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Module Ppl.Dist

Module used for defining probabilistic models

Contains a type <u>dist</u> which is used to represent probabilistic models.

- Condition Operators
- Monad Fsunctions
- Sampling
- Prior Distribution

```
exception Undefined
type prob = float
      A type for which values need to sum to 1
type likelihood = float
      A type for which values don't need to sum to 1
type 'a samples = ('a * prob) list
      A set of weighted samples, summing to one
type _ dist =
                                                                    distribution with a single
| Return : 'a -> 'a <u>dist</u>
                                                                     value
| Bind : 'a <u>dist</u> * ('a -> 'b <u>dist</u>) -> 'b <u>dist</u>
                                                                    monadic bind
| Primitive : 'a Primitive.t -> 'a dist
                                                                    primitive exact distribution
                                                                     variant that defines
| Conditional : ('a -> float) * 'a dist -> 'a dist
                                                                     likelihood model
```

Condition Operators

Type for representing distributions

```
val condition' : ('a -> likelihood) -> 'a dist -> 'a dist
val condition : bool -> 'a dist -> 'a dist
val score : likelihood -> 'a dist -> 'a dist
val observe : 'a -> 'a Primitive.t -> 'b dist -> 'b dist
```

Monad Fsunctions

```
include <u>Ppl .Monad.Monad</u> with type 'a \underline{t} := 'a <u>dist</u> type 'a t val return : 'a -> 'a \underline{t} val bind : 'a \underline{t} -> ('a -> 'b \underline{t}) -> 'b \underline{t} val (>>=) : 'a \underline{t} -> ('a -> 'b \underline{t}) -> 'b \underline{t}
```

Module Ppl.Dist

Dist (ppl.Ppl.Dist)

```
val let* : 'a \underline{t} -> ('a -> 'b \underline{t}) -> 'b \underline{t} val fmap : ('a -> 'b) -> 'a \underline{t} -> 'b \underline{t} val liftM : ('a -> 'b) -> 'a \underline{t} -> 'b \underline{t} val liftM2 : ('a -> 'b -> 'c) -> 'a \underline{t} -> 'b \underline{t} val mapM : ('a -> 'b \underline{t}) -> 'a list -> 'b list \underline{t} val sequence : 'a \underline{t} list -> 'a list \underline{t}
```

Primitives

These functions create <u>dist</u> values which correspond to primitive distributions so that they can be used in models. Ok

```
type 'a primitive
val binomial : int -> float -> int primitive
      Create a binomial distribution, the output is the number of successes from n independent trials with
      probability of success p
val normal : float -> float -> float primitive
val categorical : ('a * float) list -> 'a primitive
val discrete_uniform : 'a list -> 'a primitive
val beta : float -> float -> float primitive
val gamma : float -> float -> float primitive
val continuous_uniform : float -> float -> float primitive
val bernoulli : <u>likelihood</u> -> bool <u>dist</u>
val choice : likelihood -> 'a dist -> 'a dist -> 'a dist
Sampling
val sample : 'a dist -> 'a
val sample_n : int -> 'a dist -> 'a array
val sample_with_score : 'a dist -> 'a * likelihood
```

Prior Distribution

<u>Up</u> â <u>ppl</u> » <u>Ppl</u> » Empirical

val dist_of_n_samples : int -> 'a dist -> 'a list dist

Monad Fsunctions 2

Module Ppl. Empirical

```
module type \underline{S} = \text{sig} \dots end module \underline{\text{Discrete}} : \underline{S} module \underline{\text{Continuous}} : \text{sig} \dots end \underline{\text{Up}} \, \hat{a} \quad \underline{\text{ppl}} \, \times \, \underline{\text{Ppl}} \, \times \, \underline{\text{Evaluation}}
```

Module Ppl. Evaluation

A module for evaluating the correctness of models and inference procedures

Contains functionality to perform hypothesis tests and KL-divergences for both continuous and discrete distributions

- <u>KL-Divergence</u>
- Hypothesis Tests

```
type 'a samples = 'a Empirical.Discrete.t
type 'a dist = 'a Dist.dist
```

KL-Divergence

```
val kl_discrete : ? n:int -> 'a_Primitive.t -> 'a_dist -> float
    Find the KL divergence for two discrete distributions

val kl_continuous : ? n:int -> float_Primitive.t -> float_dist -> float
    Find the KL divergence for two continuous distributions

val kl_cum_discrete : int array -> 'a_Primitive.t -> 'a_dist_-> (int * float) array
```

