Turbomachinery: Compressor preliminary design

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Initial conditions & constraints

Inlet conditions

- $P_{T0} = 1bar$
- $T_{T0} = 300K$

Constraints

- $r_{max} = 0.45m$
- $\beta_{TT} = 1.45$
- $\dot{m} = 100\frac{kg}{s}$
- **max** η

Due to the **course track** and **preference**, the turbomachinery design will be on an **axial** compressor.

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1. Problem Description
2. Similitude
   - Problem setup
   - Main design quantities
   - $V_{t_{mean}}$, $V_{a_{mean}}$, U_{mean} & velocity triangles
3. Blade Modeling
4. Efficiency
5. CFD
6. References

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Problem setup: hypothesis

Hypothesis

- **not using** an **inlet guide vane** for simplicity of design[1]
- keeping, in the similarity/adimensional analysis of the compressor, $V_{a_{mean}}$ **constant**[2]
- keeping the blade height, b_0, **constant** both in rotor and stator[3]
- using a **free vortex** model for the velocity triangles
- neglecting inlet **entropy** generation and assuming **rotor inlet** quantities **constant**

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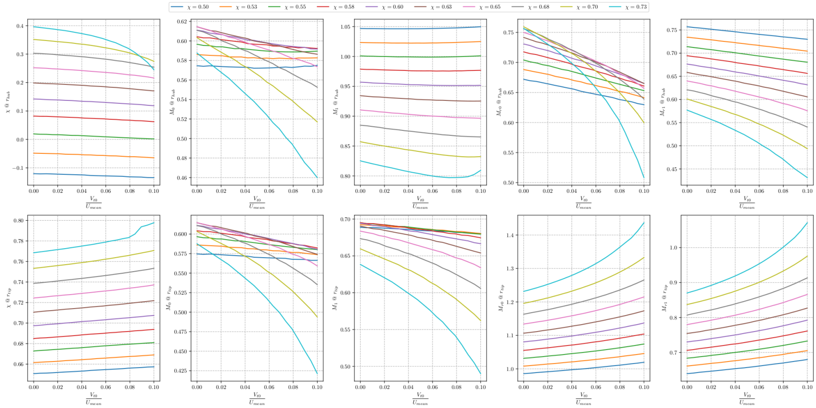
[1] $V_{t0} = 0\frac{m}{s}$ and χ dictate the behaviour of λ.
[2] $\dot{m}$ corrections will be made later on in the **radial equilibrium** solution.
[3] In order to keep each **blade streamtube section** as simple as possible.

Problem setup: solution steps

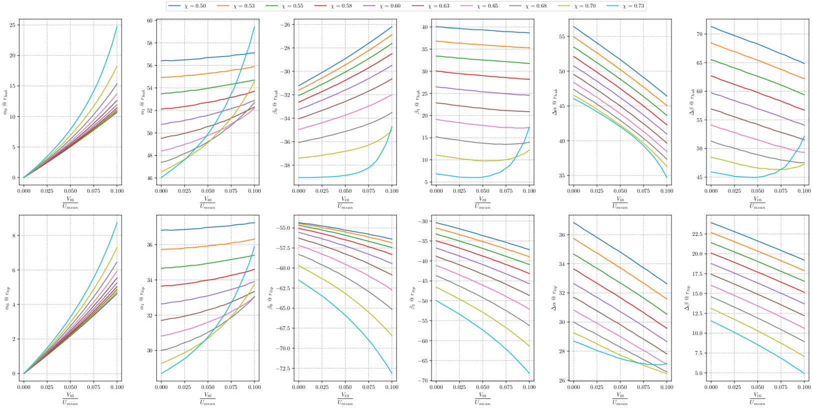
Main procedural steps:

- λ and ψ computation from χ and V_{t0}
- ϕ and η computation
- $V_{a_{mean}}$ and L_{eu} computation from ϕ, β_{TT} and η
- computing **mean** velocity triangles, using the above hypothesis
- computing **mean thermodynamic** quantities
- computing **blade height**

Graph Analysis: χ & M



Graph Analysis: α & β



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λ & ψ

From the previous **graphs**:

