

TURBOMACHINERY: COMPRESSOR PRELIMINARY DESIGN

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April 22, 2022



1 Problem Description

2 Mean Line

3 Models

4 turboLIB & Results

Initial conditions & constraints

Inlet conditions

- $P_{T0} = 1\text{bar}$
- $T_{T0} = 300K$

Constraints

- $r_{max} = 0.45m$
- $\beta_{TT} = 1.45$
- $\dot{m} = 100 \frac{\text{kg}}{\text{s}}$
- **max** η

Due to the **course track** and **preference**, the turbomachinery design will be on an **axial** compressor.

1 Problem Description

2 Mean Line

Problem setup

Main design quantities

$V_{t_{mean}}$, $V_{a_{mean}}$, U_{mean} & velocity triangles

3 Models

4 turboLIB & Results

1 Problem Description

2 Mean Line

Problem setup

Main design quantities

$V_{t_{mean}}$, $V_{a_{mean}}$, U_{mean} & velocity triangles

3 Models

4 turboLIB & Results

Problem setup: hypothesis

Hypothesis

- **not using** an **inlet guide vane** for simplicity of design¹
- keeping, in the similarity/adimensional analysis of the compressor, $V_{a\text{mean}}$ **constant**²
- keeping the blade height, b_0 , **constant** both in rotor and stator³
- using a **free vortex** model for the velocity triangles
- neglecting inlet **entropy** generation and assuming **rotor inlet** quantities **constant**
- **shrouding** at blade tip not present
- **rotor-stator** losses neglected

¹ $V_{t0} = 0 \frac{m}{s}$ and χ dictate the behaviour of λ .

² \dot{m} corrections will be made later on in the **radial equilibrium** solution.

³ In order to keep each **blade streamtube section** as simple as possible.

Problem setup: solution steps

Main procedural steps:

- λ and ψ computation from χ and V_{t0}
- ϕ and η computation
- $V_{a_{mean}}$ and L_{eu} computation from ϕ , β_{TT} and η
- computing **mean** velocity triangles, using the above hypothesis
- computing **mean thermodynamic** quantities
- computing **blade height**

