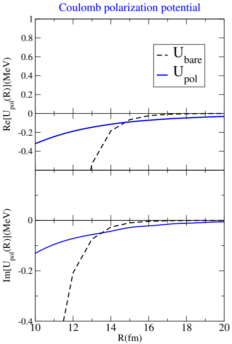
$$V_{00}(\mathbf{R}) = \int d\mathbf{r}\, \phi^*_{\mathrm{g.s.}}(\mathbf{x},\mathbf{y})\,\{V_{nt}(\mathbf{r}_1) + V_{nt}(\mathbf{r}_2) + V_{ct}(\mathbf{r}_3)\}\,\phi_{\mathrm{g.s.}}(\mathbf{x},\mathbf{y})$$

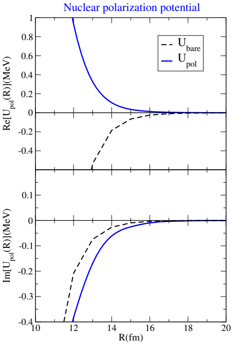
Polarization potential for ^{6}He+^{208}Pb: long-range effect

Trivially Equivalent Local Polarization potential (TELP):

$$\left[E - \varepsilon_0 - \hat{T}_{\mathbf{R}} - V_{0,0}(\mathbf{R})\right]\chi_0(\mathbf{R}) = \sum_{i\neq 0} V_{i,0}(\mathbf{R})\chi_i(\mathbf{R}) \equiv U_{\mathrm{TELP}}(\mathbf{R})\chi_0(\mathbf{R}).$$

Application to ^{6}He+^{208}Pb at 22 MeV



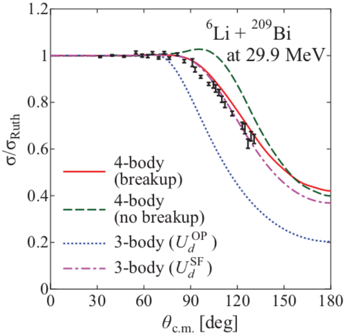


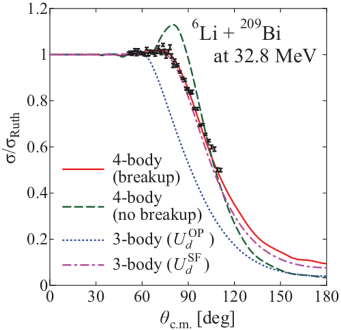
- Imaginary part of TELP is long-ranged and absorptive (explains the need for large imaginary diffuseness parameter in OM analysis)
- Real part is attractive for the Coulomb potential and repulsive for the nuclear couplings.

Application to ^{6}Li scattering

Example: ^{6}Li+^{209}Bi around Coulomb barrier

- 4-body CDCC: ^{6}Li=α+p+n
- 3-body CDCC: ^{6}Li=α+d



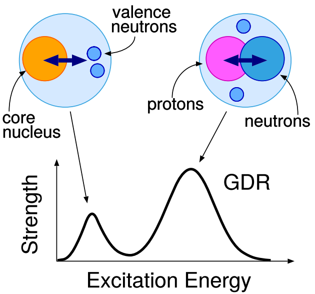


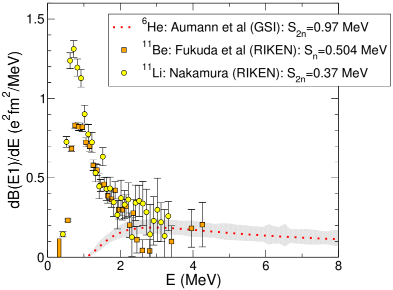
Watanabe *et al*, PRC 86, 031601(R) (2012)

Exploring the continuum with breakup reactions

1. Coulomb dissociation experiments
   - Semiclassical description: Alder and Winther
   - Quantum-mechanical description
2. Exploring continuum structures: resonances and virtual states

