Turbomachinery: Compressor preliminary design

Antonio Pucciarelli

Politecnico di Milano

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1. Problem Description
2. Similitude
3. Models
4. turboLIB & Results
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Initial conditions & constraints

Inlet conditions

- $P_{T0} = 1bar$
- $T_{T0} = 300K$

Constraints

- $r_{max} = 0.45m$
- $\beta_{TT} = 1.45$
- $\dot{m} = 100\frac{kg}{s}$
- **max** η

Due to the **course track** and **preference**, the turbomachinery design will be on an **axial** compressor.

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Problem setup: hypothesis

Hypothesis

- **not using** an **inlet guide vane** for simplicity of design[1]
- keeping, in the similarity/adimensional analysis of the compressor, $V_{a_{mean}}$ **constant**[2]
- keeping the blade height, b_0, **constant** both in rotor and stator[3]
- using a **free vortex** model for the velocity triangles
- neglecting inlet **entropy** generation and assuming **rotor inlet** quantities **constant**
- **shrouding** at blade tip not present
- **rotor-stator** losses neglected

[1] $V_{t0} = 0\frac{m}{s}$ and χ dictate the behaviour of λ.
[2] $\dot{m}$ corrections will be made later on in the **radial equilibrium** solution.
[3] In order to keep each **blade streamtube section** as simple as possible.

Problem setup: solution steps

Main procedural steps:

- λ and ψ computation from χ and V_{t0}
- ϕ and η computation
- $V_{a_{mean}}$ and L_{eu} computation from ϕ, β_{TT} and η
- computing **mean** velocity triangles, using the above hypothesis
- computing **mean thermodynamic** quantities
- computing **blade height**

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Problem setup

Main design quantities

$V_{t_{mean}}$, $V_{a_{mean}}$, U_{mean} & velocity triangles

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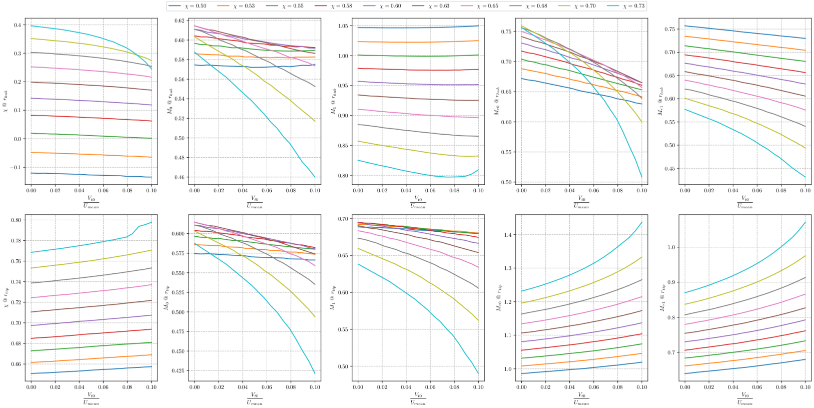
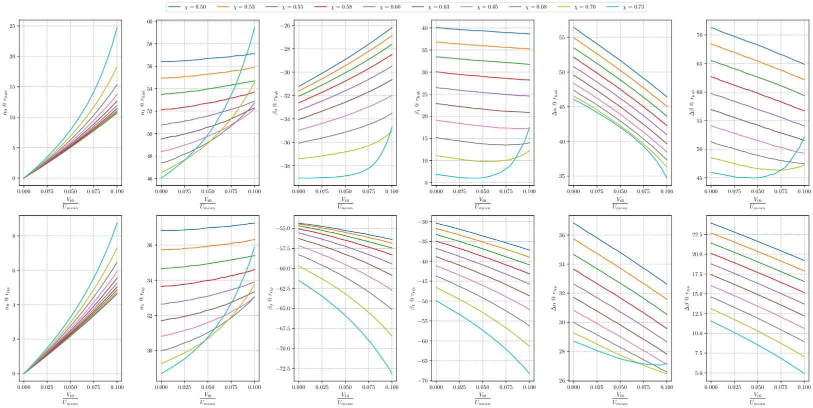
Graph Analysis: χ & M



