The Dynamic Consequences of Technology and Discount Factor Shocks in Medium-Scale RANK vs TANK Models

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Abstract

Prompted by the growing interest by macroeconomists for household heterogeneity and its importance for the dynamics in macroeconomic models, this project aims at analysing how the introduction of *hand-to-mouth* behaviour in the economy might affect these dynamics. In particular, I consider a non-linear medium-scale two-agent New Keynesian model (TANK), which features a time-invariant fraction of such hand-to-mouth households. Within this framework, I investigate the dynamics resulting from two distinct shocks – one to technology and one to the discount factor – and contrast these with the insights delivered by a representative-agent New Keynesian model (RANK). The results show that the TANK model produces aggregate dynamics that usually feature somewhat more pronounced effects on impact but are generally close to the ones obtained from RANK. Naturally, TANK's main advantage lies in delivering insights into the heterogeneity underlying these aggregate dynamics. Finally, I illustrate that *inter alia* two parameters crucially shape the responses of TANK relative to RANK.

1 Introduction and Related Literature

The prevalence of the representative agent in macroeconomic models has often been criticised on the grounds that assuming household heterogeneity away is not a valid model abstraction. However, striving to overcome this criticism, there has been significant progress towards understanding the importance of heterogeneity for macroeconomic dynamics.[1]

This project aims at contributing to this strand of research by contrasting a representative-agent model with a two-agent framework. The two models crucially differ regarding the household sector: while the former assumes a representative, consumption-smoothing household, the latter subdivides the population into a fraction of standard, consumption-smoothing agents and a fraction of *hand-to-mouth* agents, who simply consume their current-period labour income.

Both frameworks are embedded into a non-linear medium-scale setting with Keynesian features, resulting in a representative-agent New Keynesian model (RANK) and a two-agent New Keynesian model (TANK), respectively. These models are confronted with two shocks: one to technology (supply side) and one to the households' discount factor (demand side). The aim is to analyse similarities and differences of the two models' dynamics after these shocks. The `Econpizza` package by Boehl (2022) is used to solve for the models and their responses.

Conditional on the two shocks considered, the results suggest that the two economies behave qualitatively quite similarly on the aggregate level. It turns out that the responses from the TANK model are usually quantitatively more pronounced but the differences are not too large for reasonable calibrations of the share of hand-to-mouth agents and the household's inverse Frisch elasticity of labour supply. In the baseline case, aggregate consumption in TANK drops by about a percentage point more than in RANK after a discount factor shock.

Beyond the aggregate, the TANK model delivers insights on the responses of individual-level variables. For example, after a positive technology shock, consumption of the hand-to-mouth falls on impact, while unconstrained-agent consumption increases. Thus, TANK captures heterogeneous effects which are concealed by RANK.

Finally, a sensitivity analysis demonstrates that the interaction of two parameters crucially shapes the relative aggregate consumption response of TANK to RANK on impact. If both the inverse Frisch elasticity *and* the share of hand-to-mouth agents in TANK are relatively large, TANK shows a significantly different response compared to its RANK counterpart.

Related Literature

The present work is related to a range of papers which introduce hand-to-mouth agents into a New Keynesian setting. One of the early papers to do so is Galí et al. (2007). The authors find that, in US data, a government spending shock leads to an increase in consumption and show that a linearised TANK model is – in contrast to the representative-agent framework – able to account for this evidence by combining hand-to-mouth agents with other frictions such

[1] Galí (2018) includes a short overview on the recent state of the literature on household heterogeneity in New Keynesian macroeconomics.

as nominal rigidities and an imperfectly competitive labour market.

Bilbiie (2008) investigates the repercussions that limited asset-market participation by hand-to-mouth agents has on monetary policy conduct. In a log-linearised TANK model without capital, the paper shows that a high fraction of hand-to-mouth agents (and/or a high Frisch elasticity of labour supply) might reverse standard results regarding *inter alia* the elasticity of aggregate demand to the (real) interest rate and the Taylor rule.

More recently, Debortoli and Galí (2018) show that of the three aspects of heterogeneity that are present in a prototypical heterogeneous-agent New Keynesian model (HANK), the most relevant for aggregate fluctuations is the consumption gap between constrained and unconstrained households. The authors argue that, while remaining more tractable, TANK can replicate this dimension well and show that a linearised TANK, which they solve for analytically, indeed produces similar impulse responses to economy-wide shocks as HANK does.

In their seminal paper, Kaplan et al. (2018) analyse the monetary transmission mechanism in a two-asset HANK framework. The authors find that the indirect effects of interest rate changes on consumption – arising through general equilibrium mechanisms – dominate over the direct effect, which is mainly driven by intertemporal consumption substitution. This is in stark contrast to RANK and also, to a certain degree, to TANK.

The TANK framework has also been used in empirical work, e.g. in Gerke et al. (2020). This paper shows that a medium-scale TANK model, estimated on Euro Area data and featuring countercyclical transfers to hand-to-mouth agents, can dampen the overly expansionary effects that forward guidance has in RANK models.

The rest of this paper is structured as follows. Section 2 introduces the models, followed by section 3 discussing the shocks. The results are presented in section 4 and section 5 concludes.

2 The Models

The RANK and TANK models used herein draw heavily from the model employed in Gust et al. (2012), thus building on a fairly standard medium-scale New Keynesian framework with e.g. consumption habit formation, capital accumulation, investment adjustment costs, Rotemberg (1982) pricing, price indexation and a Taylor rule which is subject to the zero-lower bound.[2]

In presenting the main features of the RANK and TANK models, it is important to note that the two models differ on the household side, but are identical in all other respects, e.g. firms and monetary policy. This ensures that any variations in the dynamics across the two models result from the differential treatment of households. Subsections 2.1 and 2.2 therefore deal exclusively with the households in RANK and TANK. The full models can be found in appendix A.

2.1 The Benchmark: RANK

The RANK model assumes a continuum of households that can be represented by a single agent who maximises the infinite sum of life-time (log) utility from consumption and (CRRA-type)

[2] Gust et al. (2012) in turn rely on the models in Christiano et al. (2005) and Smets and Wouters (2007).

disutility from labour subject to standard budget constraints. The household does so by choosing consumption c_t, labour hours n_t, deposits dd_t and capital investment i_t, given the real wage w_t, the real interest rate RR_t (which equals the gross nominal rate over the expected gross inflation rate via the Fisher equation), taxes τ_t, firm and bank profits and the structural parameters.[3]

Optimal household behaviour is consequently summarised by:

$$w_t = \chi n_t^{\eta}(c_t - hc_{t-1}) \tag{2.1}$$

$$1 = \beta_{t+1} RR_t \frac{(c_t - hc_{t-1})}{(c_{t+1} - hc_t)}. \tag{2.2}$$

Equation (2.1) is a labour supply schedule describing the decision on how much labour to supply for a given wage. η in that equation is the inverse Frisch elasticity of labour supply, reflecting *ceteris paribus* how much more labour the agent would be willing to supply for a 1% increase in the real wage.

Equation (2.2) is the household's consumption Euler equation representing the trade-off between present and future consumption, taking into account the real interest rate and the household's impatience captured by β_{t+1}. Equation (A.30), spelled out in appendix A.1 for the sake of brevity, is the household's budget constraint.

Note two important aspects on households in the RANK model. First, aggregate and individual paths of consumption and labour hours correspond, implying that aggregate consumption and employment dynamics stem one-to-one from the optimisation by the representative household. Second, RANK agents can smooth consumption over time by re-allocating their resources such that they might prevent a drop in consumption in face of a contractionary shock.

2.2 The Extension: TANK

In contrast to the RANK model, the TANK model allows for household heterogeneity, albeit in a very simplistic manner. It assumes a continuum of households with unit mass, from which a share $(1 - \lambda)$ is *unconstrained*, while a fraction λ of households is *hand-to-mouth*. λ is thereby time-invariant and exogenously determined.[4] I discuss each type of household in turn.