Electric response of weakly-bound nuclei





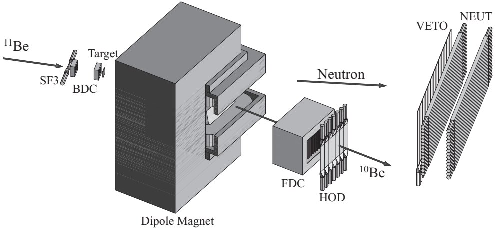
- The $E\lambda$ response can be quantified through the $B(E\lambda)$ probability:

$$\boxed{B(E\lambda; i \to f) = \frac{1}{2I_i + 1} |\langle \Psi_f \| \mathcal{M}(E\lambda) \| \Psi_i \rangle|^2}$$

- Neutron-halo nuclei have large $B(E1)$ strengths near threshold

How to probe/extract the $B(E1)$ of halo nuclei?

Example: ^{11}Be+^{208}Pb → ^{10}Be+ n+ ^{208}Pb measured at RIKEN (69 MeV/u).
Fukuda et al, PRC70, 054606 (2004))



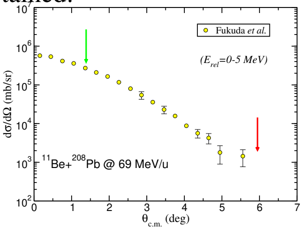
☞ ^{11}Be excitation energy can be reconstructed from core-neutron coincidences (*invariant mass method*)

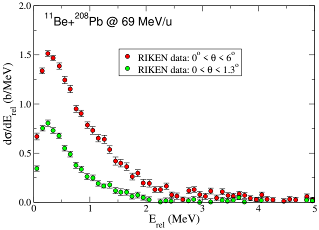
What observables are measured in Coulomb dissociation experiments?

- Experimentally, one measures angular and relative energy distribution of the ^{11}Be* system:

$$\boxed{\frac{d^2\sigma}{d\Omega\, dE}}$$

- Integrating over the angle or energy, single differential cross sections are obtained:





- In the Coulomb dominated region (i.e. small angles), the breakup cross section is expected to be dominated by the $dB(E\lambda)/dE$ distribution, but we need a theory that relates both observables.

Semiclassical 1st order Eλ excitation (Alder & Winther) (akin EPM method)

- For Eλ excitation to bound states ($0 \to n$):

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{0\to n} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{B(E\lambda, 0 \to n)}{e^2 a_0^{2\lambda-2}} f_\lambda(\theta,\xi)} \qquad \xi_{0\to n} = \frac{(E_n - E_0)}{\hbar}\frac{a_0}{v}$$

- For continuum states (breakup):

$$\boxed{\frac{d\sigma(E\lambda)}{d\Omega dE} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{1}{e^2 a_0^{2\lambda-2}} \frac{dB(E\lambda)}{dE} \frac{df_\lambda(\theta,\xi)}{d\Omega}}$$

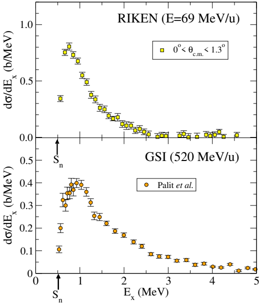
☞ $dB(E\lambda)/dE$ can be extracted from small-angle Coulomb dissociation data.

$$\boxed{\frac{d\sigma}{dE}(\theta < \theta_{\rm max}) = \int_0^{\theta_{\rm max}} \frac{d\sigma(E\lambda)}{d\Omega dE} d\Omega \propto \frac{dB(E\lambda)}{dE}}$$

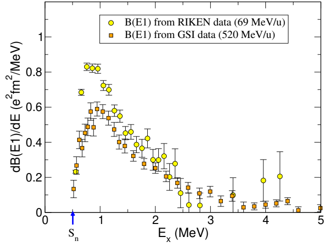
Extracting $B(E1)$ of ^{11}Be from ^{11}Be+^{208}Pb Coulomb dissociation

Common assumptions:

- Breakup dominated by Coulomb excitation (mostly E1).
- Nuclear excitation, if present, can be estimated and added incoherently
- ☞ If the assumptions above are fulfilled, the extracted $dB(E\lambda)dE$ should be independent of the incident energy and target employed, since it reflects a structure property of the projectile.



