CS 221 – Autumn 2018
Anand Natu
anatu - 06264867

<u>Homework #8: From Language to Logic</u>

Problem 4

Problem 4a

The knowledge base is

$$KB = \{(A \vee B) \rightarrow \ C, A\}$$

First, we apply the property that $P \rightarrow Q = \neg P \vee Q$, which allows us to write the knowledge base as

$$= \{\neg(A \vee B) \vee C, A\} = \{(\neg A \vee \neg B) \vee C, A\}$$

Distributing the negation connectives on A and B we have

$$= \{(\neg A \vee C) \wedge (\neg B \vee C), A\}$$

We can remove the conjunction operator to yield the equivalent group of atomic expressions

$$= \{(\neg A \vee C), (\neg B \vee C), A\}$$

Finally, we apply the same identity as in the first step to convert back into implication connectives, which then allows us to express the knowledge base into Conjunctive Normal Form:

$$= \{\boldsymbol{A} \rightarrow \boldsymbol{C}, \boldsymbol{B} \rightarrow \boldsymbol{C}, \boldsymbol{A}\}$$

To apply modus ponens to the above expression of the knowledge base, we accept the conditional statement $A \rightarrow C$, and that the antecedent A holds, such that we can infer C.

Problem 4b

The knowledge base is

$$KB = \{A \vee B, B \rightarrow C, (A \vee C) \rightarrow \ D\}$$

Once again applying the identity $P \rightarrow Q = \neg P \vee Q$, we have

$$= \{A \vee B, \neg B \vee C, (A \vee C) \rightarrow \ D\}$$

Distribution of the third rule's disjunction operation on A and C and applying the identity once again gives us the knowledge base expressed in CNF:

$$KB_{CNF} = \{A \vee B, \neg B \vee C, \neg A \vee D, \neg C \vee D\}$$

From the first two rules of the knowledge base, we obtain

$$\frac{A \vee B, \neg B \vee C}{A \vee C}$$

We can combine this result with the fourth rule so that we have

$$\frac{A \vee C, \neg C \vee D}{A \vee D}$$

Finally, applying that result to the third rule allows us to derive D directly:

$$\frac{\boldsymbol{A} \vee \boldsymbol{D}, \neg \boldsymbol{A} \vee \boldsymbol{D}}{\boldsymbol{D}}$$

Problem 5

Problem 5b

In order to prove that the resulting set of 7 constraints is not consistent for any finite, non-empty model, we can consider a sequence of $n+1$ elements in a domain of size n. For every z_i in the sequence of elements, we say that z_{i+1} is a successor of z_i. Then, the fifth constraint dictates that $i+1$ must also be larger than i, and the sixth constraint says that any z_j is greater than z_i.

However, since there are $n+1$ elements in the sequence and only n values specified in the domain, this means that there must be at least one pair of numbers that are equivalent, i.e. $z_a = z_b, b > a$. However, the transitive sixth constraint says that $z_b > z_a$, while the new seventh constraint says that a number cannot be larger than itself. **Therefore, these two constraints contradict one another in this case, which means the set of constraints is inconsistent.**