

RAON online School: Nuclear reactions with exotic nuclei

Antonio M. Moro



Universidad de Sevilla, Spain

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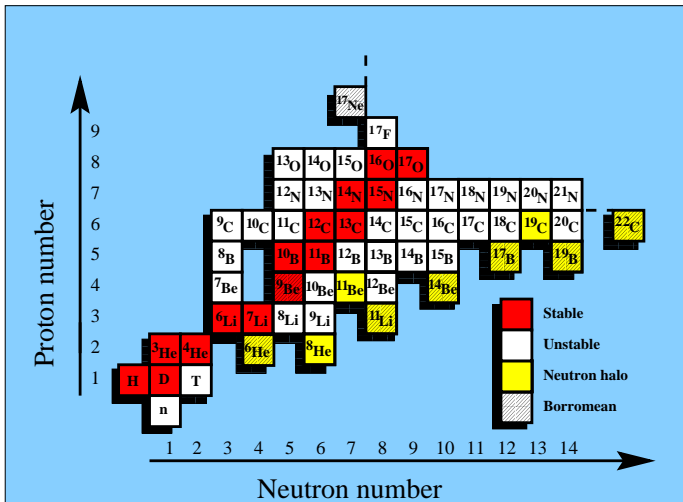
Material available at: <https://github.com/ammoro/RAON>

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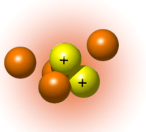
1 Reactions with weakly-bound nuclei

- Weakly-bound vs. “normal” nuclei in reaction observables
- OM analysis of weakly-bound nuclei
- The CDCC method
- Some examples of applications of the CDCC method
- Exploring the continuum with breakup reactions
- Structures in the continuum
- Transfer reactions with weakly bound nuclei
- Transfer populating unbound states
- Knock-out reactions

Light exotic nuclei: halo nuclei and Borromean systems

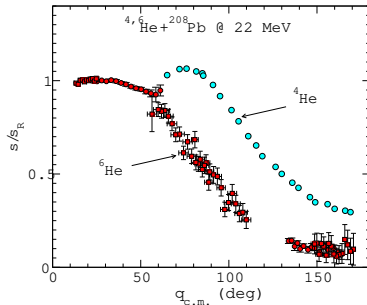


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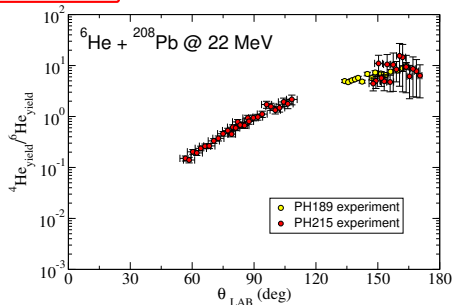
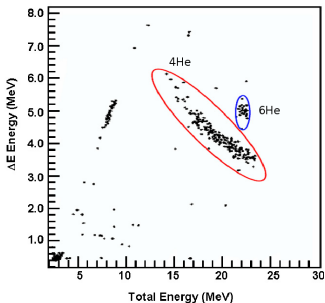
Signatures of weakly bound nuclei in reaction observables

For ${}^4\text{He}+{}^{208}\text{Pb}$, the Coulomb barrier is $V_b \approx 21$ MeV



- ^4He follows Rutherford formula at 19 MeV but not at 22 MeV.
- ^6He drastically departs from Rutherford formula at both energies!

Large fragment production



- At large angles, there are more α 's than ${}^6\text{He}$ (elastic) !
- What are the mechanisms behind the α production and how can we compute it?

Interaction cross sections of nuclei on light targets and high energies are proportional to the size of the colliding nuclei.

$$N_{out} = N_{in} e^{-\sigma t}$$

identify and count incident nuclei N_{in}

target t (cm⁻²)

identify and count the un-interacted nuclei N_{out}

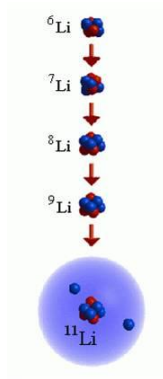
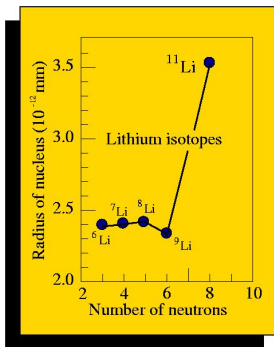
A hand-drawn graph on grid paper showing the difference in radii, $R(A) - R(^3\text{He})$ (in fm), as a function of the radius R (in fm). The x-axis represents R and ranges from 0 to 2.0 fm. The y-axis represents $R(A) - R(^3\text{He})$ and ranges from 0 to 2.0 fm. Three data series are plotted: Helium (He) as black circles, Lithium (Li) as red diamonds, and Beryllium (Be) as blue squares. A solid black curve represents the theoretical prediction, while dashed lines represent experimental data for different mass numbers A : 1.1, 1.2, and 1.3. A vertical blue arrow points to the y-axis at $R = 1.5$ fm. A box labeled 'Absolute Value R ' with an arrow points to the x-axis. Another box labeled ' $R(A) - R(^3\text{He})$ (fm)' with an arrow points to the y-axis. The graph shows that the difference in radii increases with R and is larger for higher mass numbers A .

R (fm)	$R(A) - R(^3\text{He})$ (fm) for He	$R(A) - R(^3\text{He})$ (fm) for Li	$R(A) - R(^3\text{He})$ (fm) for Be
0.5	0.0	-	-
0.8	0.2	-	-
1.0	0.5	0.5	-
1.2	0.8	0.8	-
1.4	1.0	1.0	1.0
1.6	1.2	1.2	1.2
1.8	1.4	1.4	1.4
2.0	1.6	1.6	1.6

High-energy interaction cross sections with light targets

Interaction cross sections of nuclei on light targets and high energies (hundreds MeV/nucleon) are proportional to the size of the colliding nuclei.

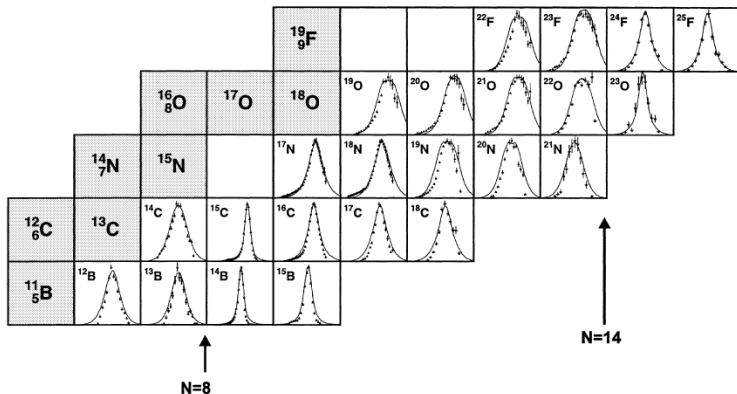
$$\sigma_I \simeq \pi(R_p + R_t)^2$$



Tanihata *et al*, PRL55, 2676 (1985)

Momentum distributions in high-energy fragmentation reactions

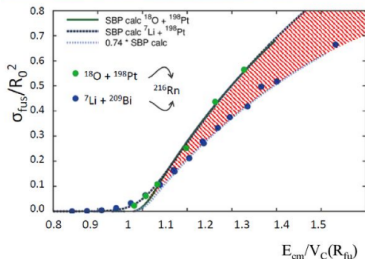
What do momentum distributions tell us about the size of the nucleus?



