CS166 Traffic Intersections

Li-Lian Ang

Minerva Schools at KGI

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Feedback and Grading

My model uses far too many for loops, are there any higher-level architectural changes I could have done to solve this? Namely, any different functions or storage methods I could have used to improve the time and memory complexity of my simulation.

I would like broader feedback on the way I designed my model. I was stuck many times on implementing different features and ended up leaving out many of the features I wanted to include because of time. Was there a better way I could have gone about creating the simulation?

I’ve been pretty poorly on the theoretical analysis front on previous assignments, so I’d like some specific pointers here on how I could continue to improve. Feedback on a broader methodology perspective is much appreciated!

In and Out of Shah Alam

Figure 1. Google Maps laid over with the relevant roads. The red dots represent traffic lights, the blue lines represent one-way traffic, and the black lines represent two-way traffic.

Shah Alam is connected to the capital, Kuala Lumpur, and other major urban areas through two major highways: the New Klang Valley Expressway (NKVE) or the Federal Highway. The former extracts a toll between MYR 1 and MYR 2 depending on where the driver exits, while the latter is free. This, combined with its more strategic placement to reduce the absolute distance travelled, makes it an appealing choice to drivers, leading to a high volume of traffic.

The route shown in Figure 1 connects drivers from the Shah Alam to the Federal Highway. There are six traffic lights between the leftmost and rightmost side of the figure, which contributes to heavy traffic buildups, especially during peak hours. This stretch of road is notorious for long waiting times between intersections, adding ten to twenty minutes of commute for a two-kilometre stretch of road. The traffic jams can stretch for kilometres and are often only relieved when traffic police arrive to help manage traffic flow.

The current strategy has been to allow one section to have an extended amount of continuous movement, which increases the consecutive waiting times for other sections. This strategy seeks to be a globally optimal solution for traffic flow.

The challenge is how to balance between minimising congestion and waiting time. Firstly, the following report will analyse the current traffic light strategy that maximises the continuous flow of traffic through a theoretical framework. Then, I will model the strategy using a simulation to determine the optimal time for the red lights, minimising the congestion and waiting time. Finally, I propose a different strategy that uses known traffic inflow to set the traffic light timing, synchronising the red and green lights across all traffic lights. I will conclude with future work and broad recommendations on traffic light solutions for Majlis Perbandaran Shah Alam (MBSA or the municipal government of Shah Alam).

Theoretical Analysis

We can theoretically determine the conditions that maximise traffic flow, minimise driver waiting time and minimise car queue length for any single lane road. We will analyse the first using mean-field approximation and the last two with queuing theory for an M/D/1 queue.

Mean-Field Approximation: Traffic Flow

We will use a mean-field approximation for traffic flow to understand how cars move in a simplified space. A mean-field approximation is used to observe macroscopic changes to a system, in this case, the traffic flow, by homogenising the relationships between cars and assuming every car moves as the average car would. We can make this assumption because each car moves by the same rules, only dependent on the distance between itself and the car in front of it. Therefore, we can think of the probability of a car being at zero velocity as equivalent to the probability of a standstill.

Instead of calculating the probability at every time step, we will assume each car has the same velocity distribution. Since the car's velocity only depends on the velocity of the car directly in front of it, we can use each car's average state to estimate how likely standstill traffic will occur as the number of cars on the road increases. Since traffic lights can be seen as cars with zero velocity, this analysis will also be helpful to understand the effects of having multiple traffic lights in series.

We will take σ as car density (the probability of a car being in a cell) and use a single lane road where the car is randomly initialised to a speed of $1 \leq v \leq v_{max}$. For ease of calculation, we will set $v_{max} = 3$, but this maximum speed can be easily scaled up or down, as we will see below.

We will use the rules from Nagel and Schreckenberg (1992), since they are both simple and a good approximation of actual driving behaviour:

- Accelerate if the car's velocity v is lower than the speed limit v_{max} and more than one larger than the distance to the car in front ($v \to v + 1$)
- Decelerate if the velocity is larger than or equal to the distance to the next car j ($v \to j - 1$)
- Randomly decelerate with a set probability p_r ($v \to v - 1$)

Current State	Next State	Probability of transition
0	0	$p_{0\to0} = \sigma + (1 - \sigma)p_r$
	1	$p_{0\to1} = (1 - \sigma)(1 - p_r)$
1	0	$p_{1\to0} = \sigma + (1 - \sigma)p_r$
	1	$p_{1\to1} = (1 - \sigma)\,[(1 - p_r)\sigma + (1 - \sigma)p_r]$
	2	$p_{1\to2} = (1 - \sigma)^2\,(1 - p_r)$
2	0	$p_{2\to0} = \sigma + (1 - \sigma)\ p_r$
	1	$p_{2\to1} = (1 - \sigma)[(1 - p_r)\sigma + (1 - \sigma)p_r]$
	2	$p_{2\to2} = (1 - \sigma)^2\,[(1 - \sigma)p_r + (1 - p_r)\sigma]$

	3	$p_{2\to3} = (1 - \sigma)^3(1 - p_r)$
3	0	$p_{3\to0} = \sigma + (1 - \sigma)\ p_r$
	1	$p_{3\to1} = (1 - \sigma)[(1 - p_r)\sigma + (1 - \sigma)p_r]$
	2	$p_{3\to2} = (1 - \sigma)[(1 - \sigma)(1 - p_r)\sigma + (1 - \sigma)^2$
	3	$p_{3\to3} = (1 - \sigma)^3(1 - p_r)$

Table 1. Probability transition for each car from one velocity to another.