1 Problem Description

2 Mean Line

Problem setup

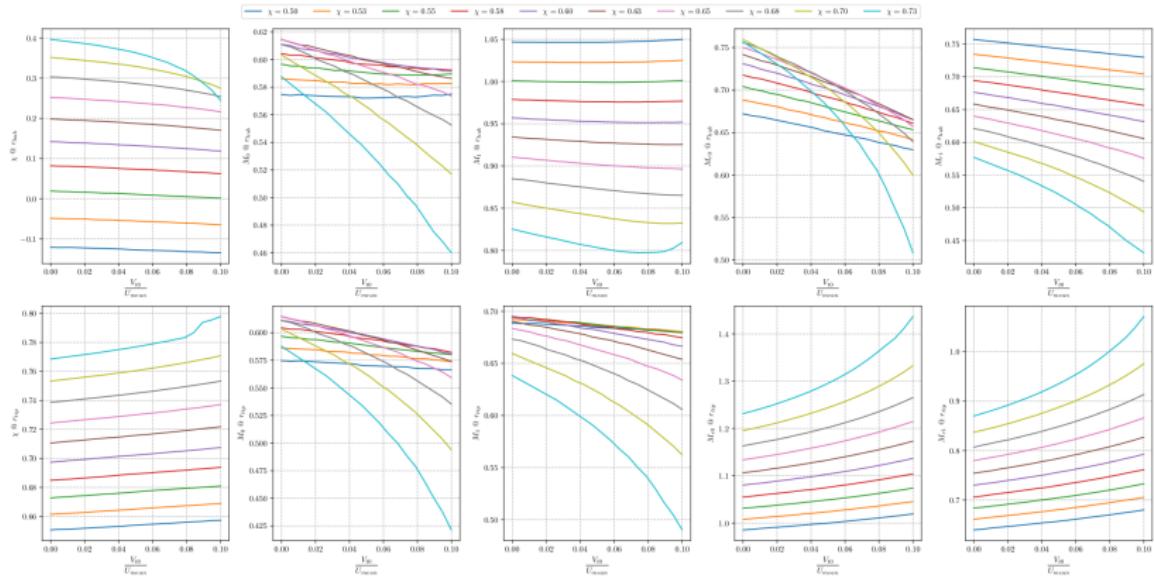
Main design quantities

$V_{t_{mean}}$, $V_{a_{mean}}$, U_{mean} & velocity triangles

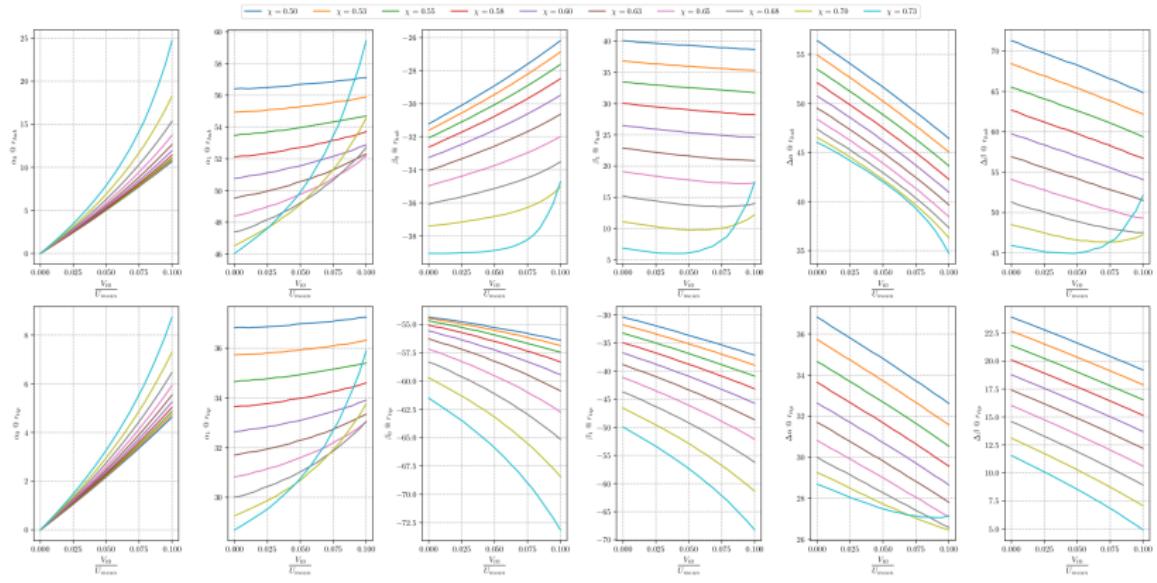
3 Models

4 turboLIB & Results

Graph analysis: χ & M



Graph analysis: α & β



λ & ψ

From the previous **graphs**:

- $\chi = 0.55$
- $r_{mean} = 0.325m$
- $\frac{V_{t0}}{U_{mean}} = 0$

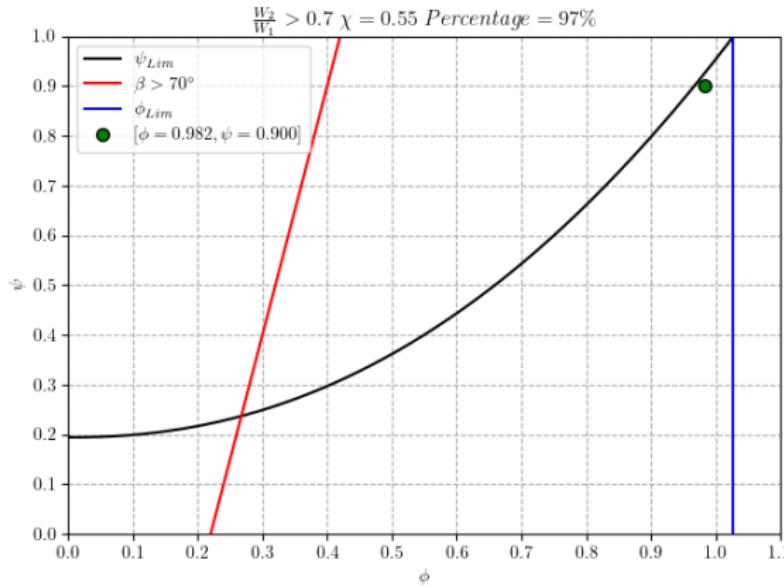
Taking into account the previous modeling **hypothesis**:

$$\lambda = \left(1 - \chi - \frac{V_{t0}}{U_{mean}}\right) \cdot 4$$

$$\psi = \frac{\lambda}{2}$$

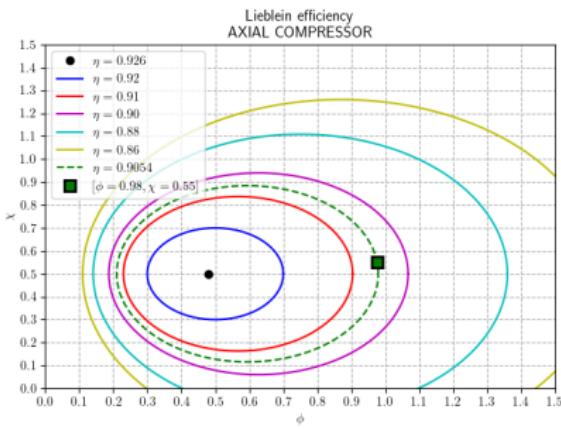
$\phi(\psi)$

From [Aun04, Sec. 10.4] it is imposed that $\frac{W_2}{W_1} \geq 0.7$ with a *safety margin* of 3%.



η & L_{eu}

η is computed from an **Lieblein** efficiency chart⁴ given ϕ and χ . This parameter will be used for the computation of L_{eu} given the β_{TT} target.



$$L_{is} = \frac{\gamma}{\gamma - 1} T_{T0} (\beta_{TT}^{\frac{\gamma-1}{\gamma}} - 1)$$

$$L_{eu} = \frac{L_{is}}{\eta}$$

⁴This chart has been interpolated from the course slides charts.

1 Problem Description

2 Mean Line

Problem setup

Main design quantities

$V_{t_{mean}}$, $V_{a_{mean}}$, U_{mean} & velocity triangles

3 Models

4 turboLIB & Results

$V_{a_{mean}}$, $V_{t_{mean}}$ & U_{mean}

$$U_{mean} = \frac{L_{eu}}{\psi}$$

$$V_{a_{mean}} = \phi \ U_{mean}$$

$$L_{eu} = U_1 \ V_{t1} - U_0 \ V_{t0}$$

$$= U_{1_{mean}} \ V_{t1_{mean}} - U_{0_{mean}} \ V_{t0_{mean}} = U_{mean} \ \Delta V_{t_{mean}}$$

$$V_{t1} = \Delta V_{t_{mean}} + V_{t0}$$

- $\Delta V_{t_{mean}}$ computation allows us to get a *first sketch* of the **velocity triangles**⁵
- The first analysis results are stored in compressor_0.55_0.325_28_28.txt

⁵Free vortex model based.

① Problem Description

② Mean Line

③ Models

Losses modeling

Radial equilibrium

Blade shape

④ turboLIB & Results

① Problem Description

② Mean Line

③ Models

Losses modeling

Profile losses

Compressibility losses

Secondary flow losses

Shock losses

Tip leakage losses

Radial equilibrium

Blade shape

④ turboLIB & Results

Profile losses

The profile losses used are related to the **Leiblein modeling approach**⁶.

The model is based on the **equivalent diffusion factor**, D_{eq} :

$$\frac{W_{max}}{W_1} = 1.12 + 0.61 \frac{\cos(\beta_1)^2}{\sigma} \cdot \frac{r_1}{r_1} \frac{V_{t1} - r_2}{V_{a1}} V_{t2}$$

$$D_{eq} = \frac{W_{max}}{W_1} \cdot \frac{W_1}{W_2}$$

D_{eq} will be used for the computation of $\bar{\omega}_{profile}$ as:

$$\bar{\omega}_{profile} = \frac{0.004 \left(1 + 3.1 (D_{eq} - 1)^2 + 0.4 (D_{eq} - 1)^8 \right) 2 \sigma}{\cos(\beta_2) \left(\frac{W_1}{W_2} \right)^2}$$

⁶The following equations are interpolated data from [Aun04, Sec. 6.4].

Compressibility losses – I

These losses can be seen as a **correction** of the **profile** losses due to the compressibility of the gas along its *journey* in the stage.

The correction refers to a **Leiblein correction** model that uses the **positive** and **negative** blade section incidence angle, i_c and i_s .

These new stall incidence angles will build a new **mean** incidence angle, i_m , that can be seen as the **optimum** incidence angle related to the inlet Mach conditions⁷.

