

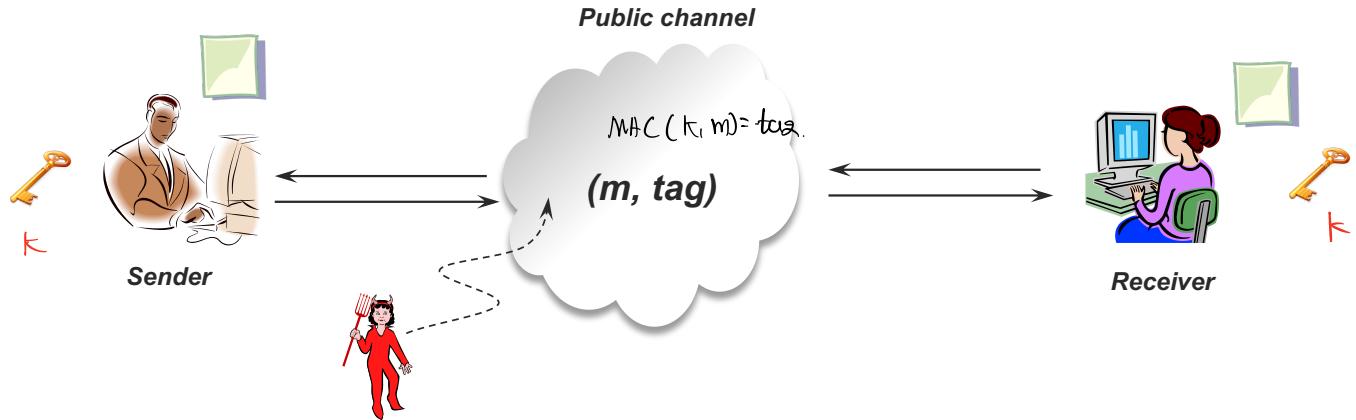
# Digital Signature Schemes

정환 박사

*Jong Hwan Park*

# What can be done with MAC?

과대광고다.

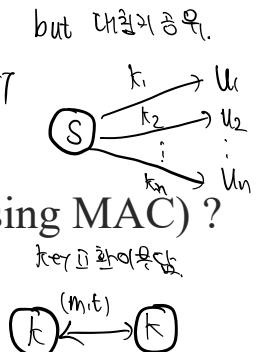


- What MAC achieved is to ensure that:

- (a) The message is sent from the very sender Sender이었을 때
  - (b) The message is not changed during transmission Integrity

- In most situations, need more than two properties:

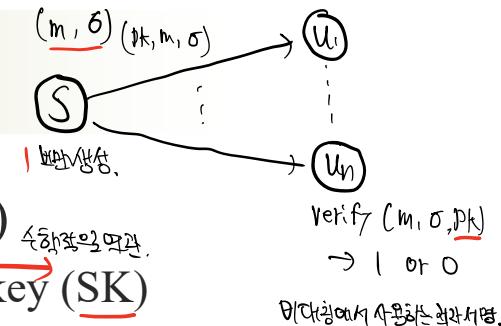
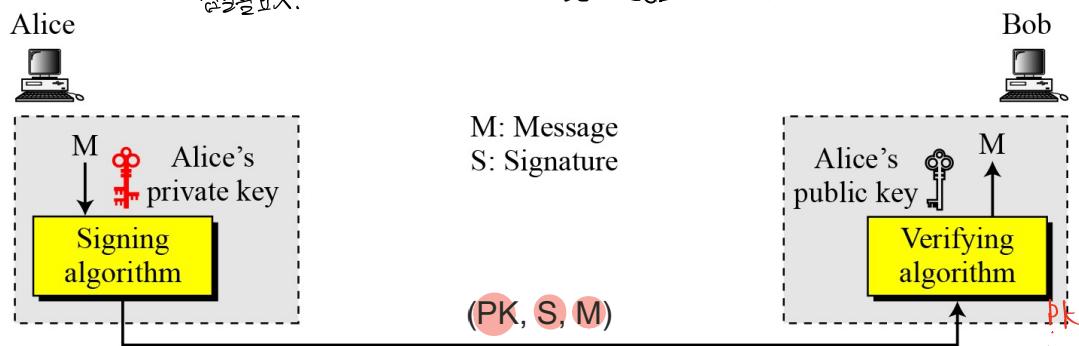
- What if server releases a software patch to all of clients (using MAC) ?
  - How can a judge be convinced of the validity of  $(m, \text{tag})$ ?



# Digital Signature Scheme (DSS)

- Definition of DSS = (KeyGen, Sign, Verify)

- KeyGen( $1^n$ )  $\rightarrow$  a public key (PK) and a private key (SK)
- • Sign(SK, M)  $\rightarrow$  a signature (S) on a message (M)
- • Verify(PK, S, M)  $\rightarrow$  output 1(valid) or 0(invalid)



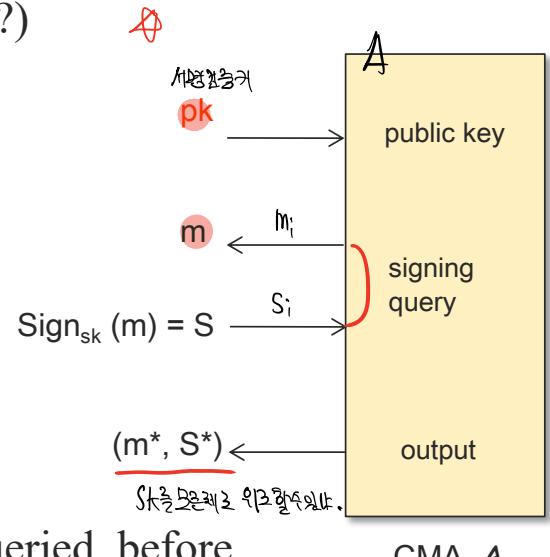
- PK is the signer(Alice)'s public key
  - How can receiver(Bob) obtain PK?  $\text{pk}$ 를 함께 전달.
  - Need authenticated channel for distributing PK (why?) Alice 개인적 키의 유통성.

# What can be done with DSS?

- As in MAC, DSS provides:
  - (a) sender authentication
  - (b) integrity
- More properties that DSS gives
  - (c) public verifiability (why?) 공개적으로 증명가능. / MAC은 대체로 누구나 확인이 가능
  - If signature S is valid, all other parties also verify it as valid
  - (d) transferability ( $\leftarrow$  public verifiability)  $(pk, s, m)$  대체로 어떤 기관에서 전달가능.
  - Signature can be copied and shown to other parties
  - ~~(e)~~ non-repudiation (보안증명) MAC과의 증명전환 가능
  - Signer cannot later deny having signed a message M
  - Meaning a receiver can prove a judge that a signer did certify M
  - Why is this not the case using MAC?
  - Not confidentiality!

# Security of DSS

- Adversary is given a public-key PK, meaning that:
  - Adversary can verify  $(m, S)$  for free (why?)
  - Defining chosen-message attack (CMA):
  - What is the goal of adversary?
    - Forgery of signature  
작성
- Adversary breaks DSS iff
  - (a) Verify( $\underbrace{m^*}_{\text{pk}}, \underbrace{S^*}_{\text{pk}})$  = 1 & (b)  $m^*$  is not queried before
- Existentially unforgeable under chosen message attacks
  - vs. selective forgery
  - Why is the existential unforgeability meaningful? (random messages)

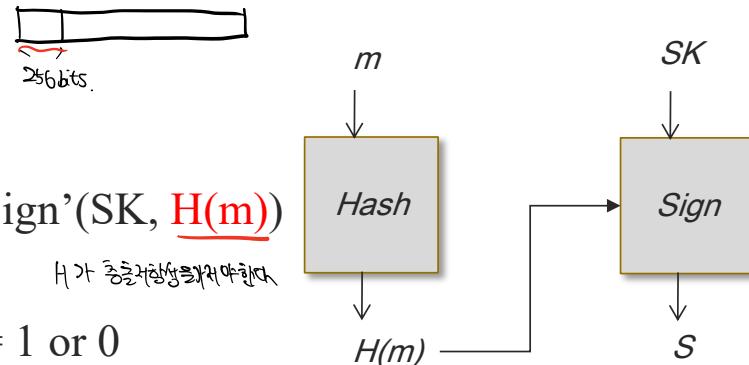


# Hash-then-Sign Paradigm

- As in PKE, DSS is very inefficient when handling long messages
  - Idea: sign a hash value  $H(m)$  instead of a long message

- How it works:

- Gen( $1^n$ ): outputs  $(PK, SK)$
- Sign( $SK, m$ ): obtain  $H(m)$  and  $S = \text{Sign}'(SK, \underline{H(m)})$
- Vrfy( $\underline{PK}, \underline{m}, S$ ): obtain  $\underline{H(m)}$  and  
 $\text{Vrfy}(PK, H(m), S) = 1 \text{ or } 0$



- Theorem: If  $H$  is collision-resistant, the  $\text{sig} = (\text{Gen}, \text{Sign}, \text{Vrfy})$  is existentially unforgeable under chosen message attacks

- What if  $H$  is not collision-resistant?

$$\text{Sign}(SK, m_1) \rightarrow S$$
$$\underline{H(m_1)} = \underline{H(m_2)}$$