- Unusually narrow momentum distributions of the fragments occur for specific isotopes (e.g. $^{23}\text{O} \rightarrow ^{22}\text{O} + n$)
- A narrow momentum distribution is a signature of an extended spatial distribution

Complete fusion suppression at above-barrier energies

CF of weakly bound nuclei suppressed at energies above the Coulomb barrier



- Observed for weakly-bound projectiles (${}^6,{}^7,{}^8\text{Li}, {}^9\text{Be}$)
- CF reduced by $\sim 30\%$ with respect to BPM or CC calculations.

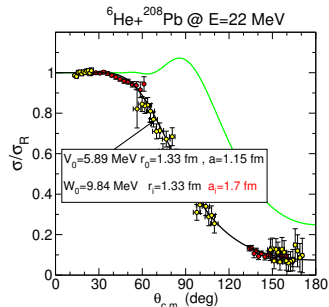
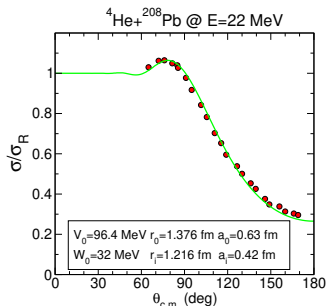
M. Dasgupta et al., PRC 70, 024606 (2004)

Common interpretation:

- ⇒ CF is mostly reduced by breakup and incomplete fusion (ICF)
- ⇒ ICF is modeled as two-step process: breakup followed by capture of one charged fragment (breakup-fusion, BF).

Normal versus halo nuclei

How does the halo structure affect the elastic scattering?



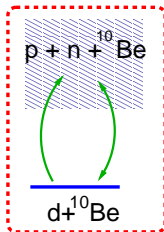
- $^4\text{He} + ^{208}\text{Pb}$ shows typical Fresnel pattern and “standard” optical model parameters
- $^6\text{He} + ^{208}\text{Pb}$ shows a prominent reduction in the elastic cross section, suggesting that part of the incident flux goes to non-elastic channels (eg. breakup)

Understanding and disentangling these non-elastic channels requires going beyond the optical model (eg. [coupled-channels method](#))

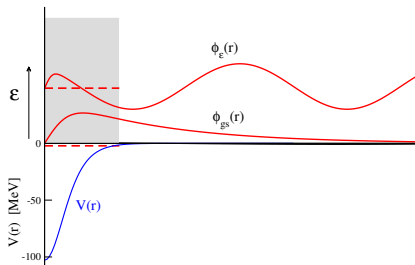
Inclusion of breakup channels: the CDCC method

Breakup modelspace

- In collisions involving weakly bound nuclei, excitation of unbound states (breakup channels) of the weakly-bound nucleus plays an important role.
- Reaction formalisms (DWBA, CC...) must be conveniently extended in order to incorporate the possibility of coupling to these breakup channels.



Bound versus scattering states

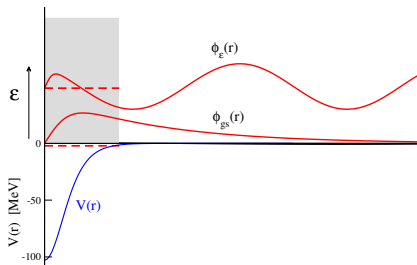


Continuum wavefunctions:

$$\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r} [Y_{\ell}(\hat{r}) \otimes \chi_s]_{jm}$$

$$\epsilon = \frac{\hbar^2 k^2}{2\mu}$$

Bound versus scattering states



Continuum wavefunctions:

$$\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r} [Y_{\ell}(\hat{r}) \otimes \chi_s]_{jm}$$

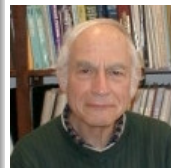
$$\varepsilon = \frac{\hbar^2 k^2}{2\mu}$$

Unbound states are not suitable for CC calculations:

- They have a continuous (infinite) distribution in energy.
- Non-normalizable: $\langle u_{k,\ell sj}(r) | u_{k',\ell sj}(r) \rangle \propto \delta(k - k')$

SOLUTION \Rightarrow continuum discretization

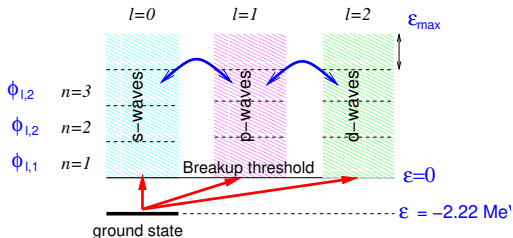
- Continuum discretization method proposed by G.H. Rawitscher [PRC9, 2210 (1974)] and Farrell, Vincent and Austern [Ann.Phys.(New York) 96, 333 (1976)] to describe deuteron scattering as an effective three-body problem $p + n + A$.



George Rawitscher
(1928-2018)

- Full numerical implementation by Kyushu group (Sakuragi, Yahiro, Kamimura, and co.):
Prog. Theor. Phys.(Kyoto) 68, 322 (1982)

Continuum discretization for deuteron scattering



- ⇒ Select a number of angular momenta ($\ell = 0, \dots, \ell_{\text{max}}$).
- ⇒ For each ℓ , set a maximum excitation energy ϵ_{max} .
- ⇒ Divide the interval $\epsilon = 0 - \epsilon_{\text{max}}$ in a set of sub-intervals (*bins*).
- ⇒ For each *bin*, calculate a representative wavefunction.

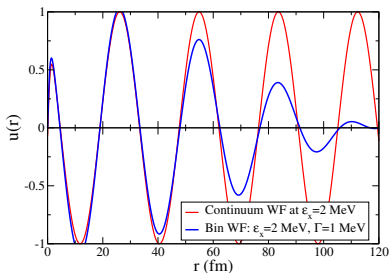
CDCC formalism: construction of the bin wavefunctions

Bin wavefunction:

$$\phi_{\ell jm}^{[k_1, k_2]}(\mathbf{r}) = \frac{u_{\ell j}^{[k_1, k_2]}(r)}{r} [Y_\ell(\hat{r}) \otimes \chi_s]_{jm} \quad [k_1, k_2] = \text{bin interval}$$

$$u_{\ell sjm}^{[k_1, k_2]}(r) = \sqrt{\frac{2}{\pi N}} \int_{k_1}^{k_2} w(k) u_{k, \ell sj}(r) dk$$

- k : linear momentum
- $u_{k, \ell sj}(r)$: scattering states (radial part)
- $w(k)$: weight function



