## Table of Contents



# HW3 - Portfolio Optimization

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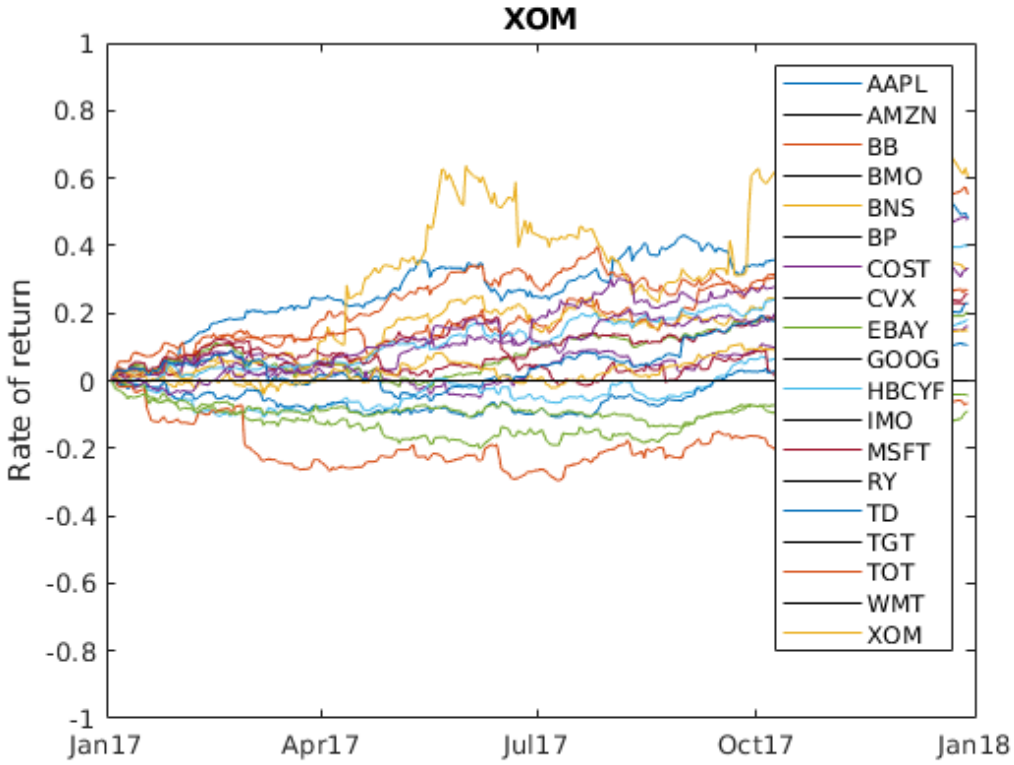
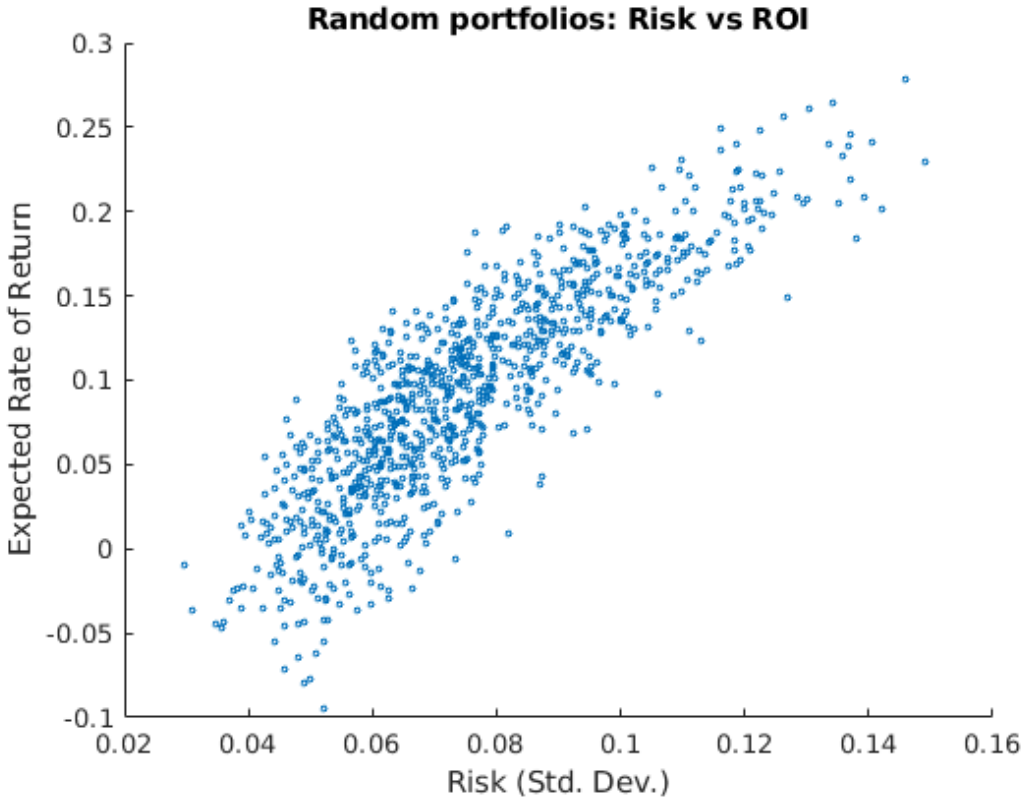
**Supplementary files:**

```
sup_code = [
"load_stocks"
"load_stock"
"portfolio_scatter"
"meancov"
"return_range"
"efficient_frontier"
"market_portfolio"
];
for i = 1:length(sup_code)
    publish(sup_code(i), 'format', 'pdf', 'evalCode', false);
end

clear all
close all

warning('off', 'MATLAB:table:ModifiedAndSavedVarnames');
% rng(1); % Random seed for testing\debugging
```