$$\frac{dB(E\lambda)}{dE} \propto \frac{d\sigma}{dE}$$



RIKEN: Fukuda et al, PRC70 (2004) 054606
GSI: Palit et al, PRC68 (2003) 034318

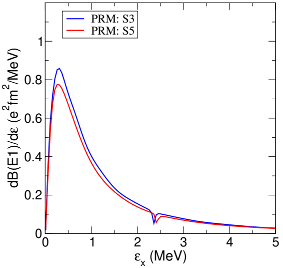
☞ *The extracted $dB(E\lambda)/dE$ distributions are reasonably compatible, but with apparent differences at the peak*

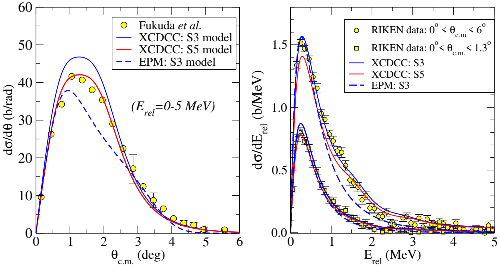
CDCC analysis of Coulomb dissociation data

- Nuclear excitation not negligible, even for small θ
- Nuclear contribution interferes with Coulomb
- Higher-order couplings can affect the cross sections (E2, E3...)

☞ These ingredients can be naturally incorporated within the CDCC method (at the expense of more complexity!)

E.g.: CDCC analysis based on two-body ^{10}Be+n model: PLB 811 (2020) 135959





☞ Different structure models yield different $B(E1)$ strengths and hence different breakup cross sections

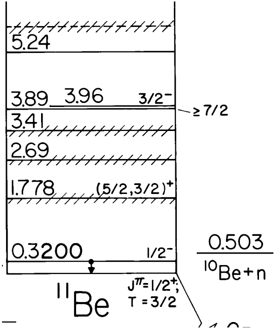
☞ Comparison with the angular distribution evidences the deficiencies of the semiclassical EPM model

Exploring structures in the continuum

1. Resonances and virtual states
2. Accessing continuum structures from breakup and transfer reactions

Exploring structures in the continuum

The continuum spectrum is not “homogeneous”; it contains in general energy regions with special structures, such as resonances and virtual states

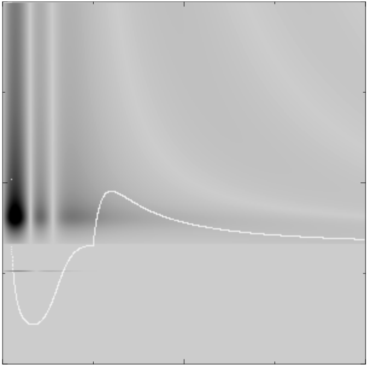


What is a resonance?

- It is a pole of the S-matrix in the complex energy plane.
- It is a structure on the continuum which may, or may not, produce a maximum in the cross section, depending on the reaction mechanism and the phase space available.
- The resonance occurs in the range of energies for which the phase shift is close to $\pi/2$.
- In this range of energies, continuum wavefunctions have a large probability of being in the radial range of the potential.
- The continuum wavefunctions are not square normalizable. For practical reasons, a normalized wave-packet (or "bin") can be constructed to represent the resonance.

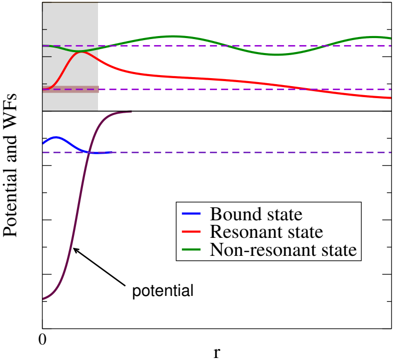
Distinctive features of a resonance

In the energy range of the resonance, the continuum wavefunctions have a large probability of being within the range of the potential.



