Turbomachinery: Compressor preliminary design

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Initial conditions & constraints

Inlet conditions

- $P_{T0} = 1bar$
- $T_{T0} = 300K$

Constraints

- $r_{max} = 0.45m$
- $\beta_{TT} = 1.45$
- $\dot{m} = 100\frac{kg}{s}$
- **max** η

Due to the **course track** and **preference**, the turbomachinery design will be on an **axial** compressor.

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Problem setup: hypothesis

Hypothesis

- **not using** an **inlet guide vane** for simplicity of design
- keeping, in the similarity/adimensional analysis of the compressor, $V_{a_{mean}}$ **constant**[1]
- keeping the blade height, b_0, **constant** both in rotor and stator
- using a **mixed vortex** model for the **rotor** velocity triangles
- using a **second order** function for the **stator** velocity triangles
- neglecting inlet **entropy** generation and assuming **rotor inlet** quantities **constant**
- **shrouding** at blade tip not present
- **rotor-stator** losses neglected

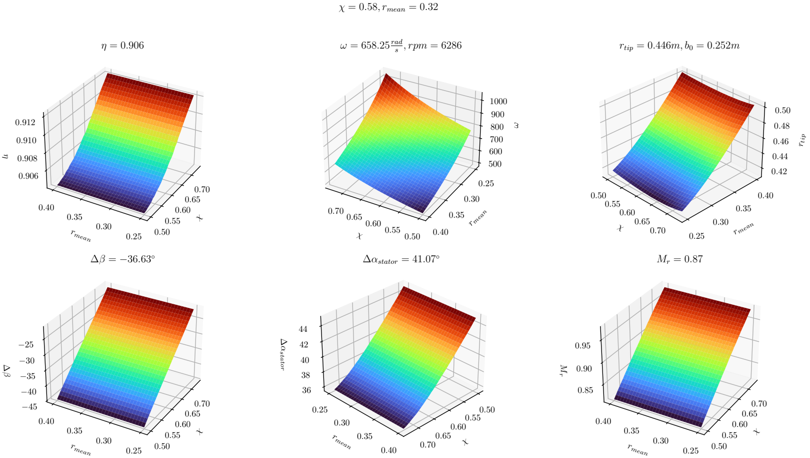
[1] $\dot{m}$ corrections will be made later on in the **radial equilibrium** solution.

Problem setup: solution steps

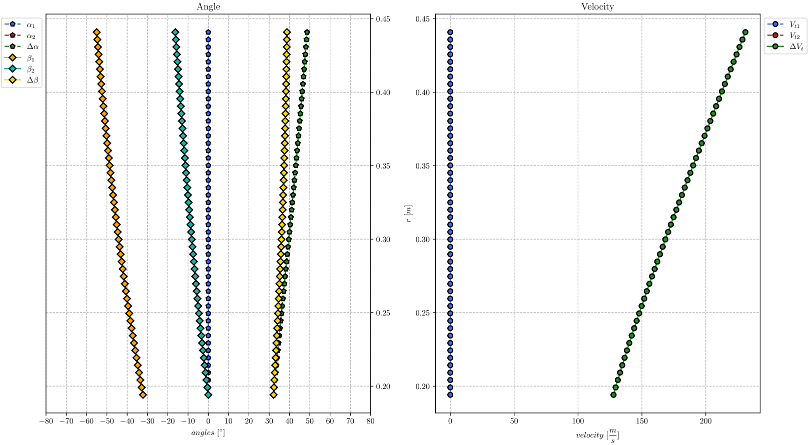
Main procedural steps:

- λ and ψ computation from χ and V_{t0}
- ϕ and η computation
- $V_{a_{mean}}$ and L_{eu} computation from ϕ, β_{TT} and η
- computing **mean** velocity triangles, using the above hypothesis
- computing **mean thermodynamic** quantities
- computing **blade height**