Unconstrained Households

A fraction $(1 - \lambda)$ of TANK's households face the same problem as the agents in RANK. They are given full access to capital investment, deposits, company ownership, etc., earning them the attribute unconstrained. Thus, these households behave analogously to those in RANK in a consumption-smoothing fashion and adhere to the conventional equations (2.1), (2.2), (A.30).

The only distinction required is in terms of notation. Let c_t^U and n_t^U denote the unconstrained household's consumption and labour hours, respectively, and thus c_t^U replaces c_t and n_t^U replaces n_t in the above equations. Appendix A.2 spells out the equations with this notation.

[3] See table 1 in appendix A for a full description of the variables' and parameters' symbols and meanings.
[4] In contrast, HANK models endogenise λ and make it time-varying, see e.g. Debortoli and Galí (2018).

Hand-to-Mouth Households

The remaining share λ of the population is very restricted in its consumption choices as, in each period, agents from this group are only allowed to consume their labour income of that period. This behaviour renders them *hand-to-mouth* or *constrained*.[5]

Formally, hand-to-mouth consumers face a budget constraint which denies them access to profits, deposits and capital investment and disregards them in tax and transfer payments by the government:

$$c_t^H = w_t n_t^H, \tag{2.3}$$

where c_t^H and n_t^H stand for hand-to-mouth consumption and labour hours, respectively, and where, in contrast to Debortoli and Galí (2018), employment may differ among agent types.

With respect to how the hand-to-mouth households choose their labour supply, the literature, e.g. Kaplan et al. (2018), usually assumes analogous behaviour to RANK-type of agents, resulting in:

$$w_t = \chi (n_t^H)^\eta (c_t^H - hc_{t-1}), \tag{2.4}$$

which is analogous to the labour supply equations (2.1) and (A.31) and where habits are formed with respect to past *aggregate* consumption, specified below.

Equation (2.3) makes clear that the constrained agents do not satisfy a conventional Euler equation. Thus, the hand-to-mouth agents' consumption does not directly depend on many parameters that do influence the unconstrained households' consumption, e.g. the discount factor, β_t, or the path of the (expected) real interest rate. The repercussions of shocks to the latter on the hand-to-mouth agents comes solely through indirect effects, in particular the labour market, as explained by Bilbiie (2008). This is in contrast to agents in RANK models, whose consumption is very sensitive to interest rate changes, as documented by Kaplan et al. (2018).

Condition (2.3) further illustrates that the hand-to-mouth agents cannot smooth their consumption in the face of shocks to their labour income, as noted by Galí et al. (2007). As I here allow for varying levels of employment, however, hand-to-mouth agents can try to supply more labour in order to (partly) shield their consumption against decreases in the wage.

The apparent sensitivity of the hand-to-mouth households to labour market developments renders this market central to their consumption. Consequently, the paths of wages and employment gain in relative importance for aggregate consumption in TANK, depending of course *inter alia* on the share of the hand-to-mouth, λ, and the households' labour supply reaction to changes in wages, η.

Aggregation

The TANK model is closed by aggregating consumption and employment to the economy-wide level, which is done as in Kaplan et al. (2018) by taking the weighted average of individual-level

[5] The literature argues along different dimensions as to why hand-to-mouth behaviour might arise. For example, Bilbiie (2008) alludes to limited asset-market participation, potentially especially relevant in developing countries, while Debortoli and Galí (2018) also refer to myopic behaviour and binding borrowing constraints.

consumption and labour, respectively, over the corresponding fractions in the population:

$$c_t = (1-\lambda)c_t^U + \lambda c_t^H \tag{2.5}$$

$$n_t = (1-\lambda)n_t^U + \lambda n_t^H, \tag{2.6}$$

where c_t is aggregate consumption and n_t is aggregate labour hours. These variables are the counterparts of consumption and employment in the RANK model.

3 The Shocks

This section discusses the shocks I confront the models with, a technology shock and a discount factor shock, and sketches some consequences for the models' dynamics one might expect.

3.1 Technology Shock

Technology, z_t, refers to the aggregate level of productivity, i.e. the economy's capacity to transform inputs to outputs. It directly enters in the aggregate production function, equation (A.8), and thus indirectly affects also other variables such as (real) marginal costs. In both models, z_t is assumed to evolve according to the following process:

$$z_t = z_{ss}\left(\frac{z_{t-1}}{z_{ss}}\right)^{\rho_z} \exp(\varepsilon_{z,t}), \tag{3.1}$$

where z_{ss} is the steady state level of technology, set to 1. $\varepsilon_{z,t}$ is the technology shock and $\rho_z = 0.8$ is its persistence.

From a standard New Keynesian perspective, it is natural to expect a positive technology shock, which can be interpreted as an expansionary supply shock, to raise production and output mechanically. This should in turn spur consumption and investment in both models. As output falls short of adjusting as much as it potentially could in the absence of price adjustment costs, inflation drops. To counteract this, the central bank is expected to cut the nominal interest rate.

Further, one would predict aggregate employment to fall in both RANK and TANK as the lower marginal utility of consumption decreases the incentive to work and the same amount of output can be sustained with less labour input.[6] In principle, $\varepsilon_{z,t} > 0$ should be accompanied by increasing wages, as workers who earn their marginal product of labour are now more productive.

Naturally, the dynamics for consumption and employment might be heterogeneous across hand-to-mouth and unconstrained agents in TANK. The latter will have consumption gains also from increased profits (as firms face higher demand and lower marginal costs) and thus further reduced incentives to work. The former, however, will be strongly affected by the changes in

[6] How technology shocks affect employment is a contentious issue in the business cycle literature. Conditional on such shocks, Real Business Cycle models predict an increase in labour hours, while New Keynesian models usually see a reduction. See e.g. Galí (1999) and Peersman and Straub (2009) for treatments.

wages and employment. Depending on the signs of these responses, these agents might potentially be urged to reduce their consumption in the short run. This could in turn give rise to differences in the aggregate responses of RANK and TANK, e.g. for consumption.

3.2 Discount Factor Shock

The second shock discussed pertains to the discount factor, β_t, which describes the weight that households attach to utility in the next period and is thus a measure of their impatience. A relatively higher value means that the households are more patient to consume.

β_t appears in the consumption Euler equations of the RANK agents, (2.2), and of the unconstrained TANK agents, (A.34), as well as in the investment Euler equation, (A.10), and the New Keynesian Phillips Curve, (A.6), describing the path of inflation. The present models assume the discount factor to follow:

$$\beta_t = \beta_{ss} \left(\frac{\beta_{t-1}}{\beta_{ss}} \right)^{\rho_\beta} \exp(\varepsilon_{\beta,t}), \tag{3.2}$$

where β_{ss} denotes the discount factor in steady state and is equal to 0.98. $\varepsilon_{\beta,t}$ is the discount factor shock with persistence $\rho_\beta = 0.8$.

Following a positive innovation $\varepsilon_{\beta,t}$, the agents in RANK and the unconstrained agents in TANK are willing to push some of their consumption into the more distant future. This leads to a reduction in present consumption and can thus be viewed as a contractionary demand shock.

In both models, this drop in demand for goods should intuitively prompt output and employment to fall, triggering reductions in wages and inflation. The monetary authority reacts to these developments by reducing the nominal interest rate to accommodate the shock. However, the Taylor rule is usually calibrated to be quite persistent (ρ is 0.8 below), so the real interest rate might indeed rise on impact of the shock, before falling below its steady state value. This might exacerbate the downturn in the short run.[7]

In the TANK model, one should expect differential effects for households as β_t is directly relevant for the unconstrained households but only indirectly affects the hand-to-mouth. The unconstrained agents will most likely react as the RANK agents do, while their increased patience will affect the hand-to-mouth agents through changes in the labour market. As the wage plummets, consumption of the constrained households will fall, most likely by more than for the unconstrained ones as they are not able to cushion the shock by smoothing consumption over time by dissaving. This will likely lead to overall different responses of the two models.

4 Results

Next, this section discusses the results obtained by implementing the two shocks into the RANK and TANK models. Note that both shocks are treated as unexpected, one-time deviations from

[7] Moreover, if the shock is large enough, the zero-lower bound might bind, thus hampering monetary easing.