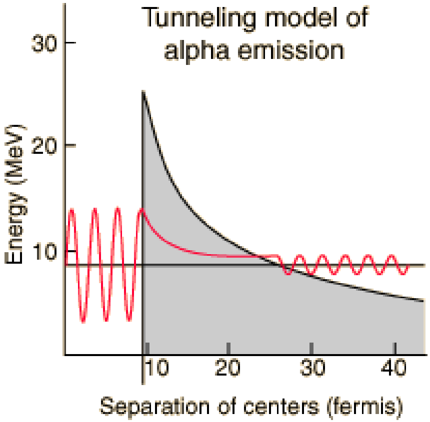
Cuts and areas ordered by size

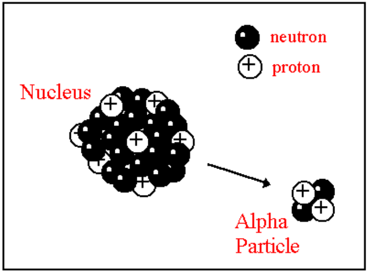
(Courtesy of C. Dasso)



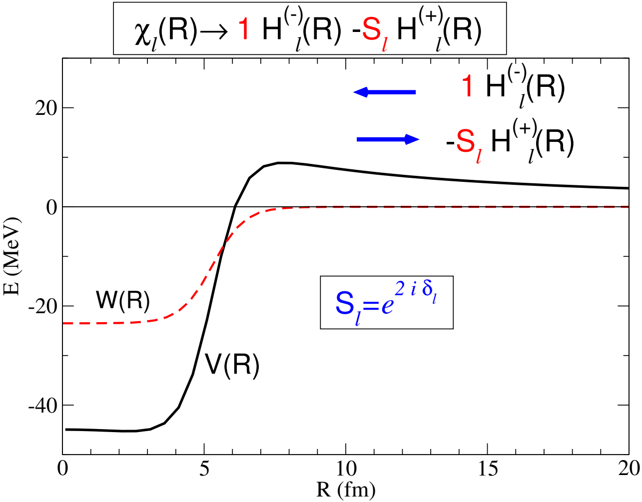
Distinctive features of a resonance

The decay of the resonance is also behind the α-decay phenomenon:

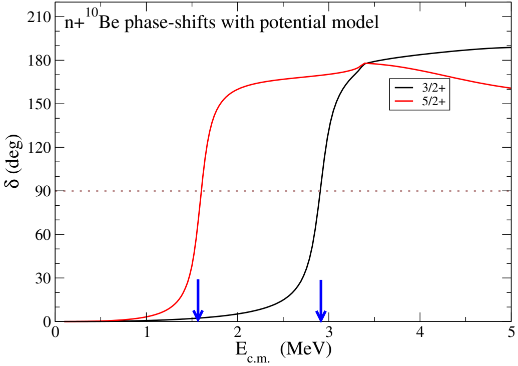


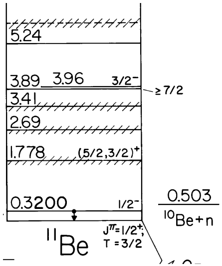


Resonances and phase-shifts



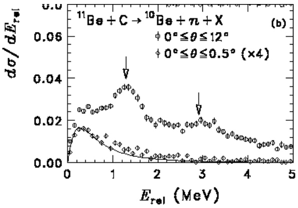
Resonances and phase-shifts

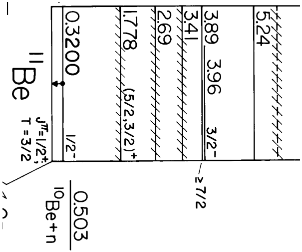




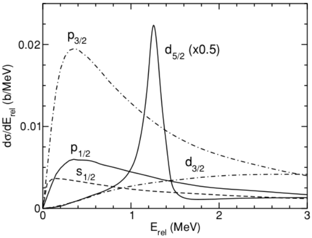
Studying resonances in nuclear breakup experiments

