Turbomachinery: Compressor preliminary design

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Initial conditions & constraints

Inlet conditions

- $P_{T0} = 1bar$
- $T_{T0} = 300K$

Constraints

- $r_{max} = 0.45m$
- $\beta_{TT} = 1.45$
- $\dot{m} = 100\frac{kg}{s}$
- **max** η

Due to the **course track** and **preference**, the turbomachinery design will be on an **axial** compressor.

1. Problem Description
2. Mean Line
   - Problem setup
   - Main design quantities
   - $V_{t_{mean}}$, $V_{a_{mean}}$, U_{mean} & velocity triangles
3. Models
4. turboLIB & Results

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Problem setup: hypothesis

Hypothesis

- **not using** an **inlet guide vane** for simplicity of design
- keeping, in the similarity/adimensional analysis of the compressor, $V_{a_{mean}}$ **constant**[1]
- keeping the blade height, b_0, **constant** both in rotor and stator
- using a **mixed vortex** model for the **rotor** velocity triangles
- using a **second order** function for the **stator** velocity triangles
- neglecting inlet **entropy** generation and assuming **rotor inlet** quantities **constant**
- **shrouding** at blade tip not present
- **rotor-stator** losses neglected

[1] $\dot{m}$ corrections will be made later on in the **radial equilibrium** solution.

Problem setup: solution steps

Main procedural steps:

- λ and ψ computation from χ and V_{t0}
- ϕ and η computation
- $V_{a_{mean}}$ and L_{eu} computation from ϕ, β_{TT} and η
- computing **mean** velocity triangles, using the above hypothesis
- computing **mean thermodynamic** quantities
- computing **blade height**

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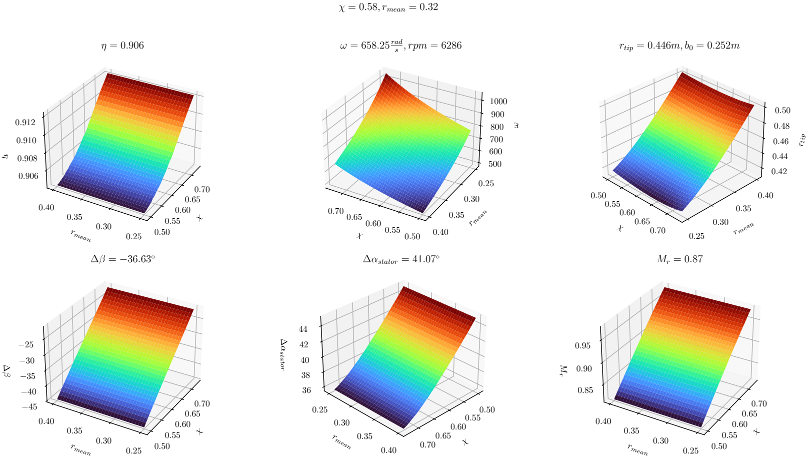
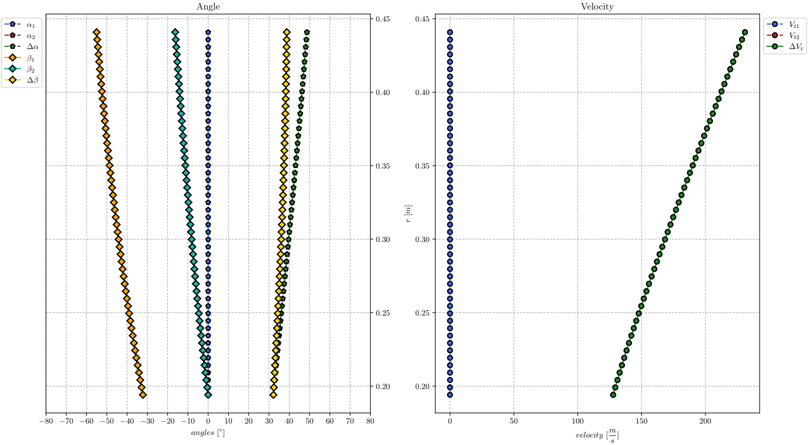
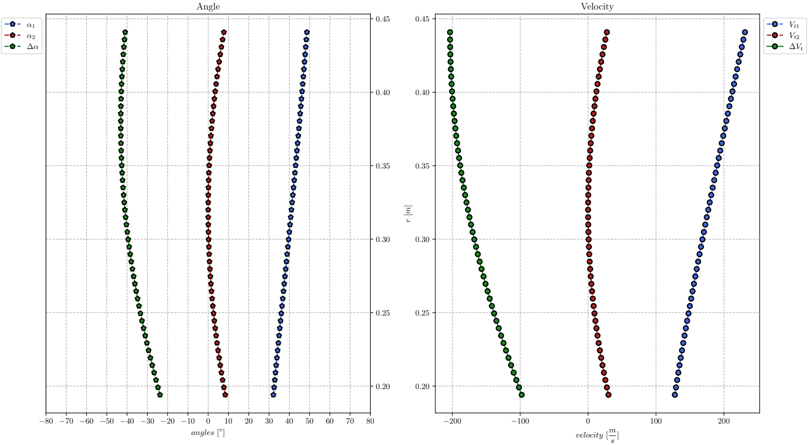
Graph analysis: χ & r_{mean}