As the next velocity of the car is dependent on the state of the car in front of it, we can see in Table 1 that the probability of transition takes on very similar forms no matter the initial velocity. Every car has the same probability of coming to a complete stop since the condition is that the next cell must be occupied (σ) or for a random deceleration to occur after acceleration, $(1 - \sigma)p_r$. Using these probabilities, we can determine the density that will result in standstill traffic (when $v = 0$ for all cars). We can set p_r to any arbitrary value between, but not including 0 and 1, to remove it as an unknown.

$$p_{t+1} = 4(\sigma + (1 - \sigma)p_r) = 4[(1 - p_r)\sigma + p_r]$$

Notice that the number four in the equation above is a scaling factor that can be replaced with any v_{max}. Below is a more general formula.

$$p_{t+1} = (v_{max} + 1)(\sigma + (1 - \sigma)p_r) = (v_{max} + 1)[(1 - p_r)\sigma + p_r]$$

There is a linear relationship between the density of cars and the probability of a standstill. Having many traffic lights on a single road, much like the scenario outlined, increases the car density by forcing other cars to stop. The probability of a standstill increases with the random braking as well, which makes intuitive sense given that more cars mean there is less buffer between cars. When one car brakes randomly, it will cause a knock-on effect to other cars behind it to brake as well. The faster that cars are allowed to travel, the higher the probability of a standstill. We can see this in the constant proportional relationship between p_{t+1} and v_{max}.

Encouraging a steady flow of traffic follows the idea of having low car density since traffic lights can be effectively treated as zero velocity cars. However, two problems arise from this strategy. Firstly, as cars travel more smoothly, they also travel faster, increasing the probability of standstill traffic. Secondly, having a steady traffic flow means increasing the waiting time for cars by allowing traffic to pile up. The road users will not be happy to wait for a long while for the light to change, especially if they arrive just when the light turns red. Therefore, the traffic light timings must balance reducing the pileup of traffic and minimising the waiting time for drivers at each intersection.

The mean-field approximation is also a massive reduction of the problem because it assumes every car as moving in tandem, which restricts considering the implications of adding traffic lights into the calculations.

Queuing Theory: Driver Standstill Time and Maximum Queue Length

We can liken a one-lane road to a single queue where each car arrives following a constant rate and is served by only one server, in this case the traffic light. Since the traffic lights change their signal according to a fixed time, we can think of the service time as how long it takes each car to pass the traffic light. The number of cars in a standstill will not be able to extend to infinity, as is assumed for queue length in the M/D/1 model, but it can grow to quite a large number.

We will apply this theory to traffic at the first intersection from the left. The length of the green and red light is four minutes. There are two lanes of traffic that feed into that road, with cars coming from both lanes at every five seconds since it is an extremely busy road. It takes one car about ten seconds to cross the intersection. With this information, we can estimate that the arrival rate of cars is two per second, $\lambda = 0.4$. The service time assumes a constant flow of cars so we will find the average time taken for a car to pass through the intersection for one cycle of green and red light. If one car takes ten seconds to cross, that means for one cycle of green, 24 cars can cross. The service time is given by:

$$\frac{1}{\mu} = \frac{t_{red} + t_{green}}{t_{green}/10} = \frac{480}{24} = 20$$

Therefore, the service rate will be $\mu = 0.05$.

With this, we can calculate the average waiting time and queue length at equilibrium with the following formulas from M/D/1 queues. $\rho = \frac{\lambda}{\mu}$ is known as the utilisation which is essentially the ratio of cars entering the system over those leaving. To model the queues, the utilisation has to be less than 1 to be able to observe an equilibrium state, otherwise the queue will go on forever.

$$t_{wait} = \frac{\rho}{2\mu(1-\rho)}$$

$$l = \frac{\rho^2}{2(1-\rho)}$$

According to our model parameters, $\rho = \frac{0.4}{0.05} = 8$ which means that eight times more cars are entering the road than there are leaving. This model's limitation is that it cannot account for instances of heavy traffic flow, which cause traffic jams to begin with. Despite this, we can still take away from the equation for service time, which is the only manipulable variable here, that the number of cars leaving the queue must be increased. This can be done either by increasing the green light length over the red light length or decreasing the time cars take to cross the intersection.

The theoretical analyses we have conducted so far massively reduce the complexity and dynamics of the problem to perform calculations. It assumes all cars enter the road from the same place and exit from the same place and fails to account for the traffic light system properly. We can overcome these shortcomings through a traffic simulation that models the scenario as closely as possible. It will allow us to test our traffic light strategy without the ethical implications, confounding variables and cost of immediately implementing the strategy.

Building a Traffic Simulation

The goal is to use simulation to determine the length of a red light that maximises traffic flow and minimises drivers' waiting time at each intersection. The former is calculated by the percentage of cars at $v > 0$ on the main street and the latter is an accumulation of time steps a driver is at $v = 0$ until they exit the system. To understand how traffic flow on the main street affects the side street, we will also be taking the maximum queue of cars coming into the main street.

To zoom in on the intersections and road layout, I have simplified the road system in Figure 2 so that the roads are straightened out and assumed them to be completely flat.

