Math for “Balance” Trading Logic

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Intro

The basic idea with this logic is that both profits and losses are distributed across all commodities (including the Fiat money). By transforming gains and losses above a threshold into less volatile commodities and thus preserving them, the overall volatility of the portfolio is reduced (gains and losses too, however). For a mathematical analysis, we start by considering a portfolio consisting of one volatile commodity and (non-volatile) fiat money.

Legend

F	Fiat money (amount)
C	Commodity (amount)
P	Price (conversion factor)
p	Percent factor
R	Profit (expressed as related factor minus 1)
K	Constant relation factor
T	Capitalization (all exchanged to fiat money)

Ruleset

We require, that value of commodity is proportional to fiat money. Relation factor K shall be constant:

1) $$F = K\,(C\,P)$$

Thus if price of commodity changes, we need to rebalance via buying / selling:

2) $$F_1 - F_0 = -\,(C_1 - C_0)\,P_1$$

Derivation

Inserting 1 in 2:

$$\Rightarrow K\,(C_1\,P_1) - F_0 = -\,(C_1 - C_0)\,P_1$$
$$\Leftrightarrow K\,(C_1\,P_1) = -\,(C_1\,P_1) + (C_0\,P_1) + F_0$$
$$\Leftrightarrow (K+1)\,(C_1\,P_1) = F0 + (C_0\,P_1)$$

3) $\Leftrightarrow C_1 = (C_0 + F_0 / P_1) / (K+1)$

Last statement 3 inserted in 1:

$$\Rightarrow F_1 = K\,(C_1\,P_1) = K\,(C_0 + F_0 / P_1) / (K+1)\,P_1$$

4) $\Leftrightarrow F_1 = K / (K+1)\,(F_0 + C_0\,P_1)$

With price function $P n$ given as:

5) $\Rightarrow P_n = P_0 * p^n$

Inserted in 3 and 4:

6) $\Rightarrow C_1 = (C_0 + (F_0 / P_0)\,p^{-1}) / (K+1)$

7) $\Rightarrow F_1 = K / (K+1)\,(F_0 + (C_0\,P_0)\,p)$

Equation 1 for $n = 0$ results to:

8) $\Rightarrow F_0 = K\,C_0\,P_0$

Using last statement 8 in 6 and 7 we can transform:

9) $\Rightarrow C_1 = (C_0 + (K\,C_0)\,p^{-1}) / (K+1) = C_0\,(K / p + 1) / (K+1)$

10) $\Rightarrow F_1 = K / (K+1)\,(F_0 + (F_0 / K)\,p) = F_0\,(K+p) / (K+1)$

Transforming 9 and 10 into function of n:

11) $\Rightarrow C_n = C_0\,((K / p + 1) / (K+1))^n$

12) $\Rightarrow F_n = F_0\,((K+p) / (K+1))^n$

Price Swing

Proof: Logic guarantees profit for price step forward and back cycle.
Using 10:

$$\Rightarrow F'_0 = F_1 = F_0\,(K + p) / (K + 1) \qquad \text{| forward step}$$

$$\Rightarrow F''_0 = F'_1 = F'_0\,(K + 1/p) / (K + 1) \qquad \text{| back step}$$

$$= F_0\,((K + p) / (K + 1))\,((K + 1/p) / (K + 1))$$

$$= F_0\,(K + p)\,(K + 1/p) / (K + 1)^2$$

We require that $F''_0 \geq F_0$

$$\Rightarrow (K + p)\,(K + 1/p) \geq (K + 1)^2$$

$$\Leftrightarrow K^2 + K(p + 1/p) + 1 \geq K^2 + 2K + 1$$

$$\Leftrightarrow K(p + 1/p) \geq 2K$$

$$\Leftrightarrow p + 1/p \geq 2$$

$$\Leftrightarrow p^2 + 1 \geq 2p$$

$$\Leftrightarrow p^2 - 2p + 1 \geq 0$$

$$\Leftrightarrow (p - 1)^2 \geq 0$$

Q.e.d

The profit of such a swing results to:

13) $$\Rightarrow R''_0 = F''_0 / F_0 - 1 = (K + p)\,(K + 1/p) / (K + 1)^2 - 1$$

Finding the local maximum with:

$$\Rightarrow \frac{d}{dK} R''_0 = \frac{(K-1)}{(K+1)^3}\,\frac{(p-1)^2}{p} = 0, \text{ for } K = 1$$

Thus max profit results for $K = 1$

The theoretical, maximum profit for a price swing would be p - 1 (p > 1), if you exchange all commodity to fiat money and back. So the relation between real and maximum profit (eg. degree of utilization) is:

14) $$\Rightarrow U''_0 = R''_0 / (p - 1)$$

[Graphical analysis of 13 and 14 as functions of K for p = 1.05, p = 1.10, p = 1.15]

Meaningful values for K are in the range $[0.5,\ 2]$.
The bigger the price step p, the better the degree of utilization. Therefore, a configured price step should not be infinitesimal, but approximately equal to the average amplitude of a swing.

If we were to apply this logic to Bitcoin, a reasonable price step would be 10%. Fluctuations of this magnitude occur more than once a month. Total assets would rise by around 0.2% for each of these swings. For a full year, the return would be around 2.5%, without the price having to rise significantly.

Price Increase

With a steady price increase ($p > 1$) the profit results to:

15) $\Rightarrow R_n = F_n / F_0 - 1 = ((K + p) / (K + 1))^n - 1$

And the theoretical, maximum profit is:

16) $\Rightarrow R^x{}_n = P_n / P_0 - 1 = p^n - 1$

Thus the degree of utilization is:

17) $\Rightarrow U_n = R_n / R^x{}_n$

R1, R2, R3 as functions of K for p = 1.02 and p = 1.08

With $p_n = 1 + (p - 1)\, n$ we can roughly say $R_n(p_1) \approx R_1(p_n)$ and $R^x{}_n(p_1) \approx R^x{}_1(p_n)$,
for $p,\ K$ around 1 and small n .
So we can simplify degree of utilization:

18) $\Rightarrow U_n = R_n / R^x{}_n \approx R_1(p_n) / R^x{}_1(p_n) = 1 / (K + 1)$

Simplified and some exakt Un with p=1.05 / 1.10 / 1.15 , n = 2 / 3 / 4

Degree of utilization is significantly better for smaller Ks. Thus the more you expect long-term price gains, the smaller should be K ($0 < K \leq 1$, see also Price Swing for meaningful values). If you do not expect price gains but rather swings in the long term, you should operate with a value of K around 1.

Usage

Collection of formulas derived in former chapters:

1) $F = K (C P)$
8) $F_0 = K C_0 P_0$
2) $F_1 - F_0 = -(C_1 - C_0) P_1$
5) $P_n = P_0 * p^n$
11) $C_n = C_0 ((K / p + 1) / (K + 1))^n$
12) $F_n = F_0 ((K + p) / (K + 1))^n$
13) $R''_0 = (K + p) (K + 1 / p) / (K + 1)^2 - 1$
15) $R_n = ((K + p) / (K + 1))^n - 1$

Placing buy / sell orders:

1. Determine suitable value of p, that is around the average amplitude of a price swing.
2. Determine suitable value of K ($0 < K \leq 1$), for which the profits of price swings and increase (according to 13 and 15) become maximum. Rule of thumb is: The more you expect price gains, the lower should be K (than 1). Determination of K might be worth another chapter (in near future). Note, that your portfolio is also expected to be balanced, such that you have enough fiat money to meet the equation $F = K (C P)$, as given by 1.
3. Place your orders according to 11 at price intervall steps given by 5. On price gains $(p > 1)$ you have to place sell orders and on price losses $(p < 1)$ you have to place buy orders at interval steps.
4. On a successful buy you have to update your sell orders and vice versa.