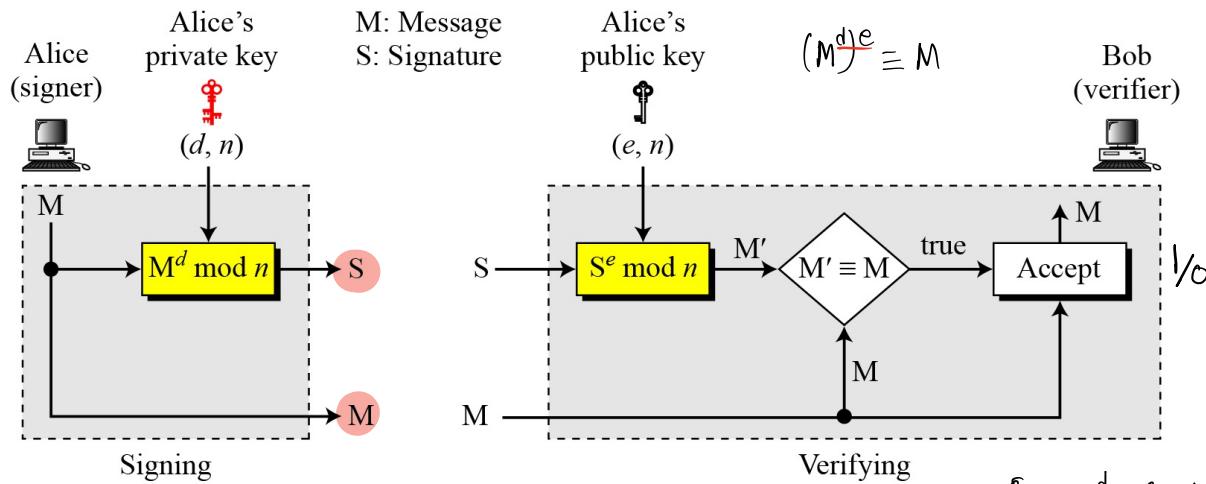
- General & standard way!

$$\text{Verify}(PK, m_2, S) \rightarrow \checkmark$$

# Textbook RSA Signature

- Key generation same as in PKE

- PK = (n, e) and SK = (d)
- How it works:



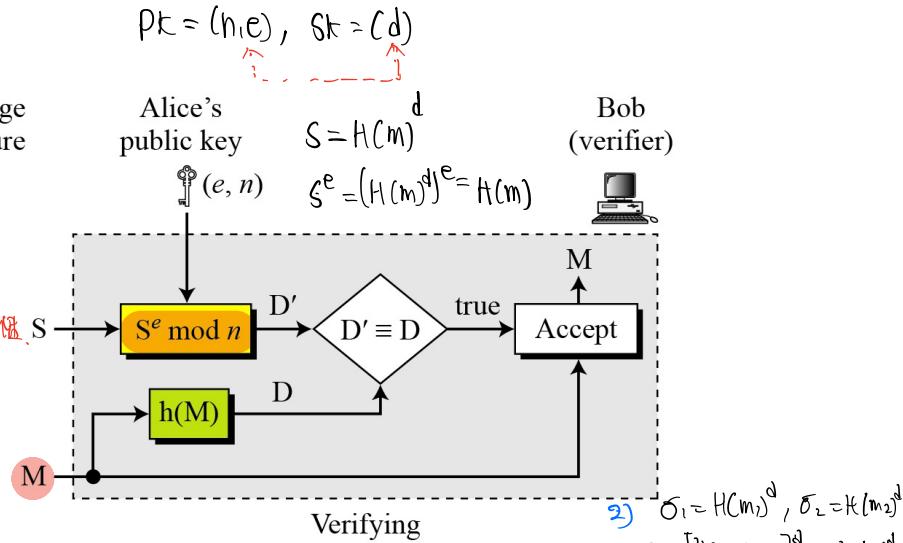
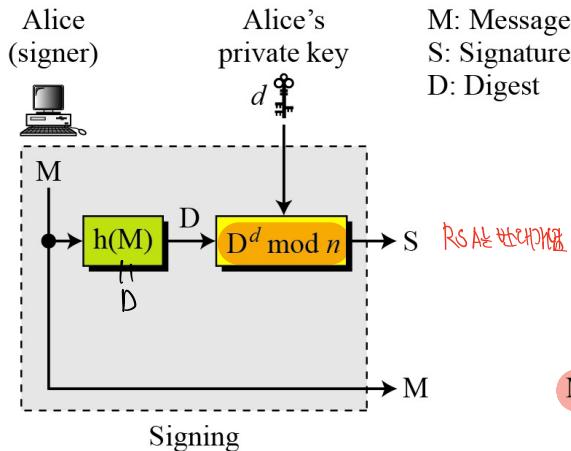
- Is it secure?

- 1) No-message attack (select  $S, S^e \text{ mod } n$ )  
 $m^{\frac{d}{e}} = S^e, \delta = S \Rightarrow (\delta)^e \equiv m^d \Rightarrow \delta^e \equiv S^e \Rightarrow \delta = S$
- 2) Known-message attack (given  $m_1^d, m_2^d \text{ mod } n \rightarrow (m_1 m_2)^d \text{ mod } n \Rightarrow m_1 \cdot m_2 \approx \delta^d$ )  
 $\delta_1 = m_1^d \pmod{n}$   
 $\delta_2 = m_2^d \pmod{n}$

# Hashed RSA Signature

$$(\underline{n = p \cdot q}, e) \rightarrow d$$

## How it works:



## How can we prevent the two previous attacks?

- No-message attack / known-message attack
  - Theorem: hashed RSA signature is existentially unforgeable under CMA
  - In PKCS#1:  $H(M) = \left[ \underbrace{00}_{4\text{bit}} \parallel \underbrace{01}_{1\text{bit}} \parallel \underbrace{FF}_{2\text{bit}} \parallel \cdots \parallel \underbrace{FF}_{2\text{bit}} \parallel \underbrace{h(m)}_{25\text{bit}} \right]^d \pmod{n}$
- Handwritten notes:
- $\sigma^* = H(m^*) \neq m^* = H(m^*)^e$
  - $\Rightarrow (\sigma^*)^e = H(m^*)^e = H(H(m^*))^e \Rightarrow$  나온다.
  - $H(m^*)^e = H(\square\square\square)$
  - 일상生活中에서 예상된다.

# Digital Signature Algorithm (DSA) → ECDSA

- In 1991, DSA was proposed by NIST
  - Has been proved secure and widely used until now
  - Advantages:
    - Shorter size of signature (why?)
    - Faster signature generation

## How it works:

- Gen:  $PK = (H, p, q, g, y = [g^x \bmod p])$  and  $SK = (H, p, q, g, x)$
- Sign: select a random  $k \in \mathbb{Z}_q^*$  and set

$$r := [\underline{[g^k \bmod p]} \bmod q] \quad \text{and} \quad s := [\underline{(H(m)+xr) \cdot k^{-1}} \bmod q]$$

\* signature =  $(r, s)$  256 bit에 대한 표기법입니다.

Vrfy: output 1 iff  $r =? [\underline{[(g^{H(m)}y^r)^{1/s}] \bmod p}] \bmod q$

$$\Rightarrow \frac{g^r \cdot y^b}{g^r} \rightarrow 1.1 \text{ 차수 과정을.}$$

$$\frac{y^b}{2^k} = r / 2^k \pmod p$$

$$2^k \cdot 2^k \cdot 2^k \cdots 2^k$$

$$(g^r \cdot y^b)^{\frac{1}{2^k}} = g^r \cdot y^{\frac{b}{2^k}}$$

$$= g^r \cdot y^{\frac{b}{2^k}} = g^r \cdot y^{\frac{b}{(H(m)+xr) \cdot k^{-1}}} = g^r \cdot y^{\frac{b}{(H(m)+xr) \cdot \frac{1}{k}}} = g^r \cdot y^{\frac{bk}{H(m)+xr}}$$

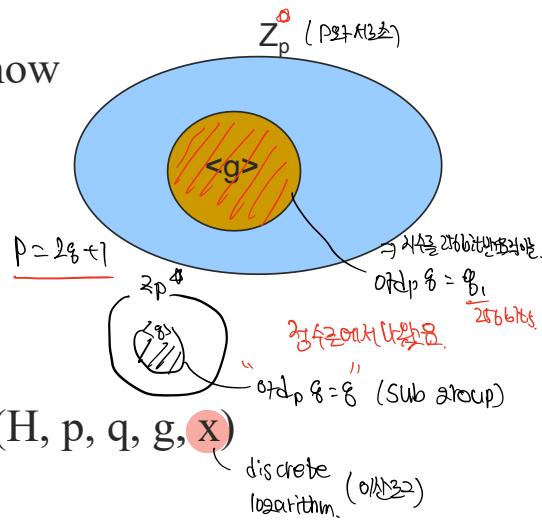
- Differently chosen  $k$  should be used at each signing (why?)

$$S_1 = \frac{H(m)+xr}{k}$$

$$S_2 = \frac{H(m)+2xr}{k}$$

$$\frac{S_1}{S_2} = \frac{H(m)+xr}{H(m)+2xr}$$

$$\Rightarrow k \text{는 고정되어야 한다.}$$



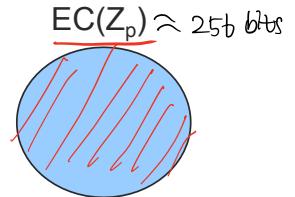
# ECDSA



$$P = 2g_1 \cdot g_2 + t$$

## ■ DSA based on Elliptic Curve

- Very efficient and widely used recently
- Suitable for resourced-constraint IoT devices



## ■ How ECDSA works:

- ECDSA-Setup( $1^n$ )
  - $PK = (G, \underline{q}, \underline{g}, \underline{y=g^x})$ ,  $sk = (\underline{x})$
- ECDSA-Sign( $sk, m$ )
  - Pick random  $\underline{k} \leftarrow Z_p$  and set  $\underline{g^k} \in G$
  - $r = F(g^k) \pmod{\underline{q}}$  and  $\underline{s = k^{-1}(H(m) + xr) \pmod{\underline{q}}}$
  - $\sigma = (r, s) \in Z_p \times Z_p$
- ECDSA-Verify( $\sigma, PK, m$ )
  - Compute  $w = s^{-1} \pmod{\underline{q}}$
  - $A = (g^{H(m)}y^r)^{1/s}$  and output 1 iff  $r = F(A) \pmod{\underline{q}}$

$$\begin{aligned} & PK(\underline{g}, \underline{y = g^x \pmod{p}}, \underline{p}) \\ & EC \end{aligned}$$
$$\begin{aligned} & r = F(g^k) \pmod{\underline{q}} \quad \text{and} \quad \underline{s = k^{-1}(H(m) + xr) \pmod{\underline{q}}} \\ & \sigma = (r, s) \in Z_p \times Z_p \\ & S = \frac{H(m) + xr}{k} \pmod{q} \end{aligned}$$

fixed base  
 $g^1, g^2, \dots, g^k$

$$\frac{1}{g^k}$$

# Digital Signature Applications

- Public key certificates
  - Public key management & PKI
- Secure file transfer
  - Integrity check before execution
- User authentication
  - Server authentication (in SSL/TLS)
- Authenticated key exchange
  - DS+DH, DS+PKE: auth. & key exchange (in SSL/TLS, IPsec, SSH)
- Online banking/shopping
  - Non-repudiation on transactions
- Authorization in organizations
  - Signing  $m=(PK||user||access\ right)$  instead of  $m=(PK||user)$
- And others ...