Graph analysis: **rotor** α, β & V_t



Graph analysis: **stator** α & V_t



λ & ψ

From the previous **graphs**:

- $\chi = 0.58$
- $r_{mean} = 0.32m$
- $\frac{V_{t0}}{U_{mean}} = 0$

Taking into account the previous modeling **hypothesis**:

$$\lambda = \left(1 - \chi - \frac{V_{t0}}{U_{mean}}\right) \cdot 4$$

$$\psi = \frac{\lambda}{2} = \frac{L_{eu_{mean}}}{U_{mean}^2}$$

$$\chi = \frac{h_1 - h_0}{h_{T1} - h_{T0}} = \frac{\frac{W_0^2}{2} - \frac{W_1^2}{2}}{U_{mean}(V_{t1} - V_{t0})} = \frac{\frac{W_{a0}^2}{2} + \frac{W_{t0}^2}{2} - \frac{W_{a1}^2}{2} - \frac{W_{t1}^2}{2}}{U_{mean}(V_{t1} - V_{t0})} = \frac{\frac{W_{t0}^2}{2} - \frac{W_{t1}^2}{2}}{U_{mean}(V_{t1} - V_{t0})}.$$

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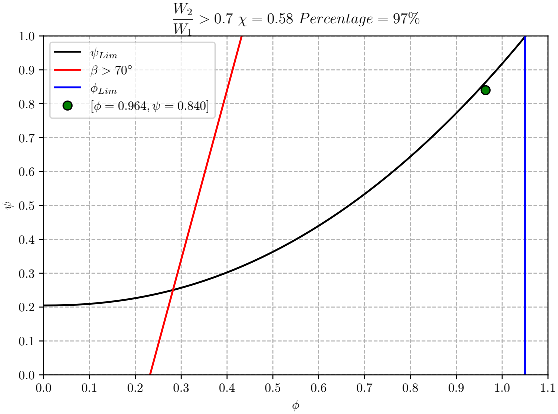
$\phi_{(\psi)}$

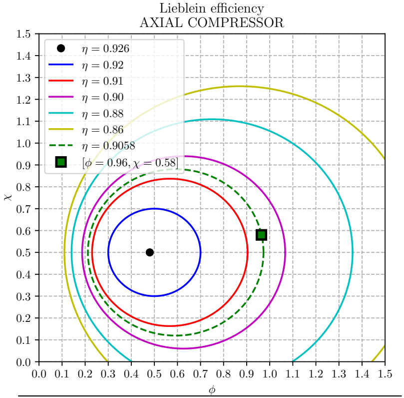
From [Aun04, Sec. 10.4] it is imposed that $\frac{W_2}{W_1} \geq 0.7$ with a *safety* margin of 3%. ϕ_{lim} line is related to the **surge safety margin**.



$$\phi = \frac{V_{a_{mean}}}{U_{mean}}.$$

η & L_{eu}

η is computed from an **Lieblein** efficiency chart[2] given ϕ and χ. This parameter will be used for the computation of L_{eu} given the β_{TT} target.



$$L_{is} = \frac{\gamma\ R}{\gamma - 1}\ T_{T0}\ (\beta_{TT}^{\frac{\gamma-1}{\gamma}} - 1)$$

$$L_{eu} = \frac{L_{is}}{\eta}$$

[2] This chart has been interpolated from the course slides charts.