⁷The implemented model follows [Aun04, Sec. 6.6]

Compressibility losses – II

$\bar{\omega}_{compressibility}$ setup:

- R_c & R_s computation:

$$R_c = 9 - \left[1 - \left(\frac{30}{\beta_1} \right)^{0.48} \right] \frac{\theta}{8.2}$$

$$R_s = 10.3 + \left(2.92 - \frac{\beta_1}{15.6} \right) \frac{\theta}{8.2}$$

- i_c & i_s computation:

$$i_c = i^* - \frac{R_c}{1 + 0.5 M_1^3}$$

$$i_s = i^* + \frac{R_s}{1 + 0.5 (K_{sh} M_1)^3}$$

Compressibility losses – III

- i_m computation: $i_m = i_c + (i_s - i_c) \frac{R_c}{R_c + R_s}$
- $\bar{\omega}_m$ computation: $\bar{\omega}_m = \bar{\omega}_{profile} \left[1 + \frac{(i_m - i^*)^2}{R_s^2} \right]$
- $\bar{\omega}_{compressibility}$ computation:

$$\bar{\omega}_{compressibility} = \bar{\omega}_m + \bar{\omega}_m \left[\frac{i - i_m}{i_c - i_m} \right]^2, \text{ if } i \leq i_m$$

$$\bar{\omega}_{compressibility} = \bar{\omega}_m + \bar{\omega}_m \left[\frac{i - i_m}{i_s - i_m} \right]^2, \text{ if } i \geq i_m$$

Secondary flow losses

Secondary flow losses

Shock losses – I

The **relative** Mach number at the rotor inlet is slightly above **sonic speed**; a **shock wave** will be present at the rotor tip. From [MF20], **shock pattern** is related to **Mach number** and **airfoil shape**.

The **shock** losses modeling is related to **König losses** modeling approach. This model describes a **2 shock waves loss** using a **single normal shock** with respect to a computed Mach number, M_{in} .

König model [Aun04, Sec. 6.7] depends mainly on **blade deflection angle**, θ , and **relative inlet Mach**, M_1 .

Shock losses – II

- computation of the **expansion wave** angle, ϕ :
$$\phi = \frac{s \cos(\psi)}{s \sin(\psi) R_u}, \text{ where } \psi = \psi_{(\beta_1, \gamma, \theta)}$$
- computation of W_s and M_s using the **Prandtl-Meyer** expansion:
$$\phi = \int_{W_1}^{W_s} \sqrt{M^2 - 1} \frac{dW}{W}$$
- M_{in} computation:
$$M_{in} = \sqrt{M_1 M_s}$$
- normal shock** solution and computation of ΔP_T
- from ΔP_T , computation of $\bar{\omega}_{shock}$

Tip leakage losses

Again these losses are computed from [Aun04, Sec. 6.9]. The main concept is: computing a **total** blade pressure loss and **assuming linear distribution** of losses from the hub to the tip.

$$\tau = \pi \delta_c \left[r_1 \rho_1 V_{a1_mean} + r_2 + \rho_2 V_{a2_mean} \right] \left[r_2 V_{t2_mean} - r_1 V_{t1_mean} \right]$$

$$\Delta P = \frac{\tau}{Z r_{tip} \delta_c c \cos(\gamma)}$$

$$U_c = 0.816 \sqrt{\frac{2\Delta P}{\rho_{mean}}} N_{row}^{0.2}$$

$$\dot{m}_c = \rho_{mean} U_c Z \delta_c c \cos(\gamma)$$

$$\Delta P_T = \frac{\Delta P \dot{m}_c}{\dot{m}}$$

ΔP_T is the **overall** total pressure loss due to **leackage** of the blade.

① Problem Description

② Mean Line

③ Models

Losses modeling

Radial equilibrium

Blade shape

④ turboLIB & Results

Radial equilibrium

The **radial equilibrium** equation: $\frac{\partial h_t}{\partial r} = T \frac{\partial s}{\partial r} + V_a \frac{\partial V_a}{\partial r} + V_t \frac{\partial rV_t}{\partial r}$ is converted into, for the exit station⁸ of the blade, a **1st order ODE**:

$$\begin{aligned} -\frac{1}{2} \frac{\partial V_{a2}^2}{\partial r} + \frac{V_{a2}^2}{2 c_P} \frac{\partial s_2}{\partial r} &= -c_P \frac{\partial T_{T1}}{\partial r} - \omega \frac{\partial rV_{t2}}{\partial r} + \omega \frac{\partial rV_{t1}}{\partial r} \\ + T_{T1} \frac{\partial s_2}{\partial r} + \frac{\omega}{c_P} rV_{t2} \frac{\partial s_2}{\partial r} - \frac{\omega}{c_P} rV_{t1} \frac{\partial s_2}{\partial r} - \frac{1}{2 c_P} V_{t2}^2 \frac{\partial s_2}{\partial r} &+ \frac{V_{t2}}{r} \frac{\partial rV_{t2}}{\partial r} \end{aligned}$$

The **ODE** will be solved for V_{a2}^2 .

s_2 is computed from $\sum_i \bar{\omega}_i$ treated earlier.

ω , V_{t1} , V_{t2} & T_{T1} are known.

- $r_{1_{mean}} V_{t1_{mean}} = r_1 V_{t1(r_1)}$
- $r_{2_{mean}} V_{t2_{mean}} = r_2 V_{t2(r_2)}$

⁸1 is the blade inlet station and 2 is the blade outlet section.

① Problem Description

② Mean Line

③ Models

Losses modeling

Radial equilibrium

Blade shape

④ turboLIB & Results

Blade shape

The **Leiblein model** from [Aun04, Ch. 6] has been used for the blade shape computation.

NACA – 65 profile has been chosen for the blade generation⁹.

The **main constraints** are: $\frac{t_b}{c} \approx 0.1$ and $\max(\sigma) = 2.2$ ¹⁰.

Due to the many possible blade configurations, an **optimization** procedure has been used for the computation of i^* , δ , θ and σ .

Each section airfoil shape is **computed** from θ and NACA – 65 C_{L0} surface coordinates.

⁹ Due to the low tip sonic Mach number it has been chosen to use this profile as well for the blade tip instead of a supersonic adapted profile shape.

¹⁰ $\frac{t_b}{c} \approx 0.1$ allows setting $K_{sh} \approx 0.1$. The upper bound on σ is made in order to limit the blade chord.

① Problem Description

② Mean Line

③ Models

④ turboLIB & Results

turboLIB

Results

① Problem Description

② Mean Line

③ Models

④ turboLIB & Results

turboLIB

NISRE results

.stl & .scad generation

Results

turboLIB

The preliminary compressor design model program [turboLIB](#) can be downloaded from GitHub.

Main objects and modules

- `turboClass.turboBlade.blade`: blade object
- `turboCoeff`: engineering coefficients module
 - `losses`: losses modeling
 - `similarity`: adimensional analysis
 - `lieblein`: blade modeling
- `geometry.bladeGeometry.geometryData`: airfoil object

NISRE setup

The NISRE is solved through a **double nested loop**:

- **continuity loop**
- **entropy loop**

Inside the **continuity loop** the `scipy.integrate.odeint` function is used for the solution of the V_{a2}^2 **ODE**.

Inside the **entropy loop** the `scipy.optimize.minimize` function is used for the computation of the blade **shape**.

.stl & .scad generation

At the end of the NISRE, all the main blade quantities are available for the **generation** of the **3D geometry**. This geometry can be converted into a **.stl** file that can be used in OpenFOAM for the flow properties study. In addition a **.scad** file is made for understanding position and checking possible contacts between rotor and stator blades.

[Bas06] suggested that a good distance between rotor and stator blades is half of the rotor chord¹¹.

¹¹In the radial equilibrium study losses between rotor and stator blades are **neglected**.

1 Problem Description

2 Mean Line

3 Models

4 turboLIB & Results

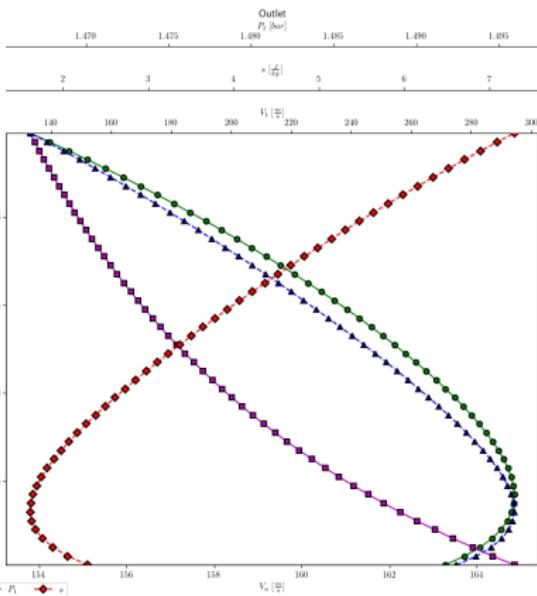
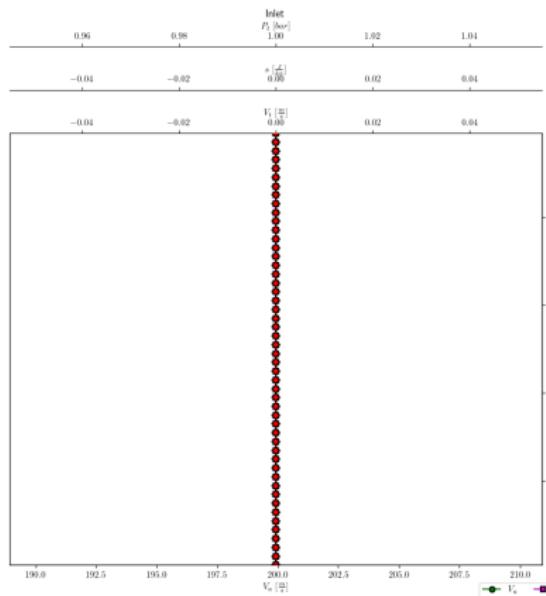
turboLIB

Results

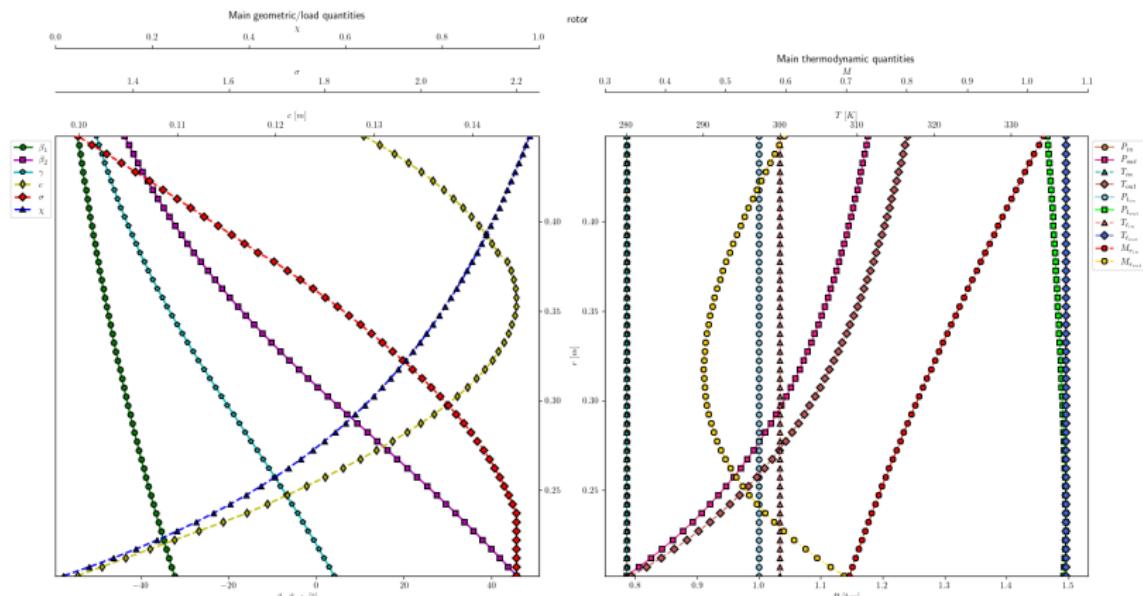
NISRE and main quantities

Efficiency

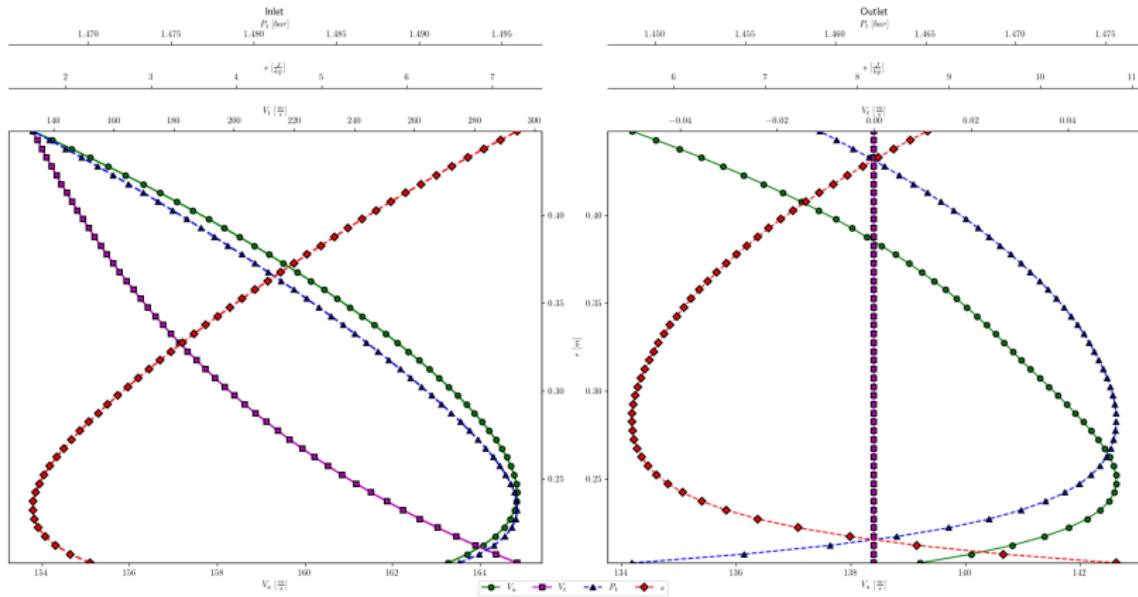
Rotor equilibrium results: NISRE



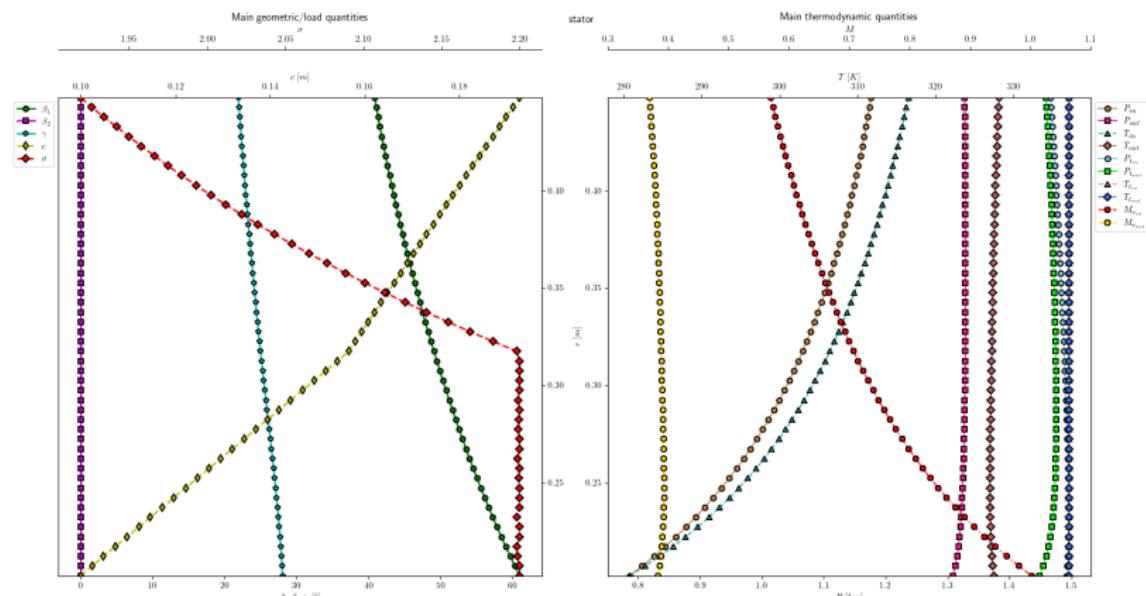
Rotor equilibrium results: main quantities



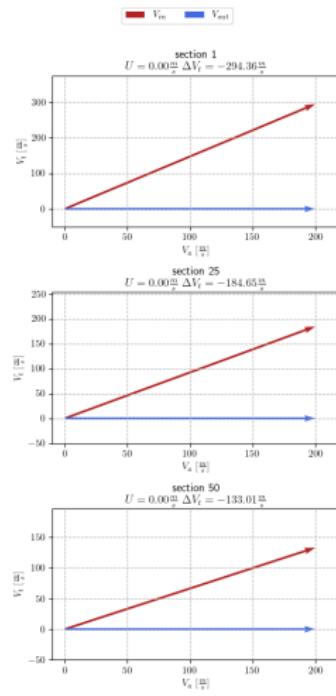
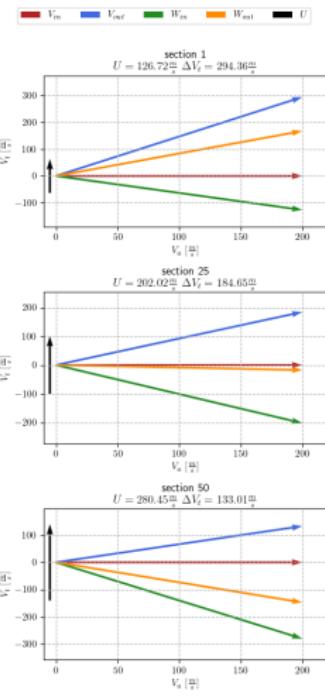
Stator equilibrium results: NISRE



Stator equilibrium results: main quantities



Velocity triangles



Rotor & stator blades

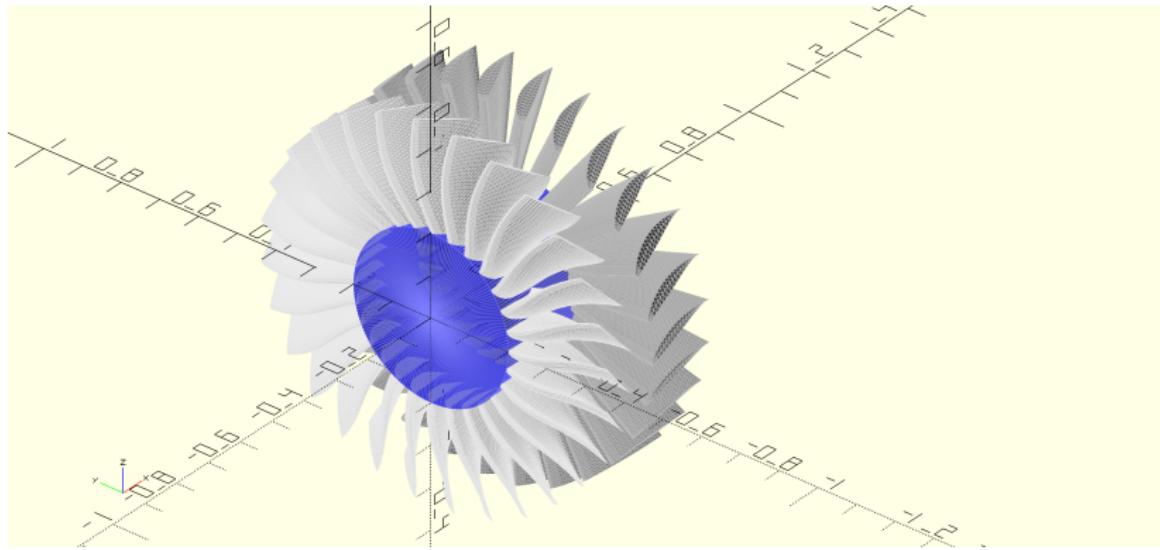


Figure 1: Rotor blade.



Figure 2: Stator blade.

Stage plot



Efficiency

The **rotor** efficiency is computed with:

$$\eta_{is_{rotor}} = \frac{W_1^2 - W_{2is}^2}{W_1^2 - W_2^2}$$

The **stator** efficiency is computed with:

$$\eta_{is_{stator}} = \frac{\Delta h_{is}}{\Delta h_{real}}$$

The modeling results are stored into
`compressor_0.55_0.325_28_28.txt`.

- [Aun04] Ronald H. Aungier. "Axial-Flow Compressors: a Strategy for Aerodynamic Design and Analysis". In: *Appl. Mech. Rev.* 57.4 (2004).
- [Bas06] Erian A. Baskharone. *Principles of turbomachinery in air-breathing engines*. Cambridge University Press, 2006.
- [MF20] Marco Manfredi and Fabrizio Fontaneto. "Transonic Axial Compressors Loss Correlations: Part I—Analysis and Update of Loss Models". In: *Turbo Expo: Power for Land, Sea, and Air*. Vol. 84065. American Society of Mechanical Engineers. 2020, V02AT32A029.

Thank you!

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