Graph Analysis: α & β



λ & ψ

From the previous **graphs**:

- $\chi = 0.55$
- $r_{mean} = 0.325m$
- $\frac{V_{t0}}{U_{mean}} = 0$

Taking into account the previous modeling **hypothesis**:

$$\lambda = \left(1 - \chi - \frac{V_{t0}}{U_{mean}}\right) \cdot 4$$

$$\psi = \frac{\lambda}{2}$$

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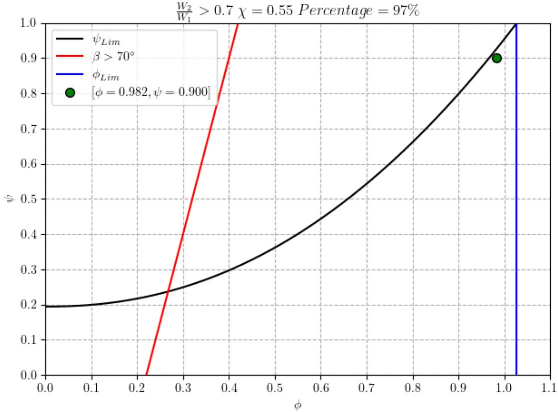
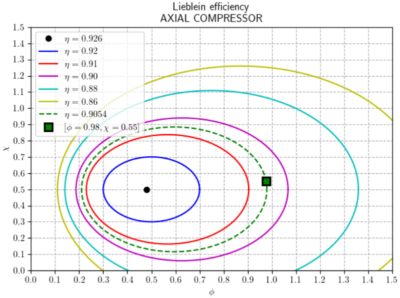
$\phi_{(\psi)}$

From [**?**, Sec. 10.4] it is imposed that $\frac{W_2}{W_1} \geq 0.7$ with a *safety* margin of 3%.



η & L_{eu}

η is computed from an **Lieblein** efficiency chart[4] given ϕ and χ. This parameter will be used for the computation of L_{eu} given the β_{TT} target.



$$L_{is} = \frac{\gamma\ R}{\gamma - 1}\ T_{in}\ (\beta_{TT}^{\frac{\gamma-1}{\gamma}} - 1)$$

$$L_{eu} = \frac{L_{is}}{\eta}$$

[4]This chart has been interpolated from the course slides charts.

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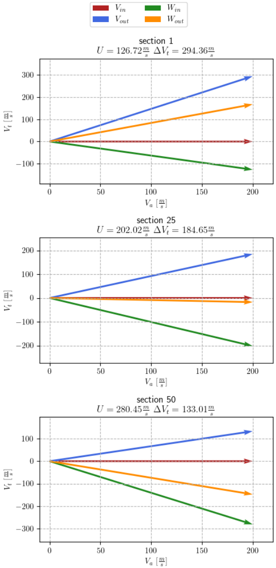
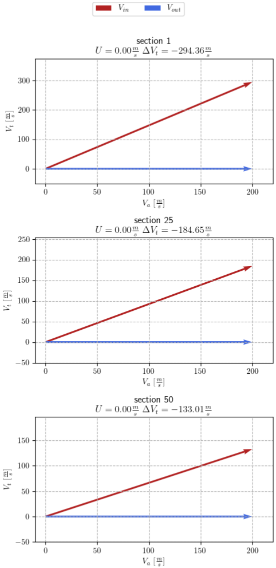
$V_{a_{mean}}$, $V_{t_{mean}}$ & U_{mean}

$$
\begin{aligned}
U_{mean} &= \frac{L_{eu}}{\psi} \\
V_{a_{mean}} &= \phi \; U_{mean} \\
L_{eu} &= U_1 \; V_{t1} - U_0 \; V_{t0} \\
&= U_{1_{mean}} \; V_{t1_{mean}} - U_{0_{mean}} \; V_{t0_{mean}} = U_{mean} \; \Delta V_{t_{mean}} \\
V_{t1} &= \Delta V_{t_{mean}} + V_{t0}
\end{aligned}
$$

- $\Delta V_{t_{mean}}$ computation allows to get a *first sketch* of the **velocity triangles**[5]
- The first analysis results are stored in `compressor_0.55_0.325_28_28.txt`

[5] **Free vortex** model based.