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$V_{a_{mean}}$, $V_{t_{mean}}$ & U_{mean}

$$U_{mean} = \sqrt{\frac{L_{eu_{mean}}}{\psi}}$$

$$V_{a_{mean}} = \phi \; U_{mean}$$

$$L_{eu_{mean}} = U_{1_{mean}} \; V_{t1_{mean}} - U_{0_{mean}} \; V_{t0_{mean}} \overset{U_1 = U_0}{=} U_{mean} \; \Delta V_{t_{mean}}$$

$$V_{t1_{mean}} = \Delta V_{t_{mean}} + V_{t0_{mean}}$$

- $\Delta V_{t_{mean}}$ computation allows us to get a *first sketch* of the **velocity triangles**[3] using ϕ, ψ and L_{eu}[4] definitions. V_a is assumed **constant** all through the stage
- The first analysis results are stored in `compressor_0.58_0.32_45_35.txt`

[3] **Mixed vortex** model and **second order** function based.

[4] $L_{eu} = U_1 \; V_{t1} - U_0 \; V_{t0}$.

Profile losses

The profile losses used are related to the **Leiblein modeling** approach[5].
The model is based on the **equivalent diffusion factor**[6], D_{eq}:

$$\frac{W_{max}}{W_1} = 1.12 + 0.61\ \frac{cos(\beta_1)^2}{\sigma} \cdot \frac{r_1\ V_{t1} - r_2\ V_{t2}}{r_1\ V_{a1}}$$

$$D_{eq} = \frac{W_{max}}{W_1} \cdot \frac{W_1}{W_2}$$

D_{eq} will be used for the computation of $\bar{\omega}_{profile}$ as:

$$\bar{\omega}_{profile} = \frac{0.004\ \left(1 + 3.1\ \left(D_{eq} - 1\right)^2 + 0.4\ \left(D_{eq} - 1\right)^8\right)\ 2\ \sigma}{cos(\beta_2)\ \left(\frac{W_1}{W_2}\right)^2}$$

[5] The following equations are interpolated data from [Aun04, Sec. 6.4].

[6] It describes how important is the **velocity change** along the blade. It can be seen as an *indicator* of the **blade loading**.

Compressibility losses – I

These losses can be seen as a **correction** of the **profile** losses due to the compressibility of the gas along its *journey* in the stage.

The correction refers to a **Leiblein correction** model that uses the **positive** and **negative** blade section incidence angle[7], i_c and i_s.

These new stall incidence angles will build a new **mean** incidence angle, i_m, that can be seen as the **optimum** incidence angle related to the inlet Mach conditions[8].

[7] i_c and i_s are related to the total pressure losses, $\bar{\omega}_c$ and $\bar{\omega}_s$, that are **twice** the minimum total pressure loss, $\bar{\omega}$, obtained at the **design incidence angle**, i^*.

[8] The implemented model follows [Aun04, Sec. 6.6].

Compressibility losses – II

$\bar{\omega}_{compressibility}$ setup:

- R_c and R_s computation[9]:

$$R_c = 9 - \left[1 - \left(\frac{30}{\beta_1}\right)^{0.48}\right] \frac{\theta}{8.2}$$

$$R_s = 10.3 + \left(2.92 - \frac{\beta_1}{15.6}\right) \frac{\theta}{8.2}$$

- i_c and i_s computation due to **compressibility** effects:

$$i_c = i^* - \frac{R_c}{1 + 0.5\ M_1^3}$$

$$i_s = i^* + \frac{R_s}{1 + 0.5\left(K_{sh}\ M_1\right)^3}$$

[9] R_c and R_s are **range indices** for the computation of i_c and i_s.

Compressibility losses – III

- i_m computation[10]: $i_m = i_c + \left(i_s - i_c\right) \frac{R_c}{R_c+R_s}$
- $\bar{\omega}_m$ computation: $\bar{\omega}_m = \bar{\omega}_{profile} \left[1 + \frac{\left(i_m - i^*\right)^2}{R_s^2}\right]$
- $\bar{\omega}_{compressibility}$ computation[11]:

$$\bar{\omega}_{compressibility} = \bar{\omega}_m + \bar{\omega}_m \left[\frac{i - i_m}{i_c - i_m}\right]^2, \text{ if } i \leq i_m$$

$$\bar{\omega}_{compressibility} = \bar{\omega}_m + \bar{\omega}_m \left[\frac{i - i_m}{i_s - i_m}\right]^2, \text{ if } i \geq i_m$$

[10] i_m is the incidence angle at wich corresponds, having accounted the **compressibility**, the **minimum pressure loss**, $\bar{\omega}_m$.