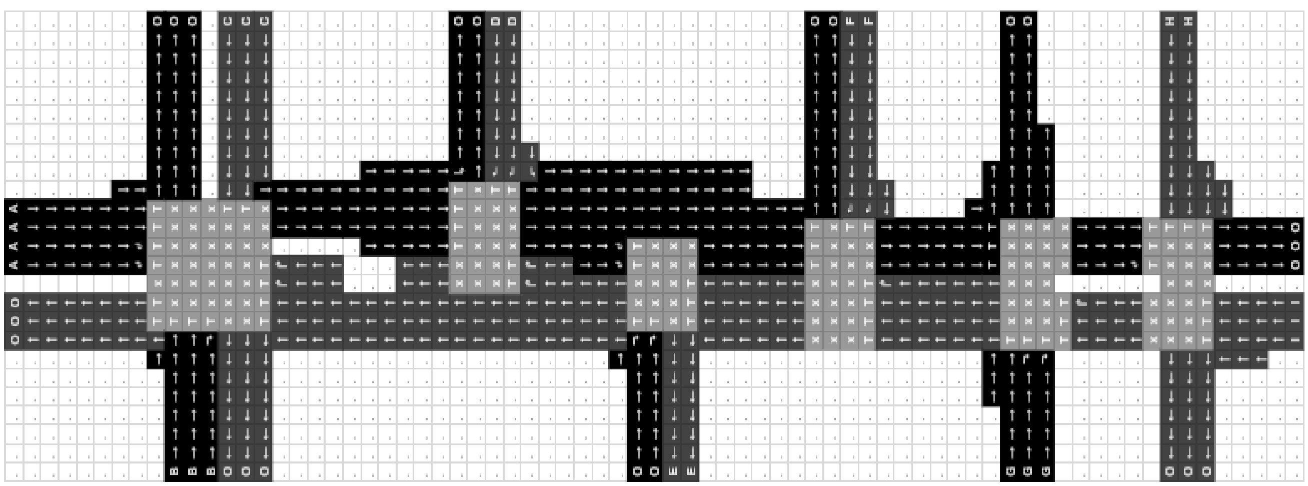

Figure 2. Grid representing the two-way road that connects drivers from Shah Alam (left) to the Federal Highway (right). The arrows represent the direction of traffic, and the 'x's represent the intersections. The O's represent the exits, and the letters represent the different points of origin.

Figure 2 shows multiple lanes of traffic in both directions that merge, split and branch off. For simplicity, we will be modelling a single lane of traffic in each direction. I believe that this is sufficient to test how different traffic light strategies affect the flow of cars. The additional complexity that multiple lanes would add is cars cutting between lanes which could potentially cause more unnecessary braking, similar to the randomised braking included in this model, or a more dynamic inflow and outflow of cars from the system. The latter allows more manipulation of how cars enter the system, which is easily adjustable for our purposes through a probability distribution.

In the simulation, the main road was compressed into 100 units, and the distance between each intersection was scaled accordingly. The roads that branch out were treated as entrances and exits into the main road, governed by the traffic lights and their estimated flow.

A driver can choose to turn left at each intersection, go straight and sometimes turn right or make a U-turn depending on the signal. All left turns in this scenario are not governed by a traffic light, while certain straight paths like in the third intersection from the right also don't follow a traffic light. The signals to go straight and turn are often asynchronous. However, for simplicity, we will assume that no matter what action a driver chooses to take, they will all follow the same signal. I believe that this generalisation should not severely affect the results since it still represents the core challenge of managing traffic flow.

Driving in the simulation

In this simulation, the cars will follow the rules laid out by Nagel and Schreckenberg, as mentioned before, with one additional rule that takes turning into an account.

- Accelerate if their velocity v is lower than the speed limit v_{max} and more than one larger than the distance to the car in front ($v \rightarrow v + 1$)

- Decelerate if the velocity is larger than or equal to the distance to the next car j ($v \to j - 1$)
- Randomly decelerate with a set probability ($v \to v - 1$)
- Slow down to turn within two-unit distance from the exit as long as $v > 1$ ($v \to v - 1$)

At every time step, the car moves by v. Once the car has reached its destination, the car will be removed from the system within the system. Cars entering the systems from one of the intersections will immediately join the main road if there is available space or wait their turn. The probability of a car entering the system is based on observational data of each road's relative business.

Traffic Flow

The inflow of traffic was simulated by a car's probability of entering the main road if the traffic light at its intersection was green. This data was gathered from impressions gained from using this route almost everyday for the last thirteen years. Each point of origin was given a probability based on how consistently it feeds cars into the main road. A probability of one means that there are always cars coming from that origin, while a probability of zero means that no cars are flowing from there. For outflow of cars, each car is assigned a destination where they will exit the main road based on a probability distribution modelled after the flow of cars out of each exit.

For traffic from Shah Alam to Federal Highway (moving right), the highest inflow of cars comes from the origin, the first intersection and the second intersection, respectively. The other intersections are smaller roads that come in from back lanes. The highest outflow of cars in that direction is the last intersection, the rightmost edge of the model and the second intersection, respectively. For traffic from Federal Highway to Shah Alam (moving left), the highest inflow of cars comes from the rightmost origin, the first intersection from the right and the second last intersection. The highest outflow is to the leftmost origin, the last and second last intersection from the right. These inflows and outflows are represented as relative probabilities in the code. Since the unit of time is arbitrary, it is more important to understand how the system handles the general flow.

Traffic Lights

There are pressure pads in front of each traffic light that detects traffic flow based on how many vehicles drive over it. The traffic lights use this information to adjust their timing. For example, if no cars are going over the pressure pads after ten seconds, the timer will drop significantly. Every traffic light has a pressure pad, so all feedback to the traffic light is localised.

Every traffic light has a display that accommodates the number one followed by two other digits, which means that the highest number is 199 seconds. By default, the countdown for waiting time begins from 199 for the first and last two intersections from the left and 60 for the remaining intersections. In the case of 199, the countdown is much longer, estimated at four or five minutes.

Based on observation, the traffic lights appear to turn green successively with a slight lag between each change which facilitates a smooth flow of traffic as cars can drive through without stopping at every intersection. To emulate and simplify this, three adjacent traffic lights in the simulation are turned red together for the length of the stopping time before moving down to turn on the next traffic light (*Diagram 1*). The model will not be implementing the feedback from the pressure sensors, since the traffic flow estimation will be for peak periods. This is not likely to make a big difference in results. The stopping time is the manipulated variable for the simulation. This model encapsulates how the traffic light system works in reality, which will help give reliable results.