V_{in}
W_{in}
V_{out}
W_{out}
section 1
$U = 126.72\frac{m}{s}$ $\Delta V_t = 294.36\frac{m}{s}$
section 25
$U = 202.02\frac{m}{s}$ $\Delta V_t = 184.65\frac{m}{s}$
section 50
$U = 280.45\frac{m}{s}$ $\Delta V_t = 133.01\frac{m}{s}$
V_t $[\frac{m}{s}]$
V_a $[\frac{m}{s}]$


V_{in}
V_{out}
section 1
$U = 0.00\frac{m}{s}$ $\Delta V_t = -294.36\frac{m}{s}$
section 25
$U = 0.00\frac{m}{s}$ $\Delta V_t = -184.65\frac{m}{s}$
section 50
$U = 0.00\frac{m}{s}$ $\Delta V_t = -133.01\frac{m}{s}$
V_t $[\frac{m}{s}]$
V_a $[\frac{m}{s}]$

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     - Compressibility losses
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     - Tip leackage losses
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Profile losses

The profile losses used are related to the **Leiblein modeling** approach[6].
The model is based on the **equivalent diffusion factor**, D_{eq}:

$$\frac{W_{max}}{W_1} = 1.12 + 0.61\ \frac{cos(\beta_1)^2}{\sigma} \cdot \frac{r_1\ V_{t1} - r_2\ V_{t2}}{r_1\ V_{a1}}$$

$$D_{eq} = \frac{W_{max}}{W_1} \cdot \frac{W_1}{W_2}$$

D_{eq} will be used for the computation of $\bar{\omega}_{profile}$ as:

$$\bar{\omega}_{profile} = \frac{0.004\ \left(1 + 3.1\ \left(D_{eq} - 1\right)^2 + 0.4\ \left(D_{eq} - 1\right)^8\right)\ 2\ \sigma}{cos(\beta_2)\ \left(\frac{W_1}{W_2}\right)^2}$$

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[6] The following equations are interpolated data from [**?**, Ch. 6].

Compressibility losses – I

This losses can be seen as a **correction** of the **profile** losses due to the compressibility of the gas along its *journey* in the stage.

The correction referres to a **Leiblein correction** model that uses the **positive** and **negative** blade section incidence angle, i_c and i_s.

These new stall incidence angles will build a new **mean** incidence angle, i_m, that can be seen as the **optimum** incidence angle related to the inlet Mach conditions[7].

[7] The implemented model follows [**?**, Ch. 10]

Compressibility losses – II

$\bar{\omega}_{compressibility}$ setup:

- R_c & R_s computation:

$$R_c = 9 - \left[1 - \left(\frac{30}{\beta_1}\right)^{0.48}\right] \frac{\theta}{8.2}$$

$$R_s = 10.3 + \left(2.92 - \frac{\beta_1}{15.6}\right) \frac{\theta}{8.2}$$

- i_c & i_s computation:

$$i_c = i^* - \frac{R_c}{1 + 0.5\ M_1^3}$$

$$i_s = i^* + \frac{R_s}{1 + 0.5\left(K_{sh}\ M_1\right)^3}$$

Compressibility losses – III

- i_m computation: $i_m = i_c + \left(i_s - i_c\right) \frac{R_c}{R_c+R_s}$
- $\bar{\omega}_m$ computation: $\bar{\omega}_m = \bar{\omega}_{profile} \left[1 + \frac{\left(i_m - i^*\right)^2}{R_s^2}\right]$
- $\bar{\omega}_{compressibility}$ computation:

$$\bar{\omega}_{compressibility} = \bar{\omega}_m + \bar{\omega}_m \left[\frac{i - i_m}{i_c - i_m}\right]^2, \text{ if } i \leq i_m$$

$$\bar{\omega}_{compressibility} = \bar{\omega}_m + \bar{\omega}_m \left[\frac{i - i_m}{i_s - i_m}\right]^2, \text{ if } i \geq i_m$$

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Shock losses – I

The **relative** Mach number at the rotor inlet is slightly above **sonic speed**; a **shock wave** will be present at the rotor tip. From [**?**], **shock pattern** are related to **Mach number** and **airfoil shape**.

