Artificial Neural Network to price Barrier Options

TDI-Project Proposal

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Business Aims | Practical objectives

Project| Practical objectives

- Reliability of ANN for out-of-sample data prediction.
- Impact of underlying volatility mechanism on the prediction.

The above objectives would help us to analyze whether a barrier option trader with a systematic behavior, learn from previous quotes and improve his/her pricing or trading performance in the future.

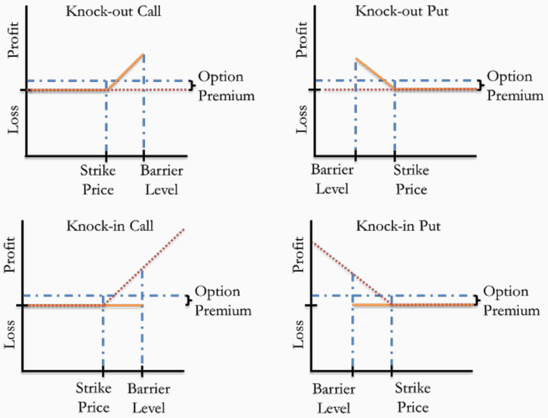
Translate Practical Objectives to Actionable Objectives

Objectives of the Project

- **Objective-1** - Model Selection - Generate simulated data. Train : Test Neural Network (80% : 20% of data samples)
- **Objective-2** - Market Data. Train the Neural Network using 80 % of market data and test using 20 % of market data.

Barrier Options | Knock-In and Knock-Out

- Characterised by a strike level and a barrier level
- Payoff depends on underlying asset's price



ANN|Adam(Adaptive moment estimation) Algorithm

Adam Algorithm - (Diederik P. Kingma,(2014))*

$M_0 =, R_0 =$ (Initialization)

For $t = 1, \ldots, T$:

$$M_t = \beta_1 M_{t-1} + (1-\beta_1)\nabla L_t(W_{t-1}) \quad \text{(1st moment estimate)}$$
$$R_t = \beta_2 R_{t-1} + (1-\beta_2)\nabla L_t(W_{t-1})^2 \quad \text{(2nd moment estimate)}$$
$$\hat{M}_t = M_t / \left(1-(\beta_1)^t\right) \quad \text{(1st moment bias correction)}$$
$$\hat{R}_t = R_t / \left(1-(\beta_2)^t\right) \quad \text{(2nd moment bias correction)}$$
$$W_t = W_{t-1} - \alpha \frac{\hat{M}_t}{\sqrt{\hat{R}_t + \epsilon}} \quad \text{(Update)}$$

Return W_T

Hyper-parameters:

$\alpha > 0$ – learning rate (typical choice: 0.001)

$\beta_1 \in [0, 1)$ – 1st moment decay rate (typical choice: 0.9)

$\beta_2 \in [0, 1)$ – 2nd moment decay rate (typical choice: 0.999)

$\epsilon > 0$ – numerical term (typical choice: 10^{-8})

*Kingma, D. P. Ba, J. (2014), 'Adam: A Method for Stochastic Op-timization' ,Conference paper at the 3rd International Conference forLearning Representations, San Diego, 2015

Softplus function

On the positive part of the input, it grows asymptotically linearly as it approaches $+\infty$ and behaves as an exponential function for the negative input.

$$\phi(x) = \ln(1 + e^x)$$

Volatility Mechanism| Historical

Historical Volatility

Historical volatility is a statistical measure of the deviation of returns over a given period of time. This method primarily depends on the usage of historical stock price data and is obtained as the standard deviation of the underlying stock's return over a given period

$$r_t = ln(S_t/S_{t-1})$$

S_t refers to the current stock price(underlying price) and S_{t-1} refers to the stock price price(underlying price) at time t-1.

Simulation Test|Approach

- Step-1 : Generate random simulated data
- Step-2 : Calculate barrier option prices using closed form formulae
- Step-3 : Train the model
- Step-4 : Test the model

Simulation Test|Pricing Barrier Option-1

Analytical formulae were provided by Reiner and Rubinstein (1991)* as follows :