# Public Key Distribution



- PK is necessary for PKE, DSS, authentication, key agreement,...
- But, how can Alice convince Bob that she is the owner of  $PK_A$ ?



- Alice and Bob cannot solve the PK distribution problem
  - Need a trusted third party (called a Certificate Authority (CA))
  - PK can then be distributed via an authenticated channel

# Public Key Distribution (1)

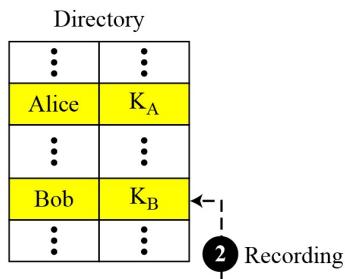
## Certificate Authority (CA)

- CA generates its own keys ( $\text{PK}_{\text{CA}}$ ,  $\text{SK}_{\text{CA}}$ )
- $\text{Cert}_{\text{CA}}(\text{PK}_{\text{Bob}}) = \text{PK}_{\text{Bob}} \parallel \text{Bob} \parallel \text{Sign}(\text{SK}_{\text{CA}}, \text{PK}_{\text{Bob}} \parallel \text{Bob})$

구성원이 알고 있다.

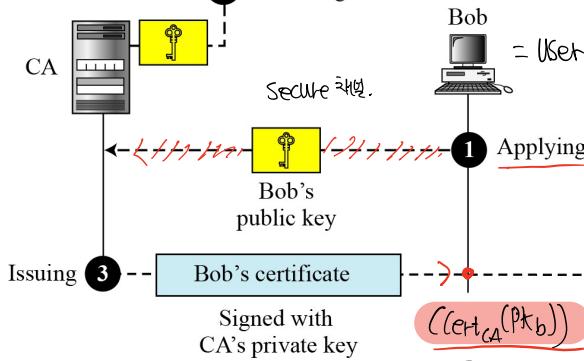
Singing

Treat it as a message signed



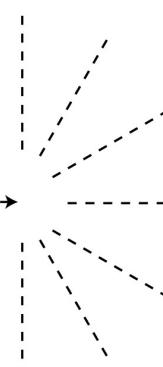
Recording

CA: Certification authority



- Bob generates ( $\text{PK}_{\text{Bob}}$ ,  $\text{SK}_{\text{Bob}}$ )
- Bob sends  $\text{PK}_{\text{Bob}}$  to CA
- CA issues  $\text{Cert}_{\text{CA}}(\text{PK}_{\text{Bob}})$  to Bob
- Later, Bob sends  $\text{Cert}_{\text{CA}}(\text{PK}_{\text{Bob}})$  to Alice

How should Alice verify  $\text{PK}_{\text{Bob}}$ ?



Distributing to public

$m, \sigma : \text{PK}_{\text{CA}}$

Alice verify  $(m, \sigma, \text{PK}_{\text{CA}})$

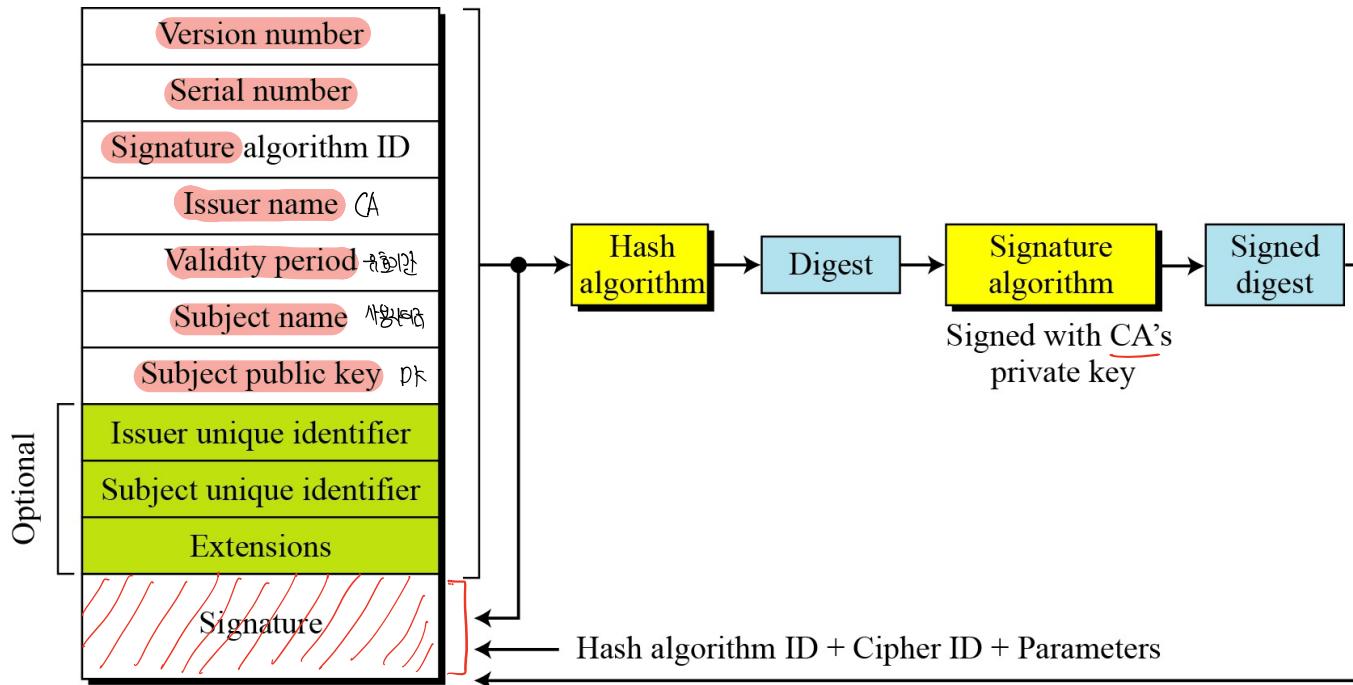
= bank server. → IoT

# Public Key Distribution (2)

## ■ X.509

- Each certificate may have a different format
- Designed by ITU, as a way to describe certificate in a structured way

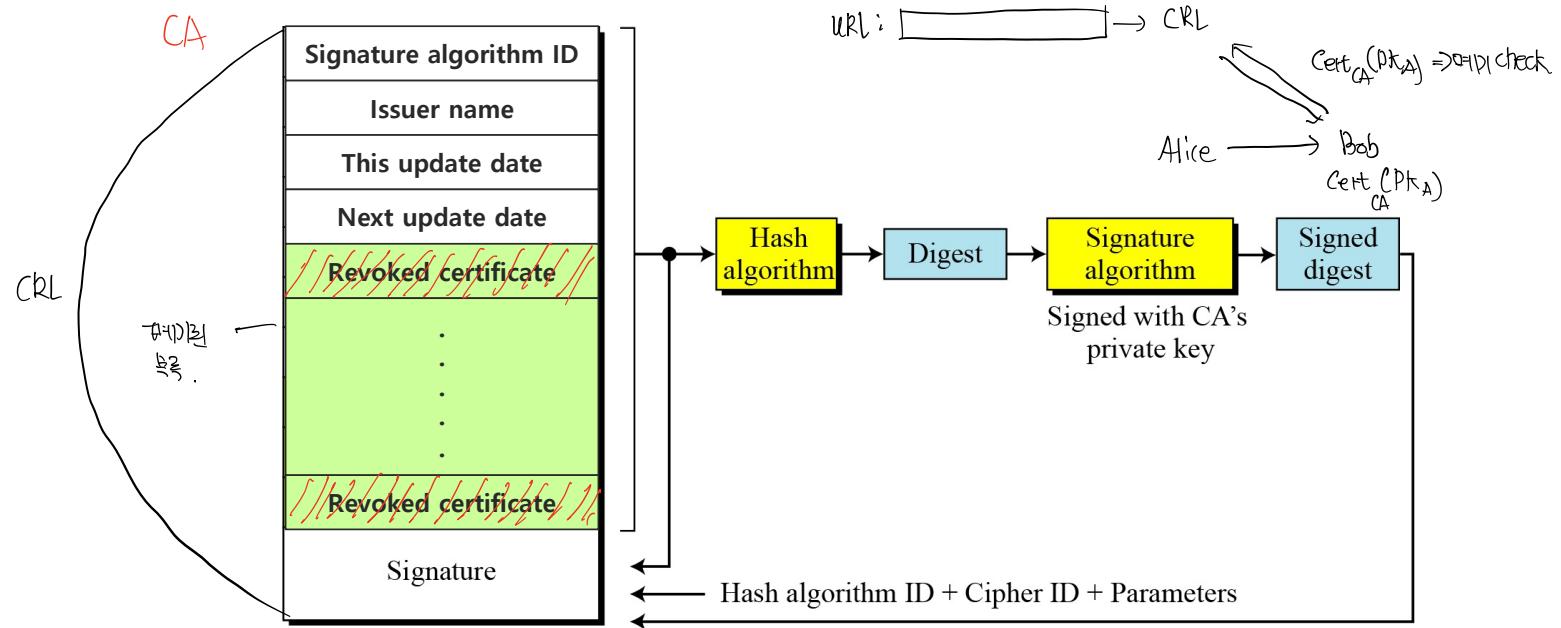
$$\text{Cert}_{CA}(PK_B) = \frac{PK_B || Bob}{m} || \text{sign}(SK_{CA}, m)$$