The **shock** losses modeling is related to **König losses** modeling approach. This model describes a **2 shock waves loss** using a **single normal shock** with respect to a computed Mach number, M_{in}.

König model depends mainly on **blade deflection angle**, θ, and **relative inlet Mach**, M_1.

Shock losses – II

- computation of the **expansion wave** angle, ϕ:
  $\phi = \frac{s\ cos(\psi)}{s\ sin(\psi)\ R_u}$, where $\psi = \psi_{(\beta_1,\gamma,\theta)}$
- computation of W_s and M_s using the **Prandtl-Meyer** expansion: $\phi = \int_{W_1}^{W_s} \sqrt{M^2 - 1}\ \frac{dW}{W}$
- M_{in} computation: $M_{in} = \sqrt{M_1\ M_s}$
- **normal shock** solution and computation of ΔP_t
- from ΔP_t, computation of $\bar{\omega}_{shock}$

Tip leackage losses

Again these losses are computed from [?]. The main concept is: computing a **total** blade pressure loss and **assume** to **distribute** the losses **linearly** from the hub to the tip.

$$\tau = \pi\ \delta_c \left[r_1 \rho_1 V_{a1_{mean}} + r_2 + \rho_2 V_{a2_{mean}} \right] \left[r_2 V_{t2_{mean}} - r_1 V_{t1_{mean}} \right]$$

$$\Delta P = \frac{\tau}{Z\ r_{tip} \delta_c\ c\ cos(\gamma)}$$

$$U_c = 0.816 \frac{\sqrt{\frac{2\Delta P}{\rho_{mean}}}}{N_{row}^{0.2}}$$

$$\dot{m}_c = \rho_{mean}\ U_c\ Z\ \delta_c\ c\ cos(\gamma)$$

$$\Delta P_t = \frac{\Delta P\ \dot{m}_c}{\dot{m}}$$

ΔP_t is the **overall** total pressure loss due to **leackage** of the blade.

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Radial equilibrium

The **radial equilibrium** equation: $\frac{\partial h_t}{\partial r} = T\frac{\partial s}{\partial r} + V_a \frac{\partial V_a}{\partial r} + V_t \frac{\partial r V_t}{\partial r}$ is converted into, for the exit station[8] of the blade, a **1st order ODE**:

$$-\frac{1}{2}\frac{\partial V_{a2}^2}{\partial r} + \frac{V_{a2}^2}{2\ c_P}\frac{\partial s_2}{\partial r} = -c_P\frac{\partial T_{t1}}{\partial r} - \omega\ \frac{\partial r V_{t2}}{\partial r} + \omega\ \frac{\partial r V_{t1}}{\partial r}$$
$$+T_{t1}\ \frac{\partial s_2}{\partial r} + \frac{\omega}{c_P} r V_{t2}\ \frac{\partial s_2}{\partial r} - \frac{\omega}{c_P} r V_{t1}\frac{\partial s_2}{\partial r} - \frac{1}{2\ c_P} V_{t2}^2 \frac{\partial s_2}{\partial r} + \frac{V_{t2}}{r}\frac{\partial r V_{t2}}{\partial r}$$

The **ODE** will be solved for V_{a2}^2.
s_2 is computed from $\sum_i \bar{\omega}_i$ treated earlier.
ω, V_{t1}, V_{t2}& T_{t1} are known.

- $r_{1_{mean}}\ \ V_{t1_{mean}} = r_1\ \ V_{t1_{(r_1)}}$
- $r_{2_{mean}}\ \ V_{t2_{mean}} = r_2\ \ V_{t2_{(r_2)}}$

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[8] 1 is the blade inlet station and 2 is the blade outlet section.

Blade shape

For the blade shape computation, it has been used the **Leiblein model** from [**?**].

$\texttt{NACA} - 65$ profile has been chosen for the blade generation[9].

The **main contraints** are: $\frac{t_b}{c} \approx 0.1$ and $max(\sigma) = 2.2$[10].

Due to the many possible blade configurations, an **optimization** procedure has been used for the computation of i^*, δ, θ and σ.

From θ and $\texttt{NACA} - 65$ C_{L0} surface coordinates, each section airfoil shape is **computed**.