# Q1

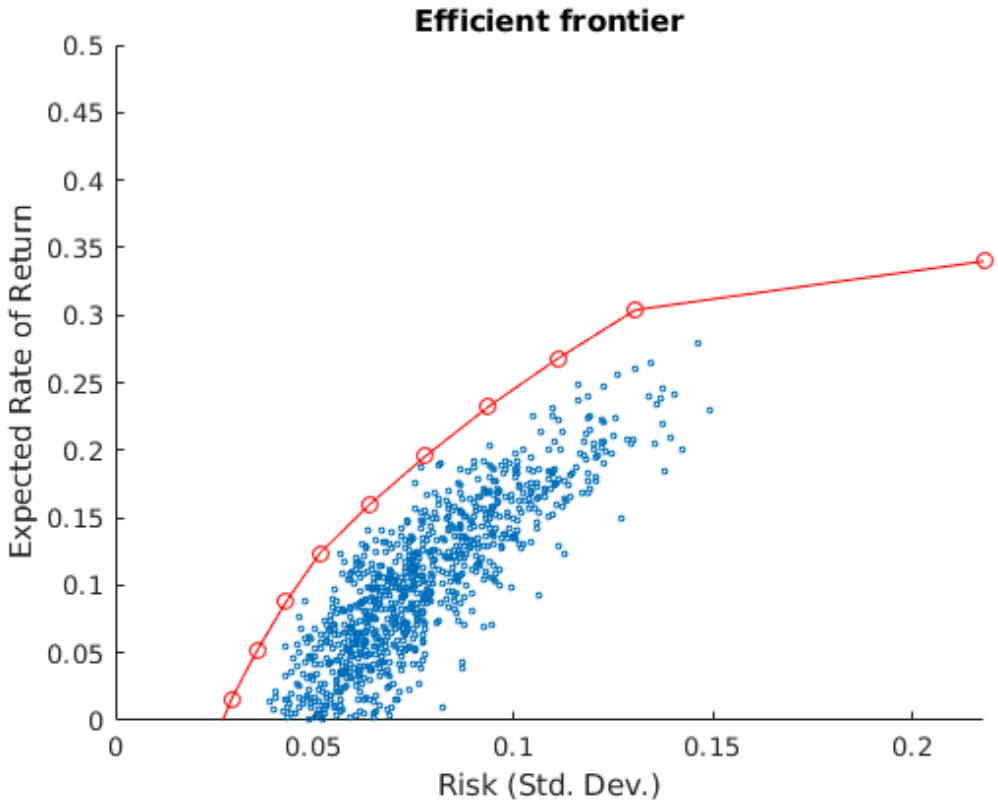
```
[X, dates, names] = load_stocks("data", "2017-01-03","2017-12-29");
disp_stocks(X, dates, names);
[r, Sig] = meancov(X);
h = portfolio_scatter(r, Sig, 1000);
```


XOM
Rate of return
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Jan17
Apr17
Jul17
Oct17
Jan18
AAPL
AMZN
BB
BMO
BNS
BP
COST
CVX
EBAY
GOOG
HBCYF
IMO
MSFT
RY
TD
TGT
TOT
WMT
XOM


Random portfolios: Risk vs ROI
Expected Rate of Return
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
Risk (Std. Dev.)