[11] After having computed i_c, i_s, i_m and $\bar{\omega}_m$. $\bar{\omega}_{compressibility}$ is a function of i. In the **total pressure losses study**: $\bar{\omega}_{compressibility} = \bar{\omega}_{compressibility_{(i^*)}}$.

Secondary flow losses

These losses are relative to the **secondary flow** inside the compressor and are *usually* **greater** than the other losses. These losses are related to **eddies** generated with the **blade-flow** interaction and **streamlines** displacement due to the presence of **pressure gradients**.

These losses are computed with **Howell**'s model [Aun04, Ch. 6][12]:

$$\bar{\beta} = \frac{arctan(tan(\beta_1) + tan(\beta_2))}{2}; \qquad \text{I}$$

$$C_L = 2\ cos(\bar{\beta}) \cdot \frac{tan(\beta_1) - tan(\beta_2)}{\sigma}; \qquad \text{II}$$

$$C_D = 0.18\ C_L^2; \qquad \text{III}$$

$$\bar{\omega}_{secondary} = C_D\ \sigma \cdot \frac{cos(\beta_1)^2}{cos(\bar{\beta})^3}; \qquad \text{loss computation}$$

[12] Howell computed a secondary flow loss model that is automatically embedded into $\bar{\omega}_{profile}$. It is used for the **estimation** of the **blade number**.

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End wall losses

These losses are related the interaction between the flow and the **compressor case**. They are *lower* than the **secondary flow** losses. It has been used a simple and fast relation made by **Howell** [Aun04, Ch. 6][13]:

$$C_D = 0.02 \, \frac{s}{b_0}$$

$$\bar{\omega}_{endWall} = C_D \, \sigma \cdot \frac{cos(\beta_1)^2}{cos(\bar{\beta})^3}; \qquad \text{loss computation}$$

[13]This loss is kept into account in the **blade numbering** study.

Shock losses – I

The **relative** Mach number at the rotor inlet is slightly above **sonic speed**; a **shock wave** will be present at the rotor tip. From [MF20], **shock pattern** is related to **Mach number** and **airfoil shape**.

The **shock** losses modeling is related to **König losses** modeling approach. This model describes a **2 shock waves loss**[14] using a **single normal shock** with respect to a computed Mach number, M_{in}.

König model depends mainly on **blade deflection angle**, θ, and **relative inlet Mach**[15], $M_{1,r}$.

[14] For a flow in **unstarted condition**: shock wave *followed by* an expansion wave and another shock wave. **Unstarted conditions** are for $M_{a,r} < 1$.

[15] **Leading edge radius** is not taken into account due to the *approximated nature* of the model.

Shock losses – II

Swan and **Miller**, [Aun04, Sec. 6.7], derived a **shock loss formulation** from **König model**.

The following are the steps made for the computation of the **shock loss**, $\bar{\omega}_{shock}$, at each **blade section**:

- computation of the **expansion wave** angle, ϕ:

$$\phi = \frac{s\ cos(\psi)}{s\ sin(\psi)\ R_u};\ \text{where } \psi = \psi_{(\beta_1,\ \gamma,\ \theta)}$$

- computation of W_s and M_s using **Prandtl-Meyer** expansion:

$$\phi = \int_{W_1}^{W_s} \sqrt{M^2 - 1}\ \frac{dW}{W}$$

- M_{in} computation: $M_{in} = \sqrt{M_{1,r}\ M_s}$
- **normal shock** solution and computation of ΔP_T
- from ΔP_T, computation of $\bar{\omega}_{shock}$

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Tip leackage losses

Again these losses are computed from [Aun04, Sec. 6.9]. The main concept is: computing a **total** blade pressure loss and **assuming linear distribution** of losses from the hub to the tip[16].