[9] Due to the low tip sonic Mach number it has been chosen to use this profile as well for the blade tip instead of a supersonic adapted profile shape.

[10] $\frac{t_b}{c} \approx 0.1$ allows setting $K_{sh} \approx 0.1$. The upper bound on σ is made in order to limit the blade chord.

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turboLIB

The preliminary compressor design model program `turboLIB` can be downloaded from github.

Main objects and modules

- `turboClass.turboBlade.blade`: blade object
- `turboCoeff`: engineering coefficients module
  - `losses`: losses modeling
  - `similarity`: adimensional analysis
  - `lieblein`: blade modeling
- `geometry.bladeGeometry.geometryData`: airfoil object

NISRE setup

The `NISRE` in solved through a **double nested** loop:

- **continuity loop**
- **entropy loop**

Inside the **continuity loop** the `scipy.integrate.odeint` function is called for the solution of the V_{a2}^2 **ODE**.

Inside the **entropy loop** the `scipy.optimize.minimize` function is called for the computation of the blade **shape**.

.stl & .scad generation

At the end of the NISRE, all the main blade quantities are avaliable for the **generation** of the **3D geometry**. This geometry can be converted into a .stl file that can be used in OpenFOAM for the flow properties study. In addition a .scad file is made for understanding the blades position and check possible contacts between rotor and stator blades.

[?] suggested that a good distance between rotor and stator blades is half of the rotor chord[11].

[11] In the radial equilibrium study losses between rotor and stator blades are **neglected**.

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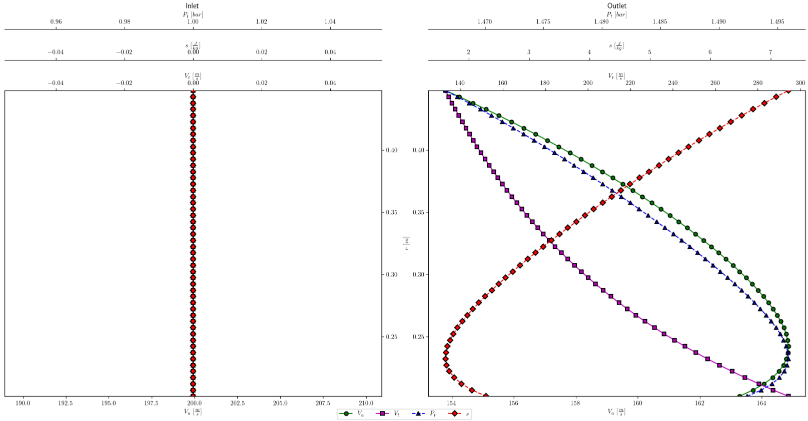
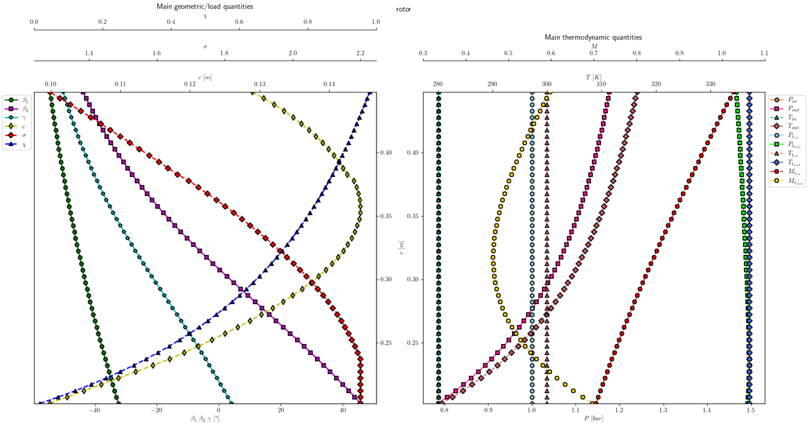
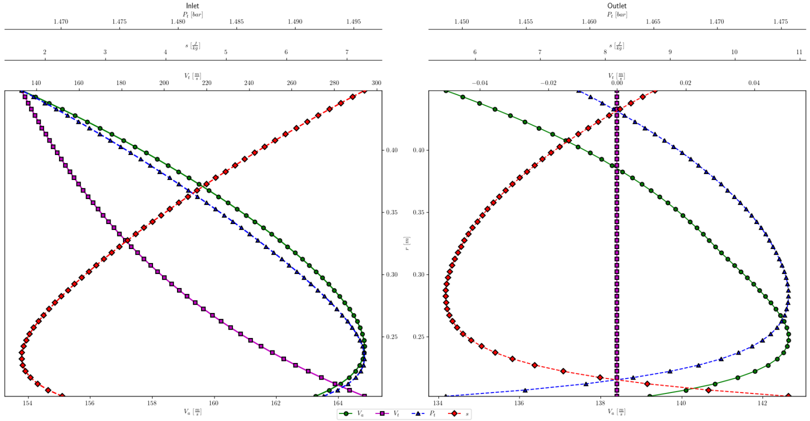
Rotor equilibrium results: NISRE