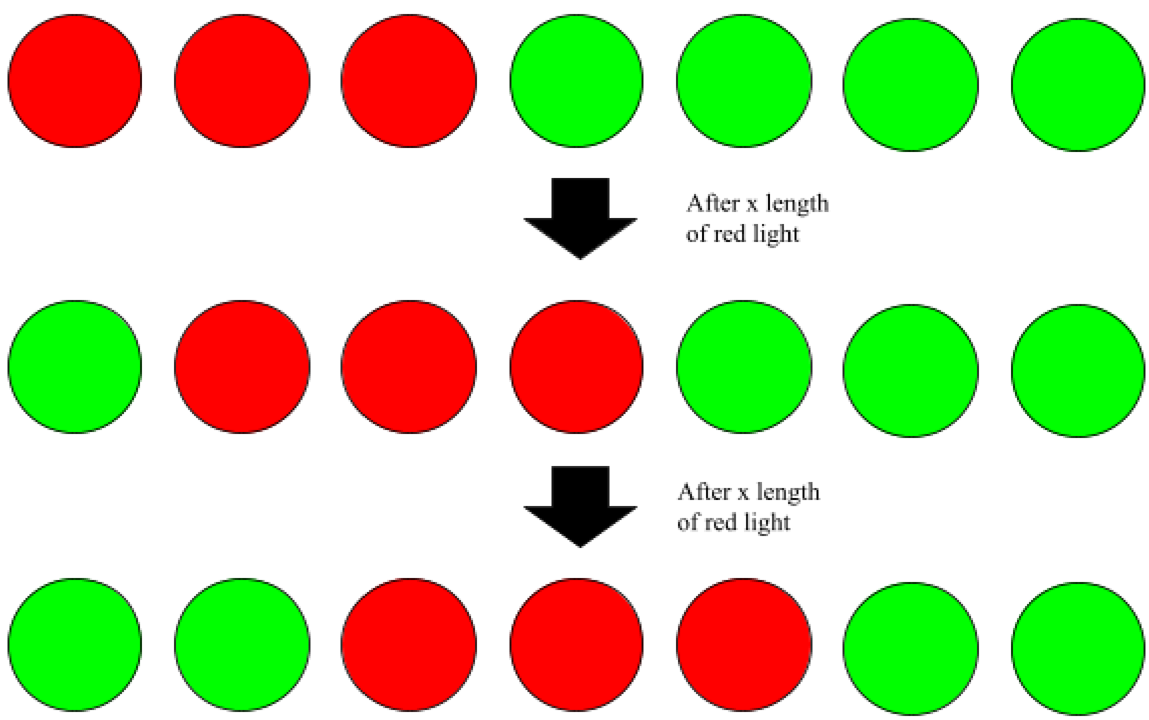

Diagram 1. Visual representation of how the traffic light system works in the original model.

Results

The following results were obtained by running the simulation over 300 time steps and performing 100 trials for every length of a red traffic light. Since there are six intersections and only three traffic lights are red at any one time, the green light length would be $t_{green} = 4t_{red}$. All the results used to tabulate the averages were taken after the 100th time step to allow the simulation time to come to equilibrium. The average traffic flow was computed as the percentage of moving cars ($v > 0$) from both sides of the road. The average waiting time was only taken from cars that had exited the system. The maximum queue length determined how long the queue of cars extended for each origin while waiting for the junction. Because of the way the simulation was coded, I will not be assigning a specific unit of time, but using the unit as a relative measure of how different variables change with each other.

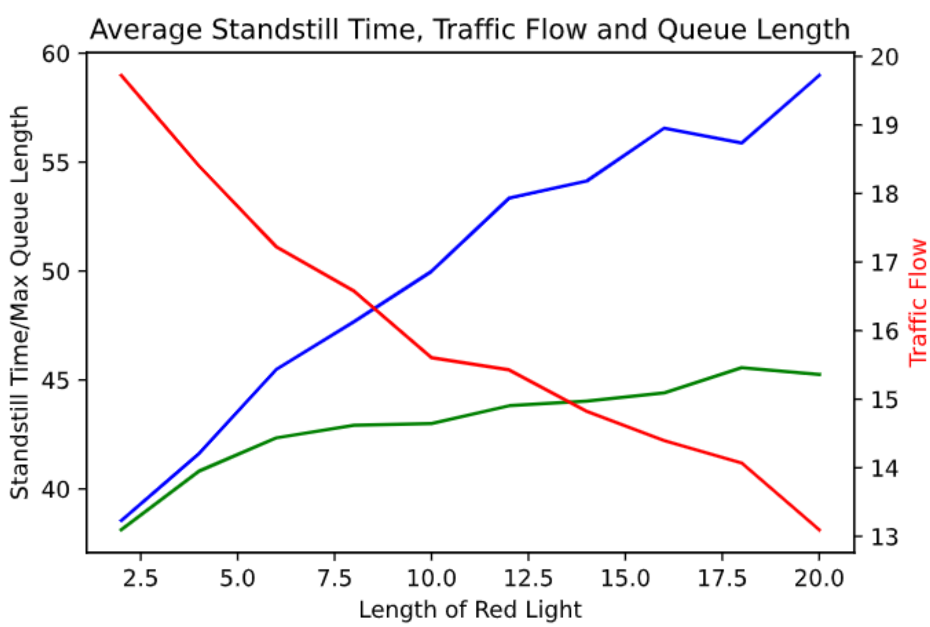

Figure 3. Averages for standstill time for each car (blue), traffic flow (red) and maximum queue length (green) for cars from side roads. The length of the red light for the opposite side of the road will be the inverse.

