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## Module Dist.PplOps

#### Operators for distributions

```
val (+~) : int dist -> int dist -> int dist
val (-~) : int dist -> int dist -> int dist
val (*~) : int dist -> int dist -> int dist
val (/~) : int dist -> int dist -> int dist
val (/~) : int dist -> int dist -> int dist
val (+.~) : float dist -> float dist -> float dist
val (-.~) : float dist -> float dist -> float dist
val (*.~) : float dist -> float dist -> float dist
val (/.~) : float dist -> float dist -> float dist
val (/.~) : float dist -> float dist -> float dist
val (&~) : bool dist -> bool dist
val (|~) : bool dist -> bool dist
val (|~) : bool dist -> bool dist
val (^~) : string dist -> string dist -> string dist
```

## Module Ppl.Dist

Module used for defining probabilistic models

Contains a type <u>dist</u> which is used to represent probabilistic models.

- Condition Operators
- Monad Fsunctions
- Sampling
- Prior Distribution

```
exception Undefined
type prob = float
      A type for which values need to sum to 1
type likelihood = float
      A type for which values don't need to sum to 1
type 'a samples = ('a * prob) list
      A set of weighted samples, summing to one
type _ dist =
                                                                    distribution with a single
| Return : 'a -> 'a <u>dist</u>
                                                                     value
| Bind : 'a <u>dist</u> * ('a -> 'b <u>dist</u>) -> 'b <u>dist</u>
                                                                    monadic bind
| Primitive : 'a Primitive.t -> 'a dist
                                                                    primitive exact distribution
                                                                     variant that defines
| Conditional : ('a -> float) * 'a dist -> 'a dist
                                                                     likelihood model
```

## **Condition Operators**

Type for representing distributions

```
val condition' : ('a -> likelihood) -> 'a dist -> 'a dist
val condition : bool -> 'a dist -> 'a dist
val score : likelihood -> 'a dist -> 'a dist
val observe : 'a -> 'a Primitive.t -> 'b dist -> 'b dist
```

#### **Monad Fsunctions**

```
include <u>Ppl .Monad.Monad</u> with type 'a <u>t</u> := 'a <u>dist</u>

type 'a t

val return : 'a -> 'a <u>t</u>

val bind : 'a <u>t</u> -> ('a -> 'b <u>t</u>) -> 'b <u>t</u>

val (>>=) : 'a <u>t</u> -> ('a -> 'b <u>t</u>) -> 'b <u>t</u>
```

Module Ppl.Dist 2

```
val let* : 'a \underline{t} -> ('a -> 'b \underline{t}) -> 'b \underline{t} val fmap : ('a -> 'b) -> 'a \underline{t} -> 'b \underline{t} val liftM : ('a -> 'b) -> 'a \underline{t} -> 'b \underline{t} val liftM2 : ('a -> 'b -> 'c) -> 'a \underline{t} -> 'b \underline{t} val mapM : ('a -> 'b \underline{t}) -> 'a list -> 'b list \underline{t} val sequence : 'a \underline{t} list -> 'a list \underline{t}
```

#### **Primitives**

These functions create <u>dist</u> values which correspond to primitive distributions so that they can be used in models. Ok

```
type 'a primitive
val binomial : int -> float -> int primitive
      Create a binomial distribution, the output is the number of successes from n independent trials with
      probability of success p
val normal : float -> float -> float primitive
val categorical : ('a * float) list -> 'a primitive
val discrete_uniform : 'a list -> 'a primitive
val beta : float -> float -> float primitive
val gamma : float -> float -> float primitive
val continuous_uniform : float -> float -> float primitive
val bernoulli : <u>likelihood</u> -> bool <u>dist</u>
val choice : likelihood -> 'a dist -> 'a dist -> 'a dist
Sampling
val sample : 'a dist -> 'a
val sample_n : int -> 'a dist -> 'a array
val sample_with_score : 'a dist -> 'a * likelihood
val dist_of_n_samples : int -> 'a dist -> 'a list dist
```