CDCC formalism for deuteron scattering

- **Hamiltonian:** $H = T_{\mathbf{R}} + h_r(\mathbf{r}) + V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt})$
- **Model wavefunction:**

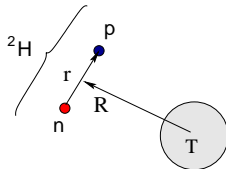
$$\Psi^{(+)}(\mathbf{R}, \mathbf{r}) = \phi_{gs}(\mathbf{r})\chi_0(\mathbf{R}) + \sum_{n>0}^N \phi_n(\mathbf{r})\chi_n(\mathbf{R})$$

- **Coupled equations:** $[H - E]\Psi(\mathbf{R}, \mathbf{r}) = 0$

$$[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})$$

- **Transition potentials:**

$$V_{n,n'}(\mathbf{R}) = \int d\mathbf{r} \phi_n^*(\mathbf{r}) \left[V_{pt}(\mathbf{R} + \frac{\mathbf{r}}{2}) + V_{nt}(\mathbf{R} - \frac{\mathbf{r}}{2}) \right] \phi_{n'}(\mathbf{r})$$



Trivially equivalent local equivalent potential (TELP)

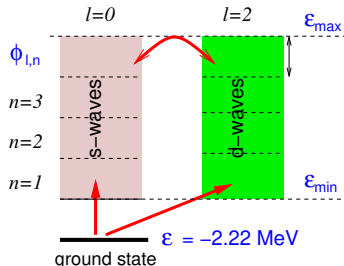
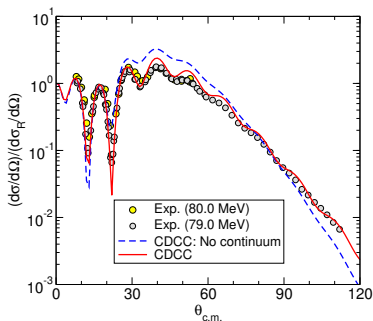
- From the elastic channel equation, a TELP can be defined as follows:

$$\left[E - \varepsilon_0 - \hat{T}_{\mathbf{R}} - V_{0,0}(\mathbf{R}) \right] \chi_0(\mathbf{R}) = \sum_{i \neq 0} V_{i,0}(\mathbf{R}) \chi_i(\mathbf{R}) \equiv U_{\text{TELP}}(\mathbf{R}) \chi_0(\mathbf{R}).$$

- In actual calculations, $U_{\text{TELP}}(R)$ will depend on the total angular momentum, but a weighted average can be performed to obtain an approximate angular-momentum independent polarization potential
- A single channel calculation with the potential $U(\mathbf{R}) = V_{0,0}(\mathbf{R}) + U_{\text{TELP}}(\mathbf{R})$ should reproduce approximately the elastic scattering cross section.

Coupling to continuum states produce:

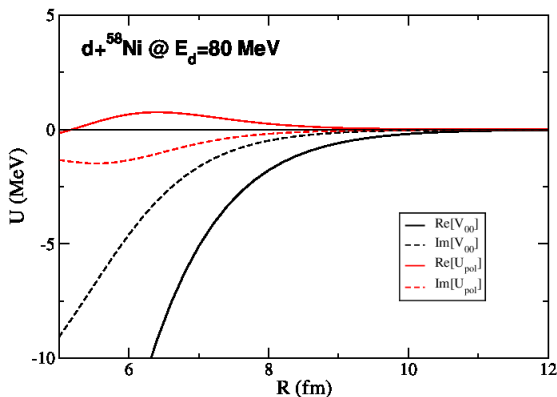
- Polarization of the projectile (modification of real part)
- Flux removal (absorption) from the elastic channel (imaginary part)



👉 No continuum \Rightarrow retain only the Watanabe potential:

$$V_{00}(\mathbf{R}) = \int d\mathbf{r} \phi_{\text{gs}}^*(\mathbf{r}) (V_{pt} + V_{nt}) \phi_{\text{gs}}(\mathbf{r})$$

Trivially equivalent local equivalent potential for $d+^{58}\text{Ni}$ @ 80 MeV



For this reaction, the TELP is complex:

- The real part is **repulsive** (reduces projectile-target attraction)
- The imaginary part is **absorptive** (flux removal)

Two- and three-body breakup observables

- CDCC scattering amplitudes readily provide **two-body breakup** observables:

$$\frac{d\sigma_n}{d\Omega_{\text{c.m.}}} = |f_{0,n}(\theta)|^2 \Rightarrow \frac{d^2\sigma}{d\Omega_{\text{c.m.}} d\epsilon_{pn}} \simeq \frac{1}{\Delta_n} \frac{d\sigma_n}{d\Omega_{\text{c.m.}}}$$

with:

- Δ_n =width of the bin containing the relative energy ϵ_{pn}
- $\Omega_{\text{c.m.}}$ =C.M. scattering angle of the projectile c.m. (not easy to measure!)
- Three-body observables** can be also calculated using a suitable combination of the scattering amplitudes and appropriate kinematical transformations ([Tostevin, PRC63, 024617 \(2001\)](#)):

$$\frac{d^3\sigma}{d\Omega_n d\Omega_p dE_p}$$

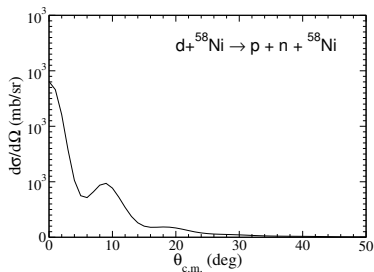
N.b.: *These 3-body observables are not directly provided by FRESKO. They must be computed separately from the calculated amplitudes.*

Two-body breakup observables: $d + {}^{58}\text{Ni} \rightarrow p + n + {}^{58}\text{Ni}$

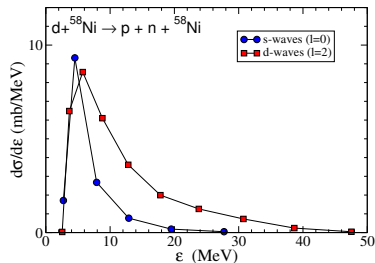
CDCC calculations for $d + {}^{58}\text{Ni}$ at 80 MeV :

- Continuum states with $\ell = 0, 2$.
- Proton and neutron intrinsic spins ignored.
- $p/n + {}^{58}\text{Ni}$ from global optical potential.
- $p+n$ simple Gaussian interaction describing deuteron g.s.

$p + n$ c.m. angular distribution



Excitation energy distribution

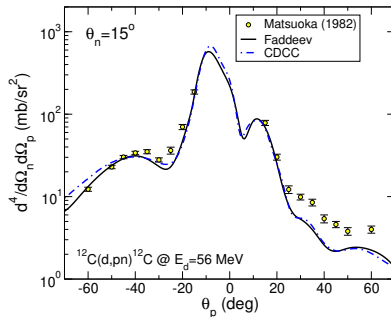
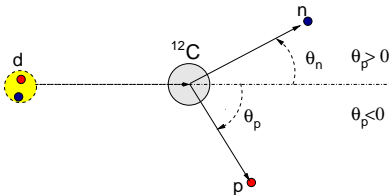


Breakup observables with CDCC: exclusive breakup of $d + {}^{12}\text{C} \rightarrow p + n + {}^{12}\text{C}$

CDCC calculations for $d + {}^{12}\text{C}$ at 56 MeV:

- Continuum states with $\ell \leq 8$ and $\varepsilon_{\text{max}} = 46$ MeV.
- Proton and neutron intrinsic spins ignored
- $p/n + {}^{58}\text{Ni}$ from Watson global optical potential
- $p+n$ simple Gaussian interaction describing deuteron g.s.