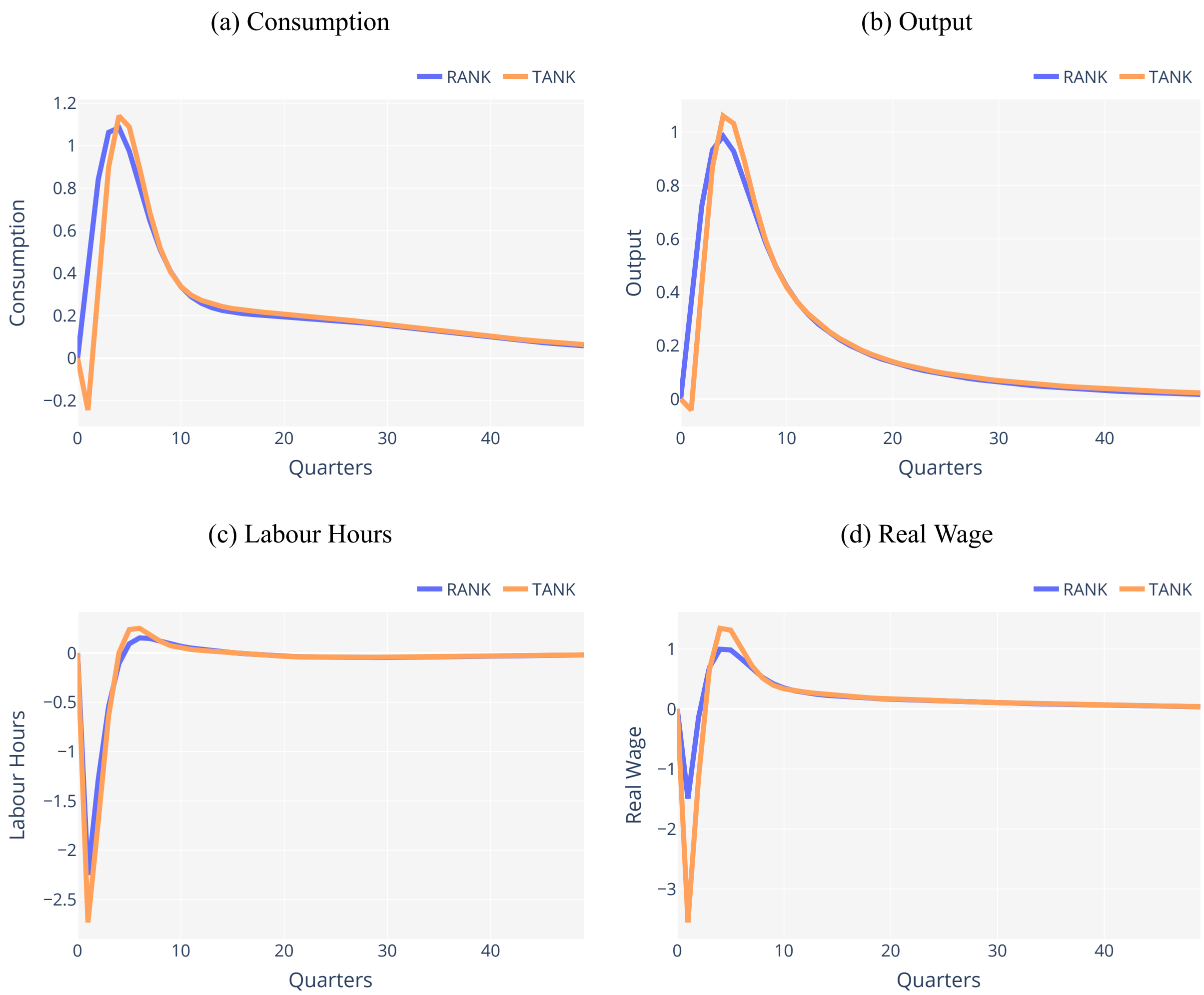
the respective variables' steady state, i.e. I set $\varepsilon_{z,t} = \varepsilon_{\beta,t} = 0.02$ in $t = 1$ and $\varepsilon_{z,t} = \varepsilon_{\beta,t} = 0$ for $t \neq 1$ in equations (3.1) and (3.2), respectively.[8]

Subsections 4.1 and 4.2 dwell separately on the results of each shock. Both subsections follow Debortoli and Galí (2018) and Kaplan et al. (2018) in using a baseline calibration of $\eta = 1$ and $\lambda = 0.3$. Section 4.3 aims at gauging the influence of η and λ on the results.

4.1 Responses to a Technology Shock

Figure 1 presents the impulse responses of some key aggregate variables (consumption, output, employment and wages) in RANK and TANK to the technology shock. Figure 6 in appendix C.1 displays further variables and figure 2 shows the individual-level impulse responses of TANK. All values are in percent deviations from the steady state of each respective variable.

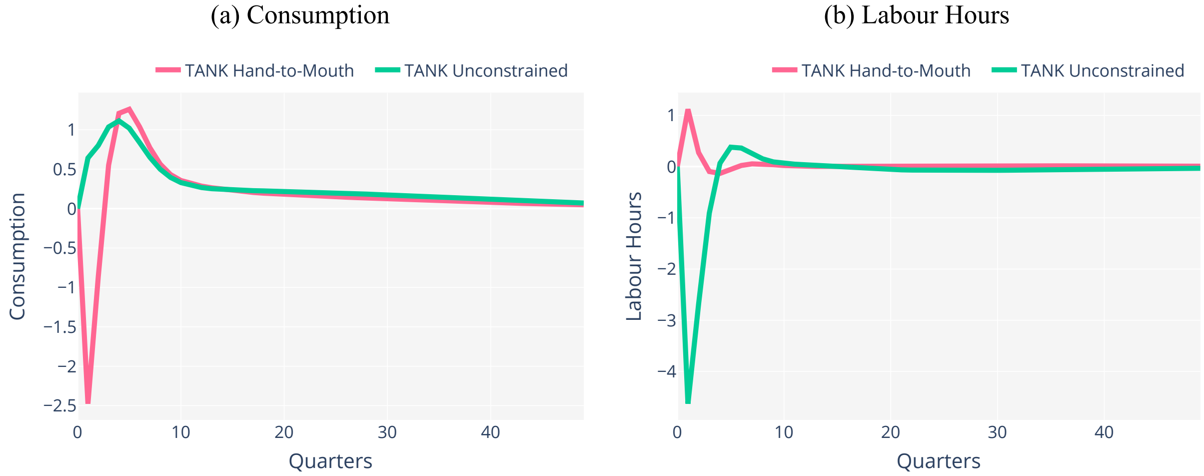
Figure 1: Aggregate Impulse Responses to a Technology Shock



Note: All values in percent deviations from the respective steady state. The time horizon is fixed to 50 quarters.

[8] The Python `Econpizza` package by Boehl (2022) is used to numerically solve for the models' steady states and the respective responses to each of these shocks. The codes for the implementation are available on GitHub (https://github.com/andkound98/macro-research-project). For further details, see appendix B.

Figure 2: Individual-Level Impulse Responses to a Technology Shock (TANK)



Note: All values in percent deviations from the respective steady state. The time horizon is fixed to 50 quarters.

Overall, the two models display similar patterns in their impulse responses. Strikingly however, the two first plots reveal some of the rather rare qualitative differences between the RANK and TANK models: on impact, aggregate consumption and output in RANK rise while they counterintuitively *fall* in TANK, albeit by not very much. After this divergence, TANK shows a strong expansion in consumption and output, which induces spikes in these two variables that lie above RANK's in $t = 3$. Thereafter, TANK's and RANK's paths are almost identical.