# Q2

```
num = 12;
[Y,rates,sigs]= efficient_frontier(r,Sig,num);
figure(h);
hold on;
title("Efficient frontier");
plot(sigs, rates, 'ro-');
ylim([0 0.5]);
xlim([0 max(sigs)]);
```

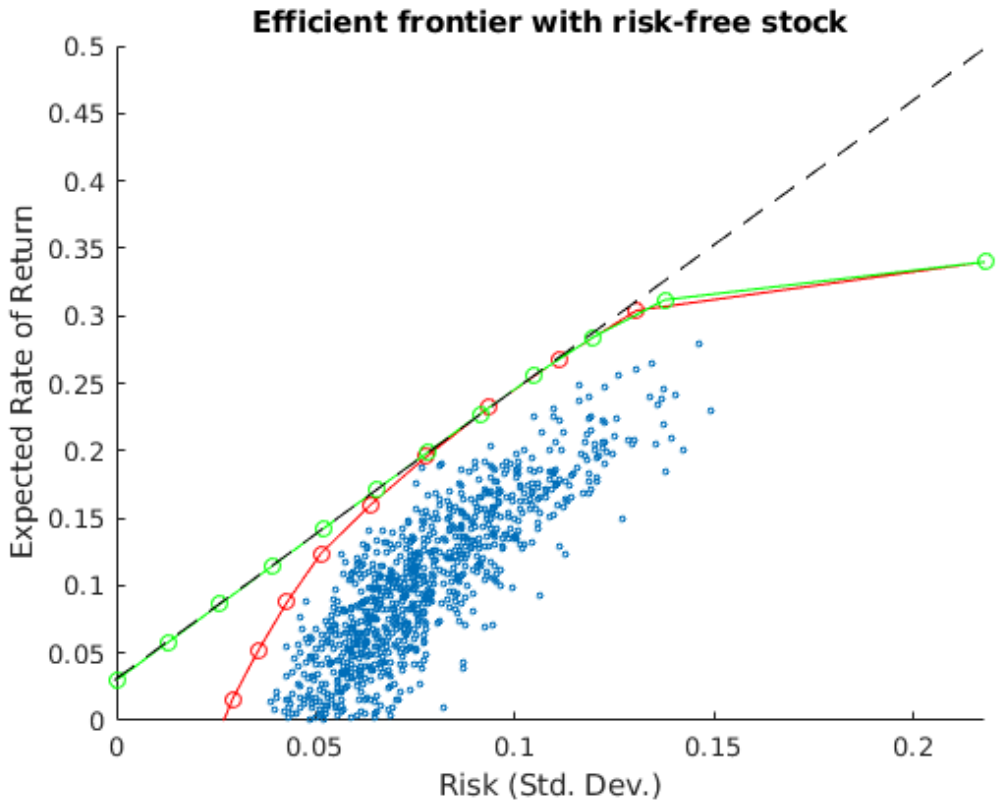



# Q3

```
f = 0.03;
r_= [r, f]; % add risk free return
Sig_ = [Sig , zeros(19,1)] ;
Sig_ = [Sig_; zeros(1,20)]; % build risk free Sig
[Y_,rates_,sigs_]=efficient_frontier(r_,Sig_,num); %calculate
 coresponding efficient frontier
plot(sigs_,rates_,'go-');
title("Efficient frontier with risk-free stock")
market_x = market_portfolio(f,r',Sig);
annotation(h,'line',[0.128070175438597 0.910526315789474],...
    [0.15852380952381 0.928571428571429],'LineStyle','--');
```