Data: Matsuoka *et al.*, NPA391, 357 (1982).

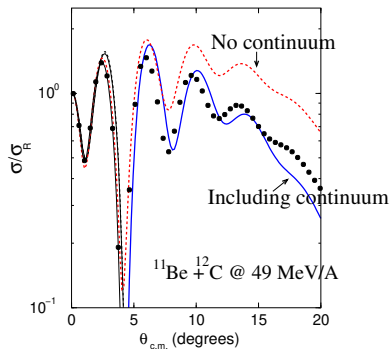
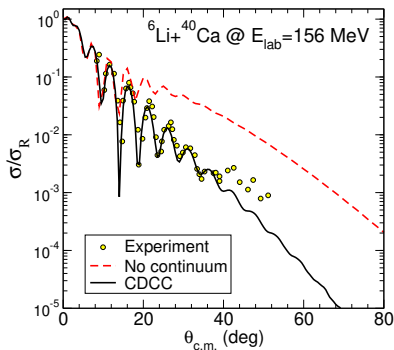


A. Deltuva *et al*, PRC 76, 064602 (2007)

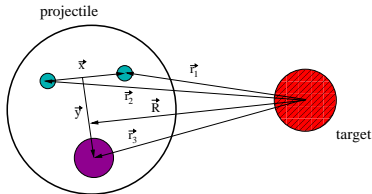
Application of the CDCC method: ${}^6\text{Li}$ and ${}^6\text{He}$ scattering

👉 The CDCC has been also applied to nuclei with a cluster structure:

- ${}^6\text{Li} = \alpha + d$ ($S_{\alpha,d} = 1.47$ MeV)
- ${}^{11}\text{Be} = {}^{10}\text{Be} + n$ ($S_n = 0.504$ MeV)



Extension to 3-body projectiles

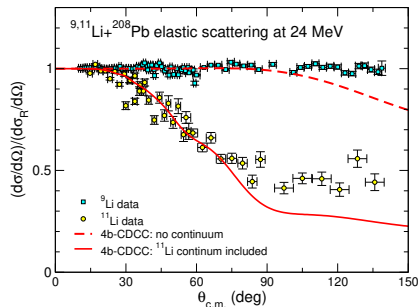
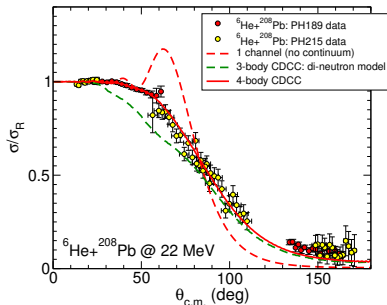


To extend the CDCC formalism, one needs to evaluate the new coupling potentials:

$$V_{n;n'}(\mathbf{R}) = \int d\mathbf{r} \phi_n^*(\mathbf{x}, \mathbf{y}) \{V_{nt}(\mathbf{r}_1) + V_{nt}(\mathbf{r}_2) + V_{at}(\mathbf{r}_3)\} \phi_{n'}(\mathbf{x}, \mathbf{y})$$

- ☞ $\phi_n(\mathbf{x}, \mathbf{y})$ three-body WFs for bound and continuum states: hyperspherical coordinates, Faddeev, etc (difficult to calculate!)
- ☞ 4b-CDCC calculations not included in FRESKO; require separate codes to compute the $\phi_n(\mathbf{x}, \mathbf{y})$ wfs (e.g. FACE) and $V_{n,n'}(\mathbf{R})$ potentials

Four-body CDCC calculations for ${}^6\text{He}$ and ${}^{11}\text{Li}$ scattering



Data (LLN): NPA803, 30 (2008); PRC 84, 044604 (2011)

M Cubero et al, PRL109, 262701 (2012)

Calculations: PRC 80, 051601 (2009)

N.b.: 1-channel potential considers only g.s. \rightarrow g.s. coupling potential:

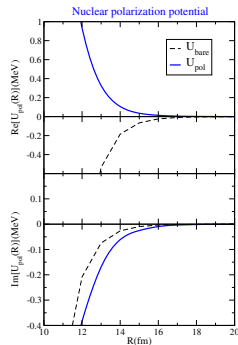
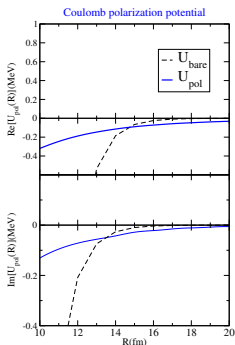
$$V_{00}(\mathbf{R}) = \int d\mathbf{r} \phi_{g.s.}^*(\mathbf{x}, \mathbf{y}) \{V_{nt}(\mathbf{r}_1) + V_{nt}(\mathbf{r}_2) + V_{ct}(\mathbf{r}_3)\} \phi_{g.s.}(\mathbf{x}, \mathbf{y})$$

Polarization potential for ${}^6\text{He}+{}^{208}\text{Pb}$: long-range effect

Trivially Equivalent Local Polarization potential (TELP):

$$\left[E - \varepsilon_0 - \hat{T}_{\mathbf{R}} - V_{i,0}(\mathbf{R}) \right] \chi_0(\mathbf{R}) = \sum_{i \neq 0} V_{i,0}(\mathbf{R}) \chi_i(\mathbf{R}) \equiv U_{\text{TELP}}(\mathbf{R}) \chi_0(\mathbf{R}).$$

Application to ${}^6\text{He}+{}^{208}\text{Pb}$ at 22 MeV

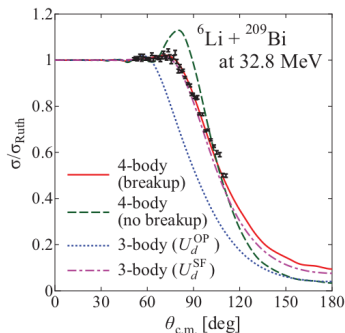
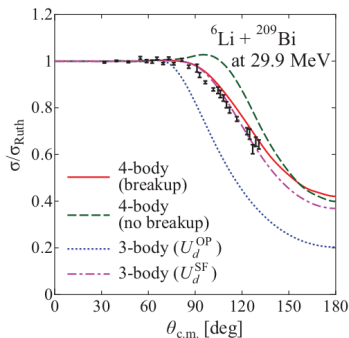


- Imaginary part of TELP is long-ranged and absorptive (explains the need for large imaginary diffuseness parameter in OM analysis)
- Real part is attractive for the Coulomb potential and repulsive for the nuclear couplings.