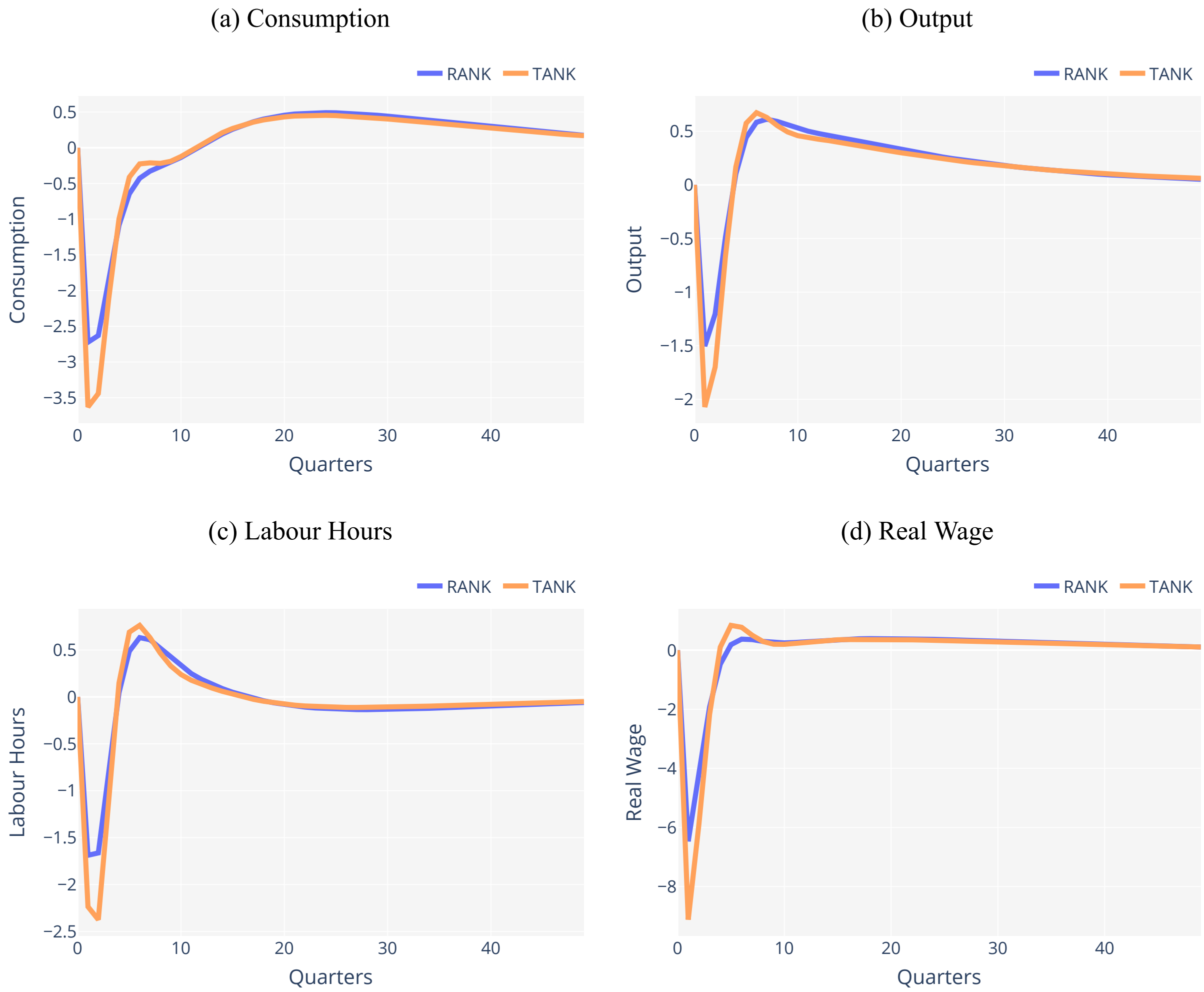
Evidently, the drop in consumption on impact by the baseline TANK model is driven by a sharp decline of about 2.5% in the hand-to-mouth's consumption (figure 2a). This contraction in turn is due to the decrease in wages (-3.5%), which the constrained households cannot cushion despite working more (graph 2b) as they do not have any savings. In contrast, the unconstrained agents' consumption increases as expected – *inter alia* because they alone gain from strongly increasing firm profits (figure 6a) – and is quantitatively close to the path in RANK.

This divergence of TANK relative to RANK illustrates that the former does have the potential to offer novel insights into the aggregate dynamics of macroeconomic models, at least conditional on technology shocks, and highlights the importance of the labour market channel for hand-to-mouth as well as for aggregate consumption.

Interestingly, the fall in wages is present in both models. It is more pronounced in the TANK model, which shows a two percentage point sharper decline compared to RANK. The reduction in wages, despite being temporary, is at odds with the intuition about a positive productivity shock and with related empirical evidence, see e.g. Peersman and Straub (2009). Potentially, this hints at the decreased labour demand by firms outweighing the drop in (aggregate) labour supply, leading to the observed drop in wages on impact.

The other figures reveal that RANK and TANK in general agree with the remaining variables' paths. Aggregate labour hours fall as is usual for New Keynesian-type of models conditional on technology shocks. In both models, inflation falls and the nominal interest rate is reduced. All these responses are somewhat more pronounced in the case of TANK.

Figure 3: Aggregate Impulse Responses to a Discount Factor Shock



Note: All values in percent deviations from the respective steady state. The time horizon is fixed to 50 quarters.

Finally, note that the impulse responses uncovered herein show some significantly more noticeable differences between RANK and TANK than those presented in Debortoli and Galí (2018) in the case of a technology shock, see figure B.3 in their appendix. Of course, this might have many reasons as their models are quite different (e.g. linear and without capital).

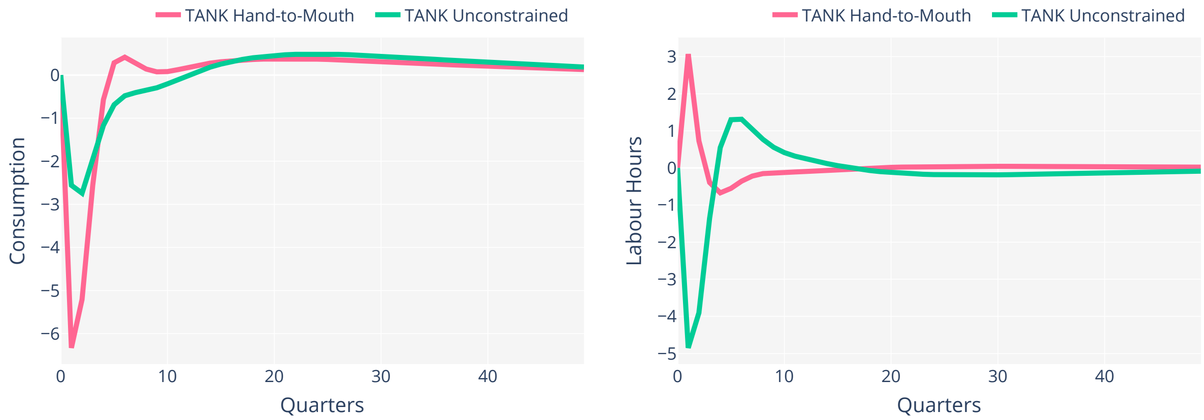
4.2 Responses to a Discount Factor Shock

Next, I turn to the results on the discount factor shock. Figure 3 plots the responses of the key aggregate variables. More impulse responses are found in figure 7 in appendix C.2. Figure 4 shows the underlying paths of consumption and employment on the household-type level.

Conditional on the discount factor shock, all responses match regarding their sign on impact and the quantitative paths of RANK and TANK are usually close to each other. Following the increased patience by the agents in RANK and by the unconstrained households in TANK, consumption and output decline in both models. Quantitatively, the responses on impact are more pronounced in the TANK model compared to RANK. In particular, consumption falls by

Figure 4: Individual-Level Impulse Responses to a Discount Factor Shock (TANK)

(a) Consumption (b) Labour Hours



Note: All values in percent deviations from the respective steady state. The time horizon is fixed to 50 quarters.

roughly one percentage point more. The difference in output amounts to half a percentage point.

Following this contraction in demand, inflation and labour hours as well as wages fall across the two models, as expected after a negative demand shock. All of these effects are more pronounced in TANK relative to RANK, following the deeper recession in the case of TANK.