Our goal is to maximise traffic flow while minimising average wait time and maximum queue length. Therefore, Figure 3 shows us that having the shortest red light length (2 units in this case) is the best solution. Each of the individual figures below also presents a narrow confidence interval, which shows that the results do not fluctuate. It indicates that enough trials have been performed to have reasonable certainty that these reflect the true results.

The waiting time and queue length increase as the length of red light increases which runs counter to the theoretical results which posited that if we kept the ratio of red light length to green light length constant, there shouldn't have been a change in either metrics. However, the theoretical results were based on the assumption that the cars were in constant motion. In the simulation, the scale of traffic buildup increases because of the prolonged length of red light, making the queue more difficult to remove. This is congruent with the idea from the queuing theory that having a service rate lower than the arrival rate will cause the queuing length to grow rapidly. Since queue length is strongly associated with waiting time, it makes sense that both metrics increase at the same rate.

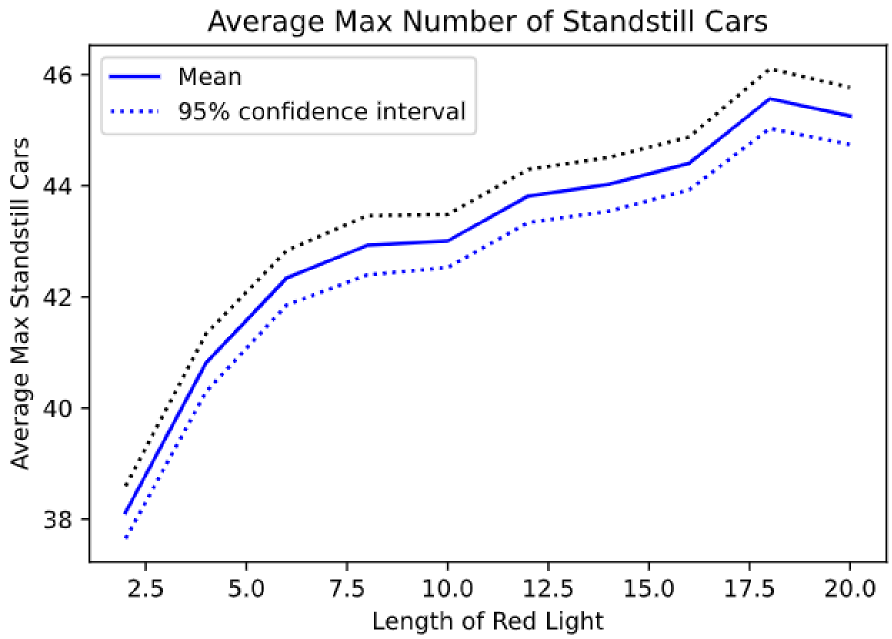

Figure 4. Average maximum queue length for side roads with a 95% confidence interval.

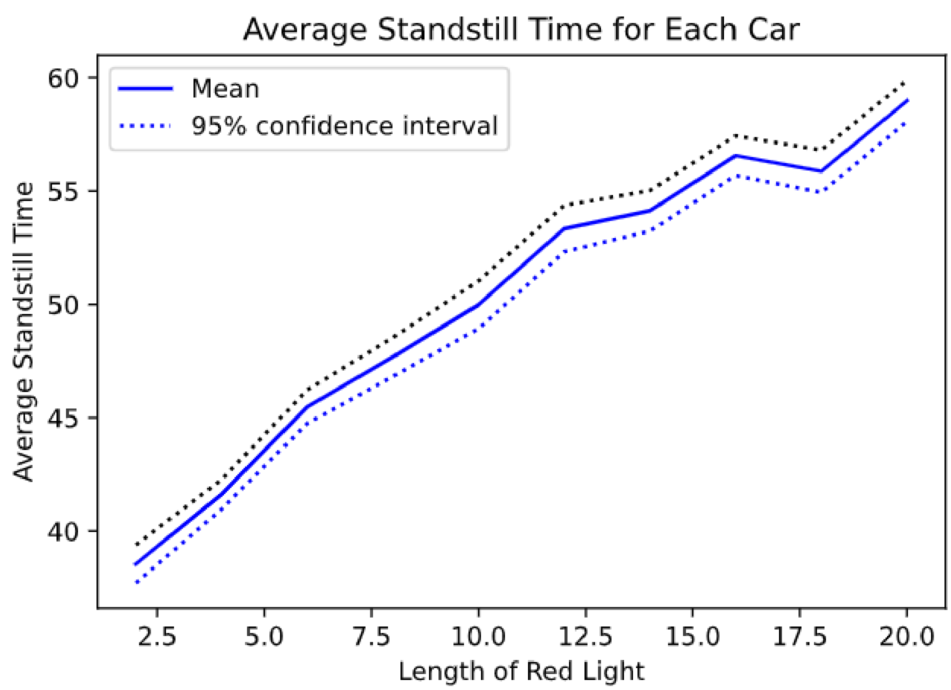

Figure 5. Average standstill time for each car with a 95% confidence interval.

The decrease in average traffic flow as the length of red light increases is also congruent with our theoretical results, showing that an increased density of cars will increase the probability of standstill traffic. Since traffic lights are essentially cars at $v = 0$, having an increased length of red light means increasing the probability of a zero velocity car being in a cell, which in turn leads to a higher probability of standstill traffic and would require a longer time to resume regular flow as the queue grows longer.