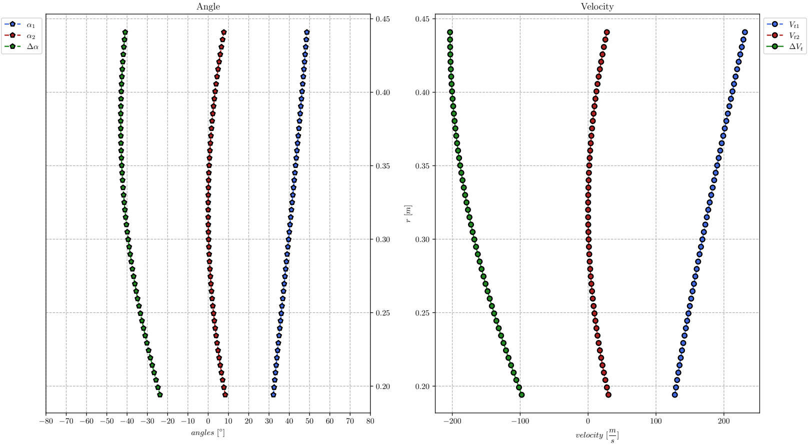
Graph analysis: χ & r_{mean}



Graph analysis: **rotor** α, β & V_t



Graph analysis: **stator** α & V_t



λ & ψ

From the previous **graphs**:

- $\chi = 0.55$
- $r_{mean} = 0.325m$
- $\frac{V_{t0}}{U_{mean}} = 0$

Taking into account the previous modeling **hypothesis**:

$$\lambda = \left(1 - \chi - \frac{V_{t0}}{U_{mean}}\right) \cdot 4$$

$$\psi = \frac{\lambda}{2}$$

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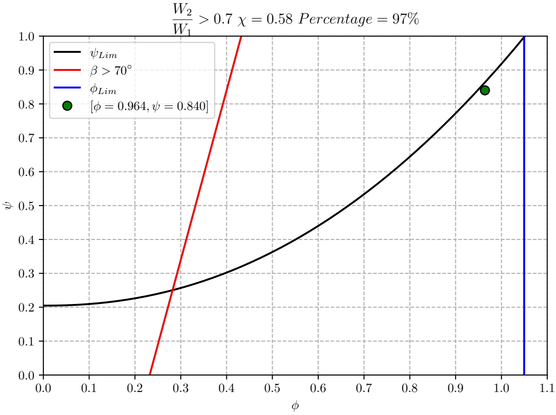
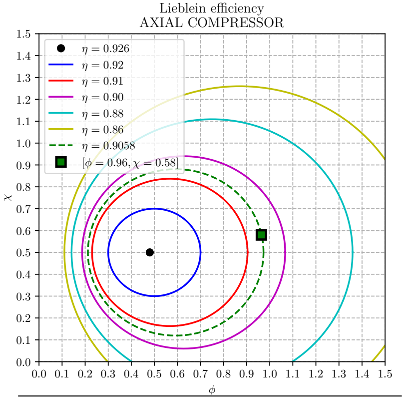
$\phi_{(\psi)}$

From [Aun04, Sec. 10.4] it is imposed that $\frac{W_2}{W_1} \geq 0.7$ with a *safety* margin of 3%.



η & L_{eu}

η is computed from an **Lieblein** efficiency chart[2] given ϕ and χ. This parameter will be used for the computation of L_{eu} given the β_{TT} target.



$$L_{is} = \frac{\gamma \ R}{\gamma - 1} \ T_{T0} \ (\beta_{TT}^{\frac{\gamma-1}{\gamma}} - 1)$$

$$L_{eu} = \frac{L_{is}}{\eta}$$

[2] This chart has been interpolated from the course slides charts.