Note that an increase in patience implies above-steady-state levels of consumption in the future, which have to be financed through savings in the present. This is reflected in an increase of investment across the models (figure 7a), whereby TANK exhibits a somewhat dampened effect compared to RANK. This could be because, after $t = 1$, the real interest rate drops by more in TANK, thus dampening the incentive to save and invest. This is due to the sharper intervention by the monetary authority, triggered by the larger drops in output and inflation.

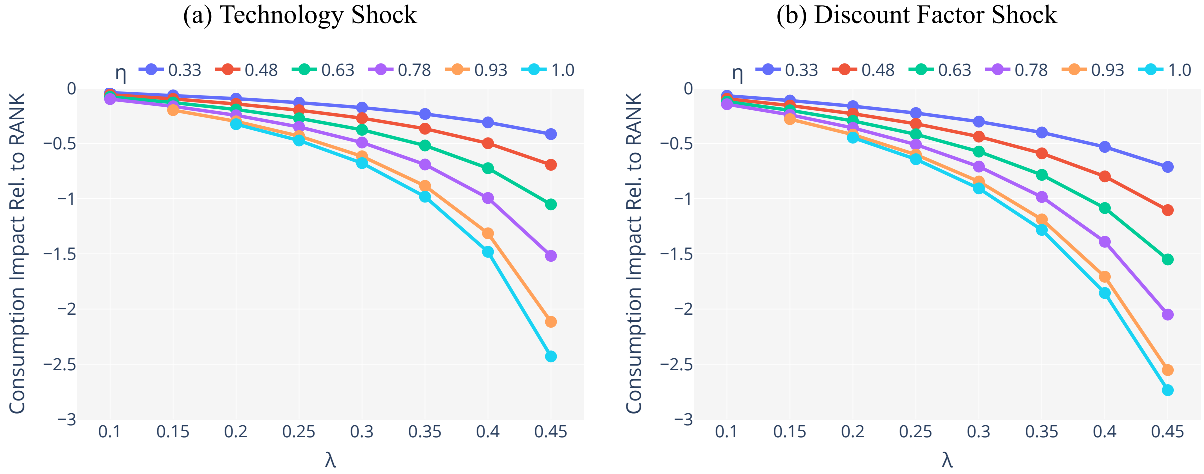
The disaggregated level is relevant as to why consumption in TANK falls by more than in RANK. It reveals that consumption of the hand-to-mouth households falls by about 6% on impact, more than double the response of the unconstrained agents' consumption, despite the fact that the former are not directly affected by the shock. However, the labour market, to which hand-to-mouth consumption is linked, reveals a strong decline in wages, which is significantly more pronounced in TANK. The hand-to-mouth try to absorb some of the shock to their consumption by working more (plot 4b) but without success. The unconstrained agents, on the other hand, reduce their labour on impact as they desire to consume less.

Hence, the TANK model again uncovers (almost by construction) interesting and plausible heterogeneous effects for the different household types, thereby underlining the importance of *indirect effects* in the language of Kaplan et al. (2018).

4.3 Sensitivity Analysis

This section aims at gauging how some of the above findings depend on the choices on λ and η as these parameters govern crucial characteristics of the models and might thus be important in

Figure 5: Aggregate Consumption Impact of TANK Relative to RANK as a Function of λ and η



Note: All values in percentage point deviations from the respective initial impact in RANK. Some parameter combinations not displayed as in these cases, `Econpizza` could not solve for the steady state in TANK.

shaping the relative dynamics between the RANK and TANK models.[9]

The literature naturally uses different values for λ and η. For example, Galí et al. (2007) set λ, the share of hand-to-mouth agents in TANK, to 0.5 in their baseline specification, while Debortoli and Galí (2018) and Gerke et al. (2020) use 0.3. A recent empirical study by Aguiar et al. (2020) finds in a sample of US households that roughly 23% of the households are hand-to-mouth.[10] Therefore, $\lambda = 0.3$ seemed to be a reasonable choice for the baseline calibration.

For η values between 0.33 and 1 are common. I followed Debortoli and Galí (2018) and Kaplan et al. (2018) by using $\eta = 1$ above. However, Smets and Wouters (2007) and Gust et al. (2012) estimate the elasticity of labour supply to be around 2, implying $\eta = 0.5$.

To get an idea of the effect that changing λ and η has, I here focus on aggregate consumption. To that end, I calculate, for each of the shocks considered in the main analysis, the difference of the consumption impact between TANK models with different values of λ, on the one hand, and the standard RANK model, on the other hand, *given* different values of η in both models. Figure 5 presents the results graphically and separately for each of the two shocks.[11]

The plot reveals that the specification of λ and η indeed has a significant impact. For any given value of η, the relative difference of the response to the shock on impact increases monotonically in absolute terms as λ gets larger. Consequently, introducing the hand-to-mouth agents and then sequentially increasing their weight in the total population renders the difference of the TANK and RANK outcomes more pronounced. Conversely, holding λ constant and increasing η also leads to larger divergence between the two models. Importantly, these statements are true for both the shocks considered.

Most strikingly, however, the relative differences are maximised (among the considered

[9] Of course, these relative differences might depend on further parameters.

[10] The number of 23% pertains to the low-net worth households. There is a separate category of the so-called "wealthy hand-to-mouth", who make up another 17% of the sample.

[11] As these are relative comparisons, one cannot infer the sign of the impact from either model from the figure.

parameter combinations) in the case of $\lambda = 0.45$ and $\eta = 1$, i.e. both parameters are chosen to be (relatively) large. In that scenario, TANK produces a quantitatively significantly different outcome relative to RANK with the same η, in the magnitude of around 2.5 percentage points after each of the shocks considered, albeit a bit higher in the case of a discount factor shock.

The intuition for this result is as follows: together, a high fraction of hand-to-mouth agents and a low Frisch elasticity of labour supply (and thus a high inverse elasticity), imply that labour market developments matter substantially for the aggregate – as there are more hand-to-mouth agents with no access to savings and only labour income – and that *simultaneously* wage changes do not translate as much into changes in hours worked, which could shield consumption from larger contractions in the face of adverse shocks. This leads to the observed strong difference of TANK relative to RANK.

5 Conclusion

This project paper analysed the introduction of hand-to-mouth agents into a non-linear medium scale New Keynesian framework. In this so-called TANK model, the hand-to-mouth agents are modeled in a very simplistic manner as they are merely allowed to consume, in each period, their current-period labour income. Having discussed the changes required to incorporate this second set of agents into an otherwise unchanged New Keynesian model, the paper investigated how the dynamics of this model behave after two different shocks. The results were compared to the responses of the corresponding RANK model.

As documented above, the responses by RANK and TANK rarely differ in their sign. Two of the exceptions are consumption and output following a technology shock. Quantitatively, RANK and TANK are shown to usually be similar. Quite strong quantitative divergence occurs, for example, in the case of the real wage on impact of a discount factor shock.

Importantly, the models' relative responses crucially depend on the parameters governing the inverse Frisch elasticity and the share of the hand-to-mouth agents in the economy. The paper shows that a combination of the former being close to 1 and the latter being above roughly 0.35 makes the quantitative differences between RANK and TANK stark. This is because, in case of such a parametrisation, changes in labour market conditions have much more of an impact in TANK compared to RANK.

Clearly, there are multiple dimensions along which the above analysis can be extended. First, it might be interesting to further augment the present setting, e.g. with an imperfectly competitive labour market as done *inter alia* in Galí et al. (2007). Also, along the lines of Debortoli and Galí (2018), it would only be natural to include a HANK model alongside RANK and TANK to investigate how fully-fledged household heterogeneity would contrast to the results herein.

Second, empirical evidence is a feature entirely absent from the present paper. Estimating the parameters, especially λ and η, would certainly be purposeful and shed light on the empirically plausible magnitude of the relative dynamics uncovered in this paper.