Hypothesis Tests

```
val kolmogorov_smirnov : ? n:int -> ? alpha:float -> float dist -> float
Primitive.t -> Owl_stats.hypothesis
```

Perform kolmogorov smirnov test, returns a hypothesis which is true if the null hypothesis is rejected

```
val chi_sq : ? n:int -> ? alpha:float -> 'a dist -> 'a Primitive.t ->
Owl_stats.hypothesis
```

Perform chi-squared test, returns a hypothesis which is true if the null hypothesis is rejected

```
<u>Up</u> â <u>ppl</u> » Ppl » Helpers
```

Module Ppl. Helpers

Utilities for working with distributions

A set of utilities for generating statistics and printing distributions

- Samples
- Printing
- Others

Samples

```
val sample_mean : ? n:int -> float_Dist.dist -> float
val sample_variance : ? n:int -> float_Dist.dist -> float
val take_k_samples : int -> 'a Dist.dist -> 'a array
val undup : ('a * float) list -> ('a, float) Core.Map.Poly.t
val weighted_dist : ? n:int -> 'a_Dist.dist -> ('a, int) Core.Map.Poly.t
```

Printing

Others

```
val time : (unit -> 'a) -> 'a
val memo : ('a -> 'b) -> 'a -> 'b
```

<u>Up</u> â <u>ppl</u> » <u>Ppl</u> » Inference

Module Ppl.Helpers 5

Module Ppl. Inference

Implementation of inference algorithms

Inference algorithms to be called on probabilistic models defined using <u>Dist</u>

- Helpers
- Exact Inference
- Importance Sampling
- Rejetion Sampling
- Sequential Monte Carlo
- Metropolis Hastings
- Particle Independent Metropolis Hastings
- Particle Cascade
- Common

```
exception Undefined
type 'a samples = ('a * float) list
```

Helpers

```
val unduplicate : 'a samples -> 'a samples
val resample : 'a samples -> 'a samples Dist.dist
val normalise : 'a samples -> 'a samples
val flatten : ('a samples * float) list -> 'a samples
```

Exact Inference

```
val enumerate : 'a <u>Dist.dist</u> -> float -> 'a <u>samples</u>
val exact_inference : 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

Importance Sampling

```
val importance : int -> 'a <u>Dist.dist</u> -> 'a <u>samples</u> <u>Dist.dist</u>
val importance' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

Rejetion Sampling

```
type rejection_type =

| Hard
| Soft

val pp_rejection_type : Stdlib.Format.formatter -> rejection type -> unit
val show_rejection_type : rejection type -> string
val create' : int -> 'a option Dist.dist -> 'a list -> 'a list
val create : int -> 'a option Dist.dist -> 'a list
```

Dist (ppl.Ppl.Dist)

```
val reject_transform_hard : ? threshold:float -> 'a_Dist.dist -> ('a *
float) Dist.dist
val reject'' : 'a_Dist.dist -> 'a_option_Dist.dist
val reject_transform_soft : 'a_Dist.dist -> ('a * float) Dist.dist
val rejection_transform : ? n:int -> rejection_type -> 'a_Dist.dist -> 'a_Dist.dist
val rejection_soft : 'a_Dist.dist -> ('a * float) option_Dist.dist
val rejection_hard : ? threshold:Core.Float.t -> 'a_Dist.dist -> ('a * float) option_Dist.dist
val rejection Dist.dist
val rejection : ? n:int -> rejection_type -> 'a_Dist.dist -> 'a_Dist.dist
```

Sequential Monte Carlo

```
val smc : int -> 'a <u>Dist.dist</u> -> 'a <u>samples Dist.dist</u>
val smc' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
val smcStandard : int -> 'a <u>Dist.dist</u> -> 'a <u>samples Dist.dist</u>
val smcStandard' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
val smcMultiple : int -> int -> 'a <u>Dist.dist</u> -> 'a <u>samples Dist.dist</u>
val smcMultiple' : int -> int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

Metropolis Hastings

```
val mh' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
val mh'' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
val mh_sampler : int -> 'a <u>Dist.dist</u> -> 'a list <u>Dist.dist</u>
val mh : burn:int -> 'a <u>Dist.dist</u> -> unit -> 'a
val mh_transform : burn:int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