# Public Key Distribution (3)

## Certificate Revocation List (CRL)

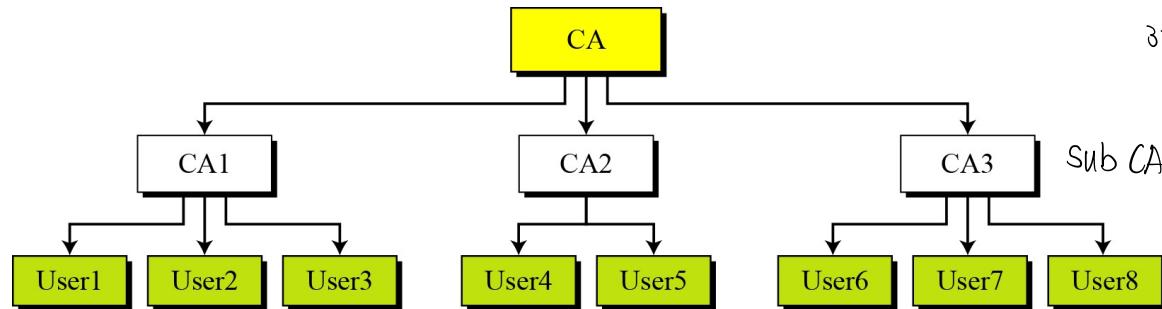
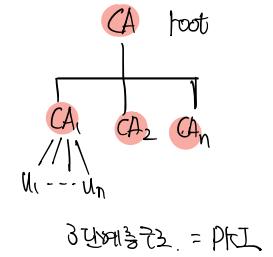
- In some cases, a certificate must be revoked before expiration
  - Compromising user's SK, losing USB, formatting hard disk
  - Being laid off in an organization,...
- Need to check whether or not a certificate is revoked before using it



# Public Key Infrastructure (PKI)



- Model for creating, distributing, and revoking certificates based on X.509
  - Single CA is unrealistic
  - In practice, there are multiple CAs in hierarchical model



$X \rightarrow Y$   
means X has signed a certificate for Y

- Delegation and Certificate chains

■  $\text{Cert}_{\text{CA1}}(\text{PK}_{\text{u1}})$ ,  $\text{Cert}_{\text{CA}}(\text{PK}_{\text{CA1}})$ ,  $\text{Cert}_{\text{CA}}(\text{PK}_{\text{CA}})$

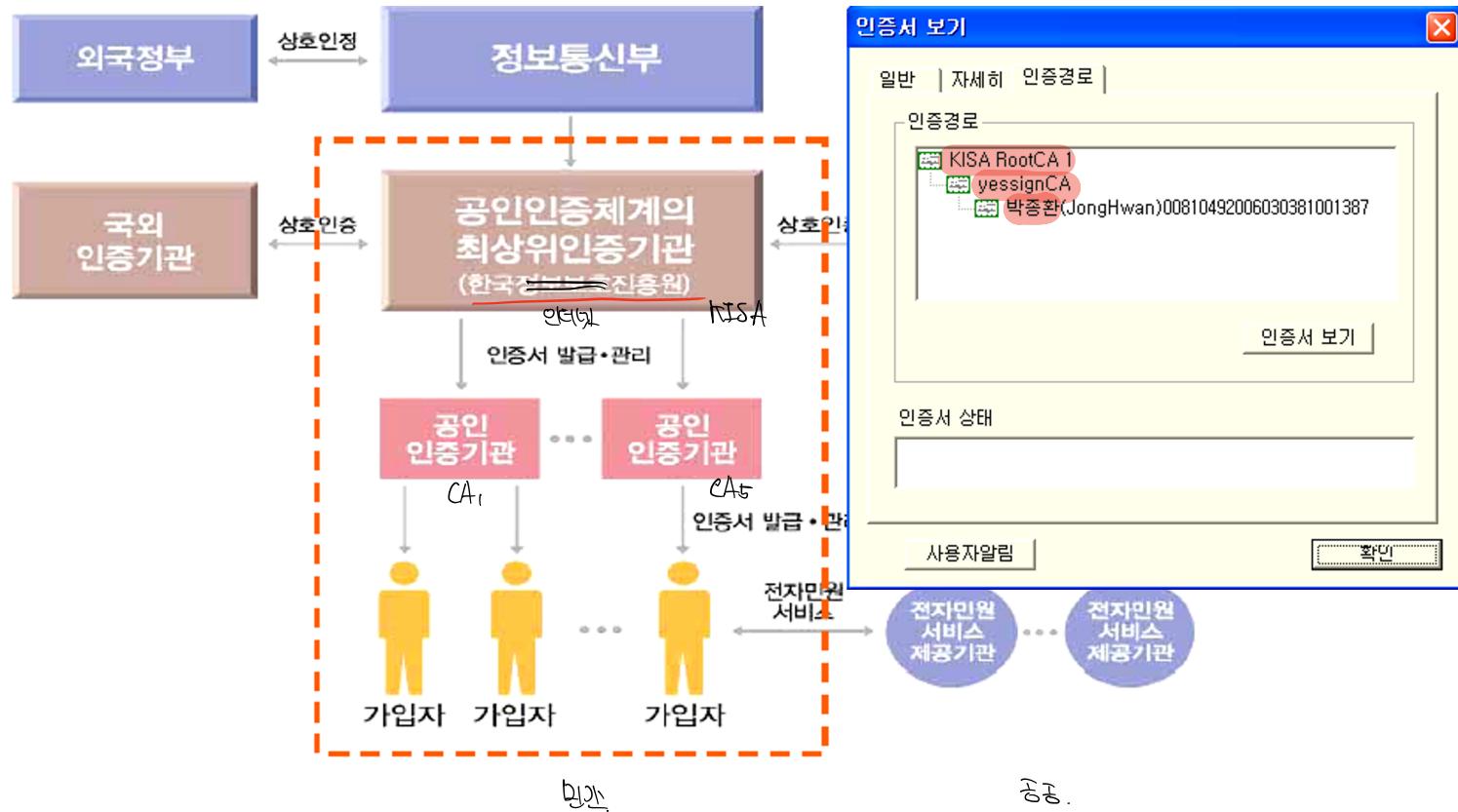
$\text{PK}_{\text{CA1}}$   $\text{PK}_{\text{CA}}$

$\Rightarrow$  3개의 Cert가 필요  
Self-Signed Cert.

Who certifies?  
→ Self-signed Cert

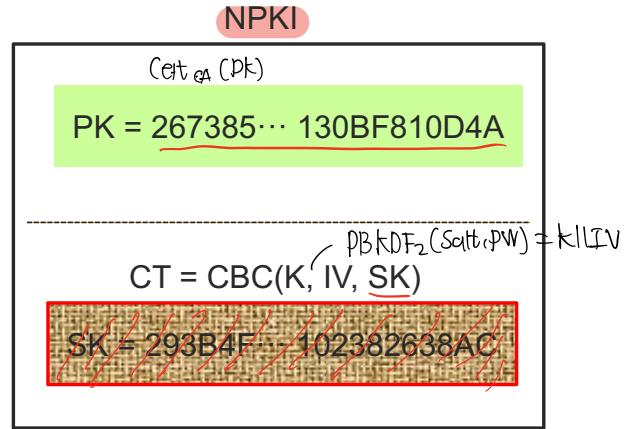
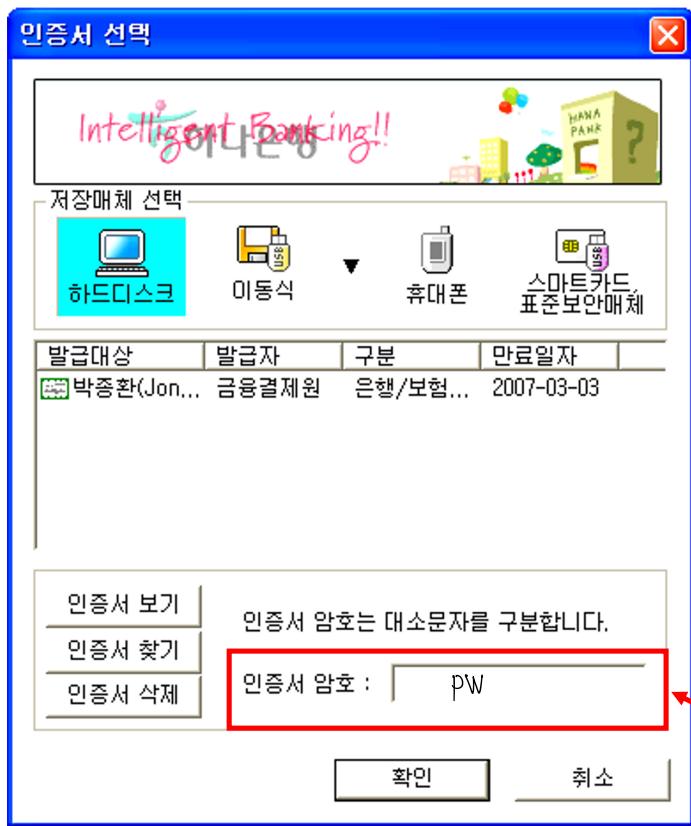
# PKI in Korea