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Appendices

A The Full Models

This appendix presents the full RANK and TANK models that are used in the main text for analysis. I state all variables, parameters and equations of the models. As RANK and TANK are identical regarding everything but the household sector, I begin by presenting those equations that are not affected by the introduction of the hand-to-mouth households. Finally, I present the household sectors of RANK and TANK separately in subsections A.1 and A.2, respectively.

Note that the structure and equations track closely the model implemented by Boehl (2022) in the `Econpizza` package, see also appendix B. Further, as stated in the main text, the models' theoretical foundations draw heavily on Gust et al. (2012).

Variables and Parameters

First, the variables and parameters that are common to RANK and TANK are presented. Afterwards, I briefly explain the consumption and labour variables.

Table 1: List of Variables and Parameters.

Variables			
Symbol	Meaning	Steady State Value	Steady State Initial Guess
y_t	(real) output		
π_t	(gross) inflation	$1.02^{0.25}$	
$\tilde{\pi}_t$	(gross) indexed inflation	π_{ss}	
R_t	(gross) nominal interest rate	$\frac{\pi_{ss}}{\beta_{ss}}$	
$R_{n,t}$	(gross) notional nominal interest rate	R_{ss}	
$R_{k,t}$	(gross) rental rate of capital	R_{ss}	
RR_t	(gross) real interest rate		
β_t	household discount factor	0.98	
$hhdf_t$	household (stochastic) discount factor	β_{ss}	
w_t	(real) wage		
τ_t	taxes/transfers by the government		
q_t	value of capital in consumption units	1	
mc_t	(real) marginal costs	$\frac{\theta-1}{\theta}$	
k_t	capital		
i_t	investment		
g_t	government consumption		
b_t	quantity of government bonds		
qb_t	price of government bonds		
$prof_t$	firm profits		

Table 1 Continued: List of Variables and Parameters.

$bprof_t$	bank profits		
dd_t	demand for bank deposits		
ds_t	supply of bank deposits		
MPK_t	marginal productivity of capital		$\frac{\alpha}{20} mc_{ss}$
cap_util_t	fraction of capital in utilisation	1	
$cap\_util\_costs_t$	capital utilisation costs		
y_prod_t	production		
z_t	technology	1	

Parameters

Symbol	Meaning	Baseline Value	Initial Guess
θ	elasticity of demand	6	
κ	decay parameter for bond coupon	0.975	
ψ	price adjustment costs	59.11	
δ	depreciation rate	0.025	
α	capital income share	0.33	
Φ	investment adjustment costs	5.6	
ϕ_π	Taylor rule coefficient on inflation	1.5	
ϕ_y	Taylor rule coefficient on output	0.1	
h	parameter on consumption habit formation	0.44	
ρ	persistence in Taylor rule	0.8	
η	inverse of the Frisch elasticity of labour supply	1	
ρ_β	persistence of discount factor shock	0.8	
ρ_z	persistence of technology shock	0.8	
χ	parameter on labour disutility		10
$par\_cap\_util\_0$	parameters for capital utilisation and	–	
$par\_cap\_util\_1$	capital utilisation costs	0.8	
ω	weight of steady state inflation in inflation indexation	0.44[12]	
λ	fraction of hand-to-mouth agents (for the TANK model)	0.3	

In the context of households, c_t and n_t are present in both RANK and TANK. In the latter case, those variables correspond to the aggregate levels of consumption and labour hours, as indicated in equations (2.5) and (2.6). For TANK, there are the additional, individual-level variables, namely: c_t^U, n_t^U, c_t^H and n_t^H, as explained in subsection 2.2 of the main text.

[12] This value is motivated by the estimated mean of this parameter in Gust et al. (2012).

For the steady state search in RANK to be successfully completed by `Econpizza`, labour hours are fixed to: $n_{ss} = 0.33$. For TANK, I need to fix steady state labour hours by the unconstrained households, i.e. $(n_t^U)_{ss} = 0.33$.

Firms

The firm side of the economy is described by seven equations. The first one pertains to the evolution of the costs to capital utilisation, depending on the level of capital utilisation. The second equation, (A.2), links capital utilisation to the marginal product of capital, which is given by equation (A.3).

The fourth equation links the rental rate on capital, the value of capital, the marginal product of capital, capital utilisation and the costs of capital utilisation. The fifth equation, equation (A.5), defines real marginal costs.

The sixth equation, i.e. equation (A.6), is the New Keynesian Phillips Curve, in a standard form derived from Rotemberg (1982) pricing. It describes the path of inflation and includes, in the present version of the model, price indexation. The latter is determined according to (A.7), where the steady state rate of inflation and actual inflation are weighted by the parameter ω.[13]

Equation (A.8) describes the economy's aggregate production function in dependence of technology, currently used capital and labour hours. The final equation is on firm profits and takes into account price adjustment costs.

$$cap\_util\_costs_t = par\_cap\_util\_0(cap\_util_t - 1) + \frac{\frac{par\_cap\_util\_1}{(1-par\_cap\_util\_1)}}{2}(cap\_util_t - 1)^2 \tag{A.1}$$

$$MPK_t = par\_cap\_util\_0 + \frac{par\_cap\_util\_1}{(1 - par\_cap\_util\_1)}(cap\_util_t - 1) \tag{A.2}$$

$$MPK_t = \alpha mc_t \frac{y_t}{(cap\_util_t k_{t-1})} \tag{A.3}$$

$$q_t \frac{R_{k,t}}{\pi_{t+1}} = MPK_{t+1} cap\_util_{t+1} + (1-\delta) q_{t+1} - cap\_util\_costs_t \tag{A.4}$$

$$w_t = (1-\alpha) mc_t \frac{y_t}{n_t} \tag{A.5}$$

$$\psi \left(\frac{\pi_t}{\tilde{\pi}_{t-1}} - 1 \right) \frac{\pi_t}{\tilde{\pi}_{t-1}} = (1-\theta) + \theta mc_t + \psi hhdf_t \left(\frac{\pi_{t+1}}{\tilde{\pi}_t} - 1 \right) \frac{\pi_{t+1}}{\tilde{\pi}_t} \frac{y_{t+1}}{y_{ss}} \tag{A.6}$$

$$\tilde{\pi}_t = \pi_{ss}^{\omega} \pi_t^{1-\omega} \tag{A.7}$$

$$y\_prod_t = z_t (k_{t-1} cap\_util_t)^{\alpha} n_t^{(1-\alpha)} \tag{A.8}$$

$$prof_t = \left(1 - mc_t - \frac{\psi}{2} \left(\frac{\pi_t}{\tilde{\pi}_{t-1}} - 1 \right)^2 \right) y_t \tag{A.9}$$

[13] The price indexation equation is taken from Gust et al. (2012).

Capital

Next, consider two equations for capital. The first one, equation (A.10), is the capital or investment Euler equation. The second one stipulates the accumulation of the capital stock. Both equations reflect that investment is subject to investment adjustment costs.

$$1 = q_t \left(1 - \frac{\Phi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 - \Phi\left(\frac{i_t}{i_{t-1}} - 1\right)\frac{i_t}{i_{t-1}}\right) + hhdf_t q_{t+1}\Phi\left(\frac{i_{t+1}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \tag{A.10}$$

$$k_t = (1-\delta)k_{t-1} + \left(1 - \frac{\Phi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right)i_t \tag{A.11}$$

Banking Sector

The next block is on the banking sector, which is modeled in a very simplistic fashion here (e.g. without bank equity). The first equation is the bank's balance sheet, equating the deposits it supplies with its bond and capital holdings. The second equation relates the price of bonds to the economy's gross nominal interest rate and thus defines a non-arbitrage condition. There is also a non-arbitrage condition for the rental rate of capital, see equation (A.14). The fourth equation, (A.15), describes how bank profits come about, namely through the inflation-adjusted spread of what the bank earns on its bond as well as capital holdings over what it pays on deposits (i.e. its financing).

$$ds_t = qb_t b_t + q_t k_t \tag{A.12}$$

$$R_t = \frac{(1+\kappa qb_{t+1})}{qb_t} \tag{A.13}$$

$$R_t = R_{k,t} \tag{A.14}$$

$$bprof_t = \frac{(1+\kappa qb_t)b_{t-1} + R_{k,t-1}q_{t-1}k_{t-1} - R_{t-1}dd_{t-1}}{\pi_t} \tag{A.15}$$