Application to ${}^6\text{Li}$ scattering

Example: ${}^6\text{Li} + {}^{209}\text{Bi}$ around Coulomb barrier

- 4-body CDCC: ${}^6\text{Li} = \alpha + p + n$
- 3-body CDCC: ${}^6\text{Li} = \alpha + d$

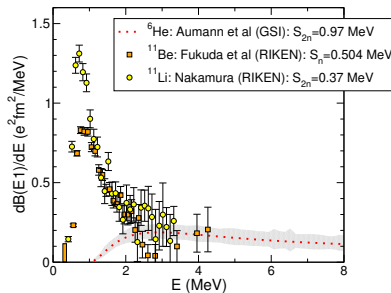
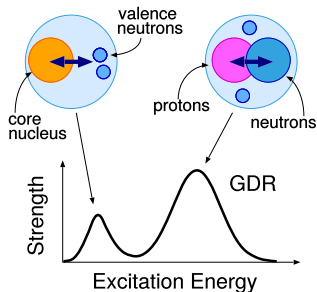


Watanabe *et al*, PRC 86, 031601(R) (2012)

[illegible]

- ➊ Coulomb dissociation experiments
 - Semiclassical description: Alder and Winther
 - Quantum-mechanical description
- ➋ Exploring continuum structures: resonances and virtual states

Electric response of weakly-bound nuclei



- The $E\lambda$ response can be quantified through the $B(E\lambda)$ probability:

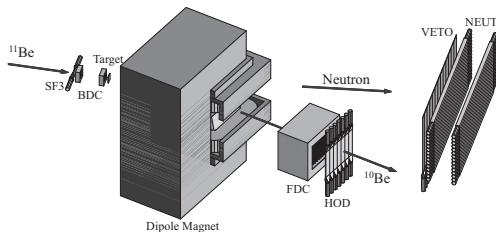
$$B(E\lambda; i \rightarrow f) = \frac{1}{2I_i + 1} |\langle \Psi_f | \mathcal{M}(E\lambda) | \Psi_i \rangle|^2$$

- Neutron-halo nuclei have large $B(E1)$ strengths near threshold

How to probe/extract the $B(E1)$ of halo nuclei?

Example: $^{11}\text{Be} + ^{208}\text{Pb} \rightarrow ^{10}\text{Be} + n + ^{208}\text{Pb}$ measured at RIKEN (69 MeV/u).

Fukuda et al, PRC70, 054606 (2004))



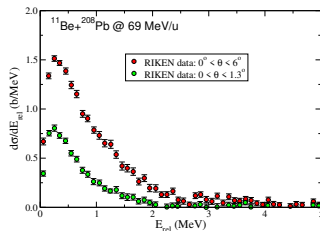
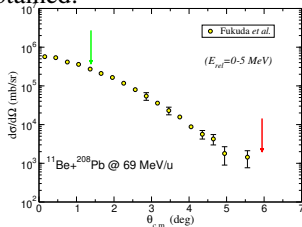
☞ ^{11}Be excitation energy can be reconstructed from core-neutron coincidences (*invariant mass method*)

What observables are measured in Coulomb dissociation experiments?

- Experimentally, one measures angular and relative energy distribution of the $^{11}\text{Be}^*$ system:

$$\frac{d^2\sigma}{d\Omega dE}$$

- Integrating over the angle or energy, single differential cross sections are obtained:



- In the Coulomb dominated region (i.e. small angles), the **breakup cross section** is expected to be dominated by the $dB(E\lambda)/dE$ distribution, but we need a theory that relates both observables.

Semiclassical 1st order $E\lambda$ excitation (Alder & Winther) (akin EPM method)

- For $E\lambda$ excitation to bound states ($0 \rightarrow n$):

$$\left(\frac{d\sigma}{d\Omega}\right)_{0 \rightarrow n} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{B(E\lambda, 0 \rightarrow n)}{e^2 a_0^{2\lambda-2}} f_\lambda(\theta, \xi) \quad \xi_{0 \rightarrow n} = \frac{(E_n - E_0)}{\hbar} \frac{a_0}{v}$$

- For continuum states (breakup):

$$\frac{d\sigma(E\lambda)}{d\Omega dE} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{1}{e^2 a_0^{2\lambda-2}} \frac{dB(E\lambda)}{dE} \frac{df_\lambda(\theta, \xi)}{d\Omega}$$

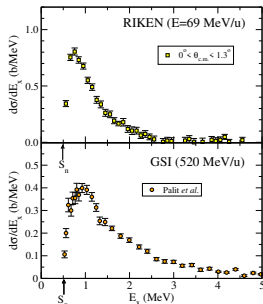
$dB(E\lambda)/dE$ can be extracted from small-angle Coulomb dissociation data.

$$\frac{d\sigma}{dE}(\theta < \theta_{\max}) = \int_0^{\theta_{\max}} \frac{d\sigma(E\lambda)}{d\Omega dE} d\Omega \propto \frac{dB(E\lambda)}{dE}$$

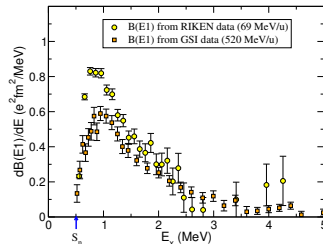
Extracting $B(E1)$ of ^{11}Be from $^{11}\text{Be} + ^{208}\text{Pb}$ Coulomb dissociation

Common assumptions:

- Breakup dominated by Coulomb excitation (mostly E1).
- Nuclear excitation, if present, can be estimated and added incoherently
- ☞ If the assumptions above are fulfilled, the extracted $dB(E\lambda)dE$ should be independent of the incident energy and target employed, since it reflects a structure property of the projectile.



$$\frac{dB(E\lambda)}{dE} \propto \frac{d\sigma}{dE}$$



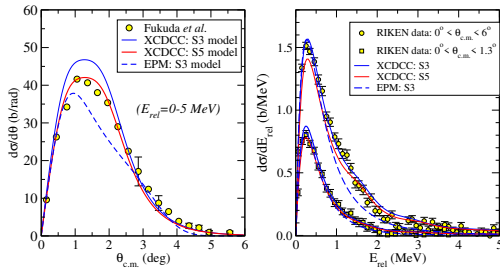
RIKEN: Fukuda et al, PRC70 (2004) 054606

GSI: Palit et al, PRC68 (2003) 034318

☞ The extracted $dB(E\lambda)/dE$ distributions are reasonably compatible, but with apparent differences at the peak

- Nuclear excitation not negligible, even for small θ
- Nuclear contribution interferes with Coulomb
- Higher-order couplings can affect the cross sections (E2, E3...)
- ☞ These ingredients can be naturally incorporated within the CDCC method (at the expense of more complexity!)