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$V_{a_{mean}}$, $V_{t_{mean}}$ & U_{mean}

$$U_{mean} = \frac{L_{eu}}{\psi}$$

$$V_{a_{mean}} = \phi \; U_{mean}$$

$$\begin{aligned} L_{eu} &= U_1 \; V_{t1} - U_0 \; V_{t0} \\ &= U_{1_{mean}} \; V_{t1_{mean}} - U_{0_{mean}} \; V_{t0_{mean}} = U_{mean} \; \Delta V_{t_{mean}} \end{aligned}$$

$$V_{t1} = \Delta V_{t_{mean}} + V_{t0}$$

- $\Delta V_{t_{mean}}$ computation allows us to get a *first sketch* of the **velocity triangles**[3]
- The first analysis results are stored in `compressor_0.58_0.32_45_35.txt`

[3] **Mixed vortex** model and **second order** function based.

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Profile losses

The profile losses used are related to the **Leiblein modeling** approach[4].
The model is based on the **equivalent diffusion factor**, D_{eq}:

$$\frac{W_{max}}{W_1} = 1.12 + 0.61\ \frac{cos(\beta_1)^2}{\sigma} \cdot \frac{r_1\ V_{t1} - r_2\ V_{t2}}{r_1\ V_{a1}}$$

$$D_{eq} = \frac{W_{max}}{W_1} \cdot \frac{W_1}{W_2}$$

D_{eq} will be used for the computation of $\bar{\omega}_{profile}$ as:

$$\bar{\omega}_{profile} = \frac{0.004\ \left(1 + 3.1\ (D_{eq} - 1)^2 + 0.4\ (D_{eq} - 1)^8\right)\ 2\ \sigma}{cos(\beta_2)\ \left(\frac{W_1}{W_2}\right)^2}$$

[4] The following equations are interpolated data from [Aun04, Sec. 6.4].

Compressibility losses – I

These losses can be seen as a **correction** of the **profile** losses due to the compressibility of the gas along its *journey* in the stage.

The correction refers to a **Leiblein correction** model that uses the **positive** and **negative** blade section incidence angle, i_c and i_s.

These new stall incidence angles will build a new **mean** incidence angle, i_m, that can be seen as the **optimum** incidence angle related to the inlet Mach conditions[5].

[5] The implemented model follows [Aun04, Sec. 6.6]

Compressibility losses – II

$\bar{\omega}_{compressibility}$ setup:

- R_c & R_s computation:

$$R_c = 9 - \left[1 - \left(\frac{30}{\beta_1}\right)^{0.48}\right] \frac{\theta}{8.2}$$

$$R_s = 10.3 + \left(2.92 - \frac{\beta_1}{15.6}\right) \frac{\theta}{8.2}$$

- i_c & i_s computation:

$$i_c = i^* - \frac{R_c}{1 + 0.5\ M_1^3}$$

$$i_s = i^* + \frac{R_s}{1 + 0.5\left(K_{sh}\ M_1\right)^3}$$

Compressibility losses – III

- i_m computation: $i_m = i_c + \left(i_s - i_c\right) \frac{R_c}{R_c+R_s}$
- $\bar{\omega}_m$ computation: $\bar{\omega}_m = \bar{\omega}_{profile} \left[1 + \frac{\left(i_m - i^*\right)^2}{R_s^2}\right]$
- $\bar{\omega}_{compressibility}$ computation:

$$\bar{\omega}_{compressibility} = \bar{\omega}_m + \bar{\omega}_m \left[\frac{i - i_m}{i_c - i_m}\right]^2, \text{ if } i \leq i_m$$

$$\bar{\omega}_{compressibility} = \bar{\omega}_m + \bar{\omega}_m \left[\frac{i - i_m}{i_s - i_m}\right]^2, \text{ if } i \geq i_m$$

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Secondary flow losses

These losses are relative to the **secondary flow** inside the compressor and are *usually* **greater** than the other losses. These losses are related to **eddies** generated with the **blade-flow** interaction and **streamlines** displacement due to the presence of **pressure gradients**.