*Binary search:*

```
Iter          x      |f(x)|
   1   1.20e-01   3.28e-02
   2   7.09e-02   3.14e-02
   3   9.54e-02   5.54e-04
   4   1.08e-01   1.99e-02
   5   1.01e-01   1.10e-02
   6   9.84e-02   5.67e-03
   7   9.69e-02   2.68e-03
   8   9.61e-02   1.09e-03
   9   9.57e-02   2.78e-04
  10   9.56e-02   1.36e-04
  11   9.57e-02   7.13e-05
  12   9.56e-02   3.23e-05
  13   9.56e-02   1.96e-05
  14   9.56e-02   6.35e-06
  15   9.56e-02   6.61e-06
  16   9.56e-02   1.29e-07
Done.
```




**Q3 - Meaning of linear part**

The linear half-line (Before the tangent point) represents the new extended region of the efficient frontier thanks to the addition of the risk free stock (RFS). The linear line is the convex combination between market portforlio (MP) and risk free stock (RFS). Since the RFS has not covariance with any other stocks, the risk of the sum of MP and RFS will increase linearly and teh return will also increase linearly as more and more portion of MP is invested. Thus, the risk and return is linearly correlated.

Prof in the next page.

Let $x_1$ be the portion of the investment allocated in the risk free stock.
let $x_2$ be the portion of the investment allocated in the market portfolio.
Return and risk of $x_1$ are $r_1$ and $\sigma_1$,
Return and risk of $x_2$ are $r_2$ and $\sigma_2$
Constrain : $x_1 + x_2 = 1$ ,$x_1 \geq 0$ and $x_2 \geq 0$

$$r_{sum} = x_1 r_1 + x_2 r_2$$

$$\sigma_{sum} = x_1 \cdot 0 + x_2 \sigma_2 = x_2 \sigma_2$$

$$\implies r_{sum} = (1 - \frac{\sigma_{sum}}{\sigma_2}) r_1 + \frac{\sigma_{sum}}{\sigma_2} r_2$$

$$\implies r_{sum} = (r_2 - r_1) \frac{\sigma_{sum}}{\sigma_2} + r_1$$

Thus $r_{sum}$ is linearly correlated with $risk_{sum}$

The top half of the tangent line represent the alfine combination of the RFS and MP. It means borrows money at the RFS rate and invest all in MP. This corresponding negetive x_1 and x_2> 1.

```
function [r, dates] = load_stock(filename, startdate, enddate)

A = readtable(filename);

dates = table2array(A(:,1));
p = table2array(A(:,6));
% Filter out the dates not between start and end date
index_array = find(dates >= startdate & dates <= enddate);
dates = dates(index_array);
p = p(index_array);

% Compute rate of return
r = (p - p(1))/p(1);

end
```

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```
function [X, dates, names] = load_stocks(dirname, startdate, enddate)
% Convert strings to datetime objects for comparisions in load_stock.m
startdate = datetime(startdate,'InputFormat','yyyy-MM-dd');
enddate = datetime(enddate,'InputFormat','yyyy-MM-dd');

files = dir(fullfile(pwd, dirname)) ;
datafiles = [];
for i = 1:length(files)
    file = files(i);
    if endsWith(file.name, ".csv")
        datafiles = [datafiles; file];
    end
end

n = length(datafiles); % n stocks
X = []; % Annoying that we do not know the number of days in
 advance...

names = cell(n, 1);
for i = 1:n
    file = datafiles(i);
    names(i) = {file.name(1:end-4)}; % Need curly brackets to convert
 to cell... Stupid..
    [roi, dates] = load_stock(fullfile(file.folder, file.name),
 startdate, enddate);
    X = [X, roi]; % Concanate new vector to matrix
end

end
```

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```
function h = portfolio_scatter(r, Sig, num)
h = figure;
randmu  = zeros(num,1);
randSig = zeros(num,1);

% For each portfolio generated a random allocation
numStocks = 5;
n = length(r);

for ii=1:num
        randomStocks = randperm(19,5); %randperm is used to avoid
 overwriting same stock
        rn = rand(numStocks, 1);
        randomAllocation = rn/sum(rn);
        weight = zeros(size(r));
        weight(randomStocks') = randomAllocation; % the selected
 stocks have some allocation, unselected stocks will be 0
        expectedReturn = r* weight';
        expectedRisk = sqrt(weight * Sig * weight'); % From modern
 portfolio theory page
        randSig(ii) = expectedRisk;
        randmu(ii) = expectedReturn;
end
scatter(randSig, randmu, 5);
title("Random portfolios: Risk vs ROI");
xlabel('Risk (Std. Dev.)');
ylabel('Expected Rate of Return');
end
```