PLB 811 (2020) 135959



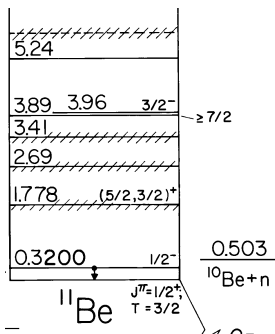
- ☞ Different structure models yield different $B(E1)$ strengths and hence different breakup cross sections
- ☞ Comparison with the angular distribution evidences the deficiencies of the semiclassical EPM model

Exploring structures in the continuum

- 1 Resonances and virtual states
- 2 Accessing continuum structures from breakup and transfer reactions

Exploring structures in the continuum

The continuum spectrum is not “homogeneous”; it contains in general energy regions with special structures, such as resonances and virtual states

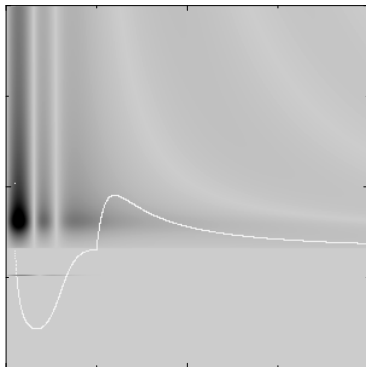


What is a resonance?

- It is a **pole** of the S-matrix in the complex energy plane.
- It is a structure on the continuum which may, or may not, produce a **maximum in the cross section**, depending on the reaction mechanism and the phase space available.
- The resonance occurs in the range of energies for which the **phase shift is close to $\pi/2$** .
- In this range of energies, continuum wavefunctions have a **large probability of being in the radial range of the potential**.
- The continuum wavefunctions are **not square normalizable**. For practical reasons, a normalized wave-packet (or “bin”) can be constructed to represent the resonance.

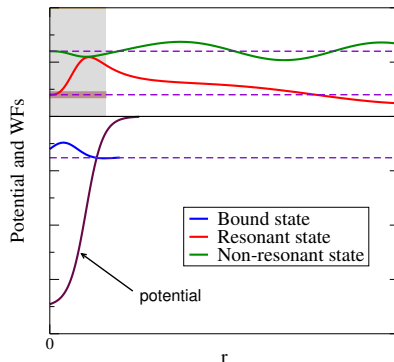
Distinctive features of a resonance

In the energy range of the resonance, the continuum wavefunctions have a large probability of being within the range of the potential.

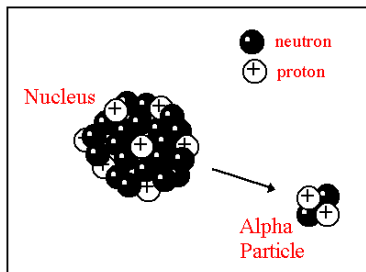


Cuts and areas ordered by size

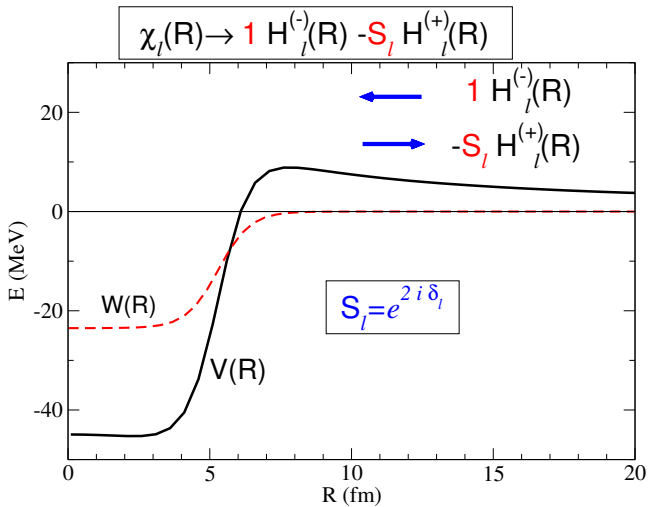
(Courtesy of C. Dasso)



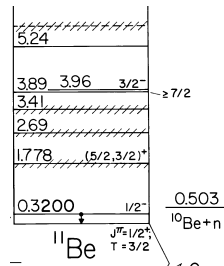
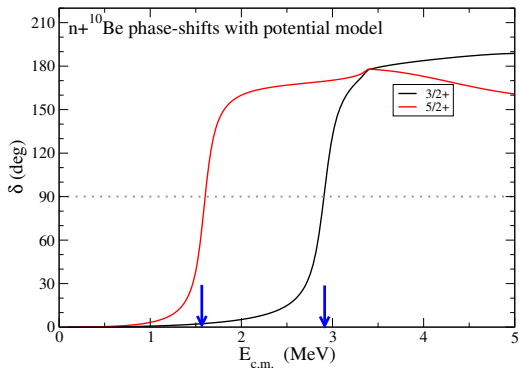
The decay of the resonance is also behind the α -decay phenomenon:



Resonances and phase-shifts

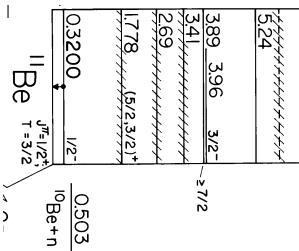
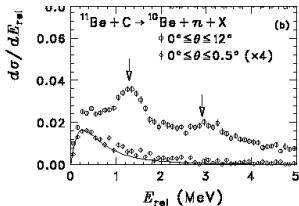


Resonances and phase-shifts

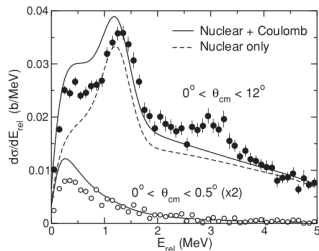
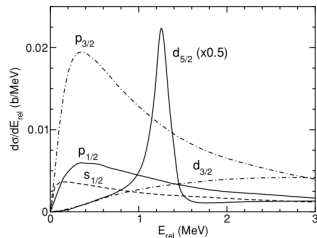


Studying resonances in nuclear breakup experiments

RIKEN data



CDCC analysis



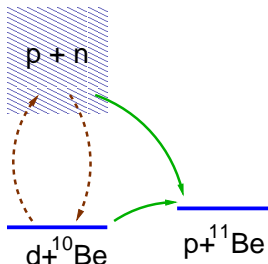
Fukuda *et al.*, PRC70 (2004) 054606

Howell *et al.*, JPG31 (2005) S1881

Transfer reactions with weakly bound nuclei

Transfer reactions with weakly bound nuclei

- DWBA approximates the total WF by the elastic channel and assumes that the transfer occurs in one step (Born approximation).
- For weakly bound projectiles (eg. deuterons), breakup is an important channel and can influence the transfer process.