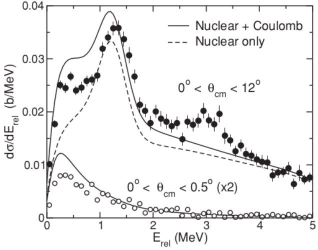
RIKEN data





CDCC analysis





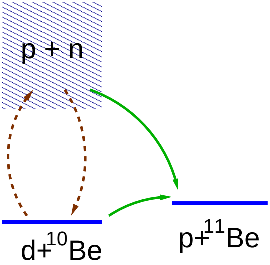
Fukuda *et al*, PRC70 (2004) 054606)

Howell *et al.*, JPG31 (2005) S1881

Transfer reactions with weakly bound nuclei

Transfer reactions with weakly bound nuclei

- DWBA approximates the total WF by the elastic channel and assumes that the transfer occurs in one step (Born approximation).
- For weakly bound projectiles (eg. deuterons), breakup is an important channel and can influence the transfer process.



- $\Psi^{(+)}_{\mathbf{K}_d}(\mathbf{R},\mathbf{r})$ includes breakup components, but these are lost when we make the DWBA approximation ($\Psi^{(+)} \approx \chi^{(+)}_d(\mathbf{K}_d,\mathbf{R})\varphi_d(\mathbf{r})$) $\Rightarrow$ need to go beyond DWBA

Adiabatic distorted wave approximation (ADWA)

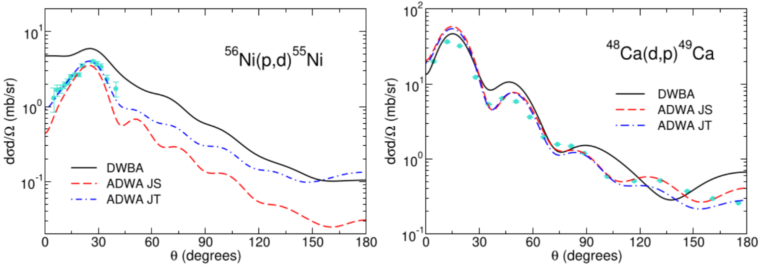
- $\chi_d^{(+)}(\mathbf{K}_d, \mathbf{R})$ describes deuteron elastic scattering but, for the (d,p) transfer matrix element, we need only $\Psi_{\mathbf{K}_d}^{(+)}(\mathbf{R}, \mathbf{r})$ for small $|\mathbf{r}|$
- R.C. Johnson and col. have derived an approximation of $\Psi_{\mathbf{K}_d}^{(+)}(\mathbf{R}, \mathbf{r})$ valid for $r \approx 0$, which includes the effect of deuteron breakup effectively (adiabatic approx.):
  1. Zero-range approximation (Johnson-Soper):[(Johnson,Soper,PRC1, 976 (1970)]

$$\boxed{U^{JS}(R) = U_{pA}(R) + U_{nA}(R)} \quad \Rightarrow \chi_d^{JS}(\mathbf{R})$$

  2. Finite-range version (Johnson–Tandy): [Johnson & Tandy, NPA235 (1974) 56]

$$\boxed{U^{JT}(R) = \frac{\langle \varphi_{pn}(\mathbf{r}) | V_{pn}(U_{nA} + U_{pA}) | \varphi_{pn}(\mathbf{r}) \rangle}{\langle \phi_{pn}(\mathbf{r}) | V_{pn} | \phi_{pn}(\mathbf{r}) \rangle}}$$

DWBA vs ADWA



From Timofeyuk and, Progress in Particle and Nuclear Physics 111 (2020) 103738

CDCC-BA approximation

- Exact transition amplitude for a general $A(d,p)B$ process:

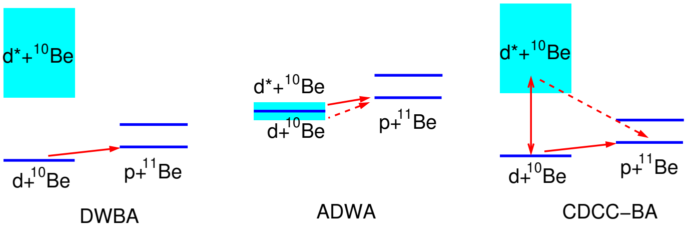
$$\boxed{\mathcal{T}^{\rm CDCC}_{d,p} = C^{\ell j}_{BA} \int\int \chi^{(-)*}_{p}(\mathbf{K}_p, \mathbf{R}')\varphi^{\ell j,*}_{nA}(\mathbf{r}')\, V_{pn}(\mathbf{r})\, \underbrace{\Psi^{(+)}_{\mathbf{K}_\alpha}(\mathbf{R}_\alpha, \xi_\alpha)}\, d\mathbf{r}'\, d\mathbf{R}'}$$

- Use CDCC approximation for $\Psi^{(+)}_{\mathbf{K}_\alpha}$:

$$\Psi^{(+)}_{\mathbf{K}_\alpha} \approx \Psi^{\rm CDCC} = \underbrace{\chi_0(\mathbf{R})\phi_0(\mathbf{r})}_{\text{elastic}} + \sum_{n',j,\pi} \underbrace{\phi^{j\pi}_{n'}(k_{n'}, \mathbf{r})\chi_{n',j,\pi}(\mathbf{R})}_{\text{breakup}}$$

- Unlike the DWBA and ADWA methods, coupling to deuteron breakup states is included explicitly.

DWBA, ADWA and CDCC-BA compared

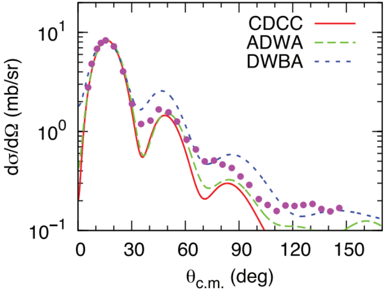


Gómez-Camacho and A.M.M., *A Pedestrian Approach to the Theory of Transfer Reactions: Application to Weakly-Bound and Unbound Exotic Nuclei*, Lecture Notes in Physics, vol 879.

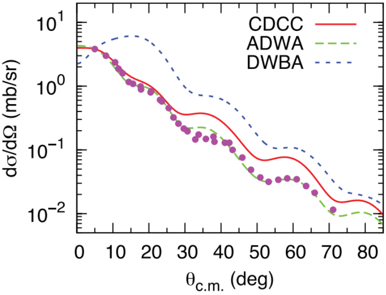
DWBA vs ADWA vs CDCC

Example: ^{58}Ni(d,p)^{59}Ni

E_d = 10 MeV



E_d = 56 MeV



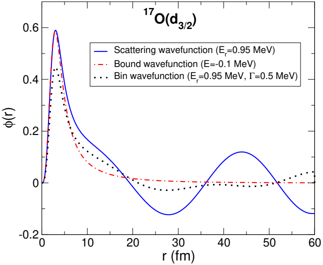
☞*CDCC and ADWBA provide better description of the data and lead also to more realistic spectroscopic information (e.g. spectroscopic factors)*

Pang *et al*, PRC 90, 044611 (2014)

Exploring resonances from transfer reactions

- Calculation of transfer to unbound states in DWBA and ADWA poses numerical problems due to the oscillatory behaviour of unbound wavefunctions.
- A regularization method must be applied:
  - Representing the resonance by a weakly bound state with the same quantum numbers
  - Vincent & Fortune contour integration in the complex radius plane (PRC2 (1970) 782)
  - Representing the resonance by a continuum *bin*

E.g.: $d_{3/2}$ resonance in ^{17}O resonance at $E_r = 0.95$ MeV

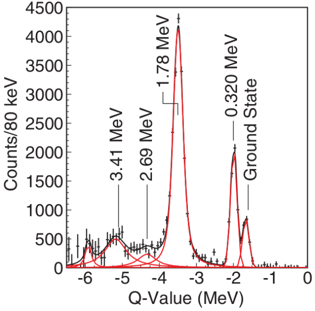


Exploring resonances from transfer reactions

- Inclusion of continuum states in DWBA poses numerical problems due to the oscillatory behaviour of unbound wavefunctions
- Regularization method must be applied, such as representing the resonance by a wavepacket (continuum *bin*, as in the CDCC method)