## ■ Example of Certificate chains

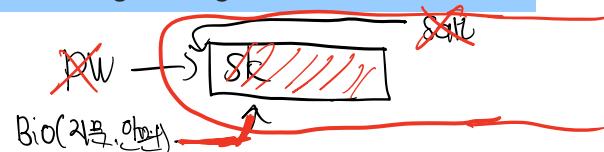


# How SK is stored (Korean Version)

- How to store a private key (SK) in NPKI 국내은행방식



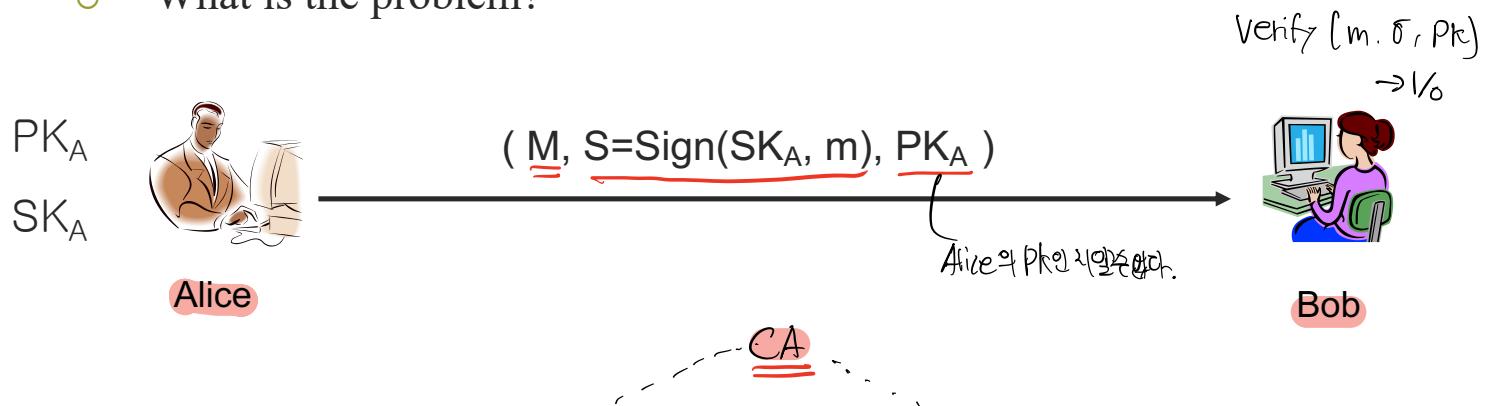
- (K, IV) = KDF(PW, salt, iteration number)
- SK = Decrypt (CT) using (K, IV)
- Sign a message using SK



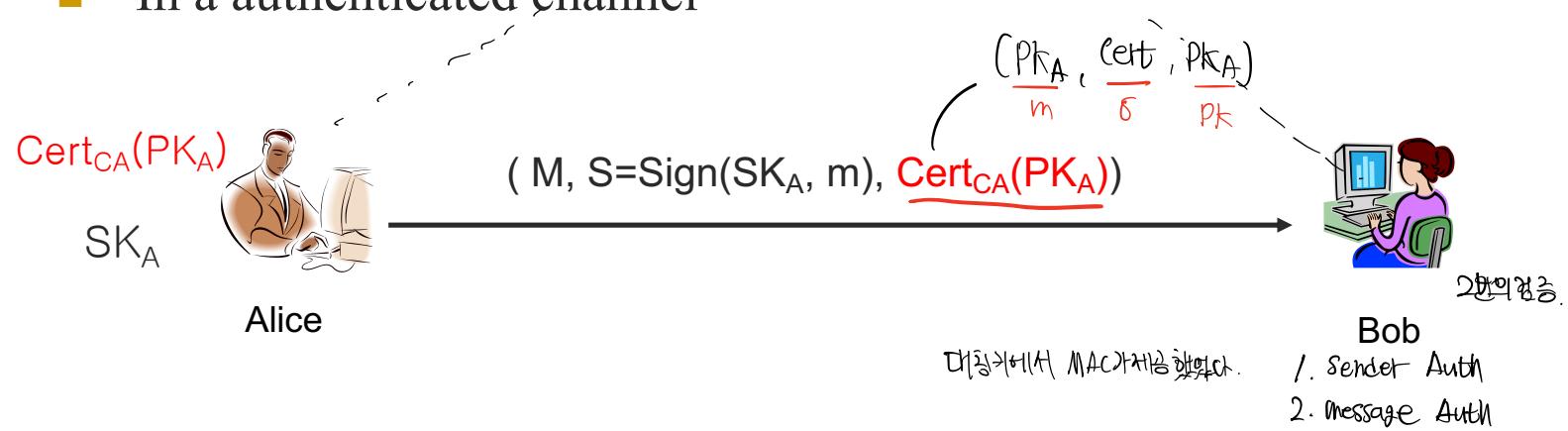
Why is that?

# File Transfer via Authenticated Channel

- Alice wants to send a file (or message) to Bob
  - What is the problem?

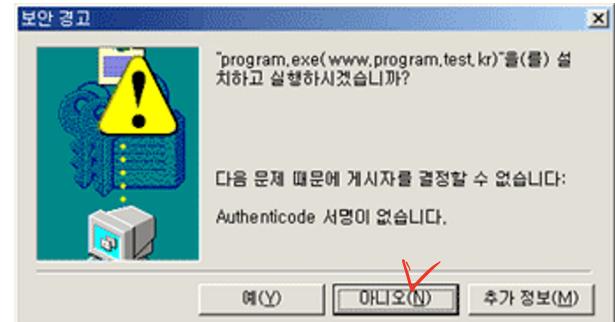
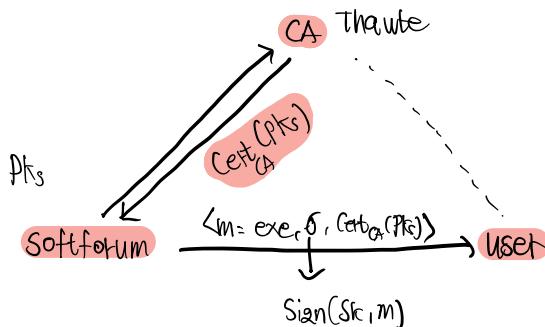
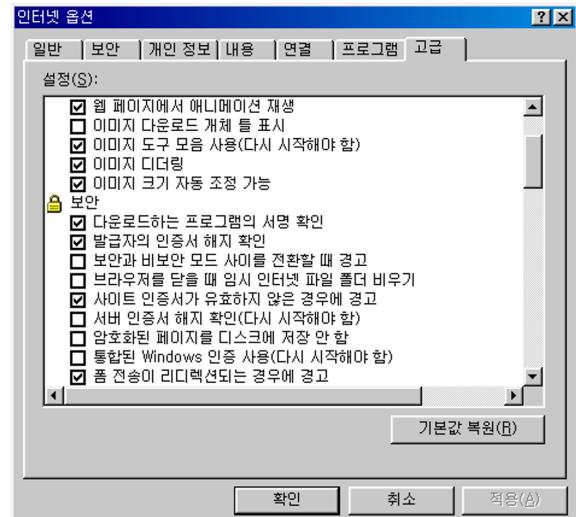
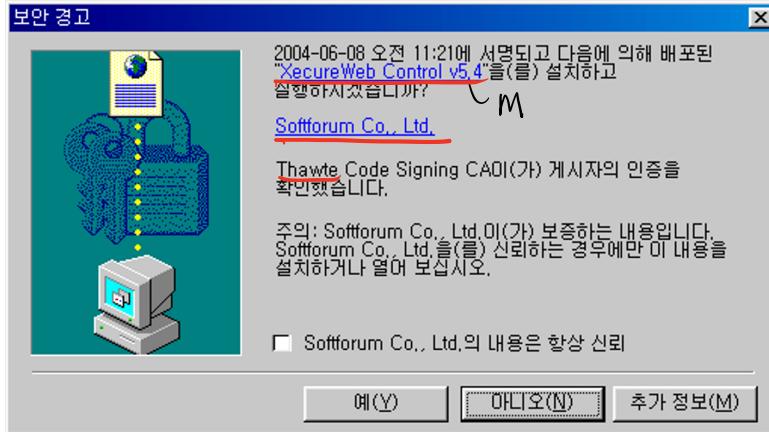