Particle Independent Metropolis Hastings

```
val pimh : int -> 'a <u>Dist.dist</u> -> 'a <u>samples</u> list <u>Dist.dist</u>
val pimh' : int -> int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

Particle Cascade

```
val resamplePC : 'a <u>samples</u> -> int -> 'a <u>samples Dist.dist</u>
val cascade : int -> 'a <u>Dist.dist</u> -> 'a <u>samples Dist.dist</u>
val cascade' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

Common

```
type infer_strat =

| MH of int
| SMC of int
| PC of int
| PIMH of int
| Importance of int
```

Rejetion Sampling 7

Dist (ppl.Ppl.Dist)

```
| Rejection of int * rejection type
| RejectionTrans of int * rejection type
| Prior
| Enum
| Forward

val pp_infer_strat : Stdlib.Format.formatter -> infer strat -> unit val show_infer_strat : infer strat -> string
val print_infer_strat : infer strat -> string
val print_infer_strat : infer strat -> string
val print_infer_strat_short : infer strat -> string
val infer : 'a Dist.dist -> infer strat -> 'a Dist.dist
val infer_sampler : 'a Dist.dist -> infer strat -> unit -> 'a

Upâ _ppl » Ppl » Plot
```

Common 8

Module Ppl.Plot

Plotting utilies

Plot provides helper functions that wrap Owl_plplot to graph PPL distributions

- <u>Histograms</u>
- Other Plots

Histograms

```
val hist_dist_continuous : ? h:Owl_plplot.Plot.handle -> ? n:int ->
? fname:string -> float_Dist.dist -> Owl_plplot.Plot.handle
val hist_dist_discrete : ? h:Owl_plplot.Plot.handle -> ? n:int ->
? fname:string -> float_Dist.dist -> Owl_plplot.Plot.handle
```

Other Plots

```
val qq_plot : ? h:Owl_plplot.Plot.handle -> ? n:int -> ? fname:string ->
float <u>Dist.dist</u> -> float <u>Primitive.t</u> -> Owl_plplot.Plot.handle
val prob_plot : ? h:Owl_plplot.Plot.handle -> ? n:int -> ? fname:string ->
float <u>Dist.dist</u> -> float <u>Primitive.t</u> -> Owl_plplot.Plot.handle
```

<u>Up</u> â <u>ppl</u> » Ppl » Primitive

Module Ppl.Plot 9

Module Ppl. Primitive

Module defining a type for primitive distributions

- New Distributions
- Predefined Distributions
- Basic Operations
- Other

```
type 'a support =
| DiscreteFinite of 'a list
                                                    A list of valid values
| DiscreteInfinite
                                                    discrete dist with infinite support e.g. poisson
| ContinuousFinite of ('a * 'a) list
                                                    set of endpoints
| Continuous Infinite
                                                    continuous dist with an infinite support e.g.
| Merged of 'a <u>support</u> * 'a <u>support</u>
                                                    combination of any of the above
       The type of supports - the values with a distribution can take
module type <a href="PRIM DIST">PRIM DIST</a> = sig ... end
       The signature for new primitives distributions
type 'a t
       Type of primitive dists wrapping a module
```

New Distributions

```
val create_primitive : sample:(unit \rightarrow 'a) \rightarrow pdf:('a \rightarrow float) \rightarrow cdf:('a \rightarrow float) \rightarrow support:'a support \rightarrow 'a \underline{t}
```

Predefined Distributions

```
val binomial : int -> float -> int t
val categorical : ('a * float) list -> 'a t
val normal : float -> float -> float t
val discrete_uniform : 'a list -> 'a t
val beta : float -> float -> float t
val gamma : float -> float -> float t
val poisson : float -> int t
val continuous_uniform : float -> float -> float t
```

Basic Operations

```
val pdf : 'a \underline{t} -> 'a -> float
val logpdf : 'a \underline{t} -> 'a -> float
val cdf : 'a \underline{t} -> 'a -> float
val sample : 'a \underline{t} -> 'a
val support : 'a \underline{t} -> 'a support
```

Other

```
val merge_supports : ('a support * 'a support) -> 'a support
Up â _ppl » Ppl » Samples
```