#### **Prior Distribution**

<u>Up</u> â <u>ppl</u> » <u>Ppl</u> » Inference

```
val prior' : 'a dist -> 'a dist
val prior : 'a dist -> ('a * likelihood) dist
val prior_with_score : 'a dist -> ('a * likelihood) dist
val support : 'a dist -> 'a list

module PplOps : Ppl __Sigs_Ops with type 'a dist := 'a dist
    Operators for distributions
```

Monad Fsunctions 3

## Module Ppl. Inference

Implementation of inference algorithms

Inference algorithms to be called on probabilistic models defined using <u>Dist</u>

- Helpers
- Exact Inference
- Importance Sampling
- Rejetion Sampling
- Sequential Monte Carlo
- Metropolis Hastings
- Particle Independent Metropolis Hastings
- Particle Cascade
- Common

```
exception Undefined
type 'a samples = ('a * float) list
```

### Helpers

```
val unduplicate : 'a samples -> 'a samples
val resample : 'a samples -> 'a samples Dist.dist
val normalise : 'a samples -> 'a samples
val flatten : ('a samples * float) list -> 'a samples
```

#### **Exact Inference**

```
val enumerate : 'a <u>Dist.dist</u> -> float -> 'a <u>samples</u>
val exact_inference : 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

### Importance Sampling

```
val importance : int -> 'a <u>Dist.dist</u> -> 'a <u>samples Dist.dist</u>
val importance' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

## Rejetion Sampling

```
type rejection_type =

| Hard
| Soft

val pp_rejection_type : Stdlib.Format.formatter -> rejection type -> unit
val show_rejection_type : rejection type -> string
val create' : int -> 'a option Dist.dist -> 'a list -> 'a list
val create : int -> 'a option Dist.dist -> 'a list
```

```
val reject_transform_hard : ? threshold:float -> 'a_Dist.dist -> ('a *
float) Dist.dist
val reject'' : 'a_Dist.dist -> 'a_option_Dist.dist
val reject_transform_soft : 'a_Dist.dist -> ('a * float) Dist.dist
val rejection_transform : ? n:int -> rejection_type -> 'a_Dist.dist -> 'a
Dist.dist
val rejection_soft : 'a_Dist.dist -> ('a * float) option_Dist.dist
val rejection_hard : ? threshold:Core.Float.t -> 'a_Dist.dist -> ('a *
float) option_Dist.dist
val rejection : ? n:int -> rejection_type -> 'a_Dist.dist -> 'a_Dist.dist
```

### **Sequential Monte Carlo**

```
val smc : int -> 'a <u>Dist.dist</u> -> 'a <u>samples Dist.dist</u>
val smc' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
val smcStandard : int -> 'a <u>Dist.dist</u> -> 'a <u>samples Dist.dist</u>
val smcStandard' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
val smcMultiple : int -> int -> 'a <u>Dist.dist</u> -> 'a <u>samples Dist.dist</u>
val smcMultiple' : int -> int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

### **Metropolis Hastings**

```
val mh' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
val mh'' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
val mh_sampler : int -> 'a <u>Dist.dist</u> -> 'a list <u>Dist.dist</u>
val mh : burn:int -> 'a <u>Dist.dist</u> -> unit -> 'a
val mh_transform : burn:int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

## **Particle Independent Metropolis Hastings**

```
val pimh : int -> 'a <u>Dist.dist</u> -> 'a <u>samples</u> list <u>Dist.dist</u>
val pimh' : int -> int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

#### **Particle Cascade**

```
val resamplePC : 'a <u>samples</u> -> int -> 'a <u>samples</u> <u>Dist.dist</u>
val cascade : int -> 'a <u>Dist.dist</u> -> 'a <u>samples</u> <u>Dist.dist</u>
val cascade' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

#### Common

```
type infer_strat =

| MH of int
| SMC of int
| PC of int
| PIMH of int
| Importance of int
```

Rejetion Sampling 5

```
| Rejection of int * rejection type

| RejectionTrans of int * rejection type

| Prior

| Enum

| Forward

val pp_infer_strat : Stdlib.Format.formatter -> infer_strat -> unit

val show_infer_strat : infer_strat -> string

val print_infer_strat : infer_strat -> string

val print_infer_strat_short : infer_strat -> string

val infer : 'a Dist.dist -> infer_strat -> 'a Dist.dist

val infer_sampler : 'a Dist.dist -> infer_strat -> unit -> 'a
```