H : Barrier level, X : Strike , σ : Volatility, T : Time to expiration, b : Cost of carry rate, r : Risk free rate, C : Price of a barrier option, S : Spot Stock price

$$\eta = \begin{cases} 1 & ifDown \\ -1 & ifUp \end{cases}$$

$$\phi = \begin{cases} 1 & ifCall \\ -1 & ifPut \end{cases}$$

$$X_1 = \frac{ln(\frac{S}{X})}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$X_2 = \frac{ln(\frac{S}{X})}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$Y_1 = \frac{ln(\frac{H^2}{SX})}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$Y_2 = \frac{ln(\frac{H^2}{S})}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$Z = \frac{ln(\frac{H}{S})}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$\mu = \frac{b - \frac{\sigma^2}{2}}{\sigma^2}$$

$$\lambda = \sqrt{\mu^2 + \frac{2r}{Q^2}}$$

Reiner ,E. Rubinstein, M. (1991) : “Breaking down the barriers”,Risk,vol4.pp. 28–35.

Simulation Test|Pricing Barrier Option-2

$$A = \phi * S * N(\phi x_1) - \phi * X * e^{-rT} N(\phi x_1 - \phi\sigma\sqrt{T})$$

$$B = \phi * S * N(\phi x_2) - \phi * X * e^{-rT} N(\phi x_2 - \phi\sigma\sqrt{T})$$

$$C = \phi * S * (\frac{H}{S})^{2(\mu+1)} N(\eta y_1) - \phi * X * e^{-rT} (\frac{H}{S})^{2(\mu)} N(\eta y_1 - \eta\sigma\sqrt{T})$$

$$D = \phi * S * (\frac{H}{S})^{2(\mu+1)} N(\eta y_2) - \phi * X * e^{-rT} (\frac{H}{S})^{2(\mu)} N(\eta y_2 - \eta\sigma\sqrt{T})$$

$$E = Ke^{-rT}[N(\eta x_2 - \eta\sigma\sqrt{T})] - (\frac{H}{S})^{2(\mu)}) - N(\eta y_2 - \eta\sigma\sqrt{T})$$

$$F = Ke^{-rT}(\frac{H}{S})^{\mu+\lambda} N(\eta z) - (\frac{H}{S})^{2(\mu-\lambda)}) - N(\eta z - 2\eta\lambda\sigma\sqrt{T})$$

Simulation Test|Pricing Barrier Option-3

Type	X < H	X > H
Down and In	A-B+D+E	C+E
Up and In	B-C+D+E	A+E
Down and Out	B-D+F	A-C+F
Up and Out	A-B+C-D+F	F

Table 1: Call Barrier Prices

Type	X < H	X > H
Down and In	A+E	B-C+D+E
Up and In	C+E	A-B+D+E
Down and Out	F	A-B+C-D+F
Up and Out	A-C+F	B-D+F

Table 2: Put Barrier Prices

Simulation Test|Pricing Barrier Option-4

We have the following parity relations between the prices of barrier options and vanilla call and put options:

$$C_{up\_in}(t) + C_{up\_out}(t) = e^{(-r(T-t))}E[(S_T - K)^+|F_t]$$

$$C_{down\_in}(t) + C_{down\_out}(t) = e^{(-r(T-t)}E[(S_T - K)^+|F_t]$$

$$P_{up\_in}(t) + P_{up\_out}(t) = e^{(-r(T-t)}E[(K - S_T)^+|F_t]$$

$$P_{down\_in}(t) + P_{down\_out}(t) = e^{(-r(T-t)}E[(K - S_T)^+|F_t]$$

Simulation Test|Artificial Neural Network - Parameters for consideration

Mean Absolute Error (MAE): MAE measures the average magnitude of the errors in a set of predictions, without considering their direction. It's the average over the test sample of the absolute differences between prediction and actual observation where all individual differences have equal weight.

Root mean squared error (RMSE): RMSE is a quadratic scoring rule that also measures the average magnitude of the error. It's the square root of the average of squared differences between prediction and actual observation.

EPOCH - It is defined as the number of times that the algorithm will work through the training dataset. Intuitively, higher EPOCH should result in better performance but my results indicated otherwise

Simulation Test|Artificial Neural Network

I randomly generate sets of data using random generation function in python. We will generate records - 1K,10K, 100K.Due to computing resources constraints, I could not execute beyond 100K. The aim is to check which data set has the lowest MAE and RMSE.