- $\chi = 0.55$
- $r_{mean} = 0.325m$
- $\frac{V_{t0}}{U_{mean}} = 0$

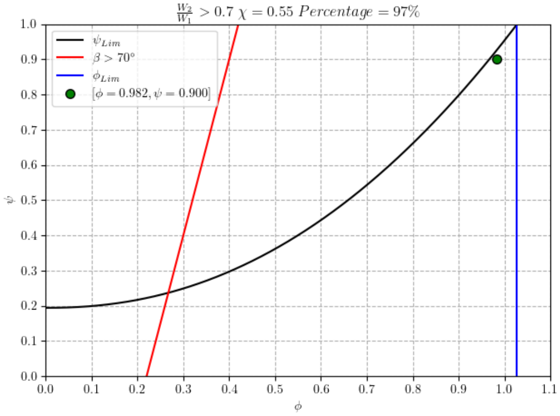
Taking into account the previous modeling **hypothesis**:

$$\lambda = \left(1 - \chi - \frac{V_{t0}}{U_{mean}}\right) \cdot 4$$

$$\psi = \frac{\lambda}{2}$$

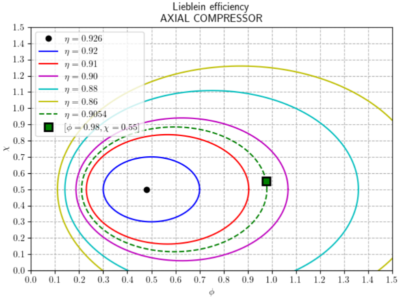
$\phi_{(\psi)}$

From [?, Sec. 10.4] it is imposed that $\frac{W_2}{W_1} \geq 0.7$ with a *safety* margin of 3%.



η & L_{eu}

η is computed from an **Lieblein** efficiency chart[4] given ϕ and χ. This parameter will be used for the computation of L_{eu} given the β_{TT} target.



$$L_{is} = \frac{\gamma\ R}{\gamma - 1}\ T_{in}\ (\beta_{TT}^{\frac{\gamma-1}{\gamma}} - 1)$$

$$L_{eu} = \frac{L_{is}}{\eta}$$

[4] This chart has been interpolated from the course slides charts.

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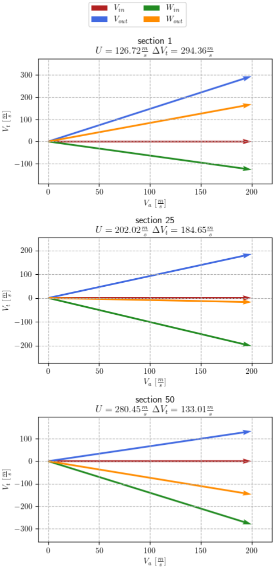
$V_{a_{mean}}$, $V_{t_{mean}}$ & U_{mean}

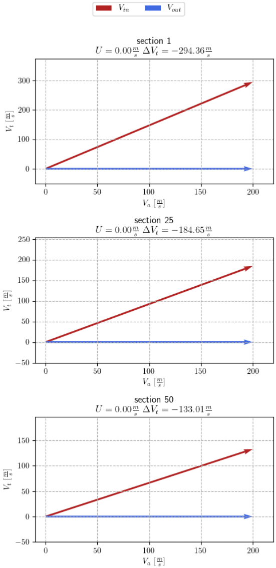
$$
\begin{aligned}
U_{mean} &= \frac{L_{eu}}{\psi} \\
V_{a_{mean}} &= \phi \; U_{mean} \\
L_{eu} &= U_1 \; V_{t1} - U_0 \; V_{t0} \\
&= U_{1_{mean}} \; V_{t1_{mean}} - U_{0_{mean}} \; V_{t0_{mean}} = U_{mean} \; \Delta V_{t_{mean}} \\
V_{t1} &= \Delta V_{t_{mean}} + V_{t0}
\end{aligned}
$$

- $\Delta V_{t_{mean}}$ computation allows to get a *first sketch* of the **velocity triangles**[5]
- The first analysis results are stored in `compressor_0.55_0.325_28_28.txt`

[5] **Free vortex** model based.



V_{in} W_{in}
V_{out} W_{out}
section 1
$U = 126.72\frac{m}{s}$ $\Delta V_t = 294.36\frac{m}{s}$
section 25
$U = 202.02\frac{m}{s}$ $\Delta V_t = 184.65\frac{m}{s}$
section 50
$U = 280.45\frac{m}{s}$ $\Delta V_t = 133.01\frac{m}{s}$
$V_t\ [\frac{m}{s}]$
$V_a\ [\frac{m}{s}]$