$$\tau = \pi\ \delta_c \left[r_1 \rho_1 V_{a1_{mean}} + r_2 \rho_2 V_{a2_{mean}} \right] \left[r_2 V_{t2_{mean}} - r_1 V_{t1_{mean}} \right]$$

$$\Delta P = \frac{\tau}{Z\ r_{tip} \delta_c\ c\ cos(\gamma)}$$

$$U_c = 0.816 \frac{\sqrt{\frac{2\Delta P}{\rho_{mean}}}}{N_{row}^{0.2}}$$

$$\dot{m}_c = \rho_{mean}\ U_c\ Z\ \delta_c\ c\ cos(\gamma)$$

$$\Delta P_T = \frac{\Delta P\ \dot{m}_c}{\dot{m}}$$

ΔP_T is the **overall** total pressure loss due to **tip leackage**.

[16] Z is the number of blades. δ_c is the tip clearance. N_{row} is the number of blade rows in the compressor.

Blade shape setup

The **Leiblein model** from [Aun04, Ch. 6] has been used for the blade shape computation.

$\texttt{NACA} - 65$ profile has been chosen for the blade generation[17].

The **main contraints** are: $\frac{t_b}{c} \approx 0.1$ and σ is related just to s[18].

Due to the many possible blade configurations, an **optimization** procedure has been used for the computation of i^*, δ and θ.

Each section airfoil shape is **computed** from θ and $\texttt{NACA} - 65$ C_{L0} surface coordinates.

[17] Due to the low tip sonic Mach number it has been chosen to use this profile as well for the blade tip instead of a supersonic adapted profile shape.

[18] $\frac{t_b}{c} \approx 0.1$ allows setting $K_{sh} \approx 0.1$. The blade chord, c, is set up as constant during the **blade numbering** study: using $AR = \frac{b_0}{c}$.

i^* computation

The **incidence angle**, i^*, is computed using:

$$K_{t,i} = \left(10\ \frac{t_b}{c}\right)^q \ ;\ \text{where } q = \frac{0.28}{0.1 + (\frac{t_b}{c})^{0.3}}$$

$$(i_0^*)_{10} = \frac{\beta_0^p}{5 + 46 \cdot e^{-2.3\sigma}} - 0.1\ \sigma^3\ e^{\frac{\beta_0 - 70}{4}} \ ;\ \text{where } p = 0.914 + \frac{\sigma^3}{160}$$

$$n = 0.025\sigma - 0.06 - \frac{\left(\frac{\beta_0}{90}\right)^{1+1.2\sigma}}{1.5 + 0.43\sigma}$$

$$i^* = K_{sh}\ K_{t,i}\ (i_0^*)_{10} + n\ \theta$$

δ computation

The **deviation angle**, δ, is computed using:

$$K_{t,\delta} = 6.25\frac{t_b}{c} + 37.5\left(\frac{t_b}{c}\right)^2$$

$$(\delta_0^*)_{10} = 0.01\sigma\beta_0 + (0.74\sigma^{1.9} + 3\sigma)\left(\frac{\beta_0}{90}\right)^{1.67+1.09\sigma}$$

$$b = 0.9625 - 0.17\frac{\beta_0}{100} - 0.85\left(\frac{\beta_0}{100}\right)^3$$

$$m = \frac{m_{1.0}}{\sigma^b} \text{ ; where } m_{1.0} = 0.17 - 0.0333\frac{\beta_0}{100} + 0.333\left(\frac{\beta_0}{100}\right)^2$$

$$\delta = K_{sh}\ K_{t,\delta}\ (\delta_0^*)_{10} + m\ \theta$$

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θ computation

Starting from the **known** flow deflection angle[19], ε, it is necessary to compute:

$$\theta = \varepsilon - i^* + \delta$$

Since i^* and δ are functions of θ, the computation of θ is made by an **iterative process**:

$$\theta = \varepsilon - i^*_{(\theta)} + \delta_{(\theta)}$$

Once found θ, the **total pressure loss** coefficients, $\bar{\omega}_*$, are computed.

[19] $\varepsilon = \beta_{inlet} - \beta_{outlet}$. β_1 and β_2 change with respect to the **axial speed**.