These losses are computed with **Howell**'s model [Aun04, Ch. 6][6]:

$$\bar{\beta} = \frac{arctan((tan(\beta_1) + tan(\beta_2)))}{2}; \qquad \mathrm{I}$$

$$C_L = 2\ cos(\bar{\beta}) \cdot \frac{tan(\beta_1) - tan(\beta_2)}{\sigma}; \qquad \mathrm{II}$$

$$C_D = 0.18\ C_L^2; \qquad \mathrm{III}$$

$$\bar{\omega}_{secondary} = C_D\ \sigma \cdot \frac{cos(\beta_1)^2}{cos(\bar{\beta})^3}; \qquad \text{loss computation}$$

[6] Howell computed a secondary flow loss model that is automatically embedded into $\bar{\omega}_{profile}$: this model is used for the **estimation** of the stator/rotor

End wall losses

These losses are related the interaction between the flow and the **compressor case**. They are *lower* than the **secondary flow** losses. It has been used a simple and fast relation made by **Howell** [Aun04, Ch. 6][7]:

$$C_D = 0.02\,\frac{s}{b_0}$$

$$\bar{\omega}_{endWall} = C_D\;\sigma \cdot \frac{cos(\beta_1)^2}{cos(\bar{\beta})^3}; \qquad \text{loss computation}$$

[7]This loss is kept into account in the **blade numbering** study.

Shock losses – I

The **relative** Mach number at the rotor inlet is slightly above **sonic speed**; a **shock wave** will be present at the rotor tip. From [MF20], **shock pattern** is related to **Mach number** and **airfoil shape**.

The **shock** losses modeling is related to **König losses** modeling approach. This model describes a **2 shock waves loss** using a **single normal shock** with respect to a computed Mach number, M_{in}.

König model [Aun04, Sec. 6.7] depends mainly on **blade deflection angle**, θ, and **relative inlet Mach**, M_1.

Shock losses – II

- computation of the **expansion wave** angle, ϕ:
  $\phi = \frac{s\ cos(\psi)}{s\ sin(\psi)\ R_u}$, where $\psi = \psi_{(\beta_1,\gamma,\theta)}$
- computation of W_s and M_s using the **Prandtl-Meyer** expansion: $\phi = \int_{W_1}^{W_s} \sqrt{M^2 - 1}\ \frac{dW}{W}$
- M_{in} computation: $M_{in} = \sqrt{M_1\ M_s}$
- **normal shock** solution and computation of ΔP_T
- from ΔP_T, computation of $\bar{\omega}_{shock}$

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Tip leackage losses

Again these losses are computed from [Aun04, Sec. 6.9]. The main concept is: computing a **total** blade pressure loss and **assuming linear distribution** of losses from the hub to the tip.

$$\tau = \pi \; \delta_c \left[r_1 \rho_1 V_{a1_{mean}} + r_2 + \rho_2 V_{a2_{mean}} \right] \left[r_2 V_{t2_{mean}} - r_1 V_{t1_{mean}} \right]$$

$$\Delta P = \frac{\tau}{Z \; r_{tip} \delta_c \; c \; cos(\gamma)}$$

$$U_c = 0.816 \frac{\sqrt{\frac{2\Delta P}{\rho_{mean}}}}{N_{row}^{0.2}}$$

$$\dot{m}_c = \rho_{mean} \; U_c \; Z \; \delta_c \; c \; cos(\gamma)$$

$$\Delta P_T = \frac{\Delta P \; \dot{m}_c}{\dot{m}}$$

ΔP_T is the **overall** total pressure loss due to **leackage** of the blade.

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Radial equilibrium

The **radial equilibrium** equation: $\frac{\partial h_t}{\partial r} = T\frac{\partial s}{\partial r} + V_a\frac{\partial V_a}{\partial r} + V_t\frac{\partial rV_t}{\partial r}$ is converted into, for the exit station[8] of the blade, a **1st order ODE**:

$$-\frac{1}{2}\frac{\partial V_{a2}^2}{\partial r} + \frac{V_{a2}^2}{2\ c_P}\frac{\partial s_2}{\partial r} = -c_P\frac{\partial T_{T1}}{\partial r} - \omega\ \frac{\partial rV_{t2}}{\partial r} + \omega\ \frac{\partial rV_{t1}}{\partial r}$$

$$+T_{T1}\ \frac{\partial s_2}{\partial r} + \frac{\omega}{c_P}rV_{t2}\ \frac{\partial s_2}{\partial r} - \frac{\omega}{c_P}rV_{t1}\frac{\partial s_2}{\partial r} - \frac{1}{2\ c_P}V_{t2}^2\frac{\partial s_2}{\partial r} + \frac{V_{t2}}{r}\frac{\partial rV_{t2}}{\partial r}$$