- In a authenticated channel



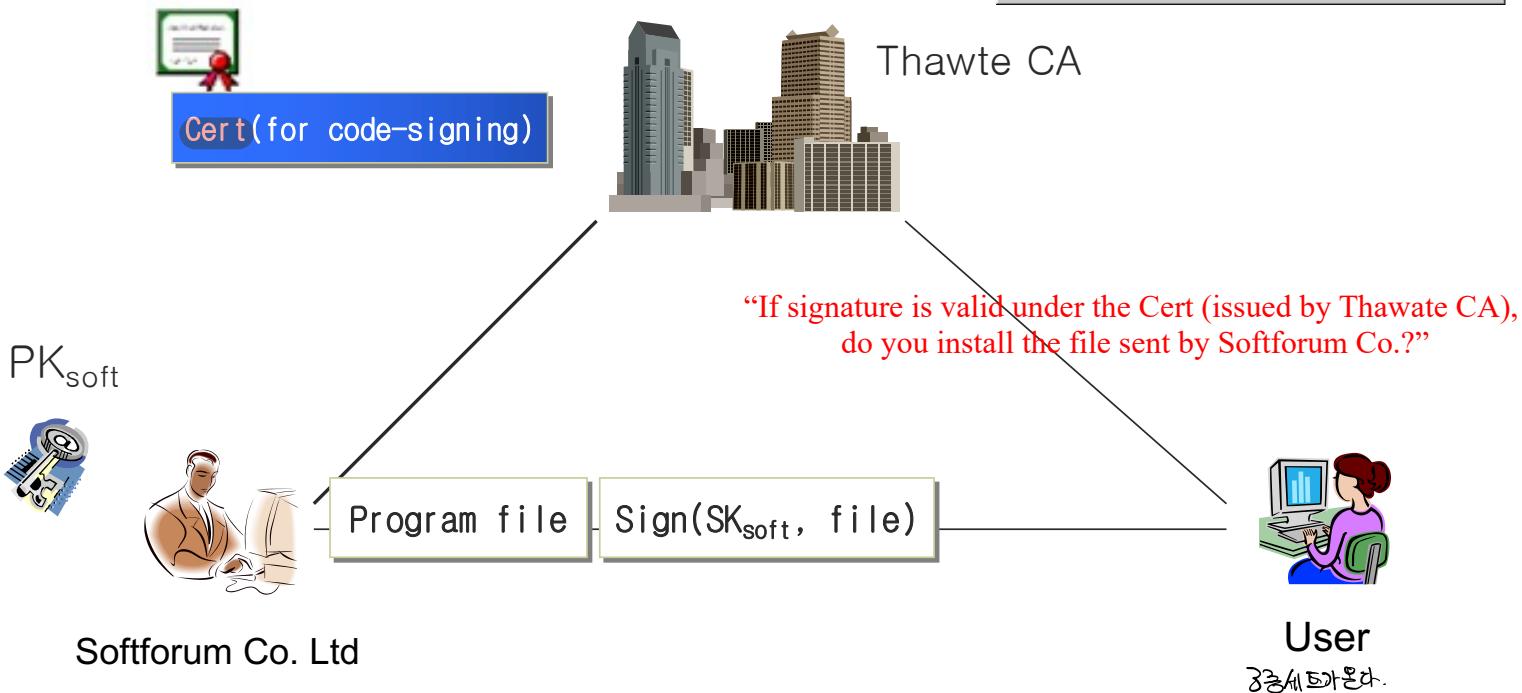
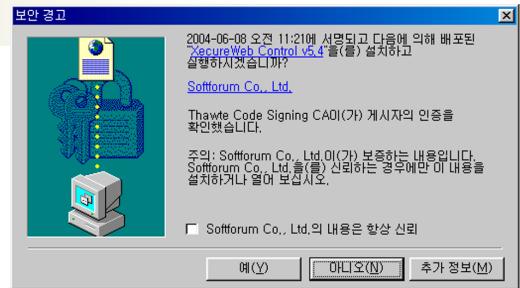
# Integrity Check using DSS (1)

## ■ Software distribution



# Integrity Check using DSS (2)

- What can we know from the notice?



# Integrity Check using DSS (3)

인터넷 옵션

내용 관리자

등급을 사용하여 볼 수 있는 인터넷 내용을 제한합니다.

사용(E)... 설정(I)...

인증서

인증서를 사용하여 자신이나 인증 기관, 그리고 게시자를 합니다.

SSL 상태 지우기(S) 인증서(C)... 게시자(E)

개인 정보

자동 완성을 저작해 두 미진 입력 내용에서 일치하는 내용을 추천해 줍니다.

자동 완성(U) Microsoft 프로필 관리자는 개인 정보를 저장합니다.

내 프로필(R)

확인 취소

인터넷 옵션

내용

인증서

용도(N): <전체>

개인 다른 사람 중개 인증 기관 신뢰된 루트 인증 기관 신뢰된 게시자 신뢰되지 않은 게시자

발급 대상: INICIS CO., LTD. 발급자: Thawte Code Sign... 만료 날짜: 2005-05... 이름: <없음>

가져오기(I)... 내보내기(E)... 제거(B) 고급(A)...

인증서 용도

보기(V) 닫기(C)

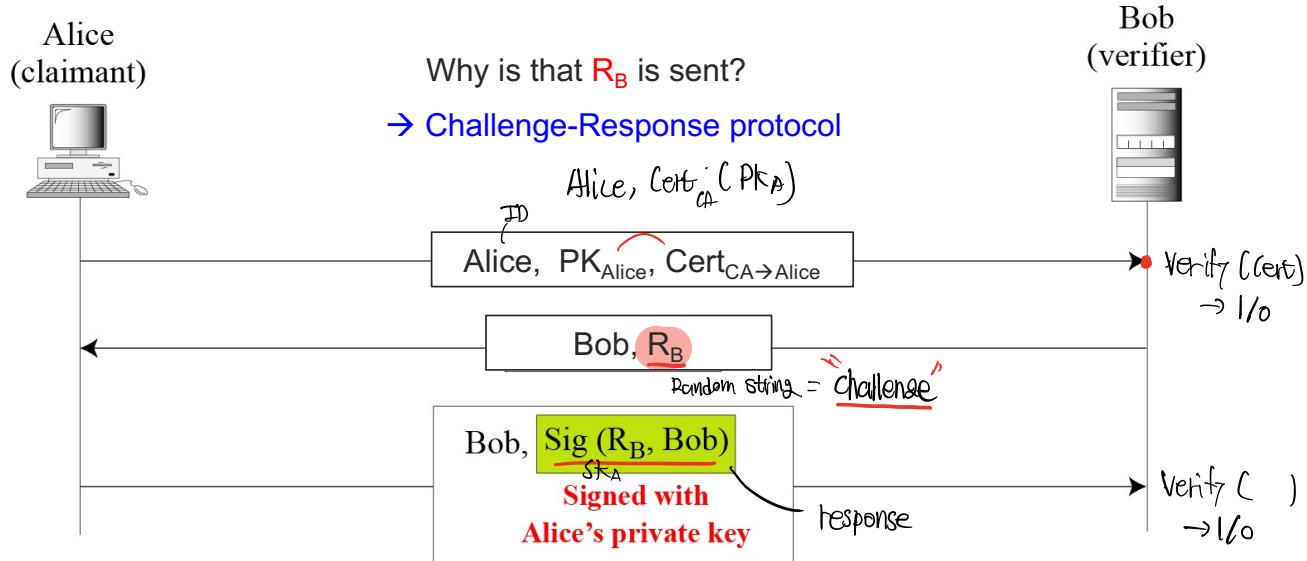
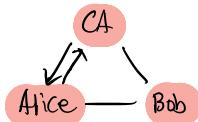
# Authentication using DSS (1) “인증서로의”

인증

- Showing certificates can be used as “digital ID-card” (why?)

- Situation:

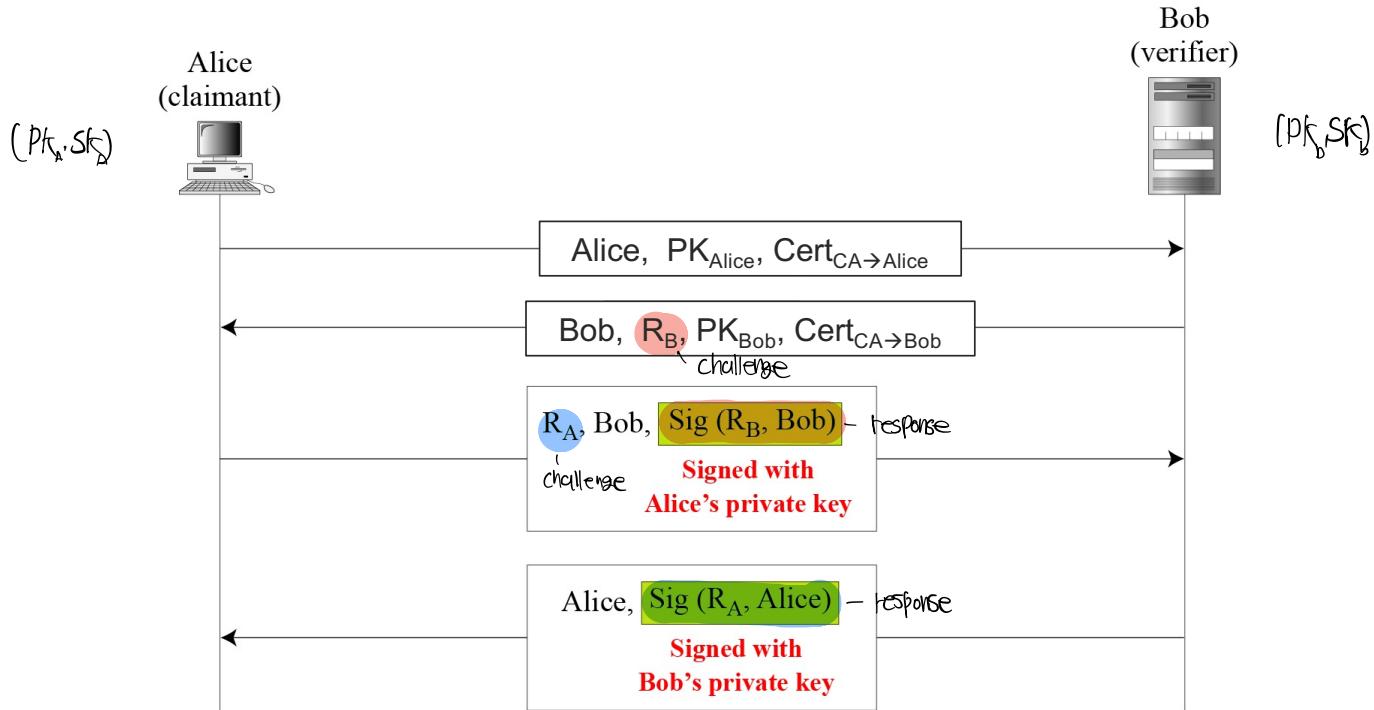
- Alice has her own (PK, SK) and certificate related to PK
- Alice is proving to Bob that “I am Alice”



# Authentication using DSS (2)

## ■ Situation:

- Alice and Bob have their own (PK, SK) and certificates
- Both want to authenticate each other in bidirectional way



# Authentication using DSS - example

## ■ Entity authentication

**01** 공인인증서 로그인  
Intelligent Banking

인증서로그인

[하나인증센터 바로가기 >](#)

유의사항  
이용하고 계시는 인증서의 발급/재발급/갱신 등 인증센터 서비스 이용 시 이용자ID/이용자 비밀번호가 필요합니다.  
이용자ID/이용자 비밀번호도 반드시 기억하시기 바랍니다.