- $\Psi_{\mathbf{K}_d}^{(+)}(\mathbf{R}, \mathbf{r})$ includes breakup components, but these are lost when we make the DWBA approximation ($\Psi^{(+)} \approx \chi_d^{(+)}(\mathbf{K}_d, \mathbf{R})\varphi_d(\mathbf{r}) \Rightarrow$ need to go beyond DWBA

Adiabatic distorted wave approximation (ADWA)

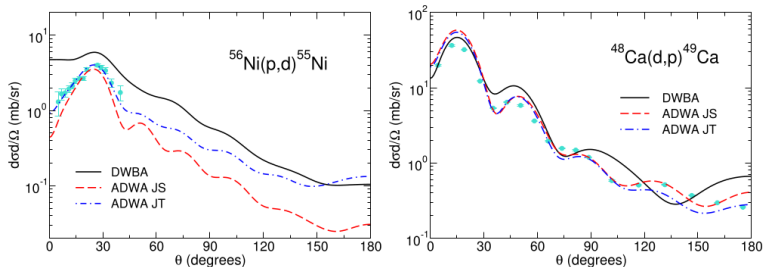
- $\chi_d^{(+)}(\mathbf{K}_d, \mathbf{R})$ describes deuteron elastic scattering but, for the (d, p) transfer matrix element, we need only $\Psi_{\mathbf{K}_d}^{(+)}(\mathbf{R}, \mathbf{r})$ for small $|\mathbf{r}|$
- R.C. Johnson and col. have derived an approximation of $\Psi_{\mathbf{K}_d}^{(+)}(\mathbf{R}, \mathbf{r})$ valid for $r \approx 0$, which includes the effect of deuteron breakup effectively (**adiabatic approx.**):
 - 1 **Zero-range** approximation (Johnson-Soper): [(Johnson, Soper, PRC1, 976 (1970))]

$$U^{JS}(R) = U_{pA}(R) + U_{nA}(R) \Rightarrow \chi_d^{JS}(\mathbf{R})$$

- 2 **Finite-range** version (Johnson–Tandy): [Johnson & Tandy, NPA235 (1974) 56]

$$U^{JT}(R) = \frac{\langle \varphi_{pn}(\mathbf{r}) | V_{pn}(U_{nA} + U_{pA}) | \varphi_{pn}(\mathbf{r}) \rangle}{\langle \phi_{pn}(\mathbf{r}) | V_{pn} | \phi_{pn}(\mathbf{r}) \rangle}$$

DWBA vs ADWA



From Timofeyuk and, Progress in Particle and Nuclear Physics 111 (2020) 103738

CDCC-BA approximation

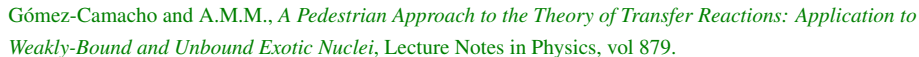
- Exact transition amplitude for a general $A(d, p)B$ process:

$$\mathcal{T}_{d,p}^{\text{CDCC}} = C_{BA}^{\ell j} \int \int \chi_p^{(-)*}(\mathbf{K}_p, \mathbf{R}') \varphi_{nA}^{\ell j,*}(\mathbf{r}') V_{pn}(\mathbf{r}) \underbrace{\Psi_{\mathbf{K}_\alpha}^{(+)}(\mathbf{R}_\alpha, \xi_\alpha)}_{\text{breakup}} d\mathbf{r}' d\mathbf{R}'$$

- Use CDCC approximation for $\Psi_{\mathbf{K}_\alpha}^{(+)}$.

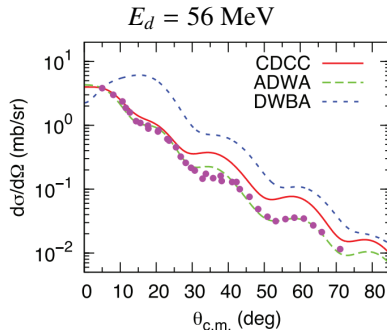
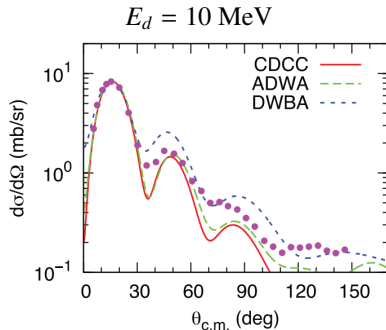
$$\Psi_{\mathbf{K}_\alpha}^{(+)} \approx \Psi^{\text{CDCC}} = \underbrace{\chi_0(\mathbf{R})\phi_0(\mathbf{r})}_{\text{elastic}} + \sum_{n'j,\pi} \underbrace{\phi_{n'}^{j\pi}(k_{n'}, \mathbf{r})\chi_{n'j,\pi}(\mathbf{R})}_{\text{breakup}}$$

- Unlike the DWBA and ADWA methods, coupling to deuteron breakup states is included explicitly.



DWBA vs ADWA vs CDCC

Example: $^{58}\text{Ni}(d,p)^{59}\text{Ni}$



☞ *CDCC and ADWA provide better description of the data and lead also to more realistic spectroscopic information (e.g. spectroscopic factors)*

Pang *et al*, PRC 90, 044611 (2014)

1. *Journal of Management Studies*, 1990, 27, 1, 1-14.

- E.g.:** $d_{3/2}$ resonance in ^{17}O resonance at $E_r = 0.95$ MeV



- Inclusion of continuum states in DWBA poses numerical problems due to the oscillatory behaviour of unbound wavefunctions
- Regularization method must be applied, such as representing the resonance by a wavepacket (continuum *bin*, as in the CDCC method)

E.g.: ^{11}Be resonance at $E_x = 1.78 \text{ MeV}$ from $^{10}\text{Be}(d,p)^{11}\text{Be}$

Schmitt et al, PRC88, 064612 (2013)

FIG. 8. (Color online) Differential cross sections are presented for transfer to the first resonance in ^{11}Be at 1.78 MeV via the $^{10}\text{Be}(d,p)$ reaction in inverse kinematics at deuteron energies of (a) 12 MeV, (b) 15 MeV, (c) 18 MeV, and (d) 21.4 MeV. The curves are from FR-ADWA calculations using (solid line) an energy bin that is the same width as for the resonance used in the calculation and (dotted line) with a width 1.5 times that value. At 12 MeV the protons were too low in energy to extract an angular distribution.

Nuclear reactions with exotic nuclei

A. M. Moro

Universidad de Sevilla

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- Inclusion of continuum states in DWBA poses numerical problems due to the oscillatory behaviour of unbound wavefunctions
- Regularization method must be applied, such as representing the resonance by a wavepacket (continuum *bin*, as in the CDCC method)

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Schmitt et al, PRC88, 064612 (2013)

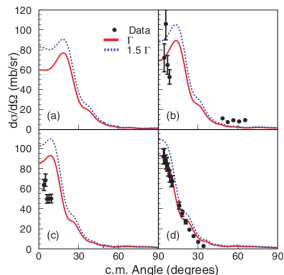
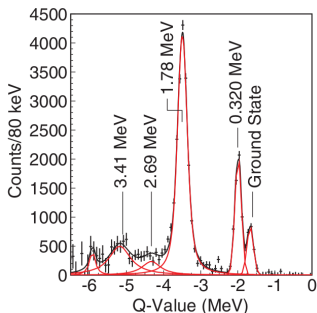
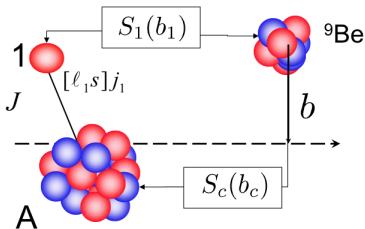