The **ODE** will be solved for V_{a2}^2.
s_2 is computed from $\sum_i \bar{\omega}_i$ treated earlier.
ω, V_{t1}, V_{t2}& T_{T1} are known.

[8] 1 is the blade inlet station and 2 is the blade outlet section.

Blade shape

The **Leiblein model** from [Aun04, Ch. 6] has been used for the blade shape computation.

$\mathtt{NACA}-65$ profile has been chosen for the blade generation[9].

The **main contraints** are: $\frac{t_b}{c} \approx 0.1$ and $max(\sigma) = 2.2$[10].

Due to the many possible blade configurations, an **optimization** procedure has been used for the computation of i^*, δ, θ and σ.

Each section airfoil shape is **computed** from θ and $\mathtt{NACA}-65$ C_{L0} surface coordinates.

[9] Due to the low tip sonic Mach number it has been chosen to use this profile as well for the blade tip instead of a supersonic adapted profile shape.

[10] $\frac{t_b}{c} \approx 0.1$ allows setting $K_{sh} \approx 0.1$. The upper bound on σ is made in order to limit the blade chord.

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turboLIB

The preliminary compressor design model program turboLIB can be downloaded from GitHub.

Main objects and modules

- `turboClass.turboBlade.blade`: blade object
- `turboCoeff`: engineering coefficients module
    - `losses`: losses modeling
    - `similarity`: adimensional analysis
    - `lieblein`: blade modeling
- `geometry.bladeGeometry.geometryData`: airfoil object

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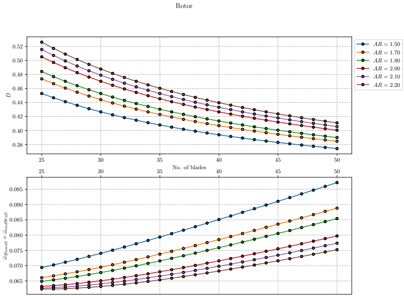
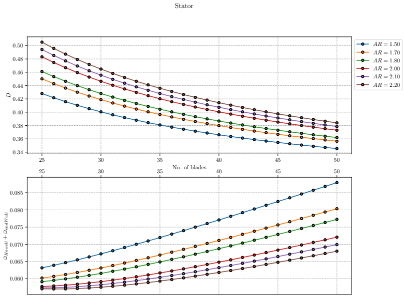
Blade number

In order to define a proper number of blades for the rotor and the stator, **Howell**'s relations have been used for the estimation of the **losses**. These relations, [Aun04, Ch. 6], are:

- $\bar{\omega}_{profile+secondary}$, this is a relation that takes into account **profile** and **3D** losses
- $\bar{\omega}_{endWall}$, this is previous expalined **end wall** loss





NISRE setup

The `NISRE` is solved through a **double nested** loop:

- **continuity loop**
- **entropy loop**

Inside the **continuity loop** the `scipy.integrate.odeint` function is used for the solution of the V_{a2}^2 **ODE**.

Inside the **entropy loop** the `scipy.optimize.minimize` function is used for the computation of the blade **shape**.