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   - Blade shape
   - Radial equilibrium
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Equation setup

The **radial equilibrium** equation: $\frac{\partial h_t}{\partial r} = T\frac{\partial s}{\partial r} + V_a\frac{\partial V_a}{\partial r} + V_t\frac{\partial rV_t}{\partial r}$ is converted into, for the exit station[20] of the blade, a **1st order ODE**:

$$-\frac{1}{2}\frac{\partial V_{a2}^2}{\partial r} + \frac{V_{a2}^2}{2\ c_P}\frac{\partial s_2}{\partial r} = -c_P\frac{\partial T_{T1}}{\partial r} - \omega\ \frac{\partial rV_{t2}}{\partial r} + \omega\ \frac{\partial rV_{t1}}{\partial r}$$
$$+T_{T1}\ \frac{\partial s_2}{\partial r} + \frac{\omega}{c_P}rV_{t2}\ \frac{\partial s_2}{\partial r} - \frac{\omega}{c_P}rV_{t1}\frac{\partial s_2}{\partial r} - \frac{1}{2\ c_P}V_{t2}^2\frac{\partial s_2}{\partial r} + \frac{V_{t2}}{r}\frac{\partial rV_{t2}}{\partial r}$$

- The **ODE** will be solved for V_{a2}^2
- s_2 is computed from[21] $\sum_i \bar{\omega}_i$
- ω, V_{t1}, V_{t2} and T_{T1} are known

[20] 1 is the blade inlet station and 2 is the blade outlet section.
[21] $\bar{\omega}_*$ computation has been treated earlier in Losses modeling.

Δs computation

From pressure loss coefficients it is possible compute the **outlet total pressure** as:

$$P_{T2,r} = P_{T1,r} + \sum_i \bar{\omega}_i \left(P_{T1,r} - P_{1,r} \right)$$

The **entropy variation** is computed as:

$$\Delta s = s_2 - s_1 = c_P \log \frac{T_{T2,r}}{T_{T1,r}} - R \log \frac{P_{T2,r}}{P_{T1,r}}$$

Frames

- $T_{T2} = T_{T1}$ in **stators** and $T_{T2,r} = T_{T1,r}$ in **rotors**
- $\bar{\omega}_*$ have to be computed using **relative** quantities for **rotors** and **absolute** quantities for **stators**
- $T_{T,r} = T + \frac{W^2}{2c_P} = T_T + \frac{W^2 - V^2}{2c_P}$
- $P_{T,r} = P_T \left(\frac{T_{T,r}}{T_T} \right)^{\frac{\gamma}{\gamma-1}}$

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     - NISRE
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turboLIB

The preliminary compressor design model program `turboLIB` can be downloaded from GitHub.

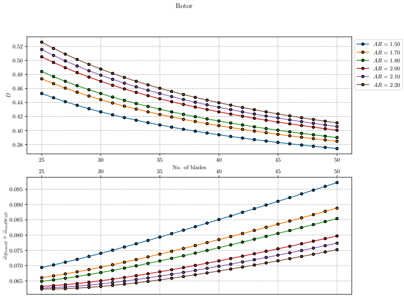
Main objects and modules

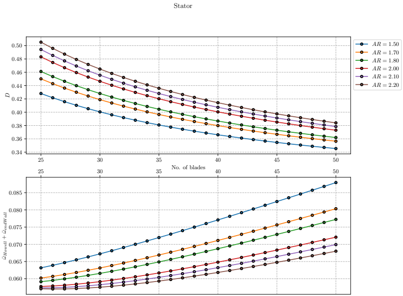
- `turboClass.turboBlade.blade`: blade object
- `turboCoeff`: engineering coefficients module
  - `losses`: losses modeling
  - `similarity`: adimensional analysis
  - `lieblein`: blade modeling
- `geometry.bladeGeometry.geometryData`: airfoil object

Blade number

In order to define a proper number of blades for the rotor and the stator, **Howell**'s relations have been used for the estimation of the **losses**. These relations, [Aun04, Ch. 6], are:

- $\bar{\omega}_{profile+secondary}$, this is a relation that takes into account **profile** and **3D** losses
- $\bar{\omega}_{endWall}$, this is previous expalined **end wall** loss





NISRE

The `NISRE` is solved through a **double nested** loop:

- **continuity loop**. Inside the **continuity loop** the `scipy.integrate.odeint` function is used for the solution of the V_{a2}^2 **ODE**
- **entropy loop**. Inside the **entropy loop** the `scipy.optimize.minimize` function is used for the computation of the blade **shape**

.stl & .scad generation

At the end of the NISRE, all the main blade quantities are avaliable for the **generation** of the **3D geometry**. This geometry can be converted into a .stl file that can be used in OpenFOAM for the flow properties study. In addition a .scad file is made for understanding position and checking possible contacts between rotor and stator blades.