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Knock-out reactions

Spectroscopic from momentum distributions

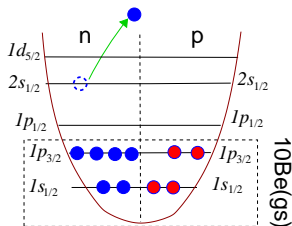
- Fast-moving projectile on a (typically) light target.
- One nucleon suddenly removed (absorbed) due to its interaction with the target.
- The remaining residue remains unchanged and is detected.
- The momentum of the core is related to that of the removed nucleon because, in the rest frame of the projectile, $\vec{P} = 0$



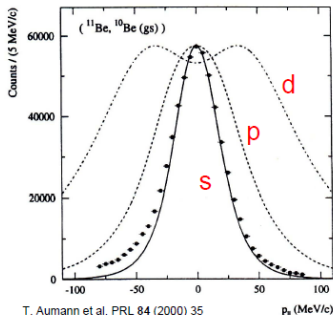
$$\vec{P} = \vec{p}_c + \vec{p}_1 = 0$$

Angular momentum sensitivity of momentum distributions

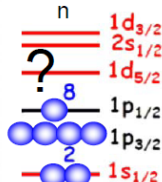
- The shape is determined by the orbital angular momentum ℓ .
- The magnitude is determined by the amount of $s_{1/2}$ (spectroscopic factor)



Residue momentum $^{11}\text{Be} \rightarrow ^{10}\text{Be}$ – halo case



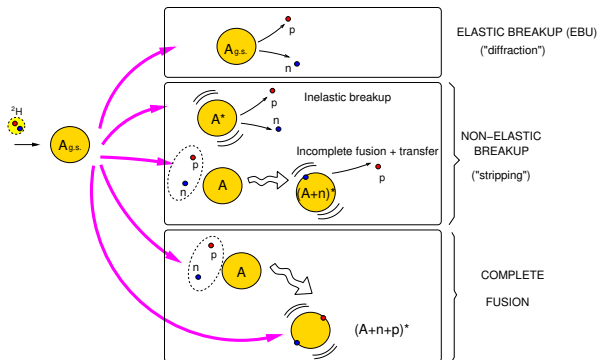
$Z = 4, N = 7$



^{11}Be

T. Aumann et al. PRL 84 (2000) 35

11. *Journal of the American Medical Association*, 1997; 278: 1039-1044.



✎ *The singles (inclusive) cross section of a given fragment will contain in general diffraction and stripping components*

Stripping cross section within a semiclassical (eikonal) theory

At high energies, one can use the sudden, eikonal approximations to obtain simple formulas for the stripping and diffraction parts of the inclusive breakup cross section for a inclusive reaction of the form $a + B \rightarrow b + X$, with $a = b + x$:

Stripping:

$$\sigma_{\text{sp}}^{\text{str}} = 2\pi \int b db \int d\mathbf{r} |\varphi_{bx}(\mathbf{r})|^2 (1 - |S_x(b_x)|)^2 |S_{bA}(b_b)|^2$$

Diffraction:

$$\sigma_{\text{sp}}^{\text{diff}} = 2\pi \int b db \left[\langle \varphi_{bx} | |S_b S_x|^2 | \varphi_{bx} \rangle - |\langle \varphi_{bx} | S_b S_x | \varphi_{bx} \rangle|^2 \right].$$

- $|S_b(b_b)|^2$ = probability of survival of the core.
- $1 - |S_x(b_x)|^2$ = probability of absorption of the valence particle.

Extraction of SFs from knockout reactions

- Agreement theory vs experiment quantified with the **reduction factor**:

$$R_s = \frac{\sigma_{\text{theor}}}{\sigma_{\text{exp}}}$$

with

$$\sigma_{\text{theor}} = \sum_{n\ell j} S_{bx}^a(I; n\ell j) \sigma_{\text{sp}}(I; n\ell j)$$

$$\sigma_{\text{sp}}(I; n\ell j) = \sigma_{\text{sp}}^{\text{EBU}} + \sigma_{\text{sp}}^{\text{NEB}}$$

- $R_s < 1 \Rightarrow$ possible correlations (long-range, short-range, tensor, ...) not included in σ_{theor} ?
- R_s strongly dependent on $\Delta S = S_p - S_n$.

Extraction of SFs from knockout reactions

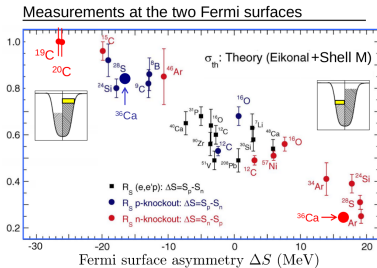
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$$R_s = \frac{\sigma_{\text{theor}}}{\sigma_{\text{exp}}}$$

with

$$\sigma_{\text{theor}} = \sum_{nlj} S_{bx}^a(I; nlj) \sigma_{\text{sp}}(I; nlj)$$

$$\sigma_{\text{sp}}(I; nlj) = \sigma_{\text{sp}}^{\text{EBU}} + \sigma_{\text{sp}}^{\text{NEB}}$$



-Gade et al, PRC 77, 044306 (2008)
Tostevin, PRC90,057602(2014)

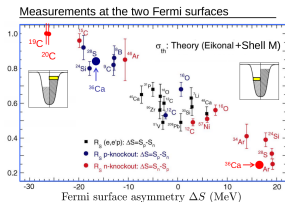
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- R_s strongly dependent on $\Delta S = S_p - S_n$.

Extraction of SFs from knockout reactions

...however, this behaviour has not been corroborated by other probes, such as transfer or proton-induced knockout reactions (p, pN)

HI knockout (~ 100 MeV/u)

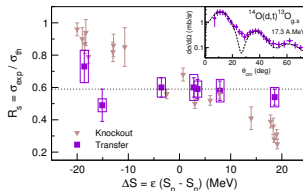
Tostevin, PRC90,057602(2014)



- Reaction model: eikonal + adiabatic
- R_s strongly dependent on $S_p - S_n$.

Low-energy transfer

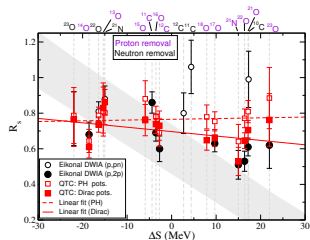
Flavigny, PRL110, 122503(2013)



- Reaction model: ADWA, DWBA, CRC
- $R_s \sim \text{constant}$.

 (p, pN) @ 200-400 MeV/u

Aumann, PPNP118,103847(2021)



- Reaction models: DWIA, TC
- $R_s \sim \text{constant}$.

Similar results from RIKEN Wakase, PTEP 021D01 (2018)

R_s from knockout disagree with those from transfer and $(p, pN) \Rightarrow$ description of reaction mechanism?