<u>Up</u> â <u>ppl</u> » <u>Ppl</u> » <u>Primitive</u> » PRIM\_DIST

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# Module type Primitive.PRIM\_DIST

The signature for new primitives distributions

```
val sample : unit -> t
val pdf : t -> float
val cdf : t -> float
val support : t support
Up â _ppl » Ppl » Primitive
```

## Module Ppl. Primitive

Module defining a type for primitive distributions

- New Distributions
- Predefined Distributions
- Basic Operations
- Other

```
type 'a support =
| DiscreteFinite of 'a list
                                                  A list of valid values
| DiscreteInfinite
                                                  discrete dist with infinite support e.g. poisson
| ContinuousFinite of ('a * 'a) list
                                                  set of endpoints
| ContinuousInfinite
                                                  continuous dist with an infinite support e.g.
| Merged of 'a <u>support</u> * 'a <u>support</u>
                                                  combination of any of the above
       The type of supports - the values with a distribution can take
module type PRIM DIST = sig ... end
       The signature for new primitives distributions
type 'a t
       Type of primitive dists wrapping a module
```

#### **New Distributions**

```
val create_primitive : sample:(unit \rightarrow 'a) \rightarrow pdf:('a \rightarrow float) \rightarrow cdf:('a \rightarrow float) \rightarrow support:'a support \rightarrow 'a \underline{t}
```

#### **Predefined Distributions**

```
val binomial : int -> float -> int t
val categorical : ('a * float) list -> 'a t
val normal : float -> float -> float t
val discrete_uniform : 'a list -> 'a t
val beta : float -> float -> float t
val gamma : float -> float -> float t
val poisson : float -> int t
val continuous_uniform : float -> float -> float t
```

## **Basic Operations**

```
val pdf : 'a \underline{t} -> 'a -> float
val logpdf : 'a \underline{t} -> 'a -> float
val cdf : 'a \underline{t} -> 'a -> float
val sample : 'a \underline{t} -> 'a
val support : 'a \underline{t} -> 'a support
```

## Other

```
val merge_supports : ('a support * 'a support) -> 'a support
Up â _ppl » Ppl » Helpers
```

Other 9

## Module Ppl. Helpers

Utilities for working with distributions

A set of utilities for generating statistics and printing distributions

- Samples
- Printing
- Others

### **Samples**

### **Printing**

```
val print_exact_exn : (module Base.Stringable.S with type t = 'a) -> 'a
    <u>Dist.dist</u> -> unit
val print_exact_bool : bool <u>Dist.dist</u> -> unit
val print_exact_int : int <u>Dist.dist</u> -> unit
val print_exact_float : float <u>Dist.dist</u> -> unit
```

#### **Others**

```
val time : (unit -> 'a) -> 'a
val memo : ('a -> 'b) -> 'a -> 'b
val memo_no_poly : (module Base__.Hashtbl_intf.Key.S with type t = 'a) ->
('a -> 'b) -> 'a -> 'b
```

<u>Up</u> â <u>ppl</u> » <u>Ppl</u> » Evaluation

## Module Ppl. Evaluation

A module for evaluating the correctness of models and inference procedures

Contains functionality to perform hypothesis tests and KL-divergences for both continuous and discrete distributions

- <u>KL-Divergence</u>
- Hypothesis Tests

```
type 'a samples = 'a Empirical.Discrete.t
type 'a dist = 'a Dist.dist
```

### **KL-Divergence**

### **Hypothesis Tests**

```
val kolmogorov_smirnov : ? n:int -> ? alpha:float -> float dist -> float
Primitive.t -> Owl_stats.hypothesis
```

Perform kolmogorov smirnov test, returns a hypothesis which is true if the null hypothesis is rejected

```
val chi_sq : ? n:int -> ? alpha:float -> 'a dist -> 'a Primitive.t ->
Owl_stats.hypothesis
```