**02** ID/PW 로그인  
Intelligent Banking

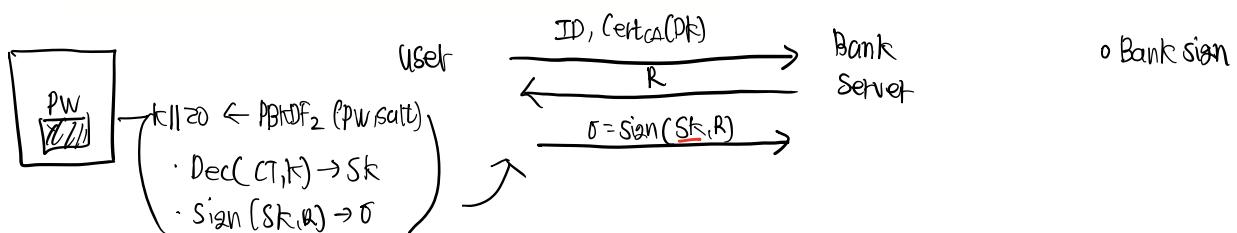
이용자 ID

이용자비밀번호

**확인**

[이용자 ID확인 >](#) [조회회원 가입 >](#)

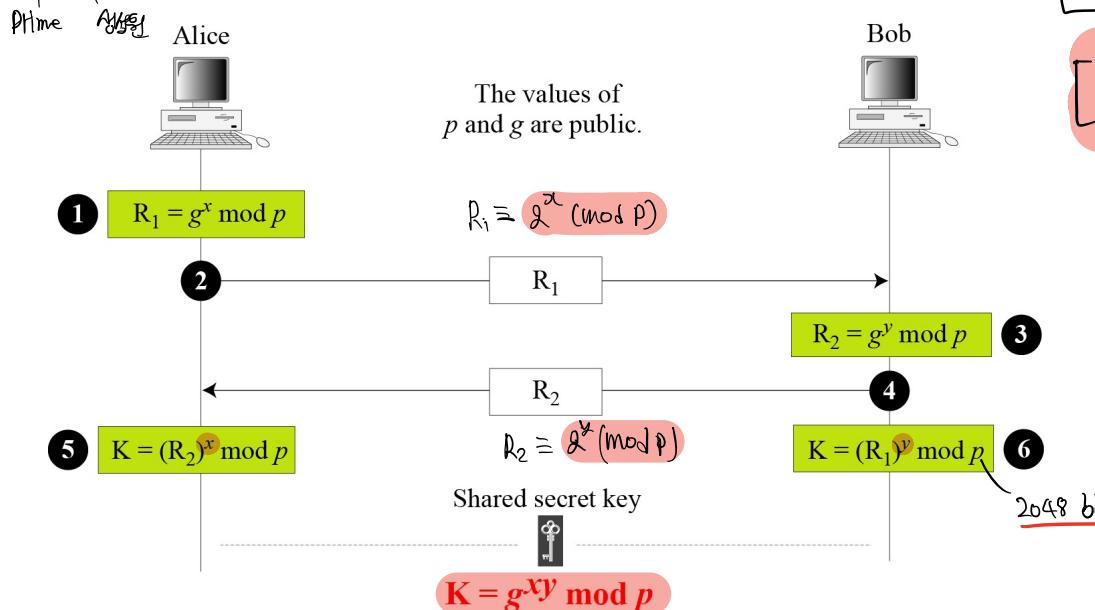
유의사항  
이용자비밀번호 5회 연속 오류 시 ID/PW 및 공인인증서 로그인 두 가지 모두 이용이 제한됩니다.



# Diffie-Hellman Key Exchange

## ■ How it works (by Diffie-Hellman, 1976)

- (p, g) should be public in an authenticated manner

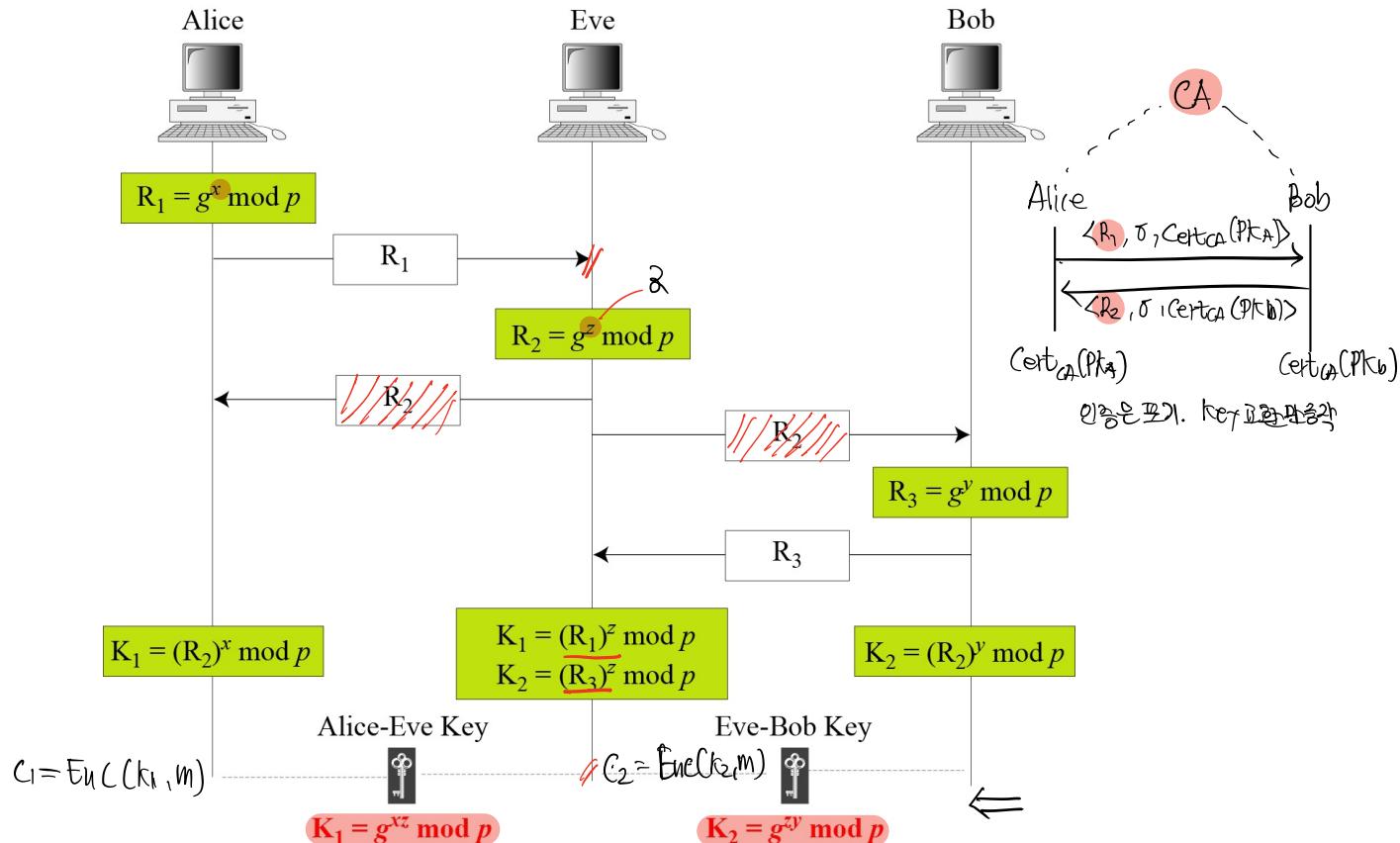


- Even if a passive attacker can see  $g^x$  and  $g^y$ , it is infeasible to get  $K=g^{xy}$
- But, insecure against active attackers → man-in-the-middle attack