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```
function [r, Sig] = meancov(X)

r = mean(X);
Sig = cov(X);

end
```

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```
function [rrange] = return_range(r,Sig,num)

n = length(r);

% Find the minimum bound for rate of return of a portfolio
cvx_begin quiet ;
    variable x1(n);
    maximize ( r * x1  );
    subject to ;
        sum(x1) == 1;
        min(x1) >= 0;
cvx_end;

maxr = x1;

% Find the minimum bound for rate of return of a portfolio
cvx_begin quiet;
    variable x2(n) ;
    minimize (quad_form(x2, Sig));
    subject to ;
        ones(1,n) * x2 == 1;
        min(x2) >= 0;
cvx_end;

minv = x2;

rrange = linspace(r*minv, r*maxr, num);
```

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```
function [Y, rates, sigs] = efficient_frontier(r, Sig, num)

n = length(r);
rrange = return_range(r, Sig, num);

Y = zeros(n, num);

for jj = 1:num
    current_rate_of_return = rrange(jj);
    cvx_begin quiet ;
        variable x1(n);
        minimize (quad_form(x1, Sig));
        subject to ;
            sum(x1) == 1;
            min(x1) >= 0;
            sum(r*x1) >= current_rate_of_return; % optimize subject to
 this annual return
    cvx_end;

    Y(:,jj) = x1;
end

rates = (r* Y)' ;
sigs = sqrt(diag(Y' * Sig * Y));
```

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```
function x = market_portfolio(f, r, Sig)

% Define function func such that
%   func(sig) = 0   when   risk_free_rate(sig, r, Sig) = f.
func = @(sig) risk_free_rate(sig,r,Sig) - f;

% Compute the minimum value of sig
cvx_begin quiet
    variable x(size(r))
    minimize (x' * Sig * x)
    subject to
        sum(x) == 1;
        x >= 0;
cvx_end

% Lower and upper bounds for sig
sig1 = sqrt( x' * Sig * x);
sig2 = sqrt(max(diag(Sig)));

% Use BinarySearch to solve func(sig) = 0
sig = BinarySearch(func, sig1, sig2);

% The market portfolio is the portfolio on the efficient frontier with
 risk
% equal to the sig satisfying   risk_free_rate(sig, r, Sig) = f.
cvx_begin quiet
    variable x(size(r))
    maximize (r'* x)
    subject to
        sum(x) == 1;
        x >= 0;
        norm(sqrtm(Sig)*x) <= sig;
cvx_end  ;
end


function rate = risk_free_rate(sig, r, Sig)
n = length(r);
[sqrtSig, resnorm] = sqrtm(Sig);

% Dual multiplier lambda gives slope of efficient frontier at the
 point
% (r'*x, sqrt(x'*Sig*x)), where x is the portfolio with maximum
 expected
% rate of return with risk at most sig.
cvx_begin quiet
    variable x(n)
    dual variable lambda
    maximize( r'*x )
    subject to
        norm(sqrtSig*x) <= sig : lambda
        sum(x) == 1
```

```
        x >= 0
cvx_end

rmax = r'*x;
% The risk-free rate is the y-intercept of the line tangent to the
% efficient frontier at the point (r'*x, sqrt(x'*Sig*x)).
rate = lambda*(-sig) + rmax;
end

function x = BinarySearch(func, x1, x2)
% This is a generic binary search routine.
% Given x1 and x2, with
%   func(x1) < 0  and  func(x2) > 0, or
%   func(x1) > 0  and  func(x2) < 0,
% returns x with abs(func(x)) < 1e-6.

disp('Binary search:');
if func(x1) > 0 && func(x2) < 0
    % Swap x1 and x2
    tmp = x1;  x1 = x2;  x2 = x1;
end
ii = 0; y = Inf;
fprintf('%4s%10s%10s\n', 'Iter', 'x', '|f(x)|');
while abs(y) > 1e-6
    ii = ii + 1;  x = (x1 + x2)/2;  y = func(x);
    fprintf('%4d%10.2e%10.2e\n', ii, x, abs(y));
    if y < 0
        x1 = x;
    else
        x2 = x;
    end
end
disp('Done.');

end
```

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