Perform chi-squared test, returns a hypothesis which is true if the null hypothesis is rejected

```
<u>Up</u> â <u>ppl » Ppl » Plot</u>
```

## Module Ppl.Plot

#### Plotting utilies

Plot provides helper functions that wrap Owl\_plplot to graph PPL distributions

- <u>Histograms</u>
- Other Plots

### **Histograms**

```
val hist_dist_continuous : ? h:Owl_plplot.Plot.handle -> ? n:int ->
? fname:string -> float_Dist.dist -> Owl_plplot.Plot.handle
val hist_dist_discrete : ? h:Owl_plplot.Plot.handle -> ? n:int ->
? fname:string -> float_Dist.dist -> Owl_plplot.Plot.handle
```

#### **Other Plots**

```
val qq_plot : ? h:Owl_plplot.Plot.handle -> ? n:int -> ? fname:string ->
float <u>Dist.dist</u> -> float <u>Primitive.t</u> -> Owl_plplot.Plot.handle
val prob_plot : ? h:Owl_plplot.Plot.handle -> ? n:int -> ? fname:string ->
float <u>Dist.dist</u> -> float <u>Primitive.t</u> -> Owl_plplot.Plot.handle
```

<u>Up</u> â <u>ppl</u> » <u>Ppl</u> » Empirical

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# Module Ppl. Empirical

```
module type \underline{S} = \text{sig} \dots \text{ end}

module \underline{\text{Discrete}} : \underline{S}

module \underline{\text{Continuous}} : \text{sig} \dots \text{ end}

\underline{\text{Up â}} \quad \underline{\text{ppl}} \times \underline{\text{Ppl}} \times \underline{\text{Empirical}} \times \underline{\text{Discrete}}
```

## Module Empirical.Discrete

```
type 'a t
val from_dist : ? n:int -> 'a_Dist.dist -> 'a_t
       Create a empirical distribution from a distribution object, using n samples to approximate it
val empty : 'a \underline{t}
       Create an empty distribution
val add_sample : 'a \underline{t} -> 'a -> 'a \underline{t}
       Add another sample to the distribution
val get_num : 'a \underline{t} -> 'a -> int
       Get the numer of samples with the value
val get_prob : 'a \underline{t} \rightarrow 'a \rightarrow float
       Get the probability of a particular value
val to_pdf : 'a \underline{t} -> 'a -> float
       Create a pdf function
val print_map : (module Core.Pretty_printer.S with type t = 'a) -> 'a t
-> unit
       print the entire distribution
val to_arr : 'a \underline{t} -> ('a * int) array
val to_norm_arr : 'a \underline{t} -> ('a * float) array
val support : 'a \underline{t} \rightarrow 'a list
       Get the set of values for the distribution
```

<u>Up</u> â <u>ppl » Ppl » Empirical</u> » Continuous

## Module Empirical.Continuous

```
type bin = float * float
type element = float
type endpoints = {
  start : float;
  finish : float;
}
type 'a t = Owl_stats.histogram

val empty : Owl_stats.histogram
val from_dist : ? n:int -> float_Dist.dist -> Owl_stats.histogram
Upâ _ppl > Ppl > Empirical > S
```

## Module type Empirical.S

```
type 'a t
val from_dist : ? n:int -> 'a_Dist.dist -> 'a_t
       Create a empirical distribution from a distribution object, using n samples to approximate it
val empty : 'a \underline{t}
       Create an empty distribution
val add_sample : 'a \underline{t} -> 'a -> 'a \underline{t}
       Add another sample to the distribution
val get_num : 'a \underline{t} -> 'a -> int
       Get the numer of samples with the value
val get_prob : 'a \underline{t} \rightarrow 'a \rightarrow float
       Get the probability of a particular value
val to_pdf : 'a \underline{t} -> 'a -> float
       Create a pdf function
val print_map : (module Core.Pretty_printer.S with type t = 'a) -> 'a t
-> unit
       print the entire distribution
val to_arr : 'a \underline{t} -> ('a * int) array
val to_norm_arr : 'a \underline{t} -> ('a * float) array
val support : 'a \underline{t} \rightarrow 'a list
       Get the set of values for the distribution
```