Government

The government is modeled with three equations. The first equation, equation (A.16), defines the intertemporal government budget constraint. It equates government expenditures (on government consumption g_t and obligations on past bonds b_{t-1}) with government income (from bond issuance b_t at market price qb_t and from taxes τ_t). The second and third equations fix the share of government consumption in the economy and the quantity of government bonds in circulation, respectively.

$$g_t + \frac{(1+\kappa qb_t)}{\pi_t}b_{t-1} = qb_t b_t + \tau_t \tag{A.16}$$

$$g_t = 0.2y_{ss} \tag{A.17}$$

$$b_t = \frac{y_{ss}}{qb_{ss}} \tag{A.18}$$

Clearing Conditions

Next, some clearing conditions. Equation (A.19) is the goods market clearing condition. The second equation defines the standard of notion of output, i.e. household consumption plus investment plus government consumption.

$$c_t + i_t + g_t + \frac{\Phi}{2}\left(\frac{i_t}{i_{t-1}}\right)^2 i_t + cap\_util\_costs_t k_{t-1} = \left(1 - \frac{\psi}{2}\left(\frac{\pi_t}{\tilde{\pi}_{t-1}} - 1\right)^2\right) y\_prod_t \tag{A.19}$$

$$c_t + i_t + g_t = y_t \tag{A.20}$$

Monetary Policy

The monetary authority is characterised by two equations. Its desired (or notional) gross nominal interest rate is the first equation, (A.21). This relationship is a standard Taylor rule which features persistence and which responds to the deviations of inflation and output from their respective steady state values. Theses responses are given by the parameters ϕ_π and ϕ_y. To model the zero-lower bound that the central bank is subject to in the present models, a second equation is required. This equation, (A.22), chooses the maximum out of the notional gross nominal interest rate and 1.

$$R_{n,t} = (R_{n,t-1})^\rho \left(R_{ss}\left(\frac{\pi_t}{\pi_{ss}}\right)^{\phi_\pi}\left(\frac{y_t}{y_{ss}}\right)^{\phi_y}\right)^{(1-\rho)} \tag{A.21}$$

$$R_t = \max\{1,\ R_{n,t}\} \tag{A.22}$$

Further Equations

An auxiliary equation, useful for finding the steady state, is equation (A.23) which sets the parameter $par\_cap\_util\_0$ equal to the marginal product of capital in steady state.

$$par\_cap\_util\_0 = MPK_{ss} \tag{A.23}$$

Exogenous Processes & Shocks

Finally, in the present models, there are two exogenous processes. The first one concerns the technology shock and the second on the discount factor shock. For a discussion of these shocks, see section 3 in the main text.

$$z_t = z_{ss}\left(\frac{z_{t-1}}{z_{ss}}\right)^{\rho_z} \exp(\varepsilon_{z,t}) \tag{A.24}$$

$$\beta_t = \beta_{ss}\left(\frac{\beta_{t-1}}{\beta_{ss}}\right)^{\rho_\beta} \exp(\varepsilon_{\beta,t}) \tag{A.25}$$

A.1 Households in RANK

The RANK household block includes five equations describing the behaviour of the representative household. These five equations can be collapsed to three in a straight forward way.

Equation (A.28) is the Fisher equation, linking the real interest rate to the nominal interest rate and the (expected) rate of inflation.

$$w_t = \chi n_t^{\eta}(c_t - hc_{t-1}) \tag{A.26}$$

$$1 = RR_t hhdf_t \tag{A.27}$$

$$RR_t = \frac{R_t}{\pi_{t+1}} \tag{A.28}$$

$$hhdf_t = \beta_{t+1}\frac{(c_t - hc_{t-1})}{(c_{t+1} - hc_t)} \tag{A.29}$$

$$\begin{aligned} dd_t + c_t + \tau_t + \frac{\Phi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 i_t &= w_t n_t + \frac{R_{t-1}}{\pi_t} dd_{t-1} + prof_t \\ &\quad + \left(q_t\left(1 - \frac{\Phi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 - 1\right) i_t + bprof_t\right. \end{aligned} \tag{A.30}$$

A.2 Households in TANK

In TANK, the household sector consists of two household types. The equations for the *unconstrained* households are analogous to the ones from the RANK agents.

$$w_t = \chi (n_t^U)^{\eta}(c_t^U - hc_{t-1}) \tag{A.31}$$

$$1 = RR_t hhdf_t \tag{A.32}$$

$$RR_t = \frac{R_t}{\pi_{t+1}} \tag{A.33}$$

$$hhdf_t = \beta_{t+1}\frac{(c_t^U - hc_{t-1})}{(c_{t+1}^U - hc_t)} \tag{A.34}$$

$$\begin{aligned} dd_t + c_t^U + \tau_t + \frac{\Phi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 i_t &= w_t n_t^U + \frac{R_{t-1}}{\pi_t} dd_{t-1} + prof_t \\ &\quad + \left(q_t\left(1 - \frac{\Phi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 - 1\right) i_t + bprof_t\right. \end{aligned} \tag{A.35}$$

The *hand-to-mouth* agents are characterised by their budget constraint and their labour supply.

$$c_t^H = w_t n_t^H \tag{A.36}$$

$$w_t = \chi (n_t^H)^{\eta}(c_t^H - hc_{t-1}) \tag{A.37}$$

Finally, there are two equations for the economy-wide aggregation of consumption and labour hours in TANK.

$$c_t = (1-\lambda)c_t^U + \lambda c_t^H \tag{A.38}$$

$$n_t = (1-\lambda)n_t^U + \lambda n_t^H \tag{A.39}$$

See subsection 2.2 for the details on the TANK household block.

B Notes on the Implementation

This appendix briefly dwells on the computational implementation of the RANK and TANK models as well as the two shocks considered in the main text. The Python codes for this implementation are provided together with the paper and are also available on a GitHub repository (https://github.com/andkound98/macro-research-project).

The RANK model described in subsection 2.1 corresponds almost exactly to the model found in the `medium_scale_nk.yaml` file from the Python package `Econpizza` by Boehl (2022).[14] The only difference is the inclusion of price indexation into the model, as clarified by the equations in appendix A. The TANK model is the RANK model with the household block modified as described in subsection 2.2.

Of course, the models need to be calibrated. This is done as indicated in column 3 of table 1. Also, for finding the model's steady state, I fix some (steady state) values as indicated in column 4 of table 1 as well as in the text below the table.

I now shortly describe the Python code files that were employed to arrive at the results in section 4.[15]

`run_models.py`

The code file `run_models.py` implements the analyses of subsections 4.1 and 4.2 and produces the plots found therein.

The code first loads the files in which the RANK and TANK models are stored, then solves for the steady state of the respective model by the means of root-finding. Thereafter, one of the shocks is initiated, which is done by setting one of the two disturbances equal to 0.02 (other values are also possible) in $t = 1$.

As a next step, the resulting (non-linear) equilibrium dynamics, i.e. the impulse responses to each of the shock are solved for, thereby guaranteeing that all variables return to their respective steady state within a prescribed period of time. The results for some key variables, on the aggregate as well as on the individual level, are then plotted. If desired, one can plot all variables' impulse responses as well.

`run_loop_eta_lambda.py`

This file conducts the analysis of subsection 4.3. In particular, the code runs a double loop, which iterates over a sequence of values for η, thereby solving in each step the RANK model with that given value of η. In each iteration, the code also loops over a sequence of values for λ in TANK and computes the corresponding TANK models. With this approach, each of the TANK models with a different value for λ can be compared to a respective RANK model with the same choice for η.

Finally, the code produces the plots for figure 5.

[14] The specific file can be found here: https://github.com/gboehl/econpizza/blob/master/econpizza/examples/med_scale_nk.yaml, last retrieved on 15th January 2023.

[15] All codes were run using the Spyder IDE 5.3.3 with Python 3.9.12 and `Econpizza` 0.4.2 on macOS 12.6.1.