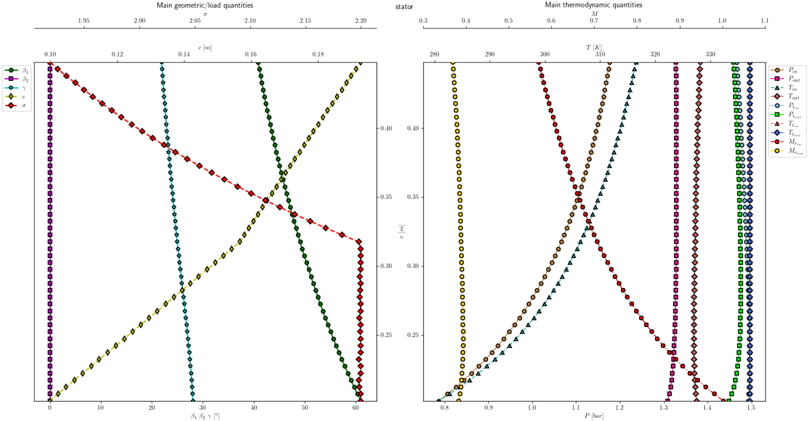
Rotor equilibrium results: main quantities



Stator equilibrium results: NISRE



Stator equilibrium results: main quantities



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Rotor & stator blades

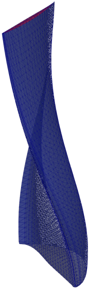


Figure 1: Rotor blade.

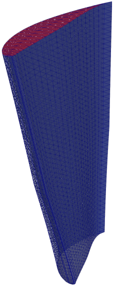
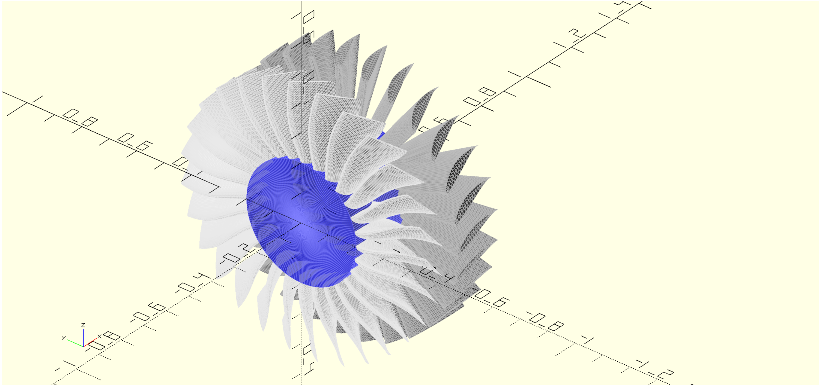


Figure 2: Stator blade.

Stage plot



Efficiency

The **rotor** efficiency is computed with:

$$\eta_{is_{rotor}} = \frac{W_1^2 - W_{2_{is}}^2}{W_1^2 - W_2}$$

The **stator** efficiency is computed with:

$$\eta_{is_{stator}} = \frac{\Delta h_{is}}{\Delta h_{real}}$$

The modeling results are stored into `compressor_0.55_0.325_28_28.txt`.

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Thank you!

Antonio Pucciarelli 974675

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