Other 11

Module Ppl. Samples

```
type 'a t
val from_dist : ? n:int -> 'a_Dist.dist -> 'a_t
        Create a empirical distribution from a distribution object, using n samples to approximate it
val empty : 'a \underline{t}
       Create an empty distribution
val add_sample : 'a \underline{t} -> 'a -> 'a \underline{t}
        Add another sample to the distribution
val get_num : 'a \underline{t} -> 'a -> int
        Get the numer of samples with the value
val get_prob : 'a \underline{t} \rightarrow 'a \rightarrow float
        Get the probability of a particular value
val to_pdf : 'a \underline{t} \rightarrow 'a \rightarrow float
        Create a pdf function
val print_map : (module Core.Pretty_printer.S with type t = 'a) -> 'a t
-> unit
       print the entire distribution
val to_arr : 'a \underline{t} -> ('a * int) array
val to_norm_arr : 'a \underline{t} -> ('a * float) array
val support : 'a \underline{t} \rightarrow 'a list
       Get the set of values for the distribution
<u>Up</u> â <u>ppl</u> » Ppl
```

Module Ppl

DSL for Probabilistic Programming

Universal PPL in OCaml

• Submodules

Submodules

```
module Plot : sig ... end
       Plotting utilies
module <a href="Primitive">Primitive</a> : sig ... end
       Module defining a type for primitive distributions
module <a>Evaluation</a> : sig ... end
       A module for evaluating the correctness of models and inference procedures
module <a>Empirical</a> : sig ... end
module <a href="Inference">Inference</a> : sig ... end
       Implementation of inference algorithms
module <u>Dist</u>: sig ... end
       Module used for defining probabilistic models
module <u>Helpers</u>: sig ... end
       Utilities for working with distributions
module <u>Samples</u>: <u>Empirical.S</u>
include <u>Dist</u>
Module used for defining probabilistic models
Contains a type <u>dist</u> which is used to represent probabilistic models.
exception Undefined
type prob = float
       A type for which values need to sum to 1
type likelihood = float
       A type for which values don't need to sum to 1
type 'a samples = ('a * prob) list
       A set of weighted samples, summing to one
type _ dist =
```

Module Ppl 13

```
| Return : 'a -> 'a dist distribution with a single value | Bind : 'a dist * ('a -> 'b dist) -> 'b dist monadic bind | Primitive : 'a Primitive.t -> 'a dist primitive exact distribution | Conditional : ('a -> float) * 'a dist variant that defines likelihood model
```

Type for representing distributions

Condition Operators

```
val condition' : ('a -> likelihood) -> 'a dist -> 'a dist
val condition : bool -> 'a dist -> 'a dist
val score : likelihood -> 'a dist -> 'a dist
val observe : 'a -> 'a Primitive.t -> 'b dist -> 'b dist
```

Monad Fsunctions

```
include Ppl .Monad.Monad with type 'a t := 'a dist

type 'a t

val return : 'a -> 'a t
val bind : 'a t -> ('a -> 'b t) -> 'b t
val (>>=) : 'a t -> ('a -> 'b t) -> 'b t
val let* : 'a t -> ('a -> 'b t) -> 'b t
val fmap : ('a -> 'b) -> 'a t -> 'b t
val liftM : ('a -> 'b) -> 'a t -> 'b t
val liftM2 : ('a -> 'b t) -> 'a list -> 'b list t
val sequence : 'a t list -> 'a list t
```

val beta : float -> float -> float primitive
val gamma : float -> float -> float primitive

Primitives

These functions create <u>dist</u> values which correspond to primitive distributions so that they can be used in models. Ok

Submodules

14

val continuous_uniform : float -> float -> float primitive

```
val bernoulli : likelihood -> bool dist
val choice : likelihood -> 'a dist -> 'a dist -> 'a dist
```

Sampling

```
val sample : 'a dist -> 'a
val sample_n : int -> 'a dist -> 'a array
val sample_with_score : 'a dist -> 'a * likelihood
val dist_of_n_samples : int -> 'a dist -> 'a list dist
```

Prior Distribution

include <u>Inference</u>

Implementation of inference algorithms

Inference algorithms to be called on probabilistic models defined using <u>Dist</u>

```
exception Undefined
type 'a samples = ('a * float) list
```

Helpers

```
val unduplicate : 'a samples -> 'a samples
val resample : 'a samples -> 'a samples Dist.dist
val normalise : 'a samples -> 'a samples
val flatten : ('a samples * float) list -> 'a samples
```

Exact Inference

```
val enumerate : 'a <u>Dist.dist</u> -> float -> 'a <u>samples</u>
val exact_inference : 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

Importance Sampling