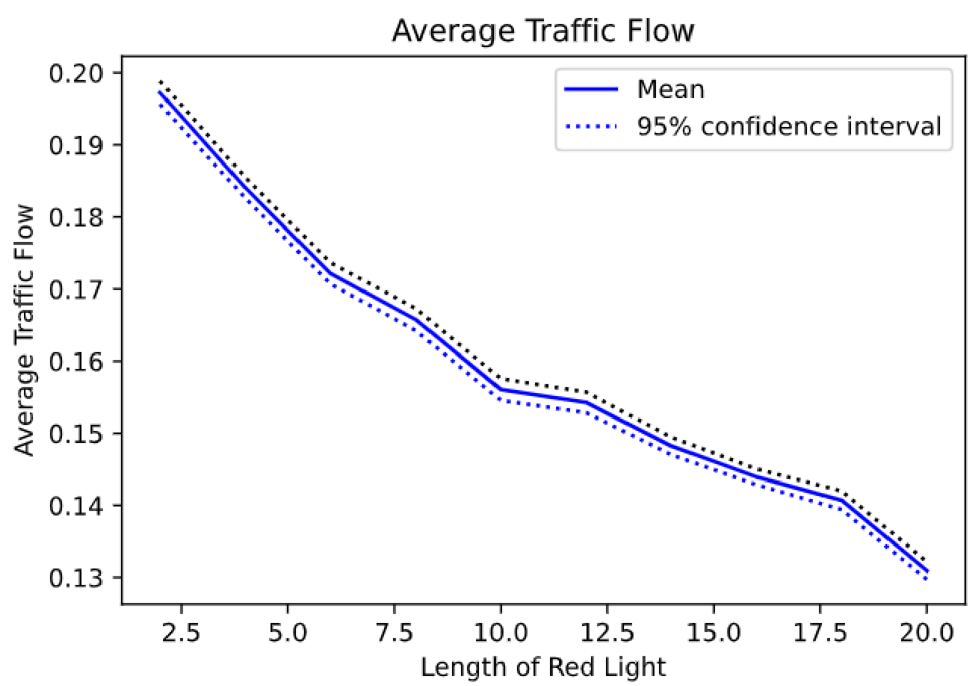

Figure 6. Average traffic flow with a 95% confidence interval.

The results seem to suggest that having the traffic lights change rapidly would give the lowest congestion, driver waiting time and car pileup. In a practical sense, the model is missing the distance taken to traverse the intersection. It assumes that once a car has passed the traffic light boundary, the next car can immediately begin moving. Solving for this would mean implementing a buffer between light changes so that no cars will cross for one unit of time to simulate the yellow light period and the natural lag between light changes. However, making the model more accurate in this sense only makes the metrics worse since fewer cars can cross the model at one time.

Another major shortcoming is that this model only simulates single lane traffic where there is no opportunity to move to a lane with fewer cars or with more space between cars. This would reduce the roads' overall congestion by reducing the dependency of a car's movement to the car in front of it. There are more options for a car to move and escape a pocket of still traffic.

Looking at the specific quantities of the simulation, even in the best case, 80% of the cars are stationary, with 38 car long queues and 20 or 40 minutes of time (depending on the unit of time being 0.5 or 1 minute) spent simply waiting to get through traffic. In the following section, I will be proposing a different traffic light strategy to improve the current metrics.

Proposed Traffic Light Strategy

The previous traffic light strategy changed the traffic lights from red to green successively to encourage the flow of traffic. In reality, an additional feature was to use feedback from pressure sensors to reduce the green light time if there were no cars. The goal was to maximise consecutive traffic flow using a staggered approach, but as we saw in the previous results, there was still poor performance.

The proposed strategy is to allow all the lights to turn green at the same time to make the road unbroken by intersections for a set period of time and turning all the traffic lights red at the same time. We will test this strategy on the same model only changing the method for the traffic light timings.

Results

There is a significant improvement in traffic as seen in Figure 7. It agrees with the original traffic light strategy that having rapid transitions between red and green lights is the optimal solution. The optimal red light length here being two units. The length of red light at four units of time appears to be a turning point for the waiting time and queue length to increase much less quickly as the length of red light increases. There are sharp peaks in traffic flow and waiting time after about six units of red light length. Each figure below has a narrow confidence interval indicating that the results can be reliably thought of as true.