E.g.: ^{11}Be resonance at $E_x = 1.78$ MeV from ^{10}Be(d,p)^{11}Be

Schmitt et al, PRC88, 064612 (2013)



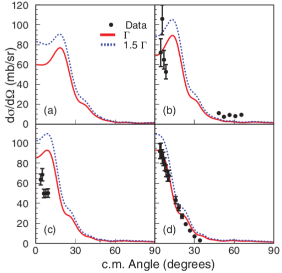
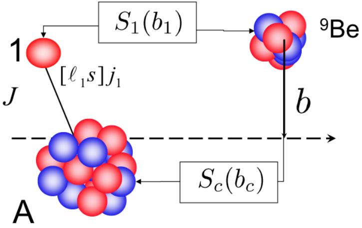


FIG. 8. (Color online) Differential cross sections are presented for transfer to the first resonance in ^{11}Be at 1.78 MeV via the ^{10}Be(d,p) reaction in inverse kinematics at deuteron energies of (a) 12 MeV, (b) 15 MeV, (c) 18 MeV, and (d) 21.4 MeV. The curves are from FR-ADWA calculations using (solid line) an energy bin that is the same width as for the resonance used in the calculation and (dotted line) with a width 1.5 times that value. At 12 MeV the protons were too low in energy to extract an angular distribution.

Knock-out reactions

Spectroscopic from momentum distributions

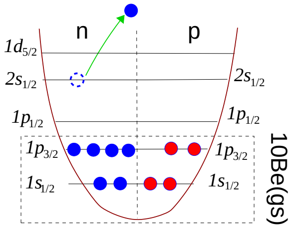
- Fast-moving projectile on a (typically) light target.
- One nucleon suddenly removed (absorbed) due to its interaction with the target.
- The remaining residue remains unchanged and is detected.
- The momentum of the core is related to that of the removed nucleon because, in the rest frame of the projectile, $\vec{P} = 0$

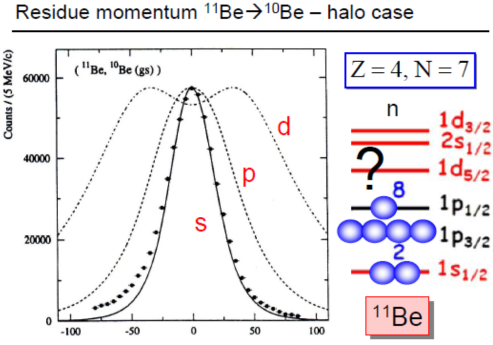


$$\boxed{\vec{P} = \vec{p}_c + \vec{p}_1 = 0}$$

Angular momentum sensitivity of momentum distributions

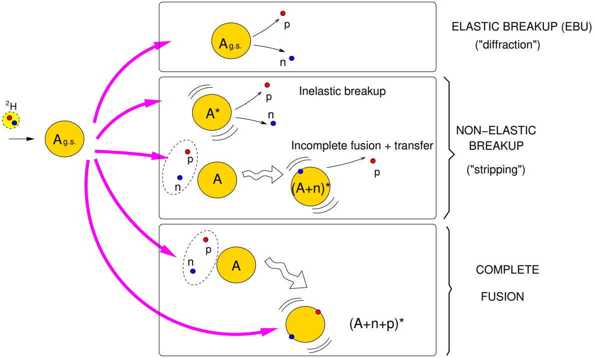
- The shape is determined by the orbital angular momentum ℓ.
- The magnitude is determined by the amount of $s_{1/2}$ (spectroscopic factor)





T. Aumann et al. PRL 84 (2000) 35

Stripping and diffraction contributions to inclusive breakup cross sections



☞*The singles (inclusive) cross section of a given fragment will contain in general diffraction and stripping components*

Stripping cross section within a semiclassical (eikonal) theory

At high energies, one can use the sudden, eikonal approximations to obtain simple formulas for the stripping and diffraction parts of the inclusive breakup cross section for a inclusive reaction of the form $a + B \rightarrow b + X$, with $a = b + x$:

Stripping:

$$\boxed{\sigma_{\rm sp}^{\rm str} = 2\pi \int bdb \int d\mathbf{r}\, |\varphi_{bx}(\mathbf{r})|^2 (1 - |S_x(b_x)|)^2 |S_{bA}(b_b)|^2}$$

Diffraction:

$$\boxed{\sigma_{\rm sp}^{\rm diff} = 2\pi \int bdb \left[\langle \varphi_{bx}|\, |S_b S_x|^2 |\varphi_{bx}\rangle - |\langle \varphi_{bx}|S_b S_x|\varphi_{bx}\rangle|^2 \right].}$$

- $|S_b(b_b)|^2$=probability of survival of the core.
- $1 - |S_x(b_x)|^2$=probability of absorption of the valence particle.

Extraction of SFs from knockout reactions

- Agreement theory vs experiment quantified with the reduction factor:

$$\boxed{R_s = \frac{\sigma_{\rm theor}}{\sigma_{\rm exp}}}$$

with

$$\boxed{\sigma_{\rm theor} = \sum_{n\ell j} S^a_{bx}(I; n\ell j)\sigma_{\rm sp}(I; n\ell j)} \qquad \boxed{\sigma_{\rm sp}(I; n\ell j) = \sigma^{\rm EBU}_{\rm sp} + \sigma^{\rm NEB}_{\rm sp}}$$

☞ $R_s < 1 \Rightarrow$ possible correlations (long-range, short-range, tensor,...) not included in $\sigma_{\rm theor}$?

☞ R_s strongly dependent on $\Delta S = S_p - S_n$.

Extraction of SFs from knockout reactions

- Agreement theory vs experiment quantified with the reduction factor:

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with

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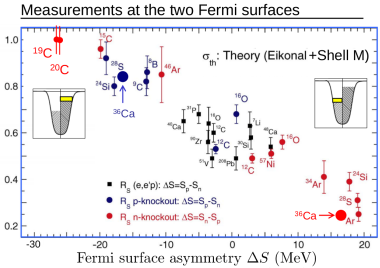


-Gade et al, PRC 77, 044306 (2008)

Tostevin, PRC90,057602(2014)

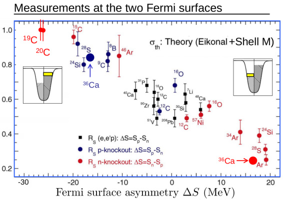
- $R_s < 1 \Rightarrow$ possible correlations (long-range, short-range, tensor,...) not included in $\sigma_{\rm theor}$?
- R_s strongly dependent on $\Delta S = S_p - S_n$.

Extraction of SFs from knockout reactions

...however, this behaviour has not been corroborated by other probes, such as transfer or proton-induced knockout reactions (p, pN)

HI knockout (∼100 MeV/u)

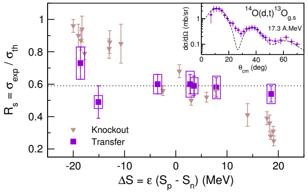
Tostevin, PRC90,057602(2014)



- Reaction model: eikonal + adiabatic
- R_s strongly dependent on $S_p - S_n$.

Low-energy transfer

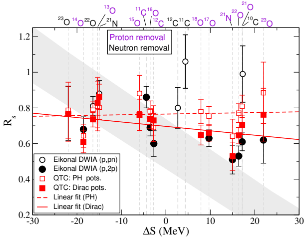
Flavigny, PRL110, 122503(2013)



- Reaction model: ADWA, DWBA, CRC
- R_s ∼ constant.

(p, pN) @ 200-400 MeV/u

Aumann, PPNP118,103847(2021)



- Reaction models: DWIA, TC
- R_s ∼ constant.

☞ Similar results from RIKEN Wakase, PTEP 021D01 (2018)

R_s from knockout disagree with those from transfer and (p, pN) ⇒ description of reaction mechanism?