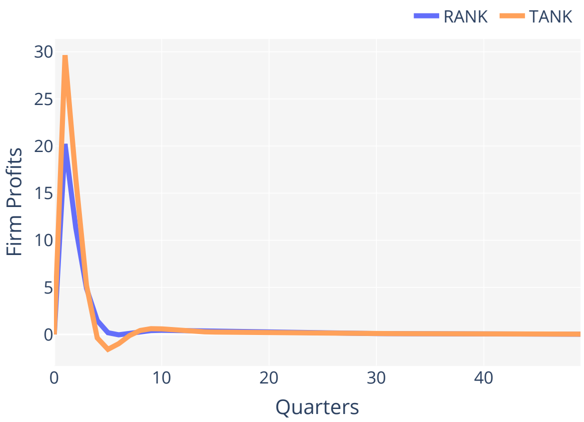
C Supplementary Impulse Responses

This appendix presents the impulse response functions of some further variables from the RANK and TANK models to each of the two shocks considered.

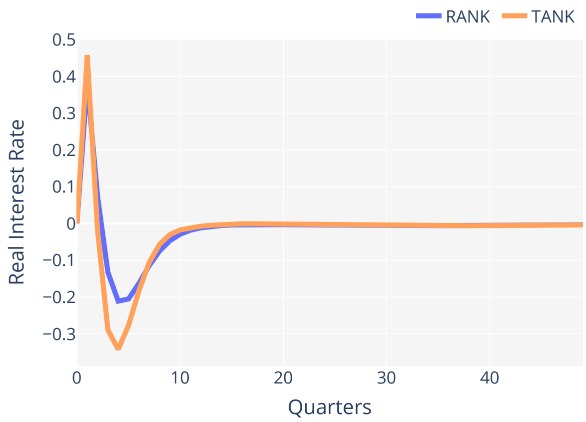
C.1 Technology Shock

Figure 6: Supplementary Aggregate Impulse Responses to a Technology Shock

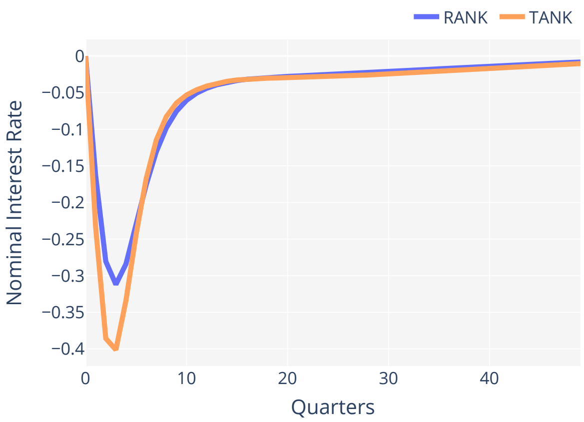
(a) Firm Profits



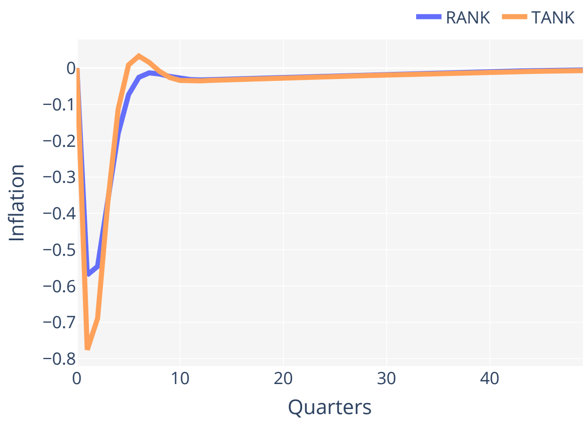
(b) Real Interest Rate



(c) Nominal Interest Rate



(d) Inflation

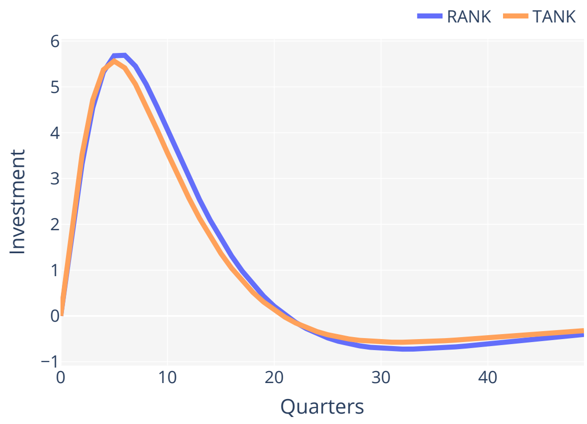


Note: Values for firm profits in percent deviations from steady state. Values for the interest rates and for inflation in percentage point deviations from the respective steady states. The time horizon is fixed to 50 quarters.

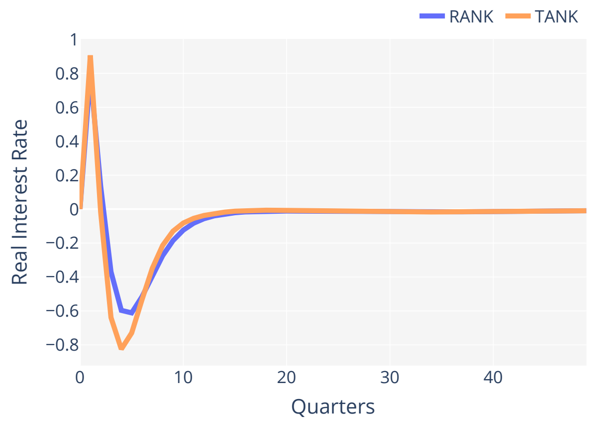
C.2 Discount Factor Shock

Figure 7: Supplementary Aggregate Impulse Responses to a Discount Factor Shock

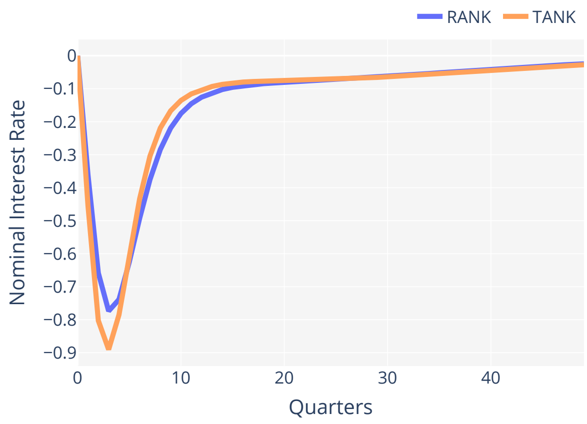
(a) Investment



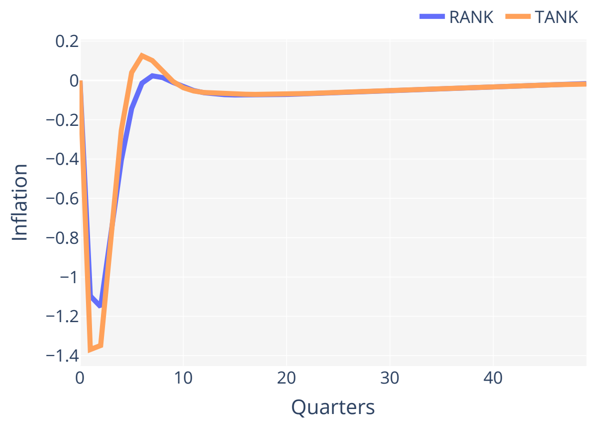
(b) Real Interest Rate



(c) Nominal Interest Rate



(d) Inflation



Note: Values for firm profits in percent deviations from steady state. Values for the interest rates and for inflation in percentage point deviations from the respective steady states. The time horizon is fixed to 50 quarters.

Statement of Authorship

I hereby confirm that the work presented has been performed and interpreted solely by myself except for where I explicitly identified the contrary. I assure that this work has not been presented in any other form for the fulfillment of any other degree or qualification. Ideas taken from other works in letter and in spirit are identified in every single case.

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