```
val importance : int -> 'a <u>Dist.dist</u> -> 'a <u>samples Dist.dist</u>
val importance' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

Primitives 15

Rejetion Sampling

```
type rejection_type =
| Hard
I Soft.
val pp_rejection_type : Stdlib.Format.formatter -> rejection type -> unit
val show_rejection_type : rejection type -> string
val create' : int -> 'a option <u>Dist.dist</u> -> 'a list -> 'a list
val create : int -> 'a option <u>Dist.dist</u> -> 'a list
val reject_transform_hard : ? threshold:float -> 'a_<u>Dist.dis</u>t -> ('a *
float) Dist.dist
val reject'' : 'a <u>Dist.dist</u> -> 'a option <u>Dist.dist</u>
val reject transform soft : 'a <u>Dist.dist</u> -> ('a * float) <u>Dist.dist</u>
val rejection_transform : ? n:int -> rejection type -> 'a_Dist.dist -> 'a
Dist.dist
val rejection_soft : 'a <u>Dist.dist</u> -> ('a * float) option <u>Dist.dist</u>
val rejection_hard : ? threshold:Core.Float.t -> 'a_Dist.dist -> ('a *
float) option Dist.dist
val rejection : ? n:int -> rejection type -> 'a Dist.dist -> 'a Dist.dist
```

Sequential Monte Carlo

```
val smc : int -> 'a <u>Dist.dist</u> -> 'a <u>samples Dist.dist</u>
val smc' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
val smcStandard : int -> 'a <u>Dist.dist</u> -> 'a <u>samples Dist.dist</u>
val smcStandard' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
val smcMultiple : int -> int -> 'a <u>Dist.dist</u> -> 'a <u>samples Dist.dist</u>
val smcMultiple' : int -> int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

Metropolis Hastings

```
val mh' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
val mh'' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
val mh_sampler : int -> 'a <u>Dist.dist</u> -> 'a list <u>Dist.dist</u>
val mh : burn:int -> 'a <u>Dist.dist</u> -> unit -> 'a
val mh_transform : burn:int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

Particle Independent Metropolis Hastings

```
val pimh : int -> 'a <u>Dist.dist</u> -> 'a <u>samples</u> list <u>Dist.dist</u>
val pimh' : int -> int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

Particle Cascade

```
val resamplePC : 'a <u>samples</u> -> int -> 'a <u>samples</u> <u>Dist.dist</u>
val cascade : int -> 'a <u>Dist.dist</u> -> 'a <u>samples</u> <u>Dist.dist</u>
val cascade' : int -> 'a <u>Dist.dist</u> -> 'a <u>Dist.dist</u>
```

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Common

```
type infer_strat =
| MH of int
| SMC of int
IPC of int
| PIMH of int
| Importance of int
| Rejection of int * rejection type
| RejectionTrans of int * rejection type
| Prior
| Enum
| Forward
val pp_infer_strat : Stdlib.Format.formatter -> infer strat -> unit
val show_infer_strat : infer strat -> string
val print_infer_strat : infer strat -> string
val print_infer_strat_short : infer_strat_ -> string
val infer : 'a <u>Dist.dist</u> -> <u>infer strat</u> -> 'a <u>Dist.dist</u>
val infer_sampler : 'a <u>Dist.dist</u> -> <u>infer strat</u> -> unit -> 'a
include <u>Helpers</u>
```

Utilities for working with distributions

A set of utilities for generating statistics and printing distributions

Samples

```
val sample_mean : ? n:int -> float_<u>Dist.dis</u>t -> float
val sample_variance : ? n:int -> float_<u>Dist.dis</u>t -> float
val take_k_samples : int -> 'a <u>Dist.dist</u> -> 'a array
val undup : ('a * float) list -> ('a, float) Core.Map.Poly.t
val weighted_dist : ? n:int -> 'a <u>Dist.dist</u> -> ('a, int) Core.Map.Poly.t
```

Printing

```
val print_exact_exn : (module Base.Stringable.S with type t = 'a) -> 'a
    <u>Dist.dist</u> -> unit
val print_exact_bool : bool <u>Dist.dist</u> -> unit
val print_exact_int : int <u>Dist.dist</u> -> unit
val print_exact_float : float <u>Dist.dist</u> -> unit
```

Others

```
val time : (unit -> 'a) -> 'a
```

Common 17

val memo : ('a -> 'b) -> 'a -> 'b

Others 18