

# Question

April 2021

## 1

What is  $z = F(x, y)$ ? Suppose we work with options.  $z$  is the total reserved value,  $x, y$  are call and put token produced respectively. Thus,  $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$  are costs to produce call and put token respectively. Option price floor is  $F$  and price ceiling is  $C$ . Here we simplify it, letting  $F = 0, C = 1$ . For  $x, y, z > 0$ ,  $\frac{\partial z}{\partial x} \in (0, 1), \frac{\partial z}{\partial y} \in (0, 1)$  and  $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$ . When  $x = y$ ,  $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 0.5$ . When  $y/x \rightarrow 0$ ,  $\frac{\partial z}{\partial x} \rightarrow 0, \frac{\partial z}{\partial y} \rightarrow 1$ . When  $y/x \rightarrow \infty$ ,  $\frac{\partial z}{\partial x} \rightarrow 1, \frac{\partial z}{\partial y} \rightarrow 0$ . We also restrict that for  $k > 0$ ,  $F(kx, ky) = kF(x, y)$ .  $\frac{\partial kz}{\partial kx} = \frac{\partial z}{\partial x}$  and  $\frac{\partial kz}{\partial ky} = \frac{\partial z}{\partial y}$ . I tried  $\frac{\partial z}{\partial x} = \frac{y}{x+y}$  and  $\frac{\partial z}{\partial y} = \frac{x}{x+y}$ . Is there a smooth solution?