Parameter Name	Parameter value(Simulation Range)
Spot Price	100-170
Strike Price	110-160
Barrier Price	90-180
Maturity	1-3 years
Interest Rate	Daily Interest - 1 Month Coupon Rate*

Table 3: Simulation Parameters

*Interest rates were picked from the US Department of Treasury website

Technical Parameters	Value
Model Parameters	Spot, Strike, Barrier, Maturity, Interest Rate
No.Hidden Layers	3
No.Hidden Layers(Nodes)	85
Input Layer(Nodes)	Softplus(12)
Hidden Layer(Nodes)	Softplus(85)
Output Layer(Nodes)	Linear(1)
Epochs	500
Batch Size	10

Table 4: Model Technical Parameters

Model Parameters(Model-1)

- Volatility Type - Historical Volatility
- Volatility - 5 Days rolling average(Benchmark)
- Instrument Type - Up and In (Barrier Option)
- Option Type - Call

No of Records	MAE	RMSE
1000	0.831579531740707	1.04124540378478
10K	0.890681742739295	1.10656617193871
100K	0.89846993974664	1.11064982930674

Table 5: 5 Days rolling average

No significant improvement across record sets

Callable Bear Contract

Callable Bear Contract(with no rebate or residual value) is also called as a turbo warrant and has the payoff of an up-and-out barrier put option on the underlying asset price.

Callable Bull Contract

Callable Bull Contract has the payoff of down and out barrier call option on the underlying asset price

The CBBC market historical data has been retrieved from the HKEX website(Hong Kong Exchanges and Clearing market website)

Parameters	Details
Underlying Securities Code	388, 700, 1810, 3690
Instrument Type	Bull and Bear
Total Number of Records	11700

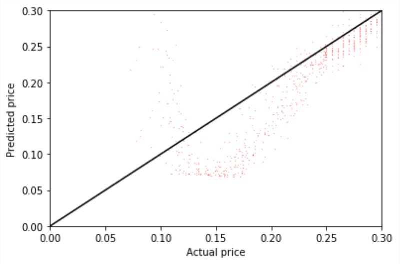
Table 6: Market Data Analysis

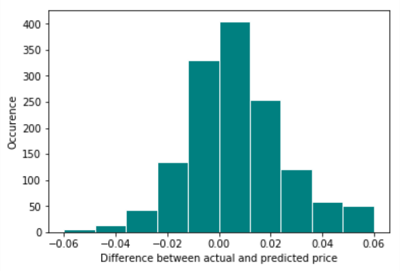
Underlying Code	Details
388	HONG KONG EXCHANGES AND CLEARING LTD
700	TENCENT HOLDINGS LTD
1810	XIAOMI CORPORATION - W
3690	MEITUAN DIANPING - W

Market Data Run - Model-1

Parameters	Details
Model Training Parameters	Spot Price,Strike Price, Barrier Price
Training Record	10K
Test Records	2K(approx)

Table 7: Model-1 Details



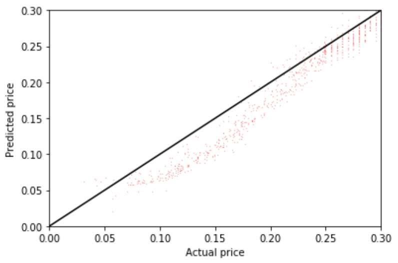


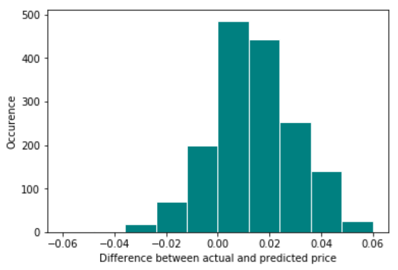
MAE : **0.0355132945101751** , RMSE : **0.0980740791381947**

Market Data Run - Model-2

Parameters	Details
Model Training Parameters	Spot,Strike,Barrier,(Strike/Barrier)ratio
Training Record	10K
Test Records	2K(approx)

Table 8: Model-2 Details





MAE : **0.017214516** , RMSE : **0.021250036**

Appendix

- Sigmoidal (or logistic) function
- Softplus function
- Rectified Linear Unit (ReLU)
- Leaky ReLU
- Exponential Linear Unit (ELU)

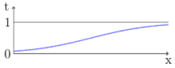


Figure 2.5: Sigmoidal Function

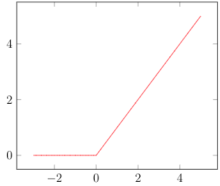


Figure 2.6: RELU Function

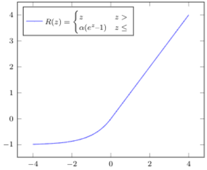


Figure 2.7: ELU