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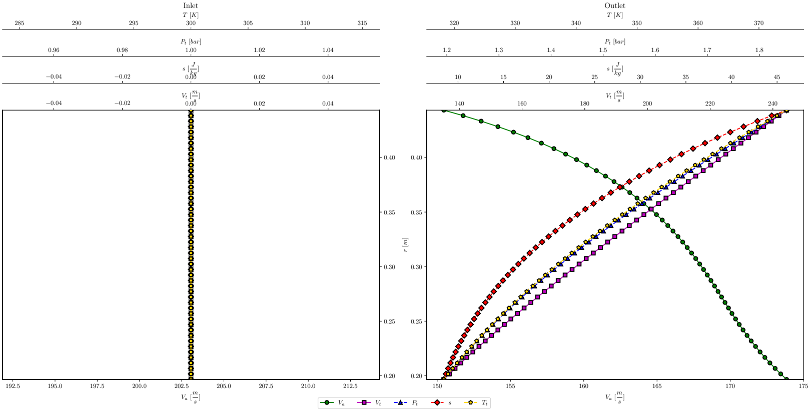
.stl & .scad generation

At the end of the NISRE, all the main blade quantities are avaliable for the **generation** of the **3D geometry**. This geometry can be converted into a .stl file that can be used in OpenFOAM for the flow properties study. In addition a .scad file is made for understanding position and checking possible contacts between rotor and stator blades.

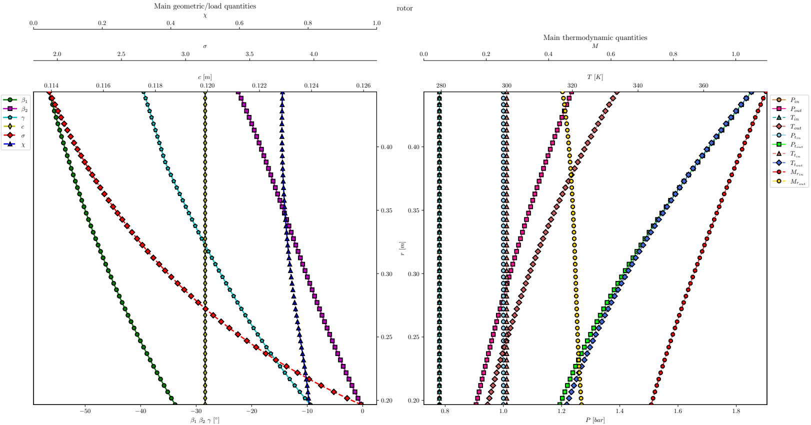
[Bas06] suggested that a good distance between rotor and stator blades is half of the rotor chord[11].

[11] In the radial equilibrium study losses between rotor and stator blades are **neglected**.