V_{in} V_{out}
section 1
$U = 0.00\frac{m}{s}$ $\Delta V_t = -294.36\frac{m}{s}$
section 25
$U = 0.00\frac{m}{s}$ $\Delta V_t = -184.65\frac{m}{s}$
section 50
$U = 0.00\frac{m}{s}$ $\Delta V_t = -133.01\frac{m}{s}$
$V_t\ [\frac{m}{s}]$
$V_a\ [\frac{m}{s}]$

1. Problem Description
2. Similitude
3. **Blade Modeling**
   - Losses modeling
   - Radial equilibrium
   - `turboLIB`
4. Efficiency
5. CFD
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Profile losses

The profile losses used are related to the **Leiblein modeling** approach[6].
The model is based on the **equivalent diffusion factor**, D_{eq}:

$$\frac{W_{max}}{W_1} = 1.12 + 0.61 \ \frac{cos(\beta_1)^2}{\sigma} \cdot \frac{r_1 \ V_{t1} - r_2 \ V_{t2}}{r_1 \ V_{a1}}$$

$$D_{eq} = \frac{W_{max}}{W_1} \cdot \frac{W_1}{W_2}$$

D_{eq} will be used for the computation of $\bar{\omega}_{profile}$ as:

$$\bar{\omega}_{profile} = \frac{0.004 \ \left(1 + 3.1 \ (D_{eq} - 1)^2 + 0.4 \ (D_{eq} - 1)^8\right) \ 2 \ \sigma}{cos(\beta_2) \ \left(\frac{W_1}{W_2}\right)^2}$$

[6] The following equations are interpolated data from [**?**, Ch. 6].

Compressibility losses – I

This losses can be seen as a **correction** of the **profile** losses due to the compressibility of the gas along its *journey* in the stage.

The correction referres to a **Leiblein correction** model that uses the **positive** and **negative** blade section incidence angle, i_c and i_s.

These new stall incidence angles will build a new **mean** incidence angle, i_m, that can be seen as the **optimum** incidence angle related to the inlet Mach conditions[7].

[7] The implemented model follows [**?**, Ch. 10]

Compressibility losses – II

$\bar{\omega}_{compressibility}$ setup:

- R_c & R_s computation:

$$R_c = 9 - \left[1 - \left(\frac{30}{\beta_1}\right)^{0.48}\right] \frac{\theta}{8.2}$$

$$R_s = 10.3 + \left(2.92 - \frac{\beta_1}{15.6}\right) \frac{\theta}{8.2}$$

- i_c & i_s computation:

$$i_c = i^* - \frac{R_c}{1 + 0.5\ M_1^3}$$

$$i_s = i^* + \frac{R_s}{1 + 0.5\left(K_{sh}\ M_1\right)^3}$$

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Compressibility losses – III

- i_m computation: $i_m = i_c + \left(i_s - i_c\right) \frac{R_c}{R_c+R_s}$
- $\bar{\omega}_m$ computation: $\bar{\omega}_m = \bar{\omega}_{profile} \left[1 + \frac{\left(i_m - i^*\right)^2}{R_s^2}\right]$
- $\bar{\omega}_{compressibility}$ computation:

$$\bar{\omega}_{compressibility} = \bar{\omega}_m + \bar{\omega}_m \left[\frac{i - i_m}{i_c - i_m}\right]^2, \text{ if } i \leq i_m$$

$$\bar{\omega}_{compressibility} = \bar{\omega}_m + \bar{\omega}_m \left[\frac{i - i_m}{i_s - i_m}\right]^2, \text{ if } i \geq i_m$$

Shock losses – I

The **relative** Mach number at the rotor inlet is slightly above **sonic speed**; a **shock wave** will be present at the rotor tip. From [**?**], **shock pattern** are related to **Mach number** and **airfoil shape**.

The **shock** losses modeling is related to **König losses** modeling approach. This model describes a **2 shock waves loss** using a **single normal shock** with respect to a computed Mach number, M_{in}.

König model depends mainly on **blade deflection angle**, θ, and **relative inlet Mach**, M_1.

Shock losses – II

- computation of the **expansion wave** angle, ϕ:
  $\phi = \frac{s\ cos(\psi)}{s\ sin(\psi)\ R_u}$, where $\psi = \psi_{(\beta_1,\gamma,\theta)}$
- computation of W_s and M_s using the **Prandtl-Meyer** expansion: $\phi = \int_{W_1}^{W_s} \sqrt{M^2 - 1}\ \frac{dW}{W}$
- M_{in} computation: $M_{in} = \sqrt{M_1\ M_s}$
- **normal shock** solution and computation of ΔP_T
- from ΔP_T, computation of $\bar{\omega}_{shock}$

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Thank you!

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