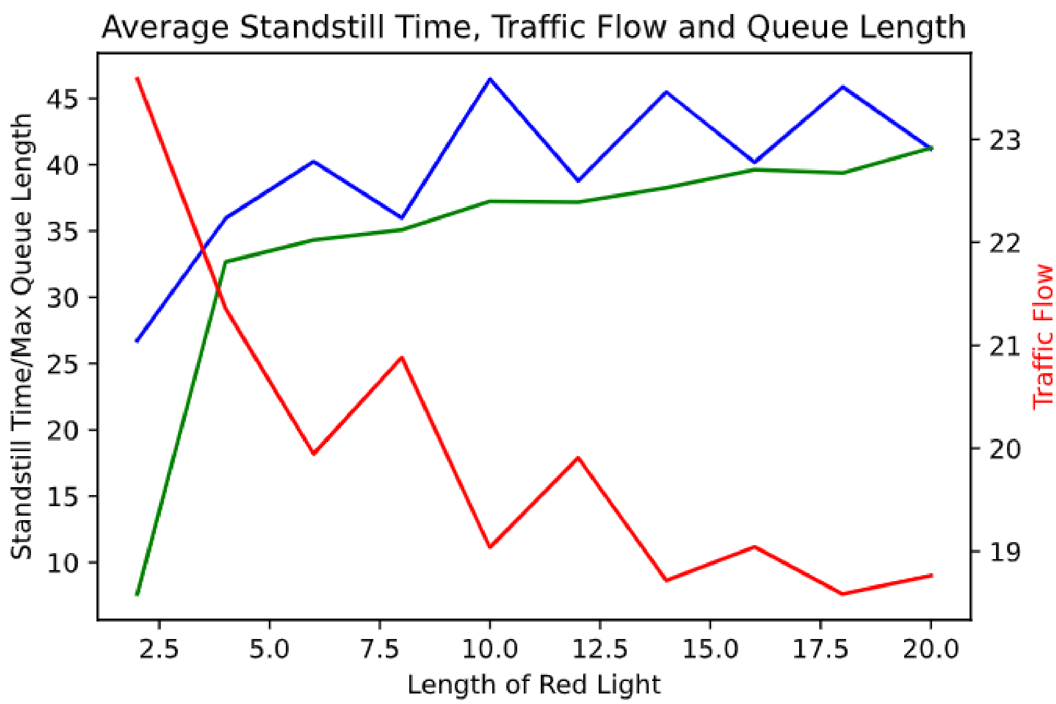

Figure 7. Averages for the standstill time for each car (blue), traffic flow (red) and maximum car queue length (green) from side roads for a variable length of red light.

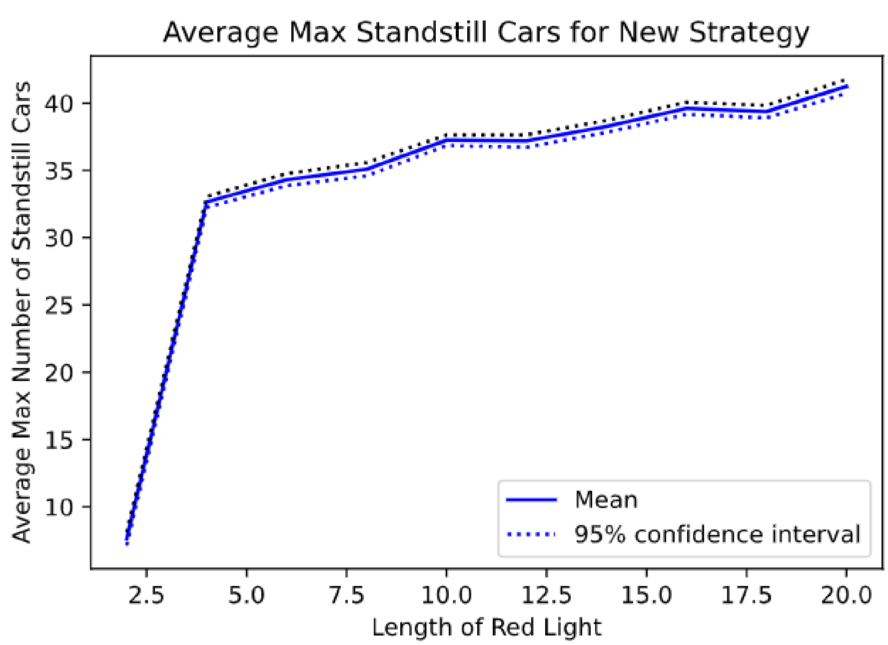

Figure 8. Average maximum number of cars in standstill with a 95% confidence interval for the new traffic strategy.

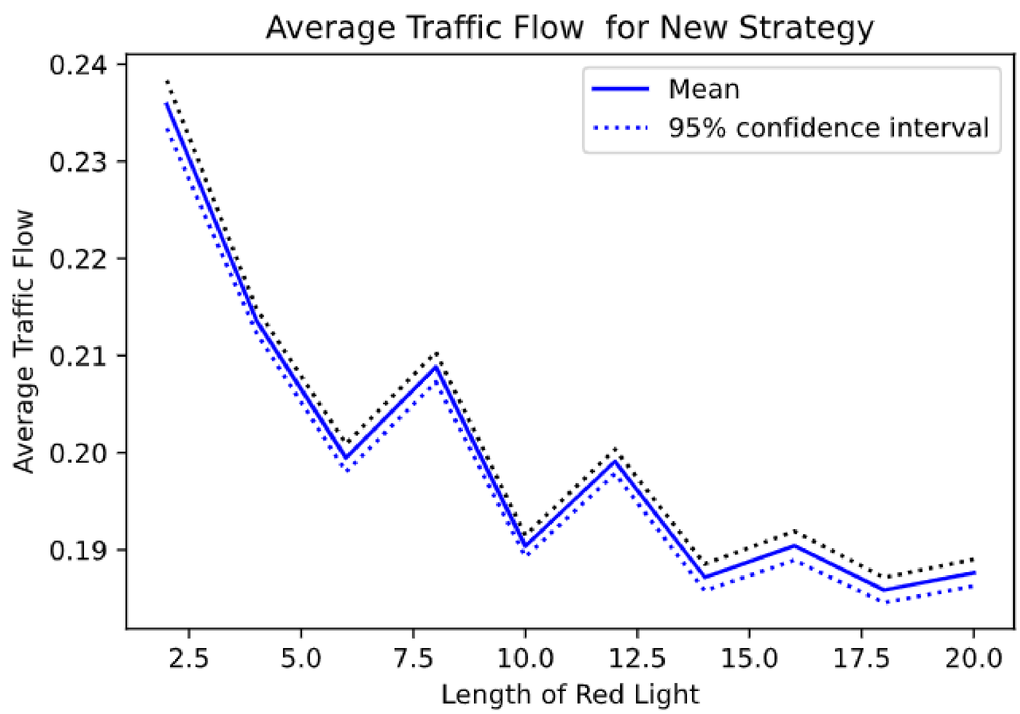

Figure 9. Average traffic flow with a 95% confidence interval for the new traffic light strategy.