[Bas06] suggested that a good distance between rotor and stator blades is half of the rotor chord[22].

[22] In the radial equilibrium study losses between rotor and stator blades are **neglected**.

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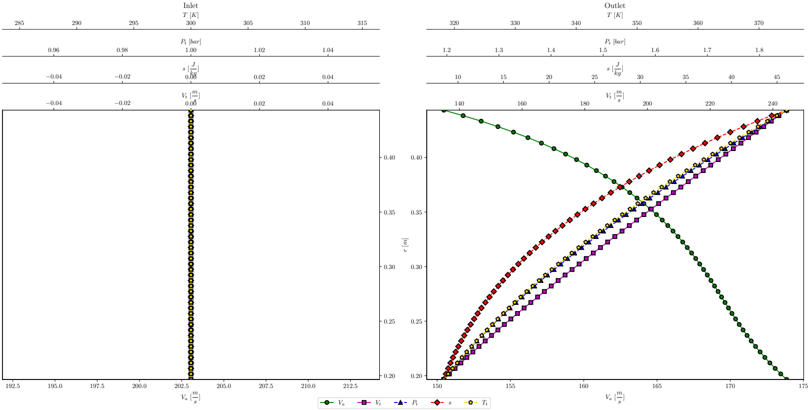
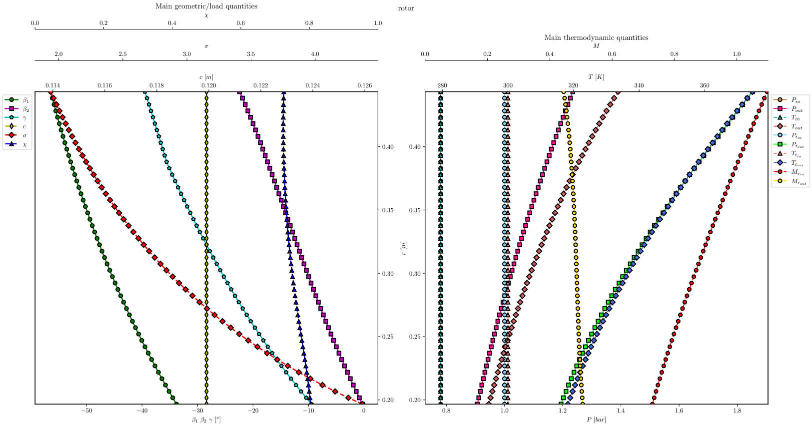
Rotor equilibrium results: NISRE



Rotor equilibrium results: main quantities



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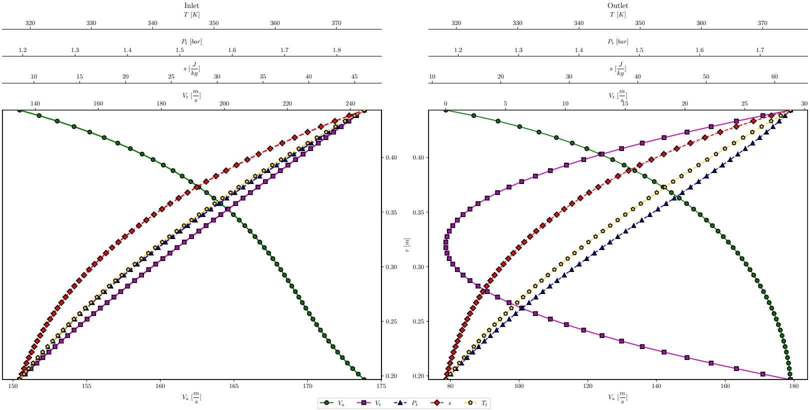
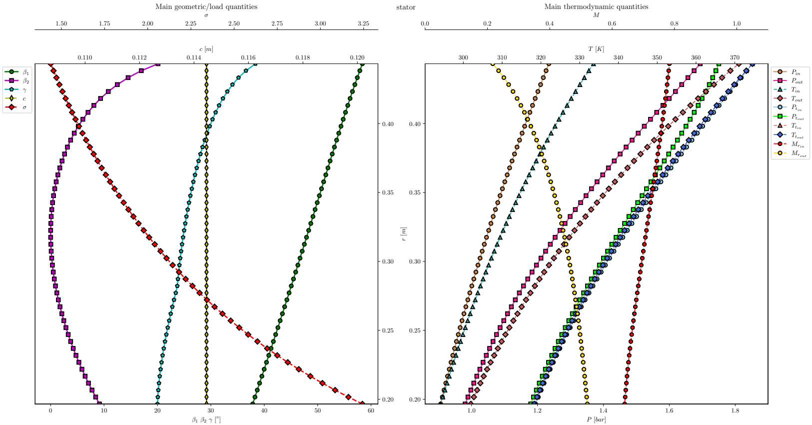
Stator equilibrium results: NISRE