# Man-In-the-Middle (MIM) Attack

## ■ Active attack

- Adversary **Eve** can intercept and modify messages being sent

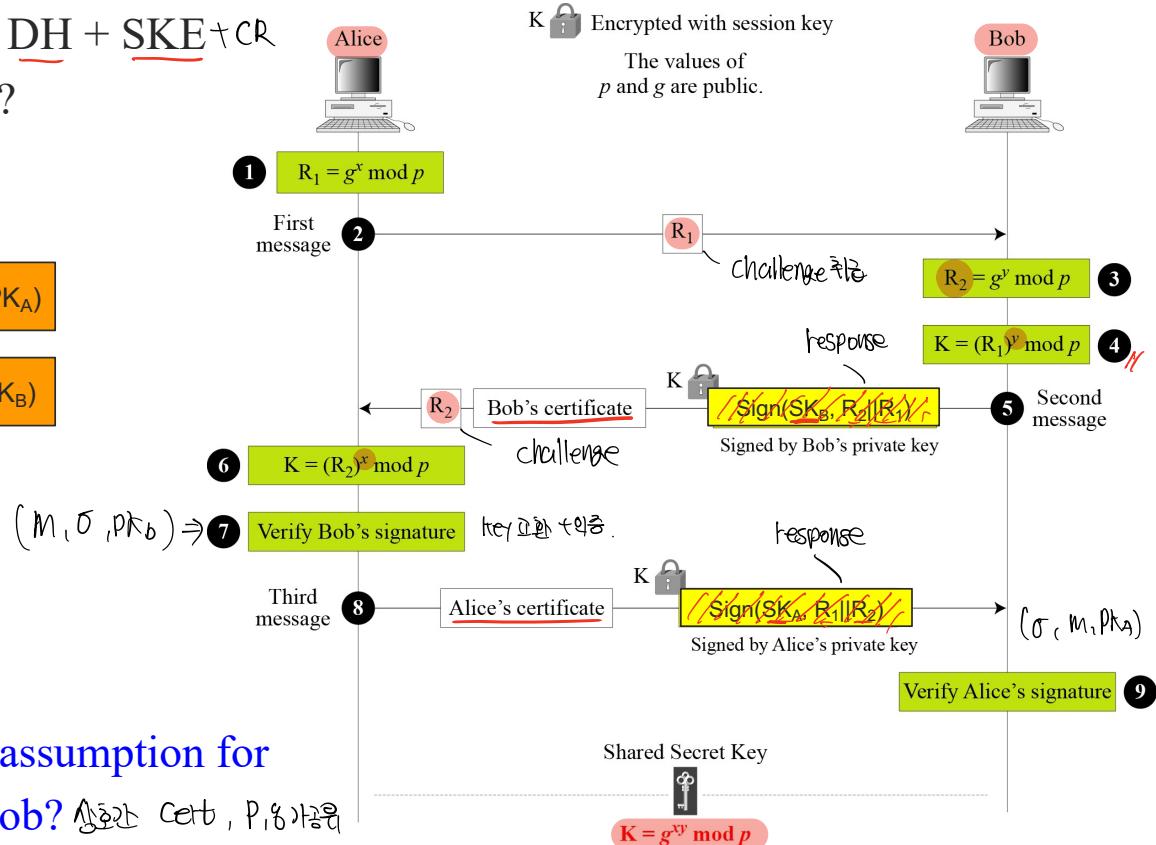


# How to prevent MIM attack (1)

- Authenticated Diffie-Hellman key-exchange protocol
  - (a.k.a) Station-to-Station (STS) protocol
  - Using DS + DH + SKE + CR
  - MIM attack?

TLS v1.2  
→ TLS v1.3  


$$\begin{aligned} \text{Cert}_A &= \text{Sign}_{\text{CA}}(\text{Alice}, \text{PK}_A) \\ \text{Cert}_B &= \text{Sign}_{\text{CA}}(\text{Bob}, \text{PK}_B) \end{aligned}$$



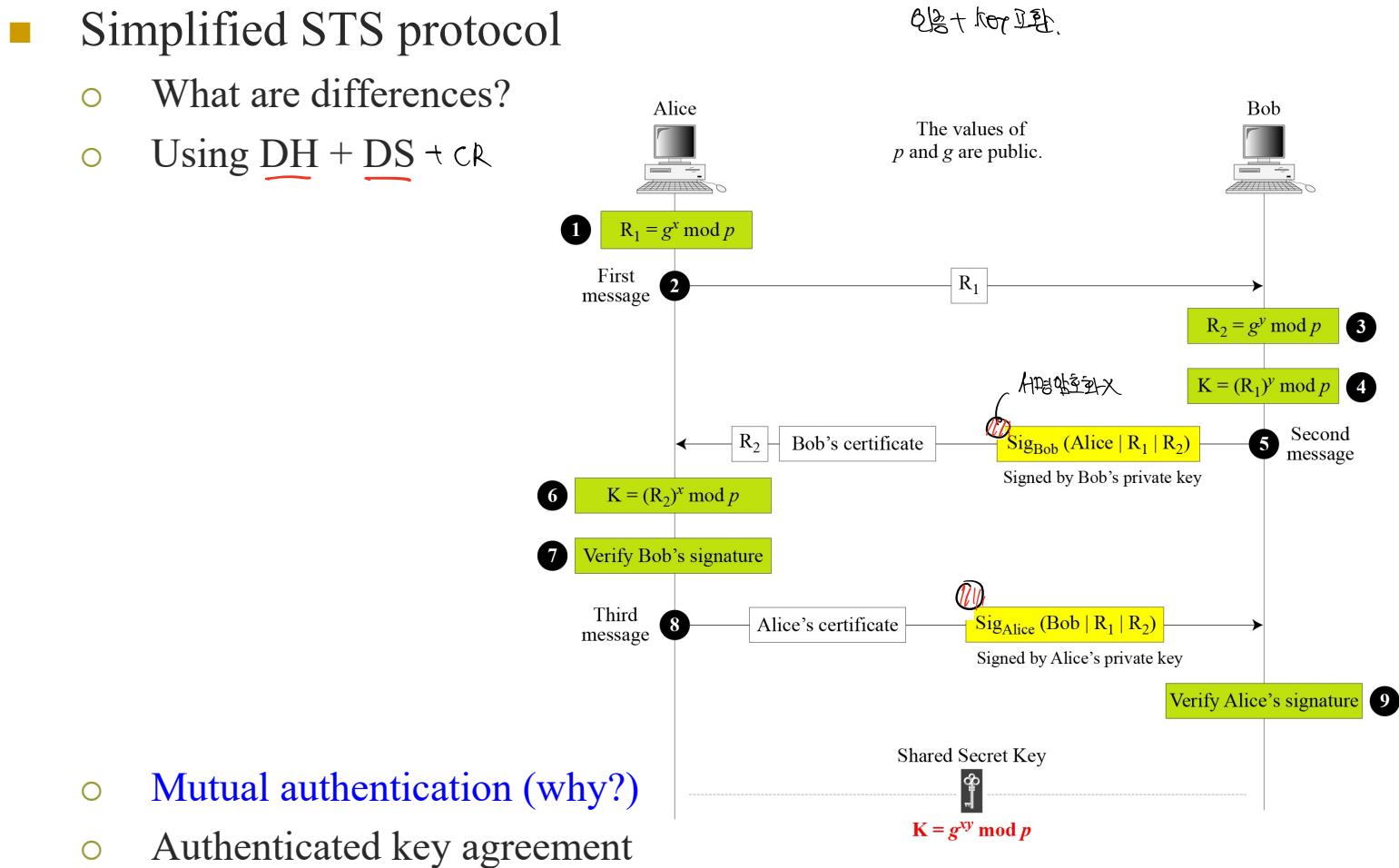
# How to prevent MIM attack (2)

↑ 악기 조작의 위험성이 있다.

## ■ Simplified STS protocol

- What are differences?
- Using DH + DS + CR

이상 + 보안 강화.



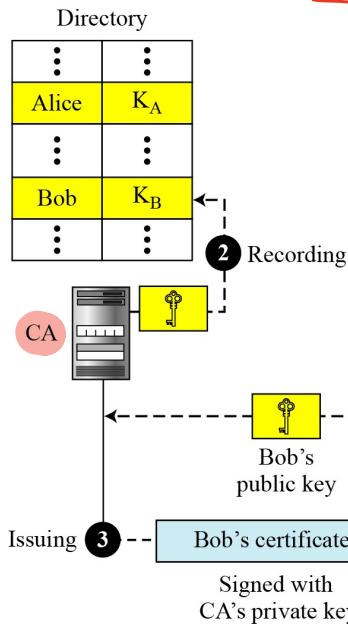
# Authorization – ID card

신분증 또는 출입증.

## ■ Certificate Authority (CA) in an organization

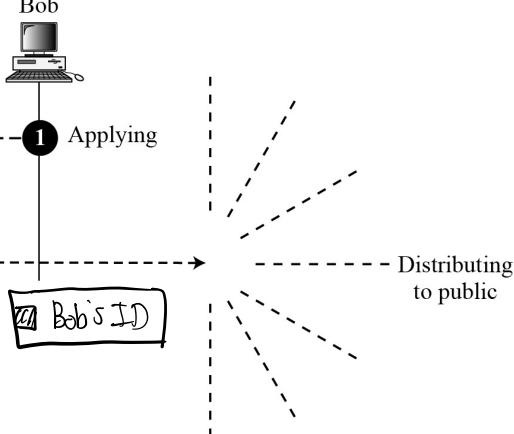
- CA generates its own keys ( $\text{PK}_{\text{CA}}$ ,  $\text{SK}_{\text{CA}}$ )
- $\text{Cert}_{\text{CA}}(\text{PK}_{\text{Bob}}) = \text{PK}_{\text{Bob}} \parallel \text{Sign}(\text{SK}_{\text{CA}}, \text{PK}_{\text{Bob}} \parallel \text{Bob} \parallel \text{access right})$

Treat it as a message signed

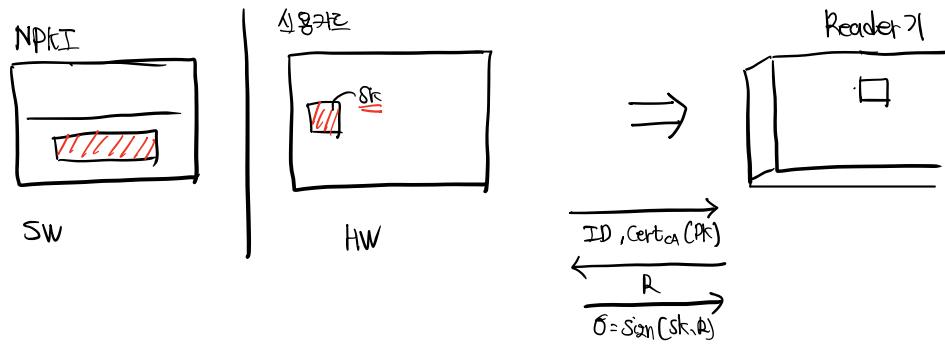


- Bob generates ( $\text{PK}_{\text{Bob}}$ ,  $\text{SK}_{\text{Bob}}$ )
- Bob sends  $\text{PK}_{\text{Bob}}$  to CA
- CA issues  $\text{Cert}_{\text{CA}}$  to Bob (considering access right)
- Later, Bob uses  $\text{Cert}_{\text{CA}}$  to authenticate himself to card reader

How should Card reader verify Bob's access right ?



Certification Access Right



# Q & A