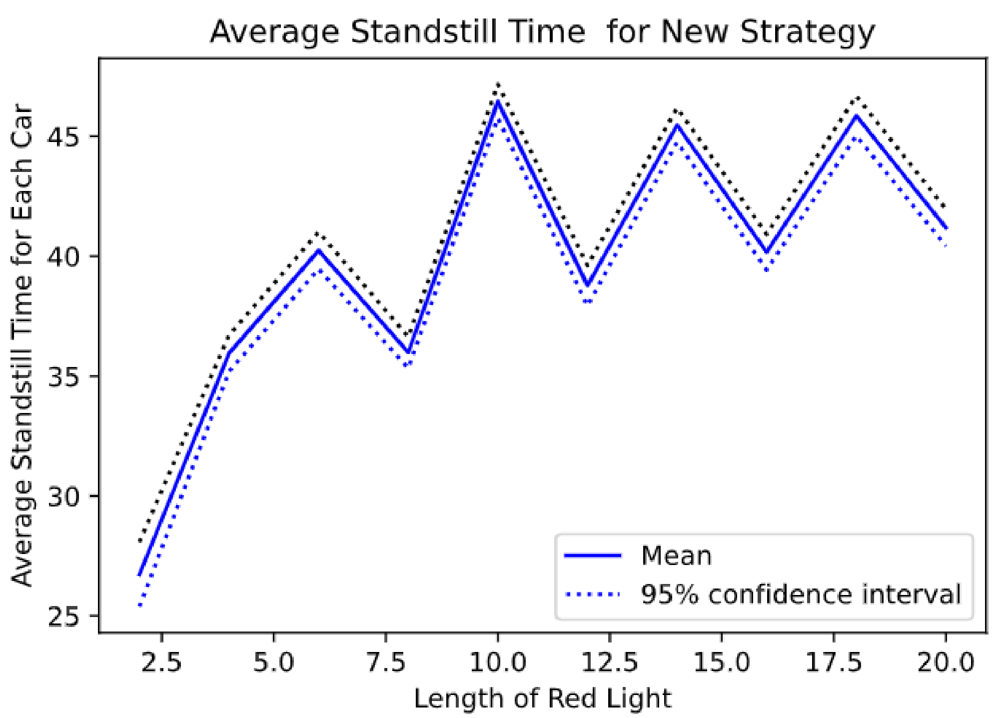

Figure 10. Average standstill time for each car with a 95% confidence interval for the new traffic light strategy.

Table 2 makes a succinct comparison between the two strategies where the proposed strategy performs better than the original strategy on all metrics even in the best and worst case scenarios.

	Original Strategy		Proposed Strategy	
Red light length	2 units	20 units	2 units	20 units
Average Standstill Time	39	60	25	40
Average Flow	0.2	0.13	0.24	0.19
Average Max Queue	38	45	12	41

Table 2. Summary table comparing the results between the original and proposed strategy. The original strategy shows the results for 2 and 20 units of length of red light.

The improvement in queue length and standstill time is likely because with an unbroken flow of cars for the length of the red light, the time to cross the intersection could have decreased. With the staggered traffic light approach, the queue of cars from an intersection further ahead would have prevented a car coming from an intersection with a green light from moving forward, thereby increasing the time taken to cross the intersection. As noted in the queuing theory, this would increase the queue length and the standstill time. In line with this reasoning, it seems that the spike in queue length and standstill time with increased length of red light is because of a queue of cars that grew too long to be removed.

Traffic flow improved because the road was treated as a regular straight road when the traffic lights were green which reduced the density introduced in the mean-field approximation, reducing the probability of a standstill.

Conclusion

Both the traffic light strategies showed what the average traffic was like for 300 units of time which if we regard a unit as half a minute or one minute would be two and a half hours and five hours of traffic respectively. The proposed model performs better on all accounts than the original model. However, these results leave much more room for improvement (refer to Future Work section) before a concrete solution can be implemented.

This proposal highlights a concern for side street traffic to be increased which may cause traffic jams for other segments of the road. Considering that this stretch of road is connected to the Federal Highway, it may contribute to the traffic jams on there and its notoriety for accidents related to sudden halting or slow down of traffic. A concrete suggestion is to allow increased sacrifice for traffic flow on the main road to relieve the fairly long queues of cars coming in from other roads.

Another main suggestion based on the theoretical analysis and corroborated by the simulation is that traffic jams are caused when there are more cars entering the system than there are leaving it. The proposed solution sought to tackle this problem by increasing the flow and getting more cars to exit the system which was successful to an extent, but MBSA should look into traffic light strategies with this quality. In even more radical measurements, this might mean removing certain intersections entirely.

Future Work

An extension of the above report would be to include multiple lanes to understand how the merging and splitting of lanes affects the traffic light strategy. It would also be beneficial to include the idea of a pressure sensor in front of the traffic lights which could change the traffic light times if there are no cars for a certain period of time.

In this simulation, I simplified side street traffic only as a point of origin for cars rather than explicitly modelling the traffic build up on those roads as a knock on effect from the main road. The metric for maximum queue length was a proxy for the length of cars on those roads, but perhaps a more specific model for each side street would give a better understanding of which exact intersections to target for improvement.

The choice for each car's origin and destination were given statically within the model, but would realistically change according to the time of day. For example, in the morning more cars would be leaving in the direction of Federal Highway to go to work, whereas in the evening more cars would be leaving in the direction of Shah Alam to return home. Simulating traffic with an increased degree of specificity may help make the results more usable for on the ground implementation.

These features would make the model more closely simulate reality and give a more accurate comparison between the two traffic light strategies.

References

Nagel, K., & Schreckenberg, M. (1992). A cellular automaton model for freeway traffic. *Journal De Physique I, 2*(12), 2221-2229. doi:10.1051/jp1:1992277