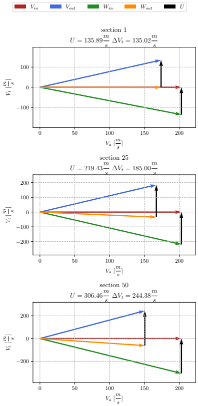


Stator equilibrium results: main quantities

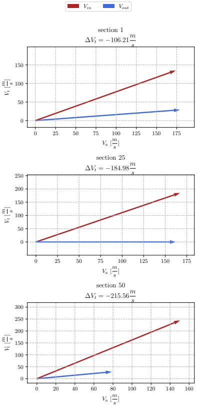


Velocity triangles

- Inlet: **axial** velocity
- Outlet: **mixed vortex** model



- Inlet: **mixed vortex** model
- Outlet: **second order** function



Rotor & stator blades

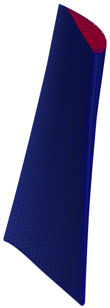


Figure 1: Rotor blade.

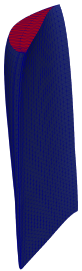
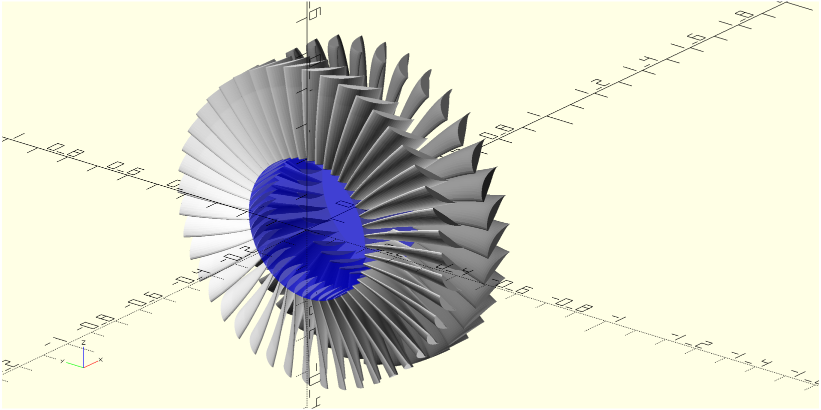


Figure 2: Stator blade.

Stage plot



Efficiency

The **rotor** efficiency is computed with:

$$\eta_{is_{rotor}} = \frac{W_1^2 - W_{2_{is}}^2}{W_1^2 - W_2^2}$$

The **stator** efficiency is computed with:

$$\eta_{is_{stator}} = \frac{\Delta h_{is}}{\Delta h_{real}}$$

The modeling results are stored into `compressor_0.58_0.32_45_35.txt`.

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[Aun04] Ronald H. Aungier. “Axial-Flow Compressors: a Strategy for Aerodynamic Design and Analysis”. In: *Appl. Mech. Rev.* 57.4 (2004).

[Bas06] Erian A. Baskharone. *Principles of turbomachinery in air-breathing engines*. Cambridge University Press, 2006.

[MF20] Marco Manfredi and Fabrizio Fontaneto. “Transonic Axial Compressors Loss Correlations: Part I—Analysis and Update of Loss Models”. In: *Turbo Expo: Power for Land, Sea, and Air*. Vol. 84065. American Society of Mechanical Engineers. 2020, V02AT32A029.

Thank you!

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