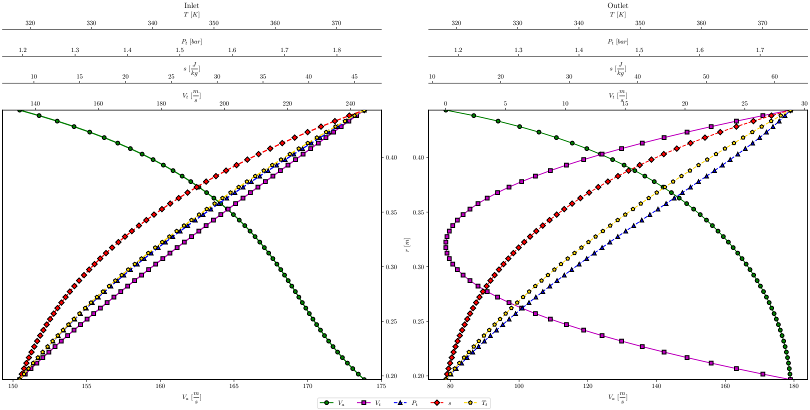
Rotor equilibrium results: NISRE



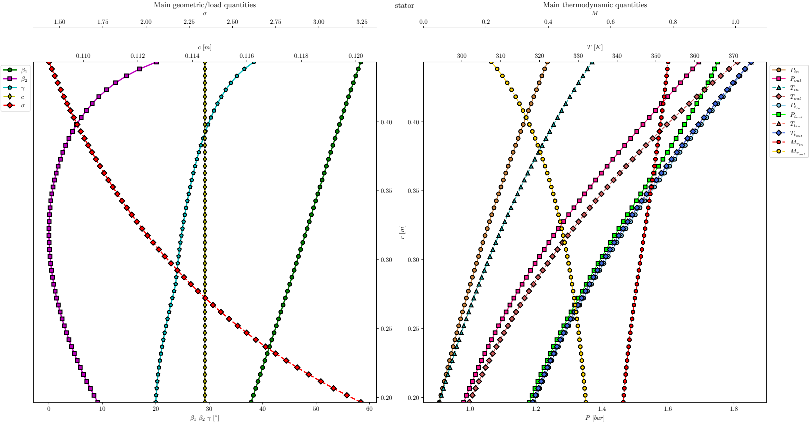
Rotor equilibrium results: main quantities



Stator equilibrium results: NISRE

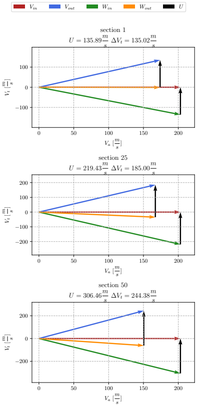


Stator equilibrium results: main quantities

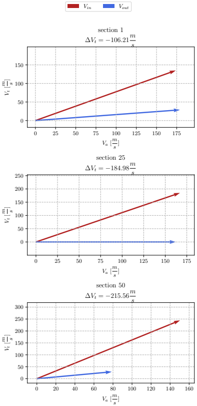


Velocity triangles

- Inlet: **axial** velocity
- Outlet: **mixed vortex** model



- Inlet: **mixed vortex** model
- Outlet: **second order** function



Rotor & stator blades

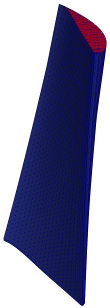


Figure 1: Rotor blade.

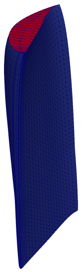
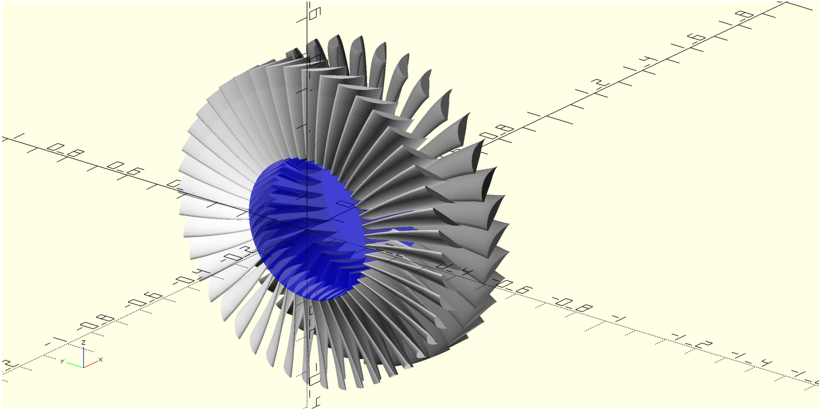


Figure 2: Stator blade.

Stage plot



Efficiency

The **rotor** efficiency is computed with:

$$\eta_{is_{rotor}} = \frac{W_1^2 - W_{2_{is}}^2}{W_1^2 - W_2}$$

The **stator** efficiency is computed with:

$$\eta_{is_{stator}} = \frac{\Delta h_{is}}{\Delta h_{real}}$$

The modeling results are stored into `compressor_0.58_0.32_45_35.txt`.

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[Aun04] Ronald H. Aungier. "Axial-Flow Compressors: a Strategy for Aerodynamic Design and Analysis". In: *Appl. Mech. Rev.* 57.4 (2004).

[Bas06] Erian A. Baskharone. *Principles of turbomachinery in air-breathing engines*. Cambridge University Press, 2006.

[MF20] Marco Manfredi and Fabrizio Fontaneto. "Transonic Axial Compressors Loss Correlations: Part I—Analysis and Update of Loss Models". In: *Turbo Expo: Power for Land, Sea, and Air*. Vol. 84065. American Society of Mechanical Engineers. 2020, V02